

The Liar Game

Dr Mark Wildon

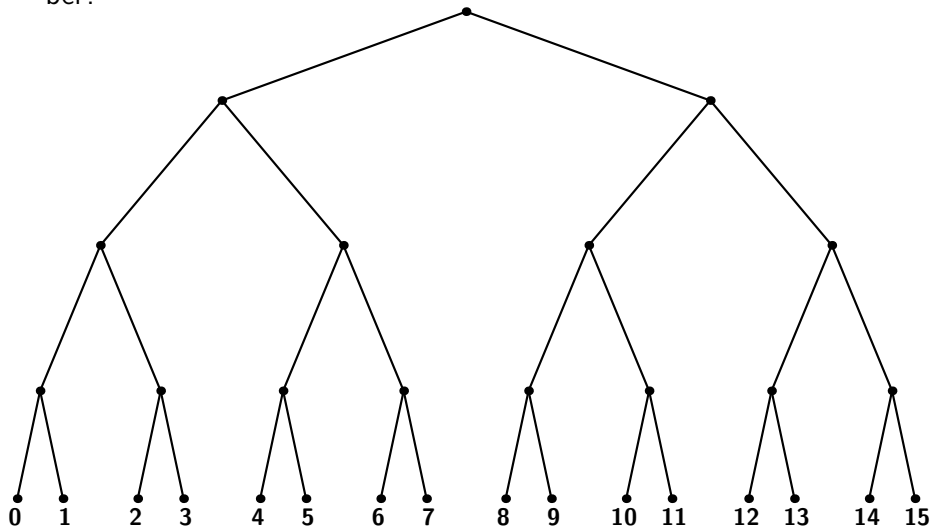


Guessing Games

Ask a friend to think of a number between 1 and 15. How many YES/NO questions do you need to ask to find out the secret number?

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$$7 \text{ (NO)} + 8 \text{ (YES)} = 15$$

- ▶ 'Is the number even?'

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- ▶ 'Is the number 8 or more?' $7 \text{ (NO)} + 8 \text{ (YES)} = 15$
- ▶ 'Is the number even?' $8 \text{ (NO)} + 7 \text{ (YES)} = 15$
- ▶ 'Is the number 12?' $14 \text{ (NO)} + 1 \text{ (YES)} = 15$

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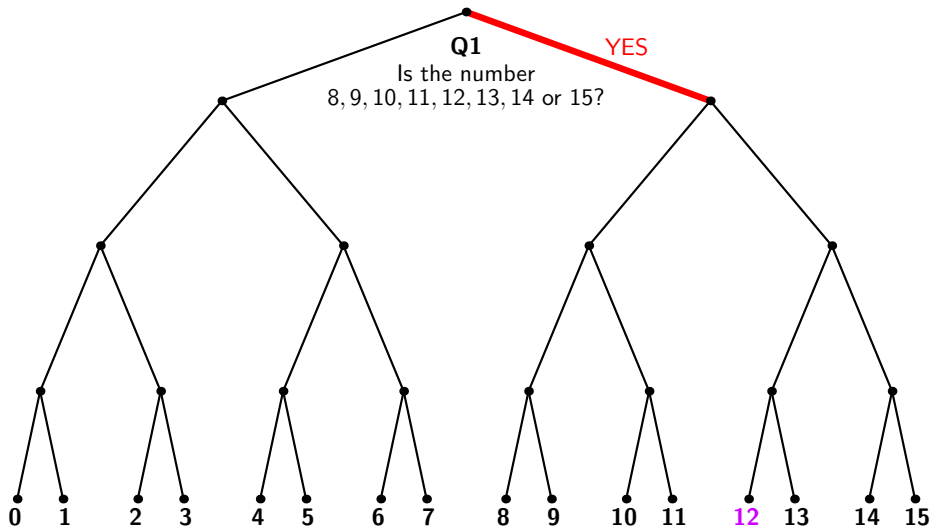
- ▶ There are **15** possible numbers.
- ▶ In the **worst case**, there are at least **8** possible numbers after the first question.
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 - ▶ 'Is the number prime?' $9 \text{ (NO)} + 6 \text{ (YES)} = 15$
- ▶ In the **worst case** there are at least **4** possible numbers after the second question.
- ▶ In the **worst case** there are at least **2** possible numbers after the third question.
- ▶ So three questions are not enough.

Binary and Computers

In a computer everything is stored as a lists of the **bits** (**b**inary **d**igits) 0 and 1.

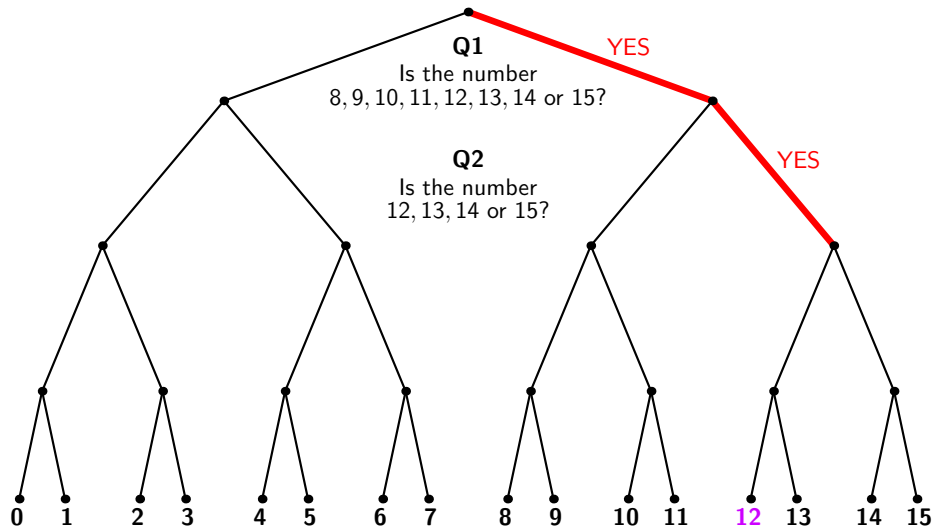
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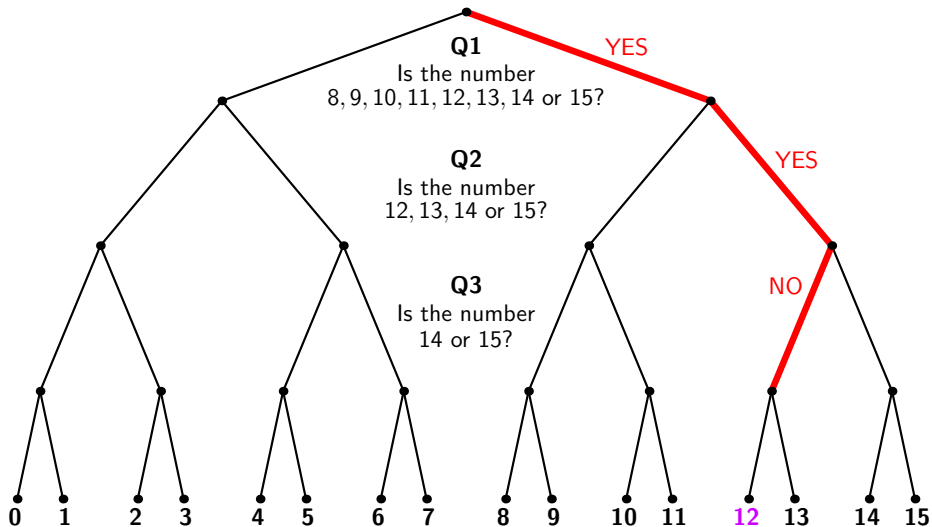
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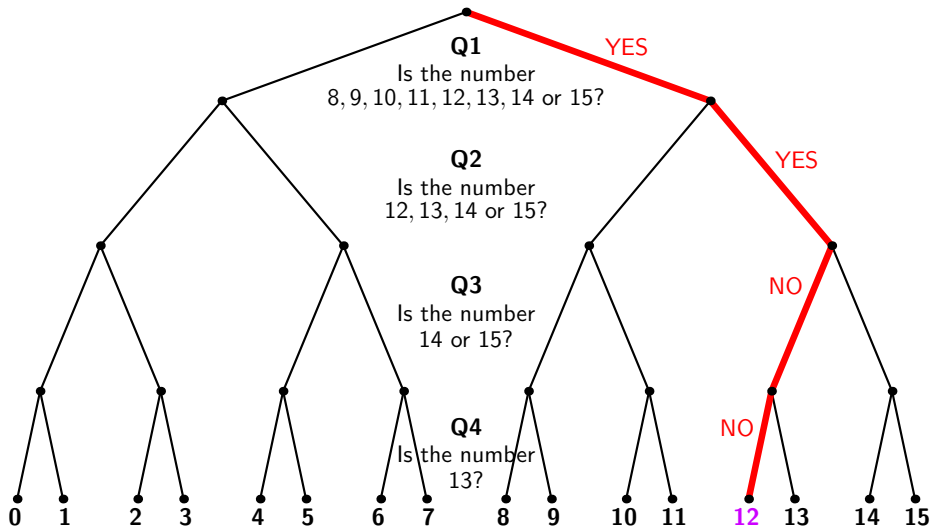
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Books, music, videos, computer programs, bitcoins . . . , all become bits.

```
01110000 11010111 00000100 00010101 11010100 01000110 00000100 11010111
00110101 00000100 01010111 11010111 01110100 00000100 01110100 11010111
00000100 00010101 11010100 00100111 00000100 01110100 00010110 10010100
01110100 00000100 10010110 10110101 00000100 01110100 00010110 11010100
00000100 10110100 11110100 11010100 10110101 01110100 10010110 11010111
01010111 00100111 00000011 11110001 00010110 11010100 01110100 00010110
11010100 00110101 00000100 11000101 01110100 10010110 10110101 00000100
01010111 11010111 00010101 01010110 11010100 00110101 00000100 10010110
01010111 00000100 01110100 00010110 11010100 00000100 11010110 10010110
01010111 01010100 00000100 01110100 11010111 00000100 10110101 11110100
01010101 01010101 11010100 00110101 00000011 01110000 00010110 11010100
00000100 10110101 01010110 10010110 01010111 11010101 10110101 00000100
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01000110
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William Shakespeare (approx 1600)

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01110100 00000100 10010110 10110101 00000100 01110100 00010110 11010100
00000100 10110100 11110100 11010100 10110101 01110100 10010110 11010111
01010111 00100111 00000011 11110001 00010110 11010100 01110100 00010110
11010100 00110101 00000100 11000101 01110100 10010110 10110101 00000100
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00110000 01110111 01000110 10000000 00011000 00000001 01011101 00011110
10101100 00000000 10101110 00001011 10101100 00101011 01101011 01101001
00001110 00101110 10101100 00101001 00101110 10001101 00100100 00100101
10101100 00101011 01101011 01101001 00001110 00001111 10001000 01001011
01100100 11001010 11001100 11001111 11001111 00001000 00000101 00010100
00001100 00110000 01000000 01011010 00110000 11000010 00110000 00110000
10000000 00011010 00111010 00110000 10000110 10111101 00011010 10101100
00000000 00001011 00101110 10101001 00101011 11101000 10101000 11001011
10001001 10100111 10101001 10101010 11001011 10100101 11001010 01001001
00001110 11001100 11001111 11001111 00001000 00010100 10000001 01011010
00110000 01000101 00010001 01111010 00110000 10100101 01011010 10101100
00000000 00001011 11101010 11101011 01101001 00101110 00101100 00101011
10101001 01101100 00001011 10101111 11101011 01101010 10101010 10101100
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Anonymous Microsoft Programmer (2010)

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10101100 00000000 10101110 00001011 10101100 00101011 01101011 01101001
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10101100 00101011 01101011 01101001 00001110 00001111 10001000 01001011
01100100 11001010 11001100 11001111 11001111 00001000 00000101 00010100
00001100 00110000 01000000 01011010 00110000 11000010 00110000 00110000
10000000 00011010 00111010 00110000 10000110 10111101 00011010 10101100
00000000 00001011 00101110 10101001 00101011 11101000 10101000 11001011
10001001 10100111 10101001 10101010 11001011 10100101 11001010 01001001
00001110 11001100 11001111 11001111 00001000 00010100 10000001 01011010
00110000 01000101 00010001 01111010 00110000 10100101 01011010 10101100
00000000 00001011 11101010 11101011 01101001 00101110 00101100 00101011
10101001 01101100 00001011 10101111 11101011 01101010 10101010 10101100
00101011 10101110 11001011 10101100 00101011 10101011 00101011 00101110
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10100101 11001010 01001001 00001110 11001100 11001111 11001111 00001000
00010100
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Anonymous Microsoft Programmer (2010)

Part of the machine code for Microsoft Word 2011.

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Why Coding Theory?

A bit gives a single piece of information: 'NO' or 'YES'; 'on' or 'off'; 0 or 1.

- ▶ A number between 0 and 15:

4 bits


- ▶ A small QR-code:



- ▶ Text on this slide
- ▶ Full text of *Hamlet*
- ▶ Pictures of Royal Holloway
- ▶ Compact disc of Beethoven 9th
- ▶ Large Hadron Collider, per second


Why Coding Theory?

A bit gives a single piece of information: 'NO' or 'YES'; 'on' or 'off'; 0 or 1.

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Errors in reading and writing are inevitable. The best we can hope is to be able to correct them when they occur.

A Simple Error Correcting Code

Number	Encoded as	Number	Encoded as
0	0000 0000 0000	8	1000 1000 1000
1	0001 0001 0001	9	1001 1001 1001
2	0010 0010 0010	10	1010 1010 1010
3	0011 0011 0011	11	1011 1011 1011
4	0100 0100 0100	12	1100 1100 1100
5	0101 0101 0101	13	1101 1101 1101
6	0110 0110 0110	14	1110 1110 1110
7	0111 0111 0111	15	1111 1111 1111

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6	0110 0110 0110	14	1110 1110 1110
7	0111 0111 0111	15	1111 1111 1111

Question. Suppose you receive 0011 0010 0011. What number was most likely sent?

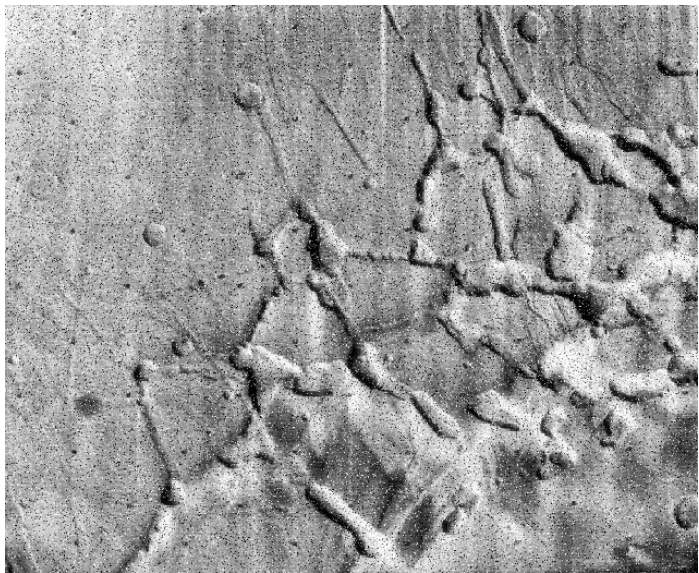
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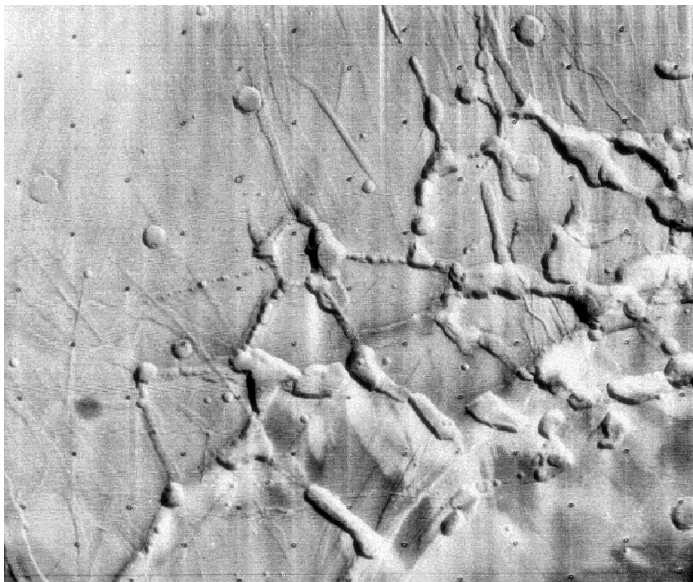
Question. Suppose you receive 0011 0010 0011. What number was most likely sent?

Answer. Since 0011 0010 0011 differs from 0011 0011 0011 in just one place, it's most likely that the number is 3.

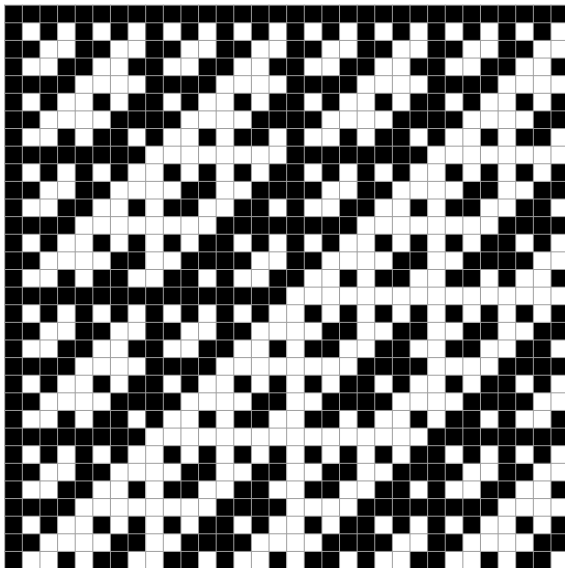
Mariner 9 Image: Improvement Due to Error Correction



Mariner 9 Image: Improvement Due to Error Correction



The Mariner 9 Code: 32 of the 64 Mariner 9 codewords:
Black Squares Show 0, White Squares Show 1



The Liar Game: Dealing with Deliberate Errors

Ask a friend to think of a number between 0 and 15. How many YES/NO questions do you need to ask, if your friend is permitted to lie **at most once**?

It is not compulsory to lie.

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Coding theory can be used to find a good strategy. Lies correspond to errors in transmission.

The Hamming Code

Richard Hamming discovered a one-error correcting binary code of length 7 with 16 codewords. He invented it because he was fed up with the paper tape reader on his early computer misreading his programs.

It gives an optimal solution to the Liar Game using 7 questions.

Remarkably, it is possible to specify all the questions in advance.



The Hamming Code

Find the binary codeword corresponding to your secret number.

0	000000	8	1110000
1	1101001	9	0011001
2	0101010	10	1011010
3	1000011	11	0110011
4	1001100	12	0111100
5	0100101	13	1010101
6	1100110	14	0010110
7	0001111	15	1111111

The questions are:

'Is there a 1 in the first position (far left) of the codeword?'

'Is there a 1 in the second position of the codeword?'

and so on. If there is one lie, then the questioner will write down one wrong bit. But because the Hamming code can correct one error, the questioner can still work out what the number is.

A Hat Game Related to Coding Theory

You and two friends are on your way to a party.

At the party a black or blue hat will be put on each person's head.

You can see your friends' hats, but not your own.

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When the host shouts 'go!', you may either say a colour or remain silent. Everyone who speaks must speak at the same time.

If everyone who speaks gets the colour of his or her hat correct, you all win some cake. If no-one speaks, or someone gets it wrong, there is no cake.

A Hat Game Related to Coding Theory

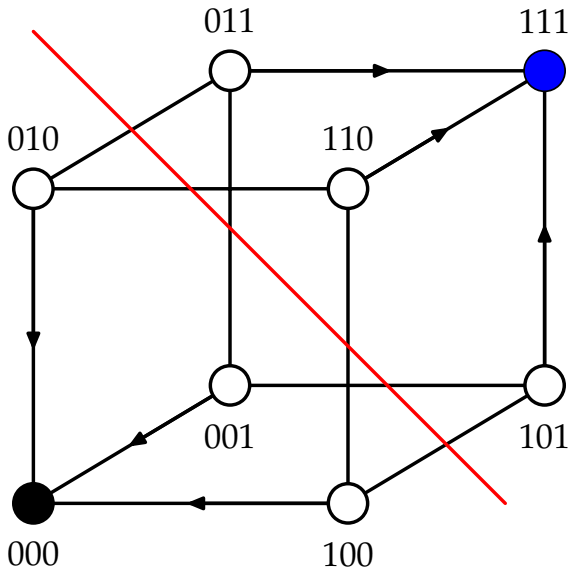
You and two friends are on your way to a party.

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Question: What is a good strategy?



Another Hat Game

You and four friends are lined up. A black or blue hat is put on each person's head. You can see all the hats in front of you, but not your own, or those behind.

So the person at the back of the line can see four hats, the next person can see three, and so on.

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Thank you! Any questions?

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- ▶ Why is maths a good subject to study?
- ▶ What do maths lecturers do all day?
- ▶ How does maths at university differ from A-level maths?
- ▶ Are women just as good as men at maths? (**Answer:** Yes!)

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- ▶ In the **worst case** there are at least **4** possible numbers after the second question.
- ▶ In the **worst case** there are at least **2** possible numbers after the third question.
- ▶ So three questions are not enough.