

# Cryptanalysis of Two Strongly Unforgeable Identity-based Signatures in the Standard Model

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## Abstract

The strong unforgeability of digital signature means that no attacker can forge a valid signature on a message, even given some previous signatures on the message, which has been widely accepted as a common security requirement. Recently, Tsai *et al.* and Hung *et al.* presented an efficient identity-based signature scheme and a revocable identity-based signature scheme, respectively. Meanwhile, they all claimed that their scheme is strongly unforgeable against chosen message attacks. In this paper, we point out that the two schemes cannot meet the requirements of strong unforgeability by giving some concrete attacks and briefly analyze the reasons why the provably-secure schemes are insecure following their security model.

*Keywords:* Cryptanalysis; Identity-Based Signature; Strong Unforgeability

## 1 Introduction

To simplify the complicated certificate management in traditional public key systems, Shamir [11] introduced the concept of identity-based public key cryptography (IB-PKC). In IB-PKC settings, an entity's public key is some unique public information such as ID card, email address, while the corresponding private key are directly derived by the private key generator (PKG) from these public identity information. Moreover, the author addressed the first identity-based signature (IBS) scheme. Since then, a few classic IBS schemes [3, 5] were presented in random oracles following Shamir's idea.

As shown in [2], however, a security proof in random oracles can only serve as a heuristic argument and does not necessarily imply the security in the real implementation. It arises interest to construct a IBS scheme provably secure without random oracles. Until 2006, the first practical IBS scheme provably secure in the standard model

was presented in [9]. Unfortunately, it does not cover the strong unforgeability [1] which is needed in a variety of applications. Afterwards, lots of improved IBS schemes were proposed to meet the requirements of strong unforgeability in the standard model [7, 8, 10].

Recently, Tsai *et al.* [12] analyzed the existing strongly unforgeable IBS schemes without using random oracles and proposed an efficient and practical IBS scheme with short signature that is secure without random oracles. In the same year, Hung *et al.* [4] introduced a revocable identity-based signature (RIBS) scheme in the standard model. They all claimed that their (R)IBS scheme is strongly unforgeable against chosen message attacks.

In this paper, we first illustrate that Tsai *et al.*'s IBS scheme cannot meet the requirements of strong unforgeability by giving some concrete attacks. Then, we demonstrate that an attacker can easily discover the difference between simulated signatures and real signatures by interacting with the challenger. Finally, we show that Hung *et al.*'s RIBS scheme is actually based on Tsai *et al.*'s IBS scheme and can give some similar cryptanalysis according to the same ideas.

The rest of this paper is organized as follows. Section 2 gives some preliminaries. In Section 3, we review Tsai *et al.*'s IBS scheme and cryptanalyze its security. In Section 4, we look back Hung *et al.*'s RIBS scheme and briefly do some cryptanalysis on it. Finally, the conclusion is given in Section 5.

## 2 Preliminaries

### 2.1 Bilinear Groups and Complexity Assumption

**Bilinear groups:** Let  $\mathbb{G}_1$  and  $\mathbb{G}_2$  be two multiplicative cyclic groups of same prime order  $p$  and  $g$  be a gen-

erator of  $\mathbb{G}_1$ . The bilinear map  $\hat{e} : \mathbb{G}_1 \times \mathbb{G}_1 \rightarrow \mathbb{G}_2$  has the following properties [6, 13]:

- Bilinearity:  $\forall g, h \in \mathbb{G}_1$  and  $\forall a, b \in \mathbb{Z}_p^*$ , we have  $\hat{e}(g^a, h^b) = \hat{e}(g, h)^{ab}$ ;
- Non-degeneracy:  $\hat{e}(g, g) \neq 1_{\mathbb{G}_2}$  for  $1_{\mathbb{G}_2}$  denotes the identity element of  $\mathbb{G}_2$ ;
- Computability: There exists an efficient algorithm to compute  $\hat{e}(g, h)$ .

### Computational Diffie-Hellman (CDH) Assumption:

Let  $(\mathbb{G}_1, \mathbb{G}_2, p, \hat{e})$  be a description of the bilinear group of prime order  $p$ .  $g$  is a generator of subgroup  $\mathbb{G}_1$ . The CDH assumption is that if the challenge tuple  $D = ((\mathbb{G}_1, \mathbb{G}_2, p, \hat{e}), g, g^a, g^b)$  is given, no probabilistic polynomial time (PPT) algorithm  $\mathcal{A}$  can output  $g^{ab} \in \mathbb{G}_1$  with more than a negligible advantage. The advantage of  $\mathcal{A}$  is defined as  $\text{Adv}_{\mathcal{A}}^{\text{CDH}}(\lambda) = \Pr[\mathcal{A}(D) = g^{ab}]$  where the probability is taken over random choices of  $a, b \in \mathbb{Z}_p$ .

## 2.2 Collision Resistant Hash (CRH) Assumption

Let  $H_k : \{0, 1\}^* \rightarrow \{0, 1\}^n$  be a collision-resistant hash family of functions, where  $n$  is a fixed length and  $k$  is an index. We say that the  $(\epsilon, t)$ -CRH assumption holds if no polynomial time adversary  $\mathcal{A}$  running in time at most  $t$  can break the collision resistance of  $H_k$  with probability  $\epsilon$ . Here, the successful probability of the adversary  $\mathcal{A}$  is presented as  $\Pr[\mathcal{A}(k) = (m_0, m_1) : m_0 \neq m_1, H_k(m_0) = H_k(m_1)]$ , where the probability is over the random choice consumed by the adversary  $\mathcal{A}$ .

## 2.3 Frameworks and Security Notions

*Identity-Based Signature (IBS) Scheme* consists of four polynomial-time algorithms as follows:

**Setup.** Given a security parameter  $\kappa$ , this algorithm produces a master secret key  $msk$  and the corresponding public parameters  $params$ . Then  $params$  are published and  $msk$  is kept by itself.

**Extract.** Given a user's identity  $ID$ , the public parameters  $params$  and the master secret key  $msk$ , this algorithm computes a private key  $D_{ID}$  for  $ID$ , which is transmitted to the user  $ID$  through a secure channel.

**Sign.** Given a private key  $D_{ID}$  of a user  $ID$  and a message  $m$ , this algorithm running by the user  $ID$  generates and outputs a signature  $\sigma$  of  $ID$  on  $m$ .

**Verify.** Given a user's identity  $ID$ , a message  $m$  and a signature  $\sigma$ , a verifier checks the validity of  $\sigma$ . More precisely, the algorithm outputs 1 if accepted, or 0 if rejected.

### Strong Unforgeability for Identity-Based Signature

Here, we denote by  $\mathcal{O}_E$  an oracle simulating the algorithm *Extract*, and by  $\mathcal{O}_S$  an oracle simulating the algorithm *Sign*. Strong unforgeability under an adaptive chosen-message attack is defined using the following game between a challenger  $\mathcal{C}$  and an adversary  $\mathcal{A}$ :

**Setup:**  $\mathcal{C}$  picks a security parameter  $\kappa$  and runs the algorithm *Setup*. It keeps the master secret key  $msk$  to itself and gives  $\mathcal{A}$  the resulting parameters  $params$ .

**Extract queries:**  $\mathcal{A}$  adaptively asks for the private key of any identity  $ID_i$ . To each extraction query of  $ID_i$ ,  $\mathcal{C}$  responds by running  $\mathcal{O}_E$  to generate the private key  $D_{ID_i}$  of  $ID_i$  and forwarding  $D_{ID_i}$  to  $\mathcal{A}$ .

**Signing queries:**  $\mathcal{A}$  adaptively asks for the signature of any identity  $ID_i$  on any message  $m_i$ . To each signing query of  $ID_i$  on  $M_i$ ,  $\mathcal{C}$  responds by running  $\mathcal{O}_S$  to generate a signature  $\sigma$ , and sending  $\sigma$  to  $\mathcal{A}$ .

**Forgery:**  $\mathcal{A}$  outputs  $(ID^*, m^*, \sigma^*)$  and wins if the followings hold:

- 1)  $\sigma^*$  is a valid signature of  $ID^*$  on  $m^*$ ;
- 2)  $ID^*$  is not queried during extract queries;
- 3)  $(ID^*, m^*, \sigma^*)$  is not queried during the sign queries.

We define  $\text{Adv}_{\mathcal{A}}$  to be the probability that  $\mathcal{A}$  wins the above game, taken over all coin toss of  $\mathcal{C}$  and  $\mathcal{A}$ . In this paper,  $\mathcal{A}$  is said to  $(t, q_e, q_s, \epsilon)$ -strongly break an identity-based signature (IBS) scheme if  $\mathcal{A}$  runs in time at most  $t$ , makes at most  $q_e$  *extract* queries, at most  $q_s$  *signing* queries, and  $\text{Adv}_{\mathcal{A}}$  is at least  $\epsilon$ . An IBS scheme is  $(t, q_e, q_s, \epsilon)$ -strongly existential unforgeable under an adaptive chosen message attack if no adversary  $(t, q_e, q_s, \epsilon)$ -strongly breaks it.

*Revocable Identity-Based Signature (RIBS) Scheme* consists of five polynomial-time algorithms as follows.

**Setup.** Given a security parameter  $\kappa$  and the total number  $z$  of all periods, this algorithm outputs a master secret key  $msk$  and the corresponding public parameters  $params$ . Then  $params$  are published and  $msk$  is kept by the PKG.

**Initial key extract.** Given an identity  $ID$ , the public parameters  $params$  and the master secret key  $msk$ , this algorithm outputs the initial key  $D_{ID}$  which is transmitted to the user  $ID$  through a secure channel.

**Time key update.** Given an identity  $ID$ , a time period  $t$ , the public parameters  $params$  and the master secret key  $msk$ , this algorithm outputs the time key  $T_{ID}$  which is transmitted to the user through a public channel. The user can combine the initial key  $D_{ID}$  and the time key  $T_{ID}$  to obtain the full private key  $S_{ID,t}$ .

**Sign.** Given an identity  $ID$ , the corresponding private key  $S_{ID,t}$ , a time period  $t$  and a message  $m$ , this algorithm outputs a signature  $\sigma$  of  $ID$  on  $m$  in  $t$ .

**Verify.** Given an identity  $ID$ , a message  $m$  and a signature  $\sigma$ , a verifier checks the validity of  $\sigma$  in the period  $t$ . More precisely, the algorithm outputs 1 if accepted, or 0 if rejected.

*Strong Unforgeability for Revocable Identity-Based Signature* Here, we denote by  $\mathcal{O}_E$  an oracle simulating the algorithm *Initial key extract*, by  $\mathcal{O}_T$  an oracle simulating the algorithm *Time key update*, and by  $\mathcal{O}_S$  an oracle simulating the algorithm *Sign*. Strong unforgeability under an adaptive chosen-message attack is defined using the following game between a challenger  $\mathcal{C}$  and an adversary  $\mathcal{A}$ :

**Setup:**  $\mathcal{C}$  picks a security parameter  $\kappa$  and runs the algorithm *Setup*. It keeps the master secret key  $msk$  to itself and gives  $\mathcal{A}$  the resulting parameters  $params$ .

**Extract queries:**  $\mathcal{A}$  adaptively asks for the initial key of any identity  $ID$ . To each extraction query of  $ID$ ,  $\mathcal{C}$  responds by running  $\mathcal{O}_E$  to generate the initial key  $D_{ID}$  and forwarding  $D_{ID}$  to  $\mathcal{A}$ .

**Update queries:**  $\mathcal{A}$  adaptively asks for the time key of any identity  $ID$  in period  $t$ . To each update query of  $ID$ ,  $\mathcal{C}$  responds by running  $\mathcal{O}_T$  to generate the time key  $T_{ID}$  and forwarding  $T_{ID}$  to  $\mathcal{A}$ .

**Signing queries:**  $\mathcal{A}$  adaptively asks for the signature of any identity  $ID$  on any message  $m$  in period  $t$ . To each signing query,  $\mathcal{C}$  responds by running  $\mathcal{O}_S$  to generate a signature  $\sigma$  and sending  $\sigma$  to  $\mathcal{A}$ .

**Forgery:**  $\mathcal{A}$  outputs  $(ID^*, m^*, t^*, \sigma^*)$  and wins if the following holds:

- 1)  $\sigma^*$  is a valid signature of  $ID^*$  on  $m^*$  in  $t^*$ ;
- 2)  $\sigma^*$  has not been outputted in the signing queries on  $(ID^*, m^*, t^*)$ ;
- 3) Either  $ID^*$  or  $(ID^*, t^*)$  has not appeared in the extract queries or the update queries, respectively.

The adversary  $\mathcal{A}$ 's advantage is defined as the probability that  $\mathcal{A}$  wins the above game. In addition, to simplify the security analysis, we consider two types of adversaries, namely, outside adversary and inside adversary (or revoked user). Note that if the adversary is an outsider, it is allowed to issue all queries in the above game except for the initial key extract query on the target identity  $ID^*$ . If the adversary is an insider, it is allowed to issue all queries in the above game except for the time key update query on  $(ID^*, t^*)$ .

### 3 Tsai *et al.*'s IBS Scheme

#### 3.1 Review of Tsai *et al.*'s Scheme

The strongly unforgeable identity-based signature scheme [12] is specified by the following four algorithms.

**Setup.** Given a security parameter  $\kappa$ , the PKG chooses two groups  $\mathbb{G}_1, \mathbb{G}_2$  of sufficiently large prime order  $p > 2^\kappa$ , a generator  $g$  of  $\mathbb{G}_1$  and an admissible bilinear map  $\hat{e} : \mathbb{G}_1 \times \mathbb{G}_1 \rightarrow \mathbb{G}_2$ . The PKG sets three collision resistant hash functions, namely,  $H_1 : \{0, 1\}^* \rightarrow \{0, 1\}^m$ ,  $H_2$  and  $H_3 : \{0, 1\}^* \rightarrow \{0, 1\}^n$ , where  $m$  and  $n$  are fixed lengths. We assume  $p > 2^m$  and  $p > 2^n$  so that the hash outputs can be viewed as the elements of  $\mathbb{Z}_p$ . The PKG chooses two random values  $u', w' \in \mathbb{G}_1$  as well as two vectors  $\vec{u} = (u_i)$  of length  $m$  and  $\vec{w} = (w_j)$  of length  $n$ , where  $u_i, w_j \in \mathbb{G}_1$  for  $i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$ . The PKG then chooses a secret random value  $\alpha \in \mathbb{Z}_p^*$  and sets  $g_1 = g^\alpha \in \mathbb{G}_1$ . Finally, the PKG randomly chooses  $g_2 \in \mathbb{G}_2$  and sets the master secret key  $msk = g_2^\alpha$  and the public parameters  $params = (\mathbb{G}_1, \mathbb{G}_2, \hat{e}, g, g_1, g_2, H_1, H_2, H_3, u', \vec{u}, w', \vec{w})$ .

**Extract.** Given a user's identity  $ID \in \{0, 1\}^*$ , the PKG sets  $v = H_1(ID)$ . Here,  $v = (v_1, v_2, \dots, v_m)$  is a bit string of length  $m$ . Let  $U \subset \{1, 2, \dots, m\}$  be the set of index  $i$  such that  $v_i = 1$ , for  $i = 1, 2, \dots, m$ . The PKG chooses a random value  $r_v \in \mathbb{Z}_p^*$  and computes the user's private key  $D_{ID} = (D_1, D_2) = (g_2^\alpha (u' \prod_{i \in U} u_i)^{r_v}, g^{r_v})$ . The PKG transmits  $D_{ID}$  to the user via a secure channel.

**Sign.** Given a message  $m \in \{0, 1\}^*$ , let  $vm = H_2(m)$  be a bit string of length  $n$  and let  $vm_j$  denote the  $j$ th bit of  $vm$ . Let  $W \subset \{1, 2, \dots, n\}$  be the set of index  $j$  such that  $vm_j = 1$ , for  $j = 1, 2, \dots, n$ . The signer with identity  $ID$  chooses a random number  $r_m \in \mathbb{Z}_p^*$  and then computes  $h = H_3(m \| g^{r_m})$ . The signer uses her/his private key  $D_{ID} = (D_1, D_2)$  to create a signature on the message  $m$  by  $\sigma = (D_1^h (w' \prod_{j \in W} w_j)^{r_m}, D_2^h, g^{r_m})$ .

**Verify.** Given a signature  $\sigma = (\sigma_1, \sigma_2, \sigma_3)$  of a signer  $ID$  on a message  $m$ , a verifier can compute  $h = H_3(m \| \sigma_3)$  to validate the signature tuple by the equation

$$\hat{e}(\sigma_1, g) \stackrel{?}{=} \hat{e}(g_1, g_2)^h \hat{e}(u' \prod_{i \in U} u_i, \sigma_2) \hat{e}(w' \prod_{i \in W} w_i, \sigma_3).$$

The algorithm outputs "accept" if the above equation holds, and "reject" otherwise.

#### 3.2 A Concrete Attack

Now, we shall in detail show how an attacker  $\mathcal{A}$  forges a new signature  $\sigma'$  for a previously signed message  $m$  by interacting with the challenger  $\mathcal{C}$  according to the security model [12].

1)  $\mathcal{C}$  takes a security parameter  $\kappa$  and runs the *Setup* algorithm to produce a master secret key  $msk = g_2^\alpha$  and public parameters  $params = (\mathbb{G}_1, \mathbb{G}_2, \hat{e}, g, g_1, g_2, H_1, H_2, H_3, u', \vec{u}, w', \vec{w})$ .  $\mathcal{C}$  then gives  $params$  to  $\mathcal{A}$  and keeps  $msk$  by itself.

2) Given any user's identity  $ID$  and any message  $m$ ,  $\mathcal{C}$  runs the *Sign* algorithms in Tsai *et al.*'s scheme and produces the corresponding signature  $\sigma$  for  $m$  under  $ID$ . The signature's concrete forms are as follows, where  $h = H_3(m||g^{r_m})$ .

$$\begin{aligned} \sigma &= (\sigma_1, \sigma_2, \sigma_3) \\ &= (D_1^h(w' \prod_{j \in W} w_j)^{r_m}, D_2^h, g^{r_m}) \\ &= ((g_2^\alpha(u' \prod_{i \in U} u_i)^{r_v})^h(w' \prod_{j \in W} w_j)^{r_m}, g^{r_v h}, g^{r_m}) \end{aligned}$$

3)  $\mathcal{A}$  picks  $r'_v \in_R \mathbb{Z}_p^*$  and computes a new signature  $\sigma'$  for  $m$  under  $ID$  with  $\sigma'_1 = \sigma_1((u' \prod_{i \in U} u_i)^{r'_v})^h = (g_2^\alpha(u' \prod_{i \in U} u_i)^{r_v+r'_v})^h(w' \prod_{j \in W} w_j)^{r_m}$ ,  $\sigma'_2 = \sigma_2 g^{r'_v h} = g^{(r_v+r'_v)h}$  and  $\sigma'_3 = \sigma_3$ , where  $h = H_3(m||g^{r_m})$ .

It is clear that  $\sigma'$  is a new valid signature for  $m$  on  $ID$  since

$$\begin{aligned} &\hat{e}(\sigma'_1, g) \\ &= \hat{e}(g_2^\alpha(u' \prod_{i \in U} u_i)^{r_v+r'_v})^h(w' \prod_{j \in W} w_j)^{r_m}, g) \\ &= \hat{e}(g_2^\alpha)^h, g) \hat{e}((u' \prod_{i \in U} u_i)^{r_v+r'_v})^h, g) \hat{e}(w' \prod_{j \in W} w_j)^{r_m}, g) \\ &= \hat{e}(g_1, g_2)^h \hat{e}(u' \prod_{i \in U} u_i, \sigma'_2) \hat{e}(w' \prod_{j \in W} w_j, \sigma'_3). \end{aligned}$$

Thus, the scheme fails to satisfy the requirement of strong unforgeability.

### 3.3 Flaws in the Security Proof

It is well known that a provably-secure cryptographic scheme should resist all attacks under the appropriate adversarial model. Then, why is Tsai *et al.*'s IBS scheme which is strictly proven under their security model not secure? In fact, there exist some fatal flaws in Tsai *et al.*'s security proof as follows.

• *Signing query* ( $ID, m$ ). Upon receiving the query along with  $(ID, m)$ , the challenger  $\mathcal{C}$  sets  $v = H_1(ID)$  and  $vm = H_2(m)$ .

**Case 1.** If  $F(v) \neq 0 \pmod{l_v}$ , the challenger  $\mathcal{C}$  can construct the private key for  $v = H_1(ID)$  as in the extract query, and then use the Signing algorithm to respond a signature  $\sigma$  on  $m$ .

**Case 2.** If  $F(v) = 0 \pmod{l_v}$  and  $K(vm) = 0 \pmod{l_m}$ , the challenger  $\mathcal{C}$  reports failure and terminates. Otherwise, if  $F(v) = 0 \pmod{l_v}$  and  $K(vm) \neq 0 \pmod{l_m}$ ,

the challenger  $\mathcal{C}$  chooses two random values  $r_v, r_m \in \mathbb{Z}_p^*$  and then computes  $h = H_3(m||g^{r_m})$  to generate the simulated signature  $\sigma = (\sigma_1, \sigma_2, \sigma_3)$ , where  $\sigma_3 = g_1^{\frac{-h}{K(vm)}} \cdot g^{r_m}$ .

From the above descriptions, we notice that if  $F(v) = 0 \pmod{l_v}$  (where  $v = H_1(ID)$ ),  $\mathcal{C}$  cannot respond a valid signature  $\sigma$  on message  $m$  under the identity  $ID$  since  $H_3(m||g^{r_m}) \neq H_3(m||\sigma_3)$ . That is to say, an adversary can easily distinguish the distribution of simulated signatures from that of real signatures by making some signing queries under the target identity  $ID^*$  and message  $m^*$ . Therefore, the security argument of Tsai *et al.*'s IBS scheme did not work out exactly as their simulated game definition.

## 4 Hung *et al.*'s RIBS Scheme

### 4.1 Review of Hung *et al.*'s Scheme

Here, we review the strongly secure revocable identity-based signature scheme [4] by the five algorithms below.

**Setup.** Given a security parameter  $\kappa$  and the total number  $z$  of all periods, the PKG chooses two cyclic groups  $\mathbb{G}_1$  and  $\mathbb{G}_2$  of sufficiently large prime order  $p > 2^\kappa$ . Let  $g$  be a generator of  $\mathbb{G}_1$  and  $\hat{e}: \mathbb{G}_1 \times \mathbb{G}_1 \rightarrow \mathbb{G}_2$  be an admissible map. The PKG sets the master secret key and the public parameters by running the following tasks.

- 1) Pick two secret values  $\alpha, \beta \in \mathbb{Z}_p^*$  at random and compute  $g_1 = g^{\alpha+\beta} \in \mathbb{G}_1$ . Select a random  $g_2 \in \mathbb{G}_1$  and compute  $g_2^\alpha$  and  $g_2^\beta$ .
- 2) Set four collision resistant hash functions  $H_1: \{0, 1\}^* \rightarrow \{0, 1\}^{n_u}$ ,  $H_2: \{0, 1\}^* \rightarrow \{0, 1\}^{n_t}$ ,  $H_3, H_4: \{0, 1\}^* \rightarrow \{0, 1\}^{n_m}$ , where  $n_u, n_t$  and  $n_m$  are fixed lengths.
- 3) Choose three random values  $u', t', w' \in \mathbb{G}_1$  and three vectors  $\vec{U} = (u_i)$ ,  $\vec{T} = (t_j)$ ,  $\vec{W} = (w_k)$ , where  $u_i, t_j, w_k \in \mathbb{G}_1$  for  $i = 1, 2, \dots, n_u$ ,  $j = 1, 2, \dots, n_t$  and  $k = 1, 2, \dots, n_m$ .
- 4) Finally, the PKG sets the master secret key  $msk = (g_2^\alpha, g_2^\beta)$  and the public parameters  $params = (\mathbb{G}_1, \mathbb{G}_2, \hat{e}, g, g_1, g_2, H_1, H_2, H_3, H_4, u', \vec{U}, t', \vec{T}, w', \vec{W})$ .

**Initial key extract.** Given a user's identity  $ID \in \{0, 1\}^*$ , the PKG sets  $vu = H_1(ID)$ . Here,  $vu = (vu_1, vu_2, \dots, vu_{n_u})$  is a bit string of length  $n_u$ . Let  $U \subset \{1, 2, \dots, n_u\}$  be the set of indices  $i$  such that  $vu_i = 1$ , for  $i = 1, 2, \dots, n_u$ . The PKG chooses a random value  $r_u \in \mathbb{Z}_p^*$  and computes the user's private key  $D_{ID} = (D_1, D_2) = (g_2^\alpha(u' \prod_{i \in U} u_i)^{r_u}, g^{r_u})$ . The PKG transmits  $D_{ID}$  to the user via a secure channel.

**Time key update.** Given a user's identity  $ID \in \{0, 1\}^*$  and a period  $t$ , the PKG sets  $vt = H_2(ID, t)$ . Here,

$vt = (vt_1, vt_2, \dots, vt_{n_t})$  is a bit string of length  $n_t$ . Let  $T \subset \{1, 2, \dots, n_t\}$  be the set of indices  $j$  such that  $vt_j = 1$ , for  $j = 1, 2, \dots, n_t$ . The PKG chooses a random value  $r_t \in \mathbb{Z}_p^*$  and computes the user's private key  $T_{ID} = (T_1, T_2) = (g_2^\beta (t' \prod_{j \in T} t_j)^{r_t}, g^{r_t})$ . The PKG sends  $T_{ID}$  to the user via a public channel. Upon receiving  $T_{ID}$ , the user combines it with his/her initial secret key  $D_{ID} = (D_1, D_2)$  to obtain the signing key  $S_{ID,t} = (S_1, S_2, S_3) = (D_1 T_1, D_2, T_2) = (g^{\alpha+\beta} (u' \prod_{i \in U} u_i)^{r_u} (t' \prod_{j \in T} t_j)^{r_t}, g^{r_u}, g^{r_t})$ .

**Sign.** For a period  $t$ , given a non-revoked user's identity  $ID \in \{0, 1\}^*$ , a message  $m \in \{0, 1\}^*$ , the user first computes a string  $vm = H_3(m)$  of length  $n_m$ . Let  $vm_k$  denote the  $k$ -th bit of the string  $vm$  and let  $W \subset \{1, 2, \dots, n_m\}$  be the set of indices  $k$  such that  $vm_k = 1$  for  $k = 1, 2, \dots, n_m$ . Then the user chooses a random number  $r_m \in \mathbb{Z}_p^*$  and computes  $g^{r_m}$  and  $h = H_4(m \| g^{r_m})$ . Finally, the user generates a signature  $\sigma$  on the message  $m$  as follows:

$$\begin{aligned}
 \sigma &= (\sigma_1, \sigma_2, \sigma_3, \sigma_4) \\
 &= (S_1^h (w' \prod_{k \in W} w_k)^{r_m}, S_2^h, S_3^h, g^{r_m}) \\
 &= (g_2^{(\alpha+\beta)h} (u' \prod_{i \in U} u_i)^{r_u h} (t' \prod_{j \in T} t_j)^{r_t h} (w' \prod_{k \in W} w_k)^{r_m}, \\
 &\quad g^{r_u h}, g^{r_t h}, g^{r_m}),
 \end{aligned}$$

where  $(S_1, S_2, S_3)$  is the signing key  $S_{ID,t}$  obtained above.

**Verify.** Given a signature  $\sigma = (\sigma_1, \sigma_2, \sigma_3, \sigma_4)$  of a signer  $ID$  on a message  $m$  in a period  $t$ , a verifier can compute  $h = H_4(m \| \sigma_4)$  to validate the signature tuple by the following equation

$$\begin{aligned}
 \hat{e}(\sigma_1, g) &\stackrel{?}{=} \hat{e}(g_1, g_2)^h \hat{e}(u' \prod_{i \in U} u_i, \sigma_2) \cdot \\
 &\quad \hat{e}(t' \prod_{j \in T} t_j, \sigma_3) \hat{e}(w' \prod_{k \in W} w_k, \sigma_4).
 \end{aligned}$$

The algorithm outputs "accept" if the above equation holds, and "reject" otherwise.

## 4.2 Some Concrete Attacks

In fact, Hung *et al.*'s RIBS scheme is based on Tsai *et al.*'s IBS scheme. Thus, we can easily give a similar cryptanalysis according to the same ideas in Subsection 3.2. Here, we shall show how an attacker  $\mathcal{A}$  forges a new signature  $\sigma'$  for a previously signed message  $m$  by interacting with the challenger  $\mathcal{C}$  under the security model defined in [4].

- 1)  $\mathcal{C}$  takes a security parameter  $\kappa$  and runs the *Setup* algorithm to produce the master secret key  $msk = (g_2^\alpha, g_2^\beta)$  and the public parameters  $params = (\mathbb{G}_1, \mathbb{G}_2, \hat{e}, g, g_1, g_2, H_1, H_2, H_3, H_4, u', \bar{U}, t', \bar{T}, w', \bar{W})$ .  $\mathcal{C}$  then gives  $params$  to  $\mathcal{A}$  and keeps  $msk$  by itself.

- 2) Given any user's identity  $ID$ , a period  $t$  and any message  $m$ ,  $\mathcal{C}$  runs the *Sign* algorithms in Hung *et al.*'s scheme and outputs the corresponding signature  $\sigma$  for  $m$  under  $ID$  in  $t$ . The signature's concrete forms are as follows, where  $h = H_3(m \| g^{r_m})$ .

$$\begin{aligned}
 \sigma &= (\sigma_1, \sigma_2, \sigma_3, \sigma_4) \\
 &= (S_1^h (w' \prod_{k \in W} w_k)^{r_m}, S_2^h, S_3^h, g^{r_m}) \\
 &= (g_2^{(\alpha+\beta)h} (u' \prod_{i \in U} u_i)^{r_u h} (t' \prod_{j \in T} t_j)^{r_t h} (w' \prod_{k \in W} w_k)^{r_m}, \\
 &\quad g^{r_u h}, g^{r_t h}, g^{r_m})
 \end{aligned}$$

- 3)  $\mathcal{A}$  picks two random values  $r'_u$  and  $r'_t$  from  $\mathbb{Z}_p^*$  and forges a new signature  $\sigma'$  on  $m$  under  $ID$  in  $t$  as follows, where  $h = H_3(m \| g^{r_m})$ .

$$\begin{aligned}
 \sigma'_1 &= \sigma_1 ((u' \prod_{i \in U} u_i)^{r'_u})^h ((t' \prod_{j \in T} t_j)^{r'_t})^h \\
 &= (g_2^{\alpha+\beta} (u' \prod_{i \in U} u_i)^{r_u+r'_u} (t' \prod_{j \in T} t_j)^{r_t+r'_t})^h (w' \prod_{j \in W} w_j)^{r_m} \\
 \sigma'_2 &= \sigma_2 g^{r'_u h} = g^{(r_u+r'_u)h}, \quad \sigma'_3 = \sigma_3 g^{r'_t h} = g^{(r_t+r'_t)h} \quad \text{and} \\
 \sigma'_4 &= \sigma_4.
 \end{aligned}$$

It is clear that  $\sigma'$  is a new valid signature for  $m$  on  $ID$  in  $t$  since

$$\begin{aligned}
 &\hat{e}(\sigma'_1, g) \\
 &= \hat{e}(g_2^{(\alpha+\beta)h} (u' \prod_{i \in U} u_i)^{(r_u+r'_u)h} (t' \prod_{j \in T} t_j)^{(r_t+r'_t)h} \\
 &\quad (w' \prod_{k \in W} w_k)^{r_m}, g) \\
 &= \hat{e}(g_2^{(\alpha+\beta)h}, g) \hat{e}((u' \prod_{i \in U} u_i)^{(r_u+r'_u)h}, g) \cdot \\
 &\quad \hat{e}((t' \prod_{j \in T} t_j)^{(r_t+r'_t)h}, g) \hat{e}((w' \prod_{k \in W} w_k)^{r_m}, g) \\
 &= \hat{e}(g_1, g_2)^h \hat{e}(u' \prod_{i \in U} u_i, \sigma'_2) \hat{e}(t' \prod_{j \in T} t_j, \sigma'_3) \hat{e}(w' \prod_{k \in W} w_k, \sigma'_4).
 \end{aligned}$$

Therefore, the RIBS scheme fails to meet the requirement of strong unforgeability under their security model. For more details about cause analysis, the interested readers are referred to Section 3.3.

## 5 Conclusion

Strong unforgeability has been widely accepted as a common security requirement for signature schemes. In this paper, we first reviewed two so-called strongly unforgeable identity-based signatures presented by Tsai *et al.* and Hung *et al.*, respectively. Then, we demonstrated that both of them are not strongly unforgeable by giving some concrete attacks. Finally, we illustrated that there exist some serious errors in their proving process.

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