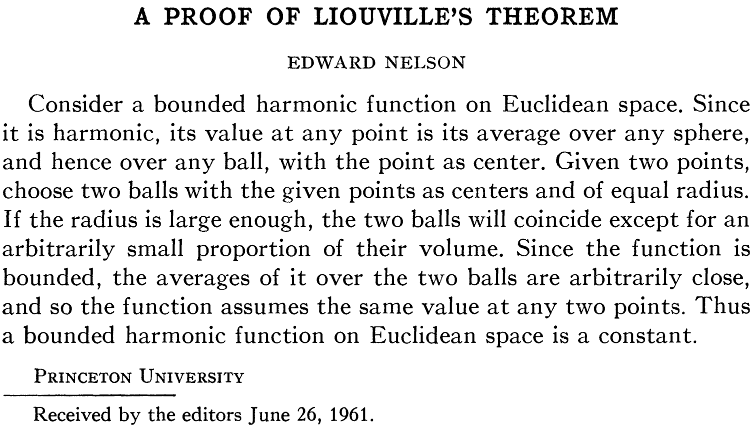
Critical evaluation of mathematical writing through peer marking and formative assessment

Mark Wildon

Writing is Important in Mathematics


A PROOF OF LIOUVILLE'S THEOREM

EDWARD NELSON

Consider a bounded harmonic function on Euclidean space. Since it is harmonic, its value at any point is its average over any sphere, and hence over any ball, with the point as center. Given two points, choose two balls with the given points as centers and of equal radius. If the radius is large enough, the two balls will coincide except for an arbitrarily small proportion of their volume. Since the function is bounded, the averages of it over the two balls are arbitrarily close, and so the function assumes the same value at any two points. Thus a bounded harmonic function on Euclidean space is a constant.

PRINCETON UNIVERSITY

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Eight letters are to be sent to eight different people. How many ways can the letters be put in addressed envelopes so that no letter reaches its intended recipient?

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d_n	0	1	2	9				
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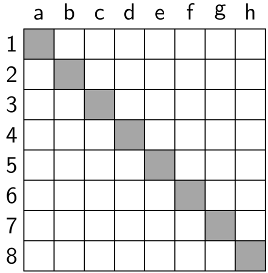
p_n	0	0.5	0.333	0.375	0.367	0.368	0.368	0.368

Crash Course on Combinatorics: (II) Rook Polynomials

Give the people letters from *a* to *h* and number their letters from 1 to 8. Placements of the eight letters into envelopes so that none of the letters is in the right envelope correspond to put eight rooks on the board below, so that:

- no two rooks attack one another, *and*
- no rook is on a shaded square.



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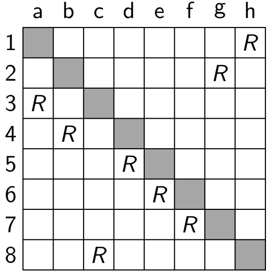

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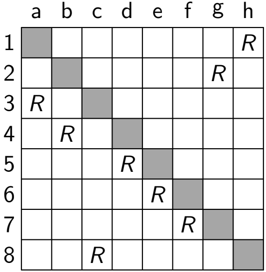


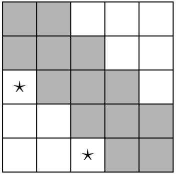
We record the number of ways to place r non-attacking rooks, subject to the two conditions above, as the coefficient of x^r in a *rook polynomial*: $1 + 56x + 1204x^2 + 12712x^3 + \cdots + 14833x^8$.

Peer Marked Question

2. Let T be the set of all derangements σ of $\{1, 2, 3, 4, 5\}$ such that

- $\sigma(i) \neq i + 1$ if $1 \leq i \leq 4$,
- $\sigma(i) \neq i - 1$ if $2 \leq i \leq 5$.

(a) Explain why $|T|$ is the number of ways to place 5 non-attacking rooks on the board B formed by the unshaded squares below. (Give an explicit example of how a permutation corresponds to a rook placement.)



(b) Find the rook polynomial of B, and hence find $|T|$. [*Hint: consider the four possibilities for the starred squares. For example, if both are occupied, the contribution to the rook polynomial is* $x^2 f_1(x) f_2(x)$ *where* $f_n(x)$ *is the rook polynomial of the* $n \times n$ *square board.*]

(c) Use Theorem 6.10 to find the number of ways to place 5 non-attacking rooks on the shaded squares.

Peer Marking Exercise

- Students were given advanced notice that Question 2 would be peer-marked. Discussed reasons for peer marking.
- A detailed model answer was provided, after work was submitted, with some advice on likely errors. Peer markers were instructed not to penalize any error more than once, so in many cases had to do some work so see if a correct method has been applied.
- Followed the peer-review model. Peer markers were identified only by anonymous letter codes. (Students were invited to submit work anonymously if they wished.)
- Timetable: answers submitted and redistributed to peer markers on Friday, marked work returned to me on Monday, and returned to students on Tuesday.

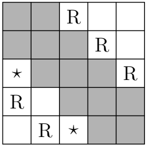
Extract from Model Answer

[*Please follow the marking scheme out of* **4 marks** *given in bold below, erring on the side of generosity. Point out errors and write helpful comments. Put your reviewer letter at the top and return* **to the lecturer** *at the Monday lecture (or earlier).*]

(a) There is a bijection between permutations of $\{1, 2, 3, 4, 5\}$ and ways to place five non-attacking rooks on the squares of a 5×5 grid: a permutation σ corresponds to the rook placement with rooks in positions $\big(i, \sigma(i)\big)$ for $i \in \{1, 2, 3, 4, 5\}$.

Derangements correspond to rook placements with no rooks on the diagonal. Then $\sigma(i) \neq i + 1$ rules out the squares above the diagonal, and $\sigma(i) \neq i - 1$ rules out the squares below the diagonal. So permutations in T correspond to rook-placements on the board of unshaded squares.

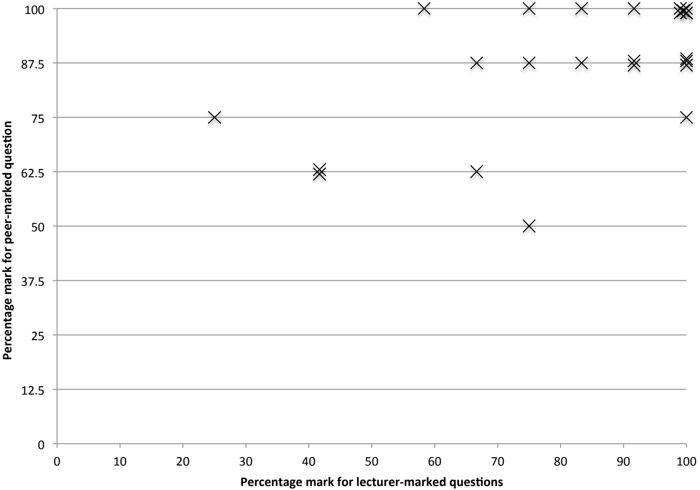
For example, the permutation σ defined by $\sigma(1) = 3$, $\sigma(2) = 4$, $\sigma(3) = 5$, $\sigma(4) = 1$, $\sigma(5) = 2$ corresponds to the placement shown below.



[**1/2 mark for a reasonable explanation covering most of the points above.** The bijection was explained in Example 6.4, so not essential to put in details of why rooks are non-attacking. There are other equally correct bijections between permutations and rook placements, for example, one could put a rook in row j and column i if $\sigma(i) = j$.]

[**1/2 mark for a correct example of the given bijection.**]

Marks for Peer-Marked Question Compared to Others 2011



Thoughts on Exercise from 2011 and 2012

- In 2011 I had 28 students and everyone submitted answers to the peer-marked question. There was a wide range of abilities and several instances where a weak student marked a very strong student's work, or *vice versa*. One comment written by a peer-marker:

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- In both years I was very impressed by the effort peer-markers put in.

Some Student Comments from 2012

I found peer-marking to be an interesting exercise. I liked how the marker only had one question to mark so they were able to spend time understanding what I had done and I appreciated the constructive feedback. **I also found it beneficial to see how someone else had tackled the question.** *Marking it meant I had to really understand the question and ways you could go wrong. I wouldn't only want to learn that way, but it was good to try something new.*

I thought the peer-marking exercises was a very good idea and would have liked to have had more of such exercises. I think it was good for us to see how others had presented their work **but we probably could have gained more from the exercise if the marked questions had been more involving***. I also think such exercises should be more widespread across the department especially in proof based courses (although it is to late for me to benefit from this!)*

Peer Marking for Other Courses?

- Detailed model answer is probably essential. But once written, I think there is a small saving of my time.
- Rather than introduce Peer-Marking in my big first year course 181 Number Systems, I instead set as a (supposedly compulsory) question 'Please set a question on any part of the 181 course.' This got some good attempts but was ignored by many students. On the whole the questions set were on the hard side, but felt by the setters to be easy, even when their supplied 'model' answer was wrong.
- Tentative conclusion: a smallish and relatively mature group probably helps. But a wide range of abilities seems not to be a problem.