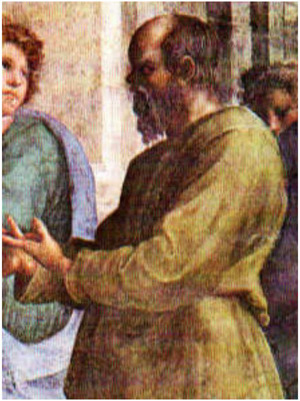
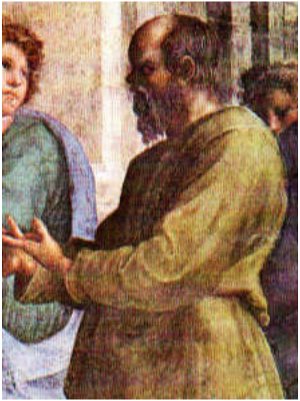
The Liar Game
Truths and Proofs from Euclid to Turing

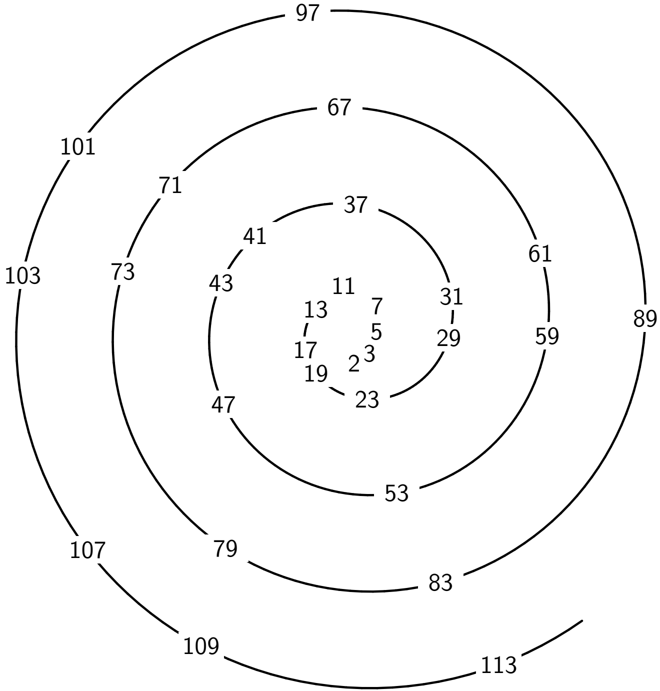
Mark Wildon



I think …



I think … therefore I am



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- Not 10 because $10 = 2 \times 5$
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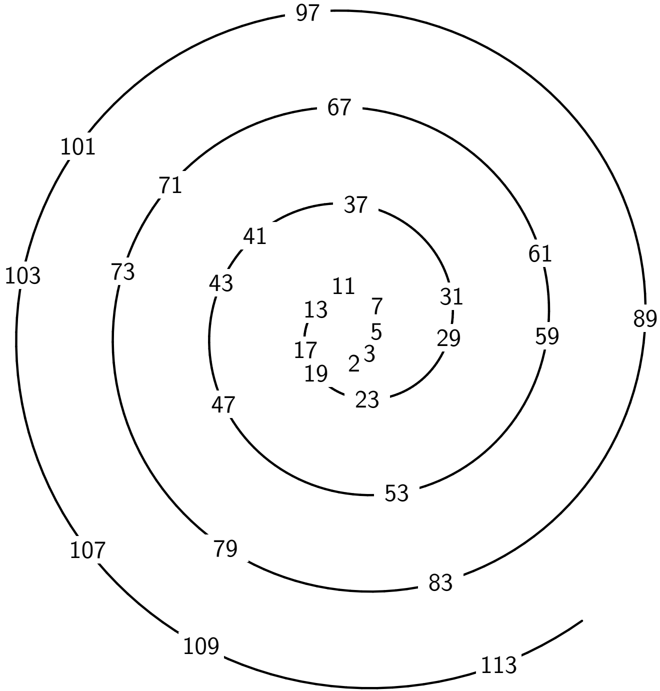
- 31 is prime


NFL
1

- 1 is not a prime

- 1 is not a prime — says who?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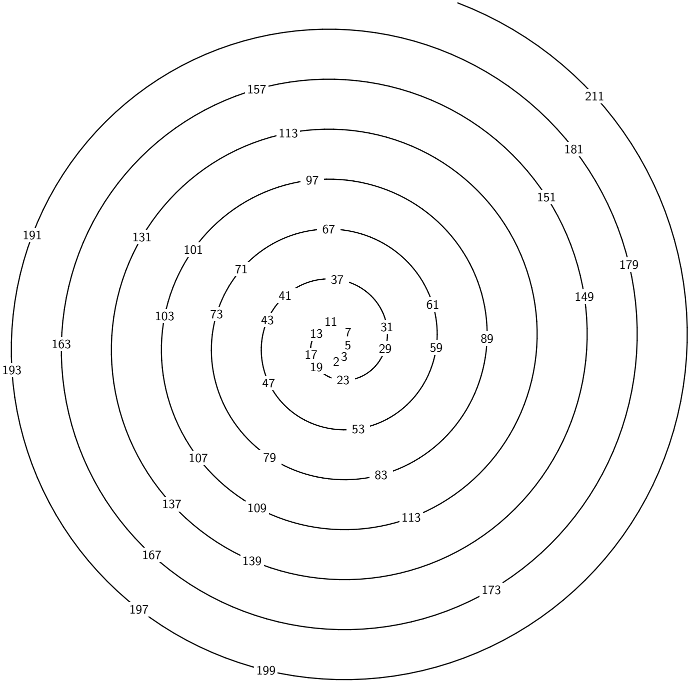

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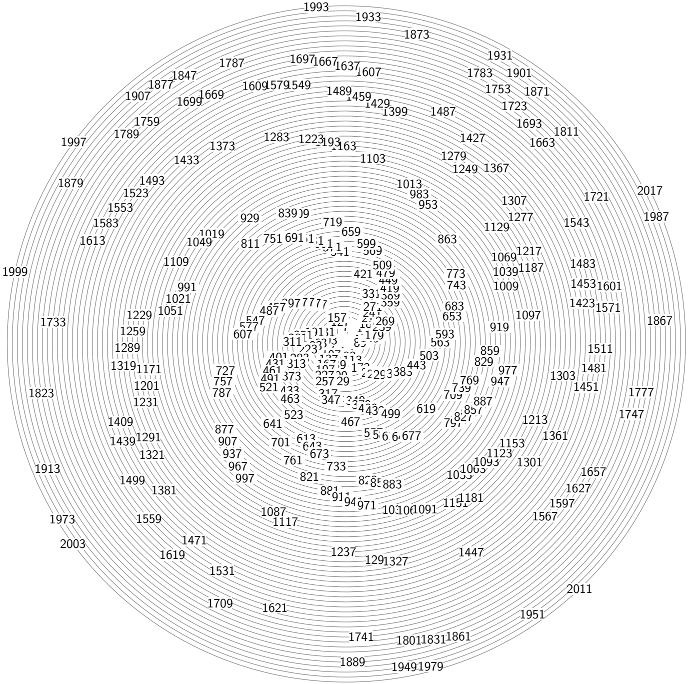

2, 3, 5, 7, 11, 13, …, 2003, 2011, 2017, 2027, 2029, …

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- Does the sequence of primes ever stop?
- Or maybe there are infinitely many primes?

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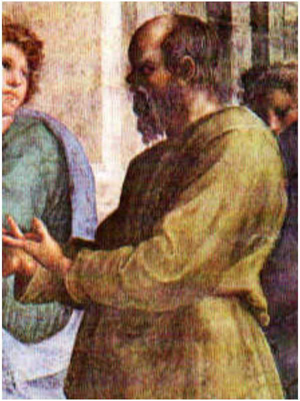
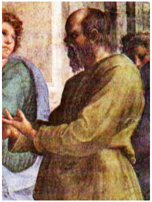
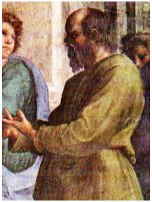


- Socrates: I think $p_1, p_2, \ldots, p_r$ might be all the primes



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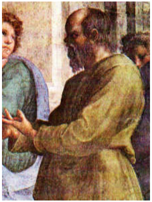

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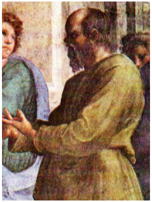

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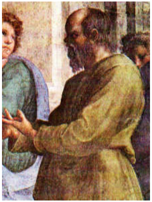

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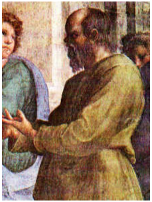

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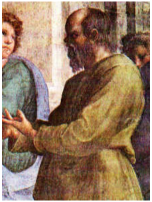

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- Socrates: Yes. So there is a prime not in my list.

Ask a friend to thinks of a number between 1 and 15. How many YES/NO questions do you need to ask to find the secret number?

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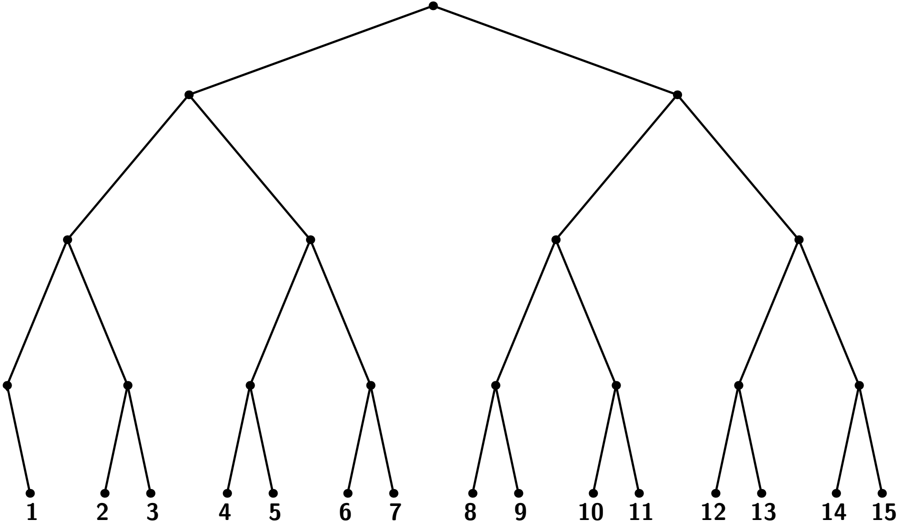

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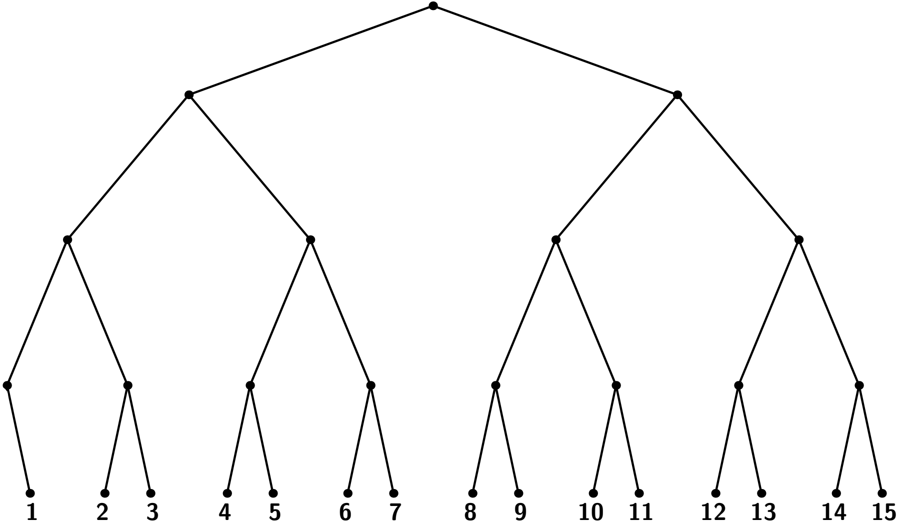

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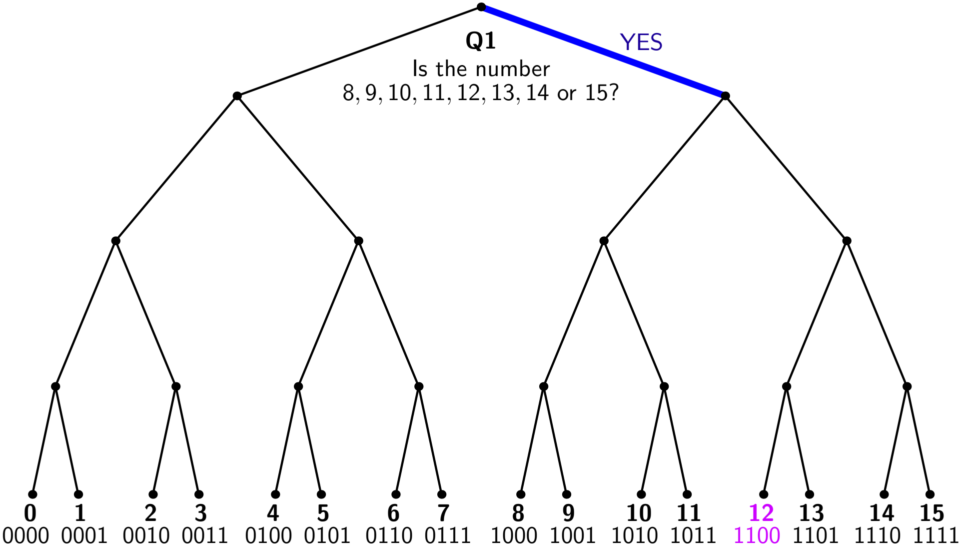

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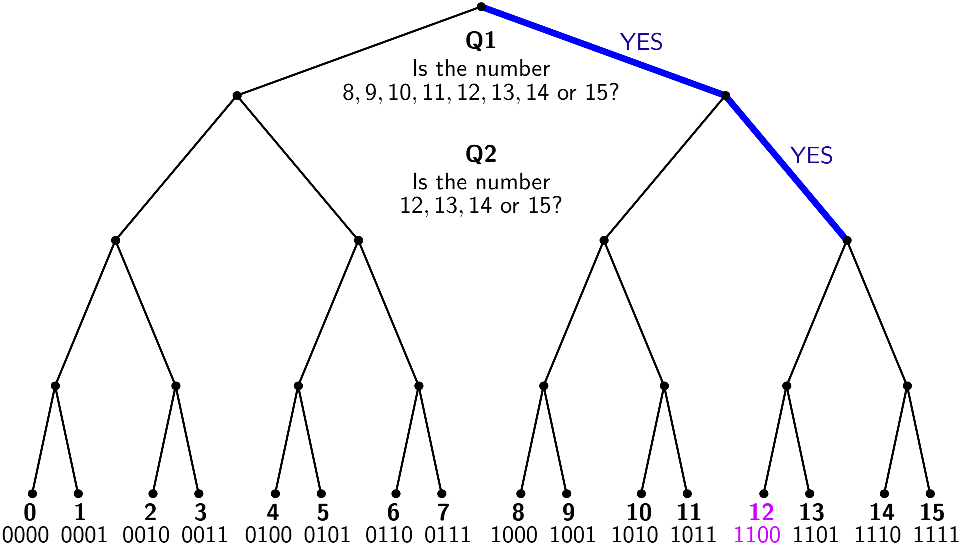

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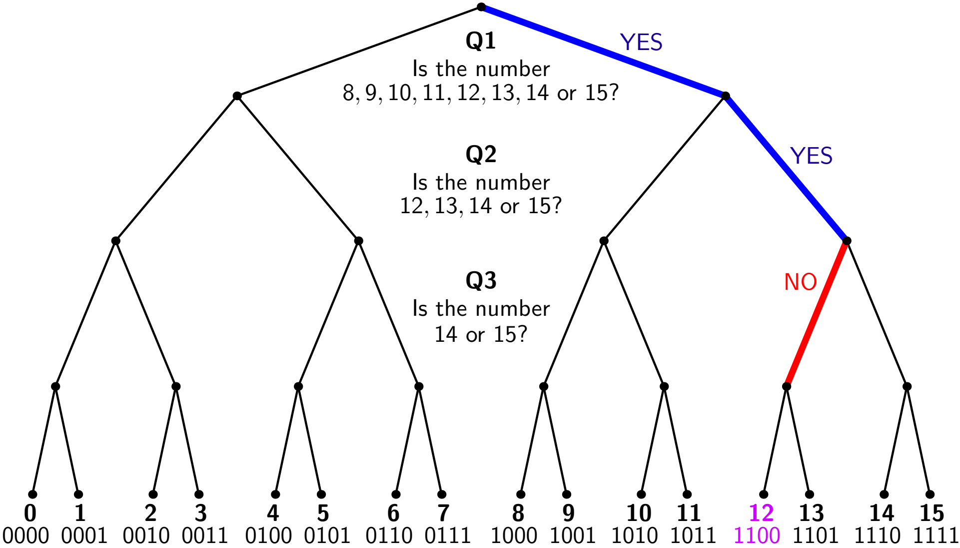

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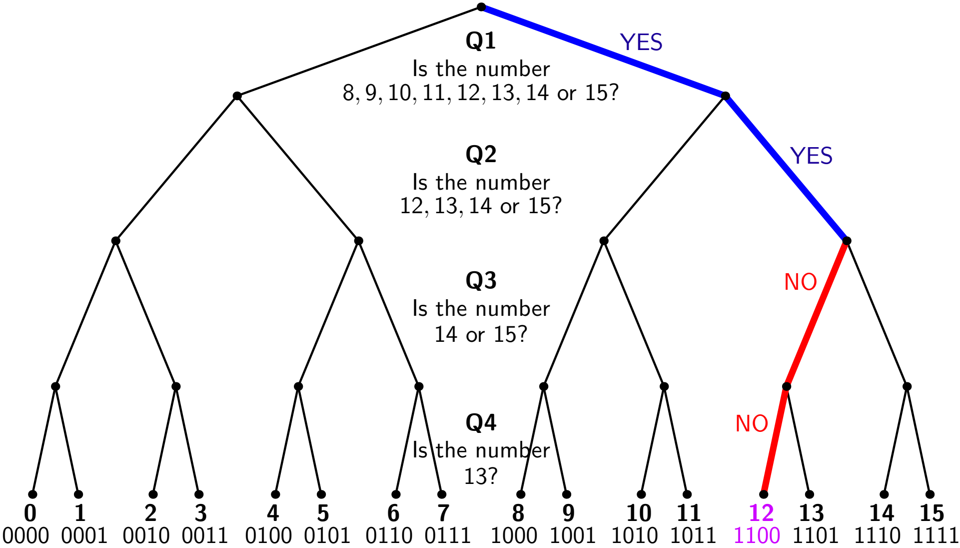

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Books, music, videos, computer programs, bitcoins …, all become bits.

01010100 01101111 00100000 01100010 01100101 00101100 00100000 01101111 01110010 00100000 01101110
01101111 01110100 00100000 01110100 01101111 00100000 01100010 01100101 00111010 00100000 01110100
01101000 01100001 01110100 00100000 01101001 01110011 00100000 01110100 01101000 01100101 00100000
01110001 01110101 01100101 01110011 01110100 01101001 01101111 01101110 00111010 00001010 01010111
01101000 01100101 01110100 01101000 01100101 01110010 00100000 00100111 01110100 01101001 01110011
00100000 01101110 01101111 01100010 01101100 01100101 01110010 00100000 01101001 01101110 00100000
01110100 01101000 01100101 00100000 01101101 01101001 01101110 01100100 00100000 01110100 01101111
00100000 01110011 01110101 01100110 01100110 01100101 01110010 00001010 01010100 01101000 01100101
00100000 01110011 01101100 01101001 01101110 01100111 01110011 00100000 01100001 01101110 01100100
00100000 01100001 01110010 01110010 01101111 01110111 01110011 00100000 01101111 01100110 00100000
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01100110 01101111 01110010 01110100 01110101 01101110 01100101 00101100

William Shakespeare (approx 1600)

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01010100 0110111 00100000 0110010 0110010 00101100 00100000 0110111 0111001 00100000 0110111
0110111 0111010 00100000 0111010 0110111 00100000 0110001 0110010 0011101 00100000 0111010
0110100 0110000 0111010 00100000 0110100 0111001 00100000 0111010 0110100 0110010 00100000
0111000 0111010 0110010 0111001 0111010 0110100 0110111 0110111 0011101 0000101 0101011
0110100 0110010 0111010 0110100 0110010 0111001 00100000 0010011 0111010 0110100 0111001
00100000 0110111 0110111 0110001 0110110 0110010 0111001 00100000 0110100 0110111 00100000
0111010 0110100 0110010 00100000 0110110 0110100 0110111 0110010 00100000 0111010 0110111
00100000 0111001 0111010 0110011 0110011 0110010 0111001 0000101 0101010 0110100 0110010
00100000 0111001 0110110 0110100 0110111 0110011 0111001 00100000 0110000 0110111 0110010
00100000 0110000 0111001 0111001 0110111 0111011 0111001 00100000 0110111 0110011 00100000
0110111 0111010 0111010 0111001 0110000 0110011 0110010 0110111 0111010 0111001 00100000
0110011 0110111 0111001 0111010 0111010 0110111 0110010 00101100

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To be, or not to be: that is the question:
Whether 'tis nobler in the mind to suffer
The slings and arrows of outrageous fortune,

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01010100 01101111 00100000 01100010 01100101 00101100 00100000 01101111 01110010 00100000 01101110
01101111 01110100 00100000 01110100 01101111 00100000 01100010 01100101 00111010 00100000 01110100
01101000 01100001 01110100 00100000 01101001 01110011 00100000 01110100 01101000 01100101 00100000
01110001 01110101 01100101 01110011 01110100 01101001 01101111 01101110 00111010 00001010 01010111
01101000 01100101 01110100 01101000 01100101 01110010 00100000 00100111 01110100 01101001 01110011
00100000 01101110 01101111 01100010 01101100 01100101 01110010 00100000 01101001 01101110 00100000
01110100 01101000 01100101 00100000 01101101 01101001 01101110 01100100 00100000 01110100 01101111
00100000 01110011 01110101 01100110 01100110 01100101 01110010 00001010 01010100 01101000 01100101
00100000 01110011 01101100 01101001 01101110 01100111 01110011 00100000 01100001 01101110 01100100
00100000 01100001 01110010 01110010 01101111 01110111 01110011 00100000 01101111 01100110 00100000
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00110000 0111011 0100110 10000000 0001100 0000001 0101101 0001110 1010100 0000000 1010110
0000101 1010100 0010101 0110101 0110100 0000110 0010110 1010100 0010100 0010110 1000101
00100100 0010010 1010100 0010101 0110101 0110100 0000110 0000111 1000100 0100101 0110010
1100101 1100100 1100111 1100111 0000100 0000101 0001010 0000110 0011000 0100000 0101101
0011000 1100001 0011000 0011000 10000000 0001101 0011101 0011000 1000011 1011101 0001101
1010100 0000000 0000101 0010111 1010100 0010101 1110100 1010100 1100101 1000100 1010011
1010100 1010101 1100101 1010010 1100101 0100100 0000111 1100110 1100111 1100111 0000100
0001010 1000000 0101101 0011000 0100010 0001000 0111101 0011000 1010010 0101101 1010110

Anonymous Microsoft Programmer (2010)

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1100101 1100100 1100111 1100111 0000100 0000010 0001010 0000110 0011000 0100000 0101101
00110000 1100001 0011000 0011000 1000000 0001101 0011101 0011000 1000011 1011110 0001101
1010100 0000000 0000101 0010111 1010100 0010101 1110100 1010100 1100101 1000100 1010011
1010100 1010101 1100101 1010010 1100101 0100100 0000111 1100110 1100111 1100111 0000100
00010100 1000000 0101101 0011000 0100010 0001000 0111101 0011000 1010010 0101101 1010110
00000000 0000101 1110101 1110101 0110100 0010111 0010110 0010101 1010100 0110110 0000101
1010111 1110101 0110101 1010101 1010110 0010101 1010111 1100101 1010110 0010101 1010101
0010101 0010111 1110101 0100100 1000100 0010011 1010010 1010100 1010101 1100101 1010010
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Part of the machine code for Microsoft Word 2011.

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  It sounds most like three repeats of 1100, which is 12

Ask a friend to think of a number between 0 and 15. How many YES/NO questions do you need to ask to find the secret number? Your friend may lie **but only once**.

It is not compulsory to lie.

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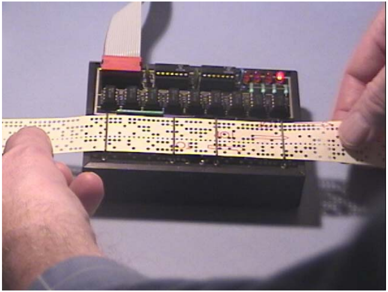
The Alice/Bob code gives a 12 question solution

Number	Encoded as	Number	Encoded as
0	000 000 000 000	8	111 000 000 000
1	000 000 000 111	9	111 000 000 111
2	000 000 111 000	10	111 000 111 000
3	000 000 111 111	11	111 000 111 111
4	000 111 000 000	12	111 111 000 000
5	000 111 000 111	13	111 111 000 111
6	000 111 111 000	14	111 111 111 000
7	000 111 111 111	15	111 111 111 111

Richard Hamming (1915 — 1998) discovered a one-error correcting binary code of length 7 with 16 codewords.

He invented it because he was fed up with the paper tape reader on his early computer misreading his programs.



Find the codeword corresponding to your secret number.

For instance if your number is 12 then the codeword is 0111100.

0	0000000	8	1110000
1	1101001	9	0011001
2	0101010	10	1011010
3	1000011	11	0110011
4	1001100	12	0111100
5	0100101	13	1010101
6	1100110	14	0010110
7	0001111	15	1111111

I'll ask you:

`What is the bit in the first position (far left) of the codeword?',

`What is the bit in the second position of the codeword?',

and so on. The Hamming code will reveal the number, even if you lie once.

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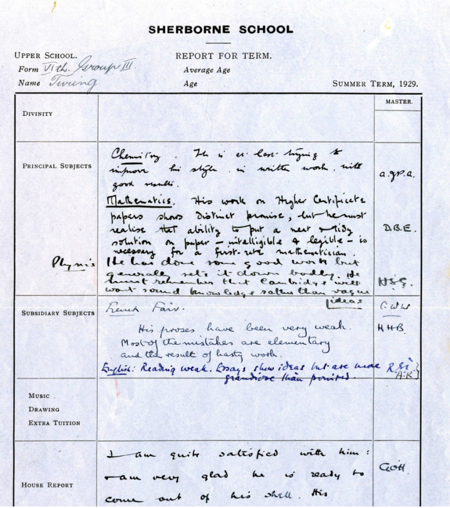
No strategy can **guarantee** to use fewer than 7 questions. So the Hamming code is optimal.

Ada Lovelace (1815 — 1857) inventor of programming

Katherine Johnson (1918 —) NASA `computer'

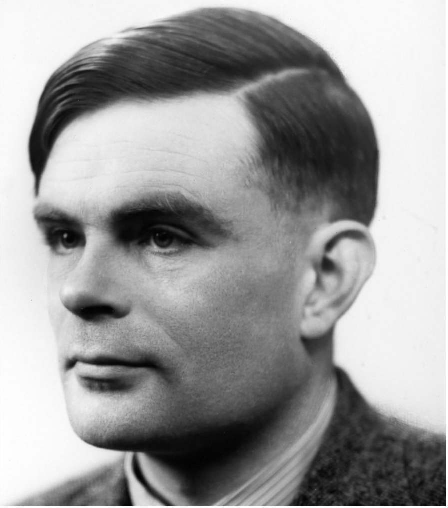
Alan Turing (1912 — 1952) was another pioneer of early computing



SHERBORNE SCHOOL

UPPER SCHOOL. REPORT FOR TERM.
Form VIth. Group III Average Age
Name Turing Age SUMMER TERM, 1929.

		MASTER.
DIVINITY		
PRINCIPAL SUBJECTS	Chemistry. He is at last trying to improve his style in written work, with good results.	a.gp.a.
	Mathematics. His work on Higher Certificate papers shows distinct promise, but he must realise that ability to put a neat and tidy solution on paper — intelligible & legible — is necessary for a first-rate mathematician.	D.B.E.
	Physics He has done some good work but generally sets it down badly. He must remember that Cambridge will want sound knowledge rather than vague ideas	H.S.S.
SUBSIDIARY SUBJECTS	French Fair.	C.W.W.
	His proses have been very weak. Most of the mistakes are elementary and the result of hasty work.	H.H.B.
	English: Reading weak. Essays show ideas but are more grandiose than pointed.	R.S.? A.R.
MUSIC DRAWING EXTRA TUITION		
HOUSE REPORT	I am quite satisfied with him: I am very glad he is ready to come out of his shell. His	G.O.H.



Mathematics papers are mostly words.

A PROOF OF LIOUVILLE'S THEOREM

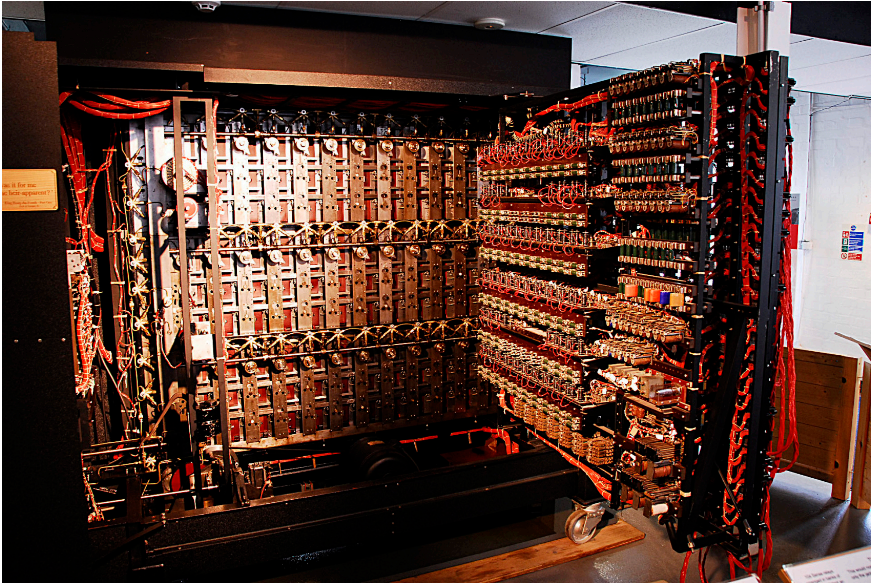
EDWARD NELSON

Consider a bounded harmonic function on Euclidean space. Since it is harmonic, its value at any point is its average over any sphere, and hence over any ball, with the point as center. Given two points, choose two balls with the given points as centers and of equal radius. If the radius is large enough, the two balls will coincide except for an arbitrarily small proportion of their volume. Since the function is bounded, the averages of it over the two balls are arbitrarily close, and so the function assumes the same value at any two points. Thus a bounded harmonic function on Euclidean space is a constant.

PRINCETON UNIVERSITY

Received by the editors June 26, 1961.

He helped crack the Enigma code used by the German Navy in the Second World War



Turing's finest mathematical achievement is the following theorem.

Theorem. There is no algorithm that will decide the truth or falsity of a mathematical statement

- There are infinitely many primes — True
- It takes 4 bits to store a number between 0 and 15 — True
- There are infinitely many primes ending 1 — True
- There is a way to win the Liar Game in 6 questions — False

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- 2^3 and 3^2 are the only consecutive integer powers — ???
- There are infinitely many twin primes such as 3, 5 or 5, 7 or 11, 13 or 17, 19 or … or 2027, 2029 or … — ???

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- There are infinitely many twin primes such as 3, 5 or 5, 7 or 11, 13 or 17, 19 or … or 2027, 2029 or … — ???
- There is a fast way to factorize large numbers into primes — ???

Thank you. Any questions?

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So the person at the back of the line can see nine hats, the next person can see eight, and so on.

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Question: What is a good strategy?