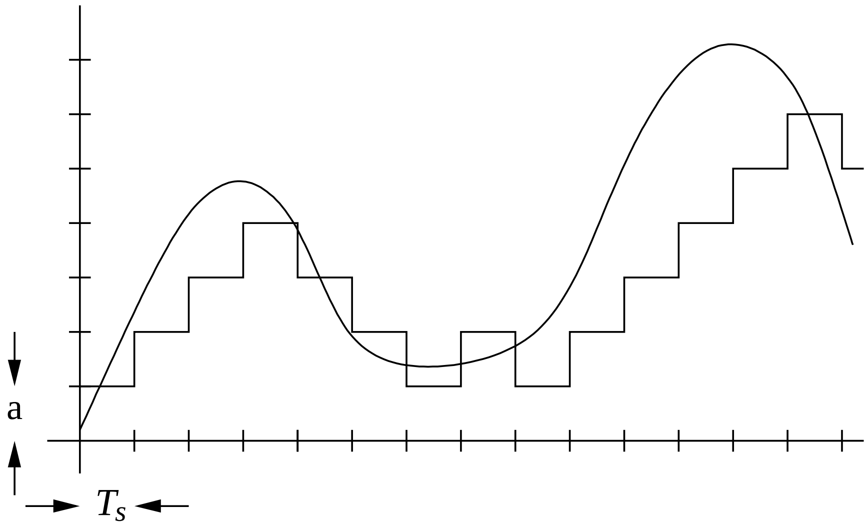
Delta modulation and DPCM

PCM is powerful, but quite complex coders and decoders are required. An increase in resolution also requires a higher number of bits per sample.

Standard PCM systems have no memory — each sample value is separately encoded into a series of binary digits. An alternative, which overcomes some limitations of PCM is to use past information in the encoding process. One way of doing this is to perform source coding using **delta modulation**:



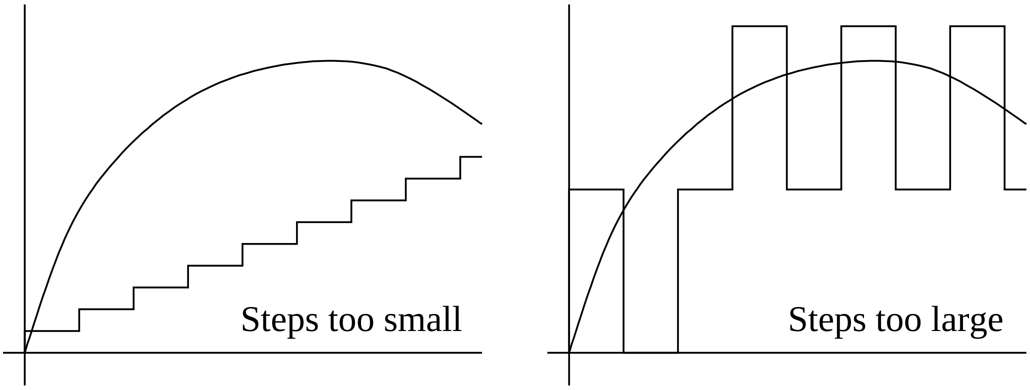
The signal is first quantised into discrete levels, but the size of the step between adjacent samples is kept constant. The signal may therefore only make a transition from one level to an adjacent one. Once the quantisation operation is performed, transmission of the signal can be achieved by sending a zero for a negative transition, and a one for a positive transition. Note that this means that the quantised signal *must* change at each sampling point.

For the above case, the transmitted bit train would be 111100010111110.

The demodulator for a delta-modulated signal is simply a staircase generator. If a one is received, the staircase increments positively, and if a zero is received, negatively. This is usually followed by a lowpass filter.

The key to using delta modulation is to make the right choice of **step size** and

sampling period — an incorrect selection will mean that the signal changes too fast for the steps to follow, a situation called **overloading**. Important parameters are therefore the **step size** and the **sampling period**.



If the signal has a known upper-frequency cutoff ω_m, then we can estimate the fastest rate at which it can change. Assuming that the signal is $f(t) = b\cos(\omega_m t)$, the maximum slope is given by

$$\left|\frac{df}{dt}\right|_{\text{max}} = b\omega_m = 2\pi b f_m.$$

For a DM system with step size a, the maximum rate of rise that can be handled is $a/T_s = af_s$, so we require

$$f_s \geq \frac{2\pi b f_m}{a} = \frac{2\pi f_m}{a/b}.$$

Making the assumption that the quantisation noise in DM is uniformly distributed over $(-a, a)$, the mean-square quantisation error power is $a^2/3$. We assume that this power is spread evenly over all frequencies up to the sampling frequency f_s. However, there is still the lowpass filter in the DM receiver — if the cutoff frequency is set to the maximum frequency f_m, then the total noise power in the reconstructed signal is

$$\overline{n_{\text{qnt}}^2(t)} = \frac{a^2}{3}\frac{f_m}{f_s}.$$

Still making the assumption of a sinusoidal signal, the SNR for DM is

$$\frac{S}{N} = \frac{3 f_s}{a^2 f_m} \overline{f^2(t)} = \frac{3 f_s}{a^2 f_m} b^2/2 = \frac{3}{8\pi^2} \left(\frac{f_s}{f_m} \right)^3,$$

when the slope overload condition is just met. The SNR therefore increases by 9dB for every doubling of the sampling frequency.

Delta modulation is extremely simple, and gives acceptible performance in many applications, but is clearly limited. One way of attempting to improve performance is to use *adaptive* DM, where the step size is not required to be constant. (The voice communication systems on the US space shuttles make use of this technique.) Another is to use delta PCM, where each desired step size is encoded as a (multiple bit) PCM signal, and transmitted to the receiver as a codeword. Differential PCM is similar, but encodes the difference between a sample and its *predicted* value — this can further reduce the number of bits required for transmission.