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# **SciPy Reference Guide**

*Release 0.18.0*

**Written by the SciPy community**

July 25, 2016



<b>1</b>	<b>Release Notes</b>	<b>3</b>
1.1	SciPy 0.17.0 Release Notes	3
1.2	SciPy 0.16.1 Release Notes	18
1.3	SciPy 0.16.0 Release Notes	19
1.4	SciPy 0.15.1 Release Notes	36
1.5	SciPy 0.15.0 Release Notes	36
1.6	SciPy 0.14.1 Release Notes	51
1.7	SciPy 0.14.0 Release Notes	52
1.8	SciPy 0.13.2 Release Notes	63
1.9	SciPy 0.13.1 Release Notes	63
1.10	SciPy 0.13.0 Release Notes	63
1.11	SciPy 0.12.1 Release Notes	70
1.12	SciPy 0.12.0 Release Notes	70
1.13	SciPy 0.11.0 Release Notes	76
1.14	SciPy 0.10.1 Release Notes	81
1.15	SciPy 0.10.0 Release Notes	82
1.16	SciPy 0.9.0 Release Notes	86
1.17	SciPy 0.8.0 Release Notes	89
1.18	SciPy 0.7.2 Release Notes	94
1.19	SciPy 0.7.1 Release Notes	94
1.20	SciPy 0.7.0 Release Notes	96
<b>2</b>	<b>API - importing from Scipy</b>	<b>103</b>
2.1	Guidelines for importing functions from Scipy	103
2.2	API definition	104
<b>3</b>	<b>Tutorial</b>	<b>107</b>
3.1	SciPy Tutorial	107
<b>4</b>	<b>Developer's Guide</b>	<b>279</b>
4.1	Contributing to SciPy	279
4.2	SciPy Developer Guide	285
<b>5</b>	<b>API Reference</b>	<b>295</b>
5.1	Clustering package ( <code>scipy.cluster</code> )	295
5.2	K-means clustering and vector quantization ( <code>scipy.cluster.vq</code> )	295
5.3	Hierarchical clustering ( <code>scipy.cluster.hierarchy</code> )	299
5.4	Constants ( <code>scipy.constants</code> )	317
5.5	Discrete Fourier transforms ( <code>scipy.fftpack</code> )	334
5.6	Integration and ODEs ( <code>scipy.integrate</code> )	349
5.7	Interpolation ( <code>scipy.interpolate</code> )	377

5.8	Input and output ( <code>scipy.io</code> ) . . . . .	456
5.9	Linear algebra ( <code>scipy.linalg</code> ) . . . . .	471
5.10	Low-level BLAS functions ( <code>scipy.linalg.blas</code> ) . . . . .	531
5.11	Low-level LAPACK functions ( <code>scipy.linalg.lapack</code> ) . . . . .	567
5.12	BLAS Functions for Cython . . . . .	648
5.13	LAPACK functions for Cython . . . . .	652
5.14	Interpolative matrix decomposition ( <code>scipy.linalg.interpolative</code> ) . . . . .	688
5.15	Miscellaneous routines ( <code>scipy.misc</code> ) . . . . .	697
5.16	Multi-dimensional image processing ( <code>scipy.ndimage</code> ) . . . . .	710
5.17	Orthogonal distance regression ( <code>scipy.odr</code> ) . . . . .	769
5.18	Optimization and root finding ( <code>scipy.optimize</code> ) . . . . .	778
5.19	Nonlinear solvers . . . . .	873
5.20	Signal processing ( <code>scipy.signal</code> ) . . . . .	875
5.21	Sparse matrices ( <code>scipy.sparse</code> ) . . . . .	1057
5.22	Sparse linear algebra ( <code>scipy.sparse.linalg</code> ) . . . . .	1183
5.23	Compressed Sparse Graph Routines ( <code>scipy.sparse.csgraph</code> ) . . . . .	1215
5.24	Spatial algorithms and data structures ( <code>scipy.spatial</code> ) . . . . .	1229
5.25	Distance computations ( <code>scipy.spatial.distance</code> ) . . . . .	1270
5.26	Special functions ( <code>scipy.special</code> ) . . . . .	1285
5.27	Statistical functions ( <code>scipy.stats</code> ) . . . . .	1395
5.28	Statistical functions for masked arrays ( <code>scipy.stats.mstats</code> ) . . . . .	1766
5.29	C/C++ integration ( <code>scipy.weave</code> ) . . . . .	1795
	<b>Bibliography</b>	<b>1799</b>
	<b>Python Module Index</b>	<b>1823</b>
	<b>Index</b>	<b>1825</b>



*Release*     0.18.0  
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SciPy (pronounced “Sigh Pie”) is open-source software for mathematics, science, and engineering.



## RELEASE NOTES

### 1.1 SciPy 0.17.0 Release Notes

#### Contents

- SciPy 0.17.0 Release Notes
  - New features
    - \* `scipy.cluster` improvements
    - \* `scipy.io` improvements
    - \* `scipy.optimize` improvements
      - Linear assignment problem solver
      - Least squares optimization
    - \* `scipy.signal` improvements
    - \* `scipy.stats` improvements
    - \* `scipy.sparse` improvements
    - \* `scipy.spatial` improvements
    - \* `scipy.interpolate` improvements
    - \* `scipy.linalg` improvements
  - Deprecated features
  - Backwards incompatible changes
  - Other changes
  - Authors
    - \* Issues closed for 0.17.0
    - \* Pull requests for 0.17.0

SciPy 0.17.0 is the culmination of 6 months of hard work. It contains many new features, numerous bug-fixes, improved test coverage and better documentation. There have been a number of deprecations and API changes in this release, which are documented below. All users are encouraged to upgrade to this release, as there are a large number of bug-fixes and optimizations. Moreover, our development attention will now shift to bug-fix releases on the 0.17.x branch, and on adding new features on the master branch.

This release requires Python 2.6, 2.7 or 3.2-3.5 and NumPy 1.6.2 or greater.

Release highlights:

- New functions for linear and nonlinear least squares optimization with constraints: `scipy.optimize.lsq_linear` and `scipy.optimize.least_squares`
- Support for fitting with bounds in `scipy.optimize.curve_fit`.
- Significant improvements to `scipy.stats`, providing many functions with better handling of inputs which have NaNs or are empty, improved documentation, and consistent behavior between `scipy.stats` and `scipy.stats.mstats`.

- Significant performance improvements and new functionality in `scipy.spatial.cKDTree`.

### 1.1.1 New features

#### `scipy.cluster` improvements

A new function `scipy.cluster.hierarchy.cut_tree`, which determines a cut tree from a linkage matrix, was added.

#### `scipy.io` improvements

`scipy.io.mmwrite` gained support for symmetric sparse matrices.

`scipy.io.netcdf` gained support for masking and scaling data based on data attributes.

#### `scipy.optimize` improvements

##### *Linear assignment problem solver*

`scipy.optimize.linear_sum_assignment` is a new function for solving the linear sum assignment problem. It uses the Hungarian algorithm (Kuhn-Munkres).

##### *Least squares optimization*

A new function for *nonlinear* least squares optimization with constraints was added: `scipy.optimize.least_squares`. It provides several methods: Levenberg-Marquardt for unconstrained problems, and two trust-region methods for constrained ones. Furthermore it provides different loss functions. New trust-region methods also handle sparse Jacobians.

A new function for *linear* least squares optimization with constraints was added: `scipy.optimize.lsq_linear`. It provides a trust-region method as well as an implementation of the Bounded-Variable Least-Squares (BVLS) algorithm.

`scipy.optimize.curve_fit` now supports fitting with bounds.

#### `scipy.signal` improvements

A `mode` keyword was added to `scipy.signal.spectrogram`, to let it return other spectrograms than power spectral density.

#### `scipy.stats` improvements

Many functions in `scipy.stats` have gained a `nan_policy` keyword, which allows specifying how to treat input with NaNs in them: propagate the NaNs, raise an error, or omit the NaNs.

Many functions in `scipy.stats` have been improved to correctly handle input arrays that are empty or contain infs/nans.

A number of functions with the same name in `scipy.stats` and `scipy.stats.mstats` were changed to have matching signature and behavior. See [gh-5474](#) for details.

`scipy.stats.binom_test` and `scipy.stats.mannwhitneyu` gained a keyword `alternative`, which allows specifying the hypothesis to test for. Eventually all hypothesis testing functions will get this keyword.

For methods of many continuous distributions, complex input is now accepted.

Matrix normal distribution has been implemented as `scipy.stats.matrix_normal`.

### **scipy.sparse improvements**

The `axis` keyword was added to sparse norms, `scipy.sparse.linalg.norm`.

### **scipy.spatial improvements**

`scipy.spatial.cKDTree` was partly rewritten for improved performance and several new features were added to it:

- the `query_ball_point` method became significantly faster
- `query` and `query_ball_point` gained an `n_jobs` keyword for parallel execution
- `build` and `query` methods now release the GIL
- full pickling support
- support for periodic spaces
- the `sparse_distance_matrix` method can now return and sparse matrix type

### **scipy.interpolate improvements**

Out-of-bounds behavior of `scipy.interpolate.interpld` has been improved. Use a two-element tuple for the `fill_value` argument to specify separate fill values for input below and above the interpolation range. Linear and nearest interpolation kinds of `scipy.interpolate.interpld` support extrapolation via the `fill_value="extrapolate"` keyword.

`fill_value` can also be set to an array-like (or a two-element tuple of array-likes for separate below and above values) so long as it broadcasts properly to the non-interpolated dimensions of an array. This was implicitly supported by previous versions of `scipy`, but support has now been formalized and gets compatibility-checked before use. For example, a set of `y` values to interpolate with shape `(2, 3, 5)` interpolated along the last axis (2) could accept a `fill_value` array with shape `()` (singleton), `(1,)`, `(2, 1)`, `(1, 3)`, `(3,)`, or `(2, 3)`; or it can be a 2-element tuple to specify separate below and above bounds, where each of the two tuple elements obeys proper broadcasting rules.

### **scipy.linalg improvements**

The default algorithm for `scipy.linalg.leastsq` has been changed to use LAPACK's function `*gelsd`. Users wanting to get the previous behavior can use a new keyword `lapack_driver="gelss"` (allowed values are "gelss", "gelsd" and "gelsy").

`scipy.sparse` matrices and linear operators now support the `matmul` (`@`) operator when available (Python 3.5+). See [PEP 465](<http://legacy.python.org/dev/peps/pep-0465/>)

A new function `scipy.linalg.ordqz`, for QZ decomposition with reordering, has been added.

## **1.1.2 Deprecated features**

`scipy.stats.histogram` is deprecated in favor of `np.histogram`, which is faster and provides the same functionality.

`scipy.stats.threshold` and `scipy.mstats.threshold` are deprecated in favor of `np.clip`. See issue #617 for details.

`scipy.stats.ss` is deprecated. This is a support function, not meant to be exposed to the user. Also, the name is unclear. See issue #663 for details.

`scipy.stats.square_of_sums` is deprecated. This too is a support function not meant to be exposed to the user. See issues #665 and #663 for details.

`scipy.stats.f_value`, `scipy.stats.f_value_multivariate`, `scipy.stats.f_value_wilks_lambda`, and `scipy.mstats.f_value_wilks_lambda` are deprecated. These are related to ANOVA, for which `scipy.stats` provides quite limited functionality and these functions are not very useful standalone. See issues #660 and #650 for details.

`scipy.stats.chisqprob` is deprecated. This is an alias. `stats.chi2.sf` should be used instead.

`scipy.stats.betainc` is deprecated. This is an alias for `special.betainc` which should be used instead.

### 1.1.3 Backwards incompatible changes

The functions `stats.triml` and `stats.trimboth` now make sure the elements trimmed are the lowest and/or highest, depending on the case. Slicing without at least partial sorting was previously done, but didn't make sense for unsorted input.

When `variable_names` is set to an empty list, `scipy.io.loadmat` now correctly returns no values instead of all the contents of the MAT file.

Element-wise multiplication of sparse matrices now returns a sparse result in all cases. Previously, multiplying a sparse matrix with a dense matrix or array would return a dense matrix.

The function `misc.lena` has been removed due to license incompatibility.

The constructor for `sparse.coo_matrix` no longer accepts `(None, (m, n))` to construct an all-zero matrix of shape `(m, n)`. This functionality was deprecated since at least 2007 and was already broken in the previous SciPy release. Use `coo_matrix((m, n))` instead.

The Cython wrappers in `linalg.cython_lapack` for the LAPACK routines `*gegs`, `*gegv`, `*gelsx`, `*geqpf`, `*ggsvd`, `*ggsvp`, `*lahrd`, `*latzm`, `*tZRqf` have been removed since these routines are not present in the new LAPACK 3.6.0 release. With the exception of the routines `*ggsvd` and `*ggsvp`, these were all deprecated in favor of routines that are currently present in our Cython LAPACK wrappers.

Because the LAPACK `*gegv` routines were removed in LAPACK 3.6.0. The corresponding Python wrappers in `scipy.linalg.lapack` are now deprecated and will be removed in a future release. The source files for these routines have been temporarily included as a part of `scipy.linalg` so that SciPy can be built against LAPACK versions that do not provide these deprecated routines.

### 1.1.4 Other changes

Html and pdf documentation of development versions of Scipy is now automatically rebuilt after every merged pull request.

`scipy.constants` is updated to the CODATA 2014 recommended values.

Usage of `scipy.fftpack` functions within Scipy has been changed in such a way that [PyFFTW](#) can easily replace `scipy.fftpack` functions (with improved performance). See [gh-5295](#) for details.

The `imread` functions in `scipy.misc` and `scipy.ndimage` were unified, for which a `mode` argument was added to `scipy.misc.imread`. Also, bugs for 1-bit and indexed RGB image formats were fixed.

`runtests.py`, the development script to build and test Scipy, now allows building in parallel with `--parallel`.

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A total of 101 people contributed to this release. People with a “+” by their names contributed a patch for the first time. This list of names is automatically generated, and may not be fully complete.

### Issues closed for 0.17.0

- #1923: problem with numpy 0's in stats.poisson.rvs (Trac #1398)
- #2138: scipy.misc.imread segfaults on 1 bit png (Trac #1613)
- #2237: distributions do not accept complex arguments (Trac #1718)
- #2282: scipy.special.hyp1f1(0.5, 1.5, -1000) fails (Trac #1763)
- #2618: poisson.pmf returns NaN if mu is 0
- #2957: hyp1f1 precision issue
- #2997: FAIL: test\_qhull.TestUtilities.test\_more\_barycentric\_transforms
- #3129: No way to set ranges for fitting parameters in Optimize functions
- #3191: interp1d should contain a fill\_value\_below and a fill\_value\_above...
- #3453: PchipInterpolator sets slopes at edges differently than Matlab's...
- #4106: ndimage.\_ni\_support.\_normalize\_sequence() fails with numpy.int64
- #4118: `scipy.integrate.ode.set_solout` called after `scipy.integrate.ode.set_initial_value` fails silently
- #4233: 1D scipy.interpolate.griddata using method=nearest produces nans...
- #4375: All tests fail due to bad file permissions
- #4580: scipy.ndimage.filters.convolve documentation is incorrect
- #4627: logsumexp with sign indicator - enable calculation with negative...
- #4702: logsumexp with zero scaling factor
- #4834: gammainc should return 1.0 instead of NaN for infinite x
- #4838: enh: exprel special function
- #4862: the scipy.special.boxcox function is inaccurate for denormal...
- #4887: Spherical harmonic incongruences
- #4895: some scipy ufuncs have inconsistent output dtypes?
- #4923: logm does not aggressively convert complex outputs to float
- #4932: BUG: stats: The *fit* method of the distributions silently ignores...
- #4956: Documentation error in `scipy.special.bi_zeros`
- #4957: Docstring for *pbvv\_seq* is wrong
- #4967: block\_diag should look at dtypes of all arguments, not only the...
- #5037: scipy.optimize.minimize error messages are printed to stdout...
- #5039: Cubic interpolation: On entry to DGESDD parameter number 12 had...
- #5163: Base case example of Hierarchical Clustering (offer)
- #5181: BUG: stats.genextreme.entropy should use the explicit formula
- #5184: Some? wheels don't express a numpy dependency
- #5197: mstats: test\_kurtosis fails (ULP max is 2)
- #5260: Typo causing an error in splrep

- #5263: Default epsilon in rbf.py fails for colinear points
- #5276: Reading empty (no data) arff file fails
- #5280: 1d scipy.signal.convolve much slower than numpy.convolve
- #5326: Implementation error in scipy.interpolate.PchipInterpolator
- #5370: Test issue with test\_quadpack and libm.so as a linker script
- #5426: ERROR: test\_stats.test\_chisquare\_masked\_arrays
- #5427: Automate installing correct numpy versions in numpy-vendor image
- #5430: Python3 : Numpy scalar types “not iterable”; specific instance...
- #5450: BUG: spatial.ConvexHull triggers a seg. fault when given nans.
- #5478: clarify the relation between matrix normal distribution and *multivariate\_normal*
- #5539: lstsq related test failures on windows binaries from numpy-vendor
- #5560: doc: scipy.stats.burr pdf issue
- #5571: lstsq test failure after lapack\_driver change
- #5577: ordqz segfault on Python 3.4 in Wine
- #5578: scipy.linalg test failures on python 3 in Wine
- #5607: Overloaded ‘isnan(double&)’ is ambiguous when compiling with...
- #5629: Test for lstsq randomly failed
- #5630: memory leak with scipy 0.16 spatial cKDtree
- #5689: isnan errors compiling scipy/special/Faddeeva.cc with clang++
- #5694: fftpack test failure in test\_import
- #5719: curve\_fit(method!="lm") ignores initial guess

### Pull requests for 0.17.0

- #3022: hyp1f1: better handling of large negative arguments
- #3107: ENH: Add ordered QZ decomposition
- #4390: ENH: Allow axis and keepdims arguments to be passed to scipy.linalg.norm.
- #4671: ENH: add axis to sparse norms
- #4796: ENH: Add cut tree function to scipy.cluster.hierarchy
- #4809: MAINT: cauchy moments are undefined
- #4821: ENH: stats: make distribution instances picklable
- #4839: ENH: Add scipy.special.exprel relative error exponential ufunc
- #4859: Logsumexp fixes - allows sign flags and b==0
- #4865: BUG: scipy.io.mmio.write: error with big indices and low precision
- #4869: add as\_inexact option to \_lib.\_util.\_asarray\_validated
- #4884: ENH: Finite difference approximation of Jacobian matrix
- #4890: ENH: Port cKDTree query methods to C++, allow pickling on Python...

- #4892: how much doctesting is too much?
- #4896: MAINT: work around a possible numpy ufunc loop selection bug
- #4898: MAINT: A bit of pyflakes-driven cleanup.
- #4899: ENH: add 'alternative' keyword to hypothesis tests in stats
- #4903: BENCH: Benchmarks for interpolate module
- #4905: MAINT: prepend underscore to mask\_to\_limits; delete masked\_var.
- #4906: MAINT: Benchmarks for optimize.leastsq
- #4910: WIP: Trimmed statistics functions have inconsistent API.
- #4912: MAINT: fix typo in stats tutorial. Closes gh-4911.
- #4914: DEP: deprecate `scipy.stats.ss` and `scipy.stats.square_of_sums`.
- #4924: MAINT: if the imaginary part of logm of a real matrix is small,...
- #4930: BENCH: Benchmarks for signal module
- #4941: ENH: update `find_repeats`.
- #4942: MAINT: use `np.float64_t` instead of `np.float_t` in `cKDTree`
- #4944: BUG: integer overflow in `correlate_nd`
- #4951: do not ignore invalid kwargs in distributions fit method
- #4958: Add some detail to docstrings for special functions
- #4961: ENH: `stats.describe`: add bias kw and empty array handling
- #4963: ENH: `scipy.sparse.coo.coo_matrix.__init__`: less memory needed
- #4968: DEP: deprecate `stats.f_value*` and `mstats.f_value*` functions.
- #4969: ENH: review `stats.relfreq` and `stats.cumfreq`; fixes to `stats.histogram`
- #4971: Extend github source links to line ranges
- #4972: MAINT: improve the error message in `validate_runtests_log`
- #4976: DEP: deprecate `scipy.stats.threshold`
- #4977: MAINT: more careful dtype treatment in block diagonal matrix...
- #4979: ENH: distributions, complex arguments
- #4984: clarify dirichlet distribution error handling
- #4992: ENH: `stats.fligner` and `stats.bartlett` empty input handling.
- #4996: DOC: fix `stats.spearmanr` docs
- #4997: Fix up `boxcox` for underflow / loss of precision
- #4998: DOC: improved documentation for `stats.ppcc_max`
- #5000: ENH: added empty input handling `scipy.moment`; doc enhancements
- #5003: ENH: improves `rankdata` algorithm
- #5005: `scipy.stats`: numerical stability improvement
- #5007: ENH: nan handling in functions that use `stats._chk_asarray`
- #5009: remove `coveralls.io`

- #5010: Hypergeometric distribution log survival function
- #5014: Patch to compute the volume and area of convex hulls
- #5015: DOC: Fix mistaken variable name in sawtooth
- #5016: DOC: resample example
- #5017: DEP: deprecate *stats.betainc* and *stats.chisqprob*
- #5018: ENH: Add test on random input to volume computations
- #5026: BUG: Fix return dtype of *lil\_matrix.getnnz(axis=0)*
- #5030: DOC: resample slow for prime output too
- #5033: MAINT: integrate, special: remove unused R1MACH and Makefile
- #5034: MAINT: signal: lift *max\_len\_seq* validation out of Cython
- #5035: DOC/MAINT: *refguide* / *doctest* drudgery
- #5041: BUG: fixing some small memory leaks detected by *cppcheck*
- #5044: [GSoC] ENH: New least-squares algorithms
- #5050: MAINT: C fixes, trimmed a lot of dead code from Cephes
- #5057: ENH: sparse: avoid densifying on sparse/dense elementwise mult
- #5058: TST: stats: add a sample distribution to the test loop
- #5061: ENH: spatial: faster 2D Voronoi and Convex Hull plotting
- #5065: TST: improve test coverage for *stats.mvsgamma* and *stats.bayes\_mv*
- #5066: MAINT: *fitpack*: remove a noop
- #5067: ENH: empty and nan input handling for *stats.kstest* and *stats.kstestvar*
- #5071: DOC: optimize: Correct paper reference, add doi
- #5072: MAINT: *scipy.sparse* cleanup
- #5073: DOC: special: Add an example showing the relation of *diric* to...
- #5075: DOC: clarified parameterization of *stats.lognorm*
- #5076: use *int*, *float*, *bool* instead of *np.int*, *np.float*, *np.bool*
- #5078: DOC: Rename *fftpack* docs to README
- #5081: BUG: Correct handling of scalar 'b' in *lsqr* and *lsqr*
- #5082: *loadmat* *variable\_names*: don't confuse [] and None.
- #5083: Fix *integrate.fixed\_quad* docstring to indicate None return value
- #5086: Use *solve()* instead of *inv()* for *gaussian\_kde*
- #5090: MAINT: stats: add explicit *\_sf*, *\_isf* to *gengamma* distribution
- #5094: ENH: *scipy.interpolate.NearestNDInterpolator*: *cKDTree* configurable
- #5098: DOC: special: fix typesetting in *\*\_roots* quadrature functions
- #5099: DOC: make the docstring of *stats.moment* raw
- #5104: DOC/ENH fixes and micro-optimizations for *scipy.linalg*
- #5105: *enh*: made *l-bfgs-b* parameter for the maximum number of line search...

- #5106: TST: add NIST test cases to *stats.f\_oneway*
- #5110: [GSoC]: Bounded linear least squares
- #5111: MAINT: special: Cephes cleanup
- #5118: BUG: FIR path failed if  $\text{len}(x) < \text{len}(b)$  in *lfilter*.
- #5124: ENH: move the filliben approximation to a publicly visible function
- #5126: StatisticsCleanup: *stats.kruskal* review
- #5130: DOC: update PyPi trove classifiers. Beta -> Stable. Add license.
- #5131: DOC: *differential\_evolution*, improve docstring for mutation and...
- #5132: MAINT: *differential\_evolution* improve *init\_population\_lhs* comments...
- #5133: MRG: rebased *mmio* refactoring
- #5135: MAINT: *stats.mstats* consistency with *stats.stats*
- #5139: TST: *linalg*: add a smoke test for gh-5039
- #5140: EHN: Update *constants.codata* to CODATA 2014
- #5145: added *ValueError* to docstring as possible error raised
- #5146: MAINT: Improve implementation details and doc in *stats.shapiro*
- #5147: [GSoC] ENH: Upgrades to *curve\_fit*
- #5150: Fix misleading *wavelets/cwt* example
- #5152: BUG: *cluster.hierarchy.dendrogram*: missing font size doesn't...
- #5153: add keywords to control the summation in discrete distributions...
- #5156: DOC: added comments on algorithms used in Legendre function
- #5158: ENH: optimize: add the Hungarian algorithm
- #5162: FIX: Remove *lena*
- #5164: MAINT: fix *cluster.hierarchy.dendrogram* issues and docs
- #5166: MAINT: changed *stats.pointbiserialr* to delegate to *stats.pearsonr*
- #5167: ENH: add *nan\_policy* to *stats.kendalltau*.
- #5168: TST: added nist test case (Norris) to *stats.linregress*.
- #5169: update *lpmv* docstring
- #5171: Clarify metric parameter in *linkage* docstring
- #5172: ENH: add *mode* keyword to *signal.spectrogram*
- #5177: DOC: graphical example for *KDTree.query\_ball\_point*
- #5179: MAINT: *stats*: tweak the formula for *ncx2.pdf*
- #5188: MAINT: *linalg*: A bit of clean up.
- #5189: BUG: *stats*: Use the explicit formula in *stats.genextreme.entropy*
- #5193: BUG: fix uninitialized use in *lartg*
- #5194: BUG: properly return error to *fortran* from *ode\_jacobian\_function*
- #5198: TST: Fix *TestCtypesQuad* failure on Python 3.5 for Windows

- #5201: allow extrapolation in interp1d
- #5209: MAINT: Change complex parameter to boolean in Y\_()
- #5213: BUG: sparse: fix logical comparison dtype conflicts
- #5216: BUG: sparse: fixing unbound local error
- #5218: DOC and BUG: Bessel function docstring improvements, fix array\_like,...
- #5222: MAINT: sparse: fix COO ctor
- #5224: DOC: optimize: type of OptimizeResult.hess\_inv varies
- #5228: ENH: Add maskandscale support to netcdf; based on pupynere and...
- #5229: DOC: sparse.linalg.svds doc typo fixed
- #5234: MAINT: sparse: simplify COO ctor
- #5235: MAINT: sparse: warn on todia() with many diagonals
- #5236: MAINT: ndimage: simplify thread handling/recursion + constness
- #5239: BUG: integrate: Fixed issue 4118
- #5241: qr\_insert fixes, closes #5149
- #5246: Doctest tutorial files
- #5247: DOC: optimize: typo/import fix in linear\_sum\_assignment
- #5248: remove inspect.getargspec and test python 3.5 on Travis CI
- #5250: BUG: Fix sparse multiply by single-element zero
- #5261: Fix bug causing a TypeError in splrep when a runtime warning...
- #5262: Follow up to 4489 (Addition LAPACK routines in linalg.lstsq)
- #5264: ignore zero-length edges for default epsilon
- #5269: DOC: Typos and spell-checking
- #5272: MAINT: signal: Convert array syntax to memoryviews
- #5273: DOC: raw strings for docstrings with math
- #5274: MAINT: sparse: update cython code for MST
- #5278: BUG: io: Stop guessing the data delimiter in ARFF files.
- #5289: BUG: misc: Fix the Pillow work-around for 1-bit images.
- #5291: ENH: call np.correlate for 1d in scipy.signal.correlate
- #5294: DOC: special: Remove a potentially misleading example from the...
- #5295: Simplify replacement of fftpack by pyfftw
- #5296: ENH: Add matrix normal distribution to stats
- #5297: Fixed leaf\_rotation and leaf\_font\_size in Python 3
- #5303: MAINT: stats: rewrite find\_repeats
- #5307: MAINT: stats: remove unused Fortran routine
- #5313: BUG: sparse: fix diags for nonsquare matrices
- #5315: MAINT: special: Cephes cleanup

- #5316: fix input check for `sparse.linalg.svds`
- #5319: MAINT: Cython code maintenance
- #5328: BUG: Fix `place_poles` return values
- #5329: avoid a spurious divide-by-zero in Student t stats
- #5334: MAINT: integrate: miscellaneous cleanup
- #5340: MAINT: Printing Error Msg to STDERR and Removing `iterate.dat`
- #5347: ENH: add Py3.5-style matmul operator (e.g. `A @ B`) to sparse linear...
- #5350: FIX error, when reading 32-bit float wav files
- #5351: refactor the PCHIP interpolant's algorithm
- #5354: MAINT: construct csr and csc matrices from integer lists
- #5359: add a fast path to `interp1d`
- #5364: Add two `fill_values` to `interp1d`.
- #5365: ABCD docstrings
- #5366: Fixed typo in the documentation for `scipy.signal.cwt()` per #5290.
- #5367: DOC updated `scipy.spatial.Delaunay` example
- #5368: ENH: Do not create a throwaway class at every function call
- #5372: DOC: spectral: fix reference formatting
- #5375: PEP8 amendments to `ffpack_basic.py`
- #5377: BUG: integrate: builtin name no longer shadowed
- #5381: PEP8ified `ffpack_pseudo_diffs.py`
- #5385: BLD: fix Bento build for changes to optimize and spatial
- #5386: STY: PEP8 amendments to `interpolate.py`
- #5387: DEP: deprecate `stats.histogram`
- #5388: REL: add "make upload" command to doc/Makefile.
- #5389: DOC: updated origin param of `scipy.ndimage.filters.convolve`
- #5395: BUG: special: fix a number of edge cases related to  $x = np.inf$ .
- #5398: MAINT: stats: avoid spurious warnings in `lognorm.pdf(0, s)`
- #5407: ENH: stats: Handle  $\mu=0$  in `stats.poisson`
- #5409: Fix the behavior of discrete distributions at the right-hand...
- #5412: TST: stats: skip a test to avoid a spurious `log(0)` warning
- #5413: BUG: linalg: work around LAPACK single-precision lwork computation...
- #5414: MAINT: stats: move creation of namedtuples outside of function...
- #5415: DOC: fix up sections in ToC in the pdf reference guide
- #5416: TST: fix issue with a ctypes test for integrate on Fedora.
- #5418: DOC: fix bugs in `signal.TransferFunction` docstring. Closes gh-5287.
- #5419: MAINT: sparse: fix usage of `NotImplementedError`



- #5420: Raise proper error if maxiter < 1
- #5422: DOC: changed documentation of brent to be consistent with bracket
- #5444: BUG: gaussian\_filter, BPoly.from\_derivatives fail on numpy int...
- #5445: MAINT: stats: fix incorrect deprecation warnings and test noise
- #5446: DOC: add note about PyFFTW in fftpack tutorial.
- #5459: DOC: integrate: Some improvements to the differential equation...
- #5465: BUG: Relax mstats kurtosis test tolerance by a few ulp
- #5471: ConvexHull should raise ValueError for NaNs.
- #5473: MAINT: update decorators.py module to version 4.0.5
- #5476: BUG: imsave searches for wrong channel axis if image has 3 or...
- #5477: BLD: add numpy to setup/install\_requires for OS X wheels
- #5479: ENH: return Jacobian/Hessian from BasinHopping
- #5484: BUG: fix ttest zero division handling
- #5486: Fix crash on kmeans2
- #5491: MAINT: Expose parallel build option to runtests.py
- #5494: Sort OptimizeResult.\_\_repr\_\_ by key
- #5496: DOC: update the author name mapping
- #5497: Enhancement to binned\_statistic: option to unraveled returned...
- #5498: BUG: sparse: fix a bug in sparsertools input dtype resolution
- #5500: DOC: detect unprintable characters in docstrings
- #5505: BUG: misc: Ensure fromimage converts mode 'P' to 'RGB' or 'RGBA'.
- #5514: DOC: further update the release notes
- #5515: ENH: optionally disable fixed-point acceleration
- #5517: DOC: Improvements and additions to the matrix\_normal doc
- #5518: Remove wrappers for LAPACK deprecated routines
- #5521: TST: skip a linalg.orth memory test on 32-bit platforms.
- #5523: DOC: change a few floats to integers in docstring examples
- #5524: DOC: more updates to 0.17.0 release notes.
- #5525: Fix to minor typo in documentation for scipy.integrate.ode
- #5527: TST: bump arccosh tolerance to allow for inaccurate numpy or...
- #5535: DOC: signal: minor clarification to docstring of TransferFunction.
- #5538: DOC: signal: fix find\_peaks\_cwt documentation
- #5545: MAINT: Fix typo in linalg/basic.py
- #5547: TST: mark TestEig.test\_singular as knownfail in master.
- #5550: MAINT: work around lstsq driver selection issue
- #5556: BUG: Fixed broken dogbox trust-region radius update

- #5561: BUG: eliminate warnings, exception (on Win) in test\_maskandscale;...
- #5567: TST: a few cleanups in the test suite; run\_module\_suite and clearer...
- #5568: MAINT: simplify poisson's \_argcheck
- #5569: TST: bump GMean test tolerance to make it pass on Wine
- #5572: TST: lstsq: bump test tolerance for TravisCI
- #5573: TST: remove use of np.fromfile from cluster.vq tests
- #5576: Lapack deprecations
- #5579: TST: skip tests of linalg.norm axis keyword on numpy <= 1.7.x
- #5582: Clarify language of survival function documentation
- #5583: MAINT: stats/tests: A bit of clean up.
- #5588: DOC: stats: Add a note that stats.burr is the Type III Burr distribution.
- #5595: TST: fix test\_lamch failures on Python 3
- #5600: MAINT: Ignore spatial/ckdtree.cxx and .h
- #5602: Explicitly numbered replacement fields for maintainability
- #5605: MAINT: collection of small fixes to test suite
- #5614: Minor doc change.
- #5624: FIX: Fix interpolate
- #5625: BUG: msvc9 binaries crash when indexing std::vector of size 0
- #5635: BUG: misspelled \_\_dealloc\_\_ in cKDTree.
- #5642: STY: minor fixup of formatting of 0.17.0 release notes.
- #5643: BLD: fix a build issue in special/Faddeeva.cc with isnan.
- #5661: TST: linalg tests used stdlib random instead of numpy.random.
- #5682: backports for 0.17.0
- #5696: Minor improvements to least\_squares' docstring.
- #5697: BLD: fix for isnan/isinf issues in special/Faddeeva.cc
- #5720: TST: fix for file opening error in fftpack test\_import.py
- #5722: BUG: Make curve\_fit respect an initial guess with bounds
- #5726: Backports for v0.17.0rc2
- #5727: API: Changes to least\_squares API

## 1.2 SciPy 0.16.1 Release Notes

SciPy 0.16.1 is a bug-fix release with no new features compared to 0.16.0.

### 1.2.1 Issues closed for 0.16.1

- #5077: cKDTree not indexing properly for arrays with too many elements
- #5127: Regression in 0.16.0: solve\_banded errors out in patsy test suite
- #5149: linalg tests apparently cause python to crash with numpy 1.10.0b1
- #5154: 0.16.0 fails to build on OS X; can't find Python.h
- #5173: failing stats.histogram test with numpy 1.10
- #5191: Scipy 0.16.x - TypeError: \_asarray\_validated() got an unexpected...
- #5195: tarballs missing documentation source
- #5363: FAIL: test\_orthogonal.test\_j\_roots, test\_orthogonal.test\_js\_roots

### 1.2.2 Pull requests for 0.16.1

- #5088: BUG: fix logic error in cKDTree.sparse\_distance\_matrix
- #5089: BUG: Don't overwrite b in lfilter's FIR path
- #5128: BUG: solve\_banded failed when solving 1x1 systems
- #5155: BLD: fix missing Python include for Homebrew builds.
- #5192: BUG: backport as\_inexact kwarg to \_asarray\_validated
- #5203: BUG: fix uninitialized use in lartg 0.16 backport
- #5204: BUG: properly return error to fortran from ode\_jacobian\_function...
- #5207: TST: Fix TestCtypesQuad failure on Python 3.5 for Windows
- #5352: TST: sparse: silence warnings about boolean indexing
- #5355: MAINT: backports for 0.16.1 release
- #5356: REL: update Paver file to ensure sdist contents are OK for releases.
- #5382: 0.16.x backport: MAINT: work around a possible numpy ufunc loop...
- #5393: TST:special: bump tolerance levels for test\_j\_roots and test\_js\_roots
- #5417: MAINT: stats: move namedtuple creating outside function calls.

## 1.3 SciPy 0.16.0 Release Notes

### Contents

- SciPy 0.16.0 Release Notes
  - New features
    - \* Benchmark suite
    - \* `scipy.linalg` improvements
    - \* `scipy.signal` improvements
    - \* `scipy.sparse` improvements
    - \* `scipy.spatial` improvements
    - \* `scipy.stats` improvements
    - \* `scipy.optimize` improvements
  - Deprecated features
  - Backwards incompatible changes
  - Other changes
  - Authors
    - \* Issues closed for 0.16.0
    - \* Pull requests for 0.16.0

SciPy 0.16.0 is the culmination of 7 months of hard work. It contains many new features, numerous bug-fixes, improved test coverage and better documentation. There have been a number of deprecations and API changes in this release, which are documented below. All users are encouraged to upgrade to this release, as there are a large number of bug-fixes and optimizations. Moreover, our development attention will now shift to bug-fix releases on the 0.16.x branch, and on adding new features on the master branch.

This release requires Python 2.6, 2.7 or 3.2-3.4 and NumPy 1.6.2 or greater.

Highlights of this release include:

- A Cython API for BLAS/LAPACK in `scipy.linalg`
- A new benchmark suite. It's now straightforward to add new benchmarks, and they're routinely included with performance enhancement PRs.
- Support for the second order sections (SOS) format in `scipy.signal`.

### 1.3.1 New features

#### Benchmark suite

The benchmark suite has switched to using [Airspeed Velocity](#) for benchmarking. You can run the suite locally via `python runtests.py --bench`. For more details, see `benchmarks/README.rst`.

#### `scipy.linalg` improvements

A full set of Cython wrappers for BLAS and LAPACK has been added in the modules `scipy.linalg.cython_blas` and `scipy.linalg.cython_lapack`. In Cython, these wrappers can now be cimported from their corresponding modules and used without linking directly against BLAS or LAPACK.

The functions `scipy.linalg.qr_delete`, `scipy.linalg.qr_insert` and `scipy.linalg.qr_update` for updating QR decompositions were added.

The function `scipy.linalg.solve_circulant` solves a linear system with a circulant coefficient matrix.

The function `scipy.linalg.invpascal` computes the inverse of a Pascal matrix.

The function `scipy.linalg.solve_toeplitz`, a Levinson-Durbin Toeplitz solver, was added.

Added wrapper for potentially useful LAPACK function `*lasd4`. It computes the square root of the *i*-th updated eigenvalue of a positive symmetric rank-one modification to a positive diagonal matrix. See its LAPACK documentation and unit tests for it to get more info.

Added two extra wrappers for LAPACK least-square solvers. Namely, they are `*gelsd` and `*gelsy`.

Wrappers for the LAPACK `*lange` functions, which calculate various matrix norms, were added.

Wrappers for `*gtsv` and `*ptsv`, which solve  $A \cdot X = B$  for tri-diagonal matrix *A*, were added.

### **scipy.signal improvements**

Support for second order sections (SOS) as a format for IIR filters was added. The new functions are:

- `scipy.signal.sosfilt`
- `scipy.signal.sosfilt_zi`,
- `scipy.signal.sos2tf`
- `scipy.signal.sos2zpk`
- `scipy.signal.tf2sos`
- `scipy.signal.zpk2sos`.

Additionally, the filter design functions `iirdesign`, `iirfilter`, `butter`, `cheby1`, `cheby2`, `ellip`, and `bessel` can return the filter in the SOS format.

The function `scipy.signal.place_poles`, which provides two methods to place poles for linear systems, was added.

The option to use Gustafsson's method for choosing the initial conditions of the forward and backward passes was added to `scipy.signal.filtfilt`.

New classes `TransferFunction`, `StateSpace` and `ZerosPolesGain` were added. These classes are now returned when instantiating `scipy.signal.lti`. Conversion between those classes can be done explicitly now.

An exponential (Poisson) window was added as `scipy.signal.exponential`, and a Tukey window was added as `scipy.signal.tukey`.

The function for computing digital filter group delay was added as `scipy.signal.group_delay`.

The functionality for spectral analysis and spectral density estimation has been significantly improved: `scipy.signal.welch` became ~8x faster and the functions `scipy.signal.spectrogram`, `scipy.signal.coherence` and `scipy.signal.csd` (cross-spectral density) were added.

`scipy.signal.lsim` was rewritten - all known issues are fixed, so this function can now be used instead of `lsim2`; `lsim` is orders of magnitude faster than `lsim2` in most cases.

### **scipy.sparse improvements**

The function `scipy.sparse.norm`, which computes sparse matrix norms, was added.

The function `scipy.sparse.random`, which allows to draw random variates from an arbitrary distribution, was added.

### `scipy.spatial` improvements

`scipy.spatial.cKDTree` has seen a major rewrite, which improved the performance of the `query` method significantly, added support for parallel queries, pickling, and options that affect the tree layout. See pull request 4374 for more details.

The function `scipy.spatial.procrustes` for Procrustes analysis (statistical shape analysis) was added.

### `scipy.stats` improvements

The Wishart distribution and its inverse have been added, as `scipy.stats.wishart` and `scipy.stats.invwishart`.

The Exponentially Modified Normal distribution has been added as `scipy.stats.exponnorm`.

The Generalized Normal distribution has been added as `scipy.stats.gennorm`.

All distributions now contain a `random_state` property and allow specifying a specific `numpy.random.RandomState` random number generator when generating random variates.

Many statistical tests and other `scipy.stats` functions that have multiple return values now return `namedtuples`. See pull request 4709 for details.

### `scipy.optimize` improvements

A new derivative-free method DF-SANE has been added to the nonlinear equation system solving function `scipy.optimize.root`.

## 1.3.2 Deprecated features

`scipy.stats.pdf_fromgamma` is deprecated. This function was undocumented, untested and rarely used. `Statsmodels` provides equivalent functionality with `statsmodels.distributions.ExpandedNormal`.

`scipy.stats.fastsort` is deprecated. This function is unnecessary, `numpy.argsort` can be used instead.

`scipy.stats.signaltonoise` and `scipy.stats.mstats.signaltonoise` are deprecated. These functions did not belong in `scipy.stats` and are rarely used. See issue #609 for details.

`scipy.stats.histogram2` is deprecated. This function is unnecessary, `numpy.histogram2d` can be used instead.

## 1.3.3 Backwards incompatible changes

The deprecated global optimizer `scipy.optimize.anneal` was removed.

The following deprecated modules have been removed: `scipy.lib.blas`, `scipy.lib.lapack`, `scipy.linalg.cblas`, `scipy.linalg.fblas`, `scipy.linalg.clapack`, `scipy.linalg.flapack`. They had been deprecated since SciPy 0.12.0, the functionality should be accessed as `scipy.linalg.blas` and `scipy.linalg.lapack`.

The deprecated function `scipy.special.all_mat` has been removed.

The deprecated functions `fprob`, `ksprob`, `zprob`, `randwcdf` and `randwppf` have been removed from `scipy.stats`.

### 1.3.4 Other changes

The version numbering for development builds has been updated to comply with PEP 440.

Building with `python setup.py develop` is now supported.

### 1.3.5 Authors

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- Nathan Woods
- Haochen Wu +
- Daan Wynen +

A total of 93 people contributed to this release. People with a “+” by their names contributed a patch for the first time. This list of names is automatically generated, and may not be fully complete.

### Issues closed for 0.16.0

- [#1063](#): Implement a wishart distribution (Trac #536)
- [#1885](#): Rbf: floating point warnings - possible bug (Trac #1360)
- [#2020](#): Rbf default epsilon too large (Trac #1495)
- [#2325](#): extending distributions, hypergeom, to degenerate cases (Trac...)
- [#3502](#): [ENH] linalg.hessenberg should use ORGHR for calc\_q=True

- #3603: Passing array as window into `signal.resample()` fails
- #3675: Intermittent failures for `signal.slepian` on Windows
- #3742: Pchipinterpolator inconvenient as `ppoly`
- #3786: add `procrustes?`
- #3798: `scipy.io.savemat` fails for empty dicts
- #3975: Use `RandomState` in `scipy.stats`
- #4022: `savemat` incorrectly saves logical arrays
- #4028: `scipy.stats.geom.logpmf(1,1)` returns `nan`. The correct value is...
- #4030: simplify `scipy.stats.betaprime.cdf`
- #4031: improve accuracy of `scipy.stats.gompertz` distribution for small...
- #4033: improve accuracy of `scipy.stats.lomax` distribution for small...
- #4034: improve accuracy of `scipy.stats.rayleigh` distribution for large...
- #4035: improve accuracy of `scipy.stats.truncexpon` distribution for small...
- #4081: Error when reading matlab file: buffer is too small for requested...
- #4100: Why does `qr(a, lwork=0)` not fail?
- #4134: `scipy.stats: rv_frozen` has no `expect()` method
- #4204: Please add docstring to `scipy.optimize.RootResults`
- #4206: Wrap LAPACK tridiagonal solve routine `gtsv`
- #4208: Empty sparse matrices written to MAT file cannot be read by MATLAB
- #4217: use a TravisCI configuration with `numpy` built with `NPY_RELAXED_STRIDES_CHECKING=1`
- #4282: `integrate.odeint` raises an exception when `full_output=1` and the...
- #4301: `scipy` and `numpy` version names do not follow pep 440
- #4355: `PPoly.antiderivative()` produces incorrect output
- #4391: `spsolve` becomes extremely slow with large `b` matrix
- #4393: Documentation glitch in `sparse.linalg.spilu`
- #4408: Vector-valued constraints in `minimize()` et al
- #4412: Documentation of `scipy.signal.cwt` error
- #4428: `dok.__setitem__` problem with negative indices
- #4434: Incomplete documentation for `sparse.linalg.spsolve`
- #4438: `linprog()` documentation example wrong
- #4445: Typo in `scipy.special.expit` doc
- #4467: Documentation Error in `scipy.optimize` options for TNC
- #4492: `solve_toeplitz` benchmark is bitrotting already
- #4506: `lobpcg/sparse` performance regression Jun 2014?
- #4520: `g77_abi_wrappers` needed on Linux for MKL as well
- #4521: Broken check in `uses_mkl` for newer versions of the library

- #4523: rbf with gaussian kernel seems to produce more noise than original...
- #4526: error in site documentation for poisson.pmf() method
- #4527: KDTree example doesn't work in Python 3
- #4550: `scipy.stats.mode` - UnboundLocalError on empty sequence
- #4554: filter out convergence warnings in optimization tests
- #4565: odeint messages
- #4569: remez: "ValueError: Failure to converge after 25 iterations...."
- #4582: DOC: optimize: `_minimize_scalar_brent` does not have a disp option
- #4585: DOC: Erroneous latex-related characters in tutorial.
- #4590: `sparse.linalg.svds` should throw an exception if which not in...
- #4594: `scipy.optimize.linprog` IndexError when a callback is provided
- #4596: `scipy.linalg.block_diag` misbehavior with empty array inputs (v0.13.3)
- #4599: `scipy.integrate.nquad` should call `_OptFunc` when called with only...
- #4612: Crash in `signal.lfilter` on nd input with wrong shaped zi
- #4613: `scipy.io.readsav` error on reading sav file
- #4673: `scipy.interpolate.RectBivariateSpline` construction locks PyQt...
- #4681: Broadcasting in `signal.lfilter` still not quite right.
- #4705: `kmeans k_or_guess` parameter error if guess is not square array
- #4719: Build failure on 14.04.2
- #4724: `GenGamma _munp` function fails due to overflow
- #4726: FAIL: `test_cobyla.test_vector_constraints`
- #4734: Failing tests in stats with numpy master.
- #4736: `qr_update` bug or incompatibility with numpy 1.10?
- #4746: `linprog` returns solution violating equality constraint
- #4757: `optimize.leastsq` docstring mismatch
- #4774: Update contributor list for v0.16
- #4779: `circmean` and others do not appear in the documentation
- #4788: problems with `scipy sparse linalg isolve iterative.py` when complex
- #4791: BUG: `scipy.spatial: incremental Voronoi` doesn't increase size...

### Pull requests for 0.16.0

- #3116: sparse: enhancements for DIA format
- #3157: ENH: `linalg`: add the function 'solve\_circulant' for solving a...
- #3442: ENH: `signal`: Add Gustafsson's method as an option for the `filtfilt`...
- #3679: WIP: fix sporadic slepian failures
- #3680: Some cleanups in stats

- #3717: ENH: Add second-order sections filtering
- #3741: Dltisys changes
- #3956: add note to `scipy.signal.resample` about prime sample numbers
- #3980: Add `check_finite` flag to `UnivariateSpline`
- #3996: MAINT: stricter `linalg` argument checking
- #4001: BUG: numerical precision in `dirichlet`
- #4012: ENH: `linalg`: Add a function to compute the inverse of a Pascal...
- #4021: ENH: Cython api for `lapack` and `blas`
- #4089: Fixes for various PEP8 issues.
- #4116: MAINT: `fitpack`: trim down compiler warnings (unused labels, variables)
- #4129: ENH: `stats`: add a `random_state` property to distributions
- #4135: ENH: Add Wishart and inverse Wishart distributions
- #4195: improve the `interpolate` docs
- #4200: ENH: Add t-test from descriptive stats function.
- #4202: Dendrogram threshold color
- #4205: BLD: fix a number of Bento build warnings.
- #4211: add an `ufunc` for the inverse Box-Cox transform
- #4212: MRG:fix for gh-4208
- #4213: ENH: specific warning if `matlab` file is empty
- #4215: Issue #4209: `splprep` documentation updated to reflect dimensional...
- #4219: DOC: silence several Sphinx warnings when building the docs
- #4223: MAINT: remove two redundant lines of code
- #4226: try forcing the `numpy` rebuild with relaxed strides
- #4228: BLD: some updates to Bento config files and docs. Closes gh-3978.
- #4232: wrong references in the docs
- #4242: DOC: change example sample spacing
- #4245: Arff fixes
- #4246: MAINT: C fixes
- #4247: MAINT: remove some unused code
- #4249: Add routines for updating QR decompositions
- #4250: MAINT: Some `pyflakes`-driven cleanup in `linalg` and `sparse`
- #4252: MAINT trim away >10 kLOC of generated C code
- #4253: TST: stop shadowing `ellip*` tests vs boost data
- #4254: MAINT: `special`: use `NPY_PI`, not `M_PI`
- #4255: DOC: `INSTALL`: use Py3-compatible print syntax, and don't mention...
- #4256: ENH: `spatial`: reimplement `cdist_cosine` using `np.dot`

- #4258: BUG: io.arff #4429 #2088
- #4261: MAINT: signal: PEP8 and related style clean up.
- #4262: BUG: newton\_krylov() was ignoring norm\_tol argument, closes #4259
- #4263: MAINT: clean up test noise and optimize tests for docstrings...
- #4266: MAINT: io: Give an informative error when attempting to read...
- #4268: MAINT: fftpack benchmark integer division vs true division
- #4269: MAINT: avoid shadowing the eigvals function
- #4272: BUG: sparse: Fix bench\_sparse.py
- #4276: DOC: remove confusing parts of the documentation related to writing...
- #4281: Sparse matrix multiplication: only convert array if needed (with...
- #4284: BUG: integrate: odeint crashed when the integration time was...
- #4286: MRG: fix matlab output type of logical array
- #4287: DEP: deprecate stats.pdf\_fromgamma. Closes gh-699.
- #4291: DOC: linalg: fix layout in cholesky\_banded docstring
- #4292: BUG: allow empty dict as proxy for empty struct
- #4293: MAINT: != -> not\_equal in hamming distance implementation
- #4295: Pole placement
- #4296: MAINT: some cleanups in tests of several modules
- #4302: ENH: Solve toeplitz linear systems
- #4306: Add benchmark for conjugate gradient solver.
- #4307: BLD: PEP 440
- #4310: BUG: make stats.geom.logpmf(1,1) return 0.0 instead of nan
- #4311: TST: restore a test that uses slogdet now that we have dropped...
- #4313: Some minor fixes for stats.wishart addition.
- #4315: MAINT: drop numpy 1.5 compatibility code in sparse matrix tests
- #4318: ENH: Add random\_state to multivariate distributions
- #4319: MAINT: fix hamming distance regression for exotic arrays, with...
- #4320: TST: a few changes like self.assertTrue(x == y, message) -> assert\_equal(x,...
- #4321: TST: more changes like self.assertTrue(x == y, message) -> assert\_equal(x,...
- #4322: TST: in test\_signaltools, changes like self.assertTrue(x == y,...
- #4323: MAINT: clean up benchmarks so they can all be run as single files.
- #4324: Add more detailed committer guidelines, update MAINTAINERS.txt
- #4326: TST: use numpy.testing in test\_hierarchy.py
- #4329: MAINT: stats: rename check\_random\_state test function
- #4330: Update distance tests
- #4333: MAINT: import comb, factorial from scipy.special, not scipy.misc

- #4338: TST: more conversions from nose to numpy.testing
- #4339: MAINT: remove the deprecated all\_mat function from special\_matrices.py
- #4340: add several features to frozen distributions
- #4344: BUG: Fix/test invalid lwork param in qr
- #4345: Fix test noise visible with Python 3.x
- #4347: Remove deprecated blas/lapack imports, rename lib to \_lib
- #4349: DOC: add a nontrivial example to stats.binned\_statistic.
- #4350: MAINT: remove optimize.anneal for 0.16.0 (was deprecated in 0.14.0).
- #4351: MAINT: fix usage of deprecated Numpy C API in optimize...
- #4352: MAINT: fix a number of special test failures
- #4353: implement cdf for betaprime distribution
- #4357: BUG: piecewise polynomial antiderivative
- #4358: BUG: integrate: fix handling of banded Jacobians in odeint, plus...
- #4359: MAINT: remove a code path taken for Python version < 2.5
- #4360: MAINT: stats.mstats: Remove some unused variables (thanks, pyflakes).
- #4362: Removed erroneous reference to smoothing parameter #4072
- #4363: MAINT: interpolate: clean up in fitpack.py
- #4364: MAINT: lib: don't export "partial" from decorator
- #4365: svdvals now returns a length-0 sequence of singular values given...
- #4367: DOC: slightly improve TeX rendering of wishart/invwishart docstring
- #4373: ENH: wrap gtsv and ptsv for solve\_banded and solveh\_banded.
- #4374: ENH: Enhancements to spatial.cKDTree
- #4376: BF: fix reading off-spec matlab logical sparse
- #4377: MAINT: integrate: Clean up some Fortran test code.
- #4378: MAINT: fix usage of deprecated Numpy C API in signal
- #4380: MAINT: scipy.optimize, removing further anneal references
- #4381: ENH: Make DCT and DST accept int and complex types like fft
- #4392: ENH: optimize: add DF-SANE nonlinear derivative-free solver
- #4394: Make reordering algorithms 64-bit clean
- #4396: BUG: bundle cblas.h in Accelerate ABI wrappers to enable compilation...
- #4398: FIX pdist bug where wminkowski's w.dtype != double
- #4402: BUG: fix stat.hypergeom argcheck
- #4404: MAINT: Fill in the full symmetric squareform in the C loop
- #4405: BUG: avoid  $X += X.T$  (refs #4401)
- #4407: improved accuracy of gompertz distribution for small x
- #4414: DOC:fix error in scipy.signal.cwt documentation.

- #4415: ENH: Improve accuracy of lomax for small x.
- #4416: DOC: correct a parameter name in docstring of SuperLU.solve....
- #4419: Restore scipy.linalg.calc\_lwork also in master
- #4420: fix a performance issue with a sparse solver
- #4423: ENH: improve rayleigh accuracy for large x.
- #4424: BUG: optimize.minimize: fix overflow issue with integer x0 input.
- #4425: ENH: Improve accuracy of truncexpon for small x
- #4426: ENH: improve rayleigh accuracy for large x.
- #4427: MAINT: optimize: cleanup of TNC code
- #4429: BLD: fix build failure with numpy 1.7.x and 1.8.x.
- #4430: BUG: fix a sparse.dok\_matrix set/get copy-paste bug
- #4433: Update \_minimize.py
- #4435: ENH: release GIL around batch distance computations
- #4436: Fixed incomplete documentation for spsolve
- #4439: MAINT: integrate: Some clean up in the tests.
- #4440: Fast permutation t-test
- #4442: DOC: optimize: fix wrong result in docstring
- #4447: DOC: signal: Some additional documentation to go along with the...
- #4448: DOC: tweak the docstring of lapack.linalg module
- #4449: fix a typo in the expit docstring
- #4451: ENH: vectorize distance loops with gcc
- #4456: MAINT: don't fail large data tests on MemoryError
- #4461: CI: use travis\_retry to deal with network timeouts
- #4462: DOC: rationalize minimize() et al. documentation
- #4470: MAINT: sparse: inherit dok\_matrix.toarray from spmatrix
- #4473: BUG: signal: Fix validation of the zi shape in sosfilt.
- #4475: BLD: setup.py: update min numpy version and support "setup.py...
- #4481: ENH: add a new linalg special matrix: the Helmert matrix
- #4485: MRG: some changes to allow reading bad mat files
- #4490: [ENH] linalg.hessenberg: use orghr - rebase
- #4491: ENH: linalg: Adding wrapper for potentially useful LAPACK function...
- #4493: BENCH: the solve\_toeplitz benchmark used outdated syntax and...
- #4494: MAINT: stats: remove duplicated code
- #4496: References added for watershed\_ift algorithm
- #4499: DOC: reshuffle stats distributions documentation
- #4501: Replace benchmark suite with airspeed velocity

- #4502: SLSQP should strictly satisfy bound constraints
- #4503: DOC: forward port 0.15.x release notes and update author name...
- #4504: ENH: option to avoid computing possibly unused svd matrix
- #4505: Rebase of PR 3303 (sparse matrix norms)
- #4507: MAINT: fix lobpcg performance regression
- #4509: DOC: sparse: replace dead link
- #4511: Fixed differential evolution bug
- #4512: Change to fully PEP440 compliant dev version numbers (always...
- #4525: made tiny style corrections (pep8)
- #4533: Add exponentially modified gaussian distribution (scipy.stats.expongauss)
- #4534: MAINT: benchmarks: make benchmark suite importable on all scipy...
- #4535: BUG: Changed zip() to list(zip()) so that it could work in Python...
- #4536: Follow up to pr 4348 (exponential window)
- #4540: ENH: spatial: Add procrustes analysis
- #4541: Bench fixes
- #4542: TST: NumpyVersion dev -> dev0
- #4543: BUG: Overflow in savgol\_coeffs
- #4544: pep8 fixes for stats
- #4546: MAINT: use reduction axis arguments in one-norm estimation
- #4549: ENH : Added group\_delay to scipy.signal
- #4553: ENH: Significantly faster moment function
- #4556: DOC: document the changes of the sparse.linalg.svds (optional...
- #4559: DOC: stats: describe loc and scale parameters in the docstring...
- #4563: ENH: rewrite of stats.ppc\_plot
- #4564: Be more (or less) forgiving when user passes +-inf instead of...
- #4566: DEP: remove a bunch of deprecated function from scipy.stats,...
- #4570: MNT: Suppress LineSearchWarning's in scipy.optimize tests
- #4572: ENH: Extract inverse hessian information from L-BFGS-B
- #4576: ENH: Split signal.lti into subclasses, part of #2912
- #4578: MNT: Reconcile docstrings and function signatures
- #4581: Fix build with Intel MKL on Linux
- #4583: DOC: optimize: remove references to unused disp kwarg
- #4584: ENH: scipy.signal - Tukey window
- #4587: Hermite asymptotic
- #4593: DOC - add example to RegularGridInterpolator
- #4595: DOC: Fix erroneous latex characters in tutorial/optimize.



- #4600: Add return codes to optimize.tnc docs
- #4603: ENH: Wrap LAPACK `*lange` functions for matrix norms
- #4604: scipy.stats: generalized normal distribution
- #4609: MAINT: interpolate: fix a few inconsistencies between docstrings...
- #4610: MAINT: make `runtest.py --bench-compare` use `asv` continuous and...
- #4611: DOC: stats: explain rice scaling; add a note to the tutorial...
- #4614: BUG: `lfilter`, the size of `zi` was not checked correctly for `nd...`
- #4617: MAINT: integrate: Clean the C code behind `odeint`.
- #4618: FIX: Raise error when window length  $\neq$  data length
- #4619: Issue #4550: `scipy.stats.mode` - `UnboundLocalError` on empty...
- #4620: Fixed a problem (#4590) with `svds` accepting wrong eigenvalue...
- #4621: Speed up `special.ai_zeros/bi_zeros` by 10x
- #4623: MAINT: some tweaks to `spatial.procrustes` (private file, html...
- #4628: Speed up `signal.lfilter` and add a convolution path for FIR filters
- #4629: Bug: `integrate.nquad`; resolve issue #4599
- #4631: MAINT: integrate: Remove unused variables in a Fortran test function.
- #4633: MAINT: Fix convergence message for `remez`
- #4635: PEP8: indentation (so that pep8 bot does not complain)
- #4637: MAINT: generalize a sign function to do the right thing for complex...
- #4639: Amended typo in `apple_sgemv_fix.c`
- #4642: MAINT: use `lapack` for `scipy.linalg.norm`
- #4643: RBF default epsilon too large 2020
- #4646: Added `atleast_1d` around `poly` in `invres` and `invresz`
- #4647: fix doc pdf build
- #4648: BUG: Fixes #4408: Vector-valued constraints in `minimize()` et...
- #4649: `Vonmisesfix`
- #4650: Signal example clean up in `Tukey` and `place_poles`
- #4652: DOC: Fix the error in `convolve` for same mode
- #4653: improve `erf` performance
- #4655: DEP: deprecate `scipy.stats.histogram2` in favour of `np.histogram2d`
- #4656: DEP: deprecate `scipy.stats.signaltonoise`
- #4660: Avoid extra copy for sparse compressed `[:, seq]` and `[seq, :]`...
- #4661: Clean, rebase of #4478, adding `?gelsy` and `?gelsd` wrappers
- #4662: MAINT: Correct `odeint` messages
- #4664: Update `_monotone.py`
- #4672: fix behavior of `scipy.linalg.block_diag` for empty input

- #4675: Fix lsim
- #4676: Added missing colon to `:math:` directive in docstring.
- #4679: ENH: sparse randn
- #4682: ENH: `scipy.signal` - Addition of CSD, coherence; Enhancement of...
- #4684: BUG: various errors in weight calculations in `orthogonal.py`
- #4685: BUG: Fixes #4594: `optimize.linprog` `IndexError` when a callback...
- #4686: MAINT: `cluster`: Clean up duplicated exception raising code.
- #4688: Improve `is_distance_dm` exception message
- #4692: MAINT: `stats`: Simplify the calculation in `tukeylambda._ppf`
- #4693: ENH: added functionality to handle scalars in `stats._chk_asarray`
- #4694: Vectorization of Anderson-Darling computations.
- #4696: Fix singleton expansion in `lfilter`.
- #4698: MAINT: quiet warnings from `cephes`.
- #4701: add `Bpoly.antiderivatives / integrals`
- #4703: Add citation of published paper
- #4706: MAINT: `special`: avoid out-of-bounds access in `specfun`
- #4707: MAINT: fix issues with `np.matrix` as input to functions related...
- #4709: ENH: `scipy.stats` now returns namedtuples.
- #4710: `scipy.io.idl`: make reader more robust to missing variables in...
- #4711: Fix crash for unknown chunks at the end of file
- #4712: Reduce `onenormest` memory usage
- #4713: MAINT: `interpolate`: no need to pass `dtype` around if it can be...
- #4714: BENCH: Add benchmarks for `stats` module
- #4715: MAINT: polish `signal.place_poles` and `signal/test_ltisys.py`
- #4716: DEP: deprecate `mstats.signaltonoise ...`
- #4717: MAINT: `basinhopping`: fix error in tests, silence `/0` warning,...
- #4718: ENH: `stats`: can specify f-shapes to fix in fitting by name
- #4721: Document that `imresize` converts the input to a PIL image
- #4722: MAINT: `PyArray_BASE` is not an lvalue unless the deprecated API...
- #4725: Fix `gengamma _numpy` failure
- #4728: DOC: add `poch` to the list of `scipy` special function descriptions
- #4735: MAINT: `stats`: avoid (a spurious) division-by-zero in `skew`
- #4738: TST: silence runtime warnings for some corner cases in `stats...`
- #4739: BLD: try to build `numpy` instead of using the one on TravisCI
- #4740: DOC: Update some docstrings with 'versionadded'.
- #4742: BLD: make sure that relaxed strides checking is in effect on...

- #4750: DOC: special: TeX typesetting of `rel ENTR`, `kl DIV` and `pseudo_huber`
- #4751: BENCH: add sparse null slice benchmark
- #4753: BUG: Fixed compilation with recent Cython versions.
- #4756: BUG: Fixes #4733: `optimize.brute` finish option is not compatible...
- #4758: DOC: `optimize.leastsq` default `maxfev` clarification
- #4759: improved stats mle fit
- #4760: MAINT: count bfgs updates more carefully
- #4762: BUGS: Fixes #4746 and #4594: `linprog` returns solution violating...
- #4763: fix small `linprog` bugs
- #4766: BENCH: add `signal.lsim` benchmark
- #4768: fix python syntax errors in docstring examples
- #4769: Fixes #4726: `test_cobyla.test_vector_constraints`
- #4770: Mark FITPACK functions as thread safe.
- #4771: edited `scipy/stats/stats.py` to fix doctest for `fisher_exact`
- #4773: DOC: update 0.16.0 release notes.
- #4775: DOC: `linalg`: add `funm_psd` as a docstring example
- #4778: Use a dictionary for function name synonyms
- #4780: Include apparently-forgotten functions in docs
- #4783: Added many missing special functions to docs
- #4784: add an axis attribute to `PPoly` and friends
- #4785: Brief note about origin of Lena image
- #4786: DOC: reformat the Methods section of the KDE docstring
- #4787: Add `rice` cdf and `ppf`.
- #4792: CI: add a kludge for detecting test failures which try to disguise...
- #4795: Make `refguide_check` smarter about false positives
- #4797: BUG/TST: `numpoints` not updated for incremental Voronoi
- #4799: BUG: `spatial`: Fix a couple edge cases for the Mahalanobis metric...
- #4801: BUG: Fix `TypeError` in `scipy.optimize._trust-region.py` when `disp=True`.
- #4803: Issues with relaxed strides in QR updating routines
- #4806: MAINT: use an informed initial guess for cauchy fit
- #4810: PEP8ify `codata.py`
- #4812: BUG: Relaxed strides cleanup in `decomp_update.pyx.in`
- #4820: BLD: update Bento build for `sgemv` fix and install cython blas/lapack...
- #4823: ENH: `scipy.signal` - Addition of spectrogram function
- #4827: DOC: add `csd` and `coherence` to `__init__.py`
- #4833: BLD: fix issue in `linalg *lange` wrappers for g77 builds.

- #4841: TST: fix test failures in `scipy.special` with mingw32 due to test...
- #4842: DOC: update `site.cfg.example`. Mostly taken over from Numpy
- #4845: BUG: signal: Make spectrogram's return values order match the...
- #4849: DOC: Fix error in ode docstring example
- #4856: BUG: fix typo causing memleak

## 1.4 SciPy 0.15.1 Release Notes

SciPy 0.15.1 is a bug-fix release with no new features compared to 0.15.0.

### 1.4.1 Issues fixed

- #4413: BUG: Tests too strict, `f2py` doesn't have to overwrite this array
- #4417: BLD: avoid using `NPY_API_VERSION` to check not using deprecated...
- #4418: Restore and deprecate `scipy.linalg.calc_work`

## 1.5 SciPy 0.15.0 Release Notes

### Contents

- SciPy 0.15.0 Release Notes
  - New features
    - \* Linear Programming Interface
    - \* Differential evolution, a global optimizer
    - \* `scipy.signal` improvements
    - \* `scipy.integrate` improvements
    - \* `scipy.linalg` improvements
    - \* `scipy.sparse` improvements
    - \* `scipy.special` improvements
    - \* `scipy.sparse.csgraph` improvements
    - \* `scipy.stats` improvements
  - Deprecated features
  - Backwards incompatible changes
    - \* `scipy.ndimage`
    - \* `scipy.integrate`
  - Authors
    - \* Issues closed
    - \* Pull requests

SciPy 0.15.0 is the culmination of 6 months of hard work. It contains several new features, numerous bug-fixes, improved test coverage and better documentation. There have been a number of deprecations and API changes in this release, which are documented below. All users are encouraged to upgrade to this release, as there are a large number of bug-fixes and optimizations. Moreover, our development attention will now shift to bug-fix releases on the 0.16.x branch, and on adding new features on the master branch.

This release requires Python 2.6, 2.7 or 3.2-3.4 and NumPy 1.5.1 or greater.

## 1.5.1 New features

### Linear Programming Interface

The new function `scipy.optimize.linprog` provides a generic linear programming similar to the way `scipy.optimize.minimize` provides a generic interface to nonlinear programming optimizers. Currently the only method supported is *simplex* which provides a two-phase, dense-matrix-based simplex algorithm. Callbacks functions are supported, allowing the user to monitor the progress of the algorithm.

### Differential evolution, a global optimizer

A new `scipy.optimize.differential_evolution` function has been added to the `optimize` module. Differential Evolution is an algorithm used for finding the global minimum of multivariate functions. It is stochastic in nature (does not use gradient methods), and can search large areas of candidate space, but often requires larger numbers of function evaluations than conventional gradient based techniques.

### `scipy.signal` improvements

The function `scipy.signal.max_len_seq` was added, which computes a Maximum Length Sequence (MLS) signal.

### `scipy.integrate` improvements

It is now possible to use `scipy.integrate` routines to integrate multivariate ctypes functions, thus avoiding calls back to Python and providing better performance.

### `scipy.linalg` improvements

The function `scipy.linalg.orthogonal_procrustes` for solving the procrustes linear algebra problem was added.

BLAS level 2 functions `her`, `syr`, `her2` and `syr2` are now wrapped in `scipy.linalg`.

### `scipy.sparse` improvements

`scipy.sparse.linalg.svds` can now take a `LinearOperator` as its main input.

### `scipy.special` improvements

Values of ellipsoidal harmonic (i.e. Lamé) functions and associated normalization constants can be now computed using `ellip_harm`, `ellip_harm_2`, and `ellip_normal`.

New convenience functions `entr`, `rel_entr`, `kl_div`, `huber`, and `pseudo_huber` were added.

### `scipy.sparse.csgraph` improvements

Routines `reverse_cuthill_mckee` and `maximum_bipartite_matching` for computing reorderings of sparse graphs were added.

### scipy.stats improvements

Added a Dirichlet multivariate distribution, `scipy.stats.dirichlet`.

The new function `scipy.stats.median_test` computes Mood's median test.

The new function `scipy.stats.combine_pvalues` implements Fisher's and Stouffer's methods for combining p-values.

`scipy.stats.describe` returns a namedtuple rather than a tuple, allowing users to access results by index or by name.

### 1.5.2 Deprecated features

The `scipy.weave` module is deprecated. It was the only module never ported to Python 3.x, and is not recommended to be used for new code - use Cython instead. In order to support existing code, `scipy.weave` has been packaged separately: <https://github.com/scipy/weave>. It is a pure Python package, and can easily be installed with `pip install weave`.

`scipy.special.bessel_diff_formula` is deprecated. It is a private function, and therefore will be removed from the public API in a following release.

`scipy.stats.nanmean`, `nanmedian` and `nanstd` functions are deprecated in favor of their numpy equivalents.

### 1.5.3 Backwards incompatible changes

The functions `scipy.ndimage.minimum_positions`, `scipy.ndimage.maximum_positions` and `scipy.ndimage.extrema` return positions as ints instead of floats.

The format of banded Jacobians in `scipy.integrate.ode` solvers is changed. Note that the previous documentation of this feature was erroneous.

### 1.5.4 Authors

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A total of 99 people contributed to this release. People with a “+” by their names contributed a patch for the first time. This list of names is automatically generated, and may not be fully complete.

### Issues closed

- #1431: `ellipk(x)` extending its domain for  $x < 0$  (Trac #904)
- #1727: consistency of `std` interface (Trac #1200)
- #1851: `Shape` parameter negated in `genextreme` (relative to R, MATLAB,...)
- #1889: `interp2d` is weird (Trac #1364)
- #2188: `splev` gives wrong values or crashes outside of support when der...
- #2343: `scipy.insterpolate`'s `splrep` function fails with certain combinations...
- #2669: `.signal.ltisys.ss2tf` should only apply to MISO systems in current...
- #2911: `interpolate.splder()` failure on Fedora
- #3171: future of `weave` in `scipy`
- #3176: Suggestion to improve error message in `scipy.integrate.odeint`
- #3198: `pdf()` and `logpdf()` methods for `scipy.stats.gaussian_kde`
- #3318: Travis CI is breaking on `test("full")`
- #3329: `scipy.stats.scoreatpercentile` backward-incompatible change not...
- #3362: Reference cycle in `scipy.sparse.linalg.eigs` with `shift-invert`...
- #3364: BUG: `linalg.hessenberg` broken (wrong results)
- #3376: `stats.f_oneway` needs floats
- #3379: Installation of `scipy 0.13.3` via `zc.buildout` fails
- #3403: `hierarchy.linkage` raises an ugly exception for a compressed `2x2...`
- #3422: `optimize.curve_fit()` handles NaN by returning all parameters...

- #3457: `linalg.fractional_matrix_power` has no docstring
- #3469: DOC: `ndimage.find_object` ignores zero-values
- #3491: `optimize.leastsq()` documentation should mention it does not work...
- #3499: `cluster.vq.whiten` return nan for all zeros column in observations
- #3503: `minimize` attempts to do vector addition when numpy arrays are...
- #3508: `exponweib.logpdf` fails for valid parameters
- #3509: `libatlas3-base-dev` does not exist
- #3550: BUG: anomalous values computed by `special.ellipkinc`
- #3555: `scipy.ndimage` positions are float instead of int
- #3557: `UnivariateSpline.__call__` should pass all relevant args through...
- #3569: No license statement for test data imported from boost?
- #3576: `mstats` test failure (too sensitive?)
- #3579: Errors on `scipy 0.14.x` branch using MKL, Ubuntu 14.04 x86\_64
- #3580: Operator overloading with sparse matrices
- #3587: Wrong alphabetical order in continuous statistical distribution...
- #3596: `scipy.signal.fftconvolve` no longer threadsafe
- #3623: BUG: `signal.convolve` takes longer than it needs to
- #3655: Integer returned from integer data in `scipy.signal.periodogram...`
- #3662: Travis failure on Numpy 1.5.1 (not reproducible?)
- #3668: `dendrogram(orientation='foo')`
- #3669: `KroghInterpolator` doesn't pass through points
- #3672: Inserting a knot in a spline
- #3682: misleading documentation of `scipy.optimize.curve_fit`
- #3699: BUG?: minor problem with `scipy.signal.lfilter` w/initial conditions
- #3700: Inconsistent exceptions raised by `scipy.io.loadmat`
- #3703: `TypeError` for `RegularGridInterpolator` with big-endian data
- #3714: Misleading error message in `eigsh`: `k` must be between 1 and `rank(A)-1`
- #3720: `coo_matrix.setdiag()` fails
- #3740: `Scipy.Spatial.KdTree (Query) Return Type?`
- #3761: Invalid result from `scipy.special.btdtri`
- #3784: DOC - Special Functions - Drum example fix for higher modes
- #3785: `minimize()` should have friendlier args=
- #3787: BUG: `signal`: Division by zero in `lombscargle`
- #3800: BUG: `scipy.sparse.csgraph.shortest_path` overwrites input matrix
- #3817: Warning in calculating moments from Binomial distribution for...
- #3821: review `scipy` usage of `np.ma.is_masked`

- #3829: Linear algebra function documentation doesn't mention default...
- #3830: A bug in Docstring of `scipy.linalg.eig`
- #3844: Issue with shape parameter returned by `genextreme`
- #3858: "ImportError: No module named Cython.Compiler.Main" on install
- #3876: `savgol_filter` not in release notes and has no versionadded
- #3884: `scipy.stats.kendalltau` empty array error
- #3895: `ValueError: illegal value in 12-th argument of internal gesdd...`
- #3898: `skimage` test broken by minmax filter change
- #3901: `scipy` sparse errors with `numpy` master
- #3905: DOC: `optimize`: `linprog` docstring has two "Returns" sections
- #3915: DOC: sphinx warnings because of `**kws` in the stats distributions...
- #3935: Split `stats.distributions` files in tutorial
- #3969: gh-3607 breaks backward compatibility in ode solver banded jacobians
- #4025: DOC: signal: The return value of `find_peaks_cwt` is not documented.
- #4029: `scipy.stats.nbinom.logpmf(0,1,1)` returns nan. Correct value is...
- #4032: ERROR: `test_imresize` (`test_pilutil.TestPILUtil`)
- #4038: errors do not propagate through `scipy.integrate.odeint` properly
- #4171: `orthogonal_procrustes` always returns scale.
- #4176: Solving the Discrete Lyapunov Equation does not work with matrix...

## Pull requests

- #3109: ENH Added Fisher's method and Stouffer's Z-score method
- #3225: Add the limiting distributions to generalized Pareto distribution...
- #3262: Implement back end of faster multivariate integration
- #3266: ENH: signal: add `type=False` as parameter for `periodogram` and...
- #3273: Add PEP8 check to Travis-CI
- #3342: ENH: `linprog` function for linear programming
- #3348: BUG: add proper error handling when using `interp2d` on regular...
- #3351: ENH: Add MLS method
- #3382: ENH: `scipy.special` information theory functions
- #3396: ENH: improve `stats.nanmedian` more by assuming nans are rare
- #3398: Added two wrappers to the `gaussian_kde` class.
- #3405: BUG: `cluster.linkage` array conversion to double dtype
- #3407: MAINT: use `assert_warns` instead of a more complicated mechanism
- #3409: ENH: change to use array view in `signal/_peak_finding.py`
- #3416: Issue 3376 : `stats.f_oneway` needs floats

- #3419: BUG: tools: Fix list of FMA instructions in `detect_cpu_extensions_wine.py`
- #3420: DOC: stats: Add 'entropy' to the stats package-level documentation.
- #3429: BUG: close intermediate file descriptor right after it is used...
- #3430: MAINT: Fix some cython variable declarations to avoid warnings...
- #3433: Correcting the normalization of chebwin window function
- #3435: Add more precise link to R's quantile documentation
- #3446: ENH: `scipy.optimize` - adding `differential_evolution`
- #3450: MAINT: remove unused function `scipy.stats.mstats._basic._kolmog1`
- #3458: Reworked version of PR-3084 (mstats-stats comparison)
- #3462: MAINT : Returning a warning for low attenuation values of chebwin...
- #3463: DOC: linalg: Add examples to functions in `matfuncs.py`
- #3477: ENH: sparse: release GIL in `sparsetools` routines
- #3480: DOC: Add more details to `deconvolve` docstring
- #3484: BLD: fix Qhull build issue with MinGW-w64. Closes gh-3237.
- #3498: MAINT: io: remove old warnings from `idl.py`
- #3504: BUG: `cluster.vq.whiten` returns nan or inf when `std==0`
- #3510: MAINT: stats: Reimplement the `pdf` and `logpdf` methods of `exponweib`.
- #3512: Fix PEP8 errors showing up on TravisCI after pep8 1.5 release
- #3514: DOC: `libatlas3-base-dev` seems to have never been a thing
- #3516: DOC improve `scipy.sparse` docstrings
- #3517: ENH: speed-up `ndimage.filters.min(max)imum_filter1d`
- #3518: Issues in `scipy.misc.logsumexp`
- #3526: DOC: graphical example for `cwt`, and use a more interesting signal
- #3527: ENH: Implement `min(max)imum_filter1d` using the MINLIST algorithm
- #3537: STY: reduce number of C compiler warnings
- #3540: DOC: linalg: add docstring to `fractional_matrix_power`
- #3542: `kde.py` Doc Typo
- #3545: BUG: stats: `stats.levy.cdf` with small arguments loses precision.
- #3547: BUG: special: `erfcinv` with small arguments loses precision.
- #3553: DOC: Convolve examples
- #3561: FIX: in `ndimage.measurements` return positions as int instead...
- #3564: Fix test failures with numpy master. Closes gh-3554
- #3565: ENH: make `interp2d` accept unsorted arrays for interpolation.
- #3566: BLD: add numpy requirement to metadata if it can't be imported.
- #3567: DOC: move `matfuncs` docstrings to user-visible functions
- #3574: Fixes multiple bugs in `mstats.theilslopes`

- #3577: TST: decrease sensitivity of an mstats test
- #3585: Cleanup of code in `scipy.constants`
- #3589: BUG: sparse: allow operator overloading
- #3594: BUG: lobpcg returned wrong values for small matrices ( $n < 10$ )
- #3598: MAINT: fix coverage and coveralls
- #3599: MAINT: `symeig` – now that’s a name I’ve not heard in a long time
- #3602: MAINT: clean up the new `optimize.linprog` and add a few more tests
- #3607: BUG: integrate: Fix some bugs and documentation errors in the...
- #3609: MAINT integrate/odepack: kill dead Fortran code
- #3616: FIX: Invalid values
- #3617: Sort `netcdf` variables in a Python-3 compatible way
- #3622: DOC: Added 0.15.0 release notes entry for `linprog` function.
- #3625: Fix documentation for `cKDTree.sparse_distance_matrix`
- #3626: MAINT: `linalg.orth` memory efficiency
- #3627: MAINT: stats: A bit of clean up
- #3628: MAINT: signal: remove a useless function from `wavelets.py`
- #3632: ENH: stats: Add Mood’s median test.
- #3636: MAINT: cluster: some clean up
- #3638: DOC: docstring of `optimize.basinhopping` confuses singular and...
- #3639: BUG: change `ddof` default to 1 in `mstats.sem`, consistent with...
- #3640: Weave: deprecate the module and disable slow tests on TravisCI
- #3641: ENH: Added support for date attributes to `io.arff.arffread`
- #3644: MAINT: stats: remove superfluous alias in `mstats_basic.py`
- #3646: ENH: adding `sum_duplicates` method to COO sparse matrix
- #3647: Fix for #3596: Make `fftconvolve` threadsafe
- #3650: BUG: sparse: smarter random index selection
- #3652: fix wrong option name in `power_divergence` docstring example
- #3654: Changing EPD to Canopy
- #3657: BUG: signal.welch: ensure floating point dtype regardless of...
- #3660: TST: mark a test as known fail
- #3661: BLD: ignore pep8 E302 (expected 2 blank lines, found 1)
- #3663: BUG: fix leaking `errstate`, and ignore `invalid=` errors in a test
- #3664: BUG: `correlate` was extremely slow when `in2.size > in1.size`
- #3667: ENH: Adds default params to pdfs of `multivariate_norm`
- #3670: ENH: Small speedup of FFT size check
- #3671: DOC: adding `differential_evolution` function to 0.15 release notes

- #3673: BUG: interpolate/fitpack: arguments to fortran routines may not...
- #3674: Add support for appending to existing netcdf files
- #3681: Speed up test('full'), solve Travis CI timeout issues
- #3683: ENH: cluster: rewrite and optimize *vq* in Cython
- #3684: Update special docs
- #3688: Spacing in special docstrings
- #3692: ENH: scipy.special: Improving *sph\_harm* function
- #3693: Update refguide entries for signal and fftpack
- #3695: Update continuous.rst
- #3696: ENH: check for valid 'orientation' kwarg in *dendrogram()*
- #3701: make 'a' and 'b' coefficients atleast\_1d array in *filtfilt*
- #3702: BUG: cluster: *\_vq* unable to handle large features
- #3704: BUG: special: *ellip(k,e)inc* nan and double expected value
- #3707: BUG: handle *fill\_value* dtype checks correctly in *RegularGridInterpolator*
- #3708: Reraise exception on failure to read mat file.
- #3709: BUG: cast 'x' to correct dtype in *KroghInterpolator.\_evaluate*
- #3712: ENH: cluster: reimplement the update-step of K-means in Cython
- #3713: FIX: Check type of *lfiltic*
- #3718: Changed INSTALL file extension to rst
- #3719: address svds returning nans for zero input matrix
- #3722: MAINT: spatial: static, unused code, *sqrt(sqeclidean)*
- #3725: ENH: use *numpy.nanmedian* if available
- #3727: TST: add a new *fixed\_point* test and change some test function...
- #3731: BUG: fix *romb* in *scipy.integrate.quadrature*
- #3734: DOC: simplify examples with *semilogx*
- #3735: DOC: Add minimal docstrings to *lti.impulse/step*
- #3736: BUG: cast *pchip* arguments to floats
- #3744: stub out inherited methods of *Akima1DInterpolator*
- #3746: DOC: Fix formatting for *Raises* section
- #3748: ENH: Added discrete Lyapunov transformation solve
- #3750: Enable automated testing with Python 3.4
- #3751: Reverse Cuthill-McKee and Maximum Bipartite Matching reorderings...
- #3759: MAINT: avoid indexing with a float array
- #3762: TST: filter out *RuntimeWarning* in *vq* tests
- #3766: TST: cluster: some cleanups in *test\_hierarchy.py*
- #3767: ENH/BUG: support negative *m* in elliptic integrals

- #3769: ENH: avoid repeated matrix inverse
- #3770: BUG: signal: In `lfilter_zi`, `b` was not rescaled correctly when...
- #3772: STY avoid unnecessary transposes in `csr_matrix.getcol/row`
- #3773: ENH: Add `ext` parameter to `UnivariateSpline` call
- #3774: BUG: in `integrate/quadpack.h`, put all declarations before statements.
- #3779: Incbet fix
- #3788: BUG: Fix `lombscargle` `ZeroDivisionError`
- #3791: Some maintenance for doc builds
- #3795: `scipy.special.legendre` docstring
- #3796: TYPO: `sheroidal` -> `spheroidal`
- #3801: BUG: `shortest_path` overwrite
- #3803: TST: `lombscargle` regression test related to `atan` vs `atan2`
- #3809: ENH: orthogonal `procrustes` solver
- #3811: ENH: `scipy.special`, Implemented Ellipsoidal harmonic function:...
- #3819: BUG: make a fully connected `csgraph` from an `ndarray` with no zeros
- #3820: MAINT: avoid spurious warnings in `binom(n, p=0).mean()` etc
- #3825: Don't claim `scipy.cluster` does distance matrix calculations.
- #3827: get and set diagonal of `coo_matrix`, and related `csgraph` `laplacian`...
- #3832: DOC: Minor additions to `integrate/nquad` docstring.
- #3845: Bug fix for #3842: Bug in `scipy.optimize.line_search`
- #3848: BUG: edge case where the covariance matrix is exactly zero
- #3850: DOC: typo
- #3851: DOC: document default argument values for some `arpack` functions
- #3860: DOC: `sparse`: add the function 'find' to the module-level docstring
- #3861: BUG: Removed unnecessary storage of args as instance variables...
- #3862: BUG: signal: fix handling of multi-output systems in `ss2tf`.
- #3865: Feature request: ability to read heterogeneous types in `FortranFile`
- #3866: MAINT: update pip wheelhouse for installs
- #3871: MAINT: `linalg`: get rid of `calc_lwork.f`
- #3872: MAINT: use `scipy.linalg` instead of `np.dual`
- #3873: BLD: show a more informative message if Cython wasn't installed.
- #3874: TST: `cluster`: cleanup the hierarchy test data
- #3877: DOC: Savitzky-Golay filter version added
- #3878: DOC: move `versionadded` to notes
- #3879: small tweaks to the docs
- #3881: FIX incorrect sorting during fancy assignment

- #3885: kendalltau function now returns a nan tuple if empty arrays used...
- #3886: BUG: fixing linprog's kwarg order to match docs
- #3888: BUG: optimize: In `_linprog_simplex`, handle the case where the...
- #3891: BUG: stats: Fix ValueError message in `chi2_contingency`.
- #3892: DOC: `sparse.linalg`: Fix `lobpcg` docstring.
- #3894: DOC: stats: Assorted docstring edits.
- #3896: Fix 2 mistakes in MatrixMarket format parsing
- #3897: BUG: associated Legendre function of second kind for  $1 < x < 1.0001$
- #3899: BUG: fix undefined behavior in `alngam`
- #3906: MAINT/DOC: Whitespace tweaks in several docstrings.
- #3907: TST: relax bounds of `interpolate` test to accomodate rounding...
- #3909: MAINT: Create a common version of `count_nonzero` for compatibility...
- #3910: Fix a couple of test errors in master
- #3911: Use MathJax for the html docs
- #3914: Rework the `_roots` functions and document them.
- #3916: Remove all `linpack_lite` code and replace with LAPACK routines
- #3917: splines, constant extrapolation
- #3918: DOC: tweak the `rv_discrete` docstring example
- #3919: Quadrature speed-up: `scipy.special.orthogonal.p_roots` with cache
- #3920: DOC: Clarify docstring for `sigma` parameter for `curve_fit`
- #3922: Fixed Docstring issues in `linprog` (Fixes #3905).
- #3924: Coerce args into tuple if necessary.
- #3926: DOC: Surround stats class methods in docstrings with backticks.
- #3927: Changed doc for `romb`'s `dx` parameter to int.
- #3928: check FITPACK conditions in `LSQUnivariateSpline`
- #3929: Added a warning about `leastsq` using with NaNs.
- #3930: ENH: optimize: `curve_fit` now warns if `pcov` is undetermined
- #3932: Clarified the  $k > n$  case.
- #3933: DOC: remove `import scipy as sp` abbreviation here and there
- #3936: Add license and copyright holders to test data imported from...
- #3938: DOC: Corrected documentation for return types.
- #3939: DOC: `fitpack`: add a note about Sch-W conditions to `splrep` docstring
- #3940: TST: `integrate`: Remove an invalid test of `odeint`.
- #3942: FIX: Corrected error message of `eigsh`.
- #3943: ENH: release GIL for filter and interpolation of `ndimage`
- #3944: FIX: Raise value error if window data-type is unsupported



- #3946: Fixed `signal.get_window` with unicode window name
- #3947: MAINT: some docstring fixes and style cleanups in `stats.mstats`
- #3949: DOC: fix a couple of issues in `stats` docstrings.
- #3950: TST: `sparse`: remove known failure that doesn't fail
- #3951: TST: switch from Rackspace wheelhouse to `numpy/cython` source...
- #3952: DOC: `stats`: Small formatting correction to the 'chi' distribution...
- #3953: DOC: `stats`: Several corrections and small additions to docstrings.
- #3955: `signal.__init__.py`: remove duplicated `get_window` entry
- #3959: TST: `sparse`: more "known failures" for DOK that don't fail
- #3960: BUG: `io.netcdf`: do not close `mmap` if there are references left...
- #3965: DOC: Fix a few more sphinx warnings that occur when building...
- #3966: DOC: add guidelines for using test generators in HACKING
- #3968: BUG: `sparse.linalg`: make `Inv` objects in `arpack` garbage-collectable...
- #3971: Remove all `linpack_lite` code and replace with LAPACK routines
- #3972: fix typo in error message
- #3973: MAINT: better error message for multivariate normal.
- #3981: turn the cryptically named `scipy.special` information theory functions...
- #3984: Wrap `her`, `syr`, `her2`, `syr2` blas routines
- #3990: improve `UnivariateSpline` docs
- #3991: ENH: `stats`: return `namedtuple` for describe output
- #3993: DOC: `stats`: `percentileofscore` references `np.percentile`
- #3997: BUG: `linalg`: `pascal(35)` was incorrect: last element overflowed...
- #3998: MAINT: use `isMaskedArray` instead of `is_masked` to check type
- #3999: TST: test against all of boost data files.
- #4000: BUG: `stats`: Fix edge-case handling in a few distributions.
- #4003: ENH: using python's warnings instead of prints in `fitpack`.
- #4004: MAINT: optimize: remove a couple unused variables in `zeros.c`
- #4006: BUG: Fix C90 compiler warnings in `NI_MinOrMaxFilter1D`
- #4007: MAINT/DOC: Fix spelling of 'decomposition' in several files.
- #4008: DOC: `stats`: Split the descriptions of the distributions in the...
- #4015: TST: `logsumexp` regression test
- #4016: MAINT: remove some inf-related warnings from `logsumexp`
- #4020: DOC: `stats`: fix whitespace in docstrings of several distributions
- #4023: Exactly one space required before assignments
- #4024: In `dendrogram()`: Correct an argument name and a grammar issue...
- #4041: BUG: misc: Ensure that the 'size' argument of PIL's 'resize' ...

- #4049: BUG: Return of `_logpmf`
- #4051: BUG: `expm` of integer matrices
- #4052: ENH: `integrate`: `odeint`: Handle exceptions in the callback functions.
- #4053: BUG: `stats`: Refactor argument validation to avoid a unicode issue.
- #4057: Added newline to `scipy.sparse.linalg.svds` documentation for correct...
- #4058: MAINT: `stats`: Add note about change to `scoreatpercentile` in release...
- #4059: ENH: `interpolate`: Allow `splev` to accept an n-dimensional array.
- #4064: Documented the return value for `scipy.signal.find_peaks_cwt`
- #4074: ENH: Support `LinearOperator` as input to `svds`
- #4084: BUG: Match exception declarations in `scipy/io/matlab/streams.pyx...`
- #4091: DOC: `special`: more clear instructions on how to evaluate polynomials
- #4105: BUG: Workaround for SGEMV segfault in Accelerate
- #4107: DOC: get rid of 'import \*' in examples
- #4113: DOC: fix typos in `distance.yule`
- #4114: MAINT C fixes
- #4117: deprecate `nanmean`, `nanmedian` and `nanstd` in favor of their `numpy...`
- #4126: `scipy.io.idl`: support description records and fix bug with null...
- #4131: ENH: release GIL in more `ndimage` functions
- #4132: MAINT: `stats`: fix a typo [skip ci]
- #4145: DOC: Fix documentation error for `nc` chi-squared dist
- #4150: Fix `_nd_image.geometric_transform` endianness bug
- #4153: MAINT: remove use of deprecated `numpy` API in `lib/lapack/f2py...`
- #4156: MAINT: `optimize`: remove dead code
- #4159: MAINT: `optimize`: clean up `Zeros` code
- #4165: DOC: add missing special functions to `__doc__`
- #4172: DOC: remove misleading `procrustes` docstring line
- #4175: DOC: `sparse`: clarify CSC and CSR constructor usage
- #4177: MAINT: enable `np.matrix` inputs to `solve_discrete_lyapunov`
- #4179: TST: fix an intermittently failing test case for `special.legendre`
- #4181: MAINT: remove unnecessary null checks before free
- #4182: Ellipsoidal harmonics
- #4183: Skip Cython build in Travis-CI
- #4184: Pr 4074
- #4187: Pr/3923
- #4190: BUG: `special`: fix up `ellip_harm` build
- #4193: BLD: fix `msvc` compiler errors

- #4194: BUG: fix buffer dtype mismatch on win-amd64
- #4199: ENH: Changed `scipy.stats.describe` output from `datalen` to `nobs`
- #4201: DOC: add `blas2` and `nan*` deprecations to the release notes
- #4243: TST: bump test tolerances

## 1.6 SciPy 0.14.1 Release Notes

SciPy 0.14.1 is a bug-fix release with no new features compared to 0.14.0.

### 1.6.1 Issues closed

- #3630: NetCDF reading results in a segfault
- #3631: SuperLU object not working as expected for complex matrices
- #3733: segfault from `map_coordinates`
- #3780: Segfault when using CSR/CSC matrix and `uint32/uint64`
- #3781: BUG: sparse: fix omitted types in `sparsetools` `typemaps`
- #3802: 0.14.0 API breakage: `_gen` generators are missing from `scipy.stats.distributions` API
- #3805: `ndimage` test failures with `numpy 1.10`
- #3812: `==` sometimes wrong on `csr_matrix`
- #3853: Many `scipy.sparse` test errors/failures with `numpy 1.9.0b2`
- #4084: fix exception declarations for `Cython 0.21.1` compatibility
- #4093: BUG: fitpack: avoid a memory error in `splev(x, tck, der=k)`
- #4104: BUG: Workaround SGEMV segfault in Accelerate (maintenance 0.14.x)
- #4143: BUG: fix `ndimage` functions for large data
- #4149: Bug in `expm` for integer arrays
- #4154: Backport gh-4041 for 0.14.1 (Ensure that the 'size' argument of PIL's 'resize' method is a tuple)
- #4163: Backport #4142 (ZeroDivisionError in `scipy.sparse.linalg.lsqr`)
- #4164: Backport gh-4153 (remove use of deprecated `numpy` API in `lib/lapack/ f2py` wrapper)
- #4180: backport pil resize support tuple fix
- #4168: Lots of `arpack` test failures on windows 32 bits with `numpy 1.9.1`
- #4203: Matrix multiplication in 0.14.x is more than 10x slower compared...
- #4218: attempt to make `ndimage` interpolation compatible with `numpy` relaxed...
- #4225: BUG: off-by-one error in `PPoly` shape checks
- #4248: BUG: optimize: fix issue with incorrect use of closure for `slsqp`.

## 1.7 SciPy 0.14.0 Release Notes

### Contents

- SciPy 0.14.0 Release Notes
  - New features
    - \* `scipy.interpolate` improvements
    - \* `scipy.linalg` improvements
    - \* `scipy.optimize` improvements
    - \* `scipy.stats` improvements
    - \* `scipy.signal` improvements
    - \* `scipy.special` improvements
    - \* `scipy.sparse` improvements
  - Deprecated features
    - \* `anneal`
    - \* `scipy.stats`
    - \* `scipy.interpolate`
  - Backwards incompatible changes
    - \* `scipy.special.lpmn`
    - \* `scipy.sparse.linalg`
    - \* `scipy.stats`
    - \* `scipy.interpolate`
  - Other changes
  - Authors
    - \* Issues closed
    - \* Pull requests

SciPy 0.14.0 is the culmination of 8 months of hard work. It contains many new features, numerous bug-fixes, improved test coverage and better documentation. There have been a number of deprecations and API changes in this release, which are documented below. All users are encouraged to upgrade to this release, as there are a large number of bug-fixes and optimizations. Moreover, our development attention will now shift to bug-fix releases on the 0.14.x branch, and on adding new features on the master branch.

This release requires Python 2.6, 2.7 or 3.2-3.4 and NumPy 1.5.1 or greater.

### 1.7.1 New features

#### **`scipy.interpolate` improvements**

A new wrapper function `scipy.interpolate.interpn` for interpolation on regular grids has been added. *interpn* supports linear and nearest-neighbor interpolation in arbitrary dimensions and spline interpolation in two dimensions.

Faster implementations of piecewise polynomials in power and Bernstein polynomial bases have been added as `scipy.interpolate.PPoly` and `scipy.interpolate.BPoly`. New users should use these in favor of `scipy.interpolate.PiecewisePolynomial`.

`scipy.interpolate.interpld` now accepts non-monotonic inputs and sorts them. If performance is critical, sorting can be turned off by using the new `assume_sorted` keyword.

Functionality for evaluation of bivariate spline derivatives in `scipy.interpolate` has been added.

The new class `scipy.interpolate.Akima1DInterpolator` implements the piecewise cubic polynomial interpolation scheme devised by H. Akima.

Functionality for fast interpolation on regular, unevenly spaced grids in arbitrary dimensions has been added as `scipy.interpolate.RegularGridInterpolator`.

### **scipy.linalg improvements**

The new function `scipy.linalg.dft` computes the matrix of the discrete Fourier transform.

A condition number estimation function for matrix exponential, `scipy.linalg.expm_cond`, has been added.

### **scipy.optimize improvements**

A set of benchmarks for `optimize`, which can be run with `optimize.bench()`, has been added.

`scipy.optimize.curve_fit` now has more controllable error estimation via the `absolute_sigma` keyword.

Support for passing custom minimization methods to `optimize.minimize()` and `optimize.minimize_scalar()` has been added, currently useful especially for combining `optimize.basinhopping()` with custom local optimizer routines.

### **scipy.stats improvements**

A new class `scipy.stats.multivariate_normal` with functionality for multivariate normal random variables has been added.

A lot of work on the `scipy.stats` distribution framework has been done. Moment calculations (skew and kurtosis mainly) are fixed and verified, all examples are now runnable, and many small accuracy and performance improvements for individual distributions were merged.

The new function `scipy.stats.anderson_ksamp` computes the k-sample Anderson-Darling test for the null hypothesis that k samples come from the same parent population.

### **scipy.signal improvements**

`scipy.signal.iirfilter` and related functions to design Butterworth, Chebyshev, elliptical and Bessel IIR filters now all use pole-zero (“zpk”) format internally instead of using transformations to numerator/denominator format. The accuracy of the produced filters, especially high-order ones, is improved significantly as a result.

The Savitzky-Golay filter was added with the new functions `scipy.signal.savgol_filter` and `scipy.signal.savgol_coeffs`.

The new function `scipy.signal.vectorstrength` computes the vector strength, a measure of phase synchrony, of a set of events.

### **scipy.special improvements**

The functions `scipy.special.boxcox` and `scipy.special.boxcox1p`, which compute the Box-Cox transformation, have been added.

### **scipy.sparse improvements**

- Significant performance improvement in CSR, CSC, and DOK indexing speed.

- When using Numpy  $\geq 1.9$  (to be released in MM 2014), sparse matrices function correctly when given to arguments of `np.dot`, `np.multiply` and other ufuncs. With earlier Numpy and Scipy versions, the results of such operations are undefined and usually unexpected.
- Sparse matrices are no longer limited to  $2^{31}$  nonzero elements. They automatically switch to using 64-bit index data type for matrices containing more elements. User code written assuming the sparse matrices use `int32` as the index data type will continue to work, except for such large matrices. Code dealing with larger matrices needs to accept either `int32` or `int64` indices.

## 1.7.2 Deprecated features

### `anneal`

The global minimization function `scipy.optimize.anneal` is deprecated. All users should use the `scipy.optimize.basinhopping` function instead.

### `scipy.stats`

`randwcdf` and `randwppf` functions are deprecated. All users should use distribution-specific `rvs` methods instead.

Probability calculation aliases `zprob`, `fprob` and `ksprob` are deprecated. Use instead the `sf` methods of the corresponding distributions or the `special` functions directly.

### `scipy.interpolate`

`PiecewisePolynomial` class is deprecated.

## 1.7.3 Backwards incompatible changes

`lpmn` no longer accepts complex-valued arguments. A new function `clpmn` with uniform complex analytic behavior has been added, and it should be used instead.

Eigenvectors in the case of generalized eigenvalue problem are normalized to unit vectors in 2-norm, rather than following the LAPACK normalization convention.

The deprecated UMFPACK wrapper in `scipy.sparse.linalg` has been removed due to license and install issues. If available, `scikits.umfpack` is still used transparently in the `spsolve` and `factorized` functions. Otherwise, SuperLU is used instead in these functions.

The deprecated functions `glm`, `oneway` and `cmedian` have been removed from `scipy.stats`.

`stats.scoreatpercentile` now returns an array instead of a list of percentiles.

The API for computing derivatives of a monotone piecewise interpolation has changed: if `p` is a `PchipInterpolator` object, `p.derivative(der)` returns a callable object representing the derivative of `p`. For in-place derivatives use the second argument of the `__call__` method: `p(0.1, der=2)` evaluates the second derivative of `p` at `x=0.1`.

The method `p.derivatives` has been removed.

## 1.7.4 Other changes

## 1.7.5 Authors

- Marc Abramowitz +
- Anders Bech Borchersen +
- Vincent Arel-Bundock +
- Petr Baudis +
- Max Bolingbroke
- François Boulogne
- Matthew Brett
- Lars Buitinck
- Evgeni Burovski
- CJ Carey +
- Thomas A Caswell +
- Pawel Chojnacki +
- Phillip Cloud +
- Stefano Costa +
- David Cournapeau
- David Menendez Hurtado +
- Matthieu Dartiailh +
- Christoph Deil +
- Jörg Dietrich +
- endolith
- Francisco de la Peña +
- Ben FrantzDale +
- Jim Garrison +
- André Gaul
- Christoph Gohlke
- Ralf Gommers
- Robert David Grant
- Alex Griffing
- Blake Griffith
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- Andreas Hilboll
- Kat Huang
- Gert-Ludwig Ingold

- James T. Webber +
- Dorota Jarecka +
- Todd Jennings +
- Thouis (Ray) Jones
- Juan Luis Cano Rodríguez
- ktritz +
- Jacques Kvam +
- Eric Larson +
- Justin Lavoie +
- Denis Laxalde
- Jussi Leinonen +
- lemonlaug +
- Tim Leslie
- Alain Leufroy +
- George Lewis +
- Max Linke +
- Brandon Liu +
- Benny Malengier +
- Matthias Kümmerer +
- Cimarron Mittelsteadt +
- Eric Moore
- Andrew Nelson +
- Niklas Hambüchen +
- Joel Nothman +
- Clemens Novak
- Emanuele Olivetti +
- Stefan Otte +
- peb +
- Josef Perktold
- pjwernerck
- poolio
- Jérôme Roy +
- Carl Sandrock +
- Andrew Sczesnak +
- Shauna +
- Fabrice Silva



- Daniel B. Smith
- Patrick Snape +
- Thomas Spura +
- Jacob Stevenson
- Julian Taylor
- Tomas Tomecek
- Richard Tsai
- Jacob Vanderplas
- Joris Vankerschaver +
- Pauli Virtanen
- Warren Weckesser

A total of 80 people contributed to this release. People with a “+” by their names contributed a patch for the first time. This list of names is automatically generated, and may not be fully complete.

### Issues closed

- #1325: add custom axis keyword to dendrogram function in `scipy.cluster.hierarchy...`
- #1437: Wrong pochhammer symbol for negative integers (Trac #910)
- #1555: `scipy.io.netcdf` leaks file descriptors (Trac #1028)
- #1569: sparse matrix failed with element-wise multiplication using `numpy.multiply()`...
- #1833: Sparse matrices are limited to  $2^{32}$  non-zero elements (Trac #1307)
- #1834: `scipy.linalg.eig` does not normalize eigenvector if B is given...
- #1866: stats for `invgamma` (Trac #1340)
- #1886: stats.zipf floating point warnings (Trac #1361)
- #1887: Stats continuous distributions - floating point warnings (Trac...
- #1897: `scoreatpercentile()` does not handle empty list inputs (Trac #1372)
- #1918: `splint` returns incorrect results (Trac #1393)
- #1949: `kurtosistest` fails in `mstats` with type error (Trac #1424)
- #2092: `scipy.test` leaves `darwin27compiled_catalog`, `cpp` and so files...
- #2106: stats ENH: shape parameters in distribution docstrings (Trac...
- #2123: Bad behavior of sparse matrices in a binary `ufunc` (Trac #1598)
- #2152: Fix `mmio/fromfile` on `gzip` on Python 3 (Trac #1627)
- #2164: `stats.rice.pdf(x, 0)` returns nan (Trac #1639)
- #2169: `scipy.optimize.fmin_bfgs` not handling functions with boundaries...
- #2177: `scipy.cluster.hierarchy.ClusterNode.pre_order` returns `IndexError`...
- #2179: `coo.todense()` segfaults (Trac #1654)
- #2185: Precision of `scipy.ndimage.gaussian_filter*()` limited (Trac #1660)

- #2186: `scipy.stats.mstats.kurtosistest` crashes on 1d input (Trac #1661)
- #2238: Negative p-value on `hypergeom.cdf` (Trac #1719)
- #2283: ascending order in interpolation routines (Trac #1764)
- #2288: `mstats.kurtosistest` is incorrectly converting to float, and fails...
- #2396: `lpmn` wrong results for  $|z| > 1$  (Trac #1877)
- #2398: `ss2tf` returns num as 2D array instead of 1D (Trac #1879)
- #2406: linkage does not take Unicode strings as method names (Trac #1887)
- #2443: IIR filter design should not transform to tf representation internally
- #2572: class method `solve` of `splu` return object corrupted or falsely...
- #2667: stats endless loop ?
- #2671: `.stats.hypergeom` documentation error in the note about pmf
- #2691: BUG `scipy.linalg.lapack: potrf/ptroi` interpret their 'lower'...
- #2721: Allow use of ellipsis in `scipy.sparse` slicing
- #2741: stats: deprecate and remove alias for special functions
- #2742: stats add rvs to rice distribution
- #2765: bugs stats entropy
- #2832: `argrextrema` returns tuple of 2 empty arrays when no peaks found...
- #2861: `scipy.stats.scoreatpercentile` broken for vector *per*
- #2891: COBYLA successful termination when constraints violated
- #2919: test failure with the current master
- #2922: `ndimage.percentile_filter` ignores origin argument for multidimensional...
- #2938: Sparse/dense matrix inplace operations fail due to `__numpy_ufunc__`
- #2944: MacPorts builds yield 40Mb worth of build warnings
- #2945: FAIL: `test_random_complex` (`test_basic.TestDet`)
- #2947: FAIL: Test some trivial edge cases for `savgol_filter()`
- #2953: Scipy Delaunay triangulation is not oriented
- #2971: `scipy.stats.mstats.winsorize` documentation error
- #2980: Problems running what seems a perfectly valid example
- #2996: entropy for `rv_discrete` is incorrect?!
- #2998: Fix numpy version comparisons
- #3002: python setup.py install fails
- #3014: Bug in `stats.fisher_exact`
- #3030: relative entropy using `scipy.stats.distribution.entropy` when...
- #3037: `scipy.optimize.curve_fit` leads to unexpected behavior when input...
- #3047: `mstats.ttest_rel` axis=None, requires masked array
- #3059: BUG: Slices of sparse matrices return incorrect dtype

- #3063: range keyword in binned\_statistics incorrect
- #3067: cumtrapz not working as expected
- #3069: sinc
- #3086: standard error calculation inconsistent between 'stats' and 'mstats'
- #3094: Add a *perm* function into `scipy.misc` and an enhancement of...
- #3111: `scipy.sparse.[hv]stack` don't respect anymore the dtype parameter
- #3172: `optimize.curve_fit` uses different nomenclature from `optimize.leastsq`
- #3196: `scipy.stats.mstats.gmean` does not actually take dtype
- #3212: Dot product of `csr_matrix` causes segmentation fault
- #3227: `ZeroDivisionError` in `broyden1` when initial guess is the right...
- #3238: `lbfgsb` output not suppressed by `disp=0`
- #3249: Sparse matrix min/max/etc don't support `axis=-1`
- #3251: `cdist` performance issue with 'sqeuclidean' metric
- #3279: `logm` fails for singular matrix
- #3285: `signal.chirp(method='hyp')` disallows hyperbolic upsweep
- #3299: MEMORY LEAK: `fmin_tnc`
- #3330: test failures with the current master
- #3345: `scipy` and/or `numpy` change is causing tests to fail in another...
- #3363: `splu` does not work for non-vector inputs
- #3385: `expit` does not handle large arguments well
- #3395: `specfun.f` doesn't compile with MinGW
- #3399: Error message bug in `scipy.cluster.hierarchy.linkage`
- #3404: `interpolate._ppoly` doesn't build with MinGW
- #3412: Test failures in `signal`
- #3466: `scipy.sparse.csgraph.shortest_path` does not work on`  
`scipy.sparse.csr_matrix` or `lil_matrix``

## Pull requests

- #442: ENH: sparse: enable 64-bit index arrays & `nnz > 2**31`
- #2766: DOC: remove `doc/seps/technology-preview.rst`
- #2772: TST: stats: Added a regression test for `stats.wilcoxon`. Closes...
- #2778: Clean up `stats._support`, close statistics review issues
- #2792: BUG io: fix file descriptor closing for `netcdf` variables
- #2847: Rice distribution: extend to `b=0`, add an explicit `rvs` method.
- #2878: [stats] fix formulas for higher moments of `dweibull` distribution
- #2904: ENH: moments for the `zipf` distribution

- #2907: ENH: add coverage info with coveralls.io for Travis runs.
- #2932: BUG+TST: setdiag implementation for dia\_matrix (Close #2931)...
- #2942: Misc fixes pointed out by Eclipse PyDev static code analysis
- #2946: ENH: allow non-monotonic input in interp1d
- #2986: BUG: runtests: chdir away from root when running tests
- #2987: DOC: linalg: don't recommend np.linalg.norm
- #2992: ENH: Add "limit" parameter to dijkstra calculation
- #2995: ENH: Use int shape
- #3006: DOC: stats: add a log base note to the docstring
- #3007: DEP: stats: Deprecate randwppf and randwcdf
- #3008: Fix mstats.kurtosistest, and test coverage for skewtest/normaltest
- #3009: Minor reST typo
- #3010: Add *scipy.optimize.Result* to API docs
- #3012: Corrects documentation error
- #3052: PEP-8 conformance improvements
- #3064: Binned statistic
- #3068: Fix Issue #3067 fix cumtrapz that was raising an exception when...
- #3073: Arff reader with nominal value of 1 character
- #3074: Some maintenance work
- #3080: Review and clean up all Box-Cox functions
- #3083: Bug: should return 0 if no regions found
- #3085: BUG: Use zpk in IIR filter design to improve accuracy
- #3101: refactor stats tests a bit
- #3112: ENH: implement Akima interpolation in 1D
- #3123: MAINT: an easier way to make ranges from slices
- #3124: File object support for imread and imsave
- #3126: pep8ify stats/distributions.py
- #3134: MAINT: split distributions.py into three files
- #3138: clean up tests for discrete distributions
- #3155: special: handle the edge case lambda=0 in pdtr, pdtrc and pdtrik
- #3156: Rename optimize.Result to OptimizeResult
- #3166: BUG: make curve\_fit() work with array\_like input. Closes gh-3037.
- #3170: Fix numpy version checks
- #3175: use numpy sinc
- #3177: Update numpy version warning, remove oldnumeric import
- #3178: DEP: remove deprecated umfpack wrapper. Closes gh-3002.

- #3179: DOC: add BPoly to the docs
- #3180: Suppress warnings when running stats.test()
- #3181: altered sem func in mstats to match stats
- #3182: Make weave tests behave
- #3183: ENH: Add k-sample Anderson-Darling test to stats module
- #3186: Fix stats.scoreatpercentile
- #3187: DOC: make curve\_fit nomenclature same as leastsq
- #3201: Added axis keyword to dendrogram function
- #3207: Make docstring examples in stats.distributions docstrings runnable
- #3218: BUG: integrate: Fix banded jacobian handling in the “vode” and...
- #3222: BUG: limit input ranges in special.nctdtr
- #3223: Fix test errors with numpy master
- #3224: Fix int32 overflows in sparsertools
- #3228: DOC: tf2ss zpk2ss note controller canonical form
- #3234: Add See Also links and Example graphs to filter design `*ord` functions
- #3235: Updated the buttord function to be consistent with the other...
- #3239: correct doc for pchip interpolation
- #3240: DOC: fix ReST errors in the BPoly docstring
- #3241: RF: check write attr of fileobject without writing
- #3243: a bit of maintenance work in stats
- #3245: BUG/ENH: stats: make frozen distributions hold separate instances
- #3247: ENH function to return nnz per row/column in some sparse matrices
- #3248: ENH much more efficient sparse min/max with axis
- #3252: Fast sgeuclidean
- #3253: FIX support axis=-1 and -2 for sparse reduce methods
- #3254: TST tests for non-canonical input to sparse matrix operations
- #3272: BUG: sparse: fix bugs in dia\_matrix.setdiag
- #3278: Also generate a tar.xz when running paver sdist
- #3286: DOC: update 0.14.0 release notes.
- #3289: TST: remove insecure mktemp use in tests
- #3292: MAINT: fix a backwards incompatible change to stats.distributions.\_\_all\_\_
- #3293: ENH: signal: Allow upsweeps of frequency in the ‘hyperbolic’...
- #3302: ENH: add dtype arg to stats.mstats.gmean and stats.mstats.hmean
- #3307: DOC: add note about different ba forms in tf2zpk
- #3309: doc enhancements to scipy.stats.mstats.winsorize
- #3310: DOC: clarify matrix vs array in mmio docstrings

- #3314: BUG: fix `scipy.io.mmread()` of gzipped files under Python3
- #3323: ENH: Efficient interpolation on regular grids in arbitrary dimensions
- #3332: DOC: clean up `scipy.special` docs
- #3335: ENH: improve `nanmedian` performance
- #3347: BUG: fix use of `np.max` in `stats.fisher_exact`
- #3356: ENH: sparse: speed up LIL indexing + assignment via Cython
- #3357: Fix “`imresize` does not work with `size = int`”
- #3358: MAINT: rename `AkimaInterpolator` to `Akima1DInterpolator`
- #3366: WHT: sparse: reindent `dsolve/*.c *.h`
- #3367: BUG: sparse/dsolve: fix dense matrix fortran order bugs in `superlu...`
- #3369: ENH `minimize`, `minimize_scalar`: Add support for user-provided...
- #3371: `scipy.stats.sigmaclip` doesn't appear in the html docs.
- #3373: BUG: sparse/dsolve: detect invalid LAPACK parameters in `superlu...`
- #3375: ENH: sparse/dsolve: make the L and U factors of `splu` and `spilu...`
- #3377: MAINT: make travis build one target against Numpy 1.5
- #3378: MAINT: `fftpack`: Remove the use of `'import *'` in a couple test...
- #3381: MAINT: replace `np.isinf(x) & (x>0)` -> `np.isposinf(x)` to avoid...
- #3383: MAINT: skip `float96` tests on platforms without `float96`
- #3384: MAINT: add `pyflakes` to Travis-CI
- #3386: BUG: stable evaluation of `expit`
- #3388: BUG: SuperLU: fix missing declaration of `dlamch`
- #3389: BUG: sparse: downcast 64-bit indices safely to `intp` when required
- #3390: BUG: nonlinear solvers are not confused by lucky guess
- #3391: TST: fix sparse test errors due to `axis=-1,-2` usage in `np.matrix.sum()`.
- #3392: BUG: sparse/lil: fix up Cython bugs in fused type lookup
- #3393: BUG: sparse/compressed: work around bug in `np.unique` in earlier...
- #3394: BUG: allow `ClusterNode.pre_order()` for non-root nodes
- #3400: BUG: `cluster.linkage` `ValueError` typo bug
- #3402: BUG: special: In `specfun.f`, replace the use of `CMPLX` with `DCMPLX`,...
- #3408: MAINT: sparse: Numpy 1.5 compatibility fixes
- #3410: MAINT: interpolate: fix blas defs in `_ppoly`
- #3411: MAINT: Numpy 1.5 fixes in `interpolate`
- #3413: Fix more test issues with older numpy versions
- #3414: TST: signal: loosen some error tolerances in the filter tests....
- #3415: MAINT: tools: automated close issue + pr listings for release...
- #3440: MAINT: wrap `sparsetools` manually instead via SWIG

- #3460: TST: open image file in binary mode
- #3467: BUG: fix validation in `csgraph.shortest_path`

## 1.8 SciPy 0.13.2 Release Notes

SciPy 0.13.2 is a bug-fix release with no new features compared to 0.13.1.

### 1.8.1 Issues fixed

- 3096: require Cython 0.19, earlier versions have memory leaks in fused types
- 3079: `ndimage.label` fix swapped 64-bitness test
- 3108: `optimize.fmin_slsqp` constraint violation

## 1.9 SciPy 0.13.1 Release Notes

SciPy 0.13.1 is a bug-fix release with no new features compared to 0.13.0. The only changes are several fixes in `ndimage`, one of which was a serious regression in `ndimage.label` (Github issue 3025), which gave incorrect results in 0.13.0.

### 1.9.1 Issues fixed

- 3025: `ndimage.label` returns incorrect results in scipy 0.13.0
- 1992: `ndimage.label` return type changed from `int32` to `uint32`
- 1992: `ndimage.find_objects` doesn't work with `int32` input in some cases

## 1.10 SciPy 0.13.0 Release Notes

**Contents**

- SciPy 0.13.0 Release Notes
  - New features
    - \* `scipy.integrate` improvements
      - N-dimensional numerical integration
      - `dopri*` improvements
    - \* `scipy.linalg` improvements
      - Interpolative decompositions
      - Polar decomposition
      - BLAS level 3 functions
      - Matrix functions
    - \* `scipy.optimize` improvements
      - Trust-region unconstrained minimization algorithms
    - \* `scipy.sparse` improvements
      - Boolean comparisons and sparse matrices
      - CSR and CSC fancy indexing
    - \* `scipy.sparse.linalg` improvements
    - \* `scipy.spatial` improvements
    - \* `scipy.signal` improvements
    - \* `scipy.special` improvements
    - \* `scipy.io` improvements
      - Unformatted Fortran file reader
      - `scipy.io.wavfile` enhancements
    - \* `scipy.interpolate` improvements
      - B-spline derivatives and antiderivatives
    - \* `scipy.stats` improvements
  - Deprecated features
    - \* `expm2` and `expm3`
    - \* `scipy.stats` functions
  - Backwards incompatible changes
    - \* LIL matrix assignment
    - \* Deprecated `radon` function removed
    - \* Removed deprecated keywords `xa` and `xb` from `stats.distributions`
    - \* Changes to MATLAB file readers / writers
  - Other changes
  - Authors

SciPy 0.13.0 is the culmination of 7 months of hard work. It contains many new features, numerous bug-fixes, improved test coverage and better documentation. There have been a number of deprecations and API changes in this release, which are documented below. All users are encouraged to upgrade to this release, as there are a large number of bug-fixes and optimizations. Moreover, our development attention will now shift to bug-fix releases on the 0.13.x branch, and on adding new features on the master branch.

This release requires Python 2.6, 2.7 or 3.1-3.3 and NumPy 1.5.1 or greater. Highlights of this release are:

- support for fancy indexing and boolean comparisons with sparse matrices
- interpolative decompositions and matrix functions in the `linalg` module
- two new trust-region solvers for unconstrained minimization



## 1.10.1 New features

### `scipy.integrate` improvements

#### *N-dimensional numerical integration*

A new function `scipy.integrate.nquad`, which provides N-dimensional integration functionality with a more flexible interface than `dblquad` and `tplquad`, has been added.

#### *dopri\* improvements*

The intermediate results from the `dopri` family of ODE solvers can now be accessed by a *solout* callback function.

### `scipy.linalg` improvements

#### *Interpolative decompositions*

Scipy now includes a new module `scipy.linalg.interpolative` containing routines for computing interpolative matrix decompositions (ID). This feature is based on the ID software package by P.G. Martinsson, V. Rokhlin, Y. Shkolnisky, and M. Tygert, previously adapted for Python in the PymatrixId package by K.L. Ho.

#### *Polar decomposition*

A new function `scipy.linalg.polar`, to compute the polar decomposition of a matrix, was added.

#### *BLAS level 3 functions*

The BLAS functions `symm`, `syrk`, `syr2k`, `hemm`, `herk` and `her2k` are now wrapped in `scipy.linalg`.

#### *Matrix functions*

Several matrix function algorithms have been implemented or updated following detailed descriptions in recent papers of Nick Higham and his co-authors. These include the matrix square root (`sqrtn`), the matrix logarithm (`logm`), the matrix exponential (`expm`) and its Frechet derivative (`expm_frechet`), and fractional matrix powers (`fractional_matrix_power`).

### `scipy.optimize` improvements

#### *Trust-region unconstrained minimization algorithms*

The `minimize` function gained two trust-region solvers for unconstrained minimization: `dogleg` and `trust-ncg`.

### `scipy.sparse` improvements

#### *Boolean comparisons and sparse matrices*

All sparse matrix types now support boolean data, and boolean operations. Two sparse matrices  $A$  and  $B$  can be compared in all the expected ways  $A < B$ ,  $A \geq B$ ,  $A \neq B$ , producing similar results as dense Numpy arrays. Comparisons with dense matrices and scalars are also supported.

#### *CSR and CSC fancy indexing*

Compressed sparse row and column sparse matrix types now support fancy indexing with boolean matrices, slices, and lists. So where  $A$  is a (CSC or CSR) sparse matrix, you can do things like:

```
>>> A[A > 0.5] = 1 # since Boolean sparse matrices work
>>> A[:,2, :3] = 2
>>> A[[1,2], 2] = 3
```

### **scipy.sparse.linalg improvements**

The new function `onenormest` provides a lower bound of the 1-norm of a linear operator and has been implemented according to Higham and Tisseur (2000). This function is not only useful for sparse matrices, but can also be used to estimate the norm of products or powers of dense matrices without explicitly building the intermediate matrix.

The multiplicative action of the matrix exponential of a linear operator (`expm_multiply`) has been implemented following the description in Al-Mohy and Higham (2011).

Abstract linear operators (`scipy.sparse.linalg.LinearOperator`) can now be multiplied, added to each other, and exponentiated, producing new linear operators. This enables easier construction of composite linear operations.

### **scipy.spatial improvements**

The vertices of a *ConvexHull* can now be accessed via the *vertices* attribute, which gives proper orientation in 2-D.

### **scipy.signal improvements**

The cosine window function `scipy.signal.cosine` was added.

### **scipy.special improvements**

New functions `scipy.special.xlogy` and `scipy.special.xlog1py` were added. These functions can simplify and speed up code that has to calculate  $x * \log(y)$  and give 0 when  $x == 0$ .

### **scipy.io improvements**

#### *Unformatted Fortran file reader*

The new class `scipy.io.FortranFile` facilitates reading unformatted sequential files written by Fortran code.

#### *scipy.io.wavfile enhancements*

`scipy.io.wavfile.write` now accepts a file buffer. Previously it only accepted a filename.

`scipy.io.wavfile.read` and `scipy.io.wavfile.write` can now handle floating point WAV files.

### **scipy.interpolate improvements**

#### *B-spline derivatives and antiderivatives*

`scipy.interpolate.splder` and `scipy.interpolate.splantider` functions for computing B-splines that represent derivatives and antiderivatives of B-splines were added. These functions are also available in the class-based FITPACK interface as `UnivariateSpline.derivative` and `UnivariateSpline.antiderivative`.

## scipy.stats improvements

Distributions now allow using keyword parameters in addition to positional parameters in all methods.

The function `scipy.stats.power_divergence` has been added for the Cressie-Read power divergence statistic and goodness of fit test. Included in this family of statistics is the “G-test” (<http://en.wikipedia.org/wiki/G-test>).

`scipy.stats.mood` now accepts multidimensional input.

An option was added to `scipy.stats.wilcoxon` for continuity correction.

`scipy.stats.chisquare` now has an *axis* argument.

`scipy.stats.mstats.chisquare` now has *axis* and *ddof* arguments.

## 1.10.2 Deprecated features

### expm2 and expm3

The matrix exponential functions `scipy.linalg.expm2` and `scipy.linalg.expm3` are deprecated. All users should use the numerically more robust `scipy.linalg.expm` function instead.

### scipy.stats functions

`scipy.stats.oneway` is deprecated; `scipy.stats.f_oneway` should be used instead.

`scipy.stats.glm` is deprecated. `scipy.stats.ttest_ind` is an equivalent function; more full-featured general (and generalized) linear model implementations can be found in statsmodels.

`scipy.stats.cmedian` is deprecated; `numpy.median` should be used instead.

## 1.10.3 Backwards incompatible changes

### LIL matrix assignment

Assigning values to LIL matrices with two index arrays now works similarly as assigning into ndarrays:

```
>>> x = lil_matrix((3, 3))
>>> x[[0,1,2],[0,1,2]]=[0,1,2]
>>> x.todense()
matrix([[ 0.,  0.,  0.],
        [ 0.,  1.,  0.],
        [ 0.,  0.,  2.]])
```

rather than giving the result:

```
>>> x.todense()
matrix([[ 0.,  1.,  2.],
        [ 0.,  1.,  2.],
        [ 0.,  1.,  2.]])
```

Users relying on the previous behavior will need to revisit their code. The previous behavior is obtained by `x[numpy.ix_([0,1,2],[0,1,2])] = ...`

### Deprecated `radon` function removed

The `misc.radon` function, which was deprecated in `scipy` 0.11.0, has been removed. Users can find a more full-featured `radon` function in `scikit-image`.

### Removed deprecated keywords `xa` and `xb` from `stats.distributions`

The keywords `xa` and `xb`, which were deprecated since 0.11.0, have been removed from the distributions in `scipy.stats`.

### Changes to MATLAB file readers / writers

The major change is that 1D arrays in `numpy` now become row vectors (shape `1, N`) when saved to a MATLAB 5 format file. Previously 1D arrays saved as column vectors (`N, 1`). This is to harmonize the behavior of writing MATLAB 4 and 5 formats, and adapt to the defaults of `numpy` and MATLAB - for example `np.atleast_2d` returns 1D arrays as row vectors.

Trying to save arrays of greater than 2 dimensions in MATLAB 4 format now raises an error instead of silently reshaping the array as 2D.

`scipy.io.loadmat('afile')` used to look for *afile* on the Python system path (`sys.path`); now `loadmat` only looks in the current directory for a relative path filename.

## 1.10.4 Other changes

Security fix: `scipy.weave` previously used temporary directories in an insecure manner under certain circumstances.

Cython is now required to build *unreleased* versions of `scipy`. The C files generated from Cython sources are not included in the git repo anymore. They are however still shipped in source releases.

The code base received a fairly large PEP8 cleanup. A `tox pep8` command has been added; new code should pass this test command.

Scipy cannot be compiled with `gfortran` 4.1 anymore (at least on RH5), likely due to that compiler version not supporting entry constructs well.

## 1.10.5 Authors

This release contains work by the following people (contributed at least one patch to this release, names in alphabetical order):

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A total of 65 people contributed to this release. People with a “+” by their names contributed a patch for the first time.

### 1.11 SciPy 0.12.1 Release Notes

SciPy 0.12.1 is a bug-fix release with no new features compared to 0.12.0. The single issue fixed by this release is a security issue in `scipy.weave`, which was previously using temporary directories in an insecure manner under certain circumstances.

### 1.12 SciPy 0.12.0 Release Notes

**Contents**

- SciPy 0.12.0 Release Notes
  - New features
    - \* `scipy.spatial` improvements
      - cKDTree feature-complete
      - Voronoi diagrams and convex hulls
      - Delaunay improvements
    - \* Spectral estimators (`scipy.signal`)
    - \* `scipy.optimize` improvements
      - Callback functions in L-BFGS-B and TNC
      - Basin hopping global optimization (`scipy.optimize.basinhopping`)
    - \* `scipy.special` improvements
      - Revised complex error functions
      - Faster orthogonal polynomials
    - \* `scipy.sparse.linalg` features
    - \* Listing Matlab(R) file contents in `scipy.io`
    - \* Documented BLAS and LAPACK low-level interfaces (`scipy.linalg`)
    - \* Polynomial interpolation improvements (`scipy.interpolate`)
  - Deprecated features
    - \* `scipy.lib.lapack`
    - \* `fblas` and `cblas`
  - Backwards incompatible changes
    - \* Removal of `scipy.io.save_as_module`
    - \* `axis` argument added to `scipy.stats.scoreatpercentile`
  - Authors

SciPy 0.12.0 is the culmination of 7 months of hard work. It contains many new features, numerous bug-fixes, improved test coverage and better documentation. There have been a number of deprecations and API changes in this release, which are documented below. All users are encouraged to upgrade to this release, as there are a large number of bug-fixes and optimizations. Moreover, our development attention will now shift to bug-fix releases on the 0.12.x branch, and on adding new features on the master branch.

Some of the highlights of this release are:

- Completed QHull wrappers in `scipy.spatial`.
- cKDTree now a drop-in replacement for KDTree.
- A new global optimizer, `basinhopping`.
- Support for Python 2 and Python 3 from the same code base (no more 2to3).

This release requires Python 2.6, 2.7 or 3.1-3.3 and NumPy 1.5.1 or greater. Support for Python 2.4 and 2.5 has been dropped as of this release.

## 1.12.1 New features

### `scipy.spatial` improvements

#### *cKDTree feature-complete*

Cython version of KDTree, cKDTree, is now feature-complete. Most operations (construction, query, `query_ball_point`, `query_pairs`, `count_neighbors` and `sparse_distance_matrix`) are between 200 and 1000 times faster

in `cKDTree` than in `KDTree`. With very minor caveats, `cKDTree` has exactly the same interface as `KDTree`, and can be used as a drop-in replacement.

### *Voronoi diagrams and convex hulls*

`scipy.spatial` now contains functionality for computing Voronoi diagrams and convex hulls using the Qhull library. (Delaunay triangulation was available since Scipy 0.9.0.)

### *Delaunay improvements*

It's now possible to pass in custom Qhull options in Delaunay triangulation. Coplanar points are now also recorded, if present. Incremental construction of Delaunay triangulations is now also possible.

### **Spectral estimators (`scipy.signal`)**

The functions `scipy.signal.periodogram` and `scipy.signal.welch` were added, providing DFT-based spectral estimators.

### **`scipy.optimize` improvements**

#### *Callback functions in L-BFGS-B and TNC*

A callback mechanism was added to L-BFGS-B and TNC minimization solvers.

#### *Basin hopping global optimization (`scipy.optimize.basinhopping`)*

A new global optimization algorithm. Basinhopping is designed to efficiently find the global minimum of a smooth function.

### **`scipy.special` improvements**

#### *Revised complex error functions*

The computation of special functions related to the error function now uses a new [Faddeeva library from MIT](#) which increases their numerical precision. The scaled and imaginary error functions `erfcx` and `erfi` were also added, and the Dawson integral `dawsn` can now be evaluated for a complex argument.

#### *Faster orthogonal polynomials*

Evaluation of orthogonal polynomials (the `eval_*` routines) is now faster in `scipy.special`, and their `out=` argument functions properly.

### **`scipy.sparse.linalg` features**

- In `scipy.sparse.linalg.spsolve`, the `b` argument can now be either a vector or a matrix.
- `scipy.sparse.linalg.inv` was added. This uses `spsolve` to compute a sparse matrix inverse.
- `scipy.sparse.linalg.expm` was added. This computes the exponential of a sparse matrix using a similar algorithm to the existing dense array implementation in `scipy.linalg.expm`.

### **Listing Matlab(R) file contents in `scipy.io`**

A new function `whosmat` is available in `scipy.io` for inspecting contents of MAT files without reading them to memory.



## Documented BLAS and LAPACK low-level interfaces (`scipy.linalg`)

The modules `scipy.linalg.blas` and `scipy.linalg.lapack` can be used to access low-level BLAS and LAPACK functions.

## Polynomial interpolation improvements (`scipy.interpolate`)

The barycentric, Krogh, piecewise and pchip polynomial interpolators in `scipy.interpolate` accept now an `axis` argument.

### 1.12.2 Deprecated features

#### *scipy.lib.lapack*

The module `scipy.lib.lapack` is deprecated. You can use `scipy.linalg.lapack` instead. The module `scipy.lib.blas` was deprecated earlier in SciPy 0.10.0.

#### *fblas* and *cblas*

Accessing the modules `scipy.linalg.fblas`, `cblas`, `flapack`, `clapack` is deprecated. Instead, use the modules `scipy.linalg.lapack` and `scipy.linalg.blas`.

### 1.12.3 Backwards incompatible changes

#### Removal of `scipy.io.save_as_module`

The function `scipy.io.save_as_module` was deprecated in SciPy 0.11.0, and is now removed.

Its private support modules `scipy.io.dumbdbm_patched` and `scipy.io.dumb_shelve` are also removed.

#### *axis* argument added to `scipy.stats.scoreatpercentile`

The function `scipy.stats.scoreatpercentile` has been given an *axis* argument. The default argument is *axis=None*, which means the calculation is done on the flattened array. Before this change, `scoreatpercentile` would act as if *axis=0* had been given. Code using `scoreatpercentile` with a multidimensional array will need to add *axis=0* to the function call to preserve the old behavior. (This API change was not noticed until long after the release of 0.12.0.)

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A total of 75 people contributed to this release. People with a “+” by their names contributed a patch for the first time.

## 1.13 SciPy 0.11.0 Release Notes

### Contents

- SciPy 0.11.0 Release Notes
  - New features
    - \* Sparse Graph Submodule
    - \* `scipy.optimize` improvements
      - Unified interfaces to minimizers
      - Unified interface to root finding algorithms
    - \* `scipy.linalg` improvements
      - New matrix equation solvers
      - QZ and QR Decomposition
      - Pascal matrices
    - \* Sparse matrix construction and operations
    - \* LSMR iterative solver
    - \* Discrete Sine Transform
    - \* `scipy.interpolate` improvements
    - \* Binned statistics (`scipy.stats`)
  - Deprecated features
  - Backwards incompatible changes
    - \* Removal of `scipy.maxentropy`
    - \* Minor change in behavior of `splev`
    - \* Behavior of `scipy.integrate.complex_ode`
    - \* Minor change in behavior of T-tests
  - Other changes
  - Authors

SciPy 0.11.0 is the culmination of 8 months of hard work. It contains many new features, numerous bug-fixes, improved test coverage and better documentation. Highlights of this release are:

- A new module has been added which provides a number of common sparse graph algorithms.
- New unified interfaces to the existing optimization and root finding functions have been added.

All users are encouraged to upgrade to this release, as there are a large number of bug-fixes and optimizations. Our development attention will now shift to bug-fix releases on the 0.11.x branch, and on adding new features on the master branch.

This release requires Python 2.4-2.7 or 3.1-3.2 and NumPy 1.5.1 or greater.

### 1.13.1 New features

#### Sparse Graph Submodule

The new submodule `scipy.sparse.csgraph` implements a number of efficient graph algorithms for graphs stored as sparse adjacency matrices. Available routines are:

- `connected_components` - determine connected components of a graph
- `laplacian` - compute the laplacian of a graph
- `shortest_path` - compute the shortest path between points on a positive graph
- `dijkstra` - use Dijkstra's algorithm for shortest path

- `floyd_warshall` - use the Floyd-Warshall algorithm for shortest path
- `breadth_first_order` - compute a breadth-first order of nodes
- `depth_first_order` - compute a depth-first order of nodes
- `breadth_first_tree` - construct the breadth-first tree from a given node
- `depth_first_tree` - construct a depth-first tree from a given node
- `minimum_spanning_tree` - construct the minimum spanning tree of a graph

### **scipy.optimize improvements**

The optimize module has received a lot of attention this release. In addition to added tests, documentation improvements, bug fixes and code clean-up, the following improvements were made:

- A unified interface to minimizers of univariate and multivariate functions has been added.
- A unified interface to root finding algorithms for multivariate functions has been added.
- The L-BFGS-B algorithm has been updated to version 3.0.

#### ***Unified interfaces to minimizers***

Two new functions `scipy.optimize.minimize` and `scipy.optimize.minimize_scalar` were added to provide a common interface to minimizers of multivariate and univariate functions respectively. For multivariate functions, `scipy.optimize.minimize` provides an interface to methods for unconstrained optimization (`fmin`, `fmin_powell`, `fmin_cg`, `fmin_ncg`, `fmin_bfgs` and `anneal`) or constrained optimization (`fmin_l_bfgs_b`, `fmin_tnc`, `fmin_cobyla` and `fmin_slsqp`). For univariate functions, `scipy.optimize.minimize_scalar` provides an interface to methods for unconstrained and bounded optimization (`brent`, `golden`, `fminbound`). This allows for easier comparing and switching between solvers.

#### ***Unified interface to root finding algorithms***

The new function `scipy.optimize.root` provides a common interface to root finding algorithms for multivariate functions, embedding `fsolve`, `leastsq` and `nonlin` solvers.

### **scipy.linalg improvements**

#### ***New matrix equation solvers***

Solvers for the Sylvester equation (`scipy.linalg.solve_sylvester`, discrete and continuous Lyapunov equations (`scipy.linalg.solve_lyapunov`, `scipy.linalg.solve_discrete_lyapunov`) and discrete and continuous algebraic Riccati equations (`scipy.linalg.solve_continuous_are`, `scipy.linalg.solve_discrete_are`) have been added to `scipy.linalg`. These solvers are often used in the field of linear control theory.

#### ***QZ and QR Decomposition***

It is now possible to calculate the QZ, or Generalized Schur, decomposition using `scipy.linalg.qz`. This function wraps the LAPACK routines `sgges`, `dgges`, `cgges`, and `zgges`.

The function `scipy.linalg.qr_multiply`, which allows efficient computation of the matrix product of Q (from a QR decomposition) and a vector, has been added.

#### ***Pascal matrices***

A function for creating Pascal matrices, `scipy.linalg.pascal`, was added.

## Sparse matrix construction and operations

Two new functions, `scipy.sparse.diags` and `scipy.sparse.block_diag`, were added to easily construct diagonal and block-diagonal sparse matrices respectively.

`scipy.sparse.csc_matrix` and `csc_matrix` now support the operations `sin`, `tan`, `arcsin`, `arctan`, `sinh`, `tanh`, `arcsinh`, `arctanh`, `rint`, `sign`, `expm1`, `log1p`, `deg2rad`, `rad2deg`, `floor`, `ceil` and `trunc`. Previously, these operations had to be performed by operating on the matrices' `data` attribute.

## LSMR iterative solver

LSMR, an iterative method for solving (sparse) linear and linear least-squares systems, was added as `scipy.sparse.linalg.lsmr`.

## Discrete Sine Transform

Bindings for the discrete sine transform functions have been added to `scipy.fftpack`.

## `scipy.interpolate` improvements

For interpolation in spherical coordinates, the three classes `scipy.interpolate.SmoothSphereBivariateSpline`, `scipy.interpolate.LSQSphereBivariateSpline`, and `scipy.interpolate.RectSphereBivariateSpline` have been added.

## Binned statistics (`scipy.stats`)

The `stats` module has gained functions to do binned statistics, which are a generalization of histograms, in 1-D, 2-D and multiple dimensions: `scipy.stats.binned_statistic`, `scipy.stats.binned_statistic_2d` and `scipy.stats.binned_statistic_dd`.

## 1.13.2 Deprecated features

`scipy.sparse.cs_graph_components` has been made a part of the sparse graph submodule, and renamed to `scipy.sparse.csgraph.connected_components`. Calling the former routine will result in a deprecation warning.

`scipy.misc.radon` has been deprecated. A more full-featured radon transform can be found in `scikits-image`.

`scipy.io.save_as_module` has been deprecated. A better way to save multiple Numpy arrays is the `numpy.savez` function.

The `xa` and `xb` parameters for all distributions in `scipy.stats.distributions` already weren't used; they have now been deprecated.

## 1.13.3 Backwards incompatible changes

### Removal of `scipy.maxentropy`

The `scipy.maxentropy` module, which was deprecated in the 0.10.0 release, has been removed. Logistic regression in `scikits.learn` is a good and modern alternative for this functionality.

### Minor change in behavior of `splev`

The spline evaluation function now behaves similarly to `interp1d` for size-1 arrays. Previous behavior:

```
>>> from scipy.interpolate import splev, splrep, interp1d
>>> x = [1,2,3,4,5]
>>> y = [4,5,6,7,8]
>>> tck = splrep(x, y)
>>> splev([1], tck)
4.
>>> splev(1, tck)
4.
```

Corrected behavior:

```
>>> splev([1], tck)
array([ 4.])
>>> splev(1, tck)
array(4.)
```

This affects also the `UnivariateSpline` classes.

### Behavior of `scipy.integrate.complex_ode`

The behavior of the `y` attribute of `complex_ode` is changed. Previously, it expressed the complex-valued solution in the form:

```
z = ode.y[:,2] + 1j * ode.y[1:,2]
```

Now, it is directly the complex-valued solution:

```
z = ode.y
```

### Minor change in behavior of T-tests

The T-tests `scipy.stats.ttest_ind`, `scipy.stats.ttest_rel` and `scipy.stats.ttest_1samp` have been changed so that `0 / 0` now returns NaN instead of 1.

## 1.13.4 Other changes

The SuperLU sources in `scipy.sparse.linalg` have been updated to version 4.3 from upstream.

The function `scipy.signal.bode`, which calculates magnitude and phase data for a continuous-time system, has been added.

The two-sample T-test `scipy.stats.ttest_ind` gained an option to compare samples with unequal variances, i.e. Welch's T-test.

`scipy.misc.logsumexp` now takes an optional `axis` keyword argument.

## 1.13.5 Authors

This release contains work by the following people (contributed at least one patch to this release, names in alphabetical order):

- Jeff Armstrong

- Chad Baker
- Brandon Beacher +
- behrisch +
- borishim +
- Matthew Brett
- Lars Buitinck
- Luis Pedro Coelho +
- Johann Cohen-Tanugi
- David Cournapeau
- dougal +
- Ali Ebrahim +
- endolith +
- Bjørn Forsman +
- Robert Gantner +
- Sebastian Gassner +
- Christoph Gohlke
- Ralf Gommers
- Yaroslav Halchenko
- Charles Harris
- Jonathan Helmus +
- Andreas Hilboll +
- Marc Honnorat +
- Jonathan Hunt +
- Maxim Ivanov +
- Thouis (Ray) Jones
- Christopher Kuster +
- Josh Lawrence +
- Denis Laxalde +
- Travis Oliphant
- Joonas Paalasmaa +
- Fabian Pedregosa
- Josef Perktold
- Gavin Price +
- Jim Radford +
- Andrew Schein +
- Skipper Seabold



- Jacob Silterra +
- Scott Sinclair
- Alexis Tabary +
- Martin Teichmann
- Matt Terry +
- Nicky van Foreest +
- Jacob Vanderplas
- Patrick Varilly +
- Pauli Virtanen
- Nils Wagner +
- Darryl Wally +
- Stefan van der Walt
- Liming Wang +
- David Warde-Farley +
- Warren Weckesser
- Sebastian Werk +
- Mike Wimmer +
- Tony S Yu +

A total of 55 people contributed to this release. People with a “+” by their names contributed a patch for the first time.

## 1.14 SciPy 0.10.1 Release Notes

### Contents

- SciPy 0.10.1 Release Notes
  - Main changes
  - Other issues fixed

SciPy 0.10.1 is a bug-fix release with no new features compared to 0.10.0.

### 1.14.1 Main changes

The most important changes are:

1. The single precision routines of `eigs` and `eigsh` in `scipy.sparse.linalg` have been disabled (they internally use double precision now).
2. A compatibility issue related to changes in NumPy macros has been fixed, in order to make `scipy 0.10.1` compile with the upcoming `numpy 1.7.0` release.

### 1.14.2 Other issues fixed

- #835: stats: nan propagation in stats.distributions
- #1202: io: netcdf segfault
- #1531: optimize: make curve\_fit work with method as callable.
- #1560: linalg: fixed mistake in eig\_banded documentation.
- #1565: ndimage: bug in ndimage.variance
- #1457: ndimage: standard\_deviation does not work with sequence of indexes
- #1562: cluster: segfault in linkage function
- #1568: stats: One-sided fisher\_exact() returns  $p < 1$  for 0 successful attempts
- #1575: stats: zscore and zmap handle the axis keyword incorrectly

## 1.15 SciPy 0.10.0 Release Notes

### Contents

- SciPy 0.10.0 Release Notes
  - New features
    - \* Bento: new optional build system
    - \* Generalized and shift-invert eigenvalue problems in `scipy.sparse.linalg`
    - \* Discrete-Time Linear Systems (`scipy.signal`)
    - \* Enhancements to `scipy.signal`
    - \* Additional decomposition options (`scipy.linalg`)
    - \* Additional special matrices (`scipy.linalg`)
    - \* Enhancements to `scipy.stats`
    - \* Enhancements to `scipy.special`
    - \* Basic support for Harwell-Boeing file format for sparse matrices
  - Deprecated features
    - \* `scipy.maxentropy`
    - \* `scipy.lib.blas`
    - \* Numsccons build system
  - Backwards-incompatible changes
  - Other changes
  - Authors

SciPy 0.10.0 is the culmination of 8 months of hard work. It contains many new features, numerous bug-fixes, improved test coverage and better documentation. There have been a limited number of deprecations and backwards-incompatible changes in this release, which are documented below. All users are encouraged to upgrade to this release, as there are a large number of bug-fixes and optimizations. Moreover, our development attention will now shift to bug-fix releases on the 0.10.x branch, and on adding new features on the development master branch.

Release highlights:

- Support for Bento as optional build system.
- Support for generalized eigenvalue problems, and all shift-invert modes available in ARPACK.

This release requires Python 2.4-2.7 or 3.1- and NumPy 1.5 or greater.

### 1.15.1 New features

#### Bento: new optional build system

SciPy can now be built with **Bento**. Bento has some nice features like parallel builds and partial rebuilds, that are not possible with the default build system (distutils). For usage instructions see BENTO\_BUILD.txt in the scipy top-level directory.

Currently SciPy has three build systems, distutils, numscs and bento. Numscs is deprecated and is planned and will likely be removed in the next release.

#### Generalized and shift-invert eigenvalue problems in `scipy.sparse.linalg`

The sparse eigenvalue problem solver functions `scipy.sparse.eigs/eigh` now support generalized eigenvalue problems, and all shift-invert modes available in ARPACK.

#### Discrete-Time Linear Systems (`scipy.signal`)

Support for simulating discrete-time linear systems, including `scipy.signal.dlsim`, `scipy.signal.dimpulse`, and `scipy.signal.dstep`, has been added to SciPy. Conversion of linear systems from continuous-time to discrete-time representations is also present via the `scipy.signal.cont2discrete` function.

#### Enhancements to `scipy.signal`

A Lomb-Scargle periodogram can now be computed with the new function `scipy.signal.lombscargle`.

The forward-backward filter function `scipy.signal.filtfilt` can now filter the data in a given axis of an n-dimensional numpy array. (Previously it only handled a 1-dimensional array.) Options have been added to allow more control over how the data is extended before filtering.

FIR filter design with `scipy.signal.firwin2` now has options to create filters of type III (zero at zero and Nyquist frequencies) and IV (zero at zero frequency).

#### Additional decomposition options (`scipy.linalg`)

A sort keyword has been added to the Schur decomposition routine (`scipy.linalg.schur`) to allow the sorting of eigenvalues in the resultant Schur form.

#### Additional special matrices (`scipy.linalg`)

The functions `hilbert` and `invhilbert` were added to `scipy.linalg`.

#### Enhancements to `scipy.stats`

- The *one-sided form* of Fisher's exact test is now also implemented in `stats.fisher_exact`.
- The function `stats.chi2_contingency` for computing the chi-square test of independence of factors in a contingency table has been added, along with the related utility functions `stats.contingency.margins` and `stats.contingency.expected_freq`.

### Enhancements to `scipy.special`

The functions  $\text{logit}(p) = \log(p/(1-p))$  and  $\text{expit}(x) = 1/(1+\exp(-x))$  have been implemented as `scipy.special.logit` and `scipy.special.expit` respectively.

### Basic support for Harwell-Boeing file format for sparse matrices

Both read and write are support through a simple function-based API, as well as a more complete API to control number format. The functions may be found in `scipy.sparse.io`.

The following features are supported:

- Read and write sparse matrices in the CSC format
- Only real, symmetric, assembled matrix are supported (RUA format)

## 1.15.2 Deprecated features

### `scipy.maxentropy`

The `maxentropy` module is unmaintained, rarely used and has not been functioning well for several releases. Therefore it has been deprecated for this release, and will be removed for `scipy 0.11`. Logistic regression in `scikits.learn` is a good alternative for this functionality. The `scipy.maxentropy.logsumexp` function has been moved to `scipy.misc`.

### `scipy.lib.blas`

There are similar BLAS wrappers in `scipy.linalg` and `scipy.lib`. These have now been consolidated as `scipy.linalg.blas`, and `scipy.lib.blas` is deprecated.

### Numscons build system

The `numscons` build system is being replaced by `Bento`, and will be removed in one of the next `scipy` releases.

## 1.15.3 Backwards-incompatible changes

The deprecated name `invnorm` was removed from `scipy.stats.distributions`, this distribution is available as `invgauss`.

The following deprecated nonlinear solvers from `scipy.optimize` have been removed:

- ```broyden_modified``` (bad performance)
- ```broyden1_modified``` (bad performance)
- ```broyden_generalized``` (equivalent to ```anderson```)
- ```anderson2``` (equivalent to ```anderson```)
- ```broyden3``` (obsoleted by new limited-memory broyden methods)
- ```vackar``` (renamed to ```diagbroyden```)

### 1.15.4 Other changes

`scipy.constants` has been updated with the CODATA 2010 constants.

`__all__` dicts have been added to all modules, which has cleaned up the namespaces (particularly useful for interactive work).

An API section has been added to the documentation, giving recommended import guidelines and specifying which submodules are public and which aren't.

### 1.15.5 Authors

This release contains work by the following people (contributed at least one patch to this release, names in alphabetical order):

- Jeff Armstrong +
- Matthew Brett
- Lars Buitinck +
- David Cournapeau
- FISH 2000 +
- Michael McNeil Forbes +
- Matty G +
- Christoph Gohlke
- Ralf Gommers
- Yaroslav Halchenko
- Charles Harris
- Thouis (Ray) Jones +
- Chris Jordan-Squire +
- Robert Kern
- Chris Lasher +
- Wes McKinney +
- Travis Oliphant
- Fabian Pedregosa
- Josef Perktold
- Thomas Robitaille +
- Pim Schellart +
- Anthony Scopatz +
- Skipper Seabold +
- Fazlul Shahriar +
- David Simcha +
- Scott Sinclair +

- Andrey Smirnov +
- Collin RM Stocks +
- Martin Teichmann +
- Jake Vanderplas +
- Gaël Varoquaux +
- Pauli Virtanen
- Stefan van der Walt
- Warren Weckesser
- Mark Wiebe +

A total of 35 people contributed to this release. People with a “+” by their names contributed a patch for the first time.

## 1.16 SciPy 0.9.0 Release Notes

### Contents

- SciPy 0.9.0 Release Notes
  - Python 3
  - Scipy source code location to be changed
  - New features
    - \* Delaunay tessellations (`scipy.spatial`)
    - \* N-dimensional interpolation (`scipy.interpolate`)
    - \* Nonlinear equation solvers (`scipy.optimize`)
    - \* New linear algebra routines (`scipy.linalg`)
    - \* Improved FIR filter design functions (`scipy.signal`)
    - \* Improved statistical tests (`scipy.stats`)
  - Deprecated features
    - \* Obsolete nonlinear solvers (in `scipy.optimize`)
  - Removed features
    - \* Old correlate/convolve behavior (in `scipy.signal`)
    - \* `scipy.stats`
    - \* `scipy.sparse`
    - \* `scipy.sparse.linalg.arpack.speigs`
  - Other changes
    - \* ARPACK interface changes

SciPy 0.9.0 is the culmination of 6 months of hard work. It contains many new features, numerous bug-fixes, improved test coverage and better documentation. There have been a number of deprecations and API changes in this release, which are documented below. All users are encouraged to upgrade to this release, as there are a large number of bug-fixes and optimizations. Moreover, our development attention will now shift to bug-fix releases on the 0.9.x branch, and on adding new features on the development trunk.

This release requires Python 2.4 - 2.7 or 3.1 - and NumPy 1.5 or greater.

Please note that SciPy is still considered to have “Beta” status, as we work toward a SciPy 1.0.0 release. The 1.0.0 release will mark a major milestone in the development of SciPy, after which changing the package structure or API will be much more difficult. Whilst these pre-1.0 releases are considered to have “Beta” status, we are committed to making them as bug-free as possible.

However, until the 1.0 release, we are aggressively reviewing and refining the functionality, organization, and interface. This is being done in an effort to make the package as coherent, intuitive, and useful as possible. To achieve this, we need help from the community of users. Specifically, we need feedback regarding all aspects of the project - everything - from which algorithms we implement, to details about our function's call signatures.

### 1.16.1 Python 3

Scipy 0.9.0 is the first SciPy release to support Python 3. The only module that is not yet ported is `scipy.weave`.

### 1.16.2 Scipy source code location to be changed

Soon after this release, Scipy will stop using SVN as the version control system, and move to Git. The development source code for Scipy can from then on be found at

<http://github.com/scipy/scipy>

### 1.16.3 New features

#### Delaunay tessellations (`scipy.spatial`)

Scipy now includes routines for computing Delaunay tessellations in N dimensions, powered by the [Qhull](#) computational geometry library. Such calculations can now make use of the new `scipy.spatial.Delaunay` interface.

#### N-dimensional interpolation (`scipy.interpolate`)

Support for scattered data interpolation is now significantly improved. This version includes a `scipy.interpolate.griddata` function that can perform linear and nearest-neighbour interpolation for N-dimensional scattered data, in addition to cubic spline (C1-smooth) interpolation in 2D and 1D. An object-oriented interface to each interpolator type is also available.

#### Nonlinear equation solvers (`scipy.optimize`)

Scipy includes new routines for large-scale nonlinear equation solving in `scipy.optimize`. The following methods are implemented:

- Newton-Krylov (`scipy.optimize.newton_krylov`)
- (Generalized) secant methods:
  - Limited-memory Broyden methods (`scipy.optimize.broyden1`, `scipy.optimize.broyden2`)
  - Anderson method (`scipy.optimize.anderson`)
- Simple iterations (`scipy.optimize.diagbroyden`, `scipy.optimize.excitingmixing`, `scipy.optimize.linearmixing`)

The `scipy.optimize.nonlin` module was completely rewritten, and some of the functions were deprecated (see above).

### New linear algebra routines (`scipy.linalg`)

SciPy now contains routines for effectively solving triangular equation systems (`scipy.linalg.solve_triangular`).

### Improved FIR filter design functions (`scipy.signal`)

The function `scipy.signal.firwin` was enhanced to allow the design of highpass, bandpass, bandstop and multi-band FIR filters.

The function `scipy.signal.firwin2` was added. This function uses the window method to create a linear phase FIR filter with an arbitrary frequency response.

The functions `scipy.signal.kaiser_atten` and `scipy.signal.kaiser_beta` were added.

### Improved statistical tests (`scipy.stats`)

A new function `scipy.stats.fisher_exact` was added, that provides Fisher's exact test for 2x2 contingency tables.

The function `scipy.stats.kendalltau` was rewritten to make it much faster ( $O(n \log(n))$  vs  $O(n^2)$ ).

## 1.16.4 Deprecated features

### Obsolete nonlinear solvers (in `scipy.optimize`)

The following nonlinear solvers from `scipy.optimize` are deprecated:

- `broyden_modified` (bad performance)
- `broyden1_modified` (bad performance)
- `broyden_generalized` (equivalent to `anderson`)
- `anderson2` (equivalent to `anderson`)
- `broyden3` (obsoleted by new limited-memory broyden methods)
- `vackar` (renamed to `diagbroyden`)

## 1.16.5 Removed features

The deprecated modules `helpmod`, `pexec` and `ppimport` were removed from `scipy.misc`.

The `output_type` keyword in many `scipy.ndimage` interpolation functions has been removed.

The `econ` keyword in `scipy.linalg.qr` has been removed. The same functionality is still available by specifying `mode='economic'`.

### Old correlate/convolve behavior (in `scipy.signal`)

The old behavior for `scipy.signal.convolve`, `scipy.signal.convolve2d`, `scipy.signal.correlate` and `scipy.signal.correlate2d` was deprecated in 0.8.0 and has now been removed. Convolve and correlate used to swap their arguments if the second argument has dimensions larger than the first one, and the mode was relative to the input with the largest dimension. The current behavior is to never swap the inputs, which is what most people expect, and is how correlation is usually defined.



## scipy.stats

Many functions in `scipy.stats` that are either available from `numpy` or have been superseded, and have been deprecated since version 0.7, have been removed: `std`, `var`, `mean`, `median`, `cov`, `corrcoef`, `z`, `zs`, `stderr`, `samplestd`, `samplevar`, `pdfapprox`, `pdf_moments` and `erfc`. These changes are mirrored in `scipy.stats.mstats`.

## scipy.sparse

Several methods of the sparse matrix classes in `scipy.sparse` which had been deprecated since version 0.7 were removed: `save`, `rowcol`, `getdata`, `listprint`, `ensure_sorted_indices`, `matvec`, `matmat` and `rmatvec`.

The functions `spkron`, `speye`, `spidentity`, `lil_eye` and `lil_diags` were removed from `scipy.sparse`. The first three functions are still available as `scipy.sparse.kron`, `scipy.sparse.eye` and `scipy.sparse.identity`.

The `dims` and `nzmax` keywords were removed from the sparse matrix constructor. The `colind` and `rowind` attributes were removed from CSR and CSC matrices respectively.

## scipy.sparse.linalg.arpack.speigs

A duplicated interface to the ARPACK library was removed.

## 1.16.6 Other changes

### ARPACK interface changes

The interface to the ARPACK eigenvalue routines in `scipy.sparse.linalg` was changed for more robustness.

The eigenvalue and SVD routines now raise `ArpackNoConvergence` if the eigenvalue iteration fails to converge. If partially converged results are desired, they can be accessed as follows:

```
import numpy as np
from scipy.sparse.linalg import eigs, ArpackNoConvergence

m = np.random.randn(30, 30)
try:
    w, v = eigs(m, 6)
except ArpackNoConvergence, err:
    partially_converged_w = err.eigenvalues
    partially_converged_v = err.eigenvectors
```

Several bugs were also fixed.

The routines were moreover renamed as follows:

- `eigen` → `eigs`
- `eigen_symmetric` → `eigsh`
- `svd` → `svds`

## 1.17 SciPy 0.8.0 Release Notes

**Contents**

- SciPy 0.8.0 Release Notes
  - Python 3
  - Major documentation improvements
  - Deprecated features
    - \* Swapping inputs for correlation functions (`scipy.signal`)
    - \* Obsolete code deprecated (`scipy.misc`)
    - \* Additional deprecations
  - New features
    - \* DCT support (`scipy.fftpack`)
    - \* Single precision support for fft functions (`scipy.fftpack`)
    - \* Correlation functions now implement the usual definition (`scipy.signal`)
    - \* Additions and modification to LTI functions (`scipy.signal`)
    - \* Improved waveform generators (`scipy.signal`)
    - \* New functions and other changes in `scipy.linalg`
    - \* New function and changes in `scipy.optimize`
    - \* New sparse least squares solver
    - \* ARPACK-based sparse SVD
    - \* Alternative behavior available for `scipy.constants.find`
    - \* Incomplete sparse LU decompositions
    - \* Faster matlab file reader and default behavior change
    - \* Faster evaluation of orthogonal polynomials
    - \* Lambert W function
    - \* Improved hypergeometric 2F1 function
    - \* More flexible interface for Radial basis function interpolation
  - Removed features
    - \* `scipy.io`

SciPy 0.8.0 is the culmination of 17 months of hard work. It contains many new features, numerous bug-fixes, improved test coverage and better documentation. There have been a number of deprecations and API changes in this release, which are documented below. All users are encouraged to upgrade to this release, as there are a large number of bug-fixes and optimizations. Moreover, our development attention will now shift to bug-fix releases on the 0.8.x branch, and on adding new features on the development trunk. This release requires Python 2.4 - 2.6 and NumPy 1.4.1 or greater.

Please note that SciPy is still considered to have “Beta” status, as we work toward a SciPy 1.0.0 release. The 1.0.0 release will mark a major milestone in the development of SciPy, after which changing the package structure or API will be much more difficult. Whilst these pre-1.0 releases are considered to have “Beta” status, we are committed to making them as bug-free as possible.

However, until the 1.0 release, we are aggressively reviewing and refining the functionality, organization, and interface. This is being done in an effort to make the package as coherent, intuitive, and useful as possible. To achieve this, we need help from the community of users. Specifically, we need feedback regarding all aspects of the project - everything - from which algorithms we implement, to details about our function’s call signatures.

### 1.17.1 Python 3

Python 3 compatibility is planned and is currently technically feasible, since Numpy has been ported. However, since the Python 3 compatible Numpy 1.5 has not been released yet, support for Python 3 in Scipy is not yet included in Scipy 0.8. SciPy 0.9, planned for fall 2010, will very likely include experimental support for Python 3.

## 1.17.2 Major documentation improvements

SciPy documentation is greatly improved.

## 1.17.3 Deprecated features

### Swapping inputs for correlation functions (`scipy.signal`)

Concern `correlate`, `correlate2d`, `convolve` and `convolve2d`. If the second input is larger than the first input, the inputs are swapped before calling the underlying computation routine. This behavior is deprecated, and will be removed in `scipy` 0.9.0.

### Obsolete code deprecated (`scipy.misc`)

The modules `helpmod`, `ppimport` and `pexec` from `scipy.misc` are deprecated. They will be removed from SciPy in version 0.9.

### Additional deprecations

- `linalg`: The function `solveh_banded` currently returns a tuple containing the Cholesky factorization and the solution to the linear system. In SciPy 0.9, the return value will be just the solution.
- The function `constants.codata.find` will generate a `DeprecationWarning`. In SciPy version 0.8.0, the keyword argument `'disp'` was added to the function, with the default value `'True'`. In 0.9.0, the default will be `'False'`.
- The `qshape` keyword argument of `signal.chirp` is deprecated. Use the argument `vertex_zero` instead.
- Passing the coefficients of a polynomial as the argument `f0` to `signal.chirp` is deprecated. Use the function `signal.sweep_poly` instead.
- The `io.recaster` module has been deprecated and will be removed in 0.9.0.

## 1.17.4 New features

### DCT support (`scipy.fftpack`)

New realtransforms have been added, namely `dct` and `idct` for Discrete Cosine Transform; type I, II and III are available.

### Single precision support for fft functions (`scipy.fftpack`)

fft functions can now handle single precision inputs as well: `fft(x)` will return a single precision array if `x` is single precision.

At the moment, for FFT sizes that are not composites of 2, 3, and 5, the transform is computed internally in double precision to avoid rounding error in FFTPACK.

### Correlation functions now implement the usual definition (`scipy.signal`)

The outputs should now correspond to their matlab and R counterparts, and do what most people expect if the `old_behavior=False` argument is passed:

- `correlate`, `convolve` and their 2d counterparts do not swap their inputs depending on their relative shape anymore;

- correlation functions now conjugate their second argument while computing the slided sum-products, which correspond to the usual definition of correlation.

### Additions and modification to LTI functions (`scipy.signal`)

- The functions `impulse2` and `step2` were added to `scipy.signal`. They use the function `scipy.signal.lsim2` to compute the impulse and step response of a system, respectively.
- The function `scipy.signal.lsim2` was changed to pass any additional keyword arguments to the ODE solver.

### Improved waveform generators (`scipy.signal`)

Several improvements to the `chirp` function in `scipy.signal` were made:

- The waveform generated when `method="logarithmic"` was corrected; it now generates a waveform that is also known as an “exponential” or “geometric” chirp. (See <http://en.wikipedia.org/wiki/Chirp>.)
- A new `chirp` method, “hyperbolic”, was added.
- Instead of the keyword `qshape`, `chirp` now uses the keyword `vertex_zero`, a boolean.
- `chirp` no longer handles an arbitrary polynomial. This functionality has been moved to a new function, `sweep_poly`.

A new function, `sweep_poly`, was added.

### New functions and other changes in `scipy.linalg`

The functions `cho_solve_banded`, `circulant`, `companion`, `hadamard` and `leslie` were added to `scipy.linalg`.

The function `block_diag` was enhanced to accept scalar and 1D arguments, along with the usual 2D arguments.

### New function and changes in `scipy.optimize`

The `curve_fit` function has been added; it takes a function and uses non-linear least squares to fit that to the provided data.

The `leastsq` and `fsolve` functions now return an array of size one instead of a scalar when solving for a single parameter.

### New sparse least squares solver

The `lsqr` function was added to `scipy.sparse`. This routine finds a least-squares solution to a large, sparse, linear system of equations.

### ARPACK-based sparse SVD

A naive implementation of SVD for sparse matrices is available in `scipy.sparse.linalg.eigen.arpack`. It is based on using an symmetric solver on  $\langle A, A \rangle$ , and as such may not be very precise.

### Alternative behavior available for `scipy.constants.find`

The keyword argument `disp` was added to the function `scipy.constants.find`, with the default value `True`. When `disp` is `True`, the behavior is the same as in SciPy version 0.7. When `False`, the function returns the list of keys instead of printing them. (In SciPy version 0.9, the default will be reversed.)

### Incomplete sparse LU decompositions

SciPy now wraps SuperLU version 4.0, which supports incomplete sparse LU decompositions. These can be accessed via `scipy.sparse.linalg.spilu`. Upgrade to SuperLU 4.0 also fixes some known bugs.

### Faster matlab file reader and default behavior change

We've rewritten the matlab file reader in Cython and it should now read matlab files at around the same speed that Matlab does.

The reader reads matlab named and anonymous functions, but it can't write them.

Until scipy 0.8.0 we have returned arrays of matlab structs as numpy object arrays, where the objects have attributes named for the struct fields. As of 0.8.0, we return matlab structs as numpy structured arrays. You can get the older behavior by using the optional `struct_as_record=False` keyword argument to `scipy.io.loadmat` and friends.

There is an inconsistency in the matlab file writer, in that it writes numpy 1D arrays as column vectors in matlab 5 files, and row vectors in matlab 4 files. We will change this in the next version, so both write row vectors. There is a `FutureWarning` when calling the writer to warn of this change; for now we suggest using the `oned_as='row'` keyword argument to `scipy.io.savemat` and friends.

### Faster evaluation of orthogonal polynomials

Values of orthogonal polynomials can be evaluated with new vectorized functions in `scipy.special`: `eval_legendre`, `eval_chebyt`, `eval_chebyu`, `eval_chebyc`, `eval_chebys`, `eval_jacobi`, `eval_laguerre`, `eval_genlaguerre`, `eval_hermite`, `eval_hermitenorm`, `eval_gegenbauer`, `eval_sh_legendre`, `eval_sh_chebyt`, `eval_sh_chebyu`, `eval_sh_jacobi`. This is faster than constructing the full coefficient representation of the polynomials, which was previously the only available way.

Note that the previous orthogonal polynomial routines will now also invoke this feature, when possible.

### Lambert W function

`scipy.special.lambertw` can now be used for evaluating the Lambert W function.

### Improved hypergeometric 2F1 function

Implementation of `scipy.special.hyp2f1` for real parameters was revised. The new version should produce accurate values for all real parameters.

### More flexible interface for Radial basis function interpolation

The `scipy.interpolate.Rbf` class now accepts a callable as input for the "function" argument, in addition to the built-in radial basis functions which can be selected with a string argument.

### 1.17.5 Removed features

scipy.stsci: the package was removed

The module *scipy.misc.limits* was removed.

The IO code in both NumPy and SciPy is being extensively reworked. NumPy will be where basic code for reading and writing NumPy arrays is located, while SciPy will house file readers and writers for various data formats (data, audio, video, images, matlab, etc.).

Several functions in `scipy.io` are removed in the 0.8.0 release including: *npfile*, *save*, *load*, *create\_module*, *create\_shelf*, *objload*, *objsave*, *fopen*, *read\_array*, *write\_array*, *fread*, *fwrite*, *bswap*, *packbits*, *unpackbits*, and *convert\_objectarray*. Some of these functions have been replaced by NumPy's raw reading and writing capabilities, memory-mapping capabilities, or array methods. Others have been moved from SciPy to NumPy, since basic array reading and writing capability is now handled by NumPy.

## 1.18 SciPy 0.7.2 Release Notes

### Contents

- [SciPy 0.7.2 Release Notes](#)

SciPy 0.7.2 is a bug-fix release with no new features compared to 0.7.1. The only change is that all C sources from Cython code have been regenerated with Cython 0.12.1. This fixes the incompatibility between binaries of SciPy 0.7.1 and NumPy 1.4.

## 1.19 SciPy 0.7.1 Release Notes

### Contents

- [SciPy 0.7.1 Release Notes](#)
  - [scipy.io](#)
  - [scipy.odr](#)
  - [scipy.signal](#)
  - [scipy.sparse](#)
  - [scipy.special](#)
  - [scipy.stats](#)
  - [Windows binaries for python 2.6](#)
  - [Universal build for scipy](#)

SciPy 0.7.1 is a bug-fix release with no new features compared to 0.7.0.

Bugs fixed:

- Several fixes in Matlab file IO

Bugs fixed:

- Work around a failure with Python 2.6

Memory leak in `lfilter` have been fixed, as well as support for array object

Bugs fixed:

- #880, #925: lfilter fixes
- #871: bicgstab fails on Win32

Bugs fixed:

- #883: `scipy.io.mmread` with `scipy.sparse.lil_matrix` broken
- `lil_matrix` and `csc_matrix` reject now unexpected sequences, cf. <http://thread.gmane.org/gmane.comp.python.scientific.user/19996>

Several bugs of varying severity were fixed in the special functions:

- #503, #640: `iv`: problems at large arguments fixed by new implementation
- #623: `jv`: fix errors at large arguments
- #679: `struve`: fix wrong output for  $v < 0$
- #803: `pbdv` produces invalid output
- #804: `lqmn`: fix crashes on some input
- #823: `betainc`: fix documentation
- #834: `exp1` strange behavior near negative integer values
- #852: `jn_zeros`: more accurate results for large  $s$ , also in `jnp/yn/ynp_zeros`
- #853: `jv`, `yv`, `iv`: invalid results for non-integer  $v < 0$ , complex  $x$
- #854: `jv`, `yv`, `iv`, : return nan more consistently when out-of-domain
- #927: `ellipj`: fix segfault on Windows
- #946: `ellpj`: fix segfault on Mac OS X/python 2.6 combination.
- `ive`, `jve`, `yve`, `kv`, `kve`: with real-valued input, return nan for out-of-domain instead of returning only the real part of the result.

Also, when `scipy.special.errprint(1)` has been enabled, warning messages are now issued as Python warnings instead of printing them to `stderr`.

- `linregress`, `mannwhitneyu`, `describe`: errors fixed
- `kstwobign`, `norm`, `expon`, `exponweib`, `exponpow`, `frechet`, `genexpon`, `rdist`, `truncexpon`, `planck`: improvements to numerical accuracy in distributions

### 1.19.1 Windows binaries for python 2.6

python 2.6 binaries for windows are now included. The binary for python 2.5 requires numpy 1.2.0 or above, and the one for python 2.6 requires numpy 1.3.0 or above.

### 1.19.2 Universal build for scipy

Mac OS X binary installer is now a proper universal build, and does not depend on gfortran anymore (libgfortran is statically linked). The python 2.5 version of scipy requires numpy 1.2.0 or above, the python 2.6 version requires numpy 1.3.0 or above.

## 1.20 SciPy 0.7.0 Release Notes

### Contents

- SciPy 0.7.0 Release Notes
  - Python 2.6 and 3.0
  - Major documentation improvements
  - Running Tests
  - Building SciPy
  - Sandbox Removed
  - Sparse Matrices
  - Statistics package
  - Reworking of IO package
  - New Hierarchical Clustering module
  - New Spatial package
  - Reworked fftpack package
  - New Constants package
  - New Radial Basis Function module
  - New complex ODE integrator
  - New generalized symmetric and hermitian eigenvalue problem solver
  - Bug fixes in the interpolation package
  - Weave clean up
  - Known problems

SciPy 0.7.0 is the culmination of 16 months of hard work. It contains many new features, numerous bug-fixes, improved test coverage and better documentation. There have been a number of deprecations and API changes in this release, which are documented below. All users are encouraged to upgrade to this release, as there are a large number of bug-fixes and optimizations. Moreover, our development attention will now shift to bug-fix releases on the 0.7.x branch, and on adding new features on the development trunk. This release requires Python 2.4 or 2.5 and NumPy 1.2 or greater.

Please note that SciPy is still considered to have “Beta” status, as we work toward a SciPy 1.0.0 release. The 1.0.0 release will mark a major milestone in the development of SciPy, after which changing the package structure or API will be much more difficult. Whilst these pre-1.0 releases are considered to have “Beta” status, we are committed to making them as bug-free as possible. For example, in addition to fixing numerous bugs in this release, we have also doubled the number of unit tests since the last release.

However, until the 1.0 release, we are aggressively reviewing and refining the functionality, organization, and interface. This is being done in an effort to make the package as coherent, intuitive, and useful as possible. To achieve this, we need help from the community of users. Specifically, we need feedback regarding all aspects of the project - everything - from which algorithms we implement, to details about our function’s call signatures.

Over the last year, we have seen a rapid increase in community involvement, and numerous infrastructure improvements to lower the barrier to contributions (e.g., more explicit coding standards, improved testing infrastructure, better documentation tools). Over the next year, we hope to see this trend continue and invite everyone to become more involved.

### 1.20.1 Python 2.6 and 3.0

A significant amount of work has gone into making SciPy compatible with Python 2.6; however, there are still some issues in this regard. The main issue with 2.6 support is NumPy. On UNIX (including Mac OS X), NumPy 1.2.1 mostly works, with a few caveats. On Windows, there are problems related to the compilation process. The upcoming



NumPy 1.3 release will fix these problems. Any remaining issues with 2.6 support for SciPy 0.7 will be addressed in a bug-fix release.

Python 3.0 is not supported at all; it requires NumPy to be ported to Python 3.0. This requires immense effort, since a lot of C code has to be ported. The transition to 3.0 is still under consideration; currently, we don't have any timeline or roadmap for this transition.

## 1.20.2 Major documentation improvements

SciPy documentation is greatly improved; you can view a HTML reference manual [online](#) or download it as a PDF file. The new reference guide was built using the popular [Sphinx tool](#).

This release also includes an updated tutorial, which hadn't been available since SciPy was ported to NumPy in 2005. Though not comprehensive, the tutorial shows how to use several essential parts of Scipy. It also includes the `ndimage` documentation from the `numarray` manual.

Nevertheless, more effort is needed on the documentation front. Luckily, contributing to Scipy documentation is now easier than before: if you find that a part of it requires improvements, and want to help us out, please register a user name in our web-based documentation editor at <https://docs.scipy.org/> and correct the issues.

## 1.20.3 Running Tests

NumPy 1.2 introduced a new testing framework based on `nose`. Starting with this release, SciPy now uses the new NumPy test framework as well. Taking advantage of the new testing framework requires `nose` version 0.10, or later. One major advantage of the new framework is that it greatly simplifies writing unit tests - which has all ready paid off, given the rapid increase in tests. To run the full test suite:

```
>>> import scipy
>>> scipy.test('full')
```

For more information, please see [The NumPy/SciPy Testing Guide](#).

We have also greatly improved our test coverage. There were just over 2,000 unit tests in the 0.6.0 release; this release nearly doubles that number, with just over 4,000 unit tests.

## 1.20.4 Building SciPy

Support for NumScons has been added. NumScons is a tentative new build system for NumPy/SciPy, using `SCons` at its core.

`SCons` is a next-generation build system, intended to replace the venerable `Make` with the integrated functionality of `autoconf/automake` and `ccache`. `Scons` is written in Python and its configuration files are Python scripts. `NumScons` is meant to replace NumPy's custom version of `distutils` providing more advanced functionality, such as `autoconf`, improved fortran support, more tools, and support for `numpy.distutils/scons` cooperation.

## 1.20.5 Sandbox Removed

While porting SciPy to NumPy in 2005, several packages and modules were moved into `scipy.sandbox`. The sandbox was a staging ground for packages that were undergoing rapid development and whose APIs were in flux. It was also a place where broken code could live. The sandbox has served its purpose well, but was starting to create confusion. Thus `scipy.sandbox` was removed. Most of the code was moved into `scipy`, some code was made into a `scikit`, and the remaining code was just deleted, as the functionality had been replaced by other code.

## 1.20.6 Sparse Matrices

Sparse matrices have seen extensive improvements. There is now support for integer dtypes such `int8`, `uint32`, etc. Two new sparse formats were added:

- new class `dia_matrix`: the sparse DIagonal format
- new class `bsr_matrix`: the Block CSR format

Several new sparse matrix construction functions were added:

- `sparse.kron`: sparse Kronecker product
- `sparse.bmat`: sparse version of `numpy.bmat`
- `sparse.vstack`: sparse version of `numpy.vstack`
- `sparse.hstack`: sparse version of `numpy.hstack`

Extraction of submatrices and nonzero values have been added:

- `sparse.tril`: extract lower triangle
- `sparse.triu`: extract upper triangle
- `sparse.find`: nonzero values and their indices

`csr_matrix` and `csc_matrix` now support slicing and fancy indexing (e.g., `A[1:3, 4:7]` and `A[[3, 2, 6, 8], :]`). Conversions among all sparse formats are now possible:

- using member functions such as `.tocsr()` and `.tolil()`
- using the `.asformat()` member function, e.g. `A.asformat('csr')`
- using constructors `A = lil_matrix([[1,2]]); B = csr_matrix(A)`

All sparse constructors now accept dense matrices and lists of lists. For example:

- `A = csr_matrix( rand(3,3) )` and `B = lil_matrix( [[1,2], [3,4]] )`

The handling of diagonals in the `spdiags` function has been changed. It now agrees with the MATLAB(TM) function of the same name.

Numerous efficiency improvements to format conversions and sparse matrix arithmetic have been made. Finally, this release contains numerous bugfixes.

## 1.20.7 Statistics package

Statistical functions for masked arrays have been added, and are accessible through `scipy.stats.mstats`. The functions are similar to their counterparts in `scipy.stats` but they have not yet been verified for identical interfaces and algorithms.

Several bugs were fixed for statistical functions, of those, `kstest` and `percentileofscore` gained new keyword arguments.

Added deprecation warning for `mean`, `median`, `var`, `std`, `cov`, and `corrcoef`. These functions should be replaced by their numpy counterparts. Note, however, that some of the default options differ between the `scipy.stats` and numpy versions of these functions.

Numerous bug fixes to `stats.distributions`: all generic methods now work correctly, several methods in individual distributions were corrected. However, a few issues remain with higher moments (`skew`, `kurtosis`) and entropy. The maximum likelihood estimator, `fit`, does not work out-of-the-box for some distributions - in some cases, starting values have to be carefully chosen, in other cases, the generic implementation of the maximum likelihood method might not be the numerically appropriate estimation method.

We expect more bugfixes, increases in numerical precision and enhancements in the next release of scipy.

### 1.20.8 Reworking of IO package

The IO code in both NumPy and SciPy is being extensively reworked. NumPy will be where basic code for reading and writing NumPy arrays is located, while SciPy will house file readers and writers for various data formats (data, audio, video, images, matlab, etc.).

Several functions in `scipy.io` have been deprecated and will be removed in the 0.8.0 release including `npfile`, `save`, `load`, `create_module`, `create_shelf`, `objload`, `objsave`, `fopen`, `read_array`, `write_array`, `fread`, `fwrite`, `bswap`, `packbits`, `unpackbits`, and `convert_objectarray`. Some of these functions have been replaced by NumPy's raw reading and writing capabilities, memory-mapping capabilities, or array methods. Others have been moved from SciPy to NumPy, since basic array reading and writing capability is now handled by NumPy.

The Matlab (TM) file readers/writers have a number of improvements:

- default version 5
- v5 writers for structures, cell arrays, and objects
- v5 readers/writers for function handles and 64-bit integers
- new `struct_as_record` keyword argument to `loadmat`, which loads struct arrays in matlab as record arrays in numpy
- string arrays have `dtype='U...'` instead of `dtype=object`
- `loadmat` no longer squeezes singleton dimensions, i.e. `squeeze_me=False` by default

### 1.20.9 New Hierarchical Clustering module

This module adds new hierarchical clustering functionality to the `scipy.cluster` package. The function interfaces are similar to the functions provided MATLAB(TM)'s Statistics Toolbox to help facilitate easier migration to the NumPy/SciPy framework. Linkage methods implemented include single, complete, average, weighted, centroid, median, and ward.

In addition, several functions are provided for computing inconsistency statistics, cophenetic distance, and maximum distance between descendants. The `fcluster` and `fclusterdata` functions transform a hierarchical clustering into a set of flat clusters. Since these flat clusters are generated by cutting the tree into a forest of trees, the `leaders` function takes a linkage and a flat clustering, and finds the root of each tree in the forest. The `ClusterNode` class represents a hierarchical clusterings as a field-navigable tree object. `to_tree` converts a matrix-encoded hierarchical clustering to a `ClusterNode` object. Routines for converting between MATLAB and SciPy linkage encodings are provided. Finally, a `dendrogram` function plots hierarchical clusterings as a dendrogram, using `matplotlib`.

### 1.20.10 New Spatial package

The new spatial package contains a collection of spatial algorithms and data structures, useful for spatial statistics and clustering applications. It includes rapidly compiled code for computing exact and approximate nearest neighbors, as well as a pure-python kd-tree with the same interface, but that supports annotation and a variety of other algorithms. The API for both modules may change somewhat, as user requirements become clearer.

It also includes a `distance` module, containing a collection of distance and dissimilarity functions for computing distances between vectors, which is useful for spatial statistics, clustering, and kd-trees. Distance and dissimilarity functions provided include Bray-Curtis, Canberra, Chebyshev, City Block, Cosine, Dice, Euclidean, Hamming,

Jaccard, Kulsinski, Mahalanobis, Matching, Minkowski, Rogers-Tanimoto, Russell-Rao, Squared Euclidean, Standardized Euclidean, Sokal-Michener, Sokal-Sneath, and Yule.

The `pdist` function computes pairwise distance between all unordered pairs of vectors in a set of vectors. The `cdist` computes the distance on all pairs of vectors in the Cartesian product of two sets of vectors. Pairwise distance matrices are stored in condensed form; only the upper triangular is stored. `squareform` converts distance matrices between square and condensed forms.

### 1.20.11 Reworked `fftpack` package

FFTW2, FFTW3, MKL and DJBFFT wrappers have been removed. Only (NETLIB) `fftpack` remains. By focusing on one backend, we hope to add new features - like float32 support - more easily.

### 1.20.12 New Constants package

`scipy.constants` provides a collection of physical constants and conversion factors. These constants are taken from CODATA Recommended Values of the Fundamental Physical Constants: 2002. They may be found at [physics.nist.gov/constants](http://physics.nist.gov/constants). The values are stored in the dictionary `physical_constants` as a tuple containing the value, the units, and the relative precision - in that order. All constants are in SI units, unless otherwise stated. Several helper functions are provided.

### 1.20.13 New Radial Basis Function module

`scipy.interpolate` now contains a Radial Basis Function module. Radial basis functions can be used for smoothing/interpolating scattered data in n-dimensions, but should be used with caution for extrapolation outside of the observed data range.

### 1.20.14 New complex ODE integrator

`scipy.integrate.ode` now contains a wrapper for the ZVODE complex-valued ordinary differential equation solver (by Peter N. Brown, Alan C. Hindmarsh, and George D. Byrne).

### 1.20.15 New generalized symmetric and hermitian eigenvalue problem solver

`scipy.linalg.eigh` now contains wrappers for more LAPACK symmetric and hermitian eigenvalue problem solvers. Users can now solve generalized problems, select a range of eigenvalues only, and choose to use a faster algorithm at the expense of increased memory usage. The signature of the `scipy.linalg.eigh` changed accordingly.

### 1.20.16 Bug fixes in the interpolation package

The shape of return values from `scipy.interpolate.interpld` used to be incorrect, if interpolated data had more than 2 dimensions and the axis keyword was set to a non-default value. This has been fixed. Moreover, `interpld` returns now a scalar (0D-array) if the input is a scalar. Users of `scipy.interpolate.interpld` may need to revise their code if it relies on the previous behavior.

### 1.20.17 Weave clean up

There were numerous improvements to `scipy.weave`. `blitz++` was relicensed by the author to be compatible with the SciPy license. `wx_spec.py` was removed.

### 1.20.18 Known problems

Here are known problems with scipy 0.7.0:

- weave test failures on windows: those are known, and are being revised.
- weave test failure with gcc 4.3 (std::labs): this is a gcc 4.3 bug. A workaround is to add `#include <cstdlib>` in `scipy/weave/blitz/blitz/funcs.h` (line 27). You can make the change in the installed scipy (in site-packages).



## API - IMPORTING FROM SCIPY

In Python the distinction between what is the public API of a library and what are private implementation details is not always clear. Unlike in other languages like Java, it is possible in Python to access “private” function or objects. Occasionally this may be convenient, but be aware that if you do so your code may break without warning in future releases. Some widely understood rules for what is and isn’t public in Python are:

- Methods / functions / classes and module attributes whose names begin with a leading underscore are private.
- If a class name begins with a leading underscore none of its members are public, whether or not they begin with a leading underscore.
- If a module name in a package begins with a leading underscore none of its members are public, whether or not they begin with a leading underscore.
- If a module or package defines `__all__` that authoritatively defines the public interface.
- If a module or package doesn’t define `__all__` then all names that don’t start with a leading underscore are public.

---

**Note:** Reading the above guidelines one could draw the conclusion that every private module or object starts with an underscore. This is not the case; the presence of underscores do mark something as private, but the absence of underscores do not mark something as public.

---

In Scipy there are modules whose names don’t start with an underscore, but that should be considered private. To clarify which modules these are we define below what the public API is for Scipy, and give some recommendations for how to import modules/functions/objects from Scipy.

### 2.1 Guidelines for importing functions from Scipy

The `scipy` namespace itself only contains functions imported from `numpy`. These functions still exist for backwards compatibility, but should be imported from `numpy` directly.

Everything in the namespaces of `scipy` submodules is public. In general, it is recommended to import functions from submodule namespaces. For example, the function `curve_fit` (defined in `scipy/optimize/minpack.py`) should be imported like this:

```
from scipy import optimize
result = optimize.curve_fit(...)
```

This form of importing submodules is preferred for all submodules except `scipy.io` (because `io` is also the name of a module in the Python stdlib):

```
from scipy import interpolate
from scipy import integrate
import scipy.io as spio
```

In some cases, the public API is one level deeper. For example the `scipy.sparse.linalg` module is public, and the functions it contains are not available in the `scipy.sparse` namespace. Sometimes it may result in more easily understandable code if functions are imported from one level deeper. For example, in the following it is immediately clear that `lomax` is a distribution if the second form is chosen:

```
# first form
from scipy import stats
stats.lomax(...)

# second form
from scipy.stats import distributions
distributions.lomax(...)
```

In that case the second form can be chosen, **if** it is documented in the next section that the submodule in question is public.

## 2.2 API definition

Every submodule listed below is public. That means that these submodules are unlikely to be renamed or changed in an incompatible way, and if that is necessary a deprecation warning will be raised for one SciPy release before the change is made.

- `scipy.cluster`
  - `vq`
  - `hierarchy`
- `scipy.constants`
- `scipy.fftpack`
- `scipy.integrate`
- `scipy.interpolate`
- `scipy.io`
  - `arff`
  - `harwell_boeing`
  - `idl`
  - `matlab`
  - `netcdf`
  - `wavfile`
- `scipy.linalg`
  - `scipy.linalg.blas`
  - `scipy.linalg.cython_blas`
  - `scipy.linalg.lapack`
  - `scipy.linalg.cython_lapack`



- `scipy.linalg.interpolative`
- `scipy.misc`
- `scipy.ndimage`
- `scipy.odr`
- `scipy.optimize`
- `scipy.signal`
- `scipy.sparse`
  - `linalg`
  - `csgraph`
- `scipy.spatial`
  - `distance`
- `scipy.special`
- `scipy.stats`
  - `distributions`
  - `mstats`
- `scipy.weave` (deprecated)



Tutorials with worked examples and background information for most SciPy submodules.

## 3.1 SciPy Tutorial

### 3.1.1 Introduction

#### Contents

- Introduction
  - SciPy Organization
  - Finding Documentation

SciPy is a collection of mathematical algorithms and convenience functions built on the Numpy extension of Python. It adds significant power to the interactive Python session by providing the user with high-level commands and classes for manipulating and visualizing data. With SciPy an interactive Python session becomes a data-processing and system-prototyping environment rivaling systems such as MATLAB, IDL, Octave, R-Lab, and SciLab.

The additional benefit of basing SciPy on Python is that this also makes a powerful programming language available for use in developing sophisticated programs and specialized applications. Scientific applications using SciPy benefit from the development of additional modules in numerous niches of the software landscape by developers across the world. Everything from parallel programming to web and data-base subroutines and classes have been made available to the Python programmer. All of this power is available in addition to the mathematical libraries in SciPy.

This tutorial will acquaint the first-time user of SciPy with some of its most important features. It assumes that the user has already installed the SciPy package. Some general Python facility is also assumed, such as could be acquired by working through the Python distribution's Tutorial. For further introductory help the user is directed to the Numpy documentation.

For brevity and convenience, we will often assume that the main packages (numpy, scipy, and matplotlib) have been imported as:

```
>>> import numpy as np
>>> import matplotlib as mpl
>>> import matplotlib.pyplot as plt
```

These are the import conventions that our community has adopted after discussion on public mailing lists. You will see these conventions used throughout NumPy and SciPy source code and documentation. While we obviously don't require you to follow these conventions in your own code, it is highly recommended.

## SciPy Organization

SciPy is organized into subpackages covering different scientific computing domains. These are summarized in the following table:

Subpackage	Description
cluster	Clustering algorithms
constants	Physical and mathematical constants
fftpack	Fast Fourier Transform routines
integrate	Integration and ordinary differential equation solvers
interpolate	Interpolation and smoothing splines
io	Input and Output
linalg	Linear algebra
ndimage	N-dimensional image processing
odr	Orthogonal distance regression
optimize	Optimization and root-finding routines
signal	Signal processing
sparse	Sparse matrices and associated routines
spatial	Spatial data structures and algorithms
special	Special functions
stats	Statistical distributions and functions
weave	C/C++ integration

Scipy sub-packages need to be imported separately, for example:

```
>>> from scipy import linalg, optimize
```

Because of their ubiquitousness, some of the functions in these subpackages are also made available in the `scipy` namespace to ease their use in interactive sessions and programs. In addition, many basic array functions from `numpy` are also available at the top-level of the `scipy` package. Before looking at the sub-packages individually, we will first look at some of these common functions.

## Finding Documentation

SciPy and NumPy have documentation versions in both HTML and PDF format available at <https://docs.scipy.org/>, that cover nearly all available functionality. However, this documentation is still work-in-progress and some parts may be incomplete or sparse. As we are a volunteer organization and depend on the community for growth, your participation - everything from providing feedback to improving the documentation and code - is welcome and actively encouraged.

Python's documentation strings are used in SciPy for on-line documentation. There are two methods for reading them and getting help. One is Python's command `help` in the `pydoc` module. Entering this command with no arguments (i.e. `>>> help`) launches an interactive help session that allows searching through the keywords and modules available to all of Python. Secondly, running the command `help(obj)` with an object as the argument displays that object's calling signature, and documentation string.

The `pydoc` method of `help` is sophisticated but uses a pager to display the text. Sometimes this can interfere with the terminal you are running the interactive session within. A `numpy/scipy`-specific help system is also available under the command `numpy.info`. The signature and documentation string for the object passed to the `help` command are printed to standard output (or to a writeable object passed as the third argument). The second keyword argument of `numpy.info` defines the maximum width of the line for printing. If a module is passed as the argument to `help` than a list of the functions and classes defined in that module is printed. For example:

```
>>> np.info(optimize.fmin)
fmin(func, x0, args=(), xtol=0.0001, ftol=0.0001, maxiter=None, maxfun=None,
      full_output=0, disp=1, retall=0, callback=None)
```

Minimize a function using the downhill simplex algorithm.

#### Parameters

-----

`func` : callable `func(x,*args)`  
 The objective function to be minimized.

`x0` : ndarray  
 Initial guess.

`args` : tuple  
 Extra arguments passed to `func`, i.e. ```f(x,*args)```.

`callback` : callable  
 Called after each iteration, as `callback(xk)`, where `xk` is the current parameter vector.

#### Returns

-----

`xopt` : ndarray  
 Parameter that minimizes function.

`fopt` : float  
 Value of function at minimum: ```fopt = func(xopt)```.

`iter` : int  
 Number of iterations performed.

`funcalls` : int  
 Number of function calls made.

`warnflag` : int  
 1 : Maximum number of function evaluations made.  
 2 : Maximum number of iterations reached.

`allvecs` : list  
 Solution at each iteration.

#### Other parameters

-----

`xtol` : float  
 Relative error in `xopt` acceptable for convergence.

`ftol` : number  
 Relative error in `func(xopt)` acceptable for convergence.

`maxiter` : int  
 Maximum number of iterations to perform.

`maxfun` : number  
 Maximum number of function evaluations to make.

`full_output` : bool  
 Set to True if `fopt` and `warnflag` outputs are desired.

`disp` : bool  
 Set to True to print convergence messages.

`retall` : bool  
 Set to True to return list of solutions at each iteration.

#### Notes

-----

Uses a Nelder-Mead simplex algorithm to find the minimum of function of one or more variables.

Another useful command is `source`. When given a function written in Python as an argument, it prints out a listing of the source code for that function. This can be helpful in learning about an algorithm or understanding exactly what a function is doing with its arguments. Also don't forget about the Python command `dir` which can be used to look at the namespace of a module or package.

## 3.1.2 Basic functions

### Contents

- Basic functions
  - Interaction with Numpy
    - \* Index Tricks
    - \* Shape manipulation
    - \* Polynomials
    - \* Vectorizing functions (vectorize)
    - \* Type handling
    - \* Other useful functions

### Interaction with Numpy

Scipy builds on Numpy, and for all basic array handling needs you can use Numpy functions:

```
>>> import numpy as np
>>> np.some_function()
```

Rather than giving a detailed description of each of these functions (which is available in the Numpy Reference Guide or by using the `help`, `info` and `source` commands), this tutorial will discuss some of the more useful commands which require a little introduction to use to their full potential.

To use functions from some of the Scipy modules, you can do:

```
>>> from scipy import some_module
>>> some_module.some_function()
```

The top level of `scipy` also contains functions from `numpy` and `numpy.lib.scimath`. However, it is better to use them directly from the `numpy` module instead.

### *Index Tricks*

There are some class instances that make special use of the slicing functionality to provide efficient means for array construction. This part will discuss the operation of `np.mgrid`, `np.ogrid`, `np.r_`, and `np.c_` for quickly constructing arrays.

For example, rather than writing something like the following

```
>>> a = np.concatenate(([3], [0]*5, np.arange(-1, 1.002, 2/9.0)))
```

with the `r_` command one can enter this as

```
>>> a = np.r_[3, [0]*5, -1:1:10j]
```

which can ease typing and make for more readable code. Notice how objects are concatenated, and the slicing syntax is (ab)used to construct ranges. The other term that deserves a little explanation is the use of the complex number `10j` as the step size in the slicing syntax. This non-standard use allows the number to be interpreted as the number of points to produce in the range rather than as a step size (note we would have used the long integer notation, `10L`, but this notation may go away in Python as the integers become unified). This non-standard usage may be unsightly to some, but it gives the user the ability to quickly construct complicated vectors in a very readable fashion. When the number of points is specified in this way, the end- point is inclusive.

The “r” stands for row concatenation because if the objects between commas are 2 dimensional arrays, they are stacked by rows (and thus must have commensurate columns). There is an equivalent command `c_` that stacks 2d arrays by columns but works identically to `r_` for 1d arrays.

Another very useful class instance which makes use of extended slicing notation is the function `mgrid`. In the simplest case, this function can be used to construct 1d ranges as a convenient substitute for `arange`. It also allows the use of complex-numbers in the step-size to indicate the number of points to place between the (inclusive) end-points. The real purpose of this function however is to produce N, N-d arrays which provide coordinate arrays for an N-dimensional volume. The easiest way to understand this is with an example of its usage:

```
>>> np.mgrid[0:5,0:5]
array([[0, 0, 0, 0, 0],
       [1, 1, 1, 1, 1],
       [2, 2, 2, 2, 2],
       [3, 3, 3, 3, 3],
       [4, 4, 4, 4, 4]],
      [[0, 1, 2, 3, 4],
       [0, 1, 2, 3, 4],
       [0, 1, 2, 3, 4],
       [0, 1, 2, 3, 4],
       [0, 1, 2, 3, 4]])
>>> np.mgrid[0:5:4j,0:5:4j]
array([[ 0.    ,  0.    ,  0.    ,  0.    ],
       [ 1.6667,  1.6667,  1.6667,  1.6667],
       [ 3.3333,  3.3333,  3.3333,  3.3333],
       [ 5.    ,  5.    ,  5.    ,  5.    ]],
      [[ 0.    ,  1.6667,  3.3333,  5.    ],
       [ 0.    ,  1.6667,  3.3333,  5.    ],
       [ 0.    ,  1.6667,  3.3333,  5.    ],
       [ 0.    ,  1.6667,  3.3333,  5.    ]])
```

Having meshed arrays like this is sometimes very useful. However, it is not always needed just to evaluate some N-dimensional function over a grid due to the array-broadcasting rules of Numpy and SciPy. If this is the only purpose for generating a meshgrid, you should instead use the function `ogrid` which generates an “open” grid using `newaxis` judiciously to create N, N-d arrays where only one dimension in each array has length greater than 1. This will save memory and create the same result if the only purpose for the meshgrid is to generate sample points for evaluation of an N-d function.

### Shape manipulation

In this category of functions are routines for squeezing out length- one dimensions from N-dimensional arrays, ensuring that an array is at least 1-, 2-, or 3-dimensional, and stacking (concatenating) arrays by rows, columns, and “pages” (in the third dimension). Routines for splitting arrays (roughly the opposite of stacking arrays) are also available.

### Polynomials

There are two (interchangeable) ways to deal with 1-d polynomials in SciPy. The first is to use the `poly1d` class from Numpy. This class accepts coefficients or polynomial roots to initialize a polynomial. The polynomial object can then be manipulated in algebraic expressions, integrated, differentiated, and evaluated. It even prints like a polynomial:

```
>>> from numpy import poly1d
>>> p = poly1d([3,4,5])
>>> print p
      2
  3 x + 4 x + 5
>>> print p*p
      4      3      2
  9 x + 24 x + 46 x + 40 x + 25
>>> print p.integ(k=6)
      3      2
  1 x + 2 x + 5 x + 6
>>> print p.deriv()
  6 x + 4
```

```
>>> p([4, 5])
array([ 69, 100])
```

The other way to handle polynomials is as an array of coefficients with the first element of the array giving the coefficient of the highest power. There are explicit functions to add, subtract, multiply, divide, integrate, differentiate, and evaluate polynomials represented as sequences of coefficients.

### Vectorizing functions (*vectorize*)

One of the features that NumPy provides is a class `vectorize` to convert an ordinary Python function which accepts scalars and returns scalars into a “vectorized-function” with the same broadcasting rules as other Numpy functions (*i.e.* the Universal functions, or ufuncs). For example, suppose you have a Python function named `addsubtract` defined as:

```
>>> def addsubtract(a,b):
...     if a > b:
...         return a - b
...     else:
...         return a + b
```

which defines a function of two scalar variables and returns a scalar result. The class `vectorize` can be used to “vectorize” this function so that

```
>>> vec_addsubtract = np.vectorize(addsubtract)
```

returns a function which takes array arguments and returns an array result:

```
>>> vec_addsubtract([0,3,6,9],[1,3,5,7])
array([1, 6, 1, 2])
```

This particular function could have been written in vector form without the use of `vectorize`. But, what if the function you have written is the result of some optimization or integration routine. Such functions can likely only be vectorized using `vectorize`.

### Type handling

Note the difference between `np.iscomplex/np.isreal` and `np.iscomplexobj/np.isrealobj`. The former command is array based and returns byte arrays of ones and zeros providing the result of the element-wise test. The latter command is object based and returns a scalar describing the result of the test on the entire object.

Often it is required to get just the real and/or imaginary part of a complex number. While complex numbers and arrays have attributes that return those values, if one is not sure whether or not the object will be complex-valued, it is better to use the functional forms `np.real` and `np.imag`. These functions succeed for anything that can be turned into a Numpy array. Consider also the function `np.real_if_close` which transforms a complex-valued number with tiny imaginary part into a real number.

Occasionally the need to check whether or not a number is a scalar (Python (long)int, Python float, Python complex, or rank-0 array) occurs in coding. This functionality is provided in the convenient function `np.isscalar` which returns a 1 or a 0.

Finally, ensuring that objects are a certain Numpy type occurs often enough that it has been given a convenient interface in SciPy through the use of the `np.cast` dictionary. The dictionary is keyed by the type it is desired to cast to and the dictionary stores functions to perform the casting. Thus, `np.cast['f'](d)` returns an array of `np.float32` from `d`. This function is also useful as an easy way to get a scalar of a certain type:

```
>>> np.cast['f'](np.pi)
array(3.1415927410125732, dtype=float32)
```



### Other useful functions

There are also several other useful functions which should be mentioned. For doing phase processing, the functions `angle`, and `unwrap` are useful. Also, the `linspace` and `logspace` functions return equally spaced samples in a linear or log scale. Finally, it's useful to be aware of the indexing capabilities of Numpy. Mention should be made of the function `select` which extends the functionality of `where` to include multiple conditions and multiple choices. The calling convention is `select(condlist, choicelist, default=0)`. `select` is a vectorized form of the multiple if-statement. It allows rapid construction of a function which returns an array of results based on a list of conditions. Each element of the return array is taken from the array in a `choicelist` corresponding to the first condition in `condlist` that is true. For example

```
>>> x = np.r_[-2:3]
>>> x
array([-2, -1,  0,  1,  2])
>>> np.select([x > 3, x >= 0], [0, x+2])
array([0, 0, 2, 3, 4])
```

Some additional useful functions can also be found in the module `scipy.misc`. For example the `factorial` and `comb` functions compute  $n!$  and  $n!/k!(n-k)!$  using either exact integer arithmetic (thanks to Python's Long integer object), or by using floating-point precision and the gamma function. Another function returns a common image used in image processing: `lena`.

Finally, two functions are provided that are useful for approximating derivatives of functions using discrete-differences. The function `central_diff_weights` returns weighting coefficients for an equally-spaced  $N$ -point approximation to the derivative of order  $o$ . These weights must be multiplied by the function corresponding to these points and the results added to obtain the derivative approximation. This function is intended for use when only samples of the function are available. When the function is an object that can be handed to a routine and evaluated, the function `derivative` can be used to automatically evaluate the object at the correct points to obtain an  $N$ -point approximation to the  $o$ -th derivative at a given point.

### 3.1.3 Special functions (`scipy.special`)

The main feature of the `scipy.special` package is the definition of numerous special functions of mathematical physics. Available functions include `airy`, `elliptic`, `bessel`, `gamma`, `beta`, `hypergeometric`, `parabolic cylinder`, `mathieu`, `spheroidal wave`, `struve`, and `kelvin`. There are also some low-level stats functions that are not intended for general use as an easier interface to these functions is provided by the `stats` module. Most of these functions can take array arguments and return array results following the same broadcasting rules as other math functions in Numerical Python. Many of these functions also accept complex numbers as input. For a complete list of the available functions with a one-line description type `>>> help(special)`. Each function also has its own documentation accessible using `help`. If you don't see a function you need, consider writing it and contributing it to the library. You can write the function in either C, Fortran, or Python. Look in the source code of the library for examples of each of these kinds of functions.

#### Bessel functions of real order(`jn`, `jn_zeros`)

Bessel functions are a family of solutions to Bessel's differential equation with real or complex order  $\alpha$ :

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (x^2 - \alpha^2)y = 0$$

Among other uses, these functions arise in wave propagation problems such as the vibrational modes of a thin drum head. Here is an example of a circular drum head anchored at the edge:

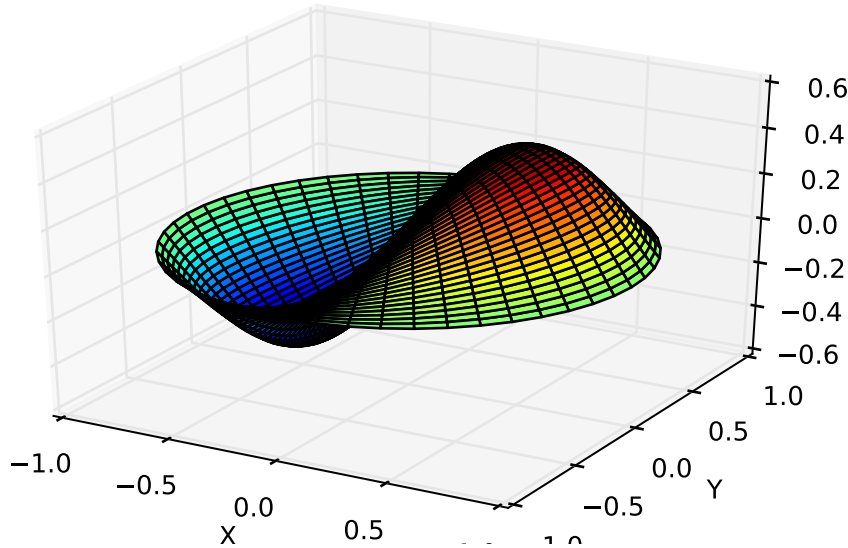
```
>>> from scipy import special
>>> def drumhead_height(n, k, distance, angle, t):
```

```

...     kth_zero = special.jn_zeros(n, k)[-1]
...     return np.cos(t) * np.cos(n*angle) * special.jn(n, distance*kth_zero)
>>> theta = np.r_[0:2*np.pi:50j]
>>> radius = np.r_[0:1:50j]
>>> x = np.array([r * np.cos(theta) for r in radius])
>>> y = np.array([r * np.sin(theta) for r in radius])
>>> z = np.array([drumhead_height(1, 1, r, theta, 0.5) for r in radius])

>>> import matplotlib.pyplot as plt
>>> from mpl_toolkits.mplot3d import Axes3D
>>> from matplotlib import cm
>>> fig = plt.figure()
>>> ax = Axes3D(fig)
>>> ax.plot_surface(x, y, z, rstride=1, cstride=1, cmap=cm.jet)
>>> ax.set_xlabel('X')
>>> ax.set_ylabel('Y')
>>> ax.set_zlabel('Z')
>>> plt.show()

```



### 3.1.4 Integration (`scipy.integrate`)

The `scipy.integrate` sub-package provides several integration techniques including an ordinary differential equation integrator. An overview of the module is provided by the help command:

```

>>> help(integrate)
Methods for Integrating Functions given function object.

quad          -- General purpose integration.
dblquad       -- General purpose double integration.
tplquad       -- General purpose triple integration.
fixed_quad    -- Integrate func(x) using Gaussian quadrature of order n.
quadrature    -- Integrate with given tolerance using Gaussian quadrature.
romberg       -- Integrate func using Romberg integration.

```

Methods for Integrating Functions given fixed samples.

```

trapz          -- Use trapezoidal rule to compute integral from samples.
cumtrapz      -- Use trapezoidal rule to cumulatively compute integral.
simps         -- Use Simpson's rule to compute integral from samples.
romb          -- Use Romberg Integration to compute integral from
              (2**k + 1) evenly-spaced samples.

```

See the special module's orthogonal polynomials (special) for Gaussian quadrature roots and weights for other weighting factors and regions.

Interface to numerical integrators of ODE systems.

```

odeint        -- General integration of ordinary differential equations.
ode           -- Integrate ODE using VODE and ZVODE routines.

```

### General integration (quad)

The function `quad` is provided to integrate a function of one variable between two points. The points can be  $\pm\infty$  ( $\pm\text{inf}$ ) to indicate infinite limits. For example, suppose you wish to integrate a `bessel` function `jv(2.5, x)` along the interval  $[0, 4.5]$ .

$$I = \int_0^{4.5} J_{2.5}(x) dx.$$

This could be computed using `quad`:

```

>>> import scipy.integrate as integrate
>>> import scipy.special as special
>>> result = integrate.quad(lambda x: special.jv(2.5,x), 0, 4.5)
>>> result
(1.1178179380783249, 7.86663172481899801e-09)

>>> from numpy import sqrt, sin, cos, pi
>>> I = sqrt(2/pi)*(18.0/27*sqrt(2)*cos(4.5) - 4.0/27*sqrt(2)*sin(4.5) +
...         sqrt(2*pi) * special.fresnel(3/sqrt(pi))[0])
>>> I
1.117817938088701

>>> print(abs(result[0]-I))
1.03761443881e-11

```

The first argument to `quad` is a “callable” Python object (*i.e.* a function, method, or class instance). Notice the use of a `lambda`-function in this case as the argument. The next two arguments are the limits of integration. The return value is a tuple, with the first element holding the estimated value of the integral and the second element holding an upper bound on the error. Notice, that in this case, the true value of this integral is

$$I = \sqrt{\frac{2}{\pi}} \left( \frac{18}{27} \sqrt{2} \cos(4.5) - \frac{4}{27} \sqrt{2} \sin(4.5) + \sqrt{2\pi} \text{Si} \left( \frac{3}{\sqrt{\pi}} \right) \right),$$

where

$$\text{Si}(x) = \int_0^x \sin\left(\frac{\pi}{2}t^2\right) dt.$$

is the Fresnel sine integral. Note that the numerically-computed integral is within  $1.04 \times 10^{-11}$  of the exact result — well below the reported error bound.

If the function to integrate takes additional parameters, they can be provided in the *args* argument. Suppose that the following integral shall be calculated:

$$I(a, b) = \int_0^1 ax^2 + b dx.$$

This integral can be evaluated by using the following code:

```
>>> from scipy.integrate import quad
>>> def integrand(x, a, b):
...     return a * x + b
>>> a = 2
>>> b = 1
>>> I = quad(integrand, 0, 1, args=(a,b))
>>> I = (2.0, 2.220446049250313e-14)
```

Infinite inputs are also allowed in `quad` by using  $\pm \text{inf}$  as one of the arguments. For example, suppose that a numerical value for the exponential integral:

$$E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n} dt.$$

is desired (and the fact that this integral can be computed as `special.expn(n, x)` is forgotten). The functionality of the function `special.expn` can be replicated by defining a new function `vec_expint` based on the routine `quad`:

```
>>> from scipy.integrate import quad
>>> def integrand(t, n, x):
...     return np.exp(-x*t) / t**n

>>> def expint(n, x):
...     return quad(integrand, 1, np.inf, args=(n, x))[0]

>>> vec_expint = np.vectorize(expint)

>>> vec_expint(3, np.arange(1.0, 4.0, 0.5))
array([ 0.1097,  0.0567,  0.0301,  0.0163,  0.0089,  0.0049])
>>> import scipy.special as special
>>> special.expn(3, np.arange(1.0, 4.0, 0.5))
array([ 0.1097,  0.0567,  0.0301,  0.0163,  0.0089,  0.0049])
```

The function which is integrated can even use the `quad` argument (though the error bound may underestimate the error due to possible numerical error in the integrand from the use of `quad`). The integral in this case is

$$I_n = \int_0^\infty \int_1^\infty \frac{e^{-xt}}{t^n} dt dx = \frac{1}{n}.$$

```
>>> result = quad(lambda x: expint(3, x), 0, np.inf)
>>> print(result)
(0.33333333324560266, 2.8548934485373678e-09)

>>> I3 = 1.0/3.0
>>> print(I3)
0.3333333333333333

>>> print(I3 - result[0])
8.77306560731e-11
```

This last example shows that multiple integration can be handled using repeated calls to `quad`.

### General multiple integration (dblquad, tplquad, nquad)

The mechanics for double and triple integration have been wrapped up into the functions `dblquad` and `tplquad`. These functions take the function to integrate and four, or six arguments, respectively. The limits of all inner integrals need to be defined as functions.

An example of using double integration to compute several values of  $I_n$  is shown below:

```
>>> from scipy.integrate import quad, dblquad
>>> def I(n):
...     return dblquad(lambda t, x: np.exp(-x*t)/t**n, 0, np.inf, lambda x: 1, lambda x: np.inf)

>>> print(I(4))
(0.2500000000043577, 1.29830334693681e-08)
>>> print(I(3))
(0.33333333325010883, 1.3888461883425516e-08)
>>> print(I(2))
(0.4999999999985751, 1.3894083651858995e-08)
```

As example for non-constant limits consider the integral

$$I = \int_{y=0}^{1/2} \int_{x=0}^{1-2y} xy \, dx \, dy = \frac{1}{96}.$$

This integral can be evaluated using the expression below (Note the use of the non-constant lambda functions for the upper limit of the inner integral):

```
>>> from scipy.integrate import dblquad
>>> area = dblquad(lambda x, y: x*y, 0, 0.5, lambda x: 0, lambda x: 1-2*x)
>>> area
(0.010416666666666668, 1.1564823173178715e-16)
```

For n-fold integration, `scipy` provides the function `nquad`. The integration bounds are an iterable object: either a list of constant bounds, or a list of functions for the non-constant integration bounds. The order of integration (and therefore the bounds) is from the innermost integral to the outermost one.

The integral from above

$$I_n = \int_0^\infty \int_1^\infty \frac{e^{-xt}}{t^n} \, dt \, dx = \frac{1}{n}$$

can be calculated as

```
>>> from scipy import integrate
>>> N = 5
>>> def f(t, x):
...     return np.exp(-x*t) / t**N
>>> integrate.nquad(f, [[1, np.inf], [0, np.inf]])
(0.20000000000002294, 1.2239614263187945e-08)
```

Note that the order of arguments for  $f$  must match the order of the integration bounds; i.e. the inner integral with respect to  $t$  is on the interval  $[1, \infty]$  and the outer integral with respect to  $x$  is on the interval  $[0, \infty]$ .

Non-constant integration bounds can be treated in a similar manner; the example from above

$$I = \int_{y=0}^{1/2} \int_{x=0}^{1-2y} xy \, dx \, dy = \frac{1}{96}.$$

can be evaluated by means of

```
>>> from scipy import integrate
>>> def f(x, y):
...     return x*y
>>> def bounds_y():
...     return [0, 0.5]
>>> def bounds_x(y):
...     return [0, 1-2*y]
>>> integrate.nquad(f, [bounds_x, bounds_y])
(0.010416666666666668, 4.101620128472366e-16)
```

which is the same result as before.

## Gaussian quadrature

A few functions are also provided in order to perform simple Gaussian quadrature over a fixed interval. The first is `fixed_quad` which performs fixed-order Gaussian quadrature. The second function is `quadrature` which performs Gaussian quadrature of multiple orders until the difference in the integral estimate is beneath some tolerance supplied by the user. These functions both use the module `special.orthogonal` which can calculate the roots and quadrature weights of a large variety of orthogonal polynomials (the polynomials themselves are available as special functions returning instances of the polynomial class — e.g. `special.legendre`).

## Romberg Integration

Romberg's method [WPR] is another method for numerically evaluating an integral. See the help function for `romberg` for further details.

## Integrating using Samples

If the samples are equally-spaced and the number of samples available is  $2^k + 1$  for some integer  $k$ , then Romberg `romb` integration can be used to obtain high-precision estimates of the integral using the available samples. Romberg integration uses the trapezoid rule at step-sizes related by a power of two and then performs Richardson extrapolation on these estimates to approximate the integral with a higher-degree of accuracy.

In case of arbitrary spaced samples, the two functions `trapz` (defined in `numpy` [NPT]) and `simps` are available. They are using Newton-Coates formulas of order 1 and 2 respectively to perform integration. The trapezoidal rule approximates the function as a straight line between adjacent points, while Simpson's rule approximates the function between three adjacent points as a parabola.

For an odd number of samples that are equally spaced Simpson's rule is exact if the function is a polynomial of order 3 or less. If the samples are not equally spaced, then the result is exact only if the function is a polynomial of order 2 or less.

```
>>> import numpy as np
>>> def f1(x):
...     return x**2
>>> def f2(x):
...     return x**3
>>> x = np.array([1, 3, 4])
>>> y1 = f1(x)
>>> from scipy.integrate import simps
>>> I1 = simps(y1, x)
>>> print(I1)
21.0
```

This corresponds exactly to

$$\int_1^4 x^2 dx = 21,$$

whereas integrating the second function

```
>>> y2 = f2(x)
>>> I2 = integrate.simps(y2, x)
>>> print(I2)
61.5
```

does not correspond to

$$\int_1^4 x^3 dx = 63.75$$

because the order of the polynomial in f2 is larger than two.

### Faster integration using Ctypes

A user desiring reduced integration times may pass a C function pointer through `ctypes` to `quad`, `dblquad`, `tplquad` or `nquad` and it will be integrated and return a result in Python. The performance increase here arises from two factors. The primary improvement is faster function evaluation, which is provided by compilation. This can also be achieved using a library like Cython or F2Py that compiles Python. Additionally we have a speedup provided by the removal of function calls between C and Python in `quad` - this cannot be achieved through Cython or F2Py. This method will provide a speed increase of ~2x for trivial functions such as sine but can produce a much more noticeable increase (10x+) for more complex functions. This feature then, is geared towards a user with numerically intensive integrations willing to write a little C to reduce computation time significantly.

`ctypes` integration can be done in a few simple steps:

1.) Write an integrand function in C with the function signature `double f(int n, double args[n])`, where `args` is an array containing the arguments of the function `f`.

```
//testlib.c
double f(int n, double args[n]){
    return args[0] - args[1] * args[2]; //corresponds to x0 - x1 * x2
}
```

2.) Now compile this file to a shared/dynamic library (a quick search will help with this as it is OS-dependent). The user must link any math libraries, etc. used. On linux this looks like:

```
$ gcc -shared -o testlib.so -fPIC testlib.c
```

The output library will be referred to as `testlib.so`, but it may have a different file extension. A library has now been created that can be loaded into Python with `ctypes`.

3.) Load shared library into Python using `ctypes` and set `restypes` and `argtypes` - this allows Scipy to interpret the function correctly:

```
>>> import ctypes
>>> from scipy import integrate
>>> lib = ctypes.CDLL('/**/testlib.so') # Use absolute path to testlib
>>> func = lib.f # Assign specific function to name func (for simplicity)
>>> func.restype = ctypes.c_double
>>> func.argtypes = (ctypes.c_int, ctypes.c_double)
```

Note that the `argtypes` will always be `(ctypes.c_int, ctypes.c_double)` regardless of the number of parameters, and `restype` will always be `ctypes.c_double`.

4.) Now integrate the library function as normally, here using `nquad`:

```
>>> integrate.nquad(func, [[0, 10], [-10, 0], [-1, 1]])
(1000.0, 1.1102230246251565e-11)
```

And the Python tuple is returned as expected in a reduced amount of time. All optional parameters can be used with this method including specifying singularities, infinite bounds, etc.

## Ordinary differential equations (`odeint`)

Integrating a set of ordinary differential equations (ODEs) given initial conditions is another useful example. The function `odeint` is available in SciPy for integrating a first-order vector differential equation:

$$\frac{dy}{dt} = \mathbf{f}(\mathbf{y}, t),$$

given initial conditions  $\mathbf{y}(0) = y_0$ , where  $\mathbf{y}$  is a length  $N$  vector and  $\mathbf{f}$  is a mapping from  $\mathcal{R}^N$  to  $\mathcal{R}^N$ . A higher-order ordinary differential equation can always be reduced to a differential equation of this type by introducing intermediate derivatives into the  $\mathbf{y}$  vector.

For example suppose it is desired to find the solution to the following second-order differential equation:

$$\frac{d^2w}{dz^2} - zw(z) = 0$$

with initial conditions  $w(0) = \frac{1}{\sqrt[3]{3^2}\Gamma(\frac{2}{3})}$  and  $\frac{dw}{dz}|_{z=0} = -\frac{1}{\sqrt[3]{3}\Gamma(\frac{1}{3})}$ . It is known that the solution to this differential equation with these boundary conditions is the Airy function

$$w = \text{Ai}(z),$$

which gives a means to check the integrator using `special.airy`.

First, convert this ODE into standard form by setting  $\mathbf{y} = [\frac{dw}{dz}, w]$  and  $t = z$ . Thus, the differential equation becomes

$$\frac{d\mathbf{y}}{dt} = \begin{bmatrix} ty_1 \\ y_0 \end{bmatrix} = \begin{bmatrix} 0 & t \\ 1 & 0 \end{bmatrix} \begin{bmatrix} y_0 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0 & t \\ 1 & 0 \end{bmatrix} \mathbf{y}.$$

In other words,

$$\mathbf{f}(\mathbf{y}, t) = \mathbf{A}(t) \mathbf{y}.$$

As an interesting reminder, if  $\mathbf{A}(t)$  commutes with  $\int_0^t \mathbf{A}(\tau) d\tau$  under matrix multiplication, then this linear differential equation has an exact solution using the matrix exponential:

$$\mathbf{y}(t) = \exp\left(\int_0^t \mathbf{A}(\tau) d\tau\right) \mathbf{y}(0),$$

However, in this case,  $\mathbf{A}(t)$  and its integral do not commute.

There are many optional inputs and outputs available when using `odeint` which can help tune the solver. These additional inputs and outputs are not needed much of the time, however, and the three required input arguments and the output solution suffice. The required inputs are the function defining the derivative, `fprime`, the initial conditions vector, `y0`, and the time points to obtain a solution, `t`, (with the initial value point as the first element of this sequence). The output to `odeint` is a matrix where each row contains the solution vector at each requested time point (thus, the initial conditions are given in the first output row).

The following example illustrates the use of `odeint` including the usage of the `Dfun` option which allows the user to specify a gradient (with respect to  $\mathbf{y}$ ) of the function,  $\mathbf{f}(\mathbf{y}, t)$ .



```

>>> from scipy.integrate import odeint
>>> from scipy.special import gamma, airy
>>> y1_0 = 1.0 / 3**(2.0/3.0) / gamma(2.0/3.0)
>>> y0_0 = -1.0 / 3**(1.0/3.0) / gamma(1.0/3.0)
>>> y0 = [y0_0, y1_0]
>>> def func(y, t):
...     return [t*y[1], y[0]]

>>> def gradient(y, t):
...     return [[0,t], [1,0]]

>>> x = np.arange(0, 4.0, 0.01)
>>> t = x
>>> ychk = airy(x)[0]
>>> y = odeint(func, y0, t)
>>> y2 = odeint(func, y0, t, Dfun=gradient)

>>> ychk[:36:6]
array([0.355028, 0.339511, 0.324068, 0.308763, 0.293658, 0.278806])

>>> y[:36:6,1]
array([0.355028, 0.339511, 0.324067, 0.308763, 0.293658, 0.278806])

>>> y2[:36:6,1]
array([0.355028, 0.339511, 0.324067, 0.308763, 0.293658, 0.278806])

```

## References

### 3.1.5 Optimization (`scipy.optimize`)

The `scipy.optimize` package provides several commonly used optimization algorithms. A detailed listing is available: `scipy.optimize` (can also be found by `help(scipy.optimize)`).

The module contains:

1. Unconstrained and constrained minimization of multivariate scalar functions (`minimize`) using a variety of algorithms (e.g. BFGS, Nelder-Mead simplex, Newton Conjugate Gradient, COBYLA or SLSQP)
2. Global (brute-force) optimization routines (e.g. `basinhopping`, `differential_evolution`)
3. Least-squares minimization (`least_squares`) and curve fitting (`curve_fit`) algorithms
4. Scalar univariate functions minimizers (`minimize_scalar`) and root finders (`newton`)
5. Multivariate equation system solvers (`root`) using a variety of algorithms (e.g. hybrid Powell, Levenberg-Marquardt or large-scale methods such as Newton-Krylov).

Below, several examples demonstrate their basic usage.

#### Unconstrained minimization of multivariate scalar functions (`minimize`)

The `minimize` function provides a common interface to unconstrained and constrained minimization algorithms for multivariate scalar functions in `scipy.optimize`. To demonstrate the minimization function consider the problem of minimizing the Rosenbrock function of  $N$  variables:  $f(\mathbf{x}) = \sum_{i=1}^{N-1} 100(x_i - x_{i-1}^2)^2 + (1 - x_{i-1})^2$ . The minimum value of this function is 0 which is achieved when  $x_i = 1$ .

Note that the Rosenbrock function and its derivatives are included in `scipy.optimize`. The implementations shown in the following sections provide examples of how to define an objective function as well as its jacobian and hessian functions.

### *Nelder-Mead Simplex algorithm (method='Nelder-Mead')*

In the example below, the `minimize` routine is used with the *Nelder-Mead* simplex algorithm (selected through the `method` parameter):

```
>>> import numpy as np
>>> from scipy.optimize import minimize

>>> def rosen(x):
...     """The Rosenbrock function"""
...     return sum(100.0*(x[1:]-x[:-1])**2.0)**2.0 + (1-x[:-1])**2.0

>>> x0 = np.array([1.3, 0.7, 0.8, 1.9, 1.2])
>>> res = minimize(rosen, x0, method='nelder-mead',
...               options={'xtol': 1e-8, 'disp': True})
Optimization terminated successfully.
      Current function value: 0.000000
      Iterations: 339
      Function evaluations: 571

>>> print(res.x)
[ 1.  1.  1.  1.  1.]
```

The simplex algorithm is probably the simplest way to minimize a fairly well-behaved function. It requires only function evaluations and is a good choice for simple minimization problems. However, because it does not use any gradient evaluations, it may take longer to find the minimum.

Another optimization algorithm that needs only function calls to find the minimum is *Powell's* method available by setting `method='powell'` in `minimize`.

### *Broyden-Fletcher-Goldfarb-Shanno algorithm (method='BFGS')*

In order to converge more quickly to the solution, this routine uses the gradient of the objective function. If the gradient is not given by the user, then it is estimated using first-differences. The Broyden-Fletcher-Goldfarb-Shanno (BFGS) method typically requires fewer function calls than the simplex algorithm even when the gradient must be estimated.

To demonstrate this algorithm, the Rosenbrock function is again used. The gradient of the Rosenbrock function is the vector:

$$\begin{aligned}\frac{\partial f}{\partial x_j} &= \sum_{i=1}^N 200 (x_i - x_{i-1}^2) (\delta_{i,j} - 2x_{i-1}\delta_{i-1,j}) - 2(1 - x_{i-1})\delta_{i-1,j} \\ &= 200 (x_j - x_{j-1}^2) - 400x_j (x_{j+1} - x_j^2) - 2(1 - x_j).\end{aligned}$$

This expression is valid for the interior derivatives. Special cases are

$$\begin{aligned}\frac{\partial f}{\partial x_0} &= -400x_0 (x_1 - x_0^2) - 2(1 - x_0), \\ \frac{\partial f}{\partial x_{N-1}} &= 200 (x_{N-1} - x_{N-2}^2).\end{aligned}$$

A Python function which computes this gradient is constructed by the code-segment:

```
>>> def rosen_der(x):
...     xm = x[1:-1]
...     xm_m1 = x[:-2]
...     xm_p1 = x[2:]
...     der = np.zeros_like(x)
...     der[1:-1] = 200*(xm-xm_m1**2) - 400*(xm_p1 - xm**2)*xm - 2*(1-xm)
...     der[0] = -400*x[0]*(x[1]-x[0]**2) - 2*(1-x[0])
```

```
...     der[-1] = 200*(x[-1]-x[-2]**2)
...     return der
```

This gradient information is specified in the `minimize` function through the `jac` parameter as illustrated below.

```
>>> res = minimize(rosen, x0, method='BFGS', jac=rosen_der,
...                 options={'disp': True})
Optimization terminated successfully.
    Current function value: 0.000000
    Iterations: 51          # may vary
    Function evaluations: 63
    Gradient evaluations: 63
>>> res.x
array([1., 1., 1., 1., 1.])
```

### Newton-Conjugate-Gradient algorithm (`method='Newton-CG'`)

The method which requires the fewest function calls and is therefore often the fastest method to minimize functions of many variables uses the Newton-Conjugate Gradient algorithm. This method is a modified Newton's method and uses a conjugate gradient algorithm to (approximately) invert the local Hessian. Newton's method is based on fitting the function locally to a quadratic form:  $f(\mathbf{x}) \approx f(\mathbf{x}_0) + \nabla f(\mathbf{x}_0) \cdot (\mathbf{x} - \mathbf{x}_0) + \frac{1}{2}(\mathbf{x} - \mathbf{x}_0)^T \mathbf{H}(\mathbf{x}_0)(\mathbf{x} - \mathbf{x}_0)$ . where  $\mathbf{H}(\mathbf{x}_0)$  is a matrix of second-derivatives (the Hessian). If the Hessian is positive definite then the local minimum of this function can be found by setting the gradient of the quadratic form to zero, resulting in  $\mathbf{x}_{\text{opt}} = \mathbf{x}_0 - \mathbf{H}^{-1} \nabla f$ . The inverse of the Hessian is evaluated using the conjugate gradient method. An example of employing this method to minimize the Rosenbrock function is given below. To take full advantage of the CG method, a function which computes the Hessian must be provided. The Hessian matrix itself does not need to be constructed, only

**Full Hessian example:** The Hessian of the Rosenbrock function is

$$\begin{aligned} H_{ij} = \frac{\partial^2 f}{\partial x_i \partial x_j} &= 200(\delta_{i,j} - 2x_{i-1}\delta_{i-1,j}) - 400x_i(\delta_{i+1,j} - 2x_i\delta_{i,j}) - 400\delta_{i,j}(x_{i+1} - x_i^2) + 2\delta_{i,j}, \\ &= (202 + 1200x_i^2 - 400x_{i+1})\delta_{i,j} - 400x_i\delta_{i+1,j} - 400x_{i-1}\delta_{i-1,j}, \end{aligned}$$

if  $i, j \in [1, N-2]$  with  $i, j \in [0, N-1]$  defining the  $N \times N$  matrix. Other non-zero entries of the matrix are

$$\begin{aligned} \frac{\partial^2 f}{\partial x_0^2} &= 1200x_0^2 - 400x_1 + 2, \\ \frac{\partial^2 f}{\partial x_0 \partial x_1} = \frac{\partial^2 f}{\partial x_1 \partial x_0} &= -400x_0, \\ \frac{\partial^2 f}{\partial x_{N-1} \partial x_{N-2}} = \frac{\partial^2 f}{\partial x_{N-2} \partial x_{N-1}} &= -400x_{N-2}, \\ \frac{\partial^2 f}{\partial x_{N-1}^2} &= 200. \end{aligned}$$

For example, the Hessian when  $N = 5$  is  $\mathbf{H} =$

1200x <sub>0</sub> <sup>2</sup> - 400x <sub>1</sub> + 2	-400x <sub>0</sub>	0	0	0
-400x <sub>0</sub>	202 + 1200x <sub>1</sub> <sup>2</sup> - 400x <sub>2</sub>	-400x <sub>1</sub>	0	0
0	-400x <sub>1</sub>	202 + 1200x <sub>2</sub> <sup>2</sup> - 400x <sub>3</sub>	-400x <sub>2</sub>	-400x <sub>2</sub>
0	0	-400x <sub>2</sub>	202 + 1200x <sub>3</sub> <sup>2</sup> - 400x <sub>4</sub>	-400x <sub>3</sub>
0	0	0	0	-400x <sub>3</sub>

CG method is shown in the following example :

```
>>> def rosen_hess(x):
...     x = np.asarray(x)
...     H = np.diag(-400*x[:-1], 1) - np.diag(400*x[:-1], -1)
...     diagonal = np.zeros_like(x)
```

```

...     diagonal[0] = 1200*x[0]**2-400*x[1]+2
...     diagonal[-1] = 200
...     diagonal[1:-1] = 202 + 1200*x[1:-1]**2 - 400*x[2:]
...     H = H + np.diag(diagonal)
...     return H

```

```

>>> res = minimize(rosen, x0, method='Newton-CG',
...                 jac=rosen_der, hess=rosen_hess,
...                 options={'xtol': 1e-8, 'disp': True})
Optimization terminated successfully.
    Current function value: 0.000000
    Iterations: 19          # may vary
    Function evaluations: 22
    Gradient evaluations: 19
    Hessian evaluations: 19
>>> res.x
array([1., 1., 1., 1., 1.])

```

**Hessian product example:** For larger minimization problems, storing the entire Hessian matrix can consume considerable time and memory. The Newton-CG algorithm only needs the product of the Hessian times an arbitrary vector. As a result, the user can supply code to compute this product rather than the full Hessian by giving a `hess` function which take the minimization vector as the first argument and the arbitrary vector as the second argument (along with extra arguments passed to the function to be minimized). If possible, using Newton-CG with the Hessian product option is probably the fastest way to minimize the function.

In this case, the product of the Rosenbrock Hessian with an arbitrary vector is not difficult to compute. If  $\mathbf{p}$  is the arbitrary vector, then  $\mathbf{H}(\mathbf{x})\mathbf{p}$  has elements:  $\mathbf{H}(\mathbf{x})\mathbf{p} =$

$$\begin{bmatrix} (1200x_0^2 - 400x_1 + 2)p_0 - 400x_0p_1 \\ \vdots \\ -400x_{i-1}p_{i-1} + (202 + 1200x_i^2 - 400x_{i+1})p_i - 400x_i p_{i+1} \\ \vdots \\ -400x_{N-2}p_{N-2} + 200p_{N-1} \end{bmatrix} . \text{Code which makes use of this Hessian product to minimize the } f$$

```

>>> def rosen_hess_p(x, p):
...     x = np.asarray(x)
...     Hp = np.zeros_like(x)
...     Hp[0] = (1200*x[0]**2 - 400*x[1] + 2)*p[0] - 400*x[0]*p[1]
...     Hp[1:-1] = -400*x[:-2]*p[:-2] + (202+1200*x[1:-1]**2-400*x[2:]) *p[1:-1] \
...               -400*x[1:-1]*p[2:]
...     Hp[-1] = -400*x[-2]*p[-2] + 200*p[-1]
...     return Hp

```

```

>>> res = minimize(rosen, x0, method='Newton-CG',
...                 jac=rosen_der, hessp=rosen_hess_p,
...                 options={'xtol': 1e-8, 'disp': True})
Optimization terminated successfully.
    Current function value: 0.000000
    Iterations: 20          # may vary
    Function evaluations: 23
    Gradient evaluations: 20
    Hessian evaluations: 44
>>> res.x
array([1., 1., 1., 1., 1.])

```

### Constrained minimization of multivariate scalar functions (`minimize`)

The `minimize` function also provides an interface to several constrained minimization algorithm. As an example, the Sequential Least Squares Programming optimization algorithm (SLSQP) will be considered here. This algorithm allows to deal with constrained minimization problems of the form:

$$\begin{aligned} & \min F(x) \\ & \text{subject to} \quad C_j(X) = 0, \quad j = 1, \dots, \text{MEQ} \\ & \quad \quad \quad C_j(x) \geq 0, \quad j = \text{MEQ} + 1, \dots, M \\ & \quad \quad \quad XL \leq x \leq XU, \quad I = 1, \dots, N. \end{aligned}$$

As an example, let us consider the problem of maximizing the function:  $f(x, y) = 2xy + 2x - x^2 - 2y^2$  subject to an equality and an inequality constraints defined as:  $x^3 - y = 0$  and  $y - 1 \geq 0$ . The objective function and its derivative are defined as follows.

```
>>> def func(x, sign=1.0):
...     """ Objective function """
...     return sign*(2*x[0]*x[1] + 2*x[0] - x[0]**2 - 2*x[1]**2)

>>> def func_deriv(x, sign=1.0):
...     """ Derivative of objective function """
...     dfdx0 = sign*(-2*x[0] + 2*x[1] + 2)
...     dfdx1 = sign*(2*x[0] - 4*x[1])
...     return np.array([ dfdx0, dfdx1 ])
```

Note that since `minimize` only minimizes functions, the `sign` parameter is introduced to multiply the objective function (and its derivative) by -1 in order to perform a maximization.

Then constraints are defined as a sequence of dictionaries, with keys `type`, `fun` and `jac`.

```
>>> cons = ({'type': 'eq',
...         'fun' : lambda x: np.array([x[0]**3 - x[1]]),
...         'jac' : lambda x: np.array([3.0*(x[0]**2.0), -1.0])},
...         {'type': 'ineq',
...         'fun' : lambda x: np.array([x[1] - 1]),
...         'jac' : lambda x: np.array([0.0, 1.0])})
```

Now an unconstrained optimization can be performed as:

```
>>> res = minimize(func, [-1.0,1.0], args=(-1.0,), jac=func_deriv,
...               method='SLSQP', options={'disp': True})
Optimization terminated successfully.      (Exit mode 0)
Current function value: -2.0
Iterations: 4                                # may vary
Function evaluations: 5
Gradient evaluations: 4

>>> print(res.x)
[ 2.  1.]
```

and a constrained optimization as:

```
>>> res = minimize(func, [-1.0,1.0], args=(-1.0,), jac=func_deriv,
...               constraints=cons, method='SLSQP', options={'disp': True})
Optimization terminated successfully.      (Exit mode 0)
Current function value: -1.00000018311
Iterations: 9                                # may vary
Function evaluations: 14
Gradient evaluations: 9
```

```
>>> print(res.x)
[ 1.00000009  1.          ]
```

### Least-squares minimization (`least_squares`)

SciPy is capable of solving robustified bound constrained nonlinear least-squares problems:

$$\min_{\mathbf{x}} \frac{1}{2} \sum_{i=1}^m \rho(f_i(\mathbf{x})^2) \tag{3.1}$$

subject to  $\mathbf{lb} \leq \mathbf{x} \leq \mathbf{ub}$

Here  $f_i(\mathbf{x})$  are smooth functions from  $R^n$  to  $R$ , we refer to them as residuals. The purpose of a scalar valued function  $\rho(\cdot)$  is to reduce the influence of outlier residuals and contribute to robustness of the solution, we refer to it as a loss function. A linear loss function gives a standard least-squares problem. Additionally, constraints in a form of lower and upper bounds on some of  $x_j$  are allowed.

All methods specific to least-squares minimization utilize a  $m \times n$  matrix of partial derivatives called Jacobian and defined as  $J_{ij} = \partial f_i / \partial x_j$ . It is highly recommended to compute this matrix analytically and pass it to `least_squares`, otherwise it will be estimated by finite differences which takes a lot of additional time and can be very inaccurate in hard cases.

Function `least_squares` can be used for fitting a function  $\varphi(t; \mathbf{x})$  to empirical data  $\{(t_i, y_i), i = 0, \dots, m - 1\}$ . To do this one should simply precompute residuals as  $f_i(\mathbf{x}) = w_i(\varphi(t_i; \mathbf{x}) - y_i)$ , where  $w_i$  are weights assigned to each observation.

#### Example of solving a fitting problem

Here we consider “Analysis of an Enzyme Reaction” problem formulated in <sup>1</sup>. There are 11 residuals defined as

$$f_i(x) = \frac{x_0(u_i^2 + u_i x_1)}{u_i^2 + u_i x_2 + x_3} - y_i, \quad i = 0, \dots, 10,$$

where  $y_i$  are measurement values and  $u_i$  are values of the independent variable. The unknown vector of parameters is  $\mathbf{x} = (x_0, x_1, x_2, x_3)^T$ . As was said previously, it is recommended to compute Jacobian matrix in a closed form:

$$J_{i0} = \frac{\partial f_i}{\partial x_0} = \frac{u_i^2 + u_i x_1}{u_i^2 + u_i x_2 + x_3} \tag{3.3}$$

$$J_{i1} = \frac{\partial f_i}{\partial x_1} = \frac{u_i x_0}{u_i^2 + u_i x_2 + x_3} \tag{3.4}$$

$$J_{i2} = \frac{\partial f_i}{\partial x_2} = -\frac{x_0(u_i^2 + u_i x_1)u_i}{(u_i^2 + u_i x_2 + x_3)^2} \tag{3.5}$$

$$J_{i3} = \frac{\partial f_i}{\partial x_3} = -\frac{x_0(u_i^2 + u_i x_1)}{(u_i^2 + u_i x_2 + x_3)^2} \tag{3.6}$$

We are going to use the “hard” starting point defined in <sup>1</sup>. To find a physically meaningful solution, avoid potential division by zero and assure convergence to the global minimum we impose constraints  $0 \leq x_j \leq 100, j = 0, 1, 2, 3$ .

The code below implements least-squares estimation of  $\mathbf{x}$  and finally plots the original data and the fitted model function:

```
>>> from scipy.optimize import least_squares
```

<sup>1</sup> Brett M. Averick et al., “The MINPACK-2 Test Problem Collection”.

```

>>> def model(x, u):
...     return x[0] * (u ** 2 + x[1] * u) / (u ** 2 + x[2] * u + x[3])

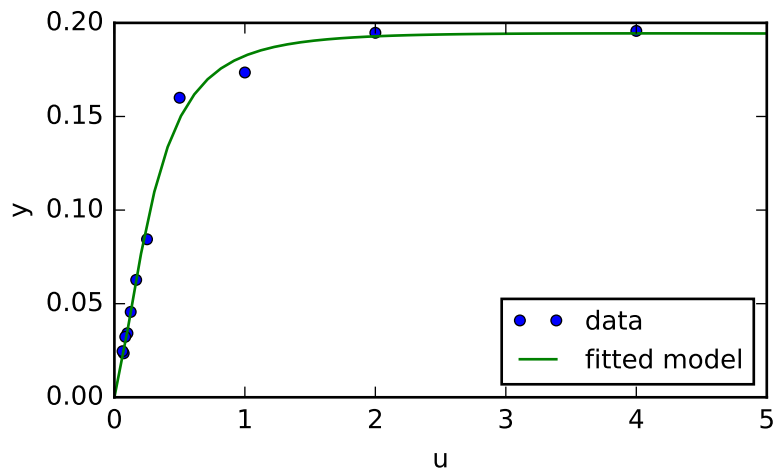
>>> def fun(x, u, y):
...     return model(x, u) - y

>>> def jac(x, u, y):
...     J = np.empty((u.size, x.size))
...     den = u ** 2 + x[2] * u + x[3]
...     num = u ** 2 + x[1] * u
...     J[:, 0] = num / den
...     J[:, 1] = x[0] * u / den
...     J[:, 2] = -x[0] * num * u / den ** 2
...     J[:, 3] = -x[0] * num / den ** 2
...     return J

>>> u = np.array([4.0, 2.0, 1.0, 5.0e-1, 2.5e-1, 1.67e-1, 1.25e-1, 1.0e-1,
...               8.33e-2, 7.14e-2, 6.25e-2])
>>> y = np.array([1.957e-1, 1.947e-1, 1.735e-1, 1.6e-1, 8.44e-2, 6.27e-2,
...               4.56e-2, 3.42e-2, 3.23e-2, 2.35e-2, 2.46e-2])
>>> x0 = np.array([2.5, 3.9, 4.15, 3.9])
>>> res = least_squares(fun, x0, jac=jac, bounds=(0, 100), args=(u, y), verbose=1)
`ftol` termination condition is satisfied.
Function evaluations 130, initial cost 4.4383e+00, final cost 1.5375e-04, first-order optimality 4.92e-08
>>> res.x
array([ 0.19280596,  0.19130423,  0.12306063,  0.13607247])

>>> import matplotlib.pyplot as plt
>>> u_test = np.linspace(0, 5)
>>> y_test = model(res.x, u_test)
>>> plt.plot(u, y, 'o', markersize=4, label='data')
>>> plt.plot(u_test, y_test, label='fitted model')
>>> plt.xlabel("u")
>>> plt.ylabel("y")
>>> plt.legend(loc='lower right')
>>> plt.show()

```



### Further examples

Three interactive examples below illustrate usage of `least_squares` in greater detail.

1. [Large-scale bundle adjustment in scipy](#) demonstrates large-scale capabilities of `least_squares` and how to efficiently compute finite difference approximation of sparse Jacobian.
2. [Robust nonlinear regression in scipy](#) shows how to handle outliers with a robust loss function in a nonlinear regression.
3. [Solving a discrete boundary-value problem in scipy](#) examines how to solve a large system of equations and use bounds to achieve desired properties of the solution.

For the details about mathematical algorithms behind the implementation refer to documentation of `least_squares`.

### Univariate function minimizers (`minimize_scalar`)

Often only the minimum of an univariate function (i.e. a function that takes a scalar as input) is needed. In these circumstances, other optimization techniques have been developed that can work faster. These are accessible from the `minimize_scalar` function which proposes several algorithms.

#### Unconstrained minimization (`method='brent'`)

There are actually two methods that can be used to minimize an univariate function: `brent` and `golden`, but `golden` is included only for academic purposes and should rarely be used. These can be respectively selected through the `method` parameter in `minimize_scalar`. The `brent` method uses Brent's algorithm for locating a minimum. Optimally a bracket (the `bracket` parameter) should be given which contains the minimum desired. A bracket is a triple  $(a, b, c)$  such that  $f(a) > f(b) < f(c)$  and  $a < b < c$ . If this is not given, then alternatively two starting points can be chosen and a bracket will be found from these points using a simple marching algorithm. If these two starting points are not provided  $0$  and  $1$  will be used (this may not be the right choice for your function and result in an unexpected minimum being returned).

Here is an example:

```
>>> from scipy.optimize import minimize_scalar
>>> f = lambda x: (x - 2) * (x + 1)**2
>>> res = minimize_scalar(f, method='brent')
>>> print(res.x)
1.0
```

#### Bounded minimization (`method='bounded'`)

Very often, there are constraints that can be placed on the solution space before minimization occurs. The `bounded` method in `minimize_scalar` is an example of a constrained minimization procedure that provides a rudimentary interval constraint for scalar functions. The interval constraint allows the minimization to occur only between two fixed endpoints, specified using the mandatory `bounds` parameter.

For example, to find the minimum of  $J_1(x)$  near  $x = 5$ , `minimize_scalar` can be called using the interval  $[4, 7]$  as a constraint. The result is  $x_{\min} = 5.3314$ :

```
>>> from scipy.special import j1
>>> res = minimize_scalar(j1, bounds=(4, 7), method='bounded')
>>> res.x
5.33144184241
```



## Custom minimizers

Sometimes, it may be useful to use a custom method as a (multivariate or univariate) minimizer, for example when using some library wrappers of `minimize` (e.g. `basinhopping`).

We can achieve that by, instead of passing a method name, we pass a callable (either a function or an object implementing a `__call__` method) as the `method` parameter.

Let us consider an (admittedly rather virtual) need to use a trivial custom multivariate minimization method that will just search the neighborhood in each dimension independently with a fixed step size:

```
>>> from scipy.optimize import OptimizeResult
>>> def custmin(fun, x0, args=(), maxfev=None, stepsize=0.1,
...           maxiter=100, callback=None, **options):
...     bestx = x0
...     besty = fun(x0)
...     funcalls = 1
...     niter = 0
...     improved = True
...     stop = False
...
...     while improved and not stop and niter < maxiter:
...         improved = False
...         niter += 1
...         for dim in range(np.size(x0)):
...             for s in [bestx[dim] - stepsize, bestx[dim] + stepsize]:
...                 testx = np.copy(bestx)
...                 testx[dim] = s
...                 testy = fun(testx, *args)
...                 funcalls += 1
...                 if testy < besty:
...                     besty = testy
...                     bestx = testx
...                     improved = True
...             if callback is not None:
...                 callback(bestx)
...             if maxfev is not None and funcalls >= maxfev:
...                 stop = True
...                 break
...
...     return OptimizeResult(fun=besty, x=bestx, nit=niter,
...                          nfev=funcalls, success=(niter > 1))
>>> x0 = [1.35, 0.9, 0.8, 1.1, 1.2]
>>> res = minimize(rosen, x0, method=custmin, options=dict(stepsize=0.05))
>>> res.x
array([1., 1., 1., 1., 1.]
```

This will work just as well in case of univariate optimization:

```
>>> def custmin(fun, bracket, args=(), maxfev=None, stepsize=0.1,
...           maxiter=100, callback=None, **options):
...     bestx = (bracket[1] + bracket[0]) / 2.0
...     besty = fun(bestx)
...     funcalls = 1
...     niter = 0
...     improved = True
...     stop = False
...
...     while improved and not stop and niter < maxiter:
```

```

...     improved = False
...     niter += 1
...     for testx in [bestx - stepsize, bestx + stepsize]:
...         testy = fun(testx, *args)
...         funcalls += 1
...         if testy < besty:
...             besty = testy
...             bestx = testx
...             improved = True
...     if callback is not None:
...         callback(bestx)
...     if maxfev is not None and funcalls >= maxfev:
...         stop = True
...         break
...
...     return OptimizeResult(fun=besty, x=bestx, nit=niter,
...                           nfev=funcalls, success=(niter > 1))
>>> def f(x):
...     return (x - 2)**2 * (x + 2)**2
>>> res = minimize_scalar(f, bracket=(-3.5, 0), method=custmin,
...                       options=dict(stepsize = 0.05))
>>> res.x
-2.0

```

## Root finding

### Scalar functions

If one has a single-variable equation, there are four different root finding algorithms that can be tried. Each of these algorithms requires the endpoints of an interval in which a root is expected (because the function changes signs). In general `brentq` is the best choice, but the other methods may be useful in certain circumstances or for academic purposes.

### Fixed-point solving

A problem closely related to finding the zeros of a function is the problem of finding a fixed-point of a function. A fixed point of a function is the point at which evaluation of the function returns the point:  $g(x) = x$ . Clearly the fixed point of  $g$  is the root of  $f(x) = g(x) - x$ . Equivalently, the root of  $f$  is the fixed\_point of  $g(x) = f(x) + x$ . The routine `fixed_point` provides a simple iterative method using Aitkens sequence acceleration to estimate the fixed point of  $g$  given a starting point.

### Sets of equations

Finding a root of a set of non-linear equations can be achieved using the `root` function. Several methods are available, amongst which `hybr` (the default) and `lm` which respectively use the hybrid method of Powell and the Levenberg-Marquardt method from MINPACK.

The following example considers the single-variable transcendental equation  $x + 2\cos(x) = 0$ , a root of which can be found as follows:

```

>>> import numpy as np
>>> from scipy.optimize import root
>>> def func(x):
...     return x + 2 * np.cos(x)
>>> sol = root(func, 0.3)
>>> sol.x
array([-1.02986653])

```

```
>>> sol.fun
array([-6.66133815e-16])
```

Consider now a set of non-linear equations

$$\begin{aligned}x_0 \cos(x_1) &= 4, \\ x_0 x_1 - x_1 &= 5.\end{aligned}$$

We define the objective function so that it also returns the Jacobian and indicate this by setting the `jac` parameter to `True`. Also, the Levenberg-Marquardt solver is used here.

```
>>> def func2(x):
...     f = [x[0] * np.cos(x[1]) - 4,
...          x[1]*x[0] - x[1] - 5]
...     df = np.array([[np.cos(x[1]), -x[0] * np.sin(x[1])],
...                    [x[1], x[0] - 1]])
...     return f, df
>>> sol = root(func2, [1, 1], jac=True, method='lm')
>>> sol.x
array([ 6.50409711,  0.90841421])
```

### Root finding for large problems

Methods `hybr` and `lm` in `root` cannot deal with a very large number of variables ( $N$ ), as they need to calculate and invert a dense  $N \times N$  Jacobian matrix on every Newton step. This becomes rather inefficient when  $N$  grows.

Consider for instance the following problem: we need to solve the following integrodifferential equation on the square  $[0, 1] \times [0, 1]$ :

$$(\partial_x^2 + \partial_y^2)P + 5 \left( \int_0^1 \int_0^1 \cosh(P) dx dy \right)^2 = 0$$

with the boundary condition  $P(x, 1) = 1$  on the upper edge and  $P = 0$  elsewhere on the boundary of the square. This can be done by approximating the continuous function  $P$  by its values on a grid,  $P_{n,m} \approx P(nh, mh)$ , with a small grid spacing  $h$ . The derivatives and integrals can then be approximated; for instance  $\partial_x^2 P(x, y) \approx (P(x+h, y) - 2P(x, y) + P(x-h, y))/h^2$ . The problem is then equivalent to finding the root of some function `residual(P)`, where `P` is a vector of length  $N_x N_y$ .

Now, because  $N_x N_y$  can be large, methods `hybr` or `lm` in `root` will take a long time to solve this problem. The solution can however be found using one of the large-scale solvers, for example `krylov`, `broyden2`, or `anderson`. These use what is known as the inexact Newton method, which instead of computing the Jacobian matrix exactly, forms an approximation for it.

The problem we have can now be solved as follows:

```
import numpy as np
from scipy.optimize import root
from numpy import cosh, zeros_like, mgrid, zeros

# parameters
nx, ny = 75, 75
hx, hy = 1./(nx-1), 1./(ny-1)

P_left, P_right = 0, 0
P_top, P_bottom = 1, 0

def residual(P):
    d2x = zeros_like(P)
    d2y = zeros_like(P)
```

```

d2x[1:-1] = (P[2:] - 2*P[1:-1] + P[:-2]) / hx/hx
d2x[0]    = (P[1] - 2*P[0] + P_left)/hx/hx
d2x[-1]   = (P_right - 2*P[-1] + P[-2])/hx/hx

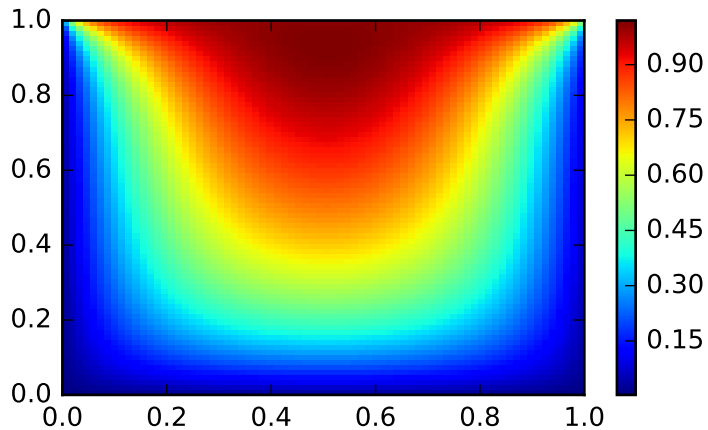
d2y[:,1:-1] = (P[:,2:] - 2*P[:,1:-1] + P[:, :-2])/hy/hy
d2y[:,0]    = (P[:,1] - 2*P[:,0] + P_bottom)/hy/hy
d2y[:, -1]  = (P_top - 2*P[:, -1] + P[:, -2])/hy/hy

return d2x + d2y + 5*cosh(P).mean()*2

# solve
guess = zeros((nx, ny), float)
sol = root(residual, guess, method='krylov', options={'disp': True})
#sol = root(residual, guess, method='broyden2', options={'disp': True, 'max_rank': 50})
#sol = root(residual, guess, method='anderson', options={'disp': True, 'M': 10})
print('Residual: %g' % abs(residual(sol.x)).max())

# visualize
import matplotlib.pyplot as plt
x, y = mgrid[0:1:(nx*1j), 0:1:(ny*1j)]
plt.pcolor(x, y, sol.x)
plt.colorbar()
plt.show()

```



### Still too slow? Preconditioning.

When looking for the zero of the functions  $f_i(\mathbf{x}) = 0$ ,  $i = 1, 2, \dots, N$ , the `krylov` solver spends most of its time inverting the Jacobian matrix,

$$J_{ij} = \frac{\partial f_i}{\partial x_j}.$$

If you have an approximation for the inverse matrix  $M \approx J^{-1}$ , you can use it for *preconditioning* the linear inversion problem. The idea is that instead of solving  $J\mathbf{s} = \mathbf{y}$  one solves  $MJ\mathbf{s} = M\mathbf{y}$ : since matrix  $MJ$  is “closer” to the identity matrix than  $J$  is, the equation should be easier for the Krylov method to deal with.

The matrix  $M$  can be passed to `root` with method `krylov` as an option `options['jac_options']['inner_M']`. It can be a (sparse) matrix or a

`scipy.sparse.linalg.LinearOperator` instance.

For the problem in the previous section, we note that the function to solve consists of two parts: the first one is application of the Laplace operator,  $[\partial_x^2 + \partial_y^2]P$ , and the second is the integral. We can actually easily compute the Jacobian corresponding to the Laplace operator part: we know that in one dimension

$$\partial_x^2 \approx \frac{1}{h_x^2} \begin{pmatrix} -2 & 1 & 0 & 0 \dots \\ 1 & -2 & 1 & 0 \dots \\ 0 & 1 & -2 & 1 \dots \\ \dots & & & \dots \end{pmatrix} = h_x^{-2} L$$

so that the whole 2-D operator is represented by

$$J_1 = \partial_x^2 + \partial_y^2 \simeq h_x^{-2} L \otimes I + h_y^{-2} I \otimes L$$

The matrix  $J_2$  of the Jacobian corresponding to the integral is more difficult to calculate, and since *all* of its entries are nonzero, it will be difficult to invert.  $J_1$  on the other hand is a relatively simple matrix, and can be inverted by `scipy.sparse.linalg.splu` (or the inverse can be approximated by `scipy.sparse.linalg.spilu`). So we are content to take  $M \approx J_1^{-1}$  and hope for the best.

In the example below, we use the preconditioner  $M = J_1^{-1}$ .

```
import numpy as np
from scipy.optimize import root
from scipy.sparse import spdiags, kron
from scipy.sparse.linalg import splu, LinearOperator
from numpy import cosh, zeros_like, mgrid, zeros, eye

# parameters
nx, ny = 75, 75
hx, hy = 1./(nx-1), 1./(ny-1)

P_left, P_right = 0, 0
P_top, P_bottom = 1, 0

def get_preconditioner():
    """Compute the preconditioner M"""
    diags_x = zeros((3, nx))
    diags_x[0,:] = 1/hx/hx
    diags_x[1,:] = -2/hx/hx
    diags_x[2,:] = 1/hx/hx
    Lx = spdiags(diags_x, [-1,0,1], nx, nx)

    diags_y = zeros((3, ny))
    diags_y[0,:] = 1/hy/hy
    diags_y[1,:] = -2/hy/hy
    diags_y[2,:] = 1/hy/hy
    Ly = spdiags(diags_y, [-1,0,1], ny, ny)

    J1 = kron(Lx, eye(ny)) + kron(eye(nx), Ly)

    # Now we have the matrix `J1`. We need to find its inverse `M` --
    # however, since an approximate inverse is enough, we can use
    # the *incomplete LU* decomposition

    J1_ilu = splu(J1)

    # This returns an object with a method .solve() that evaluates
    # the corresponding matrix-vector product. We need to wrap it into
```

```

# a LinearOperator before it can be passed to the Krylov methods:

M = LinearOperator(shape=(nx*ny, nx*ny), matvec=J1_ilu.solve)
return M

def solve(preconditioning=True):
    """Compute the solution"""
    count = [0]

    def residual(P):
        count[0] += 1

        d2x = zeros_like(P)
        d2y = zeros_like(P)

        d2x[1:-1] = (P[2:] - 2*P[1:-1] + P[:-2])/hx/hx
        d2x[0] = (P[1] - 2*P[0] + P_left)/hx/hx
        d2x[-1] = (P_right - 2*P[-1] + P[-2])/hx/hx

        d2y[:,1:-1] = (P[:,2:] - 2*P[:,1:-1] + P[:, :-2])/hy/hy
        d2y[:,0] = (P[:,1] - 2*P[:,0] + P_bottom)/hy/hy
        d2y[:, -1] = (P_top - 2*P[:, -1] + P[:, -2])/hy/hy

        return d2x + d2y + 5*cosh(P).mean()*2

    # preconditioner
    if preconditioning:
        M = get_preconditioner()
    else:
        M = None

    # solve
    guess = zeros((nx, ny), float)

    sol = root(residual, guess, method='krylov',
               options={'disp': True,
                        'jac_options': {'inner_M': M}})
    print 'Residual', abs(residual(sol.x)).max()
    print 'Evaluations', count[0]

    return sol.x

def main():
    sol = solve(preconditioning=True)

    # visualize
    import matplotlib.pyplot as plt
    x, y = mgrid[0:1:(nx*1j), 0:1:(ny*1j)]
    plt.clf()
    plt.pcolor(x, y, sol)
    plt.clim(0, 1)
    plt.colorbar()
    plt.show()

if __name__ == "__main__":
    main()

```

Resulting run, first without preconditioning:

```

0: |F(x)| = 803.614; step 1; tol 0.000257947
1: |F(x)| = 345.912; step 1; tol 0.166755
2: |F(x)| = 139.159; step 1; tol 0.145657
3: |F(x)| = 27.3682; step 1; tol 0.0348109
4: |F(x)| = 1.03303; step 1; tol 0.00128227
5: |F(x)| = 0.0406634; step 1; tol 0.00139451
6: |F(x)| = 0.00344341; step 1; tol 0.00645373
7: |F(x)| = 0.000153671; step 1; tol 0.00179246
8: |F(x)| = 6.7424e-06; step 1; tol 0.00173256
Residual 3.57078908664e-07
Evaluations 317

```

and then with preconditioning:

```

0: |F(x)| = 136.993; step 1; tol 7.49599e-06
1: |F(x)| = 4.80983; step 1; tol 0.00110945
2: |F(x)| = 0.195942; step 1; tol 0.00149362
3: |F(x)| = 0.000563597; step 1; tol 7.44604e-06
4: |F(x)| = 1.00698e-09; step 1; tol 2.87308e-12
Residual 9.29603061195e-11
Evaluations 77

```

Using a preconditioner reduced the number of evaluations of the `residual` function by a factor of 4. For problems where the residual is expensive to compute, good preconditioning can be crucial — it can even decide whether the problem is solvable in practice or not.

Preconditioning is an art, science, and industry. Here, we were lucky in making a simple choice that worked reasonably well, but there is a lot more depth to this topic than is shown here.

## References

Some further reading and related software:

### 3.1.6 Interpolation (`scipy.interpolate`)

#### Contents

- Interpolation (`scipy.interpolate`)
  - 1-D interpolation (`interp1d`)
  - Multivariate data interpolation (`griddata`)
  - Spline interpolation
    - \* Spline interpolation in 1-d: Procedural (`interpolate.splXXX`)
    - \* Spline interpolation in 1-d: Object-oriented (`UnivariateSpline`)
    - \* Two-dimensional spline representation: Procedural (`bisplrep`)
    - \* Two-dimensional spline representation: Object-oriented (`BivariateSpline`)
  - Using radial basis functions for smoothing/interpolation
    - \* 1-d Example
    - \* 2-d Example

There are several general interpolation facilities available in SciPy, for data in 1, 2, and higher dimensions:

- A class representing an interpolant (`interp1d`) in 1-D, offering several interpolation methods.
- Convenience function `griddata` offering a simple interface to interpolation in N dimensions (N = 1, 2, 3, 4, ...). Object-oriented interface for the underlying routines is also available.

- Functions for 1- and 2-dimensional (smoothed) cubic-spline interpolation, based on the FORTRAN library FITPACK. There are both procedural and object-oriented interfaces for the FITPACK library.
- Interpolation using Radial Basis Functions.

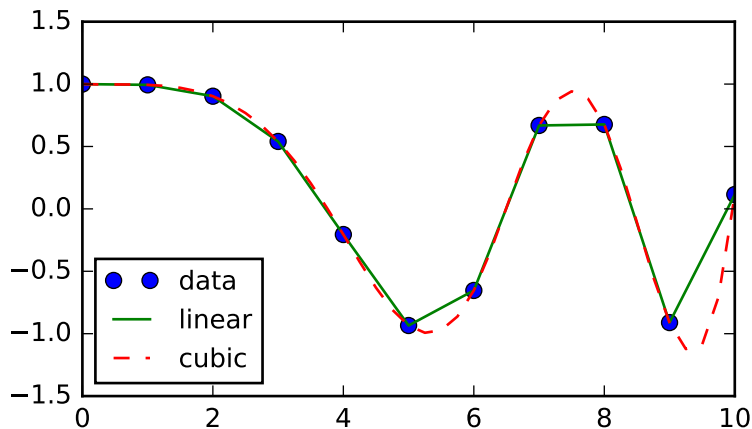
### 1-D interpolation (`interp1d`)

The `interp1d` class in `scipy.interpolate` is a convenient method to create a function based on fixed data points which can be evaluated anywhere within the domain defined by the given data using linear interpolation. An instance of this class is created by passing the 1-d vectors comprising the data. The instance of this class defines a `__call__` method and can therefore be treated like a function which interpolates between known data values to obtain unknown values (it also has a docstring for help). Behavior at the boundary can be specified at instantiation time. The following example demonstrates its use, for linear and cubic spline interpolation:

```
>>> from scipy.interpolate import interp1d

>>> x = np.linspace(0, 10, num=11, endpoint=True)
>>> y = np.cos(-x**2/9.0)
>>> f = interp1d(x, y)
>>> f2 = interp1d(x, y, kind='cubic')

>>> xnew = np.linspace(0, 10, num=41, endpoint=True)
>>> import matplotlib.pyplot as plt
>>> plt.plot(x, y, 'o', xnew, f(xnew), '-', xnew, f2(xnew), '--')
>>> plt.legend(['data', 'linear', 'cubic'], loc='best')
>>> plt.show()
```



### Multivariate data interpolation (`griddata`)

Suppose you have multidimensional data, for instance for an underlying function  $f(x, y)$  you only know the values at points  $(x[i], y[i])$  that do not form a regular grid.

Suppose we want to interpolate the 2-D function

```
>>> def func(x, y):
...     return x*(1-x)*np.cos(4*np.pi*x) * np.sin(4*np.pi*y**2)**2
```



on a grid in  $[0, 1] \times [0, 1]$

```
>>> grid_x, grid_y = np.mgrid[0:1:100j, 0:1:200j]
```

but we only know its values at 1000 data points:

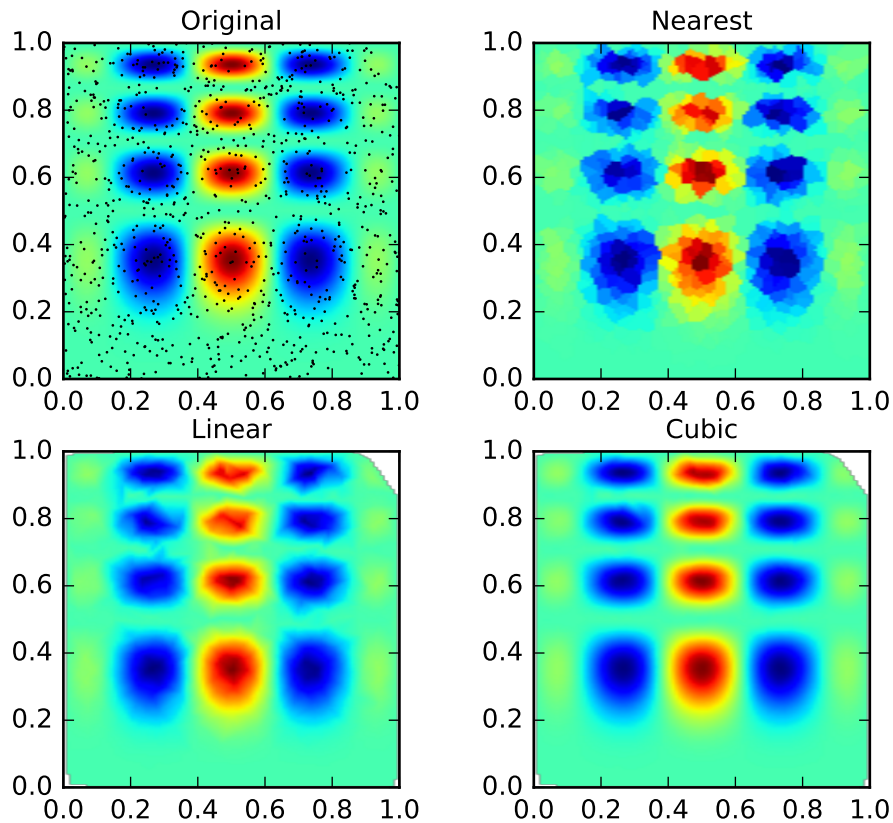
```
>>> points = np.random.rand(1000, 2)
>>> values = func(points[:,0], points[:,1])
```

This can be done with `griddata` – below we try out all of the interpolation methods:

```
>>> from scipy.interpolate import griddata
>>> grid_z0 = griddata(points, values, (grid_x, grid_y), method='nearest')
>>> grid_z1 = griddata(points, values, (grid_x, grid_y), method='linear')
>>> grid_z2 = griddata(points, values, (grid_x, grid_y), method='cubic')
```

One can see that the exact result is reproduced by all of the methods to some degree, but for this smooth function the piecewise cubic interpolant gives the best results:

```
>>> import matplotlib.pyplot as plt
>>> plt.subplot(221)
>>> plt.imshow(func(grid_x, grid_y).T, extent=(0,1,0,1), origin='lower')
>>> plt.plot(points[:,0], points[:,1], 'k.', ms=1)
>>> plt.title('Original')
>>> plt.subplot(222)
>>> plt.imshow(grid_z0.T, extent=(0,1,0,1), origin='lower')
>>> plt.title('Nearest')
>>> plt.subplot(223)
>>> plt.imshow(grid_z1.T, extent=(0,1,0,1), origin='lower')
>>> plt.title('Linear')
>>> plt.subplot(224)
>>> plt.imshow(grid_z2.T, extent=(0,1,0,1), origin='lower')
>>> plt.title('Cubic')
>>> plt.gcf().set_size_inches(6, 6)
>>> plt.show()
```



## Spline interpolation

### *Spline interpolation in 1-d: Procedural (interpolate.spIXXX)*

Spline interpolation requires two essential steps: (1) a spline representation of the curve is computed, and (2) the spline is evaluated at the desired points. In order to find the spline representation, there are two different ways to represent a curve and obtain (smoothing) spline coefficients: directly and parametrically. The direct method finds the spline representation of a curve in a two-dimensional plane using the function `splrep`. The first two arguments are the only ones required, and these provide the  $x$  and  $y$  components of the curve. The normal output is a 3-tuple,  $(t, c, k)$ , containing the knot-points  $t$ , the coefficients  $c$  and the order  $k$  of the spline. The default spline order is cubic, but this can be changed with the input keyword,  $k$ .

For curves in  $N$ -dimensional space the function `splprep` allows defining the curve parametrically. For this function only 1 input argument is required. This input is a list of  $N$ -arrays representing the curve in  $N$ -dimensional space. The length of each array is the number of curve points, and each array provides one component of the  $N$ -dimensional data point. The parameter variable is given with the keyword argument,  $u$ , which defaults to an equally-spaced monotonic sequence between 0 and 1. The default output consists of two objects: a 3-tuple,  $(t, c, k)$ , containing the spline

representation and the parameter variable  $u$ .

The keyword argument,  $s$ , is used to specify the amount of smoothing to perform during the spline fit. The default value of  $s$  is  $s = m - \sqrt{2m}$  where  $m$  is the number of data-points being fit. Therefore, **if no smoothing is desired a value of  $s = 0$  should be passed to the routines.**

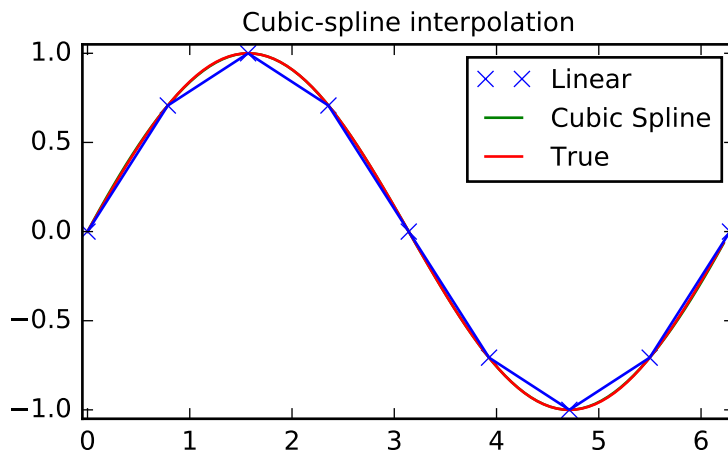
Once the spline representation of the data has been determined, functions are available for evaluating the spline (`splev`) and its derivatives (`splev`, `spalde`) at any point and the integral of the spline between any two points (`splint`). In addition, for cubic splines ( $k = 3$ ) with 8 or more knots, the roots of the spline can be estimated (`sproot`). These functions are demonstrated in the example that follows.

```
>>> import numpy as np
>>> import matplotlib.pyplot as plt
>>> from scipy import interpolate
```

Cubic-spline

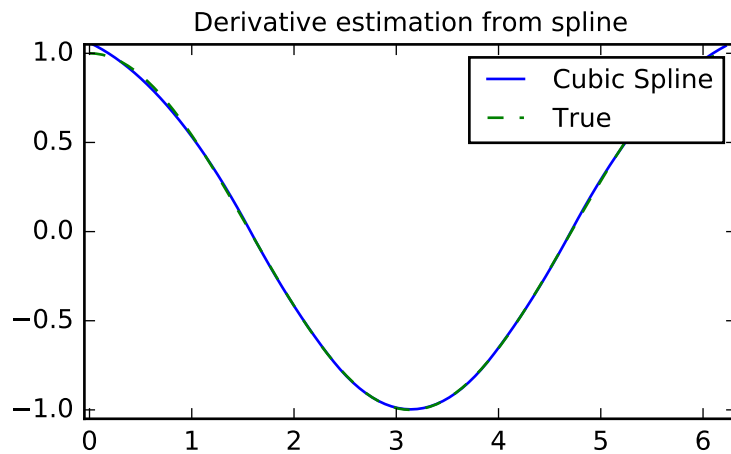
```
>>> x = np.arange(0, 2*np.pi+np.pi/4, 2*np.pi/8)
>>> y = np.sin(x)
>>> tck = interpolate.splrep(x, y, s=0)
>>> xnew = np.arange(0, 2*np.pi, np.pi/50)
>>> ynew = interpolate.splev(xnew, tck, der=0)

>>> plt.figure()
>>> plt.plot(x, y, 'x', xnew, ynew, xnew, np.sin(xnew), x, y, 'b')
>>> plt.legend(['Linear', 'Cubic Spline', 'True'])
>>> plt.axis([-0.05, 6.33, -1.05, 1.05])
>>> plt.title('Cubic-spline interpolation')
>>> plt.show()
```



Derivative of spline

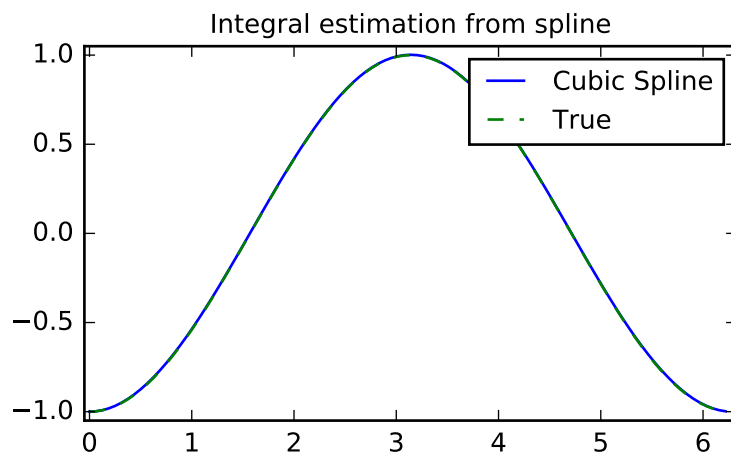
```
>>> yder = interpolate.splev(xnew, tck, der=1)
>>> plt.figure()
>>> plt.plot(xnew, yder, xnew, np.cos(xnew), '--')
>>> plt.legend(['Cubic Spline', 'True'])
>>> plt.axis([-0.05, 6.33, -1.05, 1.05])
>>> plt.title('Derivative estimation from spline')
>>> plt.show()
```



### Integral of spline

```
>>> def integ(x, tck, constant=-1):
...     x = np.atleast_1d(x)
...     out = np.zeros(x.shape, dtype=x.dtype)
...     for n in xrange(len(out)):
...         out[n] = interpolate.splint(0, x[n], tck)
...     out += constant
...     return out

>>> yint = integ(xnew, tck)
>>> plt.figure()
>>> plt.plot(xnew, yint, xnew, -np.cos(xnew), '--')
>>> plt.legend(['Cubic Spline', 'True'])
>>> plt.axis([-0.05, 6.33, -1.05, 1.05])
>>> plt.title('Integral estimation from spline')
>>> plt.show()
```



### Roots of spline

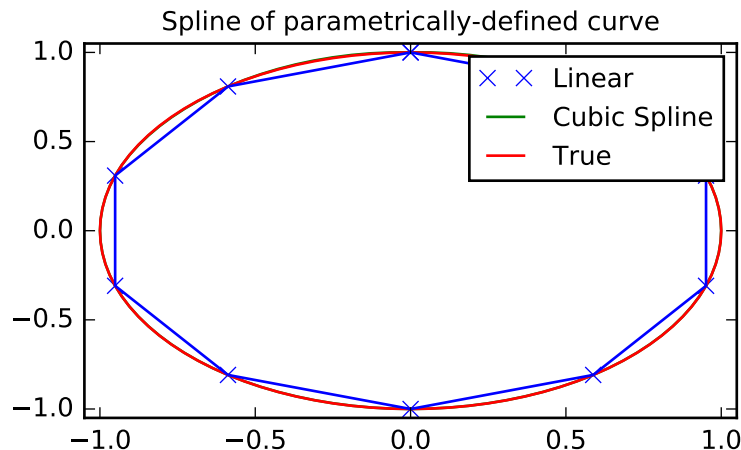
```
>>> interpolate.sproot(tck)
array([3.1416])
```

Notice that `sproot` failed to find an obvious solution at the edge of the approximation interval,  $x = 0$ . If we define the spline on a slightly larger interval, we recover both roots  $x = 0$  and  $x = 2\pi$ :

```
>>> x = np.linspace(-np.pi/4, 2.*np.pi + np.pi/4, 21)
>>> y = np.sin(x)
>>> tck = interpolate.splrep(x, y, s=0)
>>> interpolate.sproot(tck)
array([0., 3.1416])
```

### Parametric spline

```
>>> t = np.arange(0, 1.1, .1)
>>> x = np.sin(2*np.pi*t)
>>> y = np.cos(2*np.pi*t)
>>> tck, u = interpolate.splprep([x, y], s=0)
>>> unew = np.arange(0, 1.01, 0.01)
>>> out = interpolate.splev(unew, tck)
>>> plt.figure()
>>> plt.plot(x, y, 'x', out[0], out[1], np.sin(2*np.pi*unew), np.cos(2*np.pi*unew), x, y, 'b')
>>> plt.legend(['Linear', 'Cubic Spline', 'True'])
>>> plt.axis([-1.05, 1.05, -1.05, 1.05])
>>> plt.title('Spline of parametrically-defined curve')
>>> plt.show()
```



### *Spline interpolation in 1-d: Object-oriented (UnivariateSpline)*

The spline-fitting capabilities described above are also available via an object-oriented interface. The one dimensional splines are objects of the `UnivariateSpline` class, and are created with the  $x$  and  $y$  components of the curve provided as arguments to the constructor. The class defines `__call__`, allowing the object to be called with the x-axis values at which the spline should be evaluated, returning the interpolated y-values. This is shown in the example below for the subclass `InterpolatedUnivariateSpline`. The `integral`, `derivatives`, and `roots` methods are also available on `UnivariateSpline` objects, allowing definite integrals, derivatives, and roots to be computed for the spline.

The `UnivariateSpline` class can also be used to smooth data by providing a non-zero value of the smoothing parameter `s`, with the same meaning as the `s` keyword of the `splrep` function described above. This results in a spline that has fewer knots than the number of data points, and hence is no longer strictly an interpolating spline, but rather a smoothing spline. If this is not desired, the `InterpolatedUnivariateSpline` class is available. It is a subclass of `UnivariateSpline` that always passes through all points (equivalent to forcing the smoothing parameter to 0). This class is demonstrated in the example below.

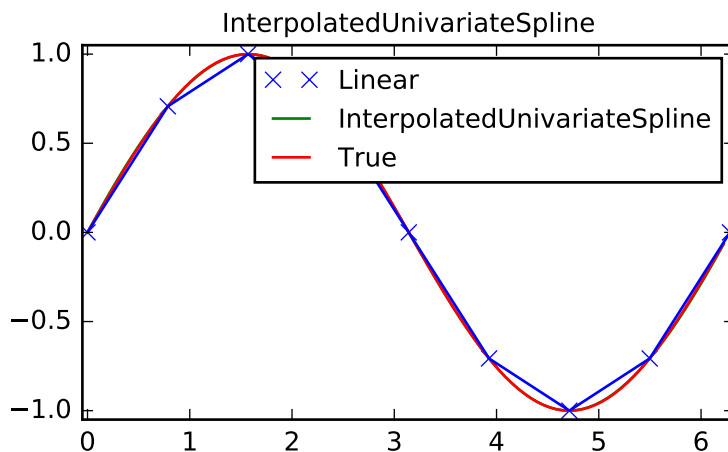
The `LSQUnivariateSpline` class is the other subclass of `UnivariateSpline`. It allows the user to specify the number and location of internal knots explicitly with the parameter `t`. This allows creation of customized splines with non-linear spacing, to interpolate in some domains and smooth in others, or change the character of the spline.

```
>>> import numpy as np
>>> import matplotlib.pyplot as plt
>>> from scipy import interpolate
```

### InterpolatedUnivariateSpline

```
>>> x = np.arange(0, 2*np.pi+np.pi/4, 2*np.pi/8)
>>> y = np.sin(x)
>>> s = interpolate.InterpolatedUnivariateSpline(x, y)
>>> xnew = np.arange(0, 2*np.pi, np.pi/50)
>>> ynew = s(xnew)

>>> plt.figure()
>>> plt.plot(x, y, 'x', xnew, ynew, xnew, np.sin(xnew), x, y, 'b')
>>> plt.legend(['Linear', 'InterpolatedUnivariateSpline', 'True'])
>>> plt.axis([-0.05, 6.33, -1.05, 1.05])
>>> plt.title('InterpolatedUnivariateSpline')
>>> plt.show()
```

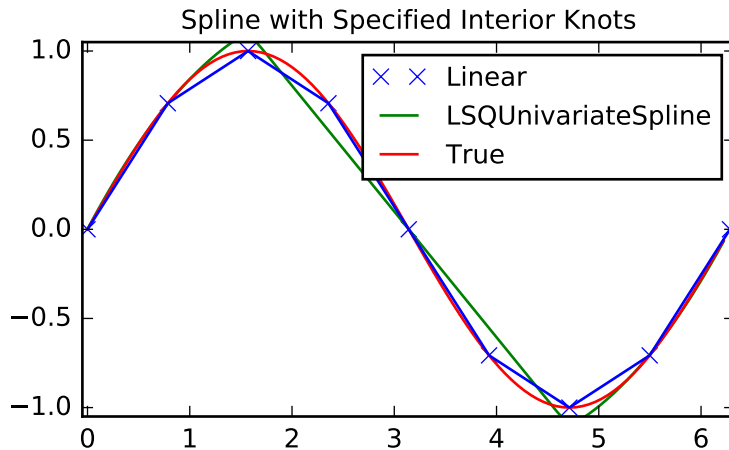


### LSQUnivariateSpline with non-uniform knots

```
>>> t = [np.pi/2-.1, np.pi/2+.1, 3*np.pi/2-.1, 3*np.pi/2+.1]
>>> s = interpolate.LSQUnivariateSpline(x, y, t, k=2)
>>> ynew = s(xnew)

>>> plt.figure()
>>> plt.plot(x, y, 'x', xnew, ynew, xnew, np.sin(xnew), x, y, 'b')
>>> plt.legend(['Linear', 'LSQUnivariateSpline', 'True'])
```

```
>>> plt.axis([-0.05, 6.33, -1.05, 1.05])
>>> plt.title('Spline with Specified Interior Knots')
>>> plt.show()
```



### Two-dimensional spline representation: Procedural (`bisplrep`)

For (smooth) spline-fitting to a two dimensional surface, the function `bisplrep` is available. This function takes as required inputs the **1-D** arrays `x`, `y`, and `z` which represent points on the surface  $z = f(x, y)$ . The default output is a list `[tx, ty, c, kx, ky]` whose entries represent respectively, the components of the knot positions, the coefficients of the spline, and the order of the spline in each coordinate. It is convenient to hold this list in a single object, `tck`, so that it can be passed easily to the function `bisplev`. The keyword, `s`, can be used to change the amount of smoothing performed on the data while determining the appropriate spline. The default value is  $s = m - \sqrt{2m}$  where  $m$  is the number of data points in the `x`, `y`, and `z` vectors. As a result, if no smoothing is desired, then  $s = 0$  should be passed to `bisplrep`.

To evaluate the two-dimensional spline and its partial derivatives (up to the order of the spline), the function `bisplev` is required. This function takes as the first two arguments **two 1-D arrays** whose cross-product specifies the domain over which to evaluate the spline. The third argument is the `tck` list returned from `bisplrep`. If desired, the fourth and fifth arguments provide the orders of the partial derivative in the `x` and `y` direction respectively.

It is important to note that two dimensional interpolation should not be used to find the spline representation of images. The algorithm used is not amenable to large numbers of input points. The signal processing toolbox contains more appropriate algorithms for finding the spline representation of an image. The two dimensional interpolation commands are intended for use when interpolating a two dimensional function as shown in the example that follows. This example uses the `mgrid` command in NumPy which is useful for defining a “mesh-grid” in many dimensions. (See also the `ogrid` command if the full-mesh is not needed). The number of output arguments and the number of dimensions of each argument is determined by the number of indexing objects passed in `mgrid`.

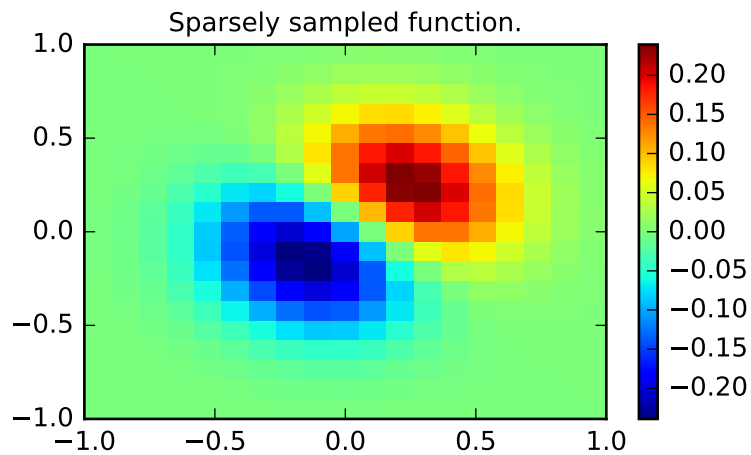
```
>>> import numpy as np
>>> from scipy import interpolate
>>> import matplotlib.pyplot as plt
```

Define function over sparse 20x20 grid

```
>>> x, y = np.mgrid[-1:1:20j, -1:1:20j]
>>> z = (x+y) * np.exp(-6.0*(x*x+y*y))
```

```

>>> plt.figure()
>>> plt.pcolor(x, y, z)
>>> plt.colorbar()
>>> plt.title("Sparsely sampled function.")
>>> plt.show()
    
```

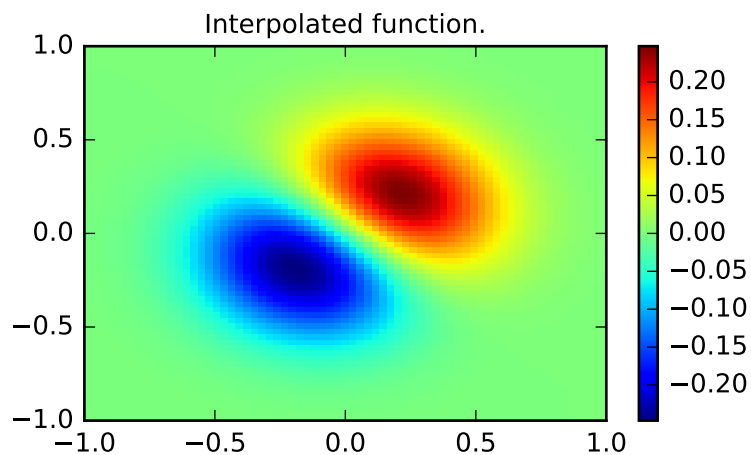


Interpolate function over new 70x70 grid

```

>>> xnew, ynew = np.mgrid[-1:1:70j, -1:1:70j]
>>> tck = interpolate.bisplrep(x, y, z, s=0)
>>> znew = interpolate.bisplev(xnew[:,0], ynew[0,:], tck)

>>> plt.figure()
>>> plt.pcolor(xnew, ynew, znew)
>>> plt.colorbar()
>>> plt.title("Interpolated function.")
>>> plt.show()
    
```





**Two-dimensional spline representation: Object-oriented (*BivariateSpline*)**

The `BivariateSpline` class is the 2-dimensional analog of the `UnivariateSpline` class. It and its subclasses implement the FITPACK functions described above in an object oriented fashion, allowing objects to be instantiated that can be called to compute the spline value by passing in the two coordinates as the two arguments.

**Using radial basis functions for smoothing/interpolation**

Radial basis functions can be used for smoothing/interpolating scattered data in n-dimensions, but should be used with caution for extrapolation outside of the observed data range.

**1-d Example**

This example compares the usage of the `Rbf` and `UnivariateSpline` classes from the `scipy.interpolate` module.

```
>>> import numpy as np
>>> from scipy.interpolate import Rbf, InterpolatedUnivariateSpline
>>> import matplotlib.pyplot as plt

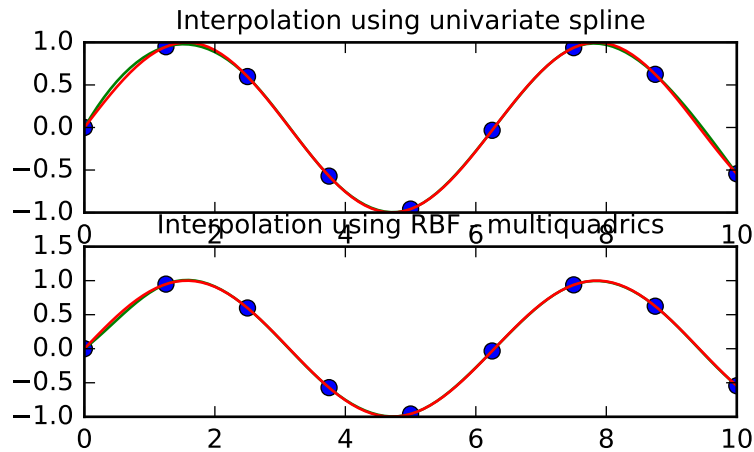
>>> # setup data
>>> x = np.linspace(0, 10, 9)
>>> y = np.sin(x)
>>> xi = np.linspace(0, 10, 101)

>>> # use fitpack2 method
>>> ius = InterpolatedUnivariateSpline(x, y)
>>> yi = ius(xi)

>>> plt.subplot(2, 1, 1)
>>> plt.plot(x, y, 'bo')
>>> plt.plot(xi, yi, 'g')
>>> plt.plot(xi, np.sin(xi), 'r')
>>> plt.title('Interpolation using univariate spline')

>>> # use RBF method
>>> rbf = Rbf(x, y)
>>> fi = rbf(xi)

>>> plt.subplot(2, 1, 2)
>>> plt.plot(x, y, 'bo')
>>> plt.plot(xi, fi, 'g')
>>> plt.plot(xi, np.sin(xi), 'r')
>>> plt.title('Interpolation using RBF - multiquadrics')
>>> plt.show()
```



### 2-d Example

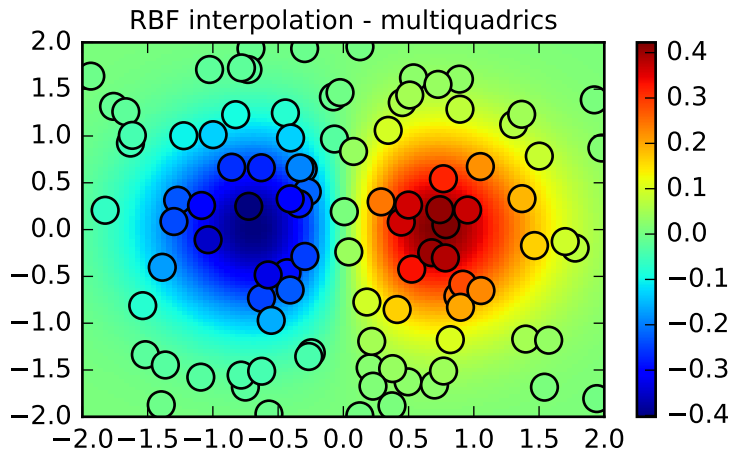
This example shows how to interpolate scattered 2d data.

```
>>> import numpy as np
>>> from scipy.interpolate import Rbf
>>> import matplotlib.pyplot as plt
>>> from matplotlib import cm

>>> # 2-d tests - setup scattered data
>>> x = np.random.rand(100)*4.0-2.0
>>> y = np.random.rand(100)*4.0-2.0
>>> z = x*np.exp(-x**2-y**2)
>>> ti = np.linspace(-2.0, 2.0, 100)
>>> XI, YI = np.meshgrid(ti, ti)

>>> # use RBF
>>> rbf = Rbf(x, y, z, epsilon=2)
>>> ZI = rbf(XI, YI)

>>> # plot the result
>>> plt.subplot(1, 1, 1)
>>> plt.pcolor(XI, YI, ZI, cmap=cm.jet)
>>> plt.scatter(x, y, 100, z, cmap=cm.jet)
>>> plt.title('RBF interpolation - multiquadrics')
>>> plt.xlim(-2, 2)
>>> plt.ylim(-2, 2)
>>> plt.colorbar()
```



### 3.1.7 Fourier Transforms (`scipy.fftpack`)

#### Contents

- Fourier Transforms (`scipy.fftpack`)
  - Fast Fourier transforms
    - \* One dimensional discrete Fourier transforms
    - \* Two and n-dimensional discrete Fourier transforms
    - \* FFT convolution
  - Discrete Cosine Transforms
    - \* Type I DCT
    - \* Type II DCT
    - \* Type III DCT
    - \* DCT and IDCT
    - \* Example
  - Discrete Sine Transforms
    - \* Type I DST
    - \* Type II DST
    - \* Type III DST
    - \* DST and IDST
  - Cache Destruction
  - References

Fourier analysis is a method for expressing a function as a sum of periodic components, and for recovering the signal from those components. When both the function and its Fourier transform are replaced with discretized counterparts, it is called the discrete Fourier transform (DFT). The DFT has become a mainstay of numerical computing in part because of a very fast algorithm for computing it, called the Fast Fourier Transform (FFT), which was known to Gauss (1805) and was brought to light in its current form by Cooley and Tukey [CT65]. Press et al. [NR] provide an accessible introduction to Fourier analysis and its applications.

**Note:** PyFFTW provides a way to replace a number of functions in `scipy.fftpack` with its own functions, which are usually significantly faster, via `pyfftw.interfaces`. Because PyFFTW relies on the GPL-licensed FFTW it cannot be included in SciPy. Users for whom the speed of FFT routines is critical should consider installing PyFFTW.

## Fast Fourier transforms

### One dimensional discrete Fourier transforms

The FFT  $y[k]$  of length  $N$  of the length- $N$  sequence  $x[n]$  is defined as

$$y[k] = \sum_{n=0}^{N-1} e^{-2\pi j \frac{kn}{N}} x[n],$$

and the inverse transform is defined as follows

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} e^{2\pi j \frac{kn}{N}} y[k].$$

These transforms can be calculated by means of `fft` and `ifft`, respectively as shown in the following example.

```
>>> from scipy.fftpack import fft, ifft
>>> x = np.array([1.0, 2.0, 1.0, -1.0, 1.5])
>>> y = fft(x)
>>> y
array([ 4.50000000+0.j          ,  2.08155948-1.65109876j,
       -1.83155948+1.60822041j, -1.83155948-1.60822041j,
        2.08155948+1.65109876j])
>>> yinv = ifft(y)
>>> yinv
array([ 1.0+0.j,  2.0+0.j,  1.0+0.j, -1.0+0.j,  1.5+0.j])
```

From the definition of the FFT it can be seen that

$$y[0] = \sum_{n=0}^{N-1} x[n].$$

In the example

```
>>> np.sum(x)
4.5
```

which corresponds to  $y[0]$ . For  $N$  even, the elements  $y[1] \dots y[N/2 - 1]$  contain the positive-frequency terms, and the elements  $y[N/2] \dots y[N - 1]$  contain the negative-frequency terms, in order of decreasingly negative frequency. For  $N$  odd, the elements  $y[1] \dots y[(N - 1)/2]$  contain the positive-frequency terms, and the elements  $y[(N + 1)/2] \dots y[N - 1]$  contain the negative-frequency terms, in order of decreasingly negative frequency.

In case the sequence  $x$  is real-valued, the values of  $y[n]$  for positive frequencies is the conjugate of the values  $y[n]$  for negative frequencies (because the spectrum is symmetric). Typically, only the FFT corresponding to positive frequencies is plotted.

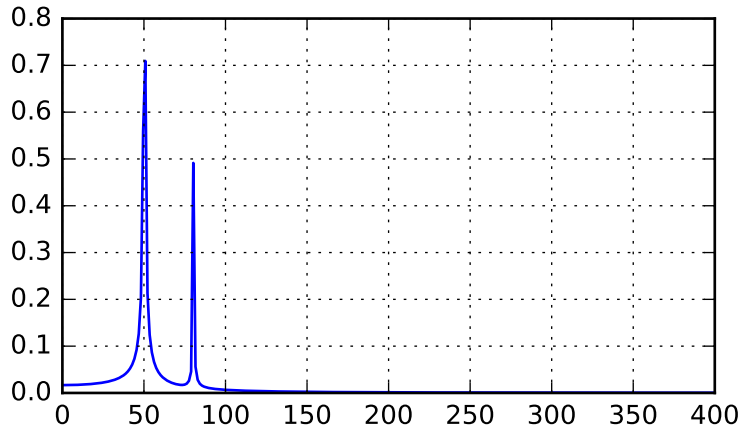
The example plots the FFT of the sum of two sines.

```
>>> from scipy.fftpack import fft
>>> # Number of sample points
>>> N = 600
>>> # sample spacing
>>> T = 1.0 / 800.0
>>> x = np.linspace(0.0, N*T, N)
>>> y = np.sin(50.0 * 2.0*np.pi*x) + 0.5*np.sin(80.0 * 2.0*np.pi*x)
>>> yf = fft(y)
```

```

>>> xf = np.linspace(0.0, 1.0/(2.0*T), N/2)
>>> import matplotlib.pyplot as plt
>>> plt.plot(xf, 2.0/N * np.abs(yf[0:N/2]))
>>> plt.grid()
>>> plt.show()

```

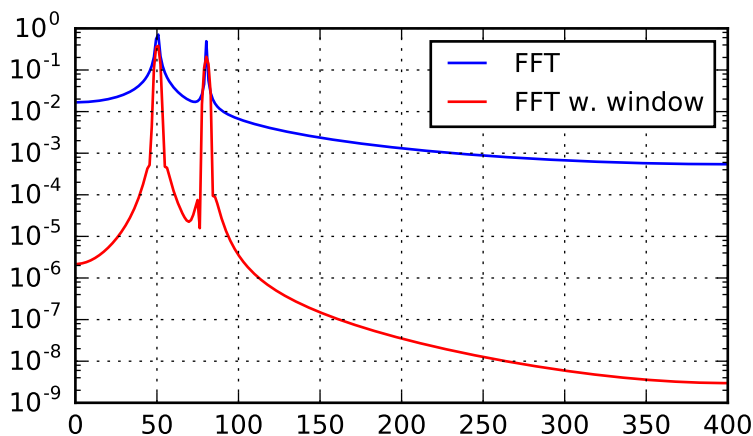


The FFT input signal is inherently truncated. This truncation can be modelled as multiplication of an infinite signal with a rectangular window function. In the spectral domain this multiplication becomes convolution of the signal spectrum with the window function spectrum, being of form  $\sin(x)/x$ . This convolution is the cause of an effect called spectral leakage (see [WPW]). Windowing the signal with a dedicated window function helps mitigate spectral leakage. The example below uses a Blackman window from `scipy.signal` and shows the effect of windowing (the zero component of the FFT has been truncated for illustrative purposes).

```

>>> from scipy.fftpack import fft
>>> # Number of sample points
>>> N = 600
>>> # sample spacing
>>> T = 1.0 / 800.0
>>> x = np.linspace(0.0, N*T, N)
>>> y = np.sin(50.0 * 2.0*np.pi*x) + 0.5*np.sin(80.0 * 2.0*np.pi*x)
>>> yf = fft(y)
>>> from scipy.signal import blackman
>>> w = blackman(N)
>>> ywf = fft(y*w)
>>> xf = np.linspace(0.0, 1.0/(2.0*T), N/2)
>>> import matplotlib.pyplot as plt
>>> plt.semilogy(xf[1:N/2], 2.0/N * np.abs(yf[1:N/2]), '-b')
>>> plt.semilogy(xf[1:N/2], 2.0/N * np.abs(ywf[1:N/2]), '-r')
>>> plt.legend(['FFT', 'FFT w. window'])
>>> plt.grid()
>>> plt.show()

```



In case the sequence  $x$  is complex-valued, the spectrum is no longer symmetric. To simplify working with the FFT functions, `scipy` provides the following two helper functions.

The function `fftfreq` returns the FFT sample frequency points.

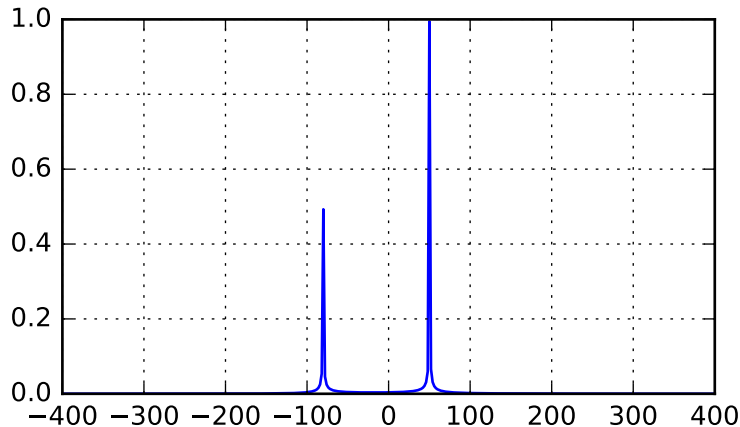
```
>>> from scipy.fftpack import fftfreq
>>> freq = fftfreq(8, 0.125)
>>> freq
array([ 0.,  1.,  2.,  3., -4., -3., -2., -1.])
```

In a similar spirit, the function `fftshift` allows swapping the lower and upper halves of a vector, so that it becomes suitable for display.

```
>>> from scipy.fftpack import fftshift
>>> x = np.arange(8)
>>> fftshift(x)
array([4, 5, 6, 7, 0, 1, 2, 3])
```

The example below plots the FFT of two complex exponentials; note the asymmetric spectrum.

```
>>> from scipy.fftpack import fft, fftfreq, fftshift
>>> # number of signal points
>>> N = 400
>>> # sample spacing
>>> T = 1.0 / 800.0
>>> x = np.linspace(0.0, N*T, N)
>>> y = np.exp(50.0 * 1.j * 2.0*np.pi*x) + 0.5*np.exp(-80.0 * 1.j * 2.0*np.pi*x)
>>> yf = fft(y)
>>> xf = fftfreq(N, T)
>>> xf = fftshift(xf)
>>> yplot = fftshift(yf)
>>> import matplotlib.pyplot as plt
>>> plt.plot(xf, 1.0/N * np.abs(yplot))
>>> plt.grid()
>>> plt.show()
```



The function `rfft` calculates the FFT of a real sequence and outputs the FFT coefficients  $y[n]$  with separate real and imaginary parts. In case of  $N$  being even:  $[y[0], \text{Re}(y[1]), \text{Im}(y[1]), \dots, \text{Re}(y[N/2])]$ ; in case  $N$  being odd  $[y[0], \text{Re}(y[1]), \text{Im}(y[1]), \dots, \text{Re}(y[N/2]), \text{Im}(y[N/2])]$ .

The corresponding function `irfft` calculates the IFFT of the FFT coefficients with this special ordering.

```
>>> from scipy.fftpack import fft, rfft, irfft
>>> x = np.array([1.0, 2.0, 1.0, -1.0, 1.5, 1.0])
>>> fft(x)
array([ 5.50+0.j          ,  2.25-0.4330127j , -2.75-1.29903811j,
        1.50+0.j          , -2.75+1.29903811j,  2.25+0.4330127j ])
>>> yr = rfft(x)
>>> yr
array([ 5.5          ,  2.25          , -0.4330127 , -2.75          , -1.29903811,
        1.5          ])
>>> irfft(yr)
array([ 1. ,  2. ,  1. , -1. ,  1.5,  1. ])
>>> x = np.array([1.0, 2.0, 1.0, -1.0, 1.5])
>>> fft(x)
array([ 4.50000000+0.j          ,  2.08155948-1.65109876j,
        -1.83155948+1.60822041j, -1.83155948-1.60822041j,
        2.08155948+1.65109876j])
>>> yr = rfft(x)
>>> yr
array([ 4.5          ,  2.08155948, -1.65109876, -1.83155948,  1.60822041])
```

### Two and n-dimensional discrete Fourier transforms

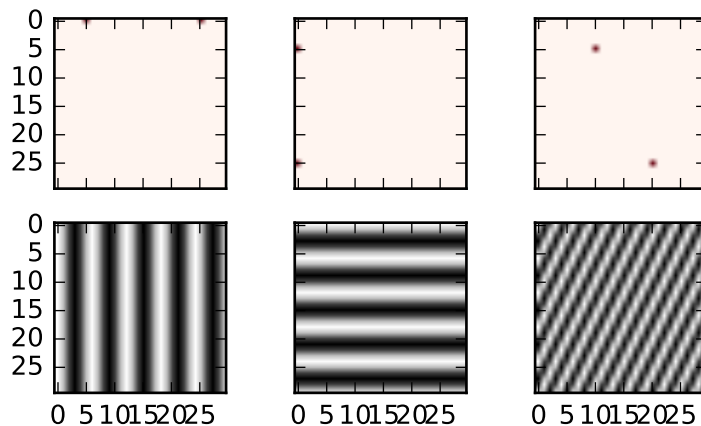
The functions `fft2` and `ifft2` provide 2-dimensional FFT, and IFFT, respectively. Similar, `fftn` and `ifftn` provide n-dimensional FFT, and IFFT, respectively.

The example below demonstrates a 2-dimensional IFFT and plots the resulting (2-dimensional) time-domain signals.

```
>>> from scipy.fftpack import ifftn
>>> import matplotlib.pyplot as plt
>>> import matplotlib.cm as cm
>>> N = 30
>>> f, ((ax1, ax2, ax3), (ax4, ax5, ax6)) = plt.subplots(2, 3, sharex='col', sharey='row')
>>> xf = np.zeros((N,N))
```

```

>>> xf[0, 5] = 1
>>> xf[0, N-5] = 1
>>> Z = ifftn(xf)
>>> ax1.imshow(xf, cmap=cm.Reds)
>>> ax4.imshow(np.real(Z), cmap=cm.gray)
>>> xf = np.zeros((N, N))
>>> xf[5, 0] = 1
>>> xf[N-5, 0] = 1
>>> Z = ifftn(xf)
>>> ax2.imshow(xf, cmap=cm.Reds)
>>> ax5.imshow(np.real(Z), cmap=cm.gray)
>>> xf = np.zeros((N, N))
>>> xf[5, 10] = 1
>>> xf[N-5, N-10] = 1
>>> Z = ifftn(xf)
>>> ax3.imshow(xf, cmap=cm.Reds)
>>> ax6.imshow(np.real(Z), cmap=cm.gray)
>>> plt.show()
    
```



### ***FFT convolution***

`scipy.fftpack.convolve` performs a convolution of two one-dimensional arrays in frequency domain.

## **Discrete Cosine Transforms**

SciPy provides a DCT with the function `dct` and a corresponding IDCT with the function `idct`. There are 8 types of the DCT [WPC], [Mak]; however, only the first 3 types are implemented in SciPy. “The” DCT generally refers to DCT type 2, and “the” Inverse DCT generally refers to DCT type 3. In addition, the DCT coefficients can be normalized differently (for most types, SciPy provides `None` and `ortho`). Two parameters of the `dct/idct` function calls allow setting the DCT type and coefficient normalization.

For a single dimension array `x`, `dct(x, norm='ortho')` is equal to MATLAB `dct(x)`.



### Type I DCT

SciPy uses the following definition of the unnormalized DCT-I (`norm='None'`):

$$y[k] = x_0 + (-1)^k x_{N-1} + 2 \sum_{n=1}^{N-2} x[n] \cos\left(\frac{\pi n k}{N-1}\right), \quad 0 \leq k < N.$$

Only `None` is supported as normalization mode for DCT-I. Note also that the DCT-I is only supported for input size  $> 1$ .

### Type II DCT

SciPy uses the following definition of the unnormalized DCT-II (`norm='None'`):

$$y[k] = 2 \sum_{n=0}^{N-1} x[n] \cos\left(\frac{\pi(2n+1)k}{2N}\right) \quad 0 \leq k < N.$$

In case of the normalized DCT (`norm='ortho'`), the DCT coefficients  $y[k]$  are multiplied by a scaling factor  $f$ :

$$f = \begin{cases} \sqrt{1/(4N)}, & \text{if } k = 0 \\ \sqrt{1/(2N)}, & \text{otherwise} \end{cases}.$$

In this case, the DCT “base functions”  $\phi_k[n] = 2f \cos\left(\frac{\pi(2n+1)k}{2N}\right)$  become orthonormal:

$$\sum_{n=0}^{N-1} \phi_k[n] \phi_l[n] = \delta_{lk}$$

### Type III DCT

SciPy uses the following definition of the unnormalized DCT-III (`norm='None'`):

$$y[k] = x_0 + 2 \sum_{n=1}^{N-1} x[n] \cos\left(\frac{\pi n(2k+1)}{2N}\right) \quad 0 \leq k < N,$$

or, for `norm='ortho'`:

$$y[k] = \frac{x_0}{\sqrt{N}} + \frac{2}{\sqrt{N}} \sum_{n=1}^{N-1} x[n] \cos\left(\frac{\pi n(2k+1)}{2N}\right) \quad 0 \leq k < N.$$

### DCT and IDCT

The (unnormalized) DCT-III is the inverse of the (unnormalized) DCT-II, up to a factor  $2N$ . The orthonormalized DCT-III is exactly the inverse of the orthonormalized DCT-II. The function `idct` performs the mappings between the DCT and IDCT types.

The example below shows the relation between DCT and IDCT for different types and normalizations.

```
>>> from scipy.fftpack import dct, idct
>>> x = np.array([1.0, 2.0, 1.0, -1.0, 1.5])
>>> dct(dct(x, type=2, norm='ortho'), type=3, norm='ortho')
[1.0, 2.0, 1.0, -1.0, 1.5]
>>> # scaling factor 2*N = 10
>>> idct(dct(x, type=2), type=2)
array([ 10., 20., 10., -10., 15.])
>>> # no scaling factor
```

```

>>> idct(dct(x, type=2, norm='ortho'), type=2, norm='ortho')
array([ 1. ,  2. ,  1. , -1. ,  1.5])
>>> # scaling factor 2*N = 10
>>> idct(dct(x, type=3), type=3)
array([ 10.,  20.,  10., -10.,  15.])
>>> # no scaling factor
>>> idct(dct(x, type=3, norm='ortho'), type=3, norm='ortho')
array([ 1. ,  2. ,  1. , -1. ,  1.5])
>>> # scaling factor 2*(N-1) = 8
>>> idct(dct(x, type=1), type=1)
array([ 8.,  16.,  8., -8.,  12.])
    
```

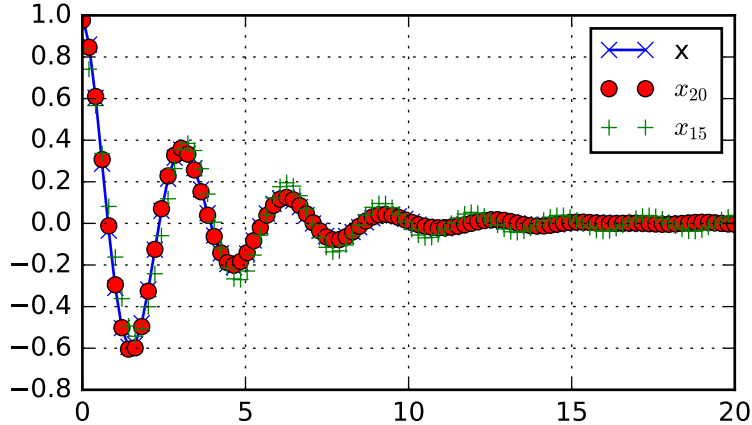
### Example

The DCT exhibits the “energy compaction property”, meaning that for many signals only the first few DCT coefficients have significant magnitude. Zeroing out the other coefficients leads to a small reconstruction error, a fact which is exploited in lossy signal compression (e.g. JPEG compression).

The example below shows a signal  $x$  and two reconstructions ( $x_{20}$  and  $x_{15}$ ) from the signal’s DCT coefficients. The signal  $x_{20}$  is reconstructed from the first 20 DCT coefficients,  $x_{15}$  is reconstructed from the first 15 DCT coefficients. It can be seen that the relative error of using 20 coefficients is still very small ( $\sim 0.1\%$ ), but provides a five-fold compression rate.

```

>>> from scipy.fftpack import dct, idct
>>> import matplotlib.pyplot as plt
>>> N = 100
>>> t = np.linspace(0, 20, N)
>>> x = np.exp(-t/3)*np.cos(2*t)
>>> y = dct(x, norm='ortho')
>>> window = np.zeros(N)
>>> window[:20] = 1
>>> yr = idct(y*window, norm='ortho')
>>> sum(abs(x-yr)**2) / sum(abs(x)**2)
0.0010901402257
>>> plt.plot(t, x, '-bx')
>>> plt.plot(t, yr, 'ro')
>>> window = np.zeros(N)
>>> window[:15] = 1
>>> yr = idct(y*window, norm='ortho')
>>> sum(abs(x-yr)**2) / sum(abs(x)**2)
0.0718818065008
>>> plt.plot(t, yr, 'g+')
>>> plt.legend(['x', '$x_{20}$', '$x_{15}$'])
>>> plt.grid()
>>> plt.show()
    
```



## Discrete Sine Transforms

SciPy provides a DST [Mak] with the function `dst` and a corresponding IDST with the function `idst`.

There are theoretically 8 types of the DST for different combinations of even/odd boundary conditions and boundary off sets [WPS], only the first 3 types are implemented in scipy.

### Type I DST

DST-I assumes the input is odd around  $n=-1$  and  $n=N$ . SciPy uses the following definition of the unnormalized DST-I (`norm='None'`):

$$y[k] = 2 \sum_{n=0}^{N-1} x[n] \sin\left(\frac{\pi(n+1)(k+1)}{N+1}\right), \quad 0 \leq k < N.$$

Only `None` is supported as normalization mode for DST-I. Note also that the DST-I is only supported for input size  $> 1$ . The (unnormalized) DST-I is its own inverse, up to a factor  $2(N+1)$ .

### Type II DST

DST-II assumes the input is odd around  $n=-1/2$  and even around  $n=N$ . SciPy uses the following definition of the unnormalized DST-II (`norm='None'`):

$$y[k] = 2 \sum_{n=0}^{N-1} x[n] \sin\left(\frac{\pi(n+1/2)(k+1)}{N}\right), \quad 0 \leq k < N.$$

### Type III DST

DST-III assumes the input is odd around  $n=-1$  and even around  $n=N-1$ . SciPy uses the following definition of the unnormalized DST-III (`norm='None'`):

$$y[k] = (-1)^k x[N-1] + 2 \sum_{n=0}^{N-2} x[n] \sin\left(\frac{\pi(n+1)(k+1/2)}{N}\right), \quad 0 \leq k < N.$$

## DST and IDST

The example below shows the relation between DST and IDST for different types and normalizations.

```

>>> from scipy.fftpack import dst, idst
>>> x = np.array([1.0, 2.0, 1.0, -1.0, 1.5])
>>> # scaling factor 2*N = 10
>>> idst(dst(x, type=2), type=2)
array([ 10.,  20.,  10., -10.,  15.])
>>> # no scaling factor
>>> idst(dst(x, type=2, norm='ortho'), type=2, norm='ortho')
array([ 1. ,  2. ,  1. , -1. ,  1.5])
>>> # scaling factor 2*N = 10
>>> idst(dst(x, type=3), type=3)
array([ 10.,  20.,  10., -10.,  15.])
>>> # no scaling factor
>>> idst(dst(x, type=3, norm='ortho'), type=3, norm='ortho')
array([ 1. ,  2. ,  1. , -1. ,  1.5])
>>> # scaling factor 2*(N+1) = 8
>>> idst(dst(x, type=1), type=1)
array([ 12.,  24.,  12., -12.,  18.])

```

## Cache Destruction

To accelerate repeat transforms on arrays of the same shape and dtype, `scipy.fftpack` keeps a cache of the prime factorization of length of the array and pre-computed trigonometric functions. These caches can be destroyed by calling the appropriate function in `scipy.fftpack._fftpack`. `dst(type=1)` and `idst(type=1)` share a cache (`*dst1_cache`). As do `dst(type=2)`, `dst(type=3)`, `idst(type=3)`, and `idst(type=3)` (`*dst2_cache`).

## References

### 3.1.8 Signal Processing (`scipy.signal`)

The signal processing toolbox currently contains some filtering functions, a limited set of filter design tools, and a few B-spline interpolation algorithms for one- and two-dimensional data. While the B-spline algorithms could technically be placed under the interpolation category, they are included here because they only work with equally-spaced data and make heavy use of filter-theory and transfer-function formalism to provide a fast B-spline transform. To understand this section you will need to understand that a signal in SciPy is an array of real or complex numbers.

#### B-splines

A B-spline is an approximation of a continuous function over a finite- domain in terms of B-spline coefficients and knot points. If the knot- points are equally spaced with spacing  $\Delta x$ , then the B-spline approximation to a 1-dimensional function is the finite-basis expansion.

$$y(x) \approx \sum_j c_j \beta^o \left( \frac{x}{\Delta x} - j \right).$$

In two dimensions with knot-spacing  $\Delta x$  and  $\Delta y$ , the function representation is

$$z(x, y) \approx \sum_j \sum_k c_{jk} \beta^o \left( \frac{x}{\Delta x} - j \right) \beta^o \left( \frac{y}{\Delta y} - k \right).$$

In these expressions,  $\beta^o(\cdot)$  is the space-limited B-spline basis function of order,  $o$ . The requirement of equally-spaced knot-points and equally-spaced data points, allows the development of fast (inverse-filtering) algorithms for

determining the coefficients,  $c_j$ , from sample-values,  $y_n$ . Unlike the general spline interpolation algorithms, these algorithms can quickly find the spline coefficients for large images.

The advantage of representing a set of samples via B-spline basis functions is that continuous-domain operators (derivatives, re- sampling, integral, etc.) which assume that the data samples are drawn from an underlying continuous function can be computed with relative ease from the spline coefficients. For example, the second-derivative of a spline is

$$y''(x) = \frac{1}{\Delta x^2} \sum_j c_j \beta^{o''} \left( \frac{x}{\Delta x} - j \right).$$

Using the property of B-splines that

$$\frac{d^2 \beta^o(w)}{dw^2} = \beta^{o-2}(w+1) - 2\beta^{o-2}(w) + \beta^{o-2}(w-1)$$

it can be seen that

$$y''(x) = \frac{1}{\Delta x^2} \sum_j c_j \left[ \beta^{o-2} \left( \frac{x}{\Delta x} - j + 1 \right) - 2\beta^{o-2} \left( \frac{x}{\Delta x} - j \right) + \beta^{o-2} \left( \frac{x}{\Delta x} - j - 1 \right) \right].$$

If  $o = 3$ , then at the sample points,

$$\begin{aligned} \Delta x^2 y'(x)|_{x=n\Delta x} &= \sum_j c_j \delta_{n-j+1} - 2c_j \delta_{n-j} + c_j \delta_{n-j-1}, \\ &= c_{n+1} - 2c_n + c_{n-1}. \end{aligned}$$

Thus, the second-derivative signal can be easily calculated from the spline fit. if desired, smoothing splines can be found to make the second-derivative less sensitive to random-errors.

The savvy reader will have already noticed that the data samples are related to the knot coefficients via a convolution operator, so that simple convolution with the sampled B-spline function recovers the original data from the spline coefficients. The output of convolutions can change depending on how boundaries are handled (this becomes increasingly more important as the number of dimensions in the data- set increases). The algorithms relating to B-splines in the signal- processing sub package assume mirror-symmetric boundary conditions. Thus, spline coefficients are computed based on that assumption, and data-samples can be recovered exactly from the spline coefficients by assuming them to be mirror-symmetric also.

Currently the package provides functions for determining second- and third- order cubic spline coefficients from equally spaced samples in one- and two- dimensions (`qspline1d`, `qspline2d`, `cspline1d`, `cspline2d`). The package also supplies a function (`bspline`) for evaluating the bspline basis function,  $\beta^o(x)$  for arbitrary order and  $x$ . For large  $o$ , the B-spline basis function can be approximated well by a zero-mean Gaussian function with standard-deviation equal to  $\sigma_o = (o + 1) / 12$ :

$$\beta^o(x) \approx \frac{1}{\sqrt{2\pi\sigma_o^2}} \exp\left(-\frac{x^2}{2\sigma_o}\right).$$

A function to compute this Gaussian for arbitrary  $x$  and  $o$  is also available (`gauss_spline`). The following code and Figure uses spline-filtering to compute an edge-image (the second-derivative of a smoothed spline) of a raccoon's face which is an array returned by the command `misc.face`. The command `sepfir2d` was used to apply a separable two-dimensional FIR filter with mirror- symmetric boundary conditions to the spline coefficients. This function is ideally suited for reconstructing samples from spline coefficients and is faster than `convolve2d` which convolves arbitrary two-dimensional filters and allows for choosing mirror-symmetric boundary conditions.

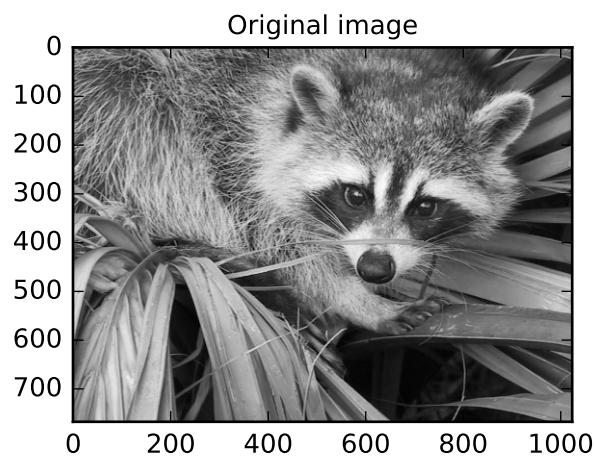
```
>>> import numpy as np
>>> from scipy import signal, misc
>>> import matplotlib.pyplot as plt
```

```
>>> image = misc.face(gray=True).astype(np.float32)
>>> derfilt = np.array([1.0, -2, 1.0], dtype=np.float32)
>>> ck = signal.cspline2d(image, 8.0)
>>> deriv = (signal.sepfir2d(ck, derfilt, [1]) +
...         signal.sepfir2d(ck, [1], derfilt))
```

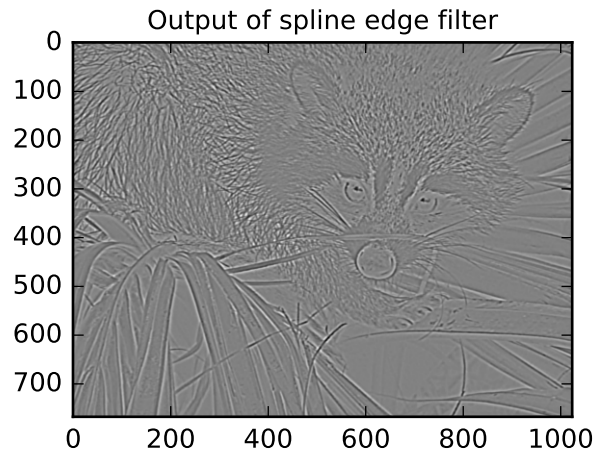
Alternatively we could have done:

```
laplacian = np.array([[0,1,0], [1,-4,1], [0,1,0]], dtype=np.float32)
deriv2 = signal.convolve2d(ck,laplacian,mode='same',boundary='symm')
```

```
>>> plt.figure()
>>> plt.imshow(image)
>>> plt.gray()
>>> plt.title('Original image')
>>> plt.show()
```



```
>>> plt.figure()
>>> plt.imshow(deriv)
>>> plt.gray()
>>> plt.title('Output of spline edge filter')
>>> plt.show()
```



## Filtering

Filtering is a generic name for any system that modifies an input signal in some way. In SciPy a signal can be thought of as a Numpy array. There are different kinds of filters for different kinds of operations. There are two broad kinds of filtering operations: linear and non-linear. Linear filters can always be reduced to multiplication of the flattened Numpy array by an appropriate matrix resulting in another flattened Numpy array. Of course, this is not usually the best way to compute the filter as the matrices and vectors involved may be huge. For example filtering a  $512 \times 512$  image with this method would require multiplication of a  $512^2 \times 512^2$  matrix with a  $512^2$  vector. Just trying to store the  $512^2 \times 512^2$  matrix using a standard Numpy array would require 68,719,476,736 elements. At 4 bytes per element this would require 256GB of memory. In most applications most of the elements of this matrix are zero and a different method for computing the output of the filter is employed.

### *Convolution/Correlation*

Many linear filters also have the property of shift-invariance. This means that the filtering operation is the same at different locations in the signal and it implies that the filtering matrix can be constructed from knowledge of one row (or column) of the matrix alone. In this case, the matrix multiplication can be accomplished using Fourier transforms.

Let  $x[n]$  define a one-dimensional signal indexed by the integer  $n$ . Full convolution of two one-dimensional signals can be expressed as

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k].$$

This equation can only be implemented directly if we limit the sequences to finite support sequences that can be stored in a computer, choose  $n = 0$  to be the starting point of both sequences, let  $K + 1$  be that value for which  $y[n] = 0$  for all  $n > K + 1$  and  $M + 1$  be that value for which  $x[n] = 0$  for all  $n > M + 1$ , then the discrete convolution expression is

$$y[n] = \sum_{k=\max(n-M,0)}^{\min(n,K)} x[k] h[n-k].$$

For convenience assume  $K \geq M$ . Then, more explicitly the output of this operation is

$$\begin{aligned}
 y[0] &= x[0]h[0] \\
 y[1] &= x[0]h[1] + x[1]h[0] \\
 y[2] &= x[0]h[2] + x[1]h[1] + x[2]h[0] \\
 &\vdots \\
 y[M] &= x[0]h[M] + x[1]h[M-1] + \cdots + x[M]h[0] \\
 y[M+1] &= x[1]h[M] + x[2]h[M-1] + \cdots + x[M+1]h[0] \\
 &\vdots \\
 y[K] &= x[K-M]h[M] + \cdots + x[K]h[0] \\
 y[K+1] &= x[K+1-M]h[M] + \cdots + x[K]h[1] \\
 &\vdots \\
 y[K+M-1] &= x[K-1]h[M] + x[K]h[M-1] \\
 y[K+M] &= x[K]h[M].
 \end{aligned}$$

Thus, the full discrete convolution of two finite sequences of lengths  $K+1$  and  $M+1$  respectively results in a finite sequence of length  $K+M+1 = (K+1) + (M+1) - 1$ .

One dimensional convolution is implemented in SciPy with the function `convolve`. This function takes as inputs the signals  $x, h$ , and an optional flag and returns the signal  $y$ . The optional flag allows for specification of which part of the output signal to return. The default value of 'full' returns the entire signal. If the flag has a value of 'same' then only the middle  $K$  values are returned starting at  $y[\lfloor \frac{M-1}{2} \rfloor]$  so that the output has the same length as the first input. If the flag has a value of 'valid' then only the middle  $K-M+1 = (K+1) - (M+1) + 1$  output values are returned where  $z$  depends on all of the values of the smallest input from  $h[0]$  to  $h[M]$ . In other words only the values  $y[M]$  to  $y[K]$  inclusive are returned.

The code below shows a simple example for convolution of 2 sequences

```

>>> x = np.array([1.0, 2.0, 3.0])
>>> h = np.array([0.0, 1.0, 0.0, 0.0, 0.0])
>>> signal.convolve(x, h)
array([ 0.,  1.,  2.,  3.,  0.,  0.,  0.])
>>> signal.convolve(x, h, 'same')
array([ 2.,  3.,  0.])
    
```

This same function `convolve` can actually take  $N$ -dimensional arrays as inputs and will return the  $N$ -dimensional convolution of the two arrays as is shown in the code example below. The same input flags are available for that case as well.

```

>>> x = np.array([[1., 1., 0., 0.], [1., 1., 0., 0.], [0., 0., 0., 0.], [0., 0., 0., 0.]])
>>> h = np.array([[1., 0., 0., 0.], [0., 0., 0., 0.], [0., 0., 1., 0.], [0., 0., 0., 0.]])
>>> signal.convolve(x, h)
array([[ 1.,  1.,  0.,  0.,  0.,  0.,  0.],
       [ 1.,  1.,  0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  1.,  1.,  0.,  0.,  0.],
       [ 0.,  0.,  1.,  1.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.,  0.,  0.]])
    
```

Correlation is very similar to convolution except for the minus sign becomes a plus sign. Thus

$$w[n] = \sum_{k=-\infty}^{\infty} y[k]x[n+k]$$



is the (cross) correlation of the signals  $y$  and  $x$ . For finite-length signals with  $y[n] = 0$  outside of the range  $[0, K]$  and  $x[n] = 0$  outside of the range  $[0, M]$ , the summation can simplify to

$$w[n] = \sum_{k=\max(0, -n)}^{\min(K, M-n)} y[k] x[n+k].$$

Assuming again that  $K \geq M$  this is

$$\begin{aligned} w[-K] &= y[K] x[0] \\ w[-K+1] &= y[K-1] x[0] + y[K] x[1] \\ &\vdots \\ w[M-K] &= y[K-M] x[0] + y[K-M+1] x[1] + \cdots + y[K] x[M] \\ w[M-K+1] &= y[K-M-1] x[0] + \cdots + y[K-1] x[M] \\ &\vdots \\ w[-1] &= y[1] x[0] + y[2] x[1] + \cdots + y[M+1] x[M] \\ w[0] &= y[0] x[0] + y[1] x[1] + \cdots + y[M] x[M] \\ w[1] &= y[0] x[1] + y[1] x[2] + \cdots + y[M-1] x[M] \\ w[2] &= y[0] x[2] + y[1] x[3] + \cdots + y[M-2] x[M] \\ &\vdots \\ w[M-1] &= y[0] x[M-1] + y[1] x[M] \\ w[M] &= y[0] x[M]. \end{aligned}$$

The SciPy function `correlate` implements this operation. Equivalent flags are available for this operation to return the full  $K + M + 1$  length sequence ('full') or a sequence with the same size as the largest sequence starting at  $w[-K + \lfloor \frac{M-1}{2} \rfloor]$  ('same') or a sequence where the values depend on all the values of the smallest sequence ('valid'). This final option returns the  $K - M + 1$  values  $w[M - K]$  to  $w[0]$  inclusive.

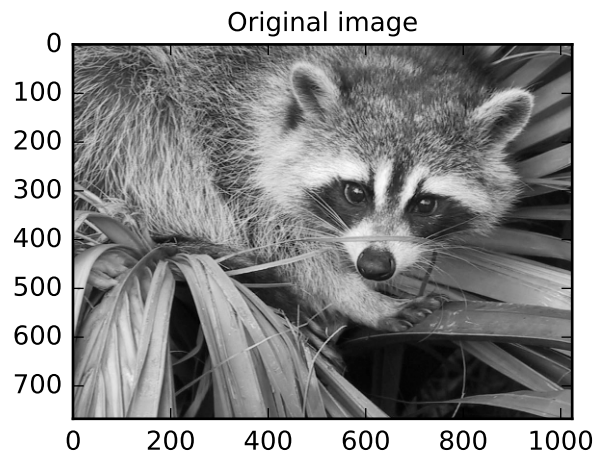
The function `correlate` can also take arbitrary  $N$ -dimensional arrays as input and return the  $N$ -dimensional convolution of the two arrays on output.

When  $N = 2$ , `correlate` and/or `convolve` can be used to construct arbitrary image filters to perform actions such as blurring, enhancing, and edge-detection for an image.

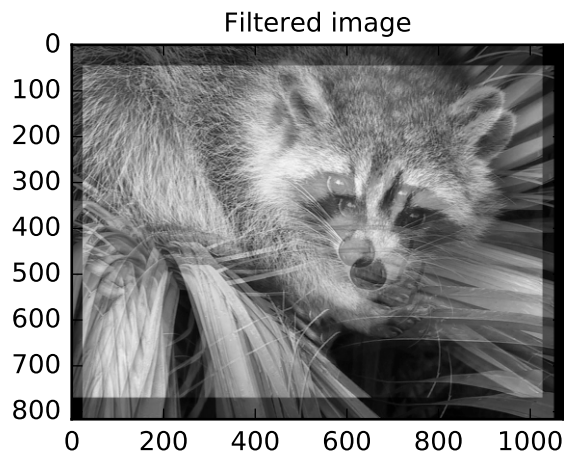
```
>>> import numpy as np
>>> from scipy import signal, misc
>>> import matplotlib.pyplot as plt

>>> image = misc.face(gray=True)
>>> w = np.zeros((50, 50))
>>> w[0][0] = 1.0
>>> w[49][25] = 1.0
>>> image_new = signal.fftconvolve(image, w)

>>> plt.figure()
>>> plt.imshow(image)
>>> plt.gray()
>>> plt.title('Original image')
>>> plt.show()
```



```
>>> plt.figure()
>>> plt.imshow(image_new)
>>> plt.gray()
>>> plt.title('Filtered image')
>>> plt.show()
```



Using `convolve` in the above example would take quite long to run. Calculating the convolution in the time domain as above is mainly used for filtering when one of the signals is much smaller than the other ( $K \gg M$ ), otherwise linear filtering is more efficiently calculated in the frequency domain provided by the function `fftconvolve`.

If the filter function  $w[n, m]$  can be factored according to

$$h[n, m] = h_1[n]h_2[m],$$

convolution can be calculated by means of the function `sepfir2d`. As an example we consider a Gaussian filter `gaussian`

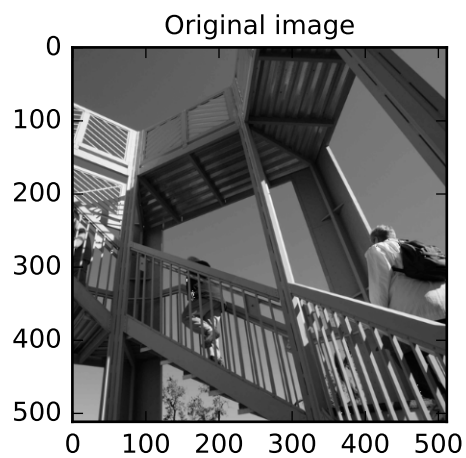
$$h[n, m] \propto e^{-x^2-y^2} = e^{-x^2} e^{-y^2}$$

which is often used for blurring.

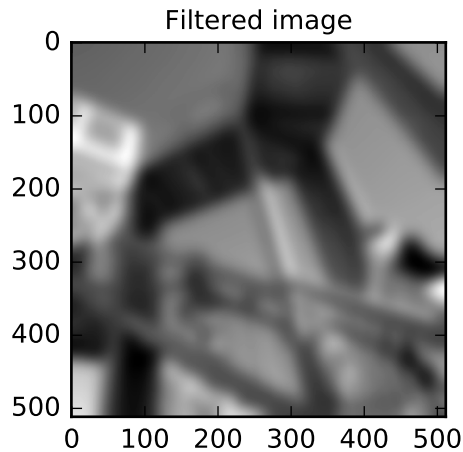
```
>>> import numpy as np
>>> from scipy import signal, misc
>>> import matplotlib.pyplot as plt

>>> image = misc.ascent()
>>> w = signal.gaussian(50, 10.0)
>>> image_new = signal.sepfir2d(image, w, w)

>>> plt.figure()
>>> plt.imshow(image)
>>> plt.gray()
>>> plt.title('Original image')
>>> plt.show()
```



```
>>> plt.figure()
>>> plt.imshow(image_new)
>>> plt.gray()
>>> plt.title('Filtered image')
>>> plt.show()
```



### Difference-equation filtering

A general class of linear one-dimensional filters (that includes convolution filters) are filters described by the difference equation

$$\sum_{k=0}^N a_k y[n-k] = \sum_{k=0}^M b_k x[n-k]$$

where  $x[n]$  is the input sequence and  $y[n]$  is the output sequence. If we assume initial rest so that  $y[n] = 0$  for  $n < 0$ , then this kind of filter can be implemented using convolution. However, the convolution filter sequence  $h[n]$  could be infinite if  $a_k \neq 0$  for  $k \geq 1$ . In addition, this general class of linear filter allows initial conditions to be placed on  $y[n]$  for  $n < 0$  resulting in a filter that cannot be expressed using convolution.

The difference equation filter can be thought of as finding  $y[n]$  recursively in terms of its previous values

$$a_0 y[n] = -a_1 y[n-1] - \dots - a_N y[n-N] + \dots + b_0 x[n] + \dots + b_M x[n-M].$$

Often  $a_0 = 1$  is chosen for normalization. The implementation in SciPy of this general difference equation filter is a little more complicated than would be implied by the previous equation. It is implemented so that only one signal needs to be delayed. The actual implementation equations are (assuming  $a_0 = 1$ ).

$$\begin{aligned} y[n] &= b_0 x[n] + z_0[n-1] \\ z_0[n] &= b_1 x[n] + z_1[n-1] - a_1 y[n] \\ z_1[n] &= b_2 x[n] + z_2[n-1] - a_2 y[n] \\ &\vdots \\ z_{K-2}[n] &= b_{K-1} x[n] + z_{K-1}[n-1] - a_{K-1} y[n] \\ z_{K-1}[n] &= b_K x[n] - a_K y[n], \end{aligned}$$

where  $K = \max(N, M)$ . Note that  $b_K = 0$  if  $K > M$  and  $a_K = 0$  if  $K > N$ . In this way, the output at time  $n$  depends only on the input at time  $n$  and the value of  $z_0$  at the previous time. This can always be calculated as long as the  $K$  values  $z_0[n-1] \dots z_{K-1}[n-1]$  are computed and stored at each time step.

The difference-equation filter is called using the command `lfilter` in SciPy. This command takes as inputs the vector  $b$ , the vector  $a$ , a signal  $x$  and returns the vector  $y$  (the same length as  $x$ ) computed using the equation given above. If  $x$  is  $N$ -dimensional, then the filter is computed along the axis provided. If, desired, initial conditions providing the values of  $z_0[-1]$  to  $z_{K-1}[-1]$  can be provided or else it will be assumed that they are all zero. If initial

conditions are provided, then the final conditions on the intermediate variables are also returned. These could be used, for example, to restart the calculation in the same state.

Sometimes it is more convenient to express the initial conditions in terms of the signals  $x[n]$  and  $y[n]$ . In other words, perhaps you have the values of  $x[-M]$  to  $x[-1]$  and the values of  $y[-N]$  to  $y[-1]$  and would like to determine what values of  $z_m[-1]$  should be delivered as initial conditions to the difference-equation filter. It is not difficult to show that for  $0 \leq m < K$ ,

$$z_m[n] = \sum_{p=0}^{K-m-1} (b_{m+p+1}x[n-p] - a_{m+p+1}y[n-p]).$$

Using this formula we can find the initial condition vector  $z_0[-1]$  to  $z_{K-1}[-1]$  given initial conditions on  $y$  (and  $x$ ). The command `lfiltic` performs this function.

As an example consider the following system:

$$y[n] = \frac{1}{2}x[n] + \frac{1}{4}x[n-1] + \frac{1}{3}y[n-1]$$

The code calculates the signal  $y[n]$  for a given signal  $x[n]$ ; first for initial conditions  $y[-1] = 0$  (default case), then for  $y[-1] = 2$  by means of `lfiltic`.

```
>>> import numpy as np
>>> from scipy import signal

>>> x = np.array([1., 0., 0., 0.])
>>> b = np.array([1.0/2, 1.0/4])
>>> a = np.array([1.0, -1.0/3])
>>> signal.lfilter(b, a, x)
array([0.5, 0.41666667, 0.13888889, 0.0462963])
>>> zi = signal.lfiltic(b, a, y=[2.])
>>> signal.lfilter(b, a, x, zi=zi)
(array([ 1.16666667,  0.63888889,  0.21296296,  0.07098765]), array([0.02366]))
```

Note that the output signal  $y[n]$  has the same length as the length as the input signal  $x[n]$ .

**Analysis of Linear Systems** Linear system described a linear difference equation can be fully described by the coefficient vectors  $a$  and  $b$  as was done above; an alternative representation is to provide a factor  $k$ ,  $N_z$  zeros  $z_k$  and  $N_p$  poles  $p_k$ , respectively, to describe the system by means of its transfer function  $H(z)$  according to

$$H(z) = k \frac{(z - z_1)(z - z_2)\dots(z - z_{N_z})}{(z - p_1)(z - p_2)\dots(z - p_{N_p})}$$

This alternative representation can be obtain with the scipy function `tf2zpk`; the inverse is provided by `zpk2tf`.

For the example from above we have

```
>>> b = np.array([1.0/2, 1.0/4])
>>> a = np.array([1.0, -1.0/3])
>>> signal.tf2zpk(b, a)
(array([-0.5]), array([ 0.33333333]), 0.5)
```

i.e. the system has a zero at  $z = -1/2$  and a pole at  $z = 1/3$ .

The scipy function `freqz` allows calculation of the frequency response of a system described by the coefficients  $a_k$  and  $b_k$ . See the help of the `freqz` function of a comprehensive example.

### Filter Design

Time-discrete filters can be classified into finite response (FIR) filters and infinite response (IIR) filters. FIR filters can provide a linear phase response, whereas IIR filters cannot. SciPy provides functions for designing both types of filters.

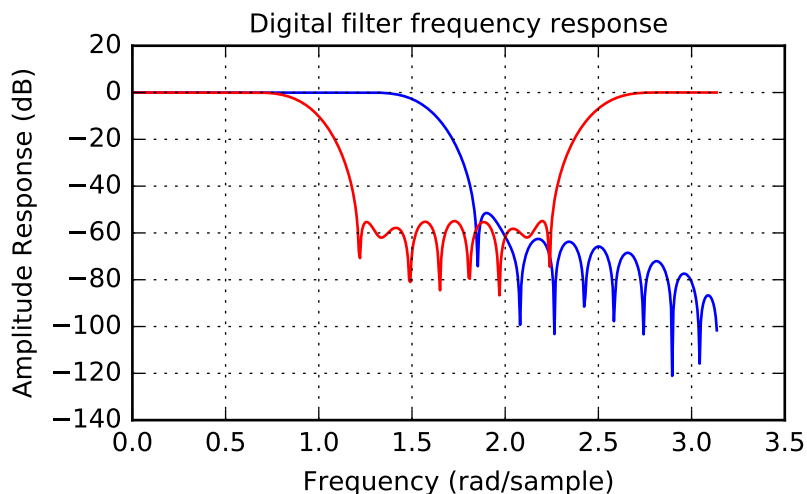
**FIR Filter** The function `firwin` designs filters according to the window method. Depending on the provided arguments, the function returns different filter types (e.g. low-pass, band-pass...).

The example below designs a low-pass and a band-stop filter, respectively.

```
>>> import numpy as np
>>> import scipy.signal as signal
>>> import matplotlib.pyplot as plt

>>> b1 = signal.firwin(40, 0.5)
>>> b2 = signal.firwin(41, [0.3, 0.8])
>>> w1, h1 = signal.freqz(b1)
>>> w2, h2 = signal.freqz(b2)

>>> plt.title('Digital filter frequency response')
>>> plt.plot(w1, 20*np.log10(np.abs(h1)), 'b')
>>> plt.plot(w2, 20*np.log10(np.abs(h2)), 'r')
>>> plt.ylabel('Amplitude Response (dB)')
>>> plt.xlabel('Frequency (rad/sample)')
>>> plt.grid()
>>> plt.show()
```



Note that `firwin` uses per default a normalized frequency defined such that the value 1 corresponds to the Nyquist frequency, whereas the function `freqz` is defined such that the value  $\pi$  corresponds to the Nyquist frequency.

The function `firwin2` allows design of almost arbitrary frequency responses by specifying an array of corner frequencies and corresponding gains, respectively.

The example below designs a filter with such an arbitrary amplitude response.

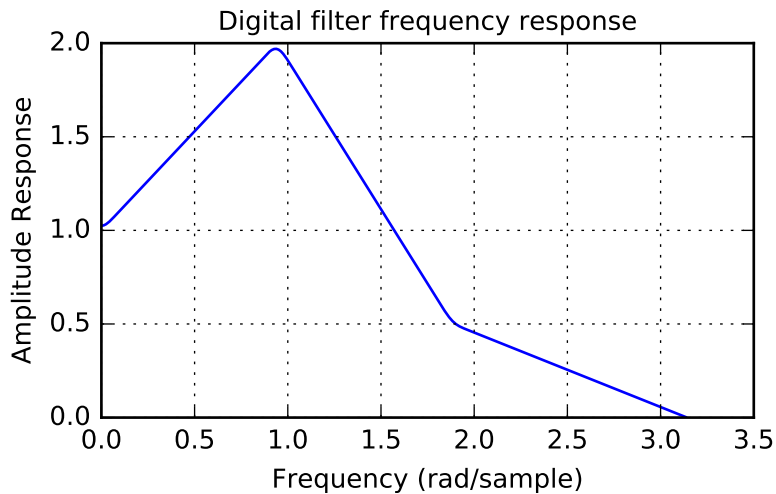
```
>>> import numpy as np
>>> import scipy.signal as signal
>>> import matplotlib.pyplot as plt
```

```

>>> b = signal.firwin2(150, [0.0, 0.3, 0.6, 1.0], [1.0, 2.0, 0.5, 0.0])
>>> w, h = signal.freqz(b)

>>> plt.title('Digital filter frequency response')
>>> plt.plot(w, np.abs(h))
>>> plt.title('Digital filter frequency response')
>>> plt.ylabel('Amplitude Response')
>>> plt.xlabel('Frequency (rad/sample)')
>>> plt.grid()
>>> plt.show()

```



Note the linear scaling of the y-axis and the different definition of the Nyquist frequency in `firwin2` and `freqz` (as explained above).

**IIR Filter** SciPy provides two functions to directly design IIR `iirdesign` and `iirfilter` where the filter type (e.g. elliptic) is passed as an argument and several more filter design functions for specific filter types; e.g. `ellip`.

The example below designs an elliptic low-pass filter with defined passband and stopband ripple, respectively. Note the much lower filter order (order 4) compared with the FIR filters from the examples above in order to reach the same stop-band attenuation of  $\approx 60$  dB.

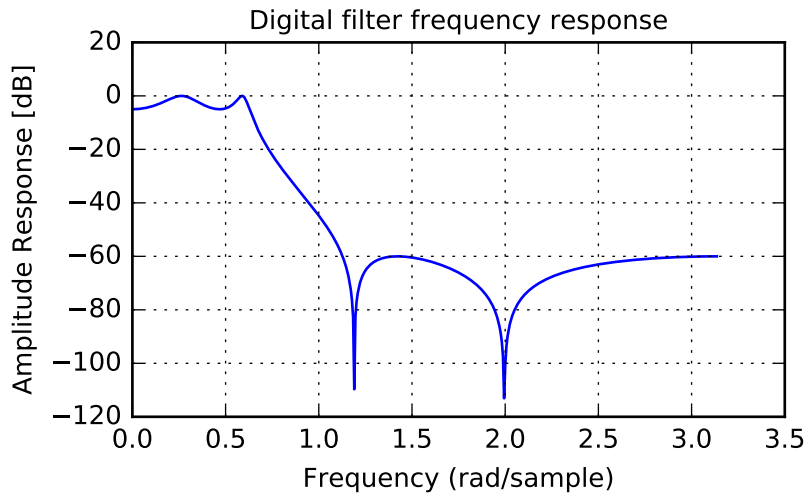
```

>>> import numpy as np
>>> import scipy.signal as signal
>>> import matplotlib.pyplot as plt

>>> b, a = signal.iirfilter(4, Wn=0.2, rp=5, rs=60, btype='lowpass', ftype='ellip')
>>> w, h = signal.freqz(b, a)

>>> plt.title('Digital filter frequency response')
>>> plt.plot(w, 20*np.log10(np.abs(h)))
>>> plt.title('Digital filter frequency response')
>>> plt.ylabel('Amplitude Response [dB]')
>>> plt.xlabel('Frequency (rad/sample)')
>>> plt.grid()
>>> plt.show()

```



**Filter Coefficients** Filter coefficients can be stored in several different formats:

- ‘ba’ or ‘tf’ = transfer function coefficients
- ‘zpk’ = zeros, poles, and overall gain
- ‘ss’ = state-space system representation
- ‘sos’ = transfer function coefficients of second-order sections

Functions such as `tf2zpk` and `zpk2ss` can convert between them.

**Transfer function representation** The `ba` or `tf` format is a 2-tuple  $(b, a)$  representing a transfer function, where  $b$  is a length  $M+1$  array of coefficients of the  $M$ -order numerator polynomial, and  $a$  is a length  $N+1$  array of coefficients of the  $N$ -order denominator, as positive, descending powers of the transfer function variable. So the tuple of  $b = [b_0, b_1, \dots, b_M]$  and  $a = [a_0, a_1, \dots, a_N]$  can represent an analog filter of the form:

$$H(s) = \frac{b_0 s^M + b_1 s^{(M-1)} + \dots + b_M}{a_0 s^N + a_1 s^{(N-1)} + \dots + a_N} = \frac{\sum_{i=0}^M b_i s^{(M-i)}}{\sum_{i=0}^N a_i s^{(N-i)}}$$

or a discrete-time filter of the form:

$$H(z) = \frac{b_0 z^M + b_1 z^{(M-1)} + \dots + b_M}{a_0 z^N + a_1 z^{(N-1)} + \dots + a_N} = \frac{\sum_{i=0}^M b_i z^{(M-i)}}{\sum_{i=0}^N a_i z^{(N-i)}}$$

This “positive powers” form is found more commonly in controls engineering. If  $M$  and  $N$  are equal (which is true for all filters generated by the bilinear transform), then this happens to be equivalent to the “negative powers” discrete-time form preferred in DSP:

$$H(z) = \frac{b_0 + b_1 z^{-1} + \dots + b_M z^{-M}}{a_0 + a_1 z^{-1} + \dots + a_N z^{-N}} = \frac{\sum_{i=0}^M b_i z^{-i}}{\sum_{i=0}^N a_i z^{-i}}$$

Although this is true for common filters, remember that this is not true in the general case. If  $M$  and  $N$  are not equal, the discrete-time transfer function coefficients must first be converted to the “positive powers” form before finding the poles and zeros.

This representation suffers from numerical error at higher orders, so other formats are preferred when possible.



**Zeros and poles representation** The `zpk` format is a 3-tuple  $(z, p, k)$ , where  $z$  is an  $M$ -length array of the complex zeros of the transfer function  $z = [z_0, z_1, \dots, z_{M-1}]$ ,  $p$  is an  $N$ -length array of the complex poles of the transfer function  $p = [p_0, p_1, \dots, p_{N-1}]$ , and  $k$  is a scalar gain. These represent the digital transfer function:

$$H(z) = k \cdot \frac{(z - z_0)(z - z_1) \cdots (z - z_{(M-1)})}{(z - p_0)(z - p_1) \cdots (z - p_{(N-1)})} = k \frac{\prod_{i=0}^{M-1} (z - z_i)}{\prod_{i=0}^{N-1} (z - p_i)}$$

or the analog transfer function:

$$H(s) = k \cdot \frac{(s - z_0)(s - z_1) \cdots (s - z_{(M-1)})}{(s - p_0)(s - p_1) \cdots (s - p_{(N-1)})} = k \frac{\prod_{i=0}^{M-1} (s - z_i)}{\prod_{i=0}^{N-1} (s - p_i)}$$

Although the sets of roots are stored as ordered NumPy arrays, their ordering does not matter;  $([-1, -2], [-3, -4], 1)$  is the same filter as  $([-2, -1], [-4, -3], 1)$ .

**State-space system representation** The `ss` format is a 4-tuple of arrays  $(A, B, C, D)$  representing the state-space of an  $N$ -order digital/discrete-time system of the form:

$$\begin{aligned} \mathbf{x}[k+1] &= \mathbf{A}\mathbf{x}[k] + \mathbf{B}\mathbf{u}[k] \\ \mathbf{y}[k] &= \mathbf{C}\mathbf{x}[k] + \mathbf{D}\mathbf{u}[k] \end{aligned}$$

or a continuous/analog system of the form:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) \end{aligned}$$

with  $P$  inputs,  $Q$  outputs and  $N$  state variables, where:

- $x$  is the state vector
- $y$  is the output vector of length  $Q$
- $u$  is the input vector of length  $P$
- $A$  is the state matrix, with shape  $(N, N)$
- $B$  is the input matrix with shape  $(N, P)$
- $C$  is the output matrix with shape  $(Q, N)$
- $D$  is the feedthrough or feedforward matrix with shape  $(Q, P)$ . (In cases where the system does not have a direct feedthrough, all values in  $D$  are zero.)

State-space is the most general representation, and the only one that allows for multiple-input, multiple-output (MIMO) systems. There are multiple state-space representations for a given transfer function. Specifically, the “controllable canonical form” and “observable canonical form” have the same coefficients as the `tf` representation, and therefore suffer from the same numerical errors.

**Second-order sections representation** The `sos` format is a single 2D array of shape  $(n\_sections, 6)$ , representing a sequence of second-order transfer functions which, when cascaded in series, realize a higher-order filter with minimal numerical error. Each row corresponds to a second-order `tf` representation, with the first three columns providing the numerator coefficients and the last three providing the denominator coefficients:

$$[b_0, b_1, b_2, a_0, a_1, a_2]$$

The coefficients are typically normalized such that  $a_0$  is always 1. The section order is usually not important with floating-point computation; the filter output will be the same regardless.

**Filter transformations** The IIR filter design functions first generate a prototype analog lowpass filter with a normalized cutoff frequency of 1 rad/sec. This is then transformed into other frequencies and band types using the following substitutions:

Type	Transformation
lp2lp	$s \rightarrow \frac{s}{\omega_0}$
lp2hp	$s \rightarrow \frac{\omega_0}{s}$
lp2bp	$s \rightarrow \frac{s^2 + \omega_0^2}{s \cdot \text{BW}}$
lp2bs	$s \rightarrow \frac{s^2 + \omega_0^2}{s}$

Here,  $\omega_0$  is the new cutoff or center frequency, and BW is the bandwidth. These preserve symmetry on a logarithmic frequency axis.

To convert the transformed analog filter into a digital filter, the `bilinear` transform is used, which makes the following substitution:

$$s \rightarrow \frac{2}{T} \frac{z - 1}{z + 1}$$

where T is the sampling time (the inverse of the sampling frequency).

### Other filters

The signal processing package provides many more filters as well.

**Median Filter** A median filter is commonly applied when noise is markedly non-Gaussian or when it is desired to preserve edges. The median filter works by sorting all of the array pixel values in a rectangular region surrounding the point of interest. The sample median of this list of neighborhood pixel values is used as the value for the output array. The sample median is the middle array value in a sorted list of neighborhood values. If there are an even number of elements in the neighborhood, then the average of the middle two values is used as the median. A general purpose median filter that works on N-dimensional arrays is `medfilt`. A specialized version that works only for two-dimensional arrays is available as `medfilt2d`.

**Order Filter** A median filter is a specific example of a more general class of filters called order filters. To compute the output at a particular pixel, all order filters use the array values in a region surrounding that pixel. These array values are sorted and then one of them is selected as the output value. For the median filter, the sample median of the list of array values is used as the output. A general order filter allows the user to select which of the sorted values will be used as the output. So, for example one could choose to pick the maximum in the list or the minimum. The order filter takes an additional argument besides the input array and the region mask that specifies which of the elements in the sorted list of neighbor array values should be used as the output. The command to perform an order filter is `order_filter`.

**Wiener filter** The Wiener filter is a simple deblurring filter for denoising images. This is not the Wiener filter commonly described in image reconstruction problems but instead it is a simple, local-mean filter. Let  $x$  be the input signal, then the output is

$$y = \begin{cases} \frac{\sigma^2}{\sigma_x^2} m_x + \left(1 - \frac{\sigma^2}{\sigma_x^2}\right) x & \sigma_x^2 \geq \sigma^2, \\ m_x & \sigma_x^2 < \sigma^2, \end{cases}$$

where  $m_x$  is the local estimate of the mean and  $\sigma_x^2$  is the local estimate of the variance. The window for these estimates is an optional input parameter (default is  $3 \times 3$ ). The parameter  $\sigma^2$  is a threshold noise parameter. If  $\sigma$  is not given then it is estimated as the average of the local variances.

**Hilbert filter** The Hilbert transform constructs the complex-valued analytic signal from a real signal. For example if  $x = \cos \omega n$  then  $y = \text{hilbert}(x)$  would return (except near the edges)  $y = \exp(j\omega n)$ . In the frequency domain, the hilbert transform performs

$$Y = X \cdot H$$

where  $H$  is 2 for positive frequencies, 0 for negative frequencies and 1 for zero-frequencies.

### Analog Filter Design

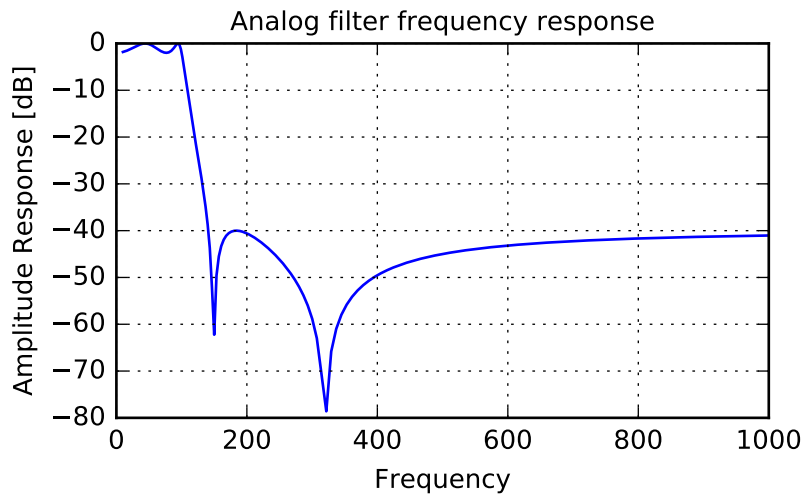
The functions `iirdesign`, `iirfilter`, and the filter design functions for specific filter types (e.g. `ellip`) all have a flag `analog` which allows design of analog filters as well.

The example below designs an analog (IIR) filter, obtains via `tf2zpk` the poles and zeros and plots them in the complex s-plane. The zeros at  $\omega \approx 150$  and  $\omega \approx 300$  can be clearly seen in the amplitude response.

```
>>> import numpy as np
>>> import scipy.signal as signal
>>> import matplotlib.pyplot as plt

>>> b, a = signal.iirdesign(wp=100, ws=200, gpass=2.0, gstop=40., analog=True)
>>> w, h = signal.freqs(b, a)

>>> plt.title('Analog filter frequency response')
>>> plt.plot(w, 20*np.log10(np.abs(h)))
>>> plt.ylabel('Amplitude Response [dB]')
>>> plt.xlabel('Frequency')
>>> plt.grid()
>>> plt.show()
```

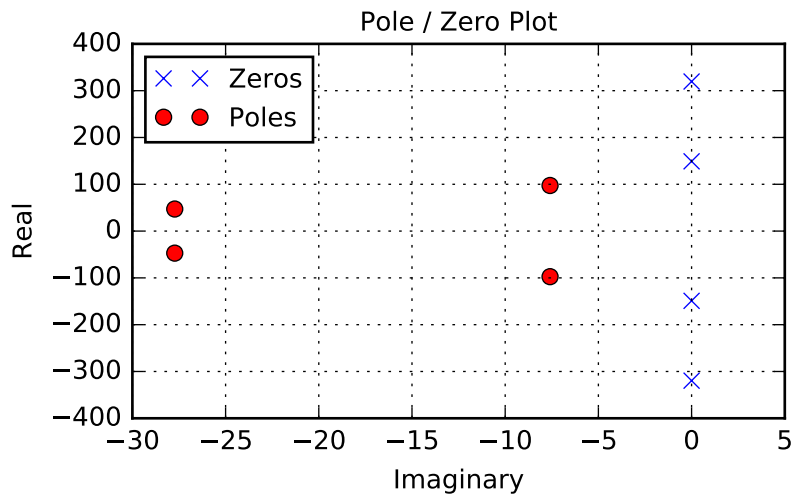


```
>>> z, p, k = signal.tf2zpk(b, a)

>>> plt.plot(np.real(z), np.imag(z), 'xb')
>>> plt.plot(np.real(p), np.imag(p), 'or')
>>> plt.legend(['Zeros', 'Poles'], loc=2)

>>> plt.title('Pole / Zero Plot')
>>> plt.ylabel('Real')
>>> plt.xlabel('Imaginary')
```

```
>>> plt.grid()
>>> plt.show()
```



## Spectral Analysis

### *Periodogram Measurements*

The scipy function `periodogram` provides a method to estimate the spectral density using the periodogram method.

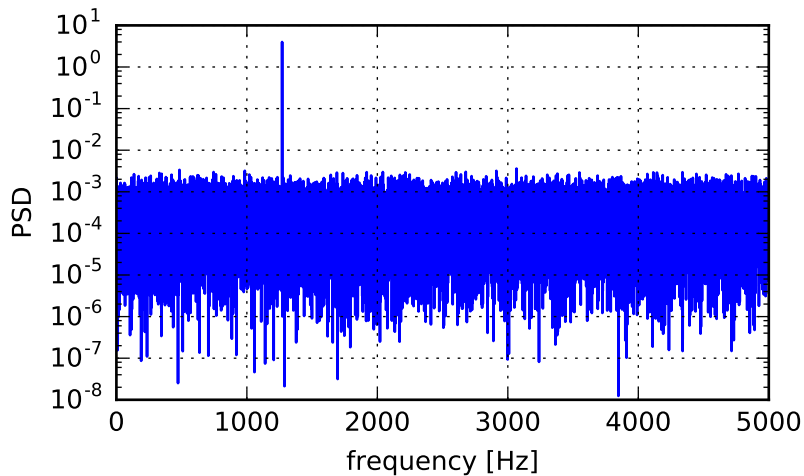
The example below calculates the periodogram of a sine signal in white Gaussian noise.

```
>>> import numpy as np
>>> import scipy.signal as signal
>>> import matplotlib.pyplot as plt

>>> fs = 10e3
>>> N = 1e5
>>> amp = 2*np.sqrt(2)
>>> freq = 1270.0
>>> noise_power = 0.001 * fs / 2
>>> time = np.arange(N) / fs
>>> x = amp*np.sin(2*np.pi*freq*time)
>>> x += np.random.normal(scale=np.sqrt(noise_power), size=time.shape)

>>> f, Pper_spec = signal.periodogram(x, fs, 'flattop', scaling='spectrum')

>>> plt.semilogy(f, Pper_spec)
>>> plt.xlabel('frequency [Hz]')
>>> plt.ylabel('PSD')
>>> plt.grid()
>>> plt.show()
```



### *Spectral Analysis using Welch's Method*

An improved method, especially with respect to noise immunity, is Welch's method which is implemented by the `scipy` function `welch`.

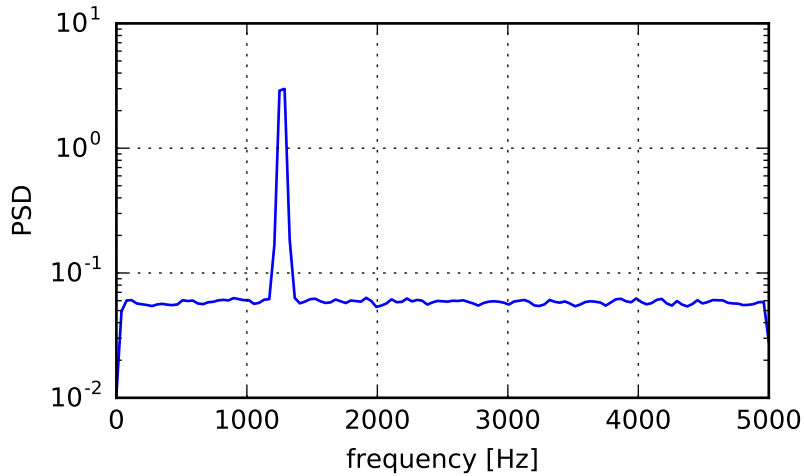
The example below estimates the spectrum using Welch's method and uses the same parameters as the example above. Note the much smoother noise floor of the spectrogram.

```
>>> import numpy as np
>>> import scipy.signal as signal
>>> import matplotlib.pyplot as plt

>>> fs = 10e3
>>> N = 1e5
>>> amp = 2*np.sqrt(2)
>>> freq = 1270.0
>>> noise_power = 0.001 * fs / 2
>>> time = np.arange(N) / fs
>>> x = amp*np.sin(2*np.pi*freq*time)
>>> x += np.random.normal(scale=np.sqrt(noise_power), size=time.shape)

>>> f, Pwelch_spec = signal.welch(x, fs, scaling='spectrum')

>>> plt.semilogy(f, Pwelch_spec)
>>> plt.xlabel('frequency [Hz]')
>>> plt.ylabel('PSD')
>>> plt.grid()
>>> plt.show()
```



### ***Lomb-Scargle Periodograms (`lombscargle`)***

Least-squares spectral analysis (LSSA) is a method of estimating a frequency spectrum, based on a least squares fit of sinusoids to data samples, similar to Fourier analysis. Fourier analysis, the most used spectral method in science, generally boosts long-periodic noise in long gapped records; LSSA mitigates such problems.

The Lomb-Scargle method performs spectral analysis on unevenly sampled data and is known to be a powerful way to find, and test the significance of, weak periodic signals.

For a time series comprising  $N_t$  measurements  $X_j \equiv X(t_j)$  sampled at times  $t_j$  where ( $j = 1, \dots, N_t$ ), assumed to have been scaled and shifted such that its mean is zero and its variance is unity, the normalized Lomb-Scargle periodogram at frequency  $f$  is

$$P_n(f) \frac{1}{2} \left\{ \frac{\left[ \sum_j^{N_t} X_j \cos \omega(t_j - \tau) \right]^2}{\sum_j^{N_t} \cos^2 \omega(t_j - \tau)} + \frac{\left[ \sum_j^{N_t} X_j \sin \omega(t_j - \tau) \right]^2}{\sum_j^{N_t} \sin^2 \omega(t_j - \tau)} \right\}.$$

Here,  $\omega \equiv 2\pi f$  is the angular frequency. The frequency dependent time offset  $\tau$  is given by

$$\tan 2\omega\tau = \frac{\sum_j^{N_t} \sin 2\omega t_j}{\sum_j^{N_t} \cos 2\omega t_j}.$$

The `lombscargle` function calculates the periodogram using a slightly modified algorithm due to Townsend<sup>2</sup> which allows the periodogram to be calculated using only a single pass through the input arrays for each frequency.

The equation is refactored as:

$$P_n(f) = \frac{1}{2} \left[ \frac{(c_\tau X C + s_\tau X S)^2}{c_\tau^2 C C + 2c_\tau s_\tau C S + s_\tau^2 S S} + \frac{(c_\tau X S - s_\tau X C)^2}{c_\tau^2 S S - 2c_\tau s_\tau C S + s_\tau^2 C C} \right]$$

and

$$\tan 2\omega\tau = \frac{2CS}{CC - SS}.$$

<sup>2</sup> R.H.D. Townsend, "Fast calculation of the Lomb-Scargle periodogram using graphics processing units.", The Astrophysical Journal Supplement Series, vol 191, pp. 247-253, 2010

Here,

$$c_\tau = \cos \omega \tau, \quad s_\tau = \sin \omega \tau$$

while the sums are

$$\begin{aligned} XC &= \sum_j^{N_t} X_j \cos \omega t_j \\ XS &= \sum_j^{N_t} X_j \sin \omega t_j \\ CC &= \sum_j^{N_t} \cos^2 \omega t_j \\ SS &= \sum_j^{N_t} \sin^2 \omega t_j \\ CS &= \sum_j^{N_t} \cos \omega t_j \sin \omega t_j. \end{aligned}$$

This requires  $N_f(2N_t + 3)$  trigonometric function evaluations giving a factor of  $\sim 2$  speed increase over the straight-forward implementation.

## Detrend

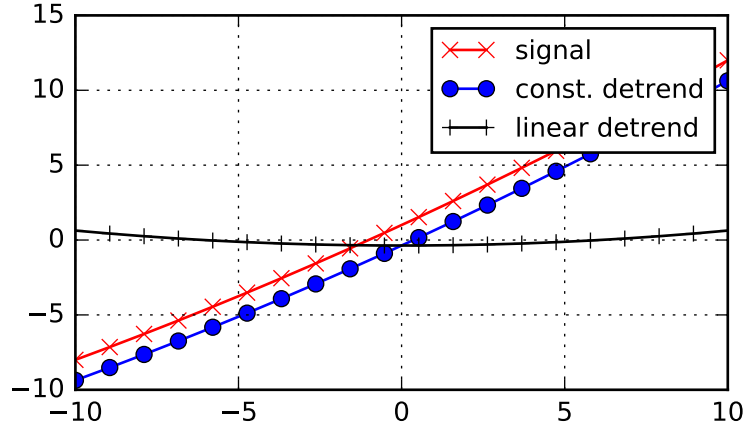
Scipy provides the function `detrend` to remove a constant or linear trend in a data series in order to see effect of higher order.

The example below removes the constant and linear trend of a 2-nd order polynomial time series and plots the remaining signal components.

```
>>> import numpy as np
>>> import scipy.signal as signal
>>> import matplotlib.pyplot as plt

>>> t = np.linspace(-10, 10, 20)
>>> y = 1 + t + 0.01*t**2
>>> yconst = signal.detrend(y, type='constant')
>>> ylin = signal.detrend(y, type='linear')

>>> plt.plot(t, y, '-rx')
>>> plt.plot(t, yconst, '-bo')
>>> plt.plot(t, ylin, '-k+')
>>> plt.grid()
>>> plt.legend(['signal', 'const. detrend', 'linear detrend'])
>>> plt.show()
```



### References

Some further reading and related software:

### 3.1.9 Linear Algebra (`scipy.linalg`)

When SciPy is built using the optimized ATLAS LAPACK and BLAS libraries, it has very fast linear algebra capabilities. If you dig deep enough, all of the raw lapack and blas libraries are available for your use for even more speed. In this section, some easier-to-use interfaces to these routines are described.

All of these linear algebra routines expect an object that can be converted into a 2-dimensional array. The output of these routines is also a two-dimensional array.

`scipy.linalg` contains all the functions in `numpy.linalg`, plus some other more advanced ones not contained in `numpy.linalg`.

Another advantage of using `scipy.linalg` over `numpy.linalg` is that it is always compiled with BLAS/LAPACK support, while for `numpy` this is optional. Therefore, the `scipy` version might be faster depending on how `numpy` was installed.

Therefore, unless you don't want to add `scipy` as a dependency to your `numpy` program, use `scipy.linalg` instead of `numpy.linalg`.

### `numpy.matrix` vs 2D `numpy.ndarray`

The classes that represent matrices, and basic operations such as matrix multiplications and transpose are a part of `numpy`. For convenience, we summarize the differences between `numpy.matrix` and `numpy.ndarray` here.

`numpy.matrix` is matrix class that has a more convenient interface than `numpy.ndarray` for matrix operations. This class supports for example MATLAB-like creation syntax via the, has matrix multiplication as default for the `*` operator, and contains `I` and `T` members that serve as shortcuts for inverse and transpose:

```
>>> import numpy as np
>>> A = np.mat('[1 2;3 4]')
>>> A
matrix([[1, 2],
        [3, 4]])
```



```

>>> A.I
matrix([[ -2. ,  1. ],
        [ 1.5, -0.5]])
>>> b = np.mat('[5 6]')
>>> b
matrix([[5, 6]])
>>> b.T
matrix([[5],
        [6]])
>>> A*b.T
matrix([[17],
        [39]])

```

Despite its convenience, the use of the `numpy.matrix` class is discouraged, since it adds nothing that cannot be accomplished with 2D `numpy.ndarray` objects, and may lead to a confusion of which class is being used. For example, the above code can be rewritten as:

```

>>> import numpy as np
>>> from scipy import linalg
>>> A = np.array([[1,2],[3,4]])
>>> A
array([[1, 2],
       [3, 4]])
>>> linalg.inv(A)
array([[ -2. ,  1. ],
       [ 1.5, -0.5]])
>>> b = np.array([[5,6]]) #2D array
>>> b
array([[5, 6]])
>>> b.T
array([[5],
       [6]])
>>> A*b #not matrix multiplication!
array([[ 5, 12],
       [15, 24]])
>>> A.dot(b.T) #matrix multiplication
array([[17],
       [39]])
>>> b = np.array([5,6]) #1D array
>>> b
array([5, 6])
>>> b.T #not matrix transpose!
array([5, 6])
>>> A.dot(b) #does not matter for multiplication
array([17, 39])

```

`scipy.linalg` operations can be applied equally to `numpy.matrix` or to 2D `numpy.ndarray` objects.

## Basic routines

### *Finding Inverse*

The inverse of a matrix **A** is the matrix **B** such that  $\mathbf{AB} = \mathbf{I}$  where **I** is the identity matrix consisting of ones down the main diagonal. Usually **B** is denoted  $\mathbf{B} = \mathbf{A}^{-1}$ . In SciPy, the matrix inverse of the Numpy array, **A**, is obtained

using `linalg.inv(A)`, or using `A.I` if `A` is a Matrix. For example, let

$$\mathbf{A} = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 5 & 1 \\ 2 & 3 & 8 \end{bmatrix}$$

then

$$\mathbf{A}^{-1} = \frac{1}{25} \begin{bmatrix} -37 & 9 & 22 \\ 14 & 2 & -9 \\ 4 & -3 & 1 \end{bmatrix} = \begin{bmatrix} -1.48 & 0.36 & 0.88 \\ 0.56 & 0.08 & -0.36 \\ 0.16 & -0.12 & 0.04 \end{bmatrix}.$$

The following example demonstrates this computation in SciPy

```
>>> import numpy as np
>>> from scipy import linalg
>>> A = np.array([[1, 3, 5], [2, 5, 1], [2, 3, 8]])
>>> A
array([[1, 3, 5],
       [2, 5, 1],
       [2, 3, 8]])
>>> linalg.inv(A)
array([[ -1.48,  0.36,  0.88],
       [ 0.56,  0.08, -0.36],
       [ 0.16, -0.12,  0.04]])
>>> A.dot(linalg.inv(A)) #double check
array([[ 1.00000000e+00, -1.11022302e-16, -5.55111512e-17],
       [ 3.05311332e-16,  1.00000000e+00,  1.87350135e-16],
       [ 2.22044605e-16, -1.11022302e-16,  1.00000000e+00]])
```

### Solving linear system

Solving linear systems of equations is straightforward using the `scipy` command `linalg.solve`. This command expects an input matrix and a right-hand-side vector. The solution vector is then computed. An option for entering a symmetric matrix is offered which can speed up the processing when applicable. As an example, suppose it is desired to solve the following simultaneous equations:

$$\begin{aligned} x + 3y + 5z &= 10 \\ 2x + 5y + z &= 8 \\ 2x + 3y + 8z &= 3 \end{aligned}$$

We could find the solution vector using a matrix inverse:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 5 & 1 \\ 2 & 3 & 8 \end{bmatrix}^{-1} \begin{bmatrix} 10 \\ 8 \\ 3 \end{bmatrix} = \frac{1}{25} \begin{bmatrix} -232 \\ 129 \\ 19 \end{bmatrix} = \begin{bmatrix} -9.28 \\ 5.16 \\ 0.76 \end{bmatrix}.$$

However, it is better to use the `linalg.solve` command which can be faster and more numerically stable. In this case it however gives the same answer as shown in the following example:

```
>>> import numpy as np
>>> from scipy import linalg
>>> A = np.array([[1, 2], [3, 4]])
>>> A
array([[1, 2],
       [3, 4]])
>>> b = np.array([[5], [6]])
>>> b
```

```

array([[5],
       [6]])
>>> linalg.inv(A).dot(b) # slow
array([[ -4. ],
       [ 4.5]])
>>> A.dot(linalg.inv(A).dot(b)) - b # check
array([[ 8.88178420e-16],
       [ 2.66453526e-15]])
>>> np.linalg.solve(A, b) # fast
array([[ -4. ],
       [ 4.5]])
>>> A.dot(np.linalg.solve(A, b)) - b # check
array([[ 0. ],
       [ 0.]])

```

### Finding Determinant

The determinant of a square matrix  $\mathbf{A}$  is often denoted  $|\mathbf{A}|$  and is a quantity often used in linear algebra. Suppose  $a_{ij}$  are the elements of the matrix  $\mathbf{A}$  and let  $M_{ij} = |\mathbf{A}_{ij}|$  be the determinant of the matrix left by removing the  $i^{\text{th}}$  row and  $j^{\text{th}}$  column from  $\mathbf{A}$ . Then for any row  $i$ ,

$$|\mathbf{A}| = \sum_j (-1)^{i+j} a_{ij} M_{ij}.$$

This is a recursive way to define the determinant where the base case is defined by accepting that the determinant of a  $1 \times 1$  matrix is the only matrix element. In SciPy the determinant can be calculated with `linalg.det`. For example, the determinant of

$$\mathbf{A} = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 5 & 1 \\ 2 & 3 & 8 \end{bmatrix}$$

is

$$\begin{aligned} |\mathbf{A}| &= 1 \begin{vmatrix} 5 & 1 \\ 3 & 8 \end{vmatrix} - 3 \begin{vmatrix} 2 & 1 \\ 2 & 8 \end{vmatrix} + 5 \begin{vmatrix} 2 & 5 \\ 2 & 3 \end{vmatrix} \\ &= 1(5 \cdot 8 - 3 \cdot 1) - 3(2 \cdot 8 - 2 \cdot 1) + 5(2 \cdot 3 - 2 \cdot 5) = -25. \end{aligned}$$

In SciPy this is computed as shown in this example:

```

>>> import numpy as np
>>> from scipy import linalg
>>> A = np.array([[1,2],[3,4]])
>>> A
array([[1, 2],
       [3, 4]])
>>> linalg.det(A)
-2.0

```

### Computing norms

Matrix and vector norms can also be computed with SciPy. A wide range of norm definitions are available using different parameters to the order argument of `linalg.norm`. This function takes a rank-1 (vectors) or a rank-2 (matrices) array and an optional order argument (default is 2). Based on these inputs a vector or matrix norm of the requested order is computed.

For vector  $x$ , the order parameter can be any real number including  $\text{inf}$  or  $-\text{inf}$ . The computed norm is

$$\|x\| = \begin{cases} \max |x_i| & \text{ord} = \text{inf} \\ \min |x_i| & \text{ord} = -\text{inf} \\ \left(\sum_i |x_i|^{\text{ord}}\right)^{1/\text{ord}} & |\text{ord}| < \infty. \end{cases}$$

For matrix  $\mathbf{A}$  the only valid values for norm are  $\pm 2, \pm 1, \pm \text{inf}$ , and 'fro' (or 'f') Thus,

$$\|\mathbf{A}\| = \begin{cases} \max_i \sum_j |a_{ij}| & \text{ord} = \text{inf} \\ \min_i \sum_j |a_{ij}| & \text{ord} = -\text{inf} \\ \max_j \sum_i |a_{ij}| & \text{ord} = 1 \\ \min_j \sum_i |a_{ij}| & \text{ord} = -1 \\ \max \sigma_i & \text{ord} = 2 \\ \min \sigma_i & \text{ord} = -2 \\ \sqrt{\text{trace}(\mathbf{A}^H \mathbf{A})} & \text{ord} = \text{'fro'}$$

where  $\sigma_i$  are the singular values of  $\mathbf{A}$ .

Examples:

```
>>> import numpy as np
>>> from scipy import linalg
>>> A=np.array([[1,2],[3,4]])
>>> A
array([[1, 2],
       [3, 4]])
>>> linalg.norm(A)
5.4772255750516612
>>> linalg.norm(A,'fro') # frobenius norm is the default
5.4772255750516612
>>> linalg.norm(A,1) # L1 norm (max column sum)
6
>>> linalg.norm(A,-1)
4
>>> linalg.norm(A,np.inf) # L inf norm (max row sum)
7
```

### Solving linear least-squares problems and pseudo-inverses

Linear least-squares problems occur in many branches of applied mathematics. In this problem a set of linear scaling coefficients is sought that allow a model to fit data. In particular it is assumed that data  $y_i$  is related to data  $x_i$  through a set of coefficients  $c_j$  and model functions  $f_j(x_i)$  via the model

$$y_i = \sum_j c_j f_j(x_i) + \epsilon_i$$

where  $\epsilon_i$  represents uncertainty in the data. The strategy of least squares is to pick the coefficients  $c_j$  to minimize

$$J(\mathbf{c}) = \sum_i \left| y_i - \sum_j c_j f_j(x_i) \right|^2.$$

Theoretically, a global minimum will occur when

$$\frac{\partial J}{\partial c_n^*} = 0 = \sum_i \left( y_i - \sum_j c_j f_j(x_i) \right) (-f_n^*(x_i))$$

or

$$\sum_j c_j \sum_i f_j(x_i) f_n^*(x_i) = \sum_i y_i f_n^*(x_i)$$

$$\mathbf{A}^H \mathbf{A} \mathbf{c} = \mathbf{A}^H \mathbf{y}$$

where

$$\{\mathbf{A}\}_{ij} = f_j(x_i).$$

When  $\mathbf{A}^H \mathbf{A}$  is invertible, then

$$\mathbf{c} = (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \mathbf{y} = \mathbf{A}^\dagger \mathbf{y}$$

where  $\mathbf{A}^\dagger$  is called the pseudo-inverse of  $\mathbf{A}$ . Notice that using this definition of  $\mathbf{A}$  the model can be written

$$\mathbf{y} = \mathbf{A} \mathbf{c} + \boldsymbol{\epsilon}.$$

The command `linalg.lstsq` will solve the linear least squares problem for  $\mathbf{c}$  given  $\mathbf{A}$  and  $\mathbf{y}$ . In addition `linalg.pinv` or `linalg.pinv2` (uses a different method based on singular value decomposition) will find  $\mathbf{A}^\dagger$  given  $\mathbf{A}$ .

The following example and figure demonstrate the use of `linalg.lstsq` and `linalg.pinv` for solving a data-fitting problem. The data shown below were generated using the model:

$$y_i = c_1 e^{-x_i} + c_2 x_i$$

where  $x_i = 0.1i$  for  $i = 1 \dots 10$ ,  $c_1 = 5$ , and  $c_2 = 4$ . Noise is added to  $y_i$  and the coefficients  $c_1$  and  $c_2$  are estimated using linear least squares.

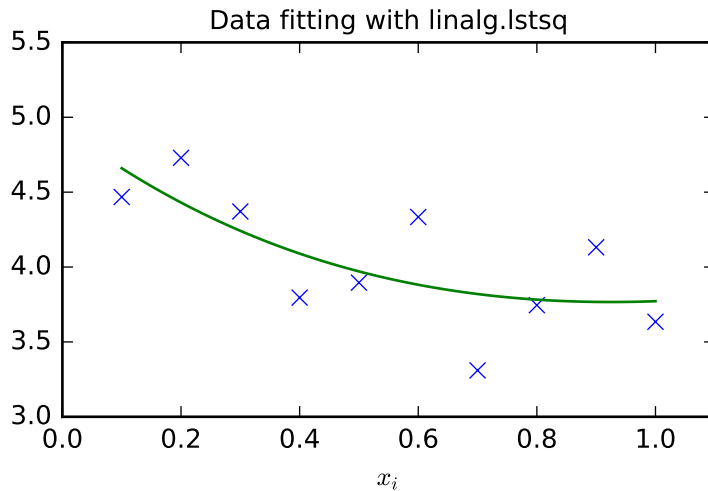
```
>>> import numpy as np
>>> from scipy import linalg
>>> import matplotlib.pyplot as plt

>>> c1, c2 = 5.0, 2.0
>>> i = np.r_[1:11]
>>> xi = 0.1*i
>>> yi = c1*np.exp(-xi) + c2*xi
>>> zi = yi + 0.05 * np.max(yi) * np.random.randn(len(yi))

>>> A = np.c_[np.exp(-xi)[:, np.newaxis], xi[:, np.newaxis]]
>>> c, resid, rank, sigma = linalg.lstsq(A, zi)

>>> xi2 = np.r_[0.1:1.0:100j]
>>> yi2 = c[0]*np.exp(-xi2) + c[1]*xi2

>>> plt.plot(xi, zi, 'x', xi2, yi2)
>>> plt.axis([0, 1.1, 3.0, 5.5])
>>> plt.xlabel('$x_i$')
>>> plt.title('Data fitting with linalg.lstsq')
>>> plt.show()
```



### Generalized inverse

The generalized inverse is calculated using the command `linalg.pinv` or `linalg.pinv2`. These two commands differ in how they compute the generalized inverse. The first uses the `linalg.lstsq` algorithm while the second uses singular value decomposition. Let  $\mathbf{A}$  be an  $M \times N$  matrix, then if  $M > N$  the generalized inverse is

$$\mathbf{A}^\dagger = (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H$$

while if  $M < N$  matrix the generalized inverse is

$$\mathbf{A}^\# = \mathbf{A}^H (\mathbf{A} \mathbf{A}^H)^{-1}.$$

In both cases for  $M = N$ , then

$$\mathbf{A}^\dagger = \mathbf{A}^\# = \mathbf{A}^{-1}$$

as long as  $\mathbf{A}$  is invertible.

## Decompositions

In many applications it is useful to decompose a matrix using other representations. There are several decompositions supported by SciPy.

### Eigenvalues and eigenvectors

The eigenvalue-eigenvector problem is one of the most commonly employed linear algebra operations. In one popular form, the eigenvalue-eigenvector problem is to find for some square matrix  $\mathbf{A}$  scalars  $\lambda$  and corresponding vectors  $\mathbf{v}$  such that

$$\mathbf{A} \mathbf{v} = \lambda \mathbf{v}.$$

For an  $N \times N$  matrix, there are  $N$  (not necessarily distinct) eigenvalues — roots of the (characteristic) polynomial

$$|\mathbf{A} - \lambda \mathbf{I}| = 0.$$

The eigenvectors,  $\mathbf{v}$ , are also sometimes called right eigenvectors to distinguish them from another set of left eigenvectors that satisfy

$$\mathbf{v}_L^H \mathbf{A} = \lambda \mathbf{v}_L^H$$

or

$$\mathbf{A}^H \mathbf{v}_L = \lambda^* \mathbf{v}_L.$$

With its default optional arguments, the command `linalg.eig` returns  $\lambda$  and  $\mathbf{v}$ . However, it can also return  $\mathbf{v}_L$  and just  $\lambda$  by itself (`linalg.eigvals` returns just  $\lambda$  as well).

In addition, `linalg.eig` can also solve the more general eigenvalue problem

$$\begin{aligned} \mathbf{A}\mathbf{v} &= \lambda\mathbf{B}\mathbf{v} \\ \mathbf{A}^H \mathbf{v}_L &= \lambda^* \mathbf{B}^H \mathbf{v}_L \end{aligned}$$

for square matrices  $\mathbf{A}$  and  $\mathbf{B}$ . The standard eigenvalue problem is an example of the general eigenvalue problem for  $\mathbf{B} = \mathbf{I}$ . When a generalized eigenvalue problem can be solved, then it provides a decomposition of  $\mathbf{A}$  as

$$\mathbf{A} = \mathbf{B}\mathbf{V}\mathbf{\Lambda}\mathbf{V}^{-1}$$

where  $\mathbf{V}$  is the collection of eigenvectors into columns and  $\mathbf{\Lambda}$  is a diagonal matrix of eigenvalues.

By definition, eigenvectors are only defined up to a constant scale factor. In SciPy, the scaling factor for the eigenvectors is chosen so that  $\|\mathbf{v}\|^2 = \sum_i v_i^2 = 1$ .

As an example, consider finding the eigenvalues and eigenvectors of the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 5 & 2 \\ 2 & 4 & 1 \\ 3 & 6 & 2 \end{bmatrix}.$$

The characteristic polynomial is

$$\begin{aligned} |\mathbf{A} - \lambda\mathbf{I}| &= (1 - \lambda)[(4 - \lambda)(2 - \lambda) - 6] - \\ &5[2(2 - \lambda) - 3] + 2[12 - 3(4 - \lambda)] \\ &= -\lambda^3 + 7\lambda^2 + 8\lambda - 3. \end{aligned}$$

The roots of this polynomial are the eigenvalues of  $\mathbf{A}$ :

$$\begin{aligned} \lambda_1 &= 7.9579 \\ \lambda_2 &= -1.2577 \\ \lambda_3 &= 0.2997. \end{aligned}$$

The eigenvectors corresponding to each eigenvalue can be found using the original equation. The eigenvectors associated with these eigenvalues can then be found.

```
>>> import numpy as np
>>> from scipy import linalg
>>> A = np.array([[1, 2], [3, 4]])
>>> la, v = linalg.eig(A)
>>> l1, l2 = la
>>> print l1, l2 # eigenvalues
(-0.372281323269+0j) (5.37228132327+0j)
>>> print v[:, 0] # first eigenvector
[-0.82456484  0.56576746]
>>> print v[:, 1] # second eigenvector
[-0.41597356 -0.90937671]
>>> print np.sum(abs(v**2), axis=0) # eigenvectors are unitary
[ 1.  1.]
>>> v1 = np.array(v[:, 0]).T
>>> print linalg.norm(A.dot(v1) - l1*v1) # check the computation
3.23682852457e-16
```

### Singular value decomposition

Singular Value Decomposition (SVD) can be thought of as an extension of the eigenvalue problem to matrices that are not square. Let  $\mathbf{A}$  be an  $M \times N$  matrix with  $M$  and  $N$  arbitrary. The matrices  $\mathbf{A}^H \mathbf{A}$  and  $\mathbf{A} \mathbf{A}^H$  are square hermitian matrices<sup>3</sup> of size  $N \times N$  and  $M \times M$  respectively. It is known that the eigenvalues of square hermitian matrices are real and non-negative. In addition, there are at most  $\min(M, N)$  identical non-zero eigenvalues of  $\mathbf{A}^H \mathbf{A}$  and  $\mathbf{A} \mathbf{A}^H$ . Define these positive eigenvalues as  $\sigma_i^2$ . The square-root of these are called singular values of  $\mathbf{A}$ . The eigenvectors of  $\mathbf{A}^H \mathbf{A}$  are collected by columns into an  $N \times N$  unitary<sup>4</sup> matrix  $\mathbf{V}$  while the eigenvectors of  $\mathbf{A} \mathbf{A}^H$  are collected by columns in the unitary matrix  $\mathbf{U}$ , the singular values are collected in an  $M \times N$  zero matrix  $\mathbf{\Sigma}$  with main diagonal entries set to the singular values. Then

$$\mathbf{A} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^H$$

is the singular-value decomposition of  $\mathbf{A}$ . Every matrix has a singular value decomposition. Sometimes, the singular values are called the spectrum of  $\mathbf{A}$ . The command `linalg.svd` will return  $\mathbf{U}$ ,  $\mathbf{V}^H$ , and  $\sigma_i$  as an array of the singular values. To obtain the matrix  $\mathbf{\Sigma}$  use `linalg.diagsvd`. The following example illustrates the use of `linalg.svd`.

```
>>> import numpy as np
>>> from scipy import linalg
>>> A = np.array([[1,2,3],[4,5,6]])
>>> A
array([[1, 2, 3],
       [4, 5, 6]])
>>> M,N = A.shape
>>> U,s,Vh = linalg.svd(A)
>>> Sig = linalg.diagsvd(s,M,N)
>>> U, Vh = U, Vh
>>> U
array([[ -0.3863177,  -0.92236578],
       [-0.92236578,   0.3863177 ]])
>>> Sig
array([[ 9.508032,  0.,  0.],
       [ 0.,  0.77286964,  0.]])
>>> Vh
array([[ -0.42866713, -0.56630692, -0.7039467 ],
       [ 0.80596391,  0.11238241, -0.58119908],
       [ 0.40824829, -0.81649658,  0.40824829]])
>>> U.dot(Sig.dot(Vh)) #check computation
array([[ 1.,  2.,  3.],
       [ 4.,  5.,  6.]])
```

### LU decomposition

The LU decomposition finds a representation for the  $M \times N$  matrix  $\mathbf{A}$  as

$$\mathbf{A} = \mathbf{P} \mathbf{L} \mathbf{U}$$

where  $\mathbf{P}$  is an  $M \times M$  permutation matrix (a permutation of the rows of the identity matrix),  $\mathbf{L}$  is in  $M \times K$  lower triangular or trapezoidal matrix ( $K = \min(M, N)$ ) with unit-diagonal, and  $\mathbf{U}$  is an upper triangular or trapezoidal matrix. The SciPy command for this decomposition is `linalg.lu`.

Such a decomposition is often useful for solving many simultaneous equations where the left-hand-side does not change but the right hand side does. For example, suppose we are going to solve

$$\mathbf{A} \mathbf{x}_i = \mathbf{b}_i$$

<sup>3</sup> A hermitian matrix  $\mathbf{D}$  satisfies  $\mathbf{D}^H = \mathbf{D}$ .

<sup>4</sup> A unitary matrix  $\mathbf{D}$  satisfies  $\mathbf{D}^H \mathbf{D} = \mathbf{I} = \mathbf{D} \mathbf{D}^H$  so that  $\mathbf{D}^{-1} = \mathbf{D}^H$ .



for many different  $\mathbf{b}_i$ . The LU decomposition allows this to be written as

$$\mathbf{PLU}\mathbf{x}_i = \mathbf{b}_i.$$

Because  $\mathbf{L}$  is lower-triangular, the equation can be solved for  $\mathbf{U}\mathbf{x}_i$  and finally  $\mathbf{x}_i$  very rapidly using forward- and back-substitution. An initial time spent factoring  $\mathbf{A}$  allows for very rapid solution of similar systems of equations in the future. If the intent for performing LU decomposition is for solving linear systems then the command `linalg.lu_factor` should be used followed by repeated applications of the command `linalg.lu_solve` to solve the system for each new right-hand-side.

### *Cholesky decomposition*

Cholesky decomposition is a special case of LU decomposition applicable to Hermitian positive definite matrices. When  $\mathbf{A} = \mathbf{A}^H$  and  $\mathbf{x}^H \mathbf{A} \mathbf{x} \geq 0$  for all  $\mathbf{x}$ , then decompositions of  $\mathbf{A}$  can be found so that

$$\begin{aligned}\mathbf{A} &= \mathbf{U}^H \mathbf{U} \\ \mathbf{A} &= \mathbf{L} \mathbf{L}^H\end{aligned}$$

where  $\mathbf{L}$  is lower-triangular and  $\mathbf{U}$  is upper triangular. Notice that  $\mathbf{L} = \mathbf{U}^H$ . The command `linalg.cholesky` computes the cholesky factorization. For using cholesky factorization to solve systems of equations there are also `linalg.cho_factor` and `linalg.cho_solve` routines that work similarly to their LU decomposition counterparts.

### *QR decomposition*

The QR decomposition (sometimes called a polar decomposition) works for any  $M \times N$  array and finds an  $M \times M$  unitary matrix  $\mathbf{Q}$  and an  $M \times N$  upper-trapezoidal matrix  $\mathbf{R}$  such that

$$\mathbf{A} = \mathbf{QR}.$$

Notice that if the SVD of  $\mathbf{A}$  is known then the QR decomposition can be found

$$\mathbf{A} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^H = \mathbf{QR}$$

implies that  $\mathbf{Q} = \mathbf{U}$  and  $\mathbf{R} = \mathbf{\Sigma} \mathbf{V}^H$ . Note, however, that in SciPy independent algorithms are used to find QR and SVD decompositions. The command for QR decomposition is `linalg.qr`.

### *Schur decomposition*

For a square  $N \times N$  matrix,  $\mathbf{A}$ , the Schur decomposition finds (not-necessarily unique) matrices  $\mathbf{T}$  and  $\mathbf{Z}$  such that

$$\mathbf{A} = \mathbf{Z} \mathbf{T} \mathbf{Z}^H$$

where  $\mathbf{Z}$  is a unitary matrix and  $\mathbf{T}$  is either upper-triangular or quasi-upper triangular depending on whether or not a real schur form or complex schur form is requested. For a real schur form both  $\mathbf{T}$  and  $\mathbf{Z}$  are real-valued when  $\mathbf{A}$  is real-valued. When  $\mathbf{A}$  is a real-valued matrix the real schur form is only quasi-upper triangular because  $2 \times 2$  blocks extrude from the main diagonal corresponding to any complex-valued eigenvalues. The command `linalg.schur` finds the Schur decomposition while the command `linalg.rsf2csf` converts  $\mathbf{T}$  and  $\mathbf{Z}$  from a real Schur form to a complex Schur form. The Schur form is especially useful in calculating functions of matrices.

The following example illustrates the schur decomposition:

```
>>> from scipy import linalg
>>> A = np.mat('[1 3 2; 1 4 5; 2 3 6]')
>>> T, Z = linalg.schur(A)
>>> T1, Z1 = linalg.schur(A, 'complex')
>>> T2, Z2 = linalg.rsf2csf(T, Z)
>>> T
array([[ 9.90012467,  1.78947961, -0.65498528],
```

```

    [ 0.          ,  0.54993766, -1.57754789],
    [ 0.          ,  0.51260928,  0.54993766]])
>>> T2
array([[ 9.90012467 +0.00000000e+00j, -0.32436598 +1.55463542e+00j,
        -0.88619748 +5.69027615e-01j],
       [ 0.00000000 +0.00000000e+00j,  0.54993766 +8.99258408e-01j,
         1.06493862 -5.80496735e-16j],
       [ 0.00000000 +0.00000000e+00j,  0.00000000 +0.00000000e+00j,
         0.54993766 -8.99258408e-01j]])
>>> abs(T1 - T2) # different
array([[ 1.06604538e-14,  2.06969555e+00,  1.69375747e+00], # may vary
       [ 0.00000000e+00,  1.33688556e-15,  4.74146496e-01],
       [ 0.00000000e+00,  0.00000000e+00,  1.13220977e-15]])
>>> abs(Z1 - Z2) # different
array([[ 0.06833781,  0.88091091,  0.79568503], # may vary
       [ 0.11857169,  0.44491892,  0.99594171],
       [ 0.12624999,  0.60264117,  0.77257633]])
>>> T, Z, T1, Z1, T2, Z2 = map(np.mat, (T, Z, T1, Z1, T2, Z2))
>>> abs(A - Z*T*Z.H) # same
matrix([[ 5.55111512e-16,  1.77635684e-15,  2.22044605e-15],
        [ 0.00000000e+00,  3.99680289e-15,  8.88178420e-16],
        [ 1.11022302e-15,  4.44089210e-16,  3.55271368e-15]])
>>> abs(A - Z1*T1*Z1.H) # same
matrix([[ 4.26993904e-15,  6.21793362e-15,  8.00007092e-15],
        [ 5.77945386e-15,  6.21798014e-15,  1.06653681e-14],
        [ 7.16681444e-15,  8.90271058e-15,  1.77635764e-14]])
>>> abs(A - Z2*T2*Z2.H) # same
matrix([[ 6.02594127e-16,  1.77648931e-15,  2.22506907e-15],
        [ 2.46275555e-16,  3.99684548e-15,  8.91642616e-16],
        [ 8.88225111e-16,  8.88312432e-16,  4.44104848e-15]])

```

### Interpolative Decomposition

`scipy.linalg.interpolative` contains routines for computing the interpolative decomposition (ID) of a matrix. For a matrix  $A \in C^{m \times n}$  of rank  $k \leq \min\{m, n\}$  this is a factorization

$$A\Pi = [A\Pi_1 \quad A\Pi_2] = A\Pi_1 [I \quad T],$$

where  $\Pi = [\Pi_1, \Pi_2]$  is a permutation matrix with  $\Pi_1 \in \{0, 1\}^{n \times k}$ , i.e.,  $A\Pi_2 = A\Pi_1 T$ . This can equivalently be written as  $A = BP$ , where  $B = A\Pi_1$  and  $P = [I, T]\Pi^T$  are the *skeleton* and *interpolation matrices*, respectively.

**See also:**

`scipy.linalg.interpolative` — for more information.

### Matrix Functions

Consider the function  $f(x)$  with Taylor series expansion

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} x^k.$$

A matrix function can be defined using this Taylor series for the square matrix  $\mathbf{A}$  as

$$f(\mathbf{A}) = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} \mathbf{A}^k.$$

While, this serves as a useful representation of a matrix function, it is rarely the best way to calculate a matrix function.

### Exponential and logarithm functions

The matrix exponential is one of the more common matrix functions. It can be defined for square matrices as

$$e^{\mathbf{A}} = \sum_{k=0}^{\infty} \frac{1}{k!} \mathbf{A}^k.$$

The command `linalg.expm3` uses this Taylor series definition to compute the matrix exponential. Due to poor convergence properties it is not often used.

Another method to compute the matrix exponential is to find an eigenvalue decomposition of  $\mathbf{A}$  :

$$\mathbf{A} = \mathbf{V} \mathbf{\Lambda} \mathbf{V}^{-1}$$

and note that

$$e^{\mathbf{A}} = \mathbf{V} e^{\mathbf{\Lambda}} \mathbf{V}^{-1}$$

where the matrix exponential of the diagonal matrix  $\mathbf{\Lambda}$  is just the exponential of its elements. This method is implemented in `linalg.expm2`.

The preferred method for implementing the matrix exponential is to use scaling and a Padé approximation for  $e^x$ . This algorithm is implemented as `linalg.expm`.

The inverse of the matrix exponential is the matrix logarithm defined as the inverse of the matrix exponential.

$$\mathbf{A} \equiv \exp(\log(\mathbf{A})).$$

The matrix logarithm can be obtained with `linalg.logm`.

### Trigonometric functions

The trigonometric functions `sin`, `cos`, and `tan` are implemented for matrices in `linalg.sinm`, `linalg.cosm`, and `linalg.tanm` respectively. The matrix `sin` and `cosine` can be defined using Euler's identity as

$$\begin{aligned} \sin(\mathbf{A}) &= \frac{e^{j\mathbf{A}} - e^{-j\mathbf{A}}}{2j} \\ \cos(\mathbf{A}) &= \frac{e^{j\mathbf{A}} + e^{-j\mathbf{A}}}{2}. \end{aligned}$$

The tangent is

$$\tan(x) = \frac{\sin(x)}{\cos(x)} = [\cos(x)]^{-1} \sin(x)$$

and so the matrix tangent is defined as

$$[\cos(\mathbf{A})]^{-1} \sin(\mathbf{A}).$$

### Hyperbolic trigonometric functions

The hyperbolic trigonometric functions `sinh`, `cosh`, and `tanh` can also be defined for matrices using the familiar definitions:

$$\begin{aligned} \sinh(\mathbf{A}) &= \frac{e^{\mathbf{A}} - e^{-\mathbf{A}}}{2} \\ \cosh(\mathbf{A}) &= \frac{e^{\mathbf{A}} + e^{-\mathbf{A}}}{2} \\ \tanh(\mathbf{A}) &= [\cosh(\mathbf{A})]^{-1} \sinh(\mathbf{A}). \end{aligned}$$

These matrix functions can be found using `linalg.sinhm`, `linalg.coshm`, and `linalg.tanhm`.

*Arbitrary function*

Finally, any arbitrary function that takes one complex number and returns a complex number can be called as a matrix function using the command `linalg.funm`. This command takes the matrix and an arbitrary Python function. It then implements an algorithm from Golub and Van Loan’s book “Matrix Computations” to compute function applied to the matrix using a Schur decomposition. Note that *the function needs to accept complex numbers* as input in order to work with this algorithm. For example the following code computes the zeroth-order Bessel function applied to a matrix.

```
>>> from scipy import special, random, linalg
>>> np.random.seed(1234)
>>> A = random.rand(3, 3)
>>> B = linalg.funm(A, lambda x: special.jv(0, x))
>>> A
array([[ 0.19151945,  0.62210877,  0.43772774],
       [ 0.78535858,  0.77997581,  0.27259261],
       [ 0.27646426,  0.80187218,  0.95813935]])
>>> B
array([[ 0.86511146, -0.19676526, -0.13856748],
       [-0.17479869,  0.7259118 , -0.16606258],
       [-0.19212044, -0.32052767,  0.73590704]])
>>> linalg.eigvals(A)
array([ 1.73881510+0.j, -0.20270676+0.j,  0.39352627+0.j])
>>> special.jv(0, linalg.eigvals(A))
array([ 0.37551908+0.j,  0.98975384+0.j,  0.96165739+0.j])
>>> linalg.eigvals(B)
array([ 0.37551908+0.j,  0.98975384+0.j,  0.96165739+0.j])
```

Note how, by virtue of how matrix analytic functions are defined, the Bessel function has acted on the matrix eigenvalues.

**Special matrices**

SciPy and NumPy provide several functions for creating special matrices that are frequently used in engineering and science.

Type	Function	Description
block diagonal	<code>scipy.linalg.block_diag</code>	Create a block diagonal matrix from the provided arrays.
circulant	<code>scipy.linalg.circulant</code>	Construct a circulant matrix.
companion	<code>scipy.linalg.companion</code>	Create a companion matrix.
Hadamard	<code>scipy.linalg.hadamard</code>	Construct a Hadamard matrix.
Hankel	<code>scipy.linalg.hankel</code>	Construct a Hankel matrix.
Hilbert	<code>scipy.linalg.hilbert</code>	Construct a Hilbert matrix.
Inverse Hilbert	<code>scipy.linalg.invhilbert</code>	Construct the inverse of a Hilbert matrix.
Leslie	<code>scipy.linalg.leslie</code>	Create a Leslie matrix.
Pascal	<code>scipy.linalg.pascal</code>	Create a Pascal matrix.
Toeplitz	<code>scipy.linalg toeplitz</code>	Construct a Toeplitz matrix.
Van der Monde	<code>numpy.vander</code>	Generate a Van der Monde matrix.

For examples of the use of these functions, see their respective docstrings.

### 3.1.10 Sparse Eigenvalue Problems with ARPACK

#### Introduction

ARPACK is a Fortran package which provides routines for quickly finding a few eigenvalues/eigenvectors of large sparse matrices. In order to find these solutions, it requires only left-multiplication by the matrix in question. This operation is performed through a *reverse-communication* interface. The result of this structure is that ARPACK is able to find eigenvalues and eigenvectors of any linear function mapping a vector to a vector.

All of the functionality provided in ARPACK is contained within the two high-level interfaces `scipy.sparse.linalg.eigs` and `scipy.sparse.linalg.eigsh`. `eigs` provides interfaces to find the eigenvalues/vectors of real or complex nonsymmetric square matrices, while `eigsh` provides interfaces for real-symmetric or complex-hermitian matrices.

#### Basic Functionality

ARPACK can solve either standard eigenvalue problems of the form

$$A\mathbf{x} = \lambda\mathbf{x}$$

or general eigenvalue problems of the form

$$A\mathbf{x} = \lambda M\mathbf{x}$$

The power of ARPACK is that it can compute only a specified subset of eigenvalue/eigenvector pairs. This is accomplished through the keyword `which`. The following values of `which` are available:

- `which = 'LM'` : Eigenvalues with largest magnitude (`eigs`, `eigsh`), that is, largest eigenvalues in the euclidean norm of complex numbers.
- `which = 'SM'` : Eigenvalues with smallest magnitude (`eigs`, `eigsh`), that is, smallest eigenvalues in the euclidean norm of complex numbers.
- `which = 'LR'` : Eigenvalues with largest real part (`eigs`)
- `which = 'SR'` : Eigenvalues with smallest real part (`eigs`)
- `which = 'LI'` : Eigenvalues with largest imaginary part (`eigs`)
- `which = 'SI'` : Eigenvalues with smallest imaginary part (`eigs`)
- `which = 'LA'` : Eigenvalues with largest algebraic value (`eigsh`), that is, largest eigenvalues inclusive of any negative sign.
- `which = 'SA'` : Eigenvalues with smallest algebraic value (`eigsh`), that is, smallest eigenvalues inclusive of any negative sign.
- `which = 'BE'` : Eigenvalues from both ends of the spectrum (`eigsh`)

Note that ARPACK is generally better at finding extremal eigenvalues: that is, eigenvalues with large magnitudes. In particular, using `which = 'SM'` may lead to slow execution time and/or anomalous results. A better approach is to use *shift-invert mode*.

#### Shift-Invert Mode

Shift invert mode relies on the following observation. For the generalized eigenvalue problem

$$A\mathbf{x} = \lambda M\mathbf{x}$$

it can be shown that

$$(A - \sigma M)^{-1} M \mathbf{x} = \nu \mathbf{x}$$

where

$$\nu = \frac{1}{\lambda - \sigma}$$

## Examples

Imagine you'd like to find the smallest and largest eigenvalues and the corresponding eigenvectors for a large matrix. ARPACK can handle many forms of input: dense matrices such as `numpy.ndarray` instances, sparse matrices such as `scipy.sparse.csr_matrix`, or a general linear operator derived from `scipy.sparse.linalg.LinearOperator`. For this example, for simplicity, we'll construct a symmetric, positive-definite matrix.

```
>>> import numpy as np
>>> from scipy.linalg import eigh
>>> from scipy.sparse.linalg import eigsh
>>> np.set_printoptions(suppress=True)
>>>
>>> np.random.seed(0)
>>> X = np.random.random((100,100)) - 0.5
>>> X = np.dot(X, X.T) #create a symmetric matrix
```

We now have a symmetric matrix `X` with which to test the routines. First compute a standard eigenvalue decomposition using `eigh`:

```
>>> evals_all, evecs_all = eigh(X)
```

As the dimension of `X` grows, this routine becomes very slow. Especially if only a few eigenvectors and eigenvalues are needed, ARPACK can be a better option. First let's compute the largest eigenvalues (which = 'LM') of `X` and compare them to the known results:

```
>>> evals_large, evecs_large = eigsh(X, 3, which='LM')
>>> print evals_all[-3:]
[ 29.1446102  30.05821805  31.19467646]
>>> print evals_large
[ 29.1446102  30.05821805  31.19467646]
>>> print np.dot(evecs_large.T, evecs_all[:, -3:])
array([[ -1.   0.   0.],          # may vary (signs)
       [  0.   1.   0.],
       [ -0.   0.  -1.]])
```

The results are as expected. ARPACK recovers the desired eigenvalues, and they match the previously known results. Furthermore, the eigenvectors are orthogonal, as we'd expect. Now let's attempt to solve for the eigenvalues with smallest magnitude:

```
>>> evals_small, evecs_small = eigsh(X, 3, which='SM')
Traceback (most recent call last):          # may vary (convergence)
...
scipy.sparse.linalg.eigen.arpack.arpack.ArpackNoConvergence:
ARPACK error -1: No convergence (1001 iterations, 0/3 eigenvectors converged)
```

Oops. We see that as mentioned above, ARPACK is not quite as adept at finding small eigenvalues. There are a few ways this problem can be addressed. We could increase the tolerance (`tol`) to lead to faster convergence:

```

>>> evals_small, evecs_small = eigsh(X, 3, which='SM', tol=1E-2)
>>> evals_all[:3]
array([0.0003783, 0.00122714, 0.00715878])
>>> evals_small
array([0.00037831, 0.00122714, 0.00715881])
>>> np.dot(evecs_small.T, evecs_all[:, :3])
array([[ 0.99999999  0.00000024 -0.00000049],      # may vary (signs)
       [-0.00000023  0.99999999  0.00000056],
       [ 0.00000031 -0.00000037  0.99999852]])

```

This works, but we lose the precision in the results. Another option is to increase the maximum number of iterations (`maxiter`) from 1000 to 5000:

```

>>> evals_small, evecs_small = eigsh(X, 3, which='SM', maxiter=5000)
>>> evals_all[:3]
array([0.0003783, 0.00122714, 0.00715878])
>>> evals_small
array([0.0003783, 0.00122714, 0.00715878])
>>> np.dot(evecs_small.T, evecs_all[:, :3])
array([[ 1.  0.  0.],      # may vary (signs)
       [-0.  1.  0.],
       [ 0.  0. -1.]])

```

We get the results we'd hoped for, but the computation time is much longer. Fortunately, ARPACK contains a mode that allows quick determination of non-external eigenvalues: *shift-invert mode*. As mentioned above, this mode involves transforming the eigenvalue problem to an equivalent problem with different eigenvalues. In this case, we hope to find eigenvalues near zero, so we'll choose `sigma = 0`. The transformed eigenvalues will then satisfy  $\nu = 1/(\sigma - \lambda) = 1/\lambda$ , so our small eigenvalues  $\lambda$  become large eigenvalues  $\nu$ .

```

>>> evals_small, evecs_small = eigsh(X, 3, sigma=0, which='LM')
>>> evals_all[:3]
array([0.0003783, 0.00122714, 0.00715878])
>>> evals_small
array([0.0003783, 0.00122714, 0.00715878])
>>> np.dot(evecs_small.T, evecs_all[:, :3])
array([[ 1.  0.  0.],      # may vary (signs)
       [ 0. -1. -0.],
       [-0. -0.  1.]])

```

We get the results we were hoping for, with much less computational time. Note that the transformation from  $\nu \rightarrow \lambda$  takes place entirely in the background. The user need not worry about the details.

The shift-invert mode provides more than just a fast way to obtain a few small eigenvalues. Say you desire to find internal eigenvalues and eigenvectors, e.g. those nearest to  $\lambda = 1$ . Simply set `sigma = 1` and ARPACK takes care of the rest:

```

>>> evals_mid, evecs_mid = eigsh(X, 3, sigma=1, which='LM')
>>> i_sort = np.argsort(abs(1. / (1 - evals_all)))[-3:]
>>> evals_all[i_sort]
array([1.16577199, 0.85081388, 1.06642272])
>>> evals_mid
array([0.85081388, 1.06642272, 1.16577199])
>>> print np.dot(evecs_mid.T, evecs_all[:, i_sort])
array([[ -0.  1.  0.],      # may vary (signs)
       [-0. -0.  1.],
       [ 1.  0.  0.]])

```

The eigenvalues come out in a different order, but they're all there. Note that the shift-invert mode requires the internal solution of a matrix inverse. This is taken care of automatically by `eigsh` and `eigs`, but the

operation can also be specified by the user. See the docstring of `scipy.sparse.linalg.eigsh` and `scipy.sparse.linalg.eigs` for details.

## References

### 3.1.11 Compressed Sparse Graph Routines (`scipy.sparse.csgraph`)

#### Example: Word Ladders

A **Word Ladder** is a word game invented by Lewis Carroll in which players find paths between words by switching one letter at a time. For example, one can link “ape” and “man” in the following way:

ape → apt → ait → bit → big → bag → mag → man

Note that each step involves changing just one letter of the word. This is just one possible path from “ape” to “man”, but is it the shortest possible path? If we desire to find the shortest word ladder path between two given words, the sparse graph submodule can help.

First we need a list of valid words. Many operating systems have such a list built-in. For example, on linux, a word list can often be found at one of the following locations:

```
/usr/share/dict
/var/lib/dict
```

Another easy source for words are the scrabble word lists available at various sites around the internet (search with your favorite search engine). We’ll first create this list. The system word lists consist of a file with one word per line. The following should be modified to use the particular word list you have available:

```
>>> word_list = open('/usr/share/dict/words').readlines()
>>> word_list = map(str.strip, word_list)
```

We want to look at words of length 3, so let’s select just those words of the correct length. We’ll also eliminate words which start with upper-case (proper nouns) or contain non alpha-numeric characters like apostrophes and hyphens. Finally, we’ll make sure everything is lower-case for comparison later:

```
>>> word_list = [word for word in word_list if len(word) == 3]
>>> word_list = [word for word in word_list if word[0].islower()]
>>> word_list = [word for word in word_list if word.isalpha()]
>>> word_list = map(str.lower, word_list)
>>> len(word_list)
586
```

Now we have a list of 586 valid three-letter words (the exact number may change depending on the particular list used). Each of these words will become a node in our graph, and we will create edges connecting the nodes associated with each pair of words which differs by only one letter.

There are efficient ways to do this, and inefficient ways to do this. To do this as efficiently as possible, we’re going to use some sophisticated numpy array manipulation:

```
>>> import numpy as np
>>> word_list = np.asarray(word_list)
>>> word_list.dtype
dtype('|S3')
>>> word_list.sort() # sort for quick searching later
```

We have an array where each entry is three bytes. We’d like to find all pairs where exactly one byte is different. We’ll start by converting each word to a three-dimensional vector:



```
>>> word_bytes = np.ndarray((word_list.size, word_list.itemsize),
...                          dtype='int8',
...                          buffer=word_list.data)
>>> word_bytes.shape
(586, 3)
```

Now we'll use the [Hamming distance](#) between each point to determine which pairs of words are connected. The Hamming distance measures the fraction of entries between two vectors which differ: any two words with a hamming distance equal to  $1/N$ , where  $N$  is the number of letters, are connected in the word ladder:

```
>>> from scipy.spatial.distance import pdist, squareform
>>> from scipy.sparse import csr_matrix
>>> hamming_dist = pdist(word_bytes, metric='hamming')
>>> graph = csr_matrix(squareform(hamming_dist < 1.5 / word_list.itemsize))
```

When comparing the distances, we don't use an equality because this can be unstable for floating point values. The inequality produces the desired result as long as no two entries of the word list are identical. Now that our graph is set up, we'll use a shortest path search to find the path between any two words in the graph:

```
>>> i1 = word_list.searchsorted('ape')
>>> i2 = word_list.searchsorted('man')
>>> word_list[i1]
'ape'
>>> word_list[i2]
'man'
```

We need to check that these match, because if the words are not in the list that will not be the case. Now all we need is to find the shortest path between these two indices in the graph. We'll use Dijkstra's algorithm, because it allows us to find the path for just one node:

```
>>> from scipy.sparse.csgraph import dijkstra
>>> distances, predecessors = dijkstra(graph, indices=i1,
...                                  return_predecessors=True)
>>> print distances[i2]
5.0
```

So we see that the shortest path between 'ape' and 'man' contains only five steps. We can use the predecessors returned by the algorithm to reconstruct this path:

```
>>> path = []
>>> i = i2
>>> while i != i1:
...     path.append(word_list[i])
...     i = predecessors[i]
>>> path.append(word_list[i1])
>>> print path[::-1]
['ape', 'apt', 'opt', 'oat', 'mat', 'man']
```

This is three fewer links than our initial example: the path from ape to man is only five steps.

Using other tools in the module, we can answer other questions. For example, are there three-letter words which are not linked in a word ladder? This is a question of connected components in the graph:

```
>>> from scipy.sparse.csgraph import connected_components
>>> N_components, component_list = connected_components(graph)
>>> print N_components
15
```

In this particular sample of three-letter words, there are 15 connected components: that is, 15 distinct sets of words with no paths between the sets. How many words are in each of these sets? We can learn this from the list of components:

```
>>> [np.sum(component_list == i) for i in range(15)]
[571, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1]
```

There is one large connected set, and 14 smaller ones. Let's look at the words in the smaller ones:

```
>>> [list(word_list[np.where(component_list == i)]) for i in range(1, 15)]
[['aha'],
 ['chi'],
 ['ebb'],
 ['ems', 'emu'],
 ['gnu'],
 ['ism'],
 ['khz'],
 ['nth'],
 ['ova'],
 ['qua'],
 ['ugh'],
 ['ups'],
 ['urn'],
 ['use']]
```

These are all the three-letter words which do not connect to others via a word ladder.

We might also be curious about which words are maximally separated. Which two words take the most links to connect? We can determine this by computing the matrix of all shortest paths. Note that by convention, the distance between two non-connected points is reported to be infinity, so we'll need to remove these before finding the maximum:

```
>>> distances, predecessors = dijkstra(graph, return_predecessors=True)
>>> np.max(distances[~np.isinf(distances)])
13.0
```

So there is at least one pair of words which takes 13 steps to get from one to the other! Let's determine which these are:

```
>>> i1, i2 = np.where(distances == 13)
>>> zip(word_list[i1], word_list[i2])
[('imp', 'ohm'),
 ('imp', 'ohs'),
 ('ohm', 'imp'),
 ('ohm', 'ump'),
 ('ohs', 'imp'),
 ('ohs', 'ump'),
 ('ump', 'ohm'),
 ('ump', 'ohs')]
```

We see that there are two pairs of words which are maximally separated from each other: 'imp' and 'ump' on one hand, and 'ohm' and 'ohs' on the other hand. We can find the connecting list in the same way as above:

```
>>> path = []
>>> i = i2[0]
>>> while i != i1[0]:
...     path.append(word_list[i])
...     i = predecessors[i1[0], i]
>>> path.append(word_list[i1[0]])
>>> print path[::-1]
['imp', 'amp', 'asp', 'ass', 'ads', 'add', 'aid', 'mid', 'mod', 'moo', 'too', 'tho', 'oho', 'ohm']
```

This gives us the path we desired to see.

Word ladders are just one potential application of scipy's fast graph algorithms for sparse matrices. Graph theory

makes appearances in many areas of mathematics, data analysis, and machine learning. The sparse graph tools are flexible enough to handle many of these situations.

### 3.1.12 Spatial data structures and algorithms (`scipy.spatial`)

`scipy.spatial` can compute triangulations, Voronoi diagrams, and convex hulls of a set of points, by leveraging the `Qhull` library.

Moreover, it contains `KDTree` implementations for nearest-neighbor point queries, and utilities for distance computations in various metrics.

#### Delaunay triangulations

The Delaunay triangulation is a subdivision of a set of points into a non-overlapping set of triangles, such that no point is inside the circumcircle of any triangle. In practice, such triangulations tend to avoid triangles with small angles.

Delaunay triangulation can be computed using `scipy.spatial` as follows:

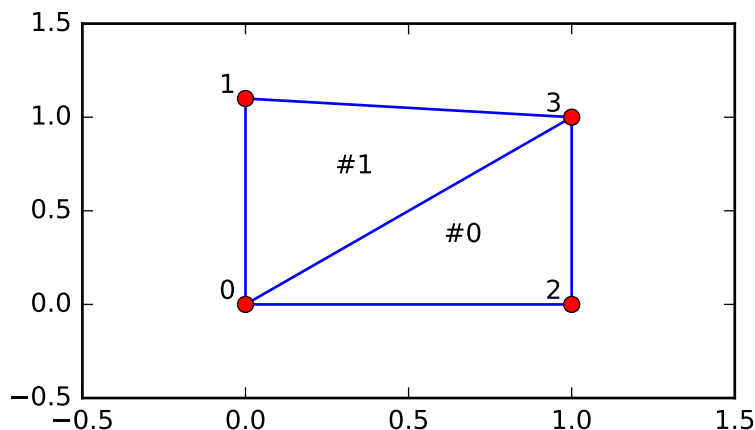
```
>>> from scipy.spatial import Delaunay
>>> points = np.array([[0, 0], [0, 1.1], [1, 0], [1, 1]])
>>> tri = Delaunay(points)
```

We can visualize it:

```
>>> import matplotlib.pyplot as plt
>>> plt.triplot(points[:,0], points[:,1], tri.simplices.copy())
>>> plt.plot(points[:,0], points[:,1], 'o')
```

And add some further decorations:

```
>>> for j, p in enumerate(points):
...     plt.text(p[0]-0.03, p[1]+0.03, j, ha='right') # label the points
>>> for j, s in enumerate(tri.simplices):
...     p = points[s].mean(axis=0)
...     plt.text(p[0], p[1], '#%d' % j, ha='center') # label triangles
>>> plt.xlim(-0.5, 1.5); plt.ylim(-0.5, 1.5)
>>> plt.show()
```



The structure of the triangulation is encoded in the following way: the `simplices` attribute contains the indices of the points in the `points` array that make up the triangle. For instance:

```
>>> i = 1
>>> tri.simplices[i,:]
array([3, 1, 0], dtype=int32)
>>> points[tri.simplices[i,:]]
array([[ 1. ,  1. ],
       [ 0. ,  1.1],
       [ 0. ,  0. ]])
```

Moreover, neighboring triangles can also be found out:

```
>>> tri.neighbors[i]
array([-1,  0, -1], dtype=int32)
```

What this tells us is that this triangle has triangle #0 as a neighbor, but no other neighbors. Moreover, it tells us that neighbor 0 is opposite the vertex 1 of the triangle:

```
>>> points[tri.simplices[i, 1]]
array([ 0. ,  1.1])
```

Indeed, from the figure we see that this is the case.

Qhull can also perform tessellations to simplices also for higher-dimensional point sets (for instance, subdivision into tetrahedra in 3-D).

### ***Coplanar points***

It is important to note that not *all* points necessarily appear as vertices of the triangulation, due to numerical precision issues in forming the triangulation. Consider the above with a duplicated point:

```
>>> points = np.array([[0, 0], [0, 1], [1, 0], [1, 1], [1, 1]])
>>> tri = Delaunay(points)
>>> np.unique(tri.simplices.ravel())
array([0, 1, 2, 3], dtype=int32)
```

Observe that point #4, which is a duplicate, does not occur as a vertex of the triangulation. That this happened is recorded:

```
>>> tri.coplanar
array([[4, 0, 3]], dtype=int32)
```

This means that point 4 resides near triangle 0 and vertex 3, but is not included in the triangulation.

Note that such degeneracies can occur not only because of duplicated points, but also for more complicated geometrical reasons, even in point sets that at first sight seem well-behaved.

However, Qhull has the “QJ” option, which instructs it to perturb the input data randomly until degeneracies are resolved:

```
>>> tri = Delaunay(points, qhull_options="QJ Pp")
>>> points[tri.simplices]
array([[1, 0],
       [1, 1],
       [0, 0]],
      [[1, 1],
       [1, 1],
       [1, 0]],
      [[1, 1],
       [0, 1],
       [0, 1]])
```

```
[0, 0]],
[[0, 1],
 [1, 1],
 [1, 1]])
```

Two new triangles appeared. However, we see that they are degenerate and have zero area.

## Convex hulls

Convex hull is the smallest convex object containing all points in a given point set.

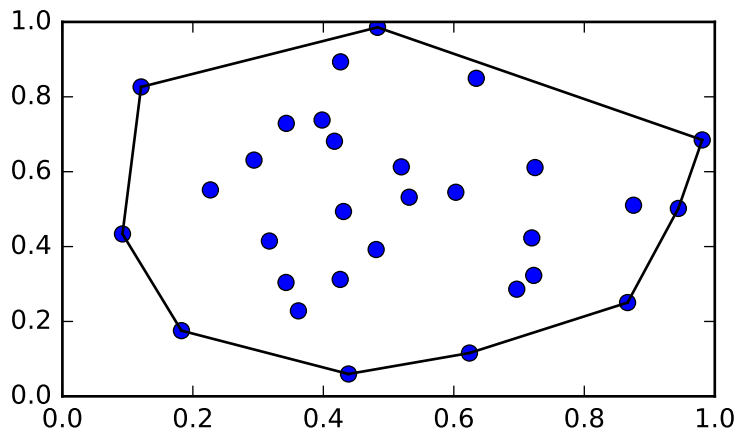
These can be computed via the Qhull wrappers in `scipy.spatial` as follows:

```
>>> from scipy.spatial import ConvexHull
>>> points = np.random.rand(30, 2) # 30 random points in 2-D
>>> hull = ConvexHull(points)
```

The convex hull is represented as a set of N-1 dimensional simplices, which in 2-D means line segments. The storage scheme is exactly the same as for the simplices in the Delaunay triangulation discussed above.

We can illustrate the above result:

```
>>> import matplotlib.pyplot as plt
>>> plt.plot(points[:,0], points[:,1], 'o')
>>> for simplex in hull.simplices:
...     plt.plot(points[simplex,0], points[simplex,1], 'k-')
>>> plt.show()
```



The same can be achieved with `scipy.spatial.convex_hull_plot_2d`.

## Voronoi diagrams

A Voronoi diagram is a subdivision of the space into the nearest neighborhoods of a given set of points.

There are two ways to approach this object using `scipy.spatial`. First, one can use the `KDTree` to answer the question “which of the points is closest to this one”, and define the regions that way:

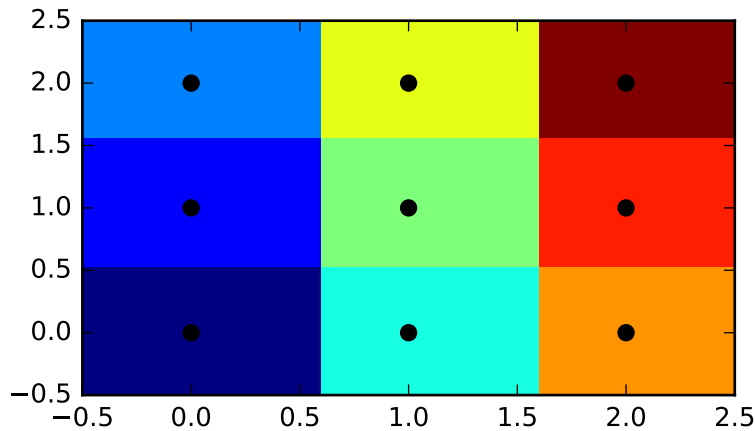
```

>>> from scipy.spatial import KDTree
>>> points = np.array([[0, 0], [0, 1], [0, 2], [1, 0], [1, 1], [1, 2],
...                   [2, 0], [2, 1], [2, 2]])
>>> tree = KDTree(points)
>>> tree.query([0.1, 0.1])
(0.14142135623730953, 0)
    
```

So the point (0.1, 0.1) belongs to region 0. In color:

```

>>> x = np.linspace(-0.5, 2.5, 31)
>>> y = np.linspace(-0.5, 2.5, 33)
>>> xx, yy = np.meshgrid(x, y)
>>> xy = np.c_[xx.ravel(), yy.ravel()]
>>> import matplotlib.pyplot as plt
>>> plt.pcolor(x, y, tree.query(xy)[1].reshape(33, 31))
>>> plt.plot(points[:,0], points[:,1], 'ko')
>>> plt.show()
    
```



This does not, however, give the Voronoi diagram as a geometrical object.

The representation in terms of lines and points can be again obtained via the Qhull wrappers in `scipy.spatial`:

```

>>> from scipy.spatial import Voronoi
>>> vor = Voronoi(points)
>>> vor.vertices
array([[ 0.5,  0.5],
       [ 1.5,  0.5],
       [ 0.5,  1.5],
       [ 1.5,  1.5]])
    
```

The Voronoi vertices denote the set of points forming the polygonal edges of the Voronoi regions. In this case, there are 9 different regions:

```

>>> vor.regions
[[], [-1, 0], [-1, 1], [1, -1, 0], [3, -1, 2], [-1, 3], [-1, 2], [3, 2, 0, 1], [2, -1, 0], [3, -1, 1]]
    
```

Negative value `-1` again indicates a point at infinity. Indeed, only one of the regions, `[3, 1, 0, 2]`, is bounded. Note here that due to similar numerical precision issues as in Delaunay triangulation above, there may be fewer Voronoi regions than input points.

The ridges (lines in 2-D) separating the regions are described as a similar collection of simplices as the convex hull pieces:

```
>>> vor.ridge_vertices
[[-1, 0], [-1, 0], [-1, 1], [-1, 1], [0, 1], [-1, 3], [-1, 2], [2, 3], [-1, 3], [-1, 2], [0, 2], [1,
```

These numbers indicate indices of the Voronoi vertices making up the line segments. -1 is again a point at infinity — only four of the 12 lines is a bounded line segment while the others extend to infinity.

The Voronoi ridges are perpendicular to lines drawn between the input points. Which two points each ridge corresponds to is also recorded:

```
>>> vor.ridge_points
array([[0, 1],
       [0, 3],
       [6, 3],
       [6, 7],
       [3, 4],
       [5, 8],
       [5, 2],
       [5, 4],
       [8, 7],
       [2, 1],
       [4, 1],
       [4, 7]], dtype=int32)
```

This information, taken together, is enough to construct the full diagram.

We can plot it as follows. First the points and the Voronoi vertices:

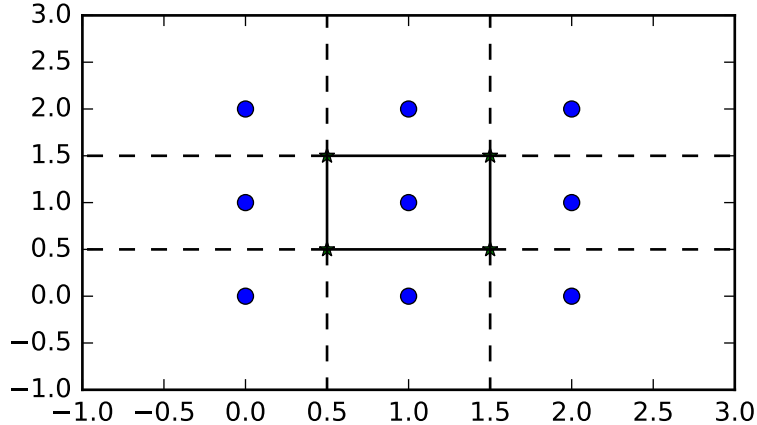
```
>>> plt.plot(points[:, 0], points[:, 1], 'o')
>>> plt.plot(vor.vertices[:, 0], vor.vertices[:, 1], '*')
>>> plt.xlim(-1, 3); plt.ylim(-1, 3)
```

Plotting the finite line segments goes as for the convex hull, but now we have to guard for the infinite edges:

```
>>> for simplex in vor.ridge_vertices:
...     simplex = np.asarray(simplex)
...     if np.all(simplex >= 0):
...         plt.plot(vor.vertices[simplex, 0], vor.vertices[simplex, 1], 'k-')
```

The ridges extending to infinity require a bit more care:

```
>>> center = points.mean(axis=0)
>>> for pointidx, simplex in zip(vor.ridge_points, vor.ridge_vertices):
...     simplex = np.asarray(simplex)
...     if np.any(simplex < 0):
...         i = simplex[simplex >= 0][0] # finite end Voronoi vertex
...         t = points[pointidx[1]] - points[pointidx[0]] # tangent
...         t = t / np.linalg.norm(t)
...         n = np.array([-t[1], t[0]]) # normal
...         midpoint = points[pointidx].mean(axis=0)
...         far_point = vor.vertices[i] + np.sign(np.dot(midpoint - center, n)) * n * 100
...         plt.plot([vor.vertices[i,0], far_point[0]],
...                  [vor.vertices[i,1], far_point[1]], 'k--')
>>> plt.show()
```



This plot can also be created using `scipy.spatial.voronoi_plot_2d`.

### 3.1.13 Statistics (`scipy.stats`)

#### Introduction

In this tutorial we discuss many, but certainly not all, features of `scipy.stats`. The intention here is to provide a user with a working knowledge of this package. We refer to the [reference manual](#) for further details.

Note: This documentation is work in progress.

#### Random Variables

There are two general distribution classes that have been implemented for encapsulating *continuous random variables* and *discrete random variables*. Over 80 continuous random variables (RVs) and 10 discrete random variables have been implemented using these classes. Besides this, new routines and distributions can easily added by the end user. (If you create one, please contribute it).

All of the statistics functions are located in the sub-package `scipy.stats` and a fairly complete listing of these functions can be obtained using `info(stats)`. The list of the random variables available can also be obtained from the docstring for the stats sub-package.

In the discussion below we mostly focus on continuous RVs. Nearly all applies to discrete variables also, but we point out some differences here: [Specific Points for Discrete Distributions](#).

In the code samples below we assume that the `scipy.stats` package is imported as

```
>>> from scipy import stats
```

and in some cases we assume that individual objects are imported as

```
>>> from scipy.stats import norm
```

#### Getting Help

First of all, all distributions are accompanied with help functions. To obtain just some basic information we print the relevant docstring: `print(stats.norm.__doc__)`.



To find the support, i.e., upper and lower bound of the distribution, call:

```
>>> print 'bounds of distribution lower: %s, upper: %s' % (norm.a, norm.b)
bounds of distribution lower: -inf, upper: inf
```

We can list all methods and properties of the distribution with `dir(norm)`. As it turns out, some of the methods are private methods although they are not named as such (their name does not start with a leading underscore), for example `veccdf`, are only available for internal calculation (those methods will give warnings when one tries to use them, and will be removed at some point).

To obtain the *real* main methods, we list the methods of the frozen distribution. (We explain the meaning of a *frozen* distribution below).

```
>>> rv = norm()
>>> dir(rv) # reformatted
['__class__', '__delattr__', '__dict__', '__doc__', '__getattr__',
 '__hash__', '__init__', '__module__', '__new__', '__reduce__', '__reduce_ex__',
 '__repr__', '__setattr__', '__str__', '__weakref__', 'args', 'cdf', 'dist',
 'entropy', 'isf', 'kws', 'moment', 'pdf', 'pmf', 'ppf', 'rvs', 'sf', 'stats']
```

Finally, we can obtain the list of available distribution through introspection:

```
>>> import warnings
>>> warnings.simplefilter('ignore', DeprecationWarning)
>>> dist_continuous = [d for d in dir(stats) if
...                   isinstance(getattr(stats,d), stats.rv_continuous)]
>>> dist_discrete = [d for d in dir(stats) if
...                 isinstance(getattr(stats,d), stats.rv_discrete)]
>>> print 'number of continuous distributions:', len(dist_continuous)
number of continuous distributions: 94
>>> print 'number of discrete distributions: ', len(dist_discrete)
number of discrete distributions: 13
```

### Common Methods

The main public methods for continuous RVs are:

- `rvs`: Random Variates
- `pdf`: Probability Density Function
- `cdf`: Cumulative Distribution Function
- `sf`: Survival Function (1-CDF)
- `ppf`: Percent Point Function (Inverse of CDF)
- `isf`: Inverse Survival Function (Inverse of SF)
- `stats`: Return mean, variance, (Fisher's) skew, or (Fisher's) kurtosis
- `moment`: non-central moments of the distribution

Let's take a normal RV as an example.

```
>>> norm.cdf(0)
0.5
```

To compute the `cdf` at a number of points, we can pass a list or a numpy array.

```
>>> norm.cdf([-1., 0, 1])
array([ 0.15865525,  0.5,  0.84134475])
>>> import numpy as np
```

```
>>> norm.cdf(np.array([-1., 0, 1]))
array([ 0.15865525,  0.5,  0.84134475])
```

Thus, the basic methods such as *pdf*, *cdf*, and so on are vectorized with `np.vectorize`.

Other generally useful methods are supported too:

```
>>> norm.mean(), norm.std(), norm.var()
(0.0, 1.0, 1.0)
>>> norm.stats(moments = "mv")
(array(0.0), array(1.0))
```

To find the median of a distribution we can use the percent point function `ppf`, which is the inverse of the `cdf`:

```
>>> norm.ppf(0.5)
0.0
```

To generate a sequence of random variates, use the `size` keyword argument:

```
>>> norm.rvs(size=3)
array([-0.35687759,  1.34347647, -0.11710531]) # random
```

Note that drawing random numbers relies on generators from `numpy.random` package. In the example above, the specific stream of random numbers is not reproducible across runs. To achieve reproducibility, you can explicitly seed a global variable

```
>>> np.random.seed(1234)
```

Relying on a global state is not recommended though. A better way is to use the `random_state` parameter which accepts an instance of `numpy.random.RandomState` class, or an integer which is then used to seed an internal `RandomState` object:

```
>>> norm.rvs(size=5, random_state=1234)
array([ 0.47143516, -1.19097569,  1.43270697, -0.3126519 , -0.72058873])
```

Don't think that `norm.rvs(5)` generates 5 variates:

```
>>> norm.rvs(5)
5.471435163732493
```

Here, 5 with no keyword is being interpreted as the first possible keyword argument, `loc`, which is the first of a pair of keyword arguments taken by all continuous distributions. This brings us to the topic of the next subsection.

### ***Shifting and Scaling***

All continuous distributions take `loc` and `scale` as keyword parameters to adjust the location and scale of the distribution, e.g. for the standard normal distribution the location is the mean and the scale is the standard deviation.

```
>>> norm.stats(loc = 3, scale = 4, moments = "mv")
(array(3.0), array(16.0))
```

In many cases the standardized distribution for a random variable  $X$  is obtained through the transformation  $(X - \text{loc}) / \text{scale}$ . The default values are `loc = 0` and `scale = 1`.

Smart use of `loc` and `scale` can help modify the standard distributions in many ways. To illustrate the scaling further, the `cdf` of an exponentially distributed RV with mean  $1/\lambda$  is given by

$$F(x) = 1 - \exp(-\lambda x)$$

By applying the scaling rule above, it can be seen that by taking `scale = 1./lambda` we get the proper scale.

```
>>> from scipy.stats import expon
>>> expon.mean(scale=3.)
3.0
```

**Note:** Distributions that take shape parameters may require more than simple application of `loc` and/or `scale` to achieve the desired form. For example, the distribution of 2-D vector lengths given a constant vector of length  $R$  perturbed by independent  $N(0, \sigma^2)$  deviations in each component is `rice( $R/\sigma$ , scale= $\sigma$ )`. The first argument is a shape parameter that needs to be scaled along with  $x$ .

The uniform distribution is also interesting:

```
>>> from scipy.stats import uniform
>>> uniform.cdf([0, 1, 2, 3, 4, 5], loc = 1, scale = 4)
array([ 0. ,  0. ,  0.25,  0.5 ,  0.75,  1.  ])
```

Finally, recall from the previous paragraph that we are left with the problem of the meaning of `norm.rvs(5)`. As it turns out, calling a distribution like this, the first argument, i.e., the 5, gets passed to set the `loc` parameter. Let's see:

```
>>> np.mean(norm.rvs(5, size=500))
4.983550784784704
```

Thus, to explain the output of the example of the last section: `norm.rvs(5)` generates a single normally distributed random variate with mean `loc=5`, because of the default `size=1`.

We recommend that you set `loc` and `scale` parameters explicitly, by passing the values as keywords rather than as arguments. Repetition can be minimized when calling more than one method of a given RV by using the technique of [Freezing a Distribution](#), as explained below.

### Shape Parameters

While a general continuous random variable can be shifted and scaled with the `loc` and `scale` parameters, some distributions require additional shape parameters. For instance, the gamma distribution, with density

$$\gamma(x, a) = \frac{\lambda(\lambda x)^{a-1}}{\Gamma(a)} e^{-\lambda x},$$

requires the shape parameter  $a$ . Observe that setting  $\lambda$  can be obtained by setting the `scale` keyword to  $1/\lambda$ .

Let's check the number and name of the shape parameters of the gamma distribution. (We know from the above that this should be 1.)

```
>>> from scipy.stats import gamma
>>> gamma.numargs
1
>>> gamma.shapes
'a'
```

Now we set the value of the shape variable to 1 to obtain the exponential distribution, so that we compare easily whether we get the results we expect.

```
>>> gamma(1, scale=2.).stats(moments="mv")
(array(2.0), array(4.0))
```

Notice that we can also specify shape parameters as keywords:

```
>>> gamma(a=1, scale=2.).stats(moments="mv")
(array(2.0), array(4.0))
```

### *Freezing a Distribution*

Passing the `loc` and `scale` keywords time and again can become quite bothersome. The concept of *freezing* a RV is used to solve such problems.

```
>>> rv = gamma(1, scale=2.)
```

By using `rv` we no longer have to include the scale or the shape parameters anymore. Thus, distributions can be used in one of two ways, either by passing all distribution parameters to each method call (such as we did earlier) or by freezing the parameters for the instance of the distribution. Let us check this:

```
>>> rv.mean(), rv.std()
(2.0, 2.0)
```

This is indeed what we should get.

### *Broadcasting*

The basic methods `pdf` and so on satisfy the usual numpy broadcasting rules. For example, we can calculate the critical values for the upper tail of the t distribution for different probabilities and degrees of freedom.

```
>>> stats.t.isf([0.1, 0.05, 0.01], [[10], [11]])
array([[ 1.37218364,  1.81246112,  2.76376946],
       [ 1.36343032,  1.79588482,  2.71807918]])
```

Here, the first row are the critical values for 10 degrees of freedom and the second row for 11 degrees of freedom (d.o.f.). Thus, the broadcasting rules give the same result of calling `isf` twice:

```
>>> stats.t.isf([0.1, 0.05, 0.01], 10)
array([ 1.37218364,  1.81246112,  2.76376946])
>>> stats.t.isf([0.1, 0.05, 0.01], 11)
array([ 1.36343032,  1.79588482,  2.71807918])
```

If the array with probabilities, i.e., `[0.1, 0.05, 0.01]` and the array of degrees of freedom i.e., `[10, 11, 12]`, have the same array shape, then element wise matching is used. As an example, we can obtain the 10% tail for 10 d.o.f., the 5% tail for 11 d.o.f. and the 1% tail for 12 d.o.f. by calling

```
>>> stats.t.isf([0.1, 0.05, 0.01], [10, 11, 12])
array([ 1.37218364,  1.79588482,  2.68099799])
```

### *Specific Points for Discrete Distributions*

Discrete distribution have mostly the same basic methods as the continuous distributions. However `pdf` is replaced the probability mass function `pmf`, no estimation methods, such as `fit`, are available, and `scale` is not a valid keyword parameter. The location parameter, keyword `loc` can still be used to shift the distribution.

The computation of the cdf requires some extra attention. In the case of continuous distribution the cumulative distribution function is in most standard cases strictly monotonic increasing in the bounds (a,b) and has therefore a unique inverse. The cdf of a discrete distribution, however, is a step function, hence the inverse cdf, i.e., the percent point function, requires a different definition:

```
ppf(q) = min{x : cdf(x) >= q, x integer}
```

For further info, see the docs [here](#).

We can look at the hypergeometric distribution as an example

```
>>> from scipy.stats import hypergeom
>>> [M, n, N] = [20, 7, 12]
```

If we use the cdf at some integer points and then evaluate the ppf at those cdf values, we get the initial integers back, for example

```
>>> x = np.arange(4)*2
>>> x
array([0, 2, 4, 6])
>>> prb = hypergeom.cdf(x, M, n, N)
>>> prb
array([ 0.0001031991744066,  0.0521155830753351,  0.6083591331269301,
        0.9897832817337386])
>>> hypergeom.ppf(prb, M, n, N)
array([ 0.,  2.,  4.,  6.]
```

If we use values that are not at the kinks of the cdf step function, we get the next higher integer back:

```
>>> hypergeom.ppf(prb + 1e-8, M, n, N)
array([ 1.,  3.,  5.,  7.])
>>> hypergeom.ppf(prb - 1e-8, M, n, N)
array([ 0.,  2.,  4.,  6.]
```

### Fitting Distributions

The main additional methods of the not frozen distribution are related to the estimation of distribution parameters:

- *fit*: maximum likelihood estimation of distribution parameters, including location and scale
- `fit_loc_scale`: estimation of location and scale when shape parameters are given
- `nnlf`: negative log likelihood function
- `expect`: Calculate the expectation of a function against the pdf or pmf

### Performance Issues and Cautionary Remarks

The performance of the individual methods, in terms of speed, varies widely by distribution and method. The results of a method are obtained in one of two ways: either by explicit calculation, or by a generic algorithm that is independent of the specific distribution.

Explicit calculation, on the one hand, requires that the method is directly specified for the given distribution, either through analytic formulas or through special functions in `scipy.special` or `numpy.random` for `rvs`. These are usually relatively fast calculations.

The generic methods, on the other hand, are used if the distribution does not specify any explicit calculation. To define a distribution, only one of pdf or cdf is necessary; all other methods can be derived using numeric integration and root finding. However, these indirect methods can be *very* slow. As an example, `rgn = stats.gausshyper.rvs(0.5, 2, 2, 2, size=100)` creates random variables in a very indirect way and takes about 19 seconds for 100 random variables on my computer, while one million random variables from the standard normal or from the t distribution take just above one second.

### Remaining Issues

The distributions in `scipy.stats` have recently been corrected and improved and gained a considerable test suite, however a few issues remain:

- the distributions have been tested over some range of parameters, however in some corner ranges, a few incorrect results may remain.
- the maximum likelihood estimation in *fit* does not work with default starting parameters for all distributions and the user needs to supply good starting parameters. Also, for some distribution using a maximum likelihood estimator might inherently not be the best choice.

## Building Specific Distributions

The next examples shows how to build your own distributions. Further examples show the usage of the distributions and some statistical tests.

### *Making a Continuous Distribution, i.e., Subclassing `rv_continuous`*

Making continuous distributions is fairly simple.

```
>>> from scipy import stats
>>> class deterministic_gen(stats.rv_continuous):
...     def _cdf(self, x):
...         return np.where(x < 0, 0., 1.)
...     def _stats(self):
...         return 0., 0., 0., 0.

>>> deterministic = deterministic_gen(name="deterministic")
>>> deterministic.cdf(np.arange(-3, 3, 0.5))
array([ 0.,  0.,  0.,  0.,  0.,  0.,  1.,  1.,  1.,  1.,  1.,  1.])
```

Interestingly, the pdf is now computed automatically:

```
>>> deterministic.pdf(np.arange(-3, 3, 0.5))
array([[ 0.00000000e+00,  0.00000000e+00,  0.00000000e+00,
         0.00000000e+00,  0.00000000e+00,  0.00000000e+00,
         5.83333333e+04,  4.16333634e-12,  4.16333634e-12,
         4.16333634e-12,  4.16333634e-12,  4.16333634e-12])
```

Be aware of the performance issues mentions in *Performance Issues and Cautionary Remarks*. The computation of unspecified common methods can become very slow, since only general methods are called which, by their very nature, cannot use any specific information about the distribution. Thus, as a cautionary example:

```
>>> from scipy.integrate import quad
>>> quad(deterministic.pdf, -1e-1, 1e-1)
(4.163336342344337e-13, 0.0)
```

But this is not correct: the integral over this pdf should be 1. Let's make the integration interval smaller:

```
>>> quad(deterministic.pdf, -1e-3, 1e-3) # warning removed
(1.000076872229173, 0.0010625571718182458)
```

This looks better. However, the problem originated from the fact that the pdf is not specified in the class definition of the deterministic distribution.

### *Subclassing `rv_discrete`*

In the following we use `stats.rv_discrete` to generate a discrete distribution that has the probabilities of the truncated normal for the intervals centered around the integers.

#### General Info

From the docstring of `rv_discrete`, `help(stats.rv_discrete)`,

“You can construct an arbitrary discrete rv where  $P\{X=x_k\} = p_k$  by passing to the `rv_discrete` initialization method (through the `values=` keyword) a tuple of sequences (`xk`, `pk`) which describes only those values of `X` (`xk`) that occur with nonzero probability (`pk`).”

Next to this, there are some further requirements for this approach to work:

- The keyword `name` is required.
- The support points of the distribution `xk` have to be integers.

- The number of significant digits (decimals) needs to be specified.

In fact, if the last two requirements are not satisfied an exception may be raised or the resulting numbers may be incorrect.

### An Example

Let's do the work. First

```
>>> npoints = 20 # number of integer support points of the distribution minus 1
>>> npointsh = npoints / 2
>>> npointsf = float(npoints)
>>> nbound = 4 # bounds for the truncated normal
>>> normbound = (1+1/npointsf) * nbound # actual bounds of truncated normal
>>> grid = np.arange(-npointsh, npointsh+2, 1) # integer grid
>>> gridlimitsnorm = (grid-0.5) / npointsh * nbound # bin limits for the truncnorm
>>> gridlimits = grid - 0.5 # used later in the analysis
>>> grid = grid[:-1]
>>> probs = np.diff(stats.truncnorm.cdf(gridlimitsnorm, -normbound, normbound))
>>> gridint = grid
```

And finally we can subclass `rv_discrete`:

```
>>> normdiscrete = stats.rv_discrete(values=(gridint,
...                                     np.round(probs, decimals=7)), name='normdiscrete')
```

Now that we have defined the distribution, we have access to all common methods of discrete distributions.

```
>>> print 'mean = %6.4f, variance = %6.4f, skew = %6.4f, kurtosis = %6.4f' % \
...      normdiscrete.stats(moments = 'mvsk')
mean = -0.0000, variance = 6.3302, skew = 0.0000, kurtosis = -0.0076

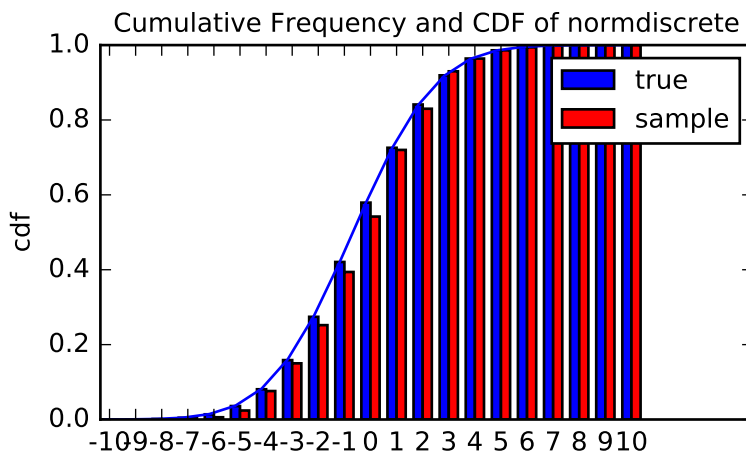
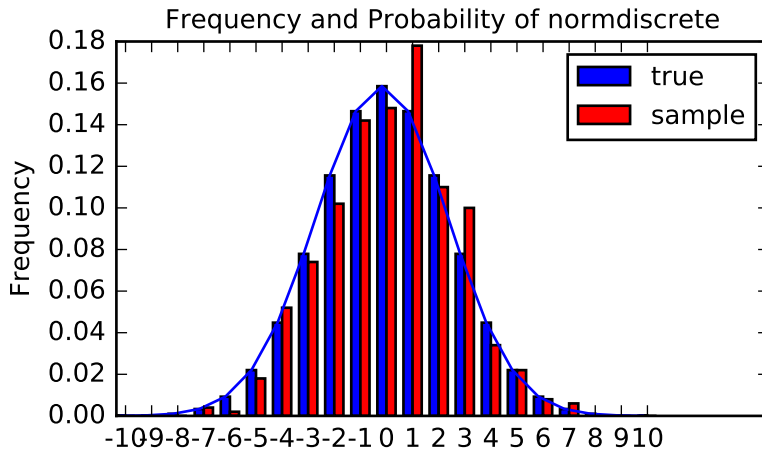
>>> nd_std = np.sqrt(normdiscrete.stats(moments='v'))
```

### Testing the Implementation

Let's generate a random sample and compare observed frequencies with the probabilities.

```
>>> n_sample = 500
>>> np.random.seed(87655678) # fix the seed for replicability
>>> rvs = normdiscrete.rvs(size=n_sample)
>>> rvsnd = rvs
>>> f, l = np.histogram(rvs, bins=gridlimits)
>>> sfreq = np.vstack([gridint, f, probs*n_sample]).T
>>> print sfreq
[[-1.00000000e+01  0.00000000e+00  2.95019349e-02]
 [ -9.00000000e+00  0.00000000e+00  1.32294142e-01]
 [ -8.00000000e+00  0.00000000e+00  5.06497902e-01]
 [ -7.00000000e+00  2.00000000e+00  1.65568919e+00]
 [ -6.00000000e+00  1.00000000e+00  4.62125309e+00]
 [ -5.00000000e+00  9.00000000e+00  1.10137298e+01]
 [ -4.00000000e+00  2.60000000e+01  2.24137683e+01]
 [ -3.00000000e+00  3.70000000e+01  3.89503370e+01]
 [ -2.00000000e+00  5.10000000e+01  5.78004747e+01]
 [ -1.00000000e+00  7.10000000e+01  7.32455414e+01]
 [  0.00000000e+00  7.40000000e+01  7.92618251e+01]
 [  1.00000000e+00  8.90000000e+01  7.32455414e+01]
 [  2.00000000e+00  5.50000000e+01  5.78004747e+01]
 [  3.00000000e+00  5.00000000e+01  3.89503370e+01]
 [  4.00000000e+00  1.70000000e+01  2.24137683e+01]
 [  5.00000000e+00  1.10000000e+01  1.10137298e+01]
```

```
[ 6.00000000e+00  4.00000000e+00  4.62125309e+00]
[ 7.00000000e+00  3.00000000e+00  1.65568919e+00]
[ 8.00000000e+00  0.00000000e+00  5.06497902e-01]
[ 9.00000000e+00  0.00000000e+00  1.32294142e-01]
[ 1.00000000e+01  0.00000000e+00  2.95019349e-02]]
```



Next, we can test, whether our sample was generated by our normdiscrete distribution. This also verifies whether the random numbers are generated correctly.

The chisquare test requires that there are a minimum number of observations in each bin. We combine the tail bins into larger bins so that they contain enough observations.

```
>>> f2 = np.hstack([[f[:5].sum(), f[5:-5], f[-5:].sum()]])
>>> p2 = np.hstack([probs[:5].sum(), probs[5:-5], probs[-5:].sum()])
>>> ch2, pval = stats.chisquare(f2, p2*n_sample)

>>> print 'chisquare for normdiscrete: chi2 = %6.3f pvalue = %6.4f' % (ch2, pval)
chisquare for normdiscrete: chi2 = 12.466 pvalue = 0.4090
```



The pvalue in this case is high, so we can be quite confident that our random sample was actually generated by the distribution.

## Analysing One Sample

First, we create some random variables. We set a seed so that in each run we get identical results to look at. As an example we take a sample from the Student t distribution:

```
>>> np.random.seed(282629734)
>>> x = stats.t.rvs(10, size=1000)
```

Here, we set the required shape parameter of the t distribution, which in statistics corresponds to the degrees of freedom, to 10. Using size=1000 means that our sample consists of 1000 independently drawn (pseudo) random numbers. Since we did not specify the keyword arguments *loc* and *scale*, those are set to their default values zero and one.

### Descriptive Statistics

*x* is a numpy array, and we have direct access to all array methods, e.g.

```
>>> print x.max(), x.min() # equivalent to np.max(x), np.min(x)
5.26327732981 -3.78975572422
>>> print x.mean(), x.var() # equivalent to np.mean(x), np.var(x)
0.0140610663985 1.28899386208
```

How do the some sample properties compare to their theoretical counterparts?

```
>>> m, v, s, k = stats.t.stats(10, moments='mvsk')
>>> n, (smin, smax), sm, sv, ss, sk = stats.describe(x)

>>> print 'distribution:',
distribution:
>>> sstr = 'mean = %6.4f, variance = %6.4f, skew = %6.4f, kurtosis = %6.4f'
>>> print sstr % (m, v, s, k)
mean = 0.0000, variance = 1.2500, skew = 0.0000, kurtosis = 1.0000
>>> print 'sample:      ',
sample:
>>> print sstr % (sm, sv, ss, sk)
mean = 0.0141, variance = 1.2903, skew = 0.2165, kurtosis = 1.0556
```

Note: stats.describe uses the unbiased estimator for the variance, while np.var is the biased estimator.

For our sample the sample statistics differ a by a small amount from their theoretical counterparts.

### T-test and KS-test

We can use the t-test to test whether the mean of our sample differs in a statistically significant way from the theoretical expectation.

```
>>> print 't-statistic = %6.3f pvalue = %6.4f' % stats.ttest_1samp(x, m)
t-statistic = 0.391 pvalue = 0.6955
```

The pvalue is 0.7, this means that with an alpha error of, for example, 10%, we cannot reject the hypothesis that the sample mean is equal to zero, the expectation of the standard t-distribution.

As an exercise, we can calculate our ttest also directly without using the provided function, which should give us the same answer, and so it does:

```
>>> tt = (sm-m)/np.sqrt(sv/float(n)) # t-statistic for mean
>>> pval = stats.t.sf(np.abs(tt), n-1)*2 # two-sided pvalue = Prob(abs(t)>tt)
>>> print 't-statistic = %6.3f pvalue = %6.4f' % (tt, pval)
t-statistic = 0.391 pvalue = 0.6955
```

The Kolmogorov-Smirnov test can be used to test the hypothesis that the sample comes from the standard t-distribution

```
>>> print 'KS-statistic D = %6.3f pvalue = %6.4f' % stats.kstest(x, 't', (10,))
KS-statistic D = 0.016 pvalue = 0.9606
```

Again the p-value is high enough that we cannot reject the hypothesis that the random sample really is distributed according to the t-distribution. In real applications, we don't know what the underlying distribution is. If we perform the Kolmogorov-Smirnov test of our sample against the standard normal distribution, then we also cannot reject the hypothesis that our sample was generated by the normal distribution given that in this example the p-value is almost 40%.

```
>>> print 'KS-statistic D = %6.3f pvalue = %6.4f' % stats.kstest(x, 'norm')
KS-statistic D = 0.028 pvalue = 0.3949
```

However, the standard normal distribution has a variance of 1, while our sample has a variance of 1.29. If we standardize our sample and test it against the normal distribution, then the p-value is again large enough that we cannot reject the hypothesis that the sample came from the normal distribution.

```
>>> d, pval = stats.kstest((x-x.mean())/x.std(), 'norm')
>>> print 'KS-statistic D = %6.3f pvalue = %6.4f' % (d, pval)
KS-statistic D = 0.032 pvalue = 0.2402
```

Note: The Kolmogorov-Smirnov test assumes that we test against a distribution with given parameters, since in the last case we estimated mean and variance, this assumption is violated, and the distribution of the test statistic on which the p-value is based, is not correct.

### Tails of the distribution

Finally, we can check the upper tail of the distribution. We can use the percent point function ppf, which is the inverse of the cdf function, to obtain the critical values, or, more directly, we can use the inverse of the survival function

```
>>> crit01, crit05, crit10 = stats.t.ppf([1-0.01, 1-0.05, 1-0.10], 10)
>>> print 'critical values from ppf at 1%, 5% and 10% %8.4f %8.4f %8.4f' % (crit01, crit05, crit10)
critical values from ppf at 1%, 5% and 10% 2.7638 1.8125 1.3722
>>> print 'critical values from isf at 1%, 5% and 10% %8.4f %8.4f %8.4f' % tuple(stats.t.isf([0.01, 0.05, 0.10]))
critical values from isf at 1%, 5% and 10% 2.7638 1.8125 1.3722

>>> freq01 = np.sum(x>crit01) / float(n) * 100
>>> freq05 = np.sum(x>crit05) / float(n) * 100
>>> freq10 = np.sum(x>crit10) / float(n) * 100
>>> print 'sample %-frequency at 1%, 5% and 10% tail %8.4f %8.4f %8.4f' % (freq01, freq05, freq10)
sample %-frequency at 1%, 5% and 10% tail 1.4000 5.8000 10.5000
```

In all three cases, our sample has more weight in the top tail than the underlying distribution. We can briefly check a larger sample to see if we get a closer match. In this case the empirical frequency is quite close to the theoretical probability, but if we repeat this several times the fluctuations are still pretty large.

```
>>> freq05l = np.sum(stats.t.rvs(10, size=10000) > crit05) / 10000.0 * 100
>>> print 'larger sample %-frequency at 5% tail %8.4f' % freq05l
larger sample %-frequency at 5% tail 4.8000
```

We can also compare it with the tail of the normal distribution, which has less weight in the tails:

```
>>> print 'tail prob. of normal at 1%%, 5%% and 10%% %8.4f %8.4f %8.4f' \
...      tuple(stats.norm.sf([crit01, crit05, crit10])*100)
tail prob. of normal at 1%, 5% and 10%    0.2857    3.4957    8.5003
```

The chisquare test can be used to test, whether for a finite number of bins, the observed frequencies differ significantly from the probabilities of the hypothesized distribution.

```
>>> quantiles = [0.0, 0.01, 0.05, 0.1, 1-0.10, 1-0.05, 1-0.01, 1.0]
>>> crit = stats.t.ppf(quantiles, 10)
>>> crit
array([-Inf, -2.76376946, -1.81246112, -1.37218364, 1.37218364, 1.81246112,
        2.76376946, Inf])
>>> n_sample = x.size
>>> freqcount = np.histogram(x, bins=crit)[0]
>>> tprob = np.diff(quantiles)
>>> nprob = np.diff(stats.norm.cdf(crit))
>>> tch, tpval = stats.chisquare(freqcount, tprob*n_sample)
>>> nch, npval = stats.chisquare(freqcount, nprob*n_sample)
>>> print 'chisquare for t:      chi2 = %6.2f pvalue = %6.4f' % (tch, tpval)
chisquare for t:      chi2 = 2.30 pvalue = 0.8901
>>> print 'chisquare for normal: chi2 = %6.2f pvalue = %6.4f' % (nch, npval)
chisquare for normal: chi2 = 64.60 pvalue = 0.0000
```

We see that the standard normal distribution is clearly rejected while the standard t-distribution cannot be rejected. Since the variance of our sample differs from both standard distribution, we can again redo the test taking the estimate for scale and location into account.

The fit method of the distributions can be used to estimate the parameters of the distribution, and the test is repeated using probabilities of the estimated distribution.

```
>>> tdof, tloc, tscale = stats.t.fit(x)
>>> nloc, nscale = stats.norm.fit(x)
>>> tprob = np.diff(stats.t.cdf(crit, tdof, loc=tloc, scale=tscale))
>>> nprob = np.diff(stats.norm.cdf(crit, loc=nloc, scale=nscale))
>>> tch, tpval = stats.chisquare(freqcount, tprob*n_sample)
>>> nch, npval = stats.chisquare(freqcount, nprob*n_sample)
>>> print 'chisquare for t:      chi2 = %6.2f pvalue = %6.4f' % (tch, tpval)
chisquare for t:      chi2 = 1.58 pvalue = 0.9542
>>> print 'chisquare for normal: chi2 = %6.2f pvalue = %6.4f' % (nch, npval)
chisquare for normal: chi2 = 11.08 pvalue = 0.0858
```

Taking account of the estimated parameters, we can still reject the hypothesis that our sample came from a normal distribution (at the 5% level), but again, with a p-value of 0.95, we cannot reject the t distribution.

### *Special tests for normal distributions*

Since the normal distribution is the most common distribution in statistics, there are several additional functions available to test whether a sample could have been drawn from a normal distribution

First we can test if skew and kurtosis of our sample differ significantly from those of a normal distribution:

```
>>> print 'normal skewtest teststat = %6.3f pvalue = %6.4f' % stats.skewtest(x)
normal skewtest teststat = 2.785 pvalue = 0.0054
>>> print 'normal kurtosistest teststat = %6.3f pvalue = %6.4f' % stats.kurtosistest(x)
normal kurtosistest teststat = 4.757 pvalue = 0.0000
```

These two tests are combined in the normality test

```
>>> print 'normaltest teststat = %6.3f pvalue = %6.4f' % stats.normaltest(x)
normaltest teststat = 30.379 pvalue = 0.0000
```

In all three tests the p-values are very low and we can reject the hypothesis that the our sample has skew and kurtosis of the normal distribution.

Since skew and kurtosis of our sample are based on central moments, we get exactly the same results if we test the standardized sample:

```
>>> print 'normaltest teststat = %6.3f pvalue = %6.4f' % \
...       stats.normaltest((x-x.mean())/x.std())
normaltest teststat = 30.379 pvalue = 0.0000
```

Because normality is rejected so strongly, we can check whether the normaltest gives reasonable results for other cases:

```
>>> print('normaltest teststat = %6.3f pvalue = %6.4f' %
...       stats.normaltest(stats.t.rvs(10, size=100)))
normaltest teststat = 4.698 pvalue = 0.0955
>>> print('normaltest teststat = %6.3f pvalue = %6.4f' %
...       stats.normaltest(stats.norm.rvs(size=1000)))
normaltest teststat = 0.613 pvalue = 0.7361
```

When testing for normality of a small sample of t-distributed observations and a large sample of normal distributed observation, then in neither case can we reject the null hypothesis that the sample comes from a normal distribution. In the first case this is because the test is not powerful enough to distinguish a t and a normally distributed random variable in a small sample.

### Comparing two samples

In the following, we are given two samples, which can come either from the same or from different distribution, and we want to test whether these samples have the same statistical properties.

#### *Comparing means*

Test with sample with identical means:

```
>>> rvs1 = stats.norm.rvs(loc=5, scale=10, size=500)
>>> rvs2 = stats.norm.rvs(loc=5, scale=10, size=500)
>>> stats.ttest_ind(rvs1, rvs2)
(-0.54890361750888583, 0.5831943748663857)
```

Test with sample with different means:

```
>>> rvs3 = stats.norm.rvs(loc=8, scale=10, size=500)
>>> stats.ttest_ind(rvs1, rvs3)
(-4.5334142901750321, 6.507128186505895e-006)
```

#### *Kolmogorov-Smirnov test for two samples ks\_2samp*

For the example where both samples are drawn from the same distribution, we cannot reject the null hypothesis since the pvalue is high

```
>>> stats.ks_2samp(rvs1, rvs2)
(0.025999999999999995, 0.99541195173064878)
```

In the second example, with different location, i.e. means, we can reject the null hypothesis since the pvalue is below 1%

```
>>> stats.ks_2samp(rvs1, rvs3)
(0.11399999999999999, 0.0027132103661283141)
```

## Kernel Density Estimation

A common task in statistics is to estimate the probability density function (PDF) of a random variable from a set of data samples. This task is called density estimation. The most well-known tool to do this is the histogram. A histogram is a useful tool for visualization (mainly because everyone understands it), but doesn't use the available data very efficiently. Kernel density estimation (KDE) is a more efficient tool for the same task. The `gaussian_kde` estimator can be used to estimate the PDF of univariate as well as multivariate data. It works best if the data is unimodal.

### Univariate estimation

We start with a minimal amount of data in order to see how `gaussian_kde` works, and what the different options for bandwidth selection do. The data sampled from the PDF is show as blue dashes at the bottom of the figure (this is called a rug plot):

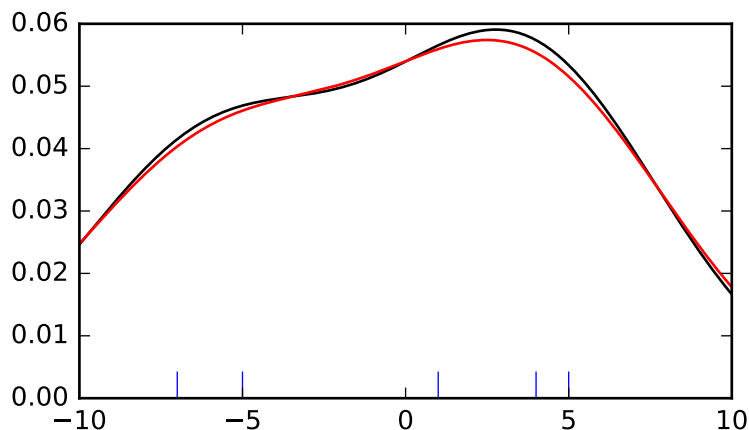
```
>>> from scipy import stats
>>> import matplotlib.pyplot as plt

>>> x1 = np.array([-7, -5, 1, 4, 5], dtype=np.float)
>>> kde1 = stats.gaussian_kde(x1)
>>> kde2 = stats.gaussian_kde(x1, bw_method='silverman')

>>> fig = plt.figure()
>>> ax = fig.add_subplot(111)

>>> ax.plot(x1, np.zeros(x1.shape), 'b+', ms=20) # rug plot
>>> x_eval = np.linspace(-10, 10, num=200)
>>> ax.plot(x_eval, kde1(x_eval), 'k-', label="Scott's Rule")
>>> ax.plot(x_eval, kde2(x_eval), 'r-', label="Silverman's Rule")

>>> plt.show()
```



We see that there is very little difference between Scott's Rule and Silverman's Rule, and that the bandwidth selection with a limited amount of data is probably a bit too wide. We can define our own bandwidth function to get a less smoothed out result.

```

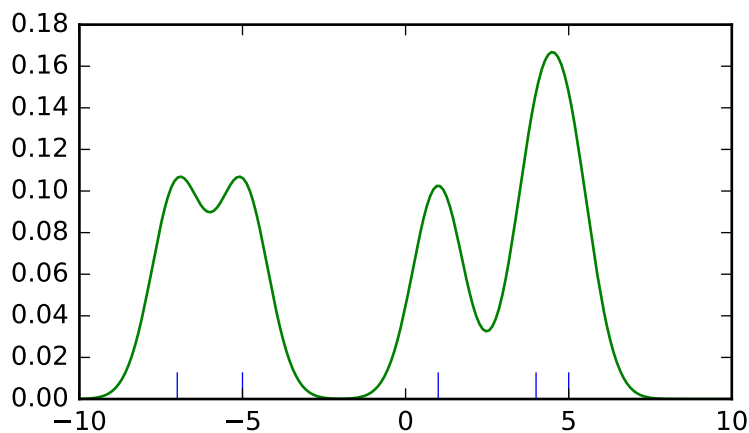
>>> def my_kde_bandwidth(obj, fac=1./5):
...     """We use Scott's Rule, multiplied by a constant factor."""
...     return np.power(obj.n, -1./(obj.d+4)) * fac

>>> fig = plt.figure()
>>> ax = fig.add_subplot(111)

>>> ax.plot(x1, np.zeros(x1.shape), 'b+', ms=20) # rug plot
>>> kde3 = stats.gaussian_kde(x1, bw_method=my_kde_bandwidth)
>>> ax.plot(x_eval, kde3(x_eval), 'g-', label="With smaller BW")

>>> plt.show()

```



We see that if we set bandwidth to be very narrow, the obtained estimate for the probability density function (PDF) is simply the sum of Gaussians around each data point.

We now take a more realistic example, and look at the difference between the two available bandwidth selection rules. Those rules are known to work well for (close to) normal distributions, but even for unimodal distributions that are quite strongly non-normal they work reasonably well. As a non-normal distribution we take a Student's T distribution with 5 degrees of freedom.

```

import numpy as np
import matplotlib.pyplot as plt
from scipy import stats

np.random.seed(12456)
x1 = np.random.normal(size=200) # random data, normal distribution
xs = np.linspace(x1.min()-1, x1.max()+1, 200)

kde1 = stats.gaussian_kde(x1)
kde2 = stats.gaussian_kde(x1, bw_method='silverman')

fig = plt.figure(figsize=(8, 6))

ax1 = fig.add_subplot(211)
ax1.plot(x1, np.zeros(x1.shape), 'b+', ms=12) # rug plot
ax1.plot(xs, kde1(xs), 'k-', label="Scott's Rule")

```

```

ax1.plot(xs, kde2(xs), 'b-', label="Silverman's Rule")
ax1.plot(xs, stats.norm.pdf(xs), 'r--', label="True PDF")

ax1.set_xlabel('x')
ax1.set_ylabel('Density')
ax1.set_title("Normal (top) and Student's T$_{df=5}$ (bottom) distributions")
ax1.legend(loc=1)

x2 = stats.t.rvs(5, size=200) # random data, T distribution
xs = np.linspace(x2.min() - 1, x2.max() + 1, 200)

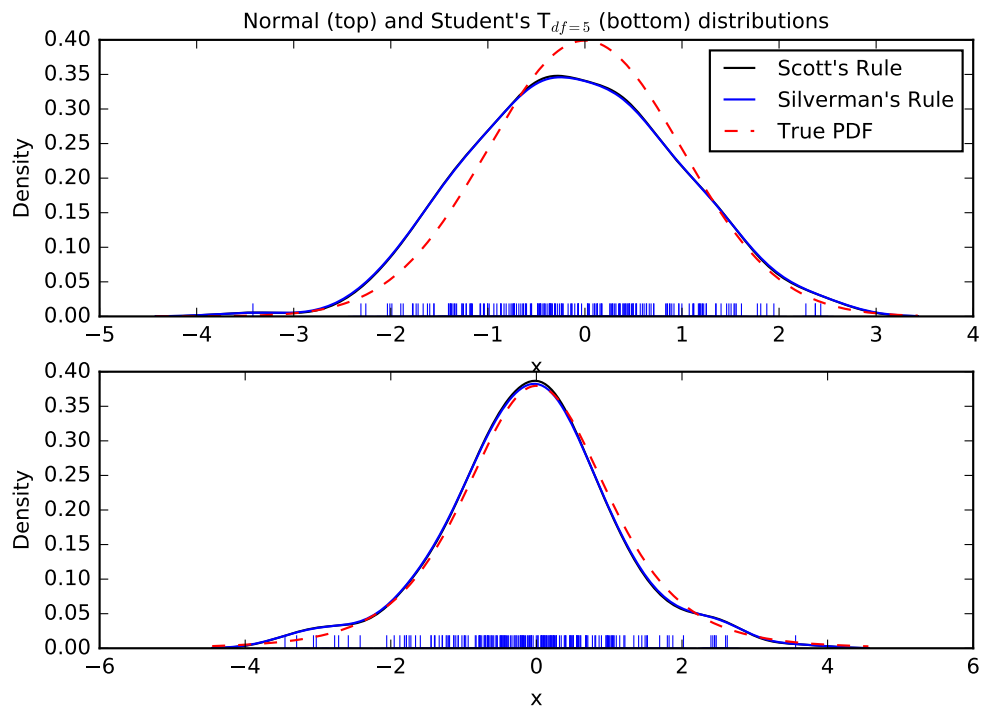
kde3 = stats.gaussian_kde(x2)
kde4 = stats.gaussian_kde(x2, bw_method='silverman')

ax2 = fig.add_subplot(212)
ax2.plot(x2, np.zeros(x2.shape), 'b+', ms=12) # rug plot
ax2.plot(xs, kde3(xs), 'k-', label="Scott's Rule")
ax2.plot(xs, kde4(xs), 'b-', label="Silverman's Rule")
ax2.plot(xs, stats.t.pdf(xs, 5), 'r--', label="True PDF")

ax2.set_xlabel('x')
ax2.set_ylabel('Density')

plt.show()

```



We now take a look at a bimodal distribution with one wider and one narrower Gaussian feature. We expect that this

will be a more difficult density to approximate, due to the different bandwidths required to accurately resolve each feature.

```
>>> from functools import partial

>>> loc1, scale1, size1 = (-2, 1, 175)
>>> loc2, scale2, size2 = (2, 0.2, 50)
>>> x2 = np.concatenate([np.random.normal(loc=loc1, scale=scale1, size=size1),
...                       np.random.normal(loc=loc2, scale=scale2, size=size2)])

>>> x_eval = np.linspace(x2.min() - 1, x2.max() + 1, 500)

>>> kde = stats.gaussian_kde(x2)
>>> kde2 = stats.gaussian_kde(x2, bw_method='silverman')
>>> kde3 = stats.gaussian_kde(x2, bw_method=partial(my_kde_bandwidth, fac=0.2))
>>> kde4 = stats.gaussian_kde(x2, bw_method=partial(my_kde_bandwidth, fac=0.5))

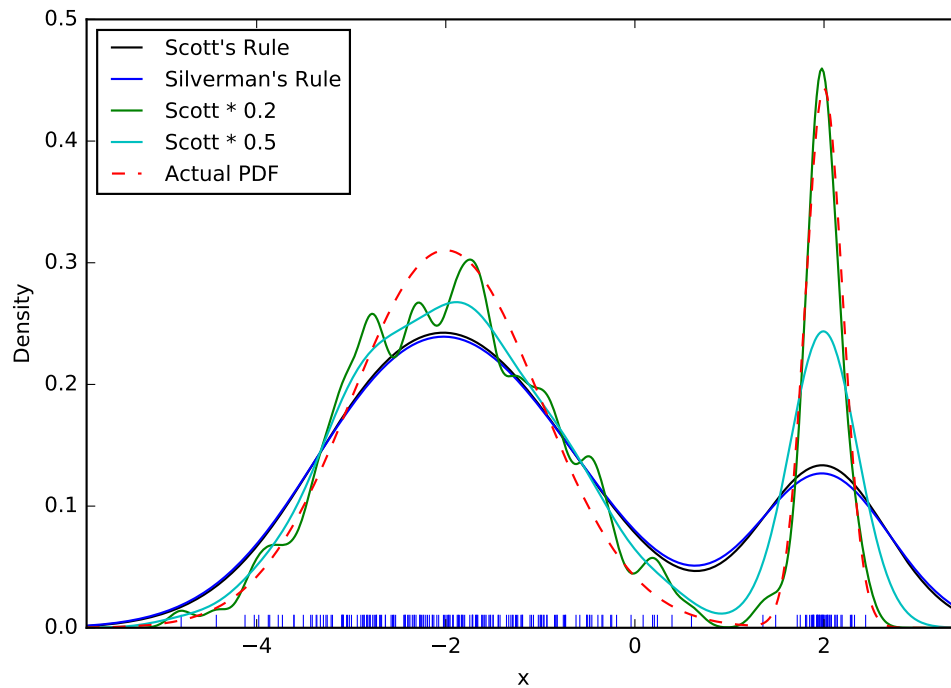
>>> pdf = stats.norm.pdf
>>> bimodal_pdf = pdf(x_eval, loc=loc1, scale=scale1) * float(size1) / x2.size + \
...               pdf(x_eval, loc=loc2, scale=scale2) * float(size2) / x2.size

>>> fig = plt.figure(figsize=(8, 6))
>>> ax = fig.add_subplot(111)

>>> ax.plot(x2, np.zeros(x2.shape), 'b+', ms=12)
>>> ax.plot(x_eval, kde(x_eval), 'k-', label="Scott's Rule")
>>> ax.plot(x_eval, kde2(x_eval), 'b-', label="Silverman's Rule")
>>> ax.plot(x_eval, kde3(x_eval), 'g-', label="Scott * 0.2")
>>> ax.plot(x_eval, kde4(x_eval), 'c-', label="Scott * 0.5")
>>> ax.plot(x_eval, bimodal_pdf, 'r--', label="Actual PDF")

>>> ax.set_xlim([x_eval.min(), x_eval.max()])
>>> ax.legend(loc=2)
>>> ax.set_xlabel('x')
>>> ax.set_ylabel('Density')
>>> plt.show()
```





As expected, the KDE is not as close to the true PDF as we would like due to the different characteristic size of the two features of the bimodal distribution. By halving the default bandwidth (`Scott * 0.5`) we can do somewhat better, while using a factor 5 smaller bandwidth than the default doesn't smooth enough. What we really need though in this case is a non-uniform (adaptive) bandwidth.

### *Multivariate estimation*

With `gaussian_kde` we can perform multivariate as well as univariate estimation. We demonstrate the bivariate case. First we generate some random data with a model in which the two variates are correlated.

```
>>> def measure(n):
...     """Measurement model, return two coupled measurements."""
...     m1 = np.random.normal(size=n)
...     m2 = np.random.normal(scale=0.5, size=n)
...     return m1+m2, m1-m2

>>> m1, m2 = measure(2000)
>>> xmin = m1.min()
>>> xmax = m1.max()
>>> ymin = m2.min()
>>> ymax = m2.max()
```

Then we apply the KDE to the data:

```
>>> X, Y = np.mgrid[xmin:xmax:100j, ymin:ymax:100j]
>>> positions = np.vstack([X.ravel(), Y.ravel()])
```

```
>>> values = np.vstack([m1, m2])
>>> kernel = stats.gaussian_kde(values)
>>> Z = np.reshape(kernel.evaluate(positions).T, X.shape)
```

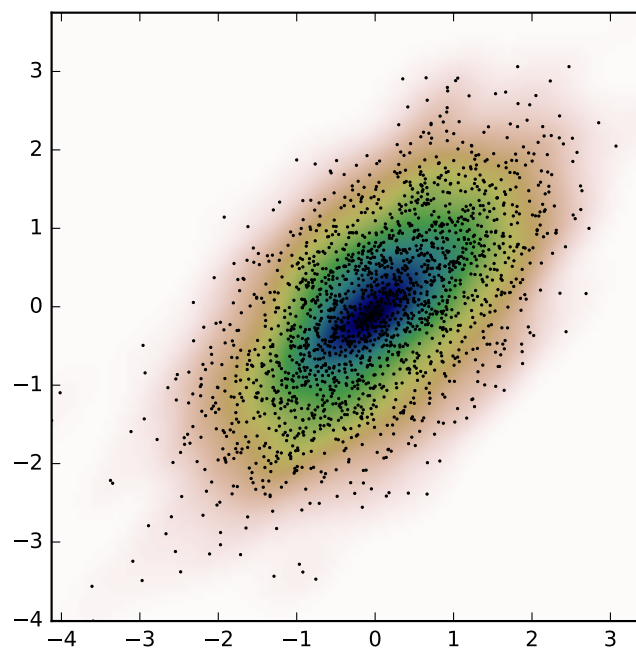
Finally we plot the estimated bivariate distribution as a colormap, and plot the individual data points on top.

```
>>> fig = plt.figure(figsize=(8, 6))
>>> ax = fig.add_subplot(111)

>>> ax.imshow(np.rot90(Z), cmap=plt.cm.gist_earth_r,
...           extent=[xmin, xmax, ymin, ymax])
>>> ax.plot(m1, m2, 'k.', markersize=2)

>>> ax.set_xlim([xmin, xmax])
>>> ax.set_ylim([ymin, ymax])

>>> plt.show()
```



### 3.1.14 Multidimensional image processing (`scipy.ndimage`)

#### Introduction

Image processing and analysis are generally seen as operations on two-dimensional arrays of values. There are however a number of fields where images of higher dimensionality must be analyzed. Good examples of these are medical

imaging and biological imaging. `numpy` is suited very well for this type of applications due its inherent multidimensional nature. The `scipy.ndimage` packages provides a number of general image processing and analysis functions that are designed to operate with arrays of arbitrary dimensionality. The packages currently includes functions for linear and non-linear filtering, binary morphology, B-spline interpolation, and object measurements.

### Properties shared by all functions

All functions share some common properties. Notably, all functions allow the specification of an output array with the `output` argument. With this argument you can specify an array that will be changed in-place with the result with the operation. In this case the result is not returned. Usually, using the `output` argument is more efficient, since an existing array is used to store the result.

The type of arrays returned is dependent on the type of operation, but it is in most cases equal to the type of the input. If, however, the `output` argument is used, the type of the result is equal to the type of the specified output argument. If no output argument is given, it is still possible to specify what the result of the output should be. This is done by simply assigning the desired `numpy` type object to the output argument. For example:

```
>>> from scipy.ndimage import correlate
>>> correlate(np.arange(10), [1, 2.5])
array([ 0,  2,  6,  9, 13, 16, 20, 23, 27, 30])
>>> correlate(np.arange(10), [1, 2.5], output=np.float64)
array([ 0. ,  2.5,  6. ,  9.5, 13. , 16.5, 20. , 23.5, 27. , 30.5])
```

### Filter functions

The functions described in this section all perform some type of spatial filtering of the input array: the elements in the output are some function of the values in the neighborhood of the corresponding input element. We refer to this neighborhood of elements as the filter kernel, which is often rectangular in shape but may also have an arbitrary footprint. Many of the functions described below allow you to define the footprint of the kernel, by passing a mask through the `footprint` parameter. For example a cross shaped kernel can be defined as follows:

```
>>> footprint = np.array([[0, 1, 0], [1, 1, 1], [0, 1, 0]])
>>> footprint
array([[0, 1, 0],
       [1, 1, 1],
       [0, 1, 0]])
```

Usually the origin of the kernel is at the center calculated by dividing the dimensions of the kernel shape by two. For instance, the origin of a one-dimensional kernel of length three is at the second element. Take for example the correlation of a one-dimensional array with a filter of length 3 consisting of ones:

```
>>> from scipy.ndimage import correlate
>>> a = [0, 0, 0, 1, 0, 0, 0]
>>> correlate(a, [1, 1, 1])
array([0, 0, 1, 1, 1, 0, 0])
```

Sometimes it is convenient to choose a different origin for the kernel. For this reason most functions support the `origin` parameter which gives the origin of the filter relative to its center. For example:

```
>>> a = [0, 0, 0, 1, 0, 0, 0]
>>> correlate(a, [1, 1, 1], origin = -1)
array([0, 1, 1, 1, 0, 0, 0])
```

The effect is a shift of the result towards the left. This feature will not be needed very often, but it may be useful especially for filters that have an even size. A good example is the calculation of backward and forward differences:

```
>>> a = [0, 0, 1, 1, 1, 0, 0]
>>> correlate1d(a, [-1, 1])           # backward difference
array([ 0,  0,  1,  0,  0, -1,  0])
>>> correlate1d(a, [-1, 1], origin = -1) # forward difference
array([ 0,  1,  0,  0, -1,  0,  0])
```

We could also have calculated the forward difference as follows:

```
>>> correlate1d(a, [0, -1, 1])
array([ 0,  1,  0,  0, -1,  0,  0])
```

However, using the *origin* parameter instead of a larger kernel is more efficient. For multidimensional kernels *origin* can be a number, in which case the origin is assumed to be equal along all axes, or a sequence giving the origin along each axis.

Since the output elements are a function of elements in the neighborhood of the input elements, the borders of the array need to be dealt with appropriately by providing the values outside the borders. This is done by assuming that the arrays are extended beyond their boundaries according certain boundary conditions. In the functions described below, the boundary conditions can be selected using the *mode* parameter which must be a string with the name of the boundary condition. Following boundary conditions are currently supported:

“nearest”	Use the value at the boundary	[1 2 3]->[1 1 2 3 3]
“wrap”	Periodically replicate the array	[1 2 3]->[3 1 2 3 1]
“reflect”	Reflect the array at the boundary	[1 2 3]->[1 1 2 3 3]
“constant”	Use a constant value, default is 0.0	[1 2 3]->[0 1 2 3 0]

The “constant” mode is special since it needs an additional parameter to specify the constant value that should be used.

---

**Note:** The easiest way to implement such boundary conditions would be to copy the data to a larger array and extend the data at the borders according to the boundary conditions. For large arrays and large filter kernels, this would be very memory consuming, and the functions described below therefore use a different approach that does not require allocating large temporary buffers.

---

### Correlation and convolution

The `correlate1d` function calculates a one-dimensional correlation along the given axis. The lines of the array along the given axis are correlated with the given *weights*. The *weights* parameter must be a one-dimensional sequences of numbers.

The function `correlate` implements multidimensional correlation of the input array with a given kernel.

The `convolve1d` function calculates a one-dimensional convolution along the given axis. The lines of the array along the given axis are convoluted with the given *weights*. The *weights* parameter must be a one-dimensional sequences of numbers.

---

**Note:** A convolution is essentially a correlation after mirroring the kernel. As a result, the *origin* parameter behaves differently than in the case of a correlation: the result is shifted in the opposite directions.

---

The function `convolve` implements multidimensional convolution of the input array with a given kernel.

---

**Note:** A convolution is essentially a correlation after mirroring the kernel. As a result, the *origin* parameter behaves differently than in the case of a correlation: the results is shifted in the opposite direction.

---

### Smoothing filters

The `gaussian_filter1d` function implements a one-dimensional Gaussian filter. The standard-deviation of the Gaussian filter is passed through the parameter *sigma*. Setting *order* = 0 corresponds to convolution with a Gaussian kernel. An order of 1, 2, or 3 corresponds to convolution with the first, second or third derivatives of a Gaussian. Higher order derivatives are not implemented.

The `gaussian_filter` function implements a multidimensional Gaussian filter. The standard-deviations of the Gaussian filter along each axis are passed through the parameter `sigma` as a sequence of numbers. If `sigma` is not a sequence but a single number, the standard deviation of the filter is equal along all directions. The order of the filter can be specified separately for each axis. An order of 0 corresponds to convolution with a Gaussian kernel. An order of 1, 2, or 3 corresponds to convolution with the first, second or third derivatives of a Gaussian. Higher order derivatives are not implemented. The `order` parameter must be a number, to specify the same order for all axes, or a sequence of numbers to specify a different order for each axis.

---

**Note:** The multidimensional filter is implemented as a sequence of one-dimensional Gaussian filters. The intermediate arrays are stored in the same data type as the output. Therefore, for output types with a lower precision, the results may be imprecise because intermediate results may be stored with insufficient precision. This can be prevented by specifying a more precise output type.

---

The `uniform_filter1d` function calculates a one-dimensional uniform filter of the given `size` along the given axis.

The `uniform_filter` implements a multidimensional uniform filter. The sizes of the uniform filter are given for each axis as a sequence of integers by the `size` parameter. If `size` is not a sequence, but a single number, the sizes along all axis are assumed to be equal.

---

**Note:** The multidimensional filter is implemented as a sequence of one-dimensional uniform filters. The intermediate arrays are stored in the same data type as the output. Therefore, for output types with a lower precision, the results may be imprecise because intermediate results may be stored with insufficient precision. This can be prevented by specifying a more precise output type.

---

### Filters based on order statistics

The `minimum_filter1d` function calculates a one-dimensional minimum filter of given `size` along the given axis.

The `maximum_filter1d` function calculates a one-dimensional maximum filter of given `size` along the given axis.

The `minimum_filter` function calculates a multidimensional minimum filter. Either the sizes of a rectangular kernel or the footprint of the kernel must be provided. The `size` parameter, if provided, must be a sequence of sizes or a single number in which case the size of the filter is assumed to be equal along each axis. The `footprint`, if provided, must be an array that defines the shape of the kernel by its non-zero elements.

The `maximum_filter` function calculates a multidimensional maximum filter. Either the sizes of a rectangular kernel or the footprint of the kernel must be provided. The `size` parameter, if provided, must be a sequence of sizes or a single number in which case the size of the filter is assumed to be equal along each axis. The `footprint`, if provided, must be an array that defines the shape of the kernel by its non-zero elements.

The `rank_filter` function calculates a multidimensional rank filter. The `rank` may be less than zero, i.e., `rank = -1` indicates the largest element. Either the sizes of a rectangular kernel or the footprint of the kernel must be provided. The `size` parameter, if provided, must be a sequence of sizes or a single number in which case the size of the filter is assumed to be equal along each axis. The `footprint`, if provided, must be an array that defines the shape of the kernel by its non-zero elements.

The `percentile_filter` function calculates a multidimensional percentile filter. The `percentile` may be less than zero, i.e., `percentile = -20` equals `percentile = 80`. Either the sizes of a rectangular kernel or the footprint of the kernel must be provided. The `size` parameter, if provided, must be a sequence of sizes or a single number in which case the size of the filter is assumed to be equal along each axis. The `footprint`, if provided, must be an array that defines the shape of the kernel by its non-zero elements.

The `median_filter` function calculates a multidimensional median filter. Either the sizes of a rectangular kernel or the footprint of the kernel must be provided. The `size` parameter, if provided, must be a sequence of sizes or a single number in which case the size of the filter is assumed to be equal along each axis. The `footprint` if provided, must be an array that defines the shape of the kernel by its non-zero elements.

## Derivatives

Derivative filters can be constructed in several ways. The function `gaussian_filter1d` described in *Smoothing filters* can be used to calculate derivatives along a given axis using the *order* parameter. Other derivative filters are the Prewitt and Sobel filters:

The `prewitt` function calculates a derivative along the given axis.

The `sobel` function calculates a derivative along the given axis.

The Laplace filter is calculated by the sum of the second derivatives along all axes. Thus, different Laplace filters can be constructed using different second derivative functions. Therefore we provide a general function that takes a function argument to calculate the second derivative along a given direction and to construct the Laplace filter:

The function `generic_laplace` calculates a laplace filter using the function passed through `derivative2` to calculate second derivatives. The function `derivative2` should have the following signature:

```
derivative2(input, axis, output, mode, cval, *extra_arguments, **extra_keywords)
```

It should calculate the second derivative along the dimension *axis*. If *output* is not None it should use that for the output and return None, otherwise it should return the result. *mode*, *cval* have the usual meaning.

The *extra\_arguments* and *extra\_keywords* arguments can be used to pass a tuple of extra arguments and a dictionary of named arguments that are passed to `derivative2` at each call.

For example:

```
>>> def d2(input, axis, output, mode, cval):
...     return correlate1d(input, [1, -2, 1], axis, output, mode, cval, 0)
...
>>> a = np.zeros((5, 5))
>>> a[2, 2] = 1
>>> from scipy.ndimage import generic_laplace
>>> generic_laplace(a, d2)
array([[ 0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  1.,  0.,  0.],
       [ 0.,  1., -4.,  1.,  0.],
       [ 0.,  0.,  1.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.]])
```

To demonstrate the use of the *extra\_arguments* argument we could do:

```
>>> def d2(input, axis, output, mode, cval, weights):
...     return correlate1d(input, weights, axis, output, mode, cval, 0)
...
>>> a = np.zeros((5, 5))
>>> a[2, 2] = 1
>>> generic_laplace(a, d2, extra_arguments = ([1, -2, 1],))
array([[ 0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  1.,  0.,  0.],
       [ 0.,  1., -4.,  1.,  0.],
       [ 0.,  0.,  1.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.]])
```

or:

```
>>> generic_laplace(a, d2, extra_keywords = {'weights': [1, -2, 1]})
array([[ 0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  1.,  0.,  0.],
       [ 0.,  1., -4.,  1.,  0.],
       [ 0.,  0.,  1.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.]])
```

The following two functions are implemented using `generic_laplace` by providing appropriate functions for the second derivative function:

The function `laplace` calculates the Laplace using discrete differentiation for the second derivative (i.e. convolution with `[1, -2, 1]`).

The function `gaussian_laplace` calculates the Laplace using `gaussian_filter` to calculate the second derivatives. The standard-deviations of the Gaussian filter along each axis are passed through the parameter `sigma` as a sequence or numbers. If `sigma` is not a sequence but a single number, the standard deviation of the filter is equal along all directions.

The gradient magnitude is defined as the square root of the sum of the squares of the gradients in all directions. Similar to the generic Laplace function there is a `generic_gradient_magnitude` function that calculated the gradient magnitude of an array:

The function `generic_gradient_magnitude` calculates a gradient magnitude using the function passed through `derivative` to calculate first derivatives. The function `derivative` should have the following signature:

```
derivative(input, axis, output, mode, cval, *extra_arguments, **extra_keywords)
```

It should calculate the derivative along the dimension `axis`. If `output` is not `None` it should use that for the output and return `None`, otherwise it should return the result. `mode`, `cval` have the usual meaning.

The `extra_arguments` and `extra_keywords` arguments can be used to pass a tuple of extra arguments and a dictionary of named arguments that are passed to `derivative` at each call.

For example, the `sobel` function fits the required signature:

```
>>> a = np.zeros((5, 5))
>>> a[2, 2] = 1
>>> from scipy.ndimage import sobel, generic_gradient_magnitude
>>> generic_gradient_magnitude(a, sobel)
array([[ 0.          ,  0.          ,  0.          ,  0.          ,  0.          ],
       [ 0.          ,  1.41421356,  2.          ,  1.41421356,  0.          ],
       [ 0.          ,  2.          ,  0.          ,  2.          ,  0.          ],
       [ 0.          ,  1.41421356,  2.          ,  1.41421356,  0.          ],
       [ 0.          ,  0.          ,  0.          ,  0.          ,  0.          ]])
```

See the documentation of `generic_laplace` for examples of using the `extra_arguments` and `extra_keywords` arguments.

The `sobel` and `prewitt` functions fit the required signature and can therefore directly be used with `generic_gradient_magnitude`. The following function implements the gradient magnitude using Gaussian derivatives:

The function `gaussian_gradient_magnitude` calculates the gradient magnitude using `gaussian_filter` to calculate the first derivatives. The standard-deviations of the Gaussian filter along each axis are passed through the parameter `sigma` as a sequence or numbers. If `sigma` is not a sequence but a single number, the standard deviation of the filter is equal along all directions.

### Generic filter functions

To implement filter functions, generic functions can be used that accept a callable object that implements the filtering operation. The iteration over the input and output arrays is handled by these generic functions, along with such details as the implementation of the boundary conditions. Only a callable object implementing a callback function that does the actual filtering work must be provided. The callback function can also be written in C and passed using a `PyCObject` (see [Extending ndimage in C](#) for more information).

The `generic_filter1d` function implements a generic one-dimensional filter function, where the actual filtering operation must be supplied as a python function (or other callable object). The `generic_filter1d` function iterates over the lines of an array and calls `function` at each line. The arguments that are passed to `function` are one-dimensional arrays of the `tFloat64` type. The first contains the values of the current line. It is extended at the beginning and the end, according to the `filter_size` and `origin` arguments. The second array should be modified in-place to provide the output values of the line. For example consider a correlation along one dimension:

```
>>> a = np.arange(12).reshape(3,4)
>>> correlate1d(a, [1, 2, 3])
array([[ 3,  8, 14, 17],
       [27, 32, 38, 41],
       [51, 56, 62, 65]])
```

The same operation can be implemented using `generic_filter1d` as follows:

```
>>> def fnc(iline, oline):
...     oline[...] = iline[:-2] + 2 * iline[1:-1] + 3 * iline[2:]
...
>>> from scipy.ndimage import generic_filter1d
>>> generic_filter1d(a, fnc, 3)
array([[ 3,  8, 14, 17],
       [27, 32, 38, 41],
       [51, 56, 62, 65]])
```

Here the origin of the kernel was (by default) assumed to be in the middle of the filter of length 3. Therefore, each input line was extended by one value at the beginning and at the end, before the function was called.

Optionally extra arguments can be defined and passed to the filter function. The *extra\_arguments* and *extra\_keywords* arguments can be used to pass a tuple of extra arguments and/or a dictionary of named arguments that are passed to derivative at each call. For example, we can pass the parameters of our filter as an argument:

```
>>> def fnc(iline, oline, a, b):
...     oline[...] = iline[:-2] + a * iline[1:-1] + b * iline[2:]
...
>>> generic_filter1d(a, fnc, 3, extra_arguments = (2, 3))
array([[ 3,  8, 14, 17],
       [27, 32, 38, 41],
       [51, 56, 62, 65]])
```

or:

```
>>> generic_filter1d(a, fnc, 3, extra_keywords = {'a':2, 'b':3})
array([[ 3,  8, 14, 17],
       [27, 32, 38, 41],
       [51, 56, 62, 65]])
```

The `generic_filter` function implements a generic filter function, where the actual filtering operation must be supplied as a python function (or other callable object). The `generic_filter` function iterates over the array and calls function at each element. The argument of function is a one-dimensional array of the `tFloat64` type, that contains the values around the current element that are within the footprint of the filter. The function should return a single value that can be converted to a double precision number. For example consider a correlation:

```
>>> a = np.arange(12).reshape(3,4)
>>> correlate(a, [[1, 0], [0, 3]])
array([[ 0,  3,  7, 11],
       [12, 15, 19, 23],
       [28, 31, 35, 39]])
```

The same operation can be implemented using `generic_filter` as follows:

```
>>> def fnc(buffer):
...     return (buffer * np.array([1, 3])).sum()
...
>>> from scipy.ndimage import generic_filter
>>> generic_filter(a, fnc, footprint = [[1, 0], [0, 1]])
array([[ 0,  3,  7, 11],
       [12, 15, 19, 23],
       [28, 31, 35, 39]])
```



Here a kernel footprint was specified that contains only two elements. Therefore the filter function receives a buffer of length equal to two, which was multiplied with the proper weights and the result summed.

When calling `generic_filter`, either the sizes of a rectangular kernel or the footprint of the kernel must be provided. The `size` parameter, if provided, must be a sequence of sizes or a single number in which case the size of the filter is assumed to be equal along each axis. The `footprint`, if provided, must be an array that defines the shape of the kernel by its non-zero elements.

Optionally extra arguments can be defined and passed to the filter function. The `extra_arguments` and `extra_keywords` arguments can be used to pass a tuple of extra arguments and/or a dictionary of named arguments that are passed to derivative at each call. For example, we can pass the parameters of our filter as an argument:

```
>>> def fnc(buffer, weights):
...     weights = np.asarray(weights)
...     return (buffer * weights).sum()
...
>>> generic_filter(a, fnc, footprint = [[1, 0], [0, 1]], extra_arguments = ([1, 3],))
array([[ 0,  3,  7, 11],
       [12, 15, 19, 23],
       [28, 31, 35, 39]])
```

or:

```
>>> generic_filter(a, fnc, footprint = [[1, 0], [0, 1]], extra_keywords= {'weights': [1, 3]})
array([[ 0,  3,  7, 11],
       [12, 15, 19, 23],
       [28, 31, 35, 39]])
```

These functions iterate over the lines or elements starting at the last axis, i.e. the last index changes the fastest. This order of iteration is guaranteed for the case that it is important to adapt the filter depending on spatial location. Here is an example of using a class that implements the filter and keeps track of the current coordinates while iterating. It performs the same filter operation as described above for `generic_filter`, but additionally prints the current coordinates:

```
>>> a = np.arange(12).reshape(3,4)
>>>
>>> class fnc_class:
...     def __init__(self, shape):
...         # store the shape:
...         self.shape = shape
...         # initialize the coordinates:
...         self.coordinates = [0] * len(shape)
...
...     def filter(self, buffer):
...         result = (buffer * np.array([1, 3])).sum()
...         print self.coordinates
...         # calculate the next coordinates:
...         axes = range(len(self.shape))
...         axes.reverse()
...         for jj in axes:
...             if self.coordinates[jj] < self.shape[jj] - 1:
...                 self.coordinates[jj] += 1
...                 break
...             else:
...                 self.coordinates[jj] = 0
...         return result
...
>>> fnc = fnc_class(shape = (3,4))
>>> generic_filter(a, fnc.filter, footprint = [[1, 0], [0, 1]])
[0, 0]
[0, 1]
```

```
[0, 2]
[0, 3]
[1, 0]
[1, 1]
[1, 2]
[1, 3]
[2, 0]
[2, 1]
[2, 2]
[2, 3]
array([[ 0,  3,  7, 11],
       [12, 15, 19, 23],
       [28, 31, 35, 39]])
```

For the `generic_filter1d` function the same approach works, except that this function does not iterate over the axis that is being filtered. The example for `generic_filter1d` then becomes this:

```
>>> a = np.arange(12).reshape(3,4)
>>>
>>> class fnc1d_class:
...     def __init__(self, shape, axis = -1):
...         # store the filter axis:
...         self.axis = axis
...         # store the shape:
...         self.shape = shape
...         # initialize the coordinates:
...         self.coordinates = [0] * len(shape)
...
...     def filter(self, iline, oline):
...         oline[...] = iline[:-2] + 2 * iline[1:-1] + 3 * iline[2:]
...         print self.coordinates
...         # calculate the next coordinates:
...         axes = range(len(self.shape))
...         # skip the filter axis:
...         del axes[self.axis]
...         axes.reverse()
...         for jj in axes:
...             if self.coordinates[jj] < self.shape[jj] - 1:
...                 self.coordinates[jj] += 1
...                 break
...             else:
...                 self.coordinates[jj] = 0
...
>>> fnc = fnc1d_class(shape = (3,4))
>>> generic_filter1d(a, fnc.filter, 3)
[0, 0]
[1, 0]
[2, 0]
array([[ 3,  8, 14, 17],
       [27, 32, 38, 41],
       [51, 56, 62, 65]])
```

### ***Fourier domain filters***

The functions described in this section perform filtering operations in the Fourier domain. Thus, the input array of such a function should be compatible with an inverse Fourier transform function, such as the functions from the `numpy.fft` module. We therefore have to deal with arrays that may be the result of a real or a complex Fourier transform. In the case of a real Fourier transform only half of the of the symmetric complex transform is stored.

Additionally, it needs to be known what the length of the axis was that was transformed by the real fft. The functions described here provide a parameter *n* that in the case of a real transform must be equal to the length of the real transform axis before transformation. If this parameter is less than zero, it is assumed that the input array was the result of a complex Fourier transform. The parameter *axis* can be used to indicate along which axis the real transform was executed.

The `fourier_shift` function multiplies the input array with the multidimensional Fourier transform of a shift operation for the given shift. The *shift* parameter is a sequences of shifts for each dimension, or a single value for all dimensions.

The `fourier_gaussian` function multiplies the input array with the multidimensional Fourier transform of a Gaussian filter with given standard-deviations *sigma*. The *sigma* parameter is a sequences of values for each dimension, or a single value for all dimensions.

The `fourier_uniform` function multiplies the input array with the multidimensional Fourier transform of a uniform filter with given sizes *size*. The *size* parameter is a sequences of values for each dimension, or a single value for all dimensions.

The `fourier_ellipsoid` function multiplies the input array with the multidimensional Fourier transform of an elliptically shaped filter with given sizes *size*. The *size* parameter is a sequences of values for each dimension, or a single value for all dimensions. This function is only implemented for dimensions 1, 2, and 3.

## Interpolation functions

This section describes various interpolation functions that are based on B-spline theory. A good introduction to B-splines can be found in: M. Unser, "Splines: A Perfect Fit for Signal and Image Processing," IEEE Signal Processing Magazine, vol. 16, no. 6, pp. 22-38, November 1999.

### *Spline pre-filters*

Interpolation using splines of an order larger than 1 requires a pre-filtering step. The interpolation functions described in section [Interpolation functions](#) apply pre-filtering by calling `spline_filter`, but they can be instructed not to do this by setting the *prefilter* keyword equal to `False`. This is useful if more than one interpolation operation is done on the same array. In this case it is more efficient to do the pre-filtering only once and use a prefiltered array as the input of the interpolation functions. The following two functions implement the pre-filtering:

The `spline_filter1d` function calculates a one-dimensional spline filter along the given axis. An output array can optionally be provided. The order of the spline must be larger than 1 and less than 6.

The `spline_filter` function calculates a multidimensional spline filter.

---

**Note:** The multidimensional filter is implemented as a sequence of one-dimensional spline filters. The intermediate arrays are stored in the same data type as the output. Therefore, if an output with a limited precision is requested, the results may be imprecise because intermediate results may be stored with insufficient precision. This can be prevented by specifying a output type of high precision.

---

### *Interpolation functions*

Following functions all employ spline interpolation to effect some type of geometric transformation of the input array. This requires a mapping of the output coordinates to the input coordinates, and therefore the possibility arises that input values outside the boundaries are needed. This problem is solved in the same way as described in [Filter functions](#) for the multidimensional filter functions. Therefore these functions all support a *mode* parameter that determines how the boundaries are handled, and a *cval* parameter that gives a constant value in case that the 'constant' mode is used.

The `geometric_transform` function applies an arbitrary geometric transform to the input. The given *mapping* function is called at each point in the output to find the corresponding coordinates in the input. *mapping* must be a callable object that accepts a tuple of length equal to the output array rank and returns the corresponding input coordinates as a tuple of length equal to the input array rank. The output shape and output type can optionally be provided. If not given they are equal to the input shape and type.

For example:

```
>>> a = np.arange(12).reshape(4,3).astype(np.float64)
>>> def shift_func(output_coordinates):
...     return (output_coordinates[0] - 0.5, output_coordinates[1] - 0.5)
...
>>> from scipy.ndimage import geometric_transform
>>> geometric_transform(a, shift_func)
array([[ 0.    ,  0.    ,  0.    ],
       [ 0.    ,  1.3625,  2.7375],
       [ 0.    ,  4.8125,  6.1875],
       [ 0.    ,  8.2625,  9.6375]])
```

Optionally extra arguments can be defined and passed to the filter function. The *extra\_arguments* and *extra\_keywords* arguments can be used to pass a tuple of extra arguments and/or a dictionary of named arguments that are passed to derivative at each call. For example, we can pass the shifts in our example as arguments:

```
>>> def shift_func(output_coordinates, s0, s1):
...     return (output_coordinates[0] - s0, output_coordinates[1] - s1)
...
>>> geometric_transform(a, shift_func, extra_arguments = (0.5, 0.5))
array([[ 0.    ,  0.    ,  0.    ],
       [ 0.    ,  1.3625,  2.7375],
       [ 0.    ,  4.8125,  6.1875],
       [ 0.    ,  8.2625,  9.6375]])
```

or:

```
>>> geometric_transform(a, shift_func, extra_keywords = {'s0': 0.5, 's1': 0.5})
array([[ 0.    ,  0.    ,  0.    ],
       [ 0.    ,  1.3625,  2.7375],
       [ 0.    ,  4.8125,  6.1875],
       [ 0.    ,  8.2625,  9.6375]])
```

---

**Note:** The mapping function can also be written in C and passed using a PyCObject. See *Extending ndimage in C* for more information.

---

The function `map_coordinates` applies an arbitrary coordinate transformation using the given array of coordinates. The shape of the output is derived from that of the coordinate array by dropping the first axis. The parameter *coordinates* is used to find for each point in the output the corresponding coordinates in the input. The values of *coordinates* along the first axis are the coordinates in the input array at which the output value is found. (See also the `numarray.coordinates` function.) Since the coordinates may be non-integer coordinates, the value of the input at these coordinates is determined by spline interpolation of the requested order. Here is an example that interpolates a 2D array at (0.5, 0.5) and (1, 2):

```
>>> a = np.arange(12).reshape(4,3).astype(np.float64)
>>> a
array([[ 0.,  1.,  2.],
       [ 3.,  4.,  5.],
       [ 6.,  7.,  8.],
       [ 9., 10., 11.]])
>>> from scipy.ndimage import map_coordinates
>>> map_coordinates(a, [[0.5, 2], [0.5, 1]])
array([ 1.3625,  7.])
```

The `affine_transform` function applies an affine transformation to the input array. The given transformation *matrix* and *offset* are used to find for each point in the output the corresponding coordinates in the input. The value of the input at the calculated coordinates is determined by spline interpolation of the requested order. The transformation *matrix* must be two-dimensional or can also be given as a one-dimensional sequence or array. In the latter case, it is assumed that the matrix is diagonal. A more efficient interpolation algorithm is then applied

that exploits the separability of the problem. The output shape and output type can optionally be provided. If not given they are equal to the input shape and type.

The `shift` function returns a shifted version of the input, using spline interpolation of the requested *order*.

The `zoom` function returns a rescaled version of the input, using spline interpolation of the requested *order*.

The `rotate` function returns the input array rotated in the plane defined by the two axes given by the parameter *axes*, using spline interpolation of the requested *order*. The angle must be given in degrees. If *reshape* is true, then the size of the output array is adapted to contain the rotated input.

## Morphology

### Binary morphology

Binary morphology (need something to put here).

The `generate_binary_structure` functions generates a binary structuring element for use in binary morphology operations. The *rank* of the structure must be provided. The size of the structure that is returned is equal to three in each direction. The value of each element is equal to one if the square of the Euclidean distance from the element to the center is less or equal to *connectivity*. For instance, two dimensional 4-connected and 8-connected structures are generated as follows:

```
>>> from scipy.ndimage import generate_binary_structure
>>> generate_binary_structure(2, 1)
array([[False,  True,  False],
       [ True,  True,  True],
       [False,  True,  False]], dtype=bool)
>>> generate_binary_structure(2, 2)
array([[ True,  True,  True],
       [ True,  True,  True],
       [ True,  True,  True]], dtype=bool)
```

Most binary morphology functions can be expressed in terms of the basic operations erosion and dilation:

The `binary_erosion` function implements binary erosion of arrays of arbitrary rank with the given structuring element. The origin parameter controls the placement of the structuring element as described in *Filter functions*. If no structuring element is provided, an element with connectivity equal to one is generated using `generate_binary_structure`. The *border\_value* parameter gives the value of the array outside boundaries. The erosion is repeated *iterations* times. If *iterations* is less than one, the erosion is repeated until the result does not change anymore. If a *mask* array is given, only those elements with a true value at the corresponding mask element are modified at each iteration.

The `binary_dilation` function implements binary dilation of arrays of arbitrary rank with the given structuring element. The origin parameter controls the placement of the structuring element as described in *Filter functions*. If no structuring element is provided, an element with connectivity equal to one is generated using `generate_binary_structure`. The *border\_value* parameter gives the value of the array outside boundaries. The dilation is repeated *iterations* times. If *iterations* is less than one, the dilation is repeated until the result does not change anymore. If a *mask* array is given, only those elements with a true value at the corresponding mask element are modified at each iteration.

Here is an example of using `binary_dilation` to find all elements that touch the border, by repeatedly dilating an empty array from the border using the data array as the mask:

```
>>> struct = np.array([[0, 1, 0], [1, 1, 1], [0, 1, 0]])
>>> a = np.array([[1,0,0,0,0], [1,1,0,1,0], [0,0,1,1,0], [0,0,0,0,0]])
>>> a
array([[1, 0, 0, 0, 0],
       [1, 1, 0, 1, 0],
       [0, 0, 1, 1, 0],
       [0, 0, 0, 0, 0]])
>>> from scipy.ndimage import binary_dilation
```

```
>>> binary_dilation(np.zeros(a.shape), struct, -1, a, border_value=1)
array([[ True, False, False, False, False],
       [ True,  True, False, False, False],
       [False, False, False, False, False],
       [False, False, False, False, False]], dtype=bool)
```

The `binary_erosion` and `binary_dilation` functions both have an `iterations` parameter which allows the erosion or dilation to be repeated a number of times. Repeating an erosion or a dilation with a given structure  $n$  times is equivalent to an erosion or a dilation with a structure that is  $n-1$  times dilated with itself. A function is provided that allows the calculation of a structure that is dilated a number of times with itself:

The `iterate_structure` function returns a structure by dilation of the input structure *iteration* - 1 times with itself. For instance:

```
>>> struct = generate_binary_structure(2, 1)
>>> struct
array([[False,  True, False],
       [ True,  True,  True],
       [False,  True, False]], dtype=bool)
>>> from scipy.ndimage import iterate_structure
>>> iterate_structure(struct, 2)
array([[False, False,  True, False, False],
       [False,  True,  True,  True, False],
       [ True,  True,  True,  True,  True],
       [False,  True,  True,  True, False],
       [False, False,  True, False, False]], dtype=bool)
```

If the origin of the original structure is equal to 0, then it is also equal to 0 for the iterated structure. If not, the origin must also be adapted if the equivalent of the *iterations* erosions or dilations must be achieved with the iterated structure. The adapted origin is simply obtained by multiplying with the number of iterations. For convenience the `iterate_structure` also returns the adapted origin if the *origin* parameter is not None:

```
>>> iterate_structure(struct, 2, -1)
(array([[False, False,  True, False, False],
       [False,  True,  True,  True, False],
       [ True,  True,  True,  True,  True],
       [False,  True,  True,  True, False],
       [False, False,  True, False, False]], dtype=bool), [-2, -2])
```

Other morphology operations can be defined in terms of erosion and dilation. Following functions provide a few of these operations for convenience:

The `binary_opening` function implements binary opening of arrays of arbitrary rank with the given structuring element. Binary opening is equivalent to a binary erosion followed by a binary dilation with the same structuring element. The origin parameter controls the placement of the structuring element as described in *Filter functions*. If no structuring element is provided, an element with connectivity equal to one is generated using `generate_binary_structure`. The *iterations* parameter gives the number of erosions that is performed followed by the same number of dilations.

The `binary_closing` function implements binary closing of arrays of arbitrary rank with the given structuring element. Binary closing is equivalent to a binary dilation followed by a binary erosion with the same structuring element. The origin parameter controls the placement of the structuring element as described in *Filter functions*. If no structuring element is provided, an element with connectivity equal to one is generated using `generate_binary_structure`. The *iterations* parameter gives the number of dilations that is performed followed by the same number of erosions.

The `binary_fill_holes` function is used to close holes in objects in a binary image, where the structure defines the connectivity of the holes. The origin parameter controls the placement of the structuring element as described in *Filter functions*. If no structuring element is provided, an element with connectivity equal to one is generated using `generate_binary_structure`.

The `binary_hit_or_miss` function implements a binary hit-or-miss transform of arrays of arbitrary rank with the given structuring elements. The hit-or-miss transform is calculated by erosion of the input with the first structure, erosion of the logical *not* of the input with the second structure, followed by the logical *and* of these two erosions. The origin parameters control the placement of the structuring elements as described in *Filter functions*. If `origin2` equals `None` it is set equal to the `origin1` parameter. If the first structuring element is not provided, a structuring element with connectivity equal to one is generated using `generate_binary_structure`, if `structure2` is not provided, it is set equal to the logical *not* of `structure1`.

### Grey-scale morphology

Grey-scale morphology operations are the equivalents of binary morphology operations that operate on arrays with arbitrary values. Below we describe the grey-scale equivalents of erosion, dilation, opening and closing. These operations are implemented in a similar fashion as the filters described in *Filter functions*, and we refer to this section for the description of filter kernels and footprints, and the handling of array borders. The grey-scale morphology operations optionally take a `structure` parameter that gives the values of the structuring element. If this parameter is not given the structuring element is assumed to be flat with a value equal to zero. The shape of the structure can optionally be defined by the `footprint` parameter. If this parameter is not given, the structure is assumed to be rectangular, with sizes equal to the dimensions of the `structure` array, or by the `size` parameter if `structure` is not given. The `size` parameter is only used if both `structure` and `footprint` are not given, in which case the structuring element is assumed to be rectangular and flat with the dimensions given by `size`. The `size` parameter, if provided, must be a sequence of sizes or a single number in which case the size of the filter is assumed to be equal along each axis. The `footprint` parameter, if provided, must be an array that defines the shape of the kernel by its non-zero elements.

Similar to binary erosion and dilation there are operations for grey-scale erosion and dilation:

The `grey_erosion` function calculates a multidimensional grey-scale erosion.

The `grey_dilation` function calculates a multidimensional grey-scale dilation.

Grey-scale opening and closing operations can be defined similar to their binary counterparts:

The `grey_opening` function implements grey-scale opening of arrays of arbitrary rank. Grey-scale opening is equivalent to a grey-scale erosion followed by a grey-scale dilation.

The `grey_closing` function implements grey-scale closing of arrays of arbitrary rank. Grey-scale opening is equivalent to a grey-scale dilation followed by a grey-scale erosion.

The `morphological_gradient` function implements a grey-scale morphological gradient of arrays of arbitrary rank. The grey-scale morphological gradient is equal to the difference of a grey-scale dilation and a grey-scale erosion.

The `morphological_laplace` function implements a grey-scale morphological laplace of arrays of arbitrary rank. The grey-scale morphological laplace is equal to the sum of a grey-scale dilation and a grey-scale erosion minus twice the input.

The `white_tophat` function implements a white top-hat filter of arrays of arbitrary rank. The white top-hat is equal to the difference of the input and a grey-scale opening.

The `black_tophat` function implements a black top-hat filter of arrays of arbitrary rank. The black top-hat is equal to the difference of a grey-scale closing and the input.

### Distance transforms

Distance transforms are used to calculate the minimum distance from each element of an object to the background. The following functions implement distance transforms for three different distance metrics: Euclidean, City Block, and Chessboard distances.

The function `distance_transform_cdt` uses a chamfer type algorithm to calculate the distance transform of the input, by replacing each object element (defined by values larger than zero) with the shortest distance to the background (all non-object elements). The structure determines the type of chamfering that is done. If the structure is equal to `'cityblock'` a structure is generated using `generate_binary_structure`



with a squared distance equal to 1. If the structure is equal to 'chessboard', a structure is generated using `generate_binary_structure` with a squared distance equal to the rank of the array. These choices correspond to the common interpretations of the cityblock and the chessboard distance metrics in two dimensions. In addition to the distance transform, the feature transform can be calculated. In this case the index of the closest background element is returned along the first axis of the result. The `return_distances`, and `return_indices` flags can be used to indicate if the distance transform, the feature transform, or both must be returned.

The `distances` and `indices` arguments can be used to give optional output arrays that must be of the correct size and type (both `Int32`).

The basics of the algorithm used to implement this function is described in: G. Borgefors, "Distance transformations in arbitrary dimensions.", *Computer Vision, Graphics, and Image Processing*, 27:321-345, 1984.

The function `distance_transform_edt` calculates the exact euclidean distance transform of the input, by replacing each object element (defined by values larger than zero) with the shortest euclidean distance to the background (all non-object elements).

In addition to the distance transform, the feature transform can be calculated. In this case the index of the closest background element is returned along the first axis of the result. The `return_distances`, and `return_indices` flags can be used to indicate if the distance transform, the feature transform, or both must be returned.

Optionally the sampling along each axis can be given by the `sampling` parameter which should be a sequence of length equal to the input rank, or a single number in which the sampling is assumed to be equal along all axes.

The `distances` and `indices` arguments can be used to give optional output arrays that must be of the correct size and type (`Float64` and `Int32`).

The algorithm used to implement this function is described in: C. R. Maurer, Jr., R. Qi, and V. Raghavan, "A linear time algorithm for computing exact euclidean distance transforms of binary images in arbitrary dimensions." *IEEE Trans. PAMI* 25, 265-270, 2003.

The function `distance_transform_bf` uses a brute-force algorithm to calculate the distance transform of the input, by replacing each object element (defined by values larger than zero) with the shortest distance to the background (all non-object elements). The metric must be one of "euclidean", "cityblock", or "chessboard".

In addition to the distance transform, the feature transform can be calculated. In this case the index of the closest background element is returned along the first axis of the result. The `return_distances`, and `return_indices` flags can be used to indicate if the distance transform, the feature transform, or both must be returned.

Optionally the sampling along each axis can be given by the `sampling` parameter which should be a sequence of length equal to the input rank, or a single number in which the sampling is assumed to be equal along all axes.

This parameter is only used in the case of the euclidean distance transform.

The `distances` and `indices` arguments can be used to give optional output arrays that must be of the correct size and type (`Float64` and `Int32`).

---

**Note:** This function uses a slow brute-force algorithm, the function `distance_transform_cdt` can be used to more efficiently calculate cityblock and chessboard distance transforms. The function `distance_transform_edt` can be used to more efficiently calculate the exact euclidean distance transform.

---

## Segmentation and labeling

Segmentation is the process of separating objects of interest from the background. The most simple approach is probably intensity thresholding, which is easily done with `numpy` functions:

```
>>> a = np.array([[1, 2, 2, 1, 1, 0],
...              [0, 2, 3, 1, 2, 0],
...              [1, 1, 1, 3, 3, 2],
...              [1, 1, 1, 1, 2, 1]])
>>> np.where(a > 1, 1, 0)
array([[0, 1, 1, 0, 0, 0],
       [0, 1, 1, 0, 1, 0],
       [0, 0, 0, 1, 1, 1],
       [0, 0, 0, 0, 1, 0]])
```



The result is a binary image, in which the individual objects still need to be identified and labeled. The function `label` generates an array where each object is assigned a unique number:

The `label` function generates an array where the objects in the input are labeled with an integer index. It returns a tuple consisting of the array of object labels and the number of objects found, unless the `output` parameter is given, in which case only the number of objects is returned. The connectivity of the objects is defined by a structuring element. For instance, in two dimensions using a four-connected structuring element gives:

```
>>> a = np.array([[0,1,1,0,0,0],[0,1,1,0,1,0],[0,0,0,1,1,1],[0,0,0,0,1,0]])
>>> s = [[0, 1, 0], [1,1,1], [0,1,0]]
>>> from scipy.ndimage import label
>>> label(a, s)
(array([[0, 1, 1, 0, 0, 0],
        [0, 1, 1, 0, 2, 0],
        [0, 0, 0, 2, 2, 2],
        [0, 0, 0, 0, 2, 0]]), 2)
```

These two objects are not connected because there is no way in which we can place the structuring element such that it overlaps with both objects. However, an 8-connected structuring element results in only a single object:

```
>>> a = np.array([[0,1,1,0,0,0],[0,1,1,0,1,0],[0,0,0,1,1,1],[0,0,0,0,1,0]])
>>> s = [[1,1,1], [1,1,1], [1,1,1]]
>>> label(a, s)[0]
array([[0, 1, 1, 0, 0, 0],
        [0, 1, 1, 0, 1, 0],
        [0, 0, 0, 1, 1, 1],
        [0, 0, 0, 0, 1, 0]])
```

If no structuring element is provided, one is generated by calling `generate_binary_structure` (see *Binary morphology*) using a connectivity of one (which in 2D is the 4-connected structure of the first example). The input can be of any type, any value not equal to zero is taken to be part of an object. This is useful if you need to ‘re-label’ an array of object indices, for instance after removing unwanted objects. Just apply the `label` function again to the index array. For instance:

```
>>> l, n = label([1, 0, 1, 0, 1])
>>> l
array([1, 0, 2, 0, 3])
>>> l = np.where(l != 2, l, 0)
>>> l
array([1, 0, 0, 0, 3])
>>> label(l)[0]
array([1, 0, 0, 0, 2])
```

---

**Note:** The structuring element used by `label` is assumed to be symmetric.

---

There is a large number of other approaches for segmentation, for instance from an estimation of the borders of the objects that can be obtained for instance by derivative filters. One such an approach is watershed segmentation. The function `watershed_ift` generates an array where each object is assigned a unique label, from an array that localizes the object borders, generated for instance by a gradient magnitude filter. It uses an array containing initial markers for the objects:

The `watershed_ift` function applies a watershed from markers algorithm, using an Iterative Forest Transform, as described in: P. Felkel, R. Wegenkittl, and M. Bruckschwaiger, “Implementation and Complexity of the Watershed-from-Markers Algorithm Computed as a Minimal Cost Forest.”, Eurographics 2001, pp. C:26-35.

The inputs of this function are the array to which the transform is applied, and an array of markers that designate the objects by a unique label, where any non-zero value is a marker. For instance:

```

>>> input = np.array([[0, 0, 0, 0, 0, 0, 0],
...                   [0, 1, 1, 1, 1, 1, 0],
...                   [0, 1, 0, 0, 0, 1, 0],
...                   [0, 1, 0, 0, 0, 1, 0],
...                   [0, 1, 0, 0, 0, 1, 0],
...                   [0, 1, 1, 1, 1, 1, 0],
...                   [0, 0, 0, 0, 0, 0, 0]], np.uint8)
>>> markers = np.array([[1, 0, 0, 0, 0, 0, 0],
...                     [0, 0, 0, 0, 0, 0, 0],
...                     [0, 0, 0, 0, 0, 0, 0],
...                     [0, 0, 0, 2, 0, 0, 0],
...                     [0, 0, 0, 0, 0, 0, 0],
...                     [0, 0, 0, 0, 0, 0, 0],
...                     [0, 0, 0, 0, 0, 0, 0]], np.int8)
>>> from scipy.ndimage import watershed_ift
>>> watershed_ift(input, markers)
array([[1, 1, 1, 1, 1, 1, 1],
       [1, 1, 2, 2, 2, 1, 1],
       [1, 2, 2, 2, 2, 2, 1],
       [1, 2, 2, 2, 2, 2, 1],
       [1, 2, 2, 2, 2, 2, 1],
       [1, 1, 2, 2, 2, 1, 1],
       [1, 1, 1, 1, 1, 1, 1]], dtype=int8)
    
```

Here two markers were used to designate an object (*marker* = 2) and the background (*marker* = 1). The order in which these are processed is arbitrary: moving the marker for the background to the lower right corner of the array yields a different result:

```

>>> markers = np.array([[0, 0, 0, 0, 0, 0, 0],
...                     [0, 0, 0, 0, 0, 0, 0],
...                     [0, 0, 0, 0, 0, 0, 0],
...                     [0, 0, 0, 2, 0, 0, 0],
...                     [0, 0, 0, 0, 0, 0, 0],
...                     [0, 0, 0, 0, 0, 0, 0],
...                     [0, 0, 0, 0, 0, 0, 1]], np.int8)
>>> watershed_ift(input, markers)
array([[1, 1, 1, 1, 1, 1, 1],
       [1, 1, 1, 1, 1, 1, 1],
       [1, 1, 2, 2, 2, 1, 1],
       [1, 1, 2, 2, 2, 1, 1],
       [1, 1, 2, 2, 2, 1, 1],
       [1, 1, 1, 1, 1, 1, 1],
       [1, 1, 1, 1, 1, 1, 1]], dtype=int8)
    
```

The result is that the object (*marker* = 2) is smaller because the second marker was processed earlier. This may not be the desired effect if the first marker was supposed to designate a background object. Therefore `watershed_ift` treats markers with a negative value explicitly as background markers and processes them after the normal markers. For instance, replacing the first marker by a negative marker gives a result similar to the first example:

```

>>> markers = np.array([[0, 0, 0, 0, 0, 0, 0],
...                     [0, 0, 0, 0, 0, 0, 0],
...                     [0, 0, 0, 0, 0, 0, 0],
...                     [0, 0, 0, 2, 0, 0, 0],
...                     [0, 0, 0, 0, 0, 0, 0],
...                     [0, 0, 0, 0, 0, 0, 0],
...                     [0, 0, 0, 0, 0, 0, -1]], np.int8)
>>> watershed_ift(input, markers)
array([[ -1, -1, -1, -1, -1, -1, -1],
    
```

```

[-1, -1, 2, 2, 2, -1, -1],
[-1, 2, 2, 2, 2, 2, -1],
[-1, 2, 2, 2, 2, 2, -1],
[-1, 2, 2, 2, 2, 2, -1],
[-1, -1, 2, 2, 2, -1, -1],
[-1, -1, -1, -1, -1, -1, -1]], dtype=int8)

```

The connectivity of the objects is defined by a structuring element. If no structuring element is provided, one is generated by calling `generate_binary_structure` (see *Binary morphology*) using a connectivity of one (which in 2D is a 4-connected structure.) For example, using an 8-connected structure with the last example yields a different object:

```

>>> watershed_ift(input, markers,
...               structure = [[1,1,1], [1,1,1], [1,1,1]])
array([[ -1,  -1,  -1,  -1,  -1,  -1,  -1],
       [-1,  2,  2,  2,  2,  2, -1],
       [-1,  2,  2,  2,  2,  2, -1],
       [-1,  2,  2,  2,  2,  2, -1],
       [-1,  2,  2,  2,  2,  2, -1],
       [-1,  2,  2,  2,  2,  2, -1],
       [-1, -1, -1, -1, -1, -1, -1]], dtype=int8)

```

---

**Note:** The implementation of `watershed_ift` limits the data types of the input to `UInt8` and `UInt16`.

---

## Object measurements

Given an array of labeled objects, the properties of the individual objects can be measured. The `find_objects` function can be used to generate a list of slices that for each object, give the smallest sub-array that fully contains the object:

The `find_objects` function finds all objects in a labeled array and returns a list of slices that correspond to the smallest regions in the array that contains the object. For instance:

```

>>> a = np.array([[0,1,1,0,0,0],[0,1,1,0,1,0],[0,0,0,1,1,1],[0,0,0,0,1,0]])
>>> l, n = label(a)
>>> from scipy.ndimage import find_objects
>>> f = find_objects(l)
>>> a[f[0]]
array([[1, 1],
       [1, 1]])
>>> a[f[1]]
array([[0, 1, 0],
       [1, 1, 1],
       [0, 1, 0]])

```

`find_objects` returns slices for all objects, unless the `max_label` parameter is larger than zero, in which case only the first `max_label` objects are returned. If an index is missing in the `label` array, `None` is returned instead of a slice. For example:

```

>>> from scipy.ndimage import find_objects
>>> find_objects([1, 0, 3, 4], max_label = 3)
[(slice(0, 1, None),), None, (slice(2, 3, None),)]

```

The list of slices generated by `find_objects` is useful to find the position and dimensions of the objects in the array, but can also be used to perform measurements on the individual objects. Say we want to find the sum of the intensities of an object in image:

```
>>> image = np.arange(4 * 6).reshape(4, 6)
>>> mask = np.array([[0,1,1,0,0,0],[0,1,1,0,1,0],[0,0,0,1,1,1],[0,0,0,0,1,0]])
>>> labels = label(mask)[0]
>>> slices = find_objects(labels)
```

Then we can calculate the sum of the elements in the second object:

```
>>> np.where(labels[slices[1]] == 2, image[slices[1]], 0).sum()
80
```

That is however not particularly efficient, and may also be more complicated for other types of measurements. Therefore a few measurements functions are defined that accept the array of object labels and the index of the object to be measured. For instance calculating the sum of the intensities can be done by:

```
>>> from scipy.ndimage import sum as ndi_sum
>>> ndi_sum(image, labels, 2)
80
```

For large arrays and small objects it is more efficient to call the measurement functions after slicing the array:

```
>>> ndi_sum(image[slices[1]], labels[slices[1]], 2)
80
```

Alternatively, we can do the measurements for a number of labels with a single function call, returning a list of results. For instance, to measure the sum of the values of the background and the second object in our example we give a list of labels:

```
>>> ndi_sum(image, labels, [0, 2])
array([178.0, 80.0])
```

The measurement functions described below all support the *index* parameter to indicate which object(s) should be measured. The default value of *index* is `None`. This indicates that all elements where the label is larger than zero should be treated as a single object and measured. Thus, in this case the *labels* array is treated as a mask defined by the elements that are larger than zero. If *index* is a number or a sequence of numbers it gives the labels of the objects that are measured. If *index* is a sequence, a list of the results is returned. Functions that return more than one result, return their result as a tuple if *index* is a single number, or as a tuple of lists, if *index* is a sequence.

The `sum` function calculates the sum of the elements of the object with label(s) given by *index*, using the *labels* array for the object labels. If *index* is `None`, all elements with a non-zero label value are treated as a single object. If *label* is `None`, all elements of *input* are used in the calculation.

The `mean` function calculates the mean of the elements of the object with label(s) given by *index*, using the *labels* array for the object labels. If *index* is `None`, all elements with a non-zero label value are treated as a single object. If *label* is `None`, all elements of *input* are used in the calculation.

The `variance` function calculates the variance of the elements of the object with label(s) given by *index*, using the *labels* array for the object labels. If *index* is `None`, all elements with a non-zero label value are treated as a single object. If *label* is `None`, all elements of *input* are used in the calculation.

The `standard_deviation` function calculates the standard deviation of the elements of the object with label(s) given by *index*, using the *labels* array for the object labels. If *index* is `None`, all elements with a non-zero label value are treated as a single object. If *label* is `None`, all elements of *input* are used in the calculation.

The `minimum` function calculates the minimum of the elements of the object with label(s) given by *index*, using the *labels* array for the object labels. If *index* is `None`, all elements with a non-zero label value are treated as a single object. If *label* is `None`, all elements of *input* are used in the calculation.

The `maximum` function calculates the maximum of the elements of the object with label(s) given by *index*, using the *labels* array for the object labels. If *index* is `None`, all elements with a non-zero label value are treated as a single object. If *label* is `None`, all elements of *input* are used in the calculation.

The `minimum_position` function calculates the position of the minimum of the elements of the object with label(s) given by *index*, using the *labels* array for the object labels. If *index* is `None`, all elements with a non-zero label value are treated as a single object. If *label* is `None`, all elements of *input* are used in the calculation.

The `maximum_position` function calculates the position of the maximum of the elements of the object with label(s) given by `index`, using the `labels` array for the object labels. If `index` is `None`, all elements with a non-zero label value are treated as a single object. If `label` is `None`, all elements of `input` are used in the calculation.

The `extrema` function calculates the minimum, the maximum, and their positions, of the elements of the object with label(s) given by `index`, using the `labels` array for the object labels. If `index` is `None`, all elements with a non-zero label value are treated as a single object. If `label` is `None`, all elements of `input` are used in the calculation. The result is a tuple giving the minimum, the maximum, the position of the minimum and the position of the maximum. The result is the same as a tuple formed by the results of the functions `minimum`, `maximum`, `minimum_position`, and `maximum_position` that are described above.

The `center_of_mass` function calculates the center of mass of the of the object with label(s) given by `index`, using the `labels` array for the object labels. If `index` is `None`, all elements with a non-zero label value are treated as a single object. If `label` is `None`, all elements of `input` are used in the calculation.

The `histogram` function calculates a histogram of the of the object with label(s) given by `index`, using the `labels` array for the object labels. If `index` is `None`, all elements with a non-zero label value are treated as a single object. If `label` is `None`, all elements of `input` are used in the calculation. Histograms are defined by their minimum (`min`), maximum (`max`) and the number of bins (`bins`). They are returned as one-dimensional arrays of type `Int32`.

## Extending ndimage in C

A few functions in the `scipy.ndimage` take a call-back argument. This can be a python function, but also a `PyCObject` containing a pointer to a C function. To use this feature, you must write your own C extension that defines the function, and define a Python function that returns a `PyCObject` containing a pointer to this function.

An example of a function that supports this is `geometric_transform` (see *Interpolation functions*). You can pass it a python callable object that defines a mapping from all output coordinates to corresponding coordinates in the input array. This mapping function can also be a C function, which generally will be much more efficient, since the overhead of calling a python function at each element is avoided.

For example to implement a simple shift function we define the following function:

```
static int
_shift_function(int *output_coordinates, double* input_coordinates,
               int output_rank, int input_rank, void *callback_data)
{
    int ii;
    /* get the shift from the callback data pointer: */
    double shift = *(double*)callback_data;
    /* calculate the coordinates: */
    for(ii = 0; ii < irank; ii++)
        icoor[ii] = ocoor[ii] - shift;
    /* return OK status: */
    return 1;
}
```

This function is called at every element of the output array, passing the current coordinates in the `output_coordinates` array. On return, the `input_coordinates` array must contain the coordinates at which the input is interpolated. The ranks of the input and output array are passed through `output_rank` and `input_rank`. The value of the shift is passed through the `callback_data` argument, which is a pointer to void. The function returns an error status, in this case always 1, since no error can occur.

A pointer to this function and a pointer to the shift value must be passed to `geometric_transform`. Both are passed by a single `PyCObject` which is created by the following python extension function:

```
static PyObject *
py_shift_function(PyObject *obj, PyObject *args)
{
```

```

double shift = 0.0;
if (!PyArg_ParseTuple(args, "d", &shift)) {
    PyErr_SetString(PyExc_RuntimeError, "invalid parameters");
    return NULL;
} else {
    /* assign the shift to a dynamically allocated location: */
    double *cdata = (double*)malloc(sizeof(double));
    *cdata = shift;
    /* wrap function and callback_data in a CObject: */
    return PyCObject_FromVoidPtrAndDesc(_shift_function, cdata,
                                        _destructor);
}
}

```

The value of the shift is obtained and then assigned to a dynamically allocated memory location. Both this data pointer and the function pointer are then wrapped in a `PyCObject`, which is returned. Additionally, a pointer to a destructor function is given, that will free the memory we allocated for the shift value when the `PyCObject` is destroyed. This destructor is very simple:

```

static void
_destructor(void* cobject, void *cdata)
{
    if (cdata)
        free(cdata);
}

```

To use these functions, an extension module is built:

```

static PyMethodDef methods[] = {
    {"shift_function", (PyCFunction)py_shift_function, METH_VARARGS, ""},
    {NULL, NULL, 0, NULL}
};

void
initexample(void)
{
    Py_InitModule("example", methods);
}

```

This extension can then be used in Python, for example:

```

>>> import example
>>> array = np.arange(12).reshape(4, 3).astype(np.float64)
>>> fnc = example.shift_function(0.5)
>>> from scipy.ndimage import geometric_transform
>>> geometric_transform(array, fnc)
array([[ 0.,    0.,    0. ],
       [ 0.,    1.3625,  2.7375],
       [ 0.,    4.8125,  6.1875],
       [ 0.,    8.2625,  9.6375]])

```

C callback functions for use with `ndimage` functions must all be written according to this scheme. The next section lists the `ndimage` functions that accept a C callback function and gives the prototype of the callback function.

### Functions that support C callback functions

The `ndimage` functions that support C callback functions are described here. Obviously, the prototype of the function that is provided to these functions must match exactly that what they expect. Therefore we give here the pro-

types of the callback functions. All these callback functions accept a void *callback\_data* pointer that must be wrapped in a `PyObject` using the Python `PyObject_FromVoidPtrAndDesc` function, which can also accept a pointer to a destructor function to free any memory allocated for *callback\_data*. If *callback\_data* is not needed, `PyObject_FromVoidPtr` may be used instead. The callback functions must return an integer error status that is equal to zero if something went wrong, or 1 otherwise. If an error occurs, you should normally set the python error status with an informative message before returning, otherwise, a default error message is set by the calling function.

The function `generic_filter` (see *Generic filter functions*) accepts a callback function with the following prototype:

The calling function iterates over the elements of the input and output arrays, calling the callback function at each element. The elements within the footprint of the filter at the current element are passed through the *buffer* parameter, and the number of elements within the footprint through *filter\_size*. The calculated value should be returned in the *return\_value* argument.

The function `generic_filter1d` (see *Generic filter functions*) accepts a callback function with the following prototype:

The calling function iterates over the lines of the input and output arrays, calling the callback function at each line. The current line is extended according to the border conditions set by the calling function, and the result is copied into the array that is passed through the *input\_line* array. The length of the input line (after extension) is passed through *input\_length*. The callback function should apply the 1D filter and store the result in the array passed through *output\_line*. The length of the output line is passed through *output\_length*.

The function `geometric_transform` (see *Interpolation functions*) expects a function with the following prototype:

The calling function iterates over the elements of the output array, calling the callback function at each element. The coordinates of the current output element are passed through *output\_coordinates*. The callback function must return the coordinates at which the input must be interpolated in *input\_coordinates*. The rank of the input and output arrays are given by *input\_rank* and *output\_rank* respectively.

### 3.1.15 File IO (`scipy.io`)

See also:

*numpy-reference.routines.io* (in `numpy`)

#### MATLAB files

---

<code>loadmat(file_name[, mdict, appendmat])</code>	Load MATLAB file.
<code>savemat(file_name, mdict[, appendmat, ...])</code>	Save a dictionary of names and arrays into a MATLAB-style <code>.mat</code> file.
<code>whosmat(file_name[, appendmat])</code>	List variables inside a MATLAB file.

---

#### The basic functions

We'll start by importing `scipy.io` and calling it `sio` for convenience:

```
>>> import scipy.io as sio
```

If you are using IPython, try tab completing on `sio`. Among the many options, you will find:

```
sio.loadmat
sio.savemat
sio.whosmat
```

These are the high-level functions you will most likely use when working with MATLAB files. You'll also find:

sio.matlab

This is the package from which `loadmat`, `savemat` and `whosmat` are imported. Within `sio.matlab`, you will find the `mio` module. This module contains the machinery that `loadmat` and `savemat` use. From time to time you may find yourself re-using this machinery.

### ***How do I start?***

You may have a `.mat` file that you want to read into Scipy. Or, you want to pass some variables from Scipy / Numpy into MATLAB.

To save us using a MATLAB license, let's start in `Octave`. `Octave` has MATLAB-compatible save and load functions. Start `Octave` (`octave` at the command line for me):

```
octave:1> a = 1:12
a =
    1    2    3    4    5    6    7    8    9   10   11   12

octave:2> a = reshape(a, [1 3 4])
a =

ans(:,:,1) =
    1    2    3

ans(:,:,2) =
    4    5    6

ans(:,:,3) =
    7    8    9

ans(:,:,4) =
   10   11   12

octave:3> save -6 octave_a.mat a % MATLAB 6 compatible
octave:4> ls octave_a.mat
octave_a.mat
```

Now, to Python:

```
>>> mat_contents = sio.loadmat('octave_a.mat')
>>> mat_contents
{'a': array([[[ 1.,  4.,  7., 10.],
              [ 2.,  5.,  8., 11.],
              [ 3.,  6.,  9., 12.]])},
 '__version__': '1.0',
 '__header__': 'MATLAB 5.0 MAT-file, written by
Octave 3.6.3, 2013-02-17 21:02:11 UTC',
 '__globals__': []}
>>> oct_a = mat_contents['a']
>>> oct_a
array([[[ 1.,  4.,  7., 10.],
         [ 2.,  5.,  8., 11.],
         [ 3.,  6.,  9., 12.]])])
>>> oct_a.shape
```



```
(1, 3, 4)
```

Now let's try the other way round:

```
>>> import numpy as np
>>> vect = np.arange(10)
>>> vect.shape
(10,)
>>> sio.savemat('np_vector.mat', {'vect':vect})
```

Then back to Octave:

```
octave:8> load np_vector.mat
octave:9> vect
vect =

    0    1    2    3    4    5    6    7    8    9

octave:10> size(vect)
ans =

     1    10
```

If you want to inspect the contents of a MATLAB file without reading the data into memory, use the `whosmat` command:

```
>>> sio.whosmat('octave_a.mat')
[('a', (1, 3, 4), 'double')]
```

`whosmat` returns a list of tuples, one for each array (or other object) in the file. Each tuple contains the name, shape and data type of the array.

### ***MATLAB structs***

MATLAB structs are a little bit like Python dicts, except the field names must be strings. Any MATLAB object can be a value of a field. As for all objects in MATLAB, structs are in fact arrays of structs, where a single struct is an array of shape (1, 1).

```
octave:11> my_struct = struct('field1', 1, 'field2', 2)
my_struct =
{
  field1 = 1
  field2 = 2
}

octave:12> save -6 octave_struct.mat my_struct
```

We can load this in Python:

```
>>> mat_contents = sio.loadmat('octave_struct.mat')
>>> mat_contents
{'my_struct': array([[[[1.0]], [[2.0]]]],
  dtype=[('field1', 'O'), ('field2', 'O')]), '__version__': '1.0', '__header__': 'MATLAB 5.0 MAT-'}
>>> oct_struct = mat_contents['my_struct']
>>> oct_struct.shape
(1, 1)
>>> val = oct_struct[0,0]
>>> val
([[1.0]], [[2.0]])
>>> val['field1']
```

```
array([[ 1.]])
>>> val['field2']
array([[ 2.]])
>>> val.dtype
dtype([('field1', '<float64>'), ('field2', '<float64>')])
```

In versions of Scipy from 0.12.0, MATLAB structs come back as numpy structured arrays, with fields named for the struct fields. You can see the field names in the `dtype` output above. Note also:

```
>>> val = oct_struct[0,0]
```

and:

```
octave:13> size(my_struct)
ans =

     1     1
```

So, in MATLAB, the struct array must be at least 2D, and we replicate that when we read into Scipy. If you want all length 1 dimensions squeezed out, try this:

```
>>> mat_contents = sio.loadmat('octave_struct.mat', squeeze_me=True)
>>> oct_struct = mat_contents['my_struct']
>>> oct_struct.shape
()
```

Sometimes, it's more convenient to load the MATLAB structs as python objects rather than numpy structured arrays - it can make the access syntax in python a bit more similar to that in MATLAB. In order to do this, use the `struct_as_record=False` parameter setting to `loadmat`.

```
>>> mat_contents = sio.loadmat('octave_struct.mat', struct_as_record=False)
>>> oct_struct = mat_contents['my_struct']
>>> oct_struct[0,0].field1
array([[ 1.]])
```

`struct_as_record=False` works nicely with `squeeze_me`:

```
>>> mat_contents = sio.loadmat('octave_struct.mat', struct_as_record=False, squeeze_me=True)
>>> oct_struct = mat_contents['my_struct']
>>> oct_struct.shape # but no - it's a scalar
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
AttributeError: 'mat_struct' object has no attribute 'shape'
>>> type(oct_struct)
<class 'scipy.io.matlab.mio5_params.mat_struct'>
>>> oct_struct.field1
1.0
```

Saving struct arrays can be done in various ways. One simple method is to use dicts:

```
>>> a_dict = {'field1': 0.5, 'field2': 'a string'}
>>> sio.savemat('saved_struct.mat', {'a_dict': a_dict})
```

loaded as:

```
octave:21> load saved_struct
octave:22> a_dict
a_dict =
```

```
    scalar structure containing the fields:
```

```
field2 = a string
field1 = 0.50000
```

You can also save structs back again to MATLAB (or Octave in our case) like this:

```
>>> dt = [('f1', 'f8'), ('f2', 'S10')]
>>> arr = np.zeros((2,), dtype=dt)
>>> arr
array([(0.0, ''), (0.0, '')],
      dtype=[('f1', '<f8'), ('f2', 'S10')])
>>> arr[0]['f1'] = 0.5
>>> arr[0]['f2'] = 'python'
>>> arr[1]['f1'] = 99
>>> arr[1]['f2'] = 'not perl'
>>> sio.savemat('np_struct_arr.mat', {'arr': arr})
```

### **MATLAB cell arrays**

Cell arrays in MATLAB are rather like python lists, in the sense that the elements in the arrays can contain any type of MATLAB object. In fact they are most similar to numpy object arrays, and that is how we load them into numpy.

```
octave:14> my_cells = {1, [2, 3]}
my_cells =
{
  [1,1] = 1
  [1,2] =
      2   3
}

octave:15> save -6 octave_cells.mat my_cells
```

Back to Python:

```
>>> mat_contents = sio.loadmat('octave_cells.mat')
>>> oct_cells = mat_contents['my_cells']
>>> print(oct_cells.dtype)
object
>>> val = oct_cells[0,0]
>>> val
array([[ 1.]])
>>> print(val.dtype)
float64
```

Saving to a MATLAB cell array just involves making a numpy object array:

```
>>> obj_arr = np.zeros((2,), dtype=np.object)
>>> obj_arr[0] = 1
>>> obj_arr[1] = 'a string'
>>> obj_arr
array([1, 'a string'], dtype=object)
>>> sio.savemat('np_cells.mat', {'obj_arr':obj_arr})
```

```
octave:16> load np_cells.mat
octave:17> obj_arr
obj_arr =
{
  [1,1] = 1
```

```
[2,1] = a string  
}
```

## IDL files

---

`readsav(file_name[, idict, python_dict, ...])` Read an IDL .sav file.

---

## Matrix Market files

---

<code>mminfo(source)</code>	Return size and storage parameters from Matrix Market file-like 'source'.
<code>mmread(source)</code>	Reads the contents of a Matrix Market file-like 'source' into a matrix.
<code>mmwrite(target, a[, comment, field, ...])</code>	Writes the sparse or dense array <i>a</i> to Matrix Market file-like <i>target</i> .

---

## Wav sound files (`scipy.io.wavfile`)

---

<code>read(filename[, mmap])</code>	Open a WAV file
<code>write(filename, rate, data)</code>	Write a numpy array as a WAV file.

---

## Arff files (`scipy.io.arff`)

Module to read ARFF files, which are the standard data format for WEKA.

ARFF is a text file format which support numerical, string and data values. The format can also represent missing data and sparse data.

See the [WEKA website](#) for more details about arff format and available datasets.

---

`loadarff(f)` Read an arff file.

---

## Netcdf (`scipy.io.netcdf`)

---

`netcdf_file(filename[, mode, mmap, version, ...])` A file object for NetCDF data.

---

Allows reading of NetCDF files (version of [pupynere](#) package)

### 3.1.16 Weave (`scipy.weave`)

#### Outline

**Contents**

- Weave (`scipy.weave`)
  - Outline
  - Introduction
  - Requirements
  - Installation
  - Testing
    - \* Testing Notes:
  - Benchmarks
  - Inline
    - \* More with `printf`
    - \* More examples
      - Binary search
      - Dictionary Sort
      - NumPy – `cast/copy/transpose`
      - wxPython
    - \* Keyword Option
    - \* Inline Arguments
    - \* Distutils keywords
      - Keyword Option Examples
      - Returning Values
      - The issue with `locals()`
      - A quick look at the code
    - \* Technical Details
    - \* Passing Variables in/out of the C/C++ code
    - \* Type Conversions
      - NumPy Argument Conversion
      - String, List, Tuple, and Dictionary Conversion
      - File Conversion
      - Callable, Instance, and Module Conversion
      - Customizing Conversions
    - \* The Catalog
      - Function Storage
      - Catalog search paths and the `PYTHONCOMPILED` variable
  - Blitz
    - \* Requirements
    - \* Limitations
    - \* NumPy efficiency issues: What compilation buys you
    - \* The Tools
      - Parser
      - Blitz and NumPy
    - \* Type definitions and coercion
    - \* Cataloging Compiled Functions
    - \* Checking Array Sizes
    - \* Creating the Extension Module
  - Extension Modules
    - \* A Simple Example
    - \* Fibonacci Example
  - Customizing Type Conversions – Type Factories
  - Things I wish `weave` did

### Introduction

The `scipy.weave` (below just `weave`) package provides tools for including C/C++ code within in Python code. This offers both another level of optimization to those who need it, and an easy way to modify and extend any supported extension libraries such as wxPython and hopefully VTK soon. Inlining C/C++ code within Python generally results in speed ups of 1.5x to 30x speed-up over algorithms written in pure Python (However, it is also possible to slow things down...). Generally algorithms that require a large number of calls to the Python API don't benefit as much from the conversion to C/C++ as algorithms that have inner loops completely convertible to C.

There are three basic ways to use `weave`. The `weave.inline()` function executes C code directly within Python, and `weave.blitz()` translates Python NumPy expressions to C++ for fast execution. `blitz()` was the original reason `weave` was built. For those interested in building extension libraries, the `ext_tools` module provides classes for building extension modules within Python.

Most of `weave`'s functionality should work on Windows and Unix, although some of its functionality requires `gcc` or a similarly modern C++ compiler that handles templates well. Up to now, most testing has been done on Windows 2000 with Microsoft's C++ compiler (MSVC) and with `gcc` (mingw32 2.95.2 and 2.95.3-6). All tests also pass on Linux (RH 7.1 with `gcc` 2.96), and I've had reports that it works on Debian also (thanks Pearu).

The `inline` and `blitz` provide new functionality to Python (although I've recently learned about the `PyInline` project which may offer similar functionality to `inline`). On the other hand, tools for building Python extension modules already exists (SWIG, SIP, pycpp, CXX, and others). As of yet, I'm not sure where `weave` fits in this spectrum. It is closest in flavor to CXX in that it makes creating new C/C++ extension modules pretty easy. However, if you're wrapping a gaggle of legacy functions or classes, SWIG and friends are definitely the better choice. `weave` is set up so that you can customize how Python types are converted to C types in `weave`. This is great for `inline()`, but, for wrapping legacy code, it is more flexible to specify things the other way around – that is how C types map to Python types. This `weave` does not do. I guess it would be possible to build such a tool on top of `weave`, but with good tools like SWIG around, I'm not sure the effort produces any new capabilities. Things like function overloading are probably easily implemented in `weave` and it might be easier to mix Python/C code in function calls, but nothing beyond this comes to mind. So, if you're developing new extension modules or optimizing Python functions in C, `weave.ext_tools()` might be the tool for you. If you're wrapping legacy code, stick with SWIG.

The next several sections give the basics of how to use `weave`. We'll discuss what's happening under the covers in more detail later on. Serious users will need to at least look at the type conversion section to understand how Python variables map to C/C++ types and how to customize this behavior. One other note. If you don't know C or C++ then these docs are probably of very little help to you. Further, it'd be helpful if you know something about writing Python extensions. `weave` does quite a bit for you, but for anything complex, you'll need to do some conversions, reference counting, etc.

---

**Note:** `weave` is actually part of the `SciPy` package. However, it also works fine as a standalone package (you can install from `scipy/weave` with `python setup.py install`). The examples here are given as if it is used as a stand alone package. If you are using from within `scipy`, you can use `from scipy import weave` and the examples will work identically.

---

### Requirements

- Python  
I use 2.1.1. Probably 2.0 or higher should work.

- C++ compiler  
`weave` uses `distutils` to actually build extension modules, so it uses whatever compiler was originally used to build Python. `weave` itself requires a C++ compiler. If you used a C++ compiler to build Python, your probably fine.

On Unix gcc is the preferred choice because I've done a little testing with it. All testing has been done with gcc, but I expect the majority of compilers should work for `inline` and `ext_tools`. The one issue I'm not sure about is that I've hard coded things so that compilations are linked with the `stdc++` library. *Is this standard across Unix compilers, or is this a gcc-ism?*

For `blitz()`, you'll need a reasonably recent version of gcc. 2.95.2 works on windows and 2.96 looks fine on Linux. Other versions are likely to work. Its likely that KAI's C++ compiler and maybe some others will work, but I haven't tried. My advice is to use gcc for now unless your willing to tinker with the code some.

On Windows, either MSVC or gcc (`mingw32`) should work. Again, you'll need gcc for `blitz()` as the MSVC compiler doesn't handle templates well.

I have not tried Cygwin, so please report success if it works for you.

- NumPy

The python NumPy module is required for `blitz()` to work and for `numpy.distutils` which is used by `weave`.

## Installation

There are currently two ways to get `weave`. First, `weave` is part of SciPy and installed automatically (as a sub-package) whenever SciPy is installed. Second, since `weave` is useful outside of the scientific community, it has been setup so that it can be used as a stand-alone module.

The stand-alone version can be downloaded from [here](#). Instructions for installing should be found there as well. `setup.py` file to simplify installation.

## Testing

Once `weave` is installed, fire up python and run its unit tests.

```
>>> import weave
>>> weave.test()
runs long time... spews tons of output and a few warnings
.
.
.
.....
.....
.....
-----
Ran 184 tests in 158.418s
OK
>>>
```

This takes a while, usually several minutes. On Unix with remote file systems, I've had it take 15 or so minutes. In the end, it should run about 180 tests and spew some speed results along the way. If you get errors, they'll be reported at the end of the output. Please report errors that you find. Some tests are known to fail at this point.

If you only want to test a single module of the package, you can do this by running `test()` for that specific module.

```
>>> import weave.scalar_spec
>>> weave.scalar_spec.test()
.....
-----
Ran 7 tests in 23.284s
```

**Testing Notes:**

- Windows 1

I've had some test fail on windows machines where I have msvc, gcc-2.95.2 (in c:gcc-2.95.2), and gcc-2.95.3-6 (in c:gcc) all installed. My environment has c:gcc in the path and does not have c:gcc-2.95.2 in the path. The test process runs very smoothly until the end where several test using gcc fail with cpp0 not found by g++. If I check `os.system('gcc -v')` before running tests, I get gcc-2.95.3-6. If I check after running tests (and after failure), I get gcc-2.95.2. ??huh??. The `os.environ['PATH']` still has c:gcc first in it and is not corrupted (msvc/distutils messes with the environment variables, so we have to undo its work in some places). If anyone else sees this, let me know - - it may just be an quirk on my machine (unlikely). Testing with the gcc- 2.95.2 installation always works.

- Windows 2

If you run the tests from PythonWin or some other GUI tool, you'll get a ton of DOS windows popping up periodically as `weave` spawns the compiler multiple times. Very annoying. Anyone know how to fix this?

- wxPython

wxPython tests are not enabled by default because importing wxPython on a Unix machine without access to a X-term will cause the program to exit. Anyone know of a safe way to detect whether wxPython can be imported and whether a display exists on a machine?

**Benchmarks**

This section has not been updated from old scipy weave and Numeric....

This section has a few benchmarks – that's all people want to see anyway right? These are mostly taken from running files in the `weave/example` directory and also from the test scripts. Without more information about what the test actually do, their value is limited. Still, their here for the curious. Look at the example scripts for more specifics about what problem was actually solved by each run. These examples are run under windows 2000 using Microsoft Visual C++ and python2.1 on a 850 MHz PIII laptop with 320 MB of RAM. Speed up is the improvement (degradation) factor of `weave` compared to conventional Python functions. The `blitz()` comparisons are shown compared to NumPy.

Table 3.7: inline and ext\_tools

Algorithm	Speed up
binary search	1.50
fibonacci (recursive)	82.10
fibonacci (loop)	9.17
return None	0.14
map	1.20
dictionary sort	2.54
vector quantization	37.40

Table 3.8: blitz – double precision

Algorithm	Speed up
<code>a = b + c</code> 512x512	3.05
<code>a = b + c + d</code> 512x512	4.59
5 pt avg. filter, 2D Image 512x512	9.01
Electromagnetics (FDTD) 100x100x100	8.61

The benchmarks shown `blitz` in the best possible light. NumPy (at least on my machine) is significantly worse for double precision than it is for single precision calculations. If your interested in single precision results, you can pretty much divide the double precision speed up by 3 and you'll be close.



## Inline

`inline()` compiles and executes C/C++ code on the fly. Variables in the local and global Python scope are also available in the C/C++ code. Values are passed to the C/C++ code by assignment much like variables are passed into a standard Python function. Values are returned from the C/C++ code through a special argument called `return_val`. Also, the contents of mutable objects can be changed within the C/C++ code and the changes remain after the C code exits and returns to Python. (more on this later)

Here's a trivial `printf` example using `inline()`:

```
>>> import weave
>>> a = 1
>>> weave.inline('printf("%d\n",a);', ['a'])
1
```

In this, its most basic form, `inline(c_code, var_list)` requires two arguments. `c_code` is a string of valid C/C++ code. `var_list` is a list of variable names that are passed from Python into C/C++. Here we have a simple `printf` statement that writes the Python variable `a` to the screen. The first time you run this, there will be a pause while the code is written to a `.cpp` file, compiled into an extension module, loaded into Python, cataloged for future use, and executed. On windows (850 MHz PIII), this takes about 1.5 seconds when using Microsoft's C++ compiler (MSVC) and 6-12 seconds using `gcc` (`mingw32 2.95.2`). All subsequent executions of the code will happen very quickly because the code only needs to be compiled once. If you kill and restart the interpreter and then execute the same code fragment again, there will be a much shorter delay in the fractions of seconds range. This is because `weave` stores a catalog of all previously compiled functions in an on disk cache. When it sees a string that has been compiled, it loads the already compiled module and executes the appropriate function.

---

**Note:** If you try the `printf` example in a GUI shell such as IDLE, PythonWin, PyShell, etc., you're unlikely to see the output. This is because the C code is writing to `stdout`, instead of to the GUI window. This doesn't mean that `inline` doesn't work in these environments – it only means that `stdout` in C is not the same as the `stdout` in Python in these cases. Non input/output functions will work as expected.

---

Although effort has been made to reduce the overhead associated with calling `inline`, it is still less efficient for simple code snippets than using equivalent Python code. The simple `printf` example is actually slower by 30% or so than using Python `print` statement. And, it is not difficult to create code fragments that are 8-10 times slower using `inline` than equivalent Python. However, for more complicated algorithms, the speedup can be worthwhile – anywhere from 1.5-30 times faster. Algorithms that have to manipulate Python objects (sorting a list) usually only see a factor of 2 or so improvement. Algorithms that are highly computational or manipulate NumPy arrays can see much larger improvements. The `examples/vq.py` file shows a factor of 30 or more improvement on the vector quantization algorithm that is used heavily in information theory and classification problems.

### *More with printf*

MSVC users will actually see a bit of compiler output that `distutils` does not suppress the first time the code executes:

```
>>> weave.inline(r'printf("%d\n",a);', ['a'])
sc_e013937dbc8c647ac62438874e5795131.cpp
  Creating library C:\DOCUME~1\eric\LOCALS~1\Temp\python21_compiled\temp
  \Release\sc_e013937dbc8c647ac62438874e5795131.lib and
  object C:\DOCUME~1\eric\LOCALS~1\Temp\python21_compiled\temp\Release\sc_e013937dbc8c647ac62438874e5795131.obj
1
```

Nothing bad is happening, its just a bit annoying. \* Anyone know how to turn this off?\*

This example also demonstrates using 'raw strings'. The `r` preceding the code string in the last example denotes that this is a 'raw string'. In raw strings, the backslash character is not interpreted as an escape character, and so it isn't necessary to use a double backslash to indicate that the 'n' is meant to be interpreted in the C `printf` statement instead of by Python. If your C code contains a lot of strings and control characters, raw strings might make things easier. Most of the time, however, standard strings work just as well.

The `printf` statement in these examples is formatted to print out integers. What happens if `a` is a string? `inline` will happily, compile a new version of the code to accept strings as input, and execute the code. The result?

```
>>> a = 'string'
>>> weave.inline(r'printf("%d\n",a);', ['a'])
32956972
```

In this case, the result is non-sensical, but also non-fatal. In other situations, it might produce a compile time error because `a` is required to be an integer at some point in the code, or it could produce a segmentation fault. Its possible to protect against passing `inline` arguments of the wrong data type by using asserts in Python.

```
>>> a = 'string'
>>> def protected_printf(a):
...     assert(type(a) == type(1))
...     weave.inline(r'printf("%d\n",a);', ['a'])
>>> protected_printf(1)
1
>>> protected_printf('string')
AssertionError...
```

For printing strings, the format statement needs to be changed. Also, `weave` doesn't convert strings to `char*`. Instead it uses `CXX Py::String` type, so you have to do a little more work. Here we convert it to a `C++ std::string` and then ask for the `char*` version.

```
>>> a = 'string'
>>> weave.inline(r'printf("%s\n",std::string(a).c_str());', ['a'])
string
```

---

### XXX

This is a little convoluted. Perhaps strings should convert to `std::string` objects instead of `CXX` objects. Or maybe to `char*`.

---

As in this case, `C/C++` code fragments often have to change to accept different types. For the given printing task, however, `C++` streams provide a way of a single statement that works for integers and strings. By default, the stream objects live in the `std` (standard) namespace and thus require the use of `std::`.

```
>>> weave.inline('std::cout << a << std::endl;', ['a'])
1
>>> a = 'string'
>>> weave.inline('std::cout << a << std::endl;', ['a'])
string
```

Examples using `printf` and `cout` are included in `examples/print_example.py`.

### *More examples*

This section shows several more advanced uses of `inline`. It includes a few algorithms from the [Python Cookbook](#) that have been re-written in inline C to improve speed as well as a couple examples using `NumPy` and `wxPython`.

**Binary search** Lets look at the example of searching a sorted list of integers for a value. For inspiration, we'll use Kalle Svensson's `binary_search()` algorithm from the [Python Cookbook](#). His recipe follows:

```
def binary_search(seq, t):
    min = 0; max = len(seq) - 1
    while 1:
        if max < min:
            return -1
```

```

m = (min + max) / 2
if seq[m] < t:
    min = m + 1
elif seq[m] > t:
    max = m - 1
else:
    return m

```

This Python version works for arbitrary Python data types. The C version below is specialized to handle integer values. There is a little type checking done in Python to assure that we're working with the correct data types before heading into C. The variables `seq` and `t` don't need to be declared because `weave` handles converting and declaring them in the C code. All other temporary variables such as `min`, `max`, etc. must be declared – it is C after all. Here's the new mixed Python/C function:

```

def c_int_binary_search(seq,t):
    # do a little type checking in Python
    assert (type(t) == type(1))
    assert (type(seq) == type([]))

    # now the C code
    code = """
        #line 29 "binary_search.py"
        int val, m, min = 0;
        int max = seq.length() - 1;
        PyObject *py_val;
        for(;;)
        {
            if (max < min )
            {
                return_val = Py::new_reference_to(Py::Int(-1));
                break;
            }
            m = (min + max) /2;
            val = py_to_int(PyList_GetItem(seq.ptr(),m), "val");
            if (val < t)
                min = m + 1;
            else if (val > t)
                max = m - 1;
            else
            {
                return_val = Py::new_reference_to(Py::Int(m));
                break;
            }
        }
        """
    return inline(code, ['seq', 't'])

```

We have two variables `seq` and `t` passed in. `t` is guaranteed (by the `assert`) to be an integer. Python integers are converted to C `int` types in the transition from Python to C. `seq` is a Python list. By default, it is translated to a CXX list object. Full documentation for the CXX library can be found at its [website](#). The basics are that the CXX provides C++ class equivalents for Python objects that simplify, or at least object orientify, working with Python objects in C/C++. For example, `seq.length()` returns the length of the list. A little more about CXX and its class methods, etc. is in the *Type Conversions* section.

**Note:** CXX uses templates and therefore may be a little less portable than another alternative by Gordan McMillan called SCXX which was inspired by CXX. It doesn't use templates so it should compile faster and be more portable. SCXX has a few less features, but it appears to me that it would mesh with the needs of `weave` quite well. Hopefully `xxx_spec` files will be written for SCXX in the future, and we'll be able to compare on a more empirical basis. Both

sets of spec files will probably stick around, it just a question of which becomes the default.

---

Most of the algorithm above looks similar in C to the original Python code. There are two main differences. The first is the setting of `return_val` instead of directly returning from the C code with a `return` statement. `return_val` is an automatically defined variable of type `PyObject*` that is returned from the C code back to Python. You'll have to handle reference counting issues when setting this variable. In this example, CXX classes and functions handle the dirty work. All CXX functions and classes live in the namespace `Py::`. The following code converts the integer `m` to a CXX `Int()` object and then to a `PyObject*` with an incremented reference count using `Py::new_reference_to()`.

```
return_val = Py::new_reference_to(Py::Int(m));
```

The second big differences shows up in the retrieval of integer values from the Python list. The simple Python `seq[i]` call balloons into a C Python API call to grab the value out of the list and then a separate call to `py_to_int()` that converts the `PyObject*` to an integer. `py_to_int()` includes both a NULL check and a `PyInt_Check()` call as well as the conversion call. If either of the checks fail, an exception is raised. The entire C++ code block is executed within a `try/catch` block that handles exceptions much like Python does. This removes the need for most error checking code.

It is worth note that CXX lists do have indexing operators that result in code that looks much like Python. However, the overhead in using them appears to be relatively high, so the standard Python API was used on the `seq.ptr()` which is the underlying `PyObject*` of the List object.

The `#line` directive that is the first line of the C code block isn't necessary, but it's nice for debugging. If the compilation fails because of the syntax error in the code, the error will be reported as an error in the Python file "binary\_search.py" with an offset from the given line number (29 here).

So what was all our effort worth in terms of efficiency? Well not a lot in this case. The examples/binary\_search.py file runs both Python and C versions of the functions As well as using the standard `bisect` module. If we run it on a 1 million element list and run the search 3000 times (for 0- 2999), here are the results we get:

```
C:\home\ej\wrk\scipy\weave\examples> python binary_search.py
Binary search for 3000 items in 1000000 length list of integers:
speed in python: 0.159999966621
speed of bisect: 0.121000051498
speed up: 1.32
speed in c: 0.110000014305
speed up: 1.45
speed in c(no asserts): 0.0900000333786
speed up: 1.78
```

So, we get roughly a 50-75% improvement depending on whether we use the Python asserts in our C version. If we move down to searching a 10000 element list, the advantage evaporates. Even smaller lists might result in the Python version being faster. I'd like to say that moving to NumPy lists (and getting rid of the `GetItem()` call) offers a substantial speed up, but my preliminary efforts didn't produce one. I think the  $\log(N)$  algorithm is to blame. Because the algorithm is nice, there just isn't much time spent computing things, so moving to C isn't that big of a win. If there are ways to reduce conversion overhead of values, this may improve the C/Python speed up. Anyone have other explanations or faster code, please let me know.

**Dictionary Sort** The demo in examples/dict\_sort.py is another example from the Python CookBook. [This submission](#), by Alex Martelli, demonstrates how to return the values from a dictionary sorted by their keys:

```
def sortedDictValues3(adict):
    keys = adict.keys()
    keys.sort()
    return map(adict.get, keys)
```

Alex provides 3 algorithms and this is the 3rd and fastest of the set. The C version of this same algorithm follows:

```
def c_sort(adict):
    assert(type(adict) == type({}))
    code = """
#line 21 "dict_sort.py"
Py::List keys = adict.keys();
Py::List items(keys.length()); keys.sort();
PyObject* item = NULL;
for(int i = 0; i < keys.length();i++)
{
    item = PyList_GET_ITEM(keys.ptr(),i);
    item = PyDict_GetItem(adict.ptr(),item);
    Py_XINCRREF(item);
    PyList_SetItem(items.ptr(),i,item);
}
return_val = Py::new_reference_to(items);
"""
    return inline_tools.inline(code,['adict'],verbose=1)
```

Like the original Python function, the C++ version can handle any Python dictionary regardless of the key/value pair types. It uses CXX objects for the most part to declare python types in C++, but uses Python API calls to manipulate their contents. Again, this choice is made for speed. The C++ version, while more complicated, is about a factor of 2 faster than Python.

```
C:\home\ej\wrk\scipy\weave\examples> python dict_sort.py
Dict sort of 1000 items for 300 iterations:
  speed in python: 0.319999933243
[0, 1, 2, 3, 4]
  speed in c: 0.151000022888
  speed up: 2.12
[0, 1, 2, 3, 4]
```

**NumPy – cast/copy/transpose** CastCopyTranspose is a function called quite heavily by Linear Algebra routines in the NumPy library. Its needed in part because of the row-major memory layout of multi-dimensional Python (and C) arrays vs. the col-major order of the underlying Fortran algorithms. For small matrices (say 100x100 or less), a significant portion of the common routines such as LU decomposition or singular value decomposition are spent in this setup routine. This shouldn't happen. Here is the Python version of the function using standard NumPy operations.

```
def _castCopyAndTranspose(type, array):
    if a.typecode() == type:
        cast_array = copy.copy(NumPy.transpose(a))
    else:
        cast_array = copy.copy(NumPy.transpose(a).astype(type))
    return cast_array
```

And the following is a inline C version of the same function:

```
from weave.blitz_tools import blitz_type_factories
from weave import scalar_spec
from weave import inline
def _cast_copy_transpose(type,a_2d):
    assert(len(shape(a_2d)) == 2)
    new_array = zeros(shape(a_2d),type)
    NumPy_type = scalar_spec.NumPy_to_blitz_type_mapping[type]
    code = \
    """
    for(int i = 0;i < _Na_2d[0]; i++)
```

```

        for(int j = 0; j < _Na_2d[1]; j++)
            new_array(i,j) = (%s) a_2d(j,i);
    """ % NumPy_type
    inline(code, ['new_array', 'a_2d'],
            type_factories = blitz_type_factories, compiler='gcc')
    return new_array
    
```

This example uses blitz++ arrays instead of the standard representation of NumPy arrays so that indexing is simpler to write. This is accomplished by passing in the blitz++ “type factories” to override the standard Python to C++ type conversions. Blitz++ arrays allow you to write clean, fast code, but they also are sloooow to compile (20 seconds or more for this snippet). This is why they aren’t the default type used for Numeric arrays (and also because most compilers can’t compile blitz arrays...). `inline()` is also forced to use ‘gcc’ as the compiler because the default compiler on Windows (MSVC) will not compile blitz code. (‘gcc’ I think will use the standard compiler on Unix machine instead of explicitly forcing gcc (check this)) Comparisons of the Python vs inline C++ code show a factor of 3 speed up. Also shown are the results of an “inplace” transpose routine that can be used if the output of the linear algebra routine can overwrite the original matrix (this is often appropriate). This provides another factor of 2 improvement.

```

#C:\home\ej\wrk\scipy\weave\examples> python cast_copy_transpose.py
# Cast/Copy/Transposing (150,150)array 1 times
# speed in python: 0.870999932289
# speed in c: 0.25
# speed up: 3.48
# inplace transpose c: 0.129999995232
# speed up: 6.70
    
```

**wxPython** `inline` knows how to handle wxPython objects. That’s nice in and of itself, but it also demonstrates that the type conversion mechanism is reasonably flexible. Chances are, it won’t take a ton of effort to support special types you might have. The examples/wx\_example.py borrows the scrolled window example from the wxPython demo, accept that it mixes inline C code in the middle of the drawing function.

```
def DoDrawing(self, dc):
```

```

    red = wxNamedColour("RED");
    blue = wxNamedColour("BLUE");
    grey_brush = wxLIGHT_GREY_BRUSH;
    code = \
    """
    #line 108 "wx_example.py"
    dc->BeginDrawing();
    dc->SetPen(wxPen(*red, 4, wxSOLID));
    dc->DrawRectangle(5, 5, 50, 50);
    dc->SetBrush(*grey_brush);
    dc->SetPen(wxPen(*blue, 4, wxSOLID));
    dc->DrawRectangle(15, 15, 50, 50);
    """
    inline(code, ['dc', 'red', 'blue', 'grey_brush'])

    dc.SetFont(wxFont(14, wxSWISS, wxNORMAL, wxNORMAL))
    dc.SetTextForeground(wxColour(0xFF, 0x20, 0xFF))
    te = dc.GetTextExtent("Hello World")
    dc.DrawText("Hello World", 60, 65)

    dc.SetPen(wxPen(wxNamedColour('VIOLET'), 4))
    dc.DrawLine(5, 65+te[1], 60+te[0], 65+te[1])
    ...
    
```

Here, some of the Python calls to wx objects were just converted to C++ calls. There isn't any benefit, it just demonstrates the capabilities. You might want to use this if you have a computationally intensive loop in your drawing code that you want to speed up. On windows, you'll have to use the MSVC compiler if you use the standard wxPython DLLs distributed by Robin Dunn. That's because MSVC and gcc, while binary compatible in C, are not binary compatible for C++. In fact, it's probably best, no matter what platform you're on, to specify that `inline` use the same compiler that was used to build wxPython to be on the safe side. There isn't currently a way to learn this info from the library – you just have to know. Also, at least on the windows platform, you'll need to install the wxWindows libraries and link to them. I think there is a way around this, but I haven't found it yet – I get some linking errors dealing with wxString. One final note. You'll probably have to tweak `weave/wx_spec.py` or `weave/wx_info.py` for your machine's configuration to point at the correct directories etc. There. That should sufficiently scare people into not even looking at this... :)

### Keyword Option

The basic definition of the `inline()` function has a slew of optional variables. It also takes keyword arguments that are passed to `distutils` as compiler options. The following is a formatted cut/paste of the argument section of `inline`'s doc-string. It explains all of the variables. Some examples using various options will follow.

```
def inline(code, arg_names, local_dict = None, global_dict = None,
          force = 0,
          compiler='',
          verbose = 0,
          support_code = None,
          customize=None,
          type_factories = None,
          auto_downcast=1,
          **kw) :
```

`inline` has quite a few options as listed below. Also, the keyword arguments for `distutils` extension modules are accepted to specify extra information needed for compiling.

### Inline Arguments

`code` string. A string of valid C++ code. It should not specify a return statement. Instead it should assign results that need to be returned to Python in the `return_val`. `arg_names` list of strings. A list of Python variable names that should be transferred from Python into the C/C++ code. `local_dict` optional. dictionary. If specified, it is a dictionary of values that should be used as the local scope for the C/C++ code. If `local_dict` is not specified the local dictionary of the calling function is used. `global_dict` optional. dictionary. If specified, it is a dictionary of values that should be used as the global scope for the C/C++ code. If `global_dict` is not specified the global dictionary of the calling function is used. `force` optional. 0 or 1. default 0. If 1, the C++ code is compiled every time `inline` is called. This is really only useful for debugging, and probably only useful if you're editing `support_code` a lot. `compiler` optional. string. The name of compiler to use when compiling. On windows, it understands 'msvc' and 'gcc' as well as all the compiler names understood by `distutils`. On Unix, it'll only understand the values understood by `distutils`. (I should add 'gcc' though to this).

On windows, the compiler defaults to the Microsoft C++ compiler. If this isn't available, it looks for `mingw32` (the gcc compiler).

On Unix, it'll probably use the same compiler that was used when compiling Python. Cygwin's behavior should be similar.

`verbose` optional. 0, 1, or 2. default 0. Specifies how much information is printed during the compile phase of inlining code. 0 is silent (except on windows with `msvc` where it still prints some garbage). 1 informs you when compiling starts, finishes, and how long it took. 2 prints out the command lines for the compilation process and can be useful if you're having problems getting code to work. Its handy for finding the name of the `.cpp` file if you need to examine it. `verbose` has no affect if the compilation isn't necessary. `support_code` optional. string. A string of valid C++ code declaring extra code that might be needed by your compiled function. This could be declarations of functions, classes, or structures. `customize` optional. `base_info.custom_info` object. An alternative way to specify `support_code`,

headers, etc. needed by the function see the `weave.base_info` module for more details. (not sure this'll be used much). `type_factories` optional. list of type specification factories. These guys are what convert Python data types to C/C++ data types. If you'd like to use a different set of type conversions than the default, specify them here. Look in the type conversions section of the main documentation for examples. `auto_downcast` optional. 0 or 1. default 1. This only affects functions that have Numeric arrays as input variables. Setting this to 1 will cause all floating point values to be cast as float instead of double if all the NumPy arrays are of type float. If even one of the arrays has type double or double complex, all variables maintain there standard types.

*Distutils keywords*

`inline()` also accepts a number of `distutils` keywords for controlling how the code is compiled. The following descriptions have been copied from Greg Ward's `distutils.extension.Extension` class doc-strings for convenience: `sources` [string] list of source filenames, relative to the distribution root (where the setup script lives), in Unix form (slash-separated) for portability. Source files may be C, C++, SWIG (.i), platform-specific resource files, or whatever else is recognized by the "build\_ext" command as source for a Python extension. Note: The `module_path` file is always appended to the front of this list `include_dirs` [string] list of directories to search for C/C++ header files (in Unix form for portability) `define_macros` [(name : string, value : string|None)] list of macros to define; each macro is defined using a 2-tuple, where 'value' is either the string to define it to or None to define it without a particular value (equivalent of "#define FOO" in source or -DFOO on Unix C compiler command line) `undef_macros` [string] list of macros to undefine explicitly `library_dirs` [string] list of directories to search for C/C++ libraries at link time `libraries` [string] list of library names (not filenames or paths) to link against `runtime_library_dirs` [string] list of directories to search for C/C++ libraries at run time (for shared extensions, this is when the extension is loaded) `extra_objects` [string] list of extra files to link with (eg. object files not implied by 'sources', static library that must be explicitly specified, binary resource files, etc.) `extra_compile_args` [string] any extra platform- and compiler-specific information to use when compiling the source files in 'sources'. For platforms and compilers where "command line" makes sense, this is typically a list of command-line arguments, but for other platforms it could be anything. `extra_link_args` [string] any extra platform- and compiler-specific information to use when linking object files together to create the extension (or to create a new static Python interpreter). Similar interpretation as for 'extra\_compile\_args'. `export_symbols` [string] list of symbols to be exported from a shared extension. Not used on all platforms, and not generally necessary for Python extensions, which typically export exactly one symbol: "init" + `extension_name`.

**Keyword Option Examples** We'll walk through several examples here to demonstrate the behavior of `inline` and also how the various arguments are used. In the simplest (most) cases, `code` and `arg_names` are the only arguments that need to be specified. Here's a simple example run on Windows machine that has Microsoft VC++ installed.

```
>>> from weave import inline
>>> a = 'string'
>>> code = """
...     int l = a.length();
...     return_val = Py::new_reference_to(Py::Int(l));
...     """
>>> inline(code, ['a'])
sc_86e98826b65b047ffd2cd5f479c627f12.cpp
Creating
  library C:\DOCUME~1\eric\LOCALS~1\Temp\python21_compiled\temp\Release\sc_86e98826b65b047ffd2cd5f479c627f12.lib
and object C:\DOCUME~1\eric\LOCALS~1\Temp\python21_compiled\temp\Release\sc_86e98826b65b047ffd2cd5f479c627f12.exp
6
>>> inline(code, ['a'])
6
```

When `inline` is first run, you'll notice that pause and some trash printed to the screen. The "trash" is actually part of the compiler's output that `distutils` does not suppress. The name of the extension file, `sc_bighonkingnumber.cpp`, is generated from the SHA-256 check sum of the C/C++ code fragment. On Unix or windows machines with only `gcc` installed, the trash will not appear. On the second call, the code fragment is not



compiled since it already exists, and only the answer is returned. Now kill the interpreter and restart, and run the same code with a different string.

```
>>> from weave import inline
>>> a = 'a longer string'
>>> code = """
...     int l = a.length();
...     return_val = Py::new_reference_to(Py::Int(l));
...     """
>>> inline(code, ['a'])
15
```

Notice this time, `inline()` did not recompile the code because it found the compiled function in the persistent catalog of functions. There is a short pause as it looks up and loads the function, but it is much shorter than compiling would require.

You can specify the local and global dictionaries if you'd like (much like `exec` or `eval()` in Python), but if they aren't specified, the "expected" ones are used – i.e. the ones from the function that called `inline()`. This is accomplished through a little call frame trickery. Here is an example where the `local_dict` is specified using the same code example from above:

```
>>> a = 'a longer string'
>>> b = 'an even longer string'
>>> my_dict = {'a':b}
>>> inline(code, ['a'])
15
>>> inline(code, ['a'], my_dict)
21
```

Every time the `code` is changed, `inline` does a recompile. However, changing any of the other options in `inline` does not force a recompile. The `force` option was added so that one could force a recompile when tinkering with other variables. In practice, it is just as easy to change the `code` by a single character (like adding a space some place) to force the recompile.

---

**Note:** It also might be nice to add some methods for purging the cache and on disk catalogs.

---

I use `verbose` sometimes for debugging. When set to 2, it'll output all the information (including the name of the `.cpp` file) that you'd expect from running a `make` file. This is nice if you need to examine the generated code to see where things are going haywire. Note that error messages from failed compiles are printed to the screen even if `verbose` is set to 0.

The following example demonstrates using `gcc` instead of the standard `msvc` compiler on windows using same code fragment as above. Because the example has already been compiled, the `force=1` flag is needed to make `inline()` ignore the previously compiled version and recompile using `gcc`. The `verbose` flag is added to show what is printed out:

```
>>>inline(code, ['a'], compiler='gcc', verbose=2, force=1)
running build_ext
building 'sc_86e98826b65b047ffd2cd5f479c627f13' extension
c:\gcc-2.95.2\bin\g++.exe -mno-cygwin -mdll -O2 -w -Wstrict-prototypes -IC:
\home\ej\wrk\scipy\weave -IC:\Python21\Include -c C:\DOCUME~1\eric\LOCAL
S~1\Temp\python21_compiled\sc_86e98826b65b047ffd2cd5f479c627f13.cpp
-o C:\DOCUME~1\eric\LOCALS~1\Temp\python21_compiled\temp\Release\sc_86e98826b65b047ffd2cd5f479c627f13
skipping C:\home\ej\wrk\scipy\weave\CXX\cxxextensions.c
(C:\DOCUME~1\eric\LOCALS~1\Temp\python21_compiled\temp\Release\cxxextensions.o up-to-date)
skipping C:\home\ej\wrk\scipy\weave\CXX\cxxsupport.cxx
(C:\DOCUME~1\eric\LOCALS~1\Temp\python21_compiled\temp\Release\cxxsupport.o up-to-date)
skipping C:\home\ej\wrk\scipy\weave\CXX\IndirectPythonInterface.cxx
(C:\DOCUME~1\eric\LOCALS~1\Temp\python21_compiled\temp\Release\indirectpythoninterface.o up-to-date)
```

```
skipping C:\home\ej\wrk\scipy\weave\CXX\cxx_extensions.cxx
(C:\DOCUME~1\eric\LOCALS~1\Temp\python21_compiled\temp\Release\cxx_extensions.o
up-to-date)
writing C:\DOCUME~1\eric\LOCALS~1\Temp\python21_compiled\temp\Release\sc_86e98826b65b047ffd2cd5f479c627f13.d
c:\gcc-2.95.2\bin\dlwrap.exe --driver-name g++ -mno-cygwin
-mdll -static --output-lib
C:\DOCUME~1\eric\LOCALS~1\Temp\python21_compiled\temp\Release\libsc_86e98826b65b047ffd2cd5f479c627f13.d
C:\DOCUME~1\eric\LOCALS~1\Temp\python21_compiled\temp\Release\sc_86e98826b65b047ffd2cd5f479c627f13.d
-sC:\DOCUME~1\eric\LOCALS~1\Temp\python21_compiled\temp\Release\sc_86e98826b65b047ffd2cd5f479c627f13.d
C:\DOCUME~1\eric\LOCALS~1\Temp\python21_compiled\temp\Release\cxxextensions.o
C:\DOCUME~1\eric\LOCALS~1\Temp\python21_compiled\temp\Release\cxxsupport.o
C:\DOCUME~1\eric\LOCALS~1\Temp\python21_compiled\temp\Release\indirectpythoninterface.o
C:\DOCUME~1\eric\LOCALS~1\Temp\python21_compiled\temp\Release\cxx_extensions.o -LC:\Python21\libs
-lpython21 -o
C:\DOCUME~1\eric\LOCALS~1\Temp\python21_compiled\sc_86e98826b65b047ffd2cd5f479c627f13.pyd
15
```

That's quite a bit of output. `verbose=1` just prints the compile time.

```
>>>inline(code,['a'],compiler='gcc',verbose=1,force=1)
Compiling code...
finished compiling (sec): 6.00800001621
15
```

---

**Note:** I've only used the `compiler` option for switching between 'msvc' and 'gcc' on windows. It may have use on Unix also, but I don't know yet.

---

The `support_code` argument is likely to be used a lot. It allows you to specify extra code fragments such as function, structure or class definitions that you want to use in the `code` string. Note that changes to `support_code` do *not* force a recompile. The catalog only relies on `code` (for performance reasons) to determine whether recompiling is necessary. So, if you make a change to `support_code`, you'll need to alter `code` in some way or use the `force` argument to get the code to recompile. I usually just add some innocuous whitespace to the end of one of the lines in `code` somewhere. Here's an example of defining a separate method for calculating the string length:

```
>>> from weave import inline
>>> a = 'a longer string'
>>> support_code = """
...     PyObject* length(Py::String a)
...     {
...         int l = a.length();
...         return Py::new_reference_to(Py::Int(l));
...     }
...     """
>>> inline("return_val = length(a);",['a'],
...       support_code = support_code)
15
```

`customize` is a left over from a previous way of specifying compiler options. It is a `custom_info` object that can specify quite a bit of information about how a file is compiled. These `info` objects are the standard way of defining compile information for type conversion classes. However, I don't think they are as handy here, especially since we've exposed all the keyword arguments that `distutils` can handle. Between these keywords, and the `support_code` option, I think `customize` may be obsolete. We'll see if anyone cares to use it. If not, it'll get axed in the next version.

The `type_factories` variable is important to people who want to customize the way arguments are converted from Python to C. We'll talk about this in the next chapter `xx` of this document when we discuss type conversions.

`auto_downcast` handles one of the big type conversion issues that is common when using NumPy arrays in con-

junction with Python scalar values. If you have an array of single precision values and multiply that array by a Python scalar, the result is upcast to a double precision array because the scalar value is double precision. This is not usually the desired behavior because it can double your memory usage. `auto_downcast` goes some distance towards changing the casting precedence of arrays and scalars. If your only using single precision arrays, it will automatically downcast all scalar values from double to single precision when they are passed into the C++ code. This is the default behavior. If you want all values to keep their default type, set `auto_downcast` to 0.

**Returning Values** Python variables in the local and global scope transfer seamlessly from Python into the C++ snippets. And, if `inline` were to completely live up to its name, any modifications to variables in the C++ code would be reflected in the Python variables when control was passed back to Python. For example, the desired behavior would be something like:

```
# THIS DOES NOT WORK
>>> a = 1
>>> weave.inline("a++;", ['a'])
>>> a
2
```

Instead you get:

```
>>> a = 1
>>> weave.inline("a++;", ['a'])
>>> a
1
```

Variables are passed into C++ as if you are calling a Python function. Python’s calling convention is sometimes called “pass by assignment”. This means it’s as if a `c_a = a` assignment is made right before `inline` call is made and the `c_a` variable is used within the C++ code. Thus, any changes made to `c_a` are not reflected in Python’s `a` variable. Things do get a little more confusing, however, when looking at variables with mutable types. Changes made in C++ to the contents of mutable types *are* reflected in the Python variables.

```
>>> a = [1, 2]
>>> weave.inline("PyList_SetItem(a.ptr(), 0, PyInt_FromLong(3));", ['a'])
>>> print a
[3, 2]
```

So modifications to the contents of mutable types in C++ are seen when control is returned to Python. Modifications to immutable types such as tuples, strings, and numbers do not alter the Python variables. If you need to make changes to an immutable variable, you’ll need to assign the new value to the “magic” variable `return_val` in C++. This value is returned by the `inline()` function:

```
>>> a = 1
>>> a = weave.inline("return_val = Py::new_reference_to(Py::Int(a+1));", ['a'])
>>> a
2
```

The `return_val` variable can also be used to return newly created values. This is possible by returning a tuple. The following trivial example illustrates how this can be done:

```
# python version
def multi_return():
    return 1, '2nd'

# C version.
def c_multi_return():
    code = """
        py::tuple results(2);
        results[0] = 1;
    """
```

```

        results[1] = "2nd";
        return_val = results;
    """
    return inline_tools.inline(code)

```

The example is available in `examples/tuple_return.py`. It also has the dubious honor of demonstrating how much `inline()` can slow things down. The C version here is about 7-10 times slower than the Python version. Of course, something so trivial has no reason to be written in C anyway.

**The issue with `locals()`** `inline` passes the `locals()` and `globals()` dictionaries from Python into the C++ function from the calling function. It extracts the variables that are used in the C++ code from these dictionaries, converts them to C++ variables, and then calculates using them. It seems like it would be trivial, then, after the calculations were finished to then insert the new values back into the `locals()` and `globals()` dictionaries so that the modified values were reflected in Python. Unfortunately, as pointed out by the Python manual, the `locals()` dictionary is not writable.

I suspect `locals()` is not writable because there are some optimizations done to speed lookups of the local namespace. I'm guessing local lookups don't always look at a dictionary to find values. Can someone "in the know" confirm or correct this? Another thing I'd like to know is whether there is a way to write to the local namespace of another stack frame from C/C++. If so, it would be possible to have some clean up code in compiled functions that wrote final values of variables in C++ back to the correct Python stack frame. I think this goes a long way toward making `inline` truly live up to its name. I don't think we'll get to the point of creating variables in Python for variables created in C – although I suppose with a C/C++ parser you could do that also.

**A quick look at the code** `weave` generates a C++ file holding an extension function for each `inline` code snippet. These file names are generated using from the SHA-256 signature of the code snippet and saved to a location specified by the `PYTHONCOMPILED` environment variable (discussed later). The `cpp` files are generally about 200-400 lines long and include quite a few functions to support type conversions, etc. However, the actual compiled function is pretty simple. Below is the familiar `printf` example:

```

>>> import weave
>>> a = 1
>>> weave.inline('printf("%d\\n",a);', ['a'])
1

```

And here is the extension function generated by `inline`:

```

static PyObject* compiled_func(PyObject*self, PyObject* args)
{
    py::object return_val;
    int exception_occurred = 0;
    PyObject *py__locals = NULL;
    PyObject *py__globals = NULL;
    PyObject *py_a;
    py_a = NULL;

    if(!PyArg_ParseTuple(args, "OO:compiled_func", &py__locals, &py__globals))
        return NULL;
    try
    {
        PyObject* raw_locals = py_to_raw_dict(py__locals, "_locals");
        PyObject* raw_globals = py_to_raw_dict(py__globals, "_globals");
        /* argument conversion code */
        py_a = get_variable("a", raw_locals, raw_globals);
        int a = convert_to_int(py_a, "a");
        /* inline code */

```

```

    /* NDARRAY API VERSION 90907 */
    printf("%d\n",a);    /*I would like to fill in changed locals and globals here...*/
}
catch(...)
{
    return_val = py::object();
    exception_occurred = 1;
}
/* cleanup code */
if(!(PyObject*)return_val && !exception_occurred)
{
    return_val = Py_None;
}
return return_val.disown();
}

```

Every inline function takes exactly two arguments – the local and global dictionaries for the current scope. All variable values are looked up out of these dictionaries. The lookups, along with all `inline` code execution, are done within a C++ `try` block. If the variables aren't found, or there is an error converting a Python variable to the appropriate type in C++, an exception is raised. The C++ exception is automatically converted to a Python exception by SCXX and returned to Python. The `py_to_int()` function illustrates how the conversions and exception handling works. `py_to_int` first checks that the given `PyObject*` pointer is not NULL and is a Python integer. If all is well, it calls the Python API to convert the value to an `int`. Otherwise, it calls `handle_bad_type()` which gathers information about what went wrong and then raises a SCXX `TypeError` which returns to Python as a `TypeError`.

```

int py_to_int(PyObject* py_obj, char* name)
{
    if (!py_obj || !PyInt_Check(py_obj))
        handle_bad_type(py_obj, "int", name);
    return (int) PyInt_AsLong(py_obj);
}

void handle_bad_type(PyObject* py_obj, char* good_type, char* var_name)
{
    char msg[500];
    sprintf(msg, "received '%s' type instead of '%s' for variable '%s'",
            find_type(py_obj), good_type, var_name);
    throw Py::TypeError(msg);
}

char* find_type(PyObject* py_obj)
{
    if(py_obj == NULL) return "C NULL value";
    if(PyCallable_Check(py_obj)) return "callable";
    if(PyString_Check(py_obj)) return "string";
    if(PyInt_Check(py_obj)) return "int";
    if(PyFloat_Check(py_obj)) return "float";
    if(PyDict_Check(py_obj)) return "dict";
    if(PyList_Check(py_obj)) return "list";
    if(PyTuple_Check(py_obj)) return "tuple";
    if(PyFile_Check(py_obj)) return "file";
    if(PyModule_Check(py_obj)) return "module";

    //should probably do more interrogation (and thinking) on these.
    if(PyCallable_Check(py_obj) && PyInstance_Check(py_obj)) return "callable";
    if(PyInstance_Check(py_obj)) return "instance";
    if(PyCallable_Check(py_obj)) return "callable";
    return "unknown type";
}

```

}

Since the `inline` is also executed within the `try/catch` block, you can use CXX exceptions within your code. It is usually a bad idea to directly `return` from your code, even if an error occurs. This skips the clean up section of the extension function. In this simple example, there isn't any clean up code, but in more complicated examples, there may be some reference counting that needs to be taken care of here on converted variables. To avoid this, either uses exceptions or set `return_val` to `NULL` and use `if/then's` to skip code after errors.

### Technical Details

There are several main steps to using C/C++ code within Python:

1. Type conversion
2. Generating C/C++ code
3. Compile the code to an extension module
4. Catalog (and cache) the function for future use

Items 1 and 2 above are related, but most easily discussed separately. Type conversions are customizable by the user if needed. Understanding them is pretty important for anything beyond trivial uses of `inline`. Generating the C/C++ code is handled by `ext_function` and `ext_module` classes and `.`. For the most part, compiling the code is handled by `distutils`. Some customizations were needed, but they were relatively minor and do not require changes to `distutils` itself. Cataloging is pretty simple in concept, but surprisingly required the most code to implement (and still likely needs some work). So, this section covers items 1 and 4 from the list. Item 2 is covered later in the chapter covering the `ext_tools` module, and `distutils` is covered by a completely separate document xxx.

### Passing Variables in/out of the C/C++ code

---

**Note:** Passing variables into the C code is pretty straight forward, but there are subtleties to how variable modifications in C are returned to Python. see [Returning Values](#) for a more thorough discussion of this issue.

---

### Type Conversions

---

**Note:** Maybe `xxx_converter` instead of `xxx_specification` is a more descriptive name. Might change in future version?

---

By default, `inline()` makes the following type conversions between Python and C++ types.

Table 3.9: Default Data Type Conversions

Python	C++
<code>int</code>	<code>int</code>
<code>float</code>	<code>double</code>
<code>complex</code>	<code>std::complex</code>
<code>string</code>	<code>py::string</code>
<code>list</code>	<code>py::list</code>
<code>dict</code>	<code>py::dict</code>
<code>tuple</code>	<code>py::tuple</code>
<code>file</code>	<code>FILE*</code>
<code>callable</code>	<code>py::object</code>
<code>instance</code>	<code>py::object</code>
<code>numpy.ndarray</code>	<code>PyArrayObject*</code>
<code>wxXXX</code>	<code>wxXXX*</code>

The `Py::` namespace is defined by the SCXX library which has C++ class equivalents for many Python types. `std::` is the namespace of the standard library in C++.

---

**Note:**

- I haven't figured out how to handle `long int` yet (I think they are currently converted to `int` - - check this).
  - Hopefully VTK will be added to the list soon
- 

Python to C++ conversions fill in code in several locations in the generated `inline` extension function. Below is the basic template for the function. This is actually the exact code that is generated by calling `weave.inline("")`.

The `/* inline code */` section is filled with the code passed to the `inline()` function call. The `/*argument conversion code*/` and `/* cleanup code */` sections are filled with code that handles conversion from Python to C++ types and code that deallocates memory or manipulates reference counts before the function returns. The following sections demonstrate how these two areas are filled in by the default conversion methods. \* Note: I'm not sure I have reference counting correct on a few of these. The only thing I increase/decrease the ref count on is NumPy arrays. If you see an issue, please let me know.

**NumPy Argument Conversion** Integer, floating point, and complex arguments are handled in a very similar fashion. Consider the following inline function that has a single integer variable passed in:

```
>>> a = 1
>>> inline("", ['a'])
```

The argument conversion code inserted for `a` is:

```
/* argument conversion code */
int a = py_to_int (get_variable("a",raw_locals,raw_globals),"a");
```

`get_variable()` reads the variable `a` from the local and global namespaces. `py_to_int()` has the following form:

```
static int py_to_int(PyObject* py_obj, char* name)
{
    if (!py_obj || !PyInt_Check(py_obj))
        handle_bad_type(py_obj, "int", name);
    return (int) PyInt_AsLong(py_obj);
}
```

Similarly, the float and complex conversion routines look like:

```
static double py_to_float(PyObject* py_obj, char* name)
{
    if (!py_obj || !PyFloat_Check(py_obj))
        handle_bad_type(py_obj, "float", name);
    return PyFloat_AsDouble(py_obj);
}

static std::complex py_to_complex(PyObject* py_obj, char* name)
{
    if (!py_obj || !PyComplex_Check(py_obj))
        handle_bad_type(py_obj, "complex", name);
    return std::complex(PyComplex_RealAsDouble(py_obj),
                       PyComplex_ImagAsDouble(py_obj));
}
```

NumPy conversions do not require any clean up code.

**String, List, Tuple, and Dictionary Conversion** Strings, Lists, Tuples and Dictionary conversions are all converted to SCXX types by default. For the following code,

```
>>> a = [1]
>>> inline("", ['a'])
```

The argument conversion code inserted for a is:

```
/* argument conversion code */
Py::List a = py_to_list(get_variable("a",raw_locals,raw_globals),"a");
```

`get_variable()` reads the variable `a` from the local and global namespaces. `py_to_list()` and its friends have the following form:

```
static Py::List py_to_list(PyObject* py_obj, char* name)
{
    if (!py_obj || !PyList_Check(py_obj))
        handle_bad_type(py_obj, "list", name);
    return Py::List(py_obj);
}

static Py::String py_to_string(PyObject* py_obj, char* name)
{
    if (!PyString_Check(py_obj))
        handle_bad_type(py_obj, "string", name);
    return Py::String(py_obj);
}

static Py::Dict py_to_dict(PyObject* py_obj, char* name)
{
    if (!py_obj || !PyDict_Check(py_obj))
        handle_bad_type(py_obj, "dict", name);
    return Py::Dict(py_obj);
}

static Py::Tuple py_to_tuple(PyObject* py_obj, char* name)
{
    if (!py_obj || !PyTuple_Check(py_obj))
        handle_bad_type(py_obj, "tuple", name);
    return Py::Tuple(py_obj);
}
```

SCXX handles reference counts on for strings, lists, tuples, and dictionaries, so clean up code isn't necessary.

**File Conversion** For the following code,

```
>>> a = open("bob", 'w')
>>> inline("", ['a'])
```

The argument conversion code is:

```
/* argument conversion code */
PyObject* py_a = get_variable("a",raw_locals,raw_globals);
FILE* a = py_to_file(py_a, "a");
```

`get_variable()` reads the variable `a` from the local and global namespaces. `py_to_file()` converts `PyObject*` to a `FILE*` and increments the reference count of the `PyObject*`:



```
FILE* py_to_file(PyObject* py_obj, char* name)
{
    if (!py_obj || !PyFile_Check(py_obj))
        handle_bad_type(py_obj, "file", name);

    Py_INCREF(py_obj);
    return PyFile_AsFile(py_obj);
}
```

Because the `PyObject*` was incremented, the clean up code needs to decrement the counter

```
/* cleanup code */
Py_XDECREF(py_a);
```

Its important to understand that file conversion only works on actual files – i.e. ones created using the `open()` command in Python. It does not support converting arbitrary objects that support the file interface into C `FILE*` pointers. This can affect many things. For example, in initial `printf()` examples, one might be tempted to solve the problem of C and Python IDE's (PythonWin, PyCrust, etc.) writing to different `stdout` and `stderr` by using `fprintf()` and passing in `sys.stdout` and `sys.stderr`. For example, instead of

```
>>> weave.inline('printf("hello\\n");')
```

You might try:

```
>>> buf = sys.stdout
>>> weave.inline('fprintf(buf, "hello\\n");', ['buf'])
```

This will work as expected from a standard python interpreter, but in PythonWin, the following occurs:

```
>>> buf = sys.stdout
>>> weave.inline('fprintf(buf, "hello\\n");', ['buf'])
Traceback (most recent call last):
  File "", line 1, in ?
  File "C:\Python21\weave\inline_tools.py", line 315, in inline
    auto_downcast = auto_downcast,
  File "C:\Python21\weave\inline_tools.py", line 386, in compile_function
    type_factories = type_factories)
  File "C:\Python21\weave\ext_tools.py", line 197, in __init__
    auto_downcast, type_factories)
  File "C:\Python21\weave\ext_tools.py", line 390, in assign_variable_types
    raise TypeError, format_error_msg(errors)
TypeError: {'buf': "Unable to convert variable 'buf' to a C++ type."}
```

The traceback tells us that `inline()` was unable to convert 'buf' to a C++ type (If instance conversion was implemented, the error would have occurred at runtime instead). Why is this? Let's look at what the `buf` object really is:

```
>>> buf
pywin.framework.interact.InteractiveView instance at 00EAD014
```

PythonWin has reassigned `sys.stdout` to a special object that implements the Python file interface. This works great in Python, but since the special object doesn't have a `FILE*` pointer underlying it, `fprintf` doesn't know what to do with it (well this will be the problem when instance conversion is implemented...).

### Callable, Instance, and Module Conversion

**Note:** Need to look into how ref counts should be handled. Also, Instance and Module conversion are not currently implemented.

```
>>> def a():
    pass
>>> inline("", ['a'])
```

Callable and instance variables are converted to PyObject\*. Nothing is done to their reference counts.

```
/* argument conversion code */
PyObject* a = py_to_callable(get_variable("a", raw_locals, raw_globals), "a");
```

get\_variable() reads the variable a from the local and global namespaces. The py\_to\_callable() and py\_to\_instance() don't currently increment the ref count.

```
PyObject* py_to_callable(PyObject* py_obj, char* name)
{
    if (!py_obj || !PyCallable_Check(py_obj))
        handle_bad_type(py_obj, "callable", name);
    return py_obj;
}
```

```
PyObject* py_to_instance(PyObject* py_obj, char* name)
{
    if (!py_obj || !PyFile_Check(py_obj))
        handle_bad_type(py_obj, "instance", name);
    return py_obj;
}
```

There is no cleanup code for callables, modules, or instances.

**Customizing Conversions** Converting from Python to C++ types is handled by xxx\_specification classes. A type specification class actually serve in two related but different roles. The first is in determining whether a Python variable that needs to be converted should be represented by the given class. The second is as a code generator that generates C++ code needed to convert from Python to C++ types for a specific variable.

When

```
>>> a = 1
>>> weave.inline('printf("%d", a);', ['a'])
```

is called for the first time, the code snippet has to be compiled. In this process, the variable 'a' is tested against a list of type specifications (the default list is stored in weave/ext\_tools.py). The first specification in the list is used to represent the variable.

Examples of xxx\_specification are scattered throughout numerous "xxx\_spec.py" files in the weave package. Closely related to the xxx\_specification classes are yyy\_info classes. These classes contain compiler, header, and support code information necessary for including a certain set of capabilities (such as blitz++ or CXX support) in a compiled module. xxx\_specification classes have one or more yyy\_info classes associated with them. If you'd like to define your own set of type specifications, the current best route is to examine some of the existing spec and info files. Maybe looking over sequence\_spec.py and cxx\_info.py are a good place to start. After defining specification classes, you'll need to pass them into inline using the type\_factories argument. A lot of times you may just want to change how a specific variable type is represented. Say you'd rather have Python strings converted to std::string or maybe char\* instead of using the CXX string object, but would like all other type conversions to have default behavior. This requires that a new specification class that handles strings is written and then prepended to a list of the default type specifications. Since it is closer to the front of the list, it effectively overrides the default string specification. The following code demonstrates how this is done: ...

## The Catalog

`catalog.py` has a class called `catalog` that helps keep track of previously compiled functions. This prevents `inline()` and related functions from having to compile functions every time they are called. Instead, `catalog` will check an in memory cache to see if the function has already been loaded into python. If it hasn't, then it starts searching through persistent catalogs on disk to see if it finds an entry for the given function. By saving information about compiled functions to disk, it isn't necessary to re-compile functions every time you stop and restart the interpreter. Functions are compiled once and stored for future use.

When `inline(cpp_code)` is called the following things happen:

1. A fast local cache of functions is checked for the last function called for `cpp_code`. If an entry for `cpp_code` doesn't exist in the cache or the cached function call fails (perhaps because the function doesn't have compatible types) then the next step is to check the catalog.
2. The catalog class also keeps an in-memory cache with a list of all the functions compiled for `cpp_code`. If `cpp_code` has ever been called, then this cache will be present (loaded from disk). If the cache isn't present, then it is loaded from disk.

If the cache is present, each function in the cache is called until one is found that was compiled for the correct argument types. If none of the functions work, a new function is compiled with the given argument types. This function is written to the on-disk catalog as well as into the in-memory cache.

3. When a lookup for `cpp_code` fails, the catalog looks through the on-disk function catalogs for the entries. The `PYTHONCOMPILED` variable determines where to search for these catalogs and in what order. If `PYTHONCOMPILED` is not present several platform dependent locations are searched. All functions found for `cpp_code` in the path are loaded into the in-memory cache with functions found earlier in the search path closer to the front of the call list.

If the function isn't found in the on-disk catalog, then the function is compiled, written to the first writable directory in the `PYTHONCOMPILED` path, and also loaded into the in-memory cache.

**Function Storage** Function caches are stored as dictionaries where the key is the entire C++ code string and the value is either a single function (as in the "level 1" cache) or a list of functions (as in the main catalog cache). On disk catalogs are stored in the same manor using standard Python shelves.

Early on, there was a question as to whether md5 checksums of the C++ code strings should be used instead of the actual code strings. I think this is the route `inline` Perl took. Some (admittedly quick) tests of the md5 vs. the entire string showed that using the entire string was at least a factor of 3 or 4 faster for Python. I think this is because it is more time consuming to compute the md5 value than it is to do look-ups of long strings in the dictionary. Look at the `examples/md5_speed.py` file for the test run.

**Catalog search paths and the PYTHONCOMPILED variable** The default location for catalog files on Unix is `~/pythonXX_compiled` where `XX` is version of Python being used. If this directory doesn't exist, it is created the first time a catalog is used. The directory must be writable. If, for any reason it isn't, then the catalog attempts to create a directory based on your user id in the `/tmp` directory. The directory permissions are set so that only you have access to the directory. If this fails, I think you're out of luck. I don't think either of these should ever fail though. On Windows, a directory called `pythonXX_compiled` is created in the user's temporary directory.

The actual catalog file that lives in this directory is a Python shelf with a platform specific name such as `"nt21compiled_catalog"` so that multiple OSes can share the same file systems without trampling on each other. Along with the catalog file, the `.cpp` and `.so` or `.pyd` files created by `inline` will live in this directory. The catalog file simply contains keys which are the C++ code strings with values that are lists of functions. The function lists point at functions within these compiled modules. Each function in the lists executes the same C++ code string, but compiled for different input variables.

You can use the `PYTHONCOMPILED` environment variable to specify alternative locations for compiled functions. On Unix this is a colon (':') separated list of directories. On windows, it is a (;') separated list of directories. These

directories will be searched prior to the default directory for a compiled function catalog. Also, the first writable directory in the list is where all new compiled function catalogs, .cpp and .so or .pyd files are written. Relative directory paths ('.' and '..') should work fine in the PYTHONCOMPILED variable as should environment variables.

There is a “special” path variable called MODULE that can be placed in the PYTHONCOMPILED variable. It specifies that the compiled catalog should reside in the same directory as the module that called it. This is useful if an admin wants to build a lot of compiled functions during the build of a package and then install them in site-packages along with the package. User’s who specify MODULE in their PYTHONCOMPILED variable will have access to these compiled functions. Note, however, that if they call the function with a set of argument types that it hasn’t previously been built for, the new function will be stored in their default directory (or some other writable directory in the PYTHONCOMPILED path) because the user will not have write access to the site-packages directory.

An example of using the PYTHONCOMPILED path on bash follows:

```
PYTHONCOMPILED=MODULE:/some/path;export PYTHONCOMPILED;
```

If you are using python21 on linux, and the module bob.py in site-packages has a compiled function in it, then the catalog search order when calling that function for the first time in a python session would be:

```
/usr/lib/python21/site-packages/linuxpython_compiled
/some/path/linuxpython_compiled
~/python21_compiled/linuxpython_compiled
```

The default location is always included in the search path.

---

**Note:** hmmm. see a possible problem here. I should probably make a sub- directory such as /usr/lib/python21/site-packages/python21\_compiled/linuxpython\_compiled so that library files compiled with python21 are tried to link with python22 files in some strange scenarios. Need to check this.

---

The in-module cache (in `weave.inline_tools` reduces the overhead of calling inline functions by about a factor of 2. It can be reduced a little more for type loop calls where the same function is called over and over again if the cache was a single value instead of a dictionary, but the benefit is very small (less than 5%) and the utility is quite a bit less. So, we’ll stick with a dictionary as the cache.

## Blitz

---

**Note:** most of this section is lifted from old documentation. It should be pretty accurate, but there may be a few discrepancies.

---

`weave.blitz()` compiles NumPy Python expressions for fast execution. For most applications, compiled expressions should provide a factor of 2-10 speed-up over NumPy arrays. Using compiled expressions is meant to be as unobtrusive as possible and works much like python’s `exec` statement. As an example, the following code fragment takes a 5 point average of the 512x512 2d image, `b`, and stores it in array, `a`:

```
from scipy import * # or from NumPy import *
a = ones((512,512), Float64)
b = ones((512,512), Float64)
# ...do some stuff to fill in b...
# now average
a[1:-1,1:-1] = (b[1:-1,1:-1] + b[2:,1:-1] + b[:-2,1:-1] \
               + b[1:-1,2:] + b[1:-1,:-2]) / 5.
```

To compile the expression, convert the expression to a string by putting quotes around it and then use `weave.blitz`:

```
import weave
expr = "a[1:-1,1:-1] = (b[1:-1,1:-1] + b[2:,1:-1] + b[:-2,1:-1]) \
```

```
weave.blitz(expr)
    "+ b[1:-1,2:] + b[1:-1,:-2]) / 5."
```

The first time `weave.blitz` is run for a given expression and set of arguments, C++ code that accomplishes the exact same task as the Python expression is generated and compiled to an extension module. This can take up to a couple of minutes depending on the complexity of the function. Subsequent calls to the function are very fast. Furthermore, the generated module is saved between program executions so that the compilation is only done once for a given expression and associated set of array types. If the given expression is executed with a new set of array types, the code must be compiled again. This does not overwrite the previously compiled function – both of them are saved and available for execution.

The following table compares the run times for standard NumPy code and compiled code for the 5 point averaging.

Method	Run Time (seconds)	Standard NumPy	0.46349	blitz (1st time compiling)	78.95526	blitz (subsequent calls)	0.05843	(factor of 8 speedup)
--------	--------------------	----------------	---------	----------------------------	----------	--------------------------	---------	-----------------------

These numbers are for a 512x512 double precision image run on a 400 MHz Celeron processor under RedHat Linux 6.2.

Because of the slow compile times, its probably most effective to develop algorithms as you usually do using the capabilities of `scipy` or the NumPy module. Once the algorithm is perfected, put quotes around it and execute it using `weave.blitz`. This provides the standard rapid prototyping strengths of Python and results in algorithms that run close to that of hand coded C or Fortran.

### Requirements

Currently, the `weave.blitz` has only been tested under Linux with `gcc-2.95-3` and on Windows with `Mingw32 (2.95.2)`. Its compiler requirements are pretty heavy duty (see the [blitz++ home page](#)), so it won't work with just any compiler. Particularly `MSVC++` isn't up to snuff. A number of other compilers such as `KAI++` will also work, but my suspicions are that `gcc` will get the most use.

### Limitations

1. Currently, `weave.blitz` handles all standard mathematical operators except for the `**` power operator. The built-in trigonometric, `log`, `floor/ceil`, and `fabs` functions might work (but haven't been tested). It also handles all types of array indexing supported by the NumPy module. `numarray`'s NumPy compatible array indexing modes are likewise supported, but `numarray`'s enhanced (array based) indexing modes are not supported.

`weave.blitz` does not currently support operations that use array broadcasting, nor have any of the special purpose functions in NumPy such as `take`, `compress`, etc. been implemented. Note that there are no obvious reasons why most of this functionality cannot be added to `scipy.weave`, so it will likely trickle into future versions. Using `slice()` objects directly instead of `start:stop:step` is also not supported.

2. Currently Python only works on expressions that include assignment such as

```
>>> result = b + c + d
```

This means that the result array must exist before calling `weave.blitz`. Future versions will allow the following:

```
>>> result = weave.blitz_eval("b + c + d")
```

3. `weave.blitz` works best when algorithms can be expressed in a "vectorized" form. Algorithms that have a large number of `if/thens` and other conditions are better hand-written in C or Fortran. Further, the restrictions imposed by requiring vectorized expressions sometimes preclude the use of more efficient data structures or algorithms. For maximum speed in these cases, hand-coded C or Fortran code is the only way to go.
4. `weave.blitz` can produce different results than NumPy in certain situations. It can happen when the array receiving the results of a calculation is also used during the calculation. The NumPy behavior is to carry out the entire calculation on the right hand side of an equation and store it in a temporary array. This temporary array is

assigned to the array on the left hand side of the equation. `blitz`, on the other hand, does a “running” calculation of the array elements assigning values from the right hand side to the elements on the left hand side immediately after they are calculated. Here is an example, provided by Prabhu Ramachandran, where this happens:

```
# 4 point average.
>>> expr = "u[1:-1, 1:-1] = (u[0:-2, 1:-1] + u[2:, 1:-1] + \
...         "u[1:-1,0:-2] + u[1:-1, 2:])*0.25"
>>> u = zeros((5, 5), 'd'); u[0,:] = 100
>>> exec (expr)
>>> u
array([[ 100.,  100.,  100.,  100.,  100.],
       [   0.,   25.,   25.,   25.,   0.],
       [   0.,   0.,   0.,   0.,   0.],
       [   0.,   0.,   0.,   0.,   0.],
       [   0.,   0.,   0.,   0.,   0.]])

>>> u = zeros((5, 5), 'd'); u[0,:] = 100
>>> weave.blitz (expr)
>>> u
array([[ 100. ,  100. ,  100. ,  100. ,  100. ],
       [   0. ,  25. ,  31.25 ,  32.8125 ,  0. ],
       [   0. ,  6.25 ,  9.375 ,  10.546875 ,  0. ],
       [   0. ,  1.5625 ,  2.734375 ,  3.3203125 ,  0. ],
       [   0. ,  0. ,  0. ,  0. ,  0. ]])
```

You can prevent this behavior by using a temporary array.

```
>>> u = zeros((5, 5), 'd'); u[0,:] = 100
>>> temp = zeros((4, 4), 'd');
>>> expr = "temp = (u[0:-2, 1:-1] + u[2:, 1:-1] + \"
...         \"u[1:-1,0:-2] + u[1:-1, 2:])*0.25;\"
...         \"u[1:-1,1:-1] = temp"
>>> weave.blitz (expr)
>>> u
array([[ 100.,  100.,  100.,  100.,  100.],
       [   0.,  25.,  25.,  25.,   0.],
       [   0.,   0.,   0.,   0.,   0.],
       [   0.,   0.,   0.,   0.,   0.],
       [   0.,   0.,   0.,   0.,   0.]])
```

5. One other point deserves mention lest people be confused. `weave.blitz` is not a general purpose Python->C compiler. It only works for expressions that contain NumPy arrays and/or Python scalar values. This focused scope concentrates effort on the computationally intensive regions of the program and sidesteps the difficult issues associated with a general purpose Python->C compiler.

### *NumPy efficiency issues: What compilation buys you*

Some might wonder why compiling NumPy expressions to C++ is beneficial since operations on NumPy array operations are already executed within C loops. The problem is that anything other than the simplest expression are executed in less than optimal fashion. Consider the following NumPy expression:

```
a = 1.2 * b + c * d
```

When NumPy calculates the value for the 2d array, `a`, it does the following steps:

```
temp1 = 1.2 * b
temp2 = c * d
a = temp1 + temp2
```

Two things to note. Since `c` is an (perhaps large) array, a large temporary array must be created to store the results of

1.2 \* b. The same is true for `temp2`. Allocation is slow. The second thing is that we have 3 loops executing, one to calculate `temp1`, one for `temp2` and one for adding them up. A C loop for the same problem might look like:

```
for(int i = 0; i < M; i++)
    for(int j = 0; j < N; j++)
        a[i,j] = 1.2 * b[i,j] + c[i,j] * d[i,j]
```

Here, the 3 loops have been fused into a single loop and there is no longer a need for a temporary array. This provides a significant speed improvement over the above example (write me and tell me what you get).

So, converting NumPy expressions into C/C++ loops that fuse the loops and eliminate temporary arrays can provide big gains. The goal, then, is to convert NumPy expression to C/C++ loops, compile them in an extension module, and then call the compiled extension function. The good news is that there is an obvious correspondence between the NumPy expression above and the C loop. The bad news is that NumPy is generally much more powerful than this simple example illustrates and handling all possible indexing possibilities results in loops that are less than straightforward to write. (Take a peek at NumPy for confirmation). Luckily, there are several available tools that simplify the process.

### The Tools

`weave.blitz` relies heavily on several remarkable tools. On the Python side, the main facilitators are Jeremy Hylton's parser module and Travis Oliphant's NumPy module. On the compiled language side, Todd Veldhuizen's `blitz++` array library, written in C++ (shhhh. don't tell David Beazley), does the heavy lifting. Don't assume that, because it's C++, it's much slower than C or Fortran. `Blitz++` uses a jaw dropping array of template techniques (metaprogramming, template expression, etc) to convert innocent-looking and readable C++ expressions into code that usually executes within a few percentage points of Fortran code for the same problem. This is good. Unfortunately all the template raz-ma-taz is very expensive to compile, so the 200 line extension modules often take 2 or more minutes to compile. This isn't so good. `weave.blitz` works to minimize this issue by remembering where compiled modules live and reusing them instead of re-compiling every time a program is re-run.

**Parser** Tearing NumPy expressions apart, examining the pieces, and then rebuilding them as C++ (`blitz`) expressions requires a parser of some sort. I can imagine someone attacking this problem with regular expressions, but it'd likely be ugly and fragile. Amazingly, Python solves this problem for us. It actually exposes its parsing engine to the world through the `parser` module. The following fragment creates an Abstract Syntax Tree (AST) object for the expression and then converts to a (rather unpleasant looking) deeply nested list representation of the tree.

```
>>> import parser
>>> import scipy.weave.misc
>>> ast = parser.suite("a = b * c + d")
>>> ast_list = ast.tolist()
>>> sym_list = scipy.weave.misc.translate_symbols(ast_list)
>>> pprint.pprint(sym_list)
['file_input',
 ['stmt',
  ['simple_stmt',
   ['small_stmt',
    ['expr_stmt',
     ['testlist',
      ['test',
       ['and_test',
        ['not_test',
         ['comparison',
          ['expr',
           ['xor_expr',
            ['and_expr',
             ['shift_expr',
              ['arith_expr',
               ['term',
```

```

                ['factor', ['power', ['atom', ['NAME', 'a']]]]]]]]]]]]]],
['EQUAL', '='],
['testlist',
 ['test',
  ['and_test',
   ['not_test',
    ['comparison',
     ['expr',
      ['xor_expr',
       ['and_expr',
        ['shift_expr',
         ['arith_expr',
          ['term',
           ['factor', ['power', ['atom', ['NAME', 'b']]]],
            ['STAR', '*'],
            ['factor', ['power', ['atom', ['NAME', 'c']]]]]],
           ['PLUS', '+'],
           ['term',
            ['factor', ['power', ['atom', ['NAME', 'd']]]]]]]]]]]]]]]],
 ['NEWLINE', '']],
 ['ENDMARKER', '']]

```

Despite its looks, with some tools developed by Jeremy H., it's possible to search these trees for specific patterns (sub-trees), extract the sub-tree, manipulate them converting python specific code fragments to blitz code fragments, and then re-insert it in the parse tree. The parser module documentation has some details on how to do this. Traversing the new blitzified tree, writing out the terminal symbols as you go, creates our new blitz++ expression string.

**Blitz and NumPy** The other nice discovery in the project is that the data structure used for NumPy arrays and blitz arrays is nearly identical. NumPy stores “strides” as byte offsets and blitz stores them as element offsets, but other than that, they are the same. Further, most of the concept and capabilities of the two libraries are remarkably similar. It is satisfying that two completely different implementations solved the problem with similar basic architectures. It is also fortuitous. The work involved in converting NumPy expressions to blitz expressions was greatly diminished. As an example, consider the code for slicing an array in Python with a stride:

```

>>> a = b[0:4:2] + c
>>> a
[0, 2, 4]

```

In Blitz it is as follows:

```

Array<2,int> b(10);
Array<2,int> c(3);
// ...
Array<2,int> a = b(Range(0,3,2)) + c;

```

Here the range object works exactly like Python slice objects with the exception that the top index (3) is inclusive where as Python's (4) is exclusive. Other differences include the type declarations in C++ and parentheses instead of brackets for indexing arrays. Currently, `weave.blitz` handles the inclusive/exclusive issue by subtracting one from upper indices during the translation. An alternative that is likely more robust/maintainable in the long run is to write a `PyRange` class that behaves like Python's `range`. This is likely very easy.

The stock blitz also doesn't handle negative indices in ranges. The current implementation of the `blitz()` has a partial solution to this problem. It calculates an index that starts with a '-' sign by subtracting it from the maximum index in the array so that:

```

upper index limit
/-----\

```



```
b[:-1] -> b(Range(0, Nb[0]-1-1))
```

This approach fails, however, when the top index is calculated from other values. In the following scenario, if `i+j` evaluates to a negative value, the compiled code will produce incorrect results and could even core-dump. Right now, all calculated indices are assumed to be positive.

```
b[:i-j] -> b(Range(0, i+j))
```

A solution is to calculate all indices up front using if/then to handle the +/- cases. This is a little work and results in more code, so it hasn't been done. I'm holding out to see if `blitz++` can be modified to handle negative indexing, but haven't looked into how much effort is involved yet. While it needs fixin', I don't think there is a ton of code where this is an issue.

The actual translation of the Python expressions to blitz expressions is currently a two part process. First, all `x:y:z` slicing expressions are removed from the AST, converted to `slice(x,y,z)` and re-inserted into the tree. Any math needed on these expressions (subtracting from the maximum index, etc.) are also performed here. `_beg` and `_end` are used as special variables that are defined as `blitz::fromBegin` and `blitz::toEnd`.

```
a[i+j:i+j+1, :] = b[2:3, :]
```

becomes a more verbose:

```
a[slice(i+j, i+j+1), slice(_beg, _end)] = b[slice(2, 3), slice(_beg, _end)]
```

The second part does a simple string search/replace to convert to a blitz expression with the following translations:

```
slice(_beg, _end) -> _all # not strictly needed, but cuts down on code.
slice             -> blitz::Range
[                -> (
]                -> )
_stp             -> 1
```

`_all` is defined in the compiled function as `blitz::Range.all()`. These translations could of course happen directly in the syntax tree. But the string replacement is slightly easier. Note that namespaces are maintained in the C++ code to lessen the likelihood of name clashes. Currently no effort is made to detect name clashes. A good rule of thumb is don't use values that start with `'_'` or `'py_'` in compiled expressions and you'll be fine.

### *Type definitions and coercion*

So far we've glossed over the dynamic vs. static typing issue between Python and C++. In Python, the type of value that a variable holds can change through the course of program execution. C/C++, on the other hand, forces you to declare the type of value a variables will hold prior at compile time. `weave.blitz` handles this issue by examining the types of the variables in the expression being executed, and compiling a function for those explicit types. For example:

```
a = ones((5,5), Float32)
b = ones((5,5), Float32)
weave.blitz("a = a + b")
```

When compiling this expression to C++, `weave.blitz` sees that the values for `a` and `b` in the local scope have type `Float32`, or 'float' on a 32 bit architecture. As a result, it compiles the function using the float type (no attempt has been made to deal with 64 bit issues).

What happens if you call a compiled function with array types that are different than the ones for which it was originally compiled? No biggie, you'll just have to wait on it to compile a new version for your new types. This doesn't overwrite the old functions, as they are still accessible. See the catalog section in the `inline()` documentation to see how this is handled. Suffice to say, the mechanism is transparent to the user and behaves like dynamic typing with the occasional wait for compiling newly typed functions.

When working with combined scalar/array operations, the type of the array is *always* used. This is similar to the `save-pace` flag that was recently added to NumPy. This prevents issues with the following expression perhaps unexpectedly being calculated at a higher (more expensive) precision that can occur in Python:

```
>>> a = array((1,2,3), dtypecode = Float32)
>>> b = a * 2.1 # results in b being a Float64 array.
```

In this example,

```
>>> a = ones((5,5), Float32)
>>> b = ones((5,5), Float32)
>>> weave.blitz("b = a * 2.1")
```

the `2.1` is cast down to a `float` before carrying out the operation. If you really want to force the calculation to be a `double`, define `a` and `b` as `double` arrays.

One other point of note. Currently, you must include both the right hand side and left hand side (assignment side) of your equation in the compiled expression. Also, the array being assigned to must be created prior to calling `weave.blitz`. I'm pretty sure this is easily changed so that a `compiled_eval` expression can be defined, but no effort has been made to allocate new arrays (and discern their type) on the fly.

### Cataloging Compiled Functions

See [The Catalog](#) section in the `weave.inline()` documentation.

### Checking Array Sizes

Surprisingly, one of the big initial problems with compiled code was making sure all the arrays in an operation were of compatible type. The following case is trivially easy:

```
a = b + c
```

It only requires that arrays `a`, `b`, and `c` have the same shape. However, expressions like:

```
a[i+j:i+j+1,:] = b[2:3,:] + c
```

are not so trivial. Since slicing is involved, the size of the slices, not the input arrays, must be checked. Broadcasting complicates things further because arrays and slices with different dimensions and shapes may be compatible for math operations (broadcasting isn't yet supported by `weave.blitz`). Reductions have a similar effect as their results are different shapes than their input operand. The binary operators in NumPy compare the shapes of their two operands just before they operate on them. This is possible because NumPy treats each operation independently. The intermediate (temporary) arrays created during sub-operations in an expression are tested for the correct shape before they are combined by another operation. Because `weave.blitz` fuses all operations into a single loop, this isn't possible. The shape comparisons must be done and guaranteed compatible before evaluating the expression.

The solution chosen converts input arrays to “dummy arrays” that only represent the dimensions of the arrays, not the data. Binary operations on dummy arrays check that input array sizes are compatible and return a dummy array with the size correct size. Evaluating an expression of dummy arrays traces the changing array sizes through all operations and fails if incompatible array sizes are ever found.

The machinery for this is housed in `weave.size_check`. It basically involves writing a new class (dummy array) and overloading its math operators to calculate the new sizes correctly. All the code is in Python and there is a fair amount of logic (mainly to handle indexing and slicing) so the operation does impose some overhead. For large arrays (ie. 50x50x50), the overhead is negligible compared to evaluating the actual expression. For small arrays (ie. 16x16), the overhead imposed for checking the shapes with this method can cause the `weave.blitz` to be slower than evaluating the expression in Python.

What can be done to reduce the overhead? (1) The size checking code could be moved into C. This would likely remove most of the overhead penalty compared to NumPy (although there is also some calling overhead), but no effort has been made to do this. (2) You can also call `weave.blitz` with `check_size=0` and the size checking isn't

done. However, if the sizes aren't compatible, it can cause a core-dump. So, foregoing `size_checking` isn't advisable until your code is well debugged.

### *Creating the Extension Module*

`weave.blitz` uses the same machinery as `weave.inline` to build the extension module. The only difference is the code included in the function is automatically generated from the NumPy array expression instead of supplied by the user.

## Extension Modules

`weave.inline` and `weave.blitz` are high level tools that generate extension modules automatically. Under the covers, they use several classes from `weave.ext_tools` to help generate the extension module. The main two classes are `ext_module` and `ext_function` (I'd like to add `ext_class` and `ext_method` also). These classes simplify the process of generating extension modules by handling most of the "boiler plate" code automatically.

---

**Note:** `inline` actually sub-classes `weave.ext_tools.ext_function` to generate slightly different code than the standard `ext_function`. The main difference is that the standard class converts function arguments to C types, while `inline` always has two arguments, the local and global dicts, and the grabs the variables that need to be converted to C from these.

---

### *A Simple Example*

The following simple example demonstrates how to build an extension module within a Python function:

```
# examples/increment_example.py
from weave import ext_tools

def build_increment_ext():
    """ Build a simple extension with functions that increment numbers.
        The extension will be built in the local directory.
    """
    mod = ext_tools.ext_module('increment_ext')

    a = 1 # effectively a type declaration for 'a' in the
          # following functions.

    ext_code = "return_val = Py::new_reference_to(Py::Int(a+1));"
    func = ext_tools.ext_function('increment', ext_code, ['a'])
    mod.add_function(func)

    ext_code = "return_val = Py::new_reference_to(Py::Int(a+2));"
    func = ext_tools.ext_function('increment_by_2', ext_code, ['a'])
    mod.add_function(func)

    mod.compile()
```

The function `build_increment_ext()` creates an extension module named `increment_ext` and compiles it to a shared library (`.so` or `.pyd`) that can be loaded into Python. `increment_ext` contains two functions, `increment` and `increment_by_2`. The first line of `build_increment_ext()`,

```
mod = ext_tools.ext_module('increment_ext')
```

creates an `ext_module` instance that is ready to have `ext_function` instances added to it. `ext_function` instances are created much with a calling convention similar to `weave.inline()`. The most common call includes a C/C++ code snippet and a list of the arguments for the function. The following:

```
ext_code = "return_val = Py::new_reference_to(Py::Int(a+1));"  
func = ext_tools.ext_function('increment', ext_code, ['a'])
```

creates a C/C++ extension function that is equivalent to the following Python function:

```
def increment(a):  
    return a + 1
```

A second method is also added to the module and then,

```
mod.compile()
```

is called to build the extension module. By default, the module is created in the current working directory. This example is available in the `examples/increment_example.py` file found in the `weave` directory. At the bottom of the file in the module's "main" program, an attempt to import `increment_ext` without building it is made. If this fails (the module doesn't exist in the `PYTHONPATH`), the module is built by calling `build_increment_ext()`. This approach only takes the time-consuming (a few seconds for this example) process of building the module if it hasn't been built before.

```
if __name__ == "__main__":  
    try:  
        import increment_ext  
    except ImportError:  
        build_increment_ext()  
        import increment_ext  
    a = 1  
    print 'a, a+1:', a, increment_ext.increment(a)  
    print 'a, a+2:', a, increment_ext.increment_by_2(a)
```

---

**Note:** If we were willing to always pay the penalty of building the C++ code for a module, we could store the SHA-256 checksum of the C++ code along with some information about the compiler, platform, etc. Then, `ext_module.compile()` could try importing the module before it actually compiles it, check the SHA-256 checksum and other meta-data in the imported module with the meta-data of the code it just produced and only compile the code if the module didn't exist or the meta-data didn't match. This would reduce the above code to:

```
if __name__ == "__main__":  
    build_increment_ext()  
  
    a = 1  
    print 'a, a+1:', a, increment_ext.increment(a)  
    print 'a, a+2:', a, increment_ext.increment_by_2(a)
```

---

**Note:** There would always be the overhead of building the C++ code, but it would only actually compile the code once. You pay a little in overhead and get cleaner "import" code. Needs some thought.

---

If you run `increment_example.py` from the command line, you get the following:

```
[eric@n0]$ python increment_example.py  
a, a+1: 1 2  
a, a+2: 1 3
```

If the module didn't exist before it was run, the module is created. If it did exist, it is just imported and used.

### *Fibonacci Example*

`examples/fibonacci.py` provides a little more complex example of how to use `ext_tools`. Fibonacci numbers are a series of numbers where each number in the series is the sum of the previous two: 1, 1, 2, 3, 5, 8, etc. Here,

the first two numbers in the series are taken to be 1. One approach to calculating Fibonacci numbers uses recursive function calls. In Python, it might be written as:

```
def fib(a):
    if a <= 2:
        return 1
    else:
        return fib(a-2) + fib(a-1)
```

In C, the same function would look something like this:

```
int fib(int a)
{
    if(a <= 2)
        return 1;
    else
        return fib(a-2) + fib(a-1);
}
```

Recursion is much faster in C than in Python, so it would be beneficial to use the C version for fibonacci number calculations instead of the Python version. We need an extension function that calls this C function to do this. This is possible by including the above code snippet as “support code” and then calling it from the extension function. Support code snippets (usually structure definitions, helper functions and the like) are inserted into the extension module C/C++ file before the extension function code. Here is how to build the C version of the fibonacci number generator:

```
def build_fibonacci():
    """ Builds an extension module with fibonacci calculators.
    """
    mod = ext_tools.ext_module('fibonacci_ext')
    a = 1 # this is effectively a type declaration

    # recursive fibonacci in C
    fib_code = """
        int fib1(int a)
        {
            if(a <= 2)
                return 1;
            else
                return fib1(a-2) + fib1(a-1);
        }
    """
    ext_code = """
        int val = fib1(a);
        return_val = Py::new_reference_to(Py::Int(val));
    """
    fib = ext_tools.ext_function('fib', ext_code, ['a'])
    fib.customize.add_support_code(fib_code)
    mod.add_function(fib)

    mod.compile()
```

XXX More about custom\_info, and what xxx\_info instances are good for.

---

**Note:** recursion is not the fastest way to calculate fibonacci numbers, but this approach serves nicely for this example.

---

## Customizing Type Conversions – Type Factories

not written

### Things I wish weave did

It is possible to get name clashes if you uses a variable name that is already defined in a header automatically included (such as `stdio.h`). For instance, if you try to pass in a variable named `stdout`, you'll get a cryptic error report due to the fact that `stdio.h` also defines the name. `weave` should probably try and handle this in some way. Other things...

## DEVELOPER'S GUIDE

Explanations of how to start contributing to SciPy, and descriptions of maintenance activities and policies.

### 4.1 Contributing to SciPy

This document aims to give an overview of how to contribute to SciPy. It tries to answer commonly asked questions, and provide some insight into how the community process works in practice. Readers who are familiar with the SciPy community and are experienced Python coders may want to jump straight to the [git workflow](#) documentation.

---

**Note:** You may want to check the latest version of this guide, which is available at: <https://github.com/scipy/scipy/blob/master/HACKING.rst.txt>

---

#### 4.1.1 Contributing new code

If you have been working with the scientific Python toolstack for a while, you probably have some code lying around of which you think “this could be useful for others too”. Perhaps it’s a good idea then to contribute it to SciPy or another open source project. The first question to ask is then, where does this code belong? That question is hard to answer here, so we start with a more specific one: *what code is suitable for putting into SciPy?* Almost all of the new code added to `scipy` has in common that it’s potentially useful in multiple scientific domains and it fits in the scope of existing `scipy` submodules. In principle new submodules can be added too, but this is far less common. For code that is specific to a single application, there may be an existing project that can use the code. Some `scikits` (`scikit-learn`, `scikit-image`, `statsmodels`, etc.) are good examples here; they have a narrower focus and because of that more domain-specific code than SciPy.

Now if you have code that you would like to see included in SciPy, how do you go about it? After checking that your code can be distributed in SciPy under a compatible license (see FAQ for details), the first step is to discuss on the `scipy-dev` mailing list. All new features, as well as changes to existing code, are discussed and decided on there. You can, and probably should, already start this discussion before your code is finished.

Assuming the outcome of the discussion on the mailing list is positive and you have a function or piece of code that does what you need it to do, what next? Before code is added to SciPy, it at least has to have good documentation, unit tests and correct code style.

1. **Unit tests** In principle you should aim to create unit tests that exercise all the code that you are adding. This gives some degree of confidence that your code runs correctly, also on Python versions and hardware or OSes that you don’t have available yourself. An extensive description of how to write unit tests is given in the NumPy [testing guidelines](#).
2. **Documentation** Clear and complete documentation is essential in order for users to be able to find and understand the code. Documentation for individual functions and classes – which includes at least a

basic description, type and meaning of all parameters and returns values, and usage examples in `doctest` format – is put in docstrings. Those docstrings can be read within the interpreter, and are compiled into a reference guide in html and pdf format. Higher-level documentation for key (areas of) functionality is provided in tutorial format and/or in module docstrings. A guide on how to write documentation is given in [how to document](#).

3. **Code style** Uniformity of style in which code is written is important to others trying to understand the code. SciPy follows the standard Python guidelines for code style, [PEP8](#). In order to check that your code conforms to PEP8, you can use the [pep8 package](#) style checker. Most IDEs and text editors have settings that can help you follow PEP8, for example by translating tabs by four spaces. Using [pyflakes](#) to check your code is also a good idea.

At the end of this document a checklist is given that may help to check if your code fulfills all requirements for inclusion in SciPy.

Another question you may have is: *where exactly do I put my code?* To answer this, it is useful to understand how the SciPy public API (application programming interface) is defined. For most modules the API is two levels deep, which means your new function should appear as `scipy.submodule.my_new_func`. `my_new_func` can be put in an existing or new file under `/scipy/<submodule>/`, its name is added to the `__all__` list in that file (which lists all public functions in the file), and those public functions are then imported in `/scipy/<submodule>/__init__.py`. Any private functions/classes should have a leading underscore (`_`) in their name. A more detailed description of what the public API of SciPy is, is given in [SciPy API](#).

Once you think your code is ready for inclusion in SciPy, you can send a pull request (PR) on Github. We won't go into the details of how to work with git here, this is described well in the [git workflow](#) section of the NumPy documentation and on the [Github help pages](#). When you send the PR for a new feature, be sure to also mention this on the `scipy-dev` mailing list. This can prompt interested people to help review your PR. Assuming that you already got positive feedback before on the general idea of your code/feature, the purpose of the code review is to ensure that the code is correct, efficient and meets the requirements outlined above. In many cases the code review happens relatively quickly, but it's possible that it stalls. If you have addressed all feedback already given, it's perfectly fine to ask on the mailing list again for review (after a reasonable amount of time, say a couple of weeks, has passed). Once the review is completed, the PR is merged into the “master” branch of SciPy.

The above describes the requirements and process for adding code to SciPy. It doesn't yet answer the question though how decisions are made exactly. The basic answer is: decisions are made by consensus, by everyone who chooses to participate in the discussion on the mailing list. This includes developers, other users and yourself. Aiming for consensus in the discussion is important – SciPy is a project by and for the scientific Python community. In those rare cases that agreement cannot be reached, the [maintainers](#) of the module in question can decide the issue.

### 4.1.2 Contributing by helping maintain existing code

The previous section talked specifically about adding new functionality to SciPy. A large part of that discussion also applies to maintenance of existing code. Maintenance means fixing bugs, improving code quality or style, documenting existing functionality better, adding missing unit tests, keeping build scripts up-to-date, etc. The [SciPy issue list](#) contains all reported bugs, build/documentation issues, etc. Fixing issues helps improve the overall quality of SciPy, and is also a good way of getting familiar with the project. You may also want to fix a bug because you ran into it and need the function in question to work correctly.

The discussion on code style and unit testing above applies equally to bug fixes. It is usually best to start by writing a unit test that shows the problem, i.e. it should pass but doesn't. Once you have that, you can fix the code so that the test does pass. That should be enough to send a PR for this issue. Unlike when adding new code, discussing this on the mailing list may not be necessary - if the old behavior of the code is clearly incorrect, no one will object to having it fixed. It may be necessary to add some warning or deprecation message for the changed behavior. This should be part of the review process.



### 4.1.3 Other ways to contribute

There are many ways to contribute other than contributing code. Participating in discussions on the `scipy-user` and `scipy-dev` *mailing lists* is a contribution in itself. The [scipy.org website](http://scipy.org) contains a lot of information on the SciPy community and can always use a new pair of hands.

### 4.1.4 Recommended development setup

Since SciPy contains parts written in C, C++, and Fortran that need to be compiled before use, make sure you have the necessary compilers and Python development headers installed. Having compiled code also means that importing SciPy from the development sources needs some additional steps, which are explained below.

First fork a copy of the main SciPy repository in Github onto your own account and then create your local repository via:

```
$ git clone git@github.com:YOURUSERNAME/scipy.git scipy
$ cd scipy
$ git remote add upstream git://github.com/scipy/scipy.git
```

To build the development version of SciPy and run tests, spawn interactive shells with the Python import paths properly set up etc., do one of:

```
$ python runtests.py -v
$ python runtests.py -v -s optimize
$ python runtests.py -v -t scipy/special/tests/test_basic.py:test_xlogy
$ python runtests.py --ipython
$ python runtests.py --python somescript.py
$ python runtests.py --bench
```

This builds SciPy first, so the first time it may take some time. If you specify `-n`, the tests are run against the version of SciPy (if any) found on current `PYTHONPATH`.

Using `runtests.py` is the recommended approach to running tests. There are also a number of alternatives to it, for example in-place build or installing to a virtualenv. See the FAQ below for details.

Some of the tests in SciPy are very slow and need to be separately enabled. See the FAQ below for details.

### 4.1.5 SciPy structure

All SciPy modules should follow the following conventions. In the following, a *SciPy module* is defined as a Python package, say `yyy`, that is located in the `scipy/` directory.

- Ideally, each SciPy module should be as self-contained as possible. That is, it should have minimal dependencies on other packages or modules. Even dependencies on other SciPy modules should be kept to a minimum. A dependency on NumPy is of course assumed.
- Directory `yyy/` contains:
  - A file `setup.py` that defines configuration (`parent_package='', top_path=None`) function for `numpy.distutils`.
  - A directory `tests/` that contains files `test_<name>.py` corresponding to modules `yyy/<name>{.py, .so, /}`.
- Private modules should be prefixed with an underscore `_`, for instance `yyy/_somemodule.py`.
- User-visible functions should have good documentation following the Numpy documentation style, see [how to document](#)

- The `__init__.py` of the module should contain the main reference documentation in its docstring. This is connected to the Sphinx documentation under `doc/` via Sphinx's automodule directive.

The reference documentation should first give a categorized list of the contents of the module using `autosummary::` directives, and after that explain points essential for understanding the use of the module.

Tutorial-style documentation with extensive examples should be separate, and put under `doc/source/tutorial/`

See the existing Scipy submodules for guidance.

For further details on Numpy distutils, see:

<https://github.com/numpy/numpy/blob/master/doc/DISTUTILS.rst.txt>

### 4.1.6 Useful links, FAQ, checklist

#### Checklist before submitting a PR

- Are there unit tests with good code coverage?
- Do all public function have docstrings including examples?
- Is the code style correct (PEP8, pyflakes)
- Is the new functionality tagged with `.. versionadded:: X.Y.Z` (with X.Y.Z the version number of the next release - can be found in `setup.py`)?
- Is the new functionality mentioned in the release notes of the next release?
- Is the new functionality added to the reference guide?
- In case of larger additions, is there a tutorial or more extensive module-level description?
- In case compiled code is added, is it integrated correctly via `setup.py` (and preferably also Bento configuration files - `bento.info` and `bscript`)?
- If you are a first-time contributor, did you add yourself to `THANKS.txt`? Please note that this is perfectly normal and desirable - the aim is to give every single contributor credit, and if you don't add yourself it's simply extra work for the reviewer (or worse, the reviewer may forget).
- Did you check that the code can be distributed under a BSD license?

#### Useful SciPy documents

- [The how to document guidelines](#)
- [NumPy/SciPy testing guidelines](#)
- [SciPy API](#)
- [SciPy maintainers](#)
- [NumPy/SciPy git workflow](#)

#### FAQ

*I based my code on existing Matlab/R/... code I found online, is this OK?*

It depends. SciPy is distributed under a BSD license, so if the code that you based your code on is also BSD licensed or has a BSD-compatible license (MIT, Apache, ...) then it's OK. Code which is GPL-licensed, has no clear license,

requires citation or is free for academic use only can't be included in SciPy. Therefore if you copied existing code with such a license or made a direct translation to Python of it, your code can't be included. See also [license compatibility](#).

*Why is SciPy under the BSD license and not, say, the GPL?*

Like Python, SciPy uses a “permissive” open source license, which allows proprietary re-use. While this allows companies to use and modify the software without giving anything back, it is felt that the larger user base results in more contributions overall, and companies often publish their modifications anyway, without being required to. See John Hunter's [BSD pitch](#).

*How do I set up a development version of SciPy in parallel to a released version that I use to do my job/research?*

One simple way to achieve this is to install the released version in site-packages, by using a binary installer or pip for example, and set up the development version in a virtualenv. First install [virtualenv](#) (optionally use [virtualenvwrapper](#)), then create your virtualenv (named `scipy-dev` here) with:

```
$ virtualenv scipy-dev
```

Now, whenever you want to switch to the virtual environment, you can use the command `source scipy-dev/bin/activate`, and `deactivate` to exit from the virtual environment and back to your previous shell. With `scipy-dev` activated, install first SciPy's dependencies:

```
$ pip install Numpy Nose Cython
```

After that, you can install a development version of SciPy, for example via:

```
$ python setup.py install
```

The installation goes to the virtual environment.

*How do I set up an in-place build for development*

For development, you can set up an in-place build so that changes made to `.py` files have effect without rebuild. First, run:

```
$ python setup.py build_ext -i
```

Then you need to point your `PYTHONPATH` environment variable to this directory. Some IDEs (Spyder for example) have utilities to manage `PYTHONPATH`. On Linux and OSX, you can run the command:

```
$ export PYTHONPATH=$PWD
```

and on Windows

```
$ set PYTHONPATH=/path/to/scipy
```

Now editing a Python source file in SciPy allows you to immediately test and use your changes (in `.py` files), by simply restarting the interpreter.

*Can I use a programming language other than Python to speed up my code?*

Yes. The languages used in SciPy are Python, Cython, C, C++ and Fortran. All of these have their pros and cons. If Python really doesn't offer enough performance, one of those languages can be used. Important concerns when using compiled languages are maintainability and portability. For maintainability, Cython is clearly preferred over C/C++/Fortran. Cython and C are more portable than C++/Fortran. A lot of the existing C and Fortran code in SciPy is older, battle-tested code that was only wrapped in (but not specifically written for) Python/SciPy. Therefore the basic advice is: use Cython. If there's specific reasons why C/C++/Fortran should be preferred, please discuss those reasons first.

*How do I debug code written in C/C++/Fortran inside SciPy?*

The easiest way to do this is to first write a Python script that invokes the C code whose execution you want to debug. For instance `mytest.py`:

```
from scipy.special import hyp2f1
print(hyp2f1(5.0, 1.0, -1.8, 0.95))
```

Now, you can run:

```
gdb --args python runtests.py -g --python mytest.py
```

If you didn't compile with debug symbols enabled before, remove the `build` directory first. While in the debugger:

```
(gdb) break cephes_hyp2f1
(gdb) run
```

The execution will now stop at the corresponding C function and you can step through it as usual. Instead of plain `gdb` you can of course use your favourite alternative debugger; run it on the `python` binary with arguments `runtests.py -g --python mytest.py`.

*How do I enable additional tests in Scipy?*

Some of the tests in Scipy's test suite are very slow and not enabled by default. You can run the full suite via:

```
$ python runtests.py -g -m full
```

This invokes the test suite `import scipy; scipy.test("full")`, enabling also slow tests.

There is an additional level of very slow tests (several minutes), which are disabled also in this case. They can be enabled by setting the environment variable `SCIPY_XSLOW=1` before running the test suite.

*How do I write tests with test generators?*

The `Nose` test framework supports so-called test generators, which can come useful if you need to have multiple tests where just a parameter changes. Using test generators so that they are more useful than harmful is tricky, and we recommend the following pattern:

```
def test_something():
    some_array = (...)

    def check(some_param):
        c = compute_result(some_array, some_param)
        known_result = (...)
        assert_allclose(c, known_result)

    for some_param in ['a', 'b', 'c']:
        yield check, some_param
```

We require the following:

- All asserts and all computation that is tested must only be reached after a `yield`. (Rationale: the generator body is part of no test, and a failure in it will show neither the test name nor for what parameters the test failed.)
- Arrays must not be passed as `yield` parameters. Either use variables from outer scope (eg. with some index passed to `yield`), or capsule test data to a class with a sensible `__repr__`. (Rationale: `Nose` truncates the printed form of arrays in test output, and this makes it impossible to know for what parameters a test failed. Arrays are big, and clutter test output unnecessarily.)
- Test generators cannot be used in test classes inheriting from `unittest.TestCase`; either use object as base class, or use standalone test functions. (Rationale: `Nose` does not run test generators in `TestCase`-inheriting classes.)

If in doubt, do not use test generators. You can track for what parameter things failed also by passing `err_msg=repr((param1, param2, ...))` to the various assert functions.

## 4.2 SciPy Developer Guide

### 4.2.1 Decision making process

This section documents the way in which decisions about various aspects of the SciPy project are made. Note that the below is only documenting the current way of working; a more formal governance model is expected to be adopted in the near future.

#### Code

Any significant decisions on adding (or not adding) new features, breaking backwards compatibility or making other significant changes to the codebase should be made on the `scipy-dev` mailing list after a discussion (preferably with full consensus).

Any non-trivial change (where trivial means a typo, or a one-liner maintenance commit) has to go in through a pull request (PR). It has to be reviewed by another developer. In case review doesn't happen quickly enough and it is important that the PR is merged quickly, the submitter of the PR should send a message to mailing list saying he/she intends to merge that PR without review at time X for reason Y unless someone reviews it before then.

Changes and new additions should be tested. Untested code is broken code.

#### Commit rights

Who gets commit rights is decided by the core development team; changes in commit rights will then be announced on the `scipy-dev` mailing list.

Who the core development team is comprised of is a little fuzzy - there are quite a few people who do have commit rights and would like to keep them but are no longer active (so they're not in the core team). To get an idea, look at the output of:

```
$ git shortlog --grep="Merge pull request" -a -c -s <current_release_minus_2>..upstream/master|sort -
```

and apply some common sense to it (and don't forget people who are still active but never merge PRs).

#### Other project aspects

All decisions are taken by the core development team on the `scipy-dev` mailing list.

### 4.2.2 Development on GitHub

SciPy development largely takes place on GitHub; this section describes the expected way of working for issues, pull requests and managing the main `scipy` repository.

#### Labels and Milestones

Each issue and pull request normally gets at least two labels: one for the topic or component (`scipy.stats`, `Documentation`, etc.), and one for the nature of the issue or pull request (`enhancement`, `maintenance`, `defect`, etc.). Other labels that may be added depending on the situation:

- `easy-fix`: for issues suitable to be tackled by new contributors.
- `needs-work`: for pull requests that have review comments that haven't been addressed for a while.

- `needs-decision`: for issues or pull requests that need a decision.
- `needs-champion`: for pull requests that were not finished by the original author, but are worth resurrecting.
- `backport-candidate`: bugfixes that should be considered for backporting by the release manager.

A milestone is created for each version number for which a release is planned. Issues that need to be addressed and pull requests that need to be merged for a particular release should be set to the corresponding milestone. After a pull request is merged, its milestone (and that of the issue it closes) should be set to the next upcoming release - this makes it easy to get an overview of changes and to add a complete list of those to the release notes.

### Dealing with pull requests

- When merging contributions, a committer is responsible for ensuring that those meet the requirements outlined in [Contributing to SciPy](#). Also check that new features and backwards compatibility breaks were discussed on the `scipy-dev` mailing list.
- New code goes in via a pull request (PR).
- Merge new code with the green button. In case of merge conflicts, ask the PR submitter to rebase (this may require providing some git instructions).
- Backports and trivial additions to finish a PR (really trivial, like a typo or PEP8 fix) can be pushed directly.
- For PRs that add new features or are in some way complex, wait at least a day or two before merging it. That way, others get a chance to comment before the code goes in.
- Squashing commits or cleaning up commit messages of a PR that you consider too messy is OK. Make sure though to retain the original author name when doing this.
- Make sure that the labels and milestone on a merged PR are set correctly.
- When you want to reject a PR: if it's very obvious you can just close it and explain why, if not obvious then it's a good idea to first explain why you think the PR is not suitable for inclusion in SciPy and then let a second committer comment or close.

### Backporting

All pull requests (whether they contain enhancements, bug fixes or something else), should be made against master. Only bug fixes are candidates for backporting to a maintenance branch. The backport strategy for SciPy is to (a) only backport fixes that are important, and (b) to only backport when it's reasonably sure that a new bugfix release on the relevant maintenance branch will be made. Typically, the developer who merges an important bugfix adds the `backport-candidate` label and pings the release manager, who decides on whether and when the backport is done. After the backport is completed, the `backport-candidate` label has to be removed again.

### Other

*PR status page:* When new commits get added to a pull request, GitHub doesn't send out any notifications. The `needs-work` label may not be justified anymore though. [This page](#) gives an overview of PRs that were updated, need review, need a decision, etc.

*Cross-referencing:* Cross-referencing issues and pull requests on GitHub is often useful. GitHub allows doing that by using `gh-xxxx` or `#xxxx` with `xxxx` the issue/PR number. The `gh-xxxx` format is strongly preferred, because it's clear that that is a GitHub link. Older issues contain `#xxxx` which is about Trac (what we used pre-GitHub) tickets.

*PR naming convention:* Pull requests, issues and commit messages usually start with a three-letter abbreviation like `ENH:` or `BUG:`. This is useful to quickly see what the nature of the commit/PR/issue is. For the full list of abbreviations, see [writing the commit message](#).

### 4.2.3 Licensing

SciPy is distributed under the modified (3-clause) BSD license. All code, documentation and other files added to SciPy by contributors is licensed under this license, unless another license is explicitly specified in the source code. Contributors keep the copyright for code they wrote and submit for inclusion to SciPy.

Other licenses that are compatible with the modified BSD license that SciPy uses are 2-clause BSD, MIT and PSF. Incompatible licenses are GPL, Apache and custom licenses that require attribution/citation or prohibit use for commercial purposes.

It regularly happens that PRs are submitted with content copied or derived from unlicensed code. Such contributions cannot be accepted for inclusion in SciPy. What is needed in such cases is to contact the original author and ask him to relicense his code under the modified BSD (or a compatible) license. If the original author agrees to this, add a comment saying so to the source files and forward the relevant email to the `scipy-dev` mailing list.

What also regularly happens is that code is translated or derived from code in R, Octave (both GPL-licensed) or a commercial application. Such code also cannot be included in SciPy. Simply implementing functionality with the same API as found in R/Octave/... is fine though, as long as the author doesn't look at the original incompatibly-licensed source code.

### 4.2.4 Version numbering

SciPy version numbering complies to [PEP 440](#). Released final versions, which are the only versions appearing on PyPI, are numbered `MAJOR.MINOR.MICRO` where:

- `MAJOR` is an integer indicating the major version. It changes very rarely; a change in `MAJOR` indicates large (possibly backwards-incompatible) changes.
- `MINOR` is an integer indicating the minor version. Minor versions are typically released twice a year and can contain new features, deprecations and bug-fixes.
- `MICRO` is an integer indicating a bug-fix version. Bug-fix versions are released when needed, typically one or two per minor version. They cannot contain new features or deprecations.

Released alpha, beta and rc (release candidate) versions are numbered like final versions but with postfixes `a#`, `b#` and `rc#` respectively, with `#` an integer. Development versions are postfixed with `.dev0+<git-commit-hash>`.

Examples of valid SciPy version strings are:

```
0.16.0
0.15.1
0.14.0a1
0.14.0b2
0.14.0rc1
0.17.0.dev0+ac53f09
```

An installed SciPy version contains these version identifiers:

```
scipy.__version__           # complete version string, including git commit hash for dev versions
scipy.version.short_version # string, only major.minor.micro
scipy.version.version       # string, same as scipy.__version__
scipy.version.full_version  # string, same as scipy.__version__
scipy.version.release       # bool, development or (alpha/beta/rc/final) released version
scipy.version.git_revision  # string, git commit hash from which scipy was built
```

## 4.2.5 Deprecations

There are various reasons for wanting to remove existing functionality: it's buggy, the API isn't understandable, it's superseded by functionality with better performance, it needs to be moved to another Scipy submodule, etc.

In general it's not a good idea to remove something without warning users about that removal first. Therefore this is what should be done before removing something from the public API:

1. Propose to deprecate the functionality on the `scipy-dev` mailing list and get agreement that that's OK.
2. Add a `DeprecationWarning` for it, which states that the functionality was deprecated, and in which release.
3. Mention the deprecation in the release notes for that release.
4. Wait till at least 6 months after the release date of the release that introduced the `DeprecationWarning` before removing the functionality.
5. Mention the removal of the functionality in the release notes.

The 6 months waiting period in practice usually means waiting two releases. When introducing the warning, also ensure that those warnings are filtered out when running the test suite so they don't pollute the output.

It's possible that there is reason to want to ignore this deprecation policy for a particular deprecation; this can always be discussed on the `scipy-dev` mailing list.

## 4.2.6 Distributing

Distributing Python packages is nontrivial - especially for a package with complex build requirements like Scipy - and subject to change. For an up-to-date overview of recommended tools and techniques, see the [Python Packaging User Guide](#). This document discusses some of the main issues and considerations for Scipy.

### Dependencies

Dependencies are things that a user has to install in order to use (or build/test) a package. They usually cause trouble, especially if they're not optional. Scipy tries to keep its dependencies to a minimum; currently they are:

*Unconditional run-time dependencies:*

- [Numpy](#)

*Conditional run-time dependencies:*

- `nose` (to run the test suite)
- `asv` (to run the benchmarks)
- `matplotlib` (for some functions that can produce plots)
- `Pillow` (for image loading/saving)
- `scikits.umfpack` (optionally used in `sparse.linalg`)
- `mpmath` (for more extended tests in `special`)

*Unconditional build-time dependencies:*

- [Numpy](#)
- A BLAS and LAPACK implementation (reference BLAS/LAPACK, ATLAS, OpenBLAS, MKL, Accelerate are all known to work)
- (for development versions) [Cython](#)



*Conditional build-time dependencies:*

- `setuptools`
- `wheel` (`python setup.py bdist_wheel`)
- **Sphinx** (docs)
- `matplotlib` (docs)
- LaTeX (pdf docs)
- `Pillow` (docs)

Furthermore of course one needs C, C++ and Fortran compilers to build Scipy, but those we don't consider to be dependencies and are therefore not discussed here. For details, see <http://scipy.org/scipylib/building/index.html>.

When a package provides useful functionality and it's proposed as a new dependency, consider also if it makes sense to vendor (i.e. ship a copy of it with scipy) the package instead. For example, `six` and `decorator` are vendored in `scipy._lib`.

The only dependency that is reported to `pip` is `Numpy`, see `install_requires` in Scipy's main `setup.py`. The other dependencies aren't needed for Scipy to function correctly, and the one unconditional build dependency that `pip` knows how to install (`Cython`) we prefer to treat like a compiler rather than a Python package that `pip` is allowed to upgrade.

### *Issues with dependency handling*

There are some serious issues with how Python packaging tools handle dependencies reported by projects. Because Scipy gets regular bug reports about this, we go in a bit of detail here.

Scipy only reports its dependency on Numpy via `install_requires` if Numpy isn't installed at all on a system. This will only change when there are either 32-bit and 64-bit Windows wheels for Numpy on PyPI or when `pip` upgrade becomes available (with sane behavior, unlike `pip install -U`, see [this PR](#)). For more details, see [this summary](#).

The situation with `setup_requires` is even worse; `pip` doesn't handle that keyword at all, while `setuptools` has issues (here's a [current one](#)) and invokes `easy_install` which comes with its own set of problems (note that Scipy doesn't support `easy_install` at all anymore; issues specific to it will be closed as "wontfix").

## Supported Python and Numpy versions

The Python versions that Scipy supports are listed in the list of PyPI classifiers in `setup.py`, and mentioned in the release notes for each release. All newly released Python versions will be supported as soon as possible. The general policy on dropping support for a Python version is that (a) usage of that version has to be quite low (say <5% of users) and (b) the version isn't included in an active long-term support release of one of the main Linux distributions anymore. Scipy typically follows Numpy, which has a similar policy. The final decision on dropping support is always taken on the `scipy-dev` mailing list.

The lowest supported Numpy version for a Scipy version is mentioned in the release notes and is encoded in `scipy/__init__.py` and the `install_requires` field of `setup.py`. Typically the latest Scipy release supports 3 or 4 minor versions of Numpy. That may become more if the frequency of Numpy releases increases (it's about 1x/year at the time of writing). Support for a particular Numpy version is typically dropped if (a) that Numpy version is several years old, and (b) the maintenance cost of keeping support is starting to outweigh the benefits. The final decision on dropping support is always taken on the `scipy-dev` mailing list.

Supported versions of optional dependencies and compilers is less clearly documented, and also isn't tested well or at all by Scipy's Continuous Integration setup. Issues regarding this are dealt with as they come up in the issue tracker or mailing list.

### Building binary installers

---

**Note:** This section is only about building Scipy binary installers to *distribute*. For info on building Scipy on the same machine as where it will be used, see [here](#).

---

There are a number of things to take into consideration when building binaries and distributing them on PyPI or elsewhere.

#### General

- A binary is specific for a single Python version (because different Python versions aren't ABI-compatible, at least up to Python 3.4).
- Build against the lowest Numpy version that you need to support, then it will work for all Numpy versions with the same major version number (Numpy does maintain backwards ABI compatibility).

#### Windows

- For 64-bit Windows installers built with a free toolchain, use the method documented at <https://github.com/numpy/numpy/wiki/Mingw-static-toolchain>. That method will likely be used for Scipy itself once it's clear that the maintenance of that toolchain is sustainable long-term. See the [MingwPy](#) project and [this thread](#) for details.
- The other way to produce 64-bit Windows installers is with `icc`, `ifort` plus MKL (or MSVC instead of `icc`). For Intel toolchain instructions see [here](#) and for (partial) MSVC instructions see [here](#).
- Older Scipy releases contained a `.exe` “superpack” installer. Those contain 3 complete builds (no SSE, SSE2, SSE3), and were built with <https://github.com/numpy/numpy-vendor>. That build setup is known to not work well anymore and is no longer supported. It used `g77` instead of `gfortran`, due to complex DLL distribution issues (see [gh-2829](#)). Because the toolchain is no longer supported, `g77` support isn't needed anymore and Scipy can now include Fortran 90/95 code.

#### OS X

- To produce OS X wheels that work with various Python versions (from [python.org](#), Homebrew, MacPython), use the build method provided by <https://github.com/MacPython/scipy-wheels>.
- DMG installers for the Python from [python.org](#) on OS X can still be produced by `tools/scipy-macosx-installer/`. Scipy doesn't distribute those installers anymore though, now that there are binary wheels on PyPi.

#### Linux

Besides PyPi not allowing Linux wheels (which is about to change with [PEP 513](#)), there are no specific issues with building binaries. To build a set of wheels for a Linux distribution and providing them in a [Wheelhouse](#), look at the wheel and [Wheelhouse](#) docs. A Wheelhouse for wheels compatible with TravisCI is <http://wheels.scipy.org>.

### 4.2.7 Making a Scipy release

At the highest level, this is what the release manager does to release a new Scipy version:

1. Propose a release schedule on the `scipy-dev` mailing list.
2. Create the maintenance branch for the release.
3. Tag the release.
4. Build all release artifacts (sources, installers, docs).
5. Upload the release artifacts.

6. Announce the release.
7. Port relevant changes to release notes and build scripts to master.

In this guide we attempt to describe in detail how to perform each of the above steps. In addition to those steps, which have to be performed by the release manager, here are descriptions of release-related activities and conventions of interest:

- [Backporting](#)
- [Labels and Milestones](#)
- [Version numbering](#)
- [Supported Python and Numpy versions](#)
- [Deprecations](#)

### Proposing a release schedule

A typical release cycle looks like:

- Create the maintenance branch
- Release a beta version
- Release a “release candidate” (RC)
- If needed, release one or more new RCs
- Release the final version once there are no issues with the last release candidate

There’s usually at least one week between each of the above steps. Experience shows that a cycle takes between 4 and 8 weeks for a new minor version. Bug-fix versions don’t need a beta or RC, and can be done much quicker.

Ideally the final release is identical to the last RC, however there may be minor difference - it’s up to the release manager to judge the risk of that. Typically, if compiled code or complex pure Python code changes then a new RC is needed, while a simple bug-fix that’s backported from master doesn’t require a new RC.

To propose a schedule, send a list with estimated dates for branching and beta/rc/final releases to `scipy-dev`. In the same email, ask everyone to check if there are important issues/PRs that need to be included and aren’t tagged with the Milestone for the release or the “backport-candidate” label.

### Creating the maintenance branch

Before branching, ensure that the release notes are updated as far as possible. Include the output of `tools/gh_lists.py` and `tools/authors.py` in the release notes.

Maintenance branches are named `maintenance/0.<minor-version>.x`. To create one, simply push a branch with the correct name to the `scipy` repo. Immediately after, push a commit where you increment the version number on the master branch and add release notes for that new version. Send an email to `scipy-dev` to let people know that you’ve done this.

### Tagging a release

First ensure that you have set up GPG correctly. See <https://github.com/scipy/scipy/issues/4919> for a discussion of signing release tags, and <http://keyring.debian.org/creating-key.html> for instructions on creating a GPG key if you do not have one.

To make your key more readily identifiable as you, consider sending your key to public key servers, with a command such as:

```
gpg --send-keys <yourkeyid>
```

Check that all relevant commits are in the branch. In particular, check issues and PRs under the Milestone for the release (<https://github.com/scipy/scipy/milestones>), PRs labeled “backport-candidate”, and that the release notes are up-to-date and included in the html docs.

Then edit `setup.py` to get the correct version number (set `ISRELEASED = True`) and commit it with a message like `REL: set version to <version-number>`. Don’t push this commit to the Scipy repo yet.

Finally tag the release locally with `git tag -s <v0.x.y>` (the `-s` ensures the tag is signed). Continue with building release artifacts (next section). Only push the release commit and tag to the scipy repo once you have built the docs and Windows installers successfully. After that push, also push a second commit which increment the version number and sets `ISRELEASED` to `False` again.

### Building release artifacts

Here is a complete list of artifacts created for a release:

- source archives (`.tar.gz`, `.zip` and `.tar.xz`)
- Binary wheels, for OS X only at the moment
- Documentation (html, pdf)
- A README file
- A Changelog file

All of these except the OS X wheel are built by running `paver release` in the repo root. Do this after you’ve created the signed tag. If this completes without issues, push the release tag to the scipy repo. This is needed because the OS X wheel build automatically builds the last tag.

To build wheels for OS X, push a commit to the master branch of <https://github.com/MacPython/scipy-wheels>. This triggers builds for all needed Python versions on TravisCI. Check in the `.travis.yml` config file what version of Python and Numpy are used for the builds (it needs to be the lowest supported Numpy version for each Python version). See the README file in the `scipy-wheels` repo for more details.

The TravisCI builds run the tests from the built wheels and if they pass upload the wheels to <http://wheels.scipy.org/>. From there you can copy them for uploading to PyPI (see next section).

### Uploading release artifacts

For a release there are currently five places on the web to upload things to:

- PyPI (tarballs, OS X wheels)
- Github releases (tarballs, release notes, Changelog)
- `scipy.org` (an announcement of the release)
- `docs.scipy.org` (html/pdf docs)

#### PyPI:

```
twine upload -s <tarballs or wheels to upload>
```

#### Github Releases:

Use GUI on <https://github.com/scipy/scipy/releases> to create release and upload all release artifacts.

**SourceForge:**

The main download sites are PyPI and Github Releases. Older releases are stored on SourceForce (<http://sourceforge.net/projects/scipy/files/scipy>). That download site has a “Latest” folder which redirects users to PyPI/GitHub, so it’s not needed to upload anything to SourceForge for new releases.

**scipy.org:**

Sources for the site are in <https://github.com/scipy/scipy.org>. Update the News section in `www/index.rst` and then do `make upload USERNAME=yourusername`.

**docs.scipy.org:**

First build the scipy docs, by running `make dist` in `scipy/doc/`. Verify that they look OK, then upload them to the doc server with `make upload USERNAME=rgommers RELEASE=0.17.0`. Note that SSH access to the doc server is needed; ask `@pv` (server admin) or `@rgommers` (can upload) if you don’t have that.

The sources for the website itself are maintained in <https://github.com/scipy/docs.scipy.org/>. Add the new Scipy version in the table of releases in `index.rst`. Push that commit, then do `make upload USERNAME=yourusername`.

**Wrapping up**

Send an email announcing the release to the following mailing lists:

- `scipy-dev`
- `scipy-user`
- `numpy-discussion`
- `python-announce` (not for beta/rc releases)

For beta and rc versions, ask people in the email to test (run the scipy tests and test against their own code) and report issues on Github or `scipy-dev`.

After the final release is done, port relevant changes to release notes, build scripts, author name mapping in `tools/authors.py` and any other changes that were only made on the maintenance branch to master.



## API REFERENCE

The exact API of all functions and classes, as given by the docstrings. The API documents expected types and allowed features for all functions, and all parameters available for the algorithms.

### 5.1 Clustering package (`scipy.cluster`)

`scipy.cluster.vq`

Clustering algorithms are useful in information theory, target detection, communications, compression, and other areas. The `vq` module only supports vector quantization and the k-means algorithms.

`scipy.cluster.hierarchy`

The `hierarchy` module provides functions for hierarchical and agglomerative clustering. Its features include generating hierarchical clusters from distance matrices, calculating statistics on clusters, cutting linkages to generate flat clusters, and visualizing clusters with dendrograms.

### 5.2 K-means clustering and vector quantization (`scipy.cluster.vq`)

Provides routines for k-means clustering, generating code books from k-means models, and quantizing vectors by comparing them with centroids in a code book.

---

<code>whiten(obs[, check_finite])</code>	Normalize a group of observations on a per feature basis.
<code>vq(obs, code_book[, check_finite])</code>	Assign codes from a code book to observations.
<code>kmeans(obs, k_or_guess[, iter, thresh, ...])</code>	Performs k-means on a set of observation vectors forming k clusters.
<code>kmeans2(data, k[, iter, thresh, minit, ...])</code>	Classify a set of observations into k clusters using the k-means algorithm.

---

`scipy.cluster.vq.whiten(obs, check_finite=True)`

Normalize a group of observations on a per feature basis.

Before running k-means, it is beneficial to rescale each feature dimension of the observation set with whitening. Each feature is divided by its standard deviation across all observations to give it unit variance.

**Parameters** `obs` : ndarray

Each row of the array is an observation. The columns are the features seen during each observation.

```
>>> #           f0      f1      f2
>>> obs = [[ 1.,    1.,    1.], #o0
...       [ 2.,    2.,    2.], #o1
```

```

...         [ 3.,  3.,  3.], #o2
...         [ 4.,  4.,  4.]] #o3
    
```

**check\_finite** : bool, optional

Whether to check that the input matrices contain only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs. Default: True

**Returns**

**result** : ndarray

Contains the values in *obs* scaled by the standard deviation of each column.

### Examples

```

>>> from scipy.cluster.vq import whiten
>>> features = np.array([[1.9, 2.3, 1.7],
...                     [1.5, 2.5, 2.2],
...                     [0.8, 0.6, 1.7,]])
>>> whiten(features)
array([[ 4.17944278,  2.69811351,  7.21248917],
       [ 3.29956009,  2.93273208,  9.33380951],
       [ 1.75976538,  0.7038557 ,  7.21248917]])
    
```

`scipy.cluster.vq.vq(obs, code_book, check_finite=True)`

Assign codes from a code book to observations.

Assigns a code from a code book to each observation. Each observation vector in the ‘M’ by ‘N’ *obs* array is compared with the centroids in the code book and assigned the code of the closest centroid.

The features in *obs* should have unit variance, which can be achieved by passing them through the `whiten` function. The code book can be created with the k-means algorithm or a different encoding algorithm.

**Parameters** **obs** : ndarray

Each row of the ‘M’ x ‘N’ array is an observation. The columns are the “features” seen during each observation. The features must be whitened first using the `whiten` function or something equivalent.

**code\_book** : ndarray

The code book is usually generated using the k-means algorithm. Each row of the array holds a different code, and the columns are the features of the code.

```

>>> #           f0    f1    f2    f3
>>> code_book = [
...         [ 1.,  2.,  3.,  4.], #c0
...         [ 1.,  2.,  3.,  4.], #c1
...         [ 1.,  2.,  3.,  4.]] #c2
    
```

**check\_finite** : bool, optional

Whether to check that the input matrices contain only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs. Default: True

**Returns**

**code** : ndarray

A length M array holding the code book index for each observation.

**dist** : ndarray

The distortion (distance) between the observation and its nearest code.

### Examples

```

>>> from numpy import array
>>> from scipy.cluster.vq import vq
>>> code_book = array([[1., 1., 1.],
...                  [2., 2., 2.]])
    
```



```
>>> features = array([[ 1.9, 2.3, 1.7],
...                  [ 1.5, 2.5, 2.2],
...                  [ 0.8, 0.6, 1.7]])
>>> vq(features, code_book)
(array([1, 1, 0], 'i'), array([ 0.43588989,  0.73484692,  0.83066239]))
```

`scipy.cluster.vq.kmeans` (*obs*, *k\_or\_guess*, *iter=20*, *thresh=1e-05*, *check\_finite=True*)

Performs k-means on a set of observation vectors forming k clusters.

The k-means algorithm adjusts the centroids until sufficient progress cannot be made, i.e. the change in distortion since the last iteration is less than some threshold. This yields a code book mapping centroids to codes and vice versa.

Distortion is defined as the sum of the squared differences between the observations and the corresponding centroid.

**Parameters** **obs** : ndarray

Each row of the M by N array is an observation vector. The columns are the features seen during each observation. The features must be whitened first with the `whiten` function.

**k\_or\_guess** : int or ndarray

The number of centroids to generate. A code is assigned to each centroid, which is also the row index of the centroid in the `code_book` matrix generated.

The initial k centroids are chosen by randomly selecting observations from the observation matrix. Alternatively, passing a k by N array specifies the initial k centroids.

**iter** : int, optional

The number of times to run k-means, returning the codebook with the lowest distortion. This argument is ignored if initial centroids are specified with an array for the `k_or_guess` parameter. This parameter does not represent the number of iterations of the k-means algorithm.

**thresh** : float, optional

Terminates the k-means algorithm if the change in distortion since the last k-means iteration is less than or equal to `thresh`.

**check\_finite** : bool, optional

Whether to check that the input matrices contain only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs. Default: True

**Returns**

**codebook** : ndarray

A k by N array of k centroids. The i`th centroid codebook[i] is represented with the code i. The centroids and codes generated represent the lowest distortion seen, not necessarily the globally minimal distortion.

**distortion** : float

The distortion between the observations passed and the centroids generated.

**See also:**

`kmeans2` a different implementation of k-means clustering with more methods for generating initial centroids but without using a distortion change threshold as a stopping criterion.

`whiten` must be called prior to passing an observation matrix to `kmeans`.

**Examples**

```
>>> from numpy import array
>>> from scipy.cluster.vq import vq, kmeans, whiten
>>> features = array([[ 1.9, 2.3],
...                  [ 1.5, 2.5],
```

```

...         [ 0.8,0.6],
...         [ 0.4,1.8],
...         [ 0.1,0.1],
...         [ 0.2,1.8],
...         [ 2.0,0.5],
...         [ 0.3,1.5],
...         [ 1.0,1.0]])
>>> whitened = whiten(features)
>>> book = array((whitened[0],whitened[2]))
>>> kmeans(whitened,book)
(array([[ 2.3110306 ,  2.86287398],          # random
        [ 0.93218041,  1.24398691]]), 0.85684700941625547)

>>> from numpy import random
>>> random.seed((1000,2000))
>>> codes = 3
>>> kmeans(whitened,codes)
(array([[ 2.3110306 ,  2.86287398],          # random
        [ 1.32544402,  0.65607529],
        [ 0.40782893,  2.02786907]]), 0.5196582527686241)
    
```

`scipy.cluster.vq.kmeans2`(*data*, *k*, *iter=10*, *thresh=1e-05*, *minit='random'*, *missing='warn'*, *check\_finite=True*)

Classify a set of observations into *k* clusters using the k-means algorithm.

The algorithm attempts to minimize the Euclidian distance between observations and centroids. Several initialization methods are included.

**Parameters**

**data** : ndarray  
A 'M' by 'N' array of 'M' observations in 'N' dimensions or a length 'M' array of 'M' one-dimensional observations.

**k** : int or ndarray  
The number of clusters to form as well as the number of centroids to generate. If *minit* initialization string is 'matrix', or if a ndarray is given instead, it is interpreted as initial cluster to use instead.

**iter** : int, optional  
Number of iterations of the k-means algorithm to run. Note that this differs in meaning from the *iters* parameter to the `kmeans` function.

**thresh** : float, optional  
(not used yet)

**minit** : str, optional  
Method for initialization. Available methods are 'random', 'points', and 'matrix':  
'random': generate *k* centroids from a Gaussian with mean and variance estimated from the data.  
'points': choose *k* observations (rows) at random from data for the initial centroids.  
'matrix': interpret the *k* parameter as a *k* by *M* (or length *k* array for one-dimensional data) array of initial centroids.

**missing** : str, optional  
Method to deal with empty clusters. Available methods are 'warn' and 'raise':  
'warn': give a warning and continue.  
'raise': raise an `ClusterError` and terminate the algorithm.

**check\_finite** : bool, optional  
Whether to check that the input matrices contain only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs. Default: True

**Returns**

**centroid** : ndarray

A 'k' by 'N' array of centroids found at the last iteration of k-means.  
**label** : ndarray  
 label[i] is the code or index of the centroid the i'th observation is closest to.

## 5.2.1 Background information

The k-means algorithm takes as input the number of clusters to generate,  $k$ , and a set of observation vectors to cluster. It returns a set of centroids, one for each of the  $k$  clusters. An observation vector is classified with the cluster number or centroid index of the centroid closest to it.

A vector  $v$  belongs to cluster  $i$  if it is closer to centroid  $i$  than any other centroids. If  $v$  belongs to  $i$ , we say centroid  $i$  is the dominating centroid of  $v$ . The k-means algorithm tries to minimize distortion, which is defined as the sum of the squared distances between each observation vector and its dominating centroid. Each step of the k-means algorithm refines the choices of centroids to reduce distortion. The change in distortion is used as a stopping criterion: when the change is lower than a threshold, the k-means algorithm is not making sufficient progress and terminates. One can also define a maximum number of iterations.

Since vector quantization is a natural application for k-means, information theory terminology is often used. The centroid index or cluster index is also referred to as a "code" and the table mapping codes to centroids and vice versa is often referred as a "code book". The result of k-means, a set of centroids, can be used to quantize vectors. Quantization aims to find an encoding of vectors that reduces the expected distortion.

All routines expect `obs` to be a  $M$  by  $N$  array where the rows are the observation vectors. The codebook is a  $k$  by  $N$  array where the  $i$ 'th row is the centroid of code word  $i$ . The observation vectors and centroids have the same feature dimension.

As an example, suppose we wish to compress a 24-bit color image (each pixel is represented by one byte for red, one for blue, and one for green) before sending it over the web. By using a smaller 8-bit encoding, we can reduce the amount of data by two thirds. Ideally, the colors for each of the 256 possible 8-bit encoding values should be chosen to minimize distortion of the color. Running k-means with  $k=256$  generates a code book of 256 codes, which fills up all possible 8-bit sequences. Instead of sending a 3-byte value for each pixel, the 8-bit centroid index (or code word) of the dominating centroid is transmitted. The code book is also sent over the wire so each 8-bit code can be translated back to a 24-bit pixel value representation. If the image of interest was of an ocean, we would expect many 24-bit blues to be represented by 8-bit codes. If it was an image of a human face, more flesh tone colors would be represented in the code book.

## 5.3 Hierarchical clustering (`scipy.cluster.hierarchy`)

These functions cut hierarchical clusterings into flat clusterings or find the roots of the forest formed by a cut by providing the flat cluster ids of each observation.

<code>fcluster(Z, t[, criterion, depth, R, monocrit])</code>	Forms flat clusters from the hierarchical clustering defined by the linkage matrix $Z$ .
<code>fclusterdata(X, t[, criterion, metric, ...])</code>	Cluster observation data using a given metric.
<code>leaders(Z, T)</code>	Returns the root nodes in a hierarchical clustering.

`scipy.cluster.hierarchy.fcluster` ( $Z$ ,  $t$ , *criteria*='inconsistent', *depth*=2, *R*=None, *monocrit*=None)

Forms flat clusters from the hierarchical clustering defined by the linkage matrix  $Z$ .

**Parameters**  $Z$  : ndarray

The hierarchical clustering encoded with the matrix returned by the `linkage` function.

$t$  : float

The threshold to apply when forming flat clusters.

**criterion** : str, optional

The criterion to use in forming flat clusters. This can be any of the following values:

***inconsistent***

[If a cluster node and all its] descendants have an inconsistent value less than or equal to  $t$  then all its leaf descendants belong to the same flat cluster. When no non-singleton cluster meets this criterion, every node is assigned to its own cluster. (Default)

***distance***

[Forms flat clusters so that the original] observations in each flat cluster have no greater a cophenetic distance than  $t$ .

***maxclust***

[Finds a minimum threshold  $r$  so that] the cophenetic distance between any two original observations in the same flat cluster is no more than  $r$  and no more than  $t$  flat clusters are formed.

***monocrit***

[Forms a flat cluster from a cluster node  $c$ ] with index  $i$  when  $\text{monocrit}[j] \leq t$ .

For example, to threshold on the maximum mean distance as computed in the inconsistency matrix  $R$  with a threshold of 0.8 do:

```
MR = maxRstat(Z, R, 3)
cluster(Z, t=0.8, criterion='monocrit', monocrit=MR)
```

***maxclust\_monocrit***

[Forms a flat cluster from a] non-singleton cluster node  $c$  when  $\text{monocrit}[i] \leq r$  for all cluster indices  $i$  below and including  $c$ .  $r$  is minimized such that no more than  $t$  flat clusters are formed.  $\text{monocrit}$  must be monotonic. For example, to minimize the threshold  $t$  on maximum inconsistency values so that no more than 3 flat clusters are formed, do:

```
MI = maxinconsts(Z, R)
cluster(Z, t=3, criterion='maxclust_monocrit', monocrit=MI)
```

**depth** : int, optional

The maximum depth to perform the inconsistency calculation. It has no meaning for the other criteria. Default is 2.

**R** : ndarray, optional

The inconsistency matrix to use for the 'inconsistent' criterion. This matrix is computed if not provided.

**monocrit** : ndarray, optional

An array of length  $n-1$ .  $\text{monocrit}[i]$  is the statistics upon which non-singleton  $i$  is thresholded. The  $\text{monocrit}$  vector must be monotonic, i.e. given a node  $c$  with index  $i$ , for all node indices  $j$  corresponding to nodes below  $c$ ,  $\text{monocrit}[i] \geq \text{monocrit}[j]$ .

**Returns**

**fcluster** : ndarray

An array of length  $n$ .  $T[i]$  is the flat cluster number to which original observation  $i$  belongs.

```
scipy.cluster.hierarchy.fclusterdata(X, t, criterion='inconsistent', metric='euclidean',
                                     depth=2, method='single', R=None)
```

Cluster observation data using a given metric.

Clusters the original observations in the  $n$ -by- $m$  data matrix  $X$  ( $n$  observations in  $m$  dimensions), using the euclidean distance metric to calculate distances between original observations, performs hierarchical clustering using the single linkage algorithm, and forms flat clusters using the inconsistency method with  $t$  as the cut-off threshold.

A one-dimensional array  $T$  of length  $n$  is returned.  $T[i]$  is the index of the flat cluster to which the original observation  $i$  belongs.

**Parameters**

- X** : (N, M) ndarray  
N by M data matrix with N observations in M dimensions.
- t** : float  
The threshold to apply when forming flat clusters.
- criterion** : str, optional  
Specifies the criterion for forming flat clusters. Valid values are ‘inconsistent’ (default), ‘distance’, or ‘maxclust’ cluster formation algorithms. See `fcluster` for descriptions.
- metric** : str, optional  
The distance metric for calculating pairwise distances. See `distance.pdist` for descriptions and linkage to verify compatibility with the linkage method.
- depth** : int, optional  
The maximum depth for the inconsistency calculation. See `inconsistent` for more information.
- method** : str, optional  
The linkage method to use (single, complete, average, weighted, median centroid, ward). See `linkage` for more information. Default is “single”.
- R** : ndarray, optional  
The inconsistency matrix. It will be computed if necessary if it is not passed.

**Returns**

- fclusterdata** : ndarray  
A vector of length n.  $T[i]$  is the flat cluster number to which original observation  $i$  belongs.

**Notes**

This function is similar to the MATLAB function `clusterdata`.

`scipy.cluster.hierarchy.leadere`s( $Z, T$ )

Returns the root nodes in a hierarchical clustering.

Returns the root nodes in a hierarchical clustering corresponding to a cut defined by a flat cluster assignment vector  $T$ . See the `fcluster` function for more information on the format of  $T$ .

For each flat cluster  $j$  of the  $k$  flat clusters represented in the  $n$ -sized flat cluster assignment vector  $T$ , this function finds the lowest cluster node  $i$  in the linkage tree  $Z$  such that:

- leaf descendents belong only to flat cluster  $j$  (i.e.  $T[p] == j$  for all  $p$  in  $S(i)$  where  $S(i)$  is the set of leaf ids of leaf nodes descendent with cluster node  $i$ )
- there does not exist a leaf that is not descendent with  $i$  that also belongs to cluster  $j$  (i.e.  $T[q] != j$  for all  $q$  not in  $S(i)$ ). If this condition is violated,  $T$  is not a valid cluster assignment vector, and an exception will be thrown.

**Parameters**

- Z** : ndarray  
The hierarchical clustering encoded as a matrix. See `linkage` for more information.
- T** : ndarray  
The flat cluster assignment vector.

**Returns**

- L** : ndarray  
The leader linkage node id’s stored as a  $k$ -element 1-D array where  $k$  is the number of flat clusters found in  $T$ .  
 $L[j] = i$  is the linkage cluster node id that is the leader of flat cluster with id  $M[j]$ . If  $i < n$ ,  $i$  corresponds to an original observation, otherwise it corresponds to a non-singleton cluster.  
For example: if  $L[3] = 2$  and  $M[3] = 8$ , the flat cluster with id 8’s leader is linkage node 2.
- M** : ndarray

The leader linkage node id's stored as a k-element 1-D array where k is the number of flat clusters found in  $\mathbb{T}$ . This allows the set of flat cluster ids to be any arbitrary set of k integers.

These are routines for agglomerative clustering.

<code>linkage(y[, method, metric])</code>	Performs hierarchical/agglomerative clustering on the condensed distance matrix $y$ .
<code>single(y)</code>	Performs single/min/nearest linkage on the condensed distance matrix $y$
<code>complete(y)</code>	Performs complete/max/farthest point linkage on a condensed distance matrix
<code>average(y)</code>	Performs average/UPGMA linkage on a condensed distance matrix
<code>weighted(y)</code>	Performs weighted/WPGMA linkage on the condensed distance matrix.
<code>centroid(y)</code>	Performs centroid/UPGMC linkage.
<code>median(y)</code>	Performs median/WPGMC linkage.
<code>ward(y)</code>	Performs Ward's linkage on a condensed or redundant distance matrix.

`scipy.cluster.hierarchy.linkage(y, method='single', metric='euclidean')`

Performs hierarchical/agglomerative clustering on the condensed distance matrix  $y$ .

$y$  must be a  $\binom{n}{2}$  sized vector where  $n$  is the number of original observations paired in the distance matrix. The behavior of this function is very similar to the MATLAB linkage function.

An  $(n - 1)$  by 4 matrix  $Z$  is returned. At the  $i$ -th iteration, clusters with indices  $Z[i, 0]$  and  $Z[i, 1]$  are combined to form cluster  $n + i$ . A cluster with an index less than  $n$  corresponds to one of the  $n$  original observations. The distance between clusters  $Z[i, 0]$  and  $Z[i, 1]$  is given by  $Z[i, 2]$ . The fourth value  $Z[i, 3]$  represents the number of original observations in the newly formed cluster.

The following linkage methods are used to compute the distance  $d(s, t)$  between two clusters  $s$  and  $t$ . The algorithm begins with a forest of clusters that have yet to be used in the hierarchy being formed. When two clusters  $s$  and  $t$  from this forest are combined into a single cluster  $u$ ,  $s$  and  $t$  are removed from the forest, and  $u$  is added to the forest. When only one cluster remains in the forest, the algorithm stops, and this cluster becomes the root.

A distance matrix is maintained at each iteration. The  $d[i, j]$  entry corresponds to the distance between cluster  $i$  and  $j$  in the original forest.

At each iteration, the algorithm must update the distance matrix to reflect the distance of the newly formed cluster  $u$  with the remaining clusters in the forest.

Suppose there are  $|u|$  original observations  $u[0], \dots, u[|u| - 1]$  in cluster  $u$  and  $|v|$  original objects  $v[0], \dots, v[|v| - 1]$  in cluster  $v$ . Recall  $s$  and  $t$  are combined to form cluster  $u$ . Let  $v$  be any remaining cluster in the forest that is not  $u$ .

The following are methods for calculating the distance between the newly formed cluster  $u$  and each  $v$ .

- `method='single'` assigns

$$d(u, v) = \min(\text{dist}(u[i], v[j]))$$

for all points  $i$  in cluster  $u$  and  $j$  in cluster  $v$ . This is also known as the Nearest Point Algorithm.

- `method='complete'` assigns

$$d(u, v) = \max(\text{dist}(u[i], v[j]))$$

for all points  $i$  in cluster  $u$  and  $j$  in cluster  $v$ . This is also known by the Farthest Point Algorithm or Voor Hees Algorithm.

- `method='average'` assigns

$$d(u, v) = \sum_{ij} \frac{d(u[i], v[j])}{(|u| * |v|)}$$

for all points  $i$  and  $j$  where  $|u|$  and  $|v|$  are the cardinalities of clusters  $u$  and  $v$ , respectively. This is also called the UPGMA algorithm.

- `method='weighted'` assigns

$$d(u, v) = (dist(s, v) + dist(t, v))/2$$

where cluster  $u$  was formed with cluster  $s$  and  $t$  and  $v$  is a remaining cluster in the forest. (also called WPGMA)

- `method='centroid'` assigns

$$dist(s, t) = \|c_s - c_t\|_2$$

where  $c_s$  and  $c_t$  are the centroids of clusters  $s$  and  $t$ , respectively. When two clusters  $s$  and  $t$  are combined into a new cluster  $u$ , the new centroid is computed over all the original objects in clusters  $s$  and  $t$ . The distance then becomes the Euclidean distance between the centroid of  $u$  and the centroid of a remaining cluster  $v$  in the forest. This is also known as the UPGMC algorithm.

- `method='median'` assigns  $d(s, t)$  like the `centroid` method. When two clusters  $s$  and  $t$  are combined into a new cluster  $u$ , the average of centroids  $s$  and  $t$  give the new centroid  $u$ . This is also known as the WPGMC algorithm.

- `method='ward'` uses the Ward variance minimization algorithm. The new entry  $d(u, v)$  is computed as follows,

$$d(u, v) = \sqrt{\frac{|v| + |s|}{T} d(v, s)^2 + \frac{|v| + |t|}{T} d(v, t)^2 - \frac{|v|}{T} d(s, t)^2}$$

where  $u$  is the newly joined cluster consisting of clusters  $s$  and  $t$ ,  $v$  is an unused cluster in the forest,  $T = |v| + |s| + |t|$ , and  $| * |$  is the cardinality of its argument. This is also known as the incremental algorithm.

Warning: When the minimum distance pair in the forest is chosen, there may be two or more pairs with the same minimum distance. This implementation may chose a different minimum than the MATLAB version.

**Parameters** `y` : ndarray

A condensed or redundant distance matrix. A condensed distance matrix is a flat array containing the upper triangular of the distance matrix. This is the form that `pdist` returns. Alternatively, a collection of  $m$  observation vectors in  $n$  dimensions may be passed as an  $m$  by  $n$  array.

**method** : str, optional

The linkage algorithm to use. See the `Linkage Methods` section below for full descriptions.

**metric** : str or function, optional

The distance metric to use in the case that `y` is a collection of observation vectors; ignored otherwise. See the `distance.pdist` function for a list of valid distance metrics. A custom distance function can also be used. See the `distance.pdist` function for details.

**Returns** `Z` : ndarray

The hierarchical clustering encoded as a linkage matrix.

### Notes

1. For method 'single' an optimized algorithm called SLINK is implemented, which has  $O(n^2)$  time complexity. For methods 'complete', 'average', 'weighted' and 'ward' an algorithm called nearest-neighbors chain is implemented, which too has time complexity  $O(n^2)$ . For other methods a naive algorithm is implemented with  $O(n^3)$  time complexity. All algorithms use  $O(n^2)$  memory. Refer to [R38] for details about the algorithms.
2. Methods 'centroid', 'median' and 'ward' are correctly defined only if Euclidean pairwise metric is used. If `y` is passed as precomputed pairwise distances, then it is a user responsibility to assure that these distances are in fact Euclidean, otherwise the produced result will be incorrect.

### References

[R38]

`scipy.cluster.hierarchy.single`(`y`)

Performs single/min/nearest linkage on the condensed distance matrix `y`

**Parameters** `y`: ndarray  
The upper triangular of the distance matrix. The result of `pdist` is returned in this form.

**Returns** `Z`: ndarray  
The linkage matrix.

**See also:**

[linkage](#) for advanced creation of hierarchical clusterings.

`scipy.cluster.hierarchy.complete`(`y`)

Performs complete/max/farthest point linkage on a condensed distance matrix

**Parameters** `y`: ndarray  
The upper triangular of the distance matrix. The result of `pdist` is returned in this form.

**Returns** `Z`: ndarray  
A linkage matrix containing the hierarchical clustering. See the `linkage` function documentation for more information on its structure.

**See also:**

[linkage](#)

`scipy.cluster.hierarchy.average`(`y`)

Performs average/UPGMA linkage on a condensed distance matrix

**Parameters** `y`: ndarray  
The upper triangular of the distance matrix. The result of `pdist` is returned in this form.

**Returns** `Z`: ndarray  
A linkage matrix containing the hierarchical clustering. See the `linkage` function documentation for more information on its structure.

**See also:**

[linkage](#) for advanced creation of hierarchical clusterings.

`scipy.cluster.hierarchy.weighted`(`y`)

Performs weighted/WPGMA linkage on the condensed distance matrix.



See `linkage` for more information on the return structure and algorithm.

**Parameters** `y` : ndarray  
The upper triangular of the distance matrix. The result of `pdist` is returned in this form.

**Returns** `Z` : ndarray  
A linkage matrix containing the hierarchical clustering. See the `linkage` function documentation for more information on its structure.

**See also:**

[`linkage`](#) for advanced creation of hierarchical clusterings.

`scipy.cluster.hierarchy.centroid`(`y`)

Performs centroid/UPGMC linkage.

See `linkage` for more information on the return structure and algorithm.

The following are common calling conventions:

1.`Z = centroid(y)`

Performs centroid/UPGMC linkage on the condensed distance matrix `y`. See `linkage` for more information on the return structure and algorithm.

2.`Z = centroid(X)`

Performs centroid/UPGMC linkage on the observation matrix `X` using Euclidean distance as the distance metric. See `linkage` for more information on the return structure and algorithm.

**Parameters** `y` : ndarray  
A condensed or redundant distance matrix. A condensed distance matrix is a flat array containing the upper triangular of the distance matrix. This is the form that `pdist` returns. Alternatively, a collection of `m` observation vectors in `n` dimensions may be passed as a `m` by `n` array.

**Returns** `Z` : ndarray  
A linkage matrix containing the hierarchical clustering. See the `linkage` function documentation for more information on its structure.

**See also:**

[`linkage`](#) for advanced creation of hierarchical clusterings.

`scipy.cluster.hierarchy.median`(`y`)

Performs median/WPGMC linkage.

See `linkage` for more information on the return structure and algorithm.

The following are common calling conventions:

1.`Z = median(y)`

Performs median/WPGMC linkage on the condensed distance matrix `y`. See `linkage` for more information on the return structure and algorithm.

2.`Z = median(X)`

Performs median/WPGMC linkage on the observation matrix `X` using Euclidean distance as the distance metric. See `linkage` for more information on the return structure and algorithm.

**Parameters** `y` : ndarray

A condensed or redundant distance matrix. A condensed distance matrix is a flat array containing the upper triangular of the distance matrix. This is the form that `pdist` returns. Alternatively, a collection of  $m$  observation vectors in  $n$  dimensions may be passed as a  $m$  by  $n$  array.

**Returns** **Z**: ndarray  
The hierarchical clustering encoded as a linkage matrix.

**See also:**

[linkage](#) for advanced creation of hierarchical clusterings.

`scipy.cluster.hierarchy.ward(y)`

Performs Ward's linkage on a condensed or redundant distance matrix.

See [linkage](#) for more information on the return structure and algorithm.

The following are common calling conventions:

1. `Z = ward(y)` Performs Ward's linkage on the condensed distance matrix  $Z$ . See [linkage](#) for more information on the return structure and algorithm.
2. `Z = ward(X)` Performs Ward's linkage on the observation matrix  $X$  using Euclidean distance as the distance metric. See [linkage](#) for more information on the return structure and algorithm.

**Parameters** **y**: ndarray  
A condensed or redundant distance matrix. A condensed distance matrix is a flat array containing the upper triangular of the distance matrix. This is the form that `pdist` returns. Alternatively, a collection of  $m$  observation vectors in  $n$  dimensions may be passed as a  $m$  by  $n$  array.

**Returns** **Z**: ndarray  
The hierarchical clustering encoded as a linkage matrix.

**See also:**

[linkage](#) for advanced creation of hierarchical clusterings.

These routines compute statistics on hierarchies.

---

<code>cophenet(Z[, Y])</code>	Calculates the cophenetic distances between each observation in the hierarchical clustering defined by
<code>from_mlab_linkage(Z)</code>	Converts a linkage matrix generated by MATLAB(TM) to a new linkage matrix compatible with this
<code>inconsistent(Z[, d])</code>	Calculates inconsistency statistics on a linkage.
<code>maxinconsts(Z, R)</code>	Returns the maximum inconsistency coefficient for each non-singleton cluster and its descendents.
<code>maxdists(Z)</code>	Returns the maximum distance between any non-singleton cluster.
<code>maxRstat(Z, R, i)</code>	Returns the maximum statistic for each non-singleton cluster and its descendents.
<code>to_mlab_linkage(Z)</code>	Converts a linkage matrix to a MATLAB(TM) compatible one.

---

`scipy.cluster.hierarchy.cophenet(Z, Y=None)`

Calculates the cophenetic distances between each observation in the hierarchical clustering defined by the linkage  $Z$ .

Suppose  $p$  and  $q$  are original observations in disjoint clusters  $s$  and  $t$ , respectively and  $s$  and  $t$  are joined by a direct parent cluster  $u$ . The cophenetic distance between observations  $i$  and  $j$  is simply the distance between clusters  $s$  and  $t$ .

**Parameters** **Z**: ndarray  
The hierarchical clustering encoded as an array (see [linkage](#) function).

**Y** : ndarray (optional)

Calculates the cophenetic correlation coefficient  $c$  of a hierarchical clustering defined by the linkage matrix  $Z$  of a set of  $n$  observations in  $m$  dimensions.  $Y$  is the condensed distance matrix from which  $Z$  was generated.

**Returns**

**c** : ndarray

The cophenetic correlation distance (if  $y$  is passed).

**d** : ndarray

The cophenetic distance matrix in condensed form. The  $ij$  th entry is the cophenetic distance between original observations  $i$  and  $j$ .

`scipy.cluster.hierarchy.from_mlab_linkage` ( $Z$ )

Converts a linkage matrix generated by MATLAB(TM) to a new linkage matrix compatible with this module.

The conversion does two things:

- the indices are converted from  $1 \dots N$  to  $0 \dots (N-1)$  form, and
- a fourth column  $Z[:,3]$  is added where  $Z[i,3]$  represents the number of original observations (leaves) in the non-singleton cluster  $i$ .

This function is useful when loading in linkages from legacy data files generated by MATLAB.

**Parameters** **Z** : ndarray

A linkage matrix generated by MATLAB(TM).

**Returns** **ZS** : ndarray

A linkage matrix compatible with this library.

`scipy.cluster.hierarchy.inconsistent` ( $Z, d=2$ )

Calculates inconsistency statistics on a linkage.

Note: This function behaves similarly to the MATLAB(TM) inconsistent function.

**Parameters** **Z** : ndarray

The  $(n-1)$  by 4 matrix encoding the linkage (hierarchical clustering). See `linkage` documentation for more information on its form.

**d** : int, optional

The number of links up to  $d$  levels below each non-singleton cluster.

**Returns**

**R** : ndarray

A  $(n-1)$  by 5 matrix where the  $i$ 'th row contains the link statistics for the non-singleton cluster  $i$ . The link statistics are computed over the link heights for links  $d$  levels below the cluster  $i$ .  $R[i, 0]$  and  $R[i, 1]$  are the mean and standard deviation of the link heights, respectively;  $R[i, 2]$  is the number of links included in the calculation; and  $R[i, 3]$  is the inconsistency coefficient,

$$\frac{Z[i, 2] - R[i, 0]}{R[i, 1]}$$

`scipy.cluster.hierarchy.maxinconsts` ( $Z, R$ )

Returns the maximum inconsistency coefficient for each non-singleton cluster and its descendents.

**Parameters** **Z** : ndarray

The hierarchical clustering encoded as a matrix. See `linkage` for more information.

**R** : ndarray

The inconsistency matrix.

**Returns**

**MI** : ndarray

A monotonic  $(n-1)$ -sized numpy array of doubles.

`scipy.cluster.hierarchy.maxdists` ( $Z$ )

Returns the maximum distance between any non-singleton cluster.

**Parameters** **Z** : ndarray

**Returns** **maxdists** : ndarray  
 The hierarchical clustering encoded as a matrix. See `linkage` for more information.  
 A  $(n-1)$  sized numpy array of doubles;  $MD[i]$  represents the maximum distance between any cluster (including singletons) below and including the node with index  $i$ . More specifically,  $MD[i] = Z[Q(i)-n, 2].max()$  where  $Q(i)$  is the set of all node indices below and including node  $i$ .

`scipy.cluster.hierarchy.maxRstat(Z, R, i)`

Returns the maximum statistic for each non-singleton cluster and its descendents.

**Parameters** **Z** : array\_like  
 The hierarchical clustering encoded as a matrix. See `linkage` for more information.  
**R** : array\_like  
 The inconsistency matrix.  
**i** : int  
 The column of  $R$  to use as the statistic.  
**Returns** **MR** : ndarray  
 Calculates the maximum statistic for the  $i$ 'th column of the inconsistency matrix  $R$  for each non-singleton cluster node.  $MR[j]$  is the maximum over  $R[Q(j)-n, i]$  where  $Q(j)$  the set of all node ids corresponding to nodes below and including  $j$ .

`scipy.cluster.hierarchy.to_mlab_linkage(Z)`

Converts a linkage matrix to a MATLAB(TM) compatible one.

Converts a linkage matrix  $Z$  generated by the linkage function of this module to a MATLAB(TM) compatible one. The return linkage matrix has the last column removed and the cluster indices are converted to  $1..N$  indexing.

**Parameters** **Z** : ndarray  
 A linkage matrix generated by this library.  
**Returns** **to\_mlab\_linkage** : ndarray  
 A linkage matrix compatible with MATLAB(TM)'s hierarchical clustering functions. The return linkage matrix has the last column removed and the cluster indices are converted to  $1..N$  indexing.

Routines for visualizing flat clusters.

---

`dendrogram(Z[, p, truncate_mode, ...])` Plots the hierarchical clustering as a dendrogram.

---

`scipy.cluster.hierarchy.dendrogram(Z, p=30, truncate_mode=None, color_threshold=None, get_leaves=True, orientation='top', labels=None, count_sort=False, distance_sort=False, show_leaf_counts=True, no_plot=False, no_labels=False, leaf_font_size=None, leaf_rotation=None, leaf_label_func=None, show_contracted=False, link_color_func=None, ax=None, above_threshold_color='b')`

Plots the hierarchical clustering as a dendrogram.

The dendrogram illustrates how each cluster is composed by drawing a U-shaped link between a non-singleton cluster and its children. The height of the top of the U-link is the distance between its children clusters. It is also the cophenetic distance between original observations in the two children clusters. It is expected that the distances in  $Z[:,2]$  be monotonic, otherwise crossings appear in the dendrogram.

**Parameters** **Z** : ndarray  
 The linkage matrix encoding the hierarchical clustering to render as a dendrogram. See the `linkage` function for more information on the format of  $Z$ .

**p** : int, optional

The `p` parameter for `truncate_mode`.

**truncate\_mode** : str, optional

The dendrogram can be hard to read when the original observation matrix from which the linkage is derived is large. Truncation is used to condense the dendrogram. There are several modes:

***None***/***'none'***

No truncation is performed (Default).

***'lastp'*** The last `p` non-singleton formed in the linkage are the only non-leaf nodes in the linkage; they correspond to rows `Z[n-p-2:end]` in `Z`. All other non-singleton clusters are contracted into leaf nodes.

***'mlab'*** This corresponds to MATLAB(TM) behavior. (not implemented yet)

***'level'***/***'mtica'***

No more than `p` levels of the dendrogram tree are displayed. This corresponds to Mathematica(TM) behavior.

**color\_threshold** : double, optional

For brevity, let  $t$  be the `color_threshold`. Colors all the descendent links below a cluster node  $k$  the same color if  $k$  is the first node below the cut threshold  $t$ . All links connecting nodes with distances greater than or equal to the threshold are colored blue. If  $t$  is less than or equal to zero, all nodes are colored blue. If `color_threshold` is `None` or `'default'`, corresponding with MATLAB(TM) behavior, the threshold is set to  $0.7 * \max(Z[:, 2])$ .

**get\_leaves** : bool, optional

Includes a list `R['leaves'] = H` in the result dictionary. For each  $i$ , `H[i] == j`, cluster node  $j$  appears in position  $i$  in the left-to-right traversal of the leaves, where  $j < 2n - 1$  and  $i < n$ .

**orientation** : str, optional

The direction to plot the dendrogram, which can be any of the following strings:

***'top'*** Plots the root at the top, and plot descendent links going downwards. (default).

***'bottom'*** Plots the root at the bottom, and plot descendent links going upwards.

***'left'*** Plots the root at the left, and plot descendent links going right.

***'right'*** Plots the root at the right, and plot descendent links going left.

**labels** : ndarray, optional

By default `labels` is `None` so the index of the original observation is used to label the leaf nodes. Otherwise, this is an  $n$ -sized list (or tuple). The `labels[i]` value is the text to put under the  $i$ th leaf node only if it corresponds to an original observation and not a non-singleton cluster.

**count\_sort** : str or bool, optional

For each node  $n$ , the order (visually, from left-to-right)  $n$ 's two descendent links are plotted is determined by this parameter, which can be any of the following values:

***False*** Nothing is done.

***'ascending'*** or ***True***

The child with the minimum number of original objects in its cluster is plotted first.

***'descendent'***

The child with the maximum number of original objects in its cluster is plotted first.

Note `distance_sort` and `count_sort` cannot both be `True`.

**distance\_sort** : str or bool, optional

For each node  $n$ , the order (visually, from left-to-right)  $n$ 's two descendent links are plotted is determined by this parameter, which can be any of the following values:

***False*** Nothing is done.

**'ascending' or True**

The child with the minimum distance between its direct descendents is plotted first.

**'descending'**

The child with the maximum distance between its direct descendents is plotted first.

Note `distance_sort` and `count_sort` cannot both be True.

**show\_leaf\_counts** : bool, optional

When True, leaf nodes representing  $k > 1$  original observation are labeled with the number of observations they contain in parentheses.

**no\_plot** : bool, optional

When True, the final rendering is not performed. This is useful if only the data structures computed for the rendering are needed or if matplotlib is not available.

**no\_labels** : bool, optional

When True, no labels appear next to the leaf nodes in the rendering of the dendrogram.

**leaf\_rotation** : double, optional

Specifies the angle (in degrees) to rotate the leaf labels. When unspecified, the rotation is based on the number of nodes in the dendrogram (default is 0).

**leaf\_font\_size** : int, optional

Specifies the font size (in points) of the leaf labels. When unspecified, the size based on the number of nodes in the dendrogram.

**leaf\_label\_func** : lambda or function, optional

When `leaf_label_func` is a callable function, for each leaf with cluster index  $k < 2n - 1$ . The function is expected to return a string with the label for the leaf.

Indices  $k < n$  correspond to original observations while indices  $k \geq n$  correspond to non-singleton clusters.

For example, to label singletons with their node id and non-singletons with their id, count, and inconsistency coefficient, simply do:

```
# First define the leaf label function.
def llf(id):
    if id < n:
        return str(id)
    else:
        return "[%d %d %1.2f]" % (id, count, R[n-id,3])
# The text for the leaf nodes is going to be big so force
# a rotation of 90 degrees.
dendrogram(Z, leaf_label_func=llf, leaf_rotation=90)
```

**show\_contracted** : bool, optional

When True the heights of non-singleton nodes contracted into a leaf node are plotted as crosses along the link connecting that leaf node. This really is only useful when truncation is used (see `truncate_mode` parameter).

**link\_color\_func** : callable, optional

If given, `link_color_function` is called with each non-singleton id corresponding to each U-shaped link it will paint. The function is expected to return the color to paint the link, encoded as a matplotlib color string code. For example:

```
dendrogram(Z, link_color_func=lambda k: colors[k])
```

`colors` the direct links below each untruncated non-singleton node `k` using `colors[k]`.

**ax** : matplotlib Axes instance, optional

If None and `no_plot` is not True, the dendrogram will be plotted on the current axes. Otherwise if `no_plot` is not True the dendrogram will be plotted on the given Axes instance. This can be useful if the dendrogram is part of a more complex figure.

**above\_threshold\_color** : str, optional

This matplotlib color string sets the color of the links above the `color_threshold`. The default is 'b'.

**Returns** **R** : dict

A dictionary of data structures computed to render the dendrogram. Its has the following keys:

- 'color\_list'**

A list of color names. The  $k$ 'th element represents the color of the  $k$ 'th link.
- 'icoord' and 'dcoord'**

Each of them is a list of lists. Let `icoord = [I1, I2, ..., Ip]` where  $I_k = [xk1, xk2, xk3, xk4]$  and `dcoord = [D1, D2, ..., Dp]` where  $D_k = [yk1, yk2, yk3, yk4]$ , then the  $k$ 'th link painted is  $(xk1, yk1) - (xk2, yk2) - (xk3, yk3) - (xk4, yk4)$ .
- 'ivl'**

A list of labels corresponding to the leaf nodes.
- 'leaves'**

For each  $i$ ,  $H[i] == j$ , cluster node  $j$  appears in position  $i$  in the left-to-right traversal of the leaves, where  $j < 2n - 1$  and  $i < n$ . If  $j$  is less than  $n$ , the  $i$ -th leaf node corresponds to an original observation. Otherwise, it corresponds to a non-singleton cluster.

See also:

`linkage`, `set_link_color_palette`

**Examples**

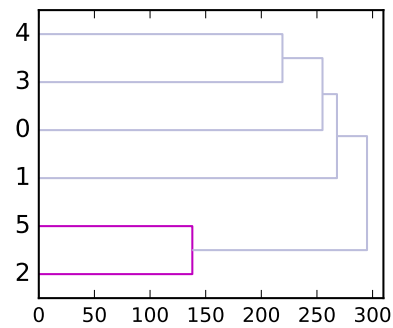
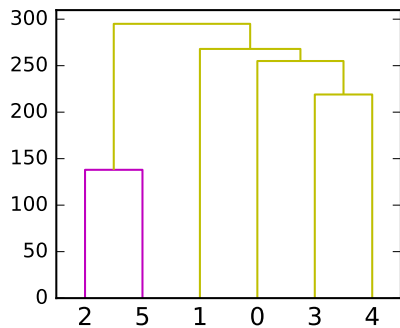
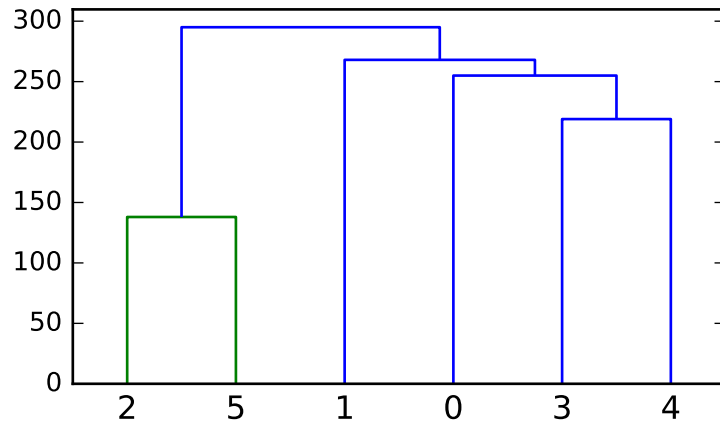
```
>>> from scipy.cluster import hierarchy
>>> import matplotlib.pyplot as plt
```

A very basic example:

```
>>> ytdist = np.array([662., 877., 255., 412., 996., 295., 468., 268.,
...                   400., 754., 564., 138., 219., 869., 669.])
>>> Z = hierarchy.linkage(ytdist, 'single')
>>> plt.figure()
>>> dn = hierarchy.dendrogram(Z)
```

Now plot in given axes, improve the color scheme and use both vertical and horizontal orientations:

```
>>> hierarchy.set_link_color_palette(['m', 'c', 'y', 'k'])
>>> fig, axes = plt.subplots(1, 2, figsize=(8, 3))
>>> dn1 = hierarchy.dendrogram(Z, ax=axes[0], above_threshold_color='y',
...                             orientation='top')
>>> dn2 = hierarchy.dendrogram(Z, ax=axes[1], above_threshold_color='#bcbddc',
...                             orientation='right')
>>> hierarchy.set_link_color_palette(None) # reset to default after use
>>> plt.show()
```



These are data structures and routines for representing hierarchies as tree objects.

<code>ClusterNode(id[, left, right, dist, count])</code>	A tree node class for representing a cluster.
<code>leaves_list(Z)</code>	Returns a list of leaf node ids
<code>to_tree(Z[, rd])</code>	Converts a hierarchical clustering encoded in the matrix Z (by linkage) into an easy-to-
<code>cut_tree(Z[, n_clusters, height])</code>	Given a linkage matrix Z, return the cut tree.

**class** `scipy.cluster.hierarchy.ClusterNode` (*id, left=None, right=None, dist=0, count=1*)  
 A tree node class for representing a cluster.

Leaf nodes correspond to original observations, while non-leaf nodes correspond to non-singleton clusters.

The `to_tree` function converts a matrix returned by the linkage function into an easy-to-use tree representation.

**See also:**

`to_tree` for converting a linkage matrix Z into a tree object.

**Methods**

<code>get_count()</code>	The number of leaf nodes (original observations) belonging to the cluster node <code>nd</code> .
--------------------------	--

Continued on next page



Table 5.7 – continued from previous page

<code>get_id()</code>	The identifier of the target node.
<code>get_left()</code>	Return a reference to the left child tree object.
<code>get_right()</code>	Returns a reference to the right child tree object.
<code>is_leaf()</code>	Returns True if the target node is a leaf.
<code>pre_order([func])</code>	Performs pre-order traversal without recursive function calls.

`ClusterNode.get_count()`

The number of leaf nodes (original observations) belonging to the cluster node `nd`. If the target node is a leaf, 1 is returned.

**Returns** `get_count` : int  
The number of leaf nodes below the target node.

`ClusterNode.get_id()`

The identifier of the target node.

For  $0 \leq i < n$ ,  $i$  corresponds to original observation  $i$ . For  $n \leq i < 2n-1$ ,  $i$  corresponds to non-singleton cluster formed at iteration  $i-n$ .

**Returns** `id` : int  
The identifier of the target node.

`ClusterNode.get_left()`

Return a reference to the left child tree object.

**Returns** `left` : ClusterNode  
The left child of the target node. If the node is a leaf, None is returned.

`ClusterNode.get_right()`

Returns a reference to the right child tree object.

**Returns** `right` : ClusterNode  
The left child of the target node. If the node is a leaf, None is returned.

`ClusterNode.is_leaf()`

Returns True if the target node is a leaf.

**Returns** `leafness` : bool  
True if the target node is a leaf node.

`ClusterNode.pre_order(func=<function <lambda> at 0x2b909c83e410>)`

Performs pre-order traversal without recursive function calls.

When a leaf node is first encountered, `func` is called with the leaf node as its argument, and its result is appended to the list.

For example, the statement:

```
ids = root.pre_order(lambda x: x.id)
```

returns a list of the node `ids` corresponding to the leaf nodes of the tree as they appear from left to right.

**Parameters** `func` : function  
Applied to each leaf ClusterNode object in the pre-order traversal. Given the  $i$ 'th leaf node in the pre-order traversal `n[i]`, the result of `func(n[i])` is stored in `L[i]`. If not provided, the index of the original observation to which the node corresponds is used.

**Returns** `L` : list  
The pre-order traversal.

`scipy.cluster.hierarchy.leaves_list` (*Z*)

Returns a list of leaf node ids

The return corresponds to the observation vector index as it appears in the tree from left to right. *Z* is a linkage matrix.

**Parameters**    *Z* : ndarray  
                   The hierarchical clustering encoded as a matrix. *Z* is a linkage matrix. See `linkage` for more information.

**Returns**        `leaves_list` : ndarray  
                   The list of leaf node ids.

`scipy.cluster.hierarchy.to_tree` (*Z*, *rd=False*)

Converts a hierarchical clustering encoded in the matrix *Z* (by linkage) into an easy-to-use tree object.

The reference *r* to the root ClusterNode object is returned.

Each ClusterNode object has a left, right, dist, id, and count attribute. The left and right attributes point to ClusterNode objects that were combined to generate the cluster. If both are None then the ClusterNode object is a leaf node, its count must be 1, and its distance is meaningless but set to 0.

Note: This function is provided for the convenience of the library user. ClusterNodes are not used as input to any of the functions in this library.

**Parameters**    *Z* : ndarray  
                   The linkage matrix in proper form (see the `linkage` function documentation).

*rd* : bool, optional  
                   When False, a reference to the root ClusterNode object is returned. Otherwise, a tuple (*r*,*d*) is returned. *r* is a reference to the root node while *d* is a dictionary mapping cluster ids to ClusterNode references. If a cluster id is less than *n*, then it corresponds to a singleton cluster (leaf node). See `linkage` for more information on the assignment of cluster ids to clusters.

**Returns**        *L* : list  
                   The pre-order traversal.

`scipy.cluster.hierarchy.cut_tree` (*Z*, *n\_clusters=None*, *height=None*)

Given a linkage matrix *Z*, return the cut tree.

**Parameters**    *Z* : scipy.cluster.linkage array  
                   The linkage matrix.

*n\_clusters* : array\_like, optional  
                   Number of clusters in the tree at the cut point.

*height* : array\_like, optional  
                   The height at which to cut the tree. Only possible for ultrametric trees.

**Returns**        `cutree` : array  
                   An array indicating group membership at each agglomeration step. I.e., for a full cut tree, in the first column each data point is in its own cluster. At the next step, two nodes are merged. Finally all singleton and non-singleton clusters are in one group. If *n\_clusters* or *height* is given, the columns correspond to the columns of *n\_clusters* or *height*.

**Examples**

```
>>> from scipy import cluster
>>> np.random.seed(23)
>>> X = np.random.randn(50, 4)
>>> Z = cluster.hierarchy.ward(X)
>>> cutree = cluster.hierarchy.cut_tree(Z, n_clusters=[5, 10])
>>> cutree[:10]
```

```
array([[0, 0],
       [1, 1],
       [2, 2],
       [3, 3],
       [3, 4],
       [2, 2],
       [0, 0],
       [1, 5],
       [3, 6],
       [4, 7]])
```

These are predicates for checking the validity of linkage and inconsistency matrices as well as for checking isomorphism of two flat cluster assignments.

---

<code>is_valid_im(R[, warning, throw, name])</code>	Returns True if the inconsistency matrix passed is valid.
<code>is_valid_linkage(Z[, warning, throw, name])</code>	Checks the validity of a linkage matrix.
<code>is_isomorphic(T1, T2)</code>	Determines if two different cluster assignments are equivalent.
<code>is_monotonic(Z)</code>	Returns True if the linkage passed is monotonic.
<code>correspond(Z, Y)</code>	Checks for correspondence between linkage and condensed distance matrices
<code>num_obs_linkage(Z)</code>	Returns the number of original observations of the linkage matrix passed.

---

`scipy.cluster.hierarchy.is_valid_im(R, warning=False, throw=False, name=None)`

Returns True if the inconsistency matrix passed is valid.

It must be a  $n$  by 4 numpy array of doubles. The standard deviations  $R[:, 1]$  must be nonnegative. The link counts  $R[:, 2]$  must be positive and no greater than  $n - 1$ .

**Parameters**

- R** : ndarray  
The inconsistency matrix to check for validity.
- warning** : bool, optional  
When True, issues a Python warning if the linkage matrix passed is invalid.
- throw** : bool, optional  
When True, throws a Python exception if the linkage matrix passed is invalid.
- name** : str, optional  
This string refers to the variable name of the invalid linkage matrix.

**Returns**

- b** : bool  
True if the inconsistency matrix is valid.

`scipy.cluster.hierarchy.is_valid_linkage(Z, warning=False, throw=False, name=None)`

Checks the validity of a linkage matrix.

A linkage matrix is valid if it is a two dimensional array (type double) with  $n$  rows and 4 columns. The first two columns must contain indices between 0 and  $2n - 1$ . For a given row  $i$ , the following two expressions have to hold:

$$0 \leq Z[i, 0] \leq i + n - 10 \leq Z[i, 1] \leq i + n - 1$$

I.e. a cluster cannot join another cluster unless the cluster being joined has been generated.

**Parameters**

- Z** : array\_like  
Linkage matrix.
- warning** : bool, optional  
When True, issues a Python warning if the linkage matrix passed is invalid.
- throw** : bool, optional  
When True, throws a Python exception if the linkage matrix passed is invalid.
- name** : str, optional  
This string refers to the variable name of the invalid linkage matrix.

**Returns** **b** : bool  
True if the inconsistency matrix is valid.

`scipy.cluster.hierarchy.is_isomorphic(T1, T2)`  
Determines if two different cluster assignments are equivalent.

**Parameters** **T1** : array\_like  
An assignment of singleton cluster ids to flat cluster ids.  
**T2** : array\_like  
An assignment of singleton cluster ids to flat cluster ids.

**Returns** **b** : bool  
Whether the flat cluster assignments *T1* and *T2* are equivalent.

`scipy.cluster.hierarchy.is_monotonic(Z)`  
Returns True if the linkage passed is monotonic.

The linkage is monotonic if for every cluster *s* and *t* joined, the distance between them is no less than the distance between any previously joined clusters.

**Parameters** **Z** : ndarray  
The linkage matrix to check for monotonicity.

**Returns** **b** : bool  
A boolean indicating whether the linkage is monotonic.

`scipy.cluster.hierarchy.correspond(Z, Y)`  
Checks for correspondence between linkage and condensed distance matrices

They must have the same number of original observations for the check to succeed.

This function is useful as a sanity check in algorithms that make extensive use of linkage and distance matrices that must correspond to the same set of original observations.

**Parameters** **Z** : array\_like  
The linkage matrix to check for correspondence.  
**Y** : array\_like  
The condensed distance matrix to check for correspondence.

**Returns** **b** : bool  
A boolean indicating whether the linkage matrix and distance matrix could possibly correspond to one another.

`scipy.cluster.hierarchy.num_obs_linkage(Z)`  
Returns the number of original observations of the linkage matrix passed.

**Parameters** **Z** : ndarray  
The linkage matrix on which to perform the operation.

**Returns** **n** : int  
The number of original observations in the linkage.

Utility routines for plotting:

---

`set_link_color_palette(palette)` Set list of matplotlib color codes for use by dendrogram.

`scipy.cluster.hierarchy.set_link_color_palette(palette)`  
Set list of matplotlib color codes for use by dendrogram.

Note that this palette is global (i.e. setting it once changes the colors for all subsequent calls to `dendrogram`) and that it affects only the the colors below `color_threshold`.

Note that `dendrogram` also accepts a custom coloring function through its `link_color_func` keyword, which is more flexible and non-global.

**Parameters** `palette` : list of str or None  
 A list of matplotlib color codes. The order of the color codes is the order in which the colors are cycled through when color thresholding in the dendrogram.  
 If None, resets the palette to its default (which is ['g', 'r', 'c', 'm', 'y', 'k']).

**Returns** None

**See also:**

`dendrogram`

**Notes**

Ability to reset the palette with None added in SciPy 0.17.0.

**Examples**

```
>>> from scipy.cluster import hierarchy
>>> ytdist = np.array([662., 877., 255., 412., 996., 295., 468., 268., 400.,
...                   754., 564., 138., 219., 869., 669.])
>>> Z = hierarchy.linkage(ytdist, 'single')
>>> dn = hierarchy.dendrogram(Z, no_plot=True)
>>> dn['color_list']
['g', 'b', 'b', 'b', 'b']
>>> hierarchy.set_link_color_palette(['c', 'm', 'y', 'k'])
>>> dn = hierarchy.dendrogram(Z, no_plot=True)
>>> dn['color_list']
['c', 'b', 'b', 'b', 'b']
>>> dn = hierarchy.dendrogram(Z, no_plot=True, color_threshold=267,
...                           above_threshold_color='k')
>>> dn['color_list']
['c', 'm', 'm', 'k', 'k']
```

Now reset the color palette to its default:

```
>>> hierarchy.set_link_color_palette(None)
```

### 5.3.1 References

- MATLAB and MathWorks are registered trademarks of The MathWorks, Inc.
- Mathematica is a registered trademark of The Wolfram Research, Inc.

## 5.4 Constants (`scipy.constants`)

Physical and mathematical constants and units.

### 5.4.1 Mathematical constants

<code>pi</code>	Pi
<code>golden</code>	Golden ratio
<code>golden_ratio</code>	Golden ratio

## 5.4.2 Physical constants

<code>c</code>	speed of light in vacuum
<code>speed_of_light</code>	speed of light in vacuum
<code>mu_0</code>	the magnetic constant $\mu_0$
<code>epsilon_0</code>	the electric constant (vacuum permittivity), $\epsilon_0$
<code>h</code>	the Planck constant $h$
<code>Planck</code>	the Planck constant $h$
<code>hbar</code>	$\hbar = h/(2\pi)$
<code>G</code>	Newtonian constant of gravitation
<code>gravitational_constant</code>	Newtonian constant of gravitation
<code>g</code>	standard acceleration of gravity
<code>e</code>	elementary charge
<code>elementary_charge</code>	elementary charge
<code>R</code>	molar gas constant
<code>gas_constant</code>	molar gas constant
<code>alpha</code>	fine-structure constant
<code>fine_structure</code>	fine-structure constant
<code>N_A</code>	Avogadro constant
<code>Avogadro</code>	Avogadro constant
<code>k</code>	Boltzmann constant
<code>Boltzmann</code>	Boltzmann constant
<code>sigma</code>	Stefan-Boltzmann constant $\sigma$
<code>Stefan_Boltzmann</code>	Stefan-Boltzmann constant $\sigma$
<code>Wien</code>	Wien displacement law constant
<code>Rydberg</code>	Rydberg constant
<code>m_e</code>	electron mass
<code>electron_mass</code>	electron mass
<code>m_p</code>	proton mass
<code>proton_mass</code>	proton mass
<code>m_n</code>	neutron mass
<code>neutron_mass</code>	neutron mass

### Constants database

In addition to the above variables, `scipy.constants` also contains the 2014 CODATA recommended values [CODATA2014] database containing more physical constants.

<code>value(key)</code>	Value in <code>physical_constants</code> indexed by key
<code>unit(key)</code>	Unit in <code>physical_constants</code> indexed by key
<code>precision(key)</code>	Relative precision in <code>physical_constants</code> indexed by key
<code>find([sub, disp])</code>	Return list of <code>physical_constant</code> keys containing a given string.
<code>ConstantWarning</code>	Accessing a constant no longer in current CODATA data set

`scipy.constants.value(key)`

Value in `physical_constants` indexed by key

**Parameters** **key** : Python string or unicode  
Key in dictionary `physical_constants`

**Returns** **value** : float  
Value in `physical_constants` corresponding to `key`

See also:

**codata** Contains the description of `physical_constants`, which, as a dictionary literal object, does not itself possess a docstring.

#### Examples

```
>>> from scipy import constants
>>> constants.value(u'elementary charge')
1.6021766208e-19
```

`scipy.constants.unit` (*key*)

Unit in `physical_constants` indexed by key

**Parameters** **key** : Python string or unicode  
Key in dictionary `physical_constants`

**Returns** **unit** : Python string  
Unit in `physical_constants` corresponding to *key*

See also:

**codata** Contains the description of `physical_constants`, which, as a dictionary literal object, does not itself possess a docstring.

#### Examples

```
>>> from scipy import constants
>>> constants.unit(u'proton mass')
'kg'
```

`scipy.constants.precision` (*key*)

Relative precision in `physical_constants` indexed by key

**Parameters** **key** : Python string or unicode  
Key in dictionary `physical_constants`

**Returns** **prec** : float  
Relative precision in `physical_constants` corresponding to *key*

See also:

**codata** Contains the description of `physical_constants`, which, as a dictionary literal object, does not itself possess a docstring.

#### Examples

```
>>> from scipy import constants
>>> constants.precision(u'proton mass')
1.2555138746605121e-08
```

`scipy.constants.find` (*sub=None, disp=False*)

Return list of `physical_constant` keys containing a given string.

**Parameters** **sub** : str, unicode  
Sub-string to search keys for. By default, return all keys.

**disp** : bool  
If True, print the keys that are found, and return None. Otherwise, return the list of keys without printing anything.

**Returns** **keys** : list or None  
If *disp* is False, the list of keys is returned. Otherwise, None is returned.

See also:

**codata** Contains the description of `physical_constants`, which, as a dictionary literal object, does not itself possess a docstring.

**exception** `scipy.constants.ConstantWarning`

Accessing a constant no longer in current CODATA data set

`scipy.constants.physical_constants`

Dictionary of physical constants, of the format `physical_constants[name] = (value, unit, uncertainty)`.

Available constants:

alpha particle mass	6.64465723e-27 kg
alpha particle mass energy equivalent	5.971920097e-10 J
alpha particle mass energy equivalent in MeV	3727.379378 MeV
alpha particle mass in u	4.00150617913 u
alpha particle molar mass	0.00400150617913 kg mol <sup>-1</sup>
alpha particle-electron mass ratio	7294.29954136
alpha particle-proton mass ratio	3.97259968907
Angstrom star	1.00001495e-10 m
atomic mass constant	1.66053904e-27 kg
atomic mass constant energy equivalent	1.492418062e-10 J
atomic mass constant energy equivalent in MeV	931.4940954 MeV
atomic mass unit-electron volt relationship	931494095.4 eV
atomic mass unit-hartree relationship	34231776.902 E <sub>h</sub>
atomic mass unit-hertz relationship	2.2523427206e+23 Hz
atomic mass unit-inverse meter relationship	7.5130066166e+14 m <sup>-1</sup>
atomic mass unit-joule relationship	1.492418062e-10 J
atomic mass unit-kelvin relationship	1.08095438e+13 K
atomic mass unit-kilogram relationship	1.66053904e-27 kg
atomic unit of 1st hyperpolarizability	3.206361329e-53 C <sup>3</sup> m <sup>3</sup> J <sup>-2</sup>
atomic unit of 2nd hyperpolarizability	6.235380085e-65 C <sup>4</sup> m <sup>4</sup> J <sup>-3</sup>
atomic unit of action	1.0545718e-34 J s
atomic unit of charge	1.6021766208e-19 C
atomic unit of charge density	1.081202377e+12 C m <sup>-3</sup>
atomic unit of current	0.006623618183 A
atomic unit of electric dipole mom.	8.478353552e-30 C m
atomic unit of electric field	5.142206707e+11 V m <sup>-1</sup>
atomic unit of electric field gradient	9.717362356e+21 V m <sup>-2</sup>
atomic unit of electric polarizability	1.6487772731e-41 C <sup>2</sup> m <sup>2</sup> J <sup>-1</sup>
atomic unit of electric potential	27.21138602 V
atomic unit of electric quadrupole mom.	4.486551484e-40 C m <sup>2</sup>
atomic unit of energy	4.35974465e-18 J
atomic unit of force	8.23872336e-08 N
atomic unit of length	5.2917721067e-11 m
atomic unit of mag. dipole mom.	1.854801999e-23 J T <sup>-1</sup>
atomic unit of mag. flux density	235051.755 T
atomic unit of magnetizability	7.8910365886e-29 J T <sup>-2</sup>
atomic unit of mass	9.10938356e-31 kg
atomic unit of mom.um	1.992851882e-24 kg m s <sup>-1</sup>
atomic unit of permittivity	1.11265005605e-10 F m <sup>-1</sup>
atomic unit of time	2.41888432651e-17 s
atomic unit of velocity	2187691.26277 m s <sup>-1</sup>
Avogadro constant	6.022140857e+23 mol <sup>-1</sup>

Continued on next page



Table 5.11 – continued from previous page

Bohr magneton	9.274009994e-24 J T <sup>-1</sup>
Bohr magneton in eV/T	5.7883818012e-05 eV T <sup>-1</sup>
Bohr magneton in Hz/T	13996245042.0 Hz T <sup>-1</sup>
Bohr magneton in inverse meters per tesla	46.68644814 m <sup>-1</sup> T <sup>-1</sup>
Bohr magneton in K/T	0.67171405 K T <sup>-1</sup>
Bohr radius	5.2917721067e-11 m
Boltzmann constant	1.38064852e-23 J K <sup>-1</sup>
Boltzmann constant in eV/K	8.6173303e-05 eV K <sup>-1</sup>
Boltzmann constant in Hz/K	20836612000.0 Hz K <sup>-1</sup>
Boltzmann constant in inverse meters per kelvin	69.503457 m <sup>-1</sup> K <sup>-1</sup>
characteristic impedance of vacuum	376.730313462 ohm
classical electron radius	2.8179403227e-15 m
Compton wavelength	2.4263102367e-12 m
Compton wavelength over 2 pi	3.8615926764e-13 m
conductance quantum	7.748091731e-05 S
conventional value of Josephson constant	4.835979e+14 Hz V <sup>-1</sup>
conventional value of von Klitzing constant	25812.807 ohm
Cu x unit	1.00207697e-13 m
deuteron g factor	0.8574382311
deuteron mag. mom.	4.33073504e-27 J T <sup>-1</sup>
deuteron mag. mom. to Bohr magneton ratio	0.0004669754554
deuteron mag. mom. to nuclear magneton ratio	0.8574382311
deuteron mass	3.343583719e-27 kg
deuteron mass energy equivalent	3.005063183e-10 J
deuteron mass energy equivalent in MeV	1875.612928 MeV
deuteron mass in u	2.01355321275 u
deuteron molar mass	0.00201355321274 kg mol <sup>-1</sup>
deuteron rms charge radius	2.1413e-15 m
deuteron-electron mag. mom. ratio	-0.0004664345535
deuteron-electron mass ratio	3670.48296785
deuteron-neutron mag. mom. ratio	-0.44820652
deuteron-proton mag. mom. ratio	0.3070122077
deuteron-proton mass ratio	1.99900750087
electric constant	8.85418781762e-12 F m <sup>-1</sup>
electron charge to mass quotient	-1.758820024e+11 C kg <sup>-1</sup>
electron g factor	-2.00231930436
electron gyromag. ratio	1.760859644e+11 s <sup>-1</sup> T <sup>-1</sup>
electron gyromag. ratio over 2 pi	28024.95164 MHz T <sup>-1</sup>
electron mag. mom.	-9.28476462e-24 J T <sup>-1</sup>
electron mag. mom. anomaly	0.00115965218091
electron mag. mom. to Bohr magneton ratio	-1.00115965218
electron mag. mom. to nuclear magneton ratio	-1838.28197234
electron mass	9.10938356e-31 kg
electron mass energy equivalent	8.18710565e-14 J
electron mass energy equivalent in MeV	0.5109989461 MeV
electron mass in u	0.00054857990907 u
electron molar mass	5.4857990907e-07 kg mol <sup>-1</sup>
electron to alpha particle mass ratio	0.000137093335548
electron to shielded helion mag. mom. ratio	864.058257
electron to shielded proton mag. mom. ratio	-658.2275971
electron volt	1.6021766208e-19 J
electron volt-atomic mass unit relationship	1.0735441105e-09 u

Continued on next page

Table 5.11 – continued from previous page

electron volt-hartree relationship	0.03674932248 E_h
electron volt-hertz relationship	2.417989262e+14 Hz
electron volt-inverse meter relationship	806554.4005 m^-1
electron volt-joule relationship	1.6021766208e-19 J
electron volt-kelvin relationship	11604.5221 K
electron volt-kilogram relationship	1.782661907e-36 kg
electron-deuteron mag. mom. ratio	-2143.923499
electron-deuteron mass ratio	0.000272443710748
electron-helion mass ratio	0.000181954307485
electron-muon mag. mom. ratio	206.766988
electron-muon mass ratio	0.0048363317
electron-neutron mag. mom. ratio	960.9205
electron-neutron mass ratio	0.00054386734428
electron-proton mag. mom. ratio	-658.2106866
electron-proton mass ratio	0.000544617021352
electron-tau mass ratio	0.000287592
electron-triton mass ratio	0.00018192000622
elementary charge	1.6021766208e-19 C
elementary charge over h	2.417989262e+14 A J^-1
Faraday constant	96485.33289 C mol^-1
Faraday constant for conventional electric current	96485.3251 C_90 mol^-1
Fermi coupling constant	1.1663787e-05 GeV^-2
fine-structure constant	0.0072973525664
first radiation constant	3.74177179e-16 W m^2
first radiation constant for spectral radiance	1.191042953e-16 W m^2 sr^-1
Hartree energy	4.35974465e-18 J
Hartree energy in eV	27.21138602 eV
hartree-atomic mass unit relationship	2.9212623197e-08 u
hartree-electron volt relationship	27.21138602 eV
hartree-hertz relationship	6.57968392071e+15 Hz
hartree-inverse meter relationship	21947463.137 m^-1
hartree-joule relationship	4.35974465e-18 J
hartree-kelvin relationship	315775.13 K
hartree-kilogram relationship	4.850870129e-35 kg
helion g factor	-4.255250616
helion mag. mom.	-1.074617522e-26 J T^-1
helion mag. mom. to Bohr magneton ratio	-0.001158740958
helion mag. mom. to nuclear magneton ratio	-2.127625308
helion mass	5.0064127e-27 kg
helion mass energy equivalent	4.499539341e-10 J
helion mass energy equivalent in MeV	2808.391586 MeV
helion mass in u	3.01493224673 u
helion molar mass	0.00301493224673 kg mol^-1
helion-electron mass ratio	5495.88527922
helion-proton mass ratio	2.99315267046
hertz-atomic mass unit relationship	4.4398216616e-24 u
hertz-electron volt relationship	4.135667662e-15 eV
hertz-hartree relationship	1.51982984601e-16 E_h
hertz-inverse meter relationship	3.33564095198e-09 m^-1
hertz-joule relationship	6.62607004e-34 J
hertz-kelvin relationship	4.7992447e-11 K
hertz-kilogram relationship	7.372497201e-51 kg

Continued on next page

Table 5.11 – continued from previous page

inverse fine-structure constant	137.035999139
inverse meter-atomic mass unit relationship	1.331025049e-15 u
inverse meter-electron volt relationship	1.2398419739e-06 eV
inverse meter-hartree relationship	4.55633525277e-08 E_h
inverse meter-hertz relationship	299792458.0 Hz
inverse meter-joule relationship	1.986445824e-25 J
inverse meter-kelvin relationship	0.0143877736 K
inverse meter-kilogram relationship	2.210219057e-42 kg
inverse of conductance quantum	12906.4037278 ohm
Josephson constant	4.835978525e+14 Hz V^-1
joule-atomic mass unit relationship	6700535363.0 u
joule-electron volt relationship	6.241509126e+18 eV
joule-hartree relationship	2.293712317e+17 E_h
joule-hertz relationship	1.509190205e+33 Hz
joule-inverse meter relationship	5.034116651e+24 m^-1
joule-kelvin relationship	7.2429731e+22 K
joule-kilogram relationship	1.11265005605e-17 kg
kelvin-atomic mass unit relationship	9.2510842e-14 u
kelvin-electron volt relationship	8.6173303e-05 eV
kelvin-hartree relationship	3.1668105e-06 E_h
kelvin-hertz relationship	20836612000.0 Hz
kelvin-inverse meter relationship	69.503457 m^-1
kelvin-joule relationship	1.38064852e-23 J
kelvin-kilogram relationship	1.53617865e-40 kg
kilogram-atomic mass unit relationship	6.022140857e+26 u
kilogram-electron volt relationship	5.60958865e+35 eV
kilogram-hartree relationship	2.061485823e+34 E_h
kilogram-hertz relationship	1.356392512e+50 Hz
kilogram-inverse meter relationship	4.524438411e+41 m^-1
kilogram-joule relationship	8.98755178737e+16 J
kilogram-kelvin relationship	6.5096595e+39 K
lattice parameter of silicon	5.431020504e-10 m
Loschmidt constant (273.15 K, 100 kPa)	2.6516467e+25 m^-3
Loschmidt constant (273.15 K, 101.325 kPa)	2.6867811e+25 m^-3
mag. constant	1.25663706144e-06 N A^-2
mag. flux quantum	2.067833831e-15 Wb
Mo x unit	1.00209952e-13 m
molar gas constant	8.3144598 J mol^-1 K^-1
molar mass constant	0.001 kg mol^-1
molar mass of carbon-12	0.012 kg mol^-1
molar Planck constant	3.990312711e-10 J s mol^-1
molar Planck constant times c	0.119626565582 J m mol^-1
molar volume of ideal gas (273.15 K, 100 kPa)	0.022710947 m^3 mol^-1
molar volume of ideal gas (273.15 K, 101.325 kPa)	0.022413962 m^3 mol^-1
molar volume of silicon	1.205883214e-05 m^3 mol^-1
muon Compton wavelength	1.173444111e-14 m
muon Compton wavelength over 2 pi	1.867594308e-15 m
muon g factor	-2.0023318418
muon mag. mom.	-4.49044826e-26 J T^-1
muon mag. mom. anomaly	0.00116592089
muon mag. mom. to Bohr magneton ratio	-0.00484197048
muon mag. mom. to nuclear magneton ratio	-8.89059705

Continued on next page

Table 5.11 – continued from previous page

muon mass	1.883531594e-28 kg
muon mass energy equivalent	1.692833774e-11 J
muon mass energy equivalent in MeV	105.6583745 MeV
muon mass in u	0.1134289257 u
muon molar mass	0.0001134289257 kg mol <sup>-1</sup>
muon-electron mass ratio	206.7682826
muon-neutron mass ratio	0.1124545167
muon-proton mag. mom. ratio	-3.183345142
muon-proton mass ratio	0.1126095262
muon-tau mass ratio	0.0594649
natural unit of action	1.0545718e-34 J s
natural unit of action in eV s	6.582119514e-16 eV s
natural unit of energy	8.18710565e-14 J
natural unit of energy in MeV	0.5109989461 MeV
natural unit of length	3.8615926764e-13 m
natural unit of mass	9.10938356e-31 kg
natural unit of mom.um	2.730924488e-22 kg m s <sup>-1</sup>
natural unit of mom.um in MeV/c	0.5109989461 MeV/c
natural unit of time	1.28808866712e-21 s
natural unit of velocity	299792458.0 m s <sup>-1</sup>
neutron Compton wavelength	1.31959090481e-15 m
neutron Compton wavelength over 2 pi	2.1001941536e-16 m
neutron g factor	-3.82608545
neutron gyromag. ratio	183247172.0 s <sup>-1</sup> T <sup>-1</sup>
neutron gyromag. ratio over 2 pi	29.1646933 MHz T <sup>-1</sup>
neutron mag. mom.	-9.662365e-27 J T <sup>-1</sup>
neutron mag. mom. to Bohr magneton ratio	-0.00104187563
neutron mag. mom. to nuclear magneton ratio	-1.91304273
neutron mass	1.674927471e-27 kg
neutron mass energy equivalent	1.505349739e-10 J
neutron mass energy equivalent in MeV	939.5654133 MeV
neutron mass in u	1.00866491588 u
neutron molar mass	0.00100866491588 kg mol <sup>-1</sup>
neutron to shielded proton mag. mom. ratio	-0.68499694
neutron-electron mag. mom. ratio	0.00104066882
neutron-electron mass ratio	1838.68366158
neutron-muon mass ratio	8.89248408
neutron-proton mag. mom. ratio	-0.68497934
neutron-proton mass difference	2.30557377e-30
neutron-proton mass difference energy equivalent	2.07214637e-13
neutron-proton mass difference energy equivalent in MeV	1.29333205
neutron-proton mass difference in u	0.001388449
neutron-proton mass ratio	1.00137841898
neutron-tau mass ratio	0.52879
Newtonian constant of gravitation	6.67408e-11 m <sup>3</sup> kg <sup>-1</sup> s <sup>-2</sup>
Newtonian constant of gravitation over h-bar c	6.70861e-39 (GeV/c <sup>2</sup> ) <sup>-2</sup>
nuclear magneton	5.050783699e-27 J T <sup>-1</sup>
nuclear magneton in eV/T	3.152451255e-08 eV T <sup>-1</sup>
nuclear magneton in inverse meters per tesla	0.02542623432 m <sup>-1</sup> T <sup>-1</sup>
nuclear magneton in K/T	0.0003658269 K T <sup>-1</sup>
nuclear magneton in MHz/T	7.622593285 MHz T <sup>-1</sup>
Planck constant	6.62607004e-34 J s

Continued on next page

Table 5.11 – continued from previous page

Planck constant in eV s	4.135667662e-15 eV s
Planck constant over 2 pi	1.0545718e-34 J s
Planck constant over 2 pi in eV s	6.582119514e-16 eV s
Planck constant over 2 pi times c in MeV fm	197.3269788 MeV fm
Planck length	1.616229e-35 m
Planck mass	2.17647e-08 kg
Planck mass energy equivalent in GeV	1.22091e+19 GeV
Planck temperature	1.416808e+32 K
Planck time	5.39116e-44 s
proton charge to mass quotient	95788332.26 C kg <sup>-1</sup>
proton Compton wavelength	1.32140985396e-15 m
proton Compton wavelength over 2 pi	2.10308910109e-16 m
proton g factor	5.585694702
proton gyromag. ratio	267522190.0 s <sup>-1</sup> T <sup>-1</sup>
proton gyromag. ratio over 2 pi	42.57747892 MHz T <sup>-1</sup>
proton mag. mom.	1.4106067873e-26 J T <sup>-1</sup>
proton mag. mom. to Bohr magneton ratio	0.0015210322053
proton mag. mom. to nuclear magneton ratio	2.7928473508
proton mag. shielding correction	2.5691e-05
proton mass	1.672621898e-27 kg
proton mass energy equivalent	1.503277593e-10 J
proton mass energy equivalent in MeV	938.2720813 MeV
proton mass in u	1.00727646688 u
proton molar mass	0.00100727646688 kg mol <sup>-1</sup>
proton rms charge radius	8.751e-16 m
proton-electron mass ratio	1836.15267389
proton-muon mass ratio	8.88024338
proton-neutron mag. mom. ratio	-1.45989805
proton-neutron mass ratio	0.99862347844
proton-tau mass ratio	0.528063
quantum of circulation	0.00036369475486 m <sup>2</sup> s <sup>-1</sup>
quantum of circulation times 2	0.00072738950972 m <sup>2</sup> s <sup>-1</sup>
Rydberg constant	10973731.5685 m <sup>-1</sup>
Rydberg constant times c in Hz	3.28984196036e+15 Hz
Rydberg constant times hc in eV	13.605693009 eV
Rydberg constant times hc in J	2.179872325e-18 J
Sackur-Tetrode constant (1 K, 100 kPa)	-1.1517084
Sackur-Tetrode constant (1 K, 101.325 kPa)	-1.1648714
second radiation constant	0.0143877736 m K
shielded helion gyromag. ratio	203789458.5 s <sup>-1</sup> T <sup>-1</sup>
shielded helion gyromag. ratio over 2 pi	32.43409966 MHz T <sup>-1</sup>
shielded helion mag. mom.	-1.07455308e-26 J T <sup>-1</sup>
shielded helion mag. mom. to Bohr magneton ratio	-0.001158671471
shielded helion mag. mom. to nuclear magneton ratio	-2.12749772
shielded helion to proton mag. mom. ratio	-0.7617665603
shielded helion to shielded proton mag. mom. ratio	-0.7617861313
shielded proton gyromag. ratio	267515317.1 s <sup>-1</sup> T <sup>-1</sup>
shielded proton gyromag. ratio over 2 pi	42.57638507 MHz T <sup>-1</sup>
shielded proton mag. mom.	1.410570547e-26 J T <sup>-1</sup>
shielded proton mag. mom. to Bohr magneton ratio	0.001520993128
shielded proton mag. mom. to nuclear magneton ratio	2.7927756
speed of light in vacuum	299792458.0 m s <sup>-1</sup>

Continued on next page

Table 5.11 – continued from previous page

standard acceleration of gravity	9.80665 m s <sup>-2</sup>
standard atmosphere	101325.0 Pa
standard-state pressure	100000.0 Pa
Stefan-Boltzmann constant	5.670367e-08 W m <sup>-2</sup> K <sup>-4</sup>
tau Compton wavelength	6.97787e-16 m
tau Compton wavelength over 2 pi	1.11056e-16 m
tau mass	3.16747e-27 kg
tau mass energy equivalent	2.84678e-10 J
tau mass energy equivalent in MeV	1776.82 MeV
tau mass in u	1.90749 u
tau molar mass	0.00190749 kg mol <sup>-1</sup>
tau-electron mass ratio	3477.15
tau-muon mass ratio	16.8167
tau-neutron mass ratio	1.89111
tau-proton mass ratio	1.89372
Thomson cross section	6.6524587158e-29 m <sup>2</sup>
triton g factor	5.95792492
triton mag. mom.	1.504609503e-26 J T <sup>-1</sup>
triton mag. mom. to Bohr magneton ratio	0.0016223936616
triton mag. mom. to nuclear magneton ratio	2.97896246
triton mass	5.007356665e-27 kg
triton mass energy equivalent	4.500387735e-10 J
triton mass energy equivalent in MeV	2808.921112 MeV
triton mass in u	3.01550071632 u
triton molar mass	0.00301550071632 kg mol <sup>-1</sup>
triton-electron mass ratio	5496.92153588
triton-proton mass ratio	2.99371703348
unified atomic mass unit	1.66053904e-27 kg
von Klitzing constant	25812.8074555 ohm
weak mixing angle	0.2223
Wien frequency displacement law constant	58789238000.0 Hz K <sup>-1</sup>
Wien wavelength displacement law constant	0.0028977729 m K
{220} lattice spacing of silicon	1.920155714e-10 m

### 5.4.3 Units

#### SI prefixes

yotta	$10^{24}$
zetta	$10^{21}$
exa	$10^{18}$
peta	$10^{15}$
tera	$10^{12}$
giga	$10^9$
mega	$10^6$
kilo	$10^3$
hecto	$10^2$
deka	$10^1$
deci	$10^{-1}$
centi	$10^{-2}$
milli	$10^{-3}$
micro	$10^{-6}$
nano	$10^{-9}$
pico	$10^{-12}$
femto	$10^{-15}$
atto	$10^{-18}$
zepto	$10^{-21}$

#### Binary prefixes

kibi	$2^{10}$
mebi	$2^{20}$
gibi	$2^{30}$
tebi	$2^{40}$
pebi	$2^{50}$
exbi	$2^{60}$
zebi	$2^{70}$
yobi	$2^{80}$

## Weight

gram	$10^{-3}$ kg
metric_ton	$10^3$ kg
grain	one grain in kg
lb	one pound (avoirdupous) in kg
pound	one pound (avoirdupous) in kg
oz	one ounce in kg
ounce	one ounce in kg
stone	one stone in kg
grain	one grain in kg
long_ton	one long ton in kg
short_ton	one short ton in kg
troy_ounce	one Troy ounce in kg
troy_pound	one Troy pound in kg
carat	one carat in kg
m_u	atomic mass constant (in kg)
u	atomic mass constant (in kg)
atomic_mass	atomic mass constant (in kg)

## Angle

degree	degree in radians
arcmin	arc minute in radians
arcminute	arc minute in radians
arcsec	arc second in radians
arcsecond	arc second in radians

## Time

minute	one minute in seconds
hour	one hour in seconds
day	one day in seconds
week	one week in seconds
year	one year (365 days) in seconds
Julian_year	one Julian year (365.25 days) in seconds



## Length

inch	one inch in meters
foot	one foot in meters
yard	one yard in meters
mile	one mile in meters
mil	one mil in meters
pt	one point in meters
point	one point in meters
survey_foot	one survey foot in meters
survey_mile	one survey mile in meters
nautical_mile	one nautical mile in meters
fermi	one Fermi in meters
angstrom	one Angstrom in meters
micron	one micron in meters
au	one astronomical unit in meters
astronomical_unit	one astronomical unit in meters
light_year	one light year in meters
parsec	one parsec in meters

## Pressure

atm	standard atmosphere in pascals
atmosphere	standard atmosphere in pascals
bar	one bar in pascals
torr	one torr (mmHg) in pascals
mmHg	one torr (mmHg) in pascals
psi	one psi in pascals

## Area

hectare	one hectare in square meters
acre	one acre in square meters

## Volume

liter	one liter in cubic meters
litre	one liter in cubic meters
gallon	one gallon (US) in cubic meters
gallon_US	one gallon (US) in cubic meters
gallon_imp	one gallon (UK) in cubic meters
fluid_ounce	one fluid ounce (US) in cubic meters
fluid_ounce_US	one fluid ounce (US) in cubic meters
fluid_ounce_imp	one fluid ounce (UK) in cubic meters
bbl	one barrel in cubic meters
barrel	one barrel in cubic meters

## Speed

kmh	kilometers per hour in meters per second
mph	miles per hour in meters per second
mach	one Mach (approx., at 15 C, 1 atm) in meters per second
speed_of_sound	one Mach (approx., at 15 C, 1 atm) in meters per second
knot	one knot in meters per second

## Temperature

zero_Celsius	zero of Celsius scale in Kelvin
degree_Fahrenheit	one Fahrenheit (only differences) in Kelvins

<code>convert_temperature(val, old_scale, new_scale)</code>	Convert from a temperature scale to another one among Celsius, Kelvin, Fahrenheit and Rankine scales.
<code>C2K(*args, **kwds)</code>	C2K is deprecated!
<code>K2C(*args, **kwds)</code>	K2C is deprecated!
<code>F2C(*args, **kwds)</code>	F2C is deprecated!
<code>C2F(*args, **kwds)</code>	C2F is deprecated!
<code>F2K(*args, **kwds)</code>	F2K is deprecated!
<code>K2F(*args, **kwds)</code>	K2F is deprecated!

`scipy.constants.convert_temperature` (*val, old\_scale, new\_scale*)

Convert from a temperature scale to another one among Celsius, Kelvin, Fahrenheit and Rankine scales.

**Parameters** **val** : array\_like  
Value(s) of the temperature(s) to be converted expressed in the original scale.

**old\_scale**: str  
Specifies as a string the original scale from which the temperature value(s) will be converted. Supported scales are Celsius ('Celsius', 'celsius', 'C' or 'c'), Kelvin ('Kelvin', 'kelvin', 'K', 'k'), Fahrenheit ('Fahrenheit', 'fahrenheit', 'F' or 'f') and Rankine ('Rankine', 'rankine', 'R', 'r').

**new\_scale**: str  
Specifies as a string the new scale to which the temperature value(s) will be converted. Supported scales are Celsius ('Celsius', 'celsius', 'C' or 'c'), Kelvin ('Kelvin', 'kelvin', 'K', 'k'), Fahrenheit ('Fahrenheit', 'fahrenheit', 'F' or 'f') and Rankine ('Rankine', 'rankine', 'R', 'r').

**Returns** **res** : float or array of floats  
Value(s) of the converted temperature(s) expressed in the new scale.

### Notes

New in version 0.18.0.

### Examples

```
>>> from scipy.constants import convert_temperature
>>> convert_temperature(np.array([-40, 40.0]), 'Celsius', 'Kelvin')
array([ 233.15,  313.15])
```

`scipy.constants.C2K` (*\*args, \*\*kwds*)

C2K is deprecated! `scipy.constants.C2K` is deprecated in `scipy 0.18.0`. Use `scipy.constants.convert_temperature` instead. Note that the new function has a different signature.

Convert Celsius to Kelvin

**Parameters** **C** : array\_like  
Celsius temperature(s) to be converted.

**Returns** **K** : float or array of floats  
Equivalent Kelvin temperature(s).

**Notes**

Computes  $K = C + \text{zero\_Celsius}$  where  $\text{zero\_Celsius} = 273.15$ , i.e., (the absolute value of) temperature “absolute zero” as measured in Celsius.

**Examples**

```
>>> from scipy.constants import C2K
>>> C2K(np.array([-40, 40.0]))
array([ 233.15,  313.15])
```

`scipy.constants.K2C(*args, **kws)`

**K2C** is deprecated! `scipy.constants.K2C` is deprecated in scipy 0.18.0. Use `scipy.constants.convert_temperature` instead. Note that the new function has a different signature.

Convert Kelvin to Celsius

**Parameters** **K** : array\_like  
Kelvin temperature(s) to be converted.

**Returns** **C** : float or array of floats  
Equivalent Celsius temperature(s).

**Notes**

Computes  $C = K - \text{zero\_Celsius}$  where  $\text{zero\_Celsius} = 273.15$ , i.e., (the absolute value of) temperature “absolute zero” as measured in Celsius.

**Examples**

```
>>> from scipy.constants import K2C
>>> K2C(np.array([233.15, 313.15]))
array([-40.,  40.])
```

`scipy.constants.F2C(*args, **kws)`

**F2C** is deprecated! `scipy.constants.F2C` is deprecated in scipy 0.18.0. Use `scipy.constants.convert_temperature` instead. Note that the new function has a different signature.

Convert Fahrenheit to Celsius

**Parameters** **F** : array\_like  
Fahrenheit temperature(s) to be converted.

**Returns** **C** : float or array of floats  
Equivalent Celsius temperature(s).

**Notes**

Computes  $C = (F - 32) / 1.8$ .

**Examples**

```
>>> from scipy.constants import F2C
>>> F2C(np.array([-40, 40.0]))
array([-40.,          ,  4.44444444])
```

`scipy.constants.C2F(*args, **kws)`

`C2F` is deprecated! `scipy.constants.C2F` is deprecated in scipy 0.18.0. Use `scipy.constants.convert_temperature` instead. Note that the new function has a different signature.

Convert Celsius to Fahrenheit

**Parameters** **C** : array\_like  
Celsius temperature(s) to be converted.

**Returns** **F** : float or array of floats  
Equivalent Fahrenheit temperature(s).

**Notes**

Computes  $F = 1.8 * C + 32$ .

**Examples**

```
>>> from scipy.constants import C2F
>>> C2F(np.array([-40, 40.0]))
array([-40., 104.]
```

`scipy.constants.F2K(*args, **kws)`

`F2K` is deprecated! `scipy.constants.F2K` is deprecated in scipy 0.18.0. Use `scipy.constants.convert_temperature` instead. Note that the new function has a different signature.

Convert Fahrenheit to Kelvin

**Parameters** **F** : array\_like  
Fahrenheit temperature(s) to be converted.

**Returns** **K** : float or array of floats  
Equivalent Kelvin temperature(s).

**Notes**

Computes  $K = (F - 32)/1.8 + \text{zero\_Celsius}$  where `zero_Celsius` = 273.15, i.e., (the absolute value of) temperature “absolute zero” as measured in Celsius.

**Examples**

```
>>> from scipy.constants import F2K
>>> F2K(np.array([-40, 104]))
array([ 233.15,  313.15])
```

`scipy.constants.K2F(*args, **kws)`

`K2F` is deprecated! `scipy.constants.K2F` is deprecated in scipy 0.18.0. Use `scipy.constants.convert_temperature` instead. Note that the new function has a different signature.

Convert Kelvin to Fahrenheit

**Parameters** **K** : array\_like  
Kelvin temperature(s) to be converted.

**Returns** **F** : float or array of floats  
Equivalent Fahrenheit temperature(s).

**Notes**

Computes  $F = 1.8 * (K - \text{zero\_Celsius}) + 32$  where `zero_Celsius` = 273.15, i.e., (the absolute value of) temperature “absolute zero” as measured in Celsius.

**Examples**

```
>>> from scipy.constants import K2F
>>> K2F(np.array([233.15, 313.15]))
array([-40., 104.]
```

**Energy**

eV	one electron volt in Joules
electron_volt	one electron volt in Joules
calorie	one calorie (thermochemical) in Joules
calorie_th	one calorie (thermochemical) in Joules
calorie_IT	one calorie (International Steam Table calorie, 1956) in Joules
erg	one erg in Joules
Btu	one British thermal unit (International Steam Table) in Joules
Btu_IT	one British thermal unit (International Steam Table) in Joules
Btu_th	one British thermal unit (thermochemical) in Joules
ton_TNT	one ton of TNT in Joules

**Power**

hp	one horsepower in watts
horsepower	one horsepower in watts

**Force**

dyn	one dyne in newtons
dyne	one dyne in newtons
lbf	one pound force in newtons
pound_force	one pound force in newtons
kgf	one kilogram force in newtons
kilogram_force	one kilogram force in newtons

**Optics**

<code>lambda2nu(lambda_)</code>	Convert wavelength to optical frequency
<code>nu2lambda(nu)</code>	Convert optical frequency to wavelength.

`scipy.constants.lambda2nu(lambda_)`

Convert wavelength to optical frequency

**Parameters** **lambda\_** : array\_like  
Wavelength(s) to be converted.

**Returns** **nu** : float or array of floats  
Equivalent optical frequency.

**Notes**

Computes  $\nu = c / \lambda$  where  $c = 299792458.0$ , i.e., the (vacuum) speed of light in meters/second.

*Examples*

```
>>> from scipy.constants import lambda2nu, speed_of_light
>>> lambda2nu(np.array([1, speed_of_light]))
array([ 2.99792458e+08,  1.00000000e+00])
```

`scipy.constants.nu2lambda` (*nu*)  
 Convert optical frequency to wavelength.

**Parameters**    **nu** : array\_like  
                   Optical frequency to be converted.

**Returns**        **lambda** : float or array of floats  
                   Equivalent wavelength(s).

*Notes*

Computes  $\lambda = c / \nu$  where  $c = 299792458.0$ , i.e., the (vacuum) speed of light in meters/second.

*Examples*

```
>>> from scipy.constants import nu2lambda, speed_of_light
>>> nu2lambda(np.array([1, speed_of_light]))
array([ 2.99792458e+08,  1.00000000e+00])
```

## 5.4.4 References

## 5.5 Discrete Fourier transforms (`scipy.fftpack`)

### 5.5.1 Fast Fourier Transforms (FFTs)

---

<code>fft(x[, n, axis, overwrite_x])</code>	Return discrete Fourier transform of real or complex sequence.
<code>ifft(x[, n, axis, overwrite_x])</code>	Return discrete inverse Fourier transform of real or complex sequence.
<code>fft2(x[, shape, axes, overwrite_x])</code>	2-D discrete Fourier transform.
<code>ifft2(x[, shape, axes, overwrite_x])</code>	2-D discrete inverse Fourier transform of real or complex sequence.
<code>fftn(x[, shape, axes, overwrite_x])</code>	Return multidimensional discrete Fourier transform.
<code>ifftn(x[, shape, axes, overwrite_x])</code>	Return inverse multi-dimensional discrete Fourier transform of arbitrary type sequence
<code>rfft(x[, n, axis, overwrite_x])</code>	Discrete Fourier transform of a real sequence.
<code>irfft(x[, n, axis, overwrite_x])</code>	Return inverse discrete Fourier transform of real sequence <i>x</i> .
<code>dct(x[, type, n, axis, norm, overwrite_x])</code>	Return the Discrete Cosine Transform of arbitrary type sequence <i>x</i> .
<code>idct(x[, type, n, axis, norm, overwrite_x])</code>	Return the Inverse Discrete Cosine Transform of an arbitrary type sequence.
<code>dst(x[, type, n, axis, norm, overwrite_x])</code>	Return the Discrete Sine Transform of arbitrary type sequence <i>x</i> .
<code>idst(x[, type, n, axis, norm, overwrite_x])</code>	Return the Inverse Discrete Sine Transform of an arbitrary type sequence.

---

`scipy.fftpack.fft` (*x*, *n=None*, *axis=-1*, *overwrite\_x=False*)

Return discrete Fourier transform of real or complex sequence.

The returned complex array contains  $y(0), y(1), \dots, y(n-1)$  where

$$y(j) = (x * \exp(-2 * \pi * \text{sqrt}(-1) * j * \text{np.arange}(n) / n)) . \text{sum}()$$

**Parameters**    **x** : array\_like  
                   Array to Fourier transform.

**n** : int, optional

Length of the Fourier transform. If  $n < x.\text{shape}[\text{axis}]$ ,  $x$  is truncated. If  $n > x.\text{shape}[\text{axis}]$ ,  $x$  is zero-padded. The default results in  $n = x.\text{shape}[\text{axis}]$ .

**axis** : int, optional

Axis along which the fft's are computed; the default is over the last axis (i.e.,  $\text{axis}=-1$ ).

**overwrite\_x** : bool, optional

If True, the contents of  $x$  can be destroyed; the default is False.

**Returns** **z** : complex ndarray

with the elements:

$$\begin{aligned} & [y(0), y(1), \dots, y(n/2), y(1-n/2), \dots, y(-1)] && \text{if } n \text{ is even} \\ & [y(0), y(1), \dots, y((n-1)/2), y(-(n-1)/2), \dots, y(-1)] && \text{if } n \text{ is odd} \end{aligned}$$

where:

$$y(j) = \sum[k=0..n-1] x[k] * \exp(-\text{sqrt}(-1)*j*k* 2*\text{pi}/n), j = 0..n-1$$

Note that  $y(-j) = y(n-j).\text{conjugate}()$ .

**See also:**

***ifft*** Inverse FFT

***rfft*** FFT of a real sequence

**Notes**

The packing of the result is “standard”: If  $A = \text{fft}(a, n)$ , then  $A[0]$  contains the zero-frequency term,  $A[1:n/2]$  contains the positive-frequency terms, and  $A[n/2:]$  contains the negative-frequency terms, in order of decreasingly negative frequency. So for an 8-point transform, the frequencies of the result are  $[0, 1, 2, 3, -4, -3, -2, -1]$ . To rearrange the fft output so that the zero-frequency component is centered, like  $[-4, -3, -2, -1, 0, 1, 2, 3]$ , use `fftshift`.

For  $n$  even,  $A[n/2]$  contains the sum of the positive and negative-frequency terms. For  $n$  even and  $x$  real,  $A[n/2]$  will always be real.

This function is most efficient when  $n$  is a power of two, and least efficient when  $n$  is prime.

If the data type of  $x$  is real, a “real FFT” algorithm is automatically used, which roughly halves the computation time. To increase efficiency a little further, use `rfft`, which does the same calculation, but only outputs half of the symmetrical spectrum. If the data is both real and symmetrical, the `dct` can again double the efficiency, by generating half of the spectrum from half of the signal.

**Examples**

```
>>> from scipy.fftpack import fft, ifft
>>> x = np.arange(5)
>>> np.allclose(fft(ifft(x)), x, atol=1e-15) # within numerical accuracy.
True
```

`scipy.fftpack.ifft(x, n=None, axis=-1, overwrite_x=False)`

Return discrete inverse Fourier transform of real or complex sequence.

The returned complex array contains  $y(0), y(1), \dots, y(n-1)$  where

$$y(j) = (x * \exp(2*\text{pi}*\text{sqrt}(-1)*j*\text{np.arange}(n)/n)).\text{mean}()$$

**Parameters** **x** : array\_like

Transformed data to invert.

**n** : int, optional

Length of the inverse Fourier transform. If  $n < x.shape[axis]$ ,  $x$  is truncated. If  $n > x.shape[axis]$ ,  $x$  is zero-padded. The default results in  $n = x.shape[axis]$ .

**axis** : int, optional

Axis along which the ifft's are computed; the default is over the last axis (i.e.,  $axis=-1$ ).

**overwrite\_x** : bool, optional

If True, the contents of  $x$  can be destroyed; the default is False.

**Returns** **ifft** : ndarray of floats

The inverse discrete Fourier transform.

**See also:**

[\*fft\*](#) Forward FFT

**Notes**

This function is most efficient when  $n$  is a power of two, and least efficient when  $n$  is prime.

If the data type of  $x$  is real, a “real IFFT” algorithm is automatically used, which roughly halves the computation time.

`scipy.fftpack.fft2(x, shape=None, axes=(-2, -1), overwrite_x=False)`  
 2-D discrete Fourier transform.

Return the two-dimensional discrete Fourier transform of the 2-D argument  $x$ .

**See also:**

[\*fftn\*](#) for detailed information.

`scipy.fftpack.ifft2(x, shape=None, axes=(-2, -1), overwrite_x=False)`  
 2-D discrete inverse Fourier transform of real or complex sequence.

Return inverse two-dimensional discrete Fourier transform of arbitrary type sequence  $x$ .

See [\*ifft\*](#) for more information.

**See also:**

[\*fft2\*](#), [\*ifft\*](#)

`scipy.fftpack.fftn(x, shape=None, axes=None, overwrite_x=False)`  
 Return multidimensional discrete Fourier transform.

The returned array contains:

$$y[j_1, \dots, j_d] = \sum_{k_1=0..n_1-1, \dots, k_d=0..n_d-1} x[k_1, \dots, k_d] * \prod_{i=1..d} \exp(-\sqrt{-1} * 2 * \pi / n_i * j_i * k_i)$$

where  $d = \text{len}(x.shape)$  and  $n = x.shape$ . Note that  $y[\dots, -j_i, \dots] = y[\dots, n_i - j_i, \dots].conjugate()$ .

**Parameters** **x** : array\_like

The (n-dimensional) array to transform.

**shape** : tuple of ints, optional

The shape of the result. If both *shape* and *axes* (see below) are None, *shape* is  $x.shape$ ; if *shape* is None but *axes* is not None, then *shape* is `scipy.take(x.shape, axes, axis=0)`. If  $shape[i] > x.shape[i]$ , the  $i$ -th dimension is padded with zeros. If  $shape[i] < x.shape[i]$ , the  $i$ -th dimension is truncated to length  $shape[i]$ .



**axes** : array\_like of ints, optional  
The axes of  $x$  (y if *shape* is not None) along which the transform is applied.

**overwrite\_x** : bool, optional  
If True, the contents of  $x$  can be destroyed. Default is False.

**Returns** **y** : complex-valued n-dimensional numpy array  
The (n-dimensional) DFT of the input array.

**See also:**

[ifftn](#)

**Examples**

```
>>> from scipy.fftpack import fftn, ifftn
>>> y = (-np.arange(16), 8 - np.arange(16), np.arange(16))
>>> np.allclose(y, fftn(ifftn(y)))
True
```

`scipy.fftpack.ifftn(x, shape=None, axes=None, overwrite_x=False)`

Return inverse multi-dimensional discrete Fourier transform of arbitrary type sequence  $x$ .

The returned array contains:

$$y[j_1, \dots, j_d] = 1/p * \sum[k_1=0..n_1-1, \dots, k_d=0..n_d-1] x[k_1, \dots, k_d] * \prod[i=1..d] \exp(\sqrt{-1} * 2 * \pi / n_i * j_i * k_i)$$

where  $d = \text{len}(x.\text{shape})$ ,  $n = x.\text{shape}$ , and  $p = \prod[i=1..d] n_i$ .

For description of parameters see [fftn](#).

**See also:**

[fftn](#) for detailed information.

`scipy.fftpack.rfft(x, n=None, axis=-1, overwrite_x=False)`

Discrete Fourier transform of a real sequence.

**Parameters** **x** : array\_like, real-valued  
The data to transform.

**n** : int, optional  
Defines the length of the Fourier transform. If  $n$  is not specified (the default) then  $n = x.\text{shape}[\text{axis}]$ . If  $n < x.\text{shape}[\text{axis}]$ ,  $x$  is truncated, if  $n > x.\text{shape}[\text{axis}]$ ,  $x$  is zero-padded.

**axis** : int, optional  
The axis along which the transform is applied. The default is the last axis.

**overwrite\_x** : bool, optional  
If set to true, the contents of  $x$  can be overwritten. Default is False.

**Returns**

**z** : real ndarray

The returned real array contains:

$$\begin{aligned} & [y(0), \text{Re}(y(1)), \text{Im}(y(1)), \dots, \text{Re}(y(n/2))] && \text{if } n \text{ is even} \\ & [y(0), \text{Re}(y(1)), \text{Im}(y(1)), \dots, \text{Re}(y(n/2)), \text{Im}(y(n/2))] && \text{if } n \text{ is odd} \end{aligned}$$

where:

$$y(j) = \sum[k=0..n-1] x[k] * \exp(-\sqrt{-1} * j * k * 2 * \pi / n)$$

$$j = 0..n-1$$

Note that  $y(-j) == y(n-j).\text{conjugate}()$ .

**See also:**

`fft`, `irfft`, `scipy.fftpack.basic`

**Notes**

Within numerical accuracy, `y == rfft(irfft(y))`.

**Examples**

```
>>> from scipy.fftpack import fft, rfft
>>> a = [9, -9, 1, 3]
>>> fft(a)
array([ 4.+0.j,  8.+12.j, 16.+0.j,  8.-12.j])
>>> rfft(a)
array([ 4.,  8., 12., 16.] )
```

`scipy.fftpack.irfft(x, n=None, axis=-1, overwrite_x=False)`

Return inverse discrete Fourier transform of real sequence `x`.

The contents of `x` are interpreted as the output of the `rfft` function.

**Parameters**

- x** : array\_like  
Transformed data to invert.
- n** : int, optional  
Length of the inverse Fourier transform. If `n < x.shape[axis]`, `x` is truncated. If `n > x.shape[axis]`, `x` is zero-padded. The default results in `n = x.shape[axis]`.
- axis** : int, optional  
Axis along which the `irfft`'s are computed; the default is over the last axis (i.e., `axis=-1`).
- overwrite\_x** : bool, optional  
If True, the contents of `x` can be destroyed; the default is False.

**Returns**

- irfft** : ndarray of floats  
The inverse discrete Fourier transform.

**See also:**

`rfft`, `ifft`

**Notes**

The returned real array contains:

`[y(0), y(1), ..., y(n-1)]`

where for `n` is even:

$$y(j) = 1/n (\text{sum}[k=1..n/2-1] (x[2*k-1]+sqrt(-1)*x[2*k]) * \exp(sqrt(-1)*j*k* 2*pi/n) + \text{c.c.} + x[0] + (-1)**(j) x[n-1])$$

and for `n` is odd:

$$y(j) = 1/n (\text{sum}[k=1..(n-1)/2] (x[2*k-1]+sqrt(-1)*x[2*k]) * \exp(sqrt(-1)*j*k* 2*pi/n) + \text{c.c.} + x[0])$$

`c.c.` denotes complex conjugate of preceding expression.

For details on input parameters, see `rfft`.

`scipy.fftpack.dct(x, type=2, n=None, axis=-1, norm=None, overwrite_x=False)`

Return the Discrete Cosine Transform of arbitrary type sequence `x`.

**Parameters**

- x** : array\_like  
The input array.
- type** : {1, 2, 3}, optional  
Type of the DCT (see Notes). Default type is 2.
- n** : int, optional  
Length of the transform. If  $n < x.shape[axis]$ ,  $x$  is truncated. If  $n > x.shape[axis]$ ,  $x$  is zero-padded. The default results in  $n = x.shape[axis]$ .
- axis** : int, optional  
Axis along which the dct is computed; the default is over the last axis (i.e., `axis=-1`).
- norm** : {None, 'ortho'}, optional  
Normalization mode (see Notes). Default is None.
- overwrite\_x** : bool, optional  
If True, the contents of  $x$  can be destroyed; the default is False.

**Returns**

- y** : ndarray of real  
The transformed input array.

**See also:**

[\*idct\*](#) Inverse DCT

### Notes

For a single dimension array  $x$ , `dct(x, norm='ortho')` is equal to MATLAB `dct(x)`.

There are theoretically 8 types of the DCT, only the first 3 types are implemented in scipy. 'The' DCT generally refers to DCT type 2, and 'the' Inverse DCT generally refers to DCT type 3.

### Type I

There are several definitions of the DCT-I; we use the following (for `norm=None`):

$$y[k] = x[0] + (-1)**k x[N-1] + 2 * \sum_{n=1}^{N-2} x[n] * \cos(\pi * k * n / (N-1))$$

Only None is supported as normalization mode for DCT-I. Note also that the DCT-I is only supported for input size  $> 1$

### Type II

There are several definitions of the DCT-II; we use the following (for `norm=None`):

$$y[k] = 2 * \sum_{n=0}^{N-1} x[n] * \cos(\pi * k * (2n+1) / (2*N)), \quad 0 \leq k < N.$$

If `norm='ortho'`,  $y[k]$  is multiplied by a scaling factor  $f$ :

$$f = \sqrt{1/(4*N)} \quad \text{if } k = 0,$$

$$f = \sqrt{1/(2*N)} \quad \text{otherwise.}$$

Which makes the corresponding matrix of coefficients orthonormal ( $OO' = Id$ ).

### Type III

There are several definitions, we use the following (for `norm=None`):

$$y[k] = x[0] + 2 * \sum_{n=1}^{N-1} x[n] * \cos(\pi * (k+0.5) * n / N), \quad 0 \leq k < N.$$

or, for `norm='ortho'` and  $0 \leq k < N$ :

$$y[k] = x[0] / \sqrt{N} + \sqrt{2/N} \sum_{n=1}^{N-1} x[n] \cos(\pi * (k+0.5) * n/N)$$

The (unnormalized) DCT-III is the inverse of the (unnormalized) DCT-II, up to a factor  $2N$ . The orthonormalized DCT-III is exactly the inverse of the orthonormalized DCT-II.

### References

[R39], [R40]

### Examples

The Type 1 DCT is equivalent to the FFT (though faster) for real, even-symmetrical inputs. The output is also real and even-symmetrical. Half of the FFT input is used to generate half of the FFT output:

```
>>> from scipy.fftpack import fft, dct
>>> fft(np.array([4., 3., 5., 10., 5., 3.])).real
array([ 30., -8., 6., -2., 6., -8.])
>>> dct(np.array([4., 3., 5., 10.]), 1)
array([ 30., -8., 6., -2.])
```

`scipy.fftpack.idct(x, type=2, n=None, axis=-1, norm=None, overwrite_x=False)`

Return the Inverse Discrete Cosine Transform of an arbitrary type sequence.

**Parameters**

- x** : array\_like  
The input array.
- type** : {1, 2, 3}, optional  
Type of the DCT (see Notes). Default type is 2.
- n** : int, optional  
Length of the transform. If  $n < x.shape[axis]$ ,  $x$  is truncated. If  $n > x.shape[axis]$ ,  $x$  is zero-padded. The default results in  $n = x.shape[axis]$ .
- axis** : int, optional  
Axis along which the `idct` is computed; the default is over the last axis (i.e., `axis=-1`).
- norm** : {None, 'ortho'}, optional  
Normalization mode (see Notes). Default is None.
- overwrite\_x** : bool, optional  
If True, the contents of  $x$  can be destroyed; the default is False.

**Returns**

- idct** : ndarray of real  
The transformed input array.

See also:

[dct](#) Forward DCT

### Notes

For a single dimension array  $x$ , `idct(x, norm='ortho')` is equal to MATLAB `idct(x)`.

'The' IDCT is the IDCT of type 2, which is the same as DCT of type 3.

IDCT of type 1 is the DCT of type 1, IDCT of type 2 is the DCT of type 3, and IDCT of type 3 is the DCT of type 2. For the definition of these types, see [dct](#).

### Examples

The Type 1 DCT is equivalent to the DFT for real, even-symmetrical inputs. The output is also real and even-symmetrical. Half of the IFFT input is used to generate half of the IFFT output:

```
>>> from scipy.fftpack import ifft, idct
>>> ifft(np.array([ 30., -8., 6., -2., 6., -8.])).real
array([ 4., 3., 5., 10., 5., 3.])
>>> idct(np.array([ 30., -8., 6., -2.]), 1) / 6
array([ 4., 3., 5., 10.]])
```

`scipy.fftpack.dst(x, type=2, n=None, axis=-1, norm=None, overwrite_x=False)`

Return the Discrete Sine Transform of arbitrary type sequence `x`.

**Parameters**

- x** : array\_like  
The input array.
- type** : {1, 2, 3}, optional  
Type of the DST (see Notes). Default type is 2.
- n** : int, optional  
Length of the transform. If  $n < x.shape[axis]$ , `x` is truncated. If  $n > x.shape[axis]$ , `x` is zero-padded. The default results in  $n = x.shape[axis]$ .
- axis** : int, optional  
Axis along which the `dst` is computed; the default is over the last axis (i.e., `axis=-1`).
- norm** : {None, 'ortho'}, optional  
Normalization mode (see Notes). Default is None.
- overwrite\_x** : bool, optional  
If True, the contents of `x` can be destroyed; the default is False.

**Returns**

- dst** : ndarray of reals  
The transformed input array.

See also:

`idst` Inverse DST

### Notes

For a single dimension array `x`.

There are theoretically 8 types of the DST for different combinations of even/odd boundary conditions and boundary off sets [R41], only the first 3 types are implemented in `scipy`.

#### Type I

There are several definitions of the DST-I; we use the following for `norm=None`. DST-I assumes the input is odd around  $n=-1$  and  $n=N$ .

$$y[k] = 2 * \sum_{n=0}^{N-1} x[n] * \sin(\pi * (k+1) * (n+1) / (N+1))$$

Only None is supported as normalization mode for DCT-I. Note also that the DCT-I is only supported for input size  $> 1$  The (unnormalized) DCT-I is its own inverse, up to a factor  $2(N+1)$ .

#### Type II

There are several definitions of the DST-II; we use the following for `norm=None`. DST-II assumes the input is odd around  $n=-1/2$  and  $n=N-1/2$ ; the output is odd around  $k=-1$  and even around  $k=N-1$

$$y[k] = 2 \sum_{n=0}^{N-1} x[n] \sin(\pi * (k+1) * (n+0.5) / N), \quad 0 \leq k < N.$$

if `norm='ortho'`,  $y[k]$  is multiplied by a scaling factor  $f$

$$f = \sqrt{1/(4*N)} \quad \text{if } k == 0$$

$$f = \sqrt{1/(2*N)} \quad \text{otherwise.}$$

### Type III

There are several definitions of the DST-III, we use the following (for `norm=None`). DST-III assumes the input is odd around  $n=-1$  and even around  $n=N-1$

$$y[k] = x[N-1] * (-1) ** k + 2 \sum_{n=0}^{N-2} x[n] \sin(\pi * (k+0.5) * (n+1) / N), \quad 0 \leq k < N.$$

The (unnormalized) DCT-III is the inverse of the (unnormalized) DCT-II, up to a factor  $2N$ . The orthonormalized DST-III is exactly the inverse of the orthonormalized DST-II.

New in version 0.11.0.

### References

[R41]

`scipy.fftpack.idst(x, type=2, n=None, axis=-1, norm=None, overwrite_x=False)`

Return the Inverse Discrete Sine Transform of an arbitrary type sequence.

**Parameters**

- x** : array\_like  
The input array.
- type** : {1, 2, 3}, optional  
Type of the DST (see Notes). Default type is 2.
- n** : int, optional  
Length of the transform. If  $n < x.shape[axis]$ ,  $x$  is truncated. If  $n > x.shape[axis]$ ,  $x$  is zero-padded. The default results in  $n = x.shape[axis]$ .
- axis** : int, optional  
Axis along which the idst is computed; the default is over the last axis (i.e., `axis=-1`).
- norm** : {None, 'ortho'}, optional  
Normalization mode (see Notes). Default is None.
- overwrite\_x** : bool, optional  
If True, the contents of  $x$  can be destroyed; the default is False.

**Returns**

- idst** : ndarray of real  
The transformed input array.

See also:

[dst](#) Forward DST

### Notes

'The' IDST is the IDST of type 2, which is the same as DST of type 3.

IDST of type 1 is the DST of type 1, IDST of type 2 is the DST of type 3, and IDST of type 3 is the DST of type 2. For the definition of these types, see [dst](#).

New in version 0.11.0.

## 5.5.2 Differential and pseudo-differential operators

---

<code>diff(x[, order, period, _cache])</code>	Return k-th derivative (or integral) of a periodic sequence x.
<code>tilbert(x, h[, period, _cache])</code>	Return h-Tilbert transform of a periodic sequence x.
<code>itilbert(x, h[, period, _cache])</code>	Return inverse h-Tilbert transform of a periodic sequence x.
<code>hilbert(x[, _cache])</code>	Return Hilbert transform of a periodic sequence x.
<code>ihilbert(x)</code>	Return inverse Hilbert transform of a periodic sequence x.
<code>cs_diff(x, a, b[, period, _cache])</code>	Return (a,b)-cosh/sinh pseudo-derivative of a periodic sequence.
<code>sc_diff(x, a, b[, period, _cache])</code>	Return (a,b)-sinh/cosh pseudo-derivative of a periodic sequence x.
<code>ss_diff(x, a, b[, period, _cache])</code>	Return (a,b)-sinh/sinh pseudo-derivative of a periodic sequence x.
<code>cc_diff(x, a, b[, period, _cache])</code>	Return (a,b)-cosh/cosh pseudo-derivative of a periodic sequence.
<code>shift(x, a[, period, _cache])</code>	Shift periodic sequence x by a: $y(u) = x(u+a)$ .

---

`scipy.fftpack.diff(x, order=1, period=None, _cache={})`

Return k-th derivative (or integral) of a periodic sequence x.

If  $x_j$  and  $y_j$  are Fourier coefficients of periodic functions x and y, respectively, then:

$$y_j = \text{pow}(\text{sqrt}(-1) * j * 2 * \text{pi} / \text{period}, \text{order}) * x_j$$

$y_0 = 0$  if order is not 0.

**Parameters** **x** : array\_like

Input array.

**order** : int, optional

The order of differentiation. Default order is 1. If order is negative, then integration is carried out under the assumption that  $x_0 == 0$ .

**period** : float, optional

The assumed period of the sequence. Default is  $2 * \text{pi}$ .

### Notes

If  $\text{sum}(x, \text{axis}=0) = 0$  then  $\text{diff}(\text{diff}(x, k), -k) == x$  (within numerical accuracy).

For odd order and even  $\text{len}(x)$ , the Nyquist mode is taken zero.

`scipy.fftpack.tilbert(x, h, period=None, _cache={})`

Return h-Tilbert transform of a periodic sequence x.

If  $x_j$  and  $y_j$  are Fourier coefficients of periodic functions x and y, respectively, then:

$$y_j = \text{sqrt}(-1) * \text{coth}(j * h * 2 * \text{pi} / \text{period}) * x_j$$

$y_0 = 0$

**Parameters** **x** : array\_like

The input array to transform.

**h** : float

Defines the parameter of the Tilbert transform.

**period** : float, optional

The assumed period of the sequence. Default period is  $2 * \text{pi}$ .

**Returns** **tilbert** : ndarray

The result of the transform.

**Notes**

If  $\text{sum}(x, \text{axis}=0) == 0$  and  $n = \text{len}(x)$  is odd then  $\text{tilbert}(\text{itilbert}(x)) == x$ .

If  $2 * \pi * h / \text{period}$  is approximately 10 or larger, then numerically  $\text{tilbert} == \text{hilbert}$  (theoretically oo-Tilbert == Hilbert).

For even  $\text{len}(x)$ , the Nyquist mode of  $x$  is taken zero.

`scipy.fftpack.itilbert(x, h, period=None, _cache={})`

Return inverse h-Tilbert transform of a periodic sequence  $x$ .

If  $x_j$  and  $y_j$  are Fourier coefficients of periodic functions  $x$  and  $y$ , respectively, then:

$$y_j = -\sqrt{-1} * \tanh(j * h * 2 * \pi / \text{period}) * x_j$$

$$y_0 = 0$$

For more details, see `tilbert`.

`scipy.fftpack.hilbert(x, _cache={})`

Return Hilbert transform of a periodic sequence  $x$ .

If  $x_j$  and  $y_j$  are Fourier coefficients of periodic functions  $x$  and  $y$ , respectively, then:

$$y_j = \sqrt{-1} * \text{sign}(j) * x_j$$

$$y_0 = 0$$

- Parameters**
- x** : array\_like  
The input array, should be periodic.
  - \_cache** : dict, optional  
Dictionary that contains the kernel used to do a convolution with.
- Returns**
- y** : ndarray  
The transformed input.

**Notes**

If  $\text{sum}(x, \text{axis}=0) == 0$  then  $\text{hilbert}(\text{ihilbert}(x)) == x$ .

For even  $\text{len}(x)$ , the Nyquist mode of  $x$  is taken zero.

The sign of the returned transform does not have a factor -1 that is more often than not found in the definition of the Hilbert transform. Note also that `scipy.signal.hilbert` does have an extra -1 factor compared to this function.

`scipy.fftpack.ihilbert(x)`

Return inverse Hilbert transform of a periodic sequence  $x$ .

If  $x_j$  and  $y_j$  are Fourier coefficients of periodic functions  $x$  and  $y$ , respectively, then:

$$y_j = -\sqrt{-1} * \text{sign}(j) * x_j$$

$$y_0 = 0$$

`scipy.fftpack.cs_diff(x, a, b, period=None, _cache={})`

Return (a,b)-cosh/sinh pseudo-derivative of a periodic sequence.

If  $x_j$  and  $y_j$  are Fourier coefficients of periodic functions  $x$  and  $y$ , respectively, then:

$$y_j = -\sqrt{-1} * \cosh(j * a * 2 * \pi / \text{period}) / \sinh(j * b * 2 * \pi / \text{period}) * x_j$$

$$y_0 = 0$$

- Parameters**
- x** : array\_like  
The array to take the pseudo-derivative from.



**a, b** : float  
 Defines the parameters of the cosh/sinh pseudo-differential operator.

**period** : float, optional  
 The period of the sequence. Default period is  $2\pi$ .

**Returns** **cs\_diff** : ndarray  
 Pseudo-derivative of periodic sequence  $x$ .

**Notes**

For even  $\text{len}(x)$ , the Nyquist mode of  $x$  is taken as zero.

`scipy.fftpack.sc_diff(x, a, b, period=None, _cache={})`

Return (a,b)-sinh/cosh pseudo-derivative of a periodic sequence  $x$ .

If  $x_j$  and  $y_j$  are Fourier coefficients of periodic functions  $x$  and  $y$ , respectively, then:

$$y_j = \sqrt{-1} * \sinh(j*a*2*\pi/\text{period}) / \cosh(j*b*2*\pi/\text{period}) * x_j$$

$$y_0 = 0$$

**Parameters** **x** : array\_like  
 Input array.

**a,b** : float  
 Defines the parameters of the sinh/cosh pseudo-differential operator.

**period** : float, optional  
 The period of the sequence  $x$ . Default is  $2\pi$ .

**Notes**

`sc_diff(sc_diff(x, a, b), b, a) == x` For even  $\text{len}(x)$ , the Nyquist mode of  $x$  is taken as zero.

`scipy.fftpack.ss_diff(x, a, b, period=None, _cache={})`

Return (a,b)-sinh/sinh pseudo-derivative of a periodic sequence  $x$ .

If  $x_j$  and  $y_j$  are Fourier coefficients of periodic functions  $x$  and  $y$ , respectively, then:

$$y_j = \sinh(j*a*2*\pi/\text{period}) / \sinh(j*b*2*\pi/\text{period}) * x_j$$

$$y_0 = a/b * x_0$$

**Parameters** **x** : array\_like  
 The array to take the pseudo-derivative from.

**a,b**  
 Defines the parameters of the sinh/sinh pseudo-differential operator.

**period** : float, optional  
 The period of the sequence  $x$ . Default is  $2\pi$ .

**Notes**

`ss_diff(ss_diff(x, a, b), b, a) == x`

`scipy.fftpack.cc_diff(x, a, b, period=None, _cache={})`

Return (a,b)-cosh/cosh pseudo-derivative of a periodic sequence.

If  $x_j$  and  $y_j$  are Fourier coefficients of periodic functions  $x$  and  $y$ , respectively, then:

$$y_j = \cosh(j*a*2*\pi/\text{period}) / \cosh(j*b*2*\pi/\text{period}) * x_j$$

**Parameters** **x** : array\_like  
 The array to take the pseudo-derivative from.

**a,b** : float

Defines the parameters of the sinh/sinh pseudo-differential operator.

**period** : float, optional

The period of the sequence *x*. Default is  $2\pi$ .

**Returns** **cc\_diff** : ndarray

Pseudo-derivative of periodic sequence *x*.

**Notes**

`cc_diff(cc_diff(x, a, b), b, a) == x`

`scipy.fftpack.shift(x, a, period=None, _cache={})`

Shift periodic sequence *x* by *a*:  $y(u) = x(u+a)$ .

If  $x_j$  and  $y_j$  are Fourier coefficients of periodic functions *x* and *y*, respectively, then:

$y_j = \exp(j*a*2*\pi/period*\sqrt{-1}) * x_j$

**Parameters** **x** : array\_like

The array to take the pseudo-derivative from.

**a** : float

Defines the parameters of the sinh/sinh pseudo-differential

**period** : float, optional

The period of the sequences *x* and *y*. Default period is  $2\pi$ .

### 5.5.3 Helper functions

---

<code>fftshift(x[, axes])</code>	Shift the zero-frequency component to the center of the spectrum.
<code>ifftshift(x[, axes])</code>	The inverse of <code>fftshift</code> .
<code>fftfreq(n[, d])</code>	Return the Discrete Fourier Transform sample frequencies.
<code>rfftfreq(n[, d])</code>	DFT sample frequencies (for usage with <code>rfft</code> , <code>irfft</code> ).

---

`scipy.fftpack.fftshift(x, axes=None)`

Shift the zero-frequency component to the center of the spectrum.

This function swaps half-spaces for all axes listed (defaults to all). Note that  $y[0]$  is the Nyquist component only if `len(x)` is even.

**Parameters** **x** : array\_like

Input array.

**axes** : int or shape tuple, optional

Axes over which to shift. Default is `None`, which shifts all axes.

**Returns** **y** : ndarray

The shifted array.

See also:

`ifftshift` The inverse of `fftshift`.

**Examples**

```
>>> freqs = np.fft.fftfreq(10, 0.1)
>>> freqs
array([ 0.,  1.,  2.,  3.,  4., -5., -4., -3., -2., -1.])
>>> np.fft.fftshift(freqs)
array([-5., -4., -3., -2., -1.,  0.,  1.,  2.,  3.,  4.])
```

Shift the zero-frequency component only along the second axis:

```
>>> freqs = np.fft.fftfreq(9, d=1./9).reshape(3, 3)
>>> freqs
array([[ 0.,  1.,  2.],
       [ 3.,  4., -4.],
       [-3., -2., -1.]])
>>> np.fft.fftshift(freqs, axes=(1,))
array([[ 2.,  0.,  1.],
       [-4.,  3.,  4.],
       [-1., -3., -2.]])
```

`scipy.fftpack.iffshift(x, axes=None)`

The inverse of `fftshift`. Although identical for even-length  $x$ , the functions differ by one sample for odd-length  $x$ .

**Parameters** **x** : array\_like  
Input array.  
**axes** : int or shape tuple, optional  
Axes over which to calculate. Defaults to None, which shifts all axes.

**Returns** **y** : ndarray  
The shifted array.

**See also:**

[`fftshift`](#) Shift zero-frequency component to the center of the spectrum.

**Examples**

```
>>> freqs = np.fft.fftfreq(9, d=1./9).reshape(3, 3)
>>> freqs
array([[ 0.,  1.,  2.],
       [ 3.,  4., -4.],
       [-3., -2., -1.]])
>>> np.fft.iffshift(np.fft.fftshift(freqs))
array([[ 0.,  1.,  2.],
       [ 3.,  4., -4.],
       [-3., -2., -1.]])
```

`scipy.fftpack.fftfreq(n, d=1.0)`

Return the Discrete Fourier Transform sample frequencies.

The returned float array  $f$  contains the frequency bin centers in cycles per unit of the sample spacing (with zero at the start). For instance, if the sample spacing is in seconds, then the frequency unit is cycles/second.

Given a window length  $n$  and a sample spacing  $d$ :

$$f = [0, 1, \dots, n/2-1, -n/2, \dots, -1] / (d*n) \quad \text{if } n \text{ is even}$$

$$f = [0, 1, \dots, (n-1)/2, -(n-1)/2, \dots, -1] / (d*n) \quad \text{if } n \text{ is odd}$$

**Parameters** **n** : int  
Window length.  
**d** : scalar, optional  
Sample spacing (inverse of the sampling rate). Defaults to 1.

**Returns** **f** : ndarray  
Array of length  $n$  containing the sample frequencies.

*Examples*

```
>>> signal = np.array([-2, 8, 6, 4, 1, 0, 3, 5], dtype=float)
>>> fourier = np.fft.fft(signal)
>>> n = signal.size
>>> timestep = 0.1
>>> freq = np.fft.fftfreq(n, d=timestep)
>>> freq
array([ 0. ,  1.25,  2.5 ,  3.75, -5.  , -3.75, -2.5 , -1.25])
```

`scipy.fftpack.rfftfreq(n, d=1.0)`

DFT sample frequencies (for usage with rfft, irfft).

The returned float array contains the frequency bins in cycles/unit (with zero at the start) given a window length *n* and a sample spacing *d*:

```
f = [0,1,1,2,2,...,n/2-1,n/2-1,n/2]/(d*n)   if n is even
f = [0,1,1,2,2,...,n/2-1,n/2-1,n/2,n/2]/(d*n)   if n is odd
```

**Parameters** **n** : int  
Window length.  
**d** : scalar, optional  
Sample spacing. Default is 1.

**Returns** **out** : ndarray  
The array of length *n*, containing the sample frequencies.

*Examples*

```
>>> from scipy import fftpack
>>> sig = np.array([-2, 8, 6, 4, 1, 0, 3, 5], dtype=float)
>>> sig_fft = fftpack.rfft(sig)
>>> n = sig_fft.size
>>> timestep = 0.1
>>> freq = fftpack.rfftfreq(n, d=timestep)
>>> freq
array([ 0. ,  1.25,  1.25,  2.5 ,  2.5 ,  3.75,  3.75,  5.  ])
```

Note that `fftshift`, `ifftshift` and `fftfreq` are numpy functions exposed by `fftpack`; importing them from `numpy` should be preferred.

### 5.5.4 Convolutions (`scipy.fftpack.convolve`)

---

<code>convolve(x,omega,[swap_real_imag,overwrite_x])</code>	Wrapper for <code>convolve</code> .
<code>convolve_z(x,omega_real,omega_imag,[overwrite_x])</code>	Wrapper for <code>convolve_z</code> .
<code>init_convolution_kernel(...)</code>	Wrapper for <code>init_convolution_kernel</code> .
<code>destroy_convolve_cache()</code>	Wrapper for <code>destroy_convolve_cache</code> .

---

`scipy.fftpack.convolve.convolve(x, omega[, swap_real_imag, overwrite_x]) = <fortran object>`

Wrapper for `convolve`.

**Parameters** **x** : input rank-1 array('d') with bounds (n)  
**omega** : input rank-1 array('d') with bounds (n)  
**Returns** **y** : rank-1 array('d') with bounds (n) and x storage  
**Other Parameters**

**overwrite\_x** : input int, optional  
Default: 0  
**swap\_real\_imag** : input int, optional  
Default: 0

`scipy.fftpack.convolve.convolve_z(x, omega_real, omega_imag[, overwrite_x]) = <fortran object>`

Wrapper for `convolve_z`.

**Parameters** **x** : input rank-1 array('d') with bounds (n)  
**omega\_real** : input rank-1 array('d') with bounds (n)  
**omega\_imag** : input rank-1 array('d') with bounds (n)  
**Returns** **y** : rank-1 array('d') with bounds (n) and x storage  
**Other Parameters**  
**overwrite\_x** : input int, optional  
Default: 0

`scipy.fftpack.convolve.init_convolution_kernel(n, kernel_func[, d, zero_nyquist, kernel_func_extra_args]) = <fortran object>`

Wrapper for `init_convolution_kernel`.

**Parameters** **n** : input int  
**kernel\_func** : call-back function  
**Returns** **omega** : rank-1 array('d') with bounds (n)  
**Other Parameters**  
**d** : input int, optional  
Default: 0  
**kernel\_func\_extra\_args** : input tuple, optional  
Default: ()  
**zero\_nyquist** : input int, optional  
Default: `d%2`

### Notes

Call-back functions:

```
def kernel_func(k): return kernel_func
Required arguments:
  k : input int
Return objects:
  kernel_func : float
```

`scipy.fftpack.convolve.destroy_convolve_cache = <fortran object>`

Wrapper for `destroy_convolve_cache`.

## 5.6 Integration and ODEs (`scipy.integrate`)

### 5.6.1 Integrating functions, given function object

<code>quad(func, a, b[, args, full_output, ...])</code>	Compute a definite integral.
<code>dblquad(func, a, b, gfun, hfun[, args, ...])</code>	Compute a double integral.
<code>tplquad(func, a, b, gfun, hfun, qfun, rfun)</code>	Compute a triple (definite) integral.
<code>nquad(func, ranges[, args, opts, full_output])</code>	Integration over multiple variables.

Continued on next page

Table 5.18 – continued from previous page

<code>fixed_quad(func, a, b[, args, n])</code>	Compute a definite integral using fixed-order Gaussian quadrature.
<code>quadrature(func, a, b[, args, tol, rtol, ...])</code>	Compute a definite integral using fixed-tolerance Gaussian quadrature.
<code>romberg(function, a, b[, args, tol, rtol, ...])</code>	Romberg integration of a callable function or method.
<code>quad_explain([output])</code>	Print extra information about <code>integrate.quad()</code> parameters and returns.
<code>newton_cotes(m[, equal])</code>	Return weights and error coefficient for Newton-Cotes integration.
<code>IntegrationWarning</code>	Warning on issues during integration.

`scipy.integrate.quad` (*func*, *a*, *b*, *args*=(), *full\_output*=0, *epsabs*=1.49e-08, *epsrel*=1.49e-08, *limit*=50, *points*=None, *weight*=None, *wvar*=None, *wopts*=None, *maxpl*=50, *limlst*=50)

Compute a definite integral.

Integrate *func* from *a* to *b* (possibly infinite interval) using a technique from the Fortran library QUADPACK.

**Parameters** **func** : function

A Python function or method to integrate. If *func* takes many arguments, it is integrated along the axis corresponding to the first argument. If the user desires improved integration performance, then *f* may instead be a `ctypes` function of the form:

`f(int n, double args[n]),`

where *args* is an array of function arguments and *n* is the length of *args*. *f.argtypes* should be set to `(c_int, c_double)`, and *f.restype* should be `(c_double,)`.

**a** : float

Lower limit of integration (use `-numpy.inf` for `-infinity`).

**b** : float

Upper limit of integration (use `numpy.inf` for `+infinity`).

**args** : tuple, optional

Extra arguments to pass to *func*.

**full\_output** : int, optional

Non-zero to return a dictionary of integration information. If non-zero, warning messages are also suppressed and the message is appended to the output tuple.

**Returns**

**y** : float

The integral of *func* from *a* to *b*.

**abserr** : float

An estimate of the absolute error in the result.

**infodict** : dict

A dictionary containing additional information. Run `scipy.integrate.quad_explain()` for more information.

**message**

A convergence message.

**explain**

Appended only with ‘cos’ or ‘sin’ weighting and infinite integration limits, it contains an explanation of the codes in `infodict[‘ierlst’]`

**Other Parameters**

**epsabs** : float or int, optional

Absolute error tolerance.

**epsrel** : float or int, optional

Relative error tolerance.

**limit** : float or int, optional

An upper bound on the number of subintervals used in the adaptive algorithm.

**points** : (sequence of floats,ints), optional

A sequence of break points in the bounded integration interval where local difficulties of the integrand may occur (e.g., singularities, discontinuities). The sequence does not have to be sorted.

**weight** : float or int, optional  
String indicating weighting function. Full explanation for this and the remaining arguments can be found below.

**wvar** : optional  
Variables for use with weighting functions.

**wopts** : optional  
Optional input for reusing Chebyshev moments.

**maxp1** : float or int, optional  
An upper bound on the number of Chebyshev moments.

**limlst** : int, optional  
Upper bound on the number of cycles ( $\geq 3$ ) for use with a sinusoidal weighting and an infinite end-point.

**See also:**

*dblquad* double integral

*tplquad* triple integral

*nquad* n-dimensional integrals (uses *quad* recursively)

*fixed\_quad* fixed-order Gaussian quadrature

*quadrature* adaptive Gaussian quadrature

*odeint* ODE integrator

*ode* ODE integrator

*simps* integrator for sampled data

*romb* integrator for sampled data

*scipy.special*  
for coefficients and roots of orthogonal polynomials

*Notes***Extra information for *quad()* inputs and outputs**

If *full\_output* is non-zero, then the third output argument (*infodict*) is a dictionary with entries as tabulated below. For infinite limits, the range is transformed to (0,1) and the optional outputs are given with respect to this transformed range. Let *M* be the input argument *limit* and let *K* be *infodict*['last']. The entries are:

**'neval'** The number of function evaluations.

**'last'** The number, *K*, of subintervals produced in the subdivision process.

**'alist'** A rank-1 array of length *M*, the first *K* elements of which are the left end points of the subintervals in the partition of the integration range.

**'blist'** A rank-1 array of length *M*, the first *K* elements of which are the right end points of the subintervals.

**'rlist'** A rank-1 array of length *M*, the first *K* elements of which are the integral approximations on the subintervals.

**'elist'** A rank-1 array of length *M*, the first *K* elements of which are the moduli of the absolute error estimates on the subintervals.

**'iord'** A rank-1 integer array of length *M*, the first *L* elements of which are pointers to the error estimates over the subintervals with  $L=K$  if  $K \leq M/2+2$  or  $L=M+1-K$  otherwise. Let *I* be the sequence

`infodict['iord']` and let  $E$  be the sequence `infodict['elist']`. Then  $E[I[1]], \dots, E[I[L]]$  forms a decreasing sequence.

If the input argument `points` is provided (i.e. it is not `None`), the following additional outputs are placed in the output dictionary. Assume the `points` sequence is of length  $P$ .

- 'pts'** A rank-1 array of length  $P+2$  containing the integration limits and the break points of the intervals in ascending order. This is an array giving the subintervals over which integration will occur.
- 'level'** A rank-1 integer array of length  $M$  ( $=limit$ ), containing the subdivision levels of the subintervals, i.e., if  $(aa,bb)$  is a subinterval of  $(pts[1], pts[2])$  where  $pts[0]$  and  $pts[2]$  are adjacent elements of `infodict['pts']`, then  $(aa,bb)$  has level  $l$  if  $|bb-aa| = |pts[2]-pts[1]| * 2^{**}(-l)$ .
- 'ndin'** A rank-1 integer array of length  $P+2$ . After the first integration over the intervals  $(pts[1], pts[2])$ , the error estimates over some of the intervals may have been increased artificially in order to put their subdivision forward. This array has ones in slots corresponding to the subintervals for which this happens.

### Weighting the integrand

The input variables, `weight` and `wvar`, are used to weight the integrand by a select list of functions. Different integration methods are used to compute the integral with these weighting functions. The possible values of `weight` and the corresponding weighting functions are.

weight	Weight function used	wvar
'cos'	$\cos(w*x)$	<code>wvar = w</code>
'sin'	$\sin(w*x)$	<code>wvar = w</code>
'alg'	$g(x) = ((x-a)**\alpha)*((b-x)**\beta)$	<code>wvar = (alpha, beta)</code>
'alg-loga'	$g(x)*\log(x-a)$	<code>wvar = (alpha, beta)</code>
'alg-logb'	$g(x)*\log(b-x)$	<code>wvar = (alpha, beta)</code>
'alg-log'	$g(x)*\log(x-a)*\log(b-x)$	<code>wvar = (alpha, beta)</code>
'cauchy'	$1/(x-c)$	<code>wvar = c</code>

`wvar` holds the parameter  $w$ ,  $(\alpha, \beta)$ , or  $c$  depending on the weight selected. In these expressions,  $a$  and  $b$  are the integration limits.

For the 'cos' and 'sin' weighting, additional inputs and outputs are available.

For finite integration limits, the integration is performed using a Clenshaw-Curtis method which uses Chebyshev moments. For repeated calculations, these moments are saved in the output dictionary:

- 'momcom'** The maximum level of Chebyshev moments that have been computed, i.e., if  $M_c$  is `infodict['momcom']` then the moments have been computed for intervals of length  $|b-a| * 2^{**}(-l), l=0, 1, \dots, M_c$ .
- 'nnlog'** A rank-1 integer array of length  $M$  ( $=limit$ ), containing the subdivision levels of the subintervals, i.e., an element of this array is equal to  $l$  if the corresponding subinterval is  $|b-a| * 2^{**}(-l)$ .
- 'chebmo'** A rank-2 array of shape  $(25, maxp1)$  containing the computed Chebyshev moments. These can be passed on to an integration over the same interval by passing this array as the second element of the sequence `wopts` and passing `infodict['momcom']` as the first element.

If one of the integration limits is infinite, then a Fourier integral is computed (assuming  $w \neq 0$ ). If `full_output` is 1 and a numerical error is encountered, besides the error message attached to the output tuple, a dictionary is also appended to the output tuple which translates the error codes in the array `info['ierlst']` to English messages. The output information dictionary contains the following entries instead of 'last', 'alist', 'blist', 'rlist', and 'elist':

- 'lst'** The number of subintervals needed for the integration (call it  $K_f$ ).



- 'rslst'** A rank-1 array of length  $M\_f=limlst$ , whose first  $K\_f$  elements contain the integral contribution over the interval  $(a+(k-1)c, a+kc)$  where  $c = (2*\text{floor}(|w|) + 1) * \text{pi} / |w|$  and  $k=1, 2, \dots, K\_f$ .
- 'erlst'** A rank-1 array of length  $M\_f$  containing the error estimate corresponding to the interval in the same position in `infodict['rslst']`.
- 'ierlst'** A rank-1 integer array of length  $M\_f$  containing an error flag corresponding to the interval in the same position in `infodict['rslst']`. See the explanation dictionary (last entry in the output tuple) for the meaning of the codes.

### Examples

Calculate  $\int_0^4 x^2 dx$  and compare with an analytic result

```
>>> from scipy import integrate
>>> x2 = lambda x: x**2
>>> integrate.quad(x2, 0, 4)
(21.333333333333332, 2.3684757858670003e-13)
>>> print(4**3 / 3.) # analytical result
21.3333333333
```

Calculate  $\int_0^\infty e^{-x} dx$

```
>>> invexp = lambda x: np.exp(-x)
>>> integrate.quad(invexp, 0, np.inf)
(1.0, 5.842605999138044e-11)

>>> f = lambda x,a : a*x
>>> y, err = integrate.quad(f, 0, 1, args=(1,))
>>> y
0.5
>>> y, err = integrate.quad(f, 0, 1, args=(3,))
>>> y
1.5
```

Calculate  $\int_0^1 x^2 + y^2 dx$  with ctypes, holding y parameter as 1:

```
testlib.c =>
    double func(int n, double args[n]){
        return args[0]*args[0] + args[1]*args[1];}
compile to library testlib.*

from scipy import integrate
import ctypes
lib = ctypes.CDLL('/home/.../testlib.*') #use absolute path
lib.func.restype = ctypes.c_double
lib.func.argtypes = (ctypes.c_int,ctypes.c_double)
integrate.quad(lib.func,0,1,(1))
#(1.3333333333333333, 1.4802973661668752e-14)
print((1.0**3/3.0 + 1.0) - (0.0**3/3.0 + 0.0)) #Analytic result
# 1.3333333333333333
```

`scipy.integrate.dblquad(func, a, b, gfun, hfun, args=(), epsabs=1.49e-08, epsrel=1.49e-08)`  
Compute a double integral.

Return the double (definite) integral of `func(y, x)` from `x = a..b` and `y = gfun(x)..hfun(x)`.

**Parameters** `func`: callable

A Python function or method of at least two variables: `y` must be the first argument and `x` the second argument.

**a, b** : float  
 The limits of integration in  $x$ :  $a < b$

**gfun** : callable  
 The lower boundary curve in  $y$  which is a function taking a single floating point argument ( $x$ ) and returning a floating point result: a lambda function can be useful here.

**hfun** : callable  
 The upper boundary curve in  $y$  (same requirements as *gfun*).

**args** : sequence, optional  
 Extra arguments to pass to *func*.

**epsabs** : float, optional  
 Absolute tolerance passed directly to the inner 1-D quadrature integration. Default is 1.49e-8.

**epsrel** : float, optional  
 Relative tolerance of the inner 1-D integrals. Default is 1.49e-8.

**Returns** **y** : float  
 The resultant integral.

**abserr** : float  
 An estimate of the error.

**See also:**

*quad* single integral

*tplquad* triple integral

*nquad* N-dimensional integrals

*fixed\_quad* fixed-order Gaussian quadrature

*quadrature* adaptive Gaussian quadrature

*odeint* ODE integrator

*ode* ODE integrator

*simps* integrator for sampled data

*romb* integrator for sampled data

*scipy.special*  
 for coefficients and roots of orthogonal polynomials

`scipy.integrate.tplquad` (*func*, *a*, *b*, *gfun*, *hfun*, *qfun*, *rfunc*, *args*=(), *epsabs*=1.49e-08, *epsrel*=1.49e-08)

Compute a triple (definite) integral.

Return the triple integral of `func(z, y, x)` from  $x = a..b$ ,  $y = gfun(x)..hfun(x)$ , and  $z = qfun(x, y)..rfunc(x, y)$ .

**Parameters** **func** : function  
 A Python function or method of at least three variables in the order ( $z, y, x$ ).

**a, b** : float  
 The limits of integration in  $x$ :  $a < b$

**gfun** : function  
 The lower boundary curve in  $y$  which is a function taking a single floating point argument ( $x$ ) and returning a floating point result: a lambda function can be useful here.

**hfun** : function  
 The upper boundary curve in  $y$  (same requirements as *gfun*).

**qfun** : function

The lower boundary surface in  $z$ . It must be a function that takes two floats in the order  $(x, y)$  and returns a float.

**rfunc** : function

The upper boundary surface in  $z$ . (Same requirements as *qfunc*.)

**args** : tuple, optional

Extra arguments to pass to *func*.

**epsabs** : float, optional

Absolute tolerance passed directly to the innermost 1-D quadrature integration. Default is  $1.49\text{e-}8$ .

**epsrel** : float, optional

Relative tolerance of the innermost 1-D integrals. Default is  $1.49\text{e-}8$ .

**Returns**

**y** : float

The resultant integral.

**abserr** : float

An estimate of the error.

**See also:**

*quad* Adaptive quadrature using QUADPACK

*quadrature* Adaptive Gaussian quadrature

*fixed\_quad* Fixed-order Gaussian quadrature

*dblquad* Double integrals

*nquad* N-dimensional integrals

*romb* Integrators for sampled data

*simps* Integrators for sampled data

*ode* ODE integrators

*odeint* ODE integrators

*scipy.special*

For coefficients and roots of orthogonal polynomials

`scipy.integrate.nquad` (*func*, *ranges*, *args=None*, *opts=None*, *full\_output=False*)

Integration over multiple variables.

Wraps *quad* to enable integration over multiple variables. Various options allow improved integration of discontinuous functions, as well as the use of weighted integration, and generally finer control of the integration process.

**Parameters** **func** : callable

The function to be integrated. Has arguments of  $x_0, \dots, x_n, t_0, t_m$ , where integration is carried out over  $x_0, \dots, x_n$ , which must be floats. Function signature should be `func(x0, x1, ..., xn, t0, t1, ..., tm)`. Integration is carried out in order. That is, integration over  $x_0$  is the innermost integral, and  $x_n$  is the outermost. If performance is a concern, this function may be a `ctypes` function of the form:

```
f(int n, double args[n])
```

where  $n$  is the number of extra parameters and `args` is an array of doubles of the additional parameters. This function may then be compiled to a dynamic/shared library then imported through `ctypes`, setting the function's `argtypes` to `(c_int, c_double)`, and the function's `restype` to `(c_double)`. Its pointer may then be passed into *nquad* normally. This allows the underlying Fortran library to evaluate

the function in the innermost integration calls without callbacks to Python, and also speeds up the evaluation of the function itself.

**ranges** : iterable object

Each element of ranges may be either a sequence of 2 numbers, or else a callable that returns such a sequence. `ranges[0]` corresponds to integration over `x0`, and so on. If an element of ranges is a callable, then it will be called with all of the integration arguments available, as well as any parametric arguments. e.g. if `func = f(x0, x1, x2, t0, t1)`, then `ranges[0]` may be defined as either `(a, b)` or else as `(a, b) = range0(x1, x2, t0, t1)`.

**args** : iterable object, optional

Additional arguments `t0, ..., tn`, required by `func`, `ranges`, and `opts`.

**opts** : iterable object or dict, optional

Options to be passed to `quad`. May be empty, a dict, or a sequence of dicts or functions that return a dict. If empty, the default options from `scipy.integrate.quad` are used. If a dict, the same options are used for all levels of integration. If a sequence, then each element of the sequence corresponds to a particular integration. e.g. `opts[0]` corresponds to integration over `x0`, and so on. If a callable, the signature must be the same as for `ranges`. The available options together with their default values are:

- `epsabs` = 1.49e-08
- `epsrel` = 1.49e-08
- `limit` = 50
- `points` = None
- `weight` = None
- `wvar` = None
- `wopts` = None

For more information on these options, see `quad` and `quad_explain`.

**full\_output** : bool, optional

Partial implementation of `full_output` from `scipy.integrate.quad`. The number of integrand function evaluations `neval` can be obtained by setting `full_output=True` when calling `nquad`.

#### Returns

**result** : float

The result of the integration.

**abserr** : float

The maximum of the estimates of the absolute error in the various integration results.

**out\_dict** : dict, optional

A dict containing additional information on the integration.

#### See also:

`quad` 1-dimensional numerical integration

`dblquad`, `tplquad`

`fixed_quad` fixed-order Gaussian quadrature

`quadrature` adaptive Gaussian quadrature

#### Examples

```
>>> from scipy import integrate
>>> func = lambda x0,x1,x2,x3 : x0**2 + x1*x2 - x3**3 + np.sin(x0) + (
...                                     1 if (x0-.2*x3-.5-.25*x1>0) else 0)
>>> points = [[lambda x1,x2,x3 : 0.2*x3 + 0.5 + 0.25*x1], [], [], []]
>>> def opts0(*args, **kwargs):
...     return {'points':[0.2*args[2] + 0.5 + 0.25*args[0]]}
>>> integrate.nquad(func, [[0,1], [-1,1], [.13,.8], [-.15,1]],
```

```

...             opts=[opts0, {}, {}, {}], full_output=True)
(1.5267454070738633, 2.9437360001402324e-14, {'neval': 388962})

>>> scale = .1
>>> def func2(x0, x1, x2, x3, t0, t1):
...     return x0*x1*x3**2 + np.sin(x2) + 1 + (1 if x0+t1*x1-t0>0 else 0)
>>> def lim0(x1, x2, x3, t0, t1):
...     return [scale * (x1**2 + x2 + np.cos(x3)*t0*t1 + 1) - 1,
...             scale * (x1**2 + x2 + np.cos(x3)*t0*t1 + 1) + 1]
>>> def lim1(x2, x3, t0, t1):
...     return [scale * (t0*x2 + t1*x3) - 1,
...             scale * (t0*x2 + t1*x3) + 1]
>>> def lim2(x3, t0, t1):
...     return [scale * (x3 + t0**2*t1**3) - 1,
...             scale * (x3 + t0**2*t1**3) + 1]
>>> def lim3(t0, t1):
...     return [scale * (t0+t1) - 1, scale * (t0+t1) + 1]
>>> def opts0(x1, x2, x3, t0, t1):
...     return {'points' : [t0 - t1*x1]}
>>> def opts1(x2, x3, t0, t1):
...     return {}
>>> def opts2(x3, t0, t1):
...     return {}
>>> def opts3(t0, t1):
...     return {}
>>> integrate.nquad(func2, [lim0, lim1, lim2, lim3], args=(0,0),
...                   opts=[opts0, opts1, opts2, opts3])
(25.066666666666666, 2.7829590483937256e-13)

```

`scipy.integrate.fixed_quad` (*func*, *a*, *b*, *args=()*, *n=5*)

Compute a definite integral using fixed-order Gaussian quadrature.

Integrate *func* from *a* to *b* using Gaussian quadrature of order *n*.

**Parameters**

- func** : callable  
A Python function or method to integrate (must accept vector inputs).
- a** : float  
Lower limit of integration.
- b** : float  
Upper limit of integration.
- args** : tuple, optional  
Extra arguments to pass to function, if any.
- n** : int, optional  
Order of quadrature integration. Default is 5.

**Returns**

- val** : float  
Gaussian quadrature approximation to the integral
- none** : None  
Statically returned value of None

See also:

- [`quad`](#) adaptive quadrature using QUADPACK
- [`dblquad`](#) double integrals
- [`tplquad`](#) triple integrals
- [`romberg`](#) adaptive Romberg quadrature

*quadrature* adaptive Gaussian quadrature  
*romb* integrators for sampled data  
*simps* integrators for sampled data  
*cumtrapz* cumulative integration for sampled data  
*ode* ODE integrator  
*odeint* ODE integrator

`scipy.integrate.quadrature` (*func*, *a*, *b*, *args=()*, *tol=1.49e-08*, *rtol=1.49e-08*, *maxiter=50*,  
*vec\_func=True*, *miniter=1*)

Compute a definite integral using fixed-tolerance Gaussian quadrature.

Integrate *func* from *a* to *b* using Gaussian quadrature with absolute tolerance *tol*.

**Parameters**

- func** : function  
A Python function or method to integrate.
- a** : float  
Lower limit of integration.
- b** : float  
Upper limit of integration.
- args** : tuple, optional  
Extra arguments to pass to function.
- tol, rtol** : float, optional  
Iteration stops when error between last two iterates is less than *tol* OR the relative change is less than *rtol*.
- maxiter** : int, optional  
Maximum order of Gaussian quadrature.
- vec\_func** : bool, optional  
True or False if *func* handles arrays as arguments (is a “vector” function). Default is True.
- miniter** : int, optional  
Minimum order of Gaussian quadrature.

**Returns**

- val** : float  
Gaussian quadrature approximation (within tolerance) to integral.
- err** : float  
Difference between last two estimates of the integral.

See also:

*romberg* adaptive Romberg quadrature  
*fixed\_quad* fixed-order Gaussian quadrature  
*quad* adaptive quadrature using QUADPACK  
*dblquad* double integrals  
*tplquad* triple integrals  
*romb* integrator for sampled data  
*simps* integrator for sampled data  
*cumtrapz* cumulative integration for sampled data  
*ode* ODE integrator  
*odeint* ODE integrator

`scipy.integrate.romberg` (*function*, *a*, *b*, *args=()*, *tol=1.48e-08*, *rtol=1.48e-08*, *show=False*, *divmax=10*, *vec\_func=False*)

Romberg integration of a callable function or method.

Returns the integral of *function* (a function of one variable) over the interval (*a*, *b*).

If *show* is 1, the triangular array of the intermediate results will be printed. If *vec\_func* is True (default is False), then *function* is assumed to support vector arguments.

**Parameters**    **function** : callable

Function to be integrated.

**a** : float

Lower limit of integration.

**b** : float

Upper limit of integration.

**Returns**        **results** : float

Result of the integration.

**Other Parameters**

**args** : tuple, optional

Extra arguments to pass to function. Each element of *args* will be passed as a single argument to *func*. Default is to pass no extra arguments.

**tol, rtol** : float, optional

The desired absolute and relative tolerances. Defaults are 1.48e-8.

**show** : bool, optional

Whether to print the results. Default is False.

**divmax** : int, optional

Maximum order of extrapolation. Default is 10.

**vec\_func** : bool, optional

Whether *func* handles arrays as arguments (i.e whether it is a “vector” function). Default is False.

**See also:**

*fixed\_quad* Fixed-order Gaussian quadrature.

*quad* Adaptive quadrature using QUADPACK.

*dblquad* Double integrals.

*tplquad* Triple integrals.

*romb* Integrators for sampled data.

*simps* Integrators for sampled data.

*cumtrapz* Cumulative integration for sampled data.

*ode* ODE integrator.

*odeint* ODE integrator.

**References**

[R42]

**Examples**

Integrate a gaussian from 0 to 1 and compare to the error function.

```
>>> from scipy import integrate
>>> from scipy.special import erf
>>> gaussian = lambda x: 1/np.sqrt(np.pi) * np.exp(-x**2)
>>> result = integrate.romberg(gaussian, 0, 1, show=True)
Romberg integration of <function vfunc at ...> from [0, 1]
```

Steps	StepSize	Results
1	1.000000	0.385872
2	0.500000	0.412631 0.421551
4	0.250000	0.419184 0.421368 0.421356
8	0.125000	0.420810 0.421352 0.421350 0.421350
16	0.062500	0.421215 0.421350 0.421350 0.421350 0.421350
32	0.031250	0.421317 0.421350 0.421350 0.421350 0.421350 0.421350 0.421350

The final result is 0.421350396475 after 33 function evaluations.

```
>>> print("%g %g" % (2*result, erf(1)))
0.842701 0.842701
```

`scipy.integrate.quad_explain` (*output*=<open file '<stdout>', mode 'w' at 0x2b9089d99150>)  
Print extra information about `integrate.quad()` parameters and returns.

**Parameters**    **output** : instance with “write” method, optional  
Information about `quad` is passed to `output.write()`. Default is `sys.stdout`.

**Returns**        None

`scipy.integrate.newton_cotes` (*rn*, *equal*=0)

Return weights and error coefficient for Newton-Cotes integration.

Suppose we have (N+1) samples of *f* at the positions  $x_0, x_1, \dots, x_N$ . Then an N-point Newton-Cotes formula for the integral between  $x_0$  and  $x_N$  is:

$$\int_{x_0}^{x_N} f(x)dx = \Delta x \sum_{i=0}^N a_i f(x_i) + B_N(\Delta x)^{N+2} f^{N+1}(\xi)$$

where  $\xi \in [x_0, x_N]$  and  $\Delta x = \frac{x_N - x_0}{N}$  is the average samples spacing.

If the samples are equally-spaced and N is even, then the error term is  $B_N(\Delta x)^{N+3} f^{N+2}(\xi)$ .

**Parameters**    **rn** : int  
The integer order for equally-spaced data or the relative positions of the samples with the first sample at 0 and the last at N, where N+1 is the length of *rn*. N is the order of the Newton-Cotes integration.

**equal** : int, optional  
Set to 1 to enforce equally spaced data.

**Returns**        **an** : ndarray  
1-D array of weights to apply to the function at the provided sample positions.

**B** : float  
Error coefficient.

### Notes

Normally, the Newton-Cotes rules are used on smaller integration regions and a composite rule is used to return the total integral.

**exception** `scipy.integrate.IntegrationWarning`

Warning on issues during integration.

## 5.6.2 Integrating functions, given fixed samples



---

<code>trapez(y[, x, dx, axis])</code>	Integrate along the given axis using the composite trapezoidal rule.
<code>cumtrapez(y[, x, dx, axis, initial])</code>	Cumulatively integrate $y(x)$ using the composite trapezoidal rule.
<code>simpson(y[, x, dx, axis, even])</code>	Integrate $y(x)$ using samples along the given axis and the composite Simpson's rule.
<code>romb(y[, dx, axis, show])</code>	Romberg integration using samples of a function.

---

`scipy.integrate.trapez` ( $y, x=None, dx=1.0, axis=-1$ )

Integrate along the given axis using the composite trapezoidal rule.

Integrate  $y(x)$  along given axis.

**Parameters**

- y** : array\_like  
Input array to integrate.
- x** : array\_like, optional  
If  $x$  is `None`, then spacing between all  $y$  elements is  $dx$ .
- dx** : scalar, optional  
If  $x$  is `None`, spacing given by  $dx$  is assumed. Default is 1.
- axis** : int, optional  
Specify the axis.

**Returns**

- trapez** : float  
Definite integral as approximated by trapezoidal rule.

**See also:**

`sum`, `cumsum`

**Notes**

Image [R48] illustrates trapezoidal rule –  $y$ -axis locations of points will be taken from  $y$  array, by default  $x$ -axis distances between points will be 1.0, alternatively they can be provided with  $x$  array or with  $dx$  scalar. Return value will be equal to combined area under the red lines.

**References**

[R47], [R48]

**Examples**

```
>>> np.trapez([1,2,3])
4.0
>>> np.trapez([1,2,3], x=[4,6,8])
8.0
>>> np.trapez([1,2,3], dx=2)
8.0
>>> a = np.arange(6).reshape(2, 3)
>>> a
array([[0, 1, 2],
       [3, 4, 5]])
>>> np.trapez(a, axis=0)
array([ 1.5,  2.5,  3.5])
>>> np.trapez(a, axis=1)
array([ 2.,  8.])
```

`scipy.integrate.cumtrapez` ( $y, x=None, dx=1.0, axis=-1, initial=None$ )

Cumulatively integrate  $y(x)$  using the composite trapezoidal rule.

**Parameters**

- y** : array\_like  
Values to integrate.
- x** : array\_like, optional

The coordinate to integrate along. If None (default), use spacing  $dx$  between consecutive elements in  $y$ .

**dx** : int, optional

Spacing between elements of  $y$ . Only used if  $x$  is None.

**axis** : int, optional

Specifies the axis to cumulate. Default is -1 (last axis).

**initial** : scalar, optional

If given, uses this value as the first value in the returned result. Typically this value should be 0. Default is None, which means no value at  $x[0]$  is returned and  $res$  has one element less than  $y$  along the axis of integration.

**Returns**

**res** : ndarray

The result of cumulative integration of  $y$  along  $axis$ . If *initial* is None, the shape is such that the axis of integration has one less value than  $y$ . If *initial* is given, the shape is equal to that of  $y$ .

**See also:**

`numpy.cumsum`, `numpy.cumprod`

**quad** adaptive quadrature using QUADPACK

**romberg** adaptive Romberg quadrature

**quadrature** adaptive Gaussian quadrature

**fixed\_quad** fixed-order Gaussian quadrature

**dblquad** double integrals

**tplquad** triple integrals

**romb** integrators for sampled data

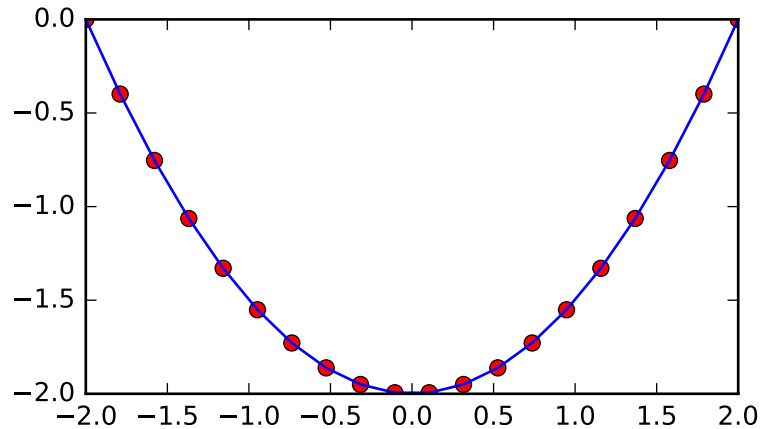
**ode** ODE integrators

**odeint** ODE integrators

**Examples**

```
>>> from scipy import integrate
>>> import matplotlib.pyplot as plt

>>> x = np.linspace(-2, 2, num=20)
>>> y = x
>>> y_int = integrate.cumtrapz(y, x, initial=0)
>>> plt.plot(x, y_int, 'ro', x, y[0] + 0.5 * x**2, 'b-')
>>> plt.show()
```



`scipy.integrate.simps` (*y*, *x=None*, *dx=1*, *axis=-1*, *even='avg'*)

Integrate  $y(x)$  using samples along the given axis and the composite Simpson's rule. If *x* is `None`, spacing of *dx* is assumed.

If there are an even number of samples, *N*, then there are an odd number of intervals (*N*-1), but Simpson's rule requires an even number of intervals. The parameter `'even'` controls how this is handled.

**Parameters**

- y** : array\_like  
Array to be integrated.
- x** : array\_like, optional  
If given, the points at which *y* is sampled.
- dx** : int, optional  
Spacing of integration points along axis of *y*. Only used when *x* is `None`. Default is 1.
- axis** : int, optional  
Axis along which to integrate. Default is the last axis.
- even** : {'avg', 'first', 'str'}, optional
  - 'avg'** [Average two results:1) use the first *N*-2 intervals with] a trapezoidal rule on the last interval and 2) use the last *N*-2 intervals with a trapezoidal rule on the first interval.
  - 'first'** [Use Simpson's rule for the first *N*-2 intervals with] a trapezoidal rule on the last interval.
  - 'last'** [Use Simpson's rule for the last *N*-2 intervals with a] trapezoidal rule on the first interval.

**See also:**

`quad` adaptive quadrature using QUADPACK

`romberg` adaptive Romberg quadrature

`quadrature` adaptive Gaussian quadrature

`fixed_quad` fixed-order Gaussian quadrature

`dblquad` double integrals

`tplquad` triple integrals

`romb` integrators for sampled data

*cumtrapz* cumulative integration for sampled data  
*ode* ODE integrators  
*odeint* ODE integrators

**Notes**

For an odd number of samples that are equally spaced the result is exact if the function is a polynomial of order 3 or less. If the samples are not equally spaced, then the result is exact only if the function is a polynomial of order 2 or less.

`scipy.integrate.romb` (*y*, *dx=1.0*, *axis=-1*, *show=False*)  
 Romberg integration using samples of a function.

**Parameters**

- y** : array\_like  
 A vector of  $2^{**k} + 1$  equally-spaced samples of a function.
- dx** : float, optional  
 The sample spacing. Default is 1.
- axis** : int, optional  
 The axis along which to integrate. Default is -1 (last axis).
- show** : bool, optional  
 When *y* is a single 1-D array, then if this argument is True print the table showing Richardson extrapolation from the samples. Default is False.

**Returns**

- romb** : ndarray  
 The integrated result for *axis*.

**See also:**

*quad* adaptive quadrature using QUADPACK  
*romberg* adaptive Romberg quadrature  
*quadrature* adaptive Gaussian quadrature  
*fixed\_quad* fixed-order Gaussian quadrature  
*dblquad* double integrals  
*tplquad* triple integrals  
*simps* integrators for sampled data  
*cumtrapz* cumulative integration for sampled data  
*ode* ODE integrators  
*odeint* ODE integrators

**See also:**

`scipy.special` for orthogonal polynomials (`special`) for Gaussian quadrature roots and weights for other weighting factors and regions.

### 5.6.3 Integrators of ODE systems

<code>odeint(func, y0, t[, args, Dfun, col_deriv, ...])</code>	Integrate a system of ordinary differential equations.
<code>ode(f[, jac])</code>	A generic interface class to numeric integrators.
<code>complex_ode(f[, jac])</code>	A wrapper of <code>ode</code> for complex systems.

Continued on next page

Table 5.20 – continued from previous page

---

<code>solve_bvp</code> ( <code>fun</code> , <code>bc</code> , <code>x</code> , <code>y</code> [, <code>p</code> , <code>S</code> , <code>fun_jac</code> , ...])	Solve a boundary-value problem for a system of ODEs.
---	--

---

`scipy.integrate.odeint` (`func`, `y0`, `t`, `args=()`, `Dfun=None`, `col_deriv=0`, `full_output=0`, `ml=None`, `mu=None`, `rtol=None`, `atol=None`, `tcrit=None`, `h0=0.0`, `hmax=0.0`, `hmin=0.0`, `ixpr=0`, `mxstep=0`, `mxhnil=0`, `mxordn=12`, `mxords=5`, `printmessg=0`)

Integrate a system of ordinary differential equations.

Solve a system of ordinary differential equations using lsoda from the FORTRAN library odepack.

Solves the initial value problem for stiff or non-stiff systems of first order ode-s:

$dy/dt = \text{func}(y, t_0, \dots)$

where `y` can be a vector.

*Note:* The first two arguments of `func(y, t0, ...)` are in the opposite order of the arguments in the system definition function used by the `scipy.integrate.ode` class.

**Parameters**

- func** : callable(`y`, `t0`, ...)  
Computes the derivative of `y` at `t0`.
- y0** : array  
Initial condition on `y` (can be a vector).
- t** : array  
A sequence of time points for which to solve for `y`. The initial value point should be the first element of this sequence.
- args** : tuple, optional  
Extra arguments to pass to function.
- Dfun** : callable(`y`, `t0`, ...)  
Gradient (Jacobian) of `func`.
- col\_deriv** : bool, optional  
True if `Dfun` defines derivatives down columns (faster), otherwise `Dfun` should define derivatives across rows.
- full\_output** : bool, optional  
True if to return a dictionary of optional outputs as the second output
- printmessg** : bool, optional  
Whether to print the convergence message

**Returns**

- y** : array, shape (len(`t`), len(`y0`))  
Array containing the value of `y` for each desired time in `t`, with the initial value `y0` in the first row.
- infodict** : dict, only returned if `full_output == True`  
Dictionary containing additional output information

key	meaning
'hu'	vector of step sizes successfully used for each time step.
'tcur'	vector with the value of t reached for each time step. (will always be at least as large as the input times).
'tolsf'	vector of tolerance scale factors, greater than 1.0, computed when a request for too much accuracy was detected.
'tsw'	value of t at the time of the last method switch (given for each time step)
'nst'	cumulative number of time steps
'nfe'	cumulative number of function evaluations for each time step
'nje'	cumulative number of jacobian evaluations for each time step
'nqu'	a vector of method orders for each successful step.
'imxer'	index of the component of largest magnitude in the weighted local error vector (e / ewt) on an error return, -1 otherwise.
'lenrw'	the length of the double work array required.
'leniw'	the length of integer work array required.
'mused'	a vector of method indicators for each successful time step: 1: adams (nonstiff), 2: bdf (stiff)

### Other Parameters

**ml, mu** : int, optional

If either of these are not None or non-negative, then the Jacobian is assumed to be banded. These give the number of lower and upper non-zero diagonals in this banded matrix. For the banded case, *Dfun* should return a matrix whose rows contain the non-zero bands (starting with the lowest diagonal). Thus, the return matrix *jac* from *Dfun* should have shape  $(ml + mu + 1, len(y0))$  when  $ml \geq 0$  or  $mu \geq 0$ . The data in *jac* must be stored such that  $jac[i - j + mu, j]$  holds the derivative of the *i*'th equation with respect to the *j*'th state variable. If *col\_deriv* is True, the transpose of this *jac* must be returned.

**rtol, atol** : float, optional

The input parameters *rtol* and *atol* determine the error control performed by the solver. The solver will control the vector, *e*, of estimated local errors in *y*, according to an inequality of the form  $\max\text{-norm of } (e / \text{ewt}) \leq 1$ , where *ewt* is a vector of positive error weights computed as  $\text{ewt} = \text{rtol} * \text{abs}(y) + \text{atol}$ . *rtol* and *atol* can be either vectors the same length as *y* or scalars. Defaults to 1.49012e-8.

**tcrit** : ndarray, optional

Vector of critical points (e.g. singularities) where integration care should be taken.

**h0** : float, (0: solver-determined), optional

The step size to be attempted on the first step.

**hmax** : float, (0: solver-determined), optional

The maximum absolute step size allowed.

**hmin** : float, (0: solver-determined), optional

The minimum absolute step size allowed.

**ixpr** : bool, optional

Whether to generate extra printing at method switches.

**mxstep** : int, (0: solver-determined), optional

Maximum number of (internally defined) steps allowed for each integration point in *t*.

**mxhnil** : int, (0: solver-determined), optional

Maximum number of messages printed.

**mxordn** : int, (0: solver-determined), optional

Maximum order to be allowed for the non-stiff (Adams) method.

**mxords** : int, (0: solver-determined), optional

Maximum order to be allowed for the stiff (BDF) method.

See also:

`ode` a more object-oriented integrator based on VODE.

`quad` for finding the area under a curve.

### Examples

The second order differential equation for the angle *theta* of a pendulum acted on by gravity with friction can be written:

$$\text{theta}''(t) + b \cdot \text{theta}'(t) + c \cdot \sin(\text{theta}(t)) = 0$$

where *b* and *c* are positive constants, and a prime (') denotes a derivative. To solve this equation with `odeint`, we must first convert it to a system of first order equations. By defining the angular velocity  $\omega(t) = \text{theta}'(t)$ , we obtain the system:

$$\begin{aligned} \text{theta}'(t) &= \omega(t) \\ \omega'(t) &= -b \cdot \omega(t) - c \cdot \sin(\text{theta}(t)) \end{aligned}$$

Let *y* be the vector [*theta*, *omega*]. We implement this system in python as:

```
>>> def pend(y, t, b, c):
...     theta, omega = y
...     dydt = [omega, -b*omega - c*np.sin(theta)]
...     return dydt
... 
```

We assume the constants are *b* = 0.25 and *c* = 5.0:

```
>>> b = 0.25
>>> c = 5.0
```

For initial conditions, we assume the pendulum is nearly vertical with  $\text{theta}(0) = \pi - 0.1$ , and it initially at rest, so  $\omega(0) = 0$ . Then the vector of initial conditions is

```
>>> y0 = [np.pi - 0.1, 0.0]
```

We generate a solution 101 evenly spaced samples in the interval  $0 \leq t \leq 10$ . So our array of times is:

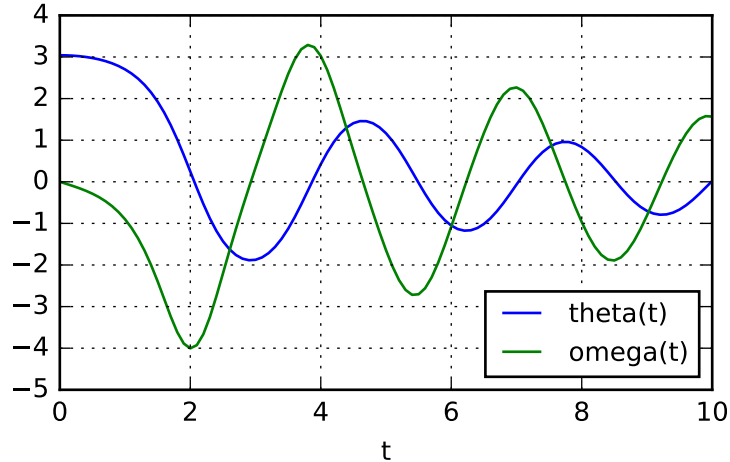
```
>>> t = np.linspace(0, 10, 101)
```

Call `odeint` to generate the solution. To pass the parameters *b* and *c* to `pend`, we give them to `odeint` using the `args` argument.

```
>>> from scipy.integrate import odeint
>>> sol = odeint(pend, y0, t, args=(b, c))
```

The solution is an array with shape (101, 2). The first column is  $\text{theta}(t)$ , and the second is  $\omega(t)$ . The following code plots both components.

```
>>> import matplotlib.pyplot as plt
>>> plt.plot(t, sol[:, 0], 'b', label='theta(t)')
>>> plt.plot(t, sol[:, 1], 'g', label='omega(t)')
>>> plt.legend(loc='best')
>>> plt.xlabel('t')
>>> plt.grid()
>>> plt.show()
```



**class** `scipy.integrate.ode` (*f*, *jac=None*)

A generic interface class to numeric integrators.

Solve an equation system  $y'(t) = f(t, y)$  with (optional) `jac = df/dy`.

*Note:* The first two arguments of `f(t, y, ...)` are in the opposite order of the arguments in the system definition function used by `scipy.integrate.odeint`.

**Parameters** **f**: callable `f(t, y, *f_args)`

Right-hand side of the differential equation. `t` is a scalar, `y.shape == (n,)`. `f_args` is set by calling `set_f_params(*args)`. `f` should return a scalar, array or list (not a tuple).

**jac**: callable `jac(t, y, *jac_args)`, optional

Jacobian of the right-hand side, `jac[i, j] = d f[i] / d y[j]`. `jac_args` is set by calling `set_jac_params(*args)`.

**See also:**

`odeint` an integrator with a simpler interface based on lsoda from ODEPACK

`quad` for finding the area under a curve

**Notes**

Available integrators are listed below. They can be selected using the `set_integrator` method.

“vode”

Real-valued Variable-coefficient Ordinary Differential Equation solver, with fixed-leading-coefficient implementation. It provides implicit Adams method (for non-stiff problems) and a method based on backward differentiation formulas (BDF) (for stiff problems).

Source: <http://www.netlib.org/ode/vode.f>

**Warning:** This integrator is not re-entrant. You cannot have two `ode` instances using the “vode” integrator at the same time.

This integrator accepts the following parameters in `set_integrator` method of the `ode` class:

- `atol` : float or sequence absolute tolerance for solution
- `rtol` : float or sequence relative tolerance for solution
- `lband` : None or int



- `uband` : None or int Jacobian band width,  $\text{jac}[i,j] \neq 0$  for  $i-\text{lband} \leq j \leq i+\text{uband}$ . Setting these requires your `jac` routine to return the jacobian in packed format,  $\text{jac\_packed}[i-j+\text{uband}, j] = \text{jac}[i,j]$ . The dimension of the matrix must be  $(\text{lband}+\text{uband}+1, \text{len}(y))$ .
- `method`: ‘adams’ or ‘bdf’ Which solver to use, Adams (non-stiff) or BDF (stiff)
- `with_jacobian` : bool This option is only considered when the user has not supplied a Jacobian function and has not indicated (by setting either band) that the Jacobian is banded. In this case, `with_jacobian` specifies whether the iteration method of the ODE solver’s correction step is chord iteration with an internally generated full Jacobian or functional iteration with no Jacobian.
- `nsteps` : int Maximum number of (internally defined) steps allowed during one call to the solver.
- `first_step` : float
- `min_step` : float
- `max_step` : float Limits for the step sizes used by the integrator.
- `order` : int Maximum order used by the integrator, order  $\leq 12$  for Adams,  $\leq 5$  for BDF.

“zvode”

Complex-valued Variable-coefficient Ordinary Differential Equation solver, with fixed-leading-coefficient implementation. It provides implicit Adams method (for non-stiff problems) and a method based on backward differentiation formulas (BDF) (for stiff problems).

Source: <http://www.netlib.org/ode/zvode.f>

**Warning:** This integrator is not re-entrant. You cannot have two `ode` instances using the “zvode” integrator at the same time.

This integrator accepts the same parameters in `set_integrator` as the “vode” solver.

**Note:** When using ZVODE for a stiff system, it should only be used for the case in which the function `f` is analytic, that is, when each `f(i)` is an analytic function of each `y(j)`. Analyticity means that the partial derivative  $df(i)/dy(j)$  is a unique complex number, and this fact is critical in the way ZVODE solves the dense or banded linear systems that arise in the stiff case. For a complex stiff ODE system in which `f` is not analytic, ZVODE is likely to have convergence failures, and for this problem one should instead use DVODE on the equivalent real system (in the real and imaginary parts of `y`).

“lsoda”

Real-valued Variable-coefficient Ordinary Differential Equation solver, with fixed-leading-coefficient implementation. It provides automatic method switching between implicit Adams method (for non-stiff problems) and a method based on backward differentiation formulas (BDF) (for stiff problems).

Source: <http://www.netlib.org/odepack>

**Warning:** This integrator is not re-entrant. You cannot have two `ode` instances using the “lsoda” integrator at the same time.

This integrator accepts the following parameters in `set_integrator` method of the `ode` class:

- `atol` : float or sequence absolute tolerance for solution
- `rtol` : float or sequence relative tolerance for solution
- `lband` : None or int
- `uband` : None or int Jacobian band width,  $\text{jac}[i,j] \neq 0$  for  $i-\text{lband} \leq j \leq i+\text{uband}$ . Setting these requires your `jac` routine to return the jacobian in packed format,  $\text{jac\_packed}[i-j+\text{uband}, j] = \text{jac}[i,j]$ .
- `with_jacobian` : bool *Not used*.
- `nsteps` : int Maximum number of (internally defined) steps allowed during one call to the solver.
- `first_step` : float
- `min_step` : float
- `max_step` : float Limits for the step sizes used by the integrator.
- `max_order_ns` : int Maximum order used in the nonstiff case (default 12).
- `max_order_s` : int Maximum order used in the stiff case (default 5).
- `max_hnil` : int Maximum number of messages reporting too small step size ( $t + h = t$ ) (default 0)

- ixpr : int Whether to generate extra printing at method switches (default False).

“dopri5”

This is an explicit runge-kutta method of order (4)5 due to Dormand & Prince (with stepsize control and dense output).

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This code is described in [HNW93].

This integrator accepts the following parameters in `set_integrator()` method of the `ode` class:

- atol : float or sequence absolute tolerance for solution
- rtol : float or sequence relative tolerance for solution
- nsteps : int Maximum number of (internally defined) steps allowed during one call to the solver.
- first\_step : float
- max\_step : float
- safety : float Safety factor on new step selection (default 0.9)
- ifactor : float
- dfactor : float Maximum factor to increase/decrease step size by in one step
- beta : float Beta parameter for stabilised step size control.
- verbosity : int Switch for printing messages (< 0 for no messages).

“dop853”

This is an explicit runge-kutta method of order 8(5,3) due to Dormand & Prince (with stepsize control and dense output).

Options and references the same as “dopri5”.

**References**

[HNW93]

**Examples**

A problem to integrate and the corresponding jacobian:

```
>>> from scipy.integrate import ode
>>>
>>> y0, t0 = [1.0j, 2.0], 0
>>>
>>> def f(t, y, arg1):
...     return [1j*arg1*y[0] + y[1], -arg1*y[1]**2]
>>> def jac(t, y, arg1):
...     return [[1j*arg1, 1], [0, -arg1*2*y[1]]]
```

The integration:

```
>>> r = ode(f, jac).set_integrator('zvode', method='bdf')
>>> r.set_initial_value(y0, t0).set_f_params(2.0).set_jac_params(2.0)
>>> t1 = 10
>>> dt = 1
>>> while r.successful() and r.t < t1:
...     print(r.t+dt, r.integrate(r.t+dt))
(1.0, array([-0.71038232+0.23749653j, 0.40000271+0.j          ]))
(2.0, array([ 0.19098503-0.52359246j, 0.22222356+0.j          ]))
(3.0, array([ 0.47153208+0.52701229j, 0.15384681+0.j          ]))
(4.0, array([-0.61905937+0.30726255j, 0.11764744+0.j          ]))
(5.0, array([ 0.02340997-0.61418799j, 0.09523835+0.j          ]))
(6.0, array([ 0.58643071+0.339819j, 0.08000018+0.j          ]))
(7.0, array([-0.52070105+0.44525141j, 0.06896565+0.j          ]))
```

```
(8.0, array([-0.15986733-0.61234476j, 0.06060616+0.j      ]))
(9.0, array([ 0.64850462+0.15048982j, 0.05405414+0.j      ]))
(10.0, array([-0.38404699+0.56382299j, 0.04878055+0.j      ]))
```

**Attributes**

<b>t</b>	(float) Current time.
<b>y</b>	(ndarray) Current variable values.

**Methods**


---

<code>integrate(t[, step, relax])</code>	Find $y=y(t)$ , set $y$ as an initial condition, and return $y$ .
<code>set_f_params(*args)</code>	Set extra parameters for user-supplied function $f$ .
<code>set_initial_value(y[, t])</code>	Set initial conditions $y(t) = y$ .
<code>set_integrator(name, **integrator_params)</code>	Set integrator by name.
<code>set_jac_params(*args)</code>	Set extra parameters for user-supplied function $jac$ .
<code>set_solout(solout)</code>	Set callable to be called at every successful integration step.
<code>successful()</code>	Check if integration was successful.

---

`ode.integrate(t, step=0, relax=0)`  
Find  $y=y(t)$ , set  $y$  as an initial condition, and return  $y$ .

`ode.set_f_params(*args)`  
Set extra parameters for user-supplied function  $f$ .

`ode.set_initial_value(y, t=0.0)`  
Set initial conditions  $y(t) = y$ .

`ode.set_integrator(name, **integrator_params)`  
Set integrator by name.

**Parameters** **name** : str  
Name of the integrator.  
**integrator\_params**  
Additional parameters for the integrator.

`ode.set_jac_params(*args)`  
Set extra parameters for user-supplied function  $jac$ .

`ode.set_solout(solout)`  
Set callable to be called at every successful integration step.

**Parameters** **solout** : callable  
 $solout(t, y)$  is called at each internal integrator step,  $t$  is a scalar providing the current independent position  $y$  is the current solution  $y.shape == (n,)$   
 $solout$  should return -1 to stop integration otherwise it should return None or 0

`ode.successful()`  
Check if integration was successful.

**class** `scipy.integrate.complex_ode(f, jac=None)`  
A wrapper of `ode` for complex systems.

This functions similarly as `ode`, but re-maps a complex-valued equation system to a real-valued one before using the integrators.

**Parameters** **f** : callable  $f(t, y, *f_args)$   
Rhs of the equation.  $t$  is a scalar,  $y.shape == (n,)$ .  $f_args$  is set by calling `set_f_params(*args)`.

**jac** : callable `jac(t, y, *jac_args)`  
 Jacobian of the rhs, `jac[i, j] = d f[i] / d y[j]`. `jac_args` is set by calling `set_f_params(*args)`.

**Examples**

For usage examples, see `ode`.

**Attributes**

<code>t</code>	(float) Current time.
<code>y</code>	(ndarray) Current variable values.

**Methods**

---

<code>integrate(t[, step, relax])</code>	Find $y=y(t)$ , set $y$ as an initial condition, and return $y$ .
<code>set_f_params(*args)</code>	Set extra parameters for user-supplied function $f$ .
<code>set_initial_value(y[, t])</code>	Set initial conditions $y(t) = y$ .
<code>set_integrator(name, **integrator_params)</code>	Set integrator by name.
<code>set_jac_params(*args)</code>	Set extra parameters for user-supplied function $jac$ .
<code>set_solout(solout)</code>	Set callable to be called at every successful integration step.
<code>successful()</code>	Check if integration was successful.

---

`complex_ode.integrate(t, step=0, relax=0)`  
 Find  $y=y(t)$ , set  $y$  as an initial condition, and return  $y$ .

`complex_ode.set_f_params(*args)`  
 Set extra parameters for user-supplied function  $f$ .

`complex_ode.set_initial_value(y, t=0.0)`  
 Set initial conditions  $y(t) = y$ .

`complex_ode.set_integrator(name, **integrator_params)`  
 Set integrator by name.

**Parameters** `name` : str  
 Name of the integrator  
**integrator\_params**  
 Additional parameters for the integrator.

`complex_ode.set_jac_params(*args)`  
 Set extra parameters for user-supplied function  $jac$ .

`complex_ode.set_solout(solout)`  
 Set callable to be called at every successful integration step.

**Parameters** `solout` : callable  
`solout(t, y)` is called at each internal integrator step,  $t$  is a scalar providing the current independent position  $y$  is the current soloution  $y.shape == (n,)$   
`solout` should return `-1` to stop integration otherwise it should return `None` or `0`

`complex_ode.successful()`  
 Check if integration was successful.

`scipy.integrate.solve_bvp(fun, bc, x, y, p=None, S=None, fun_jac=None, bc_jac=None, tol=0.001, max_nodes=1000, verbose=0)`  
 Solve a boundary-value problem for a system of ODEs.

This function numerically solves a first order system of ODEs subject to two-point boundary conditions:

$$\begin{aligned} dy / dx &= f(x, y, p) + S * y / (x - a), \quad a \leq x \leq b \\ bc(y(a), y(b), p) &= 0 \end{aligned}$$

Here  $x$  is a 1-dimensional independent variable,  $y(x)$  is a  $n$ -dimensional vector-valued function and  $p$  is a  $k$ -dimensional vector of unknown parameters which is to be found along with  $y(x)$ . For the problem to be determined there must be  $n + k$  boundary conditions, i.e.  $bc$  must be  $(n + k)$ -dimensional function.

The last singular term in the right-hand side of the system is optional. It is defined by an  $n$ -by- $n$  matrix  $S$ , such that the solution must satisfy  $S y(a) = 0$ . This condition will be forced during iterations, so it must not contradict boundary conditions. See [R44] for the explanation how this term is handled when solving BVPs numerically.

Problems in a complex domain can be solved as well. In this case  $y$  and  $p$  are considered to be complex, and  $f$  and  $bc$  are assumed to be complex-valued functions, but  $x$  stays real. Note that  $f$  and  $bc$  must be complex differentiable (satisfy Cauchy-Riemann equations [R46]), otherwise you should rewrite your problem for real and imaginary parts separately. To solve a problem in a complex domain, pass an initial guess for  $y$  with a complex data type (see below).

**Parameters** **fun** : callable

Right-hand side of the system. The calling signature is `fun(x, y)`, or `fun(x, y, p)` if parameters are present. All arguments are ndarray:  $x$  with shape  $(m)$ ,  $y$  with shape  $(n, m)$ , meaning that  $y[:, i]$  corresponds to  $x[i]$ , and  $p$  with shape  $(k)$ . The return value must be an array with shape  $(n, m)$  and with the same layout as  $y$ .

**bc** : callable

Function evaluating residuals of the boundary conditions. The calling signature is `bc(ya, yb)`, or `bc(ya, yb, p)` if parameters are present. All arguments are ndarray:  $ya$  and  $yb$  with shape  $(n)$ , and  $p$  with shape  $(k)$ . The return value must be an array with shape  $(n + k)$ .

**x** : array\_like, shape  $(m)$

Initial mesh. Must be a strictly increasing sequence of real numbers with  $x[0]=a$  and  $x[-1]=b$ .

**y** : array\_like, shape  $(n, m)$

Initial guess for the function values at the mesh nodes,  $i$ -th column corresponds to  $x[i]$ . For problems in a complex domain pass  $y$  with a complex data type (even if the initial guess is purely real).

**p** : array\_like with shape  $(k)$ , or None, optional

Initial guess for the unknown parameters. If None (default), it is assumed that the problem doesn't depend on any parameters.

**S** : array\_like with shape  $(n, n)$  or None

Matrix defining the singular term. If None (default), the problem is solved without the singular term.

**fun\_jac** : callable or None, optional

Function computing derivatives of  $f$  with respect to  $y$  and  $p$ . The calling signature is `fun_jac(x, y)`, or `fun_jac(x, y, p)` if parameters are present. The return must contain 1 or 2 elements in the following order:

- `df_dy` : array\_like with shape  $(n, n, m)$  where an element  $(i, j, q)$  equals to  $d f_i(x_q, y_q, p) / d (y_q)_j$ .
- `df_dp` : array\_like with shape  $(n, k, m)$  where an element  $(i, j, q)$  equals to  $d f_i(x_q, y_q, p) / d p_j$ .

Here  $q$  numbers nodes at which  $x$  and  $y$  are defined, whereas  $i$  and  $j$  number vector components. If the problem is solved without unknown parameters `df_dp` should not be returned.

If `fun_jac` is None (default), the derivatives will be estimated by the forward finite differences.

**bc\_jac** : callable or None, optional

Function computing derivatives of `bc` with respect to `ya`, `yb` and `p`. The calling signature is `bc_jac(ya, yb)`, or `bc_jac(ya, yb, p)` if parameters are present. The return must contain 2 or 3 elements in the following order:

- `dbc_dya` : array\_like with shape (n, n) where an element (i, j) equals to  $d bc_i(ya, yb, p) / d ya_j$ .
- `dbc_dyb` : array\_like with shape (n, n) where an element (i, j) equals to  $d bc_i(ya, yb, p) / d yb_j$ .
- `dbc_dp` : array\_like with shape (n, k) where an element (i, j) equals to  $d bc_i(ya, yb, p) / d p_j$ .

If the problem is solved without unknown parameters `dbc_dp` should not be returned. If `bc_jac` is None (default), the derivatives will be estimated by the forward finite differences.

**tol** : float, optional

Desired tolerance of the solution. If we define  $r = y' - f(x, y)$  where `y` is the found solution, then the solver tries to achieve on each mesh interval  $norm(r / (1 + abs(f))) < tol$ , where `norm` is estimated in a root mean squared sense (using a numerical quadrature formula). Default is 1e-3.

**max\_nodes** : int, optional

Maximum allowed number of the mesh nodes. If exceeded, the algorithm terminates. Default is 1000.

**verbose** : {0, 1, 2}, optional

Level of algorithm's verbosity:

- 0 (default) : work silently.
- 1 : display a termination report.
- 2 : display progress during iterations.

### Returns

Bunch object with the following fields defined:

**sol** : PPoly

Found solution for `y` as `scipy.interpolate.PPoly` instance, a C1 continuous cubic spline.

**p** : ndarray or None, shape (k,)

Found parameters. None, if the parameters were not present in the problem.

**x** : ndarray, shape (m,)

Nodes of the final mesh.

**y** : ndarray, shape (n, m)

Solution values at the mesh nodes.

**yp** : ndarray, shape (n, m)

Solution derivatives at the mesh nodes.

**rms\_residuals** : ndarray, shape (m - 1,)

RMS values of the relative residuals over each mesh interval (see the description of `tol` parameter).

**niter** : int

Number of completed iterations.

**status** : int

Reason for algorithm termination:

- 0: The algorithm converged to the desired accuracy.
- 1: The maximum number of mesh nodes is exceeded.
- 2: A singular Jacobian encountered when solving the collocation system.

**message** : string

Verbal description of the termination reason.

**success** : bool

True if the algorithm converged to the desired accuracy (`status=0`).

### Notes

This function implements a 4-th order collocation algorithm with the control of residuals similar to [R43]. A collocation system is solved by a damped Newton method with an affine-invariant criterion function as described in [R45].

Note that in [R43] integral residuals are defined without normalization by interval lengths. So their definition is different by a multiplier of  $h^{*0.5}$  ( $h$  is an interval length) from the definition used here.

New in version 0.18.0.

### References

[R43], [R44], [R45], [R46]

### Examples

In the first example we solve Bratu's problem:

$$y'' + k * \exp(y) = 0$$

$$y(0) = y(1) = 0$$

for  $k = 1$ .

We rewrite the equation as a first order system and implement its right-hand side evaluation:

$$y1' = y2$$

$$y2' = -\exp(y1)$$

```
>>> def fun(x, y):
...     return np.vstack((y[1], -np.exp(y[0])))
```

Implement evaluation of the boundary condition residuals:

```
>>> def bc(ya, yb):
...     return np.array([ya[0], yb[0]])
```

Define the initial mesh with 5 nodes:

```
>>> x = np.linspace(0, 1, 5)
```

This problem is known to have two solutions. To obtain both of them we use two different initial guesses for  $y$ . We denote them by subscripts  $a$  and  $b$ .

```
>>> y_a = np.zeros((2, x.size))
>>> y_b = np.zeros((2, x.size))
>>> y_b[0] = 3
```

Now we are ready to run the solver.

```
>>> from scipy.integrate import solve_bvp
>>> res_a = solve_bvp(fun, bc, x, y_a)
>>> res_b = solve_bvp(fun, bc, x, y_b)
```

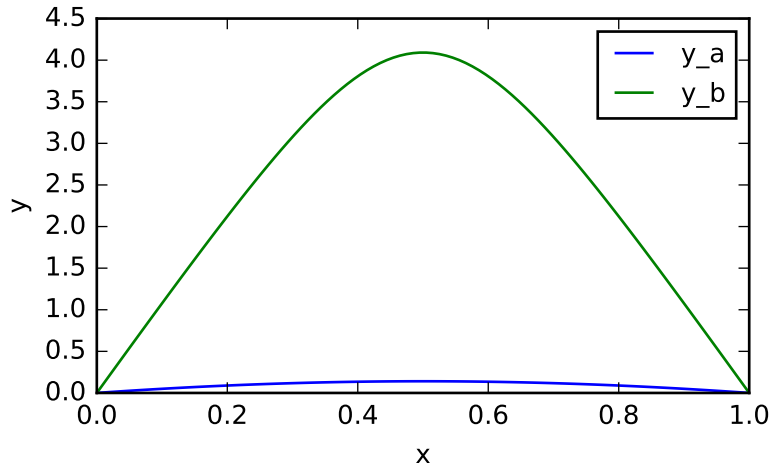
Let's plot the two found solutions. We take an advantage of having the solution in a spline form to produce a smooth plot.

```
>>> x_plot = np.linspace(0, 1, 100)
>>> y_plot_a = res_a.sol(x_plot)[0]
>>> y_plot_b = res_b.sol(x_plot)[0]
>>> import matplotlib.pyplot as plt
>>> plt.plot(x_plot, y_plot_a, label='y_a')
```

```

>>> plt.plot(x_plot, y_plot_b, label='y_b')
>>> plt.legend()
>>> plt.xlabel("x")
>>> plt.ylabel("y")
>>> plt.show()

```



We see that the two solutions have similar shape, but differ in scale significantly.

In the second example we solve a simple Sturm-Liouville problem:

$$y'' + k^2 y = 0$$

$$y(0) = y(1) = 0$$

It is known that a non-trivial solution  $y = A \sin(k x)$  is possible for  $k = \pi n$ , where  $n$  is an integer. To establish the normalization constant  $A = 1$  we add a boundary condition:

$$y'(0) = k$$

Again we rewrite our equation as a first order system and implement its right-hand side evaluation:

$$y_1' = y_2$$

$$y_2' = -k^2 y_1$$

```

>>> def fun(x, y, p):
...     k = p[0]
...     return np.vstack((y[1], -k**2 * y[0]))

```

Note that parameters  $p$  are passed as a vector (with one element in our case).

Implement the boundary conditions:

```

>>> def bc(ya, yb, p):
...     k = p[0]
...     return np.array([ya[0], yb[0], ya[1] - k])

```

Setup the initial mesh and guess for  $y$ . We aim to find the solution for  $k = 2 * \pi$ , to achieve that we set values of  $y$  to approximately follow  $\sin(2 * \pi * x)$ :

```

>>> x = np.linspace(0, 1, 5)
>>> y = np.zeros((2, x.size))

```



```
>>> y[0, 1] = 1
>>> y[0, 3] = -1
```

Run the solver with 6 as an initial guess for k.

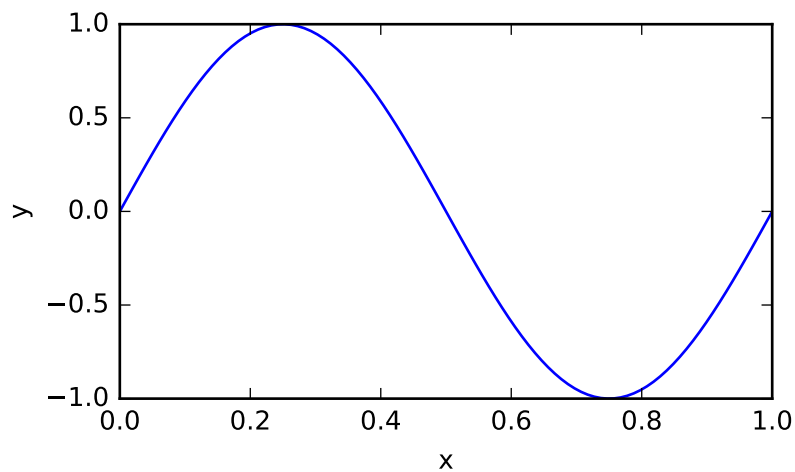
```
>>> sol = solve_bvp(fun, bc, x, y, p=[6])
```

We see that the found k is approximately correct:

```
>>> sol.p[0]
6.28329460046
```

And finally plot the solution to see the anticipated sinusoid:

```
>>> x_plot = np.linspace(0, 1, 100)
>>> y_plot = sol.sol(x_plot)[0]
>>> plt.plot(x_plot, y_plot)
>>> plt.xlabel("x")
>>> plt.ylabel("y")
>>> plt.show()
```



## 5.7 Interpolation (`scipy.interpolate`)

Sub-package for objects used in interpolation.

As listed below, this sub-package contains spline functions and classes, one-dimensional and multi-dimensional (univariate and multivariate) interpolation classes, Lagrange and Taylor polynomial interpolators, and wrappers for `FITPACK` and `DFITPACK` functions.

### 5.7.1 Univariate interpolation

<code>interp1d(x, y[, kind, axis, copy, ...])</code>	Interpolate a 1-D function.
<code>BarycentricInterpolator(xi[, yi, axis])</code>	The interpolating polynomial for a set of points
<code>KroghInterpolator(xi, yi[, axis])</code>	Interpolating polynomial for a set of points.

Continued on next page

Table 5.23 – continued from previous page

<code>PchipInterpolator(x, y[, axis, extrapolate])</code>	PCHIP 1-d monotonic cubic interpolation.
<code>barycentric_interpolate(xi, yi, x[, axis])</code>	Convenience function for polynomial interpolation.
<code>krogh_interpolate(xi, yi, x[, der, axis])</code>	Convenience function for polynomial interpolation.
<code>pchip_interpolate(xi, yi, x[, der, axis])</code>	Convenience function for pchip interpolation.
<code>Akima1DInterpolator(x, y[, axis])</code>	Akima interpolator
<code>CubicSpline(x, y[, axis, bc_type, extrapolate])</code>	Cubic spline data interpolator.
<code>PPoly(c, x[, extrapolate, axis])</code>	Piecewise polynomial in terms of coefficients and breakpoints
<code>BPoly(c, x[, extrapolate, axis])</code>	Piecewise polynomial in terms of coefficients and breakpoints.

```
class scipy.interpolate.interp1d(x, y, kind='linear', axis=-1, copy=True, bounds_error=None,
                                  fill_value=nan, assume_sorted=False)
```

Interpolate a 1-D function.

$x$  and  $y$  are arrays of values used to approximate some function  $f$ :  $y = f(x)$ . This class returns a function whose call method uses interpolation to find the value of new points.

**Parameters**

- x** : (N,) array\_like  
A 1-D array of real values.
- y** : (...N,...) array\_like  
A N-D array of real values. The length of  $y$  along the interpolation axis must be equal to the length of  $x$ .
- kind** : str or int, optional  
Specifies the kind of interpolation as a string ('linear', 'nearest', 'zero', 'slinear', 'quadratic', 'cubic' where 'slinear', 'quadratic' and 'cubic' refer to a spline interpolation of first, second or third order) or as an integer specifying the order of the spline interpolator to use. Default is 'linear'.
- axis** : int, optional  
Specifies the axis of  $y$  along which to interpolate. Interpolation defaults to the last axis of  $y$ .
- copy** : bool, optional  
If True, the class makes internal copies of  $x$  and  $y$ . If False, references to  $x$  and  $y$  are used. The default is to copy.
- bounds\_error** : bool, optional  
If True, a `ValueError` is raised any time interpolation is attempted on a value outside of the range of  $x$  (where extrapolation is necessary). If False, out of bounds values are assigned `fill_value`. By default, an error is raised unless `fill_value="extrapolate"`.
- fill\_value** : array-like or (array-like, array\_like) or "extrapolate", optional
  - if a ndarray (or float), this value will be used to fill in for requested points outside of the data range. If not provided, then the default is `NaN`. The array-like must broadcast properly to the dimensions of the non-interpolation axes.
  - If a two-element tuple, then the first element is used as a fill value for  $x_{\text{new}} < x[0]$  and the second element is used for  $x_{\text{new}} > x[-1]$ . Anything that is not a 2-element tuple (e.g., list or ndarray, regardless of shape) is taken to be a single array-like argument meant to be used for both bounds as `below, above = fill_value, fill_value`.
- New in version 0.17.0.
  - If "extrapolate", then points outside the data range will be extrapolated. ("nearest" and "linear" kinds only.)
  - New in version 0.17.0.
- assume\_sorted** : bool, optional  
If False, values of  $x$  can be in any order and they are sorted first. If True,  $x$  has to be an array of monotonically increasing values.

**See also:**

splrep, splev

**UnivariateSpline**

An object-oriented wrapper of the FITPACK routines.

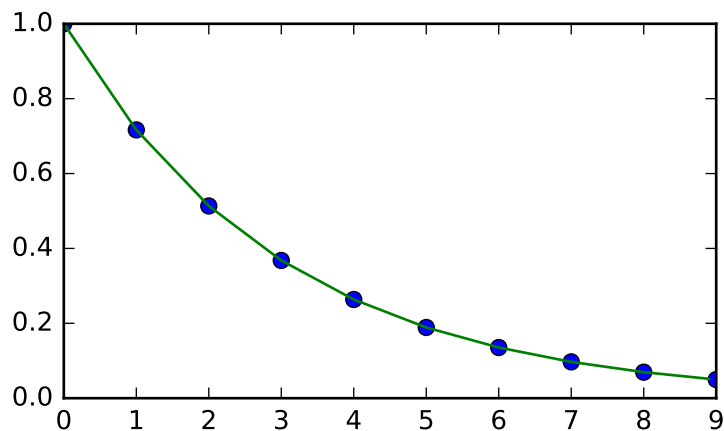
**interp2d** 2-D interpolation**Examples**

```

>>> import matplotlib.pyplot as plt
>>> from scipy import interpolate
>>> x = np.arange(0, 10)
>>> y = np.exp(-x/3.0)
>>> f = interpolate.interpld(x, y)

>>> xnew = np.arange(0, 9, 0.1)
>>> ynew = f(xnew) # use interpolation function returned by `interpld`
>>> plt.plot(x, y, 'o', xnew, ynew, '-')
>>> plt.show()

```

**Attributes**


---

`fill_value`


---

`interpld.fill_value`**Methods**


---

`__call__(x)` Evaluate the interpolant

---

```
interpld.__call__(x)
    Evaluate the interpolant
```

**Parameters** `x`: array\_like

**Returns** `y`: array\_like  
 Points to evaluate the interpolant at.  
 Interpolated values. Shape is determined by replacing the interpolation axis in the original array with the shape of `x`.

**class** `scipy.interpolate.BarycentricInterpolator` (`xi`, `yi=None`, `axis=0`)

The interpolating polynomial for a set of points

Constructs a polynomial that passes through a given set of points. Allows evaluation of the polynomial, efficient changing of the `y` values to be interpolated, and updating by adding more `x` values. For reasons of numerical stability, this function does not compute the coefficients of the polynomial.

The values `yi` need to be provided before the function is evaluated, but none of the preprocessing depends on them, so rapid updates are possible.

**Parameters** `xi`: array\_like  
 1-d array of `x` coordinates of the points the polynomial should pass through  
`yi`: array\_like, optional  
 The `y` coordinates of the points the polynomial should pass through. If `None`, the `y` values will be supplied later via the `set_y` method.  
`axis`: int, optional  
 Axis in the `yi` array corresponding to the `x`-coordinate values.

**Notes**

This class uses a “barycentric interpolation” method that treats the problem as a special case of rational function interpolation. This algorithm is quite stable, numerically, but even in a world of exact computation, unless the `x` coordinates are chosen very carefully - Chebyshev zeros (e.g.  $\cos(i*\pi/n)$ ) are a good choice - polynomial interpolation itself is a very ill-conditioned process due to the Runge phenomenon.

Based on Berrut and Trefethen 2004, “Barycentric Lagrange Interpolation”.

**Methods**

<code>__call__(x)</code>	Evaluate the interpolating polynomial at the points <code>x</code>
<code>add_xi(xi[, yi])</code>	Add more <code>x</code> values to the set to be interpolated
<code>set_yi(yi[, axis])</code>	Update the <code>y</code> values to be interpolated

`BarycentricInterpolator.__call__(x)`

Evaluate the interpolating polynomial at the points `x`

**Parameters** `x`: array\_like  
 Points to evaluate the interpolant at.  
**Returns** `y`: array\_like  
 Interpolated values. Shape is determined by replacing the interpolation axis in the original array with the shape of `x`.

**Notes**

Currently the code computes an outer product between `x` and the weights, that is, it constructs an intermediate array of size `N` by `len(x)`, where `N` is the degree of the polynomial.

`BarycentricInterpolator.add_xi` (`xi`, `yi=None`)

Add more `x` values to the set to be interpolated

The barycentric interpolation algorithm allows easy updating by adding more points for the polynomial to pass through.

**Parameters** `xi`: array\_like

The x coordinates of the points that the polynomial should pass through.

**yi** : array\_like, optional

The y coordinates of the points the polynomial should pass through. Should have shape `(xi.size, R)`; if `R > 1` then the polynomial is vector-valued. If `yi` is not given, the y values will be supplied later. `yi` should be given if and only if the interpolator has y values specified.

`BarycentricInterpolator.set_yi(yi, axis=None)`

Update the y values to be interpolated

The barycentric interpolation algorithm requires the calculation of weights, but these depend only on the `xi`. The `yi` can be changed at any time.

**Parameters** **yi** : array\_like

The y coordinates of the points the polynomial should pass through. If `None`, the y values will be supplied later.

**axis** : int, optional

Axis in the `yi` array corresponding to the x-coordinate values.

**class** `scipy.interpolate.KroghInterpolator(xi, yi, axis=0)`

Interpolating polynomial for a set of points.

The polynomial passes through all the pairs `(xi,yi)`. One may additionally specify a number of derivatives at each point `xi`; this is done by repeating the value `xi` and specifying the derivatives as successive `yi` values.

Allows evaluation of the polynomial and all its derivatives. For reasons of numerical stability, this function does not compute the coefficients of the polynomial, although they can be obtained by evaluating all the derivatives.

**Parameters** **xi** : array\_like, length N

Known x-coordinates. Must be sorted in increasing order.

**yi** : array\_like

Known y-coordinates. When an `xi` occurs two or more times in a row, the corresponding `yi`'s represent derivative values.

**axis** : int, optional

Axis in the `yi` array corresponding to the x-coordinate values.

### Notes

Be aware that the algorithms implemented here are not necessarily the most numerically stable known. Moreover, even in a world of exact computation, unless the x coordinates are chosen very carefully - Chebyshev zeros (e.g.  $\cos(i\pi/n)$ ) are a good choice - polynomial interpolation itself is a very ill-conditioned process due to the Runge phenomenon. In general, even with well-chosen x values, degrees higher than about thirty cause problems with numerical instability in this code.

Based on [R55].

### References

[R55]

### Examples

To produce a polynomial that is zero at 0 and 1 and has derivative 2 at 0, call

```
>>> from scipy.interpolate import KroghInterpolator
>>> KroghInterpolator([0, 0, 1], [0, 2, 0])
```

This constructs the quadratic  $2X^2 - 2X$ . The derivative condition is indicated by the repeated zero in the `xi` array; the corresponding `yi` values are 0, the function value, and 2, the derivative value.

For another example, given `xi`, `yi`, and a derivative `ypi` for each point, appropriate arrays can be constructed as:

```
>>> xi = np.linspace(0, 1, 5)
>>> yi, ypi = np.random.rand(2, 5)
>>> xi_k, yi_k = np.repeat(xi, 2), np.ravel(np.dstack((yi, ypi)))
>>> KroghInterpolator(xi_k, yi_k)
```

To produce a vector-valued polynomial, supply a higher-dimensional array for yi:

```
>>> KroghInterpolator([0, 1], [[2, 3], [4, 5]])
```

This constructs a linear polynomial giving (2,3) at 0 and (4,5) at 1.

### Methods

<code>__call__(x)</code>	Evaluate the interpolant
<code>derivative(x[, der])</code>	Evaluate one derivative of the polynomial at the point x
<code>derivatives(x[, der])</code>	Evaluate many derivatives of the polynomial at the point x

`KroghInterpolator.__call__(x)`

Evaluate the interpolant

- Parameters** `x`: array\_like  
Points to evaluate the interpolant at.
- Returns** `y`: array\_like  
Interpolated values. Shape is determined by replacing the interpolation axis in the original array with the shape of x.

`KroghInterpolator.derivative(x, der=1)`

Evaluate one derivative of the polynomial at the point x

- Parameters** `x`: array\_like  
Point or points at which to evaluate the derivatives
- der**: integer, optional  
Which derivative to extract. This number includes the function value as 0th derivative.
- Returns** `d`: ndarray  
Derivative interpolated at the x-points. Shape of d is determined by replacing the interpolation axis in the original array with the shape of x.

### Notes

This is computed by evaluating all derivatives up to the desired one (using `self.derivatives()`) and then discarding the rest.

`KroghInterpolator.derivatives(x, der=None)`

Evaluate many derivatives of the polynomial at the point x

Produce an array of all derivative values at the point x.

- Parameters** `x`: array\_like  
Point or points at which to evaluate the derivatives
- der**: int or None, optional  
How many derivatives to extract; None for all potentially nonzero derivatives (that is a number equal to the number of points). This number includes the function value as 0th derivative.
- Returns** `d`: ndarray  
Array with derivatives; `d[j]` contains the j-th derivative. Shape of `d[j]` is determined by replacing the interpolation axis in the original array with the shape of x.

**Examples**

```
>>> from scipy.interpolate import KroghInterpolator
>>> KroghInterpolator([0,0,0],[1,2,3]).derivatives(0)
array([1.0,2.0,3.0])
>>> KroghInterpolator([0,0,0],[1,2,3]).derivatives([0,0])
array([[1.0,1.0],
       [2.0,2.0],
       [3.0,3.0]])
```

**class** `scipy.interpolate.PchipInterpolator` (*x*, *y*, *axis=0*, *extrapolate=None*)  
 PCHIP 1-d monotonic cubic interpolation.

*x* and *y* are arrays of values used to approximate some function *f*, with  $y = f(x)$ . The interpolant uses monotonic cubic splines to find the value of new points. (PCHIP stands for Piecewise Cubic Hermite Interpolating Polynomial).

**Parameters** **x** : ndarray

A 1-D array of monotonically increasing real values. *x* cannot include duplicate values (otherwise *f* is overspecified)

**y** : ndarray

A 1-D array of real values. *y*'s length along the interpolation axis must be equal to the length of *x*. If N-D array, use *axis* parameter to select correct axis.

**axis** : int, optional

Axis in the *y* array corresponding to the *x*-coordinate values.

**extrapolate** : bool, optional

Whether to extrapolate to out-of-bounds points based on first and last intervals, or to return NaNs.

**See also:**

[Akima1DInterpolator](#), [CubicSpline](#), [BPoly](#)

**Notes**

The interpolator preserves monotonicity in the interpolation data and does not overshoot if the data is not smooth.

The first derivatives are guaranteed to be continuous, but the second derivatives may jump at  $x_k$ .

Determines the derivatives at the points  $x_k$ ,  $f'_k$ , by using PCHIP algorithm [R57].

Let  $h_k = x_{k+1} - x_k$ , and  $d_k = (y_{k+1} - y_k)/h_k$  are the slopes at internal points  $x_k$ . If the signs of  $d_k$  and  $d_{k-1}$  are different or either of them equals zero, then  $f'_k = 0$ . Otherwise, it is given by the weighted harmonic mean

$$\frac{w_1 + w_2}{f'_k} = \frac{w_1}{d_{k-1}} + \frac{w_2}{d_k}$$

where  $w_1 = 2h_k + h_{k-1}$  and  $w_2 = h_k + 2h_{k-1}$ .

The end slopes are set using a one-sided scheme [R58].

**References**

[R57], [R58]

**Methods**


---

<code>__call__(x[, nu, extrapolate])</code>	Evaluate the piecewise polynomial or its derivative.
<code>derivative([nu])</code>	Construct a new piecewise polynomial representing the derivative.
<code>antiderivative([nu])</code>	Construct a new piecewise polynomial representing the antiderivative.
<code>roots()</code>	Return the roots of the interpolated function.

---

`PchipInterpolator.__call__(x, nu=0, extrapolate=None)`

Evaluate the piecewise polynomial or its derivative.

**Parameters**

- x** : array\_like  
Points to evaluate the interpolant at.
- nu** : int, optional  
Order of derivative to evaluate. Must be non-negative.
- extrapolate** : {bool, 'periodic', None}, optional  
If bool, determines whether to extrapolate to out-of-bounds points based on first and last intervals, or to return NaNs. If 'periodic', periodic extrapolation is used. If None (default), use `self.extrapolate`.

**Returns**

- y** : array\_like  
Interpolated values. Shape is determined by replacing the interpolation axis in the original array with the shape of x.

**Notes**

Derivatives are evaluated piecewise for each polynomial segment, even if the polynomial is not differentiable at the breakpoints. The polynomial intervals are considered half-open,  $[a, b)$ , except for the last interval which is closed  $[a, b]$ .

`PchipInterpolator.derivative(nu=1)`

Construct a new piecewise polynomial representing the derivative.

**Parameters**

- nu** : int, optional  
Order of derivative to evaluate. Default is 1, i.e. compute the first derivative. If negative, the antiderivative is returned.

**Returns**

- bp** : BPoly  
Piecewise polynomial of order  $k - nu$  representing the derivative of this polynomial.

`PchipInterpolator.antiderivative(nu=1)`

Construct a new piecewise polynomial representing the antiderivative.

**Parameters**

- nu** : int, optional  
Order of antiderivative to evaluate. Default is 1, i.e. compute the first integral. If negative, the derivative is returned.

**Returns**

- bp** : BPoly  
Piecewise polynomial of order  $k + nu$  representing the antiderivative of this polynomial.

**Notes**

If antiderivative is computed and `self.extrapolate='periodic'`, it will be set to False for the returned instance. This is done because the antiderivative is no longer periodic and its correct evaluation outside of the initially given x interval is difficult.

`PchipInterpolator.roots()`

Return the roots of the interpolated function.

`scipy.interpolate.barycentric_interpolate(xi, yi, x, axis=0)`

Convenience function for polynomial interpolation.

Constructs a polynomial that passes through a given set of points, then evaluates the polynomial. For reasons of numerical stability, this function does not compute the coefficients of the polynomial.

This function uses a “barycentric interpolation” method that treats the problem as a special case of rational function interpolation. This algorithm is quite stable, numerically, but even in a world of exact computation, unless the x coordinates are chosen very carefully - Chebyshev zeros (e.g.  $\cos(i\pi/n)$ ) are a good choice - polynomial interpolation itself is a very ill-conditioned process due to the Runge phenomenon.



**Parameters**

- xi** : array\_like  
1-d array of x coordinates of the points the polynomial should pass through
- yi** : array\_like  
The y coordinates of the points the polynomial should pass through.
- x** : scalar or array\_like  
Points to evaluate the interpolator at.
- axis** : int, optional  
Axis in the yi array corresponding to the x-coordinate values.

**Returns**

- y** : scalar or array\_like  
Interpolated values. Shape is determined by replacing the interpolation axis in the original array with the shape of x.

**See also:**

[BarycentricInterpolator](#)

#### Notes

Construction of the interpolation weights is a relatively slow process. If you want to call this many times with the same xi (but possibly varying yi or x) you should use the class [BarycentricInterpolator](#). This is what this function uses internally.

`scipy.interpolate.krogh_interpolate(xi, yi, x, der=0, axis=0)`

Convenience function for polynomial interpolation.

See [KroghInterpolator](#) for more details.

**Parameters**

- xi** : array\_like  
Known x-coordinates.
- yi** : array\_like  
Known y-coordinates, of shape `(xi.size, R)`. Interpreted as vectors of length R, or scalars if R=1.
- x** : array\_like  
Point or points at which to evaluate the derivatives.
- der** : int or list, optional  
How many derivatives to extract; None for all potentially nonzero derivatives (that is a number equal to the number of points), or a list of derivatives to extract. This number includes the function value as 0th derivative.
- axis** : int, optional  
Axis in the yi array corresponding to the x-coordinate values.

**Returns**

- d** : ndarray  
If the interpolator's values are R-dimensional then the returned array will be the number of derivatives by N by R. If x is a scalar, the middle dimension will be dropped; if the yi are scalars then the last dimension will be dropped.

**See also:**

[KroghInterpolator](#)

#### Notes

Construction of the interpolating polynomial is a relatively expensive process. If you want to evaluate it repeatedly consider using the class [KroghInterpolator](#) (which is what this function uses).

`scipy.interpolate.pchip_interpolate(xi, yi, x, der=0, axis=0)`

Convenience function for pchip interpolation. xi and yi are arrays of values used to approximate some function f, with  $y_i = f(x_i)$ . The interpolant uses monotonic cubic splines to find the value of new points x and the derivatives there.

See [PchipInterpolator](#) for details.

**Parameters**

- xi** : array\_like  
A sorted list of x-coordinates, of length N.
- yi** : array\_like  
A 1-D array of real values. *yi*'s length along the interpolation axis must be equal to the length of *xi*. If N-D array, use axis parameter to select correct axis.
- x** : scalar or array\_like  
Of length M.
- der** : int or list, optional  
Derivatives to extract. The 0-th derivative can be included to return the function value.
- axis** : int, optional  
Axis in the *yi* array corresponding to the x-coordinate values.

**Returns**

- y** : scalar or array\_like  
The result, of length R or length M or M by R,

**See also:**

`PchipInterpolator`

**class** `scipy.interpolate.Akima1DInterpolator` (*x*, *y*, *axis=0*)  
Akima interpolator

Fit piecewise cubic polynomials, given vectors *x* and *y*. The interpolation method by Akima uses a continuously differentiable sub-spline built from piecewise cubic polynomials. The resultant curve passes through the given data points and will appear smooth and natural.

**Parameters**

- x** : ndarray, shape (m, )  
1-D array of monotonically increasing real values.
- y** : ndarray, shape (m, ...)  
N-D array of real values. The length of *y* along the first axis must be equal to the length of *x*.
- axis** : int, optional  
Specifies the axis of *y* along which to interpolate. Interpolation defaults to the first axis of *y*.

**See also:**

`PchipInterpolator`, `CubicSpline`, `PPoly`

**Notes**

New in version 0.14.

Use only for precise data, as the fitted curve passes through the given points exactly. This routine is useful for plotting a pleasingly smooth curve through a few given points for purposes of plotting.

**References**

[1] *A new method of interpolation and smooth curve fitting based on local procedures.* Hiroshi Akima, J. ACM, October 1970, 17(4), 589-602.

**Methods**

---

<code>__call__(x[, nu, extrapolate])</code>	Evaluate the piecewise polynomial or its derivative.
<code>derivative([nu])</code>	Construct a new piecewise polynomial representing the derivative.
<code>antiderivative([nu])</code>	Construct a new piecewise polynomial representing the antiderivative.
<code>roots([discontinuity, extrapolate])</code>	Find real roots of the the piecewise polynomial.

---

`Akima1DInterpolator.__call__(x, nu=0, extrapolate=None)`

Evaluate the piecewise polynomial or its derivative.

**Parameters**

- x** : array\_like  
Points to evaluate the interpolant at.
- nu** : int, optional  
Order of derivative to evaluate. Must be non-negative.
- extrapolate** : {bool, 'periodic', None}, optional  
If bool, determines whether to extrapolate to out-of-bounds points based on first and last intervals, or to return NaNs. If 'periodic', periodic extrapolation is used. If None (default), use `self.extrapolate`.

**Returns**

- y** : array\_like  
Interpolated values. Shape is determined by replacing the interpolation axis in the original array with the shape of x.

#### Notes

Derivatives are evaluated piecewise for each polynomial segment, even if the polynomial is not differentiable at the breakpoints. The polynomial intervals are considered half-open,  $[a, b)$ , except for the last interval which is closed  $[a, b]$ .

`Akima1DInterpolator.derivative(nu=1)`

Construct a new piecewise polynomial representing the derivative.

**Parameters**

- nu** : int, optional  
Order of derivative to evaluate. Default is 1, i.e. compute the first derivative. If negative, the antiderivative is returned.

**Returns**

- pp** : PPoly  
Piecewise polynomial of order  $k_2 = k - n$  representing the derivative of this polynomial.

#### Notes

Derivatives are evaluated piecewise for each polynomial segment, even if the polynomial is not differentiable at the breakpoints. The polynomial intervals are considered half-open,  $[a, b)$ , except for the last interval which is closed  $[a, b]$ .

`Akima1DInterpolator.antiderivative(nu=1)`

Construct a new piecewise polynomial representing the antiderivative.

Antiderivative is also the indefinite integral of the function, and derivative is its inverse operation.

**Parameters**

- nu** : int, optional  
Order of antiderivative to evaluate. Default is 1, i.e. compute the first integral. If negative, the derivative is returned.

**Returns**

- pp** : PPoly  
Piecewise polynomial of order  $k_2 = k + n$  representing the antiderivative of this polynomial.

#### Notes

The antiderivative returned by this function is continuous and continuously differentiable to order  $n-1$ , up to floating point rounding error.

If antiderivative is computed and `self.extrapolate='periodic'`, it will be set to `False` for the returned instance. This is done because the antiderivative is no longer periodic and its correct evaluation outside of the initially given x interval is difficult.

`Akima1DInterpolator.roots(discontinuity=True, extrapolate=None)`

Find real roots of the the piecewise polynomial.

**Parameters** **discontinuity** : bool, optional  
Whether to report sign changes across discontinuities at breakpoints as roots.

**extrapolate** : {bool, 'periodic', None}, optional  
If bool, determines whether to return roots from the polynomial extrapolated based on first and last intervals, 'periodic' works the same as False. If None (default), use *self.extrapolate*.

**Returns** **roots** : ndarray  
Roots of the polynomial(s).  
If the PPoly object describes multiple polynomials, the return value is an object array whose each element is an ndarray containing the roots.

**See also:**

`PPoly.solve`

**class** `scipy.interpolate.CubicSpline` (*x*, *y*, *axis=0*, *bc\_type='not-a-knot'*, *extrapolate=None*)  
Cubic spline data interpolator.

Interpolate data with a piecewise cubic polynomial which is twice continuously differentiable [R53]. The result is represented as a `PPoly` instance with breakpoints matching the given data.

**Parameters** **x** : array\_like, shape (n,)  
1-d array containing values of the independent variable. Values must be real, finite and in strictly increasing order.

**y** : array\_like  
Array containing values of the dependent variable. It can have arbitrary number of dimensions, but the length along *axis* (see below) must match the length of *x*. Values must be finite.

**axis** : int, optional  
Axis along which *y* is assumed to be varying. Meaning that for *x*[*i*] the corresponding values are `np.take(y, i, axis=axis)`. Default is 0.

**bc\_type** : string or 2-tuple, optional  
Boundary condition type. Two additional equations, given by the boundary conditions, are required to determine all coefficients of polynomials on each segment [R54]. If *bc\_type* is a string, then the specified condition will be applied at both ends of a spline. Available conditions are:

- 'not-a-knot' (default): The first and second segment at a curve end are the same polynomial. It is a good default when there is no information on boundary conditions.
- 'periodic': The interpolated functions is assumed to be periodic of period  $x[-1] - x[0]$ . The first and last value of *y* must be identical:  $y[0] == y[-1]$ . This boundary condition will result in  $y'[0] == y'[-1]$  and  $y''[0] == y''[-1]$ .
- 'clamped': The first derivative at curves ends are zero. Assuming a 1D *y*, `bc_type=((1, 0.0), (1, 0.0))` is the same condition.
- 'natural': The second derivative at curve ends are zero. Assuming a 1D *y*, `bc_type=((2, 0.0), (2, 0.0))` is the same condition.

If *bc\_type* is a 2-tuple, the first and the second value will be applied at the curve start and end respectively. The tuple values can be one of the previously mentioned strings (except 'periodic') or a tuple (*order*, *deriv\_values*) allowing to specify arbitrary derivatives at curve ends:

- order*: the derivative order, 1 or 2.
- deriv\_value*: array\_like containing derivative values, shape must be the same as *y*, excluding *axis* dimension. For example, if *y* is 1D, then *deriv\_value* must be a scalar. If *y* is 3D with the shape (n0, n1, n2) and *axis*=2, then *deriv\_value* must be 2D and have the shape (n0, n1).

**extrapolate** : {bool, 'periodic', None}, optional

If bool, determines whether to extrapolate to out-of-bounds points based on first and last intervals, or to return NaNs. If 'periodic', periodic extrapolation is used. If None (default), extrapolate is set to 'periodic' for `bc_type='periodic'` and to True otherwise.

**See also:**

[Akima1DInterpolator](#), [PchipInterpolator](#), [PPoly](#)

**Notes**

Parameters `bc_type` and `interpolate` work independently, i.e. the former controls only construction of a spline, and the latter only evaluation.

When a boundary condition is 'not-a-knot' and  $n = 2$ , it is replaced by a condition that the first derivative is equal to the linear interpolant slope. When both boundary conditions are 'not-a-knot' and  $n = 3$ , the solution is sought as a parabola passing through given points.

When 'not-a-knot' boundary conditions is applied to both ends, the resulting spline will be the same as returned by `splrep` (with `s=0`) and `InterpolatedUnivariateSpline`, but these two methods use a representation in B-spline basis.

New in version 0.18.0.

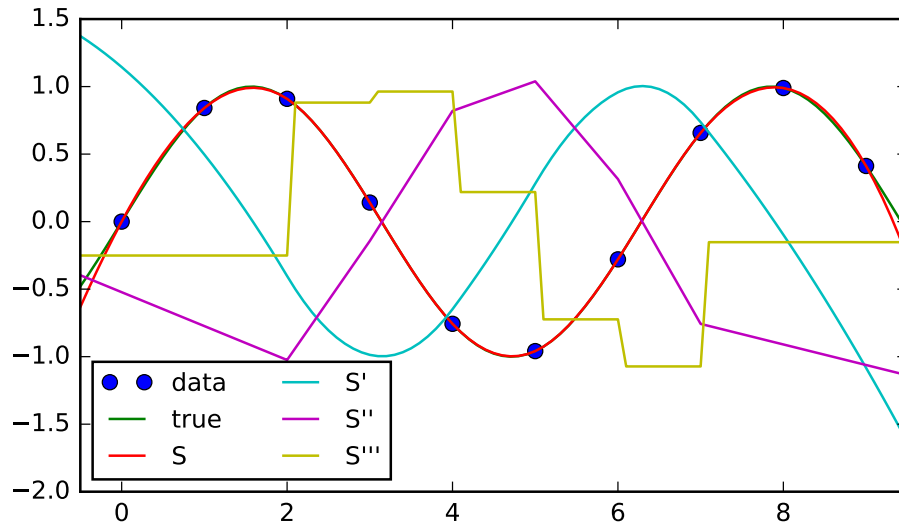
**References**

[R53], [R54]

**Examples**

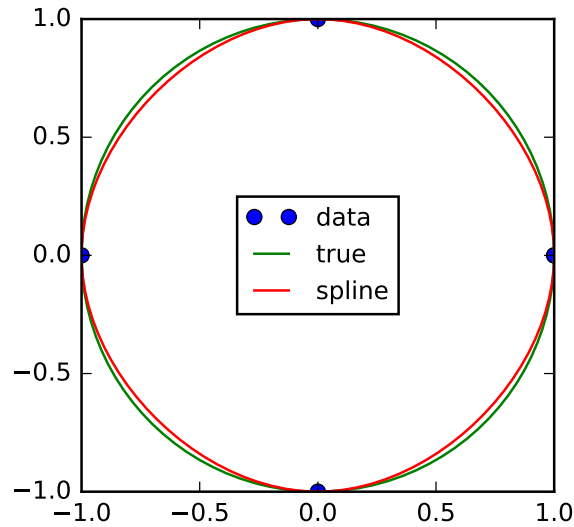
In this example the cubic spline is used to interpolate a sampled sinusoid. You can see that the spline continuity property holds for the first and second derivatives and violates only for the third derivative.

```
>>> from scipy.interpolate import CubicSpline
>>> import matplotlib.pyplot as plt
>>> x = np.arange(10)
>>> y = np.sin(x)
>>> cs = CubicSpline(x, y)
>>> xs = np.arange(-0.5, 9.6, 0.1)
>>> plt.figure(figsize=(6.5, 4))
>>> plt.plot(x, y, 'o', label='data')
>>> plt.plot(xs, np.sin(xs), label='true')
>>> plt.plot(xs, cs(xs), label="S")
>>> plt.plot(xs, cs(xs, 1), label="S'")
>>> plt.plot(xs, cs(xs, 2), label="S''")
>>> plt.plot(xs, cs(xs, 3), label="S'''")
>>> plt.xlim(-0.5, 9.5)
>>> plt.legend(loc='lower left', ncol=2)
>>> plt.show()
```



In the second example, the unit circle is interpolated with a spline. A periodic boundary condition is used. You can see that the first derivative values,  $ds/dx=0$ ,  $ds/dy=1$  at the periodic point (1, 0) are correctly computed. Note that a circle cannot be exactly represented by a cubic spline. To increase precision, more breakpoints would be required.

```
>>> theta = 2 * np.pi * np.linspace(0, 1, 5)
>>> y = np.c_[np.cos(theta), np.sin(theta)]
>>> cs = CubicSpline(theta, y, bc_type='periodic')
>>> print("ds/dx={:.1f} ds/dy={:.1f}".format(cs(0, 1)[0], cs(0, 1)[1]))
ds/dx=0.0 ds/dy=1.0
>>> xs = 2 * np.pi * np.linspace(0, 1, 100)
>>> plt.figure(figsize=(6.5, 4))
>>> plt.plot(y[:, 0], y[:, 1], 'o', label='data')
>>> plt.plot(np.cos(xs), np.sin(xs), label='true')
>>> plt.plot(cs(xs)[:, 0], cs(xs)[:, 1], label='spline')
>>> plt.axes().set_aspect('equal')
>>> plt.legend(loc='center')
>>> plt.show()
```



The third example is the interpolation of a polynomial  $y = x^{**3}$  on the interval  $0 \leq x \leq 1$ . A cubic spline can represent this function exactly. To achieve that we need to specify values and first derivatives at endpoints of the interval. Note that  $y' = 3 * x^{**2}$  and thus  $y'(0) = 0$  and  $y'(1) = 3$ .

```
>>> cs = CubicSpline([0, 1], [0, 1], bc_type=((1, 0), (1, 3)))
>>> x = np.linspace(0, 1)
>>> np.allclose(x**3, cs(x))
True
```

### Attributes

<code>x</code>	(ndarray, shape (n,)) Breakpoints. The same <code>x</code> which was passed to the constructor.
<code>c</code>	(ndarray, shape (4, n-1, ...)) Coefficients of the polynomials on each segment. The trailing dimensions match the dimensions of <code>y</code> , excluding <code>axis</code> . For example, if <code>y</code> is 1-d, then <code>c[k, i]</code> is a coefficient for $(x-x[i])^{*(3-k)}$ on the segment between <code>x[i]</code> and <code>x[i+1]</code> .
<code>axis</code>	(int) Interpolation axis. The same <code>axis</code> which was passed to the constructor.

### Methods

<code>__call__(x[, nu, extrapolate])</code>	Evaluate the piecewise polynomial or its derivative.
<code>derivative([nu])</code>	Construct a new piecewise polynomial representing the derivative.
<code>antiderivative([nu])</code>	Construct a new piecewise polynomial representing the antiderivative.
<code>integrate(a, b[, extrapolate])</code>	Compute a definite integral over a piecewise polynomial.
<code>roots([discontinuity, extrapolate])</code>	Find real roots of the the piecewise polynomial.

`CubicSpline.__call__(x, nu=0, extrapolate=None)`  
Evaluate the piecewise polynomial or its derivative.

**Parameters**

- `x`: array\_like  
Points to evaluate the interpolant at.
- `nu`: int, optional

Order of derivative to evaluate. Must be non-negative.

**extrapolate** : {bool, 'periodic', None}, optional

If bool, determines whether to extrapolate to out-of-bounds points based on first and last intervals, or to return NaNs. If 'periodic', periodic extrapolation is used. If None (default), use *self.extrapolate*.

**Returns** **y** : array\_like

Interpolated values. Shape is determined by replacing the interpolation axis in the original array with the shape of x.

**Notes**

Derivatives are evaluated piecewise for each polynomial segment, even if the polynomial is not differentiable at the breakpoints. The polynomial intervals are considered half-open,  $[a, b)$ , except for the last interval which is closed  $[a, b]$ .

`CubicSpline.derivative` (*nu=1*)

Construct a new piecewise polynomial representing the derivative.

**Parameters** **nu** : int, optional

Order of derivative to evaluate. Default is 1, i.e. compute the first derivative. If negative, the antiderivative is returned.

**Returns** **pp** : PPoly

Piecewise polynomial of order  $k_2 = k - n$  representing the derivative of this polynomial.

**Notes**

Derivatives are evaluated piecewise for each polynomial segment, even if the polynomial is not differentiable at the breakpoints. The polynomial intervals are considered half-open,  $[a, b)$ , except for the last interval which is closed  $[a, b]$ .

`CubicSpline.antiderivative` (*nu=1*)

Construct a new piecewise polynomial representing the antiderivative.

Antiderivative is also the indefinite integral of the function, and derivative is its inverse operation.

**Parameters** **nu** : int, optional

Order of antiderivative to evaluate. Default is 1, i.e. compute the first integral. If negative, the derivative is returned.

**Returns** **pp** : PPoly

Piecewise polynomial of order  $k_2 = k + n$  representing the antiderivative of this polynomial.

**Notes**

The antiderivative returned by this function is continuous and continuously differentiable to order  $n-1$ , up to floating point rounding error.

If antiderivative is computed and `self.extrapolate='periodic'`, it will be set to False for the returned instance. This is done because the antiderivative is no longer periodic and its correct evaluation outside of the initially given x interval is difficult.

`CubicSpline.integrate` (*a, b, extrapolate=None*)

Compute a definite integral over a piecewise polynomial.

**Parameters** **a** : float

Lower integration bound

**b** : float

Upper integration bound

**extrapolate** : {bool, 'periodic', None}, optional



If bool, determines whether to extrapolate to out-of-bounds points based on first and last intervals, or to return NaNs. If 'periodic', periodic extrapolation is used. If None (default), use *self.extrapolate*.

**Returns** **ig** : array\_like  
Definite integral of the piecewise polynomial over [a, b]

`CubicSpline.roots` (*discontinuity=True, extrapolate=None*)

Find real roots of the the piecewise polynomial.

**Parameters** **discontinuity** : bool, optional  
Whether to report sign changes across discontinuities at breakpoints as roots.

**extrapolate** : {bool, 'periodic', None}, optional  
If bool, determines whether to return roots from the polynomial extrapolated based on first and last intervals, 'periodic' works the same as False. If None (default), use *self.extrapolate*.

**Returns** **roots** : ndarray  
Roots of the polynomial(s).  
If the PPoly object describes multiple polynomials, the return value is an object array whose each element is an ndarray containing the roots.

**See also:**

`PPoly.solve`

**class** `scipy.interpolate.PPoly` (*c, x, extrapolate=None, axis=0*)

Piecewise polynomial in terms of coefficients and breakpoints

The polynomial in the *i*th interval is  $x[i] \leq x_p < x[i+1]$ :

$$S = \sum(c[m, i] * (x_p - x[i])^{k-m} \text{ for } m \text{ in range}(k+1))$$

where *k* is the degree of the polynomial. This representation is the local power basis.

**Parameters** **c** : ndarray, shape (k, m, ...)  
Polynomial coefficients, order *k* and *m* intervals

**x** : ndarray, shape (m+1,)  
Polynomial breakpoints. These must be sorted in increasing order.

**extrapolate** : bool or 'periodic', optional  
If bool, determines whether to extrapolate to out-of-bounds points based on first and last intervals, or to return NaNs. If 'periodic', periodic extrapolation is used. Default is True.

**axis** : int, optional  
Interpolation axis. Default is zero.

**See also:**

`BPoly` piecewise polynomials in the Bernstein basis

**Notes**

High-order polynomials in the power basis can be numerically unstable. Precision problems can start to appear for orders larger than 20-30.

**Attributes**

<code>x</code>	(ndarray) Breakpoints.
<code>c</code>	(ndarray) Coefficients of the polynomials. They are reshaped to a 3-dimensional array with the last dimension representing the trailing dimensions of the original coefficient array.
<code>axis</code>	(int) Interpolation axis.

*Methods*

<code>__call__(x[, nu, extrapolate])</code>	Evaluate the piecewise polynomial or its derivative.
<code>derivative([nu])</code>	Construct a new piecewise polynomial representing the derivative.
<code>antiderivative([nu])</code>	Construct a new piecewise polynomial representing the antiderivative.
<code>integrate(a, b[, extrapolate])</code>	Compute a definite integral over a piecewise polynomial.
<code>solve([y, discontinuity, extrapolate])</code>	Find real solutions of the the equation $pp(x) == y$ .
<code>roots([discontinuity, extrapolate])</code>	Find real roots of the the piecewise polynomial.
<code>extend(c, x[, right])</code>	Add additional breakpoints and coefficients to the polynomial.
<code>from_spline(tck[, extrapolate])</code>	Construct a piecewise polynomial from a spline
<code>from_bernstein_basis(bp[, extrapolate])</code>	Construct a piecewise polynomial in the power basis from a polynomial in Bernstein
<code>construct_fast(c, x[, extrapolate, axis])</code>	Construct the piecewise polynomial without making checks.

`PPoly.__call__(x, nu=0, extrapolate=None)`  
 Evaluate the piecewise polynomial or its derivative.

**Parameters** `x` : array\_like  
 Points to evaluate the interpolant at.  
`nu` : int, optional  
 Order of derivative to evaluate. Must be non-negative.  
`extrapolate` : {bool, 'periodic', None}, optional  
 If bool, determines whether to extrapolate to out-of-bounds points based on first and last intervals, or to return NaNs. If 'periodic', periodic extrapolation is used. If None (default), use *self.extrapolate*.  
**Returns** `y` : array\_like  
 Interpolated values. Shape is determined by replacing the interpolation axis in the original array with the shape of `x`.

*Notes*

Derivatives are evaluated piecewise for each polynomial segment, even if the polynomial is not differentiable at the breakpoints. The polynomial intervals are considered half-open,  $[a, b)$ , except for the last interval which is closed  $[a, b]$ .

`PPoly.derivative(nu=1)`  
 Construct a new piecewise polynomial representing the derivative.

**Parameters** `nu` : int, optional  
 Order of derivative to evaluate. Default is 1, i.e. compute the first derivative. If negative, the antiderivative is returned.  
**Returns** `pp` : PPoly  
 Piecewise polynomial of order  $k_2 = k - n$  representing the derivative of this polynomial.

*Notes*

Derivatives are evaluated piecewise for each polynomial segment, even if the polynomial is not differentiable at the breakpoints. The polynomial intervals are considered half-open,  $[a, b)$ , except for the last interval which is closed  $[a, b]$ .

`PPoly.antiderivative(nu=1)`  
 Construct a new piecewise polynomial representing the antiderivative.

Antiderivative is also the indefinite integral of the function, and derivative is its inverse operation.

**Parameters** `nu` : int, optional

Order of antiderivative to evaluate. Default is 1, i.e. compute the first integral. If negative, the derivative is returned.

**Returns** **pp** : PPoly  
 Piecewise polynomial of order  $k_2 = k + n$  representing the antiderivative of this polynomial.

**Notes**

The antiderivative returned by this function is continuous and continuously differentiable to order  $n-1$ , up to floating point rounding error.

If antiderivative is computed and `self.extrapolate='periodic'`, it will be set to `False` for the returned instance. This is done because the antiderivative is no longer periodic and its correct evaluation outside of the initially given  $x$  interval is difficult.

PPoly.**integrate** (*a*, *b*, *extrapolate=None*)

Compute a definite integral over a piecewise polynomial.

**Parameters** **a** : float  
 Lower integration bound

**b** : float  
 Upper integration bound

**extrapolate** : {bool, 'periodic', None}, optional  
 If bool, determines whether to extrapolate to out-of-bounds points based on first and last intervals, or to return NaNs. If 'periodic', periodic extrapolation is used. If None (default), use `self.extrapolate`.

**Returns** **ig** : array\_like  
 Definite integral of the piecewise polynomial over [a, b]

PPoly.**solve** (*y=0.0*, *discontinuity=True*, *extrapolate=None*)

Find real solutions of the the equation  $pp(x) == y$ .

**Parameters** **y** : float, optional  
 Right-hand side. Default is zero.

**discontinuity** : bool, optional  
 Whether to report sign changes across discontinuities at breakpoints as roots.

**extrapolate** : {bool, 'periodic', None}, optional  
 If bool, determines whether to return roots from the polynomial extrapolated based on first and last intervals, 'periodic' works the same as `False`. If None (default), use `self.extrapolate`.

**Returns** **roots** : ndarray  
 Roots of the polynomial(s).  
 If the PPoly object describes multiple polynomials, the return value is an object array whose each element is an ndarray containing the roots.

**Notes**

This routine works only on real-valued polynomials.

If the piecewise polynomial contains sections that are identically zero, the root list will contain the start point of the corresponding interval, followed by a `nan` value.

If the polynomial is discontinuous across a breakpoint, and there is a sign change across the breakpoint, this is reported if the `discont` parameter is `True`.

**Examples**

Finding roots of  $[x^2 - 1, (x - 1)^2]$  defined on intervals  $[-2, 1]$ ,  $[1, 2]$ :

```
>>> from scipy.interpolate import PPoly
>>> pp = PPoly(np.array([[1, -4, 3], [1, 0, 0]].T, [-2, 1, 2]))
>>> pp.roots()
array([-1.,  1.]
```

`PPoly.roots` (*discontinuity=True, extrapolate=None*)

Find real roots of the the piecewise polynomial.

**Parameters**

- discontinuity** : bool, optional  
Whether to report sign changes across discontinuities at breakpoints as roots.
- extrapolate** : {bool, 'periodic', None}, optional  
If bool, determines whether to return roots from the polynomial extrapolated based on first and last intervals, 'periodic' works the same as False. If None (default), use *self.extrapolate*.

**Returns**

- roots** : ndarray  
Roots of the polynomial(s).  
If the PPoly object describes multiple polynomials, the return value is an object array whose each element is an ndarray containing the roots.

**See also:**

`PPoly.solve`

`PPoly.extend` (*c, x, right=True*)

Add additional breakpoints and coefficients to the polynomial.

**Parameters**

- c** : ndarray, size (k, m, ...)  
Additional coefficients for polynomials in intervals `self.x[-1] <= x < x_right[0]`, `x_right[0] <= x < x_right[1]`, ..., `x_right[m-2] <= x < x_right[m-1]`
- x** : ndarray, size (m,)  
Additional breakpoints. Must be sorted and either to the right or to the left of the current breakpoints.
- right** : bool, optional  
Whether the new intervals are to the right or to the left of the current intervals.

`classmethod PPoly.from_spline` (*tck, extrapolate=None*)

Construct a piecewise polynomial from a spline

**Parameters**

- tck**  
A spline, as returned by `splrep`
- extrapolate** : bool or 'periodic', optional  
If bool, determines whether to extrapolate to out-of-bounds points based on first and last intervals, or to return NaNs. If 'periodic', periodic extrapolation is used. Default is True.

`classmethod PPoly.from_bernstein_basis` (*bp, extrapolate=None*)

Construct a piecewise polynomial in the power basis from a polynomial in Bernstein basis.

**Parameters**

- bp** : BPoly  
A Bernstein basis polynomial, as created by BPoly
- extrapolate** : bool or 'periodic', optional  
If bool, determines whether to extrapolate to out-of-bounds points based on first and last intervals, or to return NaNs. If 'periodic', periodic extrapolation is used. Default is True.

`classmethod PPoly.construct_fast` (*c, x, extrapolate=None, axis=0*)

Construct the piecewise polynomial without making checks.

Takes the same parameters as the constructor. Input arguments `c` and `x` must be arrays of the correct shape and type. The `c` array can only be of dtypes float and complex, and `x` array must have dtype float.

**class** `scipy.interpolate.BPoly` (`c, x, extrapolate=None, axis=0`)

Piecewise polynomial in terms of coefficients and breakpoints.

The polynomial in the  $i$ -th interval  $x[i] \leq x < x[i+1]$  is written in the Bernstein polynomial basis:

$$S = \sum(c[a, i] * b(a, k; x) \text{ for } a \text{ in range}(k+1)),$$

where  $k$  is the degree of the polynomial, and:

$$b(a, k; x) = \text{binom}(k, a) * t^{**k} * (1 - t)^{**}(k - a),$$

with  $t = (x - x[i]) / (x[i+1] - x[i])$  and `binom` is a binomial coefficient.

**Parameters** `c` : ndarray, shape (k, m, ...)

Polynomial coefficients, order  $k$  and  $m$  intervals

`x` : ndarray, shape (m+1,)

Polynomial breakpoints. These must be sorted in increasing order.

**extrapolate** : bool, optional

If bool, determines whether to extrapolate to out-of-bounds points based on first and last intervals, or to return NaNs. If 'periodic', periodic extrapolation is used. Default is True.

**axis** : int, optional

Interpolation axis. Default is zero.

**See also:**

**PPoly** piecewise polynomials in the power basis

### Notes

Properties of Bernstein polynomials are well documented in the literature. Here's a non-exhaustive list:

### Examples

```
>>> from scipy.interpolate import BPoly
>>> x = [0, 1]
>>> c = [[1], [2], [3]]
>>> bp = BPoly(c, x)
```

This creates a 2nd order polynomial

$$\begin{aligned} B(x) &= 1 \times b_{0,2}(x) + 2 \times b_{1,2}(x) + 3 \times b_{2,2}(x) \\ &= 1 \times (1 - x)^2 + 2 \times 2x(1 - x) + 3 \times x^2 \end{aligned}$$

### Attributes

<code>x</code>	(ndarray) Breakpoints.
<code>c</code>	(ndarray) Coefficients of the polynomials. They are reshaped to a 3-dimensional array with the last dimension representing the trailing dimensions of the original coefficient array.
<code>axis</code>	(int) Interpolation axis.

### Methods

`__call__(x[, nu, extrapolate])`

Evaluate the piecewise polynomial or its derivative.

`extend(c, x[, right])`

Add additional breakpoints and coefficients to the polynomial.

Table 5.32 – continued from previous page

<code>derivative([nu])</code>	Construct a new piecewise polynomial representing the derivative.
<code>antiderivative([nu])</code>	Construct a new piecewise polynomial representing the antiderivative.
<code>integrate(a, b[, extrapolate])</code>	Compute a definite integral over a piecewise polynomial.
<code>construct_fast(c, x[, extrapolate, axis])</code>	Construct the piecewise polynomial without making checks.
<code>from_power_basis(pp[, extrapolate])</code>	Construct a piecewise polynomial in Bernstein basis from a power basis polynomial.
<code>from_derivatives(xi, yi[, orders, extrapolate])</code>	Construct a piecewise polynomial in the Bernstein basis, compatible with the s

`BPoly.__call__(x, nu=0, extrapolate=None)`

Evaluate the piecewise polynomial or its derivative.

**Parameters** `x` : array\_like  
 Points to evaluate the interpolant at.  
`nu` : int, optional  
 Order of derivative to evaluate. Must be non-negative.  
`extrapolate` : {bool, 'periodic', None}, optional  
 If bool, determines whether to extrapolate to out-of-bounds points based on first and last intervals, or to return NaNs. If 'periodic', periodic extrapolation is used. If None (default), use `self.extrapolate`.  
**Returns** `y` : array\_like  
 Interpolated values. Shape is determined by replacing the interpolation axis in the original array with the shape of `x`.

**Notes**

Derivatives are evaluated piecewise for each polynomial segment, even if the polynomial is not differentiable at the breakpoints. The polynomial intervals are considered half-open,  $[a, b)$ , except for the last interval which is closed  $[a, b]$ .

`BPoly.extend(c, x, right=True)`

Add additional breakpoints and coefficients to the polynomial.

**Parameters** `c` : ndarray, size (k, m, ...)  
 Additional coefficients for polynomials in intervals `self.x[-1] <= x < x_right[0]`, `x_right[0] <= x < x_right[1]`, ..., `x_right[m-2] <= x < x_right[m-1]`  
`x` : ndarray, size (m,)  
 Additional breakpoints. Must be sorted and either to the right or to the left of the current breakpoints.  
`right` : bool, optional  
 Whether the new intervals are to the right or to the left of the current intervals.

`BPoly.derivative(nu=1)`

Construct a new piecewise polynomial representing the derivative.

**Parameters** `nu` : int, optional  
 Order of derivative to evaluate. Default is 1, i.e. compute the first derivative. If negative, the antiderivative is returned.  
**Returns** `bp` : BPoly  
 Piecewise polynomial of order `k - nu` representing the derivative of this polynomial.

`BPoly.antiderivative(nu=1)`

Construct a new piecewise polynomial representing the antiderivative.

**Parameters** `nu` : int, optional

Order of antiderivative to evaluate. Default is 1, i.e. compute the first integral. If negative, the derivative is returned.

**Returns** **bp** : BPoly  
Piecewise polynomial of order  $k + nu$  representing the antiderivative of this polynomial.

### Notes

If antiderivative is computed and `self.extrapolate='periodic'`, it will be set to `False` for the returned instance. This is done because the antiderivative is no longer periodic and its correct evaluation outside of the initially given `x` interval is difficult.

`BPoly.integrate(a, b, extrapolate=None)`

Compute a definite integral over a piecewise polynomial.

**Parameters** **a** : float  
Lower integration bound  
**b** : float  
Upper integration bound  
**extrapolate** : {bool, 'periodic', None}, optional  
Whether to extrapolate to out-of-bounds points based on first and last intervals, or to return NaNs. If 'periodic', periodic extrapolation is used. If None (default), use `self.extrapolate`.

**Returns** array\_like  
Definite integral of the piecewise polynomial over [a, b]

**classmethod** `BPoly.construct_fast(c, x, extrapolate=None, axis=0)`

Construct the piecewise polynomial without making checks.

Takes the same parameters as the constructor. Input arguments `c` and `x` must be arrays of the correct shape and type. The `c` array can only be of dtypes float and complex, and `x` array must have dtype float.

**classmethod** `BPoly.from_power_basis(pp, extrapolate=None)`

Construct a piecewise polynomial in Bernstein basis from a power basis polynomial.

**Parameters** **pp** : PPoly  
A piecewise polynomial in the power basis  
**extrapolate** : bool or 'periodic', optional  
If bool, determines whether to extrapolate to out-of-bounds points based on first and last intervals, or to return NaNs. If 'periodic', periodic extrapolation is used. Default is True.

**classmethod** `BPoly.from_derivatives(xi, yi, orders=None, extrapolate=None)`

Construct a piecewise polynomial in the Bernstein basis, compatible with the specified values and derivatives at breakpoints.

**Parameters** **xi** : array\_like  
sorted 1D array of x-coordinates  
**yi** : array\_like or list of array\_likes  
`yi[i][j]` is the  $j$ -th derivative known at `xi[i]`  
**orders** : None or int or array\_like of ints. Default: None.  
Specifies the degree of local polynomials. If not None, some derivatives are ignored.  
**extrapolate** : bool or 'periodic', optional  
If bool, determines whether to extrapolate to out-of-bounds points based on first and last intervals, or to return NaNs. If 'periodic', periodic extrapolation is used. Default is True.

*Notes*

If  $k$  derivatives are specified at a breakpoint  $x$ , the constructed polynomial is exactly  $k$  times continuously differentiable at  $x$ , unless the `order` is provided explicitly. In the latter case, the smoothness of the polynomial at the breakpoint is controlled by the `order`.

Deduces the number of derivatives to match at each end from `order` and the number of derivatives available. If possible it uses the same number of derivatives from each end; if the number is odd it tries to take the extra one from `y2`. In any case if not enough derivatives are available at one end or another it draws enough to make up the total from the other end.

If the order is too high and not enough derivatives are available, an exception is raised.

*Examples*

```
>>> from scipy.interpolate import BPoly
>>> BPoly.from_derivatives([0, 1], [[1, 2], [3, 4]])
```

Creates a polynomial  $f(x)$  of degree 3, defined on  $[0, 1]$  such that  $f(0) = 1$ ,  $df/dx(0) = 2$ ,  $f(1) = 3$ ,  $df/dx(1) = 4$

```
>>> BPoly.from_derivatives([0, 1, 2], [[0, 1], [0], [2]])
```

Creates a piecewise polynomial  $f(x)$ , such that  $f(0) = f(1) = 0$ ,  $f(2) = 2$ , and  $df/dx(0) = 1$ . Based on the number of derivatives provided, the order of the local polynomials is 2 on  $[0, 1]$  and 1 on  $[1, 2]$ . Notice that no restriction is imposed on the derivatives at  $x = 1$  and  $x = 2$ .

Indeed, the explicit form of the polynomial is:

$$f(x) = \begin{cases} x * (1 - x), & 0 \leq x < 1 \\ 2 * (x - 1), & 1 \leq x \leq 2 \end{cases}$$

So that  $f'(1-0) = -1$  and  $f'(1+0) = 2$

## 5.7.2 Multivariate interpolation

Unstructured data:

<code>griddata(points, values, xi[, method, ...])</code>	Interpolate unstructured D-dimensional data.
<code>LinearNDInterpolator(points, values[, ...])</code>	Piecewise linear interpolant in N dimensions.
<code>NearestNDInterpolator(points, values)</code>	Nearest-neighbour interpolation in N dimensions.
<code>CloughTocher2DInterpolator(points, values[, tol])</code>	Piecewise cubic, C1 smooth, curvature-minimizing interpolant in 2D.
<code>Rbf(*args)</code>	A class for radial basis function approximation/interpolation of n-dim
<code>interp2d(x, y, z[, kind, copy, ...])</code>	Interpolate over a 2-D grid.

`scipy.interpolate.griddata` (*points, values, xi, method='linear', fill\_value=nan, rescale=False*)  
Interpolate unstructured D-dimensional data.

- Parameters**
- points** : ndarray of floats, shape (n, D)  
Data point coordinates. Can either be an array of shape (n, D), or a tuple of *ndim* arrays.
  - values** : ndarray of float or complex, shape (n,)  
Data values.
  - xi** : ndarray of float, shape (M, D)  
Points at which to interpolate data.
  - method** : {'linear', 'nearest', 'cubic'}, optional  
Method of interpolation. One of



**nearest** return the value at the data point closest to the point of interpolation. See [NearestNDInterpolator](#) for more details.

**linear** tessellate the input point set to n-dimensional simplices, and interpolate linearly on each simplex. See [LinearNDInterpolator](#) for more details.

**cubic (1-D)** return the value determined from a cubic spline.

**cubic (2-D)** return the value determined from a piecewise cubic, continuously differentiable (C1), and approximately curvature-minimizing polynomial surface. See [CloughTocher2DInterpolator](#) for more details.

**fill\_value** : float, optional  
Value used to fill in for requested points outside of the convex hull of the input points. If not provided, then the default is nan. This option has no effect for the 'nearest' method.

**rescale** : bool, optional  
Rescale points to unit cube before performing interpolation. This is useful if some of the input dimensions have incommensurable units and differ by many orders of magnitude.  
New in version 0.14.0.

### Notes

New in version 0.9.

### Examples

Suppose we want to interpolate the 2-D function

```
>>> def func(x, y):
...     return x*(1-x)*np.cos(4*np.pi*x) * np.sin(4*np.pi*y**2)**2
```

on a grid in [0, 1]x[0, 1]

```
>>> grid_x, grid_y = np.mgrid[0:1:100j, 0:1:200j]
```

but we only know its values at 1000 data points:

```
>>> points = np.random.rand(1000, 2)
>>> values = func(points[:,0], points[:,1])
```

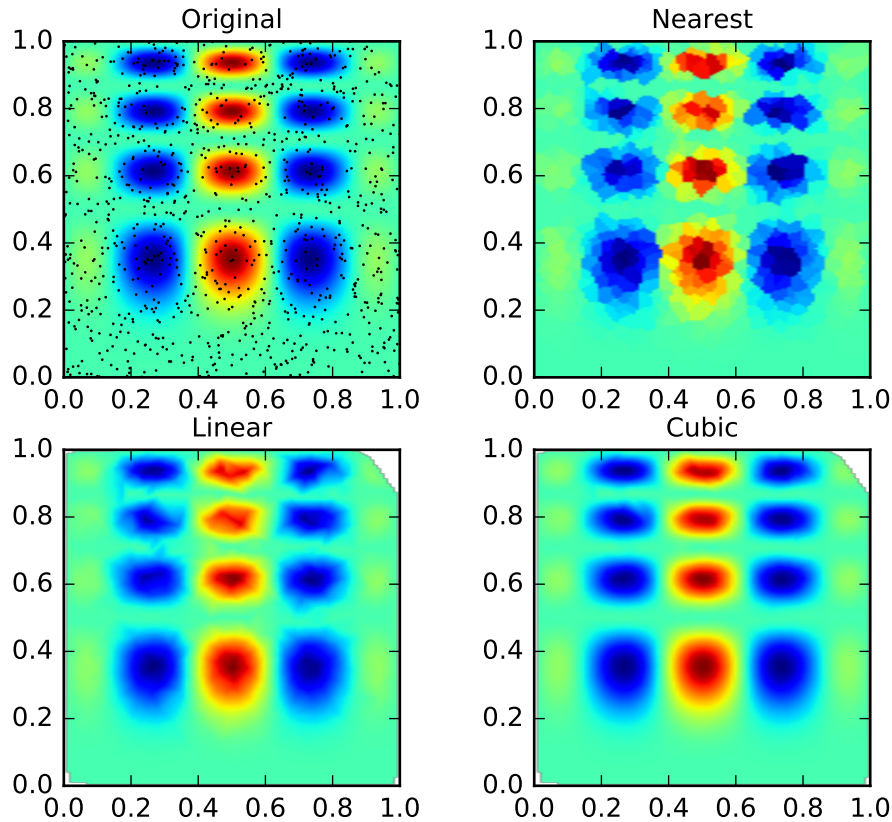
This can be done with `griddata` – below we try out all of the interpolation methods:

```
>>> from scipy.interpolate import griddata
>>> grid_z0 = griddata(points, values, (grid_x, grid_y), method='nearest')
>>> grid_z1 = griddata(points, values, (grid_x, grid_y), method='linear')
>>> grid_z2 = griddata(points, values, (grid_x, grid_y), method='cubic')
```

One can see that the exact result is reproduced by all of the methods to some degree, but for this smooth function the piecewise cubic interpolant gives the best results:

```
>>> import matplotlib.pyplot as plt
>>> plt.subplot(221)
>>> plt.imshow(func(grid_x, grid_y).T, extent=(0,1,0,1), origin='lower')
>>> plt.plot(points[:,0], points[:,1], 'k.', ms=1)
>>> plt.title('Original')
>>> plt.subplot(222)
>>> plt.imshow(grid_z0.T, extent=(0,1,0,1), origin='lower')
>>> plt.title('Nearest')
>>> plt.subplot(223)
>>> plt.imshow(grid_z1.T, extent=(0,1,0,1), origin='lower')
```

```
>>> plt.title('Linear')
>>> plt.subplot(224)
>>> plt.imshow(grid_z2.T, extent=(0,1,0,1), origin='lower')
>>> plt.title('Cubic')
>>> plt.gcf().set_size_inches(6, 6)
>>> plt.show()
```



**class** `scipy.interpolate.LinearNDInterpolator` (*points*, *values*, *fill\_value=np.nan*, *rescale=False*)

Piecewise linear interpolant in N dimensions.

New in version 0.9.

**Parameters**

- points** : ndarray of floats, shape (npoints, ndims); or Delaunay  
Data point coordinates, or a precomputed Delaunay triangulation.
- values** : ndarray of float or complex, shape (npoints, ...)  
Data values.
- fill\_value** : float, optional

Value used to fill in for requested points outside of the convex hull of the input points. If not provided, then the default is `nan`.

**rescale** : bool, optional

Rescale points to unit cube before performing interpolation. This is useful if some of the input dimensions have incommensurable units and differ by many orders of magnitude.

### Notes

The interpolant is constructed by triangulating the input data with Qhull [R56], and on each triangle performing linear barycentric interpolation.

### References

[R56]

### Methods

---

`__call__(xi)` Evaluate interpolator at given points.

---

`LinearNDInterpolator.__call__(xi)`  
Evaluate interpolator at given points.

**Parameters** **xi** : ndarray of float, shape (... , ndim)  
Points where to interpolate data at.

**class** `scipy.interpolate.NearestNDInterpolator(points, values)`  
Nearest-neighbour interpolation in N dimensions.

New in version 0.9.

**Parameters** **x** : (Npoints, Ndims) ndarray of floats  
Data point coordinates.  
**y** : (Npoints,) ndarray of float or complex  
Data values.  
**rescale** : boolean, optional  
Rescale points to unit cube before performing interpolation. This is useful if some of the input dimensions have incommensurable units and differ by many orders of magnitude.  
New in version 0.14.0.  
**tree\_options** : dict, optional  
Options passed to the underlying `cKDTree`.  
New in version 0.17.0.

### Notes

Uses `scipy.spatial.cKDTree`

### Methods

---

`__call__(*args)` Evaluate interpolator at given points.

---

`NearestNDInterpolator.__call__(*args)`  
Evaluate interpolator at given points.

**Parameters** **xi** : ndarray of float, shape (... , ndim)  
Points where to interpolate data at.

**class** `scipy.interpolate.CloughTocher2DInterpolator` (*points, values, tol=1e-6*)  
 Piecewise cubic, C1 smooth, curvature-minimizing interpolant in 2D.

New in version 0.9.

**Parameters**

- points** : ndarray of floats, shape (npoints, ndims); or Delaunay  
 Data point coordinates, or a precomputed Delaunay triangulation.
- values** : ndarray of float or complex, shape (npoints, ...)  
 Data values.
- fill\_value** : float, optional  
 Value used to fill in for requested points outside of the convex hull of the input points.  
 If not provided, then the default is `nan`.
- tol** : float, optional  
 Absolute/relative tolerance for gradient estimation.
- maxiter** : int, optional  
 Maximum number of iterations in gradient estimation.
- rescale** : bool, optional  
 Rescale points to unit cube before performing interpolation. This is useful if some  
 of the input dimensions have incommensurable units and differ by many orders of  
 magnitude.

**Notes**

The interpolant is constructed by triangulating the input data with Qhull [R52], and constructing a piecewise cubic interpolating Bezier polynomial on each triangle, using a Clough-Tocher scheme [CT]. The interpolant is guaranteed to be continuously differentiable.

The gradients of the interpolant are chosen so that the curvature of the interpolating surface is approximately minimized. The gradients necessary for this are estimated using the global algorithm described in [Nielson83, Renka84].

**References**

[R52], [CT], [Nielson83], [Renka84]

**Methods**

---

`__call__(xi)` Evaluate interpolator at given points.

---

`CloughTocher2DInterpolator.__call__(xi)`  
 Evaluate interpolator at given points.

**Parameters** **xi** : ndarray of float, shape (... , ndim)  
 Points where to interpolate data at.

**class** `scipy.interpolate.Rbf` (*\*args*)  
 A class for radial basis function approximation/interpolation of n-dimensional scattered data.

**Parameters** **\*args** : arrays  
 x, y, z, ..., d, where x, y, z, ... are the coordinates of the nodes and d is the array of values at the nodes

**function** : str or callable, optional  
 The radial basis function, based on the radius, r, given by the norm (default is Euclidean distance); the default is 'multiquadric':

```
'multiquadric': sqrt((r/self.epsilon)**2 + 1)
'inverse': 1.0/sqrt((r/self.epsilon)**2 + 1)
'gaussian': exp(-(r/self.epsilon)**2)
```

```
'linear': r
'cubic': r**3
'quintic': r**5
'thin_plate': r**2 * log(r)
```

If callable, then it must take 2 arguments (self, r). The epsilon parameter will be available as self.epsilon. Other keyword arguments passed in will be available as well.

**epsilon** : float, optional

Adjustable constant for gaussian or multiquadrics functions - defaults to approximate average distance between nodes (which is a good start).

**smooth** : float, optional

Values greater than zero increase the smoothness of the approximation. 0 is for interpolation (default), the function will always go through the nodal points in this case.

**norm** : callable, optional

A function that returns the ‘distance’ between two points, with inputs as arrays of positions (x, y, z, ...), and an output as an array of distance. E.g, the default:

```
def euclidean_norm(x1, x2):
    return sqrt( ((x1 - x2)**2).sum(axis=0) )
```

which is called with  $x1=x1[\text{ndims},\text{newaxis},:]$  and  $x2=x2[\text{ndims},:, \text{newaxis}]$  such that the result is a matrix of the distances from each point in  $x1$  to each point in  $x2$ .

### Examples

```
>>> from scipy.interpolate import Rbf
>>> x, y, z, d = np.random.rand(4, 50)
>>> rbfi = Rbf(x, y, z, d) # radial basis function interpolator instance
>>> xi = yi = zi = np.linspace(0, 1, 20)
>>> di = rbfi(xi, yi, zi) # interpolated values
>>> di.shape
(20,)
```

### Methods

---

```
__call__(*args)
```

---

Rbf.\_\_call\_\_(\*args)

```
class scipy.interpolate.interp2d(x, y, z, kind='linear', copy=True, bounds_error=False,
                                fill_value=nan)
```

Interpolate over a 2-D grid.

$x$ ,  $y$  and  $z$  are arrays of values used to approximate some function  $f$ :  $z = f(x, y)$ . This class returns a function whose call method uses spline interpolation to find the value of new points.

If  $x$  and  $y$  represent a regular grid, consider using RectBivariateSpline.

**Parameters**  $x, y$  : array\_like

Arrays defining the data point coordinates.

If the points lie on a regular grid,  $x$  can specify the column coordinates and  $y$  the row coordinates, for example:

```
>>> x = [0,1,2]; y = [0,3]; z = [[1,2,3], [4,5,6]]
```

Otherwise,  $x$  and  $y$  must specify the full coordinates for each point, for example:

```
>>> x = [0,1,2,0,1,2]; y = [0,0,0,3,3,3]; z = [1,2,3,4,5,6]
```

If *x* and *y* are multi-dimensional, they are flattened before use.

**z** : array\_like

The values of the function to interpolate at the data points. If *z* is a multi-dimensional array, it is flattened before use. The length of a flattened *z* array is either `len(x)*len(y)` if *x* and *y* specify the column and row coordinates or `len(z) == len(x) == len(y)` if *x* and *y* specify coordinates for each point.

**kind** : {'linear', 'cubic', 'quintic'}, optional

The kind of spline interpolation to use. Default is 'linear'.

**copy** : bool, optional

If True, the class makes internal copies of *x*, *y* and *z*. If False, references may be used. The default is to copy.

**bounds\_error** : bool, optional

If True, when interpolated values are requested outside of the domain of the input data (*x*,*y*), a `ValueError` is raised. If False, then *fill\_value* is used.

**fill\_value** : number, optional

If provided, the value to use for points outside of the interpolation domain. If omitted (None), values outside the domain are extrapolated.

See also:

[\*RectBivariateSpline\*](#)

Much faster 2D interpolation if your input data is on a grid

`bisplrep`, `bisplev`

[\*BivariateSpline\*](#)

a more recent wrapper of the FITPACK routines

`interp1d` one dimension version of this function

*Notes*

The minimum number of data points required along the interpolation axis is  $(k+1) ** 2$ , with  $k=1$  for linear,  $k=3$  for cubic and  $k=5$  for quintic interpolation.

The interpolator is constructed by `bisplrep`, with a smoothing factor of 0. If more control over smoothing is needed, `bisplrep` should be used directly.

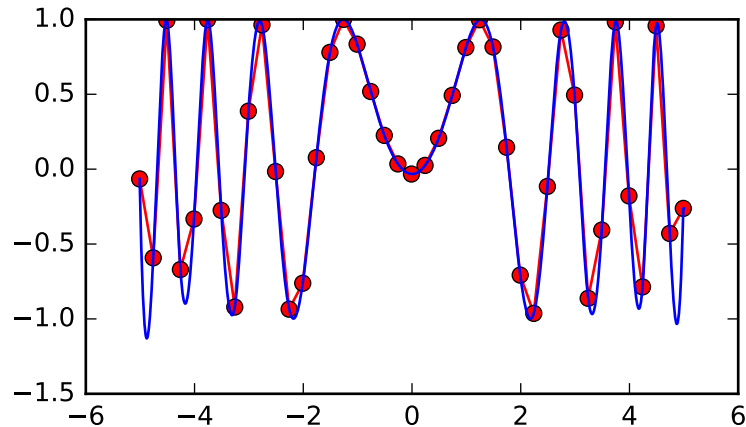
*Examples*

Construct a 2-D grid and interpolate on it:

```
>>> from scipy import interpolate
>>> x = np.arange(-5.01, 5.01, 0.25)
>>> y = np.arange(-5.01, 5.01, 0.25)
>>> xx, yy = np.meshgrid(x, y)
>>> z = np.sin(xx**2+yy**2)
>>> f = interpolate.interp2d(x, y, z, kind='cubic')
```

Now use the obtained interpolation function and plot the result:

```
>>> import matplotlib.pyplot as plt
>>> xnew = np.arange(-5.01, 5.01, 1e-2)
>>> ynew = np.arange(-5.01, 5.01, 1e-2)
>>> znew = f(xnew, ynew)
>>> plt.plot(x, z[0, :], 'ro-', xnew, znew[0, :], 'b-')
>>> plt.show()
```



### Methods

---

`__call__(x, y[, dx, dy, assume_sorted])` Interpolate the function.

---

`interp2d.__call__(x, y, dx=0, dy=0, assume_sorted=False)`  
Interpolate the function.

**Parameters**

- x** : 1D array  
x-coordinates of the mesh on which to interpolate.
- y** : 1D array  
y-coordinates of the mesh on which to interpolate.
- dx** : int  $\geq 0$ ,  $< k_x$   
Order of partial derivatives in x.
- dy** : int  $\geq 0$ ,  $< k_y$   
Order of partial derivatives in y.
- assume\_sorted** : bool, optional  
If False, values of *x* and *y* can be in any order and they are sorted first. If True, *x* and *y* have to be arrays of monotonically increasing values.

**Returns**

- z** : 2D array with shape (len(y), len(x))  
The interpolated values.

For data on a grid:

---

<code>interp</code> (points, values, xi[, method, ...])	Multidimensional interpolation on regular grids.
<code>RegularGridInterpolator</code> (points, values[, ...])	Interpolation on a regular grid in arbitrary dimensions
<code>RectBivariateSpline</code> (x, y, z[, bbox, kx, ky, s])	Bivariate spline approximation over a rectangular mesh.

---

`scipy.interpolate.interpn`(points, values, xi, method='linear', bounds\_error=True, fill\_value=nan)  
Multidimensional interpolation on regular grids.

**Parameters**

- points** : tuple of ndarray of float, with shapes (m1, ), ..., (mn, )  
The points defining the regular grid in *n* dimensions.
- values** : array\_like, shape (m1, ..., mn, ...)  
The data on the regular grid in *n* dimensions.

**xi** : ndarray of shape (... , ndim)  
 The coordinates to sample the gridded data at

**method** : str, optional  
 The method of interpolation to perform. Supported are “linear” and “nearest”, and “splinef2d”. “splinef2d” is only supported for 2-dimensional data.

**bounds\_error** : bool, optional  
 If True, when interpolated values are requested outside of the domain of the input data, a ValueError is raised. If False, then *fill\_value* is used.

**fill\_value** : number, optional  
 If provided, the value to use for points outside of the interpolation domain. If None, values outside the domain are extrapolated. Extrapolation is not supported by method “splinef2d”.

**Returns** **values\_x** : ndarray, shape xi.shape[:-1] + values.shape[ndim:]  
 Interpolated values at input coordinates.

See also:

*NearestNDInterpolator*

Nearest neighbour interpolation on unstructured data in N dimensions

*LinearNDInterpolator*

Piecewise linear interpolant on unstructured data in N dimensions

*RegularGridInterpolator*

Linear and nearest-neighbor Interpolation on a regular grid in arbitrary dimensions

*RectBivariateSpline*

Bivariate spline approximation over a rectangular mesh

*Notes*

New in version 0.14.

**class** `scipy.interpolate.RegularGridInterpolator` (*points*, *values*, *method='linear'*, *bounds\_error=True*, *fill\_value=nan*)

Interpolation on a regular grid in arbitrary dimensions

The data must be defined on a regular grid; the grid spacing however may be uneven. Linear and nearest-neighbour interpolation are supported. After setting up the interpolator object, the interpolation method (*linear* or *nearest*) may be chosen at each evaluation.

**Parameters** **points** : tuple of ndarray of float, with shapes (m1, ), ..., (mn, )  
 The points defining the regular grid in n dimensions.

**values** : array\_like, shape (m1, ..., mn, ...)  
 The data on the regular grid in n dimensions.

**method** : str, optional  
 The method of interpolation to perform. Supported are “linear” and “nearest”. This parameter will become the default for the object’s `__call__` method. Default is “linear”.

**bounds\_error** : bool, optional  
 If True, when interpolated values are requested outside of the domain of the input data, a ValueError is raised. If False, then *fill\_value* is used.

**fill\_value** : number, optional  
 If provided, the value to use for points outside of the interpolation domain. If None, values outside the domain are extrapolated.

See also:



**NearestNDInterpolator**

Nearest neighbour interpolation on unstructured data in N dimensions

**LinearNDInterpolator**

Piecewise linear interpolant on unstructured data in N dimensions

**Notes**

Contrary to `LinearNDInterpolator` and `NearestNDInterpolator`, this class avoids expensive triangulation of the input data by taking advantage of the regular grid structure.

New in version 0.14.

**References**

[R59], [R60], [R61]

**Examples**

Evaluate a simple example function on the points of a 3D grid:

```
>>> from scipy.interpolate import RegularGridInterpolator
>>> def f(x,y,z):
...     return 2 * x**3 + 3 * y**2 - z
>>> x = np.linspace(1, 4, 11)
>>> y = np.linspace(4, 7, 22)
>>> z = np.linspace(7, 9, 33)
>>> data = f(*np.meshgrid(x, y, z, indexing='ij', sparse=True))
```

data is now a 3D array with `data[i,j,k] = f(x[i], y[j], z[k])`. Next, define an interpolating function from this data:

```
>>> my_interpolating_function = RegularGridInterpolator((x, y, z), data)
```

Evaluate the interpolating function at the two points  $(x,y,z) = (2.1, 6.2, 8.3)$  and  $(3.3, 5.2, 7.1)$ :

```
>>> pts = np.array([[2.1, 6.2, 8.3], [3.3, 5.2, 7.1]])
>>> my_interpolating_function(pts)
array([ 125.80469388,  146.30069388])
```

which is indeed a close approximation to  $[f(2.1, 6.2, 8.3), f(3.3, 5.2, 7.1)]$ .

**Methods**


---

`__call__(xi[, method])` Interpolation at coordinates

---

`RegularGridInterpolator.__call__(xi, method=None)`

Interpolation at coordinates

**Parameters** **xi** : ndarray of shape (..., ndim)  
The coordinates to sample the gridded data at  
**method** : str  
The method of interpolation to perform. Supported are “linear” and “nearest”.

**class** `scipy.interpolate.RectBivariateSpline` (*x, y, z, bbox=[None, None, None, None], kx=3, ky=3, s=0*)

Bivariate spline approximation over a rectangular mesh.

Can be used for both smoothing and interpolating data.

**Parameters**

- x,y** : array\_like  
1-D arrays of coordinates in strictly ascending order.
- z** : array\_like  
2-D array of data with shape (x.size,y.size).
- bbox** : array\_like, optional  
Sequence of length 4 specifying the boundary of the rectangular approximation domain. By default, `bbox=[min(x,tx),max(x,tx),min(y,ty),max(y,ty)]`.
- kx, ky** : ints, optional  
Degrees of the bivariate spline. Default is 3.
- s** : float, optional  
Positive smoothing factor defined for estimation condition: `sum((w[i]*(z[i]-s(x[i],y[i])))**2,axis=0) <= s` Default is `s=0`, which is for interpolation.

See also:

**SmoothBivariateSpline**

a smoothing bivariate spline for scattered data

**bisplrep** an older wrapping of FITPACK

**bisplev** an older wrapping of FITPACK

**UnivariateSpline**

a similar class for univariate spline interpolation

**Methods**

---

<code>__call__(x, y[, mth, dx, dy, grid])</code>	Evaluate the spline or its derivatives at given positions.
<code>ev(xi, yi[, dx, dy])</code>	Evaluate the spline at points
<code>get_coeffs()</code>	Return spline coefficients.
<code>get_knots()</code>	Return a tuple (tx,ty) where tx,ty contain knots positions of the spline with respect to x-, y-var
<code>get_residual()</code>	Return weighted sum of squared residuals of the spline
<code>integral(xa, xb, ya, yb)</code>	Evaluate the integral of the spline over area [xa,xb] x [ya,yb].

---

`RectBivariateSpline.__call__(x, y, mth=None, dx=0, dy=0, grid=True)`

Evaluate the spline or its derivatives at given positions.

**Parameters**

- x, y** : array\_like  
Input coordinates.  
If *grid* is False, evaluate the spline at points `(x[i], y[i])`, `i=0, ..., len(x)-1`. Standard Numpy broadcasting is obeyed.  
If *grid* is True: evaluate spline at the grid points defined by the coordinate arrays `x, y`. The arrays must be sorted to increasing order.
- dx** : int  
Order of x-derivative  
New in version 0.14.0.
- dy** : int  
Order of y-derivative  
New in version 0.14.0.
- grid** : bool  
Whether to evaluate the results on a grid spanned by the input arrays, or at points specified by the input arrays.  
New in version 0.14.0.
- mth** : str

Deprecated argument. Has no effect.

`RectBivariateSpline.ev(xi, yi, dx=0, dy=0)`

Evaluate the spline at points

Returns the interpolated value at  $(xi[i], yi[i])$ ,  $i=0, \dots, len(xi)-1$ .

**Parameters**

- xi, yi** : array\_like  
Input coordinates. Standard Numpy broadcasting is obeyed.
- dx** : int, optional  
Order of x-derivative  
New in version 0.14.0.
- dy** : int, optional  
Order of y-derivative  
New in version 0.14.0.

`RectBivariateSpline.get_coeffs()`

Return spline coefficients.

`RectBivariateSpline.get_knots()`

Return a tuple (tx,ty) where tx,ty contain knots positions of the spline with respect to x-, y-variable, respectively. The position of interior and additional knots are given as  $t[k+1:-k-1]$  and  $t[:k+1]=b$ ,  $t[-k-1:]=e$ , respectively.

`RectBivariateSpline.get_residual()`

Return weighted sum of squared residuals of the spline approximation:  $\sum ((w[i]*(z[i]-s(x[i],y[i])))**2,axis=0)$

`RectBivariateSpline.integral(xa, xb, ya, yb)`

Evaluate the integral of the spline over area  $[xa,xb] \times [ya,yb]$ .

**Parameters**

- xa, xb** : float  
The end-points of the x integration interval.
- ya, yb** : float  
The end-points of the y integration interval.

**Returns**

- integ** : float  
The value of the resulting integral.

**See also:**

`scipy.ndimage.interpolation.map_coordinates`

Tensor product polynomials:

`NdPPoly`

### 5.7.3 1-D Splines

---

<code>UnivariateSpline(x, y[, w, bbox, k, s, ext, ...])</code>	One-dimensional smoothing spline fit to a given set of data points.
<code>InterpolatedUnivariateSpline(x, y[, w, ...])</code>	One-dimensional interpolating spline for a given set of data points.
<code>LSQUnivariateSpline(x, y, t[, w, bbox, k, ...])</code>	One-dimensional spline with explicit internal knots.

---

**class** `scipy.interpolate.UnivariateSpline` (*x, y, w=None, bbox=[None, None], k=3, s=None, ext=0, check\_finite=False*)

One-dimensional smoothing spline fit to a given set of data points.

Fits a spline  $y = spl(x)$  of degree  $k$  to the provided  $x, y$  data.  $s$  specifies the number of knots by specifying a smoothing condition.

**Parameters**

- x** : (N,) array\_like  
1-D array of independent input data. Must be increasing.
- y** : (N,) array\_like  
1-D array of dependent input data, of the same length as *x*.
- w** : (N,) array\_like, optional  
Weights for spline fitting. Must be positive. If None (default), weights are all equal.
- bbox** : (2,) array\_like, optional  
2-sequence specifying the boundary of the approximation interval. If None (default), `bbox=[x[0], x[-1]]`.
- k** : int, optional  
Degree of the smoothing spline. Must be  $\leq 5$ . Default is `k=3`, a cubic spline.
- s** : float or None, optional  
Positive smoothing factor used to choose the number of knots. Number of knots will be increased until the smoothing condition is satisfied:  
  

```
sum((w[i] * (y[i]-spl(x[i])))**2, axis=0) <= s
```

If None (default), `s = len(w)` which should be a good value if  $1/w[i]$  is an estimate of the standard deviation of  $y[i]$ . If 0, spline will interpolate through all data points.
- ext** : int or str, optional  
Controls the extrapolation mode for elements not in the interval defined by the knot sequence.
  - if `ext=0` or 'extrapolate', return the extrapolated value.
  - if `ext=1` or 'zeros', return 0
  - if `ext=2` or 'raise', raise a `ValueError`
  - if `ext=3` or 'const', return the boundary value.

The default value is 0.
- check\_finite** : bool, optional  
Whether to check that the input arrays contain only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination or non-sensical results) if the inputs do contain infinities or NaNs. Default is `False`.

See also:

***InterpolatedUnivariateSpline***

Subclass with smoothing forced to 0

***LSQUnivariateSpline***

Subclass in which knots are user-selected instead of being set by smoothing condition

***splrep***

An older, non object-oriented wrapping of FITPACK

`splev`, `sproot`, `splint`, `spalde`

***BivariateSpline***

A similar class for two-dimensional spline interpolation

**Notes**

The number of data points must be larger than the spline degree *k*.

**NaN handling:** If the input arrays contain `nan` values, the result is not useful, since the underlying spline fitting routines cannot deal with `nan`. A workaround is to use zero weights for not-a-number data points:

```
>>> from scipy.interpolate import UnivariateSpline
>>> x, y = np.array([1, 2, 3, 4]), np.array([1, np.nan, 3, 4])
>>> w = np.isnan(y)
```

```
>>> y[w] = 0.
>>> spl = UnivariateSpline(x, y, w=~w)
```

Notice the need to replace a nan by a numerical value (precise value does not matter as long as the corresponding weight is zero.)

### Examples

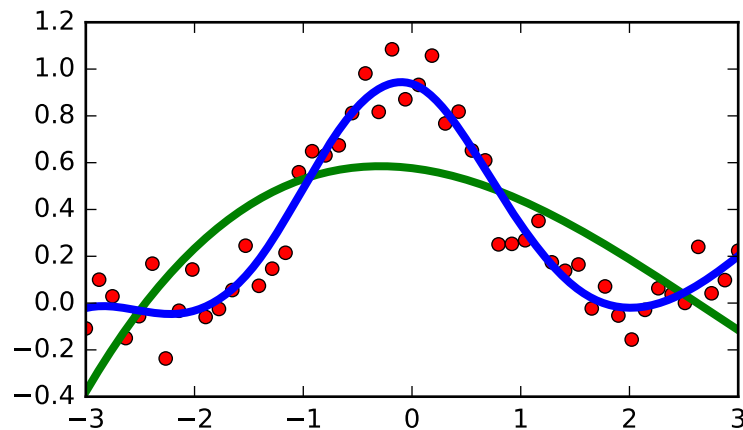
```
>>> import matplotlib.pyplot as plt
>>> from scipy.interpolate import UnivariateSpline
>>> x = np.linspace(-3, 3, 50)
>>> y = np.exp(-x**2) + 0.1 * np.random.randn(50)
>>> plt.plot(x, y, 'ro', ms=5)
```

Use the default value for the smoothing parameter:

```
>>> spl = UnivariateSpline(x, y)
>>> xs = np.linspace(-3, 3, 1000)
>>> plt.plot(xs, spl(xs), 'g', lw=3)
```

Manually change the amount of smoothing:

```
>>> spl.set_smoothing_factor(0.5)
>>> plt.plot(xs, spl(xs), 'b', lw=3)
>>> plt.show()
```



### Methods

<code>__call__(x[, nu, ext])</code>	Evaluate spline (or its nu-th derivative) at positions x.
<code>antiderivative([n])</code>	Construct a new spline representing the antiderivative of this spline.
<code>derivative([n])</code>	Construct a new spline representing the derivative of this spline.
<code>derivatives(x)</code>	Return all derivatives of the spline at the point x.
<code>get_coeffs()</code>	Return spline coefficients.
<code>get_knots()</code>	Return positions of interior knots of the spline.
<code>get_residual()</code>	Return weighted sum of squared residuals of the spline approximation.
<code>integral(a, b)</code>	Return definite integral of the spline between two given points.

Continued on next page

Table 5.43 – continued from previous page

<code>roots()</code>	Return the zeros of the spline.
<code>set_smoothing_factor(s)</code>	Continue spline computation with the given smoothing factor <code>s</code> and with the knots found at the last

`UnivariateSpline.__call__(x, nu=0, ext=None)`  
 Evaluate spline (or its `nu`-th derivative) at positions `x`.

**Parameters** `x` : array\_like  
 A 1-D array of points at which to return the value of the smoothed spline or its derivatives. Note: `x` can be unordered but the evaluation is more efficient if `x` is (partially) ordered.

`nu` : int  
 The order of derivative of the spline to compute.

`ext` : int  
 Controls the value returned for elements of `x` not in the interval defined by the knot sequence.

- if `ext=0` or ‘extrapolate’, return the extrapolated value.
- if `ext=1` or ‘zeros’, return 0
- if `ext=2` or ‘raise’, raise a `ValueError`
- if `ext=3` or ‘const’, return the boundary value.

The default value is 0, passed from the initialization of `UnivariateSpline`.

`UnivariateSpline.antiderivative(n=1)`  
 Construct a new spline representing the antiderivative of this spline.

**Parameters** `n` : int, optional  
 Order of antiderivative to evaluate. Default: 1

**Returns** `spline` : `UnivariateSpline`  
 Spline of order `k2=k+n` representing the antiderivative of this spline.

**See also:**

`splantider`, `derivative`

**Notes**

New in version 0.13.0.

**Examples**

```
>>> from scipy.interpolate import UnivariateSpline
>>> x = np.linspace(0, np.pi/2, 70)
>>> y = 1 / np.sqrt(1 - 0.8*np.sin(x)**2)
>>> spl = UnivariateSpline(x, y, s=0)
```

The derivative is the inverse operation of the antiderivative, although some floating point error accumulates:

```
>>> spl(1.7), spl.antiderivative().derivative()(1.7)
(array(2.1565429877197317), array(2.1565429877201865))
```

Antiderivative can be used to evaluate definite integrals:

```
>>> ispl = spl.antiderivative()
>>> ispl(np.pi/2) - ispl(0)
2.2572053588768486
```

This is indeed an approximation to the complete elliptic integral  $K(m) = \int_0^{\pi/2} [1 - m \sin^2 x]^{-1/2} dx$ :

```
>>> from scipy.special import ellipk
>>> ellipk(0.8)
2.2572053268208538
```

`UnivariateSpline.derivative` ( $n=1$ )

Construct a new spline representing the derivative of this spline.

**Parameters** **n** : int, optional  
Order of derivative to evaluate. Default: 1

**Returns** **spline** : `UnivariateSpline`  
Spline of order  $k_2=k-n$  representing the derivative of this spline.

**See also:**

`splder`, `antiderivative`

**Notes**

New in version 0.13.0.

**Examples**

This can be used for finding maxima of a curve:

```
>>> from scipy.interpolate import UnivariateSpline
>>> x = np.linspace(0, 10, 70)
>>> y = np.sin(x)
>>> spl = UnivariateSpline(x, y, k=4, s=0)
```

Now, differentiate the spline and find the zeros of the derivative. (NB: `sproot` only works for order 3 splines, so we fit an order 4 spline):

```
>>> spl.derivative().roots() / np.pi
array([ 0.50000001,  1.5          ,  2.49999998])
```

This agrees well with roots  $\pi/2 + n\pi$  of  $\cos(x) = \sin'(x)$ .

`UnivariateSpline.derivatives` ( $x$ )

Return all derivatives of the spline at the point  $x$ .

**Parameters** **x** : float  
The point to evaluate the derivatives at.

**Returns** **der** : `ndarray`, shape( $k+1$ ,)  
Derivatives of the orders 0 to  $k$ .

**Examples**

```
>>> from scipy.interpolate import UnivariateSpline
>>> x = np.linspace(0, 3, 11)
>>> y = x**2
>>> spl = UnivariateSpline(x, y)
>>> spl.derivatives(1.5)
array([2.25, 3.0, 2.0, 0])
```

`UnivariateSpline.get_coeffs` ()

Return spline coefficients.

`UnivariateSpline.get_knots` ()

Return positions of interior knots of the spline.

Internally, the knot vector contains  $2*k$  additional boundary knots.

`UnivariateSpline.get_residual()`  
 Return weighted sum of squared residuals of the spline approximation.

This is equivalent to:

```
sum((w[i] * (y[i]-spl(x[i])))**2, axis=0)
```

`UnivariateSpline.integral(a, b)`  
 Return definite integral of the spline between two given points.

**Parameters**

- a** : float  
Lower limit of integration.
- b** : float  
Upper limit of integration.

**Returns**

- integral** : float  
The value of the definite integral of the spline between limits.

**Examples**

```
>>> from scipy.interpolate import UnivariateSpline
>>> x = np.linspace(0, 3, 11)
>>> y = x**2
>>> spl = UnivariateSpline(x, y)
>>> spl.integral(0, 3)
9.0
```

which agrees with  $\int x^2 dx = x^3/3$  between the limits of 0 and 3.

A caveat is that this routine assumes the spline to be zero outside of the data limits:

```
>>> spl.integral(-1, 4)
9.0
>>> spl.integral(-1, 0)
0.0
```

`UnivariateSpline.roots()`  
 Return the zeros of the spline.

Restriction: only cubic splines are supported by fitpack.

`UnivariateSpline.set_smoothing_factor(s)`  
 Continue spline computation with the given smoothing factor *s* and with the knots found at the last call.

This routine modifies the spline in place.

**class** `scipy.interpolate.InterpolatedUnivariateSpline` (*x, y, w=None, bbox=[None, None], k=3, ext=0, check\_finite=False*)

One-dimensional interpolating spline for a given set of data points.

Fits a spline  $y = spl(x)$  of degree *k* to the provided *x, y* data. Spline function passes through all provided points. Equivalent to `UnivariateSpline` with *s=0*.

**Parameters**

- x** : (N,) array\_like  
Input dimension of data points – must be increasing
- y** : (N,) array\_like  
input dimension of data points
- w** : (N,) array\_like, optional  
Weights for spline fitting. Must be positive. If None (default), weights are all equal.
- bbox** : (2,) array\_like, optional  
2-sequence specifying the boundary of the approximation interval. If None (default), `bbox=[x[0], x[-1]]`.



**k** : int, optional  
Degree of the smoothing spline. Must be  $1 \leq k \leq 5$ .

**ext** : int or str, optional  
Controls the extrapolation mode for elements not in the interval defined by the knot sequence.

- if ext=0 or 'extrapolate', return the extrapolated value.
- if ext=1 or 'zeros', return 0
- if ext=2 or 'raise', raise a ValueError
- if ext=3 or 'const', return the boundary value.

The default value is 0.

**check\_finite** : bool, optional  
Whether to check that the input arrays contain only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination or non-sensical results) if the inputs do contain infinities or NaNs. Default is False.

**See also:**

#### *UnivariateSpline*

Superclass – allows knots to be selected by a smoothing condition

#### *LSQUnivariateSpline*

spline for which knots are user-selected

#### *splrep*

An older, non object-oriented wrapping of FITPACK

`splev`, `sproot`, `splint`, `spalde`

#### *BivariateSpline*

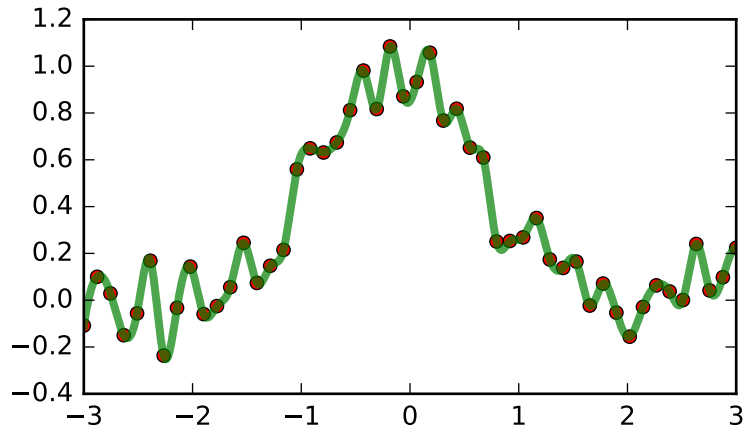
A similar class for two-dimensional spline interpolation

#### *Notes*

The number of data points must be larger than the spline degree  $k$ .

#### *Examples*

```
>>> import matplotlib.pyplot as plt
>>> from scipy.interpolate import InterpolatedUnivariateSpline
>>> x = np.linspace(-3, 3, 50)
>>> y = np.exp(-x**2) + 0.1 * np.random.randn(50)
>>> spl = InterpolatedUnivariateSpline(x, y)
>>> plt.plot(x, y, 'ro', ms=5)
>>> xs = np.linspace(-3, 3, 1000)
>>> plt.plot(xs, spl(xs), 'g', lw=3, alpha=0.7)
>>> plt.show()
```



Notice that the `spl(x)` interpolates `y`:

```
>>> spl.get_residual()
0.0
```

**Methods**

---

<code>__call__(x[, nu, ext])</code>	Evaluate spline (or its nu-th derivative) at positions <code>x</code> .
<code>antiderivative([n])</code>	Construct a new spline representing the antiderivative of this spline.
<code>derivative([n])</code>	Construct a new spline representing the derivative of this spline.
<code>derivatives(x)</code>	Return all derivatives of the spline at the point <code>x</code> .
<code>get_coeffs()</code>	Return spline coefficients.
<code>get_knots()</code>	Return positions of interior knots of the spline.
<code>get_residual()</code>	Return weighted sum of squared residuals of the spline approximation.
<code>integral(a, b)</code>	Return definite integral of the spline between two given points.
<code>roots()</code>	Return the zeros of the spline.
<code>set_smoothing_factor(s)</code>	Continue spline computation with the given smoothing factor <code>s</code> and with the knots found at the last

---

`InterpolatedUnivariateSpline.__call__(x, nu=0, ext=None)`

Evaluate spline (or its nu-th derivative) at positions `x`.

**Parameters** `x`: array\_like

A 1-D array of points at which to return the value of the smoothed spline or its derivatives. Note: `x` can be unordered but the evaluation is more efficient if `x` is (partially) ordered.

**nu**: int

The order of derivative of the spline to compute.

**ext**: int

Controls the value returned for elements of `x` not in the interval defined by the knot sequence.

- if `ext=0` or 'extrapolate', return the extrapolated value.
- if `ext=1` or 'zeros', return 0
- if `ext=2` or 'raise', raise a `ValueError`
- if `ext=3` or 'const', return the boundary value.

The default value is 0, passed from the initialization of `UnivariateSpline`.

`InterpolatedUnivariateSpline`.**antiderivative** ( $n=1$ )  
Construct a new spline representing the antiderivative of this spline.

**Parameters** **n** : int, optional  
Order of antiderivative to evaluate. Default: 1

**Returns** **spline** : `UnivariateSpline`  
Spline of order  $k_2=k+n$  representing the antiderivative of this spline.

**See also:**

`splantider`, `derivative`

**Notes**

New in version 0.13.0.

**Examples**

```
>>> from scipy.interpolate import UnivariateSpline
>>> x = np.linspace(0, np.pi/2, 70)
>>> y = 1 / np.sqrt(1 - 0.8*np.sin(x)**2)
>>> spl = UnivariateSpline(x, y, s=0)
```

The derivative is the inverse operation of the antiderivative, although some floating point error accumulates:

```
>>> spl(1.7), spl.antiderivative().derivative()(1.7)
(array(2.1565429877197317), array(2.1565429877201865))
```

Antiderivative can be used to evaluate definite integrals:

```
>>> ispl = spl.antiderivative()
>>> ispl(np.pi/2) - ispl(0)
2.2572053588768486
```

This is indeed an approximation to the complete elliptic integral  $K(m) = \int_0^{\pi/2} [1 - m \sin^2 x]^{-1/2} dx$ :

```
>>> from scipy.special import ellipk
>>> ellipk(0.8)
2.2572053268208538
```

`InterpolatedUnivariateSpline`.**derivative** ( $n=1$ )  
Construct a new spline representing the derivative of this spline.

**Parameters** **n** : int, optional  
Order of derivative to evaluate. Default: 1

**Returns** **spline** : `UnivariateSpline`  
Spline of order  $k_2=k-n$  representing the derivative of this spline.

**See also:**

`splder`, `antiderivative`

**Notes**

New in version 0.13.0.

**Examples**

This can be used for finding maxima of a curve:

```
>>> from scipy.interpolate import UnivariateSpline
>>> x = np.linspace(0, 10, 70)
>>> y = np.sin(x)
>>> spl = UnivariateSpline(x, y, k=4, s=0)
```

Now, differentiate the spline and find the zeros of the derivative. (NB: `sproot` only works for order 3 splines, so we fit an order 4 spline):

```
>>> spl.derivative().roots() / np.pi
array([ 0.50000001,  1.5          ,  2.49999998])
```

This agrees well with roots  $\pi/2 + n\pi$  of  $\cos(x) = \sin'(x)$ .

`InterpolatedUnivariateSpline.derivatives(x)`

Return all derivatives of the spline at the point x.

**Parameters** **x** : float  
The point to evaluate the derivatives at.

**Returns** **der** : ndarray, shape(k+1,)  
Derivatives of the orders 0 to k.

#### Examples

```
>>> from scipy.interpolate import UnivariateSpline
>>> x = np.linspace(0, 3, 11)
>>> y = x**2
>>> spl = UnivariateSpline(x, y)
>>> spl.derivatives(1.5)
array([2.25, 3.0, 2.0, 0])
```

`InterpolatedUnivariateSpline.get_coeffs()`

Return spline coefficients.

`InterpolatedUnivariateSpline.get_knots()`

Return positions of interior knots of the spline.

Internally, the knot vector contains  $2*k$  additional boundary knots.

`InterpolatedUnivariateSpline.get_residual()`

Return weighted sum of squared residuals of the spline approximation.

This is equivalent to:

```
sum((w[i] * (y[i]-spl(x[i])))**2, axis=0)
```

`InterpolatedUnivariateSpline.integral(a, b)`

Return definite integral of the spline between two given points.

**Parameters** **a** : float  
Lower limit of integration.

**b** : float  
Upper limit of integration.

**Returns** **integral** : float  
The value of the definite integral of the spline between limits.

#### Examples

```
>>> from scipy.interpolate import UnivariateSpline
>>> x = np.linspace(0, 3, 11)
>>> y = x**2
```

```
>>> spl = UnivariateSpline(x, y)
>>> spl.integral(0, 3)
9.0
```

which agrees with  $\int x^2 dx = x^3/3$  between the limits of 0 and 3.

A caveat is that this routine assumes the spline to be zero outside of the data limits:

```
>>> spl.integral(-1, 4)
9.0
>>> spl.integral(-1, 0)
0.0
```

`InterpolatedUnivariateSpline.roots()`  
Return the zeros of the spline.

Restriction: only cubic splines are supported by fitpack.

`InterpolatedUnivariateSpline.set_smoothing_factor(s)`  
Continue spline computation with the given smoothing factor *s* and with the knots found at the last call.

This routine modifies the spline in place.

**class** `scipy.interpolate.LSQUnivariateSpline`(*x, y, t, w=None, bbox=[None, None], k=3, ext=0, check\_finite=False*)

One-dimensional spline with explicit internal knots.

Fits a spline  $y = \text{spl}(x)$  of degree *k* to the provided *x, y* data. *t* specifies the internal knots of the spline

**Parameters**

- x** : (N,) array\_like  
Input dimension of data points – must be increasing
- y** : (N,) array\_like  
Input dimension of data points
- t** : (M,) array\_like  
interior knots of the spline. Must be in ascending order and:  
 $\text{bbox}[0] < t[0] < \dots < t[-1] < \text{bbox}[-1]$
- w** : (N,) array\_like, optional  
weights for spline fitting. Must be positive. If None (default), weights are all equal.
- bbox** : (2,) array\_like, optional  
2-sequence specifying the boundary of the approximation interval. If None (default),  $\text{bbox} = [x[0], x[-1]]$ .
- k** : int, optional  
Degree of the smoothing spline. Must be  $1 \leq k \leq 5$ . Default is  $k=3$ , a cubic spline.
- ext** : int or str, optional  
Controls the extrapolation mode for elements not in the interval defined by the knot sequence.
  - if `ext=0` or ‘`extrapolate`’, return the extrapolated value.
  - if `ext=1` or ‘`zeros`’, return 0
  - if `ext=2` or ‘`raise`’, raise a `ValueError`
  - if `ext=3` or ‘`const`’, return the boundary value.
 The default value is 0.
- check\_finite** : bool, optional  
Whether to check that the input arrays contain only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination or non-sensical results) if the inputs do contain infinities or NaNs. Default is `False`.

**Raises**

- ValueError**  
If the interior knots do not satisfy the Schoenberg-Whitney conditions

See also:

**UnivariateSpline**

Superclass – knots are specified by setting a smoothing condition

**InterpolatedUnivariateSpline**

spline passing through all points

**splrep** An older, non object-oriented wrapping of FITPACK

splev, sproot, splint, spalde

**BivariateSpline**

A similar class for two-dimensional spline interpolation

**Notes**

The number of data points must be larger than the spline degree  $k$ .

Knots  $t$  must satisfy the Schoenberg-Whitney conditions, i.e., there must be a subset of data points  $x[j]$  such that  $t[j] < x[j] < t[j+k+1]$ , for  $j=0, 1, \dots, n-k-2$ .

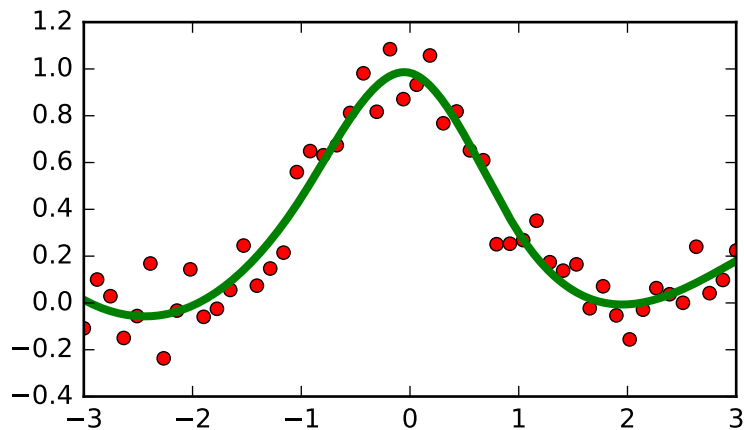
**Examples**

```
>>> from scipy.interpolate import LSQUnivariateSpline, UnivariateSpline
>>> import matplotlib.pyplot as plt
>>> x = np.linspace(-3, 3, 50)
>>> y = np.exp(-x**2) + 0.1 * np.random.randn(50)
```

Fit a smoothing spline with a pre-defined internal knots:

```
>>> t = [-1, 0, 1]
>>> spl = LSQUnivariateSpline(x, y, t)

>>> xs = np.linspace(-3, 3, 1000)
>>> plt.plot(x, y, 'ro', ms=5)
>>> plt.plot(xs, spl(xs), 'g-', lw=3)
>>> plt.show()
```



Check the knot vector:

```
>>> spl.get_knots()
array([-3., -1., 0., 1., 3.])
```

Constructing lsq spline using the knots from another spline:

```
>>> x = np.arange(10)
>>> s = UnivariateSpline(x, x, s=0)
>>> s.get_knots()
array([ 0.,  2.,  3.,  4.,  5.,  6.,  7.,  9.])
>>> knt = s.get_knots()
>>> s1 = LSQUnivariateSpline(x, x, knt[1:-1])    # Chop 1st and last knot
>>> s1.get_knots()
array([ 0.,  2.,  3.,  4.,  5.,  6.,  7.,  9.])
```

### Methods

<code>__call__(x[, nu, ext])</code>	Evaluate spline (or its nu-th derivative) at positions x.
<code>antiderivative([n])</code>	Construct a new spline representing the antiderivative of this spline.
<code>derivative([n])</code>	Construct a new spline representing the derivative of this spline.
<code>derivatives(x)</code>	Return all derivatives of the spline at the point x.
<code>get_coeffs()</code>	Return spline coefficients.
<code>get_knots()</code>	Return positions of interior knots of the spline.
<code>get_residual()</code>	Return weighted sum of squared residuals of the spline approximation.
<code>integral(a, b)</code>	Return definite integral of the spline between two given points.
<code>roots()</code>	Return the zeros of the spline.
<code>set_smoothing_factor(s)</code>	Continue spline computation with the given smoothing factor s and with the knots found at the last

`LSQUnivariateSpline.__call__(x, nu=0, ext=None)`

Evaluate spline (or its nu-th derivative) at positions x.

**Parameters** `x`: array\_like

A 1-D array of points at which to return the value of the smoothed spline or its derivatives. Note: x can be unordered but the evaluation is more efficient if x is (partially) ordered.

**nu**: int

The order of derivative of the spline to compute.

**ext**: int

Controls the value returned for elements of x not in the interval defined by the knot sequence.

- if ext=0 or 'extrapolate', return the extrapolated value.

- if ext=1 or 'zeros', return 0

- if ext=2 or 'raise', raise a ValueError

- if ext=3 or 'const', return the boundary value.

The default value is 0, passed from the initialization of UnivariateSpline.

`LSQUnivariateSpline.antiderivative(n=1)`

Construct a new spline representing the antiderivative of this spline.

**Parameters** `n`: int, optional

Order of antiderivative to evaluate. Default: 1

**Returns** `spline`: UnivariateSpline

Spline of order  $k_2=k+n$  representing the antiderivative of this spline.

**See also:**

`splantider`, `derivative`

*Notes*

New in version 0.13.0.

*Examples*

```
>>> from scipy.interpolate import UnivariateSpline
>>> x = np.linspace(0, np.pi/2, 70)
>>> y = 1 / np.sqrt(1 - 0.8*np.sin(x)**2)
>>> spl = UnivariateSpline(x, y, s=0)
```

The derivative is the inverse operation of the antiderivative, although some floating point error accumulates:

```
>>> spl(1.7), spl.antiderivative().derivative()(1.7)
(array(2.1565429877197317), array(2.1565429877201865))
```

Antiderivative can be used to evaluate definite integrals:

```
>>> ispl = spl.antiderivative()
>>> ispl(np.pi/2) - ispl(0)
2.2572053588768486
```

This is indeed an approximation to the complete elliptic integral  $K(m) = \int_0^{\pi/2} [1 - m \sin^2 x]^{-1/2} dx$ :

```
>>> from scipy.special import ellipk
>>> ellipk(0.8)
2.2572053268208538
```

`LSQUnivariateSpline.derivative(n=1)`

Construct a new spline representing the derivative of this spline.

**Parameters** **n** : int, optional  
Order of derivative to evaluate. Default: 1

**Returns** **spline** : UnivariateSpline  
Spline of order  $k_2=k-n$  representing the derivative of this spline.

**See also:**

`splder`, `antiderivative`

*Notes*

New in version 0.13.0.

*Examples*

This can be used for finding maxima of a curve:

```
>>> from scipy.interpolate import UnivariateSpline
>>> x = np.linspace(0, 10, 70)
>>> y = np.sin(x)
>>> spl = UnivariateSpline(x, y, k=4, s=0)
```

Now, differentiate the spline and find the zeros of the derivative. (NB: `sproot` only works for order 3 splines, so we fit an order 4 spline):

```
>>> spl.derivative().roots() / np.pi
array([ 0.50000001,  1.5          ,  2.49999998])
```

This agrees well with roots  $\pi/2 + n\pi$  of  $\cos(x) = \sin'(x)$ .



LSQUnivariateSpline.**derivatives**(x)  
Return all derivatives of the spline at the point x.

**Parameters** **x**: float  
The point to evaluate the derivatives at.

**Returns** **der**: ndarray, shape(k+1,)  
Derivatives of the orders 0 to k.

#### Examples

```
>>> from scipy.interpolate import UnivariateSpline
>>> x = np.linspace(0, 3, 11)
>>> y = x**2
>>> spl = UnivariateSpline(x, y)
>>> spl.derivatives(1.5)
array([2.25, 3.0, 2.0, 0])
```

LSQUnivariateSpline.**get\_coeffs**()  
Return spline coefficients.

LSQUnivariateSpline.**get\_knots**()  
Return positions of interior knots of the spline.

Internally, the knot vector contains  $2 * k$  additional boundary knots.

LSQUnivariateSpline.**get\_residual**()  
Return weighted sum of squared residuals of the spline approximation.

This is equivalent to:

```
sum((w[i] * (y[i]-spl(x[i])))**2, axis=0)
```

LSQUnivariateSpline.**integral**(a, b)  
Return definite integral of the spline between two given points.

**Parameters** **a**: float  
Lower limit of integration.

**b**: float  
Upper limit of integration.

**Returns** **integral**: float  
The value of the definite integral of the spline between limits.

#### Examples

```
>>> from scipy.interpolate import UnivariateSpline
>>> x = np.linspace(0, 3, 11)
>>> y = x**2
>>> spl = UnivariateSpline(x, y)
>>> spl.integral(0, 3)
9.0
```

which agrees with  $\int x^2 dx = x^3/3$  between the limits of 0 and 3.

A caveat is that this routine assumes the spline to be zero outside of the data limits:

```
>>> spl.integral(-1, 4)
9.0
>>> spl.integral(-1, 0)
0.0
```

`LSQUnivariateSpline.roots()`

Return the zeros of the spline.

Restriction: only cubic splines are supported by fitpack.

`LSQUnivariateSpline.set_smoothing_factor(s)`

Continue spline computation with the given smoothing factor *s* and with the knots found at the last call.

This routine modifies the spline in place.

Functional interface to FITPACK functions:

---

<code>splrep(x, y[, w, xb, xe, k, task, s, t, ...])</code>	Find the B-spline representation of 1-D curve.
<code>splprep(x[, w, u, ub, ue, k, task, s, t, ...])</code>	Find the B-spline representation of an N-dimensional curve.
<code>splev(x, tck[, der, ext])</code>	Evaluate a B-spline or its derivatives.
<code>splint(a, b, tck[, full_output])</code>	Evaluate the definite integral of a B-spline.
<code>sproot(tck[, mest])</code>	Find the roots of a cubic B-spline.
<code>spalde(x, tck)</code>	Evaluate all derivatives of a B-spline.
<code>splder(tck[, n])</code>	Compute the spline representation of the derivative of a given spline
<code>splantider(tck[, n])</code>	Compute the spline for the antiderivative (integral) of a given spline.
<code>insert(x, tck[, m, per])</code>	Insert knots into a B-spline.

---

`scipy.interpolate.splrep(x, y, w=None, xb=None, xe=None, k=3, task=0, s=None, t=None, full_output=0, per=0, quiet=1)`

Find the B-spline representation of 1-D curve.

Given the set of data points  $(x[i], y[i])$  determine a smooth spline approximation of degree *k* on the interval  $xb \leq x \leq xe$ .

**Parameters** **x, y** : array\_like

The data points defining a curve  $y = f(x)$ .

**w** : array\_like, optional

Strictly positive rank-1 array of weights the same length as *x* and *y*. The weights are used in computing the weighted least-squares spline fit. If the errors in the *y* values have standard-deviation given by the vector *d*, then *w* should be  $1/d$ . Default is `ones(len(x))`.

**xb, xe** : float, optional

The interval to fit. If *None*, these default to `x[0]` and `x[-1]` respectively.

**k** : int, optional

The degree of the spline fit. It is recommended to use cubic splines. Even values of *k* should be avoided especially with small *s* values.  $1 \leq k \leq 5$

**task** : {1, 0, -1}, optional

If `task==0` find *t* and *c* for a given smoothing factor, *s*.

If `task==1` find *t* and *c* for another value of the smoothing factor, *s*. There must have been a previous call with `task=0` or `task=1` for the same set of data (*t* will be stored and used internally)

If `task=-1` find the weighted least square spline for a given set of knots, *t*. These should be interior knots as knots on the ends will be added automatically.

**s** : float, optional

A smoothing condition. The amount of smoothness is determined by satisfying the conditions:  $\text{sum}((w * (y - g))**2, \text{axis}=0) \leq s$  where *g*(*x*) is the smoothed interpolation of (*x*,*y*). The user can use *s* to control the tradeoff between closeness and smoothness of fit. Larger *s* means more smoothing while smaller values of *s* indicate less smoothing. Recommended values of *s* depend on the weights, *w*. If the weights represent the inverse of the standard-deviation of *y*, then a good *s* value should be found in the range  $(m - \sqrt{2*m}, m + \sqrt{2*m})$  where *m* is the number of datapoints in *x*, *y*,

and  $w$ . default :  $s=m\text{-sqrt}(2*m)$  if weights are supplied.  $s = 0.0$  (interpolating) if no weights are supplied.

**t** : array\_like, optional

The knots needed for  $\text{task}=-1$ . If given then  $\text{task}$  is automatically set to  $-1$ .

**full\_output** : bool, optional

If non-zero, then return optional outputs.

**per** : bool, optional

If non-zero, data points are considered periodic with period  $x[m-1] - x[0]$  and a smooth periodic spline approximation is returned. Values of  $y[m-1]$  and  $w[m-1]$  are not used.

**quiet** : bool, optional

Non-zero to suppress messages. This parameter is deprecated; use standard Python warning filters instead.

#### Returns

**tck** : tuple

( $t,c,k$ ) a tuple containing the vector of knots, the B-spline coefficients, and the degree of the spline.

**fp** : array, optional

The weighted sum of squared residuals of the spline approximation.

**ier** : int, optional

An integer flag about `splrep` success. Success is indicated if  $\text{ier} \leq 0$ . If  $\text{ier}$  in  $[1,2,3]$  an error occurred but was not raised. Otherwise an error is raised.

**msg** : str, optional

A message corresponding to the integer flag, `ier`.

#### See also:

`UnivariateSpline`, `BivariateSpline`, `splprep`, `splev`, `sproot`, `spalde`, `splint`, `bisplrep`, `bisplev`

#### Notes

See `splev` for evaluation of the spline and its derivatives. Uses the FORTRAN routine `curfit` from FITPACK.

If provided, knots  $t$  must satisfy the Schoenberg-Whitney conditions, i.e., there must be a subset of data points  $x[j]$  such that  $t[j] < x[j] < t[j+k+1]$ , for  $j=0, 1, \dots, n-k-2$ .

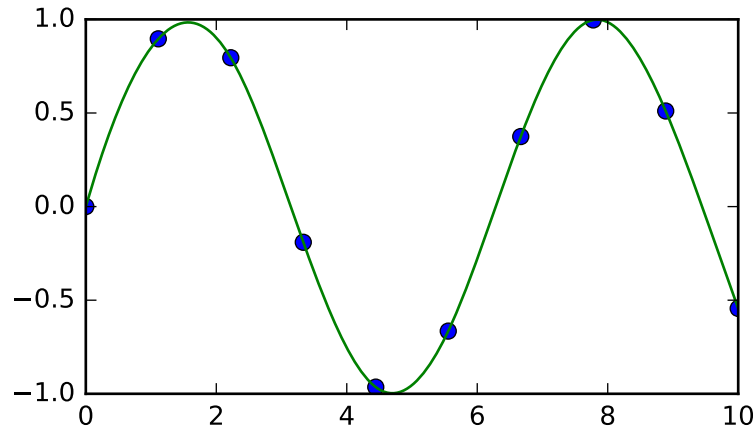
#### References

Based on algorithms described in [R81], [R82], [R83], and [R84]:

[R81], [R82], [R83], [R84]

#### Examples

```
>>> import matplotlib.pyplot as plt
>>> from scipy.interpolate import splev, splrep
>>> x = np.linspace(0, 10, 10)
>>> y = np.sin(x)
>>> tck = splrep(x, y)
>>> x2 = np.linspace(0, 10, 200)
>>> y2 = splev(x2, tck)
>>> plt.plot(x, y, 'o', x2, y2)
>>> plt.show()
```



`scipy.interpolate.splprep` ( $x$ ,  $w=None$ ,  $u=None$ ,  $ub=None$ ,  $ue=None$ ,  $k=3$ ,  $task=0$ ,  $s=None$ ,  $t=None$ ,  $full\_output=0$ ,  $nest=None$ ,  $per=0$ ,  $quiet=1$ )

Find the B-spline representation of an N-dimensional curve.

Given a list of N rank-1 arrays,  $x$ , which represent a curve in N-dimensional space parametrized by  $u$ , find a smooth approximating spline curve  $g(u)$ . Uses the FORTRAN routine `parcur` from FITPACK.

**Parameters**  $x$  : array\_like

A list of sample vector arrays representing the curve.

$w$  : array\_like, optional

Strictly positive rank-1 array of weights the same length as  $x[0]$ . The weights are used in computing the weighted least-squares spline fit. If the errors in the  $x$  values have standard-deviation given by the vector  $d$ , then  $w$  should be  $1/d$ . Default is `ones(len(x[0]))`.

$u$  : array\_like, optional

An array of parameter values. If not given, these values are calculated automatically as  $M = \text{len}(x[0])$ , where

$$v[0] = 0$$

$$v[i] = v[i-1] + \text{distance}(x[i], x[i-1])$$

$$u[i] = v[i] / v[M-1]$$

$ub, ue$  : int, optional

The end-points of the parameters interval. Defaults to  $u[0]$  and  $u[-1]$ .

$k$  : int, optional

Degree of the spline. Cubic splines are recommended. Even values of  $k$  should be avoided especially with a small  $s$ -value.  $1 \leq k \leq 5$ , default is 3.

$task$  : int, optional

If  $task==0$  (default), find  $t$  and  $c$  for a given smoothing factor,  $s$ . If  $task==1$ , find  $t$  and  $c$  for another value of the smoothing factor,  $s$ . There must have been a previous call with  $task=0$  or  $task=1$  for the same set of data. If  $task=-1$  find the weighted least square spline for a given set of knots,  $t$ .

$s$  : float, optional

A smoothing condition. The amount of smoothness is determined by satisfying the conditions:  $\text{sum}((w * (y - g)) ** 2, \text{axis}=0) \leq s$ , where  $g(x)$  is the smoothed interpolation of  $(x,y)$ . The user can use  $s$  to control the trade-off between closeness and smoothness of fit. Larger  $s$  means more smoothing while smaller values of  $s$  indicate less smoothing. Recommended values of  $s$  depend on the weights,

w. If the weights represent the inverse of the standard-deviation of y, then a good s value should be found in the range  $(m - \sqrt{2 * m}, m + \sqrt{2 * m})$ , where m is the number of data points in x, y, and w.

**t** : int, optional

The knots needed for task=-1.

**full\_output** : int, optional

If non-zero, then return optional outputs.

**nest** : int, optional

An over-estimate of the total number of knots of the spline to help in determining the storage space. By default nest=m/2. Always large enough is nest=m+k+1.

**per** : int, optional

If non-zero, data points are considered periodic with period  $x[m-1] - x[0]$  and a smooth periodic spline approximation is returned. Values of  $y[m-1]$  and  $w[m-1]$  are not used.

**quiet** : int, optional

Non-zero to suppress messages. This parameter is deprecated; use standard Python warning filters instead.

#### Returns

**tck** : tuple

A tuple (t,c,k) containing the vector of knots, the B-spline coefficients, and the degree of the spline.

**u** : array

An array of the values of the parameter.

**fp** : float

The weighted sum of squared residuals of the spline approximation.

**ier** : int

An integer flag about splprep success. Success is indicated if ier<=0. If ier in [1,2,3] an error occurred but was not raised. Otherwise an error is raised.

**msg** : str

A message corresponding to the integer flag, ier.

#### See also:

`splprep`, `splev`, `sproot`, `spalde`, `splint`, `bisplprep`, `bisplev`, `UnivariateSpline`, `BivariateSpline`

#### Notes

See `splev` for evaluation of the spline and its derivatives. The number of dimensions N must be smaller than 11.

#### References

[R78], [R79], [R80]

`scipy.interpolate.splev(x, tck, der=0, ext=0)`

Evaluate a B-spline or its derivatives.

Given the knots and coefficients of a B-spline representation, evaluate the value of the smoothing polynomial and its derivatives. This is a wrapper around the FORTRAN routines `splev` and `splder` of FITPACK.

**Parameters** **x** : array\_like

An array of points at which to return the value of the smoothed spline or its derivatives. If `tck` was returned from `splprep`, then the parameter values, u should be given.

**tck** : tuple

A sequence of length 3 returned by `splprep` or `splprep` containing the knots, coefficients, and degree of the spline.

**der** : int, optional

The order of derivative of the spline to compute (must be less than or equal to k).

**ext** : int, optional

Controls the value returned for elements of  $x$  not in the interval defined by the knot sequence.

- if `ext=0`, return the extrapolated value.
- if `ext=1`, return 0
- if `ext=2`, raise a `ValueError`
- if `ext=3`, return the boundary value.

The default value is 0.

**Returns** **y** : ndarray or list of ndarrays

An array of values representing the spline function evaluated at the points in  $x$ . If `tck` was returned from `splprep`, then this is a list of arrays representing the curve in N-dimensional space.

**See also:**

`splprep`, `splrep`, `sproot`, `spalde`, `splint`, `bisplrep`, `bisplev`

**References**

[R73], [R74], [R75]

`scipy.interpolate.splint` (*a, b, tck, full\_output=0*)

Evaluate the definite integral of a B-spline.

Given the knots and coefficients of a B-spline, evaluate the definite integral of the smoothing polynomial between two given points.

**Parameters** **a, b** : float

The end-points of the integration interval.

**tck** : tuple

A tuple (t,c,k) containing the vector of knots, the B-spline coefficients, and the degree of the spline (see `splev`).

**full\_output** : int, optional

Non-zero to return optional output.

**Returns** **integral** : float

The resulting integral.

**wrk** : ndarray

An array containing the integrals of the normalized B-splines defined on the set of knots.

**See also:**

`splprep`, `splrep`, `sproot`, `spalde`, `splev`, `bisplrep`, `bisplev`, `UnivariateSpline`, `BivariateSpline`

**Notes**

`splint` silently assumes that the spline function is zero outside the data interval (a, b).

**References**

[R76], [R77]

`scipy.interpolate.sproot` (*tck, mest=10*)

Find the roots of a cubic B-spline.

Given the knots ( $\geq 8$ ) and coefficients of a cubic B-spline return the roots of the spline.

**Parameters** **tck** : tuple

A tuple (t,c,k) containing the vector of knots, the B-spline coefficients, and the degree of the spline. The number of knots must be  $\geq 8$ , and the degree must be 3. The knots must be a monotonically increasing sequence.

**mest** : int, optional

An estimate of the number of zeros (Default is 10).

**Returns**

**zeros** : ndarray

An array giving the roots of the spline.

**See also:**

splprep, splrep, splint, spalde, splev, bisplrep, bisplev, UnivariateSpline, BivariateSpline

**References**

[R85], [R86], [R87]

scipy.interpolate.**spalde**(x, tck)

Evaluate all derivatives of a B-spline.

Given the knots and coefficients of a cubic B-spline compute all derivatives up to order k at a point (or set of points).

**Parameters** **x** : array\_like

A point or a set of points at which to evaluate the derivatives. Note that  $t(k) \leq x \leq t(n-k+1)$  must hold for each x.

**tck** : tuple

A tuple (t,c,k) containing the vector of knots, the B-spline coefficients, and the degree of the spline.

**Returns** **results** : {ndarray, list of ndarrays}

An array (or a list of arrays) containing all derivatives up to order k inclusive for each point x.

**See also:**

splprep, splrep, splint, sproot, splev, bisplrep, bisplev, UnivariateSpline, BivariateSpline

**References**

[R70], [R71], [R72]

scipy.interpolate.**splder**(tck, n=1)

Compute the spline representation of the derivative of a given spline

**Parameters** **tck** : tuple of (t, c, k)

Spline whose derivative to compute

**n** : int, optional

Order of derivative to evaluate. Default: 1

**Returns** **tck\_der** : tuple of (t2, c2, k2)

Spline of order  $k_2=k-n$  representing the derivative of the input spline.

**See also:**

splantider, splev, spalde

**Notes**

New in version 0.13.0.

### Examples

This can be used for finding maxima of a curve:

```
>>> from scipy.interpolate import splrep, splder, sproot
>>> x = np.linspace(0, 10, 70)
>>> y = np.sin(x)
>>> spl = splrep(x, y, k=4)
```

Now, differentiate the spline and find the zeros of the derivative. (NB: `sproot` only works for order 3 splines, so we fit an order 4 spline):

```
>>> dspl = splder(spl)
>>> sproot(dspl) / np.pi
array([ 0.50000001,  1.5          ,  2.49999998])
```

This agrees well with roots  $\pi/2 + n\pi$  of  $\cos(x) = \sin'(x)$ .

`scipy.interpolate.splantider` (*tck*, *n=1*)

Compute the spline for the antiderivative (integral) of a given spline.

**Parameters**    **tck** : tuple of (t, c, k)  
                   Spline whose antiderivative to compute  
                   **n** : int, optional  
                   Order of antiderivative to evaluate. Default: 1

**Returns**        **tck\_ader** : tuple of (t2, c2, k2)  
                   Spline of order  $k_2=k+n$  representing the antiderivative of the input spline.

See also:

`splder`, `splev`, `spalde`

### Notes

The `splder` function is the inverse operation of this function. Namely, `splder(splantider(tck))` is identical to *tck*, modulo rounding error.

New in version 0.13.0.

### Examples

```
>>> from scipy.interpolate import splrep, splder, splantider, splev
>>> x = np.linspace(0, np.pi/2, 70)
>>> y = 1 / np.sqrt(1 - 0.8*np.sin(x)**2)
>>> spl = splrep(x, y)
```

The derivative is the inverse operation of the antiderivative, although some floating point error accumulates:

```
>>> splev(1.7, spl), splev(1.7, splder(splantider(spl)))
(array(2.1565429877197317), array(2.1565429877201865))
```

Antiderivative can be used to evaluate definite integrals:

```
>>> ispl = splantider(spl)
>>> splev(np.pi/2, ispl) - splev(0, ispl)
2.2572053588768486
```

This is indeed an approximation to the complete elliptic integral  $K(m) = \int_0^{\pi/2} [1 - m \sin^2 x]^{-1/2} dx$ :

```
>>> from scipy.special import ellipk
>>> ellipk(0.8)
2.2572053268208538
```



`scipy.interpolate.insert(x, tck, m=1, per=0)`

Insert knots into a B-spline.

Given the knots and coefficients of a B-spline representation, create a new B-spline with a knot inserted  $m$  times at point  $x$ . This is a wrapper around the FORTRAN routine `insert` of FITPACK.

**Parameters**

- x (u)** : array\_like  
A 1-D point at which to insert a new knot(s). If `tck` was returned from `splprep`, then the parameter values, `u` should be given.
- tck** : tuple  
A tuple (t,c,k) returned by `splrep` or `splprep` containing the vector of knots, the B-spline coefficients, and the degree of the spline.
- m** : int, optional  
The number of times to insert the given knot (its multiplicity). Default is 1.
- per** : int, optional  
If non-zero, the input spline is considered periodic.

**Returns**

- tck** : tuple  
A tuple (t,c,k) containing the vector of knots, the B-spline coefficients, and the degree of the new spline.  $t_{(k+1)} \leq x \leq t_{(n-k)}$ , where  $k$  is the degree of the spline. In case of a periodic spline (`per != 0`) there must be either at least  $k$  interior knots  $t(j)$  satisfying  $t_{(k+1)} < t_{(j)} \leq x$  or at least  $k$  interior knots  $t(j)$  satisfying  $x \leq t_{(j)} < t_{(n-k)}$ .

#### Notes

Based on algorithms from [R68] and [R69].

#### References

[R68], [R69]

## 5.7.4 2-D Splines

For data on a grid:

---

<code>RectBivariateSpline(x, y, z[, bbox, kx, ky, s])</code>	Bivariate spline approximation over a rectangular mesh.
<code>RectSphereBivariateSpline(u, v, r[, s, ...])</code>	Bivariate spline approximation over a rectangular mesh on a sphere.

---

**class** `scipy.interpolate.RectBivariateSpline(x, y, z, bbox=[None, None, None, None], kx=3, ky=3, s=0)`

Bivariate spline approximation over a rectangular mesh.

Can be used for both smoothing and interpolating data.

**Parameters**

- x,y** : array\_like  
1-D arrays of coordinates in strictly ascending order.
- z** : array\_like  
2-D array of data with shape (x.size,y.size).
- bbox** : array\_like, optional  
Sequence of length 4 specifying the boundary of the rectangular approximation domain. By default, `bbox=[min(x,tx), max(x,tx), min(y,ty), max(y,ty)]`.
- kx, ky** : ints, optional  
Degrees of the bivariate spline. Default is 3.
- s** : float, optional

Positive smoothing factor defined for estimation condition:  
 $\text{sum}((w[i] * (z[i] - s(x[i], y[i]))) ** 2, \text{axis}=0) \leq s$  Default  
 is  $s=0$ , which is for interpolation.

See also:

**SmoothBivariateSpline**

a smoothing bivariate spline for scattered data

**bisplrep** an older wrapping of FITPACK

**bisplev** an older wrapping of FITPACK

**UnivariateSpline**

a similar class for univariate spline interpolation

**Methods**

---

<code>__call__(x, y[, mth, dx, dy, grid])</code>	Evaluate the spline or its derivatives at given positions.
<code>ev(xi, yi[, dx, dy])</code>	Evaluate the spline at points
<code>get_coeffs()</code>	Return spline coefficients.
<code>get_knots()</code>	Return a tuple (tx,ty) where tx,ty contain knots positions of the spline with respect to x-, y-var
<code>get_residual()</code>	Return weighted sum of squared residuals of the spline
<code>integral(xa, xb, ya, yb)</code>	Evaluate the integral of the spline over area [xa,xb] x [ya,yb].

---

`RectBivariateSpline.__call__(x, y, mth=None, dx=0, dy=0, grid=True)`

Evaluate the spline or its derivatives at given positions.

**Parameters** `x, y`: array\_like

Input coordinates.

If `grid` is False, evaluate the spline at points  $(x[i], y[i]), i=0, \dots, \text{len}(x) - 1$ . Standard Numpy broadcasting is obeyed.

If `grid` is True: evaluate spline at the grid points defined by the coordinate arrays `x, y`. The arrays must be sorted to increasing order.

**dx**: int

Order of x-derivative

New in version 0.14.0.

**dy**: int

Order of y-derivative

New in version 0.14.0.

**grid**: bool

Whether to evaluate the results on a grid spanned by the input arrays, or at points specified by the input arrays.

New in version 0.14.0.

**mth**: str

Deprecated argument. Has no effect.

`RectBivariateSpline.ev(xi, yi, dx=0, dy=0)`

Evaluate the spline at points

Returns the interpolated value at  $(xi[i], yi[i]), i=0, \dots, \text{len}(xi) - 1$ .

**Parameters** `xi, yi`: array\_like

Input coordinates. Standard Numpy broadcasting is obeyed.

**dx**: int, optional

Order of x-derivative

New in version 0.14.0.

**dy** : int, optional  
 Order of y-derivative  
 New in version 0.14.0.

`RectBivariateSpline.get_coeffs()`  
 Return spline coefficients.

`RectBivariateSpline.get_knots()`  
 Return a tuple (tx,ty) where tx,ty contain knots positions of the spline with respect to x-, y-variable, respectively. The position of interior and additional knots are given as  $t[k+1:-k-1]$  and  $t[:k+1]=b$ ,  $t[-k-1:] = e$ , respectively.

`RectBivariateSpline.get_residual()`  
 Return weighted sum of squared residuals of the spline approximation:  $\sum ((w[i]*(z[i]-s(x[i],y[i])))^2, axis=0)$

`RectBivariateSpline.integral(xa, xb, ya, yb)`  
 Evaluate the integral of the spline over area  $[xa,xb] \times [ya,yb]$ .

**Parameters** **xa, xb** : float  
 The end-points of the x integration interval.  
**ya, yb** : float  
 The end-points of the y integration interval.  
**Returns** **integ** : float  
 The value of the resulting integral.

`class scipy.interpolate.RectSphereBivariateSpline(u, v, r, s=0.0, pole_continuity=False, pole_values=None, pole_exact=False, pole_flat=False)`

Bivariate spline approximation over a rectangular mesh on a sphere.

Can be used for smoothing data.

New in version 0.11.0.

**Parameters** **u** : array\_like  
 1-D array of latitude coordinates in strictly ascending order. Coordinates must be given in radians and lie within the interval (0, pi).  
**v** : array\_like  
 1-D array of longitude coordinates in strictly ascending order. Coordinates must be given in radians, and must lie within (0, 2pi).  
**r** : array\_like  
 2-D array of data with shape (u.size, v.size).  
**s** : float, optional  
 Positive smoothing factor defined for estimation condition (s=0 is for interpolation).  
**pole\_continuity** : bool or (bool, bool), optional  
 Order of continuity at the poles  $u=0$  (`pole_continuity[0]`) and  $u=\pi$  (`pole_continuity[1]`). The order of continuity at the pole will be 1 or 0 when this is True or False, respectively. Defaults to False.  
**pole\_values** : float or (float, float), optional  
 Data values at the poles  $u=0$  and  $u=\pi$ . Either the whole parameter or each individual element can be None. Defaults to None.  
**pole\_exact** : bool or (bool, bool), optional  
 Data value exactness at the poles  $u=0$  and  $u=\pi$ . If True, the value is considered to be the right function value, and it will be fitted exactly. If False, the value will be considered to be a data value just like the other data values. Defaults to False.  
**pole\_flat** : bool or (bool, bool), optional  
 For the poles at  $u=0$  and  $u=\pi$ , specify whether or not the approximation has vanishing derivatives. Defaults to False.

See also:

### *RectBivariateSpline*

bivariate spline approximation over a rectangular mesh

### *Notes*

Currently, only the smoothing spline approximation (`iopt[0] = 0` and `iopt[0] = 1` in the FITPACK routine) is supported. The exact least-squares spline approximation is not implemented yet.

When actually performing the interpolation, the requested  $\nu$  values must lie within the same length  $2\pi$  interval that the original  $\nu$  values were chosen from.

For more information, see the [FITPACK](#) site about this function.

### *Examples*

Suppose we have global data on a coarse grid

```
>>> lats = np.linspace(10, 170, 9) * np.pi / 180.
>>> lons = np.linspace(0, 350, 18) * np.pi / 180.
>>> data = np.dot(np.atleast_2d(90. - np.linspace(-80., 80., 18)).T,
...              np.atleast_2d(180. - np.abs(np.linspace(0., 350., 9))))).T
```

We want to interpolate it to a global one-degree grid

```
>>> new_lats = np.linspace(1, 180, 180) * np.pi / 180
>>> new_lons = np.linspace(1, 360, 360) * np.pi / 180
>>> new_lats, new_lons = np.meshgrid(new_lats, new_lons)
```

We need to set up the interpolator object

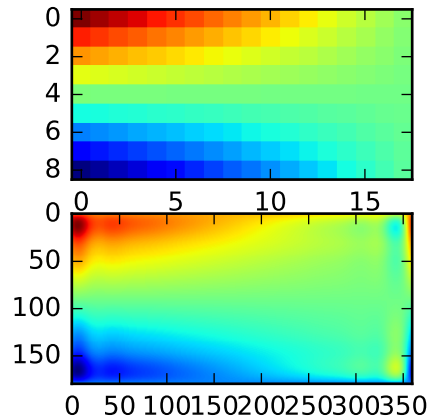
```
>>> from scipy.interpolate import RectSphereBivariateSpline
>>> lut = RectSphereBivariateSpline(lats, lons, data)
```

Finally we interpolate the data. The `RectSphereBivariateSpline` object only takes 1-D arrays as input, therefore we need to do some reshaping.

```
>>> data_interp = lut.ev(new_lats.ravel(),
...                     new_lons.ravel()).reshape((360, 180)).T
```

Looking at the original and the interpolated data, one can see that the interpolant reproduces the original data very well:

```
>>> import matplotlib.pyplot as plt
>>> fig = plt.figure()
>>> ax1 = fig.add_subplot(211)
>>> ax1.imshow(data, interpolation='nearest')
>>> ax2 = fig.add_subplot(212)
>>> ax2.imshow(data_interp, interpolation='nearest')
>>> plt.show()
```

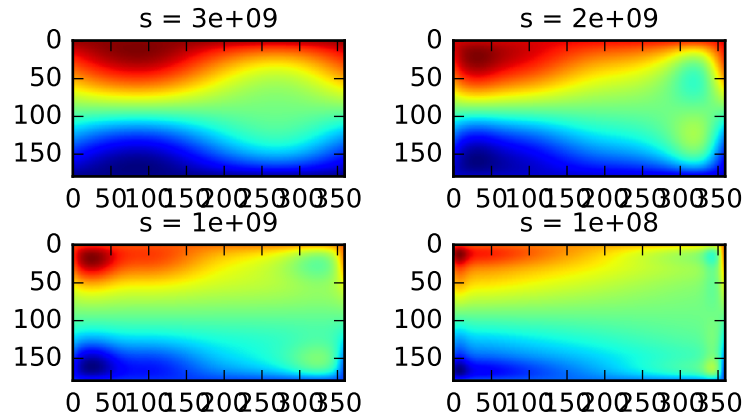


Choosing the optimal value of  $s$  can be a delicate task. Recommended values for  $s$  depend on the accuracy of the data values. If the user has an idea of the statistical errors on the data, she can also find a proper estimate for  $s$ . By assuming that, if she specifies the right  $s$ , the interpolator will use a spline  $f(u, v)$  which exactly reproduces the function underlying the data, she can evaluate  $\sum ((r(i, j) - s(u(i), v(j))) ** 2)$  to find a good estimate for this  $s$ . For example, if she knows that the statistical errors on her  $r(i, j)$ -values are not greater than 0.1, she may expect that a good  $s$  should have a value not larger than  $u.size * v.size * (0.1) ** 2$ .

If nothing is known about the statistical error in  $r(i, j)$ ,  $s$  must be determined by trial and error. The best is then to start with a very large value of  $s$  (to determine the least-squares polynomial and the corresponding upper bound  $fp0$  for  $s$ ) and then to progressively decrease the value of  $s$  (say by a factor 10 in the beginning, i.e.  $s = fp0 / 10, fp0 / 100, \dots$  and more carefully as the approximation shows more detail) to obtain closer fits.

The interpolation results for different values of  $s$  give some insight into this process:

```
>>> fig2 = plt.figure()
>>> s = [3e9, 2e9, 1e9, 1e8]
>>> for ii in xrange(len(s)):
...     lut = RectSphereBivariateSpline(lats, lons, data, s=s[ii])
...     data_interp = lut.ev(new_lats.ravel(),
...                           new_lons.ravel()).reshape((360, 180)).T
...     ax = fig2.add_subplot(2, 2, ii+1)
...     ax.imshow(data_interp, interpolation='nearest')
...     ax.set_title("s = %g" % s[ii])
>>> plt.show()
```



**Methods**

<code>__call__(theta, phi[, dtheta, dphi, grid])</code>	Evaluate the spline or its derivatives at given positions.
<code>ev(theta, phi[, dtheta, dphi])</code>	Evaluate the spline at points
<code>get_coeffs()</code>	Return spline coefficients.
<code>get_knots()</code>	Return a tuple (tx,ty) where tx,ty contain knots positions of the spline with respect to x-, y-axes.
<code>get_residual()</code>	Return weighted sum of squared residuals of the spline

`RectSphereBivariateSpline.__call__(theta, phi, dtheta=0, dphi=0, grid=True)`  
 Evaluate the spline or its derivatives at given positions.

**Parameters** **theta, phi** : array\_like  
 Input coordinates.  
 If *grid* is False, evaluate the spline at points  $(\text{theta}[i], \text{phi}[i])$ ,  $i=0, \dots, \text{len}(x)-1$ . Standard Numpy broadcasting is obeyed.  
 If *grid* is True: evaluate spline at the grid points defined by the coordinate arrays *theta*, *phi*. The arrays must be sorted to increasing order.  
**dtheta** : int, optional  
 Order of theta-derivative  
 New in version 0.14.0.  
**dphi** : int  
 Order of phi-derivative  
 New in version 0.14.0.  
**grid** : bool  
 Whether to evaluate the results on a grid spanned by the input arrays, or at points specified by the input arrays.  
 New in version 0.14.0.

`RectSphereBivariateSpline.ev(theta, phi, dtheta=0, dphi=0)`  
 Evaluate the spline at points  
 Returns the interpolated value at  $(\text{theta}[i], \text{phi}[i])$ ,  $i=0, \dots, \text{len}(\text{theta})-1$ .

**Parameters** **theta, phi** : array\_like  
 Input coordinates. Standard Numpy broadcasting is obeyed.  
**dtheta** : int, optional

Order of theta-derivative  
 New in version 0.14.0.  
**dphi** : int, optional  
 Order of phi-derivative  
 New in version 0.14.0.

`RectSphereBivariateSpline.get_coeffs()`  
 Return spline coefficients.

`RectSphereBivariateSpline.get_knots()`  
 Return a tuple (tx,ty) where tx,ty contain knots positions of the spline with respect to x-, y-variable, respectively. The position of interior and additional knots are given as `t[k+1:-k-1]` and `t[:k+1]=b`, `t[-k-1:]=e`, respectively.

`RectSphereBivariateSpline.get_residual()`  
 Return weighted sum of squared residuals of the spline approximation: `sum ((w[i]*(z[i]-s(x[i],y[i])))**2,axis=0)`

For unstructured data:

<code>BivariateSpline</code>	Base class for bivariate splines.
<code>SmoothBivariateSpline(x, y, z[, w, bbox, ...])</code>	Smooth bivariate spline approximation.
<code>SmoothSphereBivariateSpline(theta, phi, r[, ...])</code>	Smooth bivariate spline approximation in spherical coordinates.
<code>LSQBivariateSpline(x, y, z, tx, ty[, w, ...])</code>	Weighted least-squares bivariate spline approximation.
<code>LSQSphereBivariateSpline(theta, phi, r, tt, tp)</code>	Weighted least-squares bivariate spline approximation in spherical coord

**class** `scipy.interpolate.BivariateSpline`

Base class for bivariate splines.

This describes a spline  $s(x, y)$  of degrees  $k_x$  and  $k_y$  on the rectangle  $[x_b, x_e] * [y_b, y_e]$  calculated from a given set of data points  $(x, y, z)$ .

This class is meant to be subclassed, not instantiated directly. To construct these splines, call either `SmoothBivariateSpline` or `LSQBivariateSpline`.

**See also:**

***UnivariateSpline***

a similar class for univariate spline interpolation

***SmoothBivariateSpline***

to create a `BivariateSpline` through the given points

***LSQBivariateSpline***

to create a `BivariateSpline` using weighted least-squares fitting

***SphereBivariateSpline***

bivariate spline interpolation in spherical coordinates

***bisplrep*** older wrapping of FITPACK

***bisplev*** older wrapping of FITPACK

**Methods**

<code>__call__(x, y[, mth, dx, dy, grid])</code>	Evaluate the spline or its derivatives at given positions.
<code>ev(xi, yi[, dx, dy])</code>	Evaluate the spline at points
<code>get_coeffs()</code>	Return spline coefficients.

Continu

Table 5.51 – continued from previous page

<code>get_knots()</code>	Return a tuple (tx,ty) where tx,ty contain knots positions of the spline with respect to x-, y-var
<code>get_residual()</code>	Return weighted sum of squared residuals of the spline
<code>integral(xa, xb, ya, yb)</code>	Evaluate the integral of the spline over area [xa,xb] x [ya,yb].

`BivariateSpline.__call__(x, y, mth=None, dx=0, dy=0, grid=True)`

Evaluate the spline or its derivatives at given positions.

**Parameters** **x, y** : array\_like  
 Input coordinates.  
 If *grid* is False, evaluate the spline at points  $(x[i], y[i])$ ,  $i=0, \dots, \text{len}(x)-1$ . Standard Numpy broadcasting is obeyed.  
 If *grid* is True: evaluate spline at the grid points defined by the coordinate arrays *x, y*. The arrays must be sorted to increasing order.

**dx** : int  
 Order of x-derivative  
 New in version 0.14.0.

**dy** : int  
 Order of y-derivative  
 New in version 0.14.0.

**grid** : bool  
 Whether to evaluate the results on a grid spanned by the input arrays, or at points specified by the input arrays.  
 New in version 0.14.0.

**mth** : str  
 Deprecated argument. Has no effect.

`BivariateSpline.ev(xi, yi, dx=0, dy=0)`

Evaluate the spline at points

Returns the interpolated value at  $(xi[i], yi[i])$ ,  $i=0, \dots, \text{len}(xi)-1$ .

**Parameters** **xi, yi** : array\_like  
 Input coordinates. Standard Numpy broadcasting is obeyed.

**dx** : int, optional  
 Order of x-derivative  
 New in version 0.14.0.

**dy** : int, optional  
 Order of y-derivative  
 New in version 0.14.0.

`BivariateSpline.get_coeffs()`

Return spline coefficients.

`BivariateSpline.get_knots()`

Return a tuple (tx,ty) where tx,ty contain knots positions of the spline with respect to x-, y-variable, respectively. The position of interior and additional knots are given as  $t[k+1:-k-1]$  and  $t[:k+1]=b$ ,  $t[-k-1]=e$ , respectively.

`BivariateSpline.get_residual()`

Return weighted sum of squared residuals of the spline approximation:  $\text{sum}((w[i]*(z[i]-s(x[i],y[i])))**2,axis=0)$

`BivariateSpline.integral(xa, xb, ya, yb)`

Evaluate the integral of the spline over area [xa,xb] x [ya,yb].

**Parameters** **xa, xb** : float  
 The end-points of the x integration interval.



**Returns** **ya, yb** : float  
The end-points of the y integration interval.

**integ** : float  
The value of the resulting integral.

**class** `scipy.interpolate.SmoothBivariateSpline` (*x, y, z, w=None, bbox=[None, None, None, None], kx=3, ky=3, s=None, eps=None*)

Smooth bivariate spline approximation.

**Parameters** **x, y, z** : array\_like  
1-D sequences of data points (order is not important).

**w** : array\_like, optional  
Positive 1-D sequence of weights, of same length as *x*, *y* and *z*.

**bbox** : array\_like, optional  
Sequence of length 4 specifying the boundary of the rectangular approximation domain. By default, `bbox=[min(x,tx),max(x,tx),min(y,ty),max(y,ty)]`.

**kx, ky** : ints, optional  
Degrees of the bivariate spline. Default is 3.

**s** : float, optional  
Positive smoothing factor defined for estimation condition: `sum((w[i]*(z[i]-s(x[i],y[i])))**2,axis=0) <= s` Default `s=len(w)` which should be a good value if `1/w[i]` is an estimate of the standard deviation of `z[i]`.

**eps** : float, optional  
A threshold for determining the effective rank of an over-determined linear system of equations. *eps* should have a value between 0 and 1, the default is 1e-16.

**See also:**

***bisplrep*** an older wrapping of FITPACK

***bisplev*** an older wrapping of FITPACK

***UnivariateSpline***  
a similar class for univariate spline interpolation

***LSQUnivariateSpline***  
to create a BivariateSpline using weighted

**Notes**

The length of *x*, *y* and *z* should be at least  $(kx+1) * (ky+1)$ .

**Methods**

---

<code>__call__(x, y[, mth, dx, dy, grid])</code>	Evaluate the spline or its derivatives at given positions.
<code>ev(xi, yi[, dx, dy])</code>	Evaluate the spline at points
<code>get_coeffs()</code>	Return spline coefficients.
<code>get_knots()</code>	Return a tuple (tx,ty) where tx,ty contain knots positions of the spline with respect to x-, y-var
<code>get_residual()</code>	Return weighted sum of squared residuals of the spline
<code>integral(xa, xb, ya, yb)</code>	Evaluate the integral of the spline over area [xa,xb] x [ya,yb].

---

`SmoothBivariateSpline.__call__(x, y, mth=None, dx=0, dy=0, grid=True)`  
Evaluate the spline or its derivatives at given positions.

**Parameters** **x, y** : array\_like

Input coordinates.

If *grid* is False, evaluate the spline at points  $(x[i], y[i])$ ,  $i=0, \dots, \text{len}(x)-1$ . Standard Numpy broadcasting is obeyed.

If *grid* is True: evaluate spline at the grid points defined by the coordinate arrays *x*, *y*. The arrays must be sorted to increasing order.

**dx** : int

Order of x-derivative  
New in version 0.14.0.

**dy** : int

Order of y-derivative  
New in version 0.14.0.

**grid** : bool

Whether to evaluate the results on a grid spanned by the input arrays, or at points specified by the input arrays.  
New in version 0.14.0.

**mth** : str

Deprecated argument. Has no effect.

`SmoothBivariateSpline.ev(xi, yi, dx=0, dy=0)`

Evaluate the spline at points

Returns the interpolated value at  $(xi[i], yi[i])$ ,  $i=0, \dots, \text{len}(xi)-1$ .

**Parameters** **xi, yi** : array\_like

Input coordinates. Standard Numpy broadcasting is obeyed.

**dx** : int, optional

Order of x-derivative  
New in version 0.14.0.

**dy** : int, optional

Order of y-derivative  
New in version 0.14.0.

`SmoothBivariateSpline.get_coeffs()`

Return spline coefficients.

`SmoothBivariateSpline.get_knots()`

Return a tuple (tx,ty) where tx,ty contain knots positions of the spline with respect to x-, y-variable, respectively. The position of interior and additional knots are given as  $t[k+1:-k-1]$  and  $t[:k+1]=b$ ,  $t[-k-1:]=e$ , respectively.

`SmoothBivariateSpline.get_residual()`

Return weighted sum of squared residuals of the spline approximation:  $\text{sum}((w[i]*(z[i]-s(x[i],y[i])))**2,axis=0)$

`SmoothBivariateSpline.integral(xa, xb, ya, yb)`

Evaluate the integral of the spline over area  $[xa,xb] \times [ya,yb]$ .

**Parameters** **xa, xb** : float

The end-points of the x integration interval.

**ya, yb** : float

The end-points of the y integration interval.

**Returns** **integ** : float

The value of the resulting integral.

`class scipy.interpolate.SmoothSphereBivariateSpline(theta, phi, r, w=None, s=0.0, eps=1e-16)`

Smooth bivariate spline approximation in spherical coordinates.

New in version 0.11.0.

**Parameters** **theta, phi, r** : array\_like  
 1-D sequences of data points (order is not important). Coordinates must be given in radians. Theta must lie within the interval (0, pi), and phi must lie within the interval (0, 2pi).

**w** : array\_like, optional  
 Positive 1-D sequence of weights.

**s** : float, optional  
 Positive smoothing factor defined for estimation condition:  $\sum((w(i) * (r(i) - s(\theta(i), \phi(i))))^2, \text{axis}=0) \leq s$  Default  $s=\text{len}(w)$  which should be a good value if  $1/w[i]$  is an estimate of the standard deviation of  $r[i]$ .

**eps** : float, optional  
 A threshold for determining the effective rank of an over-determined linear system of equations. *eps* should have a value between 0 and 1, the default is 1e-16.

### Notes

For more information, see the [FITPACK](#) site about this function.

### Examples

Suppose we have global data on a coarse grid (the input data does not have to be on a grid):

```
>>> theta = np.linspace(0., np.pi, 7)
>>> phi = np.linspace(0., 2*np.pi, 9)
>>> data = np.empty((theta.shape[0], phi.shape[0]))
>>> data[:,0], data[0,:], data[-1,:] = 0., 0., 0.
>>> data[1:-1,1], data[1:-1,-1] = 1., 1.
>>> data[1,1:-1], data[-2,1:-1] = 1., 1.
>>> data[2:-2,2], data[2:-2,-2] = 2., 2.
>>> data[2,2:-2], data[-3,2:-2] = 2., 2.
>>> data[3,3:-2] = 3.
>>> data = np.roll(data, 4, 1)
```

We need to set up the interpolator object

```
>>> lats, lons = np.meshgrid(theta, phi)
>>> from scipy.interpolate import SmoothSphereBivariateSpline
>>> lut = SmoothSphereBivariateSpline(lats.ravel(), lons.ravel(),
...                                 data.T.ravel(), s=3.5)
```

As a first test, we'll see what the algorithm returns when run on the input coordinates

```
>>> data_orig = lut(theta, phi)
```

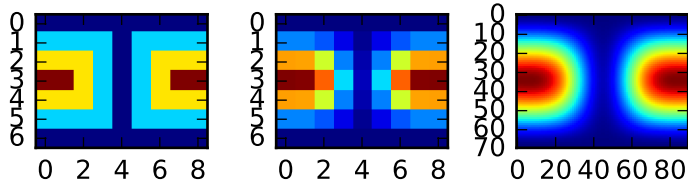
Finally we interpolate the data to a finer grid

```
>>> fine_lats = np.linspace(0., np.pi, 70)
>>> fine_lons = np.linspace(0., 2 * np.pi, 90)

>>> data_smth = lut(fine_lats, fine_lons)

>>> import matplotlib.pyplot as plt
>>> fig = plt.figure()
>>> ax1 = fig.add_subplot(131)
>>> ax1.imshow(data, interpolation='nearest')
>>> ax2 = fig.add_subplot(132)
>>> ax2.imshow(data_orig, interpolation='nearest')
>>> ax3 = fig.add_subplot(133)
```

```
>>> ax3.imshow(data_smth, interpolation='nearest')
>>> plt.show()
```



### Methods

<code>__call__(theta, phi[, dtheta, dphi, grid])</code>	Evaluate the spline or its derivatives at given positions.
<code>ev(theta, phi[, dtheta, dphi])</code>	Evaluate the spline at points
<code>get_coeffs()</code>	Return spline coefficients.
<code>get_knots()</code>	Return a tuple (tx,ty) where tx,ty contain knots positions of the spline with respect to x-, y-axes.
<code>get_residual()</code>	Return weighted sum of squared residuals of the spline

`SmoothSphereBivariateSpline.__call__(theta, phi, dtheta=0, dphi=0, grid=True)`

Evaluate the spline or its derivatives at given positions.

**Parameters** **theta, phi** : array\_like

Input coordinates.

If *grid* is False, evaluate the spline at points (theta[i], phi[i]), i=0, ..., len(x)-1. Standard Numpy broadcasting is obeyed.

If *grid* is True: evaluate spline at the grid points defined by the coordinate arrays theta, phi. The arrays must be sorted to increasing order.

**dtheta** : int, optional

Order of theta-derivative

New in version 0.14.0.

**dphi** : int

Order of phi-derivative

New in version 0.14.0.

**grid** : bool

Whether to evaluate the results on a grid spanned by the input arrays, or at points specified by the input arrays.

New in version 0.14.0.

`SmoothSphereBivariateSpline.ev(theta, phi, dtheta=0, dphi=0)`

Evaluate the spline at points

Returns the interpolated value at (theta[i], phi[i]), i=0, ..., len(theta)-1.

**Parameters** **theta, phi** : array\_like  
Input coordinates. Standard Numpy broadcasting is obeyed.

**dtheta** : int, optional  
Order of theta-derivative  
New in version 0.14.0.

**dphi** : int, optional  
Order of phi-derivative  
New in version 0.14.0.

`SmoothSphereBivariateSpline.get_coeffs()`  
Return spline coefficients.

`SmoothSphereBivariateSpline.get_knots()`  
Return a tuple (tx,ty) where tx,ty contain knots positions of the spline with respect to x-, y-variable, respectively. The position of interior and additional knots are given as  $t[k+1:-k-1]$  and  $t[:k+1]=b$ ,  $t[-k-1:]=e$ , respectively.

`SmoothSphereBivariateSpline.get_residual()`  
Return weighted sum of squared residuals of the spline approximation:  $\sum ((w[i]*(z[i]-s(x[i],y[i])))^2, axis=0)$

**class** `scipy.interpolate.LSQBivariateSpline` (*x, y, z, tx, ty, w=None, bbox=[None, None, None, None], kx=3, ky=3, eps=None*)

Weighted least-squares bivariate spline approximation.

**Parameters** **x, y, z** : array\_like  
1-D sequences of data points (order is not important).

**tx, ty** : array\_like  
Strictly ordered 1-D sequences of knots coordinates.

**w** : array\_like, optional  
Positive 1-D array of weights, of the same length as *x*, *y* and *z*.

**bbox** : (4,) array\_like, optional  
Sequence of length 4 specifying the boundary of the rectangular approximation domain. By default, `bbox=[min(x,tx), max(x,tx), min(y,ty), max(y,ty)]`.

**kx, ky** : ints, optional  
Degrees of the bivariate spline. Default is 3.

**eps** : float, optional  
A threshold for determining the effective rank of an over-determined linear system of equations. *eps* should have a value between 0 and 1, the default is 1e-16.

**See also:**

**bisplrep** an older wrapping of FITPACK

**bisplev** an older wrapping of FITPACK

**UnivariateSpline**  
a similar class for univariate spline interpolation

**SmoothBivariateSpline**  
create a smoothing BivariateSpline

**Notes**

The length of *x*, *y* and *z* should be at least  $(kx+1) * (ky+1)$ .

**Methods**

<code>__call__(x, y[, mth, dx, dy, grid])</code>	Evaluate the spline or its derivatives at given positions.
<code>ev(xi, yi[, dx, dy])</code>	Evaluate the spline at points
<code>get_coeffs()</code>	Return spline coefficients.
<code>get_knots()</code>	Return a tuple (tx,ty) where tx,ty contain knots positions of the spline with respect to x-, y-var
<code>get_residual()</code>	Return weighted sum of squared residuals of the spline
<code>integral(xa, xb, ya, yb)</code>	Evaluate the integral of the spline over area [xa,xb] x [ya,yb].

`LSQBivariateSpline.__call__(x, y, mth=None, dx=0, dy=0, grid=True)`

Evaluate the spline or its derivatives at given positions.

**Parameters** `x, y` : array\_like

Input coordinates.

If `grid` is False, evaluate the spline at points  $(x[i], y[i])$ ,  $i=0, \dots, \text{len}(x)-1$ . Standard Numpy broadcasting is obeyed.

If `grid` is True: evaluate spline at the grid points defined by the coordinate arrays `x, y`. The arrays must be sorted to increasing order.

**dx** : int

Order of x-derivative

New in version 0.14.0.

**dy** : int

Order of y-derivative

New in version 0.14.0.

**grid** : bool

Whether to evaluate the results on a grid spanned by the input arrays, or at points specified by the input arrays.

New in version 0.14.0.

**mth** : str

Deprecated argument. Has no effect.

`LSQBivariateSpline.ev(xi, yi, dx=0, dy=0)`

Evaluate the spline at points

Returns the interpolated value at  $(xi[i], yi[i])$ ,  $i=0, \dots, \text{len}(xi)-1$ .

**Parameters** `xi, yi` : array\_like

Input coordinates. Standard Numpy broadcasting is obeyed.

**dx** : int, optional

Order of x-derivative

New in version 0.14.0.

**dy** : int, optional

Order of y-derivative

New in version 0.14.0.

`LSQBivariateSpline.get_coeffs()`

Return spline coefficients.

`LSQBivariateSpline.get_knots()`

Return a tuple (tx,ty) where tx,ty contain knots positions of the spline with respect to x-, y-variable, respectively. The position of interior and additional knots are given as  $t[k+1:-k-1]$  and  $t[:k+1]=b$ ,  $t[-k-1:]=e$ , respectively.

`LSQBivariateSpline.get_residual()`

Return weighted sum of squared residuals of the spline approximation:  $\text{sum}((w[i]*(z[i]-s(x[i],y[i])))**2,axis=0)$

LSQBivariateSpline.**integral** (*xa, xb, ya, yb*)

Evaluate the integral of the spline over area [xa,xb] x [ya,yb].

**Parameters** **xa, xb** : float  
The end-points of the x integration interval.  
**ya, yb** : float  
The end-points of the y integration interval.  
**Returns** **integ** : float  
The value of the resulting integral.

**class** `scipy.interpolate.LSQSphereBivariateSpline` (*theta, phi, r, tt, tp, w=None, eps=1e-16*)  
Weighted least-squares bivariate spline approximation in spherical coordinates.

New in version 0.11.0.

**Parameters** **theta, phi, r** : array\_like  
1-D sequences of data points (order is not important). Coordinates must be given in radians. Theta must lie within the interval (0, pi), and phi must lie within the interval (0, 2pi).  
**tt, tp** : array\_like  
Strictly ordered 1-D sequences of knots coordinates. Coordinates must satisfy  $0 < tt[i] < pi, 0 < tp[i] < 2*pi$ .  
**w** : array\_like, optional  
Positive 1-D sequence of weights, of the same length as *theta, phi* and *r*.  
**eps** : float, optional  
A threshold for determining the effective rank of an over-determined linear system of equations. *eps* should have a value between 0 and 1, the default is 1e-16.

### Notes

For more information, see the [FITPACK](#) site about this function.

### Examples

Suppose we have global data on a coarse grid (the input data does not have to be on a grid):

```
>>> theta = np.linspace(0., np.pi, 7)
>>> phi = np.linspace(0., 2*np.pi, 9)
>>> data = np.empty((theta.shape[0], phi.shape[0]))
>>> data[:,0], data[0,:], data[-1,:] = 0., 0., 0.
>>> data[1:-1,1], data[1:-1,-1] = 1., 1.
>>> data[1,1:-1], data[-2,1:-1] = 1., 1.
>>> data[2:-2,2], data[2:-2,-2] = 2., 2.
>>> data[2,2:-2], data[-3,2:-2] = 2., 2.
>>> data[3,3:-2] = 3.
>>> data = np.roll(data, 4, 1)
```

We need to set up the interpolator object. Here, we must also specify the coordinates of the knots to use.

```
>>> lats, lons = np.meshgrid(theta, phi)
>>> knotst, knotsp = theta.copy(), phi.copy()
>>> knotst[0] += .0001
>>> knotst[-1] -= .0001
>>> knotsp[0] += .0001
>>> knotsp[-1] -= .0001
>>> from scipy.interpolate import LSQSphereBivariateSpline
>>> lut = LSQSphereBivariateSpline(lats.ravel(), lons.ravel(),
...                               data.T.ravel(), knotst, knotsp)
```

As a first test, we'll see what the algorithm returns when run on the input coordinates

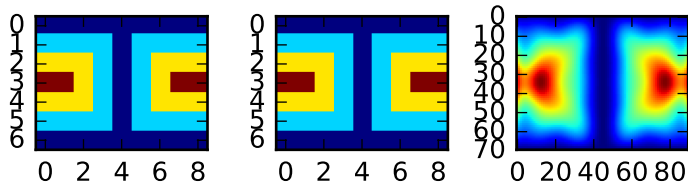
```
>>> data_orig = lut(theta, phi)
```

Finally we interpolate the data to a finer grid

```
>>> fine_lats = np.linspace(0., np.pi, 70)
>>> fine_lons = np.linspace(0., 2*np.pi, 90)

>>> data_lsq = lut(fine_lats, fine_lons)

>>> import matplotlib.pyplot as plt
>>> fig = plt.figure()
>>> ax1 = fig.add_subplot(131)
>>> ax1.imshow(data, interpolation='nearest')
>>> ax2 = fig.add_subplot(132)
>>> ax2.imshow(data_orig, interpolation='nearest')
>>> ax3 = fig.add_subplot(133)
>>> ax3.imshow(data_lsq, interpolation='nearest')
>>> plt.show()
```



### Methods

<code>__call__(theta, phi[, dtheta, dphi, grid])</code>	Evaluate the spline or its derivatives at given positions.
<code>ev(theta, phi[, dtheta, dphi])</code>	Evaluate the spline at points
<code>get_coeffs()</code>	Return spline coefficients.
<code>get_knots()</code>	Return a tuple (tx,ty) where tx,ty contain knots positions of the spline with respect to x-, y-axis.
<code>get_residual()</code>	Return weighted sum of squared residuals of the spline

`LSQSphereBivariateSpline.__call__(theta, phi, dtheta=0, dphi=0, grid=True)`

Evaluate the spline or its derivatives at given positions.

**Parameters** **theta, phi** : array\_like

Input coordinates.

If *grid* is False, evaluate the spline at points (theta[i], phi[i]), i=0, ..., len(x)-1. Standard Numpy broadcasting is obeyed.



If *grid* is True: evaluate spline at the grid points defined by the coordinate arrays *theta*, *phi*. The arrays must be sorted to increasing order.

**dtheta** : int, optional  
Order of theta-derivative  
New in version 0.14.0.

**dphi** : int  
Order of phi-derivative  
New in version 0.14.0.

**grid** : bool  
Whether to evaluate the results on a grid spanned by the input arrays, or at points specified by the input arrays.  
New in version 0.14.0.

`LSQSphereBivariateSpline.ev(theta, phi, dtheta=0, dphi=0)`

Evaluate the spline at points

Returns the interpolated value at  $(\text{theta}[i], \text{phi}[i])$ ,  $i=0, \dots, \text{len}(\text{theta})-1$ .

**Parameters** **theta, phi** : array\_like  
Input coordinates. Standard Numpy broadcasting is obeyed.  
**dtheta** : int, optional  
Order of theta-derivative  
New in version 0.14.0.  
**dphi** : int, optional  
Order of phi-derivative  
New in version 0.14.0.

`LSQSphereBivariateSpline.get_coeffs()`

Return spline coefficients.

`LSQSphereBivariateSpline.get_knots()`

Return a tuple (tx,ty) where tx,ty contain knots positions of the spline with respect to x-, y-variable, respectively. The position of interior and additional knots are given as  $t[k+1:-k-1]$  and  $t[:k+1]=b$ ,  $t[-k-1:] = e$ , respectively.

`LSQSphereBivariateSpline.get_residual()`

Return weighted sum of squared residuals of the spline approximation:  $\text{sum}((w[i]*(z[i]-s(x[i],y[i])))^2, \text{axis}=0)$

Low-level interface to FITPACK functions:

---

<code>bisplrep(x, y, z[, w, xb, xe, yb, ye, kx, ...])</code>	Find a bivariate B-spline representation of a surface.
<code>bisplev(x, y, tck[, dx, dy])</code>	Evaluate a bivariate B-spline and its derivatives.

---

`scipy.interpolate.bisplrep(x, y, z, w=None, xb=None, xe=None, yb=None, ye=None, kx=3, ky=3, task=0, s=None, eps=1e-16, tx=None, ty=None, full_output=0, nrest=None, nyest=None, quiet=1)`

Find a bivariate B-spline representation of a surface.

Given a set of data points  $(x[i], y[i], z[i])$  representing a surface  $z=f(x,y)$ , compute a B-spline representation of the surface. Based on the routine SURFIT from FITPACK.

**Parameters** **x, y, z** : ndarray  
Rank-1 arrays of data points.  
**w** : ndarray, optional  
Rank-1 array of weights. By default  $w = \text{np.ones}(\text{len}(x))$ .  
**xb, xe** : float, optional

End points of approximation interval in  $x$ . By default  $x_b = x.\min()$ ,  $x_e = x.\max()$ .

**yb, ye** : float, optional  
End points of approximation interval in  $y$ . By default  $y_b = y.\min()$ ,  $y_e = y.\max()$ .

**kx, ky** : int, optional  
The degrees of the spline ( $1 \leq k_x, k_y \leq 5$ ). Third order ( $k_x = k_y = 3$ ) is recommended.

**task** : int, optional  
If  $\text{task}=0$ , find knots in  $x$  and  $y$  and coefficients for a given smoothing factor,  $s$ . If  $\text{task}=1$ , find knots and coefficients for another value of the smoothing factor,  $s$ . `bisplprep` must have been previously called with  $\text{task}=0$  or  $\text{task}=1$ . If  $\text{task}=-1$ , find coefficients for a given set of knots  $t_x, t_y$ .

**s** : float, optional  
A non-negative smoothing factor. If weights correspond to the inverse of the standard-deviation of the errors in  $z$ , then a good  $s$ -value should be found in the range  $(m - \sqrt{2 * m}, m + \sqrt{2 * m})$  where  $m = \text{len}(x)$ .

**eps** : float, optional  
A threshold for determining the effective rank of an over-determined linear system of equations ( $0 < \text{eps} < 1$ ).  $\text{eps}$  is not likely to need changing.

**tx, ty** : ndarray, optional  
Rank-1 arrays of the knots of the spline for  $\text{task}=-1$

**full\_output** : int, optional  
Non-zero to return optional outputs.

**nxest, nyest** : int, optional  
Over-estimates of the total number of knots. If None then  $\text{nxest} = \max(k_x + \sqrt{m/2}, 2 * k_x + 3)$ ,  $\text{nyest} = \max(k_y + \sqrt{m/2}, 2 * k_y + 3)$ .

**quiet** : int, optional  
Non-zero to suppress printing of messages. This parameter is deprecated; use standard Python warning filters instead.

**tck** : array\_like  
A list  $[t_x, t_y, c, k_x, k_y]$  containing the knots ( $t_x, t_y$ ) and coefficients ( $c$ ) of the bivariate B-spline representation of the surface along with the degree of the spline.

**fp** : ndarray  
The weighted sum of squared residuals of the spline approximation.

**ier** : int  
An integer flag about `splprep` success. Success is indicated if  $\text{ier} \leq 0$ . If  $\text{ier}$  in  $[1, 2, 3]$  an error occurred but was not raised. Otherwise an error is raised.

**msg** : str  
A message corresponding to the integer flag,  $\text{ier}$ .

**Returns**

**See also:**

`splprep`, `splrep`, `splint`, `sproot`, `splev`, `UnivariateSpline`, `BivariateSpline`

**Notes**

See `bisplev` to evaluate the value of the B-spline given its `tck` representation.

**References**

[R65], [R66], [R67]

`scipy.interpolate.bisplev` ( $x, y, tck, dx=0, dy=0$ )  
Evaluate a bivariate B-spline and its derivatives.

Return a rank-2 array of spline function values (or spline derivative values) at points given by the cross-product of the rank-1 arrays  $x$  and  $y$ . In special cases, return an array or just a float if either  $x$  or  $y$  or both are floats. Based on BISPEV from FITPACK.

**Parameters**

- x, y** : ndarray  
Rank-1 arrays specifying the domain over which to evaluate the spline or its derivative.
- tck** : tuple  
A sequence of length 5 returned by `bisplprep` containing the knot locations, the coefficients, and the degree of the spline: [tx, ty, c, kx, ky].
- dx, dy** : int, optional  
The orders of the partial derivatives in  $x$  and  $y$  respectively.

**Returns**

- vals** : ndarray  
The B-spline or its derivative evaluated over the set formed by the cross-product of  $x$  and  $y$ .

**See also:**

`splprep`, `splrep`, `splint`, `sproot`, `splev`, `UnivariateSpline`, `BivariateSpline`

**Notes**

See `bisplprep` to generate the `tck` representation.

**References**

[R62], [R63], [R64]

## 5.7.5 Additional tools

---

<code>lagrange(x, w)</code>	Return a Lagrange interpolating polynomial.
<code>approximate_taylor_polynomial(f, x, degree, ...)</code>	Estimate the Taylor polynomial of $f$ at $x$ by polynomial fitting.

---

`scipy.interpolate.lagrange(x, w)`

Return a Lagrange interpolating polynomial.

Given two 1-D arrays  $x$  and  $w$ , returns the Lagrange interpolating polynomial through the points  $(x, w)$ .

Warning: This implementation is numerically unstable. Do not expect to be able to use more than about 20 points even if they are chosen optimally.

**Parameters**

- x** : array\_like  
 $x$  represents the x-coordinates of a set of datapoints.
- w** : array\_like  
 $w$  represents the y-coordinates of a set of datapoints, i.e.  $f(x)$ .

**Returns**

- lagrange** : numpy.poly1d instance  
The Lagrange interpolating polynomial.

`scipy.interpolate.approximate_taylor_polynomial(f, x, degree, scale, order=None)`

Estimate the Taylor polynomial of  $f$  at  $x$  by polynomial fitting.

**Parameters**

- f** : callable  
The function whose Taylor polynomial is sought. Should accept a vector of  $x$  values.
- x** : scalar  
The point at which the polynomial is to be evaluated.
- degree** : int  
The degree of the Taylor polynomial
- scale** : scalar

The width of the interval to use to evaluate the Taylor polynomial. Function values spread over a range this wide are used to fit the polynomial. Must be chosen carefully.

**order** : int or None, optional

The order of the polynomial to be used in the fitting;  $f$  will be evaluated `order+1` times. If None, use *degree*.

**Returns** **p** : poly1d instance

The Taylor polynomial (translated to the origin, so that for example  $p(0)=f(x)$ ).

**Notes**

The appropriate choice of “scale” is a trade-off; too large and the function differs from its Taylor polynomial too much to get a good answer, too small and round-off errors overwhelm the higher-order terms. The algorithm used becomes numerically unstable around order 30 even under ideal circumstances.

Choosing order somewhat larger than degree may improve the higher-order terms.

**See also:**

`scipy.ndimage.interpolation.map_coordinates`, `scipy.ndimage.interpolation.spline_filter`, `scipy.signal.resample`, `scipy.signal.bspline`, `scipy.signal.gauss_spline`, `scipy.signal.qspline1d`, `scipy.signal.cspline1d`, `scipy.signal.qspline1d_eval`, `scipy.signal.cspline1d_eval`, `scipy.signal.qspline2d`, `scipy.signal.cspline2d`.

Functions existing for backward compatibility (should not be used in new code):

<code>ppform(coeffs, breaks[, fill, sort])</code>	Deprecated piecewise polynomial class.
<code>spleval(xck, xnew[, deriv])</code>	Evaluate a fixed spline represented by the given tuple at the new x-values
<code>spline(xk, yk, xnew[, order, kind, conds])</code>	Interpolate a curve at new points using a spline fit
<code>splmake(xk, yk[, order, kind, conds])</code>	Return a representation of a spline given data-points at internal knots
<code>spltopp(xk, cvals, k)</code>	Return a piece-wise polynomial object from a fixed-spline tuple.
<code>pchip</code>	alias of <code>PchipInterpolator</code>

**class** `scipy.interpolate.ppform` (*coeffs, breaks, fill=0.0, sort=False*)

Deprecated piecewise polynomial class.

New code should use the `PPoly` class instead.

**Methods**

<code>__call__(x)</code>	
<code>antiderivative([nu])</code>	Construct a new piecewise polynomial representing the antiderivative.
<code>construct_fast(c, x[, extrapolate, axis])</code>	Construct the piecewise polynomial without making checks.
<code>derivative([nu])</code>	Construct a new piecewise polynomial representing the derivative.
<code>extend(c, x[, right])</code>	Add additional breakpoints and coefficients to the polynomial.
<code>from_bernstein_basis(bp[, extrapolate])</code>	Construct a piecewise polynomial in the power basis from a polynomial in Bernstein basis.
<code>from_spline(tck[, extrapolate])</code>	Construct a piecewise polynomial from a spline
<code>fromspline(xk, cvals, order[, fill])</code>	
<code>integrate(a, b[, extrapolate])</code>	Compute a definite integral over a piecewise polynomial.
<code>roots([discontinuity, extrapolate])</code>	Find real roots of the the piecewise polynomial.
<code>solve([y, discontinuity, extrapolate])</code>	Find real solutions of the the equation $pp(x) == y$ .

`ppform.__call__(x)`

`ppform.antiderivative` (*nu=1*)

Construct a new piecewise polynomial representing the antiderivative.

Antiderivative is also the indefinite integral of the function, and derivative is its inverse operation.

**Parameters** **nu** : int, optional  
Order of antiderivative to evaluate. Default is 1, i.e. compute the first integral. If negative, the derivative is returned.

**Returns** **pp** : PPoly  
Piecewise polynomial of order  $k_2 = k + n$  representing the antiderivative of this polynomial.

#### Notes

The antiderivative returned by this function is continuous and continuously differentiable to order  $n-1$ , up to floating point rounding error.

If antiderivative is computed and `self.extrapolate='periodic'`, it will be set to `False` for the returned instance. This is done because the antiderivative is no longer periodic and its correct evaluation outside of the initially given `x` interval is difficult.

**classmethod** `ppform.construct_fast` (*c*, *x*, *extrapolate=None*, *axis=0*)

Construct the piecewise polynomial without making checks.

Takes the same parameters as the constructor. Input arguments `c` and `x` must be arrays of the correct shape and type. The `c` array can only be of dtypes float and complex, and `x` array must have dtype float.

`ppform.derivative` (*nu=1*)

Construct a new piecewise polynomial representing the derivative.

**Parameters** **nu** : int, optional  
Order of derivative to evaluate. Default is 1, i.e. compute the first derivative. If negative, the antiderivative is returned.

**Returns** **pp** : PPoly  
Piecewise polynomial of order  $k_2 = k - n$  representing the derivative of this polynomial.

#### Notes

Derivatives are evaluated piecewise for each polynomial segment, even if the polynomial is not differentiable at the breakpoints. The polynomial intervals are considered half-open,  $[a, b)$ , except for the last interval which is closed  $[a, b]$ .

`ppform.extend` (*c*, *x*, *right=True*)

Add additional breakpoints and coefficients to the polynomial.

**Parameters** **c** : ndarray, size (k, m, ...)  
Additional coefficients for polynomials in intervals `self.x[-1] <= x < x_right[0]`, `x_right[0] <= x < x_right[1]`, ..., `x_right[m-2] <= x < x_right[m-1]`

**x** : ndarray, size (m)  
Additional breakpoints. Must be sorted and either to the right or to the left of the current breakpoints.

**right** : bool, optional  
Whether the new intervals are to the right or to the left of the current intervals.

**classmethod** `ppform.from_bernstein_basis` (*bp*, *extrapolate=None*)

Construct a piecewise polynomial in the power basis from a polynomial in Bernstein basis.

**Parameters** **bp** : BPoly  
A Bernstein basis polynomial, as created by BPoly

**extrapolate** : bool or 'periodic', optional

If bool, determines whether to extrapolate to out-of-bounds points based on first and last intervals, or to return NaNs. If 'periodic', periodic extrapolation is used. Default is True.

**classmethod** `ppform.from_spline` (*tck*, *extrapolate=None*)

Construct a piecewise polynomial from a spline

**Parameters** **tck**

A spline, as returned by `splrep`

**extrapolate** : bool or 'periodic', optional

If bool, determines whether to extrapolate to out-of-bounds points based on first and last intervals, or to return NaNs. If 'periodic', periodic extrapolation is used. Default is True.

**classmethod** `ppform.fromspline` (*xk*, *cvals*, *order*, *fill=0.0*)

`ppform.integrate` (*a*, *b*, *extrapolate=None*)

Compute a definite integral over a piecewise polynomial.

**Parameters** **a** : float

Lower integration bound

**b** : float

Upper integration bound

**extrapolate** : {bool, 'periodic', None}, optional

If bool, determines whether to extrapolate to out-of-bounds points based on first and last intervals, or to return NaNs. If 'periodic', periodic extrapolation is used. If None (default), use *self.extrapolate*.

**Returns** **ig** : array\_like

Definite integral of the piecewise polynomial over [a, b]

`ppform.roots` (*discontinuity=True*, *extrapolate=None*)

Find real roots of the the piecewise polynomial.

**Parameters** **discontinuity** : bool, optional

Whether to report sign changes across discontinuities at breakpoints as roots.

**extrapolate** : {bool, 'periodic', None}, optional

If bool, determines whether to return roots from the polynomial extrapolated based on first and last intervals, 'periodic' works the same as False. If None (default), use *self.extrapolate*.

**Returns** **roots** : ndarray

Roots of the polynomial(s).

If the PPoly object describes multiple polynomials, the return value is an object array whose each element is an ndarray containing the roots.

**See also:**

`PPoly.solve`

`ppform.solve` (*y=0.0*, *discontinuity=True*, *extrapolate=None*)

Find real solutions of the the equation  $pp(x) == y$ .

**Parameters** **y** : float, optional

Right-hand side. Default is zero.

**discontinuity** : bool, optional

Whether to report sign changes across discontinuities at breakpoints as roots.

**extrapolate** : {bool, 'periodic', None}, optional

If `bool`, determines whether to return roots from the polynomial extrapolated based on first and last intervals, 'periodic' works the same as `False`. If `None` (default), use `self.extrapolate`.

**Returns** `roots` : ndarray  
 Roots of the polynomial(s).  
 If the `PPoly` object describes multiple polynomials, the return value is an object array whose each element is an ndarray containing the roots.

### Notes

This routine works only on real-valued polynomials.

If the piecewise polynomial contains sections that are identically zero, the root list will contain the start point of the corresponding interval, followed by a `nan` value.

If the polynomial is discontinuous across a breakpoint, and there is a sign change across the breakpoint, this is reported if the `discont` parameter is `True`.

### Examples

Finding roots of  $[x^{**2} - 1, (x - 1)**2]$  defined on intervals  $[-2, 1], [1, 2]$ :

```
>>> from scipy.interpolate import PPoly
>>> pp = PPoly(np.array([[1, -4, 3], [1, 0, 0]]).T, [-2, 1, 2])
>>> pp.roots()
array([-1.,  1.]
```

`scipy.interpolate.spleval(xck, xnew, deriv=0)`

Evaluate a fixed spline represented by the given tuple at the new x-values

The  $x_j$  values are the interior knot points. The approximation region is  $x_j[0]$  to  $x_j[-1]$ . If  $N+1$  is the length of  $x_j$ , then  $cvals$  should have length  $N+k$  where  $k$  is the order of the spline.

**Parameters** `(xj, cvals, k)` : tuple  
 Parameters that define the fixed spline  
`xj` : array\_like  
 Interior knot points  
`cvals` : array\_like  
 Curvature  
`k` : int  
 Order of the spline  
`xnew` : array\_like  
 Locations to calculate spline  
`deriv` : int  
 Deriv

**Returns** `spleval` : ndarray  
 If `cvals` represents more than one curve (`cvals.ndim > 1`) and/or `xnew` is  $N$ -d, then the result is `xnew.shape + cvals.shape[1:]` providing the interpolation of multiple curves.

### Notes

Internally, an additional  $k-1$  knot points are added on either side of the spline.

`scipy.interpolate.spline(xk, yk, xnew, order=3, kind='smoothest', conds=None)`

Interpolate a curve at new points using a spline fit

**Parameters** `xk, yk` : array\_like  
 The x and y values that define the curve.  
`xnew` : array\_like  
 The x values where spline should estimate the y values.

**order** : int  
 Default is 3.  
**kind** : string  
 One of { 'smoothest' }  
**conds** : Don't know  
 Don't know  
**Returns** **spline** : ndarray  
 An array of y values; the spline evaluated at the positions *xnew*.

`scipy.interpolate.splmake` (*xk*, *yk*, *order=3*, *kind='smoothest'*, *conds=None*)  
 Return a representation of a spline given data-points at internal knots

**Parameters** **xk** : array\_like  
 The input array of x values of rank 1  
**yk** : array\_like  
 The input array of y values of rank N. *yk* can be an N-d array to represent more than one curve, through the same *xk* points. The first dimension is assumed to be the interpolating dimension and is the same length of *xk*.  
**order** : int, optional  
 Order of the spline  
**kind** : str, optional  
 Can be 'smoothest', 'not\_a\_knot', 'fixed', 'clamped', 'natural', 'periodic', 'symmetric', 'user', 'mixed' and it is ignored if order < 2  
**conds** : optional  
 Conds  
**Returns** **splmake** : tuple  
 Return a (*xk*, *cvals*, *k*) representation of a spline given data-points where the (internal) knots are at the data-points.

`scipy.interpolate.spltopp` (*xk*, *cvals*, *k*)  
 Return a piece-wise polynomial object from a fixed-spline tuple.

`scipy.interpolate.pchip`  
 alias of `PchipInterpolator`

## 5.8 Input and output (`scipy.io`)

SciPy has many modules, classes, and functions available to read data from and write data to a variety of file formats.

### See also:

*numpy-reference.routines.io* (in Numpy)

### 5.8.1 MATLAB® files

---

<code>loadmat</code> ( <i>file_name</i> [, <i>mdict</i> , <i>appendmat</i> ])	Load MATLAB file.
<code>savemat</code> ( <i>file_name</i> , <i>mdict</i> [, <i>appendmat</i> , ...])	Save a dictionary of names and arrays into a MATLAB-style .mat file.
<code>whosmat</code> ( <i>file_name</i> [, <i>appendmat</i> ])	List variables inside a MATLAB file.

---

`scipy.io.loadmat` (*file\_name*, *mdict=None*, *appendmat=True*, *\*\*kwargs*)  
 Load MATLAB file.

**Parameters** **file\_name** : str



Name of the mat file (do not need `.mat` extension if `appendmat==True`). Can also pass open file-like object.

**mdict** : dict, optional

Dictionary in which to insert matfile variables.

**appendmat** : bool, optional

True to append the `.mat` extension to the end of the given filename, if not already present.

**byte\_order** : str or None, optional

None by default, implying byte order guessed from mat file. Otherwise can be one of ('native', '=', 'little', '<', 'BIG', '>').

**mat\_dtype** : bool, optional

If True, return arrays in same dtype as would be loaded into MATLAB (instead of the dtype with which they are saved).

**squeeze\_me** : bool, optional

Whether to squeeze unit matrix dimensions or not.

**chars\_as\_strings** : bool, optional

Whether to convert char arrays to string arrays.

**matlab\_compatible** : bool, optional

Returns matrices as would be loaded by MATLAB (implies `squeeze_me=False`, `chars_as_strings=False`, `mat_dtype=True`, `struct_as_record=True`).

**struct\_as\_record** : bool, optional

Whether to load MATLAB structs as numpy record arrays, or as old-style numpy arrays with `dtype=object`. Setting this flag to False replicates the behavior of scipy version 0.7.x (returning numpy object arrays). The default setting is True, because it allows easier round-trip load and save of MATLAB files.

**verify\_compressed\_data\_integrity** : bool, optional

Whether the length of compressed sequences in the MATLAB file should be checked, to ensure that they are not longer than we expect. It is advisable to enable this (the default) because overlong compressed sequences in MATLAB files generally indicate that the files have experienced some sort of corruption.

**variable\_names** : None or sequence

If None (the default) - read all variables in file. Otherwise `variable_names` should be a sequence of strings, giving names of the matlab variables to read from the file. The reader will skip any variable with a name not in this sequence, possibly saving some read processing.

**Returns**

**mat\_dict** : dict

dictionary with variable names as keys, and loaded matrices as values.

### Notes

v4 (Level 1.0), v6 and v7 to 7.2 matfiles are supported.

You will need an HDF5 python library to read matlab 7.3 format mat files. Because scipy does not supply one, we do not implement the HDF5 / 7.3 interface here.

`scipy.io.savemat` (`file_name`, `mdict`, `appendmat=True`, `format='5'`, `long_field_names=False`, `do_compression=False`, `oned_as='row'`)

Save a dictionary of names and arrays into a MATLAB-style `.mat` file.

This saves the array objects in the given dictionary to a MATLAB- style `.mat` file.

**Parameters** **file\_name** : str or file-like object

Name of the `.mat` file (`.mat` extension not needed if `appendmat == True`). Can also pass open `file_like` object.

**mdict** : dict

Dictionary from which to save matfile variables.

**appendmat** : bool, optional

True (the default) to append the .mat extension to the end of the given filename, if not already present.

**format** : {'5', '4'}, string, optional

'5' (the default) for MATLAB 5 and up (to 7.2), '4' for MATLAB 4 .mat files.

**long\_field\_names** : bool, optional

False (the default) - maximum field name length in a structure is 31 characters which is the documented maximum length. True - maximum field name length in a structure is 63 characters which works for MATLAB 7.6+.

**do\_compression** : bool, optional

Whether or not to compress matrices on write. Default is False.

**oned\_as** : {'row', 'column'}, optional

If 'column', write 1-D numpy arrays as column vectors. If 'row', write 1-D numpy arrays as row vectors.

**See also:**

`mio4.MatFile4Writer`, `mio5.MatFile5Writer`

`scipy.io.whosmat` (*file\_name*, *appendmat=True*, *\*\*kwargs*)

List variables inside a MATLAB file.

**Parameters**

**file\_name** : str

Name of the mat file (do not need .mat extension if `appendmat==True`) Can also pass open file-like object.

**appendmat** : bool, optional

True to append the .mat extension to the end of the given filename, if not already present.

**byte\_order** : str or None, optional

None by default, implying byte order guessed from mat file. Otherwise can be one of ('native', '=', 'little', '<', 'BIG', '>').

**mat\_dtype** : bool, optional

If True, return arrays in same dtype as would be loaded into MATLAB (instead of the dtype with which they are saved).

**squeeze\_me** : bool, optional

Whether to squeeze unit matrix dimensions or not.

**chars\_as\_strings** : bool, optional

Whether to convert char arrays to string arrays.

**matlab\_compatible** : bool, optional

Returns matrices as would be loaded by MATLAB (implies `squeeze_me=False`, `chars_as_strings=False`, `mat_dtype=True`, `struct_as_record=True`).

**struct\_as\_record** : bool, optional

Whether to load MATLAB structs as numpy record arrays, or as old-style numpy arrays with `dtype=object`. Setting this flag to False replicates the behavior of scipy version 0.7.x (returning numpy object arrays). The default setting is True, because it allows easier round-trip load and save of MATLAB files.

**Returns**

**variables** : list of tuples

A list of tuples, where each tuple holds the matrix name (a string), its shape (tuple of ints), and its data class (a string). Possible data classes are: int8, uint8, int16, uint16, int32, uint32, int64, uint64, single, double, cell, struct, object, char, sparse, function, opaque, logical, unknown.

**Notes**

v4 (Level 1.0), v6 and v7 to 7.2 matfiles are supported.

You will need an HDF5 python library to read matlab 7.3 format mat files. Because scipy does not supply one, we do not implement the HDF5 / 7.3 interface here.

New in version 0.12.0.

## 5.8.2 IDL® files

---

`readsav(file_name[, idict, python_dict, ...])` Read an IDL .sav file.

---

`scipy.io.readsav` (*file\_name*, *idict=None*, *python\_dict=False*, *uncompressed\_file\_name=None*, *verbose=False*)  
Read an IDL .sav file.

**Parameters**

- file\_name** : str  
Name of the IDL save file.
- idict** : dict, optional  
Dictionary in which to insert .sav file variables.
- python\_dict** : bool, optional  
By default, the object return is not a Python dictionary, but a case-insensitive dictionary with item, attribute, and call access to variables. To get a standard Python dictionary, set this option to True.
- uncompressed\_file\_name** : str, optional  
This option only has an effect for .sav files written with the /compress option. If a file name is specified, compressed .sav files are uncompressed to this file. Otherwise, readsav will use the `tempfile` module to determine a temporary filename automatically, and will remove the temporary file upon successfully reading it in.
- verbose** : bool, optional  
Whether to print out information about the save file, including the records read, and available variables.

**Returns**

- idl\_dict** : AttrDict or dict  
If *python\_dict* is set to False (default), this function returns a case-insensitive dictionary with item, attribute, and call access to variables. If *python\_dict* is set to True, this function returns a Python dictionary with all variable names in lowercase. If *idict* was specified, then variables are written to the dictionary specified, and the updated dictionary is returned.

## 5.8.3 Matrix Market files

---

<code>mminfo</code> (source)	Return size and storage parameters from Matrix Market file-like 'source'.
<code>mmread</code> (source)	Reads the contents of a Matrix Market file-like 'source' into a matrix.
<code>mmwrite</code> (target, a[, comment, field, ...])	Writes the sparse or dense array <i>a</i> to Matrix Market file-like <i>target</i> .

---

`scipy.io.mminfo` (*source*)  
Return size and storage parameters from Matrix Market file-like 'source'.

**Parameters**

- source** : str or file-like  
Matrix Market filename (extension .mtx) or open file-like object

**Returns**

- rows** : int  
Number of matrix rows.
- cols** : int  
Number of matrix columns.
- entries** : int  
Number of non-zero entries of a sparse matrix or rows\*cols for a dense matrix.
- format** : str

Either 'coordinate' or 'array'.  
**field** : str  
 Either 'real', 'complex', 'pattern', or 'integer'.  
**symmetry** : str  
 Either 'general', 'symmetric', 'skew-symmetric', or 'hermitian'.

`scipy.io.mmread(source)`

Reads the contents of a Matrix Market file-like 'source' into a matrix.

**Parameters** **source** : str or file-like  
 Matrix Market filename (extensions .mtx, .mtz.gz) or open file-like object.  
**Returns** **a** : ndarray or coo\_matrix  
 Dense or sparse matrix depending on the matrix format in the Matrix Market file.

`scipy.io.mmwrite(target, a, comment='', field=None, precision=None, symmetry=None)`

Writes the sparse or dense array *a* to Matrix Market file-like *target*.

**Parameters** **target** : str or file-like  
 Matrix Market filename (extension .mtx) or open file-like object.  
**a** : array like  
 Sparse or dense 2D array.  
**comment** : str, optional  
 Comments to be prepended to the Matrix Market file.  
**field** : None or str, optional  
 Either 'real', 'complex', 'pattern', or 'integer'.  
**precision** : None or int, optional  
 Number of digits to display for real or complex values.  
**symmetry** : None or str, optional  
 Either 'general', 'symmetric', 'skew-symmetric', or 'hermitian'. If symmetry is None the symmetry type of 'a' is determined by its values.

## 5.8.4 Unformatted Fortran files

---

`FortranFile(filename[, mode, header_dtype])` A file object for unformatted sequential files from Fortran code.

---

**class** `scipy.io.FortranFile(filename, mode='r', header_dtype=<type 'numpy.uint32'>)`

A file object for unformatted sequential files from Fortran code.

**Parameters** **filename** : file or str  
 Open file object or filename.  
**mode** : {'r', 'w'}, optional  
 Read-write mode, default is 'r'.  
**header\_dtype** : dtype, optional  
 Data type of the header. Size and endianness must match the input/output file.

### Notes

These files are broken up into records of unspecified types. The size of each record is given at the start (although the size of this header is not standard) and the data is written onto disk without any formatting. Fortran compilers supporting the BACKSPACE statement will write a second copy of the size to facilitate backwards seeking.

This class only supports files written with both sizes for the record. It also does not support the subrecords used in Intel and gfortran compilers for records which are greater than 2GB with a 4-byte header.

An example of an unformatted sequential file in Fortran would be written as:

```
OPEN(1, FILE=myfilename, FORM='unformatted')

WRITE(1) myvariable
```

Since this is a non-standard file format, whose contents depend on the compiler and the endianness of the machine, caution is advised. Files from gfortran 4.8.0 and gfortran 4.1.2 on x86\_64 are known to work.

Consider using Fortran direct-access files or files from the newer Stream I/O, which can be easily read by `numpy.fromfile`.

### Examples

To create an unformatted sequential Fortran file:

```
>>> from scipy.io import FortranFile
>>> f = FortranFile('test.unf', 'w')
>>> f.write_record(np.array([1,2,3,4,5], dtype=np.int32))
>>> f.write_record(np.linspace(0,1,20).reshape((5,-1)))
>>> f.close()
```

To read this file:

```
>>> from scipy.io import FortranFile
>>> f = FortranFile('test.unf', 'r')
>>> print(f.read_ints(dtype=np.int32))
[1 2 3 4 5]
>>> print(f.read_reals(dtype=float).reshape((5,-1)))
[[ 0.          0.05263158  0.10526316  0.15789474]
 [ 0.21052632  0.26315789  0.31578947  0.36842105]
 [ 0.42105263  0.47368421  0.52631579  0.57894737]
 [ 0.63157895  0.68421053  0.73684211  0.78947368]
 [ 0.84210526  0.89473684  0.94736842  1.          ]]
>>> f.close()
```

### Methods

<code>close()</code>	Closes the file.
<code>read_ints([dtype])</code>	Reads a record of a given type from the file, defaulting to an integer
<code>read_reals([dtype])</code>	Reads a record of a given type from the file, defaulting to a floating
<code>read_record([dtype])</code>	Reads a record of a given type from the file.
<code>write_record(s)</code>	Write a record (including sizes) to the file.

`FortranFile.close()`

Closes the file. It is unsupported to call any other methods off this object after closing it. Note that this class supports the ‘with’ statement in modern versions of Python, to call this automatically

`FortranFile.read_ints(dtype='i4')`

Reads a record of a given type from the file, defaulting to an integer type (INTEGER\*4 in Fortran)

**Parameters** `dtype` : dtype, optional  
Data type specifying the size and endness of the data.

**Returns** `data` : ndarray  
A one-dimensional array object.

**See also:**

`read_reals`, `read_record`

FortranFile.**read\_reals** (*dtype='f8'*)

Reads a record of a given type from the file, defaulting to a floating point number (real\*8 in Fortran)

**Parameters** **dtype** : dtype, optional  
Data type specifying the size and endianness of the data.  
**Returns** **data** : ndarray  
A one-dimensional array object.

**See also:**

`read_ints`, `read_record`

FortranFile.**read\_record** (*dtype=None*)

Reads a record of a given type from the file.

**Parameters** **dtype** : dtype, optional  
Data type specifying the size and endianness of the data.  
**Returns** **data** : ndarray  
A one-dimensional array object.

**See also:**

`read_reals`, `read_ints`

### Notes

If the record contains a multi-dimensional array, calling `reshape` or `resize` will restructure the array to the correct size. Since Fortran multidimensional arrays are stored in column-major format, this may have some non-intuitive consequences. If the variable was declared as `'INTEGER var(5,4)'`, for example, `var` could be read with `'read_record(dtype=np.integer).reshape( (4,5) )'` since Python uses row-major ordering of indices.

One can transpose to obtain the indices in the same order as in Fortran.

For records that contain several variables or mixed types (as opposed to single scalar or array types), it is possible to specify a dtype with mixed types:

```
record = f.read_record([('a', '<f4'), ('b', '<i4')])
record['a'] # access the variable 'a'
```

and if any of the variables are arrays, the shape can be specified as the third item in the relevant tuple:

```
record = f.read_record([('a', '<f4'), ('b', '<i4', (3,3))])
```

Numpy also supports a short syntax for this kind of type:

```
record = f.read_record('<f4, (3,3)<i4')
record['f0'] # variables are called f0, f1, ...
```

FortranFile.**write\_record** (*s*)

Write a record (including sizes) to the file.

**Parameters** **s** : array\_like  
The data to write.

## 5.8.5 Netcdf

---

<code>netcdf_file(filename[, mode, mmap, version, ...])</code>	A file object for NetCDF data.
<code>netcdf_variable(data, typecode, size, shape, ...)</code>	A data object for the <code>netcdf</code> module.

---

**class** `scipy.io.netcdf_file` (*filename*, *mode*='r', *mmap*=None, *version*=1, *maskandscale*=False)  
 A file object for NetCDF data.

A `netcdf_file` object has two standard attributes: *dimensions* and *variables*. The values of both are dictionaries, mapping dimension names to their associated lengths and variable names to variables, respectively. Application programs should never modify these dictionaries.

All other attributes correspond to global attributes defined in the NetCDF file. Global file attributes are created by assigning to an attribute of the `netcdf_file` object.

**Parameters**

- filename** : string or file-like  
 string -> filename
- mode** : {'r', 'w', 'a'}, optional  
 read-write-append mode, default is 'r'
- mmap** : None or bool, optional  
 Whether to mmap *filename* when reading. Default is True when *filename* is a file name, False when *filename* is a file-like object. Note that when *mmap* is in use, data arrays returned refer directly to the mmapmed data on disk, and the file cannot be closed as long as references to it exist.
- version** : {1, 2}, optional  
 version of netcdf to read / write, where 1 means *Classic format* and 2 means *64-bit offset format*. Default is 1. See [here](#) for more info.
- maskandscale** : bool, optional  
 Whether to automatically scale and/or mask data based on attributes. Default is False.

### Notes

The major advantage of this module over other modules is that it doesn't require the code to be linked to the NetCDF libraries. This module is derived from [pupynere](#).

NetCDF files are a self-describing binary data format. The file contains metadata that describes the dimensions and variables in the file. More details about NetCDF files can be found [here](#). There are three main sections to a NetCDF data structure:

1. Dimensions
2. Variables
3. Attributes

The dimensions section records the name and length of each dimension used by the variables. The variables would then indicate which dimensions it uses and any attributes such as data units, along with containing the data values for the variable. It is good practice to include a variable that is the same name as a dimension to provide the values for that axes. Lastly, the attributes section would contain additional information such as the name of the file creator or the instrument used to collect the data.

When writing data to a NetCDF file, there is often the need to indicate the 'record dimension'. A record dimension is the unbounded dimension for a variable. For example, a temperature variable may have dimensions of latitude, longitude and time. If one wants to add more temperature data to the NetCDF file as time progresses, then the temperature variable should have the time dimension flagged as the record dimension.

In addition, the NetCDF file header contains the position of the data in the file, so access can be done in an efficient manner without loading unnecessary data into memory. It uses the `mmap` module to create Numpy arrays mapped to the data on disk, for the same purpose.

Note that when `netcdf_file` is used to open a file with `mmap=True` (default for read-only), arrays returned by it refer to data directly on the disk. The file should not be closed, and cannot be cleanly closed when asked, if such arrays are alive. You may want to copy data arrays obtained from mmapmed Netcdf file if they are to be processed after the file is closed, see the example below.

### Examples

To create a NetCDF file:

```
>>> from scipy.io import netcdf
>>> f = netcdf.netcdf_file('simple.nc', 'w')
>>> f.history = 'Created for a test'
>>> f.createDimension('time', 10)
>>> time = f.createVariable('time', 'i', ('time',))
>>> time[:] = np.arange(10)
>>> time.units = 'days since 2008-01-01'
>>> f.close()
```

Note the assignment of `range(10)` to `time[:]`. Exposing the slice of the time variable allows for the data to be set in the object, rather than letting `range(10)` overwrite the `time` variable.

To read the NetCDF file we just created:

```
>>> from scipy.io import netcdf
>>> f = netcdf.netcdf_file('simple.nc', 'r')
>>> print(f.history)
Created for a test
>>> time = f.variables['time']
>>> print(time.units)
days since 2008-01-01
>>> print(time.shape)
(10,)
>>> print(time[-1])
9
```

NetCDF files, when opened read-only, return arrays that refer directly to memory-mapped data on disk:

```
>>> data = time[:]
>>> data.base.base
<mmap.mmap object at 0x7fe753763180>
```

If the data is to be processed after the file is closed, it needs to be copied to main memory:

```
>>> data = time[:].copy()
>>> f.close()
>>> data.mean()
4.5
```

A NetCDF file can also be used as context manager:

```
>>> from scipy.io import netcdf
>>> with netcdf.netcdf_file('simple.nc', 'r') as f:
...     print(f.history)
Created for a test
```

### Methods

<code>close()</code>	Closes the NetCDF file.
<code>createDimension(name, length)</code>	Adds a dimension to the Dimension section of the NetCDF data structure.
<code>createVariable(name, type, dimensions)</code>	Create an empty variable for the <code>netcdf_file</code> object, specifying its data type and
<code>flush()</code>	Perform a sync-to-disk flush if the <code>netcdf_file</code> object is in write mode.
<code>sync()</code>	Perform a sync-to-disk flush if the <code>netcdf_file</code> object is in write mode.



`netcdf_file.close()`  
Closes the NetCDF file.

`netcdf_file.createDimension(name, length)`  
Adds a dimension to the Dimension section of the NetCDF data structure.

Note that this function merely adds a new dimension that the variables can reference. The values for the dimension, if desired, should be added as a variable using `createVariable`, referring to this dimension.

**Parameters**

- name** : str  
Name of the dimension (Eg, 'lat' or 'time').
- length** : int  
Length of the dimension.

**See also:**

`createVariable`

`netcdf_file.createVariable(name, type, dimensions)`  
Create an empty variable for the `netcdf_file` object, specifying its data type and the dimensions it uses.

**Parameters**

- name** : str  
Name of the new variable.
- type** : dtype or str  
Data type of the variable.
- dimensions** : sequence of str  
List of the dimension names used by the variable, in the desired order.

**Returns**

- variable** : `netcdf_variable`  
The newly created `netcdf_variable` object. This object has also been added to the `netcdf_file` object as well.

**See also:**

`createDimension`

**Notes**

Any dimensions to be used by the variable should already exist in the NetCDF data structure or should be created by `createDimension` prior to creating the NetCDF variable.

`netcdf_file.flush()`  
Perform a sync-to-disk flush if the `netcdf_file` object is in write mode.

**See also:**

`sync` Identical function

`netcdf_file.sync()`  
Perform a sync-to-disk flush if the `netcdf_file` object is in write mode.

**See also:**

`sync` Identical function

**class** `scipy.io.netcdf_variable`(*data, typecode, size, shape, dimensions, attributes=None, maskand-scale=False*)  
A data object for the `netcdf` module.

`netcdf_variable` objects are constructed by calling the method `netcdf_file.createVariable` on the `netcdf_file` object. `netcdf_variable` objects behave much like array objects defined in `numpy`, except that their data resides in a file. Data is read by indexing and written by assigning to an indexed subset; the entire array can be accessed by the index `[:]` or (for scalars) by using the methods `getValue` and `assignValue`. `netcdf_variable` objects also have attribute `shape` with the same meaning as for arrays, but the shape cannot be modified. There is another read-only attribute `dimensions`, whose value is the tuple of dimension names.

All other attributes correspond to variable attributes defined in the NetCDF file. Variable attributes are created by assigning to an attribute of the `netcdf_variable` object.

**Parameters**

- data** : array\_like  
The data array that holds the values for the variable. Typically, this is initialized as empty, but with the proper shape.
- typecode** : dtype character code  
Desired data-type for the data array.
- size** : int  
Desired element size for the data array.
- shape** : sequence of ints  
The shape of the array. This should match the lengths of the variable's dimensions.
- dimensions** : sequence of strings  
The names of the dimensions used by the variable. Must be in the same order of the dimension lengths given by `shape`.
- attributes** : dict, optional  
Attribute values (any type) keyed by string names. These attributes become attributes for the `netcdf_variable` object.
- maskandscale** : bool, optional  
Whether to automatically scale and/or mask data based on attributes. Default is False.

**See also:**

`isrec`, `shape`

**Attributes**

<code>dimensions</code>	(list of str) List of names of dimensions used by the variable object.
<code>isrec</code> , <code>shape</code>	Properties

**Methods**

---

<code>assignValue(value)</code>	Assign a scalar value to a <code>netcdf_variable</code> of length one.
<code>getValue()</code>	Retrieve a scalar value from a <code>netcdf_variable</code> of length one.
<code>itemsize()</code>	Return the itemsize of the variable.
<code>typecode()</code>	Return the typecode of the variable.

---

`netcdf_variable.assignValue(value)`  
Assign a scalar value to a `netcdf_variable` of length one.

**Parameters**

- value** : scalar  
Scalar value (of compatible type) to assign to a length-one netcdf variable. This value will be written to file.

**Raises**

- ValueError**  
If the input is not a scalar, or if the destination is not a length-one netcdf variable.

`netcdf_variable.getValue()`  
Retrieve a scalar value from a `netcdf_variable` of length one.

**Raises**      **ValueError**

If the netcdf variable is an array of length greater than one, this exception will be raised.

`netcdf_variable.itemsize()`  
Return the itemsize of the variable.

**Returns**      **itemsize** : int  
The element size of the variable (eg, 8 for float64).

`netcdf_variable.typecode()`  
Return the typecode of the variable.

**Returns**      **typecode** : char  
The character typecode of the variable (eg, 'i' for int).

## 5.8.6 Harwell-Boeing files

<code>hb_read(file)</code>	Read HB-format file.
<code>hb_write(file, m[, hb_info])</code>	Write HB-format file.

`scipy.io.hb_read(file)`  
Read HB-format file.

**Parameters**    **file** : str-like or file-like  
If a string-like object, file is the name of the file to read. If a file-like object, the data are read from it.

**Returns**        **data** : `scipy.sparse.csc_matrix` instance  
The data read from the HB file as a sparse matrix.

**Notes**

At the moment not the full Harwell-Boeing format is supported. Supported features are:

- assembled, non-symmetric, real matrices
- integer for pointer/indices
- exponential format for float values, and int format

`scipy.io.hb_write(file, m, hb_info=None)`  
Write HB-format file.

**Parameters**    **file** : str-like or file-like  
if a string-like object, file is the name of the file to read. If a file-like object, the data are read from it.

**m** : sparse-matrix  
the sparse matrix to write

**hb\_info** : `HBInfo`  
contains the meta-data for write

**Returns**        None

**Notes**

At the moment not the full Harwell-Boeing format is supported. Supported features are:

- assembled, non-symmetric, real matrices
- integer for pointer/indices

- exponential format for float values, and int format

## 5.8.7 Wav sound files (`scipy.io.wavfile`)

---

<code>read(filename[, mmap])</code>	Open a WAV file
<code>write(filename, rate, data)</code>	Write a numpy array as a WAV file.
<code>WavFileWarning</code>	

---

`scipy.io.wavfile.read(filename, mmap=False)`

Open a WAV file

Return the sample rate (in samples/sec) and data from a WAV file.

**Parameters**

**filename** : string or open file handle  
Input wav file.

**mmap** : bool, optional  
Whether to read data as memory-mapped. Only to be used on real files (Default: False).  
New in version 0.12.0.

**Returns**

**rate** : int  
Sample rate of wav file.

**data** : numpy array  
Data read from wav file. Data-type is determined from the file; see Notes.

### Notes

This function cannot read wav files with 24-bit data.

Common data types: [R88]

WAV format	Min	Max	NumPy dtype
32-bit floating-point	-1.0	+1.0	float32
32-bit PCM	-2147483648	+2147483647	int32
16-bit PCM	-32768	+32767	int16
8-bit PCM	0	255	uint8

Note that 8-bit PCM is unsigned.

### References

[R88]

`scipy.io.wavfile.write(filename, rate, data)`

Write a numpy array as a WAV file.

**Parameters**

**filename** : string or open file handle  
Output wav file.

**rate** : int  
The sample rate (in samples/sec).

**data** : ndarray  
A 1-D or 2-D numpy array of either integer or float data-type.

### Notes

- Writes a simple uncompressed WAV file.
- To write multiple-channels, use a 2-D array of shape (Nsamples, Nchannels).

- The bits-per-sample and PCM/float will be determined by the data-type.

Common data types: [R89]

WAV format	Min	Max	NumPy dtype
32-bit floating-point	-1.0	+1.0	float32
32-bit PCM	-2147483648	+2147483647	int32
16-bit PCM	-32768	+32767	int16
8-bit PCM	0	255	uint8

Note that 8-bit PCM is unsigned.

### References

[R89]

exception `scipy.io.wavfile.WavFileWarning`

## 5.8.8 Arff files (`scipy.io.arff`)

---

<code>loadarff(f)</code>	Read an arff file.
<code>MetaData(rel, attr)</code>	Small container to keep useful informations on a ARFF dataset.
<code>ArffError</code>	
<code>ParseArffError</code>	

---

`scipy.io.arff.loadarff(f)`

Read an arff file.

The data is returned as a record array, which can be accessed much like a dictionary of numpy arrays. For example, if one of the attributes is called ‘pressure’, then its first 10 data points can be accessed from the data record array like so: `data['pressure'][0:10]`

<b>Parameters</b>	<b>f</b> : file-like or str File-like object to read from, or filename to open.
<b>Returns</b>	<b>data</b> : record array The data of the arff file, accessible by attribute names. <b>meta</b> : <code>MetaData</code> Contains information about the arff file such as name and type of attributes, the relation (name of the dataset), etc...
<b>Raises</b>	<b>ParseArffError</b> This is raised if the given file is not ARFF-formatted. <b>NotImplementedError</b> The ARFF file has an attribute which is not supported yet.

### Notes

This function should be able to read most arff files. Not implemented functionality include:

- date type attributes
- string type attributes

It can read files with numeric and nominal attributes. It cannot read files with sparse data (`{}` in the file). However, this function can read files with missing data (`?` in the file), representing the data points as NaNs.

*Examples*

```
>>> from scipy.io import arff
>>> from StringIO import StringIO
>>> content = """
... @relation foo
... @attribute width numeric
... @attribute height numeric
... @attribute color {red,green,blue,yellow,black}
... @data
... 5.0,3.25,blue
... 4.5,3.75,green
... 3.0,4.00,red
... """
>>> f = StringIO(content)
>>> data, meta = arff.loadarff(f)
>>> data
array([(5.0, 3.25, 'blue'), (4.5, 3.75, 'green'), (3.0, 4.0, 'red')],
      dtype=[('width', '<f8'), ('height', '<f8'), ('color', '|S6')])
>>> meta
Dataset: foo
      width's type is numeric
      height's type is numeric
      color's type is nominal, range is ('red', 'green', 'blue', 'yellow', 'black')
```

**class** `scipy.io.arff.MetaData` (*rel, attr*)  
 Small container to keep useful informations on a ARFF dataset.

Knows about attributes names and types.

*Notes*

Also maintains the list of attributes in order, i.e. doing for *i* in *meta*, where *meta* is an instance of `MetaData`, will return the different attribute names in the order they were defined.

*Examples*

```
data, meta = loadarff('iris.arff')
# This will print the attributes names of the iris.arff dataset
for i in meta:
    print i
# This works too
meta.names()
# Getting attribute type
types = meta.types()
```

*Methods*

<code>names()</code>	Return the list of attribute names.
<code>types()</code>	Return the list of attribute types.

`MetaData.names()`  
 Return the list of attribute names.

`MetaData.types()`  
 Return the list of attribute types.

**exception** `scipy.io.arff.ArffError`

exception `scipy.io.arff.ParseArffError`

## 5.9 Linear algebra (`scipy.linalg`)

Linear algebra functions.

**See also:**

`numpy.linalg` for more linear algebra functions. Note that although `scipy.linalg` imports most of them, identically named functions from `scipy.linalg` may offer more or slightly differing functionality.

### 5.9.1 Basics

---

<code>inv(a[, overwrite_a, check_finite])</code>	Compute the inverse of a matrix.
<code>solve(a, b[, sym_pos, lower, overwrite_a, ...])</code>	Solve the equation $a x = b$ for $x$ .
<code>solve_banded(l_and_u, ab, b[, overwrite_ab, ...])</code>	Solve the equation $a x = b$ for $x$ , assuming $a$ is banded matrix.
<code>solveh_banded(ab, b[, overwrite_ab, ...])</code>	Solve equation $a x = b$ .
<code>solve_circulant(c, b[, singular, tol, ...])</code>	Solve $C x = b$ for $x$ , where $C$ is a circulant matrix.
<code>solve_triangular(a, b[, trans, lower, ...])</code>	Solve the equation $a x = b$ for $x$ , assuming $a$ is a triangular matrix.
<code>solve_toeplitz(c_or_cr, b[, check_finite])</code>	Solve a Toeplitz system using Levinson Recursion
<code>det(a[, overwrite_a, check_finite])</code>	Compute the determinant of a matrix
<code>norm(a[, ord, axis, keepdims])</code>	Matrix or vector norm.
<code>lstsq(a, b[, cond, overwrite_a, ...])</code>	Compute least-squares solution to equation $Ax = b$ .
<code>pinv(a[, cond, rcond, return_rank, check_finite])</code>	Compute the (Moore-Penrose) pseudo-inverse of a matrix.
<code>pinv2(a[, cond, rcond, return_rank, ...])</code>	Compute the (Moore-Penrose) pseudo-inverse of a matrix.
<code>pinvh(a[, cond, rcond, lower, return_rank, ...])</code>	Compute the (Moore-Penrose) pseudo-inverse of a Hermitian matrix.
<code>kron(a, b)</code>	Kronecker product.
<code>tril(m[, k])</code>	Make a copy of a matrix with elements above the $k$ -th diagonal zeroed.
<code>triu(m[, k])</code>	Make a copy of a matrix with elements below the $k$ -th diagonal zeroed.
<code>orthogonal_procrustes(A, B[, check_finite])</code>	Compute the matrix solution of the orthogonal Procrustes problem.
<code>LinAlgError</code>	Generic Python-exception-derived object raised by <code>linalg</code> functions.

---

`scipy.linalg.inv(a, overwrite_a=False, check_finite=True)`

Compute the inverse of a matrix.

**Parameters** **a** : array\_like

Square matrix to be inverted.

**overwrite\_a** : bool, optional

Discard data in  $a$  (may improve performance). Default is False.

**check\_finite** : bool, optional

Whether to check that the input matrix contains only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.

**Returns** **ainv** : ndarray

Inverse of the matrix  $a$ .

**Raises** **LinAlgError**

If  $a$  is singular.

**ValueError**

If  $a$  is not square, or not 2-dimensional.

### Examples

```
>>> from scipy import linalg
>>> a = np.array([[1., 2.], [3., 4.]])
>>> linalg.inv(a)
array([[ -2. ,  1. ],
       [ 1.5, -0.5]])
>>> np.dot(a, linalg.inv(a))
array([[ 1.,  0.],
       [ 0.,  1.]])
```

`scipy.linalg.solve`(*a*, *b*, *sym\_pos*=False, *lower*=False, *overwrite\_a*=False, *overwrite\_b*=False, *debug*=False, *check\_finite*=True)

Solve the equation  $a x = b$  for  $x$ .

**Parameters**

- a** : (M, M) array\_like  
A square matrix.
- b** : (M,) or (M, N) array\_like  
Right-hand side matrix in  $a x = b$ .
- sym\_pos** : bool, optional  
Assume *a* is symmetric and positive definite.
- lower** : bool, optional  
Use only data contained in the lower triangle of *a*, if *sym\_pos* is true. Default is to use upper triangle.
- overwrite\_a** : bool, optional  
Allow overwriting data in *a* (may enhance performance). Default is False.
- overwrite\_b** : bool, optional  
Allow overwriting data in *b* (may enhance performance). Default is False.
- check\_finite** : bool, optional  
Whether to check that the input matrices contain only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.

**Returns**

- x** : (M,) or (M, N) ndarray  
Solution to the system  $a x = b$ . Shape of the return matches the shape of *b*.

**Raises**

- LinAlgError**  
If *a* is singular.
- ValueError**  
If *a* is not square

### Examples

Given *a* and *b*, solve for *x*:

```
>>> a = np.array([[3, 2, 0], [1, -1, 0], [0, 5, 1]])
>>> b = np.array([2, 4, -1])
>>> from scipy import linalg
>>> x = linalg.solve(a, b)
>>> x
array([ 2., -2.,  9.])
>>> np.dot(a, x) == b
array([ True,  True,  True], dtype=bool)
```

`scipy.linalg.solve_banded`(*l\_and\_u*, *ab*, *b*, *overwrite\_ab*=False, *overwrite\_b*=False, *debug*=False, *check\_finite*=True)

Solve the equation  $a x = b$  for  $x$ , assuming *a* is banded matrix.

The matrix *a* is stored in *ab* using the matrix diagonal ordered form:



```
ab[u + i - j, j] == a[i, j]
```

Example of  $ab$  (shape of  $a$  is (6,6),  $u=1$ ,  $l=2$ ):

```
*   a01  a12  a23  a34  a45
a00  a11  a22  a33  a44  a55
a10  a21  a32  a43  a54  *
a20  a31  a42  a53  *   *
```

**Parameters**

- (l, u)** : (integer, integer)  
Number of non-zero lower and upper diagonals
- ab** : ( $l + u + 1$ , M) array\_like  
Banded matrix
- b** : (M,) or (M, K) array\_like  
Right-hand side
- overwrite\_ab** : bool, optional  
Discard data in  $ab$  (may enhance performance)
- overwrite\_b** : bool, optional  
Discard data in  $b$  (may enhance performance)
- check\_finite** : bool, optional  
Whether to check that the input matrices contain only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.

**Returns**

- x** : (M,) or (M, K) ndarray  
The solution to the system  $a x = b$ . Returned shape depends on the shape of  $b$ .

```
scipy.linalg.solveh_banded(ab, b, overwrite_ab=False, overwrite_b=False, lower=False,
                           check_finite=True)
```

Solve equation  $a x = b$ .  $a$  is Hermitian positive-definite banded matrix.

The matrix  $a$  is stored in  $ab$  either in lower diagonal or upper diagonal ordered form:

```
ab[u + i - j, j] == a[i, j] (if upper form; i <= j)
ab[ i - j, j] == a[i, j] (if lower form; i >= j)
```

Example of  $ab$  (shape of  $a$  is (6, 6),  $u=2$ ):

```
upper form:
*   *   a02  a13  a24  a35
*  a01  a12  a23  a34  a45
a00  a11  a22  a33  a44  a55
```

```
lower form:
a00  a11  a22  a33  a44  a55
a10  a21  a32  a43  a54  *
a20  a31  a42  a53  *   *
```

Cells marked with \* are not used.

**Parameters**

- ab** : ( $u + 1$ , M) array\_like  
Banded matrix
- b** : (M,) or (M, K) array\_like  
Right-hand side
- overwrite\_ab** : bool, optional  
Discard data in  $ab$  (may enhance performance)
- overwrite\_b** : bool, optional  
Discard data in  $b$  (may enhance performance)
- lower** : bool, optional  
Is the matrix in the lower form. (Default is upper form)

**check\_finite** : bool, optional

Whether to check that the input matrices contain only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.

**Returns** **x** : (M,) or (M, K) ndarray

The solution to the system  $a x = b$ . Shape of return matches shape of  $b$ .

`scipy.linalg.solve_circulant` ( $c$ ,  $b$ ,  $singular='raise'$ ,  $tol=None$ ,  $caxis=-1$ ,  $baxis=0$ ,  $outaxis=0$ )

Solve  $C x = b$  for  $x$ , where  $C$  is a circulant matrix.

$C$  is the circulant matrix associated with the vector  $c$ .

The system is solved by doing division in Fourier space. The calculation is:

```
x = ifft(fft(b) / fft(c))
```

where *fft* and *ifft* are the fast Fourier transform and its inverse, respectively. For a large vector  $c$ , this is *much* faster than solving the system with the full circulant matrix.

**Parameters** **c** : array\_like

The coefficients of the circulant matrix.

**b** : array\_like

Right-hand side matrix in  $a x = b$ .

**singular** : str, optional

This argument controls how a near singular circulant matrix is handled. If *singular* is “raise” and the circulant matrix is near singular, a `LinAlgError` is raised. If *singular* is “lstsq”, the least squares solution is returned. Default is “raise”.

**tol** : float, optional

If any eigenvalue of the circulant matrix has an absolute value that is less than or equal to *tol*, the matrix is considered to be near singular. If not given, *tol* is set to:

```
tol = abs_eigs.max() * abs_eigs.size * np.finfo(np.float64).eps
```

where *abs\_eigs* is the array of absolute values of the eigenvalues of the circulant matrix.

**caxis** : int

When  $c$  has dimension greater than 1, it is viewed as a collection of circulant vectors. In this case, *caxis* is the axis of  $c$  that holds the vectors of circulant coefficients.

**baxis** : int

When  $b$  has dimension greater than 1, it is viewed as a collection of vectors. In this case, *baxis* is the axis of  $b$  that holds the right-hand side vectors.

**outaxis** : int

When  $c$  or  $b$  are multidimensional, the value returned by `solve_circulant` is multidimensional. In this case, *outaxis* is the axis of the result that holds the solution vectors.

**Returns** **x** : ndarray

Solution to the system  $C x = b$ .

**Raises** **LinAlgError**

If the circulant matrix associated with  $c$  is near singular.

**See also:**

`circulant`

**Notes**

For a one-dimensional vector  $c$  with length  $m$ , and an array  $b$  with shape  $(m, \dots)$ ,

```
solve_circulant(c, b)
```

returns the same result as

```
solve(circulant(c), b)
```

where `solve` and `circulant` are from `scipy.linalg`.

New in version 0.16.0.

### Examples

```
>>> from scipy.linalg import solve_circulant, solve, circulant, lstsq

>>> c = np.array([2, 2, 4])
>>> b = np.array([1, 2, 3])
>>> solve_circulant(c, b)
array([ 0.75, -0.25,  0.25])
```

Compare that result to solving the system with `scipy.linalg.solve`:

```
>>> solve(circulant(c), b)
array([ 0.75, -0.25,  0.25])
```

A singular example:

```
>>> c = np.array([1, 1, 0, 0])
>>> b = np.array([1, 2, 3, 4])
```

Calling `solve_circulant(c, b)` will raise a `LinAlgError`. For the least square solution, use the option `singular='lstsq'`:

```
>>> solve_circulant(c, b, singular='lstsq')
array([ 0.25,  1.25,  2.25,  1.25])
```

Compare to `scipy.linalg.lstsq`:

```
>>> x, resid, rnk, s = lstsq(circulant(c), b)
>>> x
array([ 0.25,  1.25,  2.25,  1.25])
```

A broadcasting example:

Suppose we have the vectors of two circulant matrices stored in an array with shape (2, 5), and three *b* vectors stored in an array with shape (3, 5). For example,

```
>>> c = np.array([[1.5, 2, 3, 0, 0], [1, 1, 4, 3, 2]])
>>> b = np.arange(15).reshape(-1, 5)
```

We want to solve all combinations of circulant matrices and *b* vectors, with the result stored in an array with shape (2, 3, 5). When we disregard the axes of *c* and *b* that hold the vectors of coefficients, the shapes of the collections are (2,) and (3,), respectively, which are not compatible for broadcasting. To have a broadcast result with shape (2, 3), we add a trivial dimension to *c*: `c[:, np.newaxis, :]` has shape (2, 1, 5). The last dimension holds the coefficients of the circulant matrices, so when we call `solve_circulant`, we can use the default `caxis=-1`. The coefficients of the *b* vectors are in the last dimension of the array *b*, so we use `baxis=-1`. If we use the default `outaxis`, the result will have shape (5, 2, 3), so we'll use `outaxis=-1` to put the solution vectors in the last dimension.

```
>>> x = solve_circulant(c[:, np.newaxis, :], b, baxis=-1, outaxis=-1)
>>> x.shape
(2, 3, 5)
>>> np.set_printoptions(precision=3) # For compact output of numbers.
>>> x
```

```
array([[[-0.118,  0.22 ,  1.277, -0.142,  0.302],
        [ 0.651,  0.989,  2.046,  0.627,  1.072],
        [ 1.42 ,  1.758,  2.816,  1.396,  1.841]],
       [[ 0.401,  0.304,  0.694, -0.867,  0.377],
        [ 0.856,  0.758,  1.149, -0.412,  0.831],
        [ 1.31 ,  1.213,  1.603,  0.042,  1.286]])
```

Check by solving one pair of  $c$  and  $b$  vectors (cf. `x[1, 1, :]`):

```
>>> solve_circulant(c[1], b[1, :])
array([ 0.856,  0.758,  1.149, -0.412,  0.831])
```

`scipy.linalg.solve_triangular(a, b, trans=0, lower=False, unit_diagonal=False, overwrite_b=False, debug=False, check_finite=True)`

Solve the equation  $ax = b$  for  $x$ , assuming  $a$  is a triangular matrix.

**Parameters** **a** : (M, M) array\_like

A triangular matrix

**b** : (M,) or (M, N) array\_like

Right-hand side matrix in  $ax = b$

**lower** : bool, optional

Use only data contained in the lower triangle of  $a$ . Default is to use upper triangle.

**trans** : {0, 1, 2, 'N', 'T', 'C'}, optional

Type of system to solve:

trans	system
0 or 'N'	$ax = b$
1 or 'T'	$a^T x = b$
2 or 'C'	$a^H x = b$

**unit\_diagonal** : bool, optional

If True, diagonal elements of  $a$  are assumed to be 1 and will not be referenced.

**overwrite\_b** : bool, optional

Allow overwriting data in  $b$  (may enhance performance)

**check\_finite** : bool, optional

Whether to check that the input matrices contain only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.

**Returns** **x** : (M,) or (M, N) ndarray

Solution to the system  $ax = b$ . Shape of return matches  $b$ .

**Raises** **LinAlgError**

If  $a$  is singular

### Notes

New in version 0.9.0.

`scipy.linalg.solve_toeplitz(c_or_cr, b, check_finite=True)`

Solve a Toeplitz system using Levinson Recursion

The Toeplitz matrix has constant diagonals, with  $c$  as its first column and  $r$  as its first row. If  $r$  is not given,  $r == \text{conjugate}(c)$  is assumed.

**Parameters** **c\_or\_cr** : array\_like or tuple of (array\_like, array\_like)

The vector  $c$ , or a tuple of arrays  $(c, r)$ . Whatever the actual shape of  $c$ , it will be converted to a 1-D array. If not supplied,  $r = \text{conjugate}(c)$  is assumed; in this case, if  $c[0]$  is real, the Toeplitz matrix is Hermitian.  $r[0]$  is ignored; the first row of the Toeplitz matrix is  $[c[0], r[1:]]$ . Whatever the actual shape of  $r$ , it will be converted to a 1-D array.

**b** : (M,) or (M, K) array\_like

Right-hand side in  $T x = b$ .

**check\_finite** : bool, optional

Whether to check that the input matrices contain only finite numbers. Disabling may give a performance gain, but may result in problems (result entirely NaNs) if the inputs do contain infinities or NaNs.

**Returns** **x** : (M,) or (M, K) ndarray

The solution to the system  $T x = b$ . Shape of return matches shape of  $b$ .

### Notes

The solution is computed using Levinson-Durbin recursion, which is faster than generic least-squares methods, but can be less numerically stable.

`scipy.linalg.det(a, overwrite_a=False, check_finite=True)`

Compute the determinant of a matrix

The determinant of a square matrix is a value derived arithmetically from the coefficients of the matrix.

The determinant for a 3x3 matrix, for example, is computed as follows:

$$\begin{array}{ccc} a & b & c \\ d & e & f \\ g & h & i \end{array} = A$$

$$\det(A) = a * e * i + b * f * g + c * d * h - c * e * g - b * d * i - a * f * h$$

**Parameters** **a** : (M, M) array\_like

A square matrix.

**overwrite\_a** : bool, optional

Allow overwriting data in  $a$  (may enhance performance).

**check\_finite** : bool, optional

Whether to check that the input matrix contains only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.

**Returns** **det** : float or complex

Determinant of  $a$ .

### Notes

The determinant is computed via LU factorization, LAPACK routine `z/dgetrf`.

### Examples

```
>>> from scipy import linalg
>>> a = np.array([[1, 2, 3], [4, 5, 6], [7, 8, 9]])
>>> linalg.det(a)
0.0
>>> a = np.array([[0, 2, 3], [4, 5, 6], [7, 8, 9]])
>>> linalg.det(a)
3.0
```

`scipy.linalg.norm(a, ord=None, axis=None, keepdims=False)`

Matrix or vector norm.

This function is able to return one of seven different matrix norms, or one of an infinite number of vector norms (described below), depending on the value of the `ord` parameter.

**Parameters** **a** : (M,) or (M, N) array\_like

Input array. If `axis` is `None`,  $a$  must be 1-D or 2-D.

**ord** : {non-zero int, inf, -inf, 'fro'}, optional

Order of the norm (see table under `Notes`). `inf` means numpy's `inf` object

**axis** : {int, 2-tuple of ints, None}, optional

If `axis` is an integer, it specifies the axis of `a` along which to compute the vector norms.

If `axis` is a 2-tuple, it specifies the axes that hold 2-D matrices, and the matrix norms of these matrices are computed. If `axis` is `None` then either a vector norm (when `a` is 1-D) or a matrix norm (when `a` is 2-D) is returned.

**keepdims** : bool, optional

If this is set to `True`, the axes which are normed over are left in the result as dimensions with size one. With this option the result will broadcast correctly against the original `a`.

**Returns** **n** : float or ndarray  
Norm of the matrix or vector(s).

### Notes

For values of `ord`  $\leq 0$ , the result is, strictly speaking, not a mathematical 'norm', but it may still be useful for various numerical purposes.

The following norms can be calculated:

ord	norm for matrices	norm for vectors
None	Frobenius norm	2-norm
'fro'	Frobenius norm	-
inf	$\max(\text{sum}(\text{abs}(x), \text{axis}=1))$	$\max(\text{abs}(x))$
-inf	$\min(\text{sum}(\text{abs}(x), \text{axis}=1))$	$\min(\text{abs}(x))$
0	-	$\text{sum}(x \neq 0)$
1	$\max(\text{sum}(\text{abs}(x), \text{axis}=0))$	as below
-1	$\min(\text{sum}(\text{abs}(x), \text{axis}=0))$	as below
2	2-norm (largest sing. value)	as below
-2	smallest singular value	as below
other	-	$\text{sum}(\text{abs}(x)**\text{ord})*(1./\text{ord})$

The Frobenius norm is given by [R103]:

$$\|A\|_F = [\sum_{i,j} \text{abs}(a_{i,j})^2]^{1/2}$$

The `axis` and `keepdims` arguments are passed directly to `numpy.linalg.norm` and are only usable if they are supported by the version of numpy in use.

### References

[R103]

### Examples

```
>>> from scipy.linalg import norm
>>> a = np.arange(9) - 4.0
>>> a
array([-4., -3., -2., -1.,  0.,  1.,  2.,  3.,  4.])
>>> b = a.reshape((3, 3))
>>> b
array([[ -4.,  -3.,  -2.],
       [ -1.,   0.,   1.],
       [  2.,   3.,   4.]])

>>> norm(a)
7.7459666692414834
>>> norm(b)
```

```

7.745966692414834
>>> norm(b, 'fro')
7.745966692414834
>>> norm(a, np.inf)
4
>>> norm(b, np.inf)
9
>>> norm(a, -np.inf)
0
>>> norm(b, -np.inf)
2

>>> norm(a, 1)
20
>>> norm(b, 1)
7
>>> norm(a, -1)
-4.6566128774142013e-010
>>> norm(b, -1)
6
>>> norm(a, 2)
7.745966692414834
>>> norm(b, 2)
7.3484692283495345

>>> norm(a, -2)
0
>>> norm(b, -2)
1.8570331885190563e-016
>>> norm(a, 3)
5.8480354764257312
>>> norm(a, -3)
0

```

`scipy.linalg.lstsq(a, b, cond=None, overwrite_a=False, overwrite_b=False, check_finite=True, lapack_driver=None)`

Compute least-squares solution to equation  $Ax = b$ .

Compute a vector  $x$  such that the 2-norm  $\|b - Ax\|$  is minimized.

**Parameters** **a** : (M, N) array\_like

Left hand side matrix (2-D array).

**b** : (M,) or (M, K) array\_like

Right hand side matrix or vector (1-D or 2-D array).

**cond** : float, optional

Cutoff for ‘small’ singular values; used to determine effective rank of  $a$ . Singular values smaller than `rcond * largest_singular_value` are considered zero.

**overwrite\_a** : bool, optional

Discard data in  $a$  (may enhance performance). Default is False.

**overwrite\_b** : bool, optional

Discard data in  $b$  (may enhance performance). Default is False.

**check\_finite** : bool, optional

Whether to check that the input matrices contain only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.

**lapack\_driver**: str, optional

Which LAPACK driver is used to solve the least-squares problem. Options are ‘gelsd’, ‘gelsy’, ‘gelss’. Default (‘gelsd’) is a good choice. However,

'gelsy' can be slightly faster on many problems. 'gelss' was used historically. It is generally slow but uses less memory. New in version 0.17.0.

**Returns** **x** : (N,) or (N, K) ndarray  
Least-squares solution. Return shape matches shape of *b*.  
**residues** : () or (1,) or (K,) ndarray  
Sums of residues, squared 2-norm for each column in  $b - a x$ . If rank of matrix *a* is  $< N$  or  $> M$ , or 'gelsy' is used, this is an empty array. If *b* was 1-D, this is an (1,) shape array, otherwise the shape is (K,).  
**rank** : int  
Effective rank of matrix *a*.  
**s** : (min(M,N),) ndarray or None  
Singular values of *a*. The condition number of *a* is  $\text{abs}(s[0] / s[-1])$ . None is returned when 'gelsy' is used.

**Raises** **LinAlgError**  
If computation does not converge.  
**ValueError**  
When parameters are wrong.

See also:

**optimize.nnls**  
linear least squares with non-negativity constraint

`scipy.linalg.pinv(a, cond=None, rcond=None, return_rank=False, check_finite=True)`  
Compute the (Moore-Penrose) pseudo-inverse of a matrix.

Calculate a generalized inverse of a matrix using a least-squares solver.

**Parameters** **a** : (M, N) array\_like  
Matrix to be pseudo-inverted.  
**cond, rcond** : float, optional  
Cutoff for 'small' singular values in the least-squares solver. Singular values smaller than  $\text{rcond} * \text{largest\_singular\_value}$  are considered zero.  
**return\_rank** : bool, optional  
if True, return the effective rank of the matrix  
**check\_finite** : bool, optional  
Whether to check that the input matrix contains only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.

**Returns** **B** : (N, M) ndarray  
The pseudo-inverse of matrix *a*.  
**rank** : int  
The effective rank of the matrix. Returned if `return_rank == True`

**Raises** **LinAlgError**  
If computation does not converge.

### Examples

```
>>> from scipy import linalg
>>> a = np.random.randn(9, 6)
>>> B = linalg.pinv(a)
>>> np.allclose(a, np.dot(a, np.dot(B, a)))
True
>>> np.allclose(B, np.dot(B, np.dot(a, B)))
True
```



`scipy.linalg.pinv2(a, cond=None, rcond=None, return_rank=False, check_finite=True)`

Compute the (Moore-Penrose) pseudo-inverse of a matrix.

Calculate a generalized inverse of a matrix using its singular-value decomposition and including all ‘large’ singular values.

**Parameters**

- a** : (M, N) array\_like  
Matrix to be pseudo-inverted.
- cond, rcond** : float or None  
Cutoff for ‘small’ singular values. Singular values smaller than `rcond*largest_singular_value` are considered zero. If None or -1, suitable machine precision is used.
- return\_rank** : bool, optional  
if True, return the effective rank of the matrix
- check\_finite** : bool, optional  
Whether to check that the input matrix contains only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.

**Returns**

- B** : (N, M) ndarray  
The pseudo-inverse of matrix *a*.
- rank** : int  
The effective rank of the matrix. Returned if `return_rank == True`

**Raises**

- LinAlgError**  
If SVD computation does not converge.

#### Examples

```
>>> from scipy import linalg
>>> a = np.random.randn(9, 6)
>>> B = linalg.pinv2(a)
>>> np.allclose(a, np.dot(a, np.dot(B, a)))
True
>>> np.allclose(B, np.dot(B, np.dot(a, B)))
True
```

`scipy.linalg.pinvh(a, cond=None, rcond=None, lower=True, return_rank=False, check_finite=True)`

Compute the (Moore-Penrose) pseudo-inverse of a Hermitian matrix.

Calculate a generalized inverse of a Hermitian or real symmetric matrix using its eigenvalue decomposition and including all eigenvalues with ‘large’ absolute value.

**Parameters**

- a** : (N, N) array\_like  
Real symmetric or complex hermetian matrix to be pseudo-inverted
- cond, rcond** : float or None  
Cutoff for ‘small’ eigenvalues. Singular values smaller than `rcond * largest_eigenvalue` are considered zero.  
If None or -1, suitable machine precision is used.
- lower** : bool, optional  
Whether the pertinent array data is taken from the lower or upper triangle of *a*. (Default: lower)
- return\_rank** : bool, optional  
if True, return the effective rank of the matrix
- check\_finite** : bool, optional  
Whether to check that the input matrix contains only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.

**Returns**

- B** : (N, N) ndarray

The pseudo-inverse of matrix  $a$ .

**rank** : int  
 The effective rank of the matrix. Returned if return\_rank == True

**Raises** **LinAlgError**  
 If eigenvalue does not converge

### Examples

```
>>> from scipy.linalg import pinvh
>>> a = np.random.randn(9, 6)
>>> a = np.dot(a, a.T)
>>> B = pinvh(a)
>>> np.allclose(a, np.dot(a, np.dot(B, a)))
True
>>> np.allclose(B, np.dot(B, np.dot(a, B)))
True
```

`scipy.linalg.kron(a, b)`

Kronecker product.

The result is the block matrix:

```
a[0,0]*b   a[0,1]*b   ... a[0,-1]*b
a[1,0]*b   a[1,1]*b   ... a[1,-1]*b
...
a[-1,0]*b  a[-1,1]*b  ... a[-1,-1]*b
```

**Parameters** **a** : (M, N) ndarray  
 Input array

**b** : (P, Q) ndarray  
 Input array

**Returns** **A** : (M\*P, N\*Q) ndarray  
 Kronecker product of  $a$  and  $b$ .

### Examples

```
>>> from numpy import array
>>> from scipy.linalg import kron
>>> kron(array([[1,2],[3,4]]), array([[1,1,1]]))
array([[1, 1, 1, 2, 2, 2],
       [3, 3, 3, 4, 4, 4]])
```

`scipy.linalg.tril(m, k=0)`

Make a copy of a matrix with elements above the  $k$ -th diagonal zeroed.

**Parameters** **m** : array\_like  
 Matrix whose elements to return

**k** : int, optional  
 Diagonal above which to zero elements.  $k == 0$  is the main diagonal,  $k < 0$  subdiagonal and  $k > 0$  superdiagonal.

**Returns** **tril** : ndarray  
 Return is the same shape and type as  $m$ .

### Examples

```
>>> from scipy.linalg import tril
>>> tril([[1,2,3],[4,5,6],[7,8,9],[10,11,12]], -1)
array([[ 0,  0,  0],
```

```
[ 4,  0,  0],
 [ 7,  8,  0],
 [10, 11, 12]])
```

`scipy.linalg.triu(m, k=0)`

Make a copy of a matrix with elements below the k-th diagonal zeroed.

**Parameters**

- m** : array\_like  
Matrix whose elements to return
- k** : int, optional  
Diagonal below which to zero elements.  $k == 0$  is the main diagonal,  $k < 0$  subdiagonal and  $k > 0$  superdiagonal.

**Returns**

- triu** : ndarray  
Return matrix with zeroed elements below the k-th diagonal and has same shape and type as *m*.

### Examples

```
>>> from scipy.linalg import triu
>>> triu([[1, 2, 3], [4, 5, 6], [7, 8, 9], [10, 11, 12]], -1)
array([[ 1,  2,  3],
       [ 4,  5,  6],
       [ 0,  8,  9],
       [ 0,  0, 12]])
```

`scipy.linalg.orthogonal_procrustes(A, B, check_finite=True)`

Compute the matrix solution of the orthogonal Procrustes problem.

Given matrices A and B of equal shape, find an orthogonal matrix R that most closely maps A to B [R104]. Note that unlike higher level Procrustes analyses of spatial data, this function only uses orthogonal transformations like rotations and reflections, and it does not use scaling or translation.

**Parameters**

- A** : (M, N) array\_like  
Matrix to be mapped.
- B** : (M, N) array\_like  
Target matrix.
- check\_finite** : bool, optional  
Whether to check that the input matrices contain only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.

**Returns**

- R** : (N, N) ndarray  
The matrix solution of the orthogonal Procrustes problem. Minimizes the Frobenius norm of  $\text{dot}(A, R) - B$ , subject to  $\text{dot}(R.T, R) == I$ .
- scale** : float  
Sum of the singular values of  $\text{dot}(A.T, B)$ .

**Raises**

- ValueError**  
If the input arrays are incompatibly shaped. This may also be raised if matrix A or B contains an inf or nan and `check_finite` is True, or if the matrix product AB contains an inf or nan.

### Notes

New in version 0.15.0.

### References

[R104]

**exception** `scipy.linalg.LinAlgError`

Generic Python-exception-derived object raised by linalg functions.

General purpose exception class, derived from Python's `Exception` class, programmatically raised in linalg functions when a Linear Algebra-related condition would prevent further correct execution of the function.

**Parameters** None

**Examples**

```
>>> from numpy import linalg as LA
>>> LA.inv(np.zeros((2,2)))
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
  File "...linalg.py", line 350,
    in inv return wrap(solve(a, identity(a.shape[0], dtype=a.dtype)))
  File "...linalg.py", line 249,
    in solve
    raise LinAlgError('Singular matrix')
numpy.linalg.LinAlgError: Singular matrix
```

## 5.9.2 Eigenvalue Problems

<code>eig(a[, b, left, right, overwrite_a, ...])</code>	Solve an ordinary or generalized eigenvalue problem of a square matrix.
<code>eigvals(a[, b, overwrite_a, check_finite])</code>	Compute eigenvalues from an ordinary or generalized eigenvalue problem.
<code>eigh(a[, b, lower, eigvals_only, ...])</code>	Solve an ordinary or generalized eigenvalue problem for a complex Hermitian or real symmetric matrix.
<code>eigvalsh(a[, b, lower, overwrite_a, ...])</code>	Solve an ordinary or generalized eigenvalue problem for a complex Hermitian or real symmetric matrix.
<code>eig_banded(a_band[, lower, eigvals_only, ...])</code>	Solve real symmetric or complex hermitian band matrix eigenvalue problem.
<code>eigvals_banded(a_band[, lower, ...])</code>	Solve real symmetric or complex hermitian band matrix eigenvalue problem.

`scipy.linalg.eig(a, b=None, left=False, right=True, overwrite_a=False, overwrite_b=False, check_finite=True)`

Solve an ordinary or generalized eigenvalue problem of a square matrix.

Find eigenvalues  $w$  and right or left eigenvectors of a general matrix:

```
a vr[:,i] = w[i]          b vr[:,i]
a.H vl[:,i] = w[i].conj() b.H vl[:,i]
```

where `.H` is the Hermitian conjugation.

**Parameters**

- a** : (M, M) array\_like  
A complex or real matrix whose eigenvalues and eigenvectors will be computed.
- b** : (M, M) array\_like, optional  
Right-hand side matrix in a generalized eigenvalue problem. Default is None, identity matrix is assumed.
- left** : bool, optional  
Whether to calculate and return left eigenvectors. Default is False.
- right** : bool, optional  
Whether to calculate and return right eigenvectors. Default is True.
- overwrite\_a** : bool, optional  
Whether to overwrite *a*; may improve performance. Default is False.
- overwrite\_b** : bool, optional  
Whether to overwrite *b*; may improve performance. Default is False.
- check\_finite** : bool, optional

Whether to check that the input matrices contain only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.

**Returns**

**w** : (M,) double or complex ndarray  
The eigenvalues, each repeated according to its multiplicity.

**vl** : (M, M) double or complex ndarray  
The normalized left eigenvector corresponding to the eigenvalue  $w[i]$  is the column  $vl[:,i]$ . Only returned if `left=True`.

**vr** : (M, M) double or complex ndarray  
The normalized right eigenvector corresponding to the eigenvalue  $w[i]$  is the column  $vr[:,i]$ . Only returned if `right=True`.

**Raises** **LinAlgError**  
If eigenvalue computation does not converge.

**See also:**

**eigh** Eigenvalues and right eigenvectors for symmetric/Hermitian arrays.

`scipy.linalg.eigvals` (*a*, *b=None*, *overwrite\_a=False*, *check\_finite=True*)  
Compute eigenvalues from an ordinary or generalized eigenvalue problem.

Find eigenvalues of a general matrix:

a     $vr[:,i] = w[i]$                       b     $vr[:,i]$

**Parameters**

**a** : (M, M) array\_like  
A complex or real matrix whose eigenvalues and eigenvectors will be computed.

**b** : (M, M) array\_like, optional  
Right-hand side matrix in a generalized eigenvalue problem. If omitted, identity matrix is assumed.

**overwrite\_a** : bool, optional  
Whether to overwrite data in *a* (may improve performance)

**check\_finite** : bool, optional  
Whether to check that the input matrices contain only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.

**Returns**

**w** : (M,) double or complex ndarray  
The eigenvalues, each repeated according to its multiplicity, but not in any specific order.

**Raises** **LinAlgError**  
If eigenvalue computation does not converge

**See also:**

**eigvalsh** eigenvalues of symmetric or Hermitian arrays,

**eig** eigenvalues and right eigenvectors of general arrays.

**eigh** eigenvalues and eigenvectors of symmetric/Hermitian arrays.

`scipy.linalg.eigh` (*a*, *b=None*, *lower=True*, *eigvals\_only=False*, *overwrite\_a=False*, *overwrite\_b=False*, *turbo=True*, *eigvals=None*, *type=1*, *check\_finite=True*)  
Solve an ordinary or generalized eigenvalue problem for a complex Hermitian or real symmetric matrix.

Find eigenvalues *w* and optionally eigenvectors *v* of matrix *a*, where *b* is positive definite:

```

                a v[:,i] = w[i] b v[:,i]
v[i,:].conj() a v[:,i] = w[i]
v[i,:].conj() b v[:,i] = 1

```

- Parameters**
- a** : (M, M) array\_like  
A complex Hermitian or real symmetric matrix whose eigenvalues and eigenvectors will be computed.
  - b** : (M, M) array\_like, optional  
A complex Hermitian or real symmetric definite positive matrix in. If omitted, identity matrix is assumed.
  - lower** : bool, optional  
Whether the pertinent array data is taken from the lower or upper triangle of *a*. (Default: lower)
  - eigvals\_only** : bool, optional  
Whether to calculate only eigenvalues and no eigenvectors. (Default: both are calculated)
  - turbo** : bool, optional  
Use divide and conquer algorithm (faster but expensive in memory, only for generalized eigenvalue problem and if eigvals=None)
  - eigvals** : tuple (lo, hi), optional  
Indexes of the smallest and largest (in ascending order) eigenvalues and corresponding eigenvectors to be returned:  $0 \leq lo \leq hi \leq M-1$ . If omitted, all eigenvalues and eigenvectors are returned.
  - type** : int, optional  
Specifies the problem type to be solved:
    - type = 1:  $a v[:,i] = w[i] b v[:,i]$
    - type = 2:  $a b v[:,i] = w[i] v[:,i]$
    - type = 3:  $b a v[:,i] = w[i] v[:,i]$
  - overwrite\_a** : bool, optional  
Whether to overwrite data in *a* (may improve performance)
  - overwrite\_b** : bool, optional  
Whether to overwrite data in *b* (may improve performance)
  - check\_finite** : bool, optional  
Whether to check that the input matrices contain only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.
- Returns**
- w** : (N,) float ndarray  
The N ( $1 \leq N \leq M$ ) selected eigenvalues, in ascending order, each repeated according to its multiplicity.
  - v** : (M, N) complex ndarray  
(if eigvals\_only == False)  
The normalized selected eigenvector corresponding to the eigenvalue  $w[i]$  is the column  $v[:,i]$ .  
Normalization:
    - type 1 and 3:  $v.conj() a v = w$
    - type 2:  $inv(v).conj() a inv(v) = w$
    - type = 1 or 2:  $v.conj() b v = I$
    - type = 3:  $v.conj() inv(b) v = I$
- Raises**
- LinAlgError**  
If eigenvalue computation does not converge, an error occurred, or *b* matrix is not definite positive. Note that if input matrices are not symmetric or hermitian, no error is reported but results will be wrong.

See also:

`eig` eigenvalues and right eigenvectors for non-symmetric arrays

`scipy.linalg.eigvalsh` (*a*, *b=None*, *lower=True*, *overwrite\_a=False*, *overwrite\_b=False*, *turbo=True*, *eigvals=None*, *type=1*, *check\_finite=True*)

Solve an ordinary or generalized eigenvalue problem for a complex Hermitian or real symmetric matrix.

Find eigenvalues *w* of matrix *a*, where *b* is positive definite:

```

                a v[:,i] = w[i] b v[:,i]
v[i,:].conj() a v[:,i] = w[i]
v[i,:].conj() b v[:,i] = 1

```

**Parameters**

- a** : (M, M) array\_like  
A complex Hermitian or real symmetric matrix whose eigenvalues and eigenvectors will be computed.
- b** : (M, M) array\_like, optional  
A complex Hermitian or real symmetric definite positive matrix in. If omitted, identity matrix is assumed.
- lower** : bool, optional  
Whether the pertinent array data is taken from the lower or upper triangle of *a*. (Default: lower)
- turbo** : bool, optional  
Use divide and conquer algorithm (faster but expensive in memory, only for generalized eigenvalue problem and if *eigvals=None*)
- eigvals** : tuple (lo, hi), optional  
Indexes of the smallest and largest (in ascending order) eigenvalues and corresponding eigenvectors to be returned:  $0 \leq lo < hi \leq M-1$ . If omitted, all eigenvalues and eigenvectors are returned.
- type** : int, optional  
Specifies the problem type to be solved:  
  - type = 1:  $a v[:,i] = w[i] b v[:,i]$
  - type = 2:  $a b v[:,i] = w[i] v[:,i]$
  - type = 3:  $b a v[:,i] = w[i] v[:,i]$
- overwrite\_a** : bool, optional  
Whether to overwrite data in *a* (may improve performance)
- overwrite\_b** : bool, optional  
Whether to overwrite data in *b* (may improve performance)
- check\_finite** : bool, optional  
Whether to check that the input matrices contain only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.

**Returns**

- w** : (N,) float ndarray  
The N ( $1 \leq N \leq M$ ) selected eigenvalues, in ascending order, each repeated according to its multiplicity.

**Raises**

- LinAlgError**  
If eigenvalue computation does not converge, an error occurred, or *b* matrix is not definite positive. Note that if input matrices are not symmetric or hermitian, no error is reported but results will be wrong.

See also:

`eigvals` eigenvalues of general arrays

**eigh** eigenvalues and right eigenvectors for symmetric/Hermitian arrays

**eig** eigenvalues and right eigenvectors for non-symmetric arrays

`scipy.linalg.eig_banded(a_band, lower=False, eigvals_only=False, overwrite_a_band=False, select='a', select_range=None, max_ev=0, check_finite=True)`

Solve real symmetric or complex hermitian band matrix eigenvalue problem.

Find eigenvalues  $w$  and optionally right eigenvectors  $v$  of  $a$ :

$$a v[:,i] = w[i] v[:,i]$$

$$v.H v = \text{identity}$$

The matrix  $a$  is stored in `a_band` either in lower diagonal or upper diagonal ordered form:

$$a\_band[u + i - j, j] == a[i,j] \text{ (if upper form; } i \leq j) \quad a\_band[i - j, j] == a[i,j] \text{ (if lower form; } i \geq j)$$

where  $u$  is the number of bands above the diagonal.

Example of `a_band` (shape of  $a$  is (6,6),  $u=2$ ):

```
upper form:
* * a02 a13 a24 a35
* a01 a12 a23 a34 a45
a00 a11 a22 a33 a44 a55
```

```
lower form:
a00 a11 a22 a33 a44 a55
a10 a21 a32 a43 a54 *
a20 a31 a42 a53 * *
```

Cells marked with `*` are not used.

- Parameters**
- a\_band** : (u+1, M) array\_like  
The bands of the M by M matrix  $a$ .
  - lower** : bool, optional  
Is the matrix in the lower form. (Default is upper form)
  - eigvals\_only** : bool, optional  
Compute only the eigenvalues and no eigenvectors. (Default: calculate also eigenvectors)
  - overwrite\_a\_band** : bool, optional  
Discard data in `a_band` (may enhance performance)
  - select** : {'a', 'v', 'i'}, optional  
Which eigenvalues to calculate
- | select | calculated                                       |
|--------|--|
| 'a'    | All eigenvalues                                  |
| 'v'    | Eigenvalues in the interval (min, max]           |
| 'i'    | Eigenvalues with indices $\min \leq i \leq \max$ |
- select\_range** : (min, max), optional  
Range of selected eigenvalues
  - max\_ev** : int, optional  
For `select=='v'`, maximum number of eigenvalues expected. For other values of `select`, has no meaning.  
In doubt, leave this parameter untouched.
  - check\_finite** : bool, optional  
Whether to check that the input matrix contains only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.

**Returns**  $w$  : (M,) ndarray



The eigenvalues, in ascending order, each repeated according to its multiplicity.

**v** : (M, M) float or complex ndarray

The normalized eigenvector corresponding to the eigenvalue  $w[i]$  is the column  $v[:,i]$ .

Raises `LinAlgError` if eigenvalue computation does not converge

`scipy.linalg.eigvals_banded`(*a\_band*, *lower=False*, *overwrite\_a\_band=False*, *select='a'*, *select\_range=None*, *check\_finite=True*)

Solve real symmetric or complex hermitian band matrix eigenvalue problem.

Find eigenvalues  $w$  of  $a$ :

```
a v[:,i] = w[i] v[:,i]
v.H v    = identity
```

The matrix  $a$  is stored in *a\_band* either in lower diagonal or upper diagonal ordered form:

$a\_band[u + i - j, j] == a[i, j]$  (if upper form;  $i \leq j$ )  $a\_band[i - j, j] == a[i, j]$  (if lower form;  $i \geq j$ )

where  $u$  is the number of bands above the diagonal.

Example of *a\_band* (shape of  $a$  is (6,6),  $u=2$ ):

upper form:

```
* * a02 a13 a24 a35
* a01 a12 a23 a34 a45
a00 a11 a22 a33 a44 a55
```

lower form:

```
a00 a11 a22 a33 a44 a55
a10 a21 a32 a43 a54 *
a20 a31 a42 a53 * *
```

Cells marked with \* are not used.

**Parameters** **a\_band** : ( $u+1$ , M) array\_like

The bands of the M by M matrix  $a$ .

**lower** : bool, optional

Is the matrix in the lower form. (Default is upper form)

**overwrite\_a\_band** : bool, optional

Discard data in *a\_band* (may enhance performance)

**select** : {'a', 'v', 'i'}, optional

Which eigenvalues to calculate

select	calculated
'a'	All eigenvalues
'v'	Eigenvalues in the interval (min, max]
'i'	Eigenvalues with indices $\min \leq i \leq \max$

**select\_range** : (min, max), optional

Range of selected eigenvalues

**check\_finite** : bool, optional

Whether to check that the input matrix contains only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.

**Returns** **w** : (M,) ndarray

The eigenvalues, in ascending order, each repeated according to its multiplicity.

Raises `LinAlgError` if eigenvalue computation does not converge

See also:

[eig\\_banded](#) eigenvalues and right eigenvectors for symmetric/Hermitian band matrices

<code>eigvals</code>	eigenvalues of general arrays
<code>eigh</code>	eigenvalues and right eigenvectors for symmetric/Hermitian arrays
<code>eig</code>	eigenvalues and right eigenvectors for non-symmetric arrays

### 5.9.3 Decompositions

---

<code>lu(a[, permute_l, overwrite_a, check_finite])</code>	Compute pivoted LU decomposition of a matrix.
<code>lu_factor(a[, overwrite_a, check_finite])</code>	Compute pivoted LU decomposition of a matrix.
<code>lu_solve(lu_and_piv, b[, trans, ...])</code>	Solve an equation system, $a x = b$ , given the LU factorization of a
<code>svd(a[, full_matrices, compute_uv, ...])</code>	Singular Value Decomposition.
<code>svdvals(a[, overwrite_a, check_finite])</code>	Compute singular values of a matrix.
<code>diagsvd(s, M, N)</code>	Construct the sigma matrix in SVD from singular values and size M, N.
<code>orth(A)</code>	Construct an orthonormal basis for the range of A using SVD
<code>cholesky(a[, lower, overwrite_a, check_finite])</code>	Compute the Cholesky decomposition of a matrix.
<code>cholesky_banded(ab[, overwrite_ab, lower, ...])</code>	Cholesky decompose a banded Hermitian positive-definite matrix
<code>cho_factor(a[, lower, overwrite_a, check_finite])</code>	Compute the Cholesky decomposition of a matrix, to use in cho_solve
<code>cho_solve(c_and_lower, b[, overwrite_b, ...])</code>	Solve the linear equations $A x = b$ , given the Cholesky factorization of A.
<code>cho_solve_banded(cb_and_lower, b[, ...])</code>	Solve the linear equations $A x = b$ , given the Cholesky factorization of A.
<code>polar(a[, side])</code>	Compute the polar decomposition.
<code>qr(a[, overwrite_a, lwork, mode, pivoting, ...])</code>	Compute QR decomposition of a matrix.
<code>qr_multiply(a, c[, mode, pivoting, ...])</code>	Calculate the QR decomposition and multiply Q with a matrix.
<code>qr_update(Q, R, u, v[, overwrite_qruv, ...])</code>	Rank-k QR update
<code>qr_delete(Q, R, k, int p=1[, which, ...])</code>	QR downdate on row or column deletions
<code>qr_insert(Q, R, u, k[, which, rcond, ...])</code>	QR update on row or column insertions
<code>rq(a[, overwrite_a, lwork, mode, check_finite])</code>	Compute RQ decomposition of a matrix.
<code>qz(A, B[, output, lwork, sort, overwrite_a, ...])</code>	QZ decomposition for generalized eigenvalues of a pair of matrices.
<code>ordqz(A, B[, sort, output, overwrite_a, ...])</code>	QZ decomposition for a pair of matrices with reordering.
<code>schur(a[, output, lwork, overwrite_a, sort, ...])</code>	Compute Schur decomposition of a matrix.
<code>rsf2csf(T, Z[, check_finite])</code>	Convert real Schur form to complex Schur form.
<code>hessenberg(a[, calc_q, overwrite_a, ...])</code>	Compute Hessenberg form of a matrix.

---

`scipy.linalg.lu(a, permute_l=False, overwrite_a=False, check_finite=True)`

Compute pivoted LU decomposition of a matrix.

The decomposition is:

$$A = P L U$$

where P is a permutation matrix, L lower triangular with unit diagonal elements, and U upper triangular.

<b>Parameters</b>	<p><b>a</b> : (M, N) array_like Array to decompose</p> <p><b>permute_l</b> : bool, optional Perform the multiplication P*L (Default: do not permute)</p> <p><b>overwrite_a</b> : bool, optional Whether to overwrite data in a (may improve performance)</p> <p><b>check_finite</b> : bool, optional Whether to check that the input matrix contains only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.</p>
<b>Returns</b>	<p>(If <b>permute_l == False</b>)</p> <p><b>p</b> : (M, M) ndarray</p>

Permutation matrix  
**l**: (M, K) ndarray  
 Lower triangular or trapezoidal matrix with unit diagonal.  $K = \min(M, N)$   
**u**: (K, N) ndarray  
 Upper triangular or trapezoidal matrix  
**(If permute\_l == True)**  
**pl**: (M, K) ndarray  
 Permuted L matrix.  $K = \min(M, N)$   
**u**: (K, N) ndarray  
 Upper triangular or trapezoidal matrix

**Notes**

This is a LU factorization routine written for Scipy.

`scipy.linalg.lu_factor(a, overwrite_a=False, check_finite=True)`  
 Compute pivoted LU decomposition of a matrix.

The decomposition is:

$$A = P L U$$

where P is a permutation matrix, L lower triangular with unit diagonal elements, and U upper triangular.

**Parameters**

- a**: (M, M) array\_like  
Matrix to decompose
- overwrite\_a**: bool, optional  
Whether to overwrite data in A (may increase performance)
- check\_finite**: bool, optional  
Whether to check that the input matrix contains only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.

**Returns**

- lu**: (N, N) ndarray  
Matrix containing U in its upper triangle, and L in its lower triangle. The unit diagonal elements of L are not stored.
- piv**: (N,) ndarray  
Pivot indices representing the permutation matrix P: row i of matrix was interchanged with row piv[i].

**See also:**

[`lu\_solve`](#) solve an equation system using the LU factorization of a matrix

**Notes**

This is a wrapper to the \*GETRF routines from LAPACK.

`scipy.linalg.lu_solve(lu_and_piv, b, trans=0, overwrite_b=False, check_finite=True)`  
 Solve an equation system,  $a x = b$ , given the LU factorization of a

**Parameters**

- (lu, piv)**  
Factorization of the coefficient matrix a, as given by lu\_factor
- b**: array  
Right-hand side
- trans**: {0, 1, 2}, optional  
Type of system to solve:

trans	system
0	$a x = b$
1	$a^T x = b$
2	$a^H x = b$

**overwrite\_b** : bool, optional

Whether to overwrite data in *b* (may increase performance)

**check\_finite** : bool, optional

Whether to check that the input matrices contain only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.

**Returns** **x** : array  
Solution to the system

**See also:**

[\*lu\\_factor\*](#) LU factorize a matrix

`scipy.linalg.svd(a, full_matrices=True, compute_uv=True, overwrite_a=False, check_finite=True, lapack_driver='gesdd')`  
Singular Value Decomposition.

Factorizes the matrix *a* into two unitary matrices *U* and *Vh*, and a 1-D array *s* of singular values (real, non-negative) such that  $a == U * S * Vh$ , where *S* is a suitably shaped matrix of zeros with main diagonal *s*.

**Parameters** **a** : (M, N) array\_like

Matrix to decompose.

**full\_matrices** : bool, optional

If True, *U* and *Vh* are of shape (M, M), (N, N). If False, the shapes are (M, K) and (K, N), where  $K = \min(M, N)$ .

**compute\_uv** : bool, optional

Whether to compute also *U* and *Vh* in addition to *s*. Default is True.

**overwrite\_a** : bool, optional

Whether to overwrite *a*; may improve performance. Default is False.

**check\_finite** : bool, optional

Whether to check that the input matrix contains only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.

**lapack\_driver** : { 'gesdd', 'gesvd' }, optional

Whether to use the more efficient divide-and-conquer approach ('gesdd') or general rectangular approach ('gesvd') to compute the SVD. MATLAB and Octave use the 'gesvd' approach. Default is 'gesdd'.

New in version 0.18.

**Returns** **U** : ndarray

Unitary matrix having left singular vectors as columns. Of shape (M, M) or (M, K), depending on *full\_matrices*.

**s** : ndarray

The singular values, sorted in non-increasing order. Of shape (K,), with  $K = \min(M, N)$ .

**Vh** : ndarray

Unitary matrix having right singular vectors as rows. Of shape (N, N) or (K, N) depending on *full\_matrices*.

For *compute\_uv=False*, only *s* is returned.

**Raises** **LinAlgError**

If SVD computation does not converge.

**See also:**

**svdvals** Compute singular values of a matrix.  
**diagsvd** Construct the Sigma matrix, given the vector *s*.

**Examples**

```
>>> from scipy import linalg
>>> a = np.random.randn(9, 6) + 1.j*np.random.randn(9, 6)
>>> U, s, Vh = linalg.svd(a)
>>> U.shape, Vh.shape, s.shape
((9, 9), (6, 6), (6,))

>>> U, s, Vh = linalg.svd(a, full_matrices=False)
>>> U.shape, Vh.shape, s.shape
((9, 6), (6, 6), (6,))
>>> S = linalg.diagsvd(s, 6, 6)
>>> np.allclose(a, np.dot(U, np.dot(S, Vh)))
True

>>> s2 = linalg.svd(a, compute_uv=False)
>>> np.allclose(s, s2)
True
```

`scipy.linalg.svdvals(a, overwrite_a=False, check_finite=True)`  
 Compute singular values of a matrix.

**Parameters** **a** : (M, N) array\_like  
 Matrix to decompose.  
**overwrite\_a** : bool, optional  
 Whether to overwrite *a*; may improve performance. Default is False.  
**check\_finite** : bool, optional  
 Whether to check that the input matrix contains only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.

**Returns** **s** : (min(M, N),) ndarray  
 The singular values, sorted in decreasing order.

**Raises** **LinAlgError**  
 If SVD computation does not converge.

**See also:**

**svd** Compute the full singular value decomposition of a matrix.  
**diagsvd** Construct the Sigma matrix, given the vector *s*.

**Notes**

`svdvals(a)` only differs from `svd(a, compute_uv=False)` by its handling of the edge case of empty *a*, where it returns an empty sequence:

```
>>> a = np.empty((0, 2))
>>> from scipy.linalg import svdvals
>>> svdvals(a)
array([], dtype=float64)
```

`scipy.linalg.diagsvd(s, M, N)`  
 Construct the sigma matrix in SVD from singular values and size *M*, *N*.

**Parameters** **s** : (M,) or (N,) array\_like  
 Singular values

**M** : int  
 Size of the matrix whose singular values are *s*.  
**N** : int  
 Size of the matrix whose singular values are *s*.  
**Returns** **S** : (M, N) ndarray  
 The S-matrix in the singular value decomposition

`scipy.linalg.orth(A)`

Construct an orthonormal basis for the range of A using SVD

**Parameters** **A** : (M, N) array\_like  
 Input array  
**Returns** **Q** : (M, K) ndarray  
 Orthonormal basis for the range of A. K = effective rank of A, as determined by automatic cutoff

**See also:**

**svd** Singular value decomposition of a matrix

`scipy.linalg.cholesky(a, lower=False, overwrite_a=False, check_finite=True)`

Compute the Cholesky decomposition of a matrix.

Returns the Cholesky decomposition,  $A = LL^*$  or  $A = U^*U$  of a Hermitian positive-definite matrix A.

**Parameters** **a** : (M, M) array\_like  
 Matrix to be decomposed  
**lower** : bool, optional  
 Whether to compute the upper or lower triangular Cholesky factorization. Default is upper-triangular.  
**overwrite\_a** : bool, optional  
 Whether to overwrite data in *a* (may improve performance).  
**check\_finite** : bool, optional  
 Whether to check that the input matrix contains only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.  
**Returns** **c** : (M, M) ndarray  
 Upper- or lower-triangular Cholesky factor of *a*.  
**Raises** **LinAlgError** : if decomposition fails.

**Examples**

```
>>> from scipy import array, linalg, dot
>>> a = array([[1, -2j], [2j, 5]])
>>> L = linalg.cholesky(a, lower=True)
>>> L
array([[ 1.+0.j,  0.+0.j],
       [ 0.+2.j,  1.+0.j]])
>>> dot(L, L.T.conj())
array([[ 1.+0.j,  0.-2.j],
       [ 0.+2.j,  5.+0.j]])
```

`scipy.linalg.cholesky_banded(ab, overwrite_ab=False, lower=False, check_finite=True)`

Cholesky decompose a banded Hermitian positive-definite matrix

The matrix *a* is stored in *ab* either in lower diagonal or upper diagonal ordered form:

```

ab[u + i - j, j] == a[i, j]          (if upper form; i <= j)
ab[    i - j, j] == a[i, j]          (if lower form; i >= j)

```

Example of `ab` (shape of `a` is (6,6), `u=2`):

```

upper form:
*   *   a02 a13 a24 a35
*   a01 a12 a23 a34 a45
a00 a11 a22 a33 a44 a55

lower form:
a00 a11 a22 a33 a44 a55
a10 a21 a32 a43 a54 *
a20 a31 a42 a53 *   *

```

**Parameters**

- ab** : (u + 1, M) array\_like  
Banded matrix
- overwrite\_ab** : bool, optional  
Discard data in `ab` (may enhance performance)
- lower** : bool, optional  
Is the matrix in the lower form. (Default is upper form)
- check\_finite** : bool, optional  
Whether to check that the input matrix contains only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.

**Returns**

- c** : (u + 1, M) ndarray  
Cholesky factorization of `a`, in the same banded format as `ab`

`scipy.linalg.cho_factor` (*a*, *lower=False*, *overwrite\_a=False*, *check\_finite=True*)  
Compute the Cholesky decomposition of a matrix, to use in `cho_solve`

Returns a matrix containing the Cholesky decomposition,  $A = L L^*$  or  $A = U^* U$  of a Hermitian positive-definite matrix *a*. The return value can be directly used as the first parameter to `cho_solve`.

**Warning:** The returned matrix also contains random data in the entries not used by the Cholesky decomposition. If you need to zero these entries, use the function `cholesky` instead.

**Parameters**

- a** : (M, M) array\_like  
Matrix to be decomposed
- lower** : bool, optional  
Whether to compute the upper or lower triangular Cholesky factorization (Default: upper-triangular)
- overwrite\_a** : bool, optional  
Whether to overwrite data in `a` (may improve performance)
- check\_finite** : bool, optional  
Whether to check that the input matrix contains only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.

**Returns**

- c** : (M, M) ndarray  
Matrix whose upper or lower triangle contains the Cholesky factor of *a*. Other parts of the matrix contain random data.
- lower** : bool  
Flag indicating whether the factor is in the lower or upper triangle

**Raises**

- LinAlgError**  
Raised if decomposition fails.

See also:

[`cho\_solve`](#) Solve a linear set equations using the Cholesky factorization of a matrix.

`scipy.linalg.cho_solve(c_and_lower, b, overwrite_b=False, check_finite=True)`

Solve the linear equations  $Ax = b$ , given the Cholesky factorization of  $A$ .

**Parameters** **(c, lower)** : tuple, (array, bool)  
 Cholesky factorization of  $a$ , as given by `cho_factor`

**b** : array  
 Right-hand side

**overwrite\_b** : bool, optional  
 Whether to overwrite data in  $b$  (may improve performance)

**check\_finite** : bool, optional  
 Whether to check that the input matrices contain only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.

**Returns** **x** : array  
 The solution to the system  $Ax = b$

See also:

[`cho\_factor`](#) Cholesky factorization of a matrix

`scipy.linalg.cho_solve_banded(cb_and_lower, b, overwrite_b=False, check_finite=True)`

Solve the linear equations  $Ax = b$ , given the Cholesky factorization of  $A$ .

**Parameters** **(cb, lower)** : tuple, (array, bool)  
 $cb$  is the Cholesky factorization of  $A$ , as given by `cholesky_banded`.  $lower$  must be the same value that was given to `cholesky_banded`.

**b** : array  
 Right-hand side

**overwrite\_b** : bool, optional  
 If True, the function will overwrite the values in  $b$ .

**check\_finite** : bool, optional  
 Whether to check that the input matrices contain only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.

**Returns** **x** : array  
 The solution to the system  $Ax = b$

See also:

[`cholesky\_banded`](#)

Cholesky factorization of a banded matrix

*Notes*

New in version 0.8.0.

`scipy.linalg.polar(a, side='right')`

Compute the polar decomposition.

Returns the factors of the polar decomposition [R105]  $u$  and  $p$  such that  $a = up$  (if  $side$  is “right”) or  $a = pu$  (if  $side$  is “left”), where  $p$  is positive semidefinite. Depending on the shape of  $a$ , either the rows or columns of  $u$  are orthonormal. When  $a$  is a square array,  $u$  is a square unitary array. When  $a$  is not square, the “canonical polar decomposition” [R106] is computed.



**Parameters**

- a** : (m, n) array\_like  
The array to be factored.
- side** : {'left', 'right'}, optional  
Determines whether a right or left polar decomposition is computed. If *side* is "right", then  $a = up$ . If *side* is "left", then  $a = pu$ . The default is "right".

**Returns**

- u** : (m, n) ndarray  
If *a* is square, then *u* is unitary. If  $m > n$ , then the columns of *a* are orthonormal, and if  $m < n$ , then the rows of *u* are orthonormal.
- p** : ndarray  
*p* is Hermitian positive semidefinite. If *a* is nonsingular, *p* is positive definite. The shape of *p* is (n, n) or (m, m), depending on whether *side* is "right" or "left", respectively.

### References

[R105], [R106]

### Examples

```
>>> from scipy.linalg import polar
>>> a = np.array([[1, -1], [2, 4]])
>>> u, p = polar(a)
>>> u
array([[ 0.85749293, -0.51449576],
       [ 0.51449576,  0.85749293]])
>>> p
array([[ 1.88648444,  1.2004901 ],
       [ 1.2004901 ,  3.94446746]])
```

A non-square example, with  $m < n$ :

```
>>> b = np.array([[0.5, 1, 2], [1.5, 3, 4]])
>>> u, p = polar(b)
>>> u
array([[ -0.21196618, -0.42393237,  0.88054056],
       [ 0.39378971,  0.78757942,  0.4739708 ]])
>>> p
array([[ 0.48470147,  0.96940295,  1.15122648],
       [ 0.96940295,  1.9388059 ,  2.30245295],
       [ 1.15122648,  2.30245295,  3.65696431]])
>>> u.dot(p) # Verify the decomposition.
array([[ 0.5,  1. ,  2. ],
       [ 1.5,  3. ,  4. ]])
>>> u.dot(u.T) # The rows of u are orthonormal.
array([[ 1.00000000e+00, -2.07353665e-17],
       [-2.07353665e-17,  1.00000000e+00]])
```

Another non-square example, with  $m > n$ :

```
>>> c = b.T
>>> u, p = polar(c)
>>> u
array([[ -0.21196618,  0.39378971],
       [-0.42393237,  0.78757942],
       [ 0.88054056,  0.4739708 ]])
>>> p
array([[ 1.23116567,  1.93241587],
       [ 1.93241587,  4.84930602]])
>>> u.dot(p) # Verify the decomposition.
```

```

array([[ 0.5,  1.5],
       [ 1. ,  3. ],
       [ 2. ,  4. ]])
>>> u.T.dot(u) # The columns of u are orthonormal.
array([[ 1.00000000e+00, -1.26363763e-16],
       [-1.26363763e-16,  1.00000000e+00]])
    
```

`scipy.linalg.qr(a, overwrite_a=False, lwork=None, mode='full', pivoting=False, check_finite=True)`  
 Compute QR decomposition of a matrix.

Calculate the decomposition  $A = QR$  where  $Q$  is unitary/orthogonal and  $R$  upper triangular.

**Parameters**

- a** : (M, N) array\_like  
Matrix to be decomposed
- overwrite\_a** : bool, optional  
Whether data in a is overwritten (may improve performance)
- lwork** : int, optional  
Work array size, `lwork >= a.shape[1]`. If None or -1, an optimal size is computed.
- mode** : {'full', 'r', 'economic', 'raw'}, optional  
Determines what information is to be returned: either both  $Q$  and  $R$  ('full', default), only  $R$  ('r') or both  $Q$  and  $R$  but computed in economy-size ('economic', see Notes). The final option 'raw' (added in Scipy 0.11) makes the function return two matrices ( $Q$ ,  $\tau$ ) in the internal format used by LAPACK.
- pivoting** : bool, optional  
Whether or not factorization should include pivoting for rank-revealing qr decomposition. If pivoting, compute the decomposition  $AP = QR$  as above, but where  $P$  is chosen such that the diagonal of  $R$  is non-increasing.
- check\_finite** : bool, optional  
Whether to check that the input matrix contains only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.

**Returns**

- Q** : float or complex ndarray  
Of shape (M, M), or (M, K) for `mode='economic'`. Not returned if `mode='r'`.
- R** : float or complex ndarray  
Of shape (M, N), or (K, N) for `mode='economic'`.  $K = \min(M, N)$ .
- P** : int ndarray  
Of shape (N,) for `pivoting=True`. Not returned if `pivoting=False`.

**Raises**

- LinAlgError**  
Raised if decomposition fails

### Notes

This is an interface to the LAPACK routines `dgeqrf`, `zgeqrf`, `dorgqr`, `zungqr`, `dgeqp3`, and `zgeqp3`.

If `mode=economic`, the shapes of  $Q$  and  $R$  are (M, K) and (K, N) instead of (M,M) and (M,N), with  $K=\min(M,N)$ .

### Examples

```

>>> from scipy import random, linalg, dot, diag, all, allclose
>>> a = random.randn(9, 6)

>>> q, r = linalg.qr(a)
>>> allclose(a, np.dot(q, r))
True
>>> q.shape, r.shape
((9, 9), (9, 6))
    
```

```

>>> r2 = linalg.qr(a, mode='r')
>>> allclose(r, r2)
True

>>> q3, r3 = linalg.qr(a, mode='economic')
>>> q3.shape, r3.shape
((9, 6), (6, 6))

>>> q4, r4, p4 = linalg.qr(a, pivoting=True)
>>> d = abs(diag(r4))
>>> all(d[1:] <= d[:-1])
True
>>> allclose(a[:, p4], dot(q4, r4))
True
>>> q4.shape, r4.shape, p4.shape
((9, 9), (9, 6), (6,))

>>> q5, r5, p5 = linalg.qr(a, mode='economic', pivoting=True)
>>> q5.shape, r5.shape, p5.shape
((9, 6), (6, 6), (6,))

```

`scipy.linalg.qr_multiply(a, c, mode='right', pivoting=False, conjugate=False, overwrite_a=False, overwrite_c=False)`

Calculate the QR decomposition and multiply Q with a matrix.

Calculate the decomposition  $A = QR$  where Q is unitary/orthogonal and R upper triangular. Multiply Q with a vector or a matrix c.

**Parameters**

- a** : array\_like, shape (M, N)  
Matrix to be decomposed
- c** : array\_like, one- or two-dimensional  
calculate the product of c and q, depending on the mode:
- mode** : {'left', 'right'}, optional  
dot(Q, c) is returned if mode is 'left', dot(c, Q) is returned if mode is 'right'. The shape of c must be appropriate for the matrix multiplications, if mode is 'left',  $\min(a.shape) == c.shape[0]$ , if mode is 'right',  $a.shape[0] == c.shape[1]$ .
- pivoting** : bool, optional  
Whether or not factorization should include pivoting for rank-revealing qr decomposition, see the documentation of qr.
- conjugate** : bool, optional  
Whether Q should be complex-conjugated. This might be faster than explicit conjugation.
- overwrite\_a** : bool, optional  
Whether data in a is overwritten (may improve performance)
- overwrite\_c** : bool, optional  
Whether data in c is overwritten (may improve performance). If this is used, c must be big enough to keep the result, i.e.  $c.shape[0] = a.shape[0]$  if mode is 'left'.

**Returns**

- CQ** : float or complex ndarray  
the product of Q and c, as defined in mode
- R** : float or complex ndarray  
Of shape (K, N),  $K = \min(M, N)$ .
- P** : ndarray of ints  
Of shape (N,) for pivoting=True. Not returned if pivoting=False.

**Raises**

- LinAlgError**  
Raised if decomposition fails

**Notes**

This is an interface to the LAPACK routines `dgeqrf`, `zgeqrf`, `dormqr`, `zunmqr`, `dgeqp3`, and `zgeqp3`.

New in version 0.11.0.

`scipy.linalg.qr_update(Q, R, u, v, overwrite_qruv=False, check_finite=True)`

Rank-k QR update

If  $A = QR$  is the QR factorization of  $A$ , return the QR factorization of  $A + uv^*T$  for real  $A$  or  $A + uv^{*H}$  for complex  $A$ .

- Parameters**
- Q** : (M, M) or (M, N) array\_like  
Unitary/orthogonal matrix from the qr decomposition of  $A$ .
  - R** : (M, N) or (N, N) array\_like  
Upper triangular matrix from the qr decomposition of  $A$ .
  - u** : (M,) or (M, k) array\_like  
Left update vector
  - v** : (N,) or (N, k) array\_like  
Right update vector
  - overwrite\_qruv** : bool, optional  
If True, consume  $Q$ ,  $R$ ,  $u$ , and  $v$ , if possible, while performing the update, otherwise make copies as necessary. Defaults to False.
  - check\_finite** : bool, optional  
Whether to check that the input matrix contains only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs. Default is True.
- Returns**
- Q1** : ndarray  
Updated unitary/orthogonal factor
  - R1** : ndarray  
Updated upper triangular factor

**See also:**

`qr`, `qr_multiply`, `qr_delete`, `qr_insert`

**Notes**

This routine does not guarantee that the diagonal entries of  $R1$  are real or positive.

New in version 0.16.0.

**References**

[R113], [R114], [R115]

**Examples**

```
>>> from scipy import linalg
>>> a = np.array([[ 3., -2., -2.],
...              [ 6., -9., -3.],
...              [-3., 10.,  1.],
...              [ 6., -7.,  4.],
...              [ 7.,  8., -6.]])
>>> q, r = linalg.qr(a)
```

Given this  $q, r$  decomposition, perform a rank 1 update.

```

>>> u = np.array([7., -2., 4., 3., 5.])
>>> v = np.array([1., 3., -5.])
>>> q_up, r_up = linalg.qr_update(q, r, u, v, False)
>>> q_up
array([[ 0.54073807,  0.18645997,  0.81707661, -0.02136616,  0.06902409], # may vary (signs)
       [ 0.21629523, -0.63257324,  0.06567893,  0.34125904, -0.65749222],
       [ 0.05407381,  0.64757787, -0.12781284, -0.20031219, -0.72198188],
       [ 0.48666426, -0.30466718, -0.27487277, -0.77079214,  0.0256951 ],
       [ 0.64888568,  0.23001   , -0.4859845 ,  0.49883891,  0.20253783]])
>>> r_up
array([[ 18.49324201,  24.11691794, -44.98940746], # may vary (signs)
       [ 0.          , 31.95894662, -27.40998201],
       [ 0.          , 0.          , -9.25451794],
       [ 0.          , 0.          , 0.          ],
       [ 0.          , 0.          , 0.          ]])

```

The update is equivalent, but faster than the following.

```

>>> a_up = a + np.outer(u, v)
>>> q_direct, r_direct = linalg.qr(a_up)

```

Check that we have equivalent results:

```

>>> np.allclose(np.dot(q_up, r_up), a_up)
True

```

And the updated Q is still unitary:

```

>>> np.allclose(np.dot(q_up.T, q_up), np.eye(5))
True

```

Updating economic (reduced, thin) decompositions is also possible:

```

>>> qe, re = linalg.qr(a, mode='economic')
>>> qe_up, re_up = linalg.qr_update(qe, re, u, v, False)
>>> qe_up
array([[ 0.54073807,  0.18645997,  0.81707661], # may vary (signs)
       [ 0.21629523, -0.63257324,  0.06567893],
       [ 0.05407381,  0.64757787, -0.12781284],
       [ 0.48666426, -0.30466718, -0.27487277],
       [ 0.64888568,  0.23001   , -0.4859845 ]])
>>> re_up
array([[ 18.49324201,  24.11691794, -44.98940746], # may vary (signs)
       [ 0.          , 31.95894662, -27.40998201],
       [ 0.          , 0.          , -9.25451794]])
>>> np.allclose(np.dot(qe_up, re_up), a_up)
True
>>> np.allclose(np.dot(qe_up.T, qe_up), np.eye(3))
True

```

Similarly to the above, perform a rank 2 update.

```

>>> u2 = np.array([[ 7., -1.],
...               [-2.,  4.],
...               [ 4.,  2.],
...               [ 3., -6.],
...               [ 5.,  3.]])
>>> v2 = np.array([[ 1., 2.],
...               [ 3., 4.],
...               [-5., 2.]])

```

```

>>> q_up2, r_up2 = linalg.qr_update(q, r, u2, v2, False)
>>> q_up2
array([[ -0.33626508, -0.03477253,  0.61956287, -0.64352987, -0.29618884], # may vary (signs)
       [ -0.50439762,  0.58319694, -0.43010077, -0.33395279,  0.33008064],
       [ -0.21016568, -0.63123106,  0.0582249 , -0.13675572,  0.73163206],
       [  0.12609941,  0.49694436,  0.64590024,  0.31191919,  0.47187344],
       [ -0.75659643, -0.11517748,  0.10284903,  0.5986227 , -0.21299983]])
>>> r_up2
array([[ -23.79075451, -41.1084062 ,  24.71548348], # may vary (signs)
       [  0.          , -33.83931057,  11.02226551],
       [  0.          ,  0.          ,  48.91476811],
       [  0.          ,  0.          ,  0.          ],
       [  0.          ,  0.          ,  0.          ]])
    
```

This update is also a valid qr decomposition of  $A + U V^* T$ .

```

>>> a_up2 = a + np.dot(u2, v2.T)
>>> np.allclose(a_up2, np.dot(q_up2, r_up2))
True
>>> np.allclose(np.dot(q_up2.T, q_up2), np.eye(5))
True
    
```

`scipy.linalg.qr_delete(Q, R, k, int p=1, which='row', overwrite_qr=False, check_finite=True)`  
 QR downdate on row or column deletions

If  $A = Q R$  is the QR factorization of  $A$ , return the QR factorization of  $A$  where  $p$  rows or columns have been removed starting at row or column  $k$ .

**Parameters**

- Q** : (M, M) or (M, N) array\_like  
Unitary/orthogonal matrix from QR decomposition.
- R** : (M, N) or (N, N) array\_like  
Upper triangular matrix from QR decomposition.
- k** : int  
Index of the first row or column to delete.
- p** : int, optional  
Number of rows or columns to delete, defaults to 1.
- which** : {'row', 'col'}, optional  
Determines if rows or columns will be deleted, defaults to 'row'
- overwrite\_qr** : bool, optional  
If True, consume Q and R, overwriting their contents with their downdated versions, and returning appropriately sized views. Defaults to False.
- check\_finite** : bool, optional  
Whether to check that the input matrix contains only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs. Default is True.

**Returns**

- Q1** : ndarray  
Updated unitary/orthogonal factor
- R1** : ndarray  
Updated upper triangular factor

**See also:**

[qr](#), [qr\\_multiply](#), [qr\\_insert](#), [qr\\_update](#)

**Notes**

This routine does not guarantee that the diagonal entries of  $R1$  are positive.

New in version 0.16.0.

**References**

[R107], [R108], [R109]

**Examples**

```
>>> from scipy import linalg
>>> a = np.array([[ 3., -2., -2.],
...             [ 6., -9., -3.],
...             [-3., 10.,  1.],
...             [ 6., -7.,  4.],
...             [ 7.,  8., -6.]])
>>> q, r = linalg.qr(a)
```

Given this QR decomposition, update  $q$  and  $r$  when 2 rows are removed.

```
>>> q1, r1 = linalg.qr_delete(q, r, 2, 2, 'row', False)
>>> q1
array([[ 0.30942637,  0.15347579,  0.93845645], # may vary (signs)
       [ 0.61885275,  0.71680171, -0.32127338],
       [ 0.72199487, -0.68017681, -0.12681844]])
>>> r1
array([[ 9.69535971, -0.4125685 , -6.80738023], # may vary (signs)
       [ 0.          , -12.19958144,  1.62370412],
       [ 0.          ,  0.          , -0.15218213]])
```

The update is equivalent, but faster than the following.

```
>>> a1 = np.delete(a, slice(2,4), 0)
>>> a1
array([[ 3., -2., -2.],
       [ 6., -9., -3.],
       [ 7.,  8., -6.]])
>>> q_direct, r_direct = linalg.qr(a1)
```

Check that we have equivalent results:

```
>>> np.dot(q1, r1)
array([[ 3., -2., -2.],
       [ 6., -9., -3.],
       [ 7.,  8., -6.]])
>>> np.allclose(np.dot(q1, r1), a1)
True
```

And the updated  $Q$  is still unitary:

```
>>> np.allclose(np.dot(q1.T, q1), np.eye(3))
True
```

`scipy.linalg.qr_insert(Q, R, u, k, which='row', rcond=None, overwrite_qru=False, check_finite=True)`

QR update on row or column insertions

If  $A = QR$  is the QR factorization of  $A$ , return the QR factorization of  $A$  where rows or columns have been inserted starting at row or column  $k$ .

**Parameters**

- Q** : (M, M) array\_like  
Unitary/orthogonal matrix from the QR decomposition of  $A$ .
- R** : (M, N) array\_like  
Upper triangular matrix from the QR decomposition of  $A$ .
- u** : (N,), (p, N), (M,), or (M, p) array\_like

Rows or columns to insert

**k** : int  
Index before which  $u$  is to be inserted.

**which**: {'row', 'col'}, optional  
Determines if rows or columns will be inserted, defaults to 'row'

**rcond** : float  
Lower bound on the reciprocal condition number of  $Q$  augmented with  $u/||u||$   
Only used when updating economic mode (thin, (M,N) (N,N)) decompositions. If None, machine precision is used. Defaults to None.

**overwrite\_qru** : bool, optional  
If True, consume  $Q$ ,  $R$ , and  $u$ , if possible, while performing the update, otherwise make copies as necessary. Defaults to False.

**check\_finite** : bool, optional  
Whether to check that the input matrices contain only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs. Default is True.

**Returns**

**Q1** : ndarray  
Updated unitary/orthogonal factor

**R1** : ndarray  
Updated upper triangular factor

**Raises**

**LinAlgError** :  
If updating a (M,N) (N,N) factorization and the reciprocal condition number of  $Q$  augmented with  $u/||u||$  is smaller than **rcond**.

**See also:**

`qr`, `qr_multiply`, `qr_delete`, `qr_update`

**Notes**

This routine does not guarantee that the diagonal entries of  $R1$  are positive.

New in version 0.16.0.

**References**

[R110], [R111], [R112]

**Examples**

```
>>> from scipy import linalg
>>> a = np.array([[ 3., -2., -2.],
...              [ 6., -7.,  4.],
...              [ 7.,  8., -6.]])
>>> q, r = linalg.qr(a)
```

Given this QR decomposition, update  $q$  and  $r$  when 2 rows are inserted.

```
>>> u = np.array([[ 6., -9., -3.],
...              [-3., 10.,  1.]])
>>> q1, r1 = linalg.qr_insert(q, r, u, 2, 'row')
>>> q1
array([[ -0.25445668,  0.02246245,  0.18146236, -0.72798806,  0.60979671], # may vary (signs)
       [-0.50891336,  0.23226178, -0.82836478, -0.02837033, -0.00828114],
       [-0.50891336,  0.35715302,  0.38937158,  0.58110733,  0.35235345],
       [ 0.25445668, -0.52202743, -0.32165498,  0.36263239,  0.65404509],
       [-0.59373225, -0.73856549,  0.16065817, -0.0063658 , -0.27595554]])
>>> r1
array([[ -11.78982612,   6.44623587,   3.81685018], # may vary (signs)
```



```
[ 0.          , -16.01393278,  3.72202865],
 [ 0.          ,  0.          , -6.13010256],
 [ 0.          ,  0.          ,  0.          ],
 [ 0.          ,  0.          ,  0.          ]])
```

The update is equivalent, but faster than the following.

```
>>> a1 = np.insert(a, 2, u, 0)
>>> a1
array([[ 3., -2., -2.],
       [ 6., -7.,  4.],
       [ 6., -9., -3.],
       [-3., 10.,  1.],
       [ 7.,  8., -6.]])
>>> q_direct, r_direct = linalg.qr(a1)
```

Check that we have equivalent results:

```
>>> np.dot(q1, r1)
array([[ 3., -2., -2.],
       [ 6., -7.,  4.],
       [ 6., -9., -3.],
       [-3., 10.,  1.],
       [ 7.,  8., -6.]])

>>> np.allclose(np.dot(q1, r1), a1)
True
```

And the updated Q is still unitary:

```
>>> np.allclose(np.dot(q1.T, q1), np.eye(5))
True
```

`scipy.linalg.rq(a, overwrite_a=False, lwork=None, mode='full', check_finite=True)`

Compute RQ decomposition of a matrix.

Calculate the decomposition  $A = R Q$  where Q is unitary/orthogonal and R upper triangular.

**Parameters**

- a** : (M, N) array\_like  
Matrix to be decomposed
- overwrite\_a** : bool, optional  
Whether data in a is overwritten (may improve performance)
- lwork** : int, optional  
Work array size, `lwork >= a.shape[1]`. If None or -1, an optimal size is computed.
- mode** : {'full', 'r', 'economic'}, optional  
Determines what information is to be returned: either both Q and R ('full', default), only R ('r') or both Q and R but computed in economy-size ('economic', see Notes).
- check\_finite** : bool, optional  
Whether to check that the input matrix contains only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.

**Returns**

- R** : float or complex ndarray  
Of shape (M, N) or (M, K) for `mode='economic'`.  $K = \min(M, N)$ .
- Q** : float or complex ndarray  
Of shape (N, N) or (K, N) for `mode='economic'`. Not returned if `mode='r'`.

**Raises**

- LinAlgError**  
If decomposition fails.

### Notes

This is an interface to the LAPACK routines sgerqf, dgerqf, cgerqf, zgerqf, sorgqr, dorgqr, cungrq and zungrq.

If `mode=economic`, the shapes of Q and R are (K, N) and (M, K) instead of (N,N) and (M,N), with  $K=\min(M, N)$ .

### Examples

```
>>> from scipy import linalg
>>> from numpy import random, dot, allclose
>>> a = random.randn(6, 9)
>>> r, q = linalg.rq(a)
>>> allclose(a, dot(r, q))
True
>>> r.shape, q.shape
((6, 9), (9, 9))
>>> r2 = linalg.rq(a, mode='r')
>>> allclose(r, r2)
True
>>> r3, q3 = linalg.rq(a, mode='economic')
>>> r3.shape, q3.shape
((6, 6), (6, 9))
```

`scipy.linalg.qz(A, B, output='real', lwork=None, sort=None, overwrite_a=False, overwrite_b=False, check_finite=True)`

QZ decomposition for generalized eigenvalues of a pair of matrices.

The QZ, or generalized Schur, decomposition for a pair of N x N nonsymmetric matrices (A,B) is:

$$(A, B) = (Q * AA * Z', Q * BB * Z')$$

where AA, BB is in generalized Schur form if BB is upper-triangular with non-negative diagonal and AA is upper-triangular, or for real QZ decomposition (`output='real'`) block upper triangular with 1x1 and 2x2 blocks. In this case, the 1x1 blocks correspond to real generalized eigenvalues and 2x2 blocks are 'standardized' by making the corresponding elements of BB have the form:

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

and the pair of corresponding 2x2 blocks in AA and BB will have a complex conjugate pair of generalized eigenvalues. If (`output='complex'`) or A and B are complex matrices, Z' denotes the conjugate-transpose of Z. Q and Z are unitary matrices.

**Parameters**

- A** : (N, N) array\_like  
2d array to decompose
- B** : (N, N) array\_like  
2d array to decompose
- output** : {'real', 'complex'}, optional  
Construct the real or complex QZ decomposition for real matrices. Default is 'real'.
- lwork** : int, optional  
Work array size. If None or -1, it is automatically computed.
- sort** : {None, callable, 'lhp', 'rhp', 'iuc', 'ouc'}, optional  
NOTE: THIS INPUT IS DISABLED FOR NOW. Use ordqz instead.  
Specifies whether the upper eigenvalues should be sorted. A callable may be passed that, given a eigenvalue, returns a boolean denoting whether the eigenvalue should be sorted to the top-left (True). For real matrix pairs, the sort function takes three real arguments (alphan, alphas, beta). The eigenvalue  $x = (\text{alphan} + \text{alphai} * 1j) / \text{beta}$ . For complex matrix pairs or `output='complex'`, the

sort function takes two complex arguments (alpha, beta). The eigenvalue  $\times = (\text{alpha}/\text{beta})$ . Alternatively, string parameters may be used:

- 'lhp' Left-hand plane ( $x.\text{real} < 0.0$ )
- 'rhp' Right-hand plane ( $x.\text{real} > 0.0$ )
- 'iuc' Inside the unit circle ( $x*x.\text{conjugate}() < 1.0$ )
- 'ouc' Outside the unit circle ( $x*x.\text{conjugate}() > 1.0$ )

Defaults to None (no sorting).

**overwrite\_a** : bool, optional

Whether to overwrite data in a (may improve performance)

**overwrite\_b** : bool, optional

Whether to overwrite data in b (may improve performance)

**check\_finite** : bool, optional

If true checks the elements of *A* and *B* are finite numbers. If false does no checking and passes matrix through to underlying algorithm.

#### Returns

**AA** : (N, N) ndarray

Generalized Schur form of A.

**BB** : (N, N) ndarray

Generalized Schur form of B.

**Q** : (N, N) ndarray

The left Schur vectors.

**Z** : (N, N) ndarray

The right Schur vectors.

#### See also:

[ordqz](#)

#### Notes

Q is transposed versus the equivalent function in Matlab.

New in version 0.11.0.

#### Examples

```
>>> from scipy import linalg
>>> np.random.seed(1234)
>>> A = np.arange(9).reshape((3, 3))
>>> B = np.random.randn(3, 3)

>>> AA, BB, Q, Z = linalg.qz(A, B)
>>> AA
array([[ -13.40928183,  -4.62471562,   1.09215523],
       [  0.          ,   0.          ,   1.22805978],
       [  0.          ,   0.          ,   0.31973817]])
>>> BB
array([[ 0.33362547, -1.37393632,  0.02179805],
       [ 0.          ,  1.68144922,  0.74683866],
       [ 0.          ,  0.          ,  0.9258294 ]])
>>> Q
array([[ 0.14134727, -0.97562773,  0.16784365],
       [ 0.49835904, -0.07636948, -0.86360059],
       [ 0.85537081,  0.20571399,  0.47541828]])
>>> Z
array([[ -0.24900855, -0.51772687,  0.81850696],
       [ -0.79813178,  0.58842606,  0.12938478],
       [ -0.54861681, -0.6210585 , -0.55973739]])
```

`scipy.linalg.ordqz(A, B, sort='lhp', output='real', overwrite_a=False, overwrite_b=False, check_finite=True)`

QZ decomposition for a pair of matrices with reordering.

New in version 0.17.0.

**Parameters**

- A** : (N, N) array\_like  
2d array to decompose
- B** : (N, N) array\_like  
2d array to decompose
- sort** : {callable, 'lhp', 'rhp', 'iuc', 'ouc'}, optional  
Specifies whether the upper eigenvalues should be sorted. A callable may be passed that, given an eigenvalue, returns a boolean denoting whether the eigenvalue should be sorted to the top-left (True). For real matrix pairs, the sort function takes three real arguments (alphan, alphai, beta). The eigenvalue  $x = (\text{alphan} + \text{alphai} * 1j) / \text{beta}$ . For complex matrix pairs or `output='complex'`, the sort function takes two complex arguments (alpha, beta). The eigenvalue  $x = (\text{alpha} / \text{beta})$ . Alternatively, string parameters may be used:
  - 'lhp' Left-hand plane ( $x.\text{real} < 0.0$ )
  - 'rhp' Right-hand plane ( $x.\text{real} > 0.0$ )
  - 'iuc' Inside the unit circle ( $x * x.\text{conjugate}() < 1.0$ )
  - 'ouc' Outside the unit circle ( $x * x.\text{conjugate}() > 1.0$ )
- output** : str {'real', 'complex'}, optional  
Construct the real or complex QZ decomposition for real matrices. Default is 'real'.
- overwrite\_a** : bool, optional  
If True, the contents of A are overwritten.
- overwrite\_b** : bool, optional  
If True, the contents of B are overwritten.
- check\_finite** : bool, optional  
If true checks the elements of A and B are finite numbers. If false does no checking and passes matrix through to underlying algorithm.

**Returns**

- AA** : (N, N) ndarray  
Generalized Schur form of A.
- BB** : (N, N) ndarray  
Generalized Schur form of B.
- alpha** : (N,) ndarray  
 $\text{alpha} = \text{alphan} + \text{alphai} * 1j$ . See notes.
- beta** : (N,) ndarray  
See notes.
- Q** : (N, N) ndarray  
The left Schur vectors.
- Z** : (N, N) ndarray  
The right Schur vectors.

**See also:**

[qz](#)

**Notes**

On exit,  $(\text{ALPHAR}(j) + \text{ALPHAI}(j) * i) / \text{BETA}(j)$ ,  $j=1, \dots, N$ , will be the generalized eigenvalues.  $\text{ALPHAR}(j) + \text{ALPHAI}(j) * i$  and  $\text{BETA}(j)$ ,  $j=1, \dots, N$  are the diagonals of the complex Schur form (S,T) that would result if the 2-by-2 diagonal blocks of the real generalized Schur form of (A,B) were further reduced to triangular form using complex unitary transformations. If  $\text{ALPHAI}(j)$  is zero, then the j-th eigenvalue is real; if positive, then the j-th and (j+1)-st eigenvalues are a complex conjugate pair, with  $\text{ALPHAI}(j+1)$  negative.

```
scipy.linalg.schur(a, output='real', lwork=None, overwrite_a=False, sort=None,
                  check_finite=True)
```

Compute Schur decomposition of a matrix.

The Schur decomposition is:

$$A = Z T Z^H$$

where  $Z$  is unitary and  $T$  is either upper-triangular, or for real Schur decomposition (`output='real'`), quasi-upper triangular. In the quasi-triangular form, 2x2 blocks describing complex-valued eigenvalue pairs may extrude from the diagonal.

- Parameters**
- a** : (M, M) array\_like  
Matrix to decompose
  - output** : {'real', 'complex'}, optional  
Construct the real or complex Schur decomposition (for real matrices).
  - lwork** : int, optional  
Work array size. If None or -1, it is automatically computed.
  - overwrite\_a** : bool, optional  
Whether to overwrite data in a (may improve performance).
  - sort** : {None, callable, 'lhp', 'rhp', 'iuc', 'ouc'}, optional  
Specifies whether the upper eigenvalues should be sorted. A callable may be passed that, given a eigenvalue, returns a boolean denoting whether the eigenvalue should be sorted to the top-left (True). Alternatively, string parameters may be used:
    - 'lhp' Left-hand plane ( $x.\text{real} < 0.0$ )
    - 'rhp' Right-hand plane ( $x.\text{real} > 0.0$ )
    - 'iuc' Inside the unit circle ( $x*x.\text{conjugate}() \leq 1.0$ )
    - 'ouc' Outside the unit circle ( $x*x.\text{conjugate}() > 1.0$ )
 Defaults to None (no sorting).
  - check\_finite** : bool, optional  
Whether to check that the input matrix contains only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.
- Returns**
- T** : (M, M) ndarray  
Schur form of A. It is real-valued for the real Schur decomposition.
  - Z** : (M, M) ndarray  
An unitary Schur transformation matrix for A. It is real-valued for the real Schur decomposition.
  - sdim** : int  
If and only if sorting was requested, a third return value will contain the number of eigenvalues satisfying the sort condition.
- Raises**
- LinAlgError**
- Error raised under three conditions:
- 1.The algorithm failed due to a failure of the QR algorithm to compute all eigenvalues
  - 2.If eigenvalue sorting was requested, the eigenvalues could not be reordered due to a failure to separate eigenvalues, usually because of poor conditioning
  - 3.If eigenvalue sorting was requested, roundoff errors caused the leading eigenvalues to no longer satisfy the sorting condition

See also:

[`rsf2csf`](#) Convert real Schur form to complex Schur form

`scipy.linalg.rsf2csf` (*T*, *Z*, *check\_finite=True*)

Convert real Schur form to complex Schur form.

Convert a quasi-diagonal real-valued Schur form to the upper triangular complex-valued Schur form.

**Parameters**

- T** : (M, M) array\_like  
Real Schur form of the original matrix
- Z** : (M, M) array\_like  
Schur transformation matrix
- check\_finite** : bool, optional  
Whether to check that the input matrices contain only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.

**Returns**

- T** : (M, M) ndarray  
Complex Schur form of the original matrix
- Z** : (M, M) ndarray  
Schur transformation matrix corresponding to the complex form

**See also:**

[`schur`](#) Schur decompose a matrix

`scipy.linalg.hessenberg` (*a*, *calc\_q=False*, *overwrite\_a=False*, *check\_finite=True*)

Compute Hessenberg form of a matrix.

The Hessenberg decomposition is:

$$A = Q H Q^H$$

where *Q* is unitary/orthogonal and *H* has only zero elements below the first sub-diagonal.

**Parameters**

- a** : (M, M) array\_like  
Matrix to bring into Hessenberg form.
- calc\_q** : bool, optional  
Whether to compute the transformation matrix. Default is False.
- overwrite\_a** : bool, optional  
Whether to overwrite *a*; may improve performance. Default is False.
- check\_finite** : bool, optional  
Whether to check that the input matrix contains only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.

**Returns**

- H** : (M, M) ndarray  
Hessenberg form of *a*.
- Q** : (M, M) ndarray  
Unitary/orthogonal similarity transformation matrix  $A = Q H Q^H$ . Only returned if *calc\_q=True*.

**See also:**

[`scipy.linalg.interpolative`](#) – Interpolative matrix decompositions

## 5.9.4 Matrix Functions

<code>expm(A[, q])</code>	Compute the matrix exponential using Pade approximation.
<code>logm(A[, disp])</code>	Compute matrix logarithm.
<code>cosm(A)</code>	Compute the matrix cosine.

Continued on next page

Table 5.75 – continued from previous page

<code>sinm(A)</code>	Compute the matrix sine.
<code>tanm(A)</code>	Compute the matrix tangent.
<code>coshm(A)</code>	Compute the hyperbolic matrix cosine.
<code>sinhm(A)</code>	Compute the hyperbolic matrix sine.
<code>tanhm(A)</code>	Compute the hyperbolic matrix tangent.
<code>signm(A[, disp])</code>	Matrix sign function.
<code>sqrtn(A[, disp, blocksize])</code>	Matrix square root.
<code>funm(A, func[, disp])</code>	Evaluate a matrix function specified by a callable.
<code>expm_frechet(A, E[, method, compute_expm, ...])</code>	Frechet derivative of the matrix exponential of A in the direction E.
<code>expm_cond(A[, check_finite])</code>	Relative condition number of the matrix exponential in the Frobenius norm.
<code>fractional_matrix_power(A, t)</code>	Compute the fractional power of a matrix.

`scipy.linalg.expm(A, q=None)`

Compute the matrix exponential using Pade approximation.

**Parameters** **A** : (N, N) array\_like or sparse matrix  
Matrix to be exponentiated.

**Returns** **expm** : (N, N) ndarray  
Matrix exponential of A.

#### References

[R92]

#### Examples

```
>>> from scipy.linalg import expm, sinm, cosm
```

Matrix version of the formula  $\exp(0) = 1$ :

```
>>> expm(np.zeros((2,2)))
array([[ 1.,  0.],
       [ 0.,  1.]])
```

Euler's identity ( $\exp(i*\theta) = \cos(\theta) + i*\sin(\theta)$ ) applied to a matrix:

```
>>> a = np.array([[1.0, 2.0], [-1.0, 3.0]])
>>> expm(1j*a)
array([[ 0.42645930+1.89217551j, -2.13721484-0.97811252j],
       [ 1.06860742+0.48905626j, -1.71075555+0.91406299j]])
>>> cosm(a) + 1j*sinm(a)
array([[ 0.42645930+1.89217551j, -2.13721484-0.97811252j],
       [ 1.06860742+0.48905626j, -1.71075555+0.91406299j]])
```

`scipy.linalg.logm(A, disp=True)`

Compute matrix logarithm.

The matrix logarithm is the inverse of `expm`:  $\text{expm}(\text{logm}(A)) == A$

**Parameters** **A** : (N, N) array\_like  
Matrix whose logarithm to evaluate  
**disp** : bool, optional  
Print warning if error in the result is estimated large instead of returning estimated error. (Default: True)

**Returns** **logm** : (N, N) ndarray  
Matrix logarithm of A  
**errest** : float

(if disp == False)  
 1-norm of the estimated error,  $\|err\|_1 / \|A\|_1$

**References**

[R100], [R101], [R102]

**Examples**

```
>>> from scipy.linalg import logm, expm
>>> a = np.array([[1.0, 3.0], [1.0, 4.0]])
>>> b = logm(a)
>>> b
array([[ -1.02571087,  2.05142174],
       [ 0.68380725,  1.02571087]])
>>> expm(b) # Verify expm(logm(a)) returns a
array([[ 1.,  3.],
       [ 1.,  4.]])
```

`scipy.linalg.cosm(A)`

Compute the matrix cosine.

This routine uses `expm` to compute the matrix exponentials.

**Parameters** `A` : (N, N) array\_like  
 Input array  
**Returns** `cosm` : (N, N) ndarray  
 Matrix cosine of `A`

**Examples**

```
>>> from scipy.linalg import expm, sinm, cosm
```

Euler's identity ( $\exp(i*\theta) = \cos(\theta) + i*\sin(\theta)$ ) applied to a matrix:

```
>>> a = np.array([[1.0, 2.0], [-1.0, 3.0]])
>>> expm(1j*a)
array([[ 0.42645930+1.89217551j, -2.13721484-0.97811252j],
       [ 1.06860742+0.48905626j, -1.71075555+0.91406299j]])
>>> cosm(a) + 1j*sinm(a)
array([[ 0.42645930+1.89217551j, -2.13721484-0.97811252j],
       [ 1.06860742+0.48905626j, -1.71075555+0.91406299j]])
```

`scipy.linalg.sinm(A)`

Compute the matrix sine.

This routine uses `expm` to compute the matrix exponentials.

**Parameters** `A` : (N, N) array\_like  
 Input array.  
**Returns** `sinm` : (N, N) ndarray  
 Matrix cosine of `A`

**Examples**

```
>>> from scipy.linalg import expm, sinm, cosm
```

Euler's identity ( $\exp(i*\theta) = \cos(\theta) + i*\sin(\theta)$ ) applied to a matrix:



```

>>> a = np.array([[1.0, 2.0], [-1.0, 3.0]])
>>> expm(1j*a)
array([[ 0.42645930+1.89217551j, -2.13721484-0.97811252j],
       [ 1.06860742+0.48905626j, -1.71075555+0.91406299j]])
>>> cosm(a) + 1j*sinm(a)
array([[ 0.42645930+1.89217551j, -2.13721484-0.97811252j],
       [ 1.06860742+0.48905626j, -1.71075555+0.91406299j]])

```

`scipy.linalg.tanm(A)`

Compute the matrix tangent.

This routine uses `expm` to compute the matrix exponentials.

**Parameters** `A` : (N, N) array\_like  
Input array.

**Returns** `tanm` : (N, N) ndarray  
Matrix tangent of `A`

#### Examples

```

>>> from scipy.linalg import tanm, sinm, cosm
>>> a = np.array([[1.0, 3.0], [1.0, 4.0]])
>>> t = tanm(a)
>>> t
array([[ -2.00876993,  -8.41880636],
       [ -2.80626879, -10.42757629]])

```

Verify  $\tanm(a) = \sinm(a) \cdot \text{inv}(\cosm(a))$

```

>>> s = sinm(a)
>>> c = cosm(a)
>>> s.dot(np.linalg.inv(c))
array([[ -2.00876993,  -8.41880636],
       [ -2.80626879, -10.42757629]])

```

`scipy.linalg.coshm(A)`

Compute the hyperbolic matrix cosine.

This routine uses `expm` to compute the matrix exponentials.

**Parameters** `A` : (N, N) array\_like  
Input array.

**Returns** `coshm` : (N, N) ndarray  
Hyperbolic matrix cosine of `A`

#### Examples

```

>>> from scipy.linalg import tanhm, sinhm, coshm
>>> a = np.array([[1.0, 3.0], [1.0, 4.0]])
>>> c = coshm(a)
>>> c
array([[ 11.24592233,  38.76236492],
       [ 12.92078831,  50.00828725]])

```

Verify  $\tanhm(a) = \sinhm(a) \cdot \text{inv}(\coshm(a))$

```

>>> t = tanhm(a)
>>> s = sinhm(a)
>>> t - s.dot(np.linalg.inv(c))

```

```
array([[ 2.72004641e-15,  4.55191440e-15],
       [ 0.00000000e+00, -5.55111512e-16]])
```

`scipy.linalg.sinhm(A)`

Compute the hyperbolic matrix sine.

This routine uses `expm` to compute the matrix exponentials.

**Parameters** `A` : (N, N) array\_like  
Input array.

**Returns** `sinhm` : (N, N) ndarray  
Hyperbolic matrix sine of `A`

**Examples**

```
>>> from scipy.linalg import tanhm, sinhm, coshm
>>> a = np.array([[1.0, 3.0], [1.0, 4.0]])
>>> s = sinhm(a)
>>> s
array([[ 10.57300653,  39.28826594],
       [ 13.09608865,  49.86127247]])
```

Verify `tanhm(a) = sinhm(a).dot(inv(coshm(a)))`

```
>>> t = tanhm(a)
>>> c = coshm(a)
>>> t - s.dot(np.linalg.inv(c))
array([[ 2.72004641e-15,  4.55191440e-15],
       [ 0.00000000e+00, -5.55111512e-16]])
```

`scipy.linalg.tanhm(A)`

Compute the hyperbolic matrix tangent.

This routine uses `expm` to compute the matrix exponentials.

**Parameters** `A` : (N, N) array\_like  
Input array

**Returns** `tanhm` : (N, N) ndarray  
Hyperbolic matrix tangent of `A`

**Examples**

```
>>> from scipy.linalg import tanhm, sinhm, coshm
>>> a = np.array([[1.0, 3.0], [1.0, 4.0]])
>>> t = tanhm(a)
>>> t
array([[ 0.3428582 ,  0.51987926],
       [ 0.17329309,  0.86273746]])
```

Verify `tanhm(a) = sinhm(a).dot(inv(coshm(a)))`

```
>>> s = sinhm(a)
>>> c = coshm(a)
>>> t - s.dot(np.linalg.inv(c))
array([[ 2.72004641e-15,  4.55191440e-15],
       [ 0.00000000e+00, -5.55111512e-16]])
```

`scipy.linalg.signm(A, disp=True)`

Matrix sign function.

Extension of the scalar `sign(x)` to matrices.

**Parameters** **A** : (N, N) array\_like  
Matrix at which to evaluate the sign function

**disp** : bool, optional  
Print warning if error in the result is estimated large instead of returning estimated error. (Default: True)

**Returns** **signm** : (N, N) ndarray  
Value of the sign function at A

**errest** : float  
(if disp == False)  
1-norm of the estimated error,  $\|err\|_1 / \|A\|_1$

**Examples**

```
>>> from scipy.linalg import signm, eigvals
>>> a = [[1,2,3], [1,2,1], [1,1,1]]
>>> eigvals(a)
array([ 4.12488542+0.j, -0.76155718+0.j,  0.63667176+0.j])
>>> eigvals(signm(a))
array([-1.+0.j,  1.+0.j,  1.+0.j])
```

`scipy.linalg.sqrtm(A, disp=True, blocksize=64)`  
Matrix square root.

**Parameters** **A** : (N, N) array\_like  
Matrix whose square root to evaluate

**disp** : bool, optional  
Print warning if error in the result is estimated large instead of returning estimated error. (Default: True)

**blocksize** : integer, optional  
If the blocksize is not degenerate with respect to the size of the input array, then use a blocked algorithm. (Default: 64)

**Returns** **sqrtm** : (N, N) ndarray  
Value of the sqrt function at A

**errest** : float  
(if disp == False)  
Frobenius norm of the estimated error,  $\|err\|_F / \|A\|_F$

**References**

[R118]

**Examples**

```
>>> from scipy.linalg import sqrtm
>>> a = np.array([[1.0, 3.0], [1.0, 4.0]])
>>> r = sqrtm(a)
>>> r
array([[ 0.75592895,  1.13389342],
       [ 0.37796447,  1.88982237]])
>>> r.dot(r)
array([[ 1.,  3.],
       [ 1.,  4.]])
```

`scipy.linalg.funm(A, func, disp=True)`  
Evaluate a matrix function specified by a callable.

Returns the value of matrix-valued function  $f$  at A. The function  $f$  is an extension of the scalar-valued function *func* to matrices.

**Parameters**

- A** : (N, N) array\_like  
Matrix at which to evaluate the function
- func** : callable  
Callable object that evaluates a scalar function f. Must be vectorized (eg. using vectorize).
- disp** : bool, optional  
Print warning if error in the result is estimated large instead of returning estimated error. (Default: True)

**Returns**

- funm** : (N, N) ndarray  
Value of the matrix function specified by func evaluated at A
- errest** : float  
(if disp == False)  
1-norm of the estimated error,  $\|err\|_1 / \|A\|_1$

### Notes

This function implements the general algorithm based on Schur decomposition (Algorithm 9.1.1. in [R95]).

If the input matrix is known to be diagonalizable, then relying on the eigendecomposition is likely to be faster. For example, if your matrix is Hermitian, you can do

```
>>> from scipy.linalg import eigh
>>> def funm_herm(a, func, check_finite=False):
...     w, v = eigh(a, check_finite=check_finite)
...     ## if you further know that your matrix is positive semidefinite,
...     ## you can optionally guard against precision errors by doing
...     # w = np.maximum(w, 0)
...     w = func(w)
...     return (v * w).dot(v.conj().T)
```

### References

[R95]

### Examples

```
>>> from scipy.linalg import funm
>>> a = np.array([[1.0, 3.0], [1.0, 4.0]])
>>> funm(a, lambda x: x*x)
array([[ 4., 15.],
       [ 5., 19.]])
>>> a.dot(a)
array([[ 4., 15.],
       [ 5., 19.]])
```

`scipy.linalg.expm_frechet` (*A*, *E*, *method=None*, *compute\_expm=True*, *check\_finite=True*)  
Frechet derivative of the matrix exponential of A in the direction E.

**Parameters**

- A** : (N, N) array\_like  
Matrix of which to take the matrix exponential.
- E** : (N, N) array\_like  
Matrix direction in which to take the Frechet derivative.
- method** : str, optional  
Choice of algorithm. Should be one of
  - SPS (default)
  - blockEnlarge
- compute\_expm** : bool, optional  
Whether to compute also *expm\_A* in addition to *expm\_frechet\_AE*. Default is True.

**check\_finite** : bool, optional

Whether to check that the input matrix contains only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.

**Returns**

**expm\_A** : ndarray

Matrix exponential of A.

**expm\_frechet\_AE** : ndarray

Frechet derivative of the matrix exponential of A in the direction E.

For `compute_expm = False`, only `expm_frechet_AE` is returned.

**See also:**

[`expm`](#) Compute the exponential of a matrix.

**Notes**

This section describes the available implementations that can be selected by the *method* parameter. The default method is *SPS*.

Method *blockEnlarge* is a naive algorithm.

Method *SPS* is Scaling-Pade-Squaring [R93]. It is a sophisticated implementation which should take only about 3/8 as much time as the naive implementation. The asymptotics are the same.

New in version 0.13.0.

**References**

[R93]

**Examples**

```
>>> import scipy.linalg
>>> A = np.random.randn(3, 3)
>>> E = np.random.randn(3, 3)
>>> expm_A, expm_frechet_AE = scipy.linalg.expm_frechet(A, E)
>>> expm_A.shape, expm_frechet_AE.shape
((3, 3), (3, 3))

>>> import scipy.linalg
>>> A = np.random.randn(3, 3)
>>> E = np.random.randn(3, 3)
>>> expm_A, expm_frechet_AE = scipy.linalg.expm_frechet(A, E)
>>> M = np.zeros((6, 6))
>>> M[:3, :3] = A; M[:3, 3:] = E; M[3:, 3:] = A
>>> expm_M = scipy.linalg.expm(M)
>>> np.allclose(expm_A, expm_M[:3, :3])
True
>>> np.allclose(expm_frechet_AE, expm_M[:3, 3:])
True
```

`scipy.linalg.expm_cond(A, check_finite=True)`

Relative condition number of the matrix exponential in the Frobenius norm.

**Parameters** **A** : 2d array\_like

Square input matrix with shape (N, N).

**check\_finite** : bool, optional

Whether to check that the input matrix contains only finite numbers. Disabling may give a performance gain, but may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs.

**Returns**     **kappa** : float  
                   The relative condition number of the matrix exponential in the Frobenius norm

**See also:**

[\*expm\*](#)            Compute the exponential of a matrix.  
[\*expm\\_frechet\*](#)    Compute the Frechet derivative of the matrix exponential.

**Notes**

A faster estimate for the condition number in the 1-norm has been published but is not yet implemented in scipy.  
 New in version 0.14.0.

`scipy.linalg.fractional_matrix_power` (*A*, *t*)  
 Compute the fractional power of a matrix.

Proceeds according to the discussion in section (6) of [R94].

**Parameters**    **A** : (N, N) array\_like  
                   Matrix whose fractional power to evaluate.  
                   **t** : float  
                   Fractional power.  
**Returns**        **X** : (N, N) array\_like  
                   The fractional power of the matrix.

**References**

[R94]

**Examples**

```
>>> from scipy.linalg import fractional_matrix_power
>>> a = np.array([[1.0, 3.0], [1.0, 4.0]])
>>> b = fractional_matrix_power(a, 0.5)
>>> b
array([[ 0.75592895,  1.13389342],
       [ 0.37796447,  1.88982237]])
>>> np.dot(b, b)        # Verify square root
array([[ 1.,  3.],
       [ 1.,  4.]])
```

## 5.9.5 Matrix Equation Solvers

---

<code>solve_sylvester(a, b, q)</code>	Computes a solution ( <i>X</i> ) to the Sylvester equation $AX + XB = Q$ .
<code>solve_continuous_are(a, b, q, r)</code>	Solves the continuous algebraic Riccati equation (CARE).
<code>solve_discrete_are(a, b, q, r)</code>	Solves the discrete algebraic Riccati equation (DARE).
<code>solve_discrete_lyapunov(a, q[, method])</code>	Solves the discrete Lyapunov equation $AXA^H - X + Q = 0$ .
<code>solve_lyapunov(a, q)</code>	Solves the continuous Lyapunov equation $AX + XA^H = Q$ .

---

`scipy.linalg.solve_sylvester` (*a*, *b*, *q*)  
 Computes a solution (*X*) to the Sylvester equation  $AX + XB = Q$ .

**Parameters**    **a** : (M, M) array\_like  
                   Leading matrix of the Sylvester equation

**b** : (N, N) array\_like  
Trailing matrix of the Sylvester equation

**q** : (M, N) array\_like  
Right-hand side

**Returns** **x** : (M, N) ndarray  
The solution to the Sylvester equation.

**Raises** **LinAlgError**  
If solution was not found

**Notes**

Computes a solution to the Sylvester matrix equation via the Bartels- Stewart algorithm. The A and B matrices first undergo Schur decompositions. The resulting matrices are used to construct an alternative Sylvester equation ( $RY + YS^T = F$ ) where the R and S matrices are in quasi-triangular form (or, when R, S or F are complex, triangular form). The simplified equation is then solved using \*TRSYL from LAPACK directly.

New in version 0.11.0.

`scipy.linalg.solve_continuous_are(a, b, q, r)`  
Solves the continuous algebraic Riccati equation (CARE).

The CARE is defined as

$$(A'X + XA - XBR^{-1}B'X + Q = 0)$$

It is solved directly using a Schur decomposition method.

**Parameters** **a** : (M, M) array\_like  
Input

**b** : (M, N) array\_like  
Input

**q** : (M, M) array\_like  
Input

**r** : (N, N) array\_like  
Non-singular, square matrix

**Returns** **x** : (M, M) ndarray  
Solution to the continuous algebraic Riccati equation

**See also:**

**`solve_discrete_are`**

Solves the discrete algebraic Riccati equation

**Notes**

Method taken from: Laub, "A Schur Method for Solving Algebraic Riccati Equations." U.S. Energy Research and Development Agency under contract ERDA-E(49-18)-2087. <http://dspace.mit.edu/bitstream/handle/1721.1/1301/R-0859-05666488.pdf>

New in version 0.11.0.

`scipy.linalg.solve_discrete_are(a, b, q, r)`  
Solves the discrete algebraic Riccati equation (DARE).

The DARE is defined as

$$X = A'XA - (A'XB)(R + B'XB)^{-1}(B'XA) + Q$$

It is solved directly using a Schur decomposition method.

**Parameters** **a** : (M, M) array\_like

Non-singular, square matrix  
**b** : (M, N) array\_like  
 Input  
**q** : (M, M) array\_like  
 Input  
**r** : (N, N) array\_like  
 Non-singular, square matrix  
**Returns** **x** : ndarray  
 Solution to the continuous Lyapunov equation

**See also:**

[\*solve\\_continuous\\_are\*](#)

Solves the continuous algebraic Riccati equation

**Notes**

Method taken from: Laub, "A Schur Method for Solving Algebraic Riccati Equations." U.S. Energy Research and Development Agency under contract ERDA-E(49-18)-2087. <http://dspace.mit.edu/bitstream/handle/1721.1/1301/R-0859-05666488.pdf>

New in version 0.11.0.

`scipy.linalg.solve_discrete_lyapunov(a, q, method=None)`

Solves the discrete Lyapunov equation  $AXA^H - X + Q = 0$ .

**Parameters** **a, q** : (M, M) array\_like  
 Square matrices corresponding to A and Q in the equation above respectively. Must have the same shape.  
**method** : {'direct', 'bilinear'}, optional  
 Type of solver.  
 If not given, chosen to be `direct` if M is less than 10 and `bilinear` otherwise.  
**Returns** **x** : ndarray  
 Solution to the discrete Lyapunov equation

**See also:**

[\*solve\\_lyapunov\*](#)

computes the solution to the continuous Lyapunov equation

**Notes**

This section describes the available solvers that can be selected by the 'method' parameter. The default method is `direct` if M is less than 10 and `bilinear` otherwise.

Method `direct` uses a direct analytical solution to the discrete Lyapunov equation. The algorithm is given in, for example, [R116]. However it requires the linear solution of a system with dimension  $M^2$  so that performance degrades rapidly for even moderately sized matrices.

Method `bilinear` uses a bilinear transformation to convert the discrete Lyapunov equation to a continuous Lyapunov equation ( $BX + XB' = -C$ ) where  $B = (A - I)(A + I)^{-1}$  and  $C = 2(A' + I)^{-1}Q(A + I)^{-1}$ . The continuous equation can be efficiently solved since it is a special case of a Sylvester equation. The transformation algorithm is from Popov (1964) as described in [R117].

New in version 0.11.0.



## References

[R116], [R117]

`scipy.linalg.solve_lyapunov(a, q)`

Solves the continuous Lyapunov equation  $AX + XA^H = Q$ .

Uses the Bartels-Stewart algorithm to find  $X$ .

**Parameters**

- a** : array\_like  
A square matrix
- q** : array\_like  
Right-hand side square matrix

**Returns**

- x** : array\_like  
Solution to the continuous Lyapunov equation

**See also:**

[`solve\_sylvester`](#)

computes the solution to the Sylvester equation

## Notes

Because the continuous Lyapunov equation is just a special form of the Sylvester equation, this solver relies entirely on `solve_sylvester` for a solution.

New in version 0.11.0.

## 5.9.6 Special Matrices

---

<code>block_diag(*arrs)</code>	Create a block diagonal matrix from provided arrays.
<code>circulant(c)</code>	Construct a circulant matrix.
<code>companion(a)</code>	Create a companion matrix.
<code>dft(n[, scale])</code>	Discrete Fourier transform matrix.
<code>hadamard(n[, dtype])</code>	Construct a Hadamard matrix.
<code>hankel(c[, r])</code>	Construct a Hankel matrix.
<code>helmert(n[, full])</code>	Create a Helmert matrix of order $n$ .
<code>hilbert(n)</code>	Create a Hilbert matrix of order $n$ .
<code>invhilbert(n[, exact])</code>	Compute the inverse of the Hilbert matrix of order $n$ .
<code>leslie(f, s)</code>	Create a Leslie matrix.
<code>pascal(n[, kind, exact])</code>	Returns the $n \times n$ Pascal matrix.
<code>invpascal(n[, kind, exact])</code>	Returns the inverse of the $n \times n$ Pascal matrix.
<code>toeplitz(c[, r])</code>	Construct a Toeplitz matrix.
<code>tri(N[, M, k, dtype])</code>	Construct $(N, M)$ matrix filled with ones at and below the $k$ -th diagonal.

---

`scipy.linalg.block_diag(*arrs)`

Create a block diagonal matrix from provided arrays.

Given the inputs  $A$ ,  $B$  and  $C$ , the output will have these arrays arranged on the diagonal:

```
[[A, 0, 0],
 [0, B, 0],
 [0, 0, C]]
```

**Parameters** **A, B, C, ...** : array\_like, up to 2-D

Input arrays. A 1-D array or array\_like sequence of length  $n$  is treated as a 2-D array with shape  $(1, n)$ .

**Returns** **D** : ndarray  
 Array with  $A, B, C, \dots$  on the diagonal.  $D$  has the same dtype as  $A$ .

**Notes**

If all the input arrays are square, the output is known as a block diagonal matrix.

Empty sequences (i.e., array-likes of zero size) are ignored.

**Examples**

```
>>> from scipy.linalg import block_diag
>>> A = [[1, 0],
...      [0, 1]]
>>> B = [[3, 4, 5],
...      [6, 7, 8]]
>>> C = [[7]]
>>> block_diag(A, B, C)
array([[1, 0, 0, 0, 0, 0],
       [0, 1, 0, 0, 0, 0],
       [0, 0, 3, 4, 5, 0],
       [0, 0, 6, 7, 8, 0],
       [0, 0, 0, 0, 0, 7]])
>>> block_diag(1.0, [2, 3], [[4, 5], [6, 7]])
array([[ 1.,  0.,  0.,  0.,  0.],
       [ 0.,  2.,  3.,  0.,  0.],
       [ 0.,  0.,  0.,  4.,  5.],
       [ 0.,  0.,  0.,  6.,  7.]])
```

scipy.linalg.**circulant**(*c*)

Construct a circulant matrix.

**Parameters** **c** : (N,) array\_like  
 1-D array, the first column of the matrix.

**Returns** **A** : (N, N) ndarray  
 A circulant matrix whose first column is *c*.

**See also:**

**toeplitz** Toeplitz matrix

**hankel** Hankel matrix

**Notes**

New in version 0.8.0.

**Examples**

```
>>> from scipy.linalg import circulant
>>> circulant([1, 2, 3])
array([[1, 3, 2],
       [2, 1, 3],
       [3, 2, 1]])
```

scipy.linalg.**companion**(*a*)

Create a companion matrix.

Create the companion matrix [R90] associated with the polynomial whose coefficients are given in *a*.

**Parameters** **a** : (N,) array\_like  
1-D array of polynomial coefficients. The length of *a* must be at least two, and *a*[0] must not be zero.

**Returns** **c** : (N-1, N-1) ndarray  
The first row of *c* is  $-a[1:] / a[0]$ , and the first sub-diagonal is all ones. The data-type of the array is the same as the data-type of  $1.0 * a[0]$ .

**Raises** **ValueError**  
If any of the following are true: a) *a.ndim* != 1; b) *a.size* < 2; c) *a*[0] == 0.

**Notes**

New in version 0.8.0.

**References**

[R90]

**Examples**

```
>>> from scipy.linalg import companion
>>> companion([1, -10, 31, -30])
array([[ 10., -31.,  30.],
       [  1.,   0.,   0.],
       [  0.,   1.,   0.]])
```

`scipy.linalg.dft` (*n*, *scale=None*)

Discrete Fourier transform matrix.

Create the matrix that computes the discrete Fourier transform of a sequence [R91]. The *n*-th primitive root of unity used to generate the matrix is  $\exp(-2 * \pi * i / n)$ , where  $i = \sqrt{-1}$ .

**Parameters** **n** : int  
Size the matrix to create.

**scale** : str, optional  
Must be None, 'sqrtn', or 'n'. If *scale* is 'sqrtn', the matrix is divided by  $\sqrt{n}$ . If *scale* is 'n', the matrix is divided by *n*. If *scale* is None (the default), the matrix is not normalized, and the return value is simply the Vandermonde matrix of the roots of unity.

**Returns** **m** : (n, n) ndarray  
The DFT matrix.

**Notes**

When *scale* is None, multiplying a vector by the matrix returned by `dft` is mathematically equivalent to (but much less efficient than) the calculation performed by `scipy.fftpack.fft`.

New in version 0.14.0.

**References**

[R91]

**Examples**

```
>>> from scipy.linalg import dft
>>> np.set_printoptions(precision=5, suppress=True)
>>> x = np.array([1, 2, 3, 0, 3, 2, 1, 0])
>>> m = dft(8)
>>> m.dot(x) # Compute the DFT of x
```

```
array([[ 12.+0.j, -2.-2.j,  0.-4.j, -2.+2.j,  4.+0.j, -2.-2.j,
        -0.+4.j, -2.+2.j])
```

Verify that `m.dot(x)` is the same as `fft(x)`.

```
>>> from scipy.fftpack import fft
>>> fft(x)      # Same result as m.dot(x)
array([[ 12.+0.j, -2.-2.j,  0.-4.j, -2.+2.j,  4.+0.j, -2.-2.j,
        0.+4.j, -2.+2.j])
```

`scipy.linalg.hadamard(n, dtype=<type 'int'>)`

Construct a Hadamard matrix.

Constructs an  $n$ -by- $n$  Hadamard matrix, using Sylvester's construction.  $n$  must be a power of 2.

**Parameters**

- n** : int  
The order of the matrix.  $n$  must be a power of 2.
- dtype** : dtype, optional  
The data type of the array to be constructed.

**Returns**

- H** : (n, n) ndarray  
The Hadamard matrix.

**Notes**

New in version 0.8.0.

**Examples**

```
>>> from scipy.linalg import hadamard
>>> hadamard(2, dtype=complex)
array([[ 1.+0.j,  1.+0.j],
       [ 1.+0.j, -1.-0.j]])
>>> hadamard(4)
array([[ 1,  1,  1,  1],
       [ 1, -1,  1, -1],
       [ 1,  1, -1, -1],
       [ 1, -1, -1,  1]])
```

`scipy.linalg.hankel(c, r=None)`

Construct a Hankel matrix.

The Hankel matrix has constant anti-diagonals, with  $c$  as its first column and  $r$  as its last row. If  $r$  is not given, then  $r = \text{zeros\_like}(c)$  is assumed.

**Parameters**

- c** : array\_like  
First column of the matrix. Whatever the actual shape of  $c$ , it will be converted to a 1-D array.
- r** : array\_like, optional  
Last row of the matrix. If `None`,  $r = \text{zeros\_like}(c)$  is assumed.  $r[0]$  is ignored; the last row of the returned matrix is  $[c[-1], r[1:]]$ . Whatever the actual shape of  $r$ , it will be converted to a 1-D array.

**Returns**

- A** : (len(c), len(r)) ndarray  
The Hankel matrix. Dtype is the same as  $(c[0] + r[0]).\text{dtype}$ .

See also:

`toeplitz` Toeplitz matrix

`circulant` circulant matrix

**Examples**

```
>>> from scipy.linalg import hankel
>>> hankel([1, 17, 99])
array([[ 1, 17, 99],
       [17, 99,  0],
       [99,  0,  0]])
>>> hankel([1,2,3,4], [4,7,7,8,9])
array([[1, 2, 3, 4, 7],
       [2, 3, 4, 7, 7],
       [3, 4, 7, 7, 8],
       [4, 7, 7, 8, 9]])
```

`scipy.linalg.helmert` (*n*, *full=False*)

Create a Helmert matrix of order *n*.

This has applications in statistics, compositional or simplicial analysis, and in Aitchison geometry.

**Parameters**    **n** : int  
                   The size of the array to create.

**full** : bool, optional  
                   If True the (n, n) ndarray will be returned. Otherwise the submatrix that does not include the first row will be returned. Default: False.

**Returns**        **M** : ndarray  
                   The Helmert matrix. The shape is (n, n) or (n-1, n) depending on the *full* argument.

**Examples**

```
>>> from scipy.linalg import helmert
>>> helmert(5, full=True)
array([[ 0.4472136 ,  0.4472136 ,  0.4472136 ,  0.4472136 ,  0.4472136 ],
       [ 0.70710678, -0.70710678,  0.          ,  0.          ,  0.          ],
       [ 0.40824829,  0.40824829, -0.81649658,  0.          ,  0.          ],
       [ 0.28867513,  0.28867513,  0.28867513, -0.8660254 ,  0.          ],
       [ 0.2236068 ,  0.2236068 ,  0.2236068 ,  0.2236068 , -0.89442719]])
```

`scipy.linalg.hilbert` (*n*)

Create a Hilbert matrix of order *n*.

Returns the *n* by *n* array with entries  $h[i,j] = 1 / (i + j + 1)$ .

**Parameters**    **n** : int  
                   The size of the array to create.

**Returns**        **h** : (n, n) ndarray  
                   The Hilbert matrix.

**See also:**

[\*invhilbert\*](#) Compute the inverse of a Hilbert matrix.

**Notes**

New in version 0.10.0.

**Examples**

```
>>> from scipy.linalg import hilbert
>>> hilbert(3)
array([[ 1.          ,  0.5          ,  0.33333333],
```

```
[ 0.5          , 0.33333333, 0.25          ],
 [ 0.33333333, 0.25          , 0.2           ]])
```

`scipy.linalg.invhilbert` (*n*, *exact=False*)

Compute the inverse of the Hilbert matrix of order *n*.

The entries in the inverse of a Hilbert matrix are integers. When *n* is greater than 14, some entries in the inverse exceed the upper limit of 64 bit integers. The *exact* argument provides two options for dealing with these large integers.

**Parameters** **n** : int

The order of the Hilbert matrix.

**exact** : bool, optional

If False, the data type of the array that is returned is `np.float64`, and the array is an approximation of the inverse. If True, the array is the exact integer inverse array. To represent the exact inverse when *n* > 14, the returned array is an object array of long integers. For *n* ≤ 14, the exact inverse is returned as an array with data type `np.int64`.

**Returns** **invh** : (n, n) ndarray

The data type of the array is `np.float64` if *exact* is False. If *exact* is True, the data type is either `np.int64` (for *n* ≤ 14) or `object` (for *n* > 14). In the latter case, the objects in the array will be long integers.

**See also:**

`hilbert` Create a Hilbert matrix.

**Notes**

New in version 0.10.0.

**Examples**

```
>>> from scipy.linalg import invhilbert
>>> invhilbert(4)
array([[ 16., -120., 240., -140.],
       [-120., 1200., -2700., 1680.],
       [ 240., -2700., 6480., -4200.],
       [-140., 1680., -4200., 2800.]])
>>> invhilbert(4, exact=True)
array([[ 16, -120, 240, -140],
       [-120, 1200, -2700, 1680],
       [ 240, -2700, 6480, -4200],
       [-140, 1680, -4200, 2800]], dtype=int64)
>>> invhilbert(16) [7,7]
4.2475099528537506e+19
>>> invhilbert(16, exact=True) [7,7]
42475099528537378560L
```

`scipy.linalg.leslie` (*f*, *s*)

Create a Leslie matrix.

Given the length *n* array of fecundity coefficients *f* and the length *n*-1 array of survival coefficients *s*, return the associated Leslie matrix.

**Parameters** **f** : (N,) array\_like

The “fecundity” coefficients.

**s** : (N-1,) array\_like

The “survival” coefficients, has to be 1-D. The length of *s* must be one less than the length of *f*, and it must be at least 1.

**Returns** **L** : (N, N) ndarray  
 The array is zero except for the first row, which is  $f$ , and the first sub-diagonal, which is  $s$ . The data-type of the array will be the data-type of  $f[0]+s[0]$ .

**Notes**

New in version 0.8.0.

The Leslie matrix is used to model discrete-time, age-structured population growth [R98] [R99]. In a population with  $n$  age classes, two sets of parameters define a Leslie matrix: the  $n$  “fecundity coefficients”, which give the number of offspring per-capita produced by each age class, and the  $n - 1$  “survival coefficients”, which give the per-capita survival rate of each age class.

**References**

[R98], [R99]

**Examples**

```
>>> from scipy.linalg import leslie
>>> leslie([0.1, 2.0, 1.0, 0.1], [0.2, 0.8, 0.7])
array([[ 0.1,  2. ,  1. ,  0.1],
       [ 0.2,  0. ,  0. ,  0. ],
       [ 0. ,  0.8,  0. ,  0. ],
       [ 0. ,  0. ,  0.7,  0. ]])
```

`scipy.linalg.pascal` ( $n$ ,  $kind='symmetric'$ ,  $exact=True$ )

Returns the  $n \times n$  Pascal matrix.

The Pascal matrix is a matrix containing the binomial coefficients as its elements.

**Parameters** **n** : int  
 The size of the matrix to create; that is, the result is an  $n \times n$  matrix.  
**kind** : str, optional  
 Must be one of ‘symmetric’, ‘lower’, or ‘upper’. Default is ‘symmetric’.  
**exact** : bool, optional  
 If *exact* is True, the result is either an array of type `numpy.uint64` (if  $n < 35$ ) or an object array of Python long integers. If *exact* is False, the coefficients in the matrix are computed using `scipy.special.comb` with *exact=False*. The result will be a floating point array, and the values in the array will not be the exact coefficients, but this version is much faster than *exact=True*.

**Returns** **p** : (n, n) ndarray  
 The Pascal matrix.

**See also:**

`invpascal`

**Notes**

See [http://en.wikipedia.org/wiki/Pascal\\_matrix](http://en.wikipedia.org/wiki/Pascal_matrix) for more information about Pascal matrices.

New in version 0.11.0.

**Examples**

```
>>> from scipy.linalg import pascal
>>> pascal(4)
array([[ 1,  1,  1,  1],
       [ 1,  2,  3,  4],
       [ 1,  3,  6, 10],
```

```

        [ 1,  4, 10, 20]], dtype=uint64)
>>> pascal(4, kind='lower')
array([[1, 0, 0, 0],
       [1, 1, 0, 0],
       [1, 2, 1, 0],
       [1, 3, 3, 1]], dtype=uint64)
>>> pascal(50)[-1, -1]
25477612258980856902730428600L
>>> from scipy.special import comb
>>> comb(98, 49, exact=True)
25477612258980856902730428600L
    
```

`scipy.linalg.invpascal` (*n*, *kind*='symmetric', *exact*=True)

Returns the inverse of the *n* x *n* Pascal matrix.

The Pascal matrix is a matrix containing the binomial coefficients as its elements.

**Parameters**    **n** : int

The size of the matrix to create; that is, the result is an *n* x *n* matrix.

**kind** : str, optional

Must be one of 'symmetric', 'lower', or 'upper'. Default is 'symmetric'.

**exact** : bool, optional

If *exact* is True, the result is either an array of type `numpy.int64` (if *n* ≤ 35) or an object array of Python integers. If *exact* is False, the coefficients in the matrix are computed using `scipy.special.comb` with *exact*=False. The result will be a floating point array, and for large *n*, the values in the array will not be the exact coefficients.

**Returns**        **invp** : (n, n) ndarray

The inverse of the Pascal matrix.

**See also:**

`pascal`

**Notes**

New in version 0.16.0.

**References**

[R96], [R97]

**Examples**

```

>>> from scipy.linalg import invpascal, pascal
>>> invp = invpascal(5)
>>> invp
array([[ 5, -10, 10, -5,  1],
       [-10, 30, -35, 19, -4],
        [ 10, -35, 46, -27,  6],
        [ -5, 19, -27, 17, -4],
        [  1, -4,  6, -4,  1]])

>>> p = pascal(5)
>>> p.dot(invp)
array([[ 1.,  0.,  0.,  0.,  0.],
       [ 0.,  1.,  0.,  0.,  0.],
       [ 0.,  0.,  1.,  0.,  0.],
       [ 0.,  0.,  0.,  1.,  0.],
       [ 0.,  0.,  0.,  0.,  1.]])
    
```



An example of the use of *kind* and *exact*:

```
>>> invpascal(5, kind='lower', exact=False)
array([[ 1., -0.,  0., -0.,  0.],
       [-1.,  1., -0.,  0., -0.],
       [ 1., -2.,  1., -0.,  0.],
       [-1.,  3., -3.,  1., -0.],
       [ 1., -4.,  6., -4.,  1.]])
```

`scipy.linalg.toeplitz(c, r=None)`

Construct a Toeplitz matrix.

The Toeplitz matrix has constant diagonals, with *c* as its first column and *r* as its first row. If *r* is not given, *r* == `conjugate(c)` is assumed.

**Parameters** **c** : array\_like

First column of the matrix. Whatever the actual shape of *c*, it will be converted to a 1-D array.

**r** : array\_like, optional

First row of the matrix. If *None*, *r* = `conjugate(c)` is assumed; in this case, if *c*[0] is real, the result is a Hermitian matrix. *r*[0] is ignored; the first row of the returned matrix is [*c*[0], *r*[1:]]. Whatever the actual shape of *r*, it will be converted to a 1-D array.

**Returns** **A** : (len(c), len(r)) ndarray

The Toeplitz matrix. Dtype is the same as `(c[0] + r[0]).dtype`.

See also:

[\*circulant\*](#) circulant matrix

[\*hankel\*](#) Hankel matrix

**Notes**

The behavior when *c* or *r* is a scalar, or when *c* is complex and *r* is *None*, was changed in version 0.8.0. The behavior in previous versions was undocumented and is no longer supported.

**Examples**

```
>>> from scipy.linalg import toeplitz
>>> toeplitz([1,2,3], [1,4,5,6])
array([[1, 4, 5, 6],
       [2, 1, 4, 5],
       [3, 2, 1, 4]])
>>> toeplitz([1.0, 2+3j, 4-1j])
array([[ 1.+0.j,  2.-3.j,  4.+1.j],
       [ 2.+3.j,  1.+0.j,  2.-3.j],
       [ 4.-1.j,  2.+3.j,  1.+0.j]])
```

`scipy.linalg.tri(N, M=None, k=0, dtype=None)`

Construct (N, M) matrix filled with ones at and below the k-th diagonal.

The matrix has  $A[i,j] == 1$  for  $i \leq j + k$

**Parameters** **N** : int

The size of the first dimension of the matrix.

**M** : int or *None*, optional

The size of the second dimension of the matrix. If *M* is *None*,  $M = N$  is assumed.

**k** : int, optional  
 Number of subdiagonal below which matrix is filled with ones.  $k = 0$  is the main diagonal,  $k < 0$  subdiagonal and  $k > 0$  superdiagonal.

**dtype** : dtype, optional  
 Data type of the matrix.

**Returns** **tri** : (N, M) ndarray  
 Tri matrix.

**Examples**

```
>>> from scipy.linalg import tri
>>> tri(3, 5, 2, dtype=int)
array([[1, 1, 1, 0, 0],
       [1, 1, 1, 1, 0],
       [1, 1, 1, 1, 1]])
>>> tri(3, 5, -1, dtype=int)
array([[0, 0, 0, 0, 0],
       [1, 0, 0, 0, 0],
       [1, 1, 0, 0, 0]])
```

### 5.9.7 Low-level routines

---

<code>get_blas_funcs(names[, arrays, dtype])</code>	Return available BLAS function objects from names.
<code>get_lapack_funcs(names[, arrays, dtype])</code>	Return available LAPACK function objects from names.
<code>find_best_blas_type([arrays, dtype])</code>	Find best-matching BLAS/LAPACK type.

---

`scipy.linalg.get_blas_funcs` (*names*, *arrays=()*, *dtype=None*)  
 Return available BLAS function objects from names.

Arrays are used to determine the optimal prefix of BLAS routines.

**Parameters** **names** : str or sequence of str  
 Name(s) of BLAS functions without type prefix.

**arrays** : sequence of ndarrays, optional  
 Arrays can be given to determine optimal prefix of BLAS routines. If not given, double-precision routines will be used, otherwise the most generic type in arrays will be used.

**dtype** : str or dtype, optional  
 Data-type specifier. Not used if *arrays* is non-empty.

**Returns** **funcs** : list  
 List containing the found function(s).

**Notes**

This routine automatically chooses between Fortran/C interfaces. Fortran code is used whenever possible for arrays with column major order. In all other cases, C code is preferred.

In BLAS, the naming convention is that all functions start with a type prefix, which depends on the type of the principal matrix. These can be one of {'s', 'd', 'c', 'z'} for the numpy types {float32, float64, complex64, complex128} respectively. The code and the dtype are stored in attributes *typecode* and *dtype* of the returned functions.

`scipy.linalg.get_lapack_funcs` (*names*, *arrays=()*, *dtype=None*)  
 Return available LAPACK function objects from names.

Arrays are used to determine the optimal prefix of LAPACK routines.

<b>Parameters</b>	<b>names</b> : str or sequence of str Name(s) of LAPACK functions without type prefix.
	<b>arrays</b> : sequence of ndarrays, optional Arrays can be given to determine optimal prefix of LAPACK routines. If not given, double-precision routines will be used, otherwise the most generic type in arrays will be used.
	<b>dtype</b> : str or dtype, optional Data-type specifier. Not used if <i>arrays</i> is non-empty.
<b>Returns</b>	<b>funcs</b> : list List containing the found function(s).

**Notes**

This routine automatically chooses between Fortran/C interfaces. Fortran code is used whenever possible for arrays with column major order. In all other cases, C code is preferred.

In LAPACK, the naming convention is that all functions start with a type prefix, which depends on the type of the principal matrix. These can be one of {'s', 'd', 'c', 'z'} for the numpy types {float32, float64, complex64, complex128} respectively, and are stored in attribute *typecode* of the returned functions.

`scipy.linalg.find_best_blas_type` (*arrays=()*, *dtype=None*)

Find best-matching BLAS/LAPACK type.

Arrays are used to determine the optimal prefix of BLAS routines.

<b>Parameters</b>	<b>arrays</b> : sequence of ndarrays, optional Arrays can be given to determine optimal prefix of BLAS routines. If not given, double-precision routines will be used, otherwise the most generic type in arrays will be used.
	<b>dtype</b> : str or dtype, optional Data-type specifier. Not used if <i>arrays</i> is non-empty.
<b>Returns</b>	<b>prefix</b> : str BLAS/LAPACK prefix character.
	<b>dtype</b> : dtype Inferred Numpy data type.
	<b>prefer_fortran</b> : bool Whether to prefer Fortran order routines over C order.

**See also:**

`scipy.linalg.blas` – Low-level BLAS functions

`scipy.linalg.lapack` – Low-level LAPACK functions

`scipy.linalg.cython_blas` – Low-level BLAS functions for Cython

`scipy.linalg.cython_lapack` – Low-level LAPACK functions for Cython

## 5.10 Low-level BLAS functions (`scipy.linalg.blas`)

This module contains low-level functions from the BLAS library.

New in version 0.12.0.

**Warning:** These functions do little to no error checking. It is possible to cause crashes by mis-using them, so prefer using the higher-level routines in `scipy.linalg`.

### 5.10.1 Finding functions

<code>get_blas_funcs(names[, arrays, dtype])</code>	Return available BLAS function objects from names.
<code>find_best_blas_type([arrays, dtype])</code>	Find best-matching BLAS/LAPACK type.

`scipy.linalg.blas.get_blas_funcs` (*names*, *arrays*=(), *dtype*=None)

Return available BLAS function objects from names.

Arrays are used to determine the optimal prefix of BLAS routines.

<b>Parameters</b>	<b>names</b> : str or sequence of str Name(s) of BLAS functions without type prefix.
	<b>arrays</b> : sequence of ndarrays, optional Arrays can be given to determine optimal prefix of BLAS routines. If not given, double-precision routines will be used, otherwise the most generic type in arrays will be used.
<b>Returns</b>	<b>dtype</b> : str or dtype, optional Data-type specifier. Not used if <i>arrays</i> is non-empty.
	<b>funcs</b> : list List containing the found function(s).

#### Notes

This routine automatically chooses between Fortran/C interfaces. Fortran code is used whenever possible for arrays with column major order. In all other cases, C code is preferred.

In BLAS, the naming convention is that all functions start with a type prefix, which depends on the type of the principal matrix. These can be one of {'s', 'd', 'c', 'z'} for the numpy types {float32, float64, complex64, complex128} respectively. The code and the dtype are stored in attributes *typecode* and *dtype* of the returned functions.

`scipy.linalg.blas.find_best_blas_type` (*arrays*=(), *dtype*=None)

Find best-matching BLAS/LAPACK type.

Arrays are used to determine the optimal prefix of BLAS routines.

<b>Parameters</b>	<b>arrays</b> : sequence of ndarrays, optional Arrays can be given to determine optimal prefix of BLAS routines. If not given, double-precision routines will be used, otherwise the most generic type in arrays will be used.
	<b>dtype</b> : str or dtype, optional Data-type specifier. Not used if <i>arrays</i> is non-empty.
<b>Returns</b>	<b>prefix</b> : str BLAS/LAPACK prefix character.
	<b>dtype</b> : dtype Inferred Numpy data type.
	<b>prefer_fortran</b> : bool Whether to prefer Fortran order routines over C order.

### 5.10.2 BLAS Level 1 functions

<code>caxpy(x,y,[n,a,offx,incx,offy,incy])</code>	Wrapper for <code>caxpy</code> .
<code>ccopy(x,y,[n,offx,incx,offy,incy])</code>	Wrapper for <code>ccopy</code> .

Continued on next page

Table 5.80 – continued from previous page

<code>cdotc(x,y,[n,offx,incx,offy,incy])</code>	Wrapper for <code>cdotc</code> .
<code>cdotu(x,y,[n,offx,incx,offy,incy])</code>	Wrapper for <code>cdotu</code> .
<code>crotg(a,b)</code>	Wrapper for <code>crotg</code> .
<code>cscal(a,x,[n,offx,incx])</code>	Wrapper for <code>cscal</code> .
<code>csrot(...)</code>	Wrapper for <code>csrot</code> .
<code>csscal(a,x,[n,offx,incx,overwrite_x])</code>	Wrapper for <code>csscal</code> .
<code>cswap(x,y,[n,offx,incx,offy,incy])</code>	Wrapper for <code>cswap</code> .
<code>dasum(x,[n,offx,incx])</code>	Wrapper for <code>dasum</code> .
<code>daxpy(x,y,[n,a,offx,incx,offy,incy])</code>	Wrapper for <code>daxpy</code> .
<code>dcopy(x,y,[n,offx,incx,offy,incy])</code>	Wrapper for <code>dcopy</code> .
<code>ddot(x,y,[n,offx,incx,offy,incy])</code>	Wrapper for <code>ddot</code> .
<code>dnrm2(x,[n,offx,incx])</code>	Wrapper for <code>dnrm2</code> .
<code>drot(...)</code>	Wrapper for <code>drot</code> .
<code>drotg(a,b)</code>	Wrapper for <code>drotg</code> .
<code>drotm(...)</code>	Wrapper for <code>drotm</code> .
<code>drotmg(d1,d2,x1,y1)</code>	Wrapper for <code>drotmg</code> .
<code>dscal(a,x,[n,offx,incx])</code>	Wrapper for <code>dscal</code> .
<code>dswap(x,y,[n,offx,incx,offy,incy])</code>	Wrapper for <code>dswap</code> .
<code>dzasum(x,[n,offx,incx])</code>	Wrapper for <code>dzasum</code> .
<code>dznrm2(x,[n,offx,incx])</code>	Wrapper for <code>dznrm2</code> .
<code>icamax(x,[n,offx,incx])</code>	Wrapper for <code>icamax</code> .
<code>idamax(x,[n,offx,incx])</code>	Wrapper for <code>idamax</code> .
<code>isamax(x,[n,offx,incx])</code>	Wrapper for <code>isamax</code> .
<code>izamax(x,[n,offx,incx])</code>	Wrapper for <code>izamax</code> .
<code>sasum(x,[n,offx,incx])</code>	Wrapper for <code>sasum</code> .
<code>saxpy(x,y,[n,a,offx,incx,offy,incy])</code>	Wrapper for <code>saxpy</code> .
<code>scasum(x,[n,offx,incx])</code>	Wrapper for <code>scasum</code> .
<code>scnrm2(x,[n,offx,incx])</code>	Wrapper for <code>scnrm2</code> .
<code>scopy(x,y,[n,offx,incx,offy,incy])</code>	Wrapper for <code>scopy</code> .
<code>sdot(x,y,[n,offx,incx,offy,incy])</code>	Wrapper for <code>sdot</code> .
<code>snrm2(x,[n,offx,incx])</code>	Wrapper for <code>snrm2</code> .
<code>srot(...)</code>	Wrapper for <code>srot</code> .
<code>srotg(a,b)</code>	Wrapper for <code>srotg</code> .
<code>srotm(...)</code>	Wrapper for <code>srotm</code> .
<code>srotmg(d1,d2,x1,y1)</code>	Wrapper for <code>srotmg</code> .
<code>sscal(a,x,[n,offx,incx])</code>	Wrapper for <code>sscal</code> .
<code>sswap(x,y,[n,offx,incx,offy,incy])</code>	Wrapper for <code>sswap</code> .
<code>zaxpy(x,y,[n,a,offx,incx,offy,incy])</code>	Wrapper for <code>zaxpy</code> .
<code>zcopy(x,y,[n,offx,incx,offy,incy])</code>	Wrapper for <code>zcopy</code> .
<code>zdotc(x,y,[n,offx,incx,offy,incy])</code>	Wrapper for <code>zdotc</code> .
<code>zdotu(x,y,[n,offx,incx,offy,incy])</code>	Wrapper for <code>zdotu</code> .
<code>zdrot(...)</code>	Wrapper for <code>zdrot</code> .
<code>zdscal(a,x,[n,offx,incx,overwrite_x])</code>	Wrapper for <code>zdscal</code> .
<code>zrotg(a,b)</code>	Wrapper for <code>zrotg</code> .
<code>zscal(a,x,[n,offx,incx])</code>	Wrapper for <code>zscal</code> .
<code>zswap(x,y,[n,offx,incx,offy,incy])</code>	Wrapper for <code>zswap</code> .

`scipy.linalg.blas.caxpy(x, y[, n, a, offx, incx, offy, incy]) = <fortran object>`  
 Wrapper for `caxpy`.

**Parameters** `x`: input rank-1 array('F') with bounds (\*)  
`y`: input rank-1 array('F') with bounds (\*)  
**Returns** `z`: rank-1 array('F') with bounds (\*) and y storage

*Other Parameters*

- n** : input int, optional  
Default:  $(\text{len}(x)-\text{offx})/\text{abs}(\text{incx})$
- a** : input complex, optional  
Default: (1.0, 0.0)
- offx** : input int, optional  
Default: 0
- incx** : input int, optional  
Default: 1
- offy** : input int, optional  
Default: 0
- incy** : input int, optional  
Default: 1

`scipy.linalg.blas.ccopy(x, y[, n, offx, incx, offy, incy]) = <fortran object>`  
 Wrapper for `ccopy`.

- Parameters** **x** : input rank-1 array('F') with bounds (\*)  
**y** : input rank-1 array('F') with bounds (\*)
- Returns** **y** : rank-1 array('F') with bounds (\*)

*Other Parameters*

- n** : input int, optional  
Default:  $(\text{len}(x)-\text{offx})/\text{abs}(\text{incx})$
- offx** : input int, optional  
Default: 0
- incx** : input int, optional  
Default: 1
- offy** : input int, optional  
Default: 0
- incy** : input int, optional  
Default: 1

`scipy.linalg.blas.cdotc(x, y[, n, offx, incx, offy, incy]) = <fortran cdotc>`  
 Wrapper for `cdotc`.

- Parameters** **x** : input rank-1 array('F') with bounds (\*)  
**y** : input rank-1 array('F') with bounds (\*)
- Returns** **xy** : complex

*Other Parameters*

- n** : input int, optional  
Default:  $(\text{len}(x)-\text{offx})/\text{abs}(\text{incx})$
- offx** : input int, optional  
Default: 0
- incx** : input int, optional  
Default: 1
- offy** : input int, optional  
Default: 0
- incy** : input int, optional  
Default: 1

`scipy.linalg.blas.cdotu(x, y[, n, offx, incx, offy, incy]) = <fortran cdotu>`  
 Wrapper for `cdotu`.

- Parameters** **x** : input rank-1 array('F') with bounds (\*)  
**y** : input rank-1 array('F') with bounds (\*)
- Returns** **xy** : complex

**Other Parameters**

**n** : input int, optional  
 Default:  $(\text{len}(x) - \text{offx}) / \text{abs}(\text{incx})$   
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**offy** : input int, optional  
 Default: 0  
**incy** : input int, optional  
 Default: 1

`scipy.linalg.blas.crotg(a, b) = <fortran object>`

Wrapper for `crotg`.

**Parameters** **a** : input complex  
**b** : input complex  
**Returns** **c** : complex  
**s** : complex

`scipy.linalg.blas.cscal(a, x[, n, offx, incx]) = <fortran object>`

Wrapper for `cscal`.

**Parameters** **a** : input complex  
**x** : input rank-1 array('F') with bounds (\*)  
**Returns** **x** : rank-1 array('F') with bounds (\*)  
**Other Parameters**  
**n** : input int, optional  
 Default:  $(\text{len}(x) - \text{offx}) / \text{abs}(\text{incx})$   
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1

`scipy.linalg.blas.csrot(x, y, c, s[, n, offx, incx, offy, incy, overwrite_x, overwrite_y]) = <fortran object>`

Wrapper for `csrot`.

**Parameters** **x** : input rank-1 array('F') with bounds (\*)  
**y** : input rank-1 array('F') with bounds (\*)  
**c** : input float  
**s** : input float  
**Returns** **x** : rank-1 array('F') with bounds (\*)  
**y** : rank-1 array('F') with bounds (\*)  
**Other Parameters**  
**n** : input int, optional  
 Default:  $(\text{len}(x) - 1 - \text{offx}) / \text{abs}(\text{incx}) + 1$   
**overwrite\_x** : input int, optional  
 Default: 0  
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**overwrite\_y** : input int, optional  
 Default: 0  
**offy** : input int, optional  
 Default: 0

**incy** : input int, optional  
 Default: 1

`scipy.linalg.blas.csscal(a, x[, n, offx, incx, overwrite_x]) = <fortran object>`

Wrapper for `csscal`.

**Parameters** **a** : input float  
**x** : input rank-1 array('F') with bounds (\*)

**Returns** **x** : rank-1 array('F') with bounds (\*)

**Other Parameters**  
**n** : input int, optional  
 Default:  $(\text{len}(x) - \text{offx}) / \text{abs}(\text{incx})$   
**overwrite\_x** : input int, optional  
 Default: 0  
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1

`scipy.linalg.blas.cswap(x, y[, n, offx, incx, offy, incy]) = <fortran object>`

Wrapper for `cswap`.

**Parameters** **x** : input rank-1 array('F') with bounds (\*)  
**y** : input rank-1 array('F') with bounds (\*)

**Returns** **x** : rank-1 array('F') with bounds (\*)  
**y** : rank-1 array('F') with bounds (\*)

**Other Parameters**  
**n** : input int, optional  
 Default:  $(\text{len}(x) - \text{offx}) / \text{abs}(\text{incx})$   
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**offy** : input int, optional  
 Default: 0  
**incy** : input int, optional  
 Default: 1

`scipy.linalg.blas.dasum(x[, n, offx, incx]) = <fortran dasum>`

Wrapper for `dasum`.

**Parameters** **x** : input rank-1 array('d') with bounds (\*)

**Returns** **s** : float

**Other Parameters**  
**n** : input int, optional  
 Default:  $(\text{len}(x) - \text{offx}) / \text{abs}(\text{incx})$   
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1

`scipy.linalg.blas.daxpy(x, y[, n, a, offx, incx, offy, incy]) = <fortran object>`

Wrapper for `daxpy`.

**Parameters** **x** : input rank-1 array('d') with bounds (\*)  
**y** : input rank-1 array('d') with bounds (\*)

**Returns** **z** : rank-1 array('d') with bounds (\*) and y storage



**Other Parameters**

**n** : input int, optional  
 Default:  $(\text{len}(x) - \text{offx}) / \text{abs}(\text{incx})$   
**a** : input float, optional  
 Default: 1.0  
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**offy** : input int, optional  
 Default: 0  
**incy** : input int, optional  
 Default: 1

`scipy.linalg.blas.dcopy(x, y[, n, offx, incx, offy, incy]) = <fortran object>`  
 Wrapper for `dcopy`.

**Parameters** **x** : input rank-1 array('d') with bounds (\*)  
**y** : input rank-1 array('d') with bounds (\*)

**Returns** **y** : rank-1 array('d') with bounds (\*)

**Other Parameters**

**n** : input int, optional  
 Default:  $(\text{len}(x) - \text{offx}) / \text{abs}(\text{incx})$   
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**offy** : input int, optional  
 Default: 0  
**incy** : input int, optional  
 Default: 1

`scipy.linalg.blas.ddot(x, y[, n, offx, incx, offy, incy]) = <fortran ddot>`  
 Wrapper for `ddot`.

**Parameters** **x** : input rank-1 array('d') with bounds (\*)  
**y** : input rank-1 array('d') with bounds (\*)

**Returns** **xy** : float

**Other Parameters**

**n** : input int, optional  
 Default:  $(\text{len}(x) - \text{offx}) / \text{abs}(\text{incx})$   
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**offy** : input int, optional  
 Default: 0  
**incy** : input int, optional  
 Default: 1

`scipy.linalg.blas.dnrm2(x[, n, offx, incx]) = <fortran dnrm2>`  
 Wrapper for `dnrm2`.

**Parameters** **x** : input rank-1 array('d') with bounds (\*)

**Returns** **n2** : float

**Other Parameters**

**n** : input int, optional

Default:  $(\text{len}(x) - \text{offx}) / \text{abs}(\text{incx})$   
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1

`scipy.linalg.blas.drot(x, y, c, s[, n, offx, incx, offy, incy, overwrite_x, overwrite_y]) = <fortran object>`

Wrapper for `drot`.

**Parameters** **x** : input rank-1 array('d') with bounds (\*)  
**y** : input rank-1 array('d') with bounds (\*)  
**c** : input float  
**s** : input float

**Returns** **x** : rank-1 array('d') with bounds (\*)  
**y** : rank-1 array('d') with bounds (\*)

**Other Parameters**

**n** : input int, optional  
 Default:  $(\text{len}(x) - 1 - \text{offx}) / \text{abs}(\text{incx}) + 1$   
**overwrite\_x** : input int, optional  
 Default: 0  
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**overwrite\_y** : input int, optional  
 Default: 0  
**offy** : input int, optional  
 Default: 0  
**incy** : input int, optional  
 Default: 1

`scipy.linalg.blas.drotg(a, b) = <fortran object>`

Wrapper for `drotg`.

**Parameters** **a** : input float  
**b** : input float

**Returns** **c** : float  
**s** : float

`scipy.linalg.blas.drotm(x, y, param[, n, offx, incx, offy, incy, overwrite_x, overwrite_y]) = <fortran object>`

Wrapper for `drotm`.

**Parameters** **x** : input rank-1 array('d') with bounds (\*)  
**y** : input rank-1 array('d') with bounds (\*)  
**param** : input rank-1 array('d') with bounds (5)

**Returns** **x** : rank-1 array('d') with bounds (\*)  
**y** : rank-1 array('d') with bounds (\*)

**Other Parameters**

**n** : input int, optional  
 Default:  $(\text{len}(x) - \text{offx}) / \text{abs}(\text{incx})$   
**overwrite\_x** : input int, optional  
 Default: 0  
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional

Default: 1  
**overwrite\_y** : input int, optional  
 Default: 0  
**offy** : input int, optional  
 Default: 0  
**incy** : input int, optional  
 Default: 1

`scipy.linalg.blas.drotmg(d1, d2, x1, y1) = <fortran object>`

Wrapper for `drotmg`.

**Parameters** **d1** : input float  
**d2** : input float  
**x1** : input float  
**y1** : input float  
**Returns** **param** : rank-1 array('d') with bounds (5)

`scipy.linalg.blas.dsca1(a, x[, n, offx, incx]) = <fortran object>`

Wrapper for `dsca1`.

**Parameters** **a** : input float  
**x** : input rank-1 array('d') with bounds (\*)  
**Returns** **x** : rank-1 array('d') with bounds (\*)  
**Other Parameters**  
**n** : input int, optional  
 Default:  $(\text{len}(x) - \text{offx}) / \text{abs}(\text{incx})$   
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1

`scipy.linalg.blas.dswap(x, y[, n, offx, incx, offy, incy]) = <fortran object>`

Wrapper for `dswap`.

**Parameters** **x** : input rank-1 array('d') with bounds (\*)  
**y** : input rank-1 array('d') with bounds (\*)  
**Returns** **x** : rank-1 array('d') with bounds (\*)  
**y** : rank-1 array('d') with bounds (\*)  
**Other Parameters**  
**n** : input int, optional  
 Default:  $(\text{len}(x) - \text{offx}) / \text{abs}(\text{incx})$   
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**offy** : input int, optional  
 Default: 0  
**incy** : input int, optional  
 Default: 1

`scipy.linalg.blas.dzasum(x[, n, offx, incx]) = <fortran dzasum>`

Wrapper for `dzasum`.

**Parameters** **x** : input rank-1 array('D') with bounds (\*)  
**Returns** **s** : float  
**Other Parameters**  
**n** : input int, optional  
 Default:  $(\text{len}(x) - \text{offx}) / \text{abs}(\text{incx})$

**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1

`scipy.linalg.blas.dznrm2(x[, n, offx, incx]) = <fortran dznrm2>`  
 Wrapper for `dznrm2`.

**Parameters** **x** : input rank-1 array('D') with bounds (\*)  
**Returns** **n2** : float  
**Other Parameters**  
**n** : input int, optional  
 Default:  $(\text{len}(x) - \text{offx}) / \text{abs}(\text{incx})$   
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1

`scipy.linalg.blas.icamax(x[, n, offx, incx]) = <fortran object>`  
 Wrapper for `icamax`.

**Parameters** **x** : input rank-1 array('F') with bounds (\*)  
**Returns** **k** : int  
**Other Parameters**  
**n** : input int, optional  
 Default:  $(\text{len}(x) - \text{offx}) / \text{abs}(\text{incx})$   
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1

`scipy.linalg.blas.idamax(x[, n, offx, incx]) = <fortran object>`  
 Wrapper for `idamax`.

**Parameters** **x** : input rank-1 array('d') with bounds (\*)  
**Returns** **k** : int  
**Other Parameters**  
**n** : input int, optional  
 Default:  $(\text{len}(x) - \text{offx}) / \text{abs}(\text{incx})$   
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1

`scipy.linalg.blas.isamax(x[, n, offx, incx]) = <fortran object>`  
 Wrapper for `isamax`.

**Parameters** **x** : input rank-1 array('f') with bounds (\*)  
**Returns** **k** : int  
**Other Parameters**  
**n** : input int, optional  
 Default:  $(\text{len}(x) - \text{offx}) / \text{abs}(\text{incx})$   
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1

`scipy.linalg.blas.izamax(x[, n, offx, incx]) = <fortran object>`

Wrapper for izamax.

**Parameters** **x** : input rank-1 array('D') with bounds (\*)

**Returns** **k** : int

**Other Parameters**

**n** : input int, optional

Default:  $(\text{len}(x) - \text{offx}) / \text{abs}(\text{incx})$

**offx** : input int, optional

Default: 0

**incx** : input int, optional

Default: 1

`scipy.linalg.blas.sasum(x[, n, offx, incx]) = <fortran sasum>`

Wrapper for sasum.

**Parameters** **x** : input rank-1 array('f') with bounds (\*)

**Returns** **s** : float

**Other Parameters**

**n** : input int, optional

Default:  $(\text{len}(x) - \text{offx}) / \text{abs}(\text{incx})$

**offx** : input int, optional

Default: 0

**incx** : input int, optional

Default: 1

`scipy.linalg.blas.saxpy(x, y[, n, a, offx, incx, offy, incy]) = <fortran object>`

Wrapper for saxpy.

**Parameters** **x** : input rank-1 array('f') with bounds (\*)

**y** : input rank-1 array('f') with bounds (\*)

**Returns** **z** : rank-1 array('f') with bounds (\*) and y storage

**Other Parameters**

**n** : input int, optional

Default:  $(\text{len}(x) - \text{offx}) / \text{abs}(\text{incx})$

**a** : input float, optional

Default: 1.0

**offx** : input int, optional

Default: 0

**incx** : input int, optional

Default: 1

**offy** : input int, optional

Default: 0

**incy** : input int, optional

Default: 1

`scipy.linalg.blas.scasum(x[, n, offx, incx]) = <fortran scasum>`

Wrapper for scasum.

**Parameters** **x** : input rank-1 array('F') with bounds (\*)

**Returns** **s** : float

**Other Parameters**

**n** : input int, optional

Default:  $(\text{len}(x) - \text{offx}) / \text{abs}(\text{incx})$

**offx** : input int, optional

Default: 0

**incx** : input int, optional

Default: 1

`scipy.linalg.blas.scnrm2(x[, n, offx, incx]) = <fortran scnrm2>`  
 Wrapper for `scnrm2`.

**Parameters** `x` : input rank-1 array('F') with bounds (\*)  
**Returns** `n2` : float  
**Other Parameters**  
`n` : input int, optional  
 Default:  $(\text{len}(x) - \text{offx}) / \text{abs}(\text{incx})$   
`offx` : input int, optional  
 Default: 0  
`incx` : input int, optional  
 Default: 1

`scipy.linalg.blas.scopy(x, y[, n, offx, incx, offy, incy]) = <fortran object>`  
 Wrapper for `scopy`.

**Parameters** `x` : input rank-1 array('f') with bounds (\*)  
`y` : input rank-1 array('f') with bounds (\*)  
**Returns** `y` : rank-1 array('f') with bounds (\*)  
**Other Parameters**  
`n` : input int, optional  
 Default:  $(\text{len}(x) - \text{offx}) / \text{abs}(\text{incx})$   
`offx` : input int, optional  
 Default: 0  
`incx` : input int, optional  
 Default: 1  
`offy` : input int, optional  
 Default: 0  
`incy` : input int, optional  
 Default: 1

`scipy.linalg.blas.sdot(x, y[, n, offx, incx, offy, incy]) = <fortran sdot>`  
 Wrapper for `sdot`.

**Parameters** `x` : input rank-1 array('f') with bounds (\*)  
`y` : input rank-1 array('f') with bounds (\*)  
**Returns** `xy` : float  
**Other Parameters**  
`n` : input int, optional  
 Default:  $(\text{len}(x) - \text{offx}) / \text{abs}(\text{incx})$   
`offx` : input int, optional  
 Default: 0  
`incx` : input int, optional  
 Default: 1  
`offy` : input int, optional  
 Default: 0  
`incy` : input int, optional  
 Default: 1

`scipy.linalg.blas.snrm2(x[, n, offx, incx]) = <fortran snrm2>`  
 Wrapper for `snrm2`.

**Parameters** `x` : input rank-1 array('f') with bounds (\*)  
**Returns** `n2` : float  
**Other Parameters**  
`n` : input int, optional

Default:  $(\text{len}(x) - \text{offx}) / \text{abs}(\text{incx})$   
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1

`scipy.linalg.blas.srot(x, y, c, s[, n, offx, incx, offy, incy, overwrite_x, overwrite_y]) = <fortran object>`

Wrapper for `srot`.

**Parameters** **x** : input rank-1 array('f') with bounds (\*)  
**y** : input rank-1 array('f') with bounds (\*)  
**c** : input float  
**s** : input float

**Returns** **x** : rank-1 array('f') with bounds (\*)  
**y** : rank-1 array('f') with bounds (\*)

**Other Parameters**  
**n** : input int, optional  
 Default:  $(\text{len}(x) - 1 - \text{offx}) / \text{abs}(\text{incx}) + 1$   
**overwrite\_x** : input int, optional  
 Default: 0  
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**overwrite\_y** : input int, optional  
 Default: 0  
**offy** : input int, optional  
 Default: 0  
**incy** : input int, optional  
 Default: 1

`scipy.linalg.blas.srotg(a, b) = <fortran object>`

Wrapper for `srotg`.

**Parameters** **a** : input float  
**b** : input float

**Returns** **c** : float  
**s** : float

`scipy.linalg.blas.srotm(x, y, param[, n, offx, incx, offy, incy, overwrite_x, overwrite_y]) = <fortran object>`

Wrapper for `srotm`.

**Parameters** **x** : input rank-1 array('f') with bounds (\*)  
**y** : input rank-1 array('f') with bounds (\*)  
**param** : input rank-1 array('f') with bounds (5)

**Returns** **x** : rank-1 array('f') with bounds (\*)  
**y** : rank-1 array('f') with bounds (\*)

**Other Parameters**  
**n** : input int, optional  
 Default:  $(\text{len}(x) - \text{offx}) / \text{abs}(\text{incx})$   
**overwrite\_x** : input int, optional  
 Default: 0  
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional

Default: 1  
**overwrite\_y** : input int, optional  
 Default: 0  
**offy** : input int, optional  
 Default: 0  
**incy** : input int, optional  
 Default: 1

`scipy.linalg.blas.srotmg(d1, d2, x1, y1) = <fortran object>`  
 Wrapper for `srotmg`.

**Parameters** **d1** : input float  
**d2** : input float  
**x1** : input float  
**y1** : input float  
**Returns** **param** : rank-1 array('f') with bounds (5)

`scipy.linalg.blas.sscal(a, x[, n, offx, incx]) = <fortran object>`  
 Wrapper for `sscal`.

**Parameters** **a** : input float  
**x** : input rank-1 array('f') with bounds (\*)  
**Returns** **x** : rank-1 array('f') with bounds (\*)  
**Other Parameters**  
**n** : input int, optional  
 Default: (len(x)-offx)/abs(incx)  
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1

`scipy.linalg.blas.sswap(x, y[, n, offx, incx, offy, incy]) = <fortran object>`  
 Wrapper for `sswap`.

**Parameters** **x** : input rank-1 array('f') with bounds (\*)  
**y** : input rank-1 array('f') with bounds (\*)  
**Returns** **x** : rank-1 array('f') with bounds (\*)  
**y** : rank-1 array('f') with bounds (\*)  
**Other Parameters**  
**n** : input int, optional  
 Default: (len(x)-offx)/abs(incx)  
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**offy** : input int, optional  
 Default: 0  
**incy** : input int, optional  
 Default: 1

`scipy.linalg.blas.zaxpy(x, y[, n, a, offx, incx, offy, incy]) = <fortran object>`  
 Wrapper for `zaxpy`.

**Parameters** **x** : input rank-1 array('D') with bounds (\*)  
**y** : input rank-1 array('D') with bounds (\*)  
**Returns** **z** : rank-1 array('D') with bounds (\*) and y storage  
**Other Parameters**  
**n** : input int, optional



Default:  $(\text{len}(x)-\text{offx})/\text{abs}(\text{incx})$   
**a** : input complex, optional  
 Default: (1.0, 0.0)  
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**offy** : input int, optional  
 Default: 0  
**incy** : input int, optional  
 Default: 1

`scipy.linalg.blas.zcopy(x, y[, n, offx, incx, offy, incy]) = <fortran object>`  
 Wrapper for `zcopy`.

**Parameters** **x** : input rank-1 array('D') with bounds (\*)  
**y** : input rank-1 array('D') with bounds (\*)  
**Returns** **y** : rank-1 array('D') with bounds (\*)  
**Other Parameters**  
**n** : input int, optional  
 Default:  $(\text{len}(x)-\text{offx})/\text{abs}(\text{incx})$   
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**offy** : input int, optional  
 Default: 0  
**incy** : input int, optional  
 Default: 1

`scipy.linalg.blas.zdotc(x, y[, n, offx, incx, offy, incy]) = <fortran zdotc>`  
 Wrapper for `zdotc`.

**Parameters** **x** : input rank-1 array('D') with bounds (\*)  
**y** : input rank-1 array('D') with bounds (\*)  
**Returns** **xy** : complex  
**Other Parameters**  
**n** : input int, optional  
 Default:  $(\text{len}(x)-\text{offx})/\text{abs}(\text{incx})$   
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**offy** : input int, optional  
 Default: 0  
**incy** : input int, optional  
 Default: 1

`scipy.linalg.blas.zdotu(x, y[, n, offx, incx, offy, incy]) = <fortran zdotu>`  
 Wrapper for `zdotu`.

**Parameters** **x** : input rank-1 array('D') with bounds (\*)  
**y** : input rank-1 array('D') with bounds (\*)  
**Returns** **xy** : complex  
**Other Parameters**  
**n** : input int, optional  
 Default:  $(\text{len}(x)-\text{offx})/\text{abs}(\text{incx})$

**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**offy** : input int, optional  
 Default: 0  
**incy** : input int, optional  
 Default: 1

`scipy.linalg.blas.zdrot(x, y, c, s[, n, offx, incx, offy, incy, overwrite_x, overwrite_y]) = <fortran object>`

Wrapper for zdrot.

**Parameters** **x** : input rank-1 array('D') with bounds (\*)  
**y** : input rank-1 array('D') with bounds (\*)  
**c** : input float  
**s** : input float

**Returns** **x** : rank-1 array('D') with bounds (\*)  
**y** : rank-1 array('D') with bounds (\*)

**Other Parameters**

**n** : input int, optional  
 Default: (len(x)-1-offx)/abs(incx)+1  
**overwrite\_x** : input int, optional  
 Default: 0  
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**overwrite\_y** : input int, optional  
 Default: 0  
**offy** : input int, optional  
 Default: 0  
**incy** : input int, optional  
 Default: 1

`scipy.linalg.blas.zdscal(a, x[, n, offx, incx, overwrite_x]) = <fortran object>`

Wrapper for zdscal.

**Parameters** **a** : input float  
**x** : input rank-1 array('D') with bounds (\*)

**Returns** **x** : rank-1 array('D') with bounds (\*)

**Other Parameters**

**n** : input int, optional  
 Default: (len(x)-offx)/abs(incx)  
**overwrite\_x** : input int, optional  
 Default: 0  
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1

`scipy.linalg.blas.zrotg(a, b) = <fortran object>`

Wrapper for zrotg.

**Parameters** **a** : input complex  
**b** : input complex

**Returns** **c** : complex  
**s** : complex

`scipy.linalg.blas.zscal(a, x[, n, offx, incx]) = <fortran object>`

Wrapper for zscal.

**Parameters** **a** : input complex  
**x** : input rank-1 array('D') with bounds (\*)

**Returns** **x** : rank-1 array('D') with bounds (\*)

**Other Parameters**

**n** : input int, optional  
Default:  $(\text{len}(x) - \text{offx}) / \text{abs}(\text{incx})$

**offx** : input int, optional  
Default: 0

**incx** : input int, optional  
Default: 1

`scipy.linalg.blas.zswap(x, y[, n, offx, incx, offy, incy]) = <fortran object>`

Wrapper for zswap.

**Parameters** **x** : input rank-1 array('D') with bounds (\*)  
**y** : input rank-1 array('D') with bounds (\*)

**Returns** **x** : rank-1 array('D') with bounds (\*)  
**y** : rank-1 array('D') with bounds (\*)

**Other Parameters**

**n** : input int, optional  
Default:  $(\text{len}(x) - \text{offx}) / \text{abs}(\text{incx})$

**offx** : input int, optional  
Default: 0

**incx** : input int, optional  
Default: 1

**offy** : input int, optional  
Default: 0

**incy** : input int, optional  
Default: 1

### 5.10.3 BLAS Level 2 functions

<code>cgemv(...)</code>	Wrapper for cgemv.
<code>cgerc(...)</code>	Wrapper for cgerc.
<code>cgeru(...)</code>	Wrapper for cgeru.
<code>chemv(...)</code>	Wrapper for chemv.
<code>ctrmv(...)</code>	Wrapper for ctrmv.
<code>csyr(alpha,x,[lower,incx,offx,n,a,overwrite_a])</code>	Wrapper for csyr.
<code>cher(alpha,x,[lower,incx,offx,n,a,overwrite_a])</code>	Wrapper for cher.
<code>cher2(...)</code>	Wrapper for cher2.
<code>dgemv(...)</code>	Wrapper for dgemv.
<code>dger(...)</code>	Wrapper for dger.
<code>dsymv(...)</code>	Wrapper for dsymv.
<code>dtrmv(...)</code>	Wrapper for dtrmv.
<code>dsyr(alpha,x,[lower,incx,offx,n,a,overwrite_a])</code>	Wrapper for dsyr.
<code>dsyr2(...)</code>	Wrapper for dsyr2.
<code>sgemv(...)</code>	Wrapper for sgemv.

Continued on next page

Table 5.81 – continued from previous page

<code>sger(...)</code>	Wrapper for <code>sger</code> .
<code>ssymv(...)</code>	Wrapper for <code>ssymv</code> .
<code>strmv(...)</code>	Wrapper for <code>strmv</code> .
<code>ssyr(alpha,x,[lower,incx,offx,n,a,overwrite_a])</code>	Wrapper for <code>ssyr</code> .
<code>ssyr2(...)</code>	Wrapper for <code>ssyr2</code> .
<code>zgemv(...)</code>	Wrapper for <code>zgemv</code> .
<code>zgerc(...)</code>	Wrapper for <code>zgerc</code> .
<code>zgeru(...)</code>	Wrapper for <code>zgeru</code> .
<code>zhemv(...)</code>	Wrapper for <code>zhemv</code> .
<code>ztrmv(...)</code>	Wrapper for <code>ztrmv</code> .
<code>zsyr(alpha,x,[lower,incx,offx,n,a,overwrite_a])</code>	Wrapper for <code>zsyr</code> .
<code>zher(alpha,x,[lower,incx,offx,n,a,overwrite_a])</code>	Wrapper for <code>zher</code> .
<code>zher2(...)</code>	Wrapper for <code>zher2</code> .

`scipy.linalg.blas.cgemv(alpha, a, x[, beta, y, offx, incx, offy, incy, trans, overwrite_y]) = <fortran object>`

Wrapper for `cgemv`.

**Parameters** **alpha** : input complex  
**a** : input rank-2 array('F') with bounds (m,n)  
**x** : input rank-1 array('F') with bounds (\*)  
**Returns** **y** : rank-1 array('F') with bounds (ly)  
**Other Parameters**  
**beta** : input complex, optional  
 Default: (0.0, 0.0)  
**y** : input rank-1 array('F') with bounds (ly)  
**overwrite\_y** : input int, optional  
 Default: 0  
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**offy** : input int, optional  
 Default: 0  
**incy** : input int, optional  
 Default: 1  
**trans** : input int, optional  
 Default: 0

`scipy.linalg.blas.cgerc(alpha, x, y[, incx, incy, a, overwrite_x, overwrite_y, overwrite_a]) = <fortran object>`

Wrapper for `cgerc`.

**Parameters** **alpha** : input complex  
**x** : input rank-1 array('F') with bounds (m)  
**y** : input rank-1 array('F') with bounds (n)  
**Returns** **a** : rank-2 array('F') with bounds (m,n)  
**Other Parameters**  
**overwrite\_x** : input int, optional  
 Default: 1  
**incx** : input int, optional  
 Default: 1  
**overwrite\_y** : input int, optional  
 Default: 1

**incy** : input int, optional  
 Default: 1  
**a** : input rank-2 array('F') with bounds (m,n), optional  
 Default: (0.0,0.0)  
**overwrite\_a** : input int, optional  
 Default: 0

`scipy.linalg.blas.cgeru(alpha, x, y[, incx, incy, a, overwrite_x, overwrite_y, overwrite_a]) = <fortran object>`

Wrapper for `cgeru`.

**Parameters** **alpha** : input complex  
**x** : input rank-1 array('F') with bounds (m)  
**y** : input rank-1 array('F') with bounds (n)  
**Returns** **a** : rank-2 array('F') with bounds (m,n)  
**Other Parameters**  
**overwrite\_x** : input int, optional  
 Default: 1  
**incx** : input int, optional  
 Default: 1  
**overwrite\_y** : input int, optional  
 Default: 1  
**incy** : input int, optional  
 Default: 1  
**a** : input rank-2 array('F') with bounds (m,n), optional  
 Default: (0.0,0.0)  
**overwrite\_a** : input int, optional  
 Default: 0

`scipy.linalg.blas.chemv(alpha, a, x[, beta, y, offx, incx, offy, incy, lower, overwrite_y]) = <fortran object>`

Wrapper for `chemv`.

**Parameters** **alpha** : input complex  
**a** : input rank-2 array('F') with bounds (n,n)  
**x** : input rank-1 array('F') with bounds (\*)  
**Returns** **y** : rank-1 array('F') with bounds (1y)  
**Other Parameters**  
**beta** : input complex, optional  
 Default: (0.0, 0.0)  
**y** : input rank-1 array('F') with bounds (1y)  
**overwrite\_y** : input int, optional  
 Default: 0  
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**offy** : input int, optional  
 Default: 0  
**incy** : input int, optional  
 Default: 1  
**lower** : input int, optional  
 Default: 0

`scipy.linalg.blas.ctrmv(a, x[, offx, incx, lower, trans, unitdiag, overwrite_x]) = <fortran object>`

Wrapper for `ctrmv`.

**Parameters** **a** : input rank-2 array('F') with bounds (n,n)  
**x** : input rank-1 array('F') with bounds (\*)  
**Returns** **x** : rank-1 array('F') with bounds (\*)

**Other Parameters**  
**overwrite\_x** : input int, optional  
 Default: 0  
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**lower** : input int, optional  
 Default: 0  
**trans** : input int, optional  
 Default: 0  
**unitdiag** : input int, optional  
 Default: 0

scipy.linalg.blas.**csyr** (*alpha*, *x*[*lower*, *incx*, *offx*, *n*, *a*, *overwrite\_a* ] ) = <fortran object>  
 Wrapper for csyr.

**Parameters** **alpha** : input complex  
**x** : input rank-1 array('F') with bounds (\*)  
**Returns** **a** : rank-2 array('F') with bounds (n,n)

**Other Parameters**  
**lower** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**offx** : input int, optional  
 Default: 0  
**n** : input int, optional  
 Default: (len(x)-1-offx)/abs(incx)+1  
**a** : input rank-2 array('F') with bounds (n,n)  
**overwrite\_a** : input int, optional  
 Default: 0

scipy.linalg.blas.**cher** (*alpha*, *x*[*lower*, *incx*, *offx*, *n*, *a*, *overwrite\_a* ] ) = <fortran object>  
 Wrapper for cher.

**Parameters** **alpha** : input complex  
**x** : input rank-1 array('F') with bounds (\*)  
**Returns** **a** : rank-2 array('F') with bounds (n,n)

**Other Parameters**  
**lower** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**offx** : input int, optional  
 Default: 0  
**n** : input int, optional  
 Default: (len(x)-1-offx)/abs(incx)+1  
**a** : input rank-2 array('F') with bounds (n,n)  
**overwrite\_a** : input int, optional  
 Default: 0

`scipy.linalg.blas.cher2(alpha, x, y[, lower, incx, offx, incy, offy, n, a, overwrite_a]) = <fortran object>`

Wrapper for cher2.

**Parameters** **alpha** : input complex  
**x** : input rank-1 array('F') with bounds (\*)  
**y** : input rank-1 array('F') with bounds (\*)

**Returns** **a** : rank-2 array('F') with bounds (n,n)

**Other Parameters**  
**lower** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**offx** : input int, optional  
 Default: 0  
**incy** : input int, optional  
 Default: 1  
**offy** : input int, optional  
 Default: 0  
**n** : input int, optional  
 Default:  $((\text{len}(x)-1-\text{offx})/\text{abs}(\text{incx})+1 \leq (\text{len}(y)-1-\text{offy})/\text{abs}(\text{incy})+1 \text{ ?}(\text{len}(x)-1-\text{offx})/\text{abs}(\text{incx})+1 : (\text{len}(y)-1-\text{offy})/\text{abs}(\text{incy})+1)$   
**a** : input rank-2 array('F') with bounds (n,n)  
**overwrite\_a** : input int, optional  
 Default: 0

`scipy.linalg.blas.dgemv(alpha, a, x[, beta, y, offx, incx, offy, incy, trans, overwrite_y]) = <fortran object>`

Wrapper for dgemv.

**Parameters** **alpha** : input float  
**a** : input rank-2 array('d') with bounds (m,n)  
**x** : input rank-1 array('d') with bounds (\*)

**Returns** **y** : rank-1 array('d') with bounds (ly)

**Other Parameters**  
**beta** : input float, optional  
 Default: 0.0  
**y** : input rank-1 array('d') with bounds (ly)  
**overwrite\_y** : input int, optional  
 Default: 0  
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**offy** : input int, optional  
 Default: 0  
**incy** : input int, optional  
 Default: 1  
**trans** : input int, optional  
 Default: 0

`scipy.linalg.blas.dger(alpha, x, y[, incx, incy, a, overwrite_x, overwrite_y, overwrite_a]) = <fortran object>`

Wrapper for dger.

**Parameters** **alpha** : input float  
**x** : input rank-1 array('d') with bounds (m)  
**y** : input rank-1 array('d') with bounds (n)

**Returns** **a** : rank-2 array('d') with bounds (m,n)

**Other Parameters**  
**overwrite\_x** : input int, optional  
 Default: 1  
**incx** : input int, optional  
 Default: 1  
**overwrite\_y** : input int, optional  
 Default: 1  
**incy** : input int, optional  
 Default: 1  
**a** : input rank-2 array('d') with bounds (m,n), optional  
 Default: 0.0  
**overwrite\_a** : input int, optional  
 Default: 0

`scipy.linalg.blas.dsymv(alpha, a, x[, beta, y, offx, incx, offy, incy, lower, overwrite_y]) = <fortran object>`

Wrapper for dsymv.

**Parameters** **alpha** : input float  
**a** : input rank-2 array('d') with bounds (n,n)  
**x** : input rank-1 array('d') with bounds (\*)

**Returns** **y** : rank-1 array('d') with bounds (ly)

**Other Parameters**  
**beta** : input float, optional  
 Default: 0.0  
**y** : input rank-1 array('d') with bounds (ly)  
**overwrite\_y** : input int, optional  
 Default: 0  
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**offy** : input int, optional  
 Default: 0  
**incy** : input int, optional  
 Default: 1  
**lower** : input int, optional  
 Default: 0

`scipy.linalg.blas.dtrmv(a, x[, offx, incx, lower, trans, unitdiag, overwrite_x]) = <fortran object>`

Wrapper for dtrmv.

**Parameters** **a** : input rank-2 array('d') with bounds (n,n)  
**x** : input rank-1 array('d') with bounds (\*)

**Returns** **x** : rank-1 array('d') with bounds (\*)

**Other Parameters**  
**overwrite\_x** : input int, optional  
 Default: 0  
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1



**lower** : input int, optional  
Default: 0  
**trans** : input int, optional  
Default: 0  
**unitdiag** : input int, optional  
Default: 0

`scipy.linalg.blas.dsyr` (*alpha*, *x*[, *lower*, *incx*, *offx*, *n*, *a*, *overwrite\_a* ]) = <fortran object>  
Wrapper for dsyr.

**Parameters** **alpha** : input float  
**x** : input rank-1 array('d') with bounds (\*)  
**Returns** **a** : rank-2 array('d') with bounds (n,n)  
**Other Parameters**  
**lower** : input int, optional  
Default: 0  
**incx** : input int, optional  
Default: 1  
**offx** : input int, optional  
Default: 0  
**n** : input int, optional  
Default: (len(x)-1-offx)/abs(incx)+1  
**a** : input rank-2 array('d') with bounds (n,n)  
**overwrite\_a** : input int, optional  
Default: 0

`scipy.linalg.blas.dsyr2` (*alpha*, *x*, *y*[, *lower*, *incx*, *offx*, *incy*, *offy*, *n*, *a*, *overwrite\_a* ]) = <fortran object>  
Wrapper for dsyr2.

**Parameters** **alpha** : input float  
**x** : input rank-1 array('d') with bounds (\*)  
**y** : input rank-1 array('d') with bounds (\*)  
**Returns** **a** : rank-2 array('d') with bounds (n,n)  
**Other Parameters**  
**lower** : input int, optional  
Default: 0  
**incx** : input int, optional  
Default: 1  
**offx** : input int, optional  
Default: 0  
**incy** : input int, optional  
Default: 1  
**offy** : input int, optional  
Default: 0  
**n** : input int, optional  
Default: ((len(x)-1-offx)/abs(incx)+1 <=(len(y)-1-offy)/abs(incy)+1 ?(len(x)-1-offx)/abs(incx)+1 :(len(y)-1-offy)/abs(incy)+1)  
**a** : input rank-2 array('d') with bounds (n,n)  
**overwrite\_a** : input int, optional  
Default: 0

`scipy.linalg.blas.sgemv` (*alpha*, *a*, *x*[, *beta*, *y*, *offx*, *incx*, *offy*, *incy*, *trans*, *overwrite\_y* ]) = <fortran object>  
Wrapper for sgemv.

**Parameters** **alpha** : input float  
**a** : input rank-2 array('f') with bounds (m,n)  
**x** : input rank-1 array('f') with bounds (\*)

**Returns** **y** : rank-1 array('f') with bounds (ly)

**Other Parameters**

**beta** : input float, optional  
 Default: 0.0  
**y** : input rank-1 array('f') with bounds (ly)  
**overwrite\_y** : input int, optional  
 Default: 0  
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**offy** : input int, optional  
 Default: 0  
**incy** : input int, optional  
 Default: 1  
**trans** : input int, optional  
 Default: 0

`scipy.linalg.blas.sger(alpha, x, y[, incx, incy, a, overwrite_x, overwrite_y, overwrite_a]) = <fortran object>`

Wrapper for `sger`.

**Parameters** **alpha** : input float  
**x** : input rank-1 array('f') with bounds (m)  
**y** : input rank-1 array('f') with bounds (n)

**Returns** **a** : rank-2 array('f') with bounds (m,n)

**Other Parameters**

**overwrite\_x** : input int, optional  
 Default: 1  
**incx** : input int, optional  
 Default: 1  
**overwrite\_y** : input int, optional  
 Default: 1  
**incy** : input int, optional  
 Default: 1  
**a** : input rank-2 array('f') with bounds (m,n), optional  
 Default: 0.0  
**overwrite\_a** : input int, optional  
 Default: 0

`scipy.linalg.blas.ssymv(alpha, a, x[, beta, y, offx, incx, offy, incy, lower, overwrite_y]) = <fortran object>`

Wrapper for `ssymv`.

**Parameters** **alpha** : input float  
**a** : input rank-2 array('f') with bounds (n,n)  
**x** : input rank-1 array('f') with bounds (\*)

**Returns** **y** : rank-1 array('f') with bounds (ly)

**Other Parameters**

**beta** : input float, optional  
 Default: 0.0  
**y** : input rank-1 array('f') with bounds (ly)  
**overwrite\_y** : input int, optional

Default: 0  
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**offy** : input int, optional  
 Default: 0  
**incy** : input int, optional  
 Default: 1  
**lower** : input int, optional  
 Default: 0

`scipy.linalg.blas.strmv(a, x[, offx, incx, lower, trans, unitdiag, overwrite_x]) = <fortran object>`  
 Wrapper for `strmv`.

**Parameters** **a** : input rank-2 array('f') with bounds (n,n)  
**x** : input rank-1 array('f') with bounds (\*)

**Returns** **x** : rank-1 array('f') with bounds (\*)

**Other Parameters**

**overwrite\_x** : input int, optional  
 Default: 0  
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**lower** : input int, optional  
 Default: 0  
**trans** : input int, optional  
 Default: 0  
**unitdiag** : input int, optional  
 Default: 0

`scipy.linalg.blas.ssyrr(alpha, x[, lower, incx, offx, n, a, overwrite_a]) = <fortran object>`  
 Wrapper for `ssyr`.

**Parameters** **alpha** : input float  
**x** : input rank-1 array('f') with bounds (\*)  
**Returns** **a** : rank-2 array('f') with bounds (n,n)

**Other Parameters**

**lower** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**offx** : input int, optional  
 Default: 0  
**n** : input int, optional  
 Default:  $(\text{len}(x)-1-\text{offx})/\text{abs}(\text{incx})+1$   
**a** : input rank-2 array('f') with bounds (n,n)  
**overwrite\_a** : input int, optional  
 Default: 0

`scipy.linalg.blas.ssyrr2(alpha, x, y[, lower, incx, offx, incy, offy, n, a, overwrite_a]) = <fortran object>`

Wrapper for `ssyr2`.

**Parameters** **alpha** : input float  
**x** : input rank-1 array('f') with bounds (\*)  
**y** : input rank-1 array('f') with bounds (\*)

**Returns** **a** : rank-2 array('f') with bounds (n,n)

**Other Parameters**  
**lower** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**offx** : input int, optional  
 Default: 0  
**incy** : input int, optional  
 Default: 1  
**offy** : input int, optional  
 Default: 0  
**n** : input int, optional  
 Default: ((len(x)-1-offx)/abs(incx)+1 <=(len(y)-1-offy)/abs(incy)+1 ?(len(x)-1-offx)/abs(incx)+1 :(len(y)-1-offy)/abs(incy)+1)  
**a** : input rank-2 array('f') with bounds (n,n)  
**overwrite\_a** : input int, optional  
 Default: 0

scipy.linalg.blas.**zgemv** (*alpha*, *a*, *x*[, *beta*, *y*, *offx*, *incx*, *offy*, *incy*, *trans*, *overwrite\_y*]) = <fortran object>

Wrapper for zgemv.

**Parameters** **alpha** : input complex  
**a** : input rank-2 array('D') with bounds (m,n)  
**x** : input rank-1 array('D') with bounds (\*)

**Returns** **y** : rank-1 array('D') with bounds (1y)

**Other Parameters**  
**beta** : input complex, optional  
 Default: (0.0, 0.0)  
**y** : input rank-1 array('D') with bounds (1y)  
**overwrite\_y** : input int, optional  
 Default: 0  
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**offy** : input int, optional  
 Default: 0  
**incy** : input int, optional  
 Default: 1  
**trans** : input int, optional  
 Default: 0

scipy.linalg.blas.**zgerc** (*alpha*, *x*, *y*[, *incx*, *incy*, *a*, *overwrite\_x*, *overwrite\_y*, *overwrite\_a*]) = <fortran object>

Wrapper for zgerc.

**Parameters** **alpha** : input complex  
**x** : input rank-1 array('D') with bounds (m)  
**y** : input rank-1 array('D') with bounds (n)

**Returns** **a** : rank-2 array('D') with bounds (m,n)

**Other Parameters**

**overwrite\_x** : input int, optional  
Default: 1

**incx** : input int, optional  
Default: 1

**overwrite\_y** : input int, optional  
Default: 1

**incy** : input int, optional  
Default: 1

**a** : input rank-2 array('D') with bounds (m,n), optional  
Default: (0.0,0.0)

**overwrite\_a** : input int, optional  
Default: 0

`scipy.linalg.blas.zgeru(alpha, x, y[, incx, incy, a, overwrite_x, overwrite_y, overwrite_a]) = <fortran object>`

Wrapper for zgeru.

**Parameters** **alpha** : input complex  
**x** : input rank-1 array('D') with bounds (m)  
**y** : input rank-1 array('D') with bounds (n)

**Returns** **a** : rank-2 array('D') with bounds (m,n)

**Other Parameters**

**overwrite\_x** : input int, optional  
Default: 1

**incx** : input int, optional  
Default: 1

**overwrite\_y** : input int, optional  
Default: 1

**incy** : input int, optional  
Default: 1

**a** : input rank-2 array('D') with bounds (m,n), optional  
Default: (0.0,0.0)

**overwrite\_a** : input int, optional  
Default: 0

`scipy.linalg.blas.zhemv(alpha, a, x[, beta, y, offx, incx, offy, incy, lower, overwrite_y]) = <fortran object>`

Wrapper for zhemv.

**Parameters** **alpha** : input complex  
**a** : input rank-2 array('D') with bounds (n,n)  
**x** : input rank-1 array('D') with bounds (\*)

**Returns** **y** : rank-1 array('D') with bounds (1y)

**Other Parameters**

**beta** : input complex, optional  
Default: (0.0, 0.0)

**y** : input rank-1 array('D') with bounds (1y)

**overwrite\_y** : input int, optional  
Default: 0

**offx** : input int, optional  
Default: 0

**incx** : input int, optional  
Default: 1

**offy** : input int, optional  
Default: 0

**incy** : input int, optional  
 Default: 1  
**lower** : input int, optional  
 Default: 0

`scipy.linalg.blas.ztrmv(a, x[, offx, incx, lower, trans, unitdiag, overwrite_x]) = <fortran object>`  
 Wrapper for `ztrmv`.

**Parameters** **a** : input rank-2 array('D') with bounds (n,n)  
**x** : input rank-1 array('D') with bounds (\*)  
**Returns** **x** : rank-1 array('D') with bounds (\*)  
**Other Parameters**  
**overwrite\_x** : input int, optional  
 Default: 0  
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**lower** : input int, optional  
 Default: 0  
**trans** : input int, optional  
 Default: 0  
**unitdiag** : input int, optional  
 Default: 0

`scipy.linalg.blas.zsyr(alpha, x[, lower, incx, offx, n, a, overwrite_a]) = <fortran object>`  
 Wrapper for `zsyr`.

**Parameters** **alpha** : input complex  
**x** : input rank-1 array('D') with bounds (\*)  
**Returns** **a** : rank-2 array('D') with bounds (n,n)  
**Other Parameters**  
**lower** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**offx** : input int, optional  
 Default: 0  
**n** : input int, optional  
 Default: (len(x)-1-offx)/abs(incx)+1  
**a** : input rank-2 array('D') with bounds (n,n)  
**overwrite\_a** : input int, optional  
 Default: 0

`scipy.linalg.blas.zher(alpha, x[, lower, incx, offx, n, a, overwrite_a]) = <fortran object>`  
 Wrapper for `zher`.

**Parameters** **alpha** : input complex  
**x** : input rank-1 array('D') with bounds (\*)  
**Returns** **a** : rank-2 array('D') with bounds (n,n)  
**Other Parameters**  
**lower** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**offx** : input int, optional  
 Default: 0

**n** : input int, optional  
 Default:  $(\text{len}(x)-1-\text{offx})/\text{abs}(\text{incx})+1$   
**a** : input rank-2 array('D') with bounds (n,n)  
**overwrite\_a** : input int, optional  
 Default: 0

`scipy.linalg.blas.zher2(alpha, x, y[, lower, incx, offx, incy, offy, n, a, overwrite_a]) = <fortran object>`

Wrapper for zher2.

**Parameters** **alpha** : input complex  
**x** : input rank-1 array('D') with bounds (\*)  
**y** : input rank-1 array('D') with bounds (\*)  
**Returns** **a** : rank-2 array('D') with bounds (n,n)

**Other Parameters**

**lower** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**offx** : input int, optional  
 Default: 0  
**incy** : input int, optional  
 Default: 1  
**offy** : input int, optional  
 Default: 0  
**n** : input int, optional  
 Default:  $((\text{len}(x)-1-\text{offx})/\text{abs}(\text{incx})+1 \leq (\text{len}(y)-1-\text{offy})/\text{abs}(\text{incy})+1 ? (\text{len}(x)-1-\text{offx})/\text{abs}(\text{incx})+1 : (\text{len}(y)-1-\text{offy})/\text{abs}(\text{incy})+1)$   
**a** : input rank-2 array('D') with bounds (n,n)  
**overwrite\_a** : input int, optional  
 Default: 0

### 5.10.4 BLAS Level 3 functions

<code>cgemm(...)</code>	Wrapper for <code>cgemm</code> .
<code>chemm(alpha,a,b,[beta,c,side,lower,overwrite_c])</code>	Wrapper for <code>chemm</code> .
<code>cherk(alpha,a,[beta,c,trans,lower,overwrite_c])</code>	Wrapper for <code>cherk</code> .
<code>cher2k(...)</code>	Wrapper for <code>cher2k</code> .
<code>csymm(alpha,a,b,[beta,c,side,lower,overwrite_c])</code>	Wrapper for <code>csymm</code> .
<code>csyrk(alpha,a,[beta,c,trans,lower,overwrite_c])</code>	Wrapper for <code>csyrk</code> .
<code>csyr2k(...)</code>	Wrapper for <code>csyr2k</code> .
<code>dgemm(...)</code>	Wrapper for <code>dgemm</code> .
<code>dsymm(alpha,a,b,[beta,c,side,lower,overwrite_c])</code>	Wrapper for <code>dsymm</code> .
<code>dsyrk(alpha,a,[beta,c,trans,lower,overwrite_c])</code>	Wrapper for <code>dsyrk</code> .
<code>dsyr2k(...)</code>	Wrapper for <code>dsyr2k</code> .
<code>sgemm(...)</code>	Wrapper for <code>sgemm</code> .
<code>ssymm(alpha,a,b,[beta,c,side,lower,overwrite_c])</code>	Wrapper for <code>ssymm</code> .
<code>ssyrk(alpha,a,[beta,c,trans,lower,overwrite_c])</code>	Wrapper for <code>ssyrk</code> .
<code>ssyr2k(...)</code>	Wrapper for <code>ssyr2k</code> .
<code>zgemm(...)</code>	Wrapper for <code>zgemm</code> .
<code>zhemm(alpha,a,b,[beta,c,side,lower,overwrite_c])</code>	Wrapper for <code>zhemm</code> .
<code>zherk(alpha,a,[beta,c,trans,lower,overwrite_c])</code>	Wrapper for <code>zherk</code> .
<code>zher2k(...)</code>	Wrapper for <code>zher2k</code> .

Continued on next page

Table 5.82 – continued from previous page

<code>zsymm(alpha,a,b,[beta,c,side,lower,overwrite_c])</code>	Wrapper for <code>zsymm</code> .
<code>zsyrk(alpha,a,[beta,c,trans,lower,overwrite_c])</code>	Wrapper for <code>zsyrk</code> .
<code>zsy2k(...)</code>	Wrapper for <code>zsy2k</code> .

`scipy.linalg.blas.cgemm(alpha, a, b[, beta, c, trans_a, trans_b, overwrite_c]) = <fortran object>`  
 Wrapper for `cgemm`.

**Parameters** **alpha** : input complex  
**a** : input rank-2 array('F') with bounds (lda,ka)  
**b** : input rank-2 array('F') with bounds (ldb,kb)  
**Returns** **c** : rank-2 array('F') with bounds (m,n)  
**Other Parameters**  
**beta** : input complex, optional  
 Default: (0.0, 0.0)  
**c** : input rank-2 array('F') with bounds (m,n)  
**overwrite\_c** : input int, optional  
 Default: 0  
**trans\_a** : input int, optional  
 Default: 0  
**trans\_b** : input int, optional  
 Default: 0

`scipy.linalg.blas.chemm(alpha, a, b[, beta, c, side, lower, overwrite_c]) = <fortran object>`  
 Wrapper for `chemm`.

**Parameters** **alpha** : input complex  
**a** : input rank-2 array('F') with bounds (lda,ka)  
**b** : input rank-2 array('F') with bounds (ldb,kb)  
**Returns** **c** : rank-2 array('F') with bounds (m,n)  
**Other Parameters**  
**beta** : input complex, optional  
 Default: (0.0, 0.0)  
**c** : input rank-2 array('F') with bounds (m,n)  
**overwrite\_c** : input int, optional  
 Default: 0  
**side** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0

`scipy.linalg.blas.cherk(alpha, a[, beta, c, trans, lower, overwrite_c]) = <fortran object>`  
 Wrapper for `cherk`.

**Parameters** **alpha** : input complex  
**a** : input rank-2 array('F') with bounds (lda,ka)  
**Returns** **c** : rank-2 array('F') with bounds (n,n)  
**Other Parameters**  
**beta** : input complex, optional  
 Default: (0.0, 0.0)  
**c** : input rank-2 array('F') with bounds (n,n)  
**overwrite\_c** : input int, optional  
 Default: 0  
**trans** : input int, optional  
 Default: 0  
**lower** : input int, optional



Default: 0

`scipy.linalg.blas.cher2k(alpha, a, b[, beta, c, trans, lower, overwrite_c]) = <fortran object>`  
 Wrapper for `cher2k`.

**Parameters** **alpha** : input complex  
**a** : input rank-2 array('F') with bounds (lda,ka)  
**b** : input rank-2 array('F') with bounds (ldb,kb)

**Returns** **c** : rank-2 array('F') with bounds (n,n)

**Other Parameters**  
**beta** : input complex, optional  
 Default: (0.0, 0.0)  
**c** : input rank-2 array('F') with bounds (n,n)  
**overwrite\_c** : input int, optional  
 Default: 0  
**trans** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0

`scipy.linalg.blas.csymm(alpha, a, b[, beta, c, side, lower, overwrite_c]) = <fortran object>`  
 Wrapper for `csymm`.

**Parameters** **alpha** : input complex  
**a** : input rank-2 array('F') with bounds (lda,ka)  
**b** : input rank-2 array('F') with bounds (ldb,kb)

**Returns** **c** : rank-2 array('F') with bounds (m,n)

**Other Parameters**  
**beta** : input complex, optional  
 Default: (0.0, 0.0)  
**c** : input rank-2 array('F') with bounds (m,n)  
**overwrite\_c** : input int, optional  
 Default: 0  
**side** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0

`scipy.linalg.blas.csyrk(alpha, a[, beta, c, trans, lower, overwrite_c]) = <fortran object>`  
 Wrapper for `csyrk`.

**Parameters** **alpha** : input complex  
**a** : input rank-2 array('F') with bounds (lda,ka)

**Returns** **c** : rank-2 array('F') with bounds (n,n)

**Other Parameters**  
**beta** : input complex, optional  
 Default: (0.0, 0.0)  
**c** : input rank-2 array('F') with bounds (n,n)  
**overwrite\_c** : input int, optional  
 Default: 0  
**trans** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0

`scipy.linalg.blas.csy2k(alpha, a, b[, beta, c, trans, lower, overwrite_c]) = <fortran object>`  
 Wrapper for `csyr2k`.

**Parameters** **alpha** : input complex  
**a** : input rank-2 array('F') with bounds (lda,ka)  
**b** : input rank-2 array('F') with bounds (ldb,kb)

**Returns** **c** : rank-2 array('F') with bounds (n,n)

**Other Parameters**  
**beta** : input complex, optional  
 Default: (0.0, 0.0)  
**c** : input rank-2 array('F') with bounds (n,n)  
**overwrite\_c** : input int, optional  
 Default: 0  
**trans** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0

`scipy.linalg.blas.dgemm(alpha, a, b[, beta, c, trans_a, trans_b, overwrite_c]) = <fortran object>`  
 Wrapper for dgemm.

**Parameters** **alpha** : input float  
**a** : input rank-2 array('d') with bounds (lda,ka)  
**b** : input rank-2 array('d') with bounds (ldb,kb)

**Returns** **c** : rank-2 array('d') with bounds (m,n)

**Other Parameters**  
**beta** : input float, optional  
 Default: 0.0  
**c** : input rank-2 array('d') with bounds (m,n)  
**overwrite\_c** : input int, optional  
 Default: 0  
**trans\_a** : input int, optional  
 Default: 0  
**trans\_b** : input int, optional  
 Default: 0

`scipy.linalg.blas.dsymm(alpha, a, b[, beta, c, side, lower, overwrite_c]) = <fortran object>`  
 Wrapper for dsymm.

**Parameters** **alpha** : input float  
**a** : input rank-2 array('d') with bounds (lda,ka)  
**b** : input rank-2 array('d') with bounds (ldb,kb)

**Returns** **c** : rank-2 array('d') with bounds (m,n)

**Other Parameters**  
**beta** : input float, optional  
 Default: 0.0  
**c** : input rank-2 array('d') with bounds (m,n)  
**overwrite\_c** : input int, optional  
 Default: 0  
**side** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0

`scipy.linalg.blas.dsyrk(alpha, a[, beta, c, trans, lower, overwrite_c]) = <fortran object>`  
 Wrapper for dsyrk.

**Parameters** **alpha** : input float  
**a** : input rank-2 array('d') with bounds (lda,ka)

**Returns** **c** : rank-2 array('d') with bounds (n,n)

**Other Parameters**

**beta** : input float, optional  
Default: 0.0  
**c** : input rank-2 array('d') with bounds (n,n)  
**overwrite\_c** : input int, optional  
Default: 0  
**trans** : input int, optional  
Default: 0  
**lower** : input int, optional  
Default: 0

`scipy.linalg.blas.dsyr2k(alpha, a, b[, beta, c, trans, lower, overwrite_c]) = <fortran object>`  
 Wrapper for dsyr2k.

**Parameters** **alpha** : input float  
**a** : input rank-2 array('d') with bounds (lda,ka)  
**b** : input rank-2 array('d') with bounds (ldb,kb)  
**Returns** **c** : rank-2 array('d') with bounds (n,n)

**Other Parameters**

**beta** : input float, optional  
Default: 0.0  
**c** : input rank-2 array('d') with bounds (n,n)  
**overwrite\_c** : input int, optional  
Default: 0  
**trans** : input int, optional  
Default: 0  
**lower** : input int, optional  
Default: 0

`scipy.linalg.blas.sgemm(alpha, a, b[, beta, c, trans_a, trans_b, overwrite_c]) = <fortran object>`  
 Wrapper for sgemm.

**Parameters** **alpha** : input float  
**a** : input rank-2 array('f') with bounds (lda,ka)  
**b** : input rank-2 array('f') with bounds (ldb,kb)  
**Returns** **c** : rank-2 array('f') with bounds (m,n)

**Other Parameters**

**beta** : input float, optional  
Default: 0.0  
**c** : input rank-2 array('f') with bounds (m,n)  
**overwrite\_c** : input int, optional  
Default: 0  
**trans\_a** : input int, optional  
Default: 0  
**trans\_b** : input int, optional  
Default: 0

`scipy.linalg.blas.ssymm(alpha, a, b[, beta, c, side, lower, overwrite_c]) = <fortran object>`  
 Wrapper for ssymm.

**Parameters** **alpha** : input float  
**a** : input rank-2 array('f') with bounds (lda,ka)  
**b** : input rank-2 array('f') with bounds (ldb,kb)  
**Returns** **c** : rank-2 array('f') with bounds (m,n)

**Other Parameters**

**beta** : input float, optional  
Default: 0.0

**c** : input rank-2 array('f') with bounds (m,n)  
**overwrite\_c** : input int, optional  
 Default: 0  
**side** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0

`scipy.linalg.blas.ssyrrk(alpha, a[, beta, c, trans, lower, overwrite_c]) = <fortran object>`  
 Wrapper for `ssyrk`.

**Parameters** **alpha** : input float  
**a** : input rank-2 array('f') with bounds (lda,ka)  
**Returns** **c** : rank-2 array('f') with bounds (n,n)  
**Other Parameters**  
**beta** : input float, optional  
 Default: 0.0  
**c** : input rank-2 array('f') with bounds (n,n)  
**overwrite\_c** : input int, optional  
 Default: 0  
**trans** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0

`scipy.linalg.blas.ssyrr2k(alpha, a, b[, beta, c, trans, lower, overwrite_c]) = <fortran object>`  
 Wrapper for `ssyr2k`.

**Parameters** **alpha** : input float  
**a** : input rank-2 array('f') with bounds (lda,ka)  
**b** : input rank-2 array('f') with bounds (ldb,kb)  
**Returns** **c** : rank-2 array('f') with bounds (n,n)  
**Other Parameters**  
**beta** : input float, optional  
 Default: 0.0  
**c** : input rank-2 array('f') with bounds (n,n)  
**overwrite\_c** : input int, optional  
 Default: 0  
**trans** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0

`scipy.linalg.blas.zgemm(alpha, a, b[, beta, c, trans_a, trans_b, overwrite_c]) = <fortran object>`  
 Wrapper for `zgemm`.

**Parameters** **alpha** : input complex  
**a** : input rank-2 array('D') with bounds (lda,ka)  
**b** : input rank-2 array('D') with bounds (ldb,kb)  
**Returns** **c** : rank-2 array('D') with bounds (m,n)  
**Other Parameters**  
**beta** : input complex, optional  
 Default: (0.0, 0.0)  
**c** : input rank-2 array('D') with bounds (m,n)  
**overwrite\_c** : input int, optional  
 Default: 0  
**trans\_a** : input int, optional

Default: 0  
**trans\_b** : input int, optional  
 Default: 0

`scipy.linalg.blas.zhemm(alpha, a, b[, beta, c, side, lower, overwrite_c]) = <fortran object>`  
 Wrapper for zhemm.

**Parameters** **alpha** : input complex  
**a** : input rank-2 array('D') with bounds (lda,ka)  
**b** : input rank-2 array('D') with bounds (ldb,kb)  
**Returns** **c** : rank-2 array('D') with bounds (m,n)  
**Other Parameters**  
**beta** : input complex, optional  
 Default: (0.0, 0.0)  
**c** : input rank-2 array('D') with bounds (m,n)  
**overwrite\_c** : input int, optional  
 Default: 0  
**side** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0

`scipy.linalg.blas.zherk(alpha, a[, beta, c, trans, lower, overwrite_c]) = <fortran object>`  
 Wrapper for zherk.

**Parameters** **alpha** : input complex  
**a** : input rank-2 array('D') with bounds (lda,ka)  
**Returns** **c** : rank-2 array('D') with bounds (n,n)  
**Other Parameters**  
**beta** : input complex, optional  
 Default: (0.0, 0.0)  
**c** : input rank-2 array('D') with bounds (n,n)  
**overwrite\_c** : input int, optional  
 Default: 0  
**trans** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0

`scipy.linalg.blas.zher2k(alpha, a, b[, beta, c, trans, lower, overwrite_c]) = <fortran object>`  
 Wrapper for zher2k.

**Parameters** **alpha** : input complex  
**a** : input rank-2 array('D') with bounds (lda,ka)  
**b** : input rank-2 array('D') with bounds (ldb,kb)  
**Returns** **c** : rank-2 array('D') with bounds (n,n)  
**Other Parameters**  
**beta** : input complex, optional  
 Default: (0.0, 0.0)  
**c** : input rank-2 array('D') with bounds (n,n)  
**overwrite\_c** : input int, optional  
 Default: 0  
**trans** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0

`scipy.linalg.blas.zsymm(alpha, a, b[, beta, c, side, lower, overwrite_c]) = <fortran object>`  
 Wrapper for `zsymm`.

**Parameters** **alpha** : input complex  
**a** : input rank-2 array('D') with bounds (lda,ka)  
**b** : input rank-2 array('D') with bounds (ldb,kb)

**Returns** **c** : rank-2 array('D') with bounds (m,n)

**Other Parameters**  
**beta** : input complex, optional  
 Default: (0.0, 0.0)  
**c** : input rank-2 array('D') with bounds (m,n)  
**overwrite\_c** : input int, optional  
 Default: 0  
**side** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0

`scipy.linalg.blas.zsyrk(alpha, a[, beta, c, trans, lower, overwrite_c]) = <fortran object>`  
 Wrapper for `zsyrk`.

**Parameters** **alpha** : input complex  
**a** : input rank-2 array('D') with bounds (lda,ka)

**Returns** **c** : rank-2 array('D') with bounds (n,n)

**Other Parameters**  
**beta** : input complex, optional  
 Default: (0.0, 0.0)  
**c** : input rank-2 array('D') with bounds (n,n)  
**overwrite\_c** : input int, optional  
 Default: 0  
**trans** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0

`scipy.linalg.blas.zsyr2k(alpha, a, b[, beta, c, trans, lower, overwrite_c]) = <fortran object>`  
 Wrapper for `zsyr2k`.

**Parameters** **alpha** : input complex  
**a** : input rank-2 array('D') with bounds (lda,ka)  
**b** : input rank-2 array('D') with bounds (ldb,kb)

**Returns** **c** : rank-2 array('D') with bounds (n,n)

**Other Parameters**  
**beta** : input complex, optional  
 Default: (0.0, 0.0)  
**c** : input rank-2 array('D') with bounds (n,n)  
**overwrite\_c** : input int, optional  
 Default: 0  
**trans** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0

## 5.11 Low-level LAPACK functions (`scipy.linalg.lapack`)

This module contains low-level functions from the LAPACK library.

The *\*gegv* family of routines have been removed from LAPACK 3.6.0 and have been deprecated in SciPy 0.17.0. They will be removed in a future release.

New in version 0.12.0.

**Warning:** These functions do little to no error checking. It is possible to cause crashes by mis-using them, so prefer using the higher-level routines in `scipy.linalg`.

### 5.11.1 Finding functions

---

`get_lapack_funcs(names[, arrays, dtype])` Return available LAPACK function objects from names.

---

### 5.11.2 All functions

---

<code>sghsv(kl,ku,ab,b,[overwrite_ab,overwrite_b])</code>	Wrapper for <code>sghsv</code> .
<code>dghsv(kl,ku,ab,b,[overwrite_ab,overwrite_b])</code>	Wrapper for <code>dghsv</code> .
<code>cghsv(kl,ku,ab,b,[overwrite_ab,overwrite_b])</code>	Wrapper for <code>cghsv</code> .
<code>zghsv(kl,ku,ab,b,[overwrite_ab,overwrite_b])</code>	Wrapper for <code>zghsv</code> .
<code>sgbtrf(ab,kl,ku,[m,n,ldab,overwrite_ab])</code>	Wrapper for <code>sgbtrf</code> .
<code>dgbtrf(ab,kl,ku,[m,n,ldab,overwrite_ab])</code>	Wrapper for <code>dgbtrf</code> .
<code>cgbtrf(ab,kl,ku,[m,n,ldab,overwrite_ab])</code>	Wrapper for <code>cgbtrf</code> .
<code>zgbtrf(ab,kl,ku,[m,n,ldab,overwrite_ab])</code>	Wrapper for <code>zgbtrf</code> .
<code>sgbtrs(...)</code>	Wrapper for <code>sgbtrs</code> .
<code>dgbtrs(...)</code>	Wrapper for <code>dgbtrs</code> .
<code>cgbtrs(...)</code>	Wrapper for <code>cgbtrs</code> .
<code>zgbtrs(...)</code>	Wrapper for <code>zgbtrs</code> .
<code>sgebal(a,[scale,permute,overwrite_a])</code>	Wrapper for <code>sgebal</code> .
<code>dgebal(a,[scale,permute,overwrite_a])</code>	Wrapper for <code>dgebal</code> .
<code>cgebal(a,[scale,permute,overwrite_a])</code>	Wrapper for <code>cgebal</code> .
<code>zgebal(a,[scale,permute,overwrite_a])</code>	Wrapper for <code>zgebal</code> .
<code>sgees(...)</code>	Wrapper for <code>sgees</code> .
<code>dgees(...)</code>	Wrapper for <code>dgees</code> .
<code>cgees(...)</code>	Wrapper for <code>cgees</code> .
<code>zgees(...)</code>	Wrapper for <code>zgees</code> .
<code>sgeev(...)</code>	Wrapper for <code>sgeev</code> .
<code>dgeev(...)</code>	Wrapper for <code>dgeev</code> .
<code>cgeev(...)</code>	Wrapper for <code>cgeev</code> .
<code>zgeev(...)</code>	Wrapper for <code>zgeev</code> .
<code>sgeev_lwork(n,[compute_vl,compute_vr])</code>	Wrapper for <code>sgeev_lwork</code> .
<code>dgeev_lwork(n,[compute_vl,compute_vr])</code>	Wrapper for <code>dgeev_lwork</code> .
<code>cgeev_lwork(n,[compute_vl,compute_vr])</code>	Wrapper for <code>cgeev_lwork</code> .
<code>zgeev_lwork(n,[compute_vl,compute_vr])</code>	Wrapper for <code>zgeev_lwork</code> .
<code>sgegv(*args, **kwds)</code>	<code>sgegv</code> is deprecated!
<code>dgegv(*args, **kwds)</code>	<code>dgegv</code> is deprecated!
<code>cgegv(*args, **kwds)</code>	<code>cgegv</code> is deprecated!
<code>zgegv(*args, **kwds)</code>	<code>zgegv</code> is deprecated!

Continued on next page

Table 5.84 – continued from previous page

<code>sgehrd(a,[lo,hi,lwork,overwrite_a])</code>	Wrapper for <code>sgehrd</code> .
<code>dgehrd(a,[lo,hi,lwork,overwrite_a])</code>	Wrapper for <code>dgehrd</code> .
<code>cgehrd(a,[lo,hi,lwork,overwrite_a])</code>	Wrapper for <code>cgehrd</code> .
<code>zgehrd(a,[lo,hi,lwork,overwrite_a])</code>	Wrapper for <code>zgehrd</code> .
<code>sgehrd_lwork(n,[lo,hi])</code>	Wrapper for <code>sgehrd_lwork</code> .
<code>dgehrd_lwork(n,[lo,hi])</code>	Wrapper for <code>dgehrd_lwork</code> .
<code>cgehrd_lwork(n,[lo,hi])</code>	Wrapper for <code>cgehrd_lwork</code> .
<code>zgehrd_lwork(n,[lo,hi])</code>	Wrapper for <code>zgehrd_lwork</code> .
<code>sgelss(a,b,[cond,lwork,overwrite_a,overwrite_b])</code>	Wrapper for <code>sgelss</code> .
<code>dgelss(a,b,[cond,lwork,overwrite_a,overwrite_b])</code>	Wrapper for <code>dgelss</code> .
<code>cgelss(a,b,[cond,lwork,overwrite_a,overwrite_b])</code>	Wrapper for <code>cgelss</code> .
<code>zgelss(a,b,[cond,lwork,overwrite_a,overwrite_b])</code>	Wrapper for <code>zgelss</code> .
<code>sgelss_lwork(m,n,nrhs,[cond,lwork])</code>	Wrapper for <code>sgelss_lwork</code> .
<code>dgelss_lwork(m,n,nrhs,[cond,lwork])</code>	Wrapper for <code>dgelss_lwork</code> .
<code>cgelss_lwork(m,n,nrhs,[cond,lwork])</code>	Wrapper for <code>cgelss_lwork</code> .
<code>zgelss_lwork(m,n,nrhs,[cond,lwork])</code>	Wrapper for <code>zgelss_lwork</code> .
<code>sgelsd(...)</code>	Wrapper for <code>sgelsd</code> .
<code>dgelsd(...)</code>	Wrapper for <code>dgelsd</code> .
<code>cgelsd(...)</code>	Wrapper for <code>cgelsd</code> .
<code>zgelsd(...)</code>	Wrapper for <code>zgelsd</code> .
<code>sgelsd_lwork(m,n,nrhs,[cond,lwork])</code>	Wrapper for <code>sgelsd_lwork</code> .
<code>dgelsd_lwork(m,n,nrhs,[cond,lwork])</code>	Wrapper for <code>dgelsd_lwork</code> .
<code>cgelsd_lwork(m,n,nrhs,[cond,lwork])</code>	Wrapper for <code>cgelsd_lwork</code> .
<code>zgelsd_lwork(m,n,nrhs,[cond,lwork])</code>	Wrapper for <code>zgelsd_lwork</code> .
<code>sgelsy(...)</code>	Wrapper for <code>sgelsy</code> .
<code>dgelsy(...)</code>	Wrapper for <code>dgelsy</code> .
<code>cgelsy(...)</code>	Wrapper for <code>cgelsy</code> .
<code>zgelsy(...)</code>	Wrapper for <code>zgelsy</code> .
<code>sgelsy_lwork(m,n,nrhs,cond,[lwork])</code>	Wrapper for <code>sgelsy_lwork</code> .
<code>dgelsy_lwork(m,n,nrhs,cond,[lwork])</code>	Wrapper for <code>dgelsy_lwork</code> .
<code>cgelsy_lwork(m,n,nrhs,cond,[lwork])</code>	Wrapper for <code>cgelsy_lwork</code> .
<code>zgelsy_lwork(m,n,nrhs,cond,[lwork])</code>	Wrapper for <code>zgelsy_lwork</code> .
<code>sgeqp3(a,[lwork,overwrite_a])</code>	Wrapper for <code>sgeqp3</code> .
<code>dgeqp3(a,[lwork,overwrite_a])</code>	Wrapper for <code>dgeqp3</code> .
<code>cgeqp3(a,[lwork,overwrite_a])</code>	Wrapper for <code>cgeqp3</code> .
<code>zgeqp3(a,[lwork,overwrite_a])</code>	Wrapper for <code>zgeqp3</code> .
<code>sgeqrf(a,[lwork,overwrite_a])</code>	Wrapper for <code>sgeqrf</code> .
<code>dgeqrf(a,[lwork,overwrite_a])</code>	Wrapper for <code>dgeqrf</code> .
<code>cgeqrf(a,[lwork,overwrite_a])</code>	Wrapper for <code>cgeqrf</code> .
<code>zgeqrf(a,[lwork,overwrite_a])</code>	Wrapper for <code>zgeqrf</code> .
<code>sgerqf(a,[lwork,overwrite_a])</code>	Wrapper for <code>sgerqf</code> .
<code>dgerqf(a,[lwork,overwrite_a])</code>	Wrapper for <code>dgerqf</code> .
<code>cgerqf(a,[lwork,overwrite_a])</code>	Wrapper for <code>cgerqf</code> .
<code>zgerqf(a,[lwork,overwrite_a])</code>	Wrapper for <code>zgerqf</code> .
<code>sgesdd(...)</code>	Wrapper for <code>sgesdd</code> .
<code>dgesdd(...)</code>	Wrapper for <code>dgesdd</code> .
<code>cggesdd(...)</code>	Wrapper for <code>cggesdd</code> .
<code>zgesdd(...)</code>	Wrapper for <code>zgesdd</code> .
<code>sgesdd_lwork(m,n,[compute_uv,full_matrices])</code>	Wrapper for <code>sgesdd_lwork</code> .
<code>dgesdd_lwork(m,n,[compute_uv,full_matrices])</code>	Wrapper for <code>dgesdd_lwork</code> .
<code>cggesdd_lwork(m,n,[compute_uv,full_matrices])</code>	Wrapper for <code>cggesdd_lwork</code> .
<code>zgesdd_lwork(m,n,[compute_uv,full_matrices])</code>	Wrapper for <code>zgesdd_lwork</code> .

Continued on next page



Table 5.84 – continued from previous page

<code>sgesvd(...)</code>	Wrapper for <code>sgesvd</code> .
<code>dgesvd(...)</code>	Wrapper for <code>dgesvd</code> .
<code>cgesvd(...)</code>	Wrapper for <code>cgesvd</code> .
<code>zgesvd(...)</code>	Wrapper for <code>zgesvd</code> .
<code>sgesvd_lwork(m,n,[compute_uv,full_matrices])</code>	Wrapper for <code>sgesvd_lwork</code> .
<code>dgesvd_lwork(m,n,[compute_uv,full_matrices])</code>	Wrapper for <code>dgesvd_lwork</code> .
<code>cgesvd_lwork(m,n,[compute_uv,full_matrices])</code>	Wrapper for <code>cgesvd_lwork</code> .
<code>zgesvd_lwork(m,n,[compute_uv,full_matrices])</code>	Wrapper for <code>zgesvd_lwork</code> .
<code>sgesv(a,b,[overwrite_a,overwrite_b])</code>	Wrapper for <code>sgesv</code> .
<code>dgesv(a,b,[overwrite_a,overwrite_b])</code>	Wrapper for <code>dgesv</code> .
<code>cgesv(a,b,[overwrite_a,overwrite_b])</code>	Wrapper for <code>cgesv</code> .
<code>zgesv(a,b,[overwrite_a,overwrite_b])</code>	Wrapper for <code>zgesv</code> .
<code>sgetrf(a,[overwrite_a])</code>	Wrapper for <code>sgetrf</code> .
<code>dgetrf(a,[overwrite_a])</code>	Wrapper for <code>dgetrf</code> .
<code>cgetrf(a,[overwrite_a])</code>	Wrapper for <code>cgetrf</code> .
<code>zgetrf(a,[overwrite_a])</code>	Wrapper for <code>zgetrf</code> .
<code>sgetri(lu,piv,[lwork,overwrite_lu])</code>	Wrapper for <code>sgetri</code> .
<code>dgetri(lu,piv,[lwork,overwrite_lu])</code>	Wrapper for <code>dgetri</code> .
<code>cgetri(lu,piv,[lwork,overwrite_lu])</code>	Wrapper for <code>cgetri</code> .
<code>zgetri(lu,piv,[lwork,overwrite_lu])</code>	Wrapper for <code>zgetri</code> .
<code>sgetri_lwork(n)</code>	Wrapper for <code>sgetri_lwork</code> .
<code>dgetri_lwork(n)</code>	Wrapper for <code>dgetri_lwork</code> .
<code>cgetri_lwork(n)</code>	Wrapper for <code>cgetri_lwork</code> .
<code>zgetri_lwork(n)</code>	Wrapper for <code>zgetri_lwork</code> .
<code>sgetrs(lu,piv,b,[trans,overwrite_b])</code>	Wrapper for <code>sgetrs</code> .
<code>dgetrs(lu,piv,b,[trans,overwrite_b])</code>	Wrapper for <code>dgetrs</code> .
<code>cgetrs(lu,piv,b,[trans,overwrite_b])</code>	Wrapper for <code>cgetrs</code> .
<code>zgetrs(lu,piv,b,[trans,overwrite_b])</code>	Wrapper for <code>zgetrs</code> .
<code>sgges(...)</code>	Wrapper for <code>sgges</code> .
<code>dgges(...)</code>	Wrapper for <code>dgges</code> .
<code>cgges(...)</code>	Wrapper for <code>cgges</code> .
<code>zgges(...)</code>	Wrapper for <code>zgges</code> .
<code>sggev(...)</code>	Wrapper for <code>sggev</code> .
<code>dggev(...)</code>	Wrapper for <code>dggev</code> .
<code>cggev(...)</code>	Wrapper for <code>cggev</code> .
<code>zggev(...)</code>	Wrapper for <code>zggev</code> .
<code>chbevd(...)</code>	Wrapper for <code>chbevd</code> .
<code>zhbevd(...)</code>	Wrapper for <code>zhbevd</code> .
<code>chbevz(...)</code>	Wrapper for <code>chbevz</code> .
<code>zhbevz(...)</code>	Wrapper for <code>zhbevz</code> .
<code>cheev(a,[compute_v,lower,lwork,overwrite_a])</code>	Wrapper for <code>cheev</code> .
<code>zheev(a,[compute_v,lower,lwork,overwrite_a])</code>	Wrapper for <code>zheev</code> .
<code>cheevd(a,[compute_v,lower,lwork,overwrite_a])</code>	Wrapper for <code>cheevd</code> .
<code>zheevd(a,[compute_v,lower,lwork,overwrite_a])</code>	Wrapper for <code>zheevd</code> .
<code>cheevr(...)</code>	Wrapper for <code>cheevr</code> .
<code>zheevr(...)</code>	Wrapper for <code>zheevr</code> .
<code>chegv(...)</code>	Wrapper for <code>chegv</code> .
<code>zhhev(...)</code>	Wrapper for <code>zhhev</code> .
<code>chegvd(...)</code>	Wrapper for <code>chegvd</code> .
<code>zhhevz(...)</code>	Wrapper for <code>zhhevz</code> .
<code>chegvx(...)</code>	Wrapper for <code>chegvx</code> .
<code>zhhevz(...)</code>	Wrapper for <code>zhhevz</code> .

Continued on next page

Table 5.84 – continued from previous page

<code>slarf(v,tau,c,work,[side,incv,overwrite_c])</code>	Wrapper for <code>slarf</code> .
<code>dlarf(v,tau,c,work,[side,incv,overwrite_c])</code>	Wrapper for <code>dlarf</code> .
<code>clarf(v,tau,c,work,[side,incv,overwrite_c])</code>	Wrapper for <code>clarf</code> .
<code>zlarf(v,tau,c,work,[side,incv,overwrite_c])</code>	Wrapper for <code>zlarf</code> .
<code>slarfg(n,alpha,x,[incx,overwrite_x])</code>	Wrapper for <code>slarfg</code> .
<code>dlarfg(n,alpha,x,[incx,overwrite_x])</code>	Wrapper for <code>dlarfg</code> .
<code>clarfg(n,alpha,x,[incx,overwrite_x])</code>	Wrapper for <code>clarfg</code> .
<code>zlarfg(n,alpha,x,[incx,overwrite_x])</code>	Wrapper for <code>zlarfg</code> .
<code>slartg(f,g)</code>	Wrapper for <code>slartg</code> .
<code>dlartg(f,g)</code>	Wrapper for <code>dlartg</code> .
<code>clartg(f,g)</code>	Wrapper for <code>clartg</code> .
<code>zartg(f,g)</code>	Wrapper for <code>zartg</code> .
<code>slasd4(i,d,z,[rho])</code>	Wrapper for <code>slasd4</code> .
<code>dlasd4(i,d,z,[rho])</code>	Wrapper for <code>dlasd4</code> .
<code>slaswp(a,piv,[k1,k2,off,inc,overwrite_a])</code>	Wrapper for <code>slaswp</code> .
<code>dlaswp(a,piv,[k1,k2,off,inc,overwrite_a])</code>	Wrapper for <code>dlaswp</code> .
<code>claswp(a,piv,[k1,k2,off,inc,overwrite_a])</code>	Wrapper for <code>claswp</code> .
<code>zlaswp(a,piv,[k1,k2,off,inc,overwrite_a])</code>	Wrapper for <code>zlaswp</code> .
<code>slauum(c,[lower,overwrite_c])</code>	Wrapper for <code>slauum</code> .
<code>dlauum(c,[lower,overwrite_c])</code>	Wrapper for <code>dlauum</code> .
<code>clauum(c,[lower,overwrite_c])</code>	Wrapper for <code>clauum</code> .
<code>zlauum(c,[lower,overwrite_c])</code>	Wrapper for <code>zlauum</code> .
<code>spbsv(ab,b,[lower,ldab,overwrite_ab,overwrite_b])</code>	Wrapper for <code>spbsv</code> .
<code>dpbsv(ab,b,[lower,ldab,overwrite_ab,overwrite_b])</code>	Wrapper for <code>dpbsv</code> .
<code>cpbsv(ab,b,[lower,ldab,overwrite_ab,overwrite_b])</code>	Wrapper for <code>cpbsv</code> .
<code>zpbsv(ab,b,[lower,ldab,overwrite_ab,overwrite_b])</code>	Wrapper for <code>zpbsv</code> .
<code>spbtrf(ab,[lower,ldab,overwrite_ab])</code>	Wrapper for <code>spbtrf</code> .
<code>dpbtrf(ab,[lower,ldab,overwrite_ab])</code>	Wrapper for <code>dpbtrf</code> .
<code>cpbtrf(ab,[lower,ldab,overwrite_ab])</code>	Wrapper for <code>cpbtrf</code> .
<code>zpbtrf(ab,[lower,ldab,overwrite_ab])</code>	Wrapper for <code>zpbtrf</code> .
<code>spbtrs(ab,b,[lower,ldab,overwrite_b])</code>	Wrapper for <code>spbtrs</code> .
<code>dpbtrs(ab,b,[lower,ldab,overwrite_b])</code>	Wrapper for <code>dpbtrs</code> .
<code>cpbtrs(ab,b,[lower,ldab,overwrite_b])</code>	Wrapper for <code>cpbtrs</code> .
<code>zpbtrs(ab,b,[lower,ldab,overwrite_b])</code>	Wrapper for <code>zpbtrs</code> .
<code>sposv(a,b,[lower,overwrite_a,overwrite_b])</code>	Wrapper for <code>sposv</code> .
<code>dpovs(a,b,[lower,overwrite_a,overwrite_b])</code>	Wrapper for <code>dpovs</code> .
<code>cpovs(a,b,[lower,overwrite_a,overwrite_b])</code>	Wrapper for <code>cpovs</code> .
<code>zposv(a,b,[lower,overwrite_a,overwrite_b])</code>	Wrapper for <code>zposv</code> .
<code>spotrf(a,[lower,overwrite_a])</code>	Wrapper for <code>spotrf</code> .
<code>dpotrf(a,[lower,overwrite_a])</code>	Wrapper for <code>dpotrf</code> .
<code>cpotrf(a,[lower,overwrite_a])</code>	Wrapper for <code>cpotrf</code> .
<code>zpotrf(a,[lower,overwrite_a])</code>	Wrapper for <code>zpotrf</code> .
<code>spotri(c,[lower,overwrite_c])</code>	Wrapper for <code>spotri</code> .
<code>dpotri(c,[lower,overwrite_c])</code>	Wrapper for <code>dpotri</code> .
<code>cpotri(c,[lower,overwrite_c])</code>	Wrapper for <code>cpotri</code> .
<code>zpotri(c,[lower,overwrite_c])</code>	Wrapper for <code>zpotri</code> .
<code>spotrs(c,b,[lower,overwrite_b])</code>	Wrapper for <code>spotrs</code> .
<code>dpotrs(c,b,[lower,overwrite_b])</code>	Wrapper for <code>dpotrs</code> .
<code>cpotrs(c,b,[lower,overwrite_b])</code>	Wrapper for <code>cpotrs</code> .
<code>zpotrs(c,b,[lower,overwrite_b])</code>	Wrapper for <code>zpotrs</code> .
<code>crot(...)</code>	Wrapper for <code>crot</code> .
<code>zrot(...)</code>	Wrapper for <code>zrot</code> .

Continued on next page

Table 5.84 – continued from previous page

<code>strsyl(a,b,c,[trana,tranb,isgn,overwrite_c])</code>	Wrapper for <code>strsyl</code> .
<code>dtrsyl(a,b,c,[trana,tranb,isgn,overwrite_c])</code>	Wrapper for <code>dtrsyl</code> .
<code>ctrsyl(a,b,c,[trana,tranb,isgn,overwrite_c])</code>	Wrapper for <code>ctrsyl</code> .
<code>ztrsyl(a,b,c,[trana,tranb,isgn,overwrite_c])</code>	Wrapper for <code>ztrsyl</code> .
<code>strtri(c,[lower,unitdiag,overwrite_c])</code>	Wrapper for <code>strtri</code> .
<code>dtrtri(c,[lower,unitdiag,overwrite_c])</code>	Wrapper for <code>dtrtri</code> .
<code>ctrtri(c,[lower,unitdiag,overwrite_c])</code>	Wrapper for <code>ctrtri</code> .
<code>ztrtri(c,[lower,unitdiag,overwrite_c])</code>	Wrapper for <code>ztrtri</code> .
<code>strtrs(...)</code>	Wrapper for <code>strtrs</code> .
<code>dtrtrs(...)</code>	Wrapper for <code>dtrtrs</code> .
<code>ctrtrs(...)</code>	Wrapper for <code>ctrtrs</code> .
<code>ztrtrs(...)</code>	Wrapper for <code>ztrtrs</code> .
<code>cunghr(a,tau,[lo,hi,lwork,overwrite_a])</code>	Wrapper for <code>cunghr</code> .
<code>zunghr(a,tau,[lo,hi,lwork,overwrite_a])</code>	Wrapper for <code>zunghr</code> .
<code>cungqr(a,tau,[lwork,overwrite_a])</code>	Wrapper for <code>cungqr</code> .
<code>zungqr(a,tau,[lwork,overwrite_a])</code>	Wrapper for <code>zungqr</code> .
<code>cungrq(a,tau,[lwork,overwrite_a])</code>	Wrapper for <code>cungrq</code> .
<code>zungrq(a,tau,[lwork,overwrite_a])</code>	Wrapper for <code>zungrq</code> .
<code>cunmqr(side,trans,a,tau,c,lwork,[overwrite_c])</code>	Wrapper for <code>cunmqr</code> .
<code>zunmqr(side,trans,a,tau,c,lwork,[overwrite_c])</code>	Wrapper for <code>zunmqr</code> .
<code>sgtsv(...)</code>	Wrapper for <code>sgtsv</code> .
<code>dgtsv(...)</code>	Wrapper for <code>dgtsv</code> .
<code>cgtsv(...)</code>	Wrapper for <code>cgtsv</code> .
<code>zgtsv(...)</code>	Wrapper for <code>zgtsv</code> .
<code>sptsv(...)</code>	Wrapper for <code>sptsv</code> .
<code>dptsv(...)</code>	Wrapper for <code>dptsv</code> .
<code>cptsv(...)</code>	Wrapper for <code>cptsv</code> .
<code>zptsv(...)</code>	Wrapper for <code>zptsv</code> .
<code>slamch(cmach)</code>	Wrapper for <code>slamch</code> .
<code>dlamch(cmach)</code>	Wrapper for <code>dlamch</code> .
<code>sorghr(a,tau,[lo,hi,lwork,overwrite_a])</code>	Wrapper for <code>sorghr</code> .
<code>dorghr(a,tau,[lo,hi,lwork,overwrite_a])</code>	Wrapper for <code>dorghr</code> .
<code>sorgqr(a,tau,[lwork,overwrite_a])</code>	Wrapper for <code>sorgqr</code> .
<code>dorgqr(a,tau,[lwork,overwrite_a])</code>	Wrapper for <code>dorgqr</code> .
<code>sorgrq(a,tau,[lwork,overwrite_a])</code>	Wrapper for <code>sorgrq</code> .
<code>dorgrq(a,tau,[lwork,overwrite_a])</code>	Wrapper for <code>dorgrq</code> .
<code>sormqr(side,trans,a,tau,c,lwork,[overwrite_c])</code>	Wrapper for <code>sormqr</code> .
<code>dormqr(side,trans,a,tau,c,lwork,[overwrite_c])</code>	Wrapper for <code>dormqr</code> .
<code>ssbev(ab,[compute_v,lower,ldab,overwrite_ab])</code>	Wrapper for <code>ssbev</code> .
<code>dsbev(ab,[compute_v,lower,ldab,overwrite_ab])</code>	Wrapper for <code>dsbev</code> .
<code>ssbevd(...)</code>	Wrapper for <code>ssbevd</code> .
<code>dsbevd(...)</code>	Wrapper for <code>dsbevd</code> .
<code>ssbevx(...)</code>	Wrapper for <code>ssbevx</code> .
<code>dsbevx(...)</code>	Wrapper for <code>dsbevx</code> .
<code>ssyev(a,[compute_v,lower,lwork,overwrite_a])</code>	Wrapper for <code>ssyev</code> .
<code>dsyev(a,[compute_v,lower,lwork,overwrite_a])</code>	Wrapper for <code>dsyev</code> .
<code>ssyevd(a,[compute_v,lower,lwork,overwrite_a])</code>	Wrapper for <code>ssyevd</code> .
<code>dsyevd(a,[compute_v,lower,lwork,overwrite_a])</code>	Wrapper for <code>dsyevd</code> .
<code>ssyevr(...)</code>	Wrapper for <code>ssyevr</code> .
<code>dsyevr(...)</code>	Wrapper for <code>dsyevr</code> .
<code>ssygv(...)</code>	Wrapper for <code>ssygv</code> .
<code>dsygv(...)</code>	Wrapper for <code>dsygv</code> .

Continued on next page

Table 5.84 – continued from previous page

<code>ssygvd(...)</code>	Wrapper for <code>ssygvd</code> .
<code>dsygvd(...)</code>	Wrapper for <code>dsygvd</code> .
<code>ssygvx(...)</code>	Wrapper for <code>ssygvx</code> .
<code>dsygvx(...)</code>	Wrapper for <code>dsygvx</code> .
<code>slange(norm,a)</code>	Wrapper for <code>slange</code> .
<code>dlange(norm,a)</code>	Wrapper for <code>dlange</code> .
<code>clange(norm,a)</code>	Wrapper for <code>clange</code> .
<code>zlange(norm,a)</code>	Wrapper for <code>zlange</code> .
<code>ilaver()</code>	Wrapper for <code>ilaver</code> .

`scipy.linalg.lapack.sgbstv(kl, ku, ab, b[, overwrite_ab, overwrite_b]) = <fortran object>`

Wrapper for `sgbsv`.

**Parameters** `kl` : input int  
`ku` : input int  
`ab` : input rank-2 array('f') with bounds (2\*kl+ku+1,n)  
`b` : input rank-2 array('f') with bounds (n,nrhs)

**Returns** `lub` : rank-2 array('f') with bounds (2\*kl+ku+1,n) and ab storage  
`piv` : rank-1 array('i') with bounds (n)  
`x` : rank-2 array('f') with bounds (n,nrhs) and b storage  
`info` : int

**Other Parameters**  
`overwrite_ab` : input int, optional  
 Default: 0  
`overwrite_b` : input int, optional  
 Default: 0

`scipy.linalg.lapack.dgbstv(kl, ku, ab, b[, overwrite_ab, overwrite_b]) = <fortran object>`

Wrapper for `dgbsv`.

**Parameters** `kl` : input int  
`ku` : input int  
`ab` : input rank-2 array('d') with bounds (2\*kl+ku+1,n)  
`b` : input rank-2 array('d') with bounds (n,nrhs)

**Returns** `lub` : rank-2 array('d') with bounds (2\*kl+ku+1,n) and ab storage  
`piv` : rank-1 array('i') with bounds (n)  
`x` : rank-2 array('d') with bounds (n,nrhs) and b storage  
`info` : int

**Other Parameters**  
`overwrite_ab` : input int, optional  
 Default: 0  
`overwrite_b` : input int, optional  
 Default: 0

`scipy.linalg.lapack.cgbstv(kl, ku, ab, b[, overwrite_ab, overwrite_b]) = <fortran object>`

Wrapper for `cgbstv`.

**Parameters** `kl` : input int  
`ku` : input int  
`ab` : input rank-2 array('F') with bounds (2\*kl+ku+1,n)  
`b` : input rank-2 array('F') with bounds (n,nrhs)

**Returns** `lub` : rank-2 array('F') with bounds (2\*kl+ku+1,n) and ab storage  
`piv` : rank-1 array('i') with bounds (n)  
`x` : rank-2 array('F') with bounds (n,nrhs) and b storage  
`info` : int

**Other Parameters**

**overwrite\_ab** : input int, optional  
Default: 0  
**overwrite\_b** : input int, optional  
Default: 0

`scipy.linalg.lapack.zgbsv(kl, ku, ab, b[, overwrite_ab, overwrite_b]) = <fortran object>`  
Wrapper for zgbsv.

**Parameters** **kl** : input int  
**ku** : input int  
**ab** : input rank-2 array('D') with bounds (2\*kl+ku+1,n)  
**b** : input rank-2 array('D') with bounds (n,nrhs)  
**Returns** **lub** : rank-2 array('D') with bounds (2\*kl+ku+1,n) and ab storage  
**piv** : rank-1 array('i') with bounds (n)  
**x** : rank-2 array('D') with bounds (n,nrhs) and b storage  
**info** : int

**Other Parameters**

**overwrite\_ab** : input int, optional  
Default: 0  
**overwrite\_b** : input int, optional  
Default: 0

`scipy.linalg.lapack.sgbtrf(ab, kl, ku[, m, n, ldab, overwrite_ab]) = <fortran object>`  
Wrapper for sgbtrf.

**Parameters** **ab** : input rank-2 array('f') with bounds (ldab,\*)  
**kl** : input int  
**ku** : input int  
**Returns** **lu** : rank-2 array('f') with bounds (ldab,\*) and ab storage  
**ipiv** : rank-1 array('i') with bounds (MIN(m,n))  
**info** : int

**Other Parameters**

**m** : input int, optional  
Default: shape(ab,1)  
**n** : input int, optional  
Default: shape(ab,1)  
**overwrite\_ab** : input int, optional  
Default: 0  
**ldab** : input int, optional  
Default: shape(ab,0)

`scipy.linalg.lapack.dgbtrf(ab, kl, ku[, m, n, ldab, overwrite_ab]) = <fortran object>`  
Wrapper for dgbtrf.

**Parameters** **ab** : input rank-2 array('d') with bounds (ldab,\*)  
**kl** : input int  
**ku** : input int  
**Returns** **lu** : rank-2 array('d') with bounds (ldab,\*) and ab storage  
**ipiv** : rank-1 array('i') with bounds (MIN(m,n))  
**info** : int

**Other Parameters**

**m** : input int, optional  
Default: shape(ab,1)  
**n** : input int, optional  
Default: shape(ab,1)  
**overwrite\_ab** : input int, optional

Default: 0  
**ldab** : input int, optional  
 Default: shape(ab,0)

`scipy.linalg.lapack.cgbtrf(ab, kl, ku[, m, n, ldab, overwrite_ab]) = <fortran object>`  
 Wrapper for cgbtrf.

**Parameters** **ab** : input rank-2 array('F') with bounds (ldab,\*)  
**kl** : input int  
**ku** : input int

**Returns** **lu** : rank-2 array('F') with bounds (ldab,\*) and ab storage  
**ipiv** : rank-1 array('i') with bounds (MIN(m,n))  
**info** : int

**Other Parameters**  
**m** : input int, optional  
 Default: shape(ab,1)  
**n** : input int, optional  
 Default: shape(ab,1)  
**overwrite\_ab** : input int, optional  
 Default: 0  
**ldab** : input int, optional  
 Default: shape(ab,0)

`scipy.linalg.lapack.zgbtrf(ab, kl, ku[, m, n, ldab, overwrite_ab]) = <fortran object>`  
 Wrapper for zgbtrf.

**Parameters** **ab** : input rank-2 array('D') with bounds (ldab,\*)  
**kl** : input int  
**ku** : input int

**Returns** **lu** : rank-2 array('D') with bounds (ldab,\*) and ab storage  
**ipiv** : rank-1 array('i') with bounds (MIN(m,n))  
**info** : int

**Other Parameters**  
**m** : input int, optional  
 Default: shape(ab,1)  
**n** : input int, optional  
 Default: shape(ab,1)  
**overwrite\_ab** : input int, optional  
 Default: 0  
**ldab** : input int, optional  
 Default: shape(ab,0)

`scipy.linalg.lapack.sgbtrs(ab, kl, ku, b, ipiv[, trans, n, ldab, ldb, overwrite_b]) = <fortran object>`

Wrapper for sgbtrs.

**Parameters** **ab** : input rank-2 array('f') with bounds (ldab,\*)  
**kl** : input int  
**ku** : input int  
**b** : input rank-2 array('f') with bounds (ldb,\*)  
**ipiv** : input rank-1 array('i') with bounds (n)

**Returns** **x** : rank-2 array('f') with bounds (ldb,\*) and b storage  
**info** : int

**Other Parameters**  
**overwrite\_b** : input int, optional  
 Default: 0  
**trans** : input int, optional

Default: 0  
**n** : input int, optional  
 Default: shape(ab,1)  
**ldab** : input int, optional  
 Default: shape(ab,0)  
**ldb** : input int, optional  
 Default: shape(b,0)

`scipy.linalg.lapack.dgbtrs` (*ab, kl, ku, b, ipiv*[, *trans, n, ldab, ldb, overwrite\_b* ]) = <fortran object>

Wrapper for `dgbtrs`.

**Parameters** **ab** : input rank-2 array('d') with bounds (ldab,\*)  
**kl** : input int  
**ku** : input int  
**b** : input rank-2 array('d') with bounds (ldb,\*)  
**ipiv** : input rank-1 array('i') with bounds (n)  
**Returns** **x** : rank-2 array('d') with bounds (ldb,\*) and b storage  
**info** : int  
**Other Parameters**  
**overwrite\_b** : input int, optional  
 Default: 0  
**trans** : input int, optional  
 Default: 0  
**n** : input int, optional  
 Default: shape(ab,1)  
**ldab** : input int, optional  
 Default: shape(ab,0)  
**ldb** : input int, optional  
 Default: shape(b,0)

`scipy.linalg.lapack.cgbtrs` (*ab, kl, ku, b, ipiv*[, *trans, n, ldab, ldb, overwrite\_b* ]) = <fortran object>

Wrapper for `cgbtrs`.

**Parameters** **ab** : input rank-2 array('F') with bounds (ldab,\*)  
**kl** : input int  
**ku** : input int  
**b** : input rank-2 array('F') with bounds (ldb,\*)  
**ipiv** : input rank-1 array('i') with bounds (n)  
**Returns** **x** : rank-2 array('F') with bounds (ldb,\*) and b storage  
**info** : int  
**Other Parameters**  
**overwrite\_b** : input int, optional  
 Default: 0  
**trans** : input int, optional  
 Default: 0  
**n** : input int, optional  
 Default: shape(ab,1)  
**ldab** : input int, optional  
 Default: shape(ab,0)  
**ldb** : input int, optional  
 Default: shape(b,0)

`scipy.linalg.lapack.zgbtrs` (*ab, kl, ku, b, ipiv*[, *trans, n, ldab, ldb, overwrite\_b* ]) = <fortran object>

Wrapper for `zgbtrs`.

**Parameters** **ab** : input rank-2 array('D') with bounds (ldab,\*)  
**kl** : input int  
**ku** : input int  
**b** : input rank-2 array('D') with bounds (ldb,\*)  
**ipiv** : input rank-1 array('i') with bounds (n)

**Returns** **x** : rank-2 array('D') with bounds (ldb,\*) and b storage  
**info** : int

**Other Parameters**  
**overwrite\_b** : input int, optional  
 Default: 0  
**trans** : input int, optional  
 Default: 0  
**n** : input int, optional  
 Default: shape(ab,1)  
**ldab** : input int, optional  
 Default: shape(ab,0)  
**ldb** : input int, optional  
 Default: shape(b,0)

`scipy.linalg.lapack.sgebal(a[, scale, permute, overwrite_a]) = <fortran object>`  
 Wrapper for sgebal.

**Parameters** **a** : input rank-2 array('f') with bounds (m,n)

**Returns** **ba** : rank-2 array('f') with bounds (m,n) and a storage  
**lo** : int  
**hi** : int  
**pivscale** : rank-1 array('f') with bounds (n)  
**info** : int

**Other Parameters**  
**scale** : input int, optional  
 Default: 0  
**permute** : input int, optional  
 Default: 0  
**overwrite\_a** : input int, optional  
 Default: 0

`scipy.linalg.lapack.dgebal(a[, scale, permute, overwrite_a]) = <fortran object>`  
 Wrapper for dgebal.

**Parameters** **a** : input rank-2 array('d') with bounds (m,n)

**Returns** **ba** : rank-2 array('d') with bounds (m,n) and a storage  
**lo** : int  
**hi** : int  
**pivscale** : rank-1 array('d') with bounds (n)  
**info** : int

**Other Parameters**  
**scale** : input int, optional  
 Default: 0  
**permute** : input int, optional  
 Default: 0  
**overwrite\_a** : input int, optional  
 Default: 0

`scipy.linalg.lapack.cgebal(a[, scale, permute, overwrite_a]) = <fortran object>`  
 Wrapper for cgebal.

**Parameters** **a** : input rank-2 array('F') with bounds (m,n)



**Returns** **ba** : rank-2 array('F') with bounds (m,n) and a storage  
**lo** : int  
**hi** : int  
**pivsca**le : rank-1 array('f') with bounds (n)  
**info** : int

**Other Parameters**

**scale** : input int, optional  
 Default: 0  
**permute** : input int, optional  
 Default: 0  
**overwrite\_a** : input int, optional  
 Default: 0

`scipy.linalg.lapack.zgebal(a[, scale, permute, overwrite_a]) = <fortran object>`  
 Wrapper for zgebal.

**Parameters** **a** : input rank-2 array('D') with bounds (m,n)  
**Returns** **ba** : rank-2 array('D') with bounds (m,n) and a storage  
**lo** : int  
**hi** : int  
**pivsca**le : rank-1 array('d') with bounds (n)  
**info** : int

**Other Parameters**

**scale** : input int, optional  
 Default: 0  
**permute** : input int, optional  
 Default: 0  
**overwrite\_a** : input int, optional  
 Default: 0

`scipy.linalg.lapack.sgees(sselect, a[, compute_v, sort_t, lwork, sselect_extra_args, overwrite_a]) = <fortran object>`

Wrapper for sgees.

**Parameters** **sselect** : call-back function  
**a** : input rank-2 array('f') with bounds (n,n)  
**Returns** **t** : rank-2 array('f') with bounds (n,n) and a storage  
**sdim** : int  
**wr** : rank-1 array('f') with bounds (n)  
**wi** : rank-1 array('f') with bounds (n)  
**vs** : rank-2 array('f') with bounds (ldvs,n)  
**work** : rank-1 array('f') with bounds (MAX(lwork,1))  
**info** : int

**Other Parameters**

**compute\_v** : input int, optional  
 Default: 1  
**sort\_t** : input int, optional  
 Default: 0  
**sselect\_extra\_args** : input tuple, optional  
 Default: ()  
**overwrite\_a** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: 3\*n

**Notes**

Call-back functions:

```
def sselect(arg1,arg2): return sselect
Required arguments:
  arg1 : input float
  arg2 : input float
Return objects:
  sselect : int
```

`scipy.linalg.lapack.dgees` (*dselect*, *a*[, *compute\_v*, *sort\_t*, *lwork*, *dselect\_extra\_args*, *overwrite\_a* ] ) = <fortran object>

Wrapper for dgees.

**Parameters** **dselect** : call-back function  
**a** : input rank-2 array('d') with bounds (n,n)  
**Returns** **t** : rank-2 array('d') with bounds (n,n) and a storage  
**sdim** : int  
**wr** : rank-1 array('d') with bounds (n)  
**wi** : rank-1 array('d') with bounds (n)  
**vs** : rank-2 array('d') with bounds (ldvs,n)  
**work** : rank-1 array('d') with bounds (MAX(lwork,1))  
**info** : int

**Other Parameters**

**compute\_v** : input int, optional  
 Default: 1  
**sort\_t** : input int, optional  
 Default: 0  
**dselect\_extra\_args** : input tuple, optional  
 Default: ()  
**overwrite\_a** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: 3\*n

**Notes**

Call-back functions:

```
def dselect(arg1,arg2): return dselect
Required arguments:
  arg1 : input float
  arg2 : input float
Return objects:
  dselect : int
```

`scipy.linalg.lapack.cgees` (*cselect*, *a*[, *compute\_v*, *sort\_t*, *lwork*, *cselect\_extra\_args*, *overwrite\_a* ] ) = <fortran object>

Wrapper for cgees.

**Parameters** **cselect** : call-back function  
**a** : input rank-2 array('F') with bounds (n,n)  
**Returns** **t** : rank-2 array('F') with bounds (n,n) and a storage  
**sdim** : int  
**w** : rank-1 array('F') with bounds (n)  
**vs** : rank-2 array('F') with bounds (ldvs,n)  
**work** : rank-1 array('F') with bounds (MAX(lwork,1))  
**info** : int

**Other Parameters**

**compute\_v** : input int, optional  
Default: 1  
**sort\_t** : input int, optional  
Default: 0  
**cselect\_extra\_args** : input tuple, optional  
Default: ()  
**overwrite\_a** : input int, optional  
Default: 0  
**lwork** : input int, optional  
Default: 3\*n

**Notes**

Call-back functions:

```
def cselect(arg): return cselect
Required arguments:
  arg : input complex
Return objects:
  cselect : int
```

`scipy.linalg.lapack.zgees` (*zselect*, *a*[, *compute\_v*, *sort\_t*, *lwork*, *zselect\_extra\_args*, *overwrite\_a* ])  
= <fortran object>

Wrapper for zgees.

**Parameters** **zselect** : call-back function  
**a** : input rank-2 array('D') with bounds (n,n)  
**Returns** **t** : rank-2 array('D') with bounds (n,n) and a storage  
**sdim** : int  
**w** : rank-1 array('D') with bounds (n)  
**vs** : rank-2 array('D') with bounds (ldvs,n)  
**work** : rank-1 array('D') with bounds (MAX(lwork,1))  
**info** : int

**Other Parameters**

**compute\_v** : input int, optional  
Default: 1  
**sort\_t** : input int, optional  
Default: 0  
**zselect\_extra\_args** : input tuple, optional  
Default: ()  
**overwrite\_a** : input int, optional  
Default: 0  
**lwork** : input int, optional  
Default: 3\*n

**Notes**

Call-back functions:

```
def zselect(arg): return zselect
Required arguments:
  arg : input complex
Return objects:
  zselect : int
```

`scipy.linalg.lapack.sgeev` (*a*[, *compute\_vl*, *compute\_vr*, *lwork*, *overwrite\_a* ] ) = <fortran object>  
Wrapper for sgeev.

**Parameters** **a** : input rank-2 array('f') with bounds (n,n)  
**Returns** **wr** : rank-1 array('f') with bounds (n)  
**wi** : rank-1 array('f') with bounds (n)  
**vl** : rank-2 array('f') with bounds (ldvl,n)  
**vr** : rank-2 array('f') with bounds (ldvr,n)  
**info** : int

**Other Parameters**

**compute\_vl** : input int, optional  
 Default: 1  
**compute\_vr** : input int, optional  
 Default: 1  
**overwrite\_a** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: 4\*n

`scipy.linalg.lapack.dgeev(a[, compute_vl, compute_vr, lwork, overwrite_a]) = <fortran object>`  
 Wrapper for dgeev.

**Parameters** **a** : input rank-2 array('d') with bounds (n,n)  
**Returns** **wr** : rank-1 array('d') with bounds (n)  
**wi** : rank-1 array('d') with bounds (n)  
**vl** : rank-2 array('d') with bounds (ldvl,n)  
**vr** : rank-2 array('d') with bounds (ldvr,n)  
**info** : int

**Other Parameters**

**compute\_vl** : input int, optional  
 Default: 1  
**compute\_vr** : input int, optional  
 Default: 1  
**overwrite\_a** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: 4\*n

`scipy.linalg.lapack.cgeev(a[, compute_vl, compute_vr, lwork, overwrite_a]) = <fortran object>`  
 Wrapper for cgeev.

**Parameters** **a** : input rank-2 array('F') with bounds (n,n)  
**Returns** **w** : rank-1 array('F') with bounds (n)  
**vl** : rank-2 array('F') with bounds (ldvl,n)  
**vr** : rank-2 array('F') with bounds (ldvr,n)  
**info** : int

**Other Parameters**

**compute\_vl** : input int, optional  
 Default: 1  
**compute\_vr** : input int, optional  
 Default: 1  
**overwrite\_a** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: 2\*n

`scipy.linalg.lapack.zgeev(a[, compute_vl, compute_vr, lwork, overwrite_a]) = <fortran object>`  
 Wrapper for zgeev.

**Parameters** **a** : input rank-2 array('D') with bounds (n,n)

**Returns** **w** : rank-1 array('D') with bounds (n)  
**vl** : rank-2 array('D') with bounds (ldvl,n)  
**vr** : rank-2 array('D') with bounds (ldvr,n)  
**info** : int

**Other Parameters**

**compute\_vl** : input int, optional  
 Default: 1  
**compute\_vr** : input int, optional  
 Default: 1  
**overwrite\_a** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: 2\*n

`scipy.linalg.lapack.sgeev_lwork` ( $n$ , `compute_vl`, `compute_vr`) = <fortran object>  
 Wrapper for `sgeev_lwork`.

**Parameters** **n** : input int  
**Returns** **work** : float  
**info** : int

**Other Parameters**

**compute\_vl** : input int, optional  
 Default: 1  
**compute\_vr** : input int, optional  
 Default: 1

`scipy.linalg.lapack.dgeev_lwork` ( $n$ , `compute_vl`, `compute_vr`) = <fortran object>  
 Wrapper for `dgeev_lwork`.

**Parameters** **n** : input int  
**Returns** **work** : float  
**info** : int

**Other Parameters**

**compute\_vl** : input int, optional  
 Default: 1  
**compute\_vr** : input int, optional  
 Default: 1

`scipy.linalg.lapack.cgeev_lwork` ( $n$ , `compute_vl`, `compute_vr`) = <fortran object>  
 Wrapper for `cgeev_lwork`.

**Parameters** **n** : input int  
**Returns** **work** : complex  
**info** : int

**Other Parameters**

**compute\_vl** : input int, optional  
 Default: 1  
**compute\_vr** : input int, optional  
 Default: 1

`scipy.linalg.lapack.zgeev_lwork` ( $n$ , `compute_vl`, `compute_vr`) = <fortran object>  
 Wrapper for `zgeev_lwork`.

**Parameters** **n** : input int  
**Returns** **work** : complex  
**info** : int

**Other Parameters**

**compute\_vl** : input int, optional

Default: 1  
**compute\_vr** : input int, optional  
 Default: 1

`scipy.linalg.lapack.sgegv` (\*args, \*\*kwargs)

`sgegv` is deprecated! The `*gegv` family of routines has been deprecated in LAPACK 3.6.0 in favor of the `*ggegv` family of routines. The corresponding wrappers will be removed from SciPy in a future release.

`alphar, alphai, beta, vl, vr, info = sgegv(a, b, [compute_vl, compute_vr, lwork, overwrite_a, overwrite_b])`

Wrapper for `sgegv`.

**Parameters**    **a** : input rank-2 array('f') with bounds (n,n)  
                   **b** : input rank-2 array('f') with bounds (n,n)

**Returns**        **alphar** : rank-1 array('f') with bounds (n)  
                   **alphai** : rank-1 array('f') with bounds (n)  
                   **beta** : rank-1 array('f') with bounds (n)  
                   **vl** : rank-2 array('f') with bounds (ldvl,n)  
                   **vr** : rank-2 array('f') with bounds (ldvr,n)  
                   **info** : int

**Other Parameters**

**compute\_vl** : input int, optional  
 Default: 1  
**compute\_vr** : input int, optional  
 Default: 1  
**overwrite\_a** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: 8\*n

`scipy.linalg.lapack.dgegv` (\*args, \*\*kwargs)

`dgegv` is deprecated! The `*gegv` family of routines has been deprecated in LAPACK 3.6.0 in favor of the `*ggegv` family of routines. The corresponding wrappers will be removed from SciPy in a future release.

`alphar, alphai, beta, vl, vr, info = dgegv(a, b, [compute_vl, compute_vr, lwork, overwrite_a, overwrite_b])`

Wrapper for `dgegv`.

**Parameters**    **a** : input rank-2 array('d') with bounds (n,n)  
                   **b** : input rank-2 array('d') with bounds (n,n)

**Returns**        **alphar** : rank-1 array('d') with bounds (n)  
                   **alphai** : rank-1 array('d') with bounds (n)  
                   **beta** : rank-1 array('d') with bounds (n)  
                   **vl** : rank-2 array('d') with bounds (ldvl,n)  
                   **vr** : rank-2 array('d') with bounds (ldvr,n)  
                   **info** : int

**Other Parameters**

**compute\_vl** : input int, optional  
 Default: 1  
**compute\_vr** : input int, optional  
 Default: 1  
**overwrite\_a** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0  
**lwork** : input int, optional

Default: 8\*n

`scipy.linalg.lapack.cgegv` (\*args, \*\*kwargs)

`cgegv` is deprecated! The `*gegv` family of routines has been deprecated in LAPACK 3.6.0 in favor of the `*ggev` family of routines. The corresponding wrappers will be removed from SciPy in a future release.

`alpha, beta, vl, vr, info = cgegv(a, b, [compute_vl, compute_vr, lwork, overwrite_a, overwrite_b])`

Wrapper for `cgegv`.

**Parameters** **a** : input rank-2 array('F') with bounds (n,n)  
**b** : input rank-2 array('F') with bounds (n,n)

**Returns** **alpha** : rank-1 array('F') with bounds (n)  
**beta** : rank-1 array('F') with bounds (n)  
**vl** : rank-2 array('F') with bounds (ldvl,n)  
**vr** : rank-2 array('F') with bounds (ldvr,n)  
**info** : int

**Other Parameters**

**compute\_vl** : input int, optional  
 Default: 1  
**compute\_vr** : input int, optional  
 Default: 1  
**overwrite\_a** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: 2\*n

`scipy.linalg.lapack.zgegv` (\*args, \*\*kwargs)

`zgegv` is deprecated! The `*gegv` family of routines has been deprecated in LAPACK 3.6.0 in favor of the `*ggev` family of routines. The corresponding wrappers will be removed from SciPy in a future release.

`alpha, beta, vl, vr, info = zgegv(a, b, [compute_vl, compute_vr, lwork, overwrite_a, overwrite_b])`

Wrapper for `zgegv`.

**Parameters** **a** : input rank-2 array('D') with bounds (n,n)  
**b** : input rank-2 array('D') with bounds (n,n)

**Returns** **alpha** : rank-1 array('D') with bounds (n)  
**beta** : rank-1 array('D') with bounds (n)  
**vl** : rank-2 array('D') with bounds (ldvl,n)  
**vr** : rank-2 array('D') with bounds (ldvr,n)  
**info** : int

**Other Parameters**

**compute\_vl** : input int, optional  
 Default: 1  
**compute\_vr** : input int, optional  
 Default: 1  
**overwrite\_a** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: 2\*n

`scipy.linalg.lapack.sgehrd` (`a`[`lo`, `hi`, `lwork`, `overwrite_a`]) = <fortran object>

Wrapper for `sgehrd`.

**Parameters** **a** : input rank-2 array('f') with bounds (n,n)  
**Returns** **ht** : rank-2 array('f') with bounds (n,n) and a storage  
**tau** : rank-1 array('f') with bounds (n - 1)  
**info** : int

**Other Parameters**

**lo** : input int, optional  
 Default: 0  
**hi** : input int, optional  
 Default: n-1  
**overwrite\_a** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: MAX(n,1)

`scipy.linalg.lapack.dgehrd(a[, lo, hi, lwork, overwrite_a]) = <fortran object>`  
 Wrapper for dgehrd.

**Parameters** **a** : input rank-2 array('d') with bounds (n,n)  
**Returns** **ht** : rank-2 array('d') with bounds (n,n) and a storage  
**tau** : rank-1 array('d') with bounds (n - 1)  
**info** : int

**Other Parameters**

**lo** : input int, optional  
 Default: 0  
**hi** : input int, optional  
 Default: n-1  
**overwrite\_a** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: MAX(n,1)

`scipy.linalg.lapack.cgehrd(a[, lo, hi, lwork, overwrite_a]) = <fortran object>`  
 Wrapper for cgehrd.

**Parameters** **a** : input rank-2 array('F') with bounds (n,n)  
**Returns** **ht** : rank-2 array('F') with bounds (n,n) and a storage  
**tau** : rank-1 array('F') with bounds (n - 1)  
**info** : int

**Other Parameters**

**lo** : input int, optional  
 Default: 0  
**hi** : input int, optional  
 Default: n-1  
**overwrite\_a** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: MAX(n,1)

`scipy.linalg.lapack.zgehrd(a[, lo, hi, lwork, overwrite_a]) = <fortran object>`  
 Wrapper for zgehrd.

**Parameters** **a** : input rank-2 array('D') with bounds (n,n)  
**Returns** **ht** : rank-2 array('D') with bounds (n,n) and a storage  
**tau** : rank-1 array('D') with bounds (n - 1)  
**info** : int

**Other Parameters**

**lo** : input int, optional



Default: 0  
**hi** : input int, optional  
 Default: n-1  
**overwrite\_a** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: MAX(n,1)

`scipy.linalg.lapack.sgehrd_lwork` ( $n$ [,  $lo$ ,  $hi$ ]) = <fortran object>  
 Wrapper for `sgehrd_lwork`.

**Parameters**    **n** : input int  
**Returns**        **work** : float  
                   **info** : int  
**Other Parameters**  
                   **lo** : input int, optional  
                           Default: 0  
                   **hi** : input int, optional  
                           Default: n-1

`scipy.linalg.lapack.dgehrd_lwork` ( $n$ [,  $lo$ ,  $hi$ ]) = <fortran object>  
 Wrapper for `dgehrd_lwork`.

**Parameters**    **n** : input int  
**Returns**        **work** : float  
                   **info** : int  
**Other Parameters**  
                   **lo** : input int, optional  
                           Default: 0  
                   **hi** : input int, optional  
                           Default: n-1

`scipy.linalg.lapack.cgehrd_lwork` ( $n$ [,  $lo$ ,  $hi$ ]) = <fortran object>  
 Wrapper for `cgehrd_lwork`.

**Parameters**    **n** : input int  
**Returns**        **work** : complex  
                   **info** : int  
**Other Parameters**  
                   **lo** : input int, optional  
                           Default: 0  
                   **hi** : input int, optional  
                           Default: n-1

`scipy.linalg.lapack.zgehrd_lwork` ( $n$ [,  $lo$ ,  $hi$ ]) = <fortran object>  
 Wrapper for `zgehrd_lwork`.

**Parameters**    **n** : input int  
**Returns**        **work** : complex  
                   **info** : int  
**Other Parameters**  
                   **lo** : input int, optional  
                           Default: 0  
                   **hi** : input int, optional  
                           Default: n-1

`scipy.linalg.lapack.sgelss` ( $a$ ,  $b$ [,  $cond$ ,  $lwork$ ,  $overwrite_a$ ,  $overwrite_b$ ]) = <fortran object>  
 Wrapper for `sgelss`.

**Parameters** **a** : input rank-2 array('f') with bounds (m,n)  
**b** : input rank-2 array('f') with bounds (maxmn,nrhs)  
**Returns** **v** : rank-2 array('f') with bounds (m,n) and a storage  
**x** : rank-2 array('f') with bounds (maxmn,nrhs) and b storage  
**s** : rank-1 array('f') with bounds (minmn)  
**rank** : int  
**work** : rank-1 array('f') with bounds (MAX(lwork,1))  
**info** : int

**Other Parameters**

**overwrite\_a** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0  
**cond** : input float, optional  
 Default: -1.0  
**lwork** : input int, optional  
 Default: 3\*minmn+MAX(2\*minmn,MAX(maxmn,nrhs))

`scipy.linalg.lapack.dgelss(a, b[, cond, lwork, overwrite_a, overwrite_b]) = <fortran object>`  
 Wrapper for dgelss.

**Parameters** **a** : input rank-2 array('d') with bounds (m,n)  
**b** : input rank-2 array('d') with bounds (maxmn,nrhs)  
**Returns** **v** : rank-2 array('d') with bounds (m,n) and a storage  
**x** : rank-2 array('d') with bounds (maxmn,nrhs) and b storage  
**s** : rank-1 array('d') with bounds (minmn)  
**rank** : int  
**work** : rank-1 array('d') with bounds (MAX(lwork,1))  
**info** : int

**Other Parameters**

**overwrite\_a** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0  
**cond** : input float, optional  
 Default: -1.0  
**lwork** : input int, optional  
 Default: 3\*minmn+MAX(2\*minmn,MAX(maxmn,nrhs))

`scipy.linalg.lapack.cgelss(a, b[, cond, lwork, overwrite_a, overwrite_b]) = <fortran object>`  
 Wrapper for cgelss.

**Parameters** **a** : input rank-2 array('F') with bounds (m,n)  
**b** : input rank-2 array('F') with bounds (maxmn,nrhs)  
**Returns** **v** : rank-2 array('F') with bounds (m,n) and a storage  
**x** : rank-2 array('F') with bounds (maxmn,nrhs) and b storage  
**s** : rank-1 array('f') with bounds (minmn)  
**rank** : int  
**work** : rank-1 array('F') with bounds (MAX(lwork,1))  
**info** : int

**Other Parameters**

**overwrite\_a** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0  
**cond** : input float, optional

Default: -1.0

**lwork** : input int, optional

Default:  $2 * \text{minmn} + \text{MAX}(\text{maxmn}, \text{nrhs})$

`scipy.linalg.lapack.zgelss(a, b[, cond, lwork, overwrite_a, overwrite_b]) = <fortran object>`  
 Wrapper for zgelss.

**Parameters** **a** : input rank-2 array('D') with bounds (m,n)  
**b** : input rank-2 array('D') with bounds (maxmn,nrhs)  
**Returns** **v** : rank-2 array('D') with bounds (m,n) and a storage  
**x** : rank-2 array('D') with bounds (maxmn,nrhs) and b storage  
**s** : rank-1 array('d') with bounds (minmn)  
**rank** : int  
**work** : rank-1 array('D') with bounds (MAX(lwork,1))  
**info** : int

**Other Parameters**

**overwrite\_a** : input int, optional

Default: 0

**overwrite\_b** : input int, optional

Default: 0

**cond** : input float, optional

Default: -1.0

**lwork** : input int, optional

Default:  $2 * \text{minmn} + \text{MAX}(\text{maxmn}, \text{nrhs})$

`scipy.linalg.lapack.sgelss_lwork(m, n, nrhs[, cond, lwork]) = <fortran object>`  
 Wrapper for sgelss\_lwork.

**Parameters** **m** : input int  
**n** : input int  
**nrhs** : input int  
**Returns** **work** : float  
**info** : int

**Other Parameters**

**cond** : input float, optional

Default: -1.0

**lwork** : input int, optional

Default: -1

`scipy.linalg.lapack.dgelss_lwork(m, n, nrhs[, cond, lwork]) = <fortran object>`  
 Wrapper for dgelss\_lwork.

**Parameters** **m** : input int  
**n** : input int  
**nrhs** : input int  
**Returns** **work** : float  
**info** : int

**Other Parameters**

**cond** : input float, optional

Default: -1.0

**lwork** : input int, optional

Default: -1

`scipy.linalg.lapack.cgelss_lwork(m, n, nrhs[, cond, lwork]) = <fortran object>`  
 Wrapper for cgelss\_lwork.

**Parameters** **m** : input int  
**n** : input int  
**nrhs** : input int

**Returns** **work** : complex  
**info** : int

**Other Parameters**  
**cond** : input float, optional  
 Default: -1.0  
**lwork** : input int, optional  
 Default: -1

`scipy.linalg.lapack.zgelss_lwork(m, n, nrhs[, cond, lwork]) = <fortran object>`  
 Wrapper for zgelss\_lwork.

**Parameters** **m** : input int  
**n** : input int  
**nrhs** : input int

**Returns** **work** : complex  
**info** : int

**Other Parameters**  
**cond** : input float, optional  
 Default: -1.0  
**lwork** : input int, optional  
 Default: -1

`scipy.linalg.lapack.sgelss(a, b, lwork, size_iwork[, cond, overwrite_a, overwrite_b]) = <fortran object>`

Wrapper for sgelss.

**Parameters** **a** : input rank-2 array('f') with bounds (m,n)  
**b** : input rank-2 array('f') with bounds (maxmn,nrhs)  
**lwork** : input int  
**size\_iwork** : input int

**Returns** **x** : rank-2 array('f') with bounds (maxmn,nrhs) and b storage  
**s** : rank-1 array('f') with bounds (minmn)  
**rank** : int  
**info** : int

**Other Parameters**  
**overwrite\_a** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0  
**cond** : input float, optional  
 Default: -1.0

`scipy.linalg.lapack.dgelss(a, b, lwork, size_iwork[, cond, overwrite_a, overwrite_b]) = <fortran object>`

Wrapper for dgelss.

**Parameters** **a** : input rank-2 array('d') with bounds (m,n)  
**b** : input rank-2 array('d') with bounds (maxmn,nrhs)  
**lwork** : input int  
**size\_iwork** : input int

**Returns** **x** : rank-2 array('d') with bounds (maxmn,nrhs) and b storage  
**s** : rank-1 array('d') with bounds (minmn)  
**rank** : int  
**info** : int

**Other Parameters**

**overwrite\_a** : input int, optional  
Default: 0  
**overwrite\_b** : input int, optional  
Default: 0  
**cond** : input float, optional  
Default: -1.0

`scipy.linalg.lapack.cgelsd(a, b, lwork, size_rwork, size_iwork[, cond, overwrite_a, overwrite_b]) = <fortran object>`

Wrapper for cgelsd.

**Parameters** **a** : input rank-2 array('F') with bounds (m,n)  
**b** : input rank-2 array('F') with bounds (maxmn,nrhs)  
**lwork** : input int  
**size\_rwork** : input int  
**size\_iwork** : input int  
**Returns** **x** : rank-2 array('F') with bounds (maxmn,nrhs) and b storage  
**s** : rank-1 array('f') with bounds (minmn)  
**rank** : int  
**info** : int

**Other Parameters**

**overwrite\_a** : input int, optional  
Default: 0  
**overwrite\_b** : input int, optional  
Default: 0  
**cond** : input float, optional  
Default: -1.0

`scipy.linalg.lapack.zgelsd(a, b, lwork, size_rwork, size_iwork[, cond, overwrite_a, overwrite_b]) = <fortran object>`

Wrapper for zgelsd.

**Parameters** **a** : input rank-2 array('D') with bounds (m,n)  
**b** : input rank-2 array('D') with bounds (maxmn,nrhs)  
**lwork** : input int  
**size\_rwork** : input int  
**size\_iwork** : input int  
**Returns** **x** : rank-2 array('D') with bounds (maxmn,nrhs) and b storage  
**s** : rank-1 array('d') with bounds (minmn)  
**rank** : int  
**info** : int

**Other Parameters**

**overwrite\_a** : input int, optional  
Default: 0  
**overwrite\_b** : input int, optional  
Default: 0  
**cond** : input float, optional  
Default: -1.0

`scipy.linalg.lapack.sgelsd_lwork(m, n, nrhs[, cond, lwork]) = <fortran object>`

Wrapper for sgelsd\_lwork.

**Parameters** **m** : input int  
**n** : input int  
**nrhs** : input int

**Returns**     **work** : float  
                   **iwork** : int  
                   **info** : int

**Other Parameters**

**cond** : input float, optional  
                           Default: -1.0  
                   **lwork** : input int, optional  
                           Default: -1

`scipy.linalg.lapack.dgelsd_lwork(m, n, nrhs[, cond, lwork]) = <fortran object>`  
 Wrapper for dgelsd\_lwork.

**Parameters**   **m** : input int  
                   **n** : input int  
                   **nrhs** : input int

**Returns**     **work** : float  
                   **iwork** : int  
                   **info** : int

**Other Parameters**

**cond** : input float, optional  
                           Default: -1.0  
                   **lwork** : input int, optional  
                           Default: -1

`scipy.linalg.lapack.cgelsd_lwork(m, n, nrhs[, cond, lwork]) = <fortran object>`  
 Wrapper for cgelsd\_lwork.

**Parameters**   **m** : input int  
                   **n** : input int  
                   **nrhs** : input int

**Returns**     **work** : complex  
                   **rwork** : float  
                   **iwork** : int  
                   **info** : int

**Other Parameters**

**cond** : input float, optional  
                           Default: -1.0  
                   **lwork** : input int, optional  
                           Default: -1

`scipy.linalg.lapack.zgelsd_lwork(m, n, nrhs[, cond, lwork]) = <fortran object>`  
 Wrapper for zgelsd\_lwork.

**Parameters**   **m** : input int  
                   **n** : input int  
                   **nrhs** : input int

**Returns**     **work** : complex  
                   **rwork** : float  
                   **iwork** : int  
                   **info** : int

**Other Parameters**

**cond** : input float, optional  
                           Default: -1.0  
                   **lwork** : input int, optional  
                           Default: -1

`scipy.linalg.lapack.sgelsy(a, b, jptv, cond, lwork[, overwrite_a, overwrite_b]) = <fortran object>`

Wrapper for sgelsy.

**Parameters**

- a** : input rank-2 array('f') with bounds (m,n)
- b** : input rank-2 array('f') with bounds (maxmn,nrhs)
- jptv** : input rank-1 array('i') with bounds (n)
- cond** : input float
- lwork** : input int

**Returns**

- v** : rank-2 array('f') with bounds (m,n) and a storage
- x** : rank-2 array('f') with bounds (maxmn,nrhs) and b storage
- j** : rank-1 array('i') with bounds (n) and jptv storage
- rank** : int
- info** : int

**Other Parameters**

- overwrite\_a** : input int, optional  
Default: 0
- overwrite\_b** : input int, optional  
Default: 0

`scipy.linalg.lapack.dgelsy(a, b, jptv, cond, lwork[, overwrite_a, overwrite_b]) = <fortran object>`

Wrapper for dgelsy.

**Parameters**

- a** : input rank-2 array('d') with bounds (m,n)
- b** : input rank-2 array('d') with bounds (maxmn,nrhs)
- jptv** : input rank-1 array('i') with bounds (n)
- cond** : input float
- lwork** : input int

**Returns**

- v** : rank-2 array('d') with bounds (m,n) and a storage
- x** : rank-2 array('d') with bounds (maxmn,nrhs) and b storage
- j** : rank-1 array('i') with bounds (n) and jptv storage
- rank** : int
- info** : int

**Other Parameters**

- overwrite\_a** : input int, optional  
Default: 0
- overwrite\_b** : input int, optional  
Default: 0

`scipy.linalg.lapack.cgelsy(a, b, jptv, cond, lwork[, overwrite_a, overwrite_b]) = <fortran object>`

Wrapper for cgelsy.

**Parameters**

- a** : input rank-2 array('F') with bounds (m,n)
- b** : input rank-2 array('F') with bounds (maxmn,nrhs)
- jptv** : input rank-1 array('i') with bounds (n)
- cond** : input float
- lwork** : input int

**Returns**

- v** : rank-2 array('F') with bounds (m,n) and a storage
- x** : rank-2 array('F') with bounds (maxmn,nrhs) and b storage
- j** : rank-1 array('i') with bounds (n) and jptv storage
- rank** : int
- info** : int

**Other Parameters**

- overwrite\_a** : input int, optional  
Default: 0

**overwrite\_b** : input int, optional  
 Default: 0

`scipy.linalg.lapack.zgelsy(a, b, jptv, cond, lwork[, overwrite_a, overwrite_b]) = <fortran object>`

Wrapper for zgelsy.

**Parameters** **a** : input rank-2 array('D') with bounds (m,n)  
**b** : input rank-2 array('D') with bounds (maxmn,nrhs)  
**jptv** : input rank-1 array('i') with bounds (n)  
**cond** : input float  
**lwork** : input int

**Returns** **v** : rank-2 array('D') with bounds (m,n) and a storage  
**x** : rank-2 array('D') with bounds (maxmn,nrhs) and b storage  
**j** : rank-1 array('i') with bounds (n) and jptv storage  
**rank** : int  
**info** : int

**Other Parameters**

**overwrite\_a** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0

`scipy.linalg.lapack.sgelsy_lwork(m, n, nrhs, cond[, lwork]) = <fortran object>`

Wrapper for sgelsy\_lwork.

**Parameters** **m** : input int  
**n** : input int  
**nrhs** : input int  
**cond** : input float

**Returns** **work** : float  
**info** : int

**Other Parameters**

**lwork** : input int, optional  
 Default: -1

`scipy.linalg.lapack.dgelsy_lwork(m, n, nrhs, cond[, lwork]) = <fortran object>`

Wrapper for dgelsy\_lwork.

**Parameters** **m** : input int  
**n** : input int  
**nrhs** : input int  
**cond** : input float

**Returns** **work** : float  
**info** : int

**Other Parameters**

**lwork** : input int, optional  
 Default: -1

`scipy.linalg.lapack.cgelsy_lwork(m, n, nrhs, cond[, lwork]) = <fortran object>`

Wrapper for cgelsy\_lwork.

**Parameters** **m** : input int  
**n** : input int  
**nrhs** : input int  
**cond** : input float

**Returns** **work** : complex  
**info** : int



**Other Parameters**

**lwork** : input int, optional  
Default: -1

`scipy.linalg.lapack.zgelsy_lwork(m, n, nrhs, cond[, lwork]) = <fortran object>`  
Wrapper for `zgelsy_lwork`.

**Parameters** **m** : input int  
**n** : input int  
**nrhs** : input int  
**cond** : input float

**Returns** **work** : complex  
**info** : int

**Other Parameters**

**lwork** : input int, optional  
Default: -1

`scipy.linalg.lapack.sgeqp3(a[, lwork, overwrite_a]) = <fortran object>`  
Wrapper for `sgeqp3`.

**Parameters** **a** : input rank-2 array('f') with bounds (m,n)

**Returns** **qr** : rank-2 array('f') with bounds (m,n) and a storage  
**jpvt** : rank-1 array('i') with bounds (n)  
**tau** : rank-1 array('f') with bounds (MIN(m,n))  
**work** : rank-1 array('f') with bounds (MAX(lwork,1))  
**info** : int

**Other Parameters**

**overwrite\_a** : input int, optional  
Default: 0

**lwork** : input int, optional  
Default: 3\*(n+1)

`scipy.linalg.lapack.dgeqp3(a[, lwork, overwrite_a]) = <fortran object>`  
Wrapper for `dgeqp3`.

**Parameters** **a** : input rank-2 array('d') with bounds (m,n)

**Returns** **qr** : rank-2 array('d') with bounds (m,n) and a storage  
**jpvt** : rank-1 array('i') with bounds (n)  
**tau** : rank-1 array('d') with bounds (MIN(m,n))  
**work** : rank-1 array('d') with bounds (MAX(lwork,1))  
**info** : int

**Other Parameters**

**overwrite\_a** : input int, optional  
Default: 0

**lwork** : input int, optional  
Default: 3\*(n+1)

`scipy.linalg.lapack.cgeqp3(a[, lwork, overwrite_a]) = <fortran object>`  
Wrapper for `cgeqp3`.

**Parameters** **a** : input rank-2 array('F') with bounds (m,n)

**Returns** **qr** : rank-2 array('F') with bounds (m,n) and a storage  
**jpvt** : rank-1 array('i') with bounds (n)  
**tau** : rank-1 array('F') with bounds (MIN(m,n))  
**work** : rank-1 array('F') with bounds (MAX(lwork,1))  
**info** : int

**Other Parameters**

**overwrite\_a** : input int, optional

Default: 0  
**lwork** : input int, optional  
 Default: 3\*(n+1)

`scipy.linalg.lapack.zgeqp3(a[, lwork, overwrite_a]) = <fortran object>`  
 Wrapper for zgeqp3.

**Parameters** **a** : input rank-2 array('D') with bounds (m,n)  
**Returns** **qr** : rank-2 array('D') with bounds (m,n) and a storage  
**jpvt** : rank-1 array('i') with bounds (n)  
**tau** : rank-1 array('D') with bounds (MIN(m,n))  
**work** : rank-1 array('D') with bounds (MAX(lwork,1))  
**info** : int

**Other Parameters**  
**overwrite\_a** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: 3\*(n+1)

`scipy.linalg.lapack.sgeqrf(a[, lwork, overwrite_a]) = <fortran object>`  
 Wrapper for sgeqrf.

**Parameters** **a** : input rank-2 array('f') with bounds (m,n)  
**Returns** **qr** : rank-2 array('f') with bounds (m,n) and a storage  
**tau** : rank-1 array('f') with bounds (MIN(m,n))  
**work** : rank-1 array('f') with bounds (MAX(lwork,1))  
**info** : int

**Other Parameters**  
**overwrite\_a** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: 3\*n

`scipy.linalg.lapack.dgeqrf(a[, lwork, overwrite_a]) = <fortran object>`  
 Wrapper for dgeqrf.

**Parameters** **a** : input rank-2 array('d') with bounds (m,n)  
**Returns** **qr** : rank-2 array('d') with bounds (m,n) and a storage  
**tau** : rank-1 array('d') with bounds (MIN(m,n))  
**work** : rank-1 array('d') with bounds (MAX(lwork,1))  
**info** : int

**Other Parameters**  
**overwrite\_a** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: 3\*n

`scipy.linalg.lapack.cgeqrf(a[, lwork, overwrite_a]) = <fortran object>`  
 Wrapper for cgeqrf.

**Parameters** **a** : input rank-2 array('F') with bounds (m,n)  
**Returns** **qr** : rank-2 array('F') with bounds (m,n) and a storage  
**tau** : rank-1 array('F') with bounds (MIN(m,n))  
**work** : rank-1 array('F') with bounds (MAX(lwork,1))  
**info** : int

**Other Parameters**  
**overwrite\_a** : input int, optional  
 Default: 0

**lwork** : input int, optional  
Default: 3\*n

`scipy.linalg.lapack.zgeqrf(a[, lwork, overwrite_a]) = <fortran object>`  
Wrapper for zgeqrf.

**Parameters** **a** : input rank-2 array('D') with bounds (m,n)  
**Returns** **qr** : rank-2 array('D') with bounds (m,n) and a storage  
**tau** : rank-1 array('D') with bounds (MIN(m,n))  
**work** : rank-1 array('D') with bounds (MAX(lwork,1))  
**info** : int

**Other Parameters**

**overwrite\_a** : input int, optional  
Default: 0  
**lwork** : input int, optional  
Default: 3\*n

`scipy.linalg.lapack.sgerqf(a[, lwork, overwrite_a]) = <fortran object>`  
Wrapper for sgerqf.

**Parameters** **a** : input rank-2 array('f') with bounds (m,n)  
**Returns** **qr** : rank-2 array('f') with bounds (m,n) and a storage  
**tau** : rank-1 array('f') with bounds (MIN(m,n))  
**work** : rank-1 array('f') with bounds (MAX(lwork,1))  
**info** : int

**Other Parameters**

**overwrite\_a** : input int, optional  
Default: 0  
**lwork** : input int, optional  
Default: 3\*m

`scipy.linalg.lapack.dgerqf(a[, lwork, overwrite_a]) = <fortran object>`  
Wrapper for dgerqf.

**Parameters** **a** : input rank-2 array('d') with bounds (m,n)  
**Returns** **qr** : rank-2 array('d') with bounds (m,n) and a storage  
**tau** : rank-1 array('d') with bounds (MIN(m,n))  
**work** : rank-1 array('d') with bounds (MAX(lwork,1))  
**info** : int

**Other Parameters**

**overwrite\_a** : input int, optional  
Default: 0  
**lwork** : input int, optional  
Default: 3\*m

`scipy.linalg.lapack.cgerqf(a[, lwork, overwrite_a]) = <fortran object>`  
Wrapper for cgerqf.

**Parameters** **a** : input rank-2 array('F') with bounds (m,n)  
**Returns** **qr** : rank-2 array('F') with bounds (m,n) and a storage  
**tau** : rank-1 array('F') with bounds (MIN(m,n))  
**work** : rank-1 array('F') with bounds (MAX(lwork,1))  
**info** : int

**Other Parameters**

**overwrite\_a** : input int, optional  
Default: 0  
**lwork** : input int, optional  
Default: 3\*m

`scipy.linalg.lapack.zgerqf(a[, lwork, overwrite_a]) = <fortran object>`  
 Wrapper for `zgerqf`.

**Parameters** **a** : input rank-2 array('D') with bounds (m,n)  
**Returns** **qr** : rank-2 array('D') with bounds (m,n) and a storage  
**tau** : rank-1 array('D') with bounds (MIN(m,n))  
**work** : rank-1 array('D') with bounds (MAX(lwork,1))  
**info** : int

**Other Parameters**  
**overwrite\_a** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: 3\*m

`scipy.linalg.lapack.sgesdd(a[, compute_uv, full_matrices, lwork, overwrite_a]) = <fortran object>`

Wrapper for `sgesdd`.

**Parameters** **a** : input rank-2 array('f') with bounds (m,n)  
**Returns** **u** : rank-2 array('f') with bounds (u0,u1)  
**s** : rank-1 array('f') with bounds (minmn)  
**vt** : rank-2 array('f') with bounds (vt0,vt1)  
**info** : int

**Other Parameters**  
**overwrite\_a** : input int, optional  
 Default: 0  
**compute\_uv** : input int, optional  
 Default: 1  
**full\_matrices** : input int, optional  
 Default: 1  
**lwork** : input int, optional  
 Default: (compute\_uv?4\*minmn\*minmn+MAX(m,n)+9\*minmn:MAX(14\*minmn+4,10\*minmn+2+25

`scipy.linalg.lapack.dgesdd(a[, compute_uv, full_matrices, lwork, overwrite_a]) = <fortran object>`

Wrapper for `dgesdd`.

**Parameters** **a** : input rank-2 array('d') with bounds (m,n)  
**Returns** **u** : rank-2 array('d') with bounds (u0,u1)  
**s** : rank-1 array('d') with bounds (minmn)  
**vt** : rank-2 array('d') with bounds (vt0,vt1)  
**info** : int

**Other Parameters**  
**overwrite\_a** : input int, optional  
 Default: 0  
**compute\_uv** : input int, optional  
 Default: 1  
**full\_matrices** : input int, optional  
 Default: 1  
**lwork** : input int, optional  
 Default: (compute\_uv?4\*minmn\*minmn+MAX(m,n)+9\*minmn:MAX(14\*minmn+4,10\*minmn+2+25

`scipy.linalg.lapack.cgesdd(a[, compute_uv, full_matrices, lwork, overwrite_a]) = <fortran object>`

Wrapper for `cgesdd`.

**Parameters** **a** : input rank-2 array('F') with bounds (m,n)

**Returns** **u** : rank-2 array('F') with bounds (u0,u1)  
**s** : rank-1 array('f') with bounds (minmn)  
**vt** : rank-2 array('F') with bounds (vt0,vt1)  
**info** : int

**Other Parameters**

**overwrite\_a** : input int, optional  
 Default: 0  
**compute\_uv** : input int, optional  
 Default: 1  
**full\_matrices** : input int, optional  
 Default: 1  
**lwork** : input int, optional  
 Default: (compute\_uv?2\*minmn\*minmn+MAX(m,n)+2\*minmn:2\*minmn+MAX(m,n))

`scipy.linalg.lapack.zgesdd(a[, compute_uv, full_matrices, lwork, overwrite_a]) = <fortran object>`

Wrapper for zgesdd.

**Parameters** **a** : input rank-2 array('D') with bounds (m,n)  
**Returns** **u** : rank-2 array('D') with bounds (u0,u1)  
**s** : rank-1 array('d') with bounds (minmn)  
**vt** : rank-2 array('D') with bounds (vt0,vt1)  
**info** : int

**Other Parameters**

**overwrite\_a** : input int, optional  
 Default: 0  
**compute\_uv** : input int, optional  
 Default: 1  
**full\_matrices** : input int, optional  
 Default: 1  
**lwork** : input int, optional  
 Default: (compute\_uv?2\*minmn\*minmn+MAX(m,n)+2\*minmn:2\*minmn+MAX(m,n))

`scipy.linalg.lapack.sgesdd_lwork(m, n[, compute_uv, full_matrices]) = <fortran object>`

Wrapper for sgesdd\_lwork.

**Parameters** **m** : input int  
**n** : input int  
**Returns** **work** : float  
**info** : int

**Other Parameters**

**compute\_uv** : input int, optional  
 Default: 1  
**full\_matrices** : input int, optional  
 Default: 1

`scipy.linalg.lapack.dgesdd_lwork(m, n[, compute_uv, full_matrices]) = <fortran object>`

Wrapper for dgesdd\_lwork.

**Parameters** **m** : input int  
**n** : input int  
**Returns** **work** : float  
**info** : int

**Other Parameters**

**compute\_uv** : input int, optional  
 Default: 1  
**full\_matrices** : input int, optional

Default: 1

`scipy.linalg.lapack.cgesdd_lwork` (*m*, *n*[, *compute\_uv*, *full\_matrices*]) = <fortran object>  
 Wrapper for `cgesdd_lwork`.

**Parameters**    **m** : input int  
                   **n** : input int  
**Returns**        **work** : complex  
                   **info** : int  
**Other Parameters**  
                   **compute\_uv** : input int, optional  
                                   Default: 1  
                   **full\_matrices** : input int, optional  
                                   Default: 1

`scipy.linalg.lapack.zgesdd_lwork` (*m*, *n*[, *compute\_uv*, *full\_matrices*]) = <fortran object>  
 Wrapper for `zgesdd_lwork`.

**Parameters**    **m** : input int  
                   **n** : input int  
**Returns**        **work** : complex  
                   **info** : int  
**Other Parameters**  
                   **compute\_uv** : input int, optional  
                                   Default: 1  
                   **full\_matrices** : input int, optional  
                                   Default: 1

`scipy.linalg.lapack.sgesvd` (*a*[, *compute\_uv*, *full\_matrices*, *lwork*, *overwrite\_a*]) = <fortran object>  
 Wrapper for `sgesvd`.

**Parameters**    **a** : input rank-2 array('f') with bounds (m,n)  
**Returns**        **u** : rank-2 array('f') with bounds (u0,u1)  
                   **s** : rank-1 array('f') with bounds (minmn)  
                   **vt** : rank-2 array('f') with bounds (vt0,vt1)  
                   **info** : int  
**Other Parameters**  
                   **overwrite\_a** : input int, optional  
                                   Default: 0  
                   **compute\_uv** : input int, optional  
                                   Default: 1  
                   **full\_matrices** : input int, optional  
                                   Default: 1  
                   **lwork** : input int, optional  
                                   Default: MAX(1,MAX(3\*minmn+MAX(m,n),5\*minmn))

`scipy.linalg.lapack.dgesvd` (*a*[, *compute\_uv*, *full\_matrices*, *lwork*, *overwrite\_a*]) = <fortran object>  
 Wrapper for `dgesvd`.

**Parameters**    **a** : input rank-2 array('d') with bounds (m,n)  
**Returns**        **u** : rank-2 array('d') with bounds (u0,u1)  
                   **s** : rank-1 array('d') with bounds (minmn)  
                   **vt** : rank-2 array('d') with bounds (vt0,vt1)  
                   **info** : int  
**Other Parameters**  
                   **overwrite\_a** : input int, optional

Default: 0  
**compute\_uv** : input int, optional  
 Default: 1  
**full\_matrices** : input int, optional  
 Default: 1  
**lwork** : input int, optional  
 Default: MAX(1,MAX(3\*minmn+MAX(m,n),5\*minmn))

`scipy.linalg.lapack.cgesvd(a[, compute_uv, full_matrices, lwork, overwrite_a]) = <fortran object>`

Wrapper for cgesvd.

**Parameters** **a** : input rank-2 array('F') with bounds (m,n)  
**Returns** **u** : rank-2 array('F') with bounds (u0,u1)  
**s** : rank-1 array('f') with bounds (minmn)  
**vt** : rank-2 array('F') with bounds (vt0,vt1)  
**info** : int

**Other Parameters**

**overwrite\_a** : input int, optional  
 Default: 0  
**compute\_uv** : input int, optional  
 Default: 1  
**full\_matrices** : input int, optional  
 Default: 1  
**lwork** : input int, optional  
 Default: MAX(1,2\*minmn+MAX(m,n))

`scipy.linalg.lapack.zgesvd(a[, compute_uv, full_matrices, lwork, overwrite_a]) = <fortran object>`

Wrapper for zgesvd.

**Parameters** **a** : input rank-2 array('D') with bounds (m,n)  
**Returns** **u** : rank-2 array('D') with bounds (u0,u1)  
**s** : rank-1 array('d') with bounds (minmn)  
**vt** : rank-2 array('D') with bounds (vt0,vt1)  
**info** : int

**Other Parameters**

**overwrite\_a** : input int, optional  
 Default: 0  
**compute\_uv** : input int, optional  
 Default: 1  
**full\_matrices** : input int, optional  
 Default: 1  
**lwork** : input int, optional  
 Default: MAX(1,2\*minmn+MAX(m,n))

`scipy.linalg.lapack.sgesvd_lwork(m, n[, compute_uv, full_matrices]) = <fortran object>`

Wrapper for sgesvd\_lwork.

**Parameters** **m** : input int  
**n** : input int  
**Returns** **work** : float  
**info** : int

**Other Parameters**

**compute\_uv** : input int, optional  
 Default: 1  
**full\_matrices** : input int, optional

Default: 1

`scipy.linalg.lapack.dgesvd_lwork` (*m*, *n*[, *compute\_uv*, *full\_matrices*]) = <fortran object>  
 Wrapper for `dgesvd_lwork`.

**Parameters**    **m** : input int  
                   **n** : input int  
**Returns**        **work** : float  
                   **info** : int

**Other Parameters**  
                   **compute\_uv** : input int, optional  
                                   Default: 1  
                   **full\_matrices** : input int, optional  
                                   Default: 1

`scipy.linalg.lapack.cgesvd_lwork` (*m*, *n*[, *compute\_uv*, *full\_matrices*]) = <fortran object>  
 Wrapper for `cgesvd_lwork`.

**Parameters**    **m** : input int  
                   **n** : input int  
**Returns**        **work** : complex  
                   **info** : int

**Other Parameters**  
                   **compute\_uv** : input int, optional  
                                   Default: 1  
                   **full\_matrices** : input int, optional  
                                   Default: 1

`scipy.linalg.lapack.zgesvd_lwork` (*m*, *n*[, *compute\_uv*, *full\_matrices*]) = <fortran object>  
 Wrapper for `zgesvd_lwork`.

**Parameters**    **m** : input int  
                   **n** : input int  
**Returns**        **work** : complex  
                   **info** : int

**Other Parameters**  
                   **compute\_uv** : input int, optional  
                                   Default: 1  
                   **full\_matrices** : input int, optional  
                                   Default: 1

`scipy.linalg.lapack.sgesv` (*a*, *b*[, *overwrite\_a*, *overwrite\_b*]) = <fortran object>  
 Wrapper for `sgesv`.

**Parameters**    **a** : input rank-2 array('f') with bounds (n,n)  
                   **b** : input rank-2 array('f') with bounds (n,nrhs)  
**Returns**        **lu** : rank-2 array('f') with bounds (n,n) and a storage  
                   **piv** : rank-1 array('i') with bounds (n)  
                   **x** : rank-2 array('f') with bounds (n,nrhs) and b storage  
                   **info** : int

**Other Parameters**  
                   **overwrite\_a** : input int, optional  
                                   Default: 0  
                   **overwrite\_b** : input int, optional  
                                   Default: 0

`scipy.linalg.lapack.dgesv` (*a*, *b*[, *overwrite\_a*, *overwrite\_b*]) = <fortran object>  
 Wrapper for `dgesv`.



**Parameters** **a** : input rank-2 array('d') with bounds (n,n)  
**b** : input rank-2 array('d') with bounds (n,nrhs)  
**Returns** **lu** : rank-2 array('d') with bounds (n,n) and a storage  
**piv** : rank-1 array('i') with bounds (n)  
**x** : rank-2 array('d') with bounds (n,nrhs) and b storage  
**info** : int

**Other Parameters**

**overwrite\_a** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0

`scipy.linalg.lapack.cgesv(a, b[, overwrite_a, overwrite_b]) = <fortran object>`  
 Wrapper for `cgesv`.

**Parameters** **a** : input rank-2 array('F') with bounds (n,n)  
**b** : input rank-2 array('F') with bounds (n,nrhs)  
**Returns** **lu** : rank-2 array('F') with bounds (n,n) and a storage  
**piv** : rank-1 array('i') with bounds (n)  
**x** : rank-2 array('F') with bounds (n,nrhs) and b storage  
**info** : int

**Other Parameters**

**overwrite\_a** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0

`scipy.linalg.lapack.zgesv(a, b[, overwrite_a, overwrite_b]) = <fortran object>`  
 Wrapper for `zgesv`.

**Parameters** **a** : input rank-2 array('D') with bounds (n,n)  
**b** : input rank-2 array('D') with bounds (n,nrhs)  
**Returns** **lu** : rank-2 array('D') with bounds (n,n) and a storage  
**piv** : rank-1 array('i') with bounds (n)  
**x** : rank-2 array('D') with bounds (n,nrhs) and b storage  
**info** : int

**Other Parameters**

**overwrite\_a** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0

`scipy.linalg.lapack.sgetrf(a[, overwrite_a]) = <fortran object>`  
 Wrapper for `sgetrf`.

**Parameters** **a** : input rank-2 array('f') with bounds (m,n)  
**Returns** **lu** : rank-2 array('f') with bounds (m,n) and a storage  
**piv** : rank-1 array('i') with bounds (MIN(m,n))  
**info** : int

**Other Parameters**

**overwrite\_a** : input int, optional  
 Default: 0

`scipy.linalg.lapack.dgetrf(a[, overwrite_a]) = <fortran object>`  
 Wrapper for `dgetrf`.

**Parameters** **a** : input rank-2 array('d') with bounds (m,n)

**Returns** **lu** : rank-2 array('d') with bounds (m,n) and a storage  
**piv** : rank-1 array('i') with bounds (MIN(m,n))  
**info** : int

**Other Parameters**

**overwrite\_a** : input int, optional  
 Default: 0

`scipy.linalg.lapack.cgetrf(a[, overwrite_a]) = <fortran object>`  
 Wrapper for cgetrf.

**Parameters** **a** : input rank-2 array('F') with bounds (m,n)  
**Returns** **lu** : rank-2 array('F') with bounds (m,n) and a storage  
**piv** : rank-1 array('i') with bounds (MIN(m,n))  
**info** : int

**Other Parameters**

**overwrite\_a** : input int, optional  
 Default: 0

`scipy.linalg.lapack.zgetrf(a[, overwrite_a]) = <fortran object>`  
 Wrapper for zgetrf.

**Parameters** **a** : input rank-2 array('D') with bounds (m,n)  
**Returns** **lu** : rank-2 array('D') with bounds (m,n) and a storage  
**piv** : rank-1 array('i') with bounds (MIN(m,n))  
**info** : int

**Other Parameters**

**overwrite\_a** : input int, optional  
 Default: 0

`scipy.linalg.lapack.sgetri(lu, piv[, lwork, overwrite_lu]) = <fortran object>`  
 Wrapper for sgetri.

**Parameters** **lu** : input rank-2 array('f') with bounds (n,n)  
**piv** : input rank-1 array('i') with bounds (n)  
**Returns** **inv\_a** : rank-2 array('f') with bounds (n,n) and lu storage  
**info** : int

**Other Parameters**

**overwrite\_lu** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: 3\*n

`scipy.linalg.lapack.dgetri(lu, piv[, lwork, overwrite_lu]) = <fortran object>`  
 Wrapper for dgetri.

**Parameters** **lu** : input rank-2 array('d') with bounds (n,n)  
**piv** : input rank-1 array('i') with bounds (n)  
**Returns** **inv\_a** : rank-2 array('d') with bounds (n,n) and lu storage  
**info** : int

**Other Parameters**

**overwrite\_lu** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: 3\*n

`scipy.linalg.lapack.cgetri(lu, piv[, lwork, overwrite_lu]) = <fortran object>`  
 Wrapper for cgetri.

**Parameters** **lu** : input rank-2 array('F') with bounds (n,n)  
**piv** : input rank-1 array('i') with bounds (n)  
**Returns** **inv\_a** : rank-2 array('F') with bounds (n,n) and lu storage  
**info** : int  
**Other Parameters**  
**overwrite\_lu** : input int, optional  
Default: 0  
**lwork** : input int, optional  
Default: 3\*n

`scipy.linalg.lapack.zgetri(lu, piv[, lwork, overwrite_lu]) = <fortran object>`  
Wrapper for `zgetri`.

**Parameters** **lu** : input rank-2 array('D') with bounds (n,n)  
**piv** : input rank-1 array('i') with bounds (n)  
**Returns** **inv\_a** : rank-2 array('D') with bounds (n,n) and lu storage  
**info** : int  
**Other Parameters**  
**overwrite\_lu** : input int, optional  
Default: 0  
**lwork** : input int, optional  
Default: 3\*n

`scipy.linalg.lapack.sgetri_lwork(n) = <fortran object>`  
Wrapper for `sgetri_lwork`.

**Parameters** **n** : input int  
**Returns** **work** : float  
**info** : int

`scipy.linalg.lapack.dgetri_lwork(n) = <fortran object>`  
Wrapper for `dgetri_lwork`.

**Parameters** **n** : input int  
**Returns** **work** : float  
**info** : int

`scipy.linalg.lapack.cgetri_lwork(n) = <fortran object>`  
Wrapper for `cgetri_lwork`.

**Parameters** **n** : input int  
**Returns** **work** : complex  
**info** : int

`scipy.linalg.lapack.zgetri_lwork(n) = <fortran object>`  
Wrapper for `zgetri_lwork`.

**Parameters** **n** : input int  
**Returns** **work** : complex  
**info** : int

`scipy.linalg.lapack.sgetrs(lu, piv, b[, trans, overwrite_b]) = <fortran object>`  
Wrapper for `sgetrs`.

**Parameters** **lu** : input rank-2 array('f') with bounds (n,n)  
**piv** : input rank-1 array('i') with bounds (n)  
**b** : input rank-2 array('f') with bounds (n,nrhs)  
**Returns** **x** : rank-2 array('f') with bounds (n,nrhs) and b storage  
**info** : int

**Other Parameters**

**overwrite\_b** : input int, optional  
 Default: 0  
**trans** : input int, optional  
 Default: 0

`scipy.linalg.lapack.dgetrs (lu, piv, b[, trans, overwrite_b]) = <fortran object>`  
 Wrapper for `dgetrs`.

**Parameters** **lu** : input rank-2 array('d') with bounds (n,n)  
**piv** : input rank-1 array('i') with bounds (n)  
**b** : input rank-2 array('d') with bounds (n,nrhs)  
**Returns** **x** : rank-2 array('d') with bounds (n,nrhs) and b storage  
**info** : int

**Other Parameters**

**overwrite\_b** : input int, optional  
 Default: 0  
**trans** : input int, optional  
 Default: 0

`scipy.linalg.lapack.cgetrs (lu, piv, b[, trans, overwrite_b]) = <fortran object>`  
 Wrapper for `cgetrs`.

**Parameters** **lu** : input rank-2 array('F') with bounds (n,n)  
**piv** : input rank-1 array('i') with bounds (n)  
**b** : input rank-2 array('F') with bounds (n,nrhs)  
**Returns** **x** : rank-2 array('F') with bounds (n,nrhs) and b storage  
**info** : int

**Other Parameters**

**overwrite\_b** : input int, optional  
 Default: 0  
**trans** : input int, optional  
 Default: 0

`scipy.linalg.lapack.zgetrs (lu, piv, b[, trans, overwrite_b]) = <fortran object>`  
 Wrapper for `zgetrs`.

**Parameters** **lu** : input rank-2 array('D') with bounds (n,n)  
**piv** : input rank-1 array('i') with bounds (n)  
**b** : input rank-2 array('D') with bounds (n,nrhs)  
**Returns** **x** : rank-2 array('D') with bounds (n,nrhs) and b storage  
**info** : int

**Other Parameters**

**overwrite\_b** : input int, optional  
 Default: 0  
**trans** : input int, optional  
 Default: 0

`scipy.linalg.lapack.sgges (sselect, a, b[, jobvsl, jobvsr, sort_t, ldvsl, ldvsr, lwork, sselect_extra_args, overwrite_a, overwrite_b]) = <fortran object>`

Wrapper for `sgges`.

**Parameters** **sselect** : call-back function  
**a** : input rank-2 array('f') with bounds (lda,\*)  
**b** : input rank-2 array('f') with bounds (ldb,\*)  
**Returns** **a** : rank-2 array('f') with bounds (lda,\*)  
**b** : rank-2 array('f') with bounds (ldb,\*)  
**sdim** : int

**alphar** : rank-1 array('f') with bounds (n)  
**alphai** : rank-1 array('f') with bounds (n)  
**beta** : rank-1 array('f') with bounds (n)  
**vsl** : rank-2 array('f') with bounds (ldvsl,n)  
**vsr** : rank-2 array('f') with bounds (ldvsr,n)  
**work** : rank-1 array('f') with bounds (MAX(lwork,1))  
**info** : int

#### Other Parameters

**jobvsl** : input int, optional  
 Default: 1  
**jobvsr** : input int, optional  
 Default: 1  
**sort\_t** : input int, optional  
 Default: 0  
**sselect\_extra\_args** : input tuple, optional  
 Default: ()  
**overwrite\_a** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0  
**ldvsl** : input int, optional  
 Default: ((jobvsl==1)?n:1)  
**ldvsr** : input int, optional  
 Default: ((jobvsr==1)?n:1)  
**lwork** : input int, optional  
 Default: 8\*n+16

#### Notes

Call-back functions:

```
def sselect(alphar, alphai, beta): return sselect
Required arguments:
  alphar : input float
  alphai : input float
  beta   : input float
Return objects:
  sselect : int
```

`scipy.linalg.lapack.dgges` (*dselect*, *a*, *b*[, *jobvsl*, *jobvsr*, *sort\_t*, *ldvsl*, *ldvsr*, *lwork*, *dselect\_extra\_args*, *overwrite\_a*, *overwrite\_b*]) = <fortran object>

Wrapper for dgges.

**Parameters** **dselect** : call-back function  
**a** : input rank-2 array('d') with bounds (lda,\*)  
**b** : input rank-2 array('d') with bounds (ldb,\*)  
**Returns** **a** : rank-2 array('d') with bounds (lda,\*)  
**b** : rank-2 array('d') with bounds (ldb,\*)  
**sdim** : int  
**alphar** : rank-1 array('d') with bounds (n)  
**alphai** : rank-1 array('d') with bounds (n)  
**beta** : rank-1 array('d') with bounds (n)  
**vsl** : rank-2 array('d') with bounds (ldvsl,n)  
**vsr** : rank-2 array('d') with bounds (ldvsr,n)  
**work** : rank-1 array('d') with bounds (MAX(lwork,1))  
**info** : int

*Other Parameters*

**jobvsl** : input int, optional  
 Default: 1  
**jobvsr** : input int, optional  
 Default: 1  
**sort\_t** : input int, optional  
 Default: 0  
**dselect\_extra\_args** : input tuple, optional  
 Default: ()  
**overwrite\_a** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0  
**ldvsl** : input int, optional  
 Default: ((jobvsl==1)?n:1)  
**ldvsr** : input int, optional  
 Default: ((jobvsr==1)?n:1)  
**lwork** : input int, optional  
 Default: 8\*n+16

*Notes*

Call-back functions:

```
def dselect(alphar, alphai, beta): return dselect
Required arguments:
    alphar : input float
    alphai : input float
    beta : input float
Return objects:
    dselect : int
```

`scipy.linalg.lapack.cgges` (*cselect*, *a*, *b*[, *jobvsl*, *jobvsr*, *sort\_t*, *ldvsl*, *ldvsr*, *lwork*, *cselect\_extra\_args*, *overwrite\_a*, *overwrite\_b*]) = **<fortran object>**

Wrapper for `cgges`.

**Parameters** **cselect** : call-back function  
**a** : input rank-2 array('F') with bounds (lda,\*)  
**b** : input rank-2 array('F') with bounds (ldb,\*)  
**Returns** **a** : rank-2 array('F') with bounds (lda,\*)  
**b** : rank-2 array('F') with bounds (ldb,\*)  
**sdim** : int  
**alpha** : rank-1 array('F') with bounds (n)  
**beta** : rank-1 array('F') with bounds (n)  
**vsl** : rank-2 array('F') with bounds (ldvsl,n)  
**vsr** : rank-2 array('F') with bounds (ldvsr,n)  
**work** : rank-1 array('F') with bounds (MAX(lwork,1))  
**info** : int

*Other Parameters*

**jobvsl** : input int, optional  
 Default: 1  
**jobvsr** : input int, optional  
 Default: 1  
**sort\_t** : input int, optional  
 Default: 0  
**cselect\_extra\_args** : input tuple, optional

Default: ()  
**overwrite\_a** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0  
**ldvsl** : input int, optional  
 Default: ((jobvsl==1)?n:1)  
**ldvsr** : input int, optional  
 Default: ((jobvsr==1)?n:1)  
**lwork** : input int, optional  
 Default: 2\*n

**Notes**

Call-back functions:

```
def cselect(alpha,beta): return cselect
Required arguments:
    alpha : input complex
    beta  : input complex
Return objects:
    cselect : int
```

`scipy.linalg.lapack.zgges` (*zselect*, *a*, *b*[, *jobvsl*, *jobvsr*, *sort\_t*, *ldvsl*, *ldvsr*, *lwork*, *zselect\_extra\_args*, *overwrite\_a*, *overwrite\_b*]) = <fortran object>

Wrapper for zgges.

**Parameters** **zselect** : call-back function  
**a** : input rank-2 array('D') with bounds (lda,\*)  
**b** : input rank-2 array('D') with bounds (ldb,\*)  
**Returns** **a** : rank-2 array('D') with bounds (lda,\*)  
**b** : rank-2 array('D') with bounds (ldb,\*)  
**sdim** : int  
**alpha** : rank-1 array('D') with bounds (n)  
**beta** : rank-1 array('D') with bounds (n)  
**vsl** : rank-2 array('D') with bounds (ldvsl,n)  
**vsr** : rank-2 array('D') with bounds (ldvsr,n)  
**work** : rank-1 array('D') with bounds (MAX(lwork,1))  
**info** : int

**Other Parameters**

**jobvsl** : input int, optional  
 Default: 1  
**jobvsr** : input int, optional  
 Default: 1  
**sort\_t** : input int, optional  
 Default: 0  
**zselect\_extra\_args** : input tuple, optional  
 Default: ()  
**overwrite\_a** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0  
**ldvsl** : input int, optional  
 Default: ((jobvsl==1)?n:1)  
**ldvsr** : input int, optional  
 Default: ((jobvsr==1)?n:1)

**lwork** : input int, optional  
 Default: 2\*n

*Notes*

Call-back functions:

```
def zselect(alpha,beta): return zselect
Required arguments:
    alpha : input complex
    beta  : input complex
Return objects:
    zselect : int
```

`scipy.linalg.lapack.sggev(a, b[, compute_vl, compute_vr, lwork, overwrite_a, overwrite_b]) =`  
**<fortran object>**

Wrapper for sggev.

**Parameters** **a** : input rank-2 array('f') with bounds (n,n)  
**b** : input rank-2 array('f') with bounds (n,n)  
**Returns** **alphar** : rank-1 array('f') with bounds (n)  
**alphai** : rank-1 array('f') with bounds (n)  
**beta** : rank-1 array('f') with bounds (n)  
**vl** : rank-2 array('f') with bounds (ldvl,n)  
**vr** : rank-2 array('f') with bounds (ldvr,n)  
**work** : rank-1 array('f') with bounds (MAX(lwork,1))  
**info** : int

*Other Parameters*

**compute\_vl** : input int, optional  
 Default: 1  
**compute\_vr** : input int, optional  
 Default: 1  
**overwrite\_a** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: 8\*n

`scipy.linalg.lapack.dggev(a, b[, compute_vl, compute_vr, lwork, overwrite_a, overwrite_b]) =`  
**<fortran object>**

Wrapper for dggev.

**Parameters** **a** : input rank-2 array('d') with bounds (n,n)  
**b** : input rank-2 array('d') with bounds (n,n)  
**Returns** **alphar** : rank-1 array('d') with bounds (n)  
**alphai** : rank-1 array('d') with bounds (n)  
**beta** : rank-1 array('d') with bounds (n)  
**vl** : rank-2 array('d') with bounds (ldvl,n)  
**vr** : rank-2 array('d') with bounds (ldvr,n)  
**work** : rank-1 array('d') with bounds (MAX(lwork,1))  
**info** : int

*Other Parameters*

**compute\_vl** : input int, optional  
 Default: 1  
**compute\_vr** : input int, optional  
 Default: 1



**overwrite\_a** : input int, optional  
Default: 0  
**overwrite\_b** : input int, optional  
Default: 0  
**lwork** : input int, optional  
Default: 8\*n

`scipy.linalg.lapack.cggev(a, b[, compute_vl, compute_vr, lwork, overwrite_a, overwrite_b]) =`  
**<fortran object>**

Wrapper for cggev.

**Parameters** **a** : input rank-2 array('F') with bounds (n,n)  
**b** : input rank-2 array('F') with bounds (n,n)  
**Returns** **alpha** : rank-1 array('F') with bounds (n)  
**beta** : rank-1 array('F') with bounds (n)  
**vl** : rank-2 array('F') with bounds (ldvl,n)  
**vr** : rank-2 array('F') with bounds (ldvr,n)  
**work** : rank-1 array('F') with bounds (MAX(lwork,1))  
**info** : int

**Other Parameters**

**compute\_vl** : input int, optional  
Default: 1  
**compute\_vr** : input int, optional  
Default: 1  
**overwrite\_a** : input int, optional  
Default: 0  
**overwrite\_b** : input int, optional  
Default: 0  
**lwork** : input int, optional  
Default: 2\*n

`scipy.linalg.lapack.zggeev(a, b[, compute_vl, compute_vr, lwork, overwrite_a, overwrite_b]) =`  
**<fortran object>**

Wrapper for zggev.

**Parameters** **a** : input rank-2 array('D') with bounds (n,n)  
**b** : input rank-2 array('D') with bounds (n,n)  
**Returns** **alpha** : rank-1 array('D') with bounds (n)  
**beta** : rank-1 array('D') with bounds (n)  
**vl** : rank-2 array('D') with bounds (ldvl,n)  
**vr** : rank-2 array('D') with bounds (ldvr,n)  
**work** : rank-1 array('D') with bounds (MAX(lwork,1))  
**info** : int

**Other Parameters**

**compute\_vl** : input int, optional  
Default: 1  
**compute\_vr** : input int, optional  
Default: 1  
**overwrite\_a** : input int, optional  
Default: 0  
**overwrite\_b** : input int, optional  
Default: 0  
**lwork** : input int, optional  
Default: 2\*n

`scipy.linalg.lapack.chbevd` (*ab*[, *compute\_v*, *lower*, *ldab*, *lrwork*, *liwork*, *overwrite\_ab* ]) = **<fortran object>**

Wrapper for `chbevd`.

**Parameters** *ab* : input rank-2 array('F') with bounds (*ldab*,\*)

**Returns** *w* : rank-1 array('f') with bounds (*n*)

*z* : rank-2 array('F') with bounds (*ldz*,*ldz*)

*info* : int

**Other Parameters**

*overwrite\_ab* : input int, optional

Default: 1

*compute\_v* : input int, optional

Default: 1

*lower* : input int, optional

Default: 0

*ldab* : input int, optional

Default: `shape(ab,0)`

*lrwork* : input int, optional

Default: `(compute_v?1+5*n+2*n*n:n)`

*liwork* : input int, optional

Default: `(compute_v?3+5*n:1)`

`scipy.linalg.lapack.zhbevd` (*ab*[, *compute\_v*, *lower*, *ldab*, *lrwork*, *liwork*, *overwrite\_ab* ]) = **<fortran object>**

Wrapper for `zhbevd`.

**Parameters** *ab* : input rank-2 array('D') with bounds (*ldab*,\*)

**Returns** *w* : rank-1 array('d') with bounds (*n*)

*z* : rank-2 array('D') with bounds (*ldz*,*ldz*)

*info* : int

**Other Parameters**

*overwrite\_ab* : input int, optional

Default: 1

*compute\_v* : input int, optional

Default: 1

*lower* : input int, optional

Default: 0

*ldab* : input int, optional

Default: `shape(ab,0)`

*lrwork* : input int, optional

Default: `(compute_v?1+5*n+2*n*n:n)`

*liwork* : input int, optional

Default: `(compute_v?3+5*n:1)`

`scipy.linalg.lapack.chbevx` (*ab*, *vl*, *vu*, *il*, *iu*[, *ldab*, *compute\_v*, *range*, *lower*, *abstol*, *mmax*, *overwrite\_ab* ]) = **<fortran object>**

Wrapper for `chbevx`.

**Parameters** *ab* : input rank-2 array('F') with bounds (*ldab*,\*)

*vl* : input float

*vu* : input float

*il* : input int

*iu* : input int

**Returns** *w* : rank-1 array('f') with bounds (*n*)

*z* : rank-2 array('F') with bounds (*ldz*,*mmax*)

*m* : int

*ifail* : rank-1 array('i') with bounds (`((compute_v?n:1))`)

**info** : int**Other Parameters****overwrite\_ab** : input int, optional

Default: 1

**ldab** : input int, optional

Default: shape(ab,0)

**compute\_v** : input int, optional

Default: 1

**range** : input int, optional

Default: 0

**lower** : input int, optional

Default: 0

**abstol** : input float, optional

Default: 0.0

**mmax** : input int, optional

Default: (compute\_v?(range==2?(iu-il+1):n):1)

scipy.linalg.lapack.**zhbevx**(*ab*, *vl*, *vu*, *il*, *iu*[, *ldab*, *compute\_v*, *range*, *lower*, *abstol*, *mmax*, *overwrite\_ab*]) = <fortran object>

Wrapper for zhbevx.

**Parameters** **ab** : input rank-2 array('D') with bounds (ldab,\*)**vl** : input float**vu** : input float**il** : input int**iu** : input int**Returns** **w** : rank-1 array('d') with bounds (n)**z** : rank-2 array('D') with bounds (ldz,mmax)**m** : int**ifail** : rank-1 array('i') with bounds ((compute\_v?n:1))**info** : int**Other Parameters****overwrite\_ab** : input int, optional

Default: 1

**ldab** : input int, optional

Default: shape(ab,0)

**compute\_v** : input int, optional

Default: 1

**range** : input int, optional

Default: 0

**lower** : input int, optional

Default: 0

**abstol** : input float, optional

Default: 0.0

**mmax** : input int, optional

Default: (compute\_v?(range==2?(iu-il+1):n):1)

scipy.linalg.lapack.**cheev**(*a*[, *compute\_v*, *lower*, *lwork*, *overwrite\_a*]) = <fortran object>

Wrapper for cheev.

**Parameters** **a** : input rank-2 array('F') with bounds (n,n)**Returns** **w** : rank-1 array('f') with bounds (n)**v** : rank-2 array('F') with bounds (n,n) and a storage**info** : int**Other Parameters****compute\_v** : input int, optional

Default: 1  
**lower** : input int, optional  
 Default: 0  
**overwrite\_a** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: 2\*n-1

`scipy.linalg.lapack.zheev` ( $a$  [, *compute\_v*, *lower*, *lwork*, *overwrite\_a* ]) = <fortran object>  
 Wrapper for zheev.

**Parameters**    **a** : input rank-2 array('D') with bounds (n,n)  
**Returns**        **w** : rank-1 array('d') with bounds (n)  
                  **v** : rank-2 array('D') with bounds (n,n) and a storage  
                  **info** : int

**Other Parameters**  
**compute\_v** : input int, optional  
 Default: 1  
**lower** : input int, optional  
 Default: 0  
**overwrite\_a** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: 2\*n-1

`scipy.linalg.lapack.cheevd` ( $a$  [, *compute\_v*, *lower*, *lwork*, *overwrite\_a* ]) = <fortran object>  
 Wrapper for cheevd.

**Parameters**    **a** : input rank-2 array('F') with bounds (n,n)  
**Returns**        **w** : rank-1 array('f') with bounds (n)  
                  **v** : rank-2 array('F') with bounds (n,n) and a storage  
                  **info** : int

**Other Parameters**  
**compute\_v** : input int, optional  
 Default: 1  
**lower** : input int, optional  
 Default: 0  
**overwrite\_a** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: (compute\_v?2\*n+n\*n:n+1)

`scipy.linalg.lapack.zheevd` ( $a$  [, *compute\_v*, *lower*, *lwork*, *overwrite\_a* ]) = <fortran object>  
 Wrapper for zheevd.

**Parameters**    **a** : input rank-2 array('D') with bounds (n,n)  
**Returns**        **w** : rank-1 array('d') with bounds (n)  
                  **v** : rank-2 array('D') with bounds (n,n) and a storage  
                  **info** : int

**Other Parameters**  
**compute\_v** : input int, optional  
 Default: 1  
**lower** : input int, optional  
 Default: 0  
**overwrite\_a** : input int, optional  
 Default: 0  
**lwork** : input int, optional

Default: (compute\_v?2\*n+n\*n:n+1)

`scipy.linalg.lapack.cheevr` ( $a$ [,  $jobz$ ,  $range$ ,  $uplo$ ,  $il$ ,  $iu$ ,  $lwork$ ,  $overwrite_a$ ]) = <fortran object>  
 Wrapper for cheevr.

**Parameters**  $a$  : input rank-2 array('F') with bounds (n,n)  
**Returns**  $w$  : rank-1 array('f') with bounds (n)  
 $z$  : rank-2 array('F') with bounds (n,m)  
 $info$  : int

**Other Parameters**

$jobz$  : input string(len=1), optional  
 Default: 'V'  
 $range$  : input string(len=1), optional  
 Default: 'A'  
 $uplo$  : input string(len=1), optional  
 Default: 'L'  
 $overwrite_a$  : input int, optional  
 Default: 0  
 $il$  : input int, optional  
 Default: 1  
 $iu$  : input int, optional  
 Default: n  
 $lwork$  : input int, optional  
 Default: 18\*n

`scipy.linalg.lapack.zheevr` ( $a$ [,  $jobz$ ,  $range$ ,  $uplo$ ,  $il$ ,  $iu$ ,  $lwork$ ,  $overwrite_a$ ]) = <fortran object>  
 Wrapper for zheevr.

**Parameters**  $a$  : input rank-2 array('D') with bounds (n,n)  
**Returns**  $w$  : rank-1 array('d') with bounds (n)  
 $z$  : rank-2 array('D') with bounds (n,m)  
 $info$  : int

**Other Parameters**

$jobz$  : input string(len=1), optional  
 Default: 'V'  
 $range$  : input string(len=1), optional  
 Default: 'A'  
 $uplo$  : input string(len=1), optional  
 Default: 'L'  
 $overwrite_a$  : input int, optional  
 Default: 0  
 $il$  : input int, optional  
 Default: 1  
 $iu$  : input int, optional  
 Default: n  
 $lwork$  : input int, optional  
 Default: 18\*n

`scipy.linalg.lapack.chegv` ( $a$ ,  $b$ [,  $itype$ ,  $jobz$ ,  $uplo$ ,  $overwrite_a$ ,  $overwrite_b$ ]) = <fortran object>  
 Wrapper for chegv.

**Parameters**  $a$  : input rank-2 array('F') with bounds (n,n)  
 $b$  : input rank-2 array('F') with bounds (n,n)  
**Returns**  $a$  : rank-2 array('F') with bounds (n,n)  
 $w$  : rank-1 array('f') with bounds (n)  
 $info$  : int

*Other Parameters*

**itype** : input int, optional  
 Default: 1  
**jobz** : input string(len=1), optional  
 Default: 'V'  
**uplo** : input string(len=1), optional  
 Default: 'L'  
**overwrite\_a** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0

`scipy.linalg.lapack.zhegv(a, b[, itype, jobz, uplo, overwrite_a, overwrite_b]) = <fortran object>`  
 Wrapper for zhegv.

**Parameters** **a** : input rank-2 array('D') with bounds (n,n)  
**b** : input rank-2 array('D') with bounds (n,n)  
**Returns** **a** : rank-2 array('D') with bounds (n,n)  
**w** : rank-1 array('d') with bounds (n)  
**info** : int

*Other Parameters*

**itype** : input int, optional  
 Default: 1  
**jobz** : input string(len=1), optional  
 Default: 'V'  
**uplo** : input string(len=1), optional  
 Default: 'L'  
**overwrite\_a** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0

`scipy.linalg.lapack.chegvd(a, b[, itype, jobz, uplo, lwork, overwrite_a, overwrite_b]) = <fortran object>`

Wrapper for chegvd.

**Parameters** **a** : input rank-2 array('F') with bounds (n,n)  
**b** : input rank-2 array('F') with bounds (n,n)  
**Returns** **a** : rank-2 array('F') with bounds (n,n)  
**w** : rank-1 array('f') with bounds (n)  
**info** : int

*Other Parameters*

**itype** : input int, optional  
 Default: 1  
**jobz** : input string(len=1), optional  
 Default: 'V'  
**uplo** : input string(len=1), optional  
 Default: 'L'  
**overwrite\_a** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: 2\*n+n\*n

`scipy.linalg.lapack.zhegvd(a, b[, itype, jobz, uplo, lwork, overwrite_a, overwrite_b]) = <fortran object>`

Wrapper for zhegvd.

**Parameters** **a** : input rank-2 array('D') with bounds (n,n)  
**b** : input rank-2 array('D') with bounds (n,n)

**Returns** **a** : rank-2 array('D') with bounds (n,n)  
**w** : rank-1 array('d') with bounds (n)  
**info** : int

**Other Parameters**  
**itype** : input int, optional  
Default: 1  
**jobz** : input string(len=1), optional  
Default: 'V'  
**uplo** : input string(len=1), optional  
Default: 'L'  
**overwrite\_a** : input int, optional  
Default: 0  
**overwrite\_b** : input int, optional  
Default: 0  
**lwork** : input int, optional  
Default: 2\*n+n\*n

`scipy.linalg.lapack.chegvx(a, b, iu[, itype, jobz, uplo, il, lwork, overwrite_a, overwrite_b]) = <fortran object>`

Wrapper for chegvx.

**Parameters** **a** : input rank-2 array('F') with bounds (n,n)  
**b** : input rank-2 array('F') with bounds (n,n)  
**iu** : input int

**Returns** **w** : rank-1 array('f') with bounds (n)  
**z** : rank-2 array('F') with bounds (n,m)  
**ifail** : rank-1 array('i') with bounds (n)  
**info** : int

**Other Parameters**  
**itype** : input int, optional  
Default: 1  
**jobz** : input string(len=1), optional  
Default: 'V'  
**uplo** : input string(len=1), optional  
Default: 'L'  
**overwrite\_a** : input int, optional  
Default: 0  
**overwrite\_b** : input int, optional  
Default: 0  
**il** : input int, optional  
Default: 1  
**lwork** : input int, optional  
Default: 18\*n-1

`scipy.linalg.lapack.zhegvx(a, b, iu[, itype, jobz, uplo, il, lwork, overwrite_a, overwrite_b]) = <fortran object>`

Wrapper for zhegvx.

**Parameters** **a** : input rank-2 array('D') with bounds (n,n)  
**b** : input rank-2 array('D') with bounds (n,n)  
**iu** : input int

**Returns**     **w** : rank-1 array('d') with bounds (n)  
               **z** : rank-2 array('D') with bounds (n,m)  
               **ifail** : rank-1 array('i') with bounds (n)  
               **info** : int

**Other Parameters**

**itype** : input int, optional  
           Default: 1  
**jobz** : input string(len=1), optional  
           Default: 'V'  
**uplo** : input string(len=1), optional  
           Default: 'L'  
**overwrite\_a** : input int, optional  
           Default: 0  
**overwrite\_b** : input int, optional  
           Default: 0  
**il** : input int, optional  
       Default: 1  
**lwork** : input int, optional  
       Default: 18\*n-1

`scipy.linalg.lapack.slarf(v, tau, c, work[, side, incv, overwrite_c]) = <fortran object>`  
 Wrapper for slarf.

**Parameters**   **v** : input rank-1 array('f') with bounds (\*)  
                   **tau** : input float  
                   **c** : input rank-2 array('f') with bounds (m,n)  
                   **work** : input rank-1 array('f') with bounds (\*)

**Returns**       **c** : rank-2 array('f') with bounds (m,n)

**Other Parameters**

**side** : input string(len=1), optional  
           Default: 'L'  
**incv** : input int, optional  
           Default: 1  
**overwrite\_c** : input int, optional  
           Default: 0

`scipy.linalg.lapack.dlarf(v, tau, c, work[, side, incv, overwrite_c]) = <fortran object>`  
 Wrapper for dlarf.

**Parameters**   **v** : input rank-1 array('d') with bounds (\*)  
                   **tau** : input float  
                   **c** : input rank-2 array('d') with bounds (m,n)  
                   **work** : input rank-1 array('d') with bounds (\*)

**Returns**       **c** : rank-2 array('d') with bounds (m,n)

**Other Parameters**

**side** : input string(len=1), optional  
           Default: 'L'  
**incv** : input int, optional  
           Default: 1  
**overwrite\_c** : input int, optional  
           Default: 0

`scipy.linalg.lapack.clarf(v, tau, c, work[, side, incv, overwrite_c]) = <fortran object>`  
 Wrapper for clarf.



**Parameters** **v** : input rank-1 array('F') with bounds (\*)  
**tau** : input complex  
**c** : input rank-2 array('F') with bounds (m,n)  
**work** : input rank-1 array('F') with bounds (\*)

**Returns** **c** : rank-2 array('F') with bounds (m,n)

**Other Parameters**  
**side** : input string(len=1), optional  
Default: 'L'  
**incv** : input int, optional  
Default: 1  
**overwrite\_c** : input int, optional  
Default: 0

`scipy.linalg.lapack.zlarf(v, tau, c, work[, side, incv, overwrite_c]) = <fortran object>`  
Wrapper for zlarf.

**Parameters** **v** : input rank-1 array('D') with bounds (\*)  
**tau** : input complex  
**c** : input rank-2 array('D') with bounds (m,n)  
**work** : input rank-1 array('D') with bounds (\*)

**Returns** **c** : rank-2 array('D') with bounds (m,n)

**Other Parameters**  
**side** : input string(len=1), optional  
Default: 'L'  
**incv** : input int, optional  
Default: 1  
**overwrite\_c** : input int, optional  
Default: 0

`scipy.linalg.lapack.slarfg(n, alpha, x[, incx, overwrite_x]) = <fortran object>`  
Wrapper for slarfg.

**Parameters** **n** : input int  
**alpha** : input float  
**x** : input rank-1 array('f') with bounds (\*)

**Returns** **alpha** : float  
**x** : rank-1 array('f') with bounds (\*)  
**tau** : float

**Other Parameters**  
**overwrite\_x** : input int, optional  
Default: 0  
**incx** : input int, optional  
Default: 1

`scipy.linalg.lapack.dlarfg(n, alpha, x[, incx, overwrite_x]) = <fortran object>`  
Wrapper for dlarfg.

**Parameters** **n** : input int  
**alpha** : input float  
**x** : input rank-1 array('d') with bounds (\*)

**Returns** **alpha** : float  
**x** : rank-1 array('d') with bounds (\*)  
**tau** : float

**Other Parameters**  
**overwrite\_x** : input int, optional  
Default: 0  
**incx** : input int, optional

Default: 1

`scipy.linalg.lapack.clarfg(n, alpha, x[, incx, overwrite_x]) = <fortran object>`  
 Wrapper for clarfg.

**Parameters** **n** : input int  
**alpha** : input complex  
**x** : input rank-1 array('F') with bounds (\*)

**Returns** **alpha** : complex  
**x** : rank-1 array('F') with bounds (\*)  
**tau** : complex

**Other Parameters**  
**overwrite\_x** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1

`scipy.linalg.lapack.zlarfg(n, alpha, x[, incx, overwrite_x]) = <fortran object>`  
 Wrapper for zlarfg.

**Parameters** **n** : input int  
**alpha** : input complex  
**x** : input rank-1 array('D') with bounds (\*)

**Returns** **alpha** : complex  
**x** : rank-1 array('D') with bounds (\*)  
**tau** : complex

**Other Parameters**  
**overwrite\_x** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1

`scipy.linalg.lapack.slargt(f, g) = <fortran object>`  
 Wrapper for slartg.

**Parameters** **f** : input float  
**g** : input float

**Returns** **cs** : float  
**sn** : float  
**r** : float

`scipy.linalg.lapack.dlargt(f, g) = <fortran object>`  
 Wrapper for dlartg.

**Parameters** **f** : input float  
**g** : input float

**Returns** **cs** : float  
**sn** : float  
**r** : float

`scipy.linalg.lapack.clartg(f, g) = <fortran object>`  
 Wrapper for clartg.

**Parameters** **f** : input complex  
**g** : input complex

**Returns** **cs** : float  
**sn** : complex  
**r** : complex

`scipy.linalg.lapack.zlartg(f, g) = <fortran object>`

Wrapper for `zlartg`.

**Parameters** **f** : input complex  
**g** : input complex

**Returns** **cs** : float  
**sn** : complex  
**r** : complex

`scipy.linalg.lapack.slasd4(i, d, z[, rho]) = <fortran object>`

Wrapper for `slasd4`.

**Parameters** **i** : input int  
**d** : input rank-1 array('f') with bounds (n)  
**z** : input rank-1 array('f') with bounds (n)

**Returns** **delta** : rank-1 array('f') with bounds (n)  
**sigma** : float  
**work** : rank-1 array('f') with bounds (n)  
**info** : int

**Other Parameters**

**rho** : input float, optional  
 Default: 1.0

`scipy.linalg.lapack.dlasd4(i, d, z[, rho]) = <fortran object>`

Wrapper for `dlasd4`.

**Parameters** **i** : input int  
**d** : input rank-1 array('d') with bounds (n)  
**z** : input rank-1 array('d') with bounds (n)

**Returns** **delta** : rank-1 array('d') with bounds (n)  
**sigma** : float  
**work** : rank-1 array('d') with bounds (n)  
**info** : int

**Other Parameters**

**rho** : input float, optional  
 Default: 1.0

`scipy.linalg.lapack.slaswp(a, piv[, k1, k2, off, inc, overwrite_a]) = <fortran object>`

Wrapper for `slaswp`.

**Parameters** **a** : input rank-2 array('f') with bounds (nrows,n)  
**piv** : input rank-1 array('i') with bounds (\*)

**Returns** **a** : rank-2 array('f') with bounds (nrows,n)

**Other Parameters**

**overwrite\_a** : input int, optional  
 Default: 0  
**k1** : input int, optional  
 Default: 0  
**k2** : input int, optional  
 Default: len(piv)-1  
**off** : input int, optional  
 Default: 0  
**inc** : input int, optional  
 Default: 1

`scipy.linalg.lapack.dlaswp(a, piv[, k1, k2, off, inc, overwrite_a]) = <fortran object>`

Wrapper for `dlaswp`.

**Parameters** **a** : input rank-2 array('d') with bounds (nrows,n)  
**piv** : input rank-1 array('i') with bounds (\*)

**Returns** **a** : rank-2 array('d') with bounds (nrows,n)

**Other Parameters**

**overwrite\_a** : input int, optional

Default: 0

**k1** : input int, optional

Default: 0

**k2** : input int, optional

Default: len(piv)-1

**off** : input int, optional

Default: 0

**inc** : input int, optional

Default: 1

`scipy.linalg.lapack.claswp(a, piv[, k1, k2, off, inc, overwrite_a]) = <fortran object>`

Wrapper for claswp.

**Parameters** **a** : input rank-2 array('F') with bounds (nrows,n)

**piv** : input rank-1 array('i') with bounds (\*)

**Returns** **a** : rank-2 array('F') with bounds (nrows,n)

**Other Parameters**

**overwrite\_a** : input int, optional

Default: 0

**k1** : input int, optional

Default: 0

**k2** : input int, optional

Default: len(piv)-1

**off** : input int, optional

Default: 0

**inc** : input int, optional

Default: 1

`scipy.linalg.lapack.zlaswp(a, piv[, k1, k2, off, inc, overwrite_a]) = <fortran object>`

Wrapper for zlaswp.

**Parameters** **a** : input rank-2 array('D') with bounds (nrows,n)

**piv** : input rank-1 array('i') with bounds (\*)

**Returns** **a** : rank-2 array('D') with bounds (nrows,n)

**Other Parameters**

**overwrite\_a** : input int, optional

Default: 0

**k1** : input int, optional

Default: 0

**k2** : input int, optional

Default: len(piv)-1

**off** : input int, optional

Default: 0

**inc** : input int, optional

Default: 1

`scipy.linalg.lapack.slauum(c[, lower, overwrite_c]) = <fortran object>`

Wrapper for slauum.

**Parameters** **c** : input rank-2 array('f') with bounds (n,n)

**Returns** **a** : rank-2 array('f') with bounds (n,n) and c storage

**info** : int

**Other Parameters**

**overwrite\_c** : input int, optional  
Default: 0  
**lower** : input int, optional  
Default: 0

`scipy.linalg.lapack.dlauum(c[, lower, overwrite_c]) = <fortran object>`  
Wrapper for `dlauum`.

**Parameters** **c** : input rank-2 array('d') with bounds (n,n)  
**Returns** **a** : rank-2 array('d') with bounds (n,n) and c storage  
**info** : int

**Other Parameters**

**overwrite\_c** : input int, optional  
Default: 0  
**lower** : input int, optional  
Default: 0

`scipy.linalg.lapack.claum(c[, lower, overwrite_c]) = <fortran object>`  
Wrapper for `claum`.

**Parameters** **c** : input rank-2 array('F') with bounds (n,n)  
**Returns** **a** : rank-2 array('F') with bounds (n,n) and c storage  
**info** : int

**Other Parameters**

**overwrite\_c** : input int, optional  
Default: 0  
**lower** : input int, optional  
Default: 0

`scipy.linalg.lapack.zlauum(c[, lower, overwrite_c]) = <fortran object>`  
Wrapper for `zlauum`.

**Parameters** **c** : input rank-2 array('D') with bounds (n,n)  
**Returns** **a** : rank-2 array('D') with bounds (n,n) and c storage  
**info** : int

**Other Parameters**

**overwrite\_c** : input int, optional  
Default: 0  
**lower** : input int, optional  
Default: 0

`scipy.linalg.lapack.spbsv(ab, b[, lower, ldab, overwrite_ab, overwrite_b]) = <fortran object>`  
Wrapper for `spbsv`.

**Parameters** **ab** : input rank-2 array('f') with bounds (ldab,n)  
**b** : input rank-2 array('f') with bounds (ldb,nrhs)  
**Returns** **c** : rank-2 array('f') with bounds (ldab,n) and ab storage  
**x** : rank-2 array('f') with bounds (ldb,nrhs) and b storage  
**info** : int

**Other Parameters**

**lower** : input int, optional  
Default: 0  
**overwrite\_ab** : input int, optional  
Default: 0  
**ldab** : input int, optional  
Default: `shape(ab,0)`  
**overwrite\_b** : input int, optional

Default: 0

`scipy.linalg.lapack.dpbsv(ab, b[, lower, ldab, overwrite_ab, overwrite_b]) = <fortran object>`  
 Wrapper for `dpbsv`.

**Parameters** **ab** : input rank-2 array('d') with bounds (ldab,n)  
**b** : input rank-2 array('d') with bounds (ldb,nrhs)  
**Returns** **c** : rank-2 array('d') with bounds (ldab,n) and ab storage  
**x** : rank-2 array('d') with bounds (ldb,nrhs) and b storage  
**info** : int

**Other Parameters**  
**lower** : input int, optional  
 Default: 0  
**overwrite\_ab** : input int, optional  
 Default: 0  
**ldab** : input int, optional  
 Default: `shape(ab,0)`  
**overwrite\_b** : input int, optional  
 Default: 0

`scipy.linalg.lapack.cpbsv(ab, b[, lower, ldab, overwrite_ab, overwrite_b]) = <fortran object>`  
 Wrapper for `cpbsv`.

**Parameters** **ab** : input rank-2 array('F') with bounds (ldab,n)  
**b** : input rank-2 array('F') with bounds (ldb,nrhs)  
**Returns** **c** : rank-2 array('F') with bounds (ldab,n) and ab storage  
**x** : rank-2 array('F') with bounds (ldb,nrhs) and b storage  
**info** : int

**Other Parameters**  
**lower** : input int, optional  
 Default: 0  
**overwrite\_ab** : input int, optional  
 Default: 0  
**ldab** : input int, optional  
 Default: `shape(ab,0)`  
**overwrite\_b** : input int, optional  
 Default: 0

`scipy.linalg.lapack.zpbsv(ab, b[, lower, ldab, overwrite_ab, overwrite_b]) = <fortran object>`  
 Wrapper for `zpbsv`.

**Parameters** **ab** : input rank-2 array('D') with bounds (ldab,n)  
**b** : input rank-2 array('D') with bounds (ldb,nrhs)  
**Returns** **c** : rank-2 array('D') with bounds (ldab,n) and ab storage  
**x** : rank-2 array('D') with bounds (ldb,nrhs) and b storage  
**info** : int

**Other Parameters**  
**lower** : input int, optional  
 Default: 0  
**overwrite\_ab** : input int, optional  
 Default: 0  
**ldab** : input int, optional  
 Default: `shape(ab,0)`  
**overwrite\_b** : input int, optional  
 Default: 0

`scipy.linalg.lapack.spbtrf(ab[, lower, ldab, overwrite_ab]) = <fortran object>`  
 Wrapper for `spbtrf`.

**Parameters** **ab** : input rank-2 array('f') with bounds (ldab,n)  
**Returns** **c** : rank-2 array('f') with bounds (ldab,n) and ab storage  
**info** : int  
**Other Parameters**  
**lower** : input int, optional  
 Default: 0  
**overwrite\_ab** : input int, optional  
 Default: 0  
**ldab** : input int, optional  
 Default: shape(ab,0)

`scipy.linalg.lapack.dpbtrf(ab[, lower, ldab, overwrite_ab]) = <fortran object>`  
 Wrapper for `dpbtrf`.

**Parameters** **ab** : input rank-2 array('d') with bounds (ldab,n)  
**Returns** **c** : rank-2 array('d') with bounds (ldab,n) and ab storage  
**info** : int  
**Other Parameters**  
**lower** : input int, optional  
 Default: 0  
**overwrite\_ab** : input int, optional  
 Default: 0  
**ldab** : input int, optional  
 Default: shape(ab,0)

`scipy.linalg.lapack.cpbtrf(ab[, lower, ldab, overwrite_ab]) = <fortran object>`  
 Wrapper for `cpbtrf`.

**Parameters** **ab** : input rank-2 array('F') with bounds (ldab,n)  
**Returns** **c** : rank-2 array('F') with bounds (ldab,n) and ab storage  
**info** : int  
**Other Parameters**  
**lower** : input int, optional  
 Default: 0  
**overwrite\_ab** : input int, optional  
 Default: 0  
**ldab** : input int, optional  
 Default: shape(ab,0)

`scipy.linalg.lapack.zpbtrf(ab[, lower, ldab, overwrite_ab]) = <fortran object>`  
 Wrapper for `zpbtrf`.

**Parameters** **ab** : input rank-2 array('D') with bounds (ldab,n)  
**Returns** **c** : rank-2 array('D') with bounds (ldab,n) and ab storage  
**info** : int  
**Other Parameters**  
**lower** : input int, optional  
 Default: 0  
**overwrite\_ab** : input int, optional  
 Default: 0  
**ldab** : input int, optional  
 Default: shape(ab,0)

`scipy.linalg.lapack.spbtrs(ab, b[, lower, ldab, overwrite_b]) = <fortran object>`  
 Wrapper for `spbtrs`.

**Parameters** **ab** : input rank-2 array('f') with bounds (ldab,n)  
**b** : input rank-2 array('f') with bounds (ldb,nrhs)  
**Returns** **x** : rank-2 array('f') with bounds (ldb,nrhs) and b storage  
**info** : int  
**Other Parameters**  
**lower** : input int, optional  
Default: 0  
**ldab** : input int, optional  
Default: shape(ab,0)  
**overwrite\_b** : input int, optional  
Default: 0

`scipy.linalg.lapack.dpbttrs(ab, b[, lower, ldab, overwrite_b]) = <fortran object>`  
Wrapper for dpbttrs.

**Parameters** **ab** : input rank-2 array('d') with bounds (ldab,n)  
**b** : input rank-2 array('d') with bounds (ldb,nrhs)  
**Returns** **x** : rank-2 array('d') with bounds (ldb,nrhs) and b storage  
**info** : int  
**Other Parameters**  
**lower** : input int, optional  
Default: 0  
**ldab** : input int, optional  
Default: shape(ab,0)  
**overwrite\_b** : input int, optional  
Default: 0

`scipy.linalg.lapack.cpbttrs(ab, b[, lower, ldab, overwrite_b]) = <fortran object>`  
Wrapper for cpbttrs.

**Parameters** **ab** : input rank-2 array('F') with bounds (ldab,n)  
**b** : input rank-2 array('F') with bounds (ldb,nrhs)  
**Returns** **x** : rank-2 array('F') with bounds (ldb,nrhs) and b storage  
**info** : int  
**Other Parameters**  
**lower** : input int, optional  
Default: 0  
**ldab** : input int, optional  
Default: shape(ab,0)  
**overwrite\_b** : input int, optional  
Default: 0

`scipy.linalg.lapack.zpbttrs(ab, b[, lower, ldab, overwrite_b]) = <fortran object>`  
Wrapper for zpbttrs.

**Parameters** **ab** : input rank-2 array('D') with bounds (ldab,n)  
**b** : input rank-2 array('D') with bounds (ldb,nrhs)  
**Returns** **x** : rank-2 array('D') with bounds (ldb,nrhs) and b storage  
**info** : int  
**Other Parameters**  
**lower** : input int, optional  
Default: 0  
**ldab** : input int, optional  
Default: shape(ab,0)  
**overwrite\_b** : input int, optional  
Default: 0



`scipy.linalg.lapack.sposv(a, b[, lower, overwrite_a, overwrite_b]) = <fortran object>`  
 Wrapper for `sposv`.

**Parameters** **a** : input rank-2 array('f') with bounds (n,n)  
**b** : input rank-2 array('f') with bounds (n,nrhs)  
**Returns** **c** : rank-2 array('f') with bounds (n,n) and a storage  
**x** : rank-2 array('f') with bounds (n,nrhs) and b storage  
**info** : int  
**Other Parameters**  
**overwrite\_a** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0

`scipy.linalg.lapack.dposv(a, b[, lower, overwrite_a, overwrite_b]) = <fortran object>`  
 Wrapper for `dposv`.

**Parameters** **a** : input rank-2 array('d') with bounds (n,n)  
**b** : input rank-2 array('d') with bounds (n,nrhs)  
**Returns** **c** : rank-2 array('d') with bounds (n,n) and a storage  
**x** : rank-2 array('d') with bounds (n,nrhs) and b storage  
**info** : int  
**Other Parameters**  
**overwrite\_a** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0

`scipy.linalg.lapack.cposv(a, b[, lower, overwrite_a, overwrite_b]) = <fortran object>`  
 Wrapper for `cpovs`.

**Parameters** **a** : input rank-2 array('F') with bounds (n,n)  
**b** : input rank-2 array('F') with bounds (n,nrhs)  
**Returns** **c** : rank-2 array('F') with bounds (n,n) and a storage  
**x** : rank-2 array('F') with bounds (n,nrhs) and b storage  
**info** : int  
**Other Parameters**  
**overwrite\_a** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0

`scipy.linalg.lapack.zposv(a, b[, lower, overwrite_a, overwrite_b]) = <fortran object>`  
 Wrapper for `zposv`.

**Parameters** **a** : input rank-2 array('D') with bounds (n,n)  
**b** : input rank-2 array('D') with bounds (n,nrhs)  
**Returns** **c** : rank-2 array('D') with bounds (n,n) and a storage  
**x** : rank-2 array('D') with bounds (n,nrhs) and b storage  
**info** : int

*Other Parameters*

**overwrite\_a** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0

`scipy.linalg.lapack.spotrf(a[, lower, clean, overwrite_a]) = <fortran object>`  
 Wrapper for `spotrf`.

**Parameters** **a** : input rank-2 array('f') with bounds (n,n)  
**Returns** **c** : rank-2 array('f') with bounds (n,n) and a storage  
**info** : int

*Other Parameters*

**overwrite\_a** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0  
**clean** : input int, optional  
 Default: 1

`scipy.linalg.lapack.dpotrf(a[, lower, clean, overwrite_a]) = <fortran object>`  
 Wrapper for `dpotrf`.

**Parameters** **a** : input rank-2 array('d') with bounds (n,n)  
**Returns** **c** : rank-2 array('d') with bounds (n,n) and a storage  
**info** : int

*Other Parameters*

**overwrite\_a** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0  
**clean** : input int, optional  
 Default: 1

`scipy.linalg.lapack.cpotrf(a[, lower, clean, overwrite_a]) = <fortran object>`  
 Wrapper for `cpotrf`.

**Parameters** **a** : input rank-2 array('F') with bounds (n,n)  
**Returns** **c** : rank-2 array('F') with bounds (n,n) and a storage  
**info** : int

*Other Parameters*

**overwrite\_a** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0  
**clean** : input int, optional  
 Default: 1

`scipy.linalg.lapack.zpotrf(a[, lower, clean, overwrite_a]) = <fortran object>`  
 Wrapper for `zpotrf`.

**Parameters** **a** : input rank-2 array('D') with bounds (n,n)  
**Returns** **c** : rank-2 array('D') with bounds (n,n) and a storage  
**info** : int

*Other Parameters*

**overwrite\_a** : input int, optional

Default: 0  
**lower** : input int, optional  
 Default: 0  
**clean** : input int, optional  
 Default: 1

`scipy.linalg.lapack.spotri(c[, lower, overwrite_c]) = <fortran object>`  
 Wrapper for `spotri`.

**Parameters** **c** : input rank-2 array('f') with bounds (n,n)  
**Returns** **inv\_a** : rank-2 array('f') with bounds (n,n) and c storage  
**info** : int  
**Other Parameters**  
**overwrite\_c** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0

`scipy.linalg.lapack.dpotri(c[, lower, overwrite_c]) = <fortran object>`  
 Wrapper for `dpotri`.

**Parameters** **c** : input rank-2 array('d') with bounds (n,n)  
**Returns** **inv\_a** : rank-2 array('d') with bounds (n,n) and c storage  
**info** : int  
**Other Parameters**  
**overwrite\_c** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0

`scipy.linalg.lapack.cpotri(c[, lower, overwrite_c]) = <fortran object>`  
 Wrapper for `cpotri`.

**Parameters** **c** : input rank-2 array('F') with bounds (n,n)  
**Returns** **inv\_a** : rank-2 array('F') with bounds (n,n) and c storage  
**info** : int  
**Other Parameters**  
**overwrite\_c** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0

`scipy.linalg.lapack.zpotri(c[, lower, overwrite_c]) = <fortran object>`  
 Wrapper for `zpotri`.

**Parameters** **c** : input rank-2 array('D') with bounds (n,n)  
**Returns** **inv\_a** : rank-2 array('D') with bounds (n,n) and c storage  
**info** : int  
**Other Parameters**  
**overwrite\_c** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0

`scipy.linalg.lapack.spotrs(c, b[, lower, overwrite_b]) = <fortran object>`  
 Wrapper for `spotrs`.

**Parameters** **c** : input rank-2 array('f') with bounds (n,n)  
**b** : input rank-2 array('f') with bounds (n,nrhs)

**Returns** **x** : rank-2 array('f') with bounds (n,nrhs) and b storage  
**info** : int

**Other Parameters**

**overwrite\_b** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0

`scipy.linalg.lapack.dpotrs(c, b[, lower, overwrite_b]) = <fortran object>`  
 Wrapper for `dpotrs`.

**Parameters** **c** : input rank-2 array('d') with bounds (n,n)  
**b** : input rank-2 array('d') with bounds (n,nrhs)  
**Returns** **x** : rank-2 array('d') with bounds (n,nrhs) and b storage  
**info** : int

**Other Parameters**

**overwrite\_b** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0

`scipy.linalg.lapack.cpotrs(c, b[, lower, overwrite_b]) = <fortran object>`  
 Wrapper for `cpotrs`.

**Parameters** **c** : input rank-2 array('F') with bounds (n,n)  
**b** : input rank-2 array('F') with bounds (n,nrhs)  
**Returns** **x** : rank-2 array('F') with bounds (n,nrhs) and b storage  
**info** : int

**Other Parameters**

**overwrite\_b** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0

`scipy.linalg.lapack.zpotrs(c, b[, lower, overwrite_b]) = <fortran object>`  
 Wrapper for `zpotrs`.

**Parameters** **c** : input rank-2 array('D') with bounds (n,n)  
**b** : input rank-2 array('D') with bounds (n,nrhs)  
**Returns** **x** : rank-2 array('D') with bounds (n,nrhs) and b storage  
**info** : int

**Other Parameters**

**overwrite\_b** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0

`scipy.linalg.lapack.crot(x, y, c, s[, n, offx, incx, offy, incy, overwrite_x, overwrite_y]) = <fortran object>`

Wrapper for `crot`.

**Parameters** **x** : input rank-1 array('F') with bounds (\*)  
**y** : input rank-1 array('F') with bounds (\*)  
**c** : input float  
**s** : input complex  
**Returns** **x** : rank-1 array('F') with bounds (\*)  
**y** : rank-1 array('F') with bounds (\*)

**Other Parameters**

**n** : input int, optional  
 Default:  $(\text{len}(x)-1-\text{offx})/\text{abs}(\text{incx})+1$   
**overwrite\_x** : input int, optional  
 Default: 0  
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**overwrite\_y** : input int, optional  
 Default: 0  
**offy** : input int, optional  
 Default: 0  
**incy** : input int, optional  
 Default: 1

`scipy.linalg.lapack.zrot(x, y, c, s[, n, offx, incx, offy, incy, overwrite_x, overwrite_y]) = <fortran object>`

Wrapper for zrot.

**Parameters** **x** : input rank-1 array('D') with bounds (\*)  
**y** : input rank-1 array('D') with bounds (\*)  
**c** : input float  
**s** : input complex

**Returns** **x** : rank-1 array('D') with bounds (\*)  
**y** : rank-1 array('D') with bounds (\*)

**Other Parameters**

**n** : input int, optional  
 Default:  $(\text{len}(x)-1-\text{offx})/\text{abs}(\text{incx})+1$   
**overwrite\_x** : input int, optional  
 Default: 0  
**offx** : input int, optional  
 Default: 0  
**incx** : input int, optional  
 Default: 1  
**overwrite\_y** : input int, optional  
 Default: 0  
**offy** : input int, optional  
 Default: 0  
**incy** : input int, optional  
 Default: 1

`scipy.linalg.lapack.strsyl(a, b, c[, trana, tranb, isgn, overwrite_c]) = <fortran object>`

Wrapper for strsyl.

**Parameters** **a** : input rank-2 array('f') with bounds (m,m)  
**b** : input rank-2 array('f') with bounds (n,n)  
**c** : input rank-2 array('f') with bounds (m,n)  
**Returns** **x** : rank-2 array('f') with bounds (m,n) and c storage  
**scale** : float  
**info** : int

**Other Parameters**

**trana** : input string(len=1), optional  
 Default: 'N'  
**tranb** : input string(len=1), optional  
 Default: 'N'

**isgn** : input int, optional  
 Default: 1  
**overwrite\_c** : input int, optional  
 Default: 0

`scipy.linalg.lapack.dtrsyl(a, b, c[, trana, tranb, isgn, overwrite_c]) = <fortran object>`  
 Wrapper for dtrsyl.

**Parameters** **a** : input rank-2 array('d') with bounds (m,m)  
**b** : input rank-2 array('d') with bounds (n,n)  
**c** : input rank-2 array('d') with bounds (m,n)

**Returns** **x** : rank-2 array('d') with bounds (m,n) and c storage  
**scale** : float  
**info** : int

**Other Parameters**  
**trana** : input string(len=1), optional  
 Default: 'N'  
**tranb** : input string(len=1), optional  
 Default: 'N'  
**isgn** : input int, optional  
 Default: 1  
**overwrite\_c** : input int, optional  
 Default: 0

`scipy.linalg.lapack.ctrsyl(a, b, c[, trana, tranb, isgn, overwrite_c]) = <fortran object>`  
 Wrapper for ctrsyl.

**Parameters** **a** : input rank-2 array('F') with bounds (m,m)  
**b** : input rank-2 array('F') with bounds (n,n)  
**c** : input rank-2 array('F') with bounds (m,n)

**Returns** **x** : rank-2 array('F') with bounds (m,n) and c storage  
**scale** : float  
**info** : int

**Other Parameters**  
**trana** : input string(len=1), optional  
 Default: 'N'  
**tranb** : input string(len=1), optional  
 Default: 'N'  
**isgn** : input int, optional  
 Default: 1  
**overwrite\_c** : input int, optional  
 Default: 0

`scipy.linalg.lapack.ztrsyl(a, b, c[, trana, tranb, isgn, overwrite_c]) = <fortran object>`  
 Wrapper for ztrsyl.

**Parameters** **a** : input rank-2 array('D') with bounds (m,m)  
**b** : input rank-2 array('D') with bounds (n,n)  
**c** : input rank-2 array('D') with bounds (m,n)

**Returns** **x** : rank-2 array('D') with bounds (m,n) and c storage  
**scale** : float  
**info** : int

**Other Parameters**  
**trana** : input string(len=1), optional  
 Default: 'N'  
**tranb** : input string(len=1), optional  
 Default: 'N'

**isgn** : input int, optional  
 Default: 1  
**overwrite\_c** : input int, optional  
 Default: 0

`scipy.linalg.lapack.strtri(c[, lower, unitdiag, overwrite_c]) = <fortran object>`  
 Wrapper for `strtri`.

**Parameters** **c** : input rank-2 array('f') with bounds (n,n)  
**Returns** **inv\_c** : rank-2 array('f') with bounds (n,n) and c storage  
**info** : int

**Other Parameters**  
**overwrite\_c** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0  
**unitdiag** : input int, optional  
 Default: 0

`scipy.linalg.lapack.dstrtri(c[, lower, unitdiag, overwrite_c]) = <fortran object>`  
 Wrapper for `dstrtri`.

**Parameters** **c** : input rank-2 array('d') with bounds (n,n)  
**Returns** **inv\_c** : rank-2 array('d') with bounds (n,n) and c storage  
**info** : int

**Other Parameters**  
**overwrite\_c** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0  
**unitdiag** : input int, optional  
 Default: 0

`scipy.linalg.lapack.cstrtri(c[, lower, unitdiag, overwrite_c]) = <fortran object>`  
 Wrapper for `cstrtri`.

**Parameters** **c** : input rank-2 array('F') with bounds (n,n)  
**Returns** **inv\_c** : rank-2 array('F') with bounds (n,n) and c storage  
**info** : int

**Other Parameters**  
**overwrite\_c** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0  
**unitdiag** : input int, optional  
 Default: 0

`scipy.linalg.lapack.zstrtri(c[, lower, unitdiag, overwrite_c]) = <fortran object>`  
 Wrapper for `zstrtri`.

**Parameters** **c** : input rank-2 array('D') with bounds (n,n)  
**Returns** **inv\_c** : rank-2 array('D') with bounds (n,n) and c storage  
**info** : int

**Other Parameters**  
**overwrite\_c** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0

**unitdiag** : input int, optional  
 Default: 0

`scipy.linalg.lapack.strtrs(a, b[, lower, trans, unitdiag, lda, overwrite_b]) = <fortran object>`  
 Wrapper for `strtrs`.

**Parameters** **a** : input rank-2 array('f') with bounds (lda,n)  
**b** : input rank-2 array('f') with bounds (ldb,nrhs)  
**Returns** **x** : rank-2 array('f') with bounds (ldb,nrhs) and b storage  
**info** : int

**Other Parameters**  
**lower** : input int, optional  
 Default: 0  
**trans** : input int, optional  
 Default: 0  
**unitdiag** : input int, optional  
 Default: 0  
**lda** : input int, optional  
 Default: `shape(a,0)`  
**overwrite\_b** : input int, optional  
 Default: 0

`scipy.linalg.lapack.dtrtrs(a, b[, lower, trans, unitdiag, lda, overwrite_b]) = <fortran object>`  
 Wrapper for `dtrtrs`.

**Parameters** **a** : input rank-2 array('d') with bounds (lda,n)  
**b** : input rank-2 array('d') with bounds (ldb,nrhs)  
**Returns** **x** : rank-2 array('d') with bounds (ldb,nrhs) and b storage  
**info** : int

**Other Parameters**  
**lower** : input int, optional  
 Default: 0  
**trans** : input int, optional  
 Default: 0  
**unitdiag** : input int, optional  
 Default: 0  
**lda** : input int, optional  
 Default: `shape(a,0)`  
**overwrite\_b** : input int, optional  
 Default: 0

`scipy.linalg.lapack.ctrtrs(a, b[, lower, trans, unitdiag, lda, overwrite_b]) = <fortran object>`  
 Wrapper for `ctrtrs`.

**Parameters** **a** : input rank-2 array('F') with bounds (lda,n)  
**b** : input rank-2 array('F') with bounds (ldb,nrhs)  
**Returns** **x** : rank-2 array('F') with bounds (ldb,nrhs) and b storage  
**info** : int

**Other Parameters**  
**lower** : input int, optional  
 Default: 0  
**trans** : input int, optional  
 Default: 0  
**unitdiag** : input int, optional  
 Default: 0  
**lda** : input int, optional  
 Default: `shape(a,0)`



**overwrite\_b** : input int, optional  
Default: 0

`scipy.linalg.lapack.ztrtrs(a, b[, lower, trans, unitdiag, lda, overwrite_b]) = <fortran object>`

Wrapper for ztrtrs.

**Parameters** **a** : input rank-2 array('D') with bounds (lda,n)  
**b** : input rank-2 array('D') with bounds (ldb,nrhs)

**Returns** **x** : rank-2 array('D') with bounds (ldb,nrhs) and b storage  
**info** : int

**Other Parameters**  
**lower** : input int, optional  
Default: 0  
**trans** : input int, optional  
Default: 0  
**unitdiag** : input int, optional  
Default: 0  
**lda** : input int, optional  
Default: shape(a,0)  
**overwrite\_b** : input int, optional  
Default: 0

`scipy.linalg.lapack.cunghr(a, tau[, lo, hi, lwork, overwrite_a]) = <fortran object>`

Wrapper for cunghr.

**Parameters** **a** : input rank-2 array('F') with bounds (n,n)  
**tau** : input rank-1 array('F') with bounds (n - 1)

**Returns** **ht** : rank-2 array('F') with bounds (n,n) and a storage  
**info** : int

**Other Parameters**  
**lo** : input int, optional  
Default: 0  
**hi** : input int, optional  
Default: n-1  
**overwrite\_a** : input int, optional  
Default: 0  
**lwork** : input int, optional  
Default: hi-lo

`scipy.linalg.lapack.zunghr(a, tau[, lo, hi, lwork, overwrite_a]) = <fortran object>`

Wrapper for zunghr.

**Parameters** **a** : input rank-2 array('D') with bounds (n,n)  
**tau** : input rank-1 array('D') with bounds (n - 1)

**Returns** **ht** : rank-2 array('D') with bounds (n,n) and a storage  
**info** : int

**Other Parameters**  
**lo** : input int, optional  
Default: 0  
**hi** : input int, optional  
Default: n-1  
**overwrite\_a** : input int, optional  
Default: 0  
**lwork** : input int, optional  
Default: hi-lo

`scipy.linalg.lapack.cungqr(a, tau[, lwork, overwrite_a]) = <fortran object>`  
 Wrapper for `cungqr`.

**Parameters** **a** : input rank-2 array('F') with bounds (m,n)  
**tau** : input rank-1 array('F') with bounds (k)  
**Returns** **q** : rank-2 array('F') with bounds (m,n) and a storage  
**work** : rank-1 array('F') with bounds (MAX(lwork,1))  
**info** : int  
**Other Parameters**  
**overwrite\_a** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: 3\*n

`scipy.linalg.lapack.zungqr(a, tau[, lwork, overwrite_a]) = <fortran object>`  
 Wrapper for `zungqr`.

**Parameters** **a** : input rank-2 array('D') with bounds (m,n)  
**tau** : input rank-1 array('D') with bounds (k)  
**Returns** **q** : rank-2 array('D') with bounds (m,n) and a storage  
**work** : rank-1 array('D') with bounds (MAX(lwork,1))  
**info** : int  
**Other Parameters**  
**overwrite\_a** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: 3\*n

`scipy.linalg.lapack.cungrq(a, tau[, lwork, overwrite_a]) = <fortran object>`  
 Wrapper for `cungrq`.

**Parameters** **a** : input rank-2 array('F') with bounds (m,n)  
**tau** : input rank-1 array('F') with bounds (k)  
**Returns** **q** : rank-2 array('F') with bounds (m,n) and a storage  
**work** : rank-1 array('F') with bounds (MAX(lwork,1))  
**info** : int  
**Other Parameters**  
**overwrite\_a** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: 3\*m

`scipy.linalg.lapack.zungrq(a, tau[, lwork, overwrite_a]) = <fortran object>`  
 Wrapper for `zungrq`.

**Parameters** **a** : input rank-2 array('D') with bounds (m,n)  
**tau** : input rank-1 array('D') with bounds (k)  
**Returns** **q** : rank-2 array('D') with bounds (m,n) and a storage  
**work** : rank-1 array('D') with bounds (MAX(lwork,1))  
**info** : int  
**Other Parameters**  
**overwrite\_a** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: 3\*m

`scipy.linalg.lapack.cunmqr(side, trans, a, tau, c, lwork[, overwrite_c]) = <fortran object>`  
 Wrapper for `cunmqr`.

**Parameters** **side** : input string(len=1)  
**trans** : input string(len=1)  
**a** : input rank-2 array('F') with bounds (lda,k)  
**tau** : input rank-1 array('F') with bounds (k)  
**c** : input rank-2 array('F') with bounds (ldc,n)  
**lwork** : input int

**Returns** **cq** : rank-2 array('F') with bounds (ldc,n) and c storage  
**work** : rank-1 array('F') with bounds (MAX(lwork,1))  
**info** : int

**Other Parameters**  
**overwrite\_c** : input int, optional  
Default: 0

`scipy.linalg.lapack.zunmqr` (*side, trans, a, tau, c, lwork*[, *overwrite\_c*]) = <fortran object>  
Wrapper for zunmqr.

**Parameters** **side** : input string(len=1)  
**trans** : input string(len=1)  
**a** : input rank-2 array('D') with bounds (lda,k)  
**tau** : input rank-1 array('D') with bounds (k)  
**c** : input rank-2 array('D') with bounds (ldc,n)  
**lwork** : input int

**Returns** **cq** : rank-2 array('D') with bounds (ldc,n) and c storage  
**work** : rank-1 array('D') with bounds (MAX(lwork,1))  
**info** : int

**Other Parameters**  
**overwrite\_c** : input int, optional  
Default: 0

`scipy.linalg.lapack.sgtsv` (*dl, d, du, b*[, *overwrite\_dl, overwrite\_d, overwrite\_du, overwrite\_b*]) =  
<fortran object>

Wrapper for sgtsv.

**Parameters** **dl** : input rank-1 array('f') with bounds (n - 1)  
**d** : input rank-1 array('f') with bounds (\*)  
**du** : input rank-1 array('f') with bounds (n - 1)  
**b** : input rank-2 array('f') with bounds (,)

**Returns** **du2** : rank-1 array('f') with bounds (n - 1) and dl storage  
**d** : rank-1 array('f') with bounds (\*)  
**du** : rank-1 array('f') with bounds (n - 1)  
**x** : rank-2 array('f') with bounds (,) and b storage  
**info** : int

**Other Parameters**  
**overwrite\_dl** : input int, optional  
Default: 0  
**overwrite\_d** : input int, optional  
Default: 0  
**overwrite\_du** : input int, optional  
Default: 0  
**overwrite\_b** : input int, optional  
Default: 0

`scipy.linalg.lapack.dgtsv` (*dl, d, du, b*[, *overwrite\_dl, overwrite\_d, overwrite\_du, overwrite\_b*]) =  
<fortran object>

Wrapper for dgtsv.

**Parameters** **dl** : input rank-1 array('d') with bounds (n - 1)  
**d** : input rank-1 array('d') with bounds (\*)  
**du** : input rank-1 array('d') with bounds (n - 1)  
**b** : input rank-2 array('d') with bounds (,)  
**Returns** **du2** : rank-1 array('d') with bounds (n - 1) and dl storage  
**d** : rank-1 array('d') with bounds (\*)  
**du** : rank-1 array('d') with bounds (n - 1)  
**x** : rank-2 array('d') with bounds (,) and b storage  
**info** : int

**Other Parameters**  
**overwrite\_dl** : input int, optional  
 Default: 0  
**overwrite\_d** : input int, optional  
 Default: 0  
**overwrite\_du** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0

`scipy.linalg.lapack.cgtsv(dl, d, du, b[, overwrite_dl, overwrite_d, overwrite_du, overwrite_b]) =`  
 <fortran object>

Wrapper for cgtsv.

**Parameters** **dl** : input rank-1 array('F') with bounds (n - 1)  
**d** : input rank-1 array('F') with bounds (\*)  
**du** : input rank-1 array('F') with bounds (n - 1)  
**b** : input rank-2 array('F') with bounds (,)  
**Returns** **du2** : rank-1 array('F') with bounds (n - 1) and dl storage  
**d** : rank-1 array('F') with bounds (\*)  
**du** : rank-1 array('F') with bounds (n - 1)  
**x** : rank-2 array('F') with bounds (,) and b storage  
**info** : int

**Other Parameters**  
**overwrite\_dl** : input int, optional  
 Default: 0  
**overwrite\_d** : input int, optional  
 Default: 0  
**overwrite\_du** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0

`scipy.linalg.lapack.zgtsv(dl, d, du, b[, overwrite_dl, overwrite_d, overwrite_du, overwrite_b]) =`  
 <fortran object>

Wrapper for zgtsv.

**Parameters** **dl** : input rank-1 array('D') with bounds (n - 1)  
**d** : input rank-1 array('D') with bounds (\*)  
**du** : input rank-1 array('D') with bounds (n - 1)  
**b** : input rank-2 array('D') with bounds (,)  
**Returns** **du2** : rank-1 array('D') with bounds (n - 1) and dl storage  
**d** : rank-1 array('D') with bounds (\*)  
**du** : rank-1 array('D') with bounds (n - 1)  
**x** : rank-2 array('D') with bounds (,) and b storage  
**info** : int

**Other Parameters**

**overwrite\_dl** : input int, optional  
Default: 0  
**overwrite\_d** : input int, optional  
Default: 0  
**overwrite\_du** : input int, optional  
Default: 0  
**overwrite\_b** : input int, optional  
Default: 0

`scipy.linalg.lapack.sptsv(d, e, b[, overwrite_d, overwrite_e, overwrite_b]) = <fortran object>`  
 Wrapper for `sptsv`.

**Parameters** **d** : input rank-1 array('f') with bounds (\*)  
**e** : input rank-1 array('f') with bounds (n - 1)  
**b** : input rank-2 array('f') with bounds (,)  
**Returns** **d** : rank-1 array('f') with bounds (\*)  
**du** : rank-1 array('f') with bounds (n - 1) and e storage  
**x** : rank-2 array('f') with bounds (,) and b storage  
**info** : int

**Other Parameters**

**overwrite\_d** : input int, optional  
Default: 0  
**overwrite\_e** : input int, optional  
Default: 0  
**overwrite\_b** : input int, optional  
Default: 0

`scipy.linalg.lapack.dptsv(d, e, b[, overwrite_d, overwrite_e, overwrite_b]) = <fortran object>`  
 Wrapper for `dptsv`.

**Parameters** **d** : input rank-1 array('d') with bounds (\*)  
**e** : input rank-1 array('d') with bounds (n - 1)  
**b** : input rank-2 array('d') with bounds (,)  
**Returns** **d** : rank-1 array('d') with bounds (\*)  
**du** : rank-1 array('d') with bounds (n - 1) and e storage  
**x** : rank-2 array('d') with bounds (,) and b storage  
**info** : int

**Other Parameters**

**overwrite\_d** : input int, optional  
Default: 0  
**overwrite\_e** : input int, optional  
Default: 0  
**overwrite\_b** : input int, optional  
Default: 0

`scipy.linalg.lapack.cptsv(d, e, b[, overwrite_d, overwrite_e, overwrite_b]) = <fortran object>`  
 Wrapper for `cptsv`.

**Parameters** **d** : input rank-1 array('f') with bounds (\*)  
**e** : input rank-1 array('F') with bounds (n - 1)  
**b** : input rank-2 array('F') with bounds (,)  
**Returns** **d** : rank-1 array('f') with bounds (\*)  
**du** : rank-1 array('F') with bounds (n - 1) and e storage  
**x** : rank-2 array('F') with bounds (,) and b storage  
**info** : int

**Other Parameters**

**overwrite\_d** : input int, optional  
 Default: 0  
**overwrite\_e** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0

`scipy.linalg.lapack.zptsv(d, e, b[, overwrite_d, overwrite_e, overwrite_b]) = <fortran object>`  
 Wrapper for zptsv.

**Parameters** **d** : input rank-1 array('d') with bounds (\*)  
**e** : input rank-1 array('D') with bounds (n - 1)  
**b** : input rank-2 array('D') with bounds (,)  
**Returns** **d** : rank-1 array('d') with bounds (\*)  
**du** : rank-1 array('D') with bounds (n - 1) and e storage  
**x** : rank-2 array('D') with bounds (,) and b storage  
**info** : int

**Other Parameters**

**overwrite\_d** : input int, optional  
 Default: 0  
**overwrite\_e** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0

`scipy.linalg.lapack.slamch(cmach) = <fortran slamch>`  
 Wrapper for slamch.

**Parameters** **cmach** : input string(len=1)  
**Returns** **slamch** : float

`scipy.linalg.lapack.dlamch(cmach) = <fortran dlamch>`  
 Wrapper for dlamch.

**Parameters** **cmach** : input string(len=1)  
**Returns** **dlamch** : float

`scipy.linalg.lapack.sorghr(a, tau[, lo, hi, lwork, overwrite_a]) = <fortran object>`  
 Wrapper for sorghr.

**Parameters** **a** : input rank-2 array('f') with bounds (n,n)  
**tau** : input rank-1 array('f') with bounds (n - 1)  
**Returns** **ht** : rank-2 array('f') with bounds (n,n) and a storage  
**info** : int

**Other Parameters**

**lo** : input int, optional  
 Default: 0  
**hi** : input int, optional  
 Default: n-1  
**overwrite\_a** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: hi-lo

`scipy.linalg.lapack.dorghr(a, tau[, lo, hi, lwork, overwrite_a]) = <fortran object>`  
 Wrapper for dorghr.

**Parameters** **a** : input rank-2 array('d') with bounds (n,n)  
**tau** : input rank-1 array('d') with bounds (n - 1)  
**Returns** **ht** : rank-2 array('d') with bounds (n,n) and a storage  
**info** : int

**Other Parameters**

**lo** : input int, optional  
 Default: 0  
**hi** : input int, optional  
 Default: n-1  
**overwrite\_a** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: hi-lo

`scipy.linalg.lapack.sorgqr(a, tau[, lwork, overwrite_a]) = <fortran object>`  
 Wrapper for sorgqr.

**Parameters** **a** : input rank-2 array('f') with bounds (m,n)  
**tau** : input rank-1 array('f') with bounds (k)  
**Returns** **q** : rank-2 array('f') with bounds (m,n) and a storage  
**work** : rank-1 array('f') with bounds (MAX(lwork,1))  
**info** : int

**Other Parameters**

**overwrite\_a** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: 3\*n

`scipy.linalg.lapack.dorgqr(a, tau[, lwork, overwrite_a]) = <fortran object>`  
 Wrapper for dorgqr.

**Parameters** **a** : input rank-2 array('d') with bounds (m,n)  
**tau** : input rank-1 array('d') with bounds (k)  
**Returns** **q** : rank-2 array('d') with bounds (m,n) and a storage  
**work** : rank-1 array('d') with bounds (MAX(lwork,1))  
**info** : int

**Other Parameters**

**overwrite\_a** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: 3\*n

`scipy.linalg.lapack.sorgqr(a, tau[, lwork, overwrite_a]) = <fortran object>`  
 Wrapper for sorgqr.

**Parameters** **a** : input rank-2 array('f') with bounds (m,n)  
**tau** : input rank-1 array('f') with bounds (k)  
**Returns** **q** : rank-2 array('f') with bounds (m,n) and a storage  
**work** : rank-1 array('f') with bounds (MAX(lwork,1))  
**info** : int

**Other Parameters**

**overwrite\_a** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: 3\*m

`scipy.linalg.lapack.dorgqr(a, tau[, lwork, overwrite_a]) = <fortran object>`  
 Wrapper for `dorgqr`.

**Parameters** **a** : input rank-2 array('d') with bounds (m,n)  
**tau** : input rank-1 array('d') with bounds (k)  
**Returns** **q** : rank-2 array('d') with bounds (m,n) and a storage  
**work** : rank-1 array('d') with bounds (MAX(lwork,1))  
**info** : int  
**Other Parameters**  
**overwrite\_a** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: 3\*m

`scipy.linalg.lapack.sormqr(side, trans, a, tau, c, lwork[, overwrite_c]) = <fortran object>`  
 Wrapper for `sormqr`.

**Parameters** **side** : input string(len=1)  
**trans** : input string(len=1)  
**a** : input rank-2 array('f') with bounds (lda,k)  
**tau** : input rank-1 array('f') with bounds (k)  
**c** : input rank-2 array('f') with bounds (ldc,n)  
**lwork** : input int  
**Returns** **cq** : rank-2 array('f') with bounds (ldc,n) and c storage  
**work** : rank-1 array('f') with bounds (MAX(lwork,1))  
**info** : int  
**Other Parameters**  
**overwrite\_c** : input int, optional  
 Default: 0

`scipy.linalg.lapack.dormqr(side, trans, a, tau, c, lwork[, overwrite_c]) = <fortran object>`  
 Wrapper for `dormqr`.

**Parameters** **side** : input string(len=1)  
**trans** : input string(len=1)  
**a** : input rank-2 array('d') with bounds (lda,k)  
**tau** : input rank-1 array('d') with bounds (k)  
**c** : input rank-2 array('d') with bounds (ldc,n)  
**lwork** : input int  
**Returns** **cq** : rank-2 array('d') with bounds (ldc,n) and c storage  
**work** : rank-1 array('d') with bounds (MAX(lwork,1))  
**info** : int  
**Other Parameters**  
**overwrite\_c** : input int, optional  
 Default: 0

`scipy.linalg.lapack.ssbev(ab[, compute_v, lower, ldab, overwrite_ab]) = <fortran object>`  
 Wrapper for `ssbev`.

**Parameters** **ab** : input rank-2 array('f') with bounds (ldab,\*)  
**Returns** **w** : rank-1 array('f') with bounds (n)  
**z** : rank-2 array('f') with bounds (ldz,ldz)  
**info** : int  
**Other Parameters**  
**overwrite\_ab** : input int, optional  
 Default: 1  
**compute\_v** : input int, optional



Default: 1  
**lower** : input int, optional  
 Default: 0  
**ldab** : input int, optional  
 Default: shape(ab,0)

`scipy.linalg.lapack.dsbevd(ab[, compute_v, lower, ldab, overwrite_ab]) = <fortran object>`  
 Wrapper for dsbevd.

**Parameters** **ab** : input rank-2 array('d') with bounds (ldab,\*)  
**Returns** **w** : rank-1 array('d') with bounds (n)  
**z** : rank-2 array('d') with bounds (ldz,ldz)  
**info** : int

**Other Parameters**  
**overwrite\_ab** : input int, optional  
 Default: 1  
**compute\_v** : input int, optional  
 Default: 1  
**lower** : input int, optional  
 Default: 0  
**ldab** : input int, optional  
 Default: shape(ab,0)

`scipy.linalg.lapack.ssbevd(ab[, compute_v, lower, ldab, liwork, overwrite_ab]) = <fortran object>`

Wrapper for ssbevd.

**Parameters** **ab** : input rank-2 array('f') with bounds (ldab,\*)  
**Returns** **w** : rank-1 array('f') with bounds (n)  
**z** : rank-2 array('f') with bounds (ldz,ldz)  
**info** : int

**Other Parameters**  
**overwrite\_ab** : input int, optional  
 Default: 1  
**compute\_v** : input int, optional  
 Default: 1  
**lower** : input int, optional  
 Default: 0  
**ldab** : input int, optional  
 Default: shape(ab,0)  
**liwork** : input int, optional  
 Default: (compute\_v?3+5\*n:1)

`scipy.linalg.lapack.dsbevd(ab[, compute_v, lower, ldab, liwork, overwrite_ab]) = <fortran object>`

Wrapper for dsbevd.

**Parameters** **ab** : input rank-2 array('d') with bounds (ldab,\*)  
**Returns** **w** : rank-1 array('d') with bounds (n)  
**z** : rank-2 array('d') with bounds (ldz,ldz)  
**info** : int

**Other Parameters**  
**overwrite\_ab** : input int, optional  
 Default: 1  
**compute\_v** : input int, optional  
 Default: 1  
**lower** : input int, optional

Default: 0  
**ldab** : input int, optional  
 Default: shape(ab,0)  
**liwork** : input int, optional  
 Default: (compute\_v?3+5\*n:1)

scipy.linalg.lapack.**ssbevx** (*ab*, *vl*, *vu*, *il*, *iu*[, *ldab*, *compute\_v*, *range*, *lower*, *abstol*, *mmax*, *overwrite\_ab* ]) = <fortran object>

Wrapper for ssbevx.

**Parameters** **ab** : input rank-2 array('f') with bounds (ldab,\*)  
**vl** : input float  
**vu** : input float  
**il** : input int  
**iu** : input int  
**Returns** **w** : rank-1 array('f') with bounds (n)  
**z** : rank-2 array('f') with bounds (ldz,mmax)  
**m** : int  
**ifail** : rank-1 array('i') with bounds ((compute\_v?n:1))  
**info** : int

**Other Parameters**

**overwrite\_ab** : input int, optional  
 Default: 1  
**ldab** : input int, optional  
 Default: shape(ab,0)  
**compute\_v** : input int, optional  
 Default: 1  
**range** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0  
**abstol** : input float, optional  
 Default: 0.0  
**mmax** : input int, optional  
 Default: (compute\_v?(range==2?(iu-il+1):n):1)

scipy.linalg.lapack.**dsbevx** (*ab*, *vl*, *vu*, *il*, *iu*[, *ldab*, *compute\_v*, *range*, *lower*, *abstol*, *mmax*, *overwrite\_ab* ]) = <fortran object>

Wrapper for dsbevx.

**Parameters** **ab** : input rank-2 array('d') with bounds (ldab,\*)  
**vl** : input float  
**vu** : input float  
**il** : input int  
**iu** : input int  
**Returns** **w** : rank-1 array('d') with bounds (n)  
**z** : rank-2 array('d') with bounds (ldz,mmax)  
**m** : int  
**ifail** : rank-1 array('i') with bounds ((compute\_v?n:1))  
**info** : int

**Other Parameters**

**overwrite\_ab** : input int, optional  
 Default: 1  
**ldab** : input int, optional  
 Default: shape(ab,0)  
**compute\_v** : input int, optional

Default: 1  
**range** : input int, optional  
 Default: 0  
**lower** : input int, optional  
 Default: 0  
**abstol** : input float, optional  
 Default: 0.0  
**mmax** : input int, optional  
 Default: (compute\_v?(range==2?(iu-il+1):n):1)

`scipy.linalg.lapack.ssyev(a[, compute_v, lower, lwork, overwrite_a]) = <fortran object>`  
 Wrapper for `ssyev`.

**Parameters** **a** : input rank-2 array('f') with bounds (n,n)  
**Returns** **w** : rank-1 array('f') with bounds (n)  
**v** : rank-2 array('f') with bounds (n,n) and a storage  
**info** : int

**Other Parameters**

**compute\_v** : input int, optional  
 Default: 1  
**lower** : input int, optional  
 Default: 0  
**overwrite\_a** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: 3\*n-1

`scipy.linalg.lapack.dsyeve(a[, compute_v, lower, lwork, overwrite_a]) = <fortran object>`  
 Wrapper for `dsyev`.

**Parameters** **a** : input rank-2 array('d') with bounds (n,n)  
**Returns** **w** : rank-1 array('d') with bounds (n)  
**v** : rank-2 array('d') with bounds (n,n) and a storage  
**info** : int

**Other Parameters**

**compute\_v** : input int, optional  
 Default: 1  
**lower** : input int, optional  
 Default: 0  
**overwrite\_a** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default: 3\*n-1

`scipy.linalg.lapack.ssyevd(a[, compute_v, lower, lwork, overwrite_a]) = <fortran object>`  
 Wrapper for `ssyevd`.

**Parameters** **a** : input rank-2 array('f') with bounds (n,n)  
**Returns** **w** : rank-1 array('f') with bounds (n)  
**v** : rank-2 array('f') with bounds (n,n) and a storage  
**info** : int

**Other Parameters**

**compute\_v** : input int, optional  
 Default: 1  
**lower** : input int, optional  
 Default: 0  
**overwrite\_a** : input int, optional

Default: 0  
**lwork** : input int, optional  
 Default:  $(\text{compute\_v} ? 1 + 6 * n + 2 * n * n : 2 * n + 1)$

`scipy.linalg.lapack.dsyevd(a[, compute_v, lower, lwork, overwrite_a]) = <fortran object>`  
 Wrapper for dsyevd.

**Parameters** **a** : input rank-2 array('d') with bounds (n,n)  
**Returns** **w** : rank-1 array('d') with bounds (n)  
**v** : rank-2 array('d') with bounds (n,n) and a storage  
**info** : int

**Other Parameters**  
**compute\_v** : input int, optional  
 Default: 1  
**lower** : input int, optional  
 Default: 0  
**overwrite\_a** : input int, optional  
 Default: 0  
**lwork** : input int, optional  
 Default:  $(\text{compute\_v} ? 1 + 6 * n + 2 * n * n : 2 * n + 1)$

`scipy.linalg.lapack.ssyevr(a[, jobz, range, uplo, il, iu, lwork, overwrite_a]) = <fortran object>`  
 Wrapper for ssyevr.

**Parameters** **a** : input rank-2 array('f') with bounds (n,n)  
**Returns** **w** : rank-1 array('f') with bounds (n)  
**z** : rank-2 array('f') with bounds (n,m)  
**info** : int

**Other Parameters**  
**jobz** : input string(len=1), optional  
 Default: 'V'  
**range** : input string(len=1), optional  
 Default: 'A'  
**uplo** : input string(len=1), optional  
 Default: 'L'  
**overwrite\_a** : input int, optional  
 Default: 0  
**il** : input int, optional  
 Default: 1  
**iu** : input int, optional  
 Default: n  
**lwork** : input int, optional  
 Default:  $26 * n$

`scipy.linalg.lapack.dsyevr(a[, jobz, range, uplo, il, iu, lwork, overwrite_a]) = <fortran object>`  
 Wrapper for dsyevr.

**Parameters** **a** : input rank-2 array('d') with bounds (n,n)  
**Returns** **w** : rank-1 array('d') with bounds (n)  
**z** : rank-2 array('d') with bounds (n,m)  
**info** : int

**Other Parameters**  
**jobz** : input string(len=1), optional  
 Default: 'V'  
**range** : input string(len=1), optional  
 Default: 'A'  
**uplo** : input string(len=1), optional

Default: 'L'  
**overwrite\_a** : input int, optional  
 Default: 0  
**il** : input int, optional  
 Default: 1  
**iu** : input int, optional  
 Default: n  
**lwork** : input int, optional  
 Default: 26\*n

`scipy.linalg.lapack.ssygv(a, b[, itype, jobz, uplo, overwrite_a, overwrite_b]) = <fortran object>`  
 Wrapper for `ssygv`.

**Parameters** **a** : input rank-2 array('f') with bounds (n,n)  
**b** : input rank-2 array('f') with bounds (n,n)  
**Returns** **a** : rank-2 array('f') with bounds (n,n)  
**w** : rank-1 array('f') with bounds (n)  
**info** : int

**Other Parameters**  
**itype** : input int, optional  
 Default: 1  
**jobz** : input string(len=1), optional  
 Default: 'V'  
**uplo** : input string(len=1), optional  
 Default: 'L'  
**overwrite\_a** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0

`scipy.linalg.lapack.dsygv(a, b[, itype, jobz, uplo, overwrite_a, overwrite_b]) = <fortran object>`  
 Wrapper for `dsygv`.

**Parameters** **a** : input rank-2 array('d') with bounds (n,n)  
**b** : input rank-2 array('d') with bounds (n,n)  
**Returns** **a** : rank-2 array('d') with bounds (n,n)  
**w** : rank-1 array('d') with bounds (n)  
**info** : int

**Other Parameters**  
**itype** : input int, optional  
 Default: 1  
**jobz** : input string(len=1), optional  
 Default: 'V'  
**uplo** : input string(len=1), optional  
 Default: 'L'  
**overwrite\_a** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0

`scipy.linalg.lapack.ssygvd(a, b[, itype, jobz, uplo, lwork, overwrite_a, overwrite_b]) = <fortran object>`

Wrapper for `ssygvd`.

**Parameters** **a** : input rank-2 array('f') with bounds (n,n)  
**b** : input rank-2 array('f') with bounds (n,n)

**Returns**     **a** : rank-2 array('f') with bounds (n,n)  
                  **w** : rank-1 array('f') with bounds (n)  
                  **info** : int

**Other Parameters**

**itype** : input int, optional  
           Default: 1  
**jobz** : input string(len=1), optional  
           Default: 'V'  
**uplo** : input string(len=1), optional  
           Default: 'L'  
**overwrite\_a** : input int, optional  
           Default: 0  
**overwrite\_b** : input int, optional  
           Default: 0  
**lwork** : input int, optional  
           Default: 1+6\*n+2\*n\*n

`scipy.linalg.lapack.dsygvd(a, b[, itype, jobz, uplo, lwork, overwrite_a, overwrite_b]) = <fortran object>`

Wrapper for dsygvd.

**Parameters**   **a** : input rank-2 array('d') with bounds (n,n)  
                  **b** : input rank-2 array('d') with bounds (n,n)  
**Returns**       **a** : rank-2 array('d') with bounds (n,n)  
                  **w** : rank-1 array('d') with bounds (n)  
                  **info** : int

**Other Parameters**

**itype** : input int, optional  
           Default: 1  
**jobz** : input string(len=1), optional  
           Default: 'V'  
**uplo** : input string(len=1), optional  
           Default: 'L'  
**overwrite\_a** : input int, optional  
           Default: 0  
**overwrite\_b** : input int, optional  
           Default: 0  
**lwork** : input int, optional  
           Default: 1+6\*n+2\*n\*n

`scipy.linalg.lapack.ssygvx(a, b, iu[, itype, jobz, uplo, il, lwork, overwrite_a, overwrite_b]) = <fortran object>`

Wrapper for ssygvx.

**Parameters**   **a** : input rank-2 array('f') with bounds (n,n)  
                  **b** : input rank-2 array('f') with bounds (n,n)  
                  **iu** : input int  
**Returns**       **w** : rank-1 array('f') with bounds (n)  
                  **z** : rank-2 array('f') with bounds (n,m)  
                  **ifail** : rank-1 array('i') with bounds (n)  
                  **info** : int

**Other Parameters**

**itype** : input int, optional  
           Default: 1  
**jobz** : input string(len=1), optional  
           Default: 'V'

**uplo** : input string(len=1), optional  
 Default: 'L'  
**overwrite\_a** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0  
**il** : input int, optional  
 Default: 1  
**lwork** : input int, optional  
 Default: 8\*n

`scipy.linalg.lapack.dsygvx(a, b, iu[, itype, jobz, uplo, il, lwork, overwrite_a, overwrite_b]) =`  
**<fortran object>**

Wrapper for dsygvx.

**Parameters** **a** : input rank-2 array('d') with bounds (n,n)  
**b** : input rank-2 array('d') with bounds (n,n)  
**iu** : input int  
**Returns** **w** : rank-1 array('d') with bounds (n)  
**z** : rank-2 array('d') with bounds (n,m)  
**ifail** : rank-1 array('i') with bounds (n)  
**info** : int

**Other Parameters**

**itype** : input int, optional  
 Default: 1  
**jobz** : input string(len=1), optional  
 Default: 'V'  
**uplo** : input string(len=1), optional  
 Default: 'L'  
**overwrite\_a** : input int, optional  
 Default: 0  
**overwrite\_b** : input int, optional  
 Default: 0  
**il** : input int, optional  
 Default: 1  
**lwork** : input int, optional  
 Default: 8\*n

`scipy.linalg.lapack.slange(norm, a) = <fortran slange>`

Wrapper for slange.

**Parameters** **norm** : input string(len=1)  
**a** : input rank-2 array('f') with bounds (m,n)  
**Returns** **n2** : float

`scipy.linalg.lapack.dlange(norm, a) = <fortran dlange>`

Wrapper for dlange.

**Parameters** **norm** : input string(len=1)  
**a** : input rank-2 array('d') with bounds (m,n)  
**Returns** **n2** : float

`scipy.linalg.lapack.clange(norm, a) = <fortran clange>`

Wrapper for clange.

**Parameters** **norm** : input string(len=1)  
**a** : input rank-2 array('F') with bounds (m,n)  
**Returns** **n2** : float

`scipy.linalg.lapack.zlange` (*norm, a*) = <fortran zlange>  
 Wrapper for zlange.

**Parameters**    **norm** : input string(len=1)  
                   **a** : input rank-2 array('D') with bounds (m,n)  
**Returns**        **n2** : float

`scipy.linalg.lapack.ilaver` = <fortran object>  
 Wrapper for ilaver.

**Returns**        **major** : int  
                   **minor** : int  
                   **patch** : int

## 5.12 BLAS Functions for Cython

Usable from Cython via:

```
cimport scipy.linalg.cython_blas
```

These wrappers do not check for alignment of arrays. Alignment should be checked before these wrappers are used.

Raw function pointers (Fortran-style pointer arguments):

- caxpy
- ccopy
- cdotc
- cdotu
- cgbmv
- cgemm
- cgemv
- cgerc
- cgeru
- chbmv
- chemm
- chemv
- cher
- cher2
- cher2k
- cherk
- chpmv
- chpr
- chpr2
- crotg



- cscal
- csrot
- csscal
- cswap
- csymm
- csyr2k
- csyrk
- ctbmv
- ctbsv
- ctpmv
- ctpsv
- ctrmm
- ctrmv
- ctrsm
- ctrsv
- dasum
- daxpy
- dcabs1
- dcopy
- ddot
- dgbmv
- dgemm
- dgemv
- dger
- dnorm2
- drot
- drotg
- drotm
- drotmg
- dsbmv
- dscal
- dsdot
- dspmv
- dspr
- dspr2
- dswap

- dsymm
- dsymv
- dsyr
- dsyr2
- dsyr2k
- dsyrk
- dtbmv
- dtbsv
- dtpmv
- dtpsv
- dtrmm
- dtrmv
- dtrsm
- dtrsv
- dzasum
- dznrm2
- icamax
- idamax
- isamax
- izamax
- lsame
- sasum
- saxpy
- scasum
- scnrm2
- scopy
- sdot
- sdsdot
- sgbmv
- sgemm
- sgemv
- sger
- snrm2
- srot
- srotg
- srotm

- srotmg
- ssbmv
- sscal
- sspmv
- sspr
- sspr2
- sswap
- ssymm
- ssymv
- ssyr
- ssyr2
- ssyr2k
- ssyrk
- stbmv
- stbsv
- stpmv
- stpsv
- strmm
- strmv
- strsm
- strsv
- zaxpy
- zcopy
- zdotc
- zdotu
- zdrot
- zdscal
- zgbmv
- zgemm
- zgemv
- zgerc
- zgeru
- zhbmv
- zhemm
- zhemv
- zher

- zher2
- zher2k
- zherk
- zhpmv
- zhpr
- zhpr2
- zrotg
- zscal
- zswap
- zsymm
- zsy2k
- zsyrk
- ztbmv
- ztbsv
- ztpmv
- ztpsv
- ztrmm
- ztrmv
- ztrsm
- ztrsv

## 5.13 LAPACK functions for Cython

Usable from Cython via:

```
cimport scipy.linalg.cython_lapack
```

This module provides Cython-level wrappers for all primary routines included in LAPACK 3.1.0 except for `zcgsv` since its interface is not consistent from LAPACK 3.1.0 to 3.6.0. It also provides some of the fixed-api auxiliary routines.

These wrappers do not check for alignment of arrays. Alignment should be checked before these wrappers are used.

Raw function pointers (Fortran-style pointer arguments):

- `cbdsqr`
- `cgbbrd`
- `cgbcon`
- `cgbequ`
- `cgbrfs`
- `cgbsv`

- cgbsvx
- cgbtf2
- cgbtrf
- cgbtrs
- cgebak
- cgebal
- cgebd2
- cgebrd
- cgecon
- cgeequ
- cgees
- cgeesx
- cgeev
- cgeevx
- cgehd2
- cgehrd
- cgelq2
- cgelqf
- cgels
- cgelsd
- cgelss
- cgelsy
- cgeql2
- cgeqlf
- cgeqp3
- cgeqr2
- cgeqrf
- cgerfs
- cgerq2
- cgerqf
- cgesc2
- cgesdd
- cgesv
- cgesvd
- cgesvx
- cgetc2

- cgetf2
- cgetrf
- cgetri
- cgetrs
- cggbak
- cggbal
- cgges
- cggesx
- cggev
- cggevx
- cggglm
- cgghrd
- cgglse
- cggqrf
- cggrqf
- cgtcon
- cgtrfs
- cgtsv
- cgtsvx
- cgtrf
- cgttrs
- cgts2
- chbev
- chbevd
- chbevz
- chbgst
- chbgv
- chbgvd
- chbgvx
- chbtrd
- checon
- cheev
- cheevd
- cheevr
- cheevx
- chegs2

- `chegst`
- `chegv`
- `chegvd`
- `chegvx`
- `cherfs`
- `chesv`
- `chesvx`
- `chetd2`
- `chetf2`
- `chetrd`
- `chetrf`
- `chetri`
- `chetrs`
- `chgeqz`
- `chpcon`
- `chpev`
- `chpevd`
- `chpevx`
- `chpgst`
- `chpgv`
- `chpgvd`
- `chpgvx`
- `chprfs`
- `chpsv`
- `chpsvx`
- `chptrd`
- `chptrf`
- `chptri`
- `chptrs`
- `chsein`
- `chseqr`
- `clabrd`
- `clacgv`
- `clacn2`
- `clacon`
- `clacp2`

- `clacpy`
- `clacrm`
- `clacrt`
- `cladiv`
- `claed0`
- `claed7`
- `claed8`
- `claein`
- `claesy`
- `claev2`
- `clag2z`
- `clags2`
- `clagtm`
- `clahef`
- `clahqr`
- `clahr2`
- `claic1`
- `clals0`
- `clalsa`
- `clalsd`
- `clangb`
- `clangc`
- `clangt`
- `clanhb`
- `clanhe`
- `clanhp`
- `clanhs`
- `clanht`
- `clansb`
- `clansp`
- `clansy`
- `clantb`
- `clantp`
- `clantr`
- `clapll`
- `clapmt`



- `claqgb`
- `claqge`
- `claqhb`
- `claqhe`
- `claqhp`
- `claqp2`
- `claqps`
- `claqr0`
- `claqr1`
- `claqr2`
- `claqr3`
- `claqr4`
- `claqr5`
- `claqsb`
- `claqsp`
- `claqsy`
- `clar1v`
- `clar2v`
- `clarcn`
- `clarf`
- `clarfb`
- `clarfg`
- `clarft`
- `clarfx`
- `clargv`
- `clarlv`
- `clarrv`
- `clartg`
- `clartv`
- `clarz`
- `clarzb`
- `clarzt`
- `clascl`
- `claset`
- `clasr`
- `classq`

- claswp
- clasyf
- clatbs
- clatdf
- clatps
- clatrd
- clatrs
- clatz
- clauu2
- clauum
- cpbcon
- cpbequ
- cpbrfs
- cpbstf
- cpbsv
- cpbsvx
- cpbtf2
- cpbtrf
- cpbtrs
- cpocon
- cpoequ
- cporfs
- cposv
- cposvx
- cpotf2
- cpotrf
- cpotri
- cpotrs
- cppcon
- cppequ
- cpprfs
- cppsv
- cppsvx
- cpptrf
- cpptri
- cpptrs

- `cptcon`
- `cpteqr`
- `cptrfs`
- `cptsv`
- `cptsvx`
- `cptrfs`
- `cptrfs`
- `cpfts2`
- `crot`
- `cspcon`
- `cspmv`
- `cspr`
- `csprfs`
- `cspsv`
- `cspsvx`
- `csprtf`
- `csptri`
- `csptrs`
- `csrscl`
- `cstedc`
- `cstegr`
- `cstein`
- `cstemr`
- `csteqr`
- `csycon`
- `csymv`
- `csyr`
- `csyrfs`
- `csysv`
- `csysvx`
- `csytf2`
- `csytrf`
- `csytri`
- `csytrs`
- `ctbcon`
- `ctbrfs`

- `ctbtrs`
- `ctgevc`
- `ctgex2`
- `ctgexc`
- `ctgsen`
- `ctgsja`
- `ctgsna`
- `ctgsy2`
- `ctgsyl`
- `ctpcon`
- `ctprfs`
- `ctptri`
- `ctptrs`
- `ctrcon`
- `ctrevc`
- `ctrexc`
- `ctrfs`
- `ctrsen`
- `ctrsna`
- `ctrsyl`
- `ctrti2`
- `ctrtri`
- `ctrtrs`
- `ctzrzf`
- `cung2l`
- `cung2r`
- `cungbr`
- `cunghr`
- `cungl2`
- `cunglq`
- `cungql`
- `cungqr`
- `cungr2`
- `cungrq`
- `cungtr`
- `cunm2l`

- `cunm2r`
- `cunmbr`
- `cunmhr`
- `cunml2`
- `cunmlq`
- `cunmql`
- `cunmqr`
- `cunmr2`
- `cunmr3`
- `cunmrq`
- `cunmrz`
- `cunmtr`
- `cupgtr`
- `cupmtr`
- `dbdsdc`
- `dbdsqr`
- `ddisna`
- `dgbbrd`
- `dgbcon`
- `dgbequ`
- `dgbrfs`
- `dgbsv`
- `dgbsvx`
- `dgbtf2`
- `dgbtrf`
- `dgbtrs`
- `dgebak`
- `dgebal`
- `dgebd2`
- `dgebrd`
- `dgecon`
- `dgeequ`
- `dgees`
- `dgeesx`
- `dgeev`
- `dgeevx`

- `dgehd2`
- `dgehrd`
- `dgelq2`
- `dgelqf`
- `dgels`
- `dgelsd`
- `dgelss`
- `dgelsy`
- `dgeql2`
- `dgeqlf`
- `dgeqp3`
- `dgeqr2`
- `dgeqrf`
- `dgerfs`
- `dgerq2`
- `dgerqf`
- `dgesc2`
- `dgesdd`
- `dgesv`
- `dgesvd`
- `dgesvx`
- `dgetc2`
- `dgetf2`
- `dgetrf`
- `dgetri`
- `dgetrs`
- `dggbak`
- `dggbal`
- `dgges`
- `dggesx`
- `dggev`
- `dggevx`
- `dggglm`
- `dgghrd`
- `dgglse`
- `dggqrf`

- dggrqf
- dgtcon
- dgtrfs
- dgtsv
- dgtsvx
- dgtrfs
- dgtrfs
- dgts2
- dhgeqz
- dhsein
- dhseqr
- disnan
- dlabad
- dlabrd
- dlacn2
- dlacon
- dlacpy
- dladiv
- dlae2
- dlaebz
- dlaed0
- dlaed1
- dlaed2
- dlaed3
- dlaed4
- dlaed5
- dlaed6
- dlaed7
- dlaed8
- dlaed9
- dlaeda
- dlaein
- dlaev2
- dlaexc
- dlag2
- dlag2s

- dlags2
- dlagtf
- dlagtm
- dlagts
- dlagv2
- dlahqr
- dlahr2
- dlaic1
- dlaln2
- dlals0
- dlalsa
- dlalsd
- dlamch
- dlamrg
- dlaneg
- dlangb
- dlange
- dlangt
- dlanhs
- dlansb
- dlansp
- dlanst
- dlansy
- dlantb
- dlantp
- dlantr
- dlanv2
- dlapll
- dlapmt
- dlapy2
- dlapy3
- dlaqgb
- dlaqge
- dlaqp2
- dlaqps
- dlaqr0



- `dlaqr1`
- `dlaqr2`
- `dlaqr3`
- `dlaqr4`
- `dlaqr5`
- `dlaqsb`
- `dlaqsp`
- `dlaqsy`
- `dlaqtr`
- `dlar1v`
- `dlar2v`
- `dlarf`
- `dlarfb`
- `dlarfg`
- `dlarft`
- `dlarfx`
- `dlargv`
- `dlarnv`
- `dlarra`
- `dlarrb`
- `dlarrc`
- `dlarrd`
- `dlarre`
- `dlarrf`
- `dlarrj`
- `dlarrk`
- `dlarrl`
- `dlarrv`
- `dlartg`
- `dlartv`
- `dlaruv`
- `dlarz`
- `dlarzb`
- `dlarzt`
- `dlas2`
- `dlascl`

- dlasd0
- dlasd1
- dlasd2
- dlasd3
- dlasd4
- dlasd5
- dlasd6
- dlasd7
- dlasd8
- dlasda
- dlasdq
- dlasdt
- dlaset
- dlasq1
- dlasq2
- dlasq6
- dlasr
- dlasrt
- dlassq
- dlasv2
- dlaswp
- dlasy2
- dlasyf
- dlatbs
- dlatdf
- dlatps
- dlatrd
- dlatrs
- dlatrz
- dlauu2
- dlauum
- dopgtr
- dopmtr
- dorg2l
- dorg2r
- dorgbr

- dorghr
- dorgl2
- dorglq
- dorgql
- dorgqr
- dorgr2
- dorgrq
- dorgtr
- dorm2l
- dorm2r
- dormbr
- dormhr
- dorml2
- dormlq
- dormql
- dormqr
- dormr2
- dormr3
- dormrq
- dormrz
- dormtr
- dpbcon
- dpbequ
- dpbrfs
- dpbstf
- dpbsv
- dpbsvx
- dpbtf2
- dpbtrf
- dpbtrs
- dpocon
- dpoequ
- dporfs
- dposv
- dposvx
- dpotf2

- dpotrf
- dpotri
- dpotrs
- dppcon
- dppequ
- dpprfs
- dppsv
- dppsvx
- dpptrf
- dpptri
- dpptrs
- dptcon
- dpteqr
- dptrfs
- dptsv
- dptsvx
- dpttrf
- dpttrs
- dptts2
- drscl
- dsbev
- dsbevd
- dsbevz
- dsbgst
- dsbgv
- dsbgvd
- dsbgvx
- dsbtrd
- dsgeev
- dspcon
- dspev
- dspevd
- dspevx
- dspgst
- dspgv
- dspgvd

- `dspgvx`
- `dsprfs`
- `dspsv`
- `dspsvx`
- `dsptrd`
- `dsptrf`
- `dsptri`
- `dsptrs`
- `dstebz`
- `dstedc`
- `dstegr`
- `dstein`
- `dstemr`
- `dsteqr`
- `dsterf`
- `dstev`
- `dstevd`
- `dstevr`
- `dstevx`
- `dsycon`
- `dsyev`
- `dsyevd`
- `dsyevr`
- `dsyevx`
- `dsygs2`
- `dsygst`
- `dsygv`
- `dsygvd`
- `dsygvx`
- `dsyrfs`
- `dsysv`
- `dsysvx`
- `dsytd2`
- `dsytf2`
- `dsytrd`
- `dsytrf`

- dsytri
- dsytrs
- dtbcon
- dtbrfs
- dtbtrs
- dtgevc
- dtgex2
- dtgexc
- dtgsen
- dtgsja
- dtgsna
- dtgsy2
- dtgsyl
- dtpcon
- dtprfs
- dtptri
- dtptrs
- dtrcon
- dtrevc
- dtrexc
- dtrrfs
- dtrsen
- dtrsna
- dtrsyl
- dtrti2
- dtrtri
- dtrtrs
- dtzrzf
- dzsum1
- icmax1
- ieeck
- ilaver
- izmax1
- sbdsdc
- sbdsqr
- scsum1

- sdisna
- sgbbdd
- sgbcon
- sgbequ
- sgbrfs
- sgbsv
- sgbsvx
- sgbtf2
- sgbtrf
- sgbtrs
- sgebak
- sgebal
- sgebd2
- sgebrd
- sgecon
- sgeequ
- sgees
- sgeesx
- sgeev
- sgeevx
- sgehd2
- sgehrd
- sgelq2
- sgelqf
- sgels
- sgelsd
- sgelss
- sgelsy
- sgeql2
- sgeqlf
- sgeqp3
- sgeqr2
- sgeqrf
- sgerfs
- sgerq2
- sgerqf

- `sgesc2`
- `sgesdd`
- `sgesv`
- `sgesvd`
- `sgesvx`
- `sgetc2`
- `sgetf2`
- `sgetrf`
- `sgetri`
- `sgetrs`
- `sggbak`
- `sggbal`
- `sgges`
- `sggesx`
- `sggev`
- `sggevx`
- `sggglm`
- `sgghrd`
- `sgglse`
- `sggqrf`
- `sggrqf`
- `sgtcon`
- `sgtrfs`
- `sgtsv`
- `sgtsvx`
- `sgttrf`
- `sgttrs`
- `sgtts2`
- `shgeqz`
- `shsein`
- `shseqr`
- `slabad`
- `slabrd`
- `slacn2`
- `slacon`
- `slacpy`



- sladiv
- slae2
- slaebz
- slaed0
- slaed1
- slaed2
- slaed3
- slaed4
- slaed5
- slaed6
- slaed7
- slaed8
- slaed9
- slaeda
- slaein
- slaev2
- slaexc
- slag2
- slag2d
- slags2
- slagtf
- slagtm
- slagts
- slagv2
- slahqr
- slahr2
- slaic1
- slaln2
- slals0
- slalsa
- slalsd
- slamch
- slamrg
- slangb
- slange
- slangt

- slanh
- slansb
- slansp
- slanst
- slansy
- slantb
- slantp
- slantr
- slanv2
- slapll
- slapmt
- slapy2
- slapy3
- slaqgb
- slaqge
- slaqp2
- slaqps
- slaqr0
- slaqr1
- slaqr2
- slaqr3
- slaqr4
- slaqr5
- slaqsb
- slaqsp
- slaqsy
- slaqtr
- slar1v
- slar2v
- slarf
- slarfb
- slarfg
- slarft
- slarfx
- slargv
- slarnv

- slarra
- slarrb
- slarrc
- slarrd
- slarre
- slarrf
- slarrj
- slarrk
- slarrl
- slarrm
- slarrn
- slarrp
- slarrq
- slarrs
- slarrt
- slarrv
- slartg
- slartv
- slaruv
- slarz
- slarzb
- slarzt
- slas2
- slascl
- slasd0
- slasd1
- slasd2
- slasd3
- slasd4
- slasd5
- slasd6
- slasd7
- slasd8
- slasda
- slasdq
- slasdt
- slaset
- slasq1
- slasq2
- slasq6
- slasr
- slasrt

- `slassq`
- `slasv2`
- `slaswp`
- `slasy2`
- `slasyf`
- `slatbs`
- `slatdf`
- `slatps`
- `slatrd`
- `slatrs`
- `slatz`
- `slauu2`
- `slauum`
- `sopgtr`
- `sopmtr`
- `sorg2l`
- `sorg2r`
- `sorgbr`
- `sorghr`
- `sorgl2`
- `sorglq`
- `sorgql`
- `sorgqr`
- `sorgr2`
- `sorgrq`
- `sorgtr`
- `sorm2l`
- `sorm2r`
- `sormbr`
- `sormhr`
- `sorml2`
- `sormlq`
- `sormql`
- `sormqr`
- `sormr2`
- `sormr3`

- sormrq
- sormrz
- sormtr
- spbcon
- spbequ
- spbrfs
- spbstf
- spbsv
- spbsvx
- spbtf2
- spbtrf
- spbtrs
- spocon
- spoequ
- sporfs
- sposv
- sposvx
- spotf2
- spotrf
- spotri
- spotrs
- sppcon
- sppequ
- spprfs
- sppsv
- sppsvx
- spptrf
- spptri
- spptrs
- sptcon
- spteqr
- spttrfs
- sptsv
- sptsvx
- spttrf
- spttrs

- `sptts2`
- `srscl`
- `ssbev`
- `ssbevd`
- `ssbevz`
- `ssbgst`
- `ssbgv`
- `ssbgvd`
- `ssbgvx`
- `ssbtrd`
- `sspcon`
- `sspev`
- `sspevd`
- `sspevx`
- `sspgst`
- `sspgv`
- `sspgvd`
- `sspgvx`
- `ssprfs`
- `sspsv`
- `sspsvx`
- `ssptrd`
- `ssptrf`
- `ssptri`
- `ssptrs`
- `sstebz`
- `sstedc`
- `sstegr`
- `sstein`
- `sstemr`
- `ssteqr`
- `ssterf`
- `sstev`
- `sstevd`
- `sstevr`
- `sstevx`

- `ssycon`
- `ssyev`
- `ssyevd`
- `ssyevr`
- `ssyevx`
- `ssygs2`
- `ssygst`
- `ssygv`
- `ssygvd`
- `ssygvx`
- `ssyrfs`
- `ssysv`
- `ssysvx`
- `ssytd2`
- `ssytf2`
- `ssytrd`
- `ssytrf`
- `ssytri`
- `ssytrs`
- `stbcon`
- `stbrfs`
- `stbtrs`
- `stgevc`
- `stgex2`
- `stgexc`
- `stgsen`
- `stgsja`
- `stgsna`
- `stgsy2`
- `stgsyl`
- `stpcon`
- `stprfs`
- `stptri`
- `stptrs`
- `strcon`
- `strevc`

- strexc
- strrfs
- strsen
- strсна
- strsyl
- strti2
- strtri
- strtrs
- stzrzf
- zbdsqr
- zdrscl
- zgbbnd
- zgbcn
- zgbequ
- zgbrfs
- zgbsv
- zgbsvx
- zgbtf2
- zgbtrf
- zgbtrs
- zgebak
- zgebal
- zgebd2
- zgebrd
- zgecon
- zgeequ
- zgees
- zgeesx
- zgeev
- zgeevx
- zgehd2
- zgehrd
- zgelq2
- zgelqf
- zgels
- zgelsd



- zgelss
- zgelsy
- zgeql2
- zgeqlf
- zgeqp3
- zgeqr2
- zgeqrf
- zgerfs
- zgerq2
- zgerqf
- zgesc2
- zgesdd
- zgesv
- zgesvd
- zgesvx
- zgetc2
- zgetf2
- zgetrf
- zgetri
- zgetrs
- zggbak
- zggbal
- zgges
- zggesx
- zggev
- zggevz
- zgglm
- zgghrd
- zgglse
- zggrqf
- zggrqf
- zgtcon
- zgtrfs
- zgtsv
- zgtsvx
- zgttrf

- zgttrs
- zgts2
- zhbev
- zhbevd
- zhbevz
- zhbgst
- zhbgv
- zhbgvd
- zhbgvx
- zhbtrd
- zhecon
- zheev
- zheevd
- zheevr
- zheevz
- zhegs2
- zhegst
- zhegv
- zhegvd
- zhegvz
- zherfs
- zhesv
- zhesvx
- zhetd2
- zhetf2
- zhetrd
- zhetrf
- zhetri
- zhetrs
- zhgeqz
- zhpcon
- zhpev
- zhpevd
- zhpevx
- zhpgst
- zhpgv

- zhpgvd
- zhpgvx
- zhprfs
- zhpsv
- zhpsvx
- zhptrd
- zhptrf
- zhptri
- zhptrs
- zhsein
- zhseqr
- zlabrd
- zlacgv
- zlacn2
- zlacon
- zlap2
- zlapcy
- zlacrm
- zlacrt
- zladiv
- zlaed0
- zlaed7
- zlaed8
- zlaein
- zlaesy
- zlaev2
- zlag2c
- zlags2
- zlagtm
- zlahef
- zlahqr
- zlahr2
- zlaic1
- zlals0
- zlalsa
- zlalsd

- zlangb
- zlange
- zlangt
- zlanhb
- zlanhe
- zlanhp
- zlanhs
- zlanht
- zlansb
- zlansp
- zlansy
- zlantb
- zlantp
- zlantr
- zlapll
- zlapmt
- zlaqgb
- zlaqge
- zlaqhb
- zlaqhe
- zlaqhp
- zlaqp2
- zlaqps
- zlaqr0
- zlaqr1
- zlaqr2
- zlaqr3
- zlaqr4
- zlaqr5
- zlaqsb
- zlaqsp
- zlaqsy
- zlar1v
- zlar2v
- zlarcn
- zlarf

- zlarfb
- zlarfg
- zlarft
- zlarfx
- zlargv
- zlarnv
- zlarrv
- zlarg
- zlargv
- zlarz
- zlarzb
- zlarzt
- zlascl
- zlaset
- zlasr
- zlassq
- zlaswp
- zlasyf
- zlatbs
- zlatdf
- zlatps
- zlatrd
- zlatrs
- zlatrz
- zlauu2
- zlauum
- zpbcon
- zpbequ
- zpbrfs
- zpbstf
- zpbsv
- zpbsvx
- zpbt2
- zpbrf
- zpbrs
- zpocon

- `zpoequ`
- `zporfs`
- `zposv`
- `zposvx`
- `zpotf2`
- `zpotrf`
- `zpotri`
- `zpotrs`
- `zppcon`
- `zppequ`
- `zpprfs`
- `zppsv`
- `zppsvx`
- `zpptrf`
- `zpptri`
- `zpptrs`
- `zptcon`
- `zpteqr`
- `zptrfs`
- `zptsv`
- `zptsvx`
- `zpttrf`
- `zpttrs`
- `zptts2`
- `zrot`
- `zspcon`
- `zspmv`
- `zspr`
- `zsprfs`
- `zspsv`
- `zspsvx`
- `zsptrf`
- `zsptri`
- `zsptrs`
- `zstedc`
- `zstegr`

- zstein
- zstemr
- zsteqr
- zsycon
- zsymv
- zsyr
- zsyrf
- zsysv
- zsysvx
- zsytf2
- zsytrf
- zsytri
- zsytrs
- ztbcon
- ztbrfs
- ztbtrs
- ztgevc
- ztgex2
- ztgexc
- ztgse
- ztgsja
- ztgsna
- ztgsy2
- ztgsyl
- ztpcon
- ztprfs
- ztptri
- ztptrs
- ztrcon
- ztrevc
- ztrexc
- ztrfs
- ztrsen
- ztrsna
- ztrsyl
- ztrti2

- `ztrtri`
- `ztrtrs`
- `ztzrzf`
- `zung2l`
- `zung2r`
- `zungbr`
- `zunghr`
- `zungl2`
- `zunglq`
- `zungql`
- `zungqr`
- `zungr2`
- `zungrq`
- `zungtr`
- `zunm2l`
- `zunm2r`
- `zunmbr`
- `zunmhr`
- `zunml2`
- `zunmlq`
- `zunmql`
- `zunmqr`
- `zunmr2`
- `zunmr3`
- `zunmrq`
- `zunmrz`
- `zunmtr`
- `zupgtr`
- `zupmtr`

## 5.14 Interpolative matrix decomposition (`scipy.linalg.interpolative`)

New in version 0.13.

An interpolative decomposition (ID) of a matrix  $A \in C^{m \times n}$  of rank  $k \leq \min\{m, n\}$  is a factorization

$$A\Pi = [A\Pi_1 \quad A\Pi_2] = A\Pi_1 [I \quad T],$$



where  $\Pi = [\Pi_1, \Pi_2]$  is a permutation matrix with  $\Pi_1 \in \{0, 1\}^{n \times k}$ , i.e.,  $A\Pi_2 = A\Pi_1 T$ . This can equivalently be written as  $A = BP$ , where  $B = A\Pi_1$  and  $P = [I, T]\Pi^T$  are the *skeleton* and *interpolation matrices*, respectively.

If  $A$  does not have exact rank  $k$ , then there exists an approximation in the form of an ID such that  $A = BP + E$ , where  $\|E\| \sim \sigma_{k+1}$  is on the order of the  $(k + 1)$ -th largest singular value of  $A$ . Note that  $\sigma_{k+1}$  is the best possible error for a rank- $k$  approximation and, in fact, is achieved by the singular value decomposition (SVD)  $A \approx USV^*$ , where  $U \in C^{m \times k}$  and  $V \in C^{n \times k}$  have orthonormal columns and  $S = \text{diag}(\sigma_i) \in C^{k \times k}$  is diagonal with nonnegative entries. The principal advantages of using an ID over an SVD are that:

- it is cheaper to construct;
- it preserves the structure of  $A$ ; and
- it is more efficient to compute with in light of the identity submatrix of  $P$ .

### 5.14.1 Routines

Main functionality:

<code>interp_decomp(A, eps_or_k[, rand])</code>	Compute ID of a matrix.
<code>reconstruct_matrix_from_id(B, idx, proj)</code>	Reconstruct matrix from its ID.
<code>reconstruct_interp_matrix(idx, proj)</code>	Reconstruct interpolation matrix from ID.
<code>reconstruct_skel_matrix(A, k, idx)</code>	Reconstruct skeleton matrix from ID.
<code>id_to_svd(B, idx, proj)</code>	Convert ID to SVD.
<code>svd(A, eps_or_k[, rand])</code>	Compute SVD of a matrix via an ID.
<code>estimate_spectral_norm(A[, its])</code>	Estimate spectral norm of a matrix by the randomized power method.
<code>estimate_spectral_norm_diff(A, B[, its])</code>	Estimate spectral norm of the difference of two matrices by the randomized power method.
<code>estimate_rank(A, eps)</code>	Estimate matrix rank to a specified relative precision using randomized method.

`scipy.linalg.interpolative.interp_decomp(A, eps_or_k, rand=True)`

Compute ID of a matrix.

An ID of a matrix  $A$  is a factorization defined by a rank  $k$ , a column index array  $idx$ , and interpolation coefficients  $proj$  such that:

```
numpy.dot(A[:, idx[:k]], proj) = A[:, idx[k:]]
```

The original matrix can then be reconstructed as:

```
numpy.hstack([A[:, idx[:k]],
              numpy.dot(A[:, idx[k:]], proj)]
            )[: , numpy.argsort(idx)]
```

or via the routine `reconstruct_matrix_from_id`. This can equivalently be written as:

```
numpy.dot(A[:, idx[:k]],
          numpy.hstack([numpy.eye(k), proj])
         )[: , np.argsort(idx)]
```

in terms of the skeleton and interpolation matrices:

```
B = A[:, idx[:k]]
```

and:

```
P = numpy.hstack([numpy.eye(k), proj])[: , np.argsort(idx)]
```

respectively. See also `reconstruct_interp_matrix` and `reconstruct_skel_matrix`.

The ID can be computed to any relative precision or rank (depending on the value of *eps\_or\_k*). If a precision is specified (*eps\_or\_k* < 1), then this function has the output signature:

```
k, idx, proj = interp_decomp(A, eps_or_k)
```

Otherwise, if a rank is specified (*eps\_or\_k* >= 1), then the output signature is:

```
idx, proj = interp_decomp(A, eps_or_k)
```

**Parameters**

- A** : `numpy.ndarray` or `scipy.sparse.linalg.LinearOperator` with *rmatvec*  
Matrix to be factored
- eps\_or\_k** : float or int  
Relative error (if *eps\_or\_k* < 1) or rank (if *eps\_or\_k* >= 1) of approximation.
- rand** : bool, optional  
Whether to use random sampling if *A* is of type `numpy.ndarray` (randomized algorithms are always used if *A* is of type `scipy.sparse.linalg.LinearOperator`).

**Returns**

- k** : int  
Rank required to achieve specified relative precision if *eps\_or\_k* < 1.
- idx** : `numpy.ndarray`  
Column index array.
- proj** : `numpy.ndarray`  
Interpolation coefficients.

`scipy.linalg.interpolative.reconstruct_matrix_from_id(B, idx, proj)`

Reconstruct matrix from its ID.

A matrix *A* with skeleton matrix *B* and ID indices and coefficients *idx* and *proj*, respectively, can be reconstructed as:

```
numpy.hstack([B, numpy.dot(B, proj)][:, numpy.argsort(idx)])
```

See also `reconstruct_interp_matrix` and `reconstruct_skel_matrix`.

**Parameters**

- B** : `numpy.ndarray`  
Skeleton matrix.
- idx** : `numpy.ndarray`  
Column index array.
- proj** : `numpy.ndarray`  
Interpolation coefficients.

**Returns**

- `numpy.ndarray`  
Reconstructed matrix.

`scipy.linalg.interpolative.reconstruct_interp_matrix(idx, proj)`

Reconstruct interpolation matrix from ID.

The interpolation matrix can be reconstructed from the ID indices and coefficients *idx* and *proj*, respectively, as:

```
P = numpy.hstack([numpy.eye(proj.shape[0]), proj])[:, numpy.argsort(idx)]
```

The original matrix can then be reconstructed from its skeleton matrix *B* via:

```
numpy.dot(B, P)
```

See also `reconstruct_matrix_from_id` and `reconstruct_skel_matrix`.

**Parameters**

- idx** : `numpy.ndarray`  
Column index array.

**Returns** `proj`: `numpy.ndarray`  
Interpolation coefficients.  
`numpy.ndarray`  
Interpolation matrix.

`scipy.linalg.interpolative.reconstruct_skel_matrix(A, k, idx)`  
Reconstruct skeleton matrix from ID.

The skeleton matrix can be reconstructed from the original matrix  $A$  and its ID rank and indices  $k$  and  $idx$ , respectively, as:

```
B = A[:, idx[:k]]
```

The original matrix can then be reconstructed via:

```
numpy.hstack([B, numpy.dot(B, proj)][:, numpy.argsort(idx)])
```

See also `reconstruct_matrix_from_id` and `reconstruct_interp_matrix`.

**Parameters** `A`: `numpy.ndarray`  
Original matrix.  
`k`: `int`  
Rank of ID.  
`idx`: `numpy.ndarray`  
Column index array.  
**Returns** `numpy.ndarray`  
Skeleton matrix.

`scipy.linalg.interpolative.id_to_svd(B, idx, proj)`  
Convert ID to SVD.

The SVD reconstruction of a matrix with skeleton matrix  $B$  and ID indices and coefficients  $idx$  and  $proj$ , respectively, is:

```
U, S, V = id_to_svd(B, idx, proj)
A = numpy.dot(U, numpy.dot(numpy.diag(S), V.conj().T))
```

See also `svd`.

**Parameters** `B`: `numpy.ndarray`  
Skeleton matrix.  
`idx`: `numpy.ndarray`  
Column index array.  
`proj`: `numpy.ndarray`  
Interpolation coefficients.  
**Returns** `U`: `numpy.ndarray`  
Left singular vectors.  
`S`: `numpy.ndarray`  
Singular values.  
`V`: `numpy.ndarray`  
Right singular vectors.

`scipy.linalg.interpolative.svd(A, eps_or_k, rand=True)`  
Compute SVD of a matrix via an ID.

An SVD of a matrix  $A$  is a factorization:

```
A = numpy.dot(U, numpy.dot(numpy.diag(S), V.conj().T))
```

where  $U$  and  $V$  have orthonormal columns and  $S$  is nonnegative.

The SVD can be computed to any relative precision or rank (depending on the value of `eps_or_k`).

See also `interp_decomp` and `id_to_svd`.

**Parameters** **A**: `numpy.ndarray` or `scipy.sparse.linalg.LinearOperator`  
 Matrix to be factored, given as either a `numpy.ndarray` or a `scipy.sparse.linalg.LinearOperator` with the `matvec` and `rmatvec` methods (to apply the matrix and its adjoint).  
**eps\_or\_k**: float or int  
 Relative error (if `eps_or_k < 1`) or rank (if `eps_or_k >= 1`) of approximation.  
**rand**: bool, optional  
 Whether to use random sampling if  $A$  is of type `numpy.ndarray` (randomized algorithms are always used if  $A$  is of type `scipy.sparse.linalg.LinearOperator`).

**Returns** **U**: `numpy.ndarray`  
 Left singular vectors.  
**S**: `numpy.ndarray`  
 Singular values.  
**V**: `numpy.ndarray`  
 Right singular vectors.

`scipy.linalg.interpolative.estimate_spectral_norm(A, its=20)`

Estimate spectral norm of a matrix by the randomized power method.

**Parameters** **A**: `scipy.sparse.linalg.LinearOperator`  
 Matrix given as a `scipy.sparse.linalg.LinearOperator` with the `matvec` and `rmatvec` methods (to apply the matrix and its adjoint).  
**its**: int, optional  
 Number of power method iterations.

**Returns** float  
 Spectral norm estimate.

`scipy.linalg.interpolative.estimate_spectral_norm_diff(A, B, its=20)`

Estimate spectral norm of the difference of two matrices by the randomized power method.

**Parameters** **A**: `scipy.sparse.linalg.LinearOperator`  
 First matrix given as a `scipy.sparse.linalg.LinearOperator` with the `matvec` and `rmatvec` methods (to apply the matrix and its adjoint).  
**B**: `scipy.sparse.linalg.LinearOperator`  
 Second matrix given as a `scipy.sparse.linalg.LinearOperator` with the `matvec` and `rmatvec` methods (to apply the matrix and its adjoint).  
**its**: int, optional  
 Number of power method iterations.

**Returns** float  
 Spectral norm estimate of matrix difference.

`scipy.linalg.interpolative.estimate_rank(A, eps)`

Estimate matrix rank to a specified relative precision using randomized methods.

The matrix  $A$  can be given as either a `numpy.ndarray` or a `scipy.sparse.linalg.LinearOperator`, with different algorithms used for each case. If  $A$  is of type `numpy.ndarray`, then the output rank is typically about 8 higher than the actual numerical rank.

**Parameters** **A**: `numpy.ndarray` or `scipy.sparse.linalg.LinearOperator`  
 Matrix whose rank is to be estimated, given as either a `numpy.ndarray` or a `scipy.sparse.linalg.LinearOperator` with the `rmatvec` method (to apply the matrix adjoint).

**Returns**

**eps** : float  
Relative error for numerical rank definition.

**int**  
Estimated matrix rank.

Support functions:

---

`seed([seed])` Seed the internal random number generator used in this ID package.

`rand(*shape)` Generate standard uniform pseudorandom numbers via a very efficient lagged Fibonacci method.

---

`scipy.linalg.interpolative.seed(seed=None)`

Seed the internal random number generator used in this ID package.

The generator is a lagged Fibonacci method with 55-element internal state.

**Parameters** **seed** : int, sequence, 'default', optional

If 'default', the random seed is reset to a default value.

If `seed` is a sequence containing 55 floating-point numbers in range [0,1], these are used to set the internal state of the generator.

If the value is an integer, the internal state is obtained from `numpy.random.RandomState` (MT19937) with the integer used as the initial seed.

If `seed` is omitted (None), `numpy.random` is used to initialize the generator.

`scipy.linalg.interpolative.rand(*shape)`

Generate standard uniform pseudorandom numbers via a very efficient lagged Fibonacci method.

This routine is used for all random number generation in this package and can affect ID and SVD results.

**Parameters** **shape**

Shape of output array

## 5.14.2 References

This module uses the ID software package [R661] by Martinsson, Rokhlin, Shkolnisky, and Tygert, which is a Fortran library for computing IDs using various algorithms, including the rank-revealing QR approach of [R662] and the more recent randomized methods described in [R663], [R664], and [R665]. This module exposes its functionality in a way convenient for Python users. Note that this module does not add any functionality beyond that of organizing a simpler and more consistent interface.

We advise the user to consult also the [documentation for the ID package](#).

## 5.14.3 Tutorial

### Initializing

The first step is to import `scipy.linalg.interpolative` by issuing the command:

```
>>> import scipy.linalg.interpolative as sli
```

Now let's build a matrix. For this, we consider a Hilbert matrix, which is well know to have low rank:

```
>>> from scipy.linalg import hilbert
>>> n = 1000
>>> A = hilbert(n)
```

We can also do this explicitly via:

```
>>> import numpy as np
>>> n = 1000
>>> A = np.empty((n, n), order='F')
>>> for j in range(n):
>>>     for i in range(m):
>>>         A[i,j] = 1. / (i + j + 1)
```

Note the use of the flag `order='F'` in `numpy.empty`. This instantiates the matrix in Fortran-contiguous order and is important for avoiding data copying when passing to the backend.

We then define multiplication routines for the matrix by regarding it as a `scipy.sparse.linalg.LinearOperator`:

```
>>> from scipy.sparse.linalg import aslinearoperator
>>> L = aslinearoperator(A)
```

This automatically sets up methods describing the action of the matrix and its adjoint on a vector.

## Computing an ID

We have several choices of algorithm to compute an ID. These fall largely according to two dichotomies:

1. how the matrix is represented, i.e., via its entries or via its action on a vector; and
2. whether to approximate it to a fixed relative precision or to a fixed rank.

We step through each choice in turn below.

In all cases, the ID is represented by three parameters:

1. a rank `k`;
2. an index array `idx`; and
3. interpolation coefficients `proj`.

The ID is specified by the relation `np.dot(A[:,idx[:k]], proj) == A[:,idx[k:]]`.

### *From matrix entries*

We first consider a matrix given in terms of its entries.

To compute an ID to a fixed precision, type:

```
>>> k, idx, proj = sli.interp_decomp(A, eps)
```

where `eps < 1` is the desired precision.

To compute an ID to a fixed rank, use:

```
>>> idx, proj = sli.interp_decomp(A, k)
```

where  $k \geq 1$  is the desired rank.

Both algorithms use random sampling and are usually faster than the corresponding older, deterministic algorithms, which can be accessed via the commands:

```
>>> k, idx, proj = sli.interp_decomp(A, eps, rand=False)
```

and:

```
>>> idx, proj = sli.interp_decomp(A, k, rand=False)
```

respectively.

### *From matrix action*

Now consider a matrix given in terms of its action on a vector as a `scipy.sparse.linalg.LinearOperator`.

To compute an ID to a fixed precision, type:

```
>>> k, idx, proj = sli.interp_decomp(L, eps)
```

To compute an ID to a fixed rank, use:

```
>>> idx, proj = sli.interp_decomp(L, k)
```

These algorithms are randomized.

## Reconstructing an ID

The ID routines above do not output the skeleton and interpolation matrices explicitly but instead return the relevant information in a more compact (and sometimes more useful) form. To build these matrices, write:

```
>>> B = sli.reconstruct_skel_matrix(A, k, idx)
```

for the skeleton matrix and:

```
>>> P = sli.reconstruct_interp_matrix(idx, proj)
```

for the interpolation matrix. The ID approximation can then be computed as:

```
>>> C = np.dot(B, P)
```

This can also be constructed directly using:

```
>>> C = sli.reconstruct_matrix_from_id(B, idx, proj)
```

without having to first compute P.

Alternatively, this can be done explicitly as well using:

```
>>> B = A[:, idx[:k]]
>>> P = np.hstack([np.eye(k), proj])[:, np.argsort(idx)]
>>> C = np.dot(B, P)
```

## Computing an SVD

An ID can be converted to an SVD via the command:

```
>>> U, S, V = sli.id_to_svd(B, idx, proj)
```

The SVD approximation is then:

```
>>> C = np.dot(U, np.dot(np.diag(S), np.dot(V.conj().T)))
```

The SVD can also be computed “fresh” by combining both the ID and conversion steps into one command. Following the various ID algorithms above, there are correspondingly various SVD algorithms that one can employ.

### *From matrix entries*

We consider first SVD algorithms for a matrix given in terms of its entries.

To compute an SVD to a fixed precision, type:

```
>>> U, S, V = sli.svd(A, eps)
```

To compute an SVD to a fixed rank, use:

```
>>> U, S, V = sli.svd(A, k)
```

Both algorithms use random sampling; for the deterministic versions, issue the keyword `rand=False` as above.

### *From matrix action*

Now consider a matrix given in terms of its action on a vector.

To compute an SVD to a fixed precision, type:

```
>>> U, S, V = sli.svd(L, eps)
```

To compute an SVD to a fixed rank, use:

```
>>> U, S, V = sli.svd(L, k)
```

## Utility routines

Several utility routines are also available.

To estimate the spectral norm of a matrix, use:

```
>>> snorm = sli.estimate_spectral_norm(A)
```

This algorithm is based on the randomized power method and thus requires only matrix-vector products. The number of iterations to take can be set using the keyword `its` (default: `its=20`). The matrix is interpreted as a `scipy.sparse.linalg.LinearOperator`, but it is also valid to supply it as a `numpy.ndarray`, in which case it is trivially converted using `scipy.sparse.linalg.aslinearoperator`.

The same algorithm can also estimate the spectral norm of the difference of two matrices `A1` and `A2` as follows:



```
>>> diff = sli.estimate_spectral_norm_diff(A1, A2)
```

This is often useful for checking the accuracy of a matrix approximation.

Some routines in `scipy.linalg.interpolative` require estimating the rank of a matrix as well. This can be done with either:

```
>>> k = sli.estimate_rank(A, eps)
```

or:

```
>>> k = sli.estimate_rank(L, eps)
```

depending on the representation. The parameter `eps` controls the definition of the numerical rank.

Finally, the random number generation required for all randomized routines can be controlled via `scipy.linalg.interpolative.seed`. To reset the seed values to their original values, use:

```
>>> sli.seed('default')
```

To specify the seed values, use:

```
>>> sli.seed(s)
```

where `s` must be an integer or array of 55 floats. If an integer, the array of floats is obtained by using `np.random.rand` with the given integer seed.

To simply generate some random numbers, type:

```
>>> sli.rand(n)
```

where `n` is the number of random numbers to generate.

## Remarks

The above functions all automatically detect the appropriate interface and work with both real and complex data types, passing input arguments to the proper backend routine.

## 5.15 Miscellaneous routines (`scipy.misc`)

Various utilities that don't have another home.

Note that the Python Imaging Library (PIL) is not a dependency of SciPy and therefore the `pilutil` module is not available on systems that don't have PIL installed.

<code>ascent()</code>	Get an 8-bit grayscale bit-depth, 512 x 512 derived image for easy use in demos
<code>bytescale(data[, cmin, cmax, high, low])</code>	Byte scales an array (image).
<code>central_diff_weights(Np[, ndiv])</code>	Return weights for an Np-point central derivative.
<code>comb(N, k[, exact, repetition])</code>	The number of combinations of N things taken k at a time.
<code>derivative(func, x0[, dx, n, args, order])</code>	Find the n-th derivative of a function at a point.
<code>face([gray])</code>	Get a 1024 x 768, color image of a raccoon face.

Continued on next page

Table 5.87 – continued from previous page

<code>factorial(n[, exact])</code>	The factorial of a number or array of numbers.
<code>factorial2(n[, exact])</code>	Double factorial.
<code>factorialk(n, k[, exact])</code>	Multifactorial of n of order k, n(!...!).
<code>fromimage(im[, flatten, mode])</code>	Return a copy of a PIL image as a numpy array.
<code>imfilter(arr, ftype)</code>	Simple filtering of an image.
<code>imread(name[, flatten, mode])</code>	Read an image from a file as an array.
<code>imresize(arr, size[, interp, mode])</code>	Resize an image.
<code>imrotate(arr, angle[, interp])</code>	Rotate an image counter-clockwise by angle degrees.
<code>imsave(name, arr[, format])</code>	Save an array as an image.
<code>imshow(arr)</code>	Simple showing of an image through an external viewer.
<code>info([object, maxwidth, output, toplevel])</code>	Get help information for a function, class, or module.
<code>lena()</code>	Function that previously returned an example image
<code>logsumexp(a[, axis, b, keepdims, return_sign])</code>	Compute the log of the sum of exponentials of input elements.
<code>pade(an, m)</code>	Return Pade approximation to a polynomial as the ratio of two polynomials.
<code>toimage(arr[, high, low, cmin, cmax, pal, ...])</code>	Takes a numpy array and returns a PIL image.
<code>source(object[, output])</code>	Print or write to a file the source code for a Numpy object.
<code>who([vardict])</code>	Print the Numpy arrays in the given dictionary.

`scipy.misc.ascent()`

Get an 8-bit grayscale bit-depth, 512 x 512 derived image for easy use in demos

The image is derived from accent-to-the-top.jpg at <http://www.public-domain-image.com/people-public-domain-images-pictures/>

**Parameters**    **None**

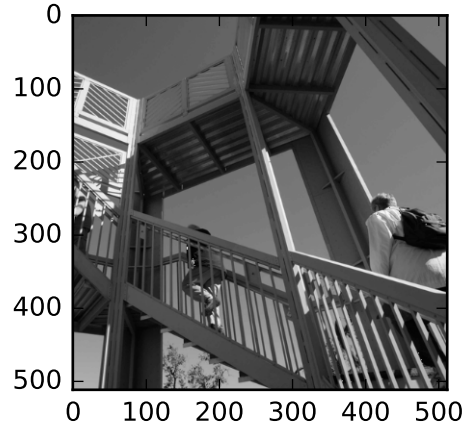
**Returns**        **ascent** : ndarray

convenient image to use for testing and demonstration

**Examples**

```
>>> import scipy.misc
>>> ascent = scipy.misc.ascent()
>>> ascent.shape
(512, 512)
>>> ascent.max()
255

>>> import matplotlib.pyplot as plt
>>> plt.gray()
>>> plt.imshow(ascent)
>>> plt.show()
```



`scipy.misc.bytescale` (*data*, *cmin=None*, *cmax=None*, *high=255*, *low=0*)

Byte scales an array (image).

Byte scaling means converting the input image to uint8 dtype and scaling the range to (*low*, *high*) (default 0-255). If the input image already has dtype uint8, no scaling is done.

**Parameters**

- data** : ndarray  
PIL image data array.
- cmin** : scalar, optional  
Bias scaling of small values. Default is `data.min()`.
- cmax** : scalar, optional  
Bias scaling of large values. Default is `data.max()`.
- high** : scalar, optional  
Scale max value to *high*. Default is 255.
- low** : scalar, optional  
Scale min value to *low*. Default is 0.

**Returns**

- img\_array** : uint8 ndarray  
The byte-scaled array.

### Examples

```
>>> from scipy.misc import bytescale
>>> img = np.array([[ 91.06794177,   3.39058326,  84.4221549 ],
...                [ 73.88003259,  80.91433048,   4.88878881],
...                [ 51.53875334,  34.45808177,  27.5873488 ]])
>>> bytescale(img)
array([[255,   0, 236],
       [205, 225,   4],
       [140,  90,  70]], dtype=uint8)
>>> bytescale(img, high=200, low=100)
array([[200, 100, 192],
       [180, 188, 102],
       [155, 135, 128]], dtype=uint8)
>>> bytescale(img, cmin=0, cmax=255)
array([[91,   3, 84],
       [74, 81,   5],
       [52, 34, 28]], dtype=uint8)
```

`scipy.misc.central_diff_weights` (*Np*, *ndiv=1*)

Return weights for an *Np*-point central derivative.

Assumes equally-spaced function points.

If weights are in the vector *w*, then derivative is  $w[0] * f(x-h_0*dx) + \dots + w[-1] * f(x+h_0*dx)$

**Parameters**

- Np** : int  
Number of points for the central derivative.
- ndiv** : int, optional  
Number of divisions. Default is 1.

**Notes**

Can be inaccurate for large number of points.

`scipy.misc.comb` (*N*, *k*, *exact=False*, *repetition=False*)

The number of combinations of *N* things taken *k* at a time.

This is often expressed as “*N* choose *k*”.

**Parameters**

- N** : int, ndarray  
Number of things.
- k** : int, ndarray  
Number of elements taken.
- exact** : bool, optional  
If *exact* is False, then floating point precision is used, otherwise exact long integer is computed.
- repetition** : bool, optional  
If *repetition* is True, then the number of combinations with repetition is computed.

**Returns**

- val** : int, ndarray  
The total number of combinations.

**Notes**

- Array arguments accepted only for *exact=False* case.
- If  $k > N$ ,  $N < 0$ , or  $k < 0$ , then a 0 is returned.

**Examples**

```
>>> from scipy.special import comb
>>> k = np.array([3, 4])
>>> n = np.array([10, 10])
>>> comb(n, k, exact=False)
array([ 120.,  210.])
>>> comb(10, 3, exact=True)
120L
>>> comb(10, 3, exact=True, repetition=True)
220L
```

`scipy.misc.derivative` (*func*, *x0*, *dx=1.0*, *n=1*, *args=()*, *order=3*)

Find the *n*-th derivative of a function at a point.

Given a function, use a central difference formula with spacing *dx* to compute the *n*-th derivative at *x0*.

**Parameters**

- func** : function  
Input function.
- x0** : float  
The point at which *n*-th derivative is found.
- dx** : float, optional

Spacing.  
**n** : int, optional  
 Order of the derivative. Default is 1.  
**args** : tuple, optional  
 Arguments  
**order** : int, optional  
 Number of points to use, must be odd.

**Notes**

Decreasing the step size too small can result in round-off error.

**Examples**

```
>>> from scipy.misc import derivative
>>> def f(x):
...     return x**3 + x**2
>>> derivative(f, 1.0, dx=1e-6)
4.999999999217337
```

`scipy.misc.face` (*gray=False*)

Get a 1024 x 768, color image of a raccoon face.

raccoon-procyon-lotor.jpg at <http://www.public-domain-image.com>

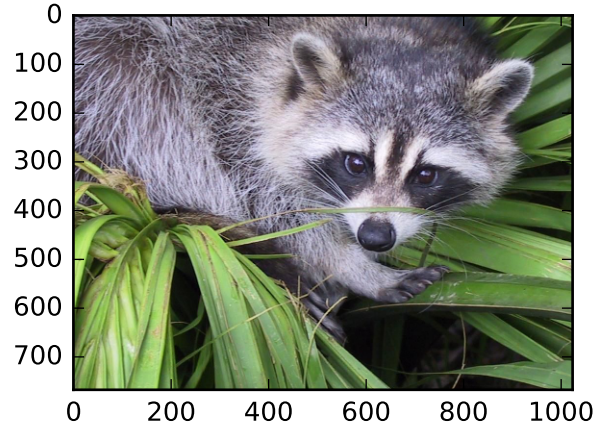
**Parameters**    **gray** : bool, optional  
 If True return 8-bit grey-scale image, otherwise return a color image

**Returns**        **face** : ndarray  
 image of a racoon face

**Examples**

```
>>> import scipy.misc
>>> face = scipy.misc.face()
>>> face.shape
(768, 1024, 3)
>>> face.max()
255
>>> face.dtype
dtype('uint8')
```

```
>>> import matplotlib.pyplot as plt
>>> plt.gray()
>>> plt.imshow(face)
>>> plt.show()
```



`scipy.misc.factorial` (*n*, *exact=False*)

The factorial of a number or array of numbers.

The factorial of non-negative integer *n* is the product of all positive integers less than or equal to *n*:

$$n! = n * (n - 1) * (n - 2) * \dots * 1$$

**Parameters**    **n** : int or array\_like of ints

Input values. If *n* < 0, the return value is 0.

**exact** : bool, optional

If True, calculate the answer exactly using long integer arithmetic. If False, result is approximated in floating point rapidly using the *gamma* function. Default is False.

**Returns**        **nf** : float or int or ndarray

Factorial of *n*, as integer or float depending on *exact*.

### Notes

For arrays with *exact=True*, the factorial is computed only once, for the largest input, with each other result computed in the process. The output dtype is increased to `int64` or `object` if necessary.

With *exact=False* the factorial is approximated using the gamma function:

$$n! = \Gamma(n + 1)$$

### Examples

```
>>> from scipy.special import factorial
>>> arr = np.array([3, 4, 5])
>>> factorial(arr, exact=False)
array([ 6., 24., 120.])
>>> factorial(arr, exact=True)
array([ 6, 24, 120])
>>> factorial(5, exact=True)
120L
```

`scipy.misc.factorial2` (*n*, *exact=False*)

Double factorial.

This is the factorial with every second value skipped. E.g.,  $7!! = 7 * 5 * 3 * 1$ . It can be approximated numerically as:

```
n!! = special.gamma(n/2+1)*2**((m+1)/2)/sqrt(pi)  n odd
     = 2**(n/2) * (n/2)!                          n even
```

**Parameters** **n** : int or array\_like  
Calculate  $n!!$ . Arrays are only supported with *exact* set to False. If  $n < 0$ , the return value is 0.

**exact** : bool, optional  
The result can be approximated rapidly using the gamma-formula above (default). If *exact* is set to True, calculate the answer exactly using integer arithmetic.

**Returns** **nff** : float or int  
Double factorial of  $n$ , as an int or a float depending on *exact*.

### Examples

```
>>> from scipy.special import factorial2
>>> factorial2(7, exact=False)
array(105.00000000000001)
>>> factorial2(7, exact=True)
105L
```

`scipy.misc.factorialk` ( $n, k, exact=True$ )  
Multifactorial of  $n$  of order  $k$ ,  $n(!!\dots!)$ .

This is the multifactorial of  $n$  skipping  $k$  values. For example,

$$\text{factorialk}(17, 4) = 17!!!! = 17 * 13 * 9 * 5 * 1$$

In particular, for any integer  $n$ , we have

```
factorialk(n, 1) = factorial(n)
factorialk(n, 2) = factorial2(n)
```

**Parameters** **n** : int  
Calculate multifactorial. If  $n < 0$ , the return value is 0.

**k** : int  
Order of multifactorial.

**exact** : bool, optional  
If *exact* is set to True, calculate the answer exactly using integer arithmetic.

**Returns** **val** : int  
Multifactorial of  $n$ .

**Raises** **NotImplementedError**  
Raises when *exact* is False

### Examples

```
>>> from scipy.special import factorialk
>>> factorialk(5, 1, exact=True)
120L
>>> factorialk(5, 3, exact=True)
10L
```

`scipy.misc.fromimage` ( $im, flatten=False, mode=None$ )  
Return a copy of a PIL image as a numpy array.

**Parameters** **im** : PIL image  
Input image.

**flatten** : bool  
 If true, convert the output to grey-scale.

**mode** : str, optional  
 Mode to convert image to, e.g. 'RGB'. See the Notes of the `imread` docstring for more details.

**Returns** **fromimage** : ndarray  
 The different colour bands/channels are stored in the third dimension, such that a grey-image is MxN, an RGB-image MxNx3 and an RGBA-image MxNx4.

`scipy.misc.imfilter(arr, ftype)`  
 Simple filtering of an image.

**Parameters** **arr** : ndarray  
 The array of Image in which the filter is to be applied.

**ftype** : str  
 The filter that has to be applied. Legal values are: 'blur', 'contour', 'detail', 'edge\_enhance', 'edge\_enhance\_more', 'emboss', 'find\_edges', 'smooth', 'smooth\_more', 'sharpen'.

**Returns** **imfilter** : ndarray  
 The array with filter applied.

**Raises** **ValueError**  
*Unknown filter type.* If the filter you are trying to apply is unsupported.

`scipy.misc.imread(name, flatten=False, mode=None)`  
 Read an image from a file as an array.

**Parameters** **name** : str or file object  
 The file name or file object to be read.

**flatten** : bool, optional  
 If True, flattens the color layers into a single gray-scale layer.

**mode** : str, optional  
 Mode to convert image to, e.g. 'RGB'. See the Notes for more details.

**Returns** **imread** : ndarray  
 The array obtained by reading the image.

**Notes**

`imread` uses the Python Imaging Library (PIL) to read an image. The following notes are from the PIL documentation.

*mode* can be one of the following strings:

- 'L' (8-bit pixels, black and white)
- 'P' (8-bit pixels, mapped to any other mode using a color palette)
- 'RGB' (3x8-bit pixels, true color)
- 'RGBA' (4x8-bit pixels, true color with transparency mask)
- 'CMYK' (4x8-bit pixels, color separation)
- 'YCbCr' (3x8-bit pixels, color video format)
- 'I' (32-bit signed integer pixels)
- 'F' (32-bit floating point pixels)

PIL also provides limited support for a few special modes, including 'LA' ('L' with alpha), 'RGBX' (true color with padding) and 'RGBA' (true color with premultiplied alpha).



When translating a color image to black and white (mode 'L', 'I' or 'F'), the library uses the ITU-R 601-2 luma transform:

$$L = R * 299/1000 + G * 587/1000 + B * 114/1000$$

When *flatten* is True, the image is converted using mode 'F'. When *mode* is not None and *flatten* is True, the image is first converted according to *mode*, and the result is then flattened using mode 'F'.

`scipy.misc.imresize` (*arr*, *size*, *interp*='bilinear', *mode*=None)

Resize an image.

**Parameters**

- arr** : ndarray  
The array of image to be resized.
- size** : int, float or tuple
  - int - Percentage of current size.
  - float - Fraction of current size.
  - tuple - Size of the output image.
- interp** : str, optional  
Interpolation to use for re-sizing ('nearest', 'lanczos', 'bilinear', 'bicubic' or 'cubic').
- mode** : str, optional  
The PIL image mode ('P', 'L', etc.) to convert *arr* before resizing.

**Returns**

- imresize** : ndarray  
The resized array of image.

See also:

[`toimage`](#) Implicitly used to convert *arr* according to *mode*.

[`scipy.ndimage.zoom`](#)

More generic implementation that does not use PIL.

`scipy.misc.imrotate` (*arr*, *angle*, *interp*='bilinear')

Rotate an image counter-clockwise by angle degrees.

**Parameters**

- arr** : ndarray  
Input array of image to be rotated.
- angle** : float  
The angle of rotation.
- interp** : str, optional  
Interpolation
  - 'nearest' : for nearest neighbor
  - 'bilinear' : for bilinear
  - 'lanczos' : for lanczos
  - 'cubic' : for bicubic
  - 'bicubic' : for bicubic

**Returns**

- imrotate** : ndarray  
The rotated array of image.

`scipy.misc.imsave` (*name*, *arr*, *format*=None)

Save an array as an image.

**Parameters**

- name** : str or file object  
Output file name or file object.
- arr** : ndarray, MxN or MxNx3 or MxNx4  
Array containing image values. If the shape is MxN, the array represents a grey-level image. Shape MxNx3 stores the red, green and blue bands along the last dimension. An alpha layer may be included, specified as the last colour band of an MxNx4 array.
- format** : str

Image format. If omitted, the format to use is determined from the file name extension. If a file object was used instead of a file name, this parameter should always be used.

### Examples

Construct an array of gradient intensity values and save to file:

```
>>> from scipy.misc import imsave
>>> x = np.zeros((255, 255))
>>> x = np.zeros((255, 255), dtype=np.uint8)
>>> x[:] = np.arange(255)
>>> imsave('gradient.png', x)
```

Construct an array with three colour bands (R, G, B) and store to file:

```
>>> rgb = np.zeros((255, 255, 3), dtype=np.uint8)
>>> rgb[..., 0] = np.arange(255)
>>> rgb[..., 1] = 55
>>> rgb[..., 2] = 1 - np.arange(255)
>>> imsave('rgb_gradient.png', rgb)
```

`scipy.misc.imshow(arr)`

Simple showing of an image through an external viewer.

Uses the image viewer specified by the environment variable `SCIPY_PIL_IMAGE_VIEWER`, or if that is not defined then *see*, to view a temporary file generated from array data.

**Parameters** **arr** : ndarray  
Array of image data to show.

**Returns** None

### Examples

```
>>> a = np.tile(np.arange(255), (255,1))
>>> from scipy import misc
>>> misc.imshow(a)
```

`scipy.misc.info(object=None, maxwidth=76, output=<open file '<stdout>', mode 'w' at 0x2b9089d99150>, toplevel='scipy')`

Get help information for a function, class, or module.

**Parameters** **object** : object or str, optional  
Input object or name to get information about. If *object* is a numpy object, its docstring is given. If it is a string, available modules are searched for matching objects. If None, information about `info` itself is returned.

**maxwidth** : int, optional  
Printing width.

**output** : file like object, optional  
File like object that the output is written to, default is `stdout`. The object has to be opened in 'w' or 'a' mode.

**toplevel** : str, optional  
Start search at this level.

### See also:

[source](#), [lookfor](#)

### Notes

When used interactively with an object, `np.info(obj)` is equivalent to `help(obj)` on the Python prompt or `obj?` on the IPython prompt.

**Examples**

```
>>> np.info(np.polyval)
polyval(p, x)
    Evaluate the polynomial p at x.
    ...
```

When using a string for *object* it is possible to get multiple results.

```
>>> np.info('fft')
*** Found in numpy ***
Core FFT routines
...
*** Found in numpy.fft ***
fft(a, n=None, axis=-1)
...
*** Repeat reference found in numpy.fft.fftpack ***
*** Total of 3 references found. ***
```

`scipy.misc.lena()`

Function that previously returned an example image

---

**Note:** Removed in 0.17

---

**Parameters**    **None**

**Returns**        **None**

**Raises**         **RuntimeError**

This functionality has been removed due to licensing reasons.

**See also:**

[face](#), [ascent](#)

**Notes**

The image previously returned by this function has an incompatible license and has been removed from SciPy. Please use [face](#) or [ascent](#) instead.

`scipy.misc.logsumexp(a, axis=None, b=None, keepdims=False, return_sign=False)`

Compute the log of the sum of exponentials of input elements.

**Parameters**    **a** : array\_like

Input array.

**axis** : None or int or tuple of ints, optional

Axis or axes over which the sum is taken. By default *axis* is None, and all elements are summed. Tuple of ints is not accepted if NumPy version is lower than 1.7.0.

New in version 0.11.0.

**keepdims** : bool, optional

If this is set to True, the axes which are reduced are left in the result as dimensions with size one. With this option, the result will broadcast correctly against the original array.

New in version 0.15.0.

**b** : array-like, optional

Scaling factor for  $\exp(a)$  must be of the same shape as *a* or broadcastable to *a*. These values may be negative in order to implement subtraction.

New in version 0.12.0.

**return\_sign** : bool, optional

If this is set to True, the result will be a pair containing sign information; if False, results that are negative will be returned as NaN. Default is False (no sign information). New in version 0.16.0.

### Returns

**res** : ndarray

The result, `np.log(np.sum(np.exp(a)))` calculated in a numerically more stable way. If *b* is given then `np.log(np.sum(b*np.exp(a)))` is returned.

**sgn** : ndarray

If `return_sign` is True, this will be an array of floating-point numbers matching `res` and +1, 0, or -1 depending on the sign of the result. If False, only one result is returned.

### See also:

`numpy.logaddexp`, `numpy.logaddexp2`

### Notes

Numpy has a `logaddexp` function which is very similar to `logsumexp`, but only handles two arguments. `logaddexp.reduce` is similar to this function, but may be less stable.

### Examples

```
>>> from scipy.misc import logsumexp
>>> a = np.arange(10)
>>> np.log(np.sum(np.exp(a)))
9.4586297444267107
>>> logsumexp(a)
9.4586297444267107
```

### With weights

```
>>> a = np.arange(10)
>>> b = np.arange(10, 0, -1)
>>> logsumexp(a, b=b)
9.9170178533034665
>>> np.log(np.sum(b*np.exp(a)))
9.9170178533034647
```

### Returning a sign flag

```
>>> logsumexp([1, 2], b=[1, -1], return_sign=True)
(1.5413248546129181, -1.0)
```

Notice that `logsumexp` does not directly support masked arrays. To use it on a masked array, convert the mask into zero weights:

```
>>> a = np.ma.array([np.log(2), 2, np.log(3)],
...                 mask=[False, True, False])
>>> b = (~a.mask).astype(int)
>>> logsumexp(a.data, b=b), np.log(5)
1.6094379124341005, 1.6094379124341005
```

`scipy.misc.pade` (*an*, *m*)

Return Padé approximation to a polynomial as the ratio of two polynomials.

**Parameters** **an** : (N,) array\_like  
Taylor series coefficients.

**m** : int  
The order of the returned approximating polynomials.

**Returns** **p, q** : Polynomial class  
The pade approximation of the polynomial defined by *an* is  $p(x)/q(x)$ .

**Examples**

```
>>> from scipy import misc
>>> e_exp = [1.0, 1.0, 1.0/2.0, 1.0/6.0, 1.0/24.0, 1.0/120.0]
>>> p, q = misc.pade(e_exp, 2)

>>> e_exp.reverse()
>>> e_poly = np.poly1d(e_exp)
```

Compare `e_poly(x)` and the pade approximation  $p(x)/q(x)$

```
>>> e_poly(1)
2.7166666666666668

>>> p(1)/q(1)
2.7179487179487181
```

`scipy.misc.toimage` (*arr*, *high*=255, *low*=0, *cmin*=None, *cmax*=None, *pal*=None, *mode*=None, *channel\_axis*=None)

Takes a numpy array and returns a PIL image.

The mode of the PIL image depends on the array shape and the *pal* and *mode* keywords.

For 2-D arrays, if *pal* is a valid (N,3) byte-array giving the RGB values (from 0 to 255) then *mode*='P', otherwise *mode*='L', unless *mode* is given as 'F' or 'I' in which case a float and/or integer array is made.

**Notes**

For 3-D arrays, the *channel\_axis* argument tells which dimension of the array holds the channel data.

For 3-D arrays if one of the dimensions is 3, the mode is 'RGB' by default or 'YCbCr' if selected.

The numpy array must be either 2 dimensional or 3 dimensional.

`scipy.misc.source` (*object*, *output*=<open file '<stdout>', mode 'w' at 0x2b9089d99150>)

Print or write to a file the source code for a Numpy object.

The source code is only returned for objects written in Python. Many functions and classes are defined in C and will therefore not return useful information.

**Parameters** **object** : numpy object  
Input object. This can be any object (function, class, module, ...).  
**output** : file object, optional  
If *output* not supplied then source code is printed to screen (sys.stdout). File object must be created with either write 'w' or append 'a' modes.

**See also:**

lookfor, info

**Examples**

```
>>> np.source(np.interp)
In file: /usr/lib/python2.6/dist-packages/numpy/lib/function_base.py
def interp(x, xp, fp, left=None, right=None):
    """... (full docstring printed)"""
    if isinstance(x, (float, int, number)):
        return compiled_interp([x], xp, fp, left, right).item()
    else:
        return compiled_interp(x, xp, fp, left, right)
```

The source code is only returned for objects written in Python.

```
>>> np.source(np.array)
Not available for this object.
```

`scipy.misc.who` (*vardict=None*)

Print the Numpy arrays in the given dictionary.

If there is no dictionary passed in or *vardict* is None then returns Numpy arrays in the `globals()` dictionary (all Numpy arrays in the namespace).

**Parameters**    **vardict** : dict, optional  
                             A dictionary possibly containing ndarrays. Default is `globals()`.

**Returns**        **out** : None  
                             Returns 'None'.

### Notes

Prints out the name, shape, bytes and type of all of the ndarrays present in *vardict*.

### Examples

```
>>> a = np.arange(10)
>>> b = np.ones(20)
>>> np.who()
Name          Shape          Bytes          Type
=====
a              10              40             int32
b              20             160            float64
Upper bound on total bytes =          200

>>> d = {'x': np.arange(2.0), 'y': np.arange(3.0), 'txt': 'Some str',
...      'idx':5}
>>> np.who(d)
Name          Shape          Bytes          Type
=====
y              3              24            float64
x              2              16            float64
Upper bound on total bytes =          40
```

## 5.16 Multi-dimensional image processing (`scipy.ndimage`)

This package contains various functions for multi-dimensional image processing.

### 5.16.1 Filters

<code>convolve(input, weights[, output, mode, ...])</code>	Multidimensional convolution.
<code>convolve1d(input, weights[, axis, output, ...])</code>	Calculate a one-dimensional convolution along the given axis.
<code>correlate(input, weights[, output, mode, ...])</code>	Multi-dimensional correlation.
<code>correlate1d(input, weights[, axis, output, ...])</code>	Calculate a one-dimensional correlation along the given axis.
<code>gaussian_filter(input, sigma[, order, ...])</code>	Multidimensional Gaussian filter.
<code>gaussian_filter1d(input, sigma[, axis, ...])</code>	One-dimensional Gaussian filter.

Continued on next page

Table 5.88 – continued from previous page

<code>gaussian_gradient_magnitude(input, sigma[, ...])</code>	Multidimensional gradient magnitude using Gaussian derivatives.
<code>gaussian_laplace(input, sigma[, output, ...])</code>	Multidimensional Laplace filter using gaussian second derivatives.
<code>generic_filter(input, function[, size, ...])</code>	Calculates a multi-dimensional filter using the given function.
<code>generic_filter1d(input, function, filter_size)</code>	Calculate a one-dimensional filter along the given axis.
<code>generic_gradient_magnitude(input, derivative)</code>	Gradient magnitude using a provided gradient function.
<code>generic_laplace(input, derivative2[, ...])</code>	N-dimensional Laplace filter using a provided second derivative function.
<code>laplace(input[, output, mode, cval])</code>	N-dimensional Laplace filter based on approximate second derivatives.
<code>maximum_filter(input[, size, footprint, ...])</code>	Calculates a multi-dimensional maximum filter.
<code>maximum_filter1d(input, size[, axis, ...])</code>	Calculate a one-dimensional maximum filter along the given axis.
<code>median_filter(input[, size, footprint, ...])</code>	Calculates a multidimensional median filter.
<code>minimum_filter(input[, size, footprint, ...])</code>	Calculates a multi-dimensional minimum filter.
<code>minimum_filter1d(input, size[, axis, ...])</code>	Calculate a one-dimensional minimum filter along the given axis.
<code>percentile_filter(input, percentile[, size, ...])</code>	Calculates a multi-dimensional percentile filter.
<code>prewitt(input[, axis, output, mode, cval])</code>	Calculate a Prewitt filter.
<code>rank_filter(input, rank[, size, footprint, ...])</code>	Calculates a multi-dimensional rank filter.
<code>sobel(input[, axis, output, mode, cval])</code>	Calculate a Sobel filter.
<code>uniform_filter(input[, size, output, mode, ...])</code>	Multi-dimensional uniform filter.
<code>uniform_filter1d(input, size[, axis, ...])</code>	Calculate a one-dimensional uniform filter along the given axis.

`scipy.ndimage.convolve(input, weights, output=None, mode='reflect', cval=0.0, origin=0)`  
Multidimensional convolution.

The array is convolved with the given kernel.

**Parameters**

- input** : array\_like  
Input array to filter.
- weights** : array\_like  
Array of weights, same number of dimensions as input
- output** : ndarray, optional  
The *output* parameter passes an array in which to store the filter output.
- mode** : { 'reflect', 'constant', 'nearest', 'mirror', 'wrap' }, optional  
the *mode* parameter determines how the array borders are handled. For 'constant' mode, values beyond borders are set to be *cval*. Default is 'reflect'.
- cval** : scalar, optional  
Value to fill past edges of input if *mode* is 'constant'. Default is 0.0
- origin** : array\_like, optional  
The *origin* parameter controls the placement of the filter, relative to the centre of the current element of the input. Default of 0 is equivalent to  $(0, ) * \text{input.ndim}$ .

**Returns**

- result** : ndarray  
The result of convolution of *input* with *weights*.

See also:

`correlate` Correlate an image with a kernel.

**Notes**

Each value in result is  $C_i = \sum_j I_{i+k-j} W_j$ , where *W* is the *weights* kernel, *j* is the n-D spatial index over *W*, *I* is the *input* and *k* is the coordinate of the center of *W*, specified by *origin* in the input parameters.

**Examples**

Perhaps the simplest case to understand is `mode='constant', cval=0.0`, because in this case borders (i.e. where the *weights* kernel, centered on any one value, extends beyond an edge of *input*).

```

>>> a = np.array([[1, 2, 0, 0],
...               [5, 3, 0, 4],
...               [0, 0, 0, 7],
...               [9, 3, 0, 0]])
>>> k = np.array([[1,1,1],[1,1,0],[1,0,0]])
>>> from scipy import ndimage
>>> ndimage.convolve(a, k, mode='constant', cval=0.0)
array([[11, 10, 7, 4],
       [10, 3, 11, 11],
       [15, 12, 14, 7],
       [12, 3, 7, 0]])
    
```

Setting `cval=1.0` is equivalent to padding the outer edge of *input* with 1.0's (and then extracting only the original region of the result).

```

>>> ndimage.convolve(a, k, mode='constant', cval=1.0)
array([[13, 11, 8, 7],
       [11, 3, 11, 14],
       [16, 12, 14, 10],
       [15, 6, 10, 5]])
    
```

With `mode='reflect'` (the default), outer values are reflected at the edge of *input* to fill in missing values.

```

>>> b = np.array([[2, 0, 0],
...               [1, 0, 0],
...               [0, 0, 0]])
>>> k = np.array([[0,1,0], [0,1,0], [0,1,0]])
>>> ndimage.convolve(b, k, mode='reflect')
array([[5, 0, 0],
       [3, 0, 0],
       [1, 0, 0]])
    
```

This includes diagonally at the corners.

```

>>> k = np.array([[1,0,0],[0,1,0],[0,0,1]])
>>> ndimage.convolve(b, k)
array([[4, 2, 0],
       [3, 2, 0],
       [1, 1, 0]])
    
```

With `mode='nearest'`, the single nearest value in to an edge in *input* is repeated as many times as needed to match the overlapping *weights*.

```

>>> c = np.array([[2, 0, 1],
...               [1, 0, 0],
...               [0, 0, 0]])
>>> k = np.array([[0, 1, 0],
...               [0, 1, 0],
...               [0, 1, 0],
...               [0, 1, 0],
...               [0, 1, 0]])
>>> ndimage.convolve(c, k, mode='nearest')
array([[7, 0, 3],
       [5, 0, 2],
       [3, 0, 1]])
    
```

`scipy.ndimage.convolve1d`(*input*, *weights*, *axis=-1*, *output=None*, *mode='reflect'*, *cval=0.0*, *origin=0*)

Calculate a one-dimensional convolution along the given axis.



The lines of the array along the given axis are convolved with the given weights.

**Parameters**

- input** : array\_like  
Input array to filter.
- weights** : ndarray  
One-dimensional sequence of numbers.
- axis** : int, optional  
The axis of *input* along which to calculate. Default is -1.
- output** : array, optional  
The *output* parameter passes an array in which to store the filter output.
- mode** : { 'reflect', 'constant', 'nearest', 'mirror', 'wrap' }, optional  
The *mode* parameter determines how the array borders are handled, where *cval* is the value when mode is equal to 'constant'. Default is 'reflect'
- cval** : scalar, optional  
Value to fill past edges of input if *mode* is 'constant'. Default is 0.0
- origin** : scalar, optional  
The *origin* parameter controls the placement of the filter. Default 0.0.

**Returns**

- convolve1d** : ndarray  
Convolved array with same shape as input

`scipy.ndimage.correlate` (*input, weights, output=None, mode='reflect', cval=0.0, origin=0*)  
Multi-dimensional correlation.

The array is correlated with the given kernel.

**Parameters**

- input** : array-like  
input array to filter
- weights** : ndarray  
array of weights, same number of dimensions as input
- output** : array, optional  
The *output* parameter passes an array in which to store the filter output.
- mode** : { 'reflect', 'constant', 'nearest', 'mirror', 'wrap' }, optional  
The *mode* parameter determines how the array borders are handled, where *cval* is the value when mode is equal to 'constant'. Default is 'reflect'
- cval** : scalar, optional  
Value to fill past edges of input if *mode* is 'constant'. Default is 0.0
- origin** : scalar, optional  
The *origin* parameter controls the placement of the filter. Default 0

**See also:**

[`convolve`](#) Convolve an image with a kernel.

`scipy.ndimage.correlate1d` (*input, weights, axis=-1, output=None, mode='reflect', cval=0.0, origin=0*)

Calculate a one-dimensional correlation along the given axis.

The lines of the array along the given axis are correlated with the given weights.

**Parameters**

- input** : array\_like  
Input array to filter.
- weights** : array  
One-dimensional sequence of numbers.
- axis** : int, optional  
The axis of *input* along which to calculate. Default is -1.
- output** : array, optional  
The *output* parameter passes an array in which to store the filter output.
- mode** : { 'reflect', 'constant', 'nearest', 'mirror', 'wrap' }, optional

The *mode* parameter determines how the array borders are handled, where *cval* is the value when mode is equal to 'constant'. Default is 'reflect'

**cval** : scalar, optional

Value to fill past edges of input if *mode* is 'constant'. Default is 0.0

**origin** : scalar, optional

The *origin* parameter controls the placement of the filter. Default 0.0.

`scipy.ndimage.gaussian_filter` (*input*, *sigma*, *order*=0, *output*=None, *mode*='reflect', *cval*=0.0, *truncate*=4.0)

Multidimensional Gaussian filter.

**Parameters** **input** : array\_like

Input array to filter.

**sigma** : scalar or sequence of scalars

Standard deviation for Gaussian kernel. The standard deviations of the Gaussian filter are given for each axis as a sequence, or as a single number, in which case it is equal for all axes.

**order** : {0, 1, 2, 3} or sequence from same set, optional

The order of the filter along each axis is given as a sequence of integers, or as a single number. An order of 0 corresponds to convolution with a Gaussian kernel. An order of 1, 2, or 3 corresponds to convolution with the first, second or third derivatives of a Gaussian. Higher order derivatives are not implemented

**output** : array, optional

The *output* parameter passes an array in which to store the filter output.

**mode** : {'reflect', 'constant', 'nearest', 'mirror', 'wrap'}, optional

The *mode* parameter determines how the array borders are handled, where *cval* is the value when mode is equal to 'constant'. Default is 'reflect'

**cval** : scalar, optional

Value to fill past edges of input if *mode* is 'constant'. Default is 0.0

**truncate** : float

Truncate the filter at this many standard deviations. Default is 4.0.

**Returns** **gaussian\_filter** : ndarray

Returned array of same shape as *input*.

### Notes

The multidimensional filter is implemented as a sequence of one-dimensional convolution filters. The intermediate arrays are stored in the same data type as the output. Therefore, for output types with a limited precision, the results may be imprecise because intermediate results may be stored with insufficient precision.

### Examples

```
>>> from scipy.ndimage import gaussian_filter
>>> a = np.arange(50, step=2).reshape((5,5))
>>> a
array([[ 0,  2,  4,  6,  8],
       [10, 12, 14, 16, 18],
       [20, 22, 24, 26, 28],
       [30, 32, 34, 36, 38],
       [40, 42, 44, 46, 48]])
>>> gaussian_filter(a, sigma=1)
array([[ 4,  6,  8,  9, 11],
       [10, 12, 14, 15, 17],
       [20, 22, 24, 25, 27],
       [29, 31, 33, 34, 36],
       [35, 37, 39, 40, 42]])
```

`scipy.ndimage.gaussian_filter1d` (*input*, *sigma*, *axis=-1*, *order=0*, *output=None*, *mode='reflect'*, *cval=0.0*, *truncate=4.0*)

One-dimensional Gaussian filter.

**Parameters**

- input** : array\_like  
Input array to filter.
- sigma** : scalar  
standard deviation for Gaussian kernel
- axis** : int, optional  
The axis of *input* along which to calculate. Default is -1.
- order** : {0, 1, 2, 3}, optional  
An order of 0 corresponds to convolution with a Gaussian kernel. An order of 1, 2, or 3 corresponds to convolution with the first, second or third derivatives of a Gaussian. Higher order derivatives are not implemented
- output** : array, optional  
The *output* parameter passes an array in which to store the filter output.
- mode** : {'reflect', 'constant', 'nearest', 'mirror', 'wrap'}, optional  
The *mode* parameter determines how the array borders are handled, where *cval* is the value when mode is equal to 'constant'. Default is 'reflect'
- cval** : scalar, optional  
Value to fill past edges of input if *mode* is 'constant'. Default is 0.0
- truncate** : float, optional  
Truncate the filter at this many standard deviations. Default is 4.0.

**Returns**

- gaussian\_filter1d** : ndarray

`scipy.ndimage.gaussian_gradient_magnitude` (*input*, *sigma*, *output=None*, *mode='reflect'*, *cval=0.0*, *\*\*kwargs*)

Multidimensional gradient magnitude using Gaussian derivatives.

**Parameters**

- input** : array\_like  
Input array to filter.
- sigma** : scalar or sequence of scalars  
The standard deviations of the Gaussian filter are given for each axis as a sequence, or as a single number, in which case it is equal for all axes..
- output** : array, optional  
The *output* parameter passes an array in which to store the filter output.
- mode** : {'reflect', 'constant', 'nearest', 'mirror', 'wrap'}, optional  
The *mode* parameter determines how the array borders are handled, where *cval* is the value when mode is equal to 'constant'. Default is 'reflect'
- cval** : scalar, optional  
Value to fill past edges of input if *mode* is 'constant'. Default is 0.0

**Extra keyword arguments will be passed to `gaussian_filter()`.**

`scipy.ndimage.gaussian_laplace` (*input*, *sigma*, *output=None*, *mode='reflect'*, *cval=0.0*, *\*\*kwargs*)

Multidimensional Laplace filter using gaussian second derivatives.

**Parameters**

- input** : array\_like  
Input array to filter.
- sigma** : scalar or sequence of scalars  
The standard deviations of the Gaussian filter are given for each axis as a sequence, or as a single number, in which case it is equal for all axes.
- output** : array, optional  
The *output* parameter passes an array in which to store the filter output.
- mode** : {'reflect', 'constant', 'nearest', 'mirror', 'wrap'}, optional  
The *mode* parameter determines how the array borders are handled, where *cval* is the value when mode is equal to 'constant'. Default is 'reflect'

**cval** : scalar, optional

Value to fill past edges of input if *mode* is 'constant'. Default is 0.0

**Extra keyword arguments will be passed to `gaussian_filter()`.**

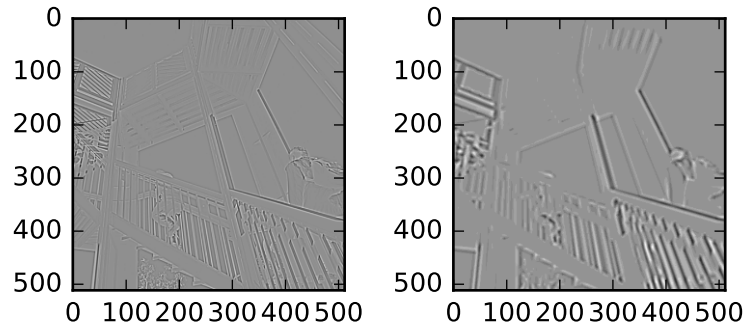
**Examples**

```
>>> from scipy import ndimage, misc
>>> import matplotlib.pyplot as plt
>>> ascent = misc.ascent()

>>> fig = plt.figure()
>>> plt.gray() # show the filtered result in grayscale
>>> ax1 = fig.add_subplot(121) # left side
>>> ax2 = fig.add_subplot(122) # right side

>>> result = ndimage.gaussian_laplace(ascent, sigma=1)
>>> ax1.imshow(result)

>>> result = ndimage.gaussian_laplace(ascent, sigma=3)
>>> ax2.imshow(result)
>>> plt.show()
```



`scipy.ndimage.generic_filter` (*input*, *function*, *size=None*, *footprint=None*, *output=None*, *mode='reflect'*, *cval=0.0*, *origin=0*, *extra\_arguments=()*, *extra\_keywords=None*)

Calculates a multi-dimensional filter using the given function.

At each element the provided function is called. The input values within the filter footprint at that element are passed to the function as a 1D array of double values.

- Parameters**
- input** : array\_like  
Input array to filter.
- function** : callable  
Function to apply at each element.
- size** : scalar or tuple, optional  
See footprint, below
- footprint** : array, optional

Either *size* or *footprint* must be defined. *size* gives the shape that is taken from the input array, at every element position, to define the input to the filter function. *footprint* is a boolean array that specifies (implicitly) a shape, but also which of the elements within this shape will get passed to the filter function. Thus `size=(n,m)` is equivalent to `footprint=np.ones((n,m))`. We adjust *size* to the number of dimensions of the input array, so that, if the input array is shape (10,10,10), and *size* is 2, then the actual size used is (2,2,2).

**output** : array, optional

The *output* parameter passes an array in which to store the filter output.

**mode** : { 'reflect', 'constant', 'nearest', 'mirror', 'wrap' }, optional

The *mode* parameter determines how the array borders are handled, where *cval* is the value when mode is equal to 'constant'. Default is 'reflect'

**cval** : scalar, optional

Value to fill past edges of input if *mode* is 'constant'. Default is 0.0

**origin** : scalar, optional

The *origin* parameter controls the placement of the filter. Default 0.0.

**extra\_arguments** : sequence, optional

Sequence of extra positional arguments to pass to passed function

**extra\_keywords** : dict, optional

dict of extra keyword arguments to pass to passed function

```
scipy.ndimage.generic_filter1d(input, function, filter_size, axis=-1, output=None,
                               mode='reflect', cval=0.0, origin=0, extra_arguments=(),
                               extra_keywords=None)
```

Calculate a one-dimensional filter along the given axis.

`generic_filter1d` iterates over the lines of the array, calling the given function at each line. The arguments of the line are the input line, and the output line. The input and output lines are 1D double arrays. The input line is extended appropriately according to the filter size and origin. The output line must be modified in-place with the result.

**Parameters** **input** : array\_like

Input array to filter.

**function** : callable

Function to apply along given axis.

**filter\_size** : scalar

Length of the filter.

**axis** : int, optional

The axis of *input* along which to calculate. Default is -1.

**output** : array, optional

The *output* parameter passes an array in which to store the filter output.

**mode** : { 'reflect', 'constant', 'nearest', 'mirror', 'wrap' }, optional

The *mode* parameter determines how the array borders are handled, where *cval* is the value when mode is equal to 'constant'. Default is 'reflect'

**cval** : scalar, optional

Value to fill past edges of input if *mode* is 'constant'. Default is 0.0

**origin** : scalar, optional

The *origin* parameter controls the placement of the filter. Default 0.0.

**extra\_arguments** : sequence, optional

Sequence of extra positional arguments to pass to passed function

**extra\_keywords** : dict, optional

dict of extra keyword arguments to pass to passed function

`scipy.ndimage.generic_gradient_magnitude` (*input*, *derivative*, *output=None*, *mode='reflect'*, *cval=0.0*, *extra\_arguments=()*, *extra\_keywords=None*)

Gradient magnitude using a provided gradient function.

**Parameters** **input** : array\_like

Input array to filter.

**derivative** : callable

Callable with the following signature:

```
derivative(input, axis, output, mode, cval,
          *extra_arguments, **extra_keywords)
```

See *extra\_arguments*, *extra\_keywords* below. *derivative* can assume that *input* and *output* are ndarrays. Note that the output from *derivative* is modified inplace; be careful to copy important inputs before returning them.

**output** : array, optional

The *output* parameter passes an array in which to store the filter output.

**mode** : {'reflect', 'constant', 'nearest', 'mirror', 'wrap'}, optional

The *mode* parameter determines how the array borders are handled, where *cval* is the value when mode is equal to 'constant'. Default is 'reflect'

**cval** : scalar, optional

Value to fill past edges of input if *mode* is 'constant'. Default is 0.0

**extra\_keywords** : dict, optional

dict of extra keyword arguments to pass to passed function

**extra\_arguments** : sequence, optional

Sequence of extra positional arguments to pass to passed function

`scipy.ndimage.generic_laplace` (*input*, *derivative2*, *output=None*, *mode='reflect'*, *cval=0.0*, *extra\_arguments=()*, *extra\_keywords=None*)

N-dimensional Laplace filter using a provided second derivative function

**Parameters** **input** : array\_like

Input array to filter.

**derivative2** : callable

Callable with the following signature:

```
derivative2(input, axis, output, mode, cval,
            *extra_arguments, **extra_keywords)
```

See *extra\_arguments*, *extra\_keywords* below.

**output** : array, optional

The *output* parameter passes an array in which to store the filter output.

**mode** : {'reflect', 'constant', 'nearest', 'mirror', 'wrap'}, optional

The *mode* parameter determines how the array borders are handled, where *cval* is the value when mode is equal to 'constant'. Default is 'reflect'

**cval** : scalar, optional

Value to fill past edges of input if *mode* is 'constant'. Default is 0.0

**extra\_keywords** : dict, optional

dict of extra keyword arguments to pass to passed function

**extra\_arguments** : sequence, optional

Sequence of extra positional arguments to pass to passed function

`scipy.ndimage.laplace` (*input*, *output=None*, *mode='reflect'*, *cval=0.0*)

N-dimensional Laplace filter based on approximate second derivatives.

**Parameters** **input** : array\_like

Input array to filter.

**output** : array, optional

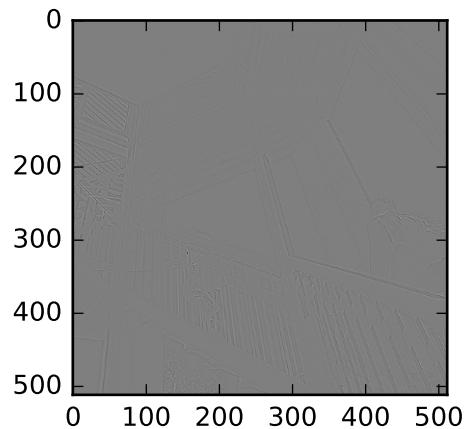
The *output* parameter passes an array in which to store the filter output.

**mode** : { 'reflect', 'constant', 'nearest', 'mirror', 'wrap' }, optional  
 The *mode* parameter determines how the array borders are handled, where *cval* is the value when mode is equal to 'constant'. Default is 'reflect'

**cval** : scalar, optional  
 Value to fill past edges of input if *mode* is 'constant'. Default is 0.0

### Examples

```
>>> from scipy import ndimage, misc
>>> import matplotlib.pyplot as plt
>>> ascent = misc.ascent()
>>> result = ndimage.laplace(ascent)
>>> plt.gray() # show the filtered result in grayscale
>>> plt.imshow(result)
```



`scipy.ndimage.maximum_filter` (*input*, *size=None*, *footprint=None*, *output=None*, *mode='reflect'*, *cval=0.0*, *origin=0*)

Calculates a multi-dimensional maximum filter.

**Parameters** **input** : array\_like

Input array to filter.

**size** : scalar or tuple, optional

See footprint, below

**footprint** : array, optional

Either *size* or *footprint* must be defined. *size* gives the shape that is taken from the input array, at every element position, to define the input to the filter function. *footprint* is a boolean array that specifies (implicitly) a shape, but also which of the elements within this shape will get passed to the filter function. Thus *size*=(*n*,*m*) is equivalent to *footprint*=`np.ones((n,m))`. We adjust *size* to the number of dimensions of the input array, so that, if the input array is shape (10,10,10), and *size* is 2, then the actual size used is (2,2,2).

**output** : array, optional

The *output* parameter passes an array in which to store the filter output.

**mode** : { 'reflect', 'constant', 'nearest', 'mirror', 'wrap' }, optional

The *mode* parameter determines how the array borders are handled, where *cval* is the value when mode is equal to 'constant'. Default is 'reflect'

**cval** : scalar, optional  
 Value to fill past edges of input if *mode* is 'constant'. Default is 0.0  
**origin** : scalar, optional  
 The *origin* parameter controls the placement of the filter. Default 0.0.

`scipy.ndimage.maximum_filter1d` (*input*, *size*, *axis*=-1, *output*=None, *mode*='reflect', *cval*=0.0, *origin*=0)

Calculate a one-dimensional maximum filter along the given axis.

The lines of the array along the given axis are filtered with a maximum filter of given size.

**Parameters**

- input** : array\_like  
Input array to filter.
- size** : int  
Length along which to calculate the 1-D maximum.
- axis** : int, optional  
The axis of *input* along which to calculate. Default is -1.
- output** : array, optional  
The *output* parameter passes an array in which to store the filter output.
- mode** : {'reflect', 'constant', 'nearest', 'mirror', 'wrap'}, optional  
The *mode* parameter determines how the array borders are handled, where *cval* is the value when mode is equal to 'constant'. Default is 'reflect'
- cval** : scalar, optional  
Value to fill past edges of input if *mode* is 'constant'. Default is 0.0
- origin** : scalar, optional  
The *origin* parameter controls the placement of the filter. Default 0.0.

**Returns**

- maximum1d** : ndarray, None  
Maximum-filtered array with same shape as input. None if *output* is not None

**Notes**

This function implements the MAXLIST algorithm [R137], as described by Richard Harter [R138], and has a guaranteed O(n) performance, *n* being the *input* length, regardless of filter size.

**References**

[R137], [R138]

`scipy.ndimage.median_filter` (*input*, *size*=None, *footprint*=None, *output*=None, *mode*='reflect', *cval*=0.0, *origin*=0)

Calculates a multidimensional median filter.

**Parameters**

- input** : array\_like  
Input array to filter.
- size** : scalar or tuple, optional  
See footprint, below
- footprint** : array, optional  
Either *size* or *footprint* must be defined. *size* gives the shape that is taken from the input array, at every element position, to define the input to the filter function. *footprint* is a boolean array that specifies (implicitly) a shape, but also which of the elements within this shape will get passed to the filter function. Thus *size*=(*n*,*m*) is equivalent to *footprint*=`np.ones((n,m))`. We adjust *size* to the number of dimensions of the input array, so that, if the input array is shape (10,10,10), and *size* is 2, then the actual size used is (2,2,2).
- output** : array, optional  
The *output* parameter passes an array in which to store the filter output.
- mode** : {'reflect', 'constant', 'nearest', 'mirror', 'wrap'}, optional



The *mode* parameter determines how the array borders are handled, where *cval* is the value when mode is equal to 'constant'. Default is 'reflect'

**cval** : scalar, optional

Value to fill past edges of input if *mode* is 'constant'. Default is 0.0

**origin** : scalar, optional

The *origin* parameter controls the placement of the filter. Default 0.0.

**Returns**

**median\_filter** : ndarray

Return of same shape as *input*.

`scipy.ndimage.minimum_filter` (*input*, *size=None*, *footprint=None*, *output=None*, *mode='reflect'*, *cval=0.0*, *origin=0*)

Calculates a multi-dimensional minimum filter.

**Parameters** **input** : array\_like

Input array to filter.

**size** : scalar or tuple, optional

See footprint, below

**footprint** : array, optional

Either *size* or *footprint* must be defined. *size* gives the shape that is taken from the input array, at every element position, to define the input to the filter function. *footprint* is a boolean array that specifies (implicitly) a shape, but also which of the elements within this shape will get passed to the filter function. Thus *size*=(*n*,*m*) is equivalent to *footprint*=`np.ones((n,m))`. We adjust *size* to the number of dimensions of the input array, so that, if the input array is shape (10,10,10), and *size* is 2, then the actual size used is (2,2,2).

**output** : array, optional

The *output* parameter passes an array in which to store the filter output.

**mode** : {'reflect', 'constant', 'nearest', 'mirror', 'wrap'}, optional

The *mode* parameter determines how the array borders are handled, where *cval* is the value when mode is equal to 'constant'. Default is 'reflect'

**cval** : scalar, optional

Value to fill past edges of input if *mode* is 'constant'. Default is 0.0

**origin** : scalar, optional

The *origin* parameter controls the placement of the filter. Default 0.0.

`scipy.ndimage.minimum_filter1d` (*input*, *size*, *axis=-1*, *output=None*, *mode='reflect'*, *cval=0.0*, *origin=0*)

Calculate a one-dimensional minimum filter along the given axis.

The lines of the array along the given axis are filtered with a minimum filter of given size.

**Parameters** **input** : array\_like

Input array to filter.

**size** : int

length along which to calculate 1D minimum

**axis** : int, optional

The axis of *input* along which to calculate. Default is -1.

**output** : array, optional

The *output* parameter passes an array in which to store the filter output.

**mode** : {'reflect', 'constant', 'nearest', 'mirror', 'wrap'}, optional

The *mode* parameter determines how the array borders are handled, where *cval* is the value when mode is equal to 'constant'. Default is 'reflect'

**cval** : scalar, optional

Value to fill past edges of input if *mode* is 'constant'. Default is 0.0

**origin** : scalar, optional

The *origin* parameter controls the placement of the filter. Default 0.0.

**Notes**

This function implements the MINLIST algorithm [R139], as described by Richard Harter [R140], and has a guaranteed  $O(n)$  performance,  $n$  being the *input* length, regardless of filter size.

**References**

[R139], [R140]

`scipy.ndimage.percentile_filter` (*input*, *percentile*, *size=None*, *footprint=None*, *output=None*, *mode='reflect'*, *cval=0.0*, *origin=0*)

Calculates a multi-dimensional percentile filter.

**Parameters**

- input** : array\_like  
Input array to filter.
- percentile** : scalar  
The percentile parameter may be less than zero, i.e., percentile = -20 equals percentile = 80
- size** : scalar or tuple, optional  
See footprint, below
- footprint** : array, optional  
Either *size* or *footprint* must be defined. *size* gives the shape that is taken from the input array, at every element position, to define the input to the filter function. *footprint* is a boolean array that specifies (implicitly) a shape, but also which of the elements within this shape will get passed to the filter function. Thus *size*=(*n*,*m*) is equivalent to *footprint*=`np.ones((n,m))`. We adjust *size* to the number of dimensions of the input array, so that, if the input array is shape (10,10,10), and *size* is 2, then the actual size used is (2,2,2).
- output** : array, optional  
The *output* parameter passes an array in which to store the filter output.
- mode** : {'reflect', 'constant', 'nearest', 'mirror', 'wrap'}, optional  
The *mode* parameter determines how the array borders are handled, where *cval* is the value when mode is equal to 'constant'. Default is 'reflect'
- cval** : scalar, optional  
Value to fill past edges of input if *mode* is 'constant'. Default is 0.0
- origin** : scalar, optional  
The *origin* parameter controls the placement of the filter. Default 0.0.

`scipy.ndimage.prewitt` (*input*, *axis=-1*, *output=None*, *mode='reflect'*, *cval=0.0*)

Calculate a Prewitt filter.

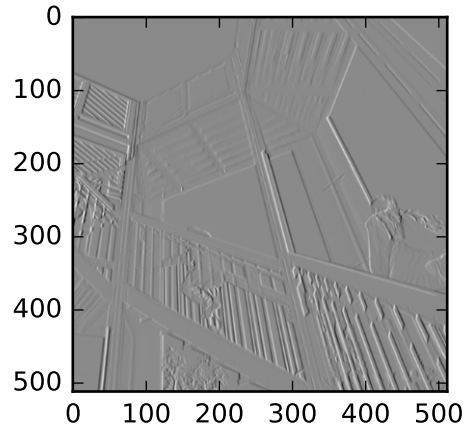
**Parameters**

- input** : array\_like  
Input array to filter.
- axis** : int, optional  
The axis of *input* along which to calculate. Default is -1.
- output** : array, optional  
The *output* parameter passes an array in which to store the filter output.
- mode** : {'reflect', 'constant', 'nearest', 'mirror', 'wrap'}, optional  
The *mode* parameter determines how the array borders are handled, where *cval* is the value when mode is equal to 'constant'. Default is 'reflect'
- cval** : scalar, optional  
Value to fill past edges of input if *mode* is 'constant'. Default is 0.0

**Examples**

```
>>> from scipy import ndimage, misc
>>> import matplotlib.pyplot as plt
>>> ascent = misc.ascent()
```

```
>>> result = ndimage.prewitt(ascent)
>>> plt.gray() # show the filtered result in grayscale
>>> plt.imshow(result)
```



`scipy.ndimage.rank_filter` (*input*, *rank*, *size=None*, *footprint=None*, *output=None*, *mode='reflect'*, *cval=0.0*, *origin=0*)

Calculates a multi-dimensional rank filter.

**Parameters**

- input** : array\_like  
Input array to filter.
- rank** : int  
The rank parameter may be less than zero, i.e., rank = -1 indicates the largest element.
- size** : scalar or tuple, optional  
See footprint, below
- footprint** : array, optional  
Either *size* or *footprint* must be defined. *size* gives the shape that is taken from the input array, at every element position, to define the input to the filter function. *footprint* is a boolean array that specifies (implicitly) a shape, but also which of the elements within this shape will get passed to the filter function. Thus *size*=(*n*,*m*) is equivalent to *footprint*=`np.ones((n,m))`. We adjust *size* to the number of dimensions of the input array, so that, if the input array is shape (10,10,10), and *size* is 2, then the actual size used is (2,2,2).
- output** : array, optional  
The *output* parameter passes an array in which to store the filter output.
- mode** : {'reflect', 'constant', 'nearest', 'mirror', 'wrap'}, optional  
The *mode* parameter determines how the array borders are handled, where *cval* is the value when mode is equal to 'constant'. Default is 'reflect'
- cval** : scalar, optional  
Value to fill past edges of input if *mode* is 'constant'. Default is 0.0
- origin** : scalar, optional  
The *origin* parameter controls the placement of the filter. Default 0.0.

`scipy.ndimage.sobel` (*input*, *axis=-1*, *output=None*, *mode='reflect'*, *cval=0.0*)

Calculate a Sobel filter.

**Parameters**

- input** : array\_like  
Input array to filter.

**axis** : int, optional  
 The axis of *input* along which to calculate. Default is -1.

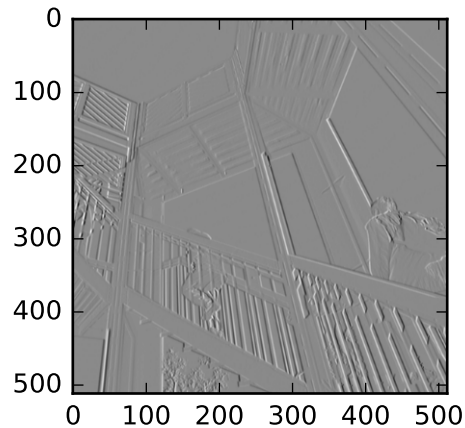
**output** : array, optional  
 The *output* parameter passes an array in which to store the filter output.

**mode** : {'reflect', 'constant', 'nearest', 'mirror', 'wrap'}, optional  
 The *mode* parameter determines how the array borders are handled, where *cval* is the value when mode is equal to 'constant'. Default is 'reflect'

**cval** : scalar, optional  
 Value to fill past edges of input if *mode* is 'constant'. Default is 0.0

### Examples

```
>>> from scipy import ndimage, misc
>>> import matplotlib.pyplot as plt
>>> ascent = misc.ascent()
>>> result = ndimage.sobel(ascent)
>>> plt.gray() # show the filtered result in grayscale
>>> plt.imshow(result)
```



`scipy.ndimage.uniform_filter` (*input*, *size=3*, *output=None*, *mode='reflect'*, *cval=0.0*, *origin=0*)  
 Multi-dimensional uniform filter.

**Parameters**

**input** : array\_like  
 Input array to filter.

**size** : int or sequence of ints, optional  
 The sizes of the uniform filter are given for each axis as a sequence, or as a single number, in which case the size is equal for all axes.

**output** : array, optional  
 The *output* parameter passes an array in which to store the filter output.

**mode** : {'reflect', 'constant', 'nearest', 'mirror', 'wrap'}, optional  
 The *mode* parameter determines how the array borders are handled, where *cval* is the value when mode is equal to 'constant'. Default is 'reflect'

**cval** : scalar, optional  
 Value to fill past edges of input if *mode* is 'constant'. Default is 0.0

**origin** : scalar, optional  
 The *origin* parameter controls the placement of the filter. Default 0.0.

### Notes

The multi-dimensional filter is implemented as a sequence of one-dimensional uniform filters. The intermediate arrays are stored in the same data type as the output. Therefore, for output types with a limited precision, the results may be imprecise because intermediate results may be stored with insufficient precision.

```
scipy.ndimage.uniform_filter1d(input, size, axis=-1, output=None, mode='reflect', cval=0.0, origin=0)
```

Calculate a one-dimensional uniform filter along the given axis.

The lines of the array along the given axis are filtered with a uniform filter of given size.

**Parameters**

- input** : array\_like  
Input array to filter.
- size** : int  
length of uniform filter
- axis** : int, optional  
The axis of *input* along which to calculate. Default is -1.
- output** : array, optional  
The *output* parameter passes an array in which to store the filter output.
- mode** : { 'reflect', 'constant', 'nearest', 'mirror', 'wrap' }, optional  
The *mode* parameter determines how the array borders are handled, where *cval* is the value when mode is equal to 'constant'. Default is 'reflect'
- cval** : scalar, optional  
Value to fill past edges of input if *mode* is 'constant'. Default is 0.0
- origin** : scalar, optional  
The *origin* parameter controls the placement of the filter. Default 0.0

## 5.16.2 Fourier filters

<code>fourier_ellipsoid(input, size[, n, axis, output])</code>	Multi-dimensional ellipsoid fourier filter.
<code>fourier_gaussian(input, sigma[, n, axis, output])</code>	Multi-dimensional Gaussian fourier filter.
<code>fourier_shift(input, shift[, n, axis, output])</code>	Multi-dimensional fourier shift filter.
<code>fourier_uniform(input, size[, n, axis, output])</code>	Multi-dimensional uniform fourier filter.

```
scipy.ndimage.fourier_ellipsoid(input, size, n=-1, axis=-1, output=None)
```

Multi-dimensional ellipsoid fourier filter.

The array is multiplied with the fourier transform of a ellipsoid of given sizes.

**Parameters**

- input** : array\_like  
The input array.
- size** : float or sequence  
The size of the box used for filtering. If a float, *size* is the same for all axes. If a sequence, *size* has to contain one value for each axis.
- n** : int, optional  
If *n* is negative (default), then the input is assumed to be the result of a complex fft. If *n* is larger than or equal to zero, the input is assumed to be the result of a real fft, and *n* gives the length of the array before transformation along the real transform direction.
- axis** : int, optional  
The axis of the real transform.
- output** : ndarray, optional  
If given, the result of filtering the input is placed in this array. None is returned in this case.

**Returns**

- fourier\_ellipsoid** : ndarray or None

The filtered input. If *output* is given as a parameter, None is returned.

**Notes**

This function is implemented for arrays of rank 1, 2, or 3.

`scipy.ndimage.fourier_gaussian` (*input*, *sigma*, *n=-1*, *axis=-1*, *output=None*)

Multi-dimensional Gaussian fourier filter.

The array is multiplied with the fourier transform of a Gaussian kernel.

**Parameters**

- input** : array\_like  
The input array.
- sigma** : float or sequence  
The sigma of the Gaussian kernel. If a float, *sigma* is the same for all axes. If a sequence, *sigma* has to contain one value for each axis.
- n** : int, optional  
If *n* is negative (default), then the input is assumed to be the result of a complex fft. If *n* is larger than or equal to zero, the input is assumed to be the result of a real fft, and *n* gives the length of the array before transformation along the real transform direction.
- axis** : int, optional  
The axis of the real transform.
- output** : ndarray, optional  
If given, the result of filtering the input is placed in this array. None is returned in this case.

**Returns**

- fourier\_gaussian** : ndarray or None  
The filtered input. If *output* is given as a parameter, None is returned.

`scipy.ndimage.fourier_shift` (*input*, *shift*, *n=-1*, *axis=-1*, *output=None*)

Multi-dimensional fourier shift filter.

The array is multiplied with the fourier transform of a shift operation.

**Parameters**

- input** : array\_like  
The input array.
- shift** : float or sequence  
The size of the box used for filtering. If a float, *shift* is the same for all axes. If a sequence, *shift* has to contain one value for each axis.
- n** : int, optional  
If *n* is negative (default), then the input is assumed to be the result of a complex fft. If *n* is larger than or equal to zero, the input is assumed to be the result of a real fft, and *n* gives the length of the array before transformation along the real transform direction.
- axis** : int, optional  
The axis of the real transform.
- output** : ndarray, optional  
If given, the result of shifting the input is placed in this array. None is returned in this case.

**Returns**

- fourier\_shift** : ndarray or None  
The shifted input. If *output* is given as a parameter, None is returned.

`scipy.ndimage.fourier_uniform` (*input*, *size*, *n=-1*, *axis=-1*, *output=None*)

Multi-dimensional uniform fourier filter.

The array is multiplied with the fourier transform of a box of given size.

**Parameters**

- input** : array\_like  
The input array.
- size** : float or sequence

The size of the box used for filtering. If a float, *size* is the same for all axes. If a sequence, *size* has to contain one value for each axis.

**n** : int, optional

If *n* is negative (default), then the input is assumed to be the result of a complex fft. If *n* is larger than or equal to zero, the input is assumed to be the result of a real fft, and *n* gives the length of the array before transformation along the real transform direction.

**axis** : int, optional

The axis of the real transform.

**output** : ndarray, optional

If given, the result of filtering the input is placed in this array. None is returned in this case.

**Returns**

**fourier\_uniform** : ndarray or None

The filtered input. If *output* is given as a parameter, None is returned.

### 5.16.3 Interpolation

---

<code>affine_transform(input, matrix[, offset, ...])</code>	Apply an affine transformation.
<code>geometric_transform(input, mapping[, ...])</code>	Apply an arbitrary geometric transform.
<code>map_coordinates(input, coordinates[, ...])</code>	Map the input array to new coordinates by interpolation.
<code>rotate(input, angle[, axes, reshape, ...])</code>	Rotate an array.
<code>shift(input, shift[, output, order, mode, ...])</code>	Shift an array.
<code>spline_filter(input[, order, output])</code>	Multi-dimensional spline filter.
<code>spline_filter1d(input[, order, axis, output])</code>	Calculates a one-dimensional spline filter along the given axis.
<code>zoom(input, zoom[, output, order, mode, ...])</code>	Zoom an array.

---

`scipy.ndimage.affine_transform` (*input*, *matrix*, *offset=0.0*, *output\_shape=None*, *output=None*, *order=3*, *mode='constant'*, *cval=0.0*, *prefilter=True*)

Apply an affine transformation.

The given matrix and offset are used to find for each point in the output the corresponding coordinates in the input by an affine transformation. The value of the input at those coordinates is determined by spline interpolation of the requested order. Points outside the boundaries of the input are filled according to the given mode.

Given an output image pixel index vector *o*, the pixel value is determined from the input image at position `np.dot(matrix, o) + offset`.

A diagonal matrix can be specified by supplying a one-dimensional array-like to the matrix parameter, in which case a more efficient algorithm is applied.

Changed in version 0.18.0: Previously, the exact interpretation of the affine transformation depended on whether the matrix was supplied as a one-dimensional or two-dimensional array. If a one-dimensional array was supplied to the matrix parameter, the output pixel value at index *o* was determined from the input image at position `matrix * (o + offset)`.

**Parameters**

**input** : ndarray

The input array.

**matrix** : ndarray

The matrix must be two-dimensional or can also be given as a one-dimensional sequence or array. In the latter case, it is assumed that the matrix is diagonal. A more efficient algorithm is then applied that exploits the separability of the problem.

**offset** : float or sequence, optional

The offset into the array where the transform is applied. If a float, *offset* is the same for each axis. If a sequence, *offset* should contain one value for each axis.

**output\_shape** : tuple of ints, optional

Shape tuple.

**output** : ndarray or dtype, optional

The array in which to place the output, or the dtype of the returned array.

**order** : int, optional

The order of the spline interpolation, default is 3. The order has to be in the range 0-5.

**mode** : str, optional

Points outside the boundaries of the input are filled according to the given mode ('constant', 'nearest', 'reflect' or 'wrap'). Default is 'constant'.

**cval** : scalar, optional

Value used for points outside the boundaries of the input if `mode='constant'`. Default is 0.0

**prefilter** : bool, optional

The parameter `prefilter` determines if the input is pre-filtered with `spline_filter` before interpolation (necessary for spline interpolation of order > 1). If False, it is assumed that the input is already filtered. Default is True.

**Returns** **affine\_transform** : ndarray or None

The transformed input. If `output` is given as a parameter, None is returned.

```
scipy.ndimage.geometric_transform(input, mapping, output_shape=None, output=None, order=3, mode='constant', cval=0.0, prefilter=True, extra_arguments=(), extra_keywords={})
```

Apply an arbitrary geometric transform.

The given mapping function is used to find, for each point in the output, the corresponding coordinates in the input. The value of the input at those coordinates is determined by spline interpolation of the requested order.

**Parameters** **input** : array\_like

The input array.

**mapping** : callable

A callable object that accepts a tuple of length equal to the output array rank, and returns the corresponding input coordinates as a tuple of length equal to the input array rank.

**output\_shape** : tuple of ints, optional

Shape tuple.

**output** : ndarray or dtype, optional

The array in which to place the output, or the dtype of the returned array.

**order** : int, optional

The order of the spline interpolation, default is 3. The order has to be in the range 0-5.

**mode** : str, optional

Points outside the boundaries of the input are filled according to the given mode ('constant', 'nearest', 'reflect' or 'wrap'). Default is 'constant'.

**cval** : scalar, optional

Value used for points outside the boundaries of the input if `mode='constant'`. Default is 0.0

**prefilter** : bool, optional

The parameter `prefilter` determines if the input is pre-filtered with `spline_filter` before interpolation (necessary for spline interpolation of order > 1). If False, it is assumed that the input is already filtered. Default is True.

**extra\_arguments** : tuple, optional

Extra arguments passed to `mapping`.

**extra\_keywords** : dict, optional

Extra keywords passed to `mapping`.

**Returns** **return\_value** : ndarray or None

The filtered input. If `output` is given as a parameter, None is returned.



**See also:**

`map_coordinates`, `affine_transform`, `spline_filter1d`

**Examples**

```
>>> from scipy import ndimage
>>> a = np.arange(12.).reshape((4, 3))
>>> def shift_func(output_coords):
...     return (output_coords[0] - 0.5, output_coords[1] - 0.5)
...
>>> ndimage.geometric_transform(a, shift_func)
array([[ 0.   ,  0.   ,  0.   ],
       [ 0.   ,  1.362,  2.738],
       [ 0.   ,  4.812,  6.187],
       [ 0.   ,  8.263,  9.637]])
```

`scipy.ndimage.map_coordinates` (*input*, *coordinates*, *output=None*, *order=3*, *mode='constant'*, *cval=0.0*, *prefilter=True*)

Map the input array to new coordinates by interpolation.

The array of coordinates is used to find, for each point in the output, the corresponding coordinates in the input. The value of the input at those coordinates is determined by spline interpolation of the requested order.

The shape of the output is derived from that of the coordinate array by dropping the first axis. The values of the array along the first axis are the coordinates in the input array at which the output value is found.

**Parameters**

- input** : ndarray  
The input array.
- coordinates** : array\_like  
The coordinates at which *input* is evaluated.
- output** : ndarray or dtype, optional  
The array in which to place the output, or the dtype of the returned array.
- order** : int, optional  
The order of the spline interpolation, default is 3. The order has to be in the range 0-5.
- mode** : str, optional  
Points outside the boundaries of the input are filled according to the given mode ('constant', 'nearest', 'reflect' or 'wrap'). Default is 'constant'.
- cval** : scalar, optional  
Value used for points outside the boundaries of the input if mode='constant'. Default is 0.0
- prefilter** : bool, optional  
The parameter prefilter determines if the input is pre-filtered with `spline_filter` before interpolation (necessary for spline interpolation of order > 1). If False, it is assumed that the input is already filtered. Default is True.

**Returns**

- map\_coordinates** : ndarray  
The result of transforming the input. The shape of the output is derived from that of *coordinates* by dropping the first axis.

**See also:**

`spline_filter`, `geometric_transform`, `scipy.interpolate`

**Examples**

```
>>> from scipy import ndimage
>>> a = np.arange(12.).reshape((4, 3))
>>> a
array([[ 0.,  1.,  2.],
```

```

    [ 3.,  4.,  5.],
    [ 6.,  7.,  8.],
    [ 9., 10., 11.]]
>>> ndimage.map_coordinates(a, [[0.5, 2], [0.5, 1]], order=1)
array([ 2.,  7.])

```

Above, the interpolated value of `a[0.5, 0.5]` gives `output[0]`, while `a[2, 1]` is `output[1]`.

```

>>> inds = np.array([[0.5, 2], [0.5, 4]])
>>> ndimage.map_coordinates(a, inds, order=1, cval=-33.3)
array([ 2. , -33.3])
>>> ndimage.map_coordinates(a, inds, order=1, mode='nearest')
array([ 2.,  8.])
>>> ndimage.map_coordinates(a, inds, order=1, cval=0, output=bool)
array([ True, False], dtype=bool)

```

`scipy.ndimage.rotate` (*input*, *angle*, *axes*=(1, 0), *reshape*=True, *output*=None, *order*=3, *mode*='constant', *cval*=0.0, *prefilter*=True)

Rotate an array.

The array is rotated in the plane defined by the two axes given by the *axes* parameter using spline interpolation of the requested order.

**Parameters**

- input** : ndarray  
The input array.
- angle** : float  
The rotation angle in degrees.
- axes** : tuple of 2 ints, optional  
The two axes that define the plane of rotation. Default is the first two axes.
- reshape** : bool, optional  
If *reshape* is true, the output shape is adapted so that the input array is contained completely in the output. Default is True.
- output** : ndarray or dtype, optional  
The array in which to place the output, or the dtype of the returned array.
- order** : int, optional  
The order of the spline interpolation, default is 3. The order has to be in the range 0-5.
- mode** : str, optional  
Points outside the boundaries of the input are filled according to the given mode ('constant', 'nearest', 'reflect' or 'wrap'). Default is 'constant'.
- cval** : scalar, optional  
Value used for points outside the boundaries of the input if *mode*='constant'. Default is 0.0
- prefilter** : bool, optional  
The parameter *prefilter* determines if the input is pre-filtered with `spline_filter` before interpolation (necessary for spline interpolation of order > 1). If False, it is assumed that the input is already filtered. Default is True.

**Returns**

- rotate** : ndarray or None  
The rotated input. If *output* is given as a parameter, None is returned.

`scipy.ndimage.shift` (*input*, *shift*, *output*=None, *order*=3, *mode*='constant', *cval*=0.0, *prefilter*=True)  
Shift an array.

The array is shifted using spline interpolation of the requested order. Points outside the boundaries of the input are filled according to the given mode.

**Parameters**

- input** : ndarray  
The input array.
- shift** : float or sequence, optional

The shift along the axes. If a float, `shift` is the same for each axis. If a sequence, `shift` should contain one value for each axis.

**output** : ndarray or dtype, optional

The array in which to place the output, or the dtype of the returned array.

**order** : int, optional

The order of the spline interpolation, default is 3. The order has to be in the range 0-5.

**mode** : str, optional

Points outside the boundaries of the input are filled according to the given mode ('constant', 'nearest', 'reflect' or 'wrap'). Default is 'constant'.

**cval** : scalar, optional

Value used for points outside the boundaries of the input if `mode='constant'`. Default is 0.0

**prefilter** : bool, optional

The parameter `prefilter` determines if the input is pre-filtered with `spline_filter` before interpolation (necessary for spline interpolation of order  $> 1$ ). If False, it is assumed that the input is already filtered. Default is True.

**Returns** **shift** : ndarray or None

The shifted input. If `output` is given as a parameter, None is returned.

`scipy.ndimage.spline_filter` (*input*, *order*=3, *output*=<type 'numpy.float64'>)

Multi-dimensional spline filter.

For more details, see `spline_filter1d`.

**See also:**

`spline_filter1d`

**Notes**

The multi-dimensional filter is implemented as a sequence of one-dimensional spline filters. The intermediate arrays are stored in the same data type as the output. Therefore, for output types with a limited precision, the results may be imprecise because intermediate results may be stored with insufficient precision.

`scipy.ndimage.spline_filter1d` (*input*, *order*=3, *axis*=-1, *output*=<type 'numpy.float64'>)

Calculates a one-dimensional spline filter along the given axis.

The lines of the array along the given axis are filtered by a spline filter. The order of the spline must be  $\geq 2$  and  $\leq 5$ .

**Parameters** **input** : array\_like

The input array.

**order** : int, optional

The order of the spline, default is 3.

**axis** : int, optional

The axis along which the spline filter is applied. Default is the last axis.

**output** : ndarray or dtype, optional

The array in which to place the output, or the dtype of the returned array. Default is `numpy.float64`.

**Returns** **spline\_filter1d** : ndarray or None

The filtered input. If `output` is given as a parameter, None is returned.

`scipy.ndimage.zoom` (*input*, *zoom*, *output*=None, *order*=3, *mode*='constant', *cval*=0.0, *prefilter*=True)

Zoom an array.

The array is zoomed using spline interpolation of the requested order.

**Parameters** **input** : ndarray

The input array.

**zoom** : float or sequence, optional  
 The zoom factor along the axes. If a float, `zoom` is the same for each axis. If a sequence, `zoom` should contain one value for each axis.

**output** : ndarray or dtype, optional  
 The array in which to place the output, or the dtype of the returned array.

**order** : int, optional  
 The order of the spline interpolation, default is 3. The order has to be in the range 0-5.

**mode** : str, optional  
 Points outside the boundaries of the input are filled according to the given mode ('constant', 'nearest', 'reflect' or 'wrap'). Default is 'constant'.

**cval** : scalar, optional  
 Value used for points outside the boundaries of the input if `mode='constant'`. Default is 0.0

**prefilter** : bool, optional  
 The parameter `prefilter` determines if the input is pre-filtered with `spline_filter` before interpolation (necessary for spline interpolation of order > 1). If False, it is assumed that the input is already filtered. Default is True.

**Returns** **zoom** : ndarray or None  
 The zoomed input. If `output` is given as a parameter, None is returned.

## 5.16.4 Measurements

<code>center_of_mass(input[, labels, index])</code>	Calculate the center of mass of the values of an array at labels.
<code>extrema(input[, labels, index])</code>	Calculate the minimums and maximums of the values of an array at labels, and
<code>find_objects(input[, max_label])</code>	Find objects in a labeled array.
<code>histogram(input, min, max, bins[, labels, index])</code>	Calculate the histogram of the values of an array, optionally at labels.
<code>label(input[, structure, output])</code>	Label features in an array.
<code>labeled_comprehension(input, labels, index, ...)</code>	Roughly equivalent to <code>[func(input[labels == i]) for i in index]</code> .
<code>maximum(input[, labels, index])</code>	Calculate the maximum of the values of an array over labeled regions.
<code>maximum_position(input[, labels, index])</code>	Find the positions of the maximums of the values of an array at labels.
<code>mean(input[, labels, index])</code>	Calculate the mean of the values of an array at labels.
<code>median(input[, labels, index])</code>	Calculate the median of the values of an array over labeled regions.
<code>minimum(input[, labels, index])</code>	Calculate the minimum of the values of an array over labeled regions.
<code>minimum_position(input[, labels, index])</code>	Find the positions of the minimums of the values of an array at labels.
<code>standard_deviation(input[, labels, index])</code>	Calculate the standard deviation of the values of an n-D image array, optionally
<code>sum(input[, labels, index])</code>	Calculate the sum of the values of the array.
<code>variance(input[, labels, index])</code>	Calculate the variance of the values of an n-D image array, optionally at spe
<code>watershed_ift(input, markers[, structure, ...])</code>	Apply watershed from markers using image foresting transform algorithm.

`scipy.ndimage.center_of_mass` (*input*, *labels=None*, *index=None*)  
 Calculate the center of mass of the values of an array at labels.

**Parameters** **input** : ndarray  
 Data from which to calculate center-of-mass.

**labels** : ndarray, optional  
 Labels for objects in *input*, as generated by `ndimage.label`. Only used with *index*. Dimensions must be the same as *input*.

**index** : int or sequence of ints, optional  
 Labels for which to calculate centers-of-mass. If not specified, all labels greater than zero are used. Only used with *labels*.

**Returns** **center\_of\_mass** : tuple, or list of tuples  
 Coordinates of centers-of-mass.

**Examples**

```
>>> a = np.array([[0, 0, 0, 0],
...               [0, 1, 1, 0],
...               [0, 1, 1, 0],
...               [0, 1, 1, 0]])
>>> from scipy import ndimage
>>> ndimage.measurements.center_of_mass(a)
(2.0, 1.5)
```

Calculation of multiple objects in an image

```
>>> b = np.array([[0, 1, 1, 0],
...               [0, 1, 0, 0],
...               [0, 0, 0, 0],
...               [0, 0, 1, 1],
...               [0, 0, 1, 1]])
>>> lbl = ndimage.label(b)[0]
>>> ndimage.measurements.center_of_mass(b, lbl, [1, 2])
[(0.33333333333333331, 1.3333333333333333), (3.5, 2.5)]
```

`scipy.ndimage.extrema` (*input*, *labels=None*, *index=None*)

Calculate the minimums and maximums of the values of an array at labels, along with their positions.

**Parameters**

- input** : ndarray  
Nd-image data to process.
- labels** : ndarray, optional  
Labels of features in input. If not None, must be same shape as *input*.
- index** : int or sequence of ints, optional  
Labels to include in output. If None (default), all values where non-zero *labels* are used.

**Returns**

- minimums, maximums** : int or ndarray  
Values of minimums and maximums in each feature.
- min\_positions, max\_positions** : tuple or list of tuples  
Each tuple gives the n-D coordinates of the corresponding minimum or maximum.

**See also:**

`maximum`, `minimum`, `maximum_position`, `minimum_position`, `center_of_mass`

**Examples**

```
>>> a = np.array([[1, 2, 0, 0],
...               [5, 3, 0, 4],
...               [0, 0, 0, 7],
...               [9, 3, 0, 0]])
>>> from scipy import ndimage
>>> ndimage.extrema(a)
(0, 9, (0, 2), (3, 0))
```

Features to process can be specified using *labels* and *index*:

```
>>> lbl, nlbl = ndimage.label(a)
>>> ndimage.extrema(a, lbl, index=np.arange(1, nlbl+1))
(array([1, 4, 3]),
 array([5, 7, 9]),
 [(0, 0), (1, 3), (3, 1)],
 [(1, 0), (2, 3), (3, 0)])
```

If no index is given, non-zero *labels* are processed:

```
>>> ndimage.extrema(a, lbl)
(1, 9, (0, 0), (3, 0))
```

`scipy.ndimage.find_objects` (*input*, *max\_label=0*)

Find objects in a labeled array.

**Parameters** **input** : ndarray of ints  
 Array containing objects defined by different labels. Labels with value 0 are ignored.

**max\_label** : int, optional  
 Maximum label to be searched for in *input*. If *max\_label* is not given, the positions of all objects are returned.

**Returns** **object\_slices** : list of tuples  
 A list of tuples, with each tuple containing N slices (with N the dimension of the input array). Slices correspond to the minimal parallelepiped that contains the object. If a number is missing, None is returned instead of a slice.

**See also:**

`label`, `center_of_mass`

**Notes**

This function is very useful for isolating a volume of interest inside a 3-D array, that cannot be “seen through”.

**Examples**

```
>>> from scipy import ndimage
>>> a = np.zeros((6,6), dtype=int)
>>> a[2:4, 2:4] = 1
>>> a[4, 4] = 1
>>> a[:2, :3] = 2
>>> a[0, 5] = 3
>>> a
array([[2, 2, 2, 0, 0, 3],
       [2, 2, 2, 0, 0, 0],
       [0, 0, 1, 1, 0, 0],
       [0, 0, 1, 1, 0, 0],
       [0, 0, 0, 0, 1, 0],
       [0, 0, 0, 0, 0, 0]])
>>> ndimage.find_objects(a)
[(slice(2, 5, None), slice(2, 5, None)), (slice(0, 2, None), slice(0, 3, None)), (slice(0, 1, None), slice(0, 5, None))]
>>> ndimage.find_objects(a, max_label=2)
[(slice(2, 5, None), slice(2, 5, None)), (slice(0, 2, None), slice(0, 3, None))]
>>> ndimage.find_objects(a == 1, max_label=2)
[(slice(2, 5, None), slice(2, 5, None)), None]

>>> loc = ndimage.find_objects(a)[0]
>>> a[loc]
array([[1, 1, 0],
       [1, 1, 0],
       [0, 0, 1]])
```

`scipy.ndimage.histogram` (*input*, *min*, *max*, *bins*, *labels=None*, *index=None*)

Calculate the histogram of the values of an array, optionally at labels.

Histogram calculates the frequency of values in an array within bins determined by *min*, *max*, and *bins*. The *labels* and *index* keywords can limit the scope of the histogram to specified sub-regions within the array.

**Parameters** **input** : array\_like  
 Data for which to calculate histogram.

**min, max** : int  
Minimum and maximum values of range of histogram bins.

**bins** : int  
Number of bins.

**labels** : array\_like, optional  
Labels for objects in *input*. If not None, must be same shape as *input*.

**index** : int or sequence of ints, optional  
Label or labels for which to calculate histogram. If None, all values where label is greater than zero are used

**Returns** **hist** : ndarray  
Histogram counts.

**Examples**

```
>>> a = np.array([[ 0.      ,  0.2146,  0.5962,  0.      ],
...              [ 0.      ,  0.7778,  0.      ,  0.      ],
...              [ 0.      ,  0.      ,  0.      ,  0.      ],
...              [ 0.      ,  0.      ,  0.7181,  0.2787],
...              [ 0.      ,  0.      ,  0.6573,  0.3094]])
>>> from scipy import ndimage
>>> ndimage.measurements.histogram(a, 0, 1, 10)
array([13,  0,  2,  1,  0,  1,  1,  2,  0,  0])
```

With labels and no indices, non-zero elements are counted:

```
>>> lbl, nlbl = ndimage.label(a)
>>> ndimage.measurements.histogram(a, 0, 1, 10, lbl)
array([0, 0, 2, 1, 0, 1, 1, 2, 0, 0])
```

Indices can be used to count only certain objects:

```
>>> ndimage.measurements.histogram(a, 0, 1, 10, lbl, 2)
array([0, 0, 1, 1, 0, 0, 1, 1, 0, 0])
```

`scipy.ndimage.label` (*input*, *structure=None*, *output=None*)

Label features in an array.

**Parameters** **input** : array\_like  
An array-like object to be labeled. Any non-zero values in *input* are counted as features and zero values are considered the background.

**structure** : array\_like, optional  
A structuring element that defines feature connections. *structure* must be symmetric. If no structuring element is provided, one is automatically generated with a squared connectivity equal to one. That is, for a 2-D *input* array, the default structuring element is:

```
[[0, 1, 0],
 [1, 1, 1],
 [0, 1, 0]]
```

**output** : (None, data-type, array\_like), optional  
If *output* is a data type, it specifies the type of the resulting labeled feature array. If *output* is an array-like object, then *output* will be updated with the labeled features from this function. This function can operate in-place, by passing *output=input*. Note that the output must be able to store the largest label, or this function will raise an Exception.

**Returns** **label** : ndarray or int  
An integer ndarray where each unique feature in *input* has a unique label in the returned array.

**num\_features** : int

How many objects were found.

If *output* is None, this function returns a tuple of (*labeled\_array*, *num\_features*).

If *output* is a ndarray, then it will be updated with values in *labeled\_array* and only *num\_features* will be returned by this function.

See also:

*find\_objects*

generate a list of slices for the labeled features (or objects); useful for finding features' position or dimensions

*Examples*

Create an image with some features, then label it using the default (cross-shaped) structuring element:

```
>>> from scipy.ndimage import label, generate_binary_structure
>>> a = np.array([[0, 0, 1, 1, 0, 0],
...              [0, 0, 0, 1, 0, 0],
...              [1, 1, 0, 0, 1, 0],
...              [0, 0, 0, 1, 0, 0]])
>>> labeled_array, num_features = label(a)
```

Each of the 4 features are labeled with a different integer:

```
>>> num_features
4
>>> labeled_array
array([[0, 0, 1, 1, 0, 0],
       [0, 0, 0, 1, 0, 0],
       [2, 2, 0, 0, 3, 0],
       [0, 0, 0, 4, 0, 0]])
```

Generate a structuring element that will consider features connected even if they touch diagonally:

```
>>> s = generate_binary_structure(2,2)
```

or,

```
>>> s = [[1,1,1],
...      [1,1,1],
...      [1,1,1]]
```

Label the image using the new structuring element:

```
>>> labeled_array, num_features = label(a, structure=s)
```

Show the 2 labeled features (note that features 1, 3, and 4 from above are now considered a single feature):

```
>>> num_features
2
>>> labeled_array
array([[0, 0, 1, 1, 0, 0],
       [0, 0, 0, 1, 0, 0],
       [2, 2, 0, 0, 1, 0],
       [0, 0, 0, 1, 0, 0]])
```

`scipy.ndimage.labeled_comprehension` (*input*, *labels*, *index*, *func*, *out\_dtype*, *default*, *pass\_positions=False*)

Roughly equivalent to `[func(input[labels == i]) for i in index]`.



Sequentially applies an arbitrary function (that works on `array_like` input) to subsets of an n-D image array specified by `labels` and `index`. The option exists to provide the function with positional parameters as the second argument.

**Parameters**

- input** : `array_like`  
Data from which to select `labels` to process.
- labels** : `array_like` or `None`  
Labels to objects in `input`. If not `None`, array must be same shape as `input`. If `None`, `func` is applied to raveled `input`.
- index** : `int`, sequence of `ints` or `None`  
Subset of `labels` to which to apply `func`. If a scalar, a single value is returned. If `None`, `func` is applied to all non-zero values of `labels`.
- func** : callable  
Python function to apply to `labels` from `input`.
- out\_dtype** : `dtype`  
Dtype to use for `result`.
- default** : `int`, `float` or `None`  
Default return value when a element of `index` does not exist in `labels`.
- pass\_positions** : `bool`, optional  
If `True`, pass linear indices to `func` as a second argument. Default is `False`.

**Returns**

- result** : `ndarray`  
Result of applying `func` to each of `labels` to `input` in `index`.

### Examples

```
>>> a = np.array([[1, 2, 0, 0],
...              [5, 3, 0, 4],
...              [0, 0, 0, 7],
...              [9, 3, 0, 0]])
>>> from scipy import ndimage
>>> lbl, nlbl = ndimage.label(a)
>>> lbls = np.arange(1, nlbl+1)
>>> ndimage.labeled_comprehension(a, lbl, lbls, np.mean, float, 0)
array([ 2.75,  5.5 ,  6.  ])
```

Falling back to `default`:

```
>>> lbls = np.arange(1, nlbl+2)
>>> ndimage.labeled_comprehension(a, lbl, lbls, np.mean, float, -1)
array([ 2.75,  5.5 ,  6.  , -1.  ])
```

Passing positions:

```
>>> def fn(val, pos):
...     print("fn says: %s : %s" % (val, pos))
...     return (val.sum()) if (pos.sum() % 2 == 0) else (-val.sum())
...
>>> ndimage.labeled_comprehension(a, lbl, lbls, fn, float, 0, True)
fn says: [1 2 5 3] : [0 1 4 5]
fn says: [4 7] : [ 7 11]
fn says: [9 3] : [12 13]
array([ 11.,  11., -12.,  0.] )
```

`scipy.ndimage.maximum` (`input`, `labels=None`, `index=None`)

Calculate the maximum of the values of an array over labeled regions.

**Parameters**

- input** : `array_like`

Array\_like of values. For each region specified by *labels*, the maximal values of *input* over the region is computed.

**labels** : array\_like, optional

An array of integers marking different regions over which the maximum value of *input* is to be computed. *labels* must have the same shape as *input*. If *labels* is not specified, the maximum over the whole array is returned.

**index** : array\_like, optional

A list of region labels that are taken into account for computing the maxima. If *index* is None, the maximum over all elements where *labels* is non-zero is returned.

**Returns**

**output** : float or list of floats

List of maxima of *input* over the regions determined by *labels* and whose index is in *index*. If *index* or *labels* are not specified, a float is returned: the maximal value of *input* if *labels* is None, and the maximal value of elements where *labels* is greater than zero if *index* is None.

**See also:**

`label`, `minimum`, `median`, `maximum_position`, `extrema`, `sum`, `mean`, `variance`, `standard_deviation`

**Notes**

The function returns a Python list and not a Numpy array, use `np.array` to convert the list to an array.

**Examples**

```
>>> a = np.arange(16).reshape((4, 4))
>>> a
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11],
       [12, 13, 14, 15]])
>>> labels = np.zeros_like(a)
>>> labels[:2, :2] = 1
>>> labels[2:, 1:3] = 2
>>> labels
array([[1, 1, 0, 0],
       [1, 1, 0, 0],
       [0, 2, 2, 0],
       [0, 2, 2, 0]])
>>> from scipy import ndimage
>>> ndimage.maximum(a)
15.0
>>> ndimage.maximum(a, labels=labels, index=[1,2])
[5.0, 14.0]
>>> ndimage.maximum(a, labels=labels)
14.0

>>> b = np.array([[1, 2, 0, 0],
...              [5, 3, 0, 4],
...              [0, 0, 0, 7],
...              [9, 3, 0, 0]])
>>> labels, labels_nb = ndimage.label(b)
>>> labels
array([[1, 1, 0, 0],
       [1, 1, 0, 2],
       [0, 0, 0, 2],
       [3, 3, 0, 0]])
```

```
>>> ndimage.maximum(b, labels=labels, index=np.arange(1, labels_nb + 1))
[5.0, 7.0, 9.0]
```

`scipy.ndimage.maximum_position` (*input*, *labels=None*, *index=None*)

Find the positions of the maximums of the values of an array at labels.

For each region specified by *labels*, the position of the maximum value of *input* within the region is returned.

**Parameters**

**input** : array\_like  
Array\_like of values.

**labels** : array\_like, optional  
An array of integers marking different regions over which the position of the maximum value of *input* is to be computed. *labels* must have the same shape as *input*. If *labels* is not specified, the location of the first maximum over the whole array is returned.  
The *labels* argument only works when *index* is specified.

**index** : array\_like, optional  
A list of region labels that are taken into account for finding the location of the maxima. If *index* is None, the first maximum over all elements where *labels* is non-zero is returned.  
The *index* argument only works when *labels* is specified.

**Returns**

**output** : list of tuples of ints  
List of tuples of ints that specify the location of maxima of *input* over the regions determined by *labels* and whose index is in *index*.  
If *index* or *labels* are not specified, a tuple of ints is returned specifying the location of the first maximal value of *input*.

**See also:**

`label`, `minimum`, `median`, `maximum_position`, `extrema`, `sum`, `mean`, `variance`, `standard_deviation`

`scipy.ndimage.mean` (*input*, *labels=None*, *index=None*)

Calculate the mean of the values of an array at labels.

**Parameters**

**input** : array\_like  
Array on which to compute the mean of elements over distinct regions.

**labels** : array\_like, optional  
Array of labels of same shape, or broadcastable to the same shape as *input*. All elements sharing the same label form one region over which the mean of the elements is computed.

**index** : int or sequence of ints, optional  
Labels of the objects over which the mean is to be computed. Default is None, in which case the mean for all values where label is greater than 0 is calculated.

**Returns**

**out** : list  
Sequence of same length as *index*, with the mean of the different regions labeled by the labels in *index*.

**See also:**

`ndimage.variance`, `ndimage.standard_deviation`, `ndimage.minimum`,  
`ndimage.maximum`, `ndimage.sum`, `ndimage.label`

**Examples**

```
>>> from scipy import ndimage
>>> a = np.arange(25).reshape((5, 5))
>>> labels = np.zeros_like(a)
```

```

>>> labels[3:5,3:5] = 1
>>> index = np.unique(labels)
>>> labels
array([[0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0],
       [0, 0, 0, 1, 1],
       [0, 0, 0, 1, 1]])
>>> index
array([0, 1])
>>> ndimage.mean(a, labels=labels, index=index)
[10.285714285714286, 21.0]
    
```

`scipy.ndimage.median` (*input*, *labels=None*, *index=None*)

Calculate the median of the values of an array over labeled regions.

**Parameters** **input** : array\_like

Array\_like of values. For each region specified by *labels*, the median value of *input* over the region is computed.

**labels** : array\_like, optional

An array\_like of integers marking different regions over which the median value of *input* is to be computed. *labels* must have the same shape as *input*. If *labels* is not specified, the median over the whole array is returned.

**index** : array\_like, optional

A list of region labels that are taken into account for computing the medians. If *index* is None, the median over all elements where *labels* is non-zero is returned.

**Returns** **median** : float or list of floats

List of medians of *input* over the regions determined by *labels* and whose index is in *index*. If *index* or *labels* are not specified, a float is returned: the median value of *input* if *labels* is None, and the median value of elements where *labels* is greater than zero if *index* is None.

**See also:**

`label`, `minimum`, `maximum`, `extrema`, `sum`, `mean`, `variance`, `standard_deviation`

**Notes**

The function returns a Python list and not a Numpy array, use `np.array` to convert the list to an array.

**Examples**

```

>>> from scipy import ndimage
>>> a = np.array([[1, 2, 0, 1],
...              [5, 3, 0, 4],
...              [0, 0, 0, 7],
...              [9, 3, 0, 0]])
>>> labels, labels_nb = ndimage.label(a)
>>> labels
array([[1, 1, 0, 2],
       [1, 1, 0, 2],
       [0, 0, 0, 2],
       [3, 3, 0, 0]])
>>> ndimage.median(a, labels=labels, index=np.arange(1, labels_nb + 1))
[2.5, 4.0, 6.0]
>>> ndimage.median(a)
1.0
    
```

```
>>> ndimage.median(a, labels=labels)
3.0
```

`scipy.ndimage.minimum` (*input*, *labels=None*, *index=None*)

Calculate the minimum of the values of an array over labeled regions.

**Parameters** **input** : array\_like

Array\_like of values. For each region specified by *labels*, the minimal values of *input* over the region is computed.

**labels** : array\_like, optional

An array\_like of integers marking different regions over which the minimum value of *input* is to be computed. *labels* must have the same shape as *input*. If *labels* is not specified, the minimum over the whole array is returned.

**index** : array\_like, optional

A list of region labels that are taken into account for computing the minima. If *index* is *None*, the minimum over all elements where *labels* is non-zero is returned.

**Returns** **minimum** : float or list of floats

List of minima of *input* over the regions determined by *labels* and whose index is in *index*. If *index* or *labels* are not specified, a float is returned: the minimal value of *input* if *labels* is *None*, and the minimal value of elements where *labels* is greater than zero if *index* is *None*.

**See also:**

`label`, `maximum`, `median`, `minimum_position`, `extrema`, `sum`, `mean`, `variance`, `standard_deviation`

**Notes**

The function returns a Python list and not a Numpy array, use `np.array` to convert the list to an array.

**Examples**

```
>>> from scipy import ndimage
>>> a = np.array([[1, 2, 0, 0],
...              [5, 3, 0, 4],
...              [0, 0, 0, 7],
...              [9, 3, 0, 0]])
>>> labels, labels_nb = ndimage.label(a)
>>> labels
array([[1, 1, 0, 0],
       [1, 1, 0, 2],
       [0, 0, 0, 2],
       [3, 3, 0, 0]])
>>> ndimage.minimum(a, labels=labels, index=np.arange(1, labels_nb + 1))
[1.0, 4.0, 3.0]
>>> ndimage.minimum(a)
0.0
>>> ndimage.minimum(a, labels=labels)
1.0
```

`scipy.ndimage.minimum_position` (*input*, *labels=None*, *index=None*)

Find the positions of the minimums of the values of an array at labels.

**Parameters** **input** : array\_like

Array\_like of values.

**labels** : array\_like, optional

An array of integers marking different regions over which the position of the minimum value of *input* is to be computed. *labels* must have the same shape as *input*. If *labels* is not specified, the location of the first minimum over the whole array is returned.

The *labels* argument only works when *index* is specified.

**index** : array\_like, optional

A list of region labels that are taken into account for finding the location of the minima. If *index* is None, the first minimum over all elements where *labels* is non-zero is returned.

The *index* argument only works when *labels* is specified.

**Returns** **output** : list of tuples of ints

Tuple of ints or list of tuples of ints that specify the location of minima of *input* over the regions determined by *labels* and whose index is in *index*.

If *index* or *labels* are not specified, a tuple of ints is returned specifying the location of the first minimal value of *input*.

**See also:**

`label`, `minimum`, `median`, `maximum_position`, `extrema`, `sum`, `mean`, `variance`, `standard_deviation`

`scipy.ndimage.standard_deviation` (*input*, *labels=None*, *index=None*)

Calculate the standard deviation of the values of an n-D image array, optionally at specified sub-regions.

**Parameters** **input** : array\_like

Nd-image data to process.

**labels** : array\_like, optional

Labels to identify sub-regions in *input*. If not None, must be same shape as *input*.

**index** : int or sequence of ints, optional

*labels* to include in output. If None (default), all values where *labels* is non-zero are used.

**Returns** **standard\_deviation** : float or ndarray

Values of standard deviation, for each sub-region if *labels* and *index* are specified.

**See also:**

`label`, `variance`, `maximum`, `minimum`, `extrema`

**Examples**

```
>>> a = np.array([[1, 2, 0, 0],
...              [5, 3, 0, 4],
...              [0, 0, 0, 7],
...              [9, 3, 0, 0]])
>>> from scipy import ndimage
>>> ndimage.standard_deviation(a)
2.7585095613392387
```

Features to process can be specified using *labels* and *index*:

```
>>> lbl, nlbl = ndimage.label(a)
>>> ndimage.standard_deviation(a, lbl, index=np.arange(1, nlbl+1))
array([ 1.479,  1.5   ,  3.   ])
```

If no *index* is given, non-zero *labels* are processed:

```
>>> ndimage.standard_deviation(a, lbl)
2.4874685927665499
```

`scipy.ndimage.sum` (*input*, *labels=None*, *index=None*)

Calculate the sum of the values of the array.

**Parameters**

- input** : array\_like  
Values of *input* inside the regions defined by *labels* are summed together.
- labels** : array\_like of ints, optional  
Assign labels to the values of the array. Has to have the same shape as *input*.
- index** : array\_like, optional  
A single label number or a sequence of label numbers of the objects to be measured.

**Returns**

- sum** : ndarray or scalar  
An array of the sums of values of *input* inside the regions defined by *labels* with the same shape as *index*. If 'index' is None or scalar, a scalar is returned.

**See also:**

`mean`, `median`

**Examples**

```
>>> from scipy import ndimage
>>> input = [0,1,2,3]
>>> labels = [1,1,2,2]
>>> ndimage.sum(input, labels, index=[1,2])
[1.0, 5.0]
>>> ndimage.sum(input, labels, index=1)
1
>>> ndimage.sum(input, labels)
6
```

`scipy.ndimage.variance` (*input*, *labels=None*, *index=None*)

Calculate the variance of the values of an n-D image array, optionally at specified sub-regions.

**Parameters**

- input** : array\_like  
Nd-image data to process.
- labels** : array\_like, optional  
Labels defining sub-regions in *input*. If not None, must be same shape as *input*.
- index** : int or sequence of ints, optional  
*labels* to include in output. If None (default), all values where *labels* is non-zero are used.

**Returns**

- variance** : float or ndarray  
Values of variance, for each sub-region if *labels* and *index* are specified.

**See also:**

`label`, `standard_deviation`, `maximum`, `minimum`, `extrema`

**Examples**

```
>>> a = np.array([[1, 2, 0, 0],
...              [5, 3, 0, 4],
...              [0, 0, 0, 7],
...              [9, 3, 0, 0]])
>>> from scipy import ndimage
>>> ndimage.variance(a)
7.609375
```

Features to process can be specified using *labels* and *index*:

```
>>> lbl, nlbl = ndimage.label(a)
>>> ndimage.variance(a, lbl, index=np.arange(1, nlbl+1))
array([ 2.1875,  2.25 ,  9.    ])
```

If no index is given, all non-zero *labels* are processed:

```
>>> ndimage.variance(a, lbl)
6.1875
```

`scipy.ndimage.watershed_ift` (*input, markers, structure=None, output=None*)

Apply watershed from markers using image foresting transform algorithm.

**Parameters**

- input** : array\_like  
Input.
- markers** : array\_like  
Markers are points within each watershed that form the beginning of the process. Negative markers are considered background markers which are processed after the other markers.
- structure** : structure element, optional  
A structuring element defining the connectivity of the object can be provided. If None, an element is generated with a squared connectivity equal to one.
- output** : ndarray, optional  
An output array can optionally be provided. The same shape as input.

**Returns**

- watershed\_ift** : ndarray  
Output. Same shape as *input*.

**References**

[R142]

### 5.16.5 Morphology

<code>binary_closing</code> (input[, structure, ...])	Multi-dimensional binary closing with the given structuring element.
<code>binary_dilation</code> (input[, structure, ...])	Multi-dimensional binary dilation with the given structuring element.
<code>binary_erosion</code> (input[, structure, ...])	Multi-dimensional binary erosion with a given structuring element.
<code>binary_fill_holes</code> (input[, structure, ...])	Fill the holes in binary objects.
<code>binary_hit_or_miss</code> (input[, structure1, ...])	Multi-dimensional binary hit-or-miss transform.
<code>binary_opening</code> (input[, structure, ...])	Multi-dimensional binary opening with the given structuring element.
<code>binary_propagation</code> (input[, structure, mask, ...])	Multi-dimensional binary propagation with the given structuring element.
<code>black_tophat</code> (input[, size, footprint, ...])	Multi-dimensional black tophat filter.
<code>distance_transform_bf</code> (input[, metric, ...])	Distance transform function by a brute force algorithm.
<code>distance_transform_cdt</code> (input[, metric, ...])	Distance transform for chamfer type of transforms.
<code>distance_transform_edt</code> (input[, sampling, ...])	Exact euclidean distance transform.
<code>generate_binary_structure</code> (rank, connectivity)	Generate a binary structure for binary morphological operations.
<code>grey_closing</code> (input[, size, footprint, ...])	Multi-dimensional greyscale closing.
<code>grey_dilation</code> (input[, size, footprint, ...])	Calculate a greyscale dilation, using either a structuring element, or a footprint.
<code>grey_erosion</code> (input[, size, footprint, ...])	Calculate a greyscale erosion, using either a structuring element, or a footprint.
<code>grey_opening</code> (input[, size, footprint, ...])	Multi-dimensional greyscale opening.
<code>iterate_structure</code> (structure, iterations[, ...])	Iterate a structure by dilating it with itself.
<code>morphological_gradient</code> (input[, size, ...])	Multi-dimensional morphological gradient.
<code>morphological_laplace</code> (input[, size, ...])	Multi-dimensional morphological laplace.
<code>white_tophat</code> (input[, size, footprint, ...])	Multi-dimensional white tophat filter.

`scipy.ndimage.binary_closing` (*input, structure=None, iterations=1, output=None, origin=0*)

Multi-dimensional binary closing with the given structuring element.

The *closing* of an input image by a structuring element is the *erosion* of the *dilation* of the image by the structuring element.

**Parameters**

- input** : array\_like



Binary array\_like to be closed. Non-zero (True) elements form the subset to be closed.  
**structure** : array\_like, optional

Structuring element used for the closing. Non-zero elements are considered True. If no structuring element is provided an element is generated with a square connectivity equal to one (i.e., only nearest neighbors are connected to the center, diagonally-connected elements are not considered neighbors).

**iterations** : {int, float}, optional

The dilation step of the closing, then the erosion step are each repeated *iterations* times (one, by default). If iterations is less than 1, each operations is repeated until the result does not change anymore.

**output** : ndarray, optional

Array of the same shape as input, into which the output is placed. By default, a new array is created.

**origin** : int or tuple of ints, optional

Placement of the filter, by default 0.

**Returns** **binary\_closing** : ndarray of bools

Closing of the input by the structuring element.

**See also:**

`grey_closing`, `binary_opening`, `binary_dilation`, `binary_erosion`,  
`generate_binary_structure`

**Notes**

*Closing* [R119] is a mathematical morphology operation [R120] that consists in the succession of a dilation and an erosion of the input with the same structuring element. Closing therefore fills holes smaller than the structuring element.

Together with *opening* (`binary_opening`), closing can be used for noise removal.

**References**

[R119], [R120]

**Examples**

```
>>> from scipy import ndimage
>>> a = np.zeros((5,5), dtype=int)
>>> a[1:-1, 1:-1] = 1; a[2,2] = 0
>>> a
array([[0, 0, 0, 0, 0],
       [0, 1, 1, 1, 0],
       [0, 1, 0, 1, 0],
       [0, 1, 1, 1, 0],
       [0, 0, 0, 0, 0]])
>>> # Closing removes small holes
>>> ndimage.binary_closing(a).astype(int)
array([[0, 0, 0, 0, 0],
       [0, 1, 1, 1, 0],
       [0, 1, 1, 1, 0],
       [0, 1, 1, 1, 0],
       [0, 0, 0, 0, 0]])
>>> # Closing is the erosion of the dilation of the input
>>> ndimage.binary_dilation(a).astype(int)
array([[0, 1, 1, 1, 0],
       [1, 1, 1, 1, 1],
       [1, 1, 1, 1, 1],
       [1, 1, 1, 1, 1],
       [1, 1, 1, 1, 1]])
```

```

        [0, 1, 1, 1, 0]])
>>> ndimage.binary_erosion(ndimage.binary_dilation(a).astype(int)
array([[0, 0, 0, 0, 0],
       [0, 1, 1, 1, 0],
       [0, 1, 1, 1, 0],
       [0, 1, 1, 1, 0],
       [0, 0, 0, 0, 0]])

>>> a = np.zeros((7,7), dtype=int)
>>> a[1:6, 2:5] = 1; a[1:3,3] = 0
>>> a
array([[0, 0, 0, 0, 0, 0, 0],
       [0, 0, 1, 0, 1, 0, 0],
       [0, 0, 1, 0, 1, 0, 0],
       [0, 0, 1, 1, 1, 0, 0],
       [0, 0, 1, 1, 1, 0, 0],
       [0, 0, 1, 1, 1, 0, 0],
       [0, 0, 0, 0, 0, 0, 0]])

>>> # In addition to removing holes, closing can also
>>> # coarsen boundaries with fine hollows.
>>> ndimage.binary_closing(a).astype(int)
array([[0, 0, 0, 0, 0, 0, 0],
       [0, 0, 1, 0, 1, 0, 0],
       [0, 0, 1, 1, 1, 0, 0],
       [0, 0, 1, 1, 1, 0, 0],
       [0, 0, 1, 1, 1, 0, 0],
       [0, 0, 1, 1, 1, 0, 0],
       [0, 0, 0, 0, 0, 0, 0]])

>>> ndimage.binary_closing(a, structure=np.ones((2,2)).astype(int)
array([[0, 0, 0, 0, 0, 0, 0],
       [0, 0, 1, 1, 1, 0, 0],
       [0, 0, 1, 1, 1, 0, 0],
       [0, 0, 1, 1, 1, 0, 0],
       [0, 0, 1, 1, 1, 0, 0],
       [0, 0, 1, 1, 1, 0, 0],
       [0, 0, 0, 0, 0, 0, 0]])
    
```

`scipy.ndimage.binary_dilation` (*input*, *structure=None*, *iterations=1*, *mask=None*, *output=None*, *border\_value=0*, *origin=0*, *brute\_force=False*)

Multi-dimensional binary dilation with the given structuring element.

**Parameters** **input** : array\_like

Binary array\_like to be dilated. Non-zero (True) elements form the subset to be dilated.

**structure** : array\_like, optional

Structuring element used for the dilation. Non-zero elements are considered True. If no structuring element is provided an element is generated with a square connectivity equal to one.

**iterations** : {int, float}, optional

The dilation is repeated *iterations* times (one, by default). If *iterations* is less than 1, the dilation is repeated until the result does not change anymore.

**mask** : array\_like, optional

If a mask is given, only those elements with a True value at the corresponding mask element are modified at each iteration.

**output** : ndarray, optional

Array of the same shape as input, into which the output is placed. By default, a new array is created.

**origin** : int or tuple of ints, optional  
 Placement of the filter, by default 0.

**border\_value** : int (cast to 0 or 1), optional  
 Value at the border in the output array.

**Returns** **binary\_dilation** : ndarray of bools  
 Dilation of the input by the structuring element.

**See also:**

`grey_dilation`, `binary_erosion`, `binary_closing`, `binary_opening`,  
`generate_binary_structure`

**Notes**

Dilation [R121] is a mathematical morphology operation [R122] that uses a structuring element for expanding the shapes in an image. The binary dilation of an image by a structuring element is the locus of the points covered by the structuring element, when its center lies within the non-zero points of the image.

**References**

[R121], [R122]

**Examples**

```
>>> from scipy import ndimage
>>> a = np.zeros((5, 5))
>>> a[2, 2] = 1
>>> a
array([[ 0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  1.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.]])
>>> ndimage.binary_dilation(a)
array([[False, False, False, False, False],
       [False, False, True, False, False],
       [False, True, True, True, False],
       [False, False, True, False, False],
       [False, False, False, False, False]], dtype=bool)
>>> ndimage.binary_dilation(a).astype(a.dtype)
array([[ 0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  1.,  0.,  0.],
       [ 0.,  1.,  1.,  1.,  0.],
       [ 0.,  0.,  1.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.]])
>>> # 3x3 structuring element with connectivity 1, used by default
>>> struct1 = ndimage.generate_binary_structure(2, 1)
>>> struct1
array([[False, True, False],
       [ True, True, True],
       [False, True, False]], dtype=bool)
>>> # 3x3 structuring element with connectivity 2
>>> struct2 = ndimage.generate_binary_structure(2, 2)
>>> struct2
array([[ True,  True,  True],
       [ True,  True,  True],
       [ True,  True,  True]], dtype=bool)
>>> ndimage.binary_dilation(a, structure=struct1).astype(a.dtype)
array([[ 0.,  0.,  0.,  0.,  0.]])
```

```

    [ 0.,  0.,  1.,  0.,  0.],
    [ 0.,  1.,  1.,  1.,  0.],
    [ 0.,  0.,  1.,  0.,  0.],
    [ 0.,  0.,  0.,  0.,  0.]]
>>> ndimage.binary_dilation(a, structure=struct2).astype(a.dtype)
array([[ 0.,  0.,  0.,  0.,  0.],
       [ 0.,  1.,  1.,  1.,  0.],
       [ 0.,  1.,  1.,  1.,  0.],
       [ 0.,  1.,  1.,  1.,  0.],
       [ 0.,  0.,  0.,  0.,  0.]])
>>> ndimage.binary_dilation(a, structure=struct1,\
... iterations=2).astype(a.dtype)
array([[ 0.,  0.,  1.,  0.,  0.],
       [ 0.,  1.,  1.,  1.,  0.],
       [ 1.,  1.,  1.,  1.,  1.],
       [ 0.,  1.,  1.,  1.,  0.],
       [ 0.,  0.,  1.,  0.,  0.]])

```

`scipy.ndimage.binary_erosion` (*input*, *structure=None*, *iterations=1*, *mask=None*, *output=None*, *border\_value=0*, *origin=0*, *brute\_force=False*)

Multi-dimensional binary erosion with a given structuring element.

Binary erosion is a mathematical morphology operation used for image processing.

**Parameters** **input** : array\_like

Binary image to be eroded. Non-zero (True) elements form the subset to be eroded.

**structure** : array\_like, optional

Structuring element used for the erosion. Non-zero elements are considered True. If no structuring element is provided, an element is generated with a square connectivity equal to one.

**iterations** : {int, float}, optional

The erosion is repeated *iterations* times (one, by default). If *iterations* is less than 1, the erosion is repeated until the result does not change anymore.

**mask** : array\_like, optional

If a mask is given, only those elements with a True value at the corresponding mask element are modified at each iteration.

**output** : ndarray, optional

Array of the same shape as *input*, into which the output is placed. By default, a new array is created.

**origin** : int or tuple of ints, optional

Placement of the filter, by default 0.

**border\_value** : int (cast to 0 or 1), optional

Value at the border in the output array.

**Returns** **binary\_erosion** : ndarray of bools

Erosion of the input by the structuring element.

**See also:**

`grey_erosion`, `binary_dilation`, `binary_closing`, `binary_opening`,  
`generate_binary_structure`

**Notes**

Erosion [R123] is a mathematical morphology operation [R124] that uses a structuring element for shrinking the shapes in an image. The binary erosion of an image by a structuring element is the locus of the points where a superimposition of the structuring element centered on the point is entirely contained in the set of non-zero elements of the image.

**References**

[R123], [R124]

**Examples**

```

>>> from scipy import ndimage
>>> a = np.zeros((7,7), dtype=int)
>>> a[1:6, 2:5] = 1
>>> a
array([[0, 0, 0, 0, 0, 0, 0],
       [0, 0, 1, 1, 1, 0, 0],
       [0, 0, 1, 1, 1, 0, 0],
       [0, 0, 1, 1, 1, 0, 0],
       [0, 0, 1, 1, 1, 0, 0],
       [0, 0, 1, 1, 1, 0, 0],
       [0, 0, 0, 0, 0, 0, 0]])
>>> ndimage.binary_erosion(a).astype(a.dtype)
array([[0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 1, 0, 0, 0],
       [0, 0, 0, 1, 0, 0, 0],
       [0, 0, 0, 1, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0]])
>>> #Erosion removes objects smaller than the structure
>>> ndimage.binary_erosion(a, structure=np.ones((5,5)).astype(a.dtype))
array([[0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0]])

```

`scipy.ndimage.binary_fill_holes` (*input*, *structure=None*, *output=None*, *origin=0*)

Fill the holes in binary objects.

**Parameters** **input** : array\_like

n-dimensional binary array with holes to be filled

**structure** : array\_like, optional

Structuring element used in the computation; large-size elements make computations faster but may miss holes separated from the background by thin regions. The default element (with a square connectivity equal to one) yields the intuitive result where all holes in the input have been filled.

**output** : ndarray, optional

Array of the same shape as *input*, into which the output is placed. By default, a new array is created.

**origin** : int, tuple of ints, optional

Position of the structuring element.

**Returns** **out** : ndarray

Transformation of the initial image *input* where holes have been filled.

**See also:**

`binary_dilation`, `binary_propagation`, `label`

### Notes

The algorithm used in this function consists in invading the complementary of the shapes in *input* from the outer boundary of the image, using binary dilations. Holes are not connected to the boundary and are therefore not invaded. The result is the complementary subset of the invaded region.

### References

[R125]

### Examples

```
>>> from scipy import ndimage
>>> a = np.zeros((5, 5), dtype=int)
>>> a[1:4, 1:4] = 1
>>> a[2,2] = 0
>>> a
array([[0, 0, 0, 0, 0],
       [0, 1, 1, 1, 0],
       [0, 1, 0, 1, 0],
       [0, 1, 1, 1, 0],
       [0, 0, 0, 0, 0]])
>>> ndimage.binary_fill_holes(a).astype(int)
array([[0, 0, 0, 0, 0],
       [0, 1, 1, 1, 0],
       [0, 1, 1, 1, 0],
       [0, 1, 1, 1, 0],
       [0, 0, 0, 0, 0]])
>>> # Too big structuring element
>>> ndimage.binary_fill_holes(a, structure=np.ones((5,5))).astype(int)
array([[0, 0, 0, 0, 0],
       [0, 1, 1, 1, 0],
       [0, 1, 0, 1, 0],
       [0, 1, 1, 1, 0],
       [0, 0, 0, 0, 0]])
```

`scipy.ndimage.binary_hit_or_miss` (*input*, *structure1=None*, *structure2=None*, *output=None*, *origin1=0*, *origin2=None*)

Multi-dimensional binary hit-or-miss transform.

The hit-or-miss transform finds the locations of a given pattern inside the input image.

**Parameters**

- input** : array\_like (cast to booleans)  
Binary image where a pattern is to be detected.
- structure1** : array\_like (cast to booleans), optional  
Part of the structuring element to be fitted to the foreground (non-zero elements) of *input*. If no value is provided, a structure of square connectivity 1 is chosen.
- structure2** : array\_like (cast to booleans), optional  
Second part of the structuring element that has to miss completely the foreground. If no value is provided, the complementary of *structure1* is taken.
- output** : ndarray, optional  
Array of the same shape as input, into which the output is placed. By default, a new array is created.
- origin1** : int or tuple of ints, optional  
Placement of the first part of the structuring element *structure1*, by default 0 for a centered structure.
- origin2** : int or tuple of ints, optional

Placement of the second part of the structuring element *structure2*, by default 0 for a centered structure. If a value is provided for *origin1* and not for *origin2*, then *origin2* is set to *origin1*.

**Returns** **binary\_hit\_or\_miss** : ndarray  
Hit-or-miss transform of *input* with the given structuring element (*structure1*, *structure2*).

**See also:**

`ndimage.morphology.binary_erosion`

**References**

[R126]

**Examples**

```
>>> from scipy import ndimage
>>> a = np.zeros((7,7), dtype=int)
>>> a[1, 1] = 1; a[2:4, 2:4] = 1; a[4:6, 4:6] = 1
>>> a
array([[0, 0, 0, 0, 0, 0, 0],
       [0, 1, 0, 0, 0, 0, 0],
       [0, 0, 1, 1, 0, 0, 0],
       [0, 0, 1, 1, 0, 0, 0],
       [0, 0, 0, 0, 1, 1, 0],
       [0, 0, 0, 0, 1, 1, 0],
       [0, 0, 0, 0, 0, 0, 0]])
>>> structure1 = np.array([[1, 0, 0], [0, 1, 1], [0, 1, 1]])
>>> structure1
array([[1, 0, 0],
       [0, 1, 1],
       [0, 1, 1]])
>>> # Find the matches of structure1 in the array a
>>> ndimage.binary_hit_or_miss(a, structure1=structure1).astype(int)
array([[0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0],
       [0, 0, 1, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 1, 0, 0],
       [0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0]])
>>> # Change the origin of the filter
>>> # origin1=1 is equivalent to origin1=(1,1) here
>>> ndimage.binary_hit_or_miss(a, structure1=structure1, \
... origin1=1).astype(int)
array([[0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 1, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 1, 0],
       [0, 0, 0, 0, 0, 0, 0]])
```

`scipy.ndimage.binary_opening` (*input*, *structure=None*, *iterations=1*, *output=None*, *origin=0*)  
Multi-dimensional binary opening with the given structuring element.

The *opening* of an input image by a structuring element is the *dilation* of the *erosion* of the image by the structuring element.

**Parameters**

- input** : array\_like  
Binary array\_like to be opened. Non-zero (True) elements form the subset to be opened.
- structure** : array\_like, optional  
Structuring element used for the opening. Non-zero elements are considered True. If no structuring element is provided an element is generated with a square connectivity equal to one (i.e., only nearest neighbors are connected to the center, diagonally-connected elements are not considered neighbors).
- iterations** : {int, float}, optional  
The erosion step of the opening, then the dilation step are each repeated *iterations* times (one, by default). If *iterations* is less than 1, each operation is repeated until the result does not change anymore.
- output** : ndarray, optional  
Array of the same shape as input, into which the output is placed. By default, a new array is created.
- origin** : int or tuple of ints, optional  
Placement of the filter, by default 0.

**Returns**

- binary\_opening** : ndarray of bools  
Opening of the input by the structuring element.

**See also:**

`grey_opening`, `binary_closing`, `binary_erosion`, `binary_dilation`,  
`generate_binary_structure`

**Notes**

*Opening* [R127] is a mathematical morphology operation [R128] that consists in the succession of an erosion and a dilation of the input with the same structuring element. Opening therefore removes objects smaller than the structuring element.

Together with *closing* (`binary_closing`), opening can be used for noise removal.

**References**

[R127], [R128]

**Examples**

```
>>> from scipy import ndimage
>>> a = np.zeros((5,5), dtype=int)
>>> a[1:4, 1:4] = 1; a[4, 4] = 1
>>> a
array([[0, 0, 0, 0, 0],
       [0, 1, 1, 1, 0],
       [0, 1, 1, 1, 0],
       [0, 1, 1, 1, 0],
       [0, 0, 0, 0, 1]])
>>> # Opening removes small objects
>>> ndimage.binary_opening(a, structure=np.ones((3,3))).astype(int)
array([[0, 0, 0, 0, 0],
       [0, 1, 1, 1, 0],
       [0, 1, 1, 1, 0],
       [0, 1, 1, 1, 0],
       [0, 0, 0, 0, 0]])
>>> # Opening can also smooth corners
>>> ndimage.binary_opening(a).astype(int)
array([[0, 0, 0, 0, 0],
       [0, 0, 1, 0, 0],
```



```

    [0, 1, 1, 1, 0],
    [0, 0, 1, 0, 0],
    [0, 0, 0, 0, 0]])
>>> # Opening is the dilation of the erosion of the input
>>> ndimage.binary_erosion(a).astype(int)
array([[0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0],
       [0, 0, 1, 0, 0],
       [0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0]])
>>> ndimage.binary_dilation(ndimage.binary_erosion(a)).astype(int)
array([[0, 0, 0, 0, 0],
       [0, 0, 1, 0, 0],
       [0, 1, 1, 1, 0],
       [0, 0, 1, 0, 0],
       [0, 0, 0, 0, 0]])

```

`scipy.ndimage.binary_propagation` (*input*, *structure=None*, *mask=None*, *output=None*, *border\_value=0*, *origin=0*)

Multi-dimensional binary propagation with the given structuring element.

**Parameters**

- input** : array\_like  
Binary image to be propagated inside *mask*.
- structure** : array\_like, optional  
Structuring element used in the successive dilations. The output may depend on the structuring element, especially if *mask* has several connex components. If no structuring element is provided, an element is generated with a squared connectivity equal to one.
- mask** : array\_like, optional  
Binary mask defining the region into which *input* is allowed to propagate.
- output** : ndarray, optional  
Array of the same shape as *input*, into which the output is placed. By default, a new array is created.
- border\_value** : int (cast to 0 or 1), optional  
Value at the border in the output array.
- origin** : int or tuple of ints, optional  
Placement of the filter, by default 0.

**Returns**

- binary\_propagation** : ndarray  
Binary propagation of *input* inside *mask*.

### Notes

This function is functionally equivalent to calling `binary_dilation` with the number of iterations less than one: iterative dilation until the result does not change anymore.

The succession of an erosion and propagation inside the original image can be used instead of an *opening* for deleting small objects while keeping the contours of larger objects untouched.

### References

[R129], [R130]

### Examples

```

>>> from scipy import ndimage
>>> input = np.zeros((8, 8), dtype=int)
>>> input[2, 2] = 1
>>> mask = np.zeros((8, 8), dtype=int)

```

```

>>> mask[1:4, 1:4] = mask[4, 4] = mask[6:8, 6:8] = 1
>>> input
array([[0, 0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 0],
       [0, 0, 1, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 0]])
>>> mask
array([[0, 0, 0, 0, 0, 0, 0, 0],
       [0, 1, 1, 1, 0, 0, 0, 0],
       [0, 1, 1, 1, 0, 0, 0, 0],
       [0, 1, 1, 1, 0, 0, 0, 0],
       [0, 0, 0, 0, 1, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 1, 1],
       [0, 0, 0, 0, 0, 0, 1, 1]])
>>> ndimage.binary_propagation(input, mask=mask).astype(int)
array([[0, 0, 0, 0, 0, 0, 0, 0],
       [0, 1, 1, 1, 0, 0, 0, 0],
       [0, 1, 1, 1, 0, 0, 0, 0],
       [0, 1, 1, 1, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 0]])
>>> ndimage.binary_propagation(input, mask=mask, \
... structure=np.ones((3,3)).astype(int))
array([[0, 0, 0, 0, 0, 0, 0, 0],
       [0, 1, 1, 1, 0, 0, 0, 0],
       [0, 1, 1, 1, 0, 0, 0, 0],
       [0, 1, 1, 1, 0, 0, 0, 0],
       [0, 0, 0, 0, 1, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 0]])

>>> # Comparison between opening and erosion+propagation
>>> a = np.zeros((6,6), dtype=int)
>>> a[2:5, 2:5] = 1; a[0, 0] = 1; a[5, 5] = 1
>>> a
array([[1, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0],
       [0, 0, 1, 1, 1, 0],
       [0, 0, 1, 1, 1, 0],
       [0, 0, 1, 1, 1, 0],
       [0, 0, 0, 0, 0, 1]])
>>> ndimage.binary_opening(a).astype(int)
array([[0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0],
       [0, 0, 0, 1, 0, 0],
       [0, 0, 1, 1, 1, 0],
       [0, 0, 0, 1, 0, 0],
       [0, 0, 0, 0, 0, 0]])
>>> b = ndimage.binary_erosion(a)
>>> b.astype(int)

```

```

array([[0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0],
       [0, 0, 0, 1, 0, 0],
       [0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0]])
>>> ndimage.binary_propagation(b, mask=a).astype(int)
array([[0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0],
       [0, 0, 1, 1, 1, 0],
       [0, 0, 1, 1, 1, 0],
       [0, 0, 1, 1, 1, 0],
       [0, 0, 0, 0, 0, 0]])

```

`scipy.ndimage.black_tophat` (*input*, *size=None*, *footprint=None*, *structure=None*, *output=None*, *mode='reflect'*, *cval=0.0*, *origin=0*)

Multi-dimensional black tophat filter.

**Parameters**

- input** : array\_like  
Input.
- size** : tuple of ints, optional  
Shape of a flat and full structuring element used for the filter. Optional if *footprint* or *structure* is provided.
- footprint** : array of ints, optional  
Positions of non-infinite elements of a flat structuring element used for the black tophat filter.
- structure** : array of ints, optional  
Structuring element used for the filter. *structure* may be a non-flat structuring element.
- output** : array, optional  
An array used for storing the output of the filter may be provided.
- mode** : {'reflect', 'constant', 'nearest', 'mirror', 'wrap'}, optional  
The *mode* parameter determines how the array borders are handled, where *cval* is the value when mode is equal to 'constant'. Default is 'reflect'
- cval** : scalar, optional  
Value to fill past edges of input if *mode* is 'constant'. Default is 0.0.
- origin** : scalar, optional  
The *origin* parameter controls the placement of the filter. Default 0

**Returns**

- black\_tophat** : ndarray  
Result of the filter of *input* with *structure*.

**See also:**

`white_tophat`, `grey_opening`, `grey_closing`

`scipy.ndimage.distance_transform_bf` (*input*, *metric='euclidean'*, *sampling=None*, *return\_distances=True*, *return\_indices=False*, *distances=None*, *indices=None*)

Distance transform function by a brute force algorithm.

This function calculates the distance transform of the *input*, by replacing each background element (zero values), with its shortest distance to the foreground (any element non-zero).

In addition to the distance transform, the feature transform can be calculated. In this case the index of the closest background element is returned along the first axis of the result.

**Parameters**

- input** : array\_like  
Input
- metric** : str, optional  
Three types of distance metric are supported: 'euclidean', 'taxicab' and 'chessboard'.

**sampling** : {int, sequence of ints}, optional

This parameter is only used in the case of the euclidean *metric* distance transform.

The sampling along each axis can be given by the *sampling* parameter which should be a sequence of length equal to the input rank, or a single number in which the *sampling* is assumed to be equal along all axes.

**return\_distances** : bool, optional

The *return\_distances* flag can be used to indicate if the distance transform is returned. The default is True.

**return\_indices** : bool, optional

The *return\_indices* flags can be used to indicate if the feature transform is returned. The default is False.

**distances** : float64 ndarray, optional

Optional output array to hold distances (if *return\_distances* is True).

**indices** : int64 ndarray, optional

Optional output array to hold indices (if *return\_indices* is True).

**Returns**

**distances** : ndarray

Distance array if *return\_distances* is True.

**indices** : ndarray

Indices array if *return\_indices* is True.

**Notes**

This function employs a slow brute force algorithm, see also the function `distance_transform_cdt` for more efficient taxicab and chessboard algorithms.

`scipy.ndimage.distance_transform_cdt` (*input*, *metric*='chessboard', *return\_distances*=True, *return\_indices*=False, *distances*=None, *indices*=None)

Distance transform for chamfer type of transforms.

**Parameters**

**input** : array\_like

Input

**metric** : {'chessboard', 'taxicab'}, optional

The *metric* determines the type of chamfering that is done. If the *metric* is equal to 'taxicab' a structure is generated using `generate_binary_structure` with a squared distance equal to 1. If the *metric* is equal to 'chessboard', a *metric* is generated using `generate_binary_structure` with a squared distance equal to the dimensionality of the array. These choices correspond to the common interpretations of the 'taxicab' and the 'chessboard' distance metrics in two dimensions.

The default for *metric* is 'chessboard'.

**return\_distances, return\_indices** : bool, optional

The *return\_distances*, and *return\_indices* flags can be used to indicate if the distance transform, the feature transform, or both must be returned.

If the feature transform is returned (*return\_indices*=True), the index of the closest background element is returned along the first axis of the result.

The *return\_distances* default is True, and the *return\_indices* default is False.

**distances, indices** : ndarrays of int32, optional

The *distances* and *indices* arguments can be used to give optional output arrays that must be the same shape as *input*.

`scipy.ndimage.distance_transform_edt` (*input*, *sampling*=None, *return\_distances*=True, *return\_indices*=False, *distances*=None, *indices*=None)

Exact euclidean distance transform.

In addition to the distance transform, the feature transform can be calculated. In this case the index of the closest background element is returned along the first axis of the result.

**Parameters**

**input** : array\_like

Input data to transform. Can be any type but will be converted into binary: 1 wherever input equates to True, 0 elsewhere.

**sampling** : float or int, or sequence of same, optional

Spacing of elements along each dimension. If a sequence, must be of length equal to the input rank; if a single number, this is used for all axes. If not specified, a grid spacing of unity is implied.

**return\_distances** : bool, optional

Whether to return distance matrix. At least one of return\_distances/return\_indices must be True. Default is True.

**return\_indices** : bool, optional

Whether to return indices matrix. Default is False.

**distances** : ndarray, optional

Used for output of distance array, must be of type float64.

**indices** : ndarray, optional

Used for output of indices, must be of type int32.

**Returns** **distance\_transform\_edt** : ndarray or list of ndarrays

Either distance matrix, index matrix, or a list of the two, depending on *return\_x* flags and *distance* and *indices* input parameters.

### Notes

The euclidean distance transform gives values of the euclidean distance:

$$y_i = \sqrt{\sum_i^n (x[i]-b[i])^2}$$

where  $b[i]$  is the background point (value 0) with the smallest Euclidean distance to input points  $x[i]$ , and  $n$  is the number of dimensions.

### Examples

```
>>> from scipy import ndimage
>>> a = np.array([[0,1,1,1,1],
...              [0,0,1,1,1],
...              [0,1,1,1,1],
...              [0,1,1,1,0],
...              [0,1,1,0,0]])
>>> ndimage.distance_transform_edt(a)
array([[ 0.   ,  1.   ,  1.4142,  2.2361,  3.   ],
       [ 0.   ,  0.   ,  1.   ,  2.   ,  2.   ],
       [ 0.   ,  1.   ,  1.4142,  1.4142,  1.   ],
       [ 0.   ,  1.   ,  1.4142,  1.   ,  0.   ],
       [ 0.   ,  1.   ,  1.   ,  0.   ,  0.   ]])
```

With a sampling of 2 units along x, 1 along y:

```
>>> ndimage.distance_transform_edt(a, sampling=[2,1])
array([[ 0.   ,  1.   ,  2.   ,  2.8284,  3.6056],
       [ 0.   ,  0.   ,  1.   ,  2.   ,  3.   ],
       [ 0.   ,  1.   ,  2.   ,  2.2361,  2.   ],
       [ 0.   ,  1.   ,  2.   ,  1.   ,  0.   ],
       [ 0.   ,  1.   ,  1.   ,  0.   ,  0.   ]])
```

Asking for indices as well:

```
>>> edt, inds = ndimage.distance_transform_edt(a, return_indices=True)
>>> inds
array([[0, 0, 1, 1, 3],
```

```
[1, 1, 1, 1, 3],
 [2, 2, 1, 3, 3],
 [3, 3, 4, 4, 3],
 [4, 4, 4, 4, 4]],
 [[0, 0, 1, 1, 4],
 [0, 1, 1, 1, 4],
 [0, 0, 1, 4, 4],
 [0, 0, 3, 3, 4],
 [0, 0, 3, 3, 4]])
```

With arrays provided for inplace outputs:

```
>>> indices = np.zeros((np.ndim(a,) + a.shape), dtype=np.int32)
>>> ndimage.distance_transform_edt(a, return_indices=True, indices=indices)
array([[ 0.   ,  1.   ,  1.4142,  2.2361,  3.   ],
       [ 0.   ,  0.   ,  1.   ,  2.   ,  2.   ],
       [ 0.   ,  1.   ,  1.4142,  1.4142,  1.   ],
       [ 0.   ,  1.   ,  1.4142,  1.   ,  0.   ],
       [ 0.   ,  1.   ,  1.   ,  0.   ,  0.   ]])
>>> indices
array([[0, 0, 1, 1, 3],
       [1, 1, 1, 1, 3],
       [2, 2, 1, 3, 3],
       [3, 3, 4, 4, 3],
       [4, 4, 4, 4, 4]],
       [[0, 0, 1, 1, 4],
       [0, 1, 1, 1, 4],
       [0, 0, 1, 4, 4],
       [0, 0, 3, 3, 4],
       [0, 0, 3, 3, 4]])
```

`scipy.ndimage.generate_binary_structure` (*rank*, *connectivity*)

Generate a binary structure for binary morphological operations.

**Parameters**    **rank** : int

Number of dimensions of the array to which the structuring element will be applied, as returned by `np.ndim`.

**connectivity** : int

*connectivity* determines which elements of the output array belong to the structure, i.e. are considered as neighbors of the central element. Elements up to a squared distance of *connectivity* from the center are considered neighbors. *connectivity* may range from 1 (no diagonal elements are neighbors) to *rank* (all elements are neighbors).

**Returns**        **output** : ndarray of bools

Structuring element which may be used for binary morphological operations, with *rank* dimensions and all dimensions equal to 3.

**See also:**

`iterate_structure`, `binary_dilation`, `binary_erosion`

**Notes**

`generate_binary_structure` can only create structuring elements with dimensions equal to 3, i.e. minimal dimensions. For larger structuring elements, that are useful e.g. for eroding large objects, one may either use `iterate_structure`, or create directly custom arrays with numpy functions such as `numpy.ones`.

**Examples**

```

>>> from scipy import ndimage
>>> struct = ndimage.generate_binary_structure(2, 1)
>>> struct
array([[False,  True,  False],
       [ True,  True,  True],
       [False,  True,  False]], dtype=bool)
>>> a = np.zeros((5,5))
>>> a[2, 2] = 1
>>> a
array([[ 0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  1.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.]])
>>> b = ndimage.binary_dilation(a, structure=struct).astype(a.dtype)
>>> b
array([[ 0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  1.,  0.,  0.],
       [ 0.,  1.,  1.,  1.,  0.],
       [ 0.,  0.,  1.,  0.,  0.],
       [ 0.,  0.,  0.,  0.,  0.]])
>>> ndimage.binary_dilation(b, structure=struct).astype(a.dtype)
array([[ 0.,  0.,  1.,  0.,  0.],
       [ 0.,  1.,  1.,  1.,  0.],
       [ 1.,  1.,  1.,  1.,  1.],
       [ 0.,  1.,  1.,  1.,  0.],
       [ 0.,  0.,  1.,  0.,  0.]])
>>> struct = ndimage.generate_binary_structure(2, 2)
>>> struct
array([[ True,  True,  True],
       [ True,  True,  True],
       [ True,  True,  True]], dtype=bool)
>>> struct = ndimage.generate_binary_structure(3, 1)
>>> struct # no diagonal elements
array([[False, False, False],
       [False,  True, False],
       [False, False, False]],
      [[False,  True, False],
       [ True,  True,  True],
       [False,  True, False]],
      [[False, False, False],
       [False,  True, False],
       [False, False, False]], dtype=bool)

```

`scipy.ndimage.grey_closing`(*input*, *size=None*, *footprint=None*, *structure=None*, *output=None*, *mode='reflect'*, *cval=0.0*, *origin=0*)

Multi-dimensional greyscale closing.

A greyscale closing consists in the succession of a greyscale dilation, and a greyscale erosion.

**Parameters** **input** : array\_like

Array over which the greyscale closing is to be computed.

**size** : tuple of ints

Shape of a flat and full structuring element used for the greyscale closing. Optional if *footprint* or *structure* is provided.

**footprint** : array of ints, optional

Positions of non-infinite elements of a flat structuring element used for the greyscale

closing.

**structure** : array of ints, optional

Structuring element used for the grayscale closing. *structure* may be a non-flat structuring element.

**output** : array, optional

An array used for storing the output of the closing may be provided.

**mode** : {'reflect', 'constant', 'nearest', 'mirror', 'wrap'}, optional

The *mode* parameter determines how the array borders are handled, where *cval* is the value when mode is equal to 'constant'. Default is 'reflect'

**cval** : scalar, optional

Value to fill past edges of input if *mode* is 'constant'. Default is 0.0.

**origin** : scalar, optional

The *origin* parameter controls the placement of the filter. Default 0

**Returns**

**grey\_closing** : ndarray

Result of the grayscale closing of *input* with *structure*.

**See also:**

`binary_closing`, `grey_dilation`, `grey_erosion`, `grey_opening`,  
`generate_binary_structure`

**Notes**

The action of a grayscale closing with a flat structuring element amounts to smoothen deep local minima, whereas binary closing fills small holes.

**References**

[R131]

**Examples**

```
>>> from scipy import ndimage
>>> a = np.arange(36).reshape((6, 6))
>>> a[3,3] = 0
>>> a
array([[ 0,  1,  2,  3,  4,  5],
       [ 6,  7,  8,  9, 10, 11],
       [12, 13, 14, 15, 16, 17],
       [18, 19, 20,  0, 22, 23],
       [24, 25, 26, 27, 28, 29],
       [30, 31, 32, 33, 34, 35]])
>>> ndimage.grey_closing(a, size=(3,3))
array([[ 7,  7,  8,  9, 10, 11],
       [ 7,  7,  8,  9, 10, 11],
       [13, 13, 14, 15, 16, 17],
       [19, 19, 20, 20, 22, 23],
       [25, 25, 26, 27, 28, 29],
       [31, 31, 32, 33, 34, 35]])
>>> # Note that the local minimum a[3,3] has disappeared
```

`scipy.ndimage.grey_dilation` (*input*, *size=None*, *footprint=None*, *structure=None*, *output=None*,  
*mode='reflect'*, *cval=0.0*, *origin=0*)

Calculate a grayscale dilation, using either a structuring element, or a footprint corresponding to a flat structuring element.

Grayscale dilation is a mathematical morphology operation. For the simple case of a full and flat structuring element, it can be viewed as a maximum filter over a sliding window.



**Parameters**

- input** : array\_like  
Array over which the grayscale dilation is to be computed.
- size** : tuple of ints  
Shape of a flat and full structuring element used for the grayscale dilation. Optional if *footprint* or *structure* is provided.
- footprint** : array of ints, optional  
Positions of non-infinite elements of a flat structuring element used for the grayscale dilation. Non-zero values give the set of neighbors of the center over which the maximum is chosen.
- structure** : array of ints, optional  
Structuring element used for the grayscale dilation. *structure* may be a non-flat structuring element.
- output** : array, optional  
An array used for storing the output of the dilation may be provided.
- mode** : { 'reflect', 'constant', 'nearest', 'mirror', 'wrap' }, optional  
The *mode* parameter determines how the array borders are handled, where *cval* is the value when mode is equal to 'constant'. Default is 'reflect'
- cval** : scalar, optional  
Value to fill past edges of input if *mode* is 'constant'. Default is 0.0.
- origin** : scalar, optional  
The *origin* parameter controls the placement of the filter. Default 0

**Returns**

- grey\_dilation** : ndarray  
Grayscale dilation of *input*.

**See also:**

`binary_dilation`, `grey_erosion`, `grey_closing`, `grey_opening`,  
`generate_binary_structure`, `ndimage.maximum_filter`

**Notes**

The grayscale dilation of an image input by a structuring element  $s$  defined over a domain  $E$  is given by:

$$(\text{input}+s)(x) = \max \{ \text{input}(y) + s(x-y), \text{ for } y \text{ in } E \}$$

In particular, for structuring elements defined as  $s(y) = 0$  for  $y$  in  $E$ , the grayscale dilation computes the maximum of the input image inside a sliding window defined by  $E$ .

Grayscale dilation [R132] is a *mathematical morphology* operation [R133].

**References**

[R132], [R133]

**Examples**

```
>>> from scipy import ndimage
>>> a = np.zeros((7,7), dtype=int)
>>> a[2:5, 2:5] = 1
>>> a[4,4] = 2; a[2,3] = 3
>>> a
array([[0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0],
       [0, 0, 1, 3, 1, 0, 0],
       [0, 0, 1, 1, 1, 0, 0],
       [0, 0, 1, 1, 2, 0, 0],
       [0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0]])
>>> ndimage.grey_dilation(a, size=(3,3))
```

```

array([[0, 0, 0, 0, 0, 0, 0],
       [0, 1, 3, 3, 3, 1, 0],
       [0, 1, 3, 3, 3, 1, 0],
       [0, 1, 3, 3, 3, 2, 0],
       [0, 1, 1, 2, 2, 2, 0],
       [0, 1, 1, 2, 2, 2, 0],
       [0, 0, 0, 0, 0, 0, 0]])
>>> ndimage.grey_dilation(a, footprint=np.ones((3,3)))
array([[0, 0, 0, 0, 0, 0, 0],
       [0, 1, 3, 3, 3, 1, 0],
       [0, 1, 3, 3, 3, 1, 0],
       [0, 1, 3, 3, 3, 2, 0],
       [0, 1, 1, 2, 2, 2, 0],
       [0, 1, 1, 2, 2, 2, 0],
       [0, 0, 0, 0, 0, 0, 0]])
>>> s = ndimage.generate_binary_structure(2,1)
>>> s
array([[False,  True,  False],
       [ True,  True,  True],
       [False,  True,  False]], dtype=bool)
>>> ndimage.grey_dilation(a, footprint=s)
array([[0, 0, 0, 0, 0, 0, 0],
       [0, 0, 1, 3, 1, 0, 0],
       [0, 1, 3, 3, 3, 1, 0],
       [0, 1, 1, 3, 2, 1, 0],
       [0, 1, 1, 2, 2, 2, 0],
       [0, 0, 1, 1, 2, 0, 0],
       [0, 0, 0, 0, 0, 0, 0]])
>>> ndimage.grey_dilation(a, size=(3,3), structure=np.ones((3,3)))
array([[1, 1, 1, 1, 1, 1, 1],
       [1, 2, 4, 4, 4, 2, 1],
       [1, 2, 4, 4, 4, 2, 1],
       [1, 2, 4, 4, 4, 3, 1],
       [1, 2, 2, 3, 3, 3, 1],
       [1, 2, 2, 3, 3, 3, 1],
       [1, 1, 1, 1, 1, 1, 1]])
    
```

`scipy.ndimage.grey_erosion` (*input*, *size=None*, *footprint=None*, *structure=None*, *output=None*, *mode='reflect'*, *cval=0.0*, *origin=0*)

Calculate a grayscale erosion, using either a structuring element, or a footprint corresponding to a flat structuring element.

Grayscale erosion is a mathematical morphology operation. For the simple case of a full and flat structuring element, it can be viewed as a minimum filter over a sliding window.

**Parameters** **input** : array\_like

Array over which the grayscale erosion is to be computed.

**size** : tuple of ints

Shape of a flat and full structuring element used for the grayscale erosion. Optional if *footprint* or *structure* is provided.

**footprint** : array of ints, optional

Positions of non-infinite elements of a flat structuring element used for the grayscale erosion. Non-zero values give the set of neighbors of the center over which the minimum is chosen.

**structure** : array of ints, optional

Structuring element used for the grayscale erosion. *structure* may be a non-flat structuring element.

**output** : array, optional

An array used for storing the output of the erosion may be provided.

**mode** : { 'reflect', 'constant', 'nearest', 'mirror', 'wrap' }, optional  
 The *mode* parameter determines how the array borders are handled, where *cval* is the value when mode is equal to 'constant'. Default is 'reflect'

**cval** : scalar, optional  
 Value to fill past edges of input if *mode* is 'constant'. Default is 0.0.

**origin** : scalar, optional  
 The *origin* parameter controls the placement of the filter. Default 0

**Returns** **output** : ndarray  
 Grayscale erosion of *input*.

**See also:**

`binary_erosion`, `grey_dilation`, `grey_opening`, `grey_closing`,  
`generate_binary_structure`, `ndimage.minimum_filter`

**Notes**

The grayscale erosion of an image input by a structuring element *s* defined over a domain *E* is given by:

$$(\text{input}+s)(x) = \min \{ \text{input}(y) - s(x-y), \text{ for } y \text{ in } E \}$$

In particular, for structuring elements defined as  $s(y) = 0$  for  $y$  in  $E$ , the grayscale erosion computes the minimum of the input image inside a sliding window defined by  $E$ .

Grayscale erosion [R134] is a *mathematical morphology* operation [R135].

**References**

[R134], [R135]

**Examples**

```
>>> from scipy import ndimage
>>> a = np.zeros((7,7), dtype=int)
>>> a[1:6, 1:6] = 3
>>> a[4,4] = 2; a[2,3] = 1
>>> a
array([[0, 0, 0, 0, 0, 0, 0],
       [0, 3, 3, 3, 3, 3, 0],
       [0, 3, 3, 1, 3, 3, 0],
       [0, 3, 3, 3, 3, 3, 0],
       [0, 3, 3, 3, 2, 3, 0],
       [0, 3, 3, 3, 3, 3, 0],
       [0, 0, 0, 0, 0, 0, 0]])
>>> ndimage.grey_erosion(a, size=(3,3))
array([[0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0],
       [0, 0, 1, 1, 1, 0, 0],
       [0, 0, 1, 1, 1, 0, 0],
       [0, 0, 3, 2, 2, 0, 0],
       [0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0]])
>>> footprint = ndimage.generate_binary_structure(2, 1)
>>> footprint
array([[False,  True,  False],
       [ True,  True,  True],
       [False,  True,  False]], dtype=bool)
>>> # Diagonally-connected elements are not considered neighbors
>>> ndimage.grey_erosion(a, size=(3,3), footprint=footprint)
```

```
array([[0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0],
       [0, 0, 1, 1, 1, 0, 0],
       [0, 0, 3, 1, 2, 0, 0],
       [0, 0, 3, 2, 2, 0, 0],
       [0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0]])
```

`scipy.ndimage.grey_opening` (*input*, *size=None*, *footprint=None*, *structure=None*, *output=None*, *mode='reflect'*, *cval=0.0*, *origin=0*)

Multi-dimensional greyscale opening.

A greyscale opening consists in the succession of a greyscale erosion, and a greyscale dilation.

**Parameters**

- input** : array\_like  
Array over which the grayscale opening is to be computed.
- size** : tuple of ints  
Shape of a flat and full structuring element used for the grayscale opening. Optional if *footprint* or *structure* is provided.
- footprint** : array of ints, optional  
Positions of non-infinite elements of a flat structuring element used for the grayscale opening.
- structure** : array of ints, optional  
Structuring element used for the grayscale opening. *structure* may be a non-flat structuring element.
- output** : array, optional  
An array used for storing the output of the opening may be provided.
- mode** : {'reflect', 'constant', 'nearest', 'mirror', 'wrap'}, optional  
The *mode* parameter determines how the array borders are handled, where *cval* is the value when mode is equal to 'constant'. Default is 'reflect'
- cval** : scalar, optional  
Value to fill past edges of input if *mode* is 'constant'. Default is 0.0.
- origin** : scalar, optional  
The *origin* parameter controls the placement of the filter. Default 0

**Returns**

- grey\_opening** : ndarray  
Result of the grayscale opening of *input* with *structure*.

**See also:**

`binary_opening`, `grey_dilation`, `grey_erosion`, `grey_closing`,  
`generate_binary_structure`

**Notes**

The action of a grayscale opening with a flat structuring element amounts to smoothen high local maxima, whereas binary opening erases small objects.

**References**

[R136]

**Examples**

```
>>> from scipy import ndimage
>>> a = np.arange(36).reshape((6, 6))
>>> a[3, 3] = 50
>>> a
array([[ 0,  1,  2,  3,  4,  5],
```

```

    [ 6,  7,  8,  9, 10, 11],
    [12, 13, 14, 15, 16, 17],
    [18, 19, 20, 50, 22, 23],
    [24, 25, 26, 27, 28, 29],
    [30, 31, 32, 33, 34, 35]])
>>> ndimage.grey_opening(a, size=(3,3))
array([[ 0,  1,  2,  3,  4,  4],
       [ 6,  7,  8,  9, 10, 10],
       [12, 13, 14, 15, 16, 16],
       [18, 19, 20, 22, 22, 22],
       [24, 25, 26, 27, 28, 28],
       [24, 25, 26, 27, 28, 28]])
>>> # Note that the local maximum a[3,3] has disappeared

```

`scipy.ndimage.iterate_structure` (*structure*, *iterations*, *origin=None*)

Iterate a structure by dilating it with itself.

**Parameters**    **structure** : array\_like

Structuring element (an array of bools, for example), to be dilated with itself.

**iterations** : int

number of dilations performed on the structure with itself

**origin** : optional

If origin is None, only the iterated structure is returned. If not, a tuple of the iterated structure and the modified origin is returned.

**Returns**        **iterate\_structure** : ndarray of bools

A new structuring element obtained by dilating *structure* (*iterations* - 1) times with itself.

**See also:**

`generate_binary_structure`

**Examples**

```

>>> from scipy import ndimage
>>> struct = ndimage.generate_binary_structure(2, 1)
>>> struct.astype(int)
array([[0, 1, 0],
       [1, 1, 1],
       [0, 1, 0]])
>>> ndimage.iterate_structure(struct, 2).astype(int)
array([[0, 0, 1, 0, 0],
       [0, 1, 1, 1, 0],
       [1, 1, 1, 1, 1],
       [0, 1, 1, 1, 0],
       [0, 0, 1, 0, 0]])
>>> ndimage.iterate_structure(struct, 3).astype(int)
array([[0, 0, 0, 1, 0, 0, 0],
       [0, 0, 1, 1, 1, 0, 0],
       [0, 1, 1, 1, 1, 1, 0],
       [1, 1, 1, 1, 1, 1, 1],
       [0, 1, 1, 1, 1, 1, 0],
       [0, 0, 1, 1, 1, 0, 0],
       [0, 0, 0, 1, 0, 0, 0]])

```

`scipy.ndimage.morphological_gradient` (*input*, *size=None*, *footprint=None*, *structure=None*, *output=None*, *mode='reflect'*, *cval=0.0*, *origin=0*)

Multi-dimensional morphological gradient.

The morphological gradient is calculated as the difference between a dilation and an erosion of the input with a given structuring element.

**Parameters**

- input** : array\_like  
Array over which to compute the morphological gradient.
- size** : tuple of ints  
Shape of a flat and full structuring element used for the mathematical morphology operations. Optional if *footprint* or *structure* is provided. A larger *size* yields a more blurred gradient.
- footprint** : array of ints, optional  
Positions of non-infinite elements of a flat structuring element used for the morphology operations. Larger footprints give a more blurred morphological gradient.
- structure** : array of ints, optional  
Structuring element used for the morphology operations. *structure* may be a non-flat structuring element.
- output** : array, optional  
An array used for storing the output of the morphological gradient may be provided.
- mode** : { 'reflect', 'constant', 'nearest', 'mirror', 'wrap' }, optional  
The *mode* parameter determines how the array borders are handled, where *cval* is the value when mode is equal to 'constant'. Default is 'reflect'
- cval** : scalar, optional  
Value to fill past edges of input if *mode* is 'constant'. Default is 0.0.
- origin** : scalar, optional  
The *origin* parameter controls the placement of the filter. Default 0

**Returns**

- morphological\_gradient** : ndarray  
Morphological gradient of *input*.

**See also:**

`grey_dilation`, `grey_erosion`, `ndimage.gaussian_gradient_magnitude`

### Notes

For a flat structuring element, the morphological gradient computed at a given point corresponds to the maximal difference between elements of the input among the elements covered by the structuring element centered on the point.

### References

[R141]

### Examples

```
>>> from scipy import ndimage
>>> a = np.zeros((7,7), dtype=int)
>>> a[2:5, 2:5] = 1
>>> ndimage.morphological_gradient(a, size=(3,3))
array([[0, 0, 0, 0, 0, 0, 0],
       [0, 1, 1, 1, 1, 1, 0],
       [0, 1, 1, 1, 1, 1, 0],
       [0, 1, 1, 0, 1, 1, 0],
       [0, 1, 1, 1, 1, 1, 0],
       [0, 1, 1, 1, 1, 1, 0],
       [0, 0, 0, 0, 0, 0, 0]])
>>> # The morphological gradient is computed as the difference
>>> # between a dilation and an erosion
>>> ndimage.grey_dilation(a, size=(3,3)) -\
... ndimage.grey_erosion(a, size=(3,3))
```

```

array([[0, 0, 0, 0, 0, 0, 0],
       [0, 1, 1, 1, 1, 1, 0],
       [0, 1, 1, 1, 1, 1, 0],
       [0, 1, 1, 0, 1, 1, 0],
       [0, 1, 1, 1, 1, 1, 0],
       [0, 1, 1, 1, 1, 1, 0],
       [0, 0, 0, 0, 0, 0, 0]])
>>> a = np.zeros((7,7), dtype=int)
>>> a[2:5, 2:5] = 1
>>> a[4,4] = 2; a[2,3] = 3
>>> a
array([[0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0],
       [0, 0, 1, 3, 1, 0, 0],
       [0, 0, 1, 1, 1, 0, 0],
       [0, 0, 1, 1, 2, 0, 0],
       [0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0]])
>>> ndimage.morphological_gradient(a, size=(3,3))
array([[0, 0, 0, 0, 0, 0, 0],
       [0, 1, 3, 3, 3, 1, 0],
       [0, 1, 3, 3, 3, 1, 0],
       [0, 1, 3, 2, 3, 2, 0],
       [0, 1, 1, 2, 2, 2, 0],
       [0, 1, 1, 2, 2, 2, 0],
       [0, 0, 0, 0, 0, 0, 0]])

```

`scipy.ndimage.morphological_laplace` (*input*, *size=None*, *footprint=None*, *structure=None*, *output=None*, *mode='reflect'*, *cval=0.0*, *origin=0*)

Multi-dimensional morphological laplace.

**Parameters**

- input** : array\_like  
Input.
- size** : int or sequence of ints, optional  
See *structure*.
- footprint** : bool or ndarray, optional  
See *structure*.
- structure** : structure, optional  
Either *size*, *footprint*, or the *structure* must be provided.
- output** : ndarray, optional  
An output array can optionally be provided.
- mode** : { 'reflect', 'constant', 'nearest', 'mirror', 'wrap' }, optional  
The mode parameter determines how the array borders are handled. For 'constant' mode, values beyond borders are set to be *cval*. Default is 'reflect'.
- cval** : scalar, optional  
Value to fill past edges of input if mode is 'constant'. Default is 0.0
- origin** : origin, optional  
The origin parameter controls the placement of the filter.

**Returns**

- morphological\_laplace** : ndarray  
Output

`scipy.ndimage.white_tophat` (*input*, *size=None*, *footprint=None*, *structure=None*, *output=None*, *mode='reflect'*, *cval=0.0*, *origin=0*)

Multi-dimensional white tophat filter.

**Parameters**

- input** : array\_like  
Input.
- size** : tuple of ints

Shape of a flat and full structuring element used for the filter. Optional if *footprint* or *structure* is provided.

**footprint** : array of ints, optional

Positions of elements of a flat structuring element used for the white tophat filter.

**structure** : array of ints, optional

Structuring element used for the filter. *structure* may be a non-flat structuring element.

**output** : array, optional

An array used for storing the output of the filter may be provided.

**mode** : { 'reflect', 'constant', 'nearest', 'mirror', 'wrap' }, optional

The *mode* parameter determines how the array borders are handled, where *cval* is the value when mode is equal to 'constant'. Default is 'reflect'

**cval** : scalar, optional

Value to fill past edges of input if *mode* is 'constant'. Default is 0.0.

**origin** : scalar, optional

The *origin* parameter controls the placement of the filter. Default is 0.

**Returns**

**output** : ndarray

Result of the filter of *input* with *structure*.

**See also:**

`black_tophat`

## 5.16.6 Utility

---

`imread(fname[, flatten, mode])` Read an image from a file as an array.

---

`scipy.ndimage.imread(fname, flatten=False, mode=None)`

Read an image from a file as an array.

**Parameters**

**fname** : str or file object

The file name or file object to be read.

**flatten** : bool, optional

If True, flattens the color layers into a single gray-scale layer.

**mode** : str, optional

Mode to convert image to, e.g. 'RGB'. See the Notes for more details.

**Returns**

**imread** : ndarray

The array obtained by reading the image.

**Notes**

`imread` uses the Python Imaging Library (PIL) to read an image. The following notes are from the PIL documentation.

*mode* can be one of the following strings:

- 'L' (8-bit pixels, black and white)
- 'P' (8-bit pixels, mapped to any other mode using a color palette)
- 'RGB' (3x8-bit pixels, true color)
- 'RGBA' (4x8-bit pixels, true color with transparency mask)
- 'CMYK' (4x8-bit pixels, color separation)
- 'YCbCr' (3x8-bit pixels, color video format)
- 'I' (32-bit signed integer pixels)



- ‘F’ (32-bit floating point pixels)

PIL also provides limited support for a few special modes, including ‘LA’ (‘L’ with alpha), ‘RGBX’ (true color with padding) and ‘RGBa’ (true color with premultiplied alpha).

When translating a color image to black and white (mode ‘L’, ‘I’ or ‘F’), the library uses the ITU-R 601-2 luma transform:

$$L = R * 299/1000 + G * 587/1000 + B * 114/1000$$

When *flatten* is True, the image is converted using mode ‘F’. When *mode* is not None and *flatten* is True, the image is first converted according to *mode*, and the result is then flattened using mode ‘F’.

## 5.17 Orthogonal distance regression (`scipy.odr`)

### 5.17.1 Package Content

<code>Data(x[, y, we, wd, fix, meta])</code>	The data to fit.
<code>RealData(x[, y, sx, sy, covx, covy, fix, meta])</code>	The data, with weightings as actual standard deviations and/or covariances.
<code>Model(fcn[, fjacb, fjacd, extra_args, ...])</code>	The Model class stores information about the function you wish to fit.
<code>ODR(data, model[, beta0, delta0, ifixb, ...])</code>	The ODR class gathers all information and coordinates the running of the main fitting.
<code>Output(output)</code>	The Output class stores the output of an ODR run.
<code>odr(fcn, beta0, y, x[, we, wd, fjacb, ...])</code>	Low-level function for ODR.
<code>OdrWarning</code>	Warning indicating that the data passed into ODR will cause problems when passed to the fitting function.
<code>OdrError</code>	Exception indicating an error in fitting.
<code>OdrStop</code>	Exception stopping fitting.
<code>odr_error</code>	alias of <code>OdrError</code>
<code>odr_stop</code>	alias of <code>OdrStop</code>

**class** `scipy.odr.Data` (*x*, *y=None*, *we=None*, *wd=None*, *fix=None*, *meta={}*)

The data to fit.

**Parameters** **x** : array\_like

Observed data for the independent variable of the regression

**y** : array\_like, optional

If array-like, observed data for the dependent variable of the regression. A scalar input implies that the model to be used on the data is implicit.

**we** : array\_like, optional

If *we* is a scalar, then that value is used for all data points (and all dimensions of the response variable). If *we* is a rank-1 array of length *q* (the dimensionality of the response variable), then this vector is the diagonal of the covariant weighting matrix for all data points. If *we* is a rank-1 array of length *n* (the number of data points), then the *i*'th element is the weight for the *i*'th response variable observation (single-dimensional only). If *we* is a rank-2 array of shape (*q*, *q*), then this is the full covariant weighting matrix broadcast to each observation. If *we* is a rank-2 array of shape (*q*, *n*), then *we[:,i]* is the diagonal of the covariant weighting matrix for the *i*'th observation. If *we* is a rank-3 array of shape (*q*, *q*, *n*), then *we[:, :, i]* is the full specification of the covariant weighting matrix for each observation. If the fit is implicit, then only a positive scalar value is used.

**wd** : array\_like, optional

If *wd* is a scalar, then that value is used for all data points (and all dimensions of the input variable). If *wd* = 0, then the covariant weighting matrix for each observation is set to the identity matrix (so each dimension of each observation has the same

weight). If *wd* is a rank-1 array of length *m* (the dimensionality of the input variable), then this vector is the diagonal of the covariant weighting matrix for all data points. If *wd* is a rank-1 array of length *n* (the number of data points), then the *i*'th element is the weight for the *i*'th input variable observation (single-dimensional only). If *wd* is a rank-2 array of shape (*m*, *m*), then this is the full covariant weighting matrix broadcast to each observation. If *wd* is a rank-2 array of shape (*m*, *n*), then *wd[:,i]* is the diagonal of the covariant weighting matrix for the *i*'th observation. If *wd* is a rank-3 array of shape (*m*, *m*, *n*), then *wd[:, :, i]* is the full specification of the covariant weighting matrix for each observation.

**fix** : array\_like of ints, optional

The *fix* argument is the same as *ifixx* in the class ODR. It is an array of integers with the same shape as *data.x* that determines which input observations are treated as fixed. One can use a sequence of length *m* (the dimensionality of the input observations) to fix some dimensions for all observations. A value of 0 fixes the observation, a value > 0 makes it free.

**meta** : dict, optional

Free-form dictionary for metadata.

### Notes

Each argument is attached to the member of the instance of the same name. The structures of *x* and *y* are described in the Model class docstring. If *y* is an integer, then the Data instance can only be used to fit with implicit models where the dimensionality of the response is equal to the specified value of *y*.

The *we* argument weights the effect a deviation in the response variable has on the fit. The *wd* argument weights the effect a deviation in the input variable has on the fit. To handle multidimensional inputs and responses easily, the structure of these arguments has the *n*'th dimensional axis first. These arguments heavily use the structured arguments feature of ODRPACK to conveniently and flexibly support all options. See the ODRPACK User's Guide for a full explanation of how these weights are used in the algorithm. Basically, a higher value of the weight for a particular data point makes a deviation at that point more detrimental to the fit.

### Methods

---

`set_meta(**kwds)` Update the metadata dictionary with the keywords and data provided by keywords.

---

`Data.set_meta(**kwds)`

Update the metadata dictionary with the keywords and data provided by keywords.

### Examples

```
data.set_meta(lab="Ph 7; Lab 26", title="Ag110 + Ag108 Decay")
```

```
class scipy.odr.RealData(x, y=None, sx=None, sy=None, covx=None, covy=None, fix=None,
                        meta={})
```

The data, with weightings as actual standard deviations and/or covariances.

**Parameters** **x** : array\_like

Observed data for the independent variable of the regression

**y** : array\_like, optional

If array-like, observed data for the dependent variable of the regression. A scalar input implies that the model to be used on the data is implicit.

**sx, sy** : array\_like, optional

Standard deviations of *x*. *sx* are standard deviations of *x* and are converted to weights by dividing 1.0 by their squares.

**sy** : array\_like, optional

Standard deviations of  $y$ .  $sy$  are standard deviations of  $y$  and are converted to weights by dividing 1.0 by their squares.

**covx** : array\_like, optional

Covariance of  $x$   $covx$  is an array of covariance matrices of  $x$  and are converted to weights by performing a matrix inversion on each observation's covariance matrix.

**covy** : array\_like, optional

Covariance of  $y$   $covy$  is an array of covariance matrices and are converted to weights by performing a matrix inversion on each observation's covariance matrix.

**fix** : array\_like, optional

The argument and member `fix` is the same as `Data.fix` and `ODR.ifixx`: It is an array of integers with the same shape as  $x$  that determines which input observations are treated as fixed. One can use a sequence of length  $m$  (the dimensionality of the input observations) to fix some dimensions for all observations. A value of 0 fixes the observation, a value  $> 0$  makes it free.

**meta** : dict, optional

Free-form dictionary for metadata.

### Notes

The weights  $wd$  and  $we$  are computed from provided values as follows:

$sx$  and  $sy$  are converted to weights by dividing 1.0 by their squares. For example, `wd = 1./numpy.power('sx', 2)`.

$covx$  and  $covy$  are arrays of covariance matrices and are converted to weights by performing a matrix inversion on each observation's covariance matrix. For example, `we[i] = numpy.linalg.inv(covy[i])`.

These arguments follow the same structured argument conventions as  $wd$  and  $we$  only restricted by their natures:  $sx$  and  $sy$  can't be rank-3, but  $covx$  and  $covy$  can be.

Only set *either*  $sx$  or  $covx$  (not both). Setting both will raise an exception. Same with  $sy$  and  $covy$ .

### Methods

---

`set_meta(**kwds)` Update the metadata dictionary with the keywords and data provided by keywords.

---

`RealData.set_meta(**kwds)`

Update the metadata dictionary with the keywords and data provided by keywords.

### Examples

```
data.set_meta(lab="Ph 7; Lab 26", title="Ag110 + Ag108 Decay")
```

**class** `scipy.odr.Model1` (*fcn*, *fjacb*=None, *fjacd*=None, *extra\_args*=None, *estimate*=None, *implicit*=0, *meta*=None)

The `Model` class stores information about the function you wish to fit.

It stores the function itself, at the least, and optionally stores functions which compute the Jacobians used during fitting. Also, one can provide a function that will provide reasonable starting values for the fit parameters possibly given the set of data.

**Parameters**

- fcn** : function  
`fcn(beta, x) -> y`
- fjacb** : function  
 Jacobian of `fcn` wrt the fit parameters `beta`.  
`fjacb(beta, x) -> @f_i(x,B)/@B_j`
- fjacd** : function

Jacobian of *fcn* wrt the (possibly multidimensional) input variable.  
`fjacd(beta, x) -> @f_i(x,B)/@x_j`  
**extra\_args** : tuple, optional  
 If specified, *extra\_args* should be a tuple of extra arguments to pass to *fcn*, *fjacb*, and *fjacd*. Each will be called by *apply(fcn, (beta, x) + extra\_args)*  
**estimate** : array\_like of rank-1  
 Provides estimates of the fit parameters from the data  
`estimate(data) -> estbeta`  
**implicit** : boolean  
 If TRUE, specifies that the model is implicit; i.e *fcn(beta, x) ~ 0* and there is no *y* data to fit against  
**meta** : dict, optional  
 freeform dictionary of metadata for the model

**Notes**

Note that the *fcn*, *fjacb*, and *fjacd* operate on NumPy arrays and return a NumPy array. The *estimate* object takes an instance of the Data class.

Here are the rules for the shapes of the argument and return arrays of the callback functions:

- x** if the input data is single-dimensional, then *x* is rank-1 array; i.e. `x = array([1, 2, 3, ...])`; `x.shape = (n,)` If the input data is multi-dimensional, then *x* is a rank-2 array; i.e., `x = array([[1, 2, ...], [2, 4, ...]])`; `x.shape = (m, n)`. In all cases, it has the same shape as the input data array passed to `odr`. *m* is the dimensionality of the input data, *n* is the number of observations.
- y** if the response variable is single-dimensional, then *y* is a rank-1 array, i.e., `y = array([2, 4, ...])`; `y.shape = (n,)`. If the response variable is multi-dimensional, then *y* is a rank-2 array, i.e., `y = array([[2, 4, ...], [3, 6, ...]])`; `y.shape = (q, n)` where *q* is the dimensionality of the response variable.
- beta** rank-1 array of length *p* where *p* is the number of parameters; i.e. `beta = array([B_1, B_2, ..., B_p])`
- fjacb** if the response variable is multi-dimensional, then the return array's shape is (*q, p, n*) such that `fjacb(x, beta)[1, k, i] = d f_l(X, B) / d B_k` evaluated at the *i*'th data point. If *q* == 1, then the return array is only rank-2 and with shape (*p, n*).
- fjacd** as with *fjacb*, only the return array's shape is (*q, m, n*) such that `fjacd(x, beta)[1, j, i] = d f_l(X, B) / d X_j` at the *i*'th data point. If *q* == 1, then the return array's shape is (*m, n*). If *m* == 1, the shape is (*q, n*). If *m* == *q* == 1, the shape is (*n*).

**Methods**

---

`set_meta(**kwds)` Update the metadata dictionary with the keywords and data provided here.

---

`Model.set_meta (**kwds)`  
 Update the metadata dictionary with the keywords and data provided here.

**Examples**

`set_meta(name="Exponential", equation="y = a exp(b x) + c")`

```
class scipy.odr.ODR(data, model, beta0=None, delta0=None, ifixb=None, ifixx=None, job=None,
                    iprint=None, errfile=None, rptfile=None, ndigit=None, taufac=None, sstol=None,
                    partol=None, maxit=None, stpb=None, stpd=None, sclb=None, scld=None,
                    work=None, iwork=None)
```

The ODR class gathers all information and coordinates the running of the main fitting routine.

Members of instances of the ODR class have the same names as the arguments to the initialization routine.

**Parameters** **data** : Data class instance

instance of the Data class

**model** : Model class instance

instance of the Model class

**Other Parameters**

**beta0** : array\_like of rank-1

a rank-1 sequence of initial parameter values. Optional if model provides an “estimate” function to estimate these values.

**delta0** : array\_like of floats of rank-1, optional

a (double-precision) float array to hold the initial values of the errors in the input variables. Must be same shape as data.x

**ifixb** : array\_like of ints of rank-1, optional

sequence of integers with the same length as beta0 that determines which parameters are held fixed. A value of 0 fixes the parameter, a value > 0 makes the parameter free.

**ifixx** : array\_like of ints with same shape as data.x, optional

an array of integers with the same shape as data.x that determines which input observations are treated as fixed. One can use a sequence of length m (the dimensionality of the input observations) to fix some dimensions for all observations. A value of 0 fixes the observation, a value > 0 makes it free.

**job** : int, optional

an integer telling ODRPACK what tasks to perform. See p. 31 of the ODRPACK User’s Guide if you absolutely must set the value here. Use the method set\_job post-initialization for a more readable interface.

**iprint** : int, optional

an integer telling ODRPACK what to print. See pp. 33-34 of the ODRPACK User’s Guide if you absolutely must set the value here. Use the method set\_iprint post-initialization for a more readable interface.

**errfile** : str, optional

string with the filename to print ODRPACK errors to. *Do Not Open This File Yourself!*

**rptfile** : str, optional

string with the filename to print ODRPACK summaries to. *Do Not Open This File Yourself!*

**ndigit** : int, optional

integer specifying the number of reliable digits in the computation of the function.

**taufac** : float, optional

float specifying the initial trust region. The default value is 1. The initial trust region is equal to taufac times the length of the first computed Gauss-Newton step. taufac must be less than 1.

**sstol** : float, optional

float specifying the tolerance for convergence based on the relative change in the sum-of-squares. The default value is  $\text{eps}^{**}(1/2)$  where eps is the smallest value such that  $1 + \text{eps} > 1$  for double precision computation on the machine. sstol must be less than 1.

**partol** : float, optional

float specifying the tolerance for convergence based on the relative change in the estimated parameters. The default value is  $\text{eps}^{**}(2/3)$  for explicit models and  $\text{eps}^{**}(1/3)$  for implicit models. partol must be less than 1.

**maxit** : int, optional

integer specifying the maximum number of iterations to perform. For first runs, maxit is the total number of iterations performed and defaults to 50. For restarts, maxit is the number of additional iterations to perform and defaults to 10.

**stpb** : array\_like, optional

sequence (`len(stpb) == len(beta0)`) of relative step sizes to compute finite difference derivatives wrt the parameters.

**stpd** : optional

array (`stpd.shape == data.x.shape` or `stpd.shape == (m,)`) of relative step sizes to compute finite difference derivatives wrt the input variable errors. If `stpd` is a rank-1 array with length `m` (the dimensionality of the input variable), then the values are broadcast to all observations.

**sclb** : array\_like, optional

sequence (`len(stpb) == len(beta0)`) of scaling factors for the parameters. The purpose of these scaling factors are to scale all of the parameters to around unity. Normally appropriate scaling factors are computed if this argument is not specified. Specify them yourself if the automatic procedure goes awry.

**scld** : array\_like, optional

array (`scld.shape == data.x.shape` or `scld.shape == (m,)`) of scaling factors for the errors in the input variables. Again, these factors are automatically computed if you do not provide them. If `scld.shape == (m,)`, then the scaling factors are broadcast to all observations.

**work** : ndarray, optional

array to hold the double-valued working data for ODRPACK. When restarting, takes the value of `self.output.work`.

**iwork** : ndarray, optional

array to hold the integer-valued working data for ODRPACK. When restarting, takes the value of `self.output.iwork`.

**Attributes**

<code>data</code>	(Data) The data for this fit
<code>model</code>	(Model) The model used in fit
<code>output</code>	(Output) An instance of the Output class containing all of the returned data from an invocation of <code>ODR.run()</code> or <code>ODR.restart()</code>

**Methods**

---

<code>restart(iter)</code>	Restarts the run with <code>iter</code> more iterations.
<code>run()</code>	Run the fitting routine with all of the information given.
<code>set_iprint([init, so_init, iter, so_iter, ...])</code>	Set the <code>iprint</code> parameter for the printing of computation reports.
<code>set_job([fit_type, deriv, var_calc, ...])</code>	Sets the “job” parameter in a hopefully comprehensible way.

---

`ODR.restart(iter=None)`

Restarts the run with `iter` more iterations.

**Parameters** `iter` : int, optional  
 ODRPACK’s default for the number of new iterations is 10.

**Returns** `output` : Output instance  
 This object is also assigned to the attribute `.output`.

`ODR.run()`

Run the fitting routine with all of the information given.

**Returns** `output` : Output instance  
 This object is also assigned to the attribute `.output`.

`ODR.set_iprint(init=None, so_init=None, iter=None, so_iter=None, iter_step=None, final=None, so_final=None)`

Set the `iprint` parameter for the printing of computation reports.

If any of the arguments are specified here, then they are set in the `iprint` member. If `iprint` is not set manually or with this method, then ODRPACK defaults to no printing. If no filename is specified with the member `rptfile`, then ODRPACK prints to `stdout`. One can tell ODRPACK to print to `stdout` in addition to the specified filename by setting the `so_*` arguments to this function, but one cannot specify to print to `stdout` but not a file since one can do that by not specifying a `rptfile` filename.

There are three reports: initialization, iteration, and final reports. They are represented by the arguments `init`, `iter`, and `final` respectively. The permissible values are 0, 1, and 2 representing “no report”, “short report”, and “long report” respectively.

The argument `iter_step` ( $0 \leq \text{iter\_step} \leq 9$ ) specifies how often to make the iteration report; the report will be made for every `iter_step`'th iteration starting with iteration one. If `iter_step == 0`, then no iteration report is made, regardless of the other arguments.

If the `rptfile` is `None`, then any `so_*` arguments supplied will raise an exception.

ODR. **set\_job** (*fit\_type=None, deriv=None, var\_calc=None, del\_init=None, restart=None*)  
Sets the “job” parameter in a hopefully comprehensible way.

If an argument is not specified, then the value is left as is. The default value from class initialization is for all of these options set to 0.

**Parameters**

- fit\_type** : {0, 1, 2} int
  - 0 -> explicit ODR
  - 1 -> implicit ODR
  - 2 -> ordinary least-squares
- deriv** : {0, 1, 2, 3} int
  - 0 -> forward finite differences
  - 1 -> central finite differences
  - 2 -> *user-supplied derivatives (Jacobians) with results* checked by ODRPACK
  - 3 -> user-supplied derivatives, no checking
- var\_calc** : {0, 1, 2} int
  - 0 -> *calculate asymptotic covariance matrix and fit* parameter uncertainties (`V_B`, `s_B`) using derivatives recomputed at the final solution
  - 1 -> calculate `V_B` and `s_B` using derivatives from last iteration
  - 2 -> do not calculate `V_B` and `s_B`
- del\_init** : {0, 1} int
  - 0 -> initial input variable offsets set to 0
  - 1 -> initial offsets provided by user in variable “work”
- restart** : {0, 1} int
  - 0 -> fit is not a restart
  - 1 -> fit is a restart

#### Notes

The permissible values are different from those given on pg. 31 of the ODRPACK User’s Guide only in that one cannot specify numbers greater than the last value for each variable.

If one does not supply functions to compute the Jacobians, the fitting procedure will change `deriv` to 0, finite differences, as a default. To initialize the input variable offsets by yourself, set `del_init` to 1 and put the offsets into the “work” variable correctly.

**class** `scipy.odr.Output` (*output*)

The Output class stores the output of an ODR run.

*Notes*

Takes one argument for initialization, the return value from the function `odr`. The attributes listed as “optional” above are only present if `odr` was run with `full_output=1`.

*Attributes*

<code>beta</code>	(ndarray) Estimated parameter values, of shape (q).
<code>sd_beta</code>	(ndarray) Standard errors of the estimated parameters, of shape (p).
<code>cov_beta</code>	(ndarray) Covariance matrix of the estimated parameters, of shape (p,p).
<code>delta</code>	(ndarray, optional) Array of estimated errors in input variables, of same shape as <i>x</i> .
<code>eps</code>	(ndarray, optional) Array of estimated errors in response variables, of same shape as <i>y</i> .
<code>xplus</code>	(ndarray, optional) Array of <i>x</i> + <i>delta</i> .
<code>y</code>	(ndarray, optional) Array $y = fcn(x + delta)$ .
<code>res_var</code>	(float, optional) Residual variance.
<code>sum_square</code>	(float, optional) Sum of squares error.
<code>sum_square_delta</code>	(float, optional) Sum of squares of delta error.
<code>sum_square_eps</code>	(float, optional) Sum of squares of eps error.
<code>inv_condnum</code>	(float, optional) Inverse condition number (cf. ODRPACK UG p. 77).
<code>rel_error</code>	(float, optional) Relative error in function values computed within <i>fcn</i> .
<code>work</code>	(ndarray, optional) Final work array.
<code>work_ind</code>	(dict, optional) Indices into work for drawing out values (cf. ODRPACK UG p. 83).
<code>info</code>	(int, optional) Reason for returning, as output by ODRPACK (cf. ODRPACK UG p. 38).
<code>stopreason</code>	(list of str, optional) <i>info</i> interpreted into English.

*Methods*

---

`pprint()` Pretty-print important results.

---

Output `.pprint()`

Pretty-print important results.

`scipy.odr.odr` (*fcn, beta0, y, x, we=None, wd=None, fjacb=None, fjacd=None, extra\_args=None, ifixx=None, ifixb=None, job=0, iprint=0, errfile=None, rptfile=None, ndigit=0, tau-fac=0.0, sstol=-1.0, partol=-1.0, maxit=-1, stpb=None, stpd=None, sclb=None, scld=None, work=None, iwork=None, full\_output=0*)

Low-level function for ODR.

**See also:**

[ODR](#), [Model](#), [Data](#), [RealData](#)

*Notes*

This is a function performing the same operation as the `ODR`, `Model` and `Data` classes together. The parameters of this function are explained in the class documentation.

**exception** `scipy.odr.OdrWarning`

Warning indicating that the data passed into ODR will cause problems when passed into ‘odr’ that the user should be aware of.

**exception** `scipy.odr.OdrError`

Exception indicating an error in fitting.

This is raised by `scipy.odr` if an error occurs during fitting.



**exception** `scipy.odr.OdrStop`

Exception stopping fitting.

You can raise this exception in your objective function to tell `scipy.odr` to stop fitting.

`scipy.odr.odr_error`

alias of `OdrError`

`scipy.odr.odr_stop`

alias of `OdrStop`

Prebuilt models:

---

`polynomial(order)` Factory function for a general polynomial model.

---

`scipy.odr.polynomial` (*order*)

Factory function for a general polynomial model.

**Parameters**    **order** : int or sequence

If an integer, it becomes the order of the polynomial to fit. If a sequence of numbers, then these are the explicit powers in the polynomial. A constant term (power 0) is always included, so don't include 0. Thus, `polynomial(n)` is equivalent to `polynomial(range(1, n+1))`.

**Returns**        **polynomial** : Model instance

Model instance.

`scipy.odr.exponential`

`scipy.odr.multilinear`

`scipy.odr.unilinear`

`scipy.odr.quadratic`

`scipy.odr.polynomial`

## 5.17.2 Usage information

### Introduction

Why Orthogonal Distance Regression (ODR)? Sometimes one has measurement errors in the explanatory (a.k.a., “independent”) variable(s), not just the response (a.k.a., “dependent”) variable(s). Ordinary Least Squares (OLS) fitting procedures treat the data for explanatory variables as fixed, i.e., not subject to error of any kind. Furthermore, OLS procedures require that the response variables be an explicit function of the explanatory variables; sometimes making the equation explicit is impractical and/or introduces errors. ODR can handle both of these cases with ease, and can even reduce to the OLS case if that is sufficient for the problem.

ODRPACK is a FORTRAN-77 library for performing ODR with possibly non-linear fitting functions. It uses a modified trust-region Levenberg-Marquardt-type algorithm [R690] to estimate the function parameters. The fitting functions are provided by Python functions operating on NumPy arrays. The required derivatives may be provided by Python functions as well, or may be estimated numerically. ODRPACK can do explicit or implicit ODR fits, or it can do OLS. Input and output variables may be multi-dimensional. Weights can be provided to account for different variances of the observations, and even covariances between dimensions of the variables.

The `scipy.odr` package offers an object-oriented interface to ODRPACK, in addition to the low-level `odr` function. Additional background information about ODRPACK can be found in the [ODRPACK User's Guide](#), reading which is recommended.

## Basic usage

1. Define the function you want to fit against.:

```
def f(B, x):
    '''Linear function y = m*x + b'''
    # B is a vector of the parameters.
    # x is an array of the current x values.
    # x is in the same format as the x passed to Data or RealData.
    #
    # Return an array in the same format as y passed to Data or RealData.
    return B[0]*x + B[1]
```

2. Create a Model.:

```
linear = Model(f)
```

3. Create a Data or RealData instance.:

```
mydata = Data(x, y, wd=1./power(sx,2), we=1./power(sy,2))
```

or, when the actual covariances are known:

```
mydata = RealData(x, y, sx=sx, sy=sy)
```

4. Instantiate ODR with your data, model and initial parameter estimate.:

```
myodr = ODR(mydata, linear, beta0=[1., 2.])
```

5. Run the fit.:

```
myoutput = myodr.run()
```

6. Examine output.:

```
myoutput.pprint()
```

## References

# 5.18 Optimization and root finding (`scipy.optimize`)

## 5.18.1 Optimization

### Local Optimization

---

<code>minimize(fun, x0[, args, method, jac, hess, ...])</code>	Minimization of scalar function of one or more variables.
<code>minimize_scalar(fun[, bracket, bounds, ...])</code>	Minimization of scalar function of one variable.
<code>OptimizeResult</code>	Represents the optimization result.
<code>OptimizeWarning</code>	

---

`scipy.optimize.minimize` (*fun*, *x0*, *args=()*, *method=None*, *jac=None*, *hess=None*, *hessp=None*, *bounds=None*, *constraints=()*, *tol=None*, *callback=None*, *options=None*)

Minimization of scalar function of one or more variables.

In general, the optimization problems are of the form:

minimize  $f(x)$  subject to

$$\begin{aligned} g_i(x) &\geq 0, & i &= 1, \dots, m \\ h_j(x) &= 0, & j &= 1, \dots, p \end{aligned}$$

where  $x$  is a vector of one or more variables.  $g_i(x)$  are the inequality constraints.  $h_j(x)$  are the equality constraints.

Optionally, the lower and upper bounds for each element in  $x$  can also be specified using the *bounds* argument.

- Parameters**
- fun** : callable  
Objective function.
  - x0** : ndarray  
Initial guess.
  - args** : tuple, optional  
Extra arguments passed to the objective function and its derivatives (Jacobian, Hessian).
  - method** : str or callable, optional  
Type of solver. Should be one of
    - ‘Nelder-Mead’ ([see here](#))
    - ‘Powell’ ([see here](#))
    - ‘CG’ ([see here](#))
    - ‘BFGS’ ([see here](#))
    - ‘Newton-CG’ ([see here](#))
    - ‘L-BFGS-B’ ([see here](#))
    - ‘TNC’ ([see here](#))
    - ‘COBYLA’ ([see here](#))
    - ‘SLSQP’ ([see here](#))
    - ‘dogleg’ ([see here](#))
    - ‘trust-ncg’ ([see here](#))
    - custom - a callable object (added in version 0.14.0), see below for description.
 If not given, chosen to be one of BFGS, L-BFGS-B, SLSQP, depending if the problem has constraints or bounds.
  - jac** : bool or callable, optional  
Jacobian (gradient) of objective function. Only for CG, BFGS, Newton-CG, L-BFGS-B, TNC, SLSQP, dogleg, trust-ncg. If *jac* is a Boolean and is True, *fun* is assumed to return the gradient along with the objective function. If False, the gradient will be estimated numerically. *jac* can also be a callable returning the gradient of the objective. In this case, it must accept the same arguments as *fun*.
  - hess, hessp** : callable, optional  
Hessian (matrix of second-order derivatives) of objective function or Hessian of objective function times an arbitrary vector  $p$ . Only for Newton-CG, dogleg, trust-ncg. Only one of *hessp* or *hess* needs to be given. If *hess* is provided, then *hessp* will be ignored. If neither *hess* nor *hessp* is provided, then the Hessian product will be approximated using finite differences on *jac*. *hessp* must compute the Hessian times an arbitrary vector.
  - bounds** : sequence, optional  
Bounds for variables (only for L-BFGS-B, TNC and SLSQP). (*min*, *max*) pairs for each element in  $x$ , defining the bounds on that parameter. Use None for one of *min* or *max* when there is no bound in that direction.

**constraints** : dict or sequence of dict, optional

Constraints definition (only for COBYLA and SLSQP). Each constraint is defined in a dictionary with fields:

<b>type</b>	[str] Constraint type: 'eq' for equality, 'ineq' for inequality.
<b>fun</b>	[callable] The function defining the constraint.
<b>jac</b>	[callable, optional] The Jacobian of <i>fun</i> (only for SLSQP).
<b>args</b>	[sequence, optional] Extra arguments to be passed to the function and Jacobian.

Equality constraint means that the constraint function result is to be zero whereas inequality means that it is to be non-negative. Note that COBYLA only supports inequality constraints.

**tol** : float, optional

Tolerance for termination. For detailed control, use solver-specific options.

**options** : dict, optional

A dictionary of solver options. All methods accept the following generic options:

<b>maxiter</b>	[int] Maximum number of iterations to perform.
<b>disp</b>	[bool] Set to True to print convergence messages.

For method-specific options, see `show_options`.

**callback** : callable, optional

Called after each iteration, as `callback(xk)`, where `xk` is the current parameter vector.

**Returns** **res** : OptimizeResult

The optimization result represented as a `OptimizeResult` object. Important attributes are: `x` the solution array, `success` a Boolean flag indicating if the optimizer exited successfully and `message` which describes the cause of the termination. See `OptimizeResult` for a description of other attributes.

See also:

`minimize_scalar`

Interface to minimization algorithms for scalar univariate functions

`show_options`

Additional options accepted by the solvers

Notes

This section describes the available solvers that can be selected by the 'method' parameter. The default method is *BFGS*.

### Unconstrained minimization

Method *Nelder-Mead* uses the Simplex algorithm [R160], [R161]. This algorithm is robust in many applications. However, if numerical computation of derivative can be trusted, other algorithms using the first and/or second derivatives information might be preferred for their better performance in general.

Method *Powell* is a modification of Powell's method [R162], [R163] which is a conjugate direction method. It performs sequential one-dimensional minimizations along each vector of the directions set (*direc* field in *options* and *info*), which is updated at each iteration of the main minimization loop. The function need not be differentiable, and no derivatives are taken.

Method *CG* uses a nonlinear conjugate gradient algorithm by Polak and Ribiere, a variant of the Fletcher-Reeves method described in [R164] pp. 120-122. Only the first derivatives are used.

Method *BFGS* uses the quasi-Newton method of Broyden, Fletcher, Goldfarb, and Shanno (BFGS) [R164] pp. 136. It uses the first derivatives only. BFGS has proven good performance even for non-smooth optimizations. This method also returns an approximation of the Hessian inverse, stored as *hess\_inv* in the `OptimizeResult` object.

Method *Newton-CG* uses a Newton-CG algorithm [R164] pp. 168 (also known as the truncated Newton method). It uses a CG method to compute the search direction. See also *TNC* method for a box-constrained minimization with a similar algorithm.

Method *dogleg* uses the dog-leg trust-region algorithm [R164] for unconstrained minimization. This algorithm requires the gradient and Hessian; furthermore the Hessian is required to be positive definite.

Method *trust-ncg* uses the Newton conjugate gradient trust-region algorithm [R164] for unconstrained minimization. This algorithm requires the gradient and either the Hessian or a function that computes the product of the Hessian with a given vector.

### Constrained minimization

Method *L-BFGS-B* uses the L-BFGS-B algorithm [R165], [R166] for bound constrained minimization.

Method *TNC* uses a truncated Newton algorithm [R164], [R167] to minimize a function with variables subject to bounds. This algorithm uses gradient information; it is also called Newton Conjugate-Gradient. It differs from the *Newton-CG* method described above as it wraps a C implementation and allows each variable to be given upper and lower bounds.

Method *COBYLA* uses the Constrained Optimization BY Linear Approximation (COBYLA) method [R168], <sup>1</sup>, <sup>2</sup>. The algorithm is based on linear approximations to the objective function and each constraint. The method wraps a FORTRAN implementation of the algorithm. The constraints functions ‘fun’ may return either a single number or an array or list of numbers.

Method *SLSQP* uses Sequential Least Squares Programming to minimize a function of several variables with any combination of bounds, equality and inequality constraints. The method wraps the SLSQP Optimization subroutine originally implemented by Dieter Kraft <sup>3</sup>. Note that the wrapper handles infinite values in bounds by converting them into large floating values.

### Custom minimizers

It may be useful to pass a custom minimization method, for example when using a frontend to this method such as `scipy.optimize.basinhopping` or a different library. You can simply pass a callable as the `method` parameter.

The callable is called as `method(fun, x0, args, **kwargs, **options)` where `kwargs` corresponds to any other parameters passed to `minimize` (such as `callback`, `hess`, etc.), except the `options` dict, which has its contents also passed as `method` parameters pair by pair. Also, if `jac` has been passed as a bool type, `jac` and `fun` are mangled so that `fun` returns just the function values and `jac` is converted to a function returning the Jacobian. The method shall return an `OptimizeResult` object.

The provided `method` callable must be able to accept (and possibly ignore) arbitrary parameters; the set of parameters accepted by `minimize` may expand in future versions and then these parameters will be passed to the method. You can find an example in the `scipy.optimize` tutorial.

New in version 0.11.0.

### References

[R160], [R161], [R162], [R163], [R164], [R165], [R166], [R167], [R168], <sup>10</sup>, <sup>11</sup>, <sup>12</sup>

### Examples

Let us consider the problem of minimizing the Rosenbrock function. This function (and its respective derivatives) is implemented in `rosen` (resp. `rosen_der`, `rosen_hess`) in the `scipy.optimize`.

<sup>1</sup> Powell M J D. Direct search algorithms for optimization calculations. 1998. Acta Numerica 7: 287-336.

<sup>2</sup> Powell M J D. A view of algorithms for optimization without derivatives. 2007. Cambridge University Technical Report DAMTP 2007/NA03

<sup>3</sup> Kraft, D. A software package for sequential quadratic programming. 1988. Tech. Rep. DFVLR-FB 88-28, DLR German Aerospace Center – Institute for Flight Mechanics, Koln, Germany.

```
>>> from scipy.optimize import minimize, rosen, rosen_der
```

A simple application of the *Nelder-Mead* method is:

```
>>> x0 = [1.3, 0.7, 0.8, 1.9, 1.2]
>>> res = minimize(rosen, x0, method='Nelder-Mead', tol=1e-6)
>>> res.x
array([ 1.,  1.,  1.,  1.,  1.]
```

Now using the *BFGS* algorithm, using the first derivative and a few options:

```
>>> res = minimize(rosen, x0, method='BFGS', jac=rosen_der,
...               options={'gtol': 1e-6, 'disp': True})
Optimization terminated successfully.
    Current function value: 0.000000
    Iterations: 26
    Function evaluations: 31
    Gradient evaluations: 31
>>> res.x
array([ 1.,  1.,  1.,  1.,  1.]
>>> print(res.message)
Optimization terminated successfully.
>>> res.hess_inv
array([[ 0.00749589,  0.01255155,  0.02396251,  0.04750988,  0.09495377], # may vary
       [ 0.01255155,  0.02510441,  0.04794055,  0.09502834,  0.18996269],
       [ 0.02396251,  0.04794055,  0.09631614,  0.19092151,  0.38165151],
       [ 0.04750988,  0.09502834,  0.19092151,  0.38341252,  0.7664427 ],
       [ 0.09495377,  0.18996269,  0.38165151,  0.7664427,   1.53713523]])
```

Next, consider a minimization problem with several constraints (namely Example 16.4 from [R164]). The objective function is:

```
>>> fun = lambda x: (x[0] - 1)**2 + (x[1] - 2.5)**2
```

There are three constraints defined as:

```
>>> cons = ({'type': 'ineq', 'fun': lambda x: x[0] - 2 * x[1] + 2},
...        {'type': 'ineq', 'fun': lambda x: -x[0] - 2 * x[1] + 6},
...        {'type': 'ineq', 'fun': lambda x: -x[0] + 2 * x[1] + 2})
```

And variables must be positive, hence the following bounds:

```
>>> bnds = ((0, None), (0, None))
```

The optimization problem is solved using the *SLSQP* method as:

```
>>> res = minimize(fun, (2, 0), method='SLSQP', bounds=bnds,
...               constraints=cons)
```

It should converge to the theoretical solution (1.4, 1.7).

```
scipy.optimize.minimize_scalar(fun, bracket=None, bounds=None, args=(), method='brent',
                              tol=None, options=None)
```

Minimization of scalar function of one variable.

**Parameters** **fun** : callable

Objective function. Scalar function, must return a scalar.

**bracket** : sequence, optional

For methods 'brent' and 'golden', **bracket** defines the bracketing interval and can either have three items (*a*, *b*, *c*) so that  $a < b < c$  and  $fun(b) < fun(a)$ ,  $fun(c)$  or two items *a* and *c* which are assumed to be a starting interval for a downhill bracket search

(see `bracket`); it doesn't always mean that the obtained solution will satisfy  $a \leq x \leq c$ .

**bounds** : sequence, optional

For method 'bounded', *bounds* is mandatory and must have two items corresponding to the optimization bounds.

**args** : tuple, optional

Extra arguments passed to the objective function.

**method** : str or callable, optional

Type of solver. Should be one of

- 'Brent' ([see here](#))
- 'Bounded' ([see here](#))
- 'Golden' ([see here](#))
- custom - a callable object (added in version 0.14.0), see below

**tol** : float, optional

Tolerance for termination. For detailed control, use solver-specific options.

**options** : dict, optional

*A dictionary of solver options.*

*maxiter* [int] Maximum number of iterations to perform.

*disp* [bool] Set to True to print convergence messages.

See `show_options` for solver-specific options.

**Returns** **res** : OptimizeResult

The optimization result represented as a `OptimizeResult` object. Important attributes are: `x` the solution array, `success` a Boolean flag indicating if the optimizer exited successfully and `message` which describes the cause of the termination. See `OptimizeResult` for a description of other attributes.

**See also:**

`minimize` Interface to minimization algorithms for scalar multivariate functions

`show_options`

Additional options accepted by the solvers

**Notes**

This section describes the available solvers that can be selected by the 'method' parameter. The default method is *Brent*.

Method *Brent* uses Brent's algorithm to find a local minimum. The algorithm uses inverse parabolic interpolation when possible to speed up convergence of the golden section method.

Method *Golden* uses the golden section search technique. It uses analog of the bisection method to decrease the bracketed interval. It is usually preferable to use the *Brent* method.

Method *Bounded* can perform bounded minimization. It uses the Brent method to find a local minimum in the interval  $x_1 < x_{opt} < x_2$ .

**Custom minimizers**

It may be useful to pass a custom minimization method, for example when using some library frontend to `minimize_scalar`. You can simply pass a callable as the `method` parameter.

The callable is called as `method(fun, args, **kwargs, **options)` where `kwargs` corresponds to any other parameters passed to `minimize` (such as `bracket`, `tol`, etc.), except the `options` dict, which has its contents also passed as `method` parameters pair by pair. The method shall return an `OptimizeResult` object.

The provided *method* callable must be able to accept (and possibly ignore) arbitrary parameters; the set of parameters accepted by `minimize` may expand in future versions and then these parameters will be passed to the method. You can find an example in the `scipy.optimize` tutorial.

New in version 0.11.0.

**Examples**

Consider the problem of minimizing the following function.

```
>>> def f(x):
...     return (x - 2) * x * (x + 2)**2
```

Using the *Brent* method, we find the local minimum as:

```
>>> from scipy.optimize import minimize_scalar
>>> res = minimize_scalar(f)
>>> res.x
1.28077640403
```

Using the *Bounded* method, we find a local minimum with specified bounds as:

```
>>> res = minimize_scalar(f, bounds=(-3, -1), method='bounded')
>>> res.x
-2.0000002026
```

**class** `scipy.optimize.OptimizeResult`

Represents the optimization result.

**Notes**

There may be additional attributes not listed above depending of the specific solver. Since this class is essentially a subclass of dict with attribute accessors, one can see which attributes are available using the `keys()` method.

**Attributes**

<code>x</code>	(ndarray) The solution of the optimization.
<code>success</code>	(bool) Whether or not the optimizer exited successfully.
<code>status</code>	(int) Termination status of the optimizer. Its value depends on the underlying solver. Refer to <i>message</i> for details.
<code>message</code>	(str) Description of the cause of the termination.
<code>fun, jac, hess:</code>	Values of objective function, its Jacobian and its Hessian (if available). The Hessians may be approximations, see the documentation of the function in question.
<code>ndarray</code>	
<code>hess_inv</code>	(object) Inverse of the objective function's Hessian; may be an approximation. Not available for all solvers. The type of this attribute may be either <code>np.ndarray</code> or <code>scipy.sparse.linalg.LinearOperator</code> .
<code>nfev, njev, nhev</code>	(int) Number of evaluations of the objective functions and of its Jacobian and Hessian.
<code>nit</code>	(int) Number of iterations performed by the optimizer.
<code>maxcv</code>	(float) The maximum constraint violation.

**Methods**

- `clear()` -> None. Remove all items from D.)
- `copy()` -> a shallow copy of D)
- `fromkeys(...)` v defaults to None.
- `get((k[,d])` -> D[k] if k in D, ...)

Continued on next page



Table 5.102 – continued from previous page

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<code>has_key(k)</code> -> True if D has a key k, else False)	
<code>items()</code> -> list of D's (key, value) pairs, ...)	
<code>iteritems()</code> -> an iterator over the (key, ...)	
<code>iterkeys()</code> -> an iterator over the keys of D)	
<code>itervalues(...)</code>	
<code>keys()</code> -> list of D's keys)	
<code>pop((k[,d])</code> -> v, ...)	If key is not found, d is returned if given, otherwise <code>KeyError</code> is raised
<code>popitem()</code> -> (k, v), ...)	2-tuple; but raise <code>KeyError</code> if D is empty.
<code>setdefault((k[,d])</code> -> D.get(k,d), ...)	
<code>update((E, ...)</code>	If E present and has a <code>.keys()</code> method, does: for k in E: D[k] = E[k]
<code>values()</code> -> list of D's values)	
<code>viewitems(...)</code>	
<code>viewkeys(...)</code>	
<code>viewvalues(...)</code>	

---

`OptimizeResult.clear()` → None. Remove all items from D.

`OptimizeResult.copy()` → a shallow copy of D

**static** `OptimizeResult.fromkeys(S[, v])` → New dict with keys from S and values equal to v.  
v defaults to None.

`OptimizeResult.get(k[, d])` → D[k] if k in D, else d. d defaults to None.

`OptimizeResult.has_key(k)` → True if D has a key k, else False

`OptimizeResult.items()` → list of D's (key, value) pairs, as 2-tuples

`OptimizeResult.iteritems()` → an iterator over the (key, value) items of D

`OptimizeResult.iterkeys()` → an iterator over the keys of D

`OptimizeResult.itervalues()` → an iterator over the values of D

`OptimizeResult.keys()` → list of D's keys

`OptimizeResult.pop(k[, d])` → v, remove specified key and return the corresponding value.  
If key is not found, d is returned if given, otherwise `KeyError` is raised

`OptimizeResult.popitem()` → (k, v), remove and return some (key, value) pair as a  
2-tuple; but raise `KeyError` if D is empty.

`OptimizeResult.setdefault(k[, d])` → D.get(k,d), also set D[k]=d if k not in D

`OptimizeResult.update([E], **F)` → None. Update D from dict/iterable E and F.  
If E present and has a `.keys()` method, does: for k in E: D[k] = E[k] If E present and lacks `.keys()` method,  
does: for (k, v) in E: D[k] = v In either case, this is followed by: for k in F: D[k] = F[k]

`OptimizeResult.values()` → list of D's values

`OptimizeResult.viewitems()` → a set-like object providing a view on D's items

`OptimizeResult.viewkeys()` → a set-like object providing a view on D's keys

`OptimizeResult.viewvalues()` → an object providing a view on D's values

**exception** `scipy.optimize.OptimizeWarning`

The `minimize` function supports the following methods:

*minimize(method='Nelder-Mead')*

`scipy.optimize.minimize` (*fun, x0, args=(), method='Nelder-Mead', tol=None, callback=None, options={'disp': False, 'initial\_simplex': None, 'maxiter': None, 'xatol': 0.0001, 'return\_all': False, 'fatol': 0.0001, 'func': None, 'maxfev': None}*)

Minimization of scalar function of one or more variables using the Nelder-Mead algorithm.

**See also:**

For documentation for the rest of the parameters, see `scipy.optimize.minimize`

- Options**
- disp** : bool  
Set to True to print convergence messages.
- maxiter, maxfev** : int  
Maximum allowed number of iterations and function evaluations. Will default to  $N*200$ , where  $N$  is the number of variables, if neither *maxiter* or *maxfev* is set. If both *maxiter* and *maxfev* are set, minimization will stop at the first reached.
- initial\_simplex** : array\_like of shape  $(N + 1, N)$   
Initial simplex. If given, overrides *x0*. `initial_simplex[j, :]` should contain the coordinates of the  $j$ -th vertex of the  $N+1$  vertices in the simplex, where  $N$  is the dimension.
- xatol** : float, optional  
Absolute error in `xopt` between iterations that is acceptable for convergence.
- fatol** : number, optional  
Absolute error in `func(xopt)` between iterations that is acceptable for convergence.

*minimize(method='Powell')*

`scipy.optimize.minimize` (*fun, x0, args=(), method='Powell', tol=None, callback=None, options={'disp': False, 'return\_all': False, 'maxiter': None, 'direc': None, 'func': None, 'maxfev': None, 'xtol': 0.0001, 'ftol': 0.0001}*)

Minimization of scalar function of one or more variables using the modified Powell algorithm.

**See also:**

For documentation for the rest of the parameters, see `scipy.optimize.minimize`

- Options**
- disp** : bool  
Set to True to print convergence messages.
- xtol** : float  
Relative error in solution `xopt` acceptable for convergence.
- ftol** : float

Relative error in `fun(xopt)` acceptable for convergence.

**maxiter, maxfev** : int

Maximum allowed number of iterations and function evaluations. Will default to  $N*1000$ , where  $N$  is the number of variables, if neither *maxiter* or *maxfev* is set. If both *maxiter* and *maxfev* are set, minimization will stop at the first reached.

**direc** : ndarray

Initial set of direction vectors for the Powell method.

### *minimize(method='CG')*

```
scipy.optimize.minimize(fun, x0, args=(), method='CG', jac=None, tol=None, callback=None, options={'disp': False, 'gtol': 1e-05, 'eps': 1.4901161193847656e-08, 'return_all': False, 'maxiter': None, 'norm': inf})
```

Minimization of scalar function of one or more variables using the conjugate gradient algorithm.

#### See also:

For documentation for the rest of the parameters, see `scipy.optimize.minimize`

<i>Options</i>	<p><b>disp</b> : bool Set to True to print convergence messages.</p> <p><b>maxiter</b> : int Maximum number of iterations to perform.</p> <p><b>gtol</b> : float Gradient norm must be less than <i>gtol</i> before successful termination.</p> <p><b>norm</b> : float Order of norm (Inf is max, -Inf is min).</p> <p><b>eps</b> : float or ndarray If <i>jac</i> is approximated, use this value for the step size.</p>
----------------	---

### *minimize(method='BFGS')*

```
scipy.optimize.minimize(fun, x0, args=(), method='BFGS', jac=None, tol=None, callback=None, options={'disp': False, 'gtol': 1e-05, 'eps': 1.4901161193847656e-08, 'return_all': False, 'maxiter': None, 'norm': inf})
```

Minimization of scalar function of one or more variables using the BFGS algorithm.

#### See also:

For documentation for the rest of the parameters, see `scipy.optimize.minimize`

<i>Options</i>	<p><b>disp</b> : bool Set to True to print convergence messages.</p> <p><b>maxiter</b> : int Maximum number of iterations to perform.</p> <p><b>gtol</b> : float Gradient norm must be less than <i>gtol</i> before successful termination.</p> <p><b>norm</b> : float Order of norm (Inf is max, -Inf is min).</p> <p><b>eps</b> : float or ndarray If <i>jac</i> is approximated, use this value for the step size.</p>
----------------	---

*minimize(method='Newton-CG')*

```
scipy.optimize.minimize(fun, x0, args=(), method='Newton-CG', jac=None, hess=None,
                        hessp=None, tol=None, callback=None, options={'disp': False, 'xtol':
                        1e-05, 'eps': 1.4901161193847656e-08, 'return_all': False, 'maxiter':
                        None})
```

Minimization of scalar function of one or more variables using the Newton-CG algorithm.

Note that the *jac* parameter (Jacobian) is required.

**See also:**

For documentation for the rest of the parameters, see [scipy.optimize.minimize](#)

**Options**

- disp** : bool  
Set to True to print convergence messages.
- xtol** : float  
Average relative error in solution *xopt* acceptable for convergence.
- maxiter** : int  
Maximum number of iterations to perform.
- eps** : float or ndarray  
If *jac* is approximated, use this value for the step size.

*minimize(method='L-BFGS-B')*

```
scipy.optimize.minimize(fun, x0, args=(), method='L-BFGS-B', jac=None, bounds=None,
                        tol=None, callback=None, options={'disp': None, 'maxls': 20,
                        'iprint': -1, 'gtol': 1e-05, 'eps': 1e-08, 'maxiter': 15000, 'ftol':
                        2.220446049250313e-09, 'maxcor': 10, 'maxfun': 15000})
```

Minimize a scalar function of one or more variables using the L-BFGS-B algorithm.

**See also:**

For documentation for the rest of the parameters, see [scipy.optimize.minimize](#)

**Options**

- disp** : bool  
Set to True to print convergence messages.
- maxcor** : int  
The maximum number of variable metric corrections used to define the limited memory matrix. (The limited memory BFGS method does not store the full hessian but uses this many terms in an approximation to it.)
- factr** : float  
The iteration stops when  $(f^k - f^{k+1}) / \max\{|f^k|, |f^{k+1}|, 1\} \leq \text{factr} * \text{eps}$ , where *eps* is the machine precision, which is automatically generated by the code. Typical values for *factr* are: 1e12 for low accuracy; 1e7 for moderate accuracy; 10.0 for extremely high accuracy.
- ftol** : float  
The iteration stops when  $(f^k - f^{k+1}) / \max\{|f^k|, |f^{k+1}|, 1\} \leq \text{ftol}$ .
- gtol** : float  
The iteration will stop when  $\max\{|\text{proj } g_i| \mid i = 1, \dots, n\} \leq \text{gtol}$  where *pg<sub>i</sub>* is the *i*-th component of the projected gradient.
- eps** : float  
Step size used for numerical approximation of the jacobian.
- disp** : int  
Set to True to print convergence messages.
- maxfun** : int  
Maximum number of function evaluations.

**maxiter** : int

Maximum number of iterations.

**maxls** : int, optional

Maximum number of line search steps (per iteration). Default is 20.

*minimize(method='TNC')*

```
scipy.optimize.minimize(fun, x0, args=(), method='TNC', jac=None, bounds=None, tol=None, call-
back=None, options={'disp': False, 'minfev': 0, 'scale': None, 'rescale':
-1, 'offset': None, 'gtol': -1, 'eps': 1e-08, 'eta': -1, 'maxiter': None, 'max-
CGit': -1, 'mesg_num': None, 'ftol': -1, 'xtol': -1, 'stepmx': 0, 'accuracy':
0})
```

Minimize a scalar function of one or more variables using a truncated Newton (TNC) algorithm.

**See also:**

For documentation for the rest of the parameters, see `scipy.optimize.minimize`

#### Options

**eps** : float

Step size used for numerical approximation of the jacobian.

**scale** : list of floats

Scaling factors to apply to each variable. If None, the factors are up-low for interval bounded variables and  $1+|x|$  for the others. Defaults to None

**offset** : float

Value to subtract from each variable. If None, the offsets are  $(up+low)/2$  for interval bounded variables and  $x$  for the others.

**disp** : bool

Set to True to print convergence messages.

**maxCGit** : int

Maximum number of hessian\*vector evaluations per main iteration. If maxCGit == 0, the direction chosen is -gradient if maxCGit < 0, maxCGit is set to  $\max(1, \min(50, n/2))$ . Defaults to -1.

**maxiter** : int

Maximum number of function evaluation. if None, *maxiter* is set to  $\max(100, 10*\text{len}(x0))$ . Defaults to None.

**eta** : float

Severity of the line search. if < 0 or > 1, set to 0.25. Defaults to -1.

**stepmx** : float

Maximum step for the line search. May be increased during call. If too small, it will be set to 10.0. Defaults to 0.

**accuracy** : float

Relative precision for finite difference calculations. If  $\leq$  machine\_precision, set to  $\sqrt{\text{machine\_precision}}$ . Defaults to 0.

**minfev** : float

Minimum function value estimate. Defaults to 0.

**ftol** : float

Precision goal for the value of f in the stopping criterion. If ftol < 0.0, ftol is set to 0.0 defaults to -1.

**xtol** : float

Precision goal for the value of x in the stopping criterion (after applying x scaling factors). If xtol < 0.0, xtol is set to  $\sqrt{\text{machine\_precision}}$ . Defaults to -1.

**gtol** : float

Precision goal for the value of the projected gradient in the stopping criterion (after applying x scaling factors). If gtol < 0.0, gtol is set to  $1e-2 * \sqrt{\text{accuracy}}$ . Setting it to 0.0 is not recommended. Defaults to -1.

**rescale** : float

Scaling factor (in log10) used to trigger f value rescaling. If 0, rescale at each iteration. If a large value, never rescale. If < 0, rescale is set to 1.3.

***minimize(method='COBYLA')***

`scipy.optimize.minimize` (*fun*, *x0*, *args=()*, *method='COBYLA'*, *constraints=()*, *tol=None*, *callback=None*, *options={'iprint': 1, 'disp': False, 'maxiter': 1000, 'catol': 0.0002, 'rhobeg': 1.0}*)

Minimize a scalar function of one or more variables using the Constrained Optimization BY Linear Approximation (COBYLA) algorithm.

**See also:**

For documentation for the rest of the parameters, see `scipy.optimize.minimize`

<b>Options</b>	<b>rhobeg</b> : float Reasonable initial changes to the variables.
	<b>tol</b> : float Final accuracy in the optimization (not precisely guaranteed). This is a lower bound on the size of the trust region.
	<b>disp</b> : bool Set to True to print convergence messages. If False, <i>verbosity</i> is ignored as set to 0.
	<b>maxiter</b> : int Maximum number of function evaluations.
	<b>catol</b> : float Tolerance (absolute) for constraint violations

***minimize(method='SLSQP')***

`scipy.optimize.minimize` (*fun*, *x0*, *args=()*, *method='SLSQP'*, *jac=None*, *bounds=None*, *constraints=()*, *tol=None*, *callback=None*, *options={'disp': False, 'iprint': 1, 'eps': 1.4901161193847656e-08, 'func': None, 'maxiter': 100, 'ftol': 1e-06}*)

Minimize a scalar function of one or more variables using Sequential Least Squares Programming (SLSQP).

**See also:**

For documentation for the rest of the parameters, see `scipy.optimize.minimize`

<b>Options</b>	<b>ftol</b> : float Precision goal for the value of f in the stopping criterion.
	<b>eps</b> : float Step size used for numerical approximation of the jacobian.
	<b>disp</b> : bool Set to True to print convergence messages. If False, <i>verbosity</i> is ignored and set to 0.
	<b>maxiter</b> : int Maximum number of iterations.

***minimize(method='dogleg')***

`scipy.optimize.minimize` (*fun*, *x0*, *args=()*, *method='dogleg'*, *jac=None*, *hess=None*, *tol=None*, *callback=None*, *options={}*)

Minimization of scalar function of one or more variables using the dog-leg trust-region algorithm.

**See also:**

For documentation for the rest of the parameters, see `scipy.optimize.minimize`

<b>Options</b>	<b>initial_trust_radius</b> : float
----------------	-------------------------------------

Initial trust-region radius.  
**max\_trust\_radius** : float  
 Maximum value of the trust-region radius. No steps that are longer than this value will be proposed.  
**eta** : float  
 Trust region related acceptance stringency for proposed steps.  
**gtol** : float  
 Gradient norm must be less than *gtol* before successful termination.

### *minimize(method='trust-ncg')*

`scipy.optimize.minimize` (*fun*, *x0*, *args=()*, *method='trust-ncg'*, *jac=None*, *hess=None*, *hessp=None*, *tol=None*, *callback=None*, *options={}*)

Minimization of scalar function of one or more variables using the Newton conjugate gradient trust-region algorithm.

#### See also:

For documentation for the rest of the parameters, see `scipy.optimize.minimize`

**Options**

**initial\_trust\_radius** : float  
 Initial trust-region radius.  
**max\_trust\_radius** : float  
 Maximum value of the trust-region radius. No steps that are longer than this value will be proposed.  
**eta** : float  
 Trust region related acceptance stringency for proposed steps.  
**gtol** : float  
 Gradient norm must be less than *gtol* before successful termination.

The `minimize_scalar` function supports the following methods:

### *minimize\_scalar(method='brent')*

`scipy.optimize.minimize_scalar` (*fun*, *args=()*, *method='brent'*, *tol=None*, *options={'xtol': 1.48e-08, 'brack': None, 'func': None, 'maxiter': 500}*)

#### See also:

For documentation for the rest of the parameters, see `scipy.optimize.minimize_scalar`

**Options**

**maxiter** : int  
 Maximum number of iterations to perform.  
**xtol** : float  
 Relative error in solution *xopt* acceptable for convergence.

#### Notes

Uses inverse parabolic interpolation when possible to speed up convergence of golden section method.

### *minimize\_scalar(method='bounded')*

`scipy.optimize.minimize_scalar` (*fun*, *bounds=None*, *args=()*, *method='bounded'*, *tol=None*, *options={'disp': 0, 'maxiter': 500, 'func': None, 'xatol': 1e-05}*)

#### See also:

For documentation for the rest of the parameters, see `scipy.optimize.minimize_scalar`

**Options**

- maxiter** : int  
Maximum number of iterations to perform.
- disp** : bool  
Set to True to print convergence messages.
- xatol** : float  
Absolute error in solution *xopt* acceptable for convergence.

*minimize\_scalar(method='golden')*

`scipy.optimize.minimize_scalar` (*func*, *args*=(), *method*='golden', *tol*=None, *options*={'xtol': 1.4901161193847656e-08, 'brack': None, 'func': None})

**See also:**

For documentation for the rest of the parameters, see `scipy.optimize.minimize_scalar`

**Options**

- maxiter** : int  
Maximum number of iterations to perform.
- xtol** : float  
Relative error in solution *xopt* acceptable for convergence.

The specific optimization method interfaces below in this subsection are not recommended for use in new scripts; all of these methods are accessible via a newer, more consistent interface provided by the functions above.

General-purpose multivariate methods:

---

<code>fmin(func, x0[, args, xtol, ftol, maxiter, ...])</code>	Minimize a function using the downhill simplex algorithm.
<code>fmin_powell(func, x0[, args, xtol, ftol, ...])</code>	Minimize a function using modified Powell's method.
<code>fmin_cg(f, x0[, fprime, args, gtol, norm, ...])</code>	Minimize a function using a nonlinear conjugate gradient algorithm.
<code>fmin_bfgs(f, x0[, fprime, args, gtol, norm, ...])</code>	Minimize a function using the BFGS algorithm.
<code>fmin_ncg(f, x0, fprime[, fhess_p, fhess, ...])</code>	Unconstrained minimization of a function using the Newton-CG method.

---

`scipy.optimize.fmin` (*func*, *x0*, *args*=(), *xtol*=0.0001, *ftol*=0.0001, *maxiter*=None, *maxfun*=None, *full\_output*=0, *disp*=1, *retall*=0, *callback*=None, *initial\_simplex*=None)

Minimize a function using the downhill simplex algorithm.

This algorithm only uses function values, not derivatives or second derivatives.

**Parameters**

- func** : callable `func(x,*args)`  
The objective function to be minimized.
- x0** : ndarray  
Initial guess.
- args** : tuple, optional  
Extra arguments passed to `func`, i.e. `f(x,*args)`.
- xtol** : float, optional  
Absolute error in *xopt* between iterations that is acceptable for convergence.
- ftol** : number, optional  
Absolute error in `func(xopt)` between iterations that is acceptable for convergence.
- maxiter** : int, optional  
Maximum number of iterations to perform.
- maxfun** : number, optional  
Maximum number of function evaluations to make.
- full\_output** : bool, optional  
Set to True if `fopt` and `warnflag` outputs are desired.
- disp** : bool, optional  
Set to True to print convergence messages.



**retall** : bool, optional  
Set to True to return list of solutions at each iteration.

**callback** : callable, optional  
Called after each iteration, as `callback(xk)`, where `xk` is the current parameter vector.

**initial\_simplex** : array\_like of shape (N + 1, N), optional  
Initial simplex. If given, overrides `x0`. `initial_simplex[j, :]` should contain the coordinates of the `j`-th vertex of the `N+1` vertices in the simplex, where `N` is the dimension.

**Returns**

**xopt** : ndarray  
Parameter that minimizes function.

**fopt** : float  
Value of function at minimum: `fopt = func(xopt)`.

**iter** : int  
Number of iterations performed.

**funcalls** : int  
Number of function calls made.

**warnflag** : int  
1 : Maximum number of function evaluations made. 2 : Maximum number of iterations reached.

**allvecs** : list  
Solution at each iteration.

**See also:**

**minimize** Interface to minimization algorithms for multivariate functions. See the ‘Nelder-Mead’ *method* in particular.

**Notes**

Uses a Nelder-Mead simplex algorithm to find the minimum of function of one or more variables.

This algorithm has a long history of successful use in applications. But it will usually be slower than an algorithm that uses first or second derivative information. In practice it can have poor performance in high-dimensional problems and is not robust to minimizing complicated functions. Additionally, there currently is no complete theory describing when the algorithm will successfully converge to the minimum, or how fast it will if it does. Both the `ftol` and `xtol` criteria must be met for convergence.

**References**

[R154], [R155]

`scipy.optimize.fmin_powell` (*func*, *x0*, *args=()*, *xtol=0.0001*, *ftol=0.0001*, *maxiter=None*, *maxfun=None*, *full\_output=0*, *disp=1*, *retall=0*, *callback=None*, *direc=None*)

Minimize a function using modified Powell’s method. This method only uses function values, not derivatives.

**Parameters**

**func** : callable `f(x,*args)`  
Objective function to be minimized.

**x0** : ndarray  
Initial guess.

**args** : tuple, optional  
Extra arguments passed to `func`.

**callback** : callable, optional  
An optional user-supplied function, called after each iteration. Called as `callback(xk)`, where `xk` is the current parameter vector.

**direc** : ndarray, optional  
Initial direction set.

**xtol** : float, optional  
Line-search error tolerance.

**ftol** : float, optional  
Relative error in `func(xopt)` acceptable for convergence.

**maxiter** : int, optional  
Maximum number of iterations to perform.

**maxfun** : int, optional  
Maximum number of function evaluations to make.

**full\_output** : bool, optional  
If True, `fopt`, `xi`, `direc`, `iter`, `funcalls`, and `warnflag` are returned.

**disp** : bool, optional  
If True, print convergence messages.

**retall** : bool, optional  
If True, return a list of the solution at each iteration.

**Returns**

**xopt** : ndarray  
Parameter which minimizes *func*.

**fopt** : number  
Value of function at minimum: `fopt = func(xopt)`.

**direc** : ndarray  
Current direction set.

**iter** : int  
Number of iterations.

**funcalls** : int  
Number of function calls made.

**warnflag** : int  
*Integer warning flag:*  
1 : Maximum number of function evaluations. 2 : Maximum number of iterations.

**allvecs** : list  
List of solutions at each iteration.

**See also:**

**`minimize`** Interface to unconstrained minimization algorithms for multivariate functions. See the ‘Powell’ *method* in particular.

**Notes**

Uses a modification of Powell’s method to find the minimum of a function of N variables. Powell’s method is a conjugate direction method.

The algorithm has two loops. The outer loop merely iterates over the inner loop. The inner loop minimizes over each current direction in the direction set. At the end of the inner loop, if certain conditions are met, the direction that gave the largest decrease is dropped and replaced with the difference between the current estimated x and the estimated x from the beginning of the inner-loop.

The technical conditions for replacing the direction of greatest increase amount to checking that

- 1.No further gain can be made along the direction of greatest increase from that iteration.
- 2.The direction of greatest increase accounted for a large sufficient fraction of the decrease in the function value from that iteration of the inner loop.

**References**

Powell M.J.D. (1964) An efficient method for finding the minimum of a function of several variables without calculating derivatives, *Computer Journal*, 7 (2):155-162.

Press W., Teukolsky S.A., Vetterling W.T., and Flannery B.P.: *Numerical Recipes* (any edition), Cambridge University Press

```
scipy.optimize.fmin_cg(f, x0, fprime=None, args=(), gtol=1e-05, norm=inf,
                      epsilon=1.4901161193847656e-08, maxiter=None, full_output=0, disp=1,
                      retall=0, callback=None)
```

Minimize a function using a nonlinear conjugate gradient algorithm.

**Parameters** **f**: callable,  $f(x, *args)$

Objective function to be minimized. Here  $x$  must be a 1-D array of the variables that are to be changed in the search for a minimum, and  $args$  are the other (fixed) parameters of  $f$ .

**x0**: ndarray

A user-supplied initial estimate of  $x_{opt}$ , the optimal value of  $x$ . It must be a 1-D array of values.

**fprime**: callable,  $fprime(x, *args)$ , optional

A function that returns the gradient of  $f$  at  $x$ . Here  $x$  and  $args$  are as described above for  $f$ . The returned value must be a 1-D array. Defaults to None, in which case the gradient is approximated numerically (see *epsilon*, below).

**args**: tuple, optional

Parameter values passed to  $f$  and  $fprime$ . Must be supplied whenever additional fixed parameters are needed to completely specify the functions  $f$  and  $fprime$ .

**gtol**: float, optional

Stop when the norm of the gradient is less than *gtol*.

**norm**: float, optional

Order to use for the norm of the gradient ( $-\text{np}.\text{Inf}$  is min,  $\text{np}.\text{Inf}$  is max).

**epsilon**: float or ndarray, optional

Step size(s) to use when  $fprime$  is approximated numerically. Can be a scalar or a 1-D array. Defaults to  $\text{sqrt}(\text{eps})$ , with  $\text{eps}$  the floating point machine precision. Usually  $\text{sqrt}(\text{eps})$  is about  $1.5\text{e-}8$ .

**maxiter**: int, optional

Maximum number of iterations to perform. Default is  $200 * \text{len}(x0)$ .

**full\_output**: bool, optional

If True, return *fopt*, *func\_calls*, *grad\_calls*, and *warnflag* in addition to *xopt*. See the Returns section below for additional information on optional return values.

**disp**: bool, optional

If True, return a convergence message, followed by *xopt*.

**retall**: bool, optional

If True, add to the returned values the results of each iteration.

**callback**: callable, optional

An optional user-supplied function, called after each iteration. Called as  $\text{callback}(xk)$ , where  $xk$  is the current value of  $x0$ .

**Returns**

**xopt**: ndarray

Parameters which minimize  $f$ , i.e.  $f(xopt) == fopt$ .

**fopt**: float, optional

Minimum value found,  $f(xopt)$ . Only returned if *full\_output* is True.

**func\_calls**: int, optional

The number of function\_calls made. Only returned if *full\_output* is True.

**grad\_calls**: int, optional

The number of gradient calls made. Only returned if *full\_output* is True.

**warnflag**: int, optional

Integer value with warning status, only returned if *full\_output* is True.

0 : Success.

1 : The maximum number of iterations was exceeded.

2 [Gradient and/or function calls were not changing. May indicate] that precision was lost, i.e., the routine did not converge.

**allvecs** : list of ndarray, optional

List of arrays, containing the results at each iteration. Only returned if *retall* is True.

#### See also:

**minimize** common interface to all `scipy.optimize` algorithms for unconstrained and constrained minimization of multivariate functions. It provides an alternative way to call `fmin_cg`, by specifying `method='CG'`.

#### Notes

This conjugate gradient algorithm is based on that of Polak and Ribiere [R156].

Conjugate gradient methods tend to work better when:

1.  $f$  has a unique global minimizing point, and no local minima or other stationary points,
2.  $f$  is, at least locally, reasonably well approximated by a quadratic function of the variables,
3.  $f$  is continuous and has a continuous gradient,
4.  $f$ 's prime is not too large, e.g., has a norm less than 1000,
5. The initial guess,  $x_0$ , is reasonably close to  $f$ 's global minimizing point,  $x_{opt}$ .

#### References

[R156]

#### Examples

Example 1: seek the minimum value of the expression  $a*u**2 + b*u*v + c*v**2 + d*u + e*v + f$  for given values of the parameters and an initial guess  $(u, v) = (0, 0)$ .

```
>>> args = (2, 3, 7, 8, 9, 10) # parameter values
>>> def f(x, *args):
...     u, v = x
...     a, b, c, d, e, f = args
...     return a*u**2 + b*u*v + c*v**2 + d*u + e*v + f
>>> def gradf(x, *args):
...     u, v = x
...     a, b, c, d, e, f = args
...     gu = 2*a*u + b*v + d # u-component of the gradient
...     gv = b*u + 2*c*v + e # v-component of the gradient
...     return np.asarray((gu, gv))
>>> x0 = np.asarray((0, 0)) # Initial guess.
>>> from scipy import optimize
>>> res1 = optimize.fmin_cg(f, x0, fprime=gradf, args=args)
Optimization terminated successfully.
    Current function value: 1.617021
    Iterations: 4
    Function evaluations: 8
    Gradient evaluations: 8
>>> res1
array([-1.80851064, -0.25531915])
```

Example 2: solve the same problem using the `minimize` function. (This `myopts` dictionary shows all of the available options, although in practice only non-default values would be needed. The returned value will be a dictionary.)

```
>>> opts = {'maxiter' : None,      # default value.
...         'disp' : True,        # non-default value.
...         'gtol' : 1e-5,       # default value.
...         'norm' : np.inf,     # default value.
...         'eps' : 1.4901161193847656e-08} # default value.
>>> res2 = optimize.minimize(f, x0, jac=gradf, args=args,
...                          method='CG', options=opts)
Optimization terminated successfully.
      Current function value: 1.617021
      Iterations: 4
      Function evaluations: 8
      Gradient evaluations: 8
>>> res2.x # minimum found
array([-1.80851064, -0.25531915])
```

```
scipy.optimize.fmin_bfgs(f, x0, fprime=None, args=(), gtol=1e-05, norm=inf,
                        epsilon=1.4901161193847656e-08, maxiter=None, full_output=0,
                        disp=1, retall=0, callback=None)
```

Minimize a function using the BFGS algorithm.

**Parameters**

- f** : callable f(x,\*args)  
Objective function to be minimized.
- x0** : ndarray  
Initial guess.
- fprime** : callable f'(x,\*args), optional  
Gradient of f.
- args** : tuple, optional  
Extra arguments passed to f and fprime.
- gtol** : float, optional  
Gradient norm must be less than gtol before successful termination.
- norm** : float, optional  
Order of norm (Inf is max, -Inf is min)
- epsilon** : int or ndarray, optional  
If fprime is approximated, use this value for the step size.
- callback** : callable, optional  
An optional user-supplied function to call after each iteration. Called as `callback(xk)`, where `xk` is the current parameter vector.
- maxiter** : int, optional  
Maximum number of iterations to perform.
- full\_output** : bool, optional  
If True, return `fopt`, `func_calls`, `grad_calls`, and `warnflag` in addition to `xopt`.
- disp** : bool, optional  
Print convergence message if True.
- retall** : bool, optional  
Return a list of results at each iteration if True.

**Returns**

- xopt** : ndarray  
Parameters which minimize f, i.e. `f(xopt) == fopt`.
- fopt** : float  
Minimum value.
- gopt** : ndarray  
Value of gradient at minimum, `f'(xopt)`, which should be near 0.
- Bopt** : ndarray

Value of  $1/f''(x_{opt})$ , i.e. the inverse hessian matrix.  
**func\_calls** : int  
 Number of function\_calls made.  
**grad\_calls** : int  
 Number of gradient calls made.  
**warnflag** : integer  
 1 : Maximum number of iterations exceeded. 2 : Gradient and/or function calls not changing.  
**allvecs** : list  
`OptimizeResult` at each iteration. Only returned if `retall` is True.

**See also:**

`minimize` Interface to minimization algorithms for multivariate functions. See the ‘BFGS’ *method* in particular.

**Notes**

Optimize the function, *f*, whose gradient is given by *fprime* using the quasi-Newton method of Broyden, Fletcher, Goldfarb, and Shanno (BFGS)

**References**

Wright, and Nocedal ‘Numerical Optimization’, 1999, pg. 198.

```
scipy.optimize.fmin_ncg(f, x0, fprime, fhess_p=None, fhess=None, args=(), avextol=1e-05,
                        epsilon=1.4901161193847656e-08, maxiter=None, full_output=0, disp=1,
                        retall=0, callback=None)
```

Unconstrained minimization of a function using the Newton-CG method.

**Parameters**

- f** : callable  $f(x, *args)$   
Objective function to be minimized.
- x0** : ndarray  
Initial guess.
- fprime** : callable  $f'(x, *args)$   
Gradient of *f*.
- fhess\_p** : callable  $fhess\_p(x, p, *args)$ , optional  
Function which computes the Hessian of *f* times an arbitrary vector, *p*.
- fhess** : callable  $fhess(x, *args)$ , optional  
Function to compute the Hessian matrix of *f*.
- args** : tuple, optional  
Extra arguments passed to *f*, *fprime*, *fhess\_p*, and *fhess* (the same set of extra arguments is supplied to all of these functions).
- epsilon** : float or ndarray, optional  
If *fhess* is approximated, use this value for the step size.
- callback** : callable, optional  
An optional user-supplied function which is called after each iteration. Called as `callback(xk)`, where *xk* is the current parameter vector.
- avextol** : float, optional  
Convergence is assumed when the average relative error in the minimizer falls below this amount.
- maxiter** : int, optional  
Maximum number of iterations to perform.
- full\_output** : bool, optional  
If True, return the optional outputs.
- disp** : bool, optional  
If True, print convergence message.

**Returns**

- retall** : bool, optional  
If True, return a list of results at each iteration.
- xopt** : ndarray  
Parameters which minimize  $f$ , i.e.  $f(x_{opt}) == f_{opt}$ .
- fopt** : float  
Value of the function at  $x_{opt}$ , i.e.  $f_{opt} = f(x_{opt})$ .
- fcalls** : int  
Number of function calls made.
- gcalls** : int  
Number of gradient calls made.
- hcalls** : int  
Number of hessian calls made.
- warnflag** : int  
Warnings generated by the algorithm. 1 : Maximum number of iterations exceeded.
- allvecs** : list  
The result at each iteration, if `retall` is True (see below).

**See also:**

**minimize** Interface to minimization algorithms for multivariate functions. See the ‘Newton-CG’ *method* in particular.

**Notes**

Only one of `fhess_p` or `fhess` need to be given. If `fhess` is provided, then `fhess_p` will be ignored. If neither `fhess` nor `fhess_p` is provided, then the hessian product will be approximated using finite differences on `fprime`. `fhess_p` must compute the hessian times an arbitrary vector. If it is not given, finite-differences on `fprime` are used to compute it.

Newton-CG methods are also called truncated Newton methods. This function differs from `scipy.optimize.fmin_tnc` because

1. `scipy.optimize.fmin_ncg` is written purely in python using `numpy` and `scipy` while `scipy.optimize.fmin_tnc` calls a C function.
2. `scipy.optimize.fmin_ncg` is only for unconstrained minimization while `scipy.optimize.fmin_tnc` is for unconstrained minimization or box constrained minimization. (Box constraints give lower and upper bounds for each variable separately.)

**References**

Wright & Nocedal, ‘Numerical Optimization’, 1999, pg. 140.

Constrained multivariate methods:

<code>fmin_l_bfgs_b(func, x0[, fprime, args, ...])</code>	Minimize a function <code>func</code> using the L-BFGS-B algorithm.
<code>fmin_tnc(func, x0[, fprime, args, ...])</code>	Minimize a function with variables subject to bounds, using gradient info
<code>fmin_cobyla(func, x0, cons[, args, ...])</code>	Minimize a function using the Constrained Optimization BY Linear Appr
<code>fmin_slsqp(func, x0[, eqcons, f_eqcons, ...])</code>	Minimize a function using Sequential Least Squares Programming
<code>differential_evolution(func, bounds[, args, ...])</code>	Finds the global minimum of a multivariate function.

`scipy.optimize.fmin_l_bfgs_b` (`func`, `x0`, `fprime=None`, `args=()`, `approx_grad=0`, `bounds=None`, `m=10`, `factr=10000000.0`, `pgtol=1e-05`, `epsilon=1e-08`, `iprint=-1`, `maxfun=15000`, `maxiter=15000`, `disp=None`, `callback=None`, `maxls=20`)

Minimize a function `func` using the L-BFGS-B algorithm.

**Parameters** `func` : callable  $f(x, *args)$

Function to minimise.

**x0** : ndarray  
Initial guess.

**fprime** : callable  $f_{\text{prime}}(x, *args)$ , optional  
The gradient of *func*. If None, then *func* returns the function value and the gradient ( $f, g = \text{func}(x, *args)$ ), unless *approx\_grad* is True in which case *func* returns only *f*.

**args** : sequence, optional  
Arguments to pass to *func* and *fprime*.

**approx\_grad** : bool, optional  
Whether to approximate the gradient numerically (in which case *func* returns only the function value).

**bounds** : list, optional  
(min, max) pairs for each element in *x*, defining the bounds on that parameter. Use None or +-inf for one of min or max when there is no bound in that direction.

**m** : int, optional  
The maximum number of variable metric corrections used to define the limited memory matrix. (The limited memory BFGS method does not store the full hessian but uses this many terms in an approximation to it.)

**factr** : float, optional  
The iteration stops when  $(f^k - f^{k+1}) / \max\{|f^k|, |f^{k+1}|, 1\} \leq \text{factr} * \text{eps}$ , where *eps* is the machine precision, which is automatically generated by the code. Typical values for *factr* are: 1e12 for low accuracy; 1e7 for moderate accuracy; 10.0 for extremely high accuracy.

**pgtol** : float, optional  
The iteration will stop when  $\max\{|\text{proj } g_i| \mid i = 1, \dots, n\} \leq \text{pgtol}$  where *pg<sub>i</sub>* is the *i*-th component of the projected gradient.

**epsilon** : float, optional  
Step size used when *approx\_grad* is True, for numerically calculating the gradient

**iprint** : int, optional  
Controls the frequency of output. *iprint* < 0 means no output; *iprint* = 0 print only one line at the last iteration;  $0 < \text{iprint} < 99$  print also *f* and  $|\text{proj } g|$  every *iprint* iterations; *iprint* = 99 print details of every iteration except *n*-vectors; *iprint* = 100 print also the changes of active set and final *x*; *iprint* > 100 print details of every iteration including *x* and *g*.

**disp** : int, optional  
If zero, then no output. If a positive number, then this over-rides *iprint* (i.e., *iprint* gets the value of *disp*).

**maxfun** : int, optional  
Maximum number of function evaluations.

**maxiter** : int, optional  
Maximum number of iterations.

**callback** : callable, optional  
Called after each iteration, as *callback*(*xk*), where *xk* is the current parameter vector.

**maxls** : int, optional  
Maximum number of line search steps (per iteration). Default is 20.

**Returns**

**x** : array\_like  
Estimated position of the minimum.

**f** : float  
Value of *func* at the minimum.

**d** : dict  
Information dictionary.  
•d['warnflag'] is



- 0 if converged,
- 1 if too many function evaluations or too many iterations,
- 2 if stopped for another reason, given in `d['task']`
- `d['grad']` is the gradient at the minimum (should be 0 ish)
- `d['funcalls']` is the number of function calls made.
- `d['nit']` is the number of iterations.

**See also:**

**`minimize`** Interface to minimization algorithms for multivariate functions. See the ‘L-BFGS-B’ *method* in particular.

**Notes**

License of L-BFGS-B (FORTRAN code):

The version included here (in fortran code) is 3.0 (released April 25, 2011). It was written by Ciyou Zhu, Richard Byrd, and Jorge Nocedal <[nocedal@ece.nwu.edu](mailto:nocedal@ece.nwu.edu)>. It carries the following condition for use:

This software is freely available, but we expect that all publications describing work using this software, or all commercial products using it, quote at least one of the references given below. This software is released under the BSD License.

**References**

- R. H. Byrd, P. Lu and J. Nocedal. A Limited Memory Algorithm for Bound Constrained Optimization, (1995), SIAM Journal on Scientific and Statistical Computing, 16, 5, pp. 1190-1208.
- C. Zhu, R. H. Byrd and J. Nocedal. L-BFGS-B: Algorithm 778: L-BFGS-B, FORTRAN routines for large scale bound constrained optimization (1997), ACM Transactions on Mathematical Software, 23, 4, pp. 550 - 560.
- J.L. Morales and J. Nocedal. L-BFGS-B: Remark on Algorithm 778: L-BFGS-B, FORTRAN routines for large scale bound constrained optimization (2011), ACM Transactions on Mathematical Software, 38, 1.

`scipy.optimize.fmin_tnc` (*func*, *x0*, *fprime=None*, *args=()*, *approx\_grad=0*, *bounds=None*, *epsilon=1e-08*, *scale=None*, *offset=None*, *messages=15*, *maxCGit=-1*, *maxfun=None*, *eta=-1*, *stepmx=0*, *accuracy=0*, *fmin=0*, *ftol=-1*, *xtol=-1*, *pgtol=-1*, *rescale=-1*, *disp=None*, *callback=None*)

Minimize a function with variables subject to bounds, using gradient information in a truncated Newton algorithm. This method wraps a C implementation of the algorithm.

**Parameters** **func** : callable `func(x, *args)`

Function to minimize. Must do one of:

- 1.Return `f` and `g`, where `f` is the value of the function and `g` its gradient (a list of floats).
- 2.Return the function value but supply gradient function separately as *fprime*.
- 3.Return the function value and set `approx_grad=True`.

If the function returns `None`, the minimization is aborted.

**x0** : array\_like

Initial estimate of minimum.

**fprime** : callable `fprime(x, *args)`, optional

Gradient of *func*. If `None`, then either *func* must return the function value and the gradient (`f, g = func(x, *args)`) or *approx\_grad* must be `True`.

**args** : tuple, optional

Arguments to pass to function.

**approx\_grad** : bool, optional

If true, approximate the gradient numerically.

**bounds** : list, optional

(min, max) pairs for each element in x0, defining the bounds on that parameter. Use None or +/-inf for one of min or max when there is no bound in that direction.

**epsilon** : float, optional

Used if approx\_grad is True. The stepsize in a finite difference approximation for fprime.

**scale** : array\_like, optional

Scaling factors to apply to each variable. If None, the factors are up-low for interval bounded variables and 1+|x| for the others. Defaults to None.

**offset** : array\_like, optional

Value to subtract from each variable. If None, the offsets are (up+low)/2 for interval bounded variables and x for the others.

**messages** : int, optional

Bit mask used to select messages display during minimization values defined in the MSGS dict. Defaults to MGS\_ALL.

**disp** : int, optional

Integer interface to messages. 0 = no message, 5 = all messages

**maxCGit** : int, optional

Maximum number of hessian\*vector evaluations per main iteration. If maxCGit == 0, the direction chosen is -gradient if maxCGit < 0, maxCGit is set to max(1,min(50,n/2)). Defaults to -1.

**maxfun** : int, optional

Maximum number of function evaluation. if None, maxfun is set to max(100, 10\*len(x0)). Defaults to None.

**eta** : float, optional

Severity of the line search. if < 0 or > 1, set to 0.25. Defaults to -1.

**stepmx** : float, optional

Maximum step for the line search. May be increased during call. If too small, it will be set to 10.0. Defaults to 0.

**accuracy** : float, optional

Relative precision for finite difference calculations. If <= machine\_precision, set to sqrt(machine\_precision). Defaults to 0.

**fmin** : float, optional

Minimum function value estimate. Defaults to 0.

**ftol** : float, optional

Precision goal for the value of f in the stopping criterion. If ftol < 0.0, ftol is set to 0.0 defaults to -1.

**xtol** : float, optional

Precision goal for the value of x in the stopping criterion (after applying x scaling factors). If xtol < 0.0, xtol is set to sqrt(machine\_precision). Defaults to -1.

**pgtol** : float, optional

Precision goal for the value of the projected gradient in the stopping criterion (after applying x scaling factors). If pgtol < 0.0, pgtol is set to 1e-2 \* sqrt(accuracy). Setting it to 0.0 is not recommended. Defaults to -1.

**rescale** : float, optional

Scaling factor (in log10) used to trigger f value rescaling. If 0, rescale at each iteration. If a large value, never rescale. If < 0, rescale is set to 1.3.

**callback** : callable, optional

Called after each iteration, as callback(xk), where xk is the current parameter vector.

**Returns**

**x** : ndarray

The solution.

**nfeval** : int

The number of function evaluations.

**rc** : int  
Return code, see below

**See also:**

**`minimize`** Interface to minimization algorithms for multivariate functions. See the ‘TNC’ *method* in particular.

**Notes**

The underlying algorithm is truncated Newton, also called Newton Conjugate-Gradient. This method differs from `scipy.optimize.fmin_ncg` in that

- 1.It wraps a C implementation of the algorithm
- 2.It allows each variable to be given an upper and lower bound.

The algorithm incorporates the bound constraints by determining the descent direction as in an unconstrained truncated Newton, but never taking a step-size large enough to leave the space of feasible  $x$ 's. The algorithm keeps track of a set of currently active constraints, and ignores them when computing the minimum allowable step size. (The  $x$ 's associated with the active constraint are kept fixed.) If the maximum allowable step size is zero then a new constraint is added. At the end of each iteration one of the constraints may be deemed no longer active and removed. A constraint is considered no longer active is if it is currently active but the gradient for that variable points inward from the constraint. The specific constraint removed is the one associated with the variable of largest index whose constraint is no longer active.

Return codes are defined as follows:

```
-1 : Infeasible (lower bound > upper bound)
0 : Local minimum reached (|pg| ~= 0)
1 : Converged (|f_n-f_(n-1)| ~= 0)
2 : Converged (|x_n-x_(n-1)| ~= 0)
3 : Max. number of function evaluations reached
4 : Linear search failed
5 : All lower bounds are equal to the upper bounds
6 : Unable to progress
7 : User requested end of minimization
```

**References**

Wright S., Nocedal J. (2006), ‘Numerical Optimization’

Nash S.G. (1984), “Newton-Type Minimization Via the Lanczos Method”, SIAM Journal of Numerical Analysis 21, pp. 770-778

`scipy.optimize.fmin_cobyla` (*func*, *x0*, *cons*, *args=()*, *consargs=None*, *rhobeg=1.0*, *rhoend=0.0001*, *iprint=1*, *maxfun=1000*, *disp=None*, *catol=0.0002*)

Minimize a function using the Constrained Optimization BY Linear Approximation (COBYLA) method. This method wraps a FORTRAN implementation of the algorithm.

**Parameters** **func** : callable

Function to minimize. In the form `func(x, *args)`.

**x0** : ndarray

Initial guess.

**cons** : sequence

Constraint functions; must all be  $\geq 0$  (a single function if only 1 constraint). Each function takes the parameters  $x$  as its first argument, and it can return either a single number or an array or list of numbers.

**args** : tuple, optional

Extra arguments to pass to function.

**consargs** : tuple, optional

Extra arguments to pass to constraint functions (default of None means use same extra arguments as those passed to func). Use () for no extra arguments.

**rhobeg** : float, optional

Reasonable initial changes to the variables.

**rhoend** : float, optional

Final accuracy in the optimization (not precisely guaranteed). This is a lower bound on the size of the trust region.

**iprint** : {0, 1, 2, 3}, optional

Controls the frequency of output; 0 implies no output. Deprecated.

**disp** : {0, 1, 2, 3}, optional

Over-rides the iprint interface. Preferred.

**maxfun** : int, optional

Maximum number of function evaluations.

**catol** : float, optional

Absolute tolerance for constraint violations.

**Returns**

**x** : ndarray

The argument that minimises  $f$ .

**See also:**

**minimize** Interface to minimization algorithms for multivariate functions. See the ‘COBYLA’ *method* in particular.

**Notes**

This algorithm is based on linear approximations to the objective function and each constraint. We briefly describe the algorithm.

Suppose the function is being minimized over  $k$  variables. At the  $j$ th iteration the algorithm has  $k+1$  points  $v_1, \dots, v_{k+1}$ , an approximate solution  $x_j$ , and a radius  $RHO_j$ . (i.e. linear plus a constant) approximations to the objective function and constraint functions such that their function values agree with the linear approximation on the  $k+1$  points  $v_1, \dots, v_{k+1}$ . This gives a linear program to solve (where the linear approximations of the constraint functions are constrained to be non-negative).

However the linear approximations are likely only good approximations near the current simplex, so the linear program is given the further requirement that the solution, which will become  $x_{j+1}$ , must be within  $RHO_j$  from  $x_j$ .  $RHO_j$  only decreases, never increases. The initial  $RHO_j$  is  $rhobeg$  and the final  $RHO_j$  is  $rhoend$ . In this way COBYLA’s iterations behave like a trust region algorithm.

Additionally, the linear program may be inconsistent, or the approximation may give poor improvement. For details about how these issues are resolved, as well as how the points  $v_i$  are updated, refer to the source code or the references below.

**References**

Powell M.J.D. (1994), “A direct search optimization method that models the objective and constraint functions by linear interpolation.”, in *Advances in Optimization and Numerical Analysis*, eds. S. Gomez and J-P Hennart, Kluwer Academic (Dordrecht), pp. 51-67

Powell M.J.D. (1998), “Direct search algorithms for optimization calculations”, *Acta Numerica* 7, 287-336

Powell M.J.D. (2007), “A view of algorithms for optimization without derivatives”, Cambridge University Technical Report DAMTP 2007/NA03

**Examples**

Minimize the objective function  $f(x,y) = x*y$  subject to the constraints  $x**2 + y**2 < 1$  and  $y > 0$ :

```
>>> def objective(x):
...     return x[0]*x[1]
...
>>> def constr1(x):
...     return 1 - (x[0]**2 + x[1]**2)
...
>>> def constr2(x):
...     return x[1]
...
>>> from scipy.optimize import fmin_cobyla
>>> fmin_cobyla(objective, [0.0, 0.1], [constr1, constr2], rhoend=1e-7)
array([-0.70710685,  0.70710671])
```

The exact solution is  $(-\sqrt{2}/2, \sqrt{2}/2)$ .

`scipy.optimize.fmin_slsqp` (*func*, *x0*, *eqcons*=(), *f\_eqcons*=None, *ieqcons*=(), *f\_ieqcons*=None, *bounds*=(), *fprime*=None, *fprime\_eqcons*=None, *fprime\_ieqcons*=None, *args*=(), *iter*=100, *acc*=1e-06, *iprint*=1, *disp*=None, *full\_output*=0, *epsilon*=1.4901161193847656e-08, *callback*=None)

Minimize a function using Sequential Least Squares Programming

Python interface function for the SLSQP Optimization subroutine originally implemented by Dieter Kraft.

**Parameters**

- func** : callable  $f(x, *args)$   
Objective function.
- x0** : 1-D ndarray of float  
Initial guess for the independent variable(s).
- eqcons** : list, optional  
A list of functions of length  $n$  such that  $eqcons[j](x, *args) == 0.0$  in a successfully optimized problem.
- f\_eqcons** : callable  $f(x, *args)$ , optional  
Returns a 1-D array in which each element must equal 0.0 in a successfully optimized problem. If *f\_eqcons* is specified, *eqcons* is ignored.
- ieqcons** : list, optional  
A list of functions of length  $n$  such that  $ieqcons[j](x, *args) >= 0.0$  in a successfully optimized problem.
- f\_ieqcons** : callable  $f(x, *args)$ , optional  
Returns a 1-D ndarray in which each element must be greater or equal to 0.0 in a successfully optimized problem. If *f\_ieqcons* is specified, *ieqcons* is ignored.
- bounds** : list, optional  
A list of tuples specifying the lower and upper bound for each independent variable [(x0, xu0), (x1, xu1), ...] Infinite values will be interpreted as large floating values.
- fprime** : callable  $f(x, *args)$ , optional  
A function that evaluates the partial derivatives of *func*.
- fprime\_eqcons** : callable  $f(x, *args)$ , optional  
A function of the form  $f(x, *args)$  that returns the  $m$  by  $n$  array of equality constraint normals. If not provided, the normals will be approximated. The array returned by *fprime\_eqcons* should be sized as ( len(eqcons), len(x0) ).
- fprime\_ieqcons** : callable  $f(x, *args)$ , optional  
A function of the form  $f(x, *args)$  that returns the  $m$  by  $n$  array of inequality constraint normals. If not provided, the normals will be approximated. The array returned by *fprime\_ieqcons* should be sized as ( len(ieqcons), len(x0) ).
- args** : sequence, optional  
Additional arguments passed to *func* and *fprime*.

**iter** : int, optional  
The maximum number of iterations.

**acc** : float, optional  
Requested accuracy.

**iprint** : int, optional  
The verbosity of `fmin_slsqp` :  

- `iprint <= 0` : Silent operation
- `iprint == 1` : Print summary upon completion (default)
- `iprint >= 2` : Print status of each iterate and summary

**disp** : int, optional  
Over-rides the `iprint` interface (preferred).

**full\_output** : bool, optional  
If False, return only the minimizer of `func` (default). Otherwise, output final objective function and summary information.

**epsilon** : float, optional  
The step size for finite-difference derivative estimates.

**callback** : callable, optional  
Called after each iteration, as `callback(x)`, where `x` is the current parameter vector.

**Returns**

**out** : ndarray of float  
The final minimizer of `func`.

**fx** : ndarray of float, if `full_output` is true  
The final value of the objective function.

**its** : int, if `full_output` is true  
The number of iterations.

**imode** : int, if `full_output` is true  
The exit mode from the optimizer (see below).

**smode** : string, if `full_output` is true  
Message describing the exit mode from the optimizer.

**See also:**

***minimize*** Interface to minimization algorithms for multivariate functions. See the ‘SLSQP’ *method* in particular.

**Notes**

Exit modes are defined as follows

```
-1 : Gradient evaluation required (g & a)
0 : Optimization terminated successfully.
1 : Function evaluation required (f & c)
2 : More equality constraints than independent variables
3 : More than 3*n iterations in LSQ subproblem
4 : Inequality constraints incompatible
5 : Singular matrix E in LSQ subproblem
6 : Singular matrix C in LSQ subproblem
7 : Rank-deficient equality constraint subproblem HFTI
8 : Positive directional derivative for linesearch
9 : Iteration limit exceeded
```

**Examples**

Examples are given *in the tutorial*.

`scipy.optimize.differential_evolution` (*func*, *bounds*, *args=()*, *strategy='best1bin'*, *maxiter=1000*, *popsize=15*, *tol=0.01*, *mutation=(0.5, 1)*, *recombination=0.7*, *seed=None*, *callback=None*, *disp=False*, *polish=True*, *init='latinhypercube'*)

Finds the global minimum of a multivariate function. Differential Evolution is stochastic in nature (does not use gradient methods) to find the minimum, and can search large areas of candidate space, but often requires larger numbers of function evaluations than conventional gradient based techniques.

The algorithm is due to Storn and Price [R150].

**Parameters** **func** : callable

The objective function to be minimized. Must be in the form  $f(x, *args)$ , where  $x$  is the argument in the form of a 1-D array and  $args$  is a tuple of any additional fixed parameters needed to completely specify the function.

**bounds** : sequence

Bounds for variables.  $(min, max)$  pairs for each element in  $x$ , defining the lower and upper bounds for the optimizing argument of *func*. It is required to have  $len(bounds) == len(x)$ .  $len(bounds)$  is used to determine the number of parameters in  $x$ .

**args** : tuple, optional

Any additional fixed parameters needed to completely specify the objective function.

**strategy** : str, optional

The differential evolution strategy to use. Should be one of:

- 'best1bin'
- 'best1exp'
- 'rand1exp'
- 'randtobest1exp'
- 'best2exp'
- 'rand2exp'
- 'randtobest1bin'
- 'best2bin'
- 'rand2bin'
- 'rand1bin'

The default is 'best1bin'.

**maxiter** : int, optional

The maximum number of generations over which the entire population is evolved. The maximum number of function evaluations (with no polishing) is:  $(maxiter + 1) * popsize * len(x)$

**popsize** : int, optional

A multiplier for setting the total population size. The population has  $popsize * len(x)$  individuals.

**tol** : float, optional

When the mean of the population energies, multiplied by *tol*, divided by the standard deviation of the population energies is greater than 1 the solving process terminates:  $convergence = mean(pop) * tol / stdev(pop) > 1$

**mutation** : float or tuple(float, float), optional

The mutation constant. In the literature this is also known as differential weight, being denoted by  $F$ . If specified as a float it should be in the range  $[0, 2]$ . If specified as a tuple  $(min, max)$  dithering is employed. Dithering randomly changes the mutation constant on a generation by generation basis. The mutation constant for that generation is taken from  $U[min, max)$ . Dithering can help speed convergence significantly. Increasing the mutation constant increases the search radius, but will slow down convergence.

**recombination** : float, optional

The recombination constant, should be in the range [0, 1]. In the literature this is also known as the crossover probability, being denoted by CR. Increasing this value allows a larger number of mutants to progress into the next generation, but at the risk of population stability.

**seed** : int or `np.random.RandomState`, optional

If *seed* is not specified the `np.RandomState` singleton is used. If *seed* is an int, a new `np.random.RandomState` instance is used, seeded with *seed*. If *seed* is already a `np.random.RandomState` instance, then that `np.random.RandomState` instance is used. Specify *seed* for repeatable minimizations.

**disp** : bool, optional

Display status messages

**callback** : callable, `callback(xk, convergence=val)`, optional

A function to follow the progress of the minimization. *xk* is the current value of *x0*. *val* represents the fractional value of the population convergence. When *val* is greater than one the function halts. If callback returns `True`, then the minimization is halted (any polishing is still carried out).

**polish** : bool, optional

If `True` (default), then `scipy.optimize.minimize` with the *L-BFGS-B* method is used to polish the best population member at the end, which can improve the minimization slightly.

**init** : string, optional

Specify how the population initialization is performed. Should be one of:

- 'latinhypercube'
- 'random'

The default is 'latinhypercube'. Latin Hypercube sampling tries to maximize coverage of the available parameter space. 'random' initializes the population randomly - this has the drawback that clustering can occur, preventing the whole of parameter space being covered.

#### Returns

**res** : `OptimizeResult`

The optimization result represented as a `OptimizeResult` object. Important attributes are: *x* the solution array, *success* a Boolean flag indicating if the optimizer exited successfully and *message* which describes the cause of the termination. See `OptimizeResult` for a description of other attributes. If *polish* was employed, and a lower minimum was obtained by the polishing, then `OptimizeResult` also contains the *jac* attribute.

#### Notes

Differential evolution is a stochastic population based method that is useful for global optimization problems. At each pass through the population the algorithm mutates each candidate solution by mixing with other candidate solutions to create a trial candidate. There are several strategies [R151] for creating trial candidates, which suit some problems more than others. The 'best1bin' strategy is a good starting point for many systems. In this strategy two members of the population are randomly chosen. Their difference is used to mutate the best member (the *best* in *best1bin*),  $b_0$ , so far:

$$b' = b_0 + mutation * (population[rand0] - population[rand1])$$

A trial vector is then constructed. Starting with a randomly chosen 'i'th parameter the trial is sequentially filled (in modulo) with parameters from  $b'$  or the original candidate. The choice of whether to use  $b'$  or the original candidate is made with a binomial distribution (the 'bin' in 'best1bin') - a random number in [0, 1) is generated. If this number is less than the *recombination* constant then the parameter is loaded from  $b'$ , otherwise it is loaded from the original candidate. The final parameter is always loaded from  $b'$ . Once the trial candidate is built its fitness is assessed. If the trial is better than the original candidate then it takes its place. If it is also better than the best overall candidate it also replaces that. To improve your chances of finding a global minimum use higher



*popsize* values, with higher *mutation* and (dithering), but lower *recombination* values. This has the effect of widening the search radius, but slowing convergence.

New in version 0.15.0.

### References

[R150], [R151], [R152]

### Examples

Let us consider the problem of minimizing the Rosenbrock function. This function is implemented in `rosen` in `scipy.optimize`.

```
>>> from scipy.optimize import rosen, differential_evolution
>>> bounds = [(0,2), (0, 2), (0, 2), (0, 2), (0, 2)]
>>> result = differential_evolution(rosen, bounds)
>>> result.x, result.fun
(array([1., 1., 1., 1., 1.]), 1.9216496320061384e-19)
```

Next find the minimum of the Ackley function ([http://en.wikipedia.org/wiki/Test\\_functions\\_for\\_optimization](http://en.wikipedia.org/wiki/Test_functions_for_optimization)).

```
>>> from scipy.optimize import differential_evolution
>>> import numpy as np
>>> def ackley(x):
...     arg1 = -0.2 * np.sqrt(0.5 * (x[0] ** 2 + x[1] ** 2))
...     arg2 = 0.5 * (np.cos(2. * np.pi * x[0]) + np.cos(2. * np.pi * x[1]))
...     return -20. * np.exp(arg1) - np.exp(arg2) + 20. + np.e
>>> bounds = [(-5, 5), (-5, 5)]
>>> result = differential_evolution(ackley, bounds)
>>> result.x, result.fun
(array([ 0.,  0.]), 4.4408920985006262e-16)
```

Univariate (scalar) minimization methods:

<code>fminbound(func, x1, x2[, args, xtol, ...])</code>	Bounded minimization for scalar functions.
<code>brent(func[, args, brack, tol, full_output, ...])</code>	Given a function of one-variable and a possible bracketing interval, return the minimum.
<code>golden(func[, args, brack, tol, full_output])</code>	Return the minimum of a function of one variable.

`scipy.optimize.fminbound` (*func*, *x1*, *x2*, *args*=(), *xtol*=1e-05, *maxfun*=500, *full\_output*=0, *disp*=1)

Bounded minimization for scalar functions.

**Parameters**

- func** : callable f(x,\*args)  
Objective function to be minimized (must accept and return scalars).
- x1, x2** : float or array scalar  
The optimization bounds.
- args** : tuple, optional  
Extra arguments passed to function.
- xtol** : float, optional  
The convergence tolerance.
- maxfun** : int, optional  
Maximum number of function evaluations allowed.
- full\_output** : bool, optional  
If True, return optional outputs.
- disp** : int, optional  
*If non-zero, print messages.*  
0 : no message printing. 1 : non-convergence notification messages only. 2 : print a message on convergence too. 3 : print iteration results.

**Returns**

- xopt** : ndarray  
Parameters (over given interval) which minimize the objective function.
- fval** : number  
The function value at the minimum point.
- ierr** : int  
An error flag (0 if converged, 1 if maximum number of function calls reached).
- numfunc** : int  
The number of function calls made.

**See also:**

[\*minimize\\_scalar\*](#)

Interface to minimization algorithms for scalar univariate functions. See the ‘Bounded’ *method* in particular.

**Notes**

Finds a local minimizer of the scalar function *func* in the interval  $x_1 < x_{opt} < x_2$  using Brent’s method. (See [\*brent\*](#) for auto-bracketing).

`scipy.optimize.brent` (*func*, *args*=(), *brack*=None, *tol*=1.48e-08, *full\_output*=0, *maxiter*=500)

Given a function of one-variable and a possible bracketing interval, return the minimum of the function isolated to a fractional precision of *tol*.

**Parameters**

- func** : callable f(x,\*args)  
Objective function.
- args** : tuple, optional  
Additional arguments (if present).
- brack** : tuple, optional  
Either a triple (xa,xb,xc) where  $x_a < x_b < x_c$  and  $func(x_b) < func(x_a), func(x_c)$  or a pair (xa,xb) which are used as a starting interval for a downhill bracket search (see [\*bracket\*](#)). Providing the pair (xa,xb) does not always mean the obtained solution will satisfy  $x_a \leq x \leq x_b$ .
- tol** : float, optional  
Stop if between iteration change is less than *tol*.
- full\_output** : bool, optional  
If True, return all output args (xmin, fval, iter, funcalls).
- maxiter** : int, optional  
Maximum number of iterations in solution.

**Returns**

- xmin** : ndarray  
Optimum point.
- fval** : float  
Optimum value.
- iter** : int  
Number of iterations.
- funcalls** : int  
Number of objective function evaluations made.

**See also:**

[\*minimize\\_scalar\*](#)

Interface to minimization algorithms for scalar univariate functions. See the ‘Brent’ *method* in particular.

**Notes**

Uses inverse parabolic interpolation when possible to speed up convergence of golden section method.

`scipy.optimize.golden` (*func*, *args*=(), *brack*=None, *tol*=1.4901161193847656e-08, *full\_output*=0)

Return the minimum of a function of one variable.

Given a function of one variable and a possible bracketing interval, return the minimum of the function isolated to a fractional precision of *tol*.

**Parameters**

- func** : callable func(x,\*args)  
Objective function to minimize.
- args** : tuple, optional  
Additional arguments (if present), passed to func.
- brack** : tuple, optional  
Triple (a,b,c), where (a<b<c) and func(b) < func(a),func(c). If bracket consists of two numbers (a, c), then they are assumed to be a starting interval for a downhill bracket search (see `bracket`); it doesn't always mean that obtained solution will satisfy  $a \leq x \leq c$ .
- tol** : float, optional  
x tolerance stop criterion
- full\_output** : bool, optional  
If True, return optional outputs.

See also:

**`minimize_scalar`**

Interface to minimization algorithms for scalar univariate functions. See the 'Golden' *method* in particular.

**Notes**

Uses analog of bisection method to decrease the bracketed interval.

**Equation (Local) Minimizers**


---

<code>leastsq</code> (func, x0[, args, Dfun, full_output, ...])	Minimize the sum of squares of a set of equations.
<code>least_squares</code> (fun, x0[, jac, bounds, ...])	Solve a nonlinear least-squares problem with bounds on the variables.
<code>nls</code> (A, b)	Solve $\operatorname{argmin}_x   Ax - b  _2$ for $x \geq 0$ .
<code>lsq_linear</code> (A, b[, bounds, method, tol, ...])	Solve a linear least-squares problem with bounds on the variables.

---

`scipy.optimize.leastsq` (*func*, *x0*, *args*=(), *Dfun*=None, *full\_output*=0, *col\_deriv*=0, *ftol*=1.49012e-08, *xtol*=1.49012e-08, *gtol*=0.0, *maxfev*=0, *epsfcn*=None, *factor*=100, *diag*=None)

Minimize the sum of squares of a set of equations.

$$x = \operatorname{arg\,min}_y (\operatorname{sum}(\operatorname{func}(y) ** 2, \operatorname{axis}=0))$$

**Parameters**

- func** : callable  
should take at least one (possibly length N vector) argument and returns M floating point numbers. It must not return NaNs or fitting might fail.
- x0** : ndarray  
The starting estimate for the minimization.
- args** : tuple, optional

Any extra arguments to `func` are placed in this tuple.

**Dfun** : callable, optional

A function or method to compute the Jacobian of `func` with derivatives across the rows. If this is `None`, the Jacobian will be estimated.

**full\_output** : bool, optional

non-zero to return all optional outputs.

**col\_deriv** : bool, optional

non-zero to specify that the Jacobian function computes derivatives down the columns (faster, because there is no transpose operation).

**ftol** : float, optional

Relative error desired in the sum of squares.

**xtol** : float, optional

Relative error desired in the approximate solution.

**gtol** : float, optional

Orthogonality desired between the function vector and the columns of the Jacobian.

**maxfev** : int, optional

The maximum number of calls to the function. If `Dfun` is provided then the default `maxfev` is  $100 \cdot (N+1)$  where  $N$  is the number of elements in `x0`, otherwise the default `maxfev` is  $200 \cdot (N+1)$ .

**epsfcn** : float, optional

A variable used in determining a suitable step length for the forward- difference approximation of the Jacobian (for `Dfun=None`). Normally the actual step length will be  $\sqrt{\text{epsfcn}} \cdot x$ . If `epsfcn` is less than the machine precision, it is assumed that the relative errors are of the order of the machine precision.

**factor** : float, optional

A parameter determining the initial step bound (`factor * ||diag * x||`). Should be in interval  $(0.1, 100)$ .

**diag** : sequence, optional

$N$  positive entries that serve as a scale factors for the variables.

**Returns**

**x** : ndarray

The solution (or the result of the last iteration for an unsuccessful call).

**cov\_x** : ndarray

Uses the `fjac` and `ipvt` optional outputs to construct an estimate of the jacobian around the solution. `None` if a singular matrix encountered (indicates very flat curvature in some direction). This matrix must be multiplied by the residual variance to get the covariance of the parameter estimates – see `curve_fit`.

**infodict** : dict

a dictionary of optional outputs with the key s:

**nfev** The number of function calls

**fvec** The function evaluated at the output

**fjac** A permutation of the  $R$  matrix of a QR factorization of the final approximate Jacobian matrix, stored column wise. Together with `ipvt`, the covariance of the estimate can be approximated.

**ipvt** An integer array of length  $N$  which defines a permutation matrix,  $p$ , such that  $fjac \cdot p = q \cdot r$ , where  $r$  is upper triangular with diagonal elements of nonincreasing magnitude. Column  $j$  of  $p$  is column `ipvt(j)` of the identity matrix.

**qtf** The vector  $(\text{transpose}(q) * fvec)$ .

**mesg** : str

A string message giving information about the cause of failure.

**ier** : int

An integer flag. If it is equal to 1, 2, 3 or 4, the solution was found. Otherwise, the solution was not found. In either case, the optional output variable ‘`mesg`’ gives more information.

**Notes**

“leastsq” is a wrapper around MINPACK’s lmdif and lmdcr algorithms.

cov\_x is a Jacobian approximation to the Hessian of the least squares objective function. This approximation assumes that the objective function is based on the difference between some observed target data (ydata) and a (non-linear) function of the parameters  $f(xdata, params)$

```
func(params) = ydata - f(xdata, params)
```

so that the objective function is

```
min params sum((ydata - f(xdata, params))**2, axis=0)
```

```
scipy.optimize.least_squares(fun, x0, jac='2-point', bounds=(-inf, inf), method='trf',
                             ftol=1e-08, xtol=1e-08, gtol=1e-08, x_scale=1.0, loss='linear',
                             f_scale=1.0, diff_step=None, tr_solver=None, tr_options={},
                             jac_sparsity=None, max_nfev=None, verbose=0, args=(),
                             kwargs={})
```

Solve a nonlinear least-squares problem with bounds on the variables.

Given the residuals  $f(x)$  (an  $m$ -dimensional function of  $n$  variables) and the loss function  $\rho(s)$  (a scalar function), `least_squares` finds a local minimum of the cost function  $F(x)$ :

```
minimize F(x) = 0.5 * sum(rho(f_i(x)**2), i = 0, ..., m - 1)
subject to lb <= x <= ub
```

The purpose of the loss function  $\rho(s)$  is to reduce the influence of outliers on the solution.

**Parameters** **fun** : callable

Function which computes the vector of residuals, with the signature `fun(x, *args, **kwargs)`, i.e., the minimization proceeds with respect to its first argument. The argument  $x$  passed to this function is an ndarray of shape  $(n,)$  (never a scalar, even for  $n=1$ ). It must return a 1-d array\_like of shape  $(m,)$  or a scalar.

**x0** : array\_like with shape  $(n,)$  or float

Initial guess on independent variables. If float, it will be treated as a 1-d array with one element.

**jac** : {'2-point', '3-point', 'cs', callable}, optional

Method of computing the Jacobian matrix (an  $m$ -by- $n$  matrix, where element  $(i, j)$  is the partial derivative of  $f[i]$  with respect to  $x[j]$ ). The keywords select a finite difference scheme for numerical estimation. The scheme '3-point' is more accurate, but requires twice as much operations compared to '2-point' (default). The scheme 'cs' uses complex steps, and while potentially the most accurate, it is applicable only when *fun* correctly handles complex inputs and can be analytically continued to the complex plane. Method 'lm' always uses the '2-point' scheme. If callable, it is used as `jac(x, *args, **kwargs)` and should return a good approximation (or the exact value) for the Jacobian as an array\_like (np.atleast\_2d is applied), a sparse matrix or a `scipy.sparse.linalg.LinearOperator`.

**bounds** : 2-tuple of array\_like, optional

Lower and upper bounds on independent variables. Defaults to no bounds. Each array must match the size of *x0* or be a scalar, in the latter case a bound will be the same for all variables. Use `np.inf` with an appropriate sign to disable bounds on all or some variables.

**method** : {'trf', 'dogbox', 'lm'}, optional

Algorithm to perform minimization.

- 'trf' : Trust Region Reflective algorithm, particularly suitable for large sparse problems with bounds. Generally robust method.

- ‘dogbox’ : dogleg algorithm with rectangular trust regions, typical use case is small problems with bounds. Not recommended for problems with rank-deficient Jacobian.
- ‘lm’ : Levenberg-Marquardt algorithm as implemented in MINPACK. Doesn’t handle bounds and sparse Jacobians. Usually the most efficient method for small unconstrained problems.

Default is ‘trf’. See Notes for more information.

**ftol** : float, optional

Tolerance for termination by the change of the cost function. Default is 1e-8. The optimization process is stopped when  $dF < ftol * F$ , and there was an adequate agreement between a local quadratic model and the true model in the last step.

**xtol** : float, optional

Tolerance for termination by the change of the independent variables. Default is 1e-8. The exact condition depends on the *method* used:

- For ‘trf’ and ‘dogbox’ :  $norm(dx) < xtol * (xtol + norm(x))$
- For ‘lm’ :  $\Delta < xtol * norm(xs)$ , where  $\Delta$  is a trust-region radius and  $xs$  is the value of  $x$  scaled according to  $x\_scale$  parameter (see below).

**gtol** : float, optional

Tolerance for termination by the norm of the gradient. Default is 1e-8. The exact condition depends on a *method* used:

- For ‘trf’ :  $norm(g\_scaled, ord=np.inf) < gtol$ , where  $g\_scaled$  is the value of the gradient scaled to account for the presence of the bounds [STIR].
- For ‘dogbox’ :  $norm(g\_free, ord=np.inf) < gtol$ , where  $g\_free$  is the gradient with respect to the variables which are not in the optimal state on the boundary.
- For ‘lm’ : the maximum absolute value of the cosine of angles between columns of the Jacobian and the residual vector is less than  $gtol$ , or the residual vector is zero.

**x\_scale** : array\_like or ‘jac’, optional

Characteristic scale of each variable. Setting  $x\_scale$  is equivalent to reformulating the problem in scaled variables  $xs = x / x\_scale$ . An alternative view is that the size of a trust region along  $j$ -th dimension is proportional to  $x\_scale[j]$ . Improved convergence may be achieved by setting  $x\_scale$  such that a step of a given size along any of the scaled variables has a similar effect on the cost function. If set to ‘jac’, the scale is iteratively updated using the inverse norms of the columns of the Jacobian matrix (as described in [JJMore]).

**loss** : str or callable, optional

Determines the loss function. The following keyword values are allowed:

- ‘linear’ (default) :  $\rho(z) = z$ . Gives a standard least-squares problem.
- ‘soft\_l1’ :  $\rho(z) = 2 * ((1 + z)**0.5 - 1)$ . The smooth approximation of l1 (absolute value) loss. Usually a good choice for robust least squares.
- ‘huber’ :  $\rho(z) = z$  if  $z \leq 1$  else  $2*z**0.5 - 1$ . Works similarly to ‘soft\_l1’.
- ‘cauchy’ :  $\rho(z) = \ln(1 + z)$ . Severely weakens outliers influence, but may cause difficulties in optimization process.
- ‘arctan’ :  $\rho(z) = \arctan(z)$ . Limits a maximum loss on a single residual, has properties similar to ‘cauchy’.

If callable, it must take a 1-d ndarray  $z=f**2$  and return an array\_like with shape (3, m) where row 0 contains function values, row 1 contains first derivatives and row 2 contains second derivatives. Method ‘lm’ supports only ‘linear’ loss.

**f\_scale** : float, optional

Value of soft margin between inlier and outlier residuals, default is 1.0. The loss function is evaluated as follows  $\rho_0(f**2) = C**2 * \rho(f**2 / C**2)$ ,

where  $C$  is `f_scale`, and `rho` is determined by `loss` parameter. This parameter has no effect with `loss='linear'`, but for other `loss` values it is of crucial importance.

**max\_nfev** : None or int, optional

Maximum number of function evaluations before the termination. If None (default), the value is chosen automatically:

- For 'trf' and 'dogbox' :  $100 * n$ .
- For 'lm' :  $100 * n$  if `jac` is callable and  $100 * n * (n + 1)$  otherwise (because 'lm' counts function calls in Jacobian estimation).

**diff\_step** : None or array\_like, optional

Determines the relative step size for the finite difference approximation of the Jacobian. The actual step is computed as  $x * \text{diff\_step}$ . If None (default), then `diff_step` is taken to be a conventional "optimal" power of machine epsilon for the finite difference scheme used [NR].

**tr\_solver** : {None, 'exact', 'lsmr'}, optional

Method for solving trust-region subproblems, relevant only for 'trf' and 'dogbox' methods.

- 'exact' is suitable for not very large problems with dense Jacobian matrices. The computational complexity per iteration is comparable to a singular value decomposition of the Jacobian matrix.
- 'lsmr' is suitable for problems with sparse and large Jacobian matrices. It uses the iterative procedure `scipy.sparse.linalg.lsmr` for finding a solution of a linear least-squares problem and only requires matrix-vector product evaluations.

If None (default) the solver is chosen based on the type of Jacobian returned on the first iteration.

**tr\_options** : dict, optional

Keyword options passed to trust-region solver.

- `tr_solver='exact'` : `tr_options` are ignored.
- `tr_solver='lsmr'` : options for `scipy.sparse.linalg.lsmr`. Additionally `method='trf'` supports 'regularize' option (bool, default is True) which adds a regularization term to the normal equation, which improves convergence if the Jacobian is rank-deficient [Byrd] (eq. 3.4).

**jac\_sparsity** : {None, array\_like, sparse matrix}, optional

Defines the sparsity structure of the Jacobian matrix for finite difference estimation, its shape must be (m, n). If the Jacobian has only few non-zero elements in *each* row, providing the sparsity structure will greatly speed up the computations [Curtis]. A zero entry means that a corresponding element in the Jacobian is identically zero. If provided, forces the use of 'lsmr' trust-region solver. If None (default) then dense differencing will be used. Has no effect for 'lm' method.

**verbose** : {0, 1, 2}, optional

Level of algorithm's verbosity:

- 0 (default) : work silently.
- 1 : display a termination report.
- 2 : display progress during iterations (not supported by 'lm' method).

**args, kwargs** : tuple and dict, optional

Additional arguments passed to `fun` and `jac`. Both empty by default. The calling signature is `fun(x, *args, **kwargs)` and the same for `jac`.

#### Returns

`OptimizeResult` with the following fields defined:

**x** : ndarray, shape (n,)

Solution found.

**cost** : float

Value of the cost function at the solution.

**fun** : ndarray, shape (m,)

Vector of residuals at the solution.

**jac** : ndarray, sparse matrix or LinearOperator, shape (m, n)

Modified Jacobian matrix at the solution, in the sense that  $J^T J$  is a Gauss-Newton approximation of the Hessian of the cost function. The type is the same as the one used by the algorithm.

**grad** : ndarray, shape (m,)

Gradient of the cost function at the solution.

**optimality** : float

First-order optimality measure. In unconstrained problems, it is always the uniform norm of the gradient. In constrained problems, it is the quantity which was compared with *gtol* during iterations.

**active\_mask** : ndarray of int, shape (n,)

Each component shows whether a corresponding constraint is active (that is, whether a variable is at the bound):

- 0 : a constraint is not active.
- 1 : a lower bound is active.
- 1 : an upper bound is active.

Might be somewhat arbitrary for ‘trf’ method as it generates a sequence of strictly feasible iterates and *active\_mask* is determined within a tolerance threshold.

**nfev** : int

Number of function evaluations done. Methods ‘trf’ and ‘dogbox’ do not count function calls for numerical Jacobian approximation, as opposed to ‘lm’ method.

**njev** : int or None

Number of Jacobian evaluations done. If numerical Jacobian approximation is used in ‘lm’ method, it is set to None.

**status** : int

The reason for algorithm termination:

- 1 : improper input parameters status returned from MINPACK.
- 0 : the maximum number of function evaluations is exceeded.
- 1 : *gtol* termination condition is satisfied.
- 2 : *ftol* termination condition is satisfied.
- 3 : *xtol* termination condition is satisfied.
- 4 : Both *ftol* and *xtol* termination conditions are satisfied.

**message** : str

Verbal description of the termination reason.

**success** : bool

True if one of the convergence criteria is satisfied (*status* > 0).

**See also:**

[\*leastsq\*](#) A legacy wrapper for the MINPACK implementation of the Levenberg-Marquadt algorithm.

[\*curve\\_fit\*](#) Least-squares minimization applied to a curve fitting problem.

### Notes

Method ‘lm’ (Levenberg-Marquardt) calls a wrapper over least-squares algorithms implemented in MINPACK (lmdr, lmdif). It runs the Levenberg-Marquardt algorithm formulated as a trust-region type algorithm. The implementation is based on paper [JJMore], it is very robust and efficient with a lot of smart tricks. It should be your first choice for unconstrained problems. Note that it doesn’t support bounds. Also it doesn’t work when  $m < n$ .

Method ‘trf’ (Trust Region Reflective) is motivated by the process of solving a system of equations, which constitute the first-order optimality condition for a bound-constrained minimization problem as formulated in [STIR]. The algorithm iteratively solves trust-region subproblems augmented by a special diagonal quadratic term and with trust-region shape determined by the distance from the bounds and the direction of the gradient. This enhancements help to avoid making steps directly into bounds and efficiently explore the whole space of variables. To further improve convergence, the algorithm considers search directions reflected from the bounds.



To obey theoretical requirements, the algorithm keeps iterates strictly feasible. With dense Jacobians trust-region subproblems are solved by an exact method very similar to the one described in [JJMore] (and implemented in MINPACK). The difference from the MINPACK implementation is that a singular value decomposition of a Jacobian matrix is done once per iteration, instead of a QR decomposition and series of Givens rotation eliminations. For large sparse Jacobians a 2-d subspace approach of solving trust-region subproblems is used [STIR], [Byrd]. The subspace is spanned by a scaled gradient and an approximate Gauss-Newton solution delivered by `scipy.sparse.linalg.lsmr`. When no constraints are imposed the algorithm is very similar to MINPACK and has generally comparable performance. The algorithm works quite robust in unbounded and bounded problems, thus it is chosen as a default algorithm.

Method ‘dogbox’ operates in a trust-region framework, but considers rectangular trust regions as opposed to conventional ellipsoids [Voglis]. The intersection of a current trust region and initial bounds is again rectangular, so on each iteration a quadratic minimization problem subject to bound constraints is solved approximately by Powell’s dogleg method [NumOpt]. The required Gauss-Newton step can be computed exactly for dense Jacobians or approximately by `scipy.sparse.linalg.lsmr` for large sparse Jacobians. The algorithm is likely to exhibit slow convergence when the rank of Jacobian is less than the number of variables. The algorithm often outperforms ‘trf’ in bounded problems with a small number of variables.

Robust loss functions are implemented as described in [BA]. The idea is to modify a residual vector and a Jacobian matrix on each iteration such that computed gradient and Gauss-Newton Hessian approximation match the true gradient and Hessian approximation of the cost function. Then the algorithm proceeds in a normal way, i.e. robust loss functions are implemented as a simple wrapper over standard least-squares algorithms.

New in version 0.17.0.

### References

[STIR], [NR], [Byrd], [Curtis], [JJMore], [Voglis], [NumOpt], [BA]

### Examples

In this example we find a minimum of the Rosenbrock function without bounds on independent variables.

```
>>> def fun_rosenbrock(x):
...     return np.array([10 * (x[1] - x[0]**2), (1 - x[0])])
```

Notice that we only provide the vector of the residuals. The algorithm constructs the cost function as a sum of squares of the residuals, which gives the Rosenbrock function. The exact minimum is at  $x = [1.0, 1.0]$ .

```
>>> from scipy.optimize import least_squares
>>> x0_rosenbrock = np.array([2, 2])
>>> res_1 = least_squares(fun_rosenbrock, x0_rosenbrock)
>>> res_1.x
array([ 1.,  1.])
>>> res_1.cost
9.8669242910846867e-30
>>> res_1.optimality
8.8928864934219529e-14
```

We now constrain the variables, in such a way that the previous solution becomes infeasible. Specifically, we require that  $x[1] \geq 1.5$ , and  $x[0]$  left unconstrained. To this end, we specify the *bounds* parameter to `least_squares` in the form `bounds=(-np.inf, 1.5], np.inf)`.

We also provide the analytic Jacobian:

```
>>> def jac_rosenbrock(x):
...     return np.array([
...         [-20 * x[0], 10],
...         [-1, 0]])
```

Putting this all together, we see that the new solution lies on the bound:

```
>>> res_2 = least_squares(fun_rosenbrock, x0_rosenbrock, jac_rosenbrock,
...                       bounds=(-np.inf, 1.5], np.inf))
...
>>> res_2.x
array([ 1.22437075,  1.5          ])
>>> res_2.cost
0.025213093946805685
>>> res_2.optimality
1.5885401433157753e-07
```

Now we solve a system of equations (i.e., the cost function should be zero at a minimum) for a Broyden tridiagonal vector-valued function of 100000 variables:

```
>>> def fun_broyden(x):
...     f = (3 - x) * x + 1
...     f[1:] -= x[:-1]
...     f[:-1] -= 2 * x[1:]
...     return f
```

The corresponding Jacobian matrix is sparse. We tell the algorithm to estimate it by finite differences and provide the sparsity structure of Jacobian to significantly speed up this process.

```
>>> from scipy.sparse import lil_matrix
>>> def sparsity_broyden(n):
...     sparsity = lil_matrix((n, n), dtype=int)
...     i = np.arange(n)
...     sparsity[i, i] = 1
...     i = np.arange(1, n)
...     sparsity[i, i - 1] = 1
...     i = np.arange(n - 1)
...     sparsity[i, i + 1] = 1
...     return sparsity
...
>>> n = 100000
>>> x0_broyden = -np.ones(n)
...
>>> res_3 = least_squares(fun_broyden, x0_broyden,
...                       jac_sparsity=sparsity_broyden(n))
...
>>> res_3.cost
4.5687069299604613e-23
>>> res_3.optimality
1.1650454296851518e-11
```

Let's also solve a curve fitting problem using robust loss function to take care of outliers in the data. Define the model function as  $y = a + b * \exp(c * t)$ , where  $t$  is a predictor variable,  $y$  is an observation and  $a$ ,  $b$ ,  $c$  are parameters to estimate.

First, define the function which generates the data with noise and outliers, define the model parameters, and generate data:

```
>>> def gen_data(t, a, b, c, noise=0, n_outliers=0, random_state=0):
...     y = a + b * np.exp(t * c)
...
...     rnd = np.random.RandomState(random_state)
...     error = noise * rnd.randn(t.size)
...     outliers = rnd.randint(0, t.size, n_outliers)
...     error[outliers] *= 10
...
...     return y + error
```

```

...
>>> a = 0.5
>>> b = 2.0
>>> c = -1
>>> t_min = 0
>>> t_max = 10
>>> n_points = 15
...
>>> t_train = np.linspace(t_min, t_max, n_points)
>>> y_train = gen_data(t_train, a, b, c, noise=0.1, n_outliers=3)

```

Define function for computing residuals and initial estimate of parameters.

```

>>> def fun(x, t, y):
...     return x[0] + x[1] * np.exp(x[2] * t) - y
...
>>> x0 = np.array([1.0, 1.0, 0.0])

```

Compute a standard least-squares solution:

```

>>> res_lsq = least_squares(fun, x0, args=(t_train, y_train))

```

Now compute two solutions with two different robust loss functions. The parameter *f\_scale* is set to 0.1, meaning that inlier residuals should not significantly exceed 0.1 (the noise level used).

```

>>> res_soft_l1 = least_squares(fun, x0, loss='soft_l1', f_scale=0.1,
...                             args=(t_train, y_train))
>>> res_log = least_squares(fun, x0, loss='cauchy', f_scale=0.1,
...                          args=(t_train, y_train))

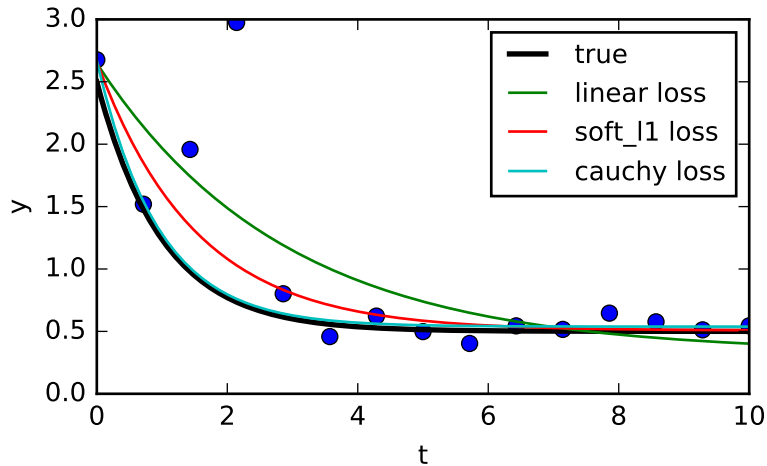
```

And finally plot all the curves. We see that by selecting an appropriate *loss* we can get estimates close to optimal even in the presence of strong outliers. But keep in mind that generally it is recommended to try ‘soft\_l1’ or ‘huber’ losses first (if at all necessary) as the other two options may cause difficulties in optimization process.

```

>>> t_test = np.linspace(t_min, t_max, n_points * 10)
>>> y_true = gen_data(t_test, a, b, c)
>>> y_lsq = gen_data(t_test, *res_lsq.x)
>>> y_soft_l1 = gen_data(t_test, *res_soft_l1.x)
>>> y_log = gen_data(t_test, *res_log.x)
...
>>> import matplotlib.pyplot as plt
>>> plt.plot(t_train, y_train, 'o')
>>> plt.plot(t_test, y_true, 'k', linewidth=2, label='true')
>>> plt.plot(t_test, y_lsq, label='linear loss')
>>> plt.plot(t_test, y_soft_l1, label='soft_l1 loss')
>>> plt.plot(t_test, y_log, label='cauchy loss')
>>> plt.xlabel("t")
>>> plt.ylabel("y")
>>> plt.legend()
>>> plt.show()

```



`scipy.optimize.nnls(A, b)`

Solve  $\operatorname{argmin}_x \|Ax - b\|_2$  for  $x \geq 0$ . This is a wrapper for a FORTRAN non-negative least squares solver.

**Parameters**

- A** : ndarray  
Matrix A as shown above.
- b** : ndarray  
Right-hand side vector.

**Returns**

- x** : ndarray  
Solution vector.
- rnorm** : float  
The residual,  $\|Ax - b\|_2$ .

#### Notes

The FORTRAN code was published in the book below. The algorithm is an active set method. It solves the KKT (Karush-Kuhn-Tucker) conditions for the non-negative least squares problem.

#### References

Lawson C., Hanson R.J., (1987) Solving Least Squares Problems, SIAM

`scipy.optimize.lsqr_linear(A, b, bounds=(-inf, inf), method='trf', tol=1e-10, lsq_solver=None, lsqr_tol=None, max_iter=None, verbose=0)`  
Solve a linear least-squares problem with bounds on the variables.

Given a m-by-n design matrix A and a target vector b with m elements, `lsqr_linear` solves the following optimization problem:

```
minimize 0.5 * ||A x - b||**2
subject to lb <= x <= ub
```

This optimization problem is convex, hence a found minimum (if iterations have converged) is guaranteed to be global.

**Parameters**

- A** : array\_like, sparse matrix of LinearOperator, shape (m, n)  
Design matrix. Can be `scipy.sparse.linalg.LinearOperator`.
- b** : array\_like, shape (m,)  
Target vector.

**bounds** : 2-tuple of array\_like, optional

Lower and upper bounds on independent variables. Defaults to no bounds. Each array must have shape (n,) or be a scalar, in the latter case a bound will be the same for all variables. Use `np.inf` with an appropriate sign to disable bounds on all or some variables.

**method** : 'trf' or 'bvls', optional

Method to perform minimization.

- 'trf' : Trust Region Reflective algorithm adapted for a linear least-squares problem. This is an interior-point-like method and the required number of iterations is weakly correlated with the number of variables.
- 'bvls' : Bounded-Variable Least-Squares algorithm. This is an active set method, which requires the number of iterations comparable to the number of variables. Can't be used when *A* is sparse or LinearOperator.

Default is 'trf'.

**tol** : float, optional

Tolerance parameter. The algorithm terminates if a relative change of the cost function is less than *tol* on the last iteration. Additionally the first-order optimality measure is considered:

- `method='trf'` terminates if the uniform norm of the gradient, scaled to account for the presence of the bounds, is less than *tol*.
- `method='bvls'` terminates if Karush-Kuhn-Tucker conditions are satisfied within *tol* tolerance.

**lsq\_solver** : {None, 'exact', 'lsmr'}, optional

Method of solving unbounded least-squares problems throughout iterations:

- 'exact' : Use dense QR or SVD decomposition approach. Can't be used when *A* is sparse or LinearOperator.
- 'lsmr' : Use `scipy.sparse.linalg.lsmr` iterative procedure which requires only matrix-vector product evaluations. Can't be used with `method='bvls'`.

If None (default) the solver is chosen based on type of *A*.

**lsmr\_tol** : None, float or 'auto', optional

Tolerance parameters 'atol' and 'btol' for `scipy.sparse.linalg.lsmr`. If None (default), it is set to  $1e-2 * tol$ . If 'auto', the tolerance will be adjusted based on the optimality of the current iterate, which can speed up the optimization process, but is not always reliable.

**max\_iter** : None or int, optional

Maximum number of iterations before termination. If None (default), it is set to 100 for `method='trf'` or to the number of variables for `method='bvls'` (not counting iterations for 'bvls' initialization).

**verbose** : {0, 1, 2}, optional

Level of algorithm's verbosity:

- 0 : work silently (default).
- 1 : display a termination report.
- 2 : display progress during iterations.

### Returns

OptimizeResult with the following fields defined:

**x** : ndarray, shape (n,)

Solution found.

**cost** : float

Value of the cost function at the solution.

**fun** : ndarray, shape (m,)

Vector of residuals at the solution.

**optimality** : float

First-order optimality measure. The exact meaning depends on *method*, refer to the description of *tol* parameter.

**active\_mask** : ndarray of int, shape (n,)

Each component shows whether a corresponding constraint is active (that is, whether a variable is at the bound):

- 0 : a constraint is not active.
- 1 : a lower bound is active.
- 1 : an upper bound is active.

Might be somewhat arbitrary for the *trf* method as it generates a sequence of strictly feasible iterates and *active\_mask* is determined within a tolerance threshold.

**nit** : int

Number of iterations. Zero if the unconstrained solution is optimal.

**status** : int

Reason for algorithm termination:

- 1 : the algorithm was not able to make progress on the last iteration.
- 0 : the maximum number of iterations is exceeded.
- 1 : the first-order optimality measure is less than *tol*.
- 2 : the relative change of the cost function is less than *tol*.
- 3 : the unconstrained solution is optimal.

**message** : str

Verbal description of the termination reason.

**success** : bool

True if one of the convergence criteria is satisfied (*status* > 0).

**See also:**

[\*nnls\*](#) Linear least squares with non-negativity constraint.

[\*least\\_squares\*](#)  
Nonlinear least squares with bounds on the variables.

### Notes

The algorithm first computes the unconstrained least-squares solution by `numpy.linalg.lstsq` or `scipy.sparse.linalg.lsqr` depending on *lsq\_solver*. This solution is returned as optimal if it lies within the bounds.

Method ‘trf’ runs the adaptation of the algorithm described in [STIR] for a linear least-squares problem. The iterations are essentially the same as in the nonlinear least-squares algorithm, but as the quadratic function model is always accurate, we don’t need to track or modify the radius of a trust region. The line search (backtracking) is used as a safety net when a selected step does not decrease the cost function. Read more detailed description of the algorithm in `scipy.optimize.least_squares`.

Method ‘bvls’ runs a Python implementation of the algorithm described in [BVLS]. The algorithm maintains active and free sets of variables, on each iteration chooses a new variable to move from the active set to the free set and then solves the unconstrained least-squares problem on free variables. This algorithm is guaranteed to give an accurate solution eventually, but may require up to n iterations for a problem with n variables. Additionally, an ad-hoc initialization procedure is implemented, that determines which variables to set free or active initially. It takes some number of iterations before actual BVLS starts, but can significantly reduce the number of further iterations.

### References

[STIR], [BVLS]

### Examples

In this example a problem with a large sparse matrix and bounds on the variables is solved.

```

>>> from scipy.sparse import rand
>>> from scipy.optimize import lsq_linear
...
>>> np.random.seed(0)
...
>>> m = 20000
>>> n = 10000
...
>>> A = rand(m, n, density=1e-4)
>>> b = np.random.randn(m)
...
>>> lb = np.random.randn(n)
>>> ub = lb + 1
...
>>> res = lsq_linear(A, b, bounds=(lb, ub), lsqr_tol='auto', verbose=1)
# may vary
The relative change of the cost function is less than `tol`.
Number of iterations 16, initial cost 1.5039e+04, final cost 1.1112e+04,
first-order optimality 4.66e-08.

```

## Global Optimization

<code>basinhopping(func, x0[, niter, T, stepsize, ...])</code>	Find the global minimum of a function using the basin-hopping algorithm
<code>brute(func, ranges[, args, Ns, full_output, ...])</code>	Minimize a function over a given range by brute force.
<code>differential_evolution(func, bounds[, args, ...])</code>	Finds the global minimum of a multivariate function.

`scipy.optimize.basinhopping` (*func*, *x0*, *niter=100*, *T=1.0*, *stepsize=0.5*, *minimizer\_kwargs=None*, *take\_step=None*, *accept\_test=None*, *callback=None*, *interval=50*, *disp=False*, *niter\_success=None*)

Find the global minimum of a function using the basin-hopping algorithm

**Parameters**

- func** : callable  $f(x, *args)$   
Function to be optimized. *args* can be passed as an optional item in the dict `minimizer_kwargs`
- x0** : ndarray  
Initial guess.
- niter** : integer, optional  
The number of basin hopping iterations
- T** : float, optional  
The “temperature” parameter for the accept or reject criterion. Higher “temperatures” mean that larger jumps in function value will be accepted. For best results *T* should be comparable to the separation (in function value) between local minima.
- stepsize** : float, optional  
initial step size for use in the random displacement.
- minimizer\_kwargs** : dict, optional  
Extra keyword arguments to be passed to the minimizer `scipy.optimize.minimize()` Some important options could be:
  - method** [str] The minimization method (e.g. "L-BFGS-B")
  - args** [tuple] Extra arguments passed to the objective function (*func*) and its derivatives (Jacobian, Hessian).
- take\_step** : callable `take_step(x)`, optional  
Replace the default step taking routine with this routine. The default step taking routine is a random displacement of the coordinates, but other step taking algorithms

may be better for some systems. `take_step` can optionally have the attribute `take_step.stepsize`. If this attribute exists, then `basinhopping` will adjust `take_step.stepsize` in order to try to optimize the global minimum search.

**accept\_test** : callable, `accept_test(f_new=f_new, x_new=x_new, f_old=fold, x_old=x_old)`, optional

Define a test which will be used to judge whether or not to accept the step. This will be used in addition to the Metropolis test based on “temperature” `T`. The acceptable return values are `True`, `False`, or `"force accept"`. If any of the tests return `False` then the step is rejected. If the latter, then this will override any other tests in order to accept the step. This can be used, for example, to forcefully escape from a local minimum that `basinhopping` is trapped in.

**callback** : callable, `callback(x, f, accept)`, optional

A callback function which will be called for all minima found. `x` and `f` are the coordinates and function value of the trial minimum, and `accept` is whether or not that minimum was accepted. This can be used, for example, to save the lowest `N` minima found. Also, `callback` can be used to specify a user defined stop criterion by optionally returning `True` to stop the `basinhopping` routine.

**interval** : integer, optional

interval for how often to update the `stepsize`

**disp** : bool, optional

Set to `True` to print status messages

**niter\_success** : integer, optional

Stop the run if the global minimum candidate remains the same for this number of iterations.

#### Returns

**res** : `OptimizeResult`

The optimization result represented as a `OptimizeResult` object. Important attributes are: `x` the solution array, `fun` the value of the function at the solution, and `message` which describes the cause of the termination. The `OptimizeResult` object returned by the selected minimizer at the lowest minimum is also contained within this object and can be accessed through the `lowest_optimization_result` attribute. See `OptimizeResult` for a description of other attributes.

#### See also:

**`minimize`** The local minimization function called once for each `basinhopping` step. `minimizer_kwargs` is passed to this routine.

#### Notes

Basin-hopping is a stochastic algorithm which attempts to find the global minimum of a smooth scalar function of one or more variables [R144] [R145] [R146] [R147]. The algorithm in its current form was described by David Wales and Jonathan Doye [R145] <http://www-wales.ch.cam.ac.uk/>.

The algorithm is iterative with each cycle composed of the following features

- 1.random perturbation of the coordinates
- 2.local minimization
- 3.accept or reject the new coordinates based on the minimized function value

The acceptance test used here is the Metropolis criterion of standard Monte Carlo algorithms, although there are many other possibilities [R146].

This global minimization method has been shown to be extremely efficient for a wide variety of problems in physics and chemistry. It is particularly useful when the function has many minima separated by large barriers.



See the Cambridge Cluster Database <http://www-wales.ch.cam.ac.uk/CCD.html> for databases of molecular systems that have been optimized primarily using basin-hopping. This database includes minimization problems exceeding 300 degrees of freedom.

See the free software program GMIN (<http://www-wales.ch.cam.ac.uk/GMIN>) for a Fortran implementation of basin-hopping. This implementation has many different variations of the procedure described above, including more advanced step taking algorithms and alternate acceptance criterion.

For stochastic global optimization there is no way to determine if the true global minimum has actually been found. Instead, as a consistency check, the algorithm can be run from a number of different random starting points to ensure the lowest minimum found in each example has converged to the global minimum. For this reason `basinhopping` will by default simply run for the number of iterations `niter` and return the lowest minimum found. It is left to the user to ensure that this is in fact the global minimum.

Choosing `stepsize`: This is a crucial parameter in `basinhopping` and depends on the problem being solved. Ideally it should be comparable to the typical separation between local minima of the function being optimized. `basinhopping` will, by default, adjust `stepsize` to find an optimal value, but this may take many iterations. You will get quicker results if you set a sensible value for `stepsize`.

Choosing `T`: The parameter `T` is the temperature used in the metropolis criterion. Basinhopping steps are accepted with probability 1 if `func(xnew) < func(xold)`, or otherwise with probability:

$$\exp(-(\text{func}(x_{\text{new}}) - \text{func}(x_{\text{old}})) / T)$$

So, for best results, `T` should be comparable to the typical difference in function values between local minima.

New in version 0.12.0.

## References

[R144], [R145], [R146], [R147]

## Examples

The following example is a one-dimensional minimization problem, with many local minima superimposed on a parabola.

```
>>> from scipy.optimize import basinhopping
>>> func = lambda x: np.cos(14.5 * x - 0.3) + (x + 0.2) * x
>>> x0=[1.]
```

`Basinhopping`, internally, uses a local minimization algorithm. We will use the parameter `minimizer_kwargs` to tell `basinhopping` which algorithm to use and how to set up that minimizer. This parameter will be passed to `scipy.optimize.minimize()`.

```
>>> minimizer_kwargs = {"method": "BFGS"}
>>> ret = basinhopping(func, x0, minimizer_kwargs=minimizer_kwargs,
...                   niter=200)
>>> print("global minimum: x = %.4f, f(x0) = %.4f" % (ret.x, ret.fun))
global minimum: x = -0.1951, f(x0) = -1.0009
```

Next consider a two-dimensional minimization problem. Also, this time we will use gradient information to significantly speed up the search.

```
>>> def func2d(x):
...     f = np.cos(14.5 * x[0] - 0.3) + (x[1] + 0.2) * x[1] + (x[0] +
...                                                         0.2) * x[0]
...     df = np.zeros(2)
...     df[0] = -14.5 * np.sin(14.5 * x[0] - 0.3) + 2. * x[0] + 0.2
...     df[1] = 2. * x[1] + 0.2
...     return f, df
```

We'll also use a different local minimization algorithm. Also we must tell the minimizer that our function returns both energy and gradient (jacobian)

```
>>> minimizer_kwargs = {"method": "L-BFGS-B", "jac": True}
>>> x0 = [1.0, 1.0]
>>> ret = basinhopping(func2d, x0, minimizer_kwargs=minimizer_kwargs,
...                   niter=200)
>>> print("global minimum: x = [%.4f, %.4f], f(x0) = %.4f" % (ret.x[0],
...                                                         ret.x[1],
...                                                         ret.fun))
global minimum: x = [-0.1951, -0.1000], f(x0) = -1.0109
```

Here is an example using a custom step taking routine. Imagine you want the first coordinate to take larger steps than the rest of the coordinates. This can be implemented like so:

```
>>> class MyTakeStep(object):
...     def __init__(self, stepsize=0.5):
...         self.stepsize = stepsize
...     def __call__(self, x):
...         s = self.stepsize
...         x[0] += np.random.uniform(-2.*s, 2.*s)
...         x[1:] += np.random.uniform(-s, s, x[1:].shape)
...         return x
```

Since `MyTakeStep.stepsize` exists `basinhopping` will adjust the magnitude of `stepsize` to optimize the search. We'll use the same 2-D function as before

```
>>> mytakestep = MyTakeStep()
>>> ret = basinhopping(func2d, x0, minimizer_kwargs=minimizer_kwargs,
...                   niter=200, take_step=mytakestep)
>>> print("global minimum: x = [%.4f, %.4f], f(x0) = %.4f" % (ret.x[0],
...                                                         ret.x[1],
...                                                         ret.fun))
global minimum: x = [-0.1951, -0.1000], f(x0) = -1.0109
```

Now let's do an example using a custom callback function which prints the value of every minimum found

```
>>> def print_fun(x, f, accepted):
...     print("at minimum %.4f accepted %d" % (f, int(accepted)))
```

We'll run it for only 10 `basinhopping` steps this time.

```
>>> np.random.seed(1)
>>> ret = basinhopping(func2d, x0, minimizer_kwargs=minimizer_kwargs,
...                   niter=10, callback=print_fun)
at minimum 0.4159 accepted 1
at minimum -0.9073 accepted 1
at minimum -0.1021 accepted 1
at minimum -0.1021 accepted 1
at minimum 0.9102 accepted 1
at minimum 0.9102 accepted 1
at minimum 2.2945 accepted 0
at minimum -0.1021 accepted 1
at minimum -1.0109 accepted 1
at minimum -1.0109 accepted 1
```

The minimum at `-1.0109` is actually the global minimum, found already on the 8th iteration.

Now let's implement bounds on the problem using a custom `accept_test`:

```

>>> class MyBounds(object):
...     def __init__(self, xmax=[1.1,1.1], xmin=[-1.1,-1.1]):
...         self.xmax = np.array(xmax)
...         self.xmin = np.array(xmin)
...     def __call__(self, **kwargs):
...         x = kwargs["x_new"]
...         tmax = bool(np.all(x <= self.xmax))
...         tmin = bool(np.all(x >= self.xmin))
...         return tmax and tmin

>>> mybounds = MyBounds()
>>> ret = basinhopping(func2d, x0, minimizer_kwargs=minimizer_kwargs,
...                   niter=10, accept_test=mybounds)

```

`scipy.optimize.brute` (*func*, *ranges*, *args*=(), *Ns*=20, *full\_output*=0, *finish*=<function *fmin* at 0x2b9098a1fe60>, *disp*=False)

Minimize a function over a given range by brute force.

Uses the “brute force” method, i.e. computes the function’s value at each point of a multidimensional grid of points, to find the global minimum of the function.

The function is evaluated everywhere in the range with the datatype of the first call to the function, as enforced by the `vectorize` NumPy function. The value and type of the function evaluation returned when `full_output=True` are affected in addition by the `finish` argument (see Notes).

**Parameters** **func** : callable

The objective function to be minimized. Must be in the form `f(x, *args)`, where `x` is the argument in the form of a 1-D array and `args` is a tuple of any additional fixed parameters needed to completely specify the function.

**ranges** : tuple

Each component of the *ranges* tuple must be either a “slice object” or a range tuple of the form `(low, high)`. The program uses these to create the grid of points on which the objective function will be computed. See *Note 2* for more detail.

**args** : tuple, optional

Any additional fixed parameters needed to completely specify the function.

**Ns** : int, optional

Number of grid points along the axes, if not otherwise specified. See *Note 2*.

**full\_output** : bool, optional

If True, return the evaluation grid and the objective function’s values on it.

**finish** : callable, optional

An optimization function that is called with the result of brute force minimization as initial guess. *finish* should take *func* and the initial guess as positional arguments, and take *args* as keyword arguments. It may additionally take *full\_output* and/or *disp* as keyword arguments. Use None if no “polishing” function is to be used. See Notes for more details.

**disp** : bool, optional

Set to True to print convergence messages.

**Returns**

**x0** : ndarray

A 1-D array containing the coordinates of a point at which the objective function had its minimum value. (See *Note 1* for which point is returned.)

**fval** : float

Function value at the point *x0*. (Returned when *full\_output* is True.)

**grid** : tuple

Representation of the evaluation grid. It has the same length as *x0*. (Returned when *full\_output* is True.)

**Jout** : ndarray

Function values at each point of the evaluation grid, *i.e.*, `Jout = func(*grid)`.  
(Returned when `full_output` is `True`.)

**See also:**

`basinhopping`, `differential_evolution`

**Notes**

*Note 1:* The program finds the gridpoint at which the lowest value of the objective function occurs. If `finish` is `None`, that is the point returned. When the global minimum occurs within (or not very far outside) the grid's boundaries, and the grid is fine enough, that point will be in the neighborhood of the global minimum.

However, users often employ some other optimization program to “polish” the gridpoint values, *i.e.*, to seek a more precise (local) minimum near `brute`'s best gridpoint. The `brute` function's `finish` option provides a convenient way to do that. Any polishing program used must take `brute`'s output as its initial guess as a positional argument, and take `brute`'s input values for `args` as keyword arguments, otherwise an error will be raised. It may additionally take `full_output` and/or `disp` as keyword arguments.

`brute` assumes that the `finish` function returns either an `OptimizeResult` object or a tuple in the form: `(xmin, Jmin, ..., statuscode)`, where `xmin` is the minimizing value of the argument, `Jmin` is the minimum value of the objective function, “...” may be some other returned values (which are not used by `brute`), and `statuscode` is the status code of the `finish` program.

Note that when `finish` is not `None`, the values returned are those of the `finish` program, *not* the gridpoint ones. Consequently, while `brute` confines its search to the input grid points, the `finish` program's results usually will not coincide with any gridpoint, and may fall outside the grid's boundary. Thus, if a minimum only needs to be found over the provided grid points, make sure to pass in `finish=None`.

*Note 2:* The grid of points is a `numpy.mgrid` object. For `brute` the `ranges` and `Ns` inputs have the following effect. Each component of the `ranges` tuple can be either a slice object or a two-tuple giving a range of values, such as `(0, 5)`. If the component is a slice object, `brute` uses it directly. If the component is a two-tuple range, `brute` internally converts it to a slice object that interpolates `Ns` points from its low-value to its high-value, inclusive.

**Examples**

We illustrate the use of `brute` to seek the global minimum of a function of two variables that is given as the sum of a positive-definite quadratic and two deep “Gaussian-shaped” craters. Specifically, define the objective function `f` as the sum of three other functions, `f = f1 + f2 + f3`. We suppose each of these has a signature `(z, *params)`, where `z = (x, y)`, and `params` and the functions are as defined below.

```
>>> params = (2, 3, 7, 8, 9, 10, 44, -1, 2, 26, 1, -2, 0.5)
>>> def f1(z, *params):
...     x, y = z
...     a, b, c, d, e, f, g, h, i, j, k, l, scale = params
...     return (a * x**2 + b * x * y + c * y**2 + d*x + e*y + f)

>>> def f2(z, *params):
...     x, y = z
...     a, b, c, d, e, f, g, h, i, j, k, l, scale = params
...     return (-g*np.exp(-((x-h)**2 + (y-i)**2) / scale))

>>> def f3(z, *params):
...     x, y = z
...     a, b, c, d, e, f, g, h, i, j, k, l, scale = params
...     return (-j*np.exp(-((x-k)**2 + (y-l)**2) / scale))
```

```
>>> def f(z, *params):
...     return f1(z, *params) + f2(z, *params) + f3(z, *params)
```

Thus, the objective function may have local minima near the minimum of each of the three functions of which it is composed. To use `fmin` to polish its gridpoint result, we may then continue as follows:

```
>>> rranges = (slice(-4, 4, 0.25), slice(-4, 4, 0.25))
>>> from scipy import optimize
>>> resbrute = optimize.brute(f, rranges, args=params, full_output=True,
...                          finish=optimize.fmin)
>>> resbrute[0] # global minimum
array([-1.05665192,  1.80834843])
>>> resbrute[1] # function value at global minimum
-3.4085818767
```

Note that if `finish` had been set to `None`, we would have gotten the gridpoint `[-1.0 1.75]` where the rounded function value is `-2.892`.

```
scipy.optimize.differential_evolution(func, bounds, args=(), strategy='best1bin', max-
                                     iter=1000, popsize=15, tol=0.01, mutation=(0.5,
                                     1), recombination=0.7, seed=None, callback=None,
                                     disp=False, polish=True, init='latinhypercube')
```

Finds the global minimum of a multivariate function. Differential Evolution is stochastic in nature (does not use gradient methods) to find the minimum, and can search large areas of candidate space, but often requires larger numbers of function evaluations than conventional gradient based techniques.

The algorithm is due to Storn and Price [R150].

**Parameters** `func` : callable

The objective function to be minimized. Must be in the form `f(x, *args)`, where `x` is the argument in the form of a 1-D array and `args` is a tuple of any additional fixed parameters needed to completely specify the function.

**bounds** : sequence

Bounds for variables. `(min, max)` pairs for each element in `x`, defining the lower and upper bounds for the optimizing argument of `func`. It is required to have `len(bounds) == len(x)`. `len(bounds)` is used to determine the number of parameters in `x`.

**args** : tuple, optional

Any additional fixed parameters needed to completely specify the objective function.

**strategy** : str, optional

The differential evolution strategy to use. Should be one of:

- 'best1bin'
- 'best1exp'
- 'rand1exp'
- 'randtobest1exp'
- 'best2exp'
- 'rand2exp'
- 'randtobest1bin'
- 'best2bin'
- 'rand2bin'
- 'rand1bin'

The default is 'best1bin'.

**maxiter** : int, optional

The maximum number of generations over which the entire population is evolved. The maximum number of function evaluations (with no polishing) is: `(maxiter + 1) * popsize * len(x)`

**popsize** : int, optional

A multiplier for setting the total population size. The population has `popsize * len(x)` individuals.

**tol** : float, optional

When the mean of the population energies, multiplied by `tol`, divided by the standard deviation of the population energies is greater than 1 the solving process terminates:  $\text{convergence} = \text{mean}(\text{pop}) * \text{tol} / \text{stdev}(\text{pop}) > 1$

**mutation** : float or tuple(float, float), optional

The mutation constant. In the literature this is also known as differential weight, being denoted by  $F$ . If specified as a float it should be in the range  $[0, 2]$ . If specified as a tuple  $(\text{min}, \text{max})$  dithering is employed. Dithering randomly changes the mutation constant on a generation by generation basis. The mutation constant for that generation is taken from  $U[\text{min}, \text{max})$ . Dithering can help speed convergence significantly. Increasing the mutation constant increases the search radius, but will slow down convergence.

**recombination** : float, optional

The recombination constant, should be in the range  $[0, 1]$ . In the literature this is also known as the crossover probability, being denoted by  $CR$ . Increasing this value allows a larger number of mutants to progress into the next generation, but at the risk of population stability.

**seed** : int or `np.random.RandomState`, optional

If `seed` is not specified the `np.RandomState` singleton is used. If `seed` is an int, a new `np.random.RandomState` instance is used, seeded with `seed`. If `seed` is already a `np.random.RandomState` instance, then that `np.random.RandomState` instance is used. Specify `seed` for repeatable minimizations.

**disp** : bool, optional

Display status messages

**callback** : callable, `callback(xk, convergence=val)`, optional

A function to follow the progress of the minimization. `xk` is the current value of `x0`. `val` represents the fractional value of the population convergence. When `val` is greater than one the function halts. If `callback` returns `True`, then the minimization is halted (any polishing is still carried out).

**polish** : bool, optional

If `True` (default), then `scipy.optimize.minimize` with the *L-BFGS-B* method is used to polish the best population member at the end, which can improve the minimization slightly.

**init** : string, optional

Specify how the population initialization is performed. Should be one of:

- 'latinhypercube'
- 'random'

The default is 'latinhypercube'. Latin Hypercube sampling tries to maximize coverage of the available parameter space. 'random' initializes the population randomly - this has the drawback that clustering can occur, preventing the whole of parameter space being covered.

### Returns

**res** : `OptimizeResult`

The optimization result represented as a `OptimizeResult` object. Important attributes are: `x` the solution array, `success` a Boolean flag indicating if the optimizer exited successfully and `message` which describes the cause of the termination. See `OptimizeResult` for a description of other attributes. If `polish` was employed, and a lower minimum was obtained by the polishing, then `OptimizeResult` also contains the `jac` attribute.

### Notes

Differential evolution is a stochastic population based method that is useful for global optimization problems. At each pass through the population the algorithm mutates each candidate solution by mixing with other candidate solutions to create a trial candidate. There are several strategies [R151] for creating trial candidates, which suit some problems more than others. The ‘best1bin’ strategy is a good starting point for many systems. In this strategy two members of the population are randomly chosen. Their difference is used to mutate the best member (the *best* in *best1bin*),  $b_0$ , so far:

$$b' = b_0 + mutation * (population[rand0] - population[rand1])$$

A trial vector is then constructed. Starting with a randomly chosen ‘i’th parameter the trial is sequentially filled (in modulo) with parameters from  $b'$  or the original candidate. The choice of whether to use  $b'$  or the original candidate is made with a binomial distribution (the ‘bin’ in ‘best1bin’) - a random number in  $[0, 1)$  is generated. If this number is less than the *recombination* constant then the parameter is loaded from  $b'$ , otherwise it is loaded from the original candidate. The final parameter is always loaded from  $b'$ . Once the trial candidate is built its fitness is assessed. If the trial is better than the original candidate then it takes its place. If it is also better than the best overall candidate it also replaces that. To improve your chances of finding a global minimum use higher *popsize* values, with higher *mutation* and (dithering), but lower *recombination* values. This has the effect of widening the search radius, but slowing convergence.

New in version 0.15.0.

### References

[R150], [R151], [R152]

### Examples

Let us consider the problem of minimizing the Rosenbrock function. This function is implemented in `rosen` in `scipy.optimize`.

```
>>> from scipy.optimize import rosen, differential_evolution
>>> bounds = [(0,2), (0, 2), (0, 2), (0, 2), (0, 2)]
>>> result = differential_evolution(rosen, bounds)
>>> result.x, result.fun
(array([1., 1., 1., 1., 1.]), 1.9216496320061384e-19)
```

Next find the minimum of the Ackley function ([http://en.wikipedia.org/wiki/Test\\_functions\\_for\\_optimization](http://en.wikipedia.org/wiki/Test_functions_for_optimization)).

```
>>> from scipy.optimize import differential_evolution
>>> import numpy as np
>>> def ackley(x):
...     arg1 = -0.2 * np.sqrt(0.5 * (x[0] ** 2 + x[1] ** 2))
...     arg2 = 0.5 * (np.cos(2. * np.pi * x[0]) + np.cos(2. * np.pi * x[1]))
...     return -20. * np.exp(arg1) - np.exp(arg2) + 20. + np.e
>>> bounds = [(-5, 5), (-5, 5)]
>>> result = differential_evolution(ackley, bounds)
>>> result.x, result.fun
(array([ 0.,  0.]), 4.4408920985006262e-16)
```

## Rosenbrock function

<code>rosen(x)</code>	The Rosenbrock function.
<code>rosen_der(x)</code>	The derivative (i.e.
<code>rosen_hess(x)</code>	The Hessian matrix of the Rosenbrock function.
<code>rosen_hess_prod(x, p)</code>	Product of the Hessian matrix of the Rosenbrock function with a vector.

`scipy.optimize.rosen(x)`

The Rosenbrock function.

The function computed is:

```
sum(100.0*(x[1:] - x[:-1])**2.0)**2.0 + (1 - x[:-1])**2.0)
```

**Parameters** `x`: array\_like  
 1-D array of points at which the Rosenbrock function is to be computed.

**Returns** `f`: float  
 The value of the Rosenbrock function.

**See also:**

`rosen_der`, `rosen_hess`, `rosen_hess_prod`

`scipy.optimize.rosen_der(x)`

The derivative (i.e. gradient) of the Rosenbrock function.

**Parameters** `x`: array\_like  
 1-D array of points at which the derivative is to be computed.

**Returns** `rosen_der`: (N,) ndarray  
 The gradient of the Rosenbrock function at  $x$ .

**See also:**

`rosen`, `rosen_hess`, `rosen_hess_prod`

`scipy.optimize.rosen_hess(x)`

The Hessian matrix of the Rosenbrock function.

**Parameters** `x`: array\_like  
 1-D array of points at which the Hessian matrix is to be computed.

**Returns** `rosen_hess`: ndarray  
 The Hessian matrix of the Rosenbrock function at  $x$ .

**See also:**

`rosen`, `rosen_der`, `rosen_hess_prod`

`scipy.optimize.rosen_hess_prod(x, p)`

Product of the Hessian matrix of the Rosenbrock function with a vector.

**Parameters** `x`: array\_like  
 1-D array of points at which the Hessian matrix is to be computed.

`p`: array\_like  
 1-D array, the vector to be multiplied by the Hessian matrix.

**Returns** `rosen_hess_prod`: ndarray  
 The Hessian matrix of the Rosenbrock function at  $x$  multiplied by the vector  $p$ .

**See also:**

`rosen`, `rosen_der`, `rosen_hess`

## 5.18.2 Fitting

---

`curve_fit(f, xdata, ydata[, p0, sigma, ...])` Use non-linear least squares to fit a function,  $f$ , to data.

---



```
scipy.optimize.curve_fit(f, xdata, ydata, p0=None, sigma=None, absolute_sigma=False,
                        check_finite=True, bounds=(-inf, inf), method=None, jac=None,
                        **kwargs)
```

Use non-linear least squares to fit a function, *f*, to data.

Assumes  $ydata = f(xdata, *params) + \epsilon$

**Parameters** **f**: callable

The model function,  $f(x, \dots)$ . It must take the independent variable as the first argument and the parameters to fit as separate remaining arguments.

**xdata**: An M-length sequence or an (k,M)-shaped array

for functions with *k* predictors. The independent variable where the data is measured.

**ydata**: M-length sequence

The dependent data — nominally  $f(xdata, \dots)$

**p0**: None, scalar, or N-length sequence, optional

Initial guess for the parameters. If None, then the initial values will all be 1 (if the number of parameters for the function can be determined using introspection, otherwise a `ValueError` is raised).

**sigma**: None or M-length sequence, optional

If not None, the uncertainties in the *ydata* array. These are used as weights in the least-squares problem i.e. minimising  $\text{np.sum}((f(xdata, *popt) - ydata) / \text{sigma})^{**2}$  ) If None, the uncertainties are assumed to be 1.

**absolute\_sigma**: bool, optional

If False, *sigma* denotes relative weights of the data points. The returned covariance matrix *pcov* is based on *estimated* errors in the data, and is not affected by the overall magnitude of the values in *sigma*. Only the relative magnitudes of the *sigma* values matter.

If True, *sigma* describes one standard deviation errors of the input data points. The estimated covariance in *pcov* is based on these values.

**check\_finite**: bool, optional

If True, check that the input arrays do not contain nans or infs, and raise a `ValueError` if they do. Setting this parameter to False may silently produce nonsensical results if the input arrays do contain nans. Default is True.

**bounds**: 2-tuple of array\_like, optional

Lower and upper bounds on independent variables. Defaults to no bounds. Each element of the tuple must be either an array with the length equal to the number of parameters, or a scalar (in which case the bound is taken to be the same for all parameters.) Use `np.inf` with an appropriate sign to disable bounds on all or some parameters.

New in version 0.17.

**method**: {'lm', 'trf', 'dogbox'}, optional

Method to use for optimization. See `least_squares` for more details. Default is 'lm' for unconstrained problems and 'trf' if *bounds* are provided. The method 'lm' won't work when the number of observations is less than the number of variables, use 'trf' or 'dogbox' in this case.

New in version 0.17.

**jac**: callable, string or None, optional

Function with signature `jac(x, ...)` which computes the Jacobian matrix of the model function with respect to parameters as a dense array\_like structure. It will be scaled according to provided *sigma*. If None (default), the Jacobian will be estimated numerically. String keywords for 'trf' and 'dogbox' methods can be used to select a finite difference scheme, see `least_squares`.

New in version 0.18.

**kwargs**

	<p>Keyword arguments passed to <code>leastsq</code> for <code>method='lm'</code> or <code>least_squares</code> otherwise.</p>
<b>Returns</b>	<p><b>popt</b> : array                  Optimal values for the parameters so that the sum of the squared error of <math>f(xdata, *popt) - ydata</math> is minimized</p> <p><b>pcov</b> : 2d array                  The estimated covariance of <code>popt</code>. The diagonals provide the variance of the parameter estimate. To compute one standard deviation errors on the parameters use <code>perr = np.sqrt(np.diag(pcov))</code>.                  How the <i>sigma</i> parameter affects the estimated covariance depends on <i>absolute_sigma</i> argument, as described above.                  If the Jacobian matrix at the solution doesn't have a full rank, then 'lm' method returns a matrix filled with <code>np.inf</code>, on the other hand 'trf' and 'dogbox' methods use Moore-Penrose pseudoinverse to compute the covariance matrix.</p>
<b>Raises</b>	<p><b>ValueError</b>                  if either <i>ydata</i> or <i>xdata</i> contain NaNs, or if incompatible options are used.</p> <p><b>RuntimeError</b>                  if the least-squares minimization fails.</p> <p><b>OptimizeWarning</b>                  if covariance of the parameters can not be estimated.</p>

See also:

#### `least_squares`

Minimize the sum of squares of nonlinear functions.

#### `stats.linregress`

Calculate a linear least squares regression for two sets of measurements.

#### Notes

With `method='lm'`, the algorithm uses the Levenberg-Marquardt algorithm through `leastsq`. Note that this algorithm can only deal with unconstrained problems.

Box constraints can be handled by methods 'trf' and 'dogbox'. Refer to the docstring of `least_squares` for more information.

#### Examples

```
>>> import numpy as np
>>> from scipy.optimize import curve_fit
>>> def func(x, a, b, c):
...     return a * np.exp(-b * x) + c

>>> xdata = np.linspace(0, 4, 50)
>>> y = func(xdata, 2.5, 1.3, 0.5)
>>> ydata = y + 0.2 * np.random.normal(size=len(xdata))

>>> popt, pcov = curve_fit(func, xdata, ydata)
```

Constrain the optimization to the region of  $0 < a < 3, 0 < b < 2$  and  $0 < c < 1$ :

```
>>> popt, pcov = curve_fit(func, xdata, ydata, bounds=(0, [3., 2., 1.]))
```

### 5.18.3 Root finding

#### Scalar functions

---

<code>brentq(f, a, b[, args, xtol, rtol, maxiter, ...])</code>	Find a root of a function in a bracketing interval using Brent's method.
<code>brenth(f, a, b[, args, xtol, rtol, maxiter, ...])</code>	Find root of $f$ in $[a, b]$ .
<code>ridder(f, a, b[, args, xtol, rtol, maxiter, ...])</code>	Find a root of a function in an interval.
<code>bisect(f, a, b[, args, xtol, rtol, maxiter, ...])</code>	Find root of a function within an interval.
<code>newton(func, x0[, fprime, args, tol, ...])</code>	Find a zero using the Newton-Raphson or secant method.

---

`scipy.optimize.brentq(f, a, b, args=(), xtol=2e-12, rtol=8.8817841970012523e-16, maxiter=100, full_output=False, disp=True)`

Find a root of a function in a bracketing interval using Brent's method.

Uses the classic Brent's method to find a zero of the function  $f$  on the sign changing interval  $[a, b]$ . Generally considered the best of the rootfinding routines here. It is a safe version of the secant method that uses inverse quadratic extrapolation. Brent's method combines root bracketing, interval bisection, and inverse quadratic interpolation. It is sometimes known as the van Wijngaarden-Dekker-Brent method. Brent (1973) claims convergence is guaranteed for functions computable within  $[a, b]$ .

[Brent1973] provides the classic description of the algorithm. Another description can be found in a recent edition of Numerical Recipes, including [PressEtal1992]. Another description is at <http://mathworld.wolfram.com/BrentsMethod.html>. It should be easy to understand the algorithm just by reading our code. Our code diverges a bit from standard presentations: we choose a different formula for the extrapolation step.

**Parameters** **f** : function

Python function returning a number. The function  $f$  must be continuous, and  $f(a)$  and  $f(b)$  must have opposite signs.

**a** : number

One end of the bracketing interval  $[a, b]$ .

**b** : number

The other end of the bracketing interval  $[a, b]$ .

**xtol** : number, optional

The computed root  $x_0$  will satisfy `np.allclose(x, x0, atol=xtol, rtol=rtol)`, where  $x$  is the exact root. The parameter must be nonnegative. For nice functions, Brent's method will often satisfy the above condition will  $xtol/2$  and  $rtol/2$ . [Brent1973]

**rtol** : number, optional

The computed root  $x_0$  will satisfy `np.allclose(x, x0, atol=xtol, rtol=rtol)`, where  $x$  is the exact root. The parameter cannot be smaller than its default value of `4*np.finfo(float).eps`. For nice functions, Brent's method will often satisfy the above condition will  $xtol/2$  and  $rtol/2$ . [Brent1973]

**maxiter** : number, optional

if convergence is not achieved in `maxiter` iterations, an error is raised. Must be  $\geq 0$ .

**args** : tuple, optional

containing extra arguments for the function  $f$ .  $f$  is called by `apply(f, (x)+args)`.

**full\_output** : bool, optional

If `full_output` is `False`, the root is returned. If `full_output` is `True`, the return value is `(x, r)`, where  $x$  is the root, and  $r$  is a `RootResults` object.

**disp** : bool, optional

If `True`, raise `RuntimeError` if the algorithm didn't converge.

**Returns**

**x0** : float

Zero of  $f$  between  $a$  and  $b$ .  
**r** : RootResults (present if `full_output = True`)  
 Object containing information about the convergence. In particular, `r.converged` is True if the routine converged.

See also:

**multivariate**

`fmin`, `fmin_powell`, `fmin_cg`, `fmin_bfgs`, `fmin_ncg`

**nonlinear** `leastsq`

**constrained**

`fmin_l_bfgs_b`, `fmin_tnc`, `fmin_cobyla`

**global** `basinhopping`, `brute`, `differential_evolution`

**local** `fminbound`, `brent`, `golden`, `bracket`

**n-dimensional**

`fsolve`

**one-dimensional**

`brentq`, `brenth`, `ridder`, `bisect`, `newton`

**scalar** `fixed_point`

**Notes**

$f$  must be continuous.  $f(a)$  and  $f(b)$  must have opposite signs.

**References**

[Brent1973], [PressEtal1992]

`scipy.optimize.brenth` ( $f$ ,  $a$ ,  $b$ , `args=()`, `xtol=2e-12`, `rtol=8.8817841970012523e-16`, `maxiter=100`, `full_output=False`, `disp=True`)

Find root of  $f$  in  $[a,b]$ .

A variation on the classic Brent routine to find a zero of the function  $f$  between the arguments  $a$  and  $b$  that uses hyperbolic extrapolation instead of inverse quadratic extrapolation. There was a paper back in the 1980's ...  $f(a)$  and  $f(b)$  cannot have the same signs. Generally on a par with the `brent` routine, but not as heavily tested. It is a safe version of the secant method that uses hyperbolic extrapolation. The version here is by Chuck Harris.

**Parameters** **f** : function

Python function returning a number.  $f$  must be continuous, and  $f(a)$  and  $f(b)$  must have opposite signs.

**a** : number

One end of the bracketing interval  $[a,b]$ .

**b** : number

The other end of the bracketing interval  $[a,b]$ .

**xtol** : number, optional

The computed root  $x_0$  will satisfy `np.allclose(x, x0, atol=xtol, rtol=rtol)`, where  $x$  is the exact root. The parameter must be nonnegative. As with `brentq`, for nice functions the method will often satisfy the above condition will `xtol/2` and `rtol/2`.

**rtol** : number, optional

The computed root  $x_0$  will satisfy `np.allclose(x, x0, atol=xtol, rtol=rtol)`, where  $x$  is the exact root. The parameter cannot be smaller than its default value of `4*np.finfo(float).eps`. As with `brentq`, for nice functions the method will often satisfy the above condition will `xtol/2` and `rtol/2`.

**maxiter** : number, optional  
if convergence is not achieved in maxiter iterations, an error is raised. Must be  $\geq 0$ .

**args** : tuple, optional  
containing extra arguments for the function  $f$ .  $f$  is called by `apply(f, (x)+args)`.

**full\_output** : bool, optional  
If `full_output` is False, the root is returned. If `full_output` is True, the return value is  $(x, r)$ , where  $x$  is the root, and  $r$  is a RootResults object.

**disp** : bool, optional  
If True, raise RuntimeError if the algorithm didn't converge.

**Returns**

**x0** : float  
Zero of  $f$  between  $a$  and  $b$ .

**r** : RootResults (present if `full_output = True`)  
Object containing information about the convergence. In particular, `r.converged` is True if the routine converged.

**See also:**

`fmin`, `fmin_powell`, `fmin_cg`

**leastsq** nonlinear least squares minimizer

`fmin_l_bfgs_b`, `fmin_tnc`, `fmin_cobyla`, `basinhopping`, `differential_evolution`, `brute`, `fminbound`, `brent`, `golden`, `bracket`

**fsolve** n-dimensional root-finding

`brentq`, `brenth`, `ridder`, `bisect`, `newton`

**fixed\_point**

scalar fixed-point finder

`scipy.optimize.ridder` ( $f$ ,  $a$ ,  $b$ ,  $args=()$ ,  $xtol=2e-12$ ,  $rtol=8.8817841970012523e-16$ ,  $maxiter=100$ ,  $full_output=False$ ,  $disp=True$ )

Find a root of a function in an interval.

**Parameters** **f** : function

Python function returning a number.  $f$  must be continuous, and  $f(a)$  and  $f(b)$  must have opposite signs.

**a** : number

One end of the bracketing interval  $[a,b]$ .

**b** : number

The other end of the bracketing interval  $[a,b]$ .

**xtol** : number, optional

The computed root  $x_0$  will satisfy `np.allclose(x, x0, atol=xtol, rtol=rtol)`, where  $x$  is the exact root. The parameter must be nonnegative.

**rtol** : number, optional

The computed root  $x_0$  will satisfy `np.allclose(x, x0, atol=xtol, rtol=rtol)`, where  $x$  is the exact root. The parameter cannot be smaller than its default value of `4*np.finfo(float).eps`.

**maxiter** : number, optional

if convergence is not achieved in maxiter iterations, an error is raised. Must be  $\geq 0$ .

**args** : tuple, optional

containing extra arguments for the function  $f$ .  $f$  is called by `apply(f, (x)+args)`.

**full\_output** : bool, optional

If `full_output` is False, the root is returned. If `full_output` is True, the return value is  $(x, r)$ , where  $x$  is the root, and  $r$  is a RootResults object.

**disp** : bool, optional

**Returns**

- If True, raise RuntimeError if the algorithm didn't converge.
- x0** : float  
Zero of  $f$  between  $a$  and  $b$ .
- r** : RootResults (present if `full_output = True`)  
Object containing information about the convergence. In particular, `r.converged` is True if the routine converged.

**See also:**

`brentq`, `brenth`, `bisect`, `newton`

***fixed\_point***

scalar fixed-point finder

**Notes**

Uses [Ridders1979] method to find a zero of the function  $f$  between the arguments  $a$  and  $b$ . Ridders' method is faster than bisection, but not generally as fast as the Brent routines. [Ridders1979] provides the classic description and source of the algorithm. A description can also be found in any recent edition of Numerical Recipes.

The routine used here diverges slightly from standard presentations in order to be a bit more careful of tolerance.

**References**

[Ridders1979]

`scipy.optimize.bisect` ( $f$ ,  $a$ ,  $b$ ,  $args=()$ ,  $xtol=2e-12$ ,  $rtol=8.8817841970012523e-16$ ,  $maxiter=100$ ,  $full_output=False$ ,  $disp=True$ )

Find root of a function within an interval.

Basic bisection routine to find a zero of the function  $f$  between the arguments  $a$  and  $b$ .  $f(a)$  and  $f(b)$  cannot have the same signs. Slow but sure.

**Parameters**

- f** : function  
Python function returning a number.  $f$  must be continuous, and  $f(a)$  and  $f(b)$  must have opposite signs.
- a** : number  
One end of the bracketing interval  $[a,b]$ .
- b** : number  
The other end of the bracketing interval  $[a,b]$ .
- xtol** : number, optional  
The computed root  $x_0$  will satisfy `np.allclose(x, x0, atol=xtol, rtol=rtol)`, where  $x$  is the exact root. The parameter must be nonnegative.
- rtol** : number, optional  
The computed root  $x_0$  will satisfy `np.allclose(x, x0, atol=xtol, rtol=rtol)`, where  $x$  is the exact root. The parameter cannot be smaller than its default value of `4*np.finfo(float).eps`.
- maxiter** : number, optional  
if convergence is not achieved in  $maxiter$  iterations, an error is raised. Must be  $\geq 0$ .
- args** : tuple, optional  
containing extra arguments for the function  $f$ .  $f$  is called by `apply(f, (x)+args)`.
- full\_output** : bool, optional  
If `full_output` is False, the root is returned. If `full_output` is True, the return value is  $(x, r)$ , where  $x$  is the root, and  $r$  is a *RootResults* object.
- disp** : bool, optional  
If True, raise RuntimeError if the algorithm didn't converge.

**Returns** **x0** : float

Zero of  $f$  between  $a$  and  $b$ .  
**r** : RootResults (present if `full_output = True`)  
 Object containing information about the convergence. In particular, `r.converged` is True if the routine converged.

**See also:**

`brentq`, `brenth`, `bisect`, `newton`

**`fixed_point`**

scalar fixed-point finder

**`fsolve`**

n-dimensional root-finding

`scipy.optimize.newton` (*func*, *x0*, *fprime=None*, *args=()*, *tol=1.48e-08*, *maxiter=50*, *fprime2=None*)

Find a zero using the Newton-Raphson or secant method.

Find a zero of the function *func* given a nearby starting point *x0*. The Newton-Raphson method is used if the derivative *fprime* of *func* is provided, otherwise the secant method is used. If the second order derivative *fprime2* of *func* is provided, parabolic Halley's method is used.

**Parameters**

**func** : function

The function whose zero is wanted. It must be a function of a single variable of the form  $f(x,a,b,c,\dots)$ , where  $a,b,c,\dots$  are extra arguments that can be passed in the *args* parameter.

**x0** : float

An initial estimate of the zero that should be somewhere near the actual zero.

**fprime** : function, optional

The derivative of the function when available and convenient. If it is None (default), then the secant method is used.

**args** : tuple, optional

Extra arguments to be used in the function call.

**tol** : float, optional

The allowable error of the zero value.

**maxiter** : int, optional

Maximum number of iterations.

**fprime2** : function, optional

The second order derivative of the function when available and convenient. If it is None (default), then the normal Newton-Raphson or the secant method is used. If it is given, parabolic Halley's method is used.

**Returns**

**zero** : float

Estimated location where function is zero.

**See also:**

`brentq`, `brenth`, `ridder`, `bisect`

**`fsolve`**

find zeroes in n dimensions.

**Notes**

The convergence rate of the Newton-Raphson method is quadratic, the Halley method is cubic, and the secant method is sub-quadratic. This means that if the function is well behaved the actual error in the estimated zero is approximately the square (cube for Halley) of the requested tolerance up to roundoff error. However, the stopping criterion used here is the step size and there is no guarantee that a zero has been found. Consequently the result should be verified. Safer algorithms are `brentq`, `brenth`, `ridder`, and `bisect`, but they all require that the root first be bracketed in an interval where the function changes sign. The `brentq` algorithm is recommended for general use in one dimensional problems when such an interval has been found.

Fixed point finding:



---

`fixed_point(func, x0[, args, xtol, maxiter, ...])` Find a fixed point of the function.

---

`scipy.optimize.fixed_point` (*func*, *x0*, *args*=(), *xtol*=1e-08, *maxiter*=500, *method*='del2')

Find a fixed point of the function.

Given a function of one or more variables and a starting point, find a fixed-point of the function: i.e. where `func(x0) == x0`.

**Parameters**

- func** : function  
Function to evaluate.
- x0** : array\_like  
Fixed point of function.
- args** : tuple, optional  
Extra arguments to *func*.
- xtol** : float, optional  
Convergence tolerance, defaults to 1e-08.
- maxiter** : int, optional  
Maximum number of iterations, defaults to 500.
- method** : {"del2", "iteration"}, optional  
Method of finding the fixed-point, defaults to "del2" which uses Steffensen's Method with Aitken's  $\text{Del}^2$  convergence acceleration [R153]. The "iteration" method simply iterates the function until convergence is detected, without attempting to accelerate the convergence.

### References

[R153]

### Examples

```
>>> from scipy import optimize
>>> def func(x, c1, c2):
...     return np.sqrt(c1/(x+c2))
>>> c1 = np.array([10, 12.])
>>> c2 = np.array([3, 5.])
>>> optimize.fixed_point(func, [1.2, 1.3], args=(c1,c2))
array([ 1.4920333 ,  1.37228132])
```

## Multidimensional

General nonlinear solvers:

---

<code>root</code> ( <i>fun</i> , <i>x0</i> [, <i>args</i> , <i>method</i> , <i>jac</i> , <i>tol</i> , ...])	Find a root of a vector function.
<code>fsolve</code> ( <i>func</i> , <i>x0</i> [, <i>args</i> , <i>fprime</i> , ...])	Find the roots of a function.
<code>broyden1</code> ( <i>F</i> , <i>xin</i> [, <i>iter</i> , <i>alpha</i> , ...])	Find a root of a function, using Broyden's first Jacobian approximation.
<code>broyden2</code> ( <i>F</i> , <i>xin</i> [, <i>iter</i> , <i>alpha</i> , ...])	Find a root of a function, using Broyden's second Jacobian approximation.

---

`scipy.optimize.root` (*fun*, *x0*, *args*=(), *method*='hybr', *jac*=None, *tol*=None, *callback*=None, *options*=None)

Find a root of a vector function.

**Parameters**

- fun** : callable  
A vector function to find a root of.
- x0** : ndarray

Initial guess.

**args** : tuple, optional  
 Extra arguments passed to the objective function and its Jacobian.

**method** : str, optional  
 Type of solver. Should be one of

- ‘hybr’ ([see here](#))
- ‘lm’ ([see here](#))
- ‘broyden1’ ([see here](#))
- ‘broyden2’ ([see here](#))
- ‘anderson’ ([see here](#))
- ‘linearmixing’ ([see here](#))
- ‘diagbroyden’ ([see here](#))
- ‘excitingmixing’ ([see here](#))
- ‘krylov’ ([see here](#))
- ‘df-sane’ ([see here](#))

**jac** : bool or callable, optional  
 If *jac* is a Boolean and is True, *fun* is assumed to return the value of Jacobian along with the objective function. If False, the Jacobian will be estimated numerically. *jac* can also be a callable returning the Jacobian of *fun*. In this case, it must accept the same arguments as *fun*.

**tol** : float, optional  
 Tolerance for termination. For detailed control, use solver-specific options.

**callback** : function, optional  
 Optional callback function. It is called on every iteration as `callback(x, f)` where *x* is the current solution and *f* the corresponding residual. For all methods but ‘hybr’ and ‘lm’.

**options** : dict, optional  
 A dictionary of solver options. E.g. *xtol* or *maxiter*, see `show_options()` for details.

**Returns** **sol** : OptimizeResult  
 The solution represented as a `OptimizeResult` object. Important attributes are: *x* the solution array, *success* a Boolean flag indicating if the algorithm exited successfully and *message* which describes the cause of the termination. See `OptimizeResult` for a description of other attributes.

**See also:**

[`show\_options`](#)

Additional options accepted by the solvers

**Notes**

This section describes the available solvers that can be selected by the ‘method’ parameter. The default method is *hybr*.

Method *hybr* uses a modification of the Powell hybrid method as implemented in MINPACK [R171].

Method *lm* solves the system of nonlinear equations in a least squares sense using a modification of the Levenberg-Marquardt algorithm as implemented in MINPACK [R171].

Method *df-sane* is a derivative-free spectral method. [R173]

Methods *broyden1*, *broyden2*, *anderson*, *linearmixing*, *diagbroyden*, *excitingmixing*, *krylov* are inexact Newton methods, with backtracking or full line searches [R172]. Each method corresponds to a particular Jacobian approximations. See `nonlin` for details.

- Method *broyden1* uses Broyden’s first Jacobian approximation, it is known as Broyden’s good method.

- Method *broyden2* uses Broyden's second Jacobian approximation, it is known as Broyden's bad method.
- Method *anderson* uses (extended) Anderson mixing.
- Method *Krylov* uses Krylov approximation for inverse Jacobian. It is suitable for large-scale problem.
- Method *diagbroyden* uses diagonal Broyden Jacobian approximation.
- Method *linearmixing* uses a scalar Jacobian approximation.
- Method *excitingmixing* uses a tuned diagonal Jacobian approximation.

**Warning:** The algorithms implemented for methods *diagbroyden*, *linearmixing* and *excitingmixing* may be useful for specific problems, but whether they will work may depend strongly on the problem.

New in version 0.11.0.

### References

[R171], [R172], [R173]

### Examples

The following functions define a system of nonlinear equations and its jacobian.

```
>>> def fun(x):
...     return [x[0] + 0.5 * (x[0] - x[1])**3 - 1.0,
...            0.5 * (x[1] - x[0])**3 + x[1]]

>>> def jac(x):
...     return np.array([[1 + 1.5 * (x[0] - x[1])**2,
...                      -1.5 * (x[0] - x[1])**2],
...                     [-1.5 * (x[1] - x[0])**2,
...                      1 + 1.5 * (x[1] - x[0])**2]])
```

A solution can be obtained as follows.

```
>>> from scipy import optimize
>>> sol = optimize.root(fun, [0, 0], jac=jac, method='hybr')
>>> sol.x
array([ 0.8411639,  0.1588361])
```

`scipy.optimize.fsolve` (*func*, *x0*, *args=()*, *fprime=None*, *full\_output=0*, *col\_deriv=0*, *xtol=1.49012e-08*, *maxfev=0*, *band=None*, *epsfcn=None*, *factor=100*, *diag=None*)

Find the roots of a function.

Return the roots of the (non-linear) equations defined by  $\text{func}(x) = 0$  given a starting estimate.

**Parameters**

- func** : callable  $f(x, *args)$   
A function that takes at least one (possibly vector) argument.
- x0** : ndarray  
The starting estimate for the roots of  $\text{func}(x) = 0$ .
- args** : tuple, optional  
Any extra arguments to *func*.
- fprime** : callable(x), optional  
A function to compute the Jacobian of *func* with derivatives across the rows. By default, the Jacobian will be estimated.
- full\_output** : bool, optional  
If True, return optional outputs.
- col\_deriv** : bool, optional

Specify whether the Jacobian function computes derivatives down the columns (faster, because there is no transpose operation).

**xtol** : float, optional

The calculation will terminate if the relative error between two consecutive iterates is at most *xtol*.

**maxfev** : int, optional

The maximum number of calls to the function. If zero, then  $100 * (N+1)$  is the maximum where *N* is the number of elements in *x0*.

**band** : tuple, optional

If set to a two-sequence containing the number of sub- and super-diagonals within the band of the Jacobi matrix, the Jacobi matrix is considered banded (only for *fprime=None*).

**epsfcn** : float, optional

A suitable step length for the forward-difference approximation of the Jacobian (for *fprime=None*). If *epsfcn* is less than the machine precision, it is assumed that the relative errors in the functions are of the order of the machine precision.

**factor** : float, optional

A parameter determining the initial step bound ( $\text{factor} * || \text{diag} * x ||$ ). Should be in the interval (0.1, 100).

**diag** : sequence, optional

*N* positive entries that serve as a scale factors for the variables.

#### Returns

**x** : ndarray

The solution (or the result of the last iteration for an unsuccessful call).

**infodict** : dict

A dictionary of optional outputs with the keys:

**nfev**        number of function calls

**njev**        number of Jacobian calls

**fvec**        function evaluated at the output

**fjac**        the orthogonal matrix, *q*, produced by the QR factorization of the final approximate Jacobian matrix, stored column wise

**r**            upper triangular matrix produced by QR factorization of the same matrix

**qtf**        the vector ( $\text{transpose}(q) * \text{fvec}$ )

**ier** : int

An integer flag. Set to 1 if a solution was found, otherwise refer to *mesg* for more information.

**mesg** : str

If no solution is found, *mesg* details the cause of failure.

#### See also:

[root](#)        Interface to root finding algorithms for multivariate

functions.

#### Notes

`fsolve` is a wrapper around MINPACK's `hybrd` and `hybrj` algorithms.

```
scipy.optimize.broyden1(F, xin, iter=None, alpha=None, reduction_method='restart',
                        max_rank=None, verbose=False, maxiter=None, f_tol=None,
                        f_rtol=None, x_tol=None, x_rtol=None, tol_norm=None,
                        line_search='armijo', callback=None, **kw)
```

Find a root of a function, using Broyden's first Jacobian approximation.

This method is also known as "Broyden's good method".

<b>Parameters</b>	<p><b>F</b> : function(x) -&gt; f Function whose root to find; should take and return an array-like object.</p> <p><b>x0</b> : array_like Initial guess for the solution</p> <p><b>alpha</b> : float, optional Initial guess for the Jacobian is <math>(-1/\text{alpha})</math>.</p> <p><b>reduction_method</b> : str or tuple, optional Method used in ensuring that the rank of the Broyden matrix stays low. Can either be a string giving the name of the method, or a tuple of the form <code>(method, param1, param2, ...)</code> that gives the name of the method and values for additional parameters. Methods available:</p> <ul style="list-style-type: none"> <li>• <code>restart</code>: drop all matrix columns. Has no extra parameters.</li> <li>• <code>simple</code>: drop oldest matrix column. Has no extra parameters.</li> <li>• <code>svd</code>: keep only the most significant SVD components. Takes an extra parameter, <code>to_retain</code>, which determines the number of SVD components to retain when rank reduction is done. Default is <code>max_rank - 2</code>.</li> </ul> <p><b>max_rank</b> : int, optional Maximum rank for the Broyden matrix. Default is infinity (ie., no rank reduction).</p> <p><b>iter</b> : int, optional Number of iterations to make. If omitted (default), make as many as required to meet tolerances.</p> <p><b>verbose</b> : bool, optional Print status to stdout on every iteration.</p> <p><b>maxiter</b> : int, optional Maximum number of iterations to make. If more are needed to meet convergence, <code>NoConvergence</code> is raised.</p> <p><b>f_tol</b> : float, optional Absolute tolerance (in max-norm) for the residual. If omitted, default is 6e-6.</p> <p><b>f_rtol</b> : float, optional Relative tolerance for the residual. If omitted, not used.</p> <p><b>x_tol</b> : float, optional Absolute minimum step size, as determined from the Jacobian approximation. If the step size is smaller than this, optimization is terminated as successful. If omitted, not used.</p> <p><b>x_rtol</b> : float, optional Relative minimum step size. If omitted, not used.</p> <p><b>tol_norm</b> : function(vector) -&gt; scalar, optional Norm to use in convergence check. Default is the maximum norm.</p> <p><b>line_search</b> : {None, 'armijo' (default), 'wolfe'}, optional Which type of a line search to use to determine the step size in the direction given by the Jacobian approximation. Defaults to 'armijo'.</p> <p><b>callback</b> : function, optional Optional callback function. It is called on every iteration as <code>callback(x, f)</code> where <code>x</code> is the current solution and <code>f</code> the corresponding residual.</p>
<b>Returns</b>	<p><b>sol</b> : ndarray An array (of similar array type as <code>x0</code>) containing the final solution.</p>
<b>Raises</b>	<p><b>NoConvergence</b> When a solution was not found.</p>

**Notes**

This algorithm implements the inverse Jacobian Quasi-Newton update

$$H_+ = H + (dx - Hdf)dx^\dagger H / (dx^\dagger Hdf)$$

which corresponds to Broyden's first Jacobian update

$$J_+ = J + (df - Jdx)dx^\dagger / dx^\dagger dx$$

### References

[R148]

`scipy.optimize.broyden2(F, xin, iter=None, alpha=None, reduction_method='restart', max_rank=None, verbose=False, maxiter=None, f_tol=None, f_rtol=None, x_tol=None, x_rtol=None, tol_norm=None, line_search='armijo', callback=None, **kw)`

Find a root of a function, using Broyden's second Jacobian approximation.

This method is also known as "Broyden's bad method".

**Parameters**

- F** : function(x) -> f  
Function whose root to find; should take and return an array-like object.
- x0** : array\_like  
Initial guess for the solution
- alpha** : float, optional  
Initial guess for the Jacobian is  $(-1/\text{alpha})$ .
- reduction\_method** : str or tuple, optional  
Method used in ensuring that the rank of the Broyden matrix stays low. Can either be a string giving the name of the method, or a tuple of the form `(method, param1, param2, ...)` that gives the name of the method and values for additional parameters.  
Methods available:
  - `restart`: drop all matrix columns. Has no extra parameters.
  - `simple`: drop oldest matrix column. Has no extra parameters.
  - `svd`: keep only the most significant SVD components. Takes an extra parameter, `to_retain`, which determines the number of SVD components to retain when rank reduction is done. Default is `max_rank - 2`.
- max\_rank** : int, optional  
Maximum rank for the Broyden matrix. Default is infinity (ie., no rank reduction).
- iter** : int, optional  
Number of iterations to make. If omitted (default), make as many as required to meet tolerances.
- verbose** : bool, optional  
Print status to stdout on every iteration.
- maxiter** : int, optional  
Maximum number of iterations to make. If more are needed to meet convergence, `NoConvergence` is raised.
- f\_tol** : float, optional  
Absolute tolerance (in max-norm) for the residual. If omitted, default is 6e-6.
- f\_rtol** : float, optional  
Relative tolerance for the residual. If omitted, not used.
- x\_tol** : float, optional  
Absolute minimum step size, as determined from the Jacobian approximation. If the step size is smaller than this, optimization is terminated as successful. If omitted, not used.
- x\_rtol** : float, optional  
Relative minimum step size. If omitted, not used.
- tol\_norm** : function(vector) -> scalar, optional  
Norm to use in convergence check. Default is the maximum norm.
- line\_search** : {None, 'armijo' (default), 'wolfe'}, optional

Which type of a line search to use to determine the step size in the direction given by the Jacobian approximation. Defaults to 'armijo'.

**callback** : function, optional

Optional callback function. It is called on every iteration as `callback(x, f)` where  $x$  is the current solution and  $f$  the corresponding residual.

**Returns**

**sol** : ndarray

An array (of similar array type as  $x0$ ) containing the final solution.

**Raises**

**NoConvergence**

When a solution was not found.

### Notes

This algorithm implements the inverse Jacobian Quasi-Newton update

$$H_+ = H + (dx - Hdf)df^\dagger / (df^\dagger df)$$

corresponding to Broyden's second method.

### References

[R149]

The `root` function supports the following methods:

**root(method='hybr')**

`scipy.optimize.root` (*fun*, *x0*, *args=()*, *method='hybr'*, *jac=None*, *tol=None*, *callback=None*, *options={'col\_deriv': 0, 'diag': None, 'factor': 100, 'eps': None, 'band': None, 'func': None, 'maxfev': 0, 'xtol': 1.49012e-08}*)

Find the roots of a multivariate function using MINPACK's `hybrd` and `hybrj` routines (modified Powell method).

**See also:**

For documentation for the rest of the parameters, see `scipy.optimize.root`

**Options**

**col\_deriv** : bool

Specify whether the Jacobian function computes derivatives down the columns (faster, because there is no transpose operation).

**xtol** : float

The calculation will terminate if the relative error between two consecutive iterates is at most *xtol*.

**maxfev** : int

The maximum number of calls to the function. If zero, then  $100 * (N+1)$  is the maximum where  $N$  is the number of elements in  $x0$ .

**band** : tuple

If set to a two-sequence containing the number of sub- and super-diagonals within the band of the Jacobi matrix, the Jacobi matrix is considered banded (only for `fprime=None`).

**eps** : float

A suitable step length for the forward-difference approximation of the Jacobian (for `fprime=None`). If *eps* is less than the machine precision, it is assumed that the relative errors in the functions are of the order of the machine precision.

**factor** : float

A parameter determining the initial step bound (`factor * ||diag * x||`). Should be in the interval (0.1, 100).

**diag** : sequence

$N$  positive entries that serve as a scale factors for the variables.

***root(method='lm')***

`scipy.optimize.root` (*fun*, *x0*, *args=()*, *method='lm'*, *jac=None*, *tol=None*, *callback=None*, *options={'col\_deriv': 0, 'diag': None, 'factor': 100, 'gtol': 0.0, 'eps': 0.0, 'func': None, 'maxiter': 0, 'xtol': 1.49012e-08, 'ftol': 1.49012e-08}*)

Solve for least squares with Levenberg-Marquardt

**See also:**

For documentation for the rest of the parameters, see `scipy.optimize.root`

<b>Options</b>	<p><b>col_deriv</b> : bool non-zero to specify that the Jacobian function computes derivatives down the columns (faster, because there is no transpose operation).</p> <p><b>ftol</b> : float Relative error desired in the sum of squares.</p> <p><b>xtol</b> : float Relative error desired in the approximate solution.</p> <p><b>gtol</b> : float Orthogonality desired between the function vector and the columns of the Jacobian.</p> <p><b>maxiter</b> : int The maximum number of calls to the function. If zero, then 100*(N+1) is the maximum where N is the number of elements in <i>x0</i>.</p> <p><b>epsfcn</b> : float A suitable step length for the forward-difference approximation of the Jacobian (for <i>Dfun=None</i>). If <i>epsfcn</i> is less than the machine precision, it is assumed that the relative errors in the functions are of the order of the machine precision.</p> <p><b>factor</b> : float A parameter determining the initial step bound (<i>factor</i> *    <i>diag</i> * <i>x</i>  ). Should be in interval (0.1, 100).</p> <p><b>diag</b> : sequence N positive entries that serve as a scale factors for the variables.</p>
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***root(method='broyden1')***

`scipy.optimize.root` (*fun*, *x0*, *args=()*, *method='broyden1'*, *tol=None*, *callback=None*, *options={}*)

**See also:**

For documentation for the rest of the parameters, see `scipy.optimize.root`

<b>Options</b>	<p><b>nit</b> : int, optional Number of iterations to make. If omitted (default), make as many as required to meet tolerances.</p> <p><b>disp</b> : bool, optional Print status to stdout on every iteration.</p> <p><b>maxiter</b> : int, optional Maximum number of iterations to make. If more are needed to meet convergence, <i>NoConvergence</i> is raised.</p> <p><b>ftol</b> : float, optional Relative tolerance for the residual. If omitted, not used.</p> <p><b>fatol</b> : float, optional Absolute tolerance (in max-norm) for the residual. If omitted, default is 6e-6.</p> <p><b>xtol</b> : float, optional Relative minimum step size. If omitted, not used.</p> <p><b>xatol</b> : float, optional</p>
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Absolute minimum step size, as determined from the Jacobian approximation. If the step size is smaller than this, optimization is terminated as successful. If omitted, not used.

**tol\_norm** : function(vector) -> scalar, optional

Norm to use in convergence check. Default is the maximum norm.

**line\_search** : {None, 'armijo' (default), 'wolfe'}, optional

Which type of a line search to use to determine the step size in the direction given by the Jacobian approximation. Defaults to 'armijo'.

**jac\_options** : dict, optional

*Options for the respective Jacobian approximation.*

**alpha** [float, optional] Initial guess for the Jacobian is (-1/alpha).

**reduction\_method** [str or tuple, optional] Method used in ensuring that the rank of the Broyden matrix stays low. Can either be a string giving the name of the method, or a tuple of the form (method, param1, param2, ...) that gives the name of the method and values for additional parameters.

*Methods available:*

- **restart**: drop all matrix columns. Has no extra parameters.

- **simple**: drop oldest matrix column. Has no extra parameters.

- **svd**: keep only the most significant SVD components.

*Extra parameters:*

**-to\_retain**: number of SVD compo

retain when rank reduction is done. Default is max\_rank - 2.

**max\_rank** [int, optional] Maximum rank for the Broyden matrix. Default is infinity (ie., no rank reduction).

*root(method='broyden2')*

`scipy.optimize.root` (*fun*, *x0*, *args=()*, *method='broyden2'*, *tol=None*, *callback=None*, *options={}*)

**See also:**

For documentation for the rest of the parameters, see `scipy.optimize.root`

**Options**

- nit** : int, optional  
Number of iterations to make. If omitted (default), make as many as required to meet tolerances.
- disp** : bool, optional  
Print status to stdout on every iteration.
- maxiter** : int, optional  
Maximum number of iterations to make. If more are needed to meet convergence, *NoConvergence* is raised.
- ftol** : float, optional  
Relative tolerance for the residual. If omitted, not used.
- fatol** : float, optional  
Absolute tolerance (in max-norm) for the residual. If omitted, default is 6e-6.
- xtol** : float, optional  
Relative minimum step size. If omitted, not used.
- xatol** : float, optional  
Absolute minimum step size, as determined from the Jacobian approximation. If the step size is smaller than this, optimization is terminated as successful. If omitted, not used.
- tol\_norm** : function(vector) -> scalar, optional  
Norm to use in convergence check. Default is the maximum norm.
- line\_search** : {None, 'armijo' (default), 'wolfe'}, optional  
Which type of a line search to use to determine the step size in the direction given by the Jacobian approximation. Defaults to 'armijo'.
- jac\_options** : dict, optional  
Options for the respective Jacobian approximation.
  - alpha** [float, optional] Initial guess for the Jacobian is (-1/alpha).
  - reduction\_method** [str or tuple, optional] Method used in ensuring that the rank of the Broyden matrix stays low. Can either be a string giving the name of the method, or a tuple of the form (method, param1, param2, ...) that gives the name of the method and values for additional parameters.

**Methods available:**

- **restart**: drop all matrix columns. Has no extra parameters.
- **simple**: drop oldest matrix column. Has no extra parameters.
- **svd**: keep only the most significant SVD components.

**Extra parameters:**

*-to\_retain: number of SVD compo*

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**max\_rank** [int, optional] Maximum rank for the Broyden matrix. De-  
fault is infinity (ie., no rank reduction).

**root(method='anderson')**

`scipy.optimize.root` (*fun*, *x0*, *args=()*, *method='anderson'*, *tol=None*, *callback=None*, *options={}*)

**See also:**

For documentation for the rest of the parameters, see `scipy.optimize.root`

**Options**

- nit** : int, optional  
Number of iterations to make. If omitted (default), make as many as required to meet tolerances.
- disp** : bool, optional  
Print status to stdout on every iteration.
- maxiter** : int, optional  
Maximum number of iterations to make. If more are needed to meet convergence, *NoConvergence* is raised.
- ftol** : float, optional  
Relative tolerance for the residual. If omitted, not used.
- fatol** : float, optional  
Absolute tolerance (in max-norm) for the residual. If omitted, default is 6e-6.
- xtol** : float, optional  
Relative minimum step size. If omitted, not used.
- xatol** : float, optional  
Absolute minimum step size, as determined from the Jacobian approximation. If the step size is smaller than this, optimization is terminated as successful. If omitted, not used.
- tol\_norm** : function(vector) -> scalar, optional  
Norm to use in convergence check. Default is the maximum norm.
- line\_search** : {None, 'armijo' (default), 'wolfe'}, optional  
Which type of a line search to use to determine the step size in the direction given by the Jacobian approximation. Defaults to 'armijo'.
- jac\_options** : dict, optional  
Options for the respective Jacobian approximation.
- alpha** [float, optional] Initial guess for the Jacobian is (-1/alpha).
- M** [float, optional] Number of previous vectors to retain. Defaults to 5.

*w0* [float, optional] Regularization parameter for numerical stability. Compared to unity, good values of the order of 0.01.

*root(method='linear mixing')*

`scipy.optimize.root` (*fun*, *x0*, *args=()*, *method='linear mixing'*, *tol=None*, *callback=None*, *options={}*)

**See also:**

For documentation for the rest of the parameters, see `scipy.optimize.root`

**Options**

- nit** : int, optional  
Number of iterations to make. If omitted (default), make as many as required to meet tolerances.
- disp** : bool, optional  
Print status to stdout on every iteration.
- maxiter** : int, optional  
Maximum number of iterations to make. If more are needed to meet convergence, *NoConvergence* is raised.
- ftol** : float, optional  
Relative tolerance for the residual. If omitted, not used.
- fatol** : float, optional  
Absolute tolerance (in max-norm) for the residual. If omitted, default is 6e-6.
- xtol** : float, optional  
Relative minimum step size. If omitted, not used.
- xatol** : float, optional  
Absolute minimum step size, as determined from the Jacobian approximation. If the step size is smaller than this, optimization is terminated as successful. If omitted, not used.
- tol\_norm** : function(vector) -> scalar, optional  
Norm to use in convergence check. Default is the maximum norm.
- line\_search** : {None, 'armijo' (default), 'wolfe' }, optional  
Which type of a line search to use to determine the step size in the direction given by the Jacobian approximation. Defaults to 'armijo'.
- jac\_options** : dict, optional  
Options for the respective Jacobian approximation.
  - alpha* [float, optional] initial guess for the jacobian is (-1/alpha).

*root(method='diagbroyden')*

`scipy.optimize.root` (*fun*, *x0*, *args=()*, *method='diagbroyden'*, *tol=None*, *callback=None*, *options={}*)

**See also:**

For documentation for the rest of the parameters, see `scipy.optimize.root`

**Options**

- nit** : int, optional  
Number of iterations to make. If omitted (default), make as many as required to meet tolerances.
- disp** : bool, optional  
Print status to stdout on every iteration.
- maxiter** : int, optional  
Maximum number of iterations to make. If more are needed to meet convergence, *NoConvergence* is raised.

**ftol** : float, optional  
Relative tolerance for the residual. If omitted, not used.

**fatol** : float, optional  
Absolute tolerance (in max-norm) for the residual. If omitted, default is 6e-6.

**xtol** : float, optional  
Relative minimum step size. If omitted, not used.

**xatol** : float, optional  
Absolute minimum step size, as determined from the Jacobian approximation. If the step size is smaller than this, optimization is terminated as successful. If omitted, not used.

**tol\_norm** : function(vector) -> scalar, optional  
Norm to use in convergence check. Default is the maximum norm.

**line\_search** : {None, 'armijo' (default), 'wolfe'}, optional  
Which type of a line search to use to determine the step size in the direction given by the Jacobian approximation. Defaults to 'armijo'.

**jac\_options** : dict, optional  
Options for the respective Jacobian approximation.  
**alpha** [float, optional] initial guess for the jacobian is (-1/alpha).

*root(method='excitingmixing')*

`scipy.optimize.root` (*fun*, *x0*, *args=()*, *method='excitingmixing'*, *tol=None*, *callback=None*, *options={}*)

**See also:**

For documentation for the rest of the parameters, see `scipy.optimize.root`

**Options**

**nit** : int, optional  
Number of iterations to make. If omitted (default), make as many as required to meet tolerances.

**disp** : bool, optional  
Print status to stdout on every iteration.

**maxiter** : int, optional  
Maximum number of iterations to make. If more are needed to meet convergence, *NoConvergence* is raised.

**ftol** : float, optional  
Relative tolerance for the residual. If omitted, not used.

**fatol** : float, optional  
Absolute tolerance (in max-norm) for the residual. If omitted, default is 6e-6.

**xtol** : float, optional  
Relative minimum step size. If omitted, not used.

**xatol** : float, optional  
Absolute minimum step size, as determined from the Jacobian approximation. If the step size is smaller than this, optimization is terminated as successful. If omitted, not used.

**tol\_norm** : function(vector) -> scalar, optional  
Norm to use in convergence check. Default is the maximum norm.

**line\_search** : {None, 'armijo' (default), 'wolfe'}, optional  
Which type of a line search to use to determine the step size in the direction given by the Jacobian approximation. Defaults to 'armijo'.

**jac\_options** : dict, optional  
Options for the respective Jacobian approximation.

***alpha*** [float, optional] Initial Jacobian approximation is  $(-1/\alpha)$ ,  
***alphamax*** [float, optional] The entries of the diagonal Jacobian are kept in the range  $[\alpha, \text{alphamax}]$ .

***root(method='krylov')***

`scipy.optimize.root` (*fun*, *x0*, *args=()*, *method='krylov'*, *tol=None*, *callback=None*, *options={}*)

**See also:**

For documentation for the rest of the parameters, see `scipy.optimize.root`

**Options**

- nit** : int, optional  
Number of iterations to make. If omitted (default), make as many as required to meet tolerances.
- disp** : bool, optional  
Print status to stdout on every iteration.
- maxiter** : int, optional  
Maximum number of iterations to make. If more are needed to meet convergence, *NoConvergence* is raised.
- ftol** : float, optional  
Relative tolerance for the residual. If omitted, not used.
- fatol** : float, optional  
Absolute tolerance (in max-norm) for the residual. If omitted, default is  $6e-6$ .
- xtol** : float, optional  
Relative minimum step size. If omitted, not used.
- xatol** : float, optional  
Absolute minimum step size, as determined from the Jacobian approximation. If the step size is smaller than this, optimization is terminated as successful. If omitted, not used.
- tol\_norm** : function(vector) -> scalar, optional  
Norm to use in convergence check. Default is the maximum norm.
- line\_search** : {None, 'armijo' (default), 'wolfe'}, optional  
Which type of a line search to use to determine the step size in the direction given by the Jacobian approximation. Defaults to 'armijo'.
- jac\_options** : dict, optional  
Options for the respective Jacobian approximation.
- rdiff*** [float, optional] Relative step size to use in numerical differentiation.
- method*** [{ 'lgmres', 'gmres', 'bicgstab', 'cgs', 'minres' } or function]  
Krylov method to use to approximate the Jacobian. Can be a string, or a function implementing the same interface as the iterative solvers in `scipy.sparse.linalg`.
- inner\_M*** [LinearOperator or InverseJacobian]  
The default is `scipy.sparse.linalg.lgmres`.  
Preconditioner for the inner Krylov iteration. Note that you can use also inverse Jacobians as (adaptive) preconditioners. For example,  

```
>>> jac = BroydenFirst()
>>> kjac = KrylovJacobian(inner_M=jac.inverse)
```
- inner\_tol*, *inner\_maxiter*, ...**  
Parameters to pass on to the "inner" Krylov solver. See `scipy.sparse.linalg.gmres` for details.

**outer\_k** [int, optional] Size of the subspace kept across LGMRES nonlinear iterations.  
See `scipy.sparse.linalg.lgmres` for details.

### `root(method='df-sane')`

`scipy.optimize.root` (*fun*, *x0*, *args=()*, *method='df-sane'*, *tol=None*, *callback=None*, *options={'disp': False, 'fnorm': None, 'sigma\_0': 1.0, 'eta\_strategy': None, 'sigma\_eps': 1e-10, 'M': 10, 'line\_search': 'cruz', 'fatol': 1e-300, 'func': None, 'maxfev': 1000, 'ftol': 1e-08}*)

Solve nonlinear equation with the DF-SANE method

#### See also:

For documentation for the rest of the parameters, see `scipy.optimize.root`

**Options**

- ftol** : float, optional  
Relative norm tolerance.
- fatol** : float, optional  
Absolute norm tolerance. Algorithm terminates when  $\|func(x)\| < fatol + ftol \|func(x_0)\|$ .
- fnorm** : callable, optional  
Norm to use in the convergence check. If None, 2-norm is used.
- maxfev** : int, optional  
Maximum number of function evaluations.
- disp** : bool, optional  
Whether to print convergence process to stdout.
- eta\_strategy** : callable, optional  
Choice of the `eta_k` parameter, which gives slack for growth of  $\|F\|^{**2}$ . Called as `eta_k = eta_strategy(k, x, F)` with *k* the iteration number, *x* the current iterate and *F* the current residual. Should satisfy `eta_k > 0` and `sum(eta, k=0..inf) < inf`. Default:  $\|F\|^{**2} / (1 + k)^{**2}$ .
- sigma\_eps** : float, optional  
The spectral coefficient is constrained to `sigma_eps < sigma < 1/sigma_eps`. Default: 1e-10
- sigma\_0** : float, optional  
Initial spectral coefficient. Default: 1.0
- M** : int, optional  
Number of iterates to include in the nonmonotonic line search. Default: 10
- line\_search** : {'cruz', 'cheng'}  
Type of line search to employ. 'cruz' is the original one defined in [Martinez & Raydan. Math. Comp. 75, 1429 (2006)], 'cheng' is a modified search defined in [Cheng & Li. IMA J. Numer. Anal. 29, 814 (2009)]. Default: 'cruz'

#### References

[R732], [R733], [R734]

Large-scale nonlinear solvers:

---

<code>newton_krylov</code> ( <i>F</i> , <i>xin</i> [, <i>iter</i> , <i>rdiff</i> , <i>method</i> , ...])	Find a root of a function, using Krylov approximation for inverse Jacobian.
<code>anderson</code> ( <i>F</i> , <i>xin</i> [, <i>iter</i> , <i>alpha</i> , <i>w0</i> , <i>M</i> , ...])	Find a root of a function, using (extended) Anderson mixing.

---

```
scipy.optimize.newton_krylov(F, xin, iter=None, rdiff=None, method='lgmres',
                             inner_maxiter=20, inner_M=None, outer_k=10, verbose=False,
                             maxiter=None, f_tol=None, f_rtol=None, x_tol=None,
                             x_rtol=None, tol_norm=None, line_search='armijo', call-
                             back=None, **kw)
```

Find a root of a function, using Krylov approximation for inverse Jacobian.

This method is suitable for solving large-scale problems.

**Parameters** **F** : function(x) -> f

Function whose root to find; should take and return an array-like object.

**x0** : array\_like

Initial guess for the solution

**rdiff** : float, optional

Relative step size to use in numerical differentiation.

**method** : {'lgmres', 'gmres', 'bicgstab', 'cgs', 'minres'} or function

Krylov method to use to approximate the Jacobian. Can be a string, or a function implementing the same interface as the iterative solvers in `scipy.sparse.linalg`.

The default is `scipy.sparse.linalg.lgmres`.

**inner\_M** : LinearOperator or InverseJacobian

Preconditioner for the inner Krylov iteration. Note that you can use also inverse Jacobians as (adaptive) preconditioners. For example,

```
>>> from scipy.optimize.nonlin import BroydenFirst, KrylovJacobian
>>> from scipy.optimize.nonlin import InverseJacobian
>>> jac = BroydenFirst()
>>> kjac = KrylovJacobian(inner_M=InverseJacobian(jac))
```

If the preconditioner has a method named 'update', it will be called as `update(x, f)` after each nonlinear step, with `x` giving the current point, and `f` the current function value.

**inner\_tol, inner\_maxiter, ...**

Parameters to pass on to the "inner" Krylov solver. See `scipy.sparse.linalg.gmres` for details.

**outer\_k** : int, optional

Size of the subspace kept across LGMRES nonlinear iterations. See `scipy.sparse.linalg.lgmres` for details.

**iter** : int, optional

Number of iterations to make. If omitted (default), make as many as required to meet tolerances.

**verbose** : bool, optional

Print status to stdout on every iteration.

**maxiter** : int, optional

Maximum number of iterations to make. If more are needed to meet convergence, `NoConvergence` is raised.

**f\_tol** : float, optional

Absolute tolerance (in max-norm) for the residual. If omitted, default is 6e-6.

**f\_rtol** : float, optional

Relative tolerance for the residual. If omitted, not used.

**x\_tol** : float, optional

Absolute minimum step size, as determined from the Jacobian approximation. If the step size is smaller than this, optimization is terminated as successful. If omitted, not used.

**x\_rtol** : float, optional



Relative minimum step size. If omitted, not used.

**tol\_norm** : function(vector) -> scalar, optional  
Norm to use in convergence check. Default is the maximum norm.

**line\_search** : {None, 'armijo' (default), 'wolfe'}, optional  
Which type of a line search to use to determine the step size in the direction given by the Jacobian approximation. Defaults to 'armijo'.

**callback** : function, optional  
Optional callback function. It is called on every iteration as `callback(x, f)` where  $x$  is the current solution and  $f$  the corresponding residual.

**Returns** **sol** : ndarray  
An array (of similar array type as  $x_0$ ) containing the final solution.

**Raises** **NoConvergence**  
When a solution was not found.

**See also:**

`scipy.sparse.linalg.gmres`, `scipy.sparse.linalg.lgmres`

**Notes**

This function implements a Newton-Krylov solver. The basic idea is to compute the inverse of the Jacobian with an iterative Krylov method. These methods require only evaluating the Jacobian-vector products, which are conveniently approximated by a finite difference:

$$Jv \approx (f(x + \omega * v/|v|) - f(x))/\omega$$

Due to the use of iterative matrix inverses, these methods can deal with large nonlinear problems.

Scipy's `scipy.sparse.linalg` module offers a selection of Krylov solvers to choose from. The default here is `lgmres`, which is a variant of restarted GMRES iteration that reuses some of the information obtained in the previous Newton steps to invert Jacobians in subsequent steps.

For a review on Newton-Krylov methods, see for example [R169], and for the LGMRES sparse inverse method, see [R170].

**References**

[R169], [R170]

`scipy.optimize.anderson`( $F$ ,  $xin$ , *iter=None*, *alpha=None*, *w0=0.01*, *M=5*, *verbose=False*, *maxiter=None*, *f\_tol=None*, *f\_rtol=None*, *x\_tol=None*, *x\_rtol=None*, *tol\_norm=None*, *line\_search='armijo'*, *callback=None*, *\*\*kw*)

Find a root of a function, using (extended) Anderson mixing.

The Jacobian is formed by for a 'best' solution in the space spanned by last  $M$  vectors. As a result, only a  $M \times M$  matrix inversions and  $M \times N$  multiplications are required. [Ey]

**Parameters**

**F** : function(x) -> f  
Function whose root to find; should take and return an array-like object.

**x0** : array\_like  
Initial guess for the solution

**alpha** : float, optional  
Initial guess for the Jacobian is (-1/alpha).

**M** : float, optional  
Number of previous vectors to retain. Defaults to 5.

**w0** : float, optional  
Regularization parameter for numerical stability. Compared to unity, good values of the order of 0.01.

**iter** : int, optional

Number of iterations to make. If omitted (default), make as many as required to meet tolerances.

**verbose** : bool, optional  
Print status to stdout on every iteration.

**maxiter** : int, optional  
Maximum number of iterations to make. If more are needed to meet convergence, *NoConvergence* is raised.

**f\_tol** : float, optional  
Absolute tolerance (in max-norm) for the residual. If omitted, default is 6e-6.

**f\_rtol** : float, optional  
Relative tolerance for the residual. If omitted, not used.

**x\_tol** : float, optional  
Absolute minimum step size, as determined from the Jacobian approximation. If the step size is smaller than this, optimization is terminated as successful. If omitted, not used.

**x\_rtol** : float, optional  
Relative minimum step size. If omitted, not used.

**tol\_norm** : function(vector) -> scalar, optional  
Norm to use in convergence check. Default is the maximum norm.

**line\_search** : {None, 'armijo' (default), 'wolfe' }, optional  
Which type of a line search to use to determine the step size in the direction given by the Jacobian approximation. Defaults to 'armijo'.

**callback** : function, optional  
Optional callback function. It is called on every iteration as `callback(x, f)` where  $x$  is the current solution and  $f$  the corresponding residual.

**Returns** **sol** : ndarray  
An array (of similar array type as  $x0$ ) containing the final solution.

**Raises** **NoConvergence**  
When a solution was not found.

### References

[Ey]

Simple iterations:

<code>excitingmixing(F, xin[, iter, alpha, ...])</code>	Find a root of a function, using a tuned diagonal Jacobian approximation.
<code>linearmixing(F, xin[, iter, alpha, verbose, ...])</code>	Find a root of a function, using a scalar Jacobian approximation.
<code>diagbroyden(F, xin[, iter, alpha, verbose, ...])</code>	Find a root of a function, using diagonal Broyden Jacobian approximation.

`scipy.optimize.excitingmixing` ( $F$ ,  $xin$ ,  $iter=None$ ,  $alpha=None$ ,  $alphamax=1.0$ ,  $verbose=False$ ,  $maxiter=None$ ,  $f\_tol=None$ ,  $f\_rtol=None$ ,  $x\_tol=None$ ,  $x\_rtol=None$ ,  $tol\_norm=None$ ,  $line\_search='armijo'$ ,  $callback=None$ ,  $**kw$ )

Find a root of a function, using a tuned diagonal Jacobian approximation.

The Jacobian matrix is diagonal and is tuned on each iteration.

**Warning:** This algorithm may be useful for specific problems, but whether it will work may depend strongly on the problem.

**Parameters** **F** : function( $x$ ) ->  $f$   
Function whose root to find; should take and return an array-like object.

**x0** : array\_like

Initial guess for the solution

**alpha** : float, optional  
Initial Jacobian approximation is  $(-1/\text{alpha})$ .

**alphamax** : float, optional  
The entries of the diagonal Jacobian are kept in the range  $[\text{alpha}, \text{alphamax}]$ .

**iter** : int, optional  
Number of iterations to make. If omitted (default), make as many as required to meet tolerances.

**verbose** : bool, optional  
Print status to stdout on every iteration.

**maxiter** : int, optional  
Maximum number of iterations to make. If more are needed to meet convergence, *NoConvergence* is raised.

**f\_tol** : float, optional  
Absolute tolerance (in max-norm) for the residual. If omitted, default is  $6e-6$ .

**f\_rtol** : float, optional  
Relative tolerance for the residual. If omitted, not used.

**x\_tol** : float, optional  
Absolute minimum step size, as determined from the Jacobian approximation. If the step size is smaller than this, optimization is terminated as successful. If omitted, not used.

**x\_rtol** : float, optional  
Relative minimum step size. If omitted, not used.

**tol\_norm** : function(vector) -> scalar, optional  
Norm to use in convergence check. Default is the maximum norm.

**line\_search** : {None, 'armijo' (default), 'wolfe'}, optional  
Which type of a line search to use to determine the step size in the direction given by the Jacobian approximation. Defaults to 'armijo'.

**callback** : function, optional  
Optional callback function. It is called on every iteration as `callback(x, f)` where  $x$  is the current solution and  $f$  the corresponding residual.

**Returns** **sol** : ndarray  
An array (of similar array type as  $x_0$ ) containing the final solution.

**Raises** **NoConvergence**  
When a solution was not found.

`scipy.optimize.linarmixing(F, xin, iter=None, alpha=None, verbose=False, maxiter=None, f_tol=None, f_rtol=None, x_tol=None, x_rtol=None, tol_norm=None, line_search='armijo', callback=None, **kw)`

Find a root of a function, using a scalar Jacobian approximation.

**Warning:** This algorithm may be useful for specific problems, but whether it will work may depend strongly on the problem.

**Parameters** **F** : function(x) -> f  
Function whose root to find; should take and return an array-like object.

**x0** : array\_like  
Initial guess for the solution

**alpha** : float, optional  
The Jacobian approximation is  $(-1/\text{alpha})$ .

**iter** : int, optional  
Number of iterations to make. If omitted (default), make as many as required to meet tolerances.

**verbose** : bool, optional  
Print status to stdout on every iteration.

**maxiter** : int, optional  
Maximum number of iterations to make. If more are needed to meet convergence, *NoConvergence* is raised.

**f\_tol** : float, optional  
Absolute tolerance (in max-norm) for the residual. If omitted, default is 6e-6.

**f\_rtol** : float, optional  
Relative tolerance for the residual. If omitted, not used.

**x\_tol** : float, optional  
Absolute minimum step size, as determined from the Jacobian approximation. If the step size is smaller than this, optimization is terminated as successful. If omitted, not used.

**x\_rtol** : float, optional  
Relative minimum step size. If omitted, not used.

**tol\_norm** : function(vector) -> scalar, optional  
Norm to use in convergence check. Default is the maximum norm.

**line\_search** : {None, 'armijo' (default), 'wolfe' }, optional  
Which type of a line search to use to determine the step size in the direction given by the Jacobian approximation. Defaults to 'armijo'.

**callback** : function, optional  
Optional callback function. It is called on every iteration as `callback(x, f)` where  $x$  is the current solution and  $f$  the corresponding residual.

**Returns** **sol** : ndarray  
An array (of similar array type as  $x_0$ ) containing the final solution.

**Raises** **NoConvergence**  
When a solution was not found.

`scipy.optimize.diagbroyden(F, xin, iter=None, alpha=None, verbose=False, maxiter=None, f_tol=None, f_rtol=None, x_tol=None, x_rtol=None, tol_norm=None, line_search='armijo', callback=None, **kw)`

Find a root of a function, using diagonal Broyden Jacobian approximation.

The Jacobian approximation is derived from previous iterations, by retaining only the diagonal of Broyden matrices.

**Warning:** This algorithm may be useful for specific problems, but whether it will work may depend strongly on the problem.

**Parameters** **F** : function(x) -> f  
Function whose root to find; should take and return an array-like object.

**x0** : array\_like  
Initial guess for the solution

**alpha** : float, optional  
Initial guess for the Jacobian is (-1/alpha).

**iter** : int, optional  
Number of iterations to make. If omitted (default), make as many as required to meet tolerances.

**verbose** : bool, optional  
Print status to stdout on every iteration.

**maxiter** : int, optional  
Maximum number of iterations to make. If more are needed to meet convergence, *NoConvergence* is raised.

**f\_tol** : float, optional

Absolute tolerance (in max-norm) for the residual. If omitted, default is 6e-6.

**f\_rtol** : float, optional  
Relative tolerance for the residual. If omitted, not used.

**x\_tol** : float, optional  
Absolute minimum step size, as determined from the Jacobian approximation. If the step size is smaller than this, optimization is terminated as successful. If omitted, not used.

**x\_rtol** : float, optional  
Relative minimum step size. If omitted, not used.

**tol\_norm** : function(vector) -> scalar, optional  
Norm to use in convergence check. Default is the maximum norm.

**line\_search** : {None, 'armijo' (default), 'wolfe' }, optional  
Which type of a line search to use to determine the step size in the direction given by the Jacobian approximation. Defaults to 'armijo'.

**callback** : function, optional  
Optional callback function. It is called on every iteration as `callback(x, f)` where  $x$  is the current solution and  $f$  the corresponding residual.

**Returns** **sol** : ndarray  
An array (of similar array type as  $x_0$ ) containing the final solution.

**Raises** **NoConvergence**  
When a solution was not found.

[Additional information on the nonlinear solvers](#)

## 5.18.4 Linear Programming

Simplex Algorithm:

<code>linprog(c[, A_ub, b_ub, A_eq, b_eq, bounds, ...])</code>	Minimize a linear objective function subject to linear equality and inequality constraints.
<code>linprog_verbose_callback(xk, **kwargs)</code>	A sample callback function demonstrating the linprog callback interface.

`scipy.optimize.linprog(c, A_ub=None, b_ub=None, A_eq=None, b_eq=None, bounds=None, method='simplex', callback=None, options=None)`

Minimize a linear objective function subject to linear equality and inequality constraints.

Linear Programming is intended to solve the following problem form:

Minimize:  $c^T * x$

**Subject to:**  $A\_ub * x \leq b\_ub$

$A\_eq * x == b\_eq$

**Parameters** **c** : array\_like

Coefficients of the linear objective function to be minimized.

**A\_ub** : array\_like, optional

2-D array which, when matrix-multiplied by  $x$ , gives the values of the upper-bound inequality constraints at  $x$ .

**b\_ub** : array\_like, optional

1-D array of values representing the upper-bound of each inequality constraint (row) in  $A\_ub$ .

**A\_eq** : array\_like, optional

2-D array which, when matrix-multiplied by  $x$ , gives the values of the equality constraints at  $x$ .

**b\_eq** : array\_like, optional

1-D array of values representing the RHS of each equality constraint (row) in `A_eq`.

**bounds** : sequence, optional

(`min`, `max`) pairs for each element in `x`, defining the bounds on that parameter. Use `None` for one of `min` or `max` when there is no bound in that direction. By default bounds are (`0`, `None`) (non-negative) If a sequence containing a single tuple is provided, then `min` and `max` will be applied to all variables in the problem.

**method** : str, optional

Type of solver. At this time only ‘simplex’ is supported (*see here*).

**callback** : callable, optional

If a callback function is provide, it will be called within each iteration of the simplex algorithm. The callback must have the signature `callback(xk, **kwargs)` where `xk` is the current solution vector and `kwargs` is a dictionary containing the following:

```
"tableau" : The current Simplex algorithm tableau
"nit" : The current iteration.
"pivot" : The pivot (row, column) used for the next iteration.
"phase" : Whether the algorithm is in Phase 1 or Phase 2.
"basis" : The indices of the columns of the basic variables.
```

**options** : dict, optional

A dictionary of solver options. All methods accept the following generic options:

```
maxiter [int] Maximum number of iterations to perform.
disp [bool] Set to True to print convergence messages.
```

**Returns** A `scipy.optimize.OptimizeResult` consisting of the following fields:

```
x [ndarray] The independent variable vector which optimizes the linear programming problem.
fun [float] Value of the objective function.
slack [ndarray] The values of the slack variables. Each slack variable corresponds to an inequality constraint. If the slack is zero, then the corresponding constraint is active.
success [bool] Returns True if the algorithm succeeded in finding an optimal solution.
status [int] An integer representing the exit status of the optimization:
    0 : Optimization terminated successfully
    1 : Iteration limit reached
    2 : Problem appears to be infeasible
    3 : Problem appears to be unbounded
nit [int] The number of iterations performed.
message [str] A string descriptor of the exit status of the optimization.
```

**See also:**

[`show\_options`](#)

Additional options accepted by the solvers

**Notes**

This section describes the available solvers that can be selected by the ‘method’ parameter. The default method is *Simplex*.

Method *Simplex* uses the Simplex algorithm (as it relates to Linear Programming, NOT the Nelder-Mead Simplex) [R157], [R158]. This algorithm should be reasonably reliable and fast.

New in version 0.15.0.

### References

[R157], [R158], [R159]

### Examples

Consider the following problem:

Minimize:  $f = -1*x[0] + 4*x[1]$

**Subject to:**  $-3*x[0] + 1*x[1] \leq 6$

$$\begin{aligned} 1*x[0] + 2*x[1] &\leq 4 \\ x[1] &\geq -3 \end{aligned}$$

where:  $-\text{inf} \leq x[0] \leq \text{inf}$

This problem deviates from the standard linear programming problem. In standard form, linear programming problems assume the variables  $x$  are non-negative. Since the variables don't have standard bounds where  $0 \leq x \leq \text{inf}$ , the bounds of the variables must be explicitly set.

There are two upper-bound constraints, which can be expressed as

$$\text{dot}(A\_ub, x) \leq b\_ub$$

The input for this problem is as follows:

```
>>> c = [-1, 4]
>>> A = [[-3, 1], [1, 2]]
>>> b = [6, 4]
>>> x0_bounds = (None, None)
>>> x1_bounds = (-3, None)
>>> from scipy.optimize import linprog
>>> res = linprog(c, A_ub=A, b_ub=b, bounds=(x0_bounds, x1_bounds),
...             options={"disp": True})
Optimization terminated successfully.
    Current function value: -22.000000
    Iterations: 1
>>> print(res)
fun: -22.0
message: 'Optimization terminated successfully.'
nit: 1
slack: array([ 39.,  0.])
status: 0
success: True
x: array([ 10., -3.])
```

Note the actual objective value is 11.428571. In this case we minimized the negative of the objective function.

`scipy.optimize.linprog_verbose_callback` ( $xk, **kwargs$ )

A sample callback function demonstrating the `linprog` callback interface. This callback produces detailed output to `sys.stdout` before each iteration and after the final iteration of the simplex algorithm.

**Parameters** **xk** : array\_like

The current solution vector.

**\*\*kwargs** : dict

A dictionary containing the following parameters:

**tableau** [array\_like] The current tableau of the simplex algorithm.

**phase** Its structure is defined in `solve_simplex`.

**nit** [int] The current Phase of the simplex algorithm (1 or 2)

[int] The current iteration number.

***pivot*** [tuple(int, int)] The index of the tableau selected as the next pivot, or nan if no pivot exists

***basis*** [array(int)] A list of the current basic variables. Each element contains the name of a basic variable and its value

***complete*** [bool] True if the simplex algorithm has completed (and this is the final call to callback), otherwise False.

The `linprog` function supports the following methods:

### linprog(method='Simplex')

```
scipy.optimize.linprog(c, A_ub=None, b_ub=None, A_eq=None, b_eq=None, bounds=None,
                       method='simplex', callback=None, options={'disp': False, 'bland': False,
                       'tol': 1e-12, 'maxiter': 1000})
```

Solve the following linear programming problem via a two-phase simplex algorithm.

```
minimize: cT * x
subject to: A_ub * x <= b_ub
           A_eq * x == b_eq
```

**Parameters** **c** : array\_like

Coefficients of the linear objective function to be minimized.

**A\_ub** : array\_like

2-D array which, when matrix-multiplied by x, gives the values of the upper-bound inequality constraints at x.

**b\_ub** : array\_like

1-D array of values representing the upper-bound of each inequality constraint (row) in A\_ub.

**A\_eq** : array\_like

2-D array which, when matrix-multiplied by x, gives the values of the equality constraints at x.

**b\_eq** : array\_like

1-D array of values representing the RHS of each equality constraint (row) in A\_eq.

**bounds** : array\_like

The bounds for each independent variable in the solution, which can take one of three forms:: None : The default bounds, all variables are non-negative. (lb, ub) : If a 2-element sequence is provided, the same lower bound (lb) and upper bound (ub) will be applied to all variables.

*[(lb\_0, ub\_0), (lb\_1, ub\_1), ...]*

[If an n x 2 sequence is provided,] each variable x<sub>i</sub> will be bounded by lb[i] and ub[i].

Infinite bounds are specified using -np.inf (negative) or np.inf (positive).

**callback** : callable

If a callback function is provide, it will be called within each iteration of the simplex algorithm. The callback must have the signature *callback(xk, \*\*kwargs)* where xk is the current solution vector and kwargs is a dictionary containing the following:: "tableau" : The current Simplex algorithm tableau "nit" : The current iteration. "pivot" : The pivot (row, column) used for the next iteration. "phase" : Whether the algorithm is in Phase 1 or Phase 2. "bv" : A structured array containing a string representation of each basic variable and its current value.



**Returns** A `scipy.optimize.OptimizeResult` consisting of the following fields:

```

x : ndarray
    The independent variable vector which optimizes the linear
    programming problem.
fun : float
    Value of the objective function.
slack : ndarray
    The values of the slack variables. Each slack variable corresponds
    to an inequality constraint. If the slack is zero, then the
    corresponding constraint is active.
success : bool
    Returns True if the algorithm succeeded in finding an optimal
    solution.
status : int
    An integer representing the exit status of the optimization::
    0 : Optimization terminated successfully
    1 : Iteration limit reached
    2 : Problem appears to be infeasible
    3 : Problem appears to be unbounded
nit : int
    The number of iterations performed.
message : str
    A string descriptor of the exit status of the optimization.

```

**See also:**

For documentation for the rest of the parameters, see `scipy.optimize.linprog`

**Options**

```

maxiter : int
    The maximum number of iterations to perform.
disp : bool
    If True, print exit status message to sys.stdout
tol : float
    The tolerance which determines when a solution is “close enough” to zero
    in Phase 1 to be considered a basic feasible solution or close enough to
    positive to serve as an optimal solution.
bland : bool
    If True, use Bland’s anti-cycling rule [3] to choose pivots to prevent cycling.
    If False, choose pivots which should lead to a converged solution more
    quickly. The latter method is subject to cycling (non-convergence) in rare
    instances.

```

**References**

[R725], [R726], [R727]

**Examples**

Consider the following problem:

Minimize:  $f = -1*x[0] + 4*x[1]$

**Subject to:**  $-3*x[0] + 1*x[1] \leq 6$

$$1*x[0] + 2*x[1] \leq 4$$

$$x[1] \geq -3$$

where:  $-\text{inf} \leq x[0] \leq \text{inf}$

This problem deviates from the standard linear programming problem. In standard form, linear programming problems assume the variables  $x$  are non-negative. Since the variables don't have standard bounds where  $0 \leq x \leq \text{inf}$ , the bounds of the variables must be explicitly set.

There are two upper-bound constraints, which can be expressed as

$$\text{dot}(A\_ub, x) \leq b\_ub$$

The input for this problem is as follows:

```
>>> from scipy.optimize import linprog
>>> c = [-1, 4]
>>> A = [[-3, 1], [1, 2]]
>>> b = [6, 4]
>>> x0_bnds = (None, None)
>>> x1_bnds = (-3, None)
>>> res = linprog(c, A, b, bounds=(x0_bnds, x1_bnds))
>>> print(res)
      fun: -22.0
  message: 'Optimization terminated successfully.'
         nit: 1
      slack: array([ 39.,  0.])
      status: 0
  success: True
         x: array([ 10., -3.]
```

Assignment problems:

---

`linear_sum_assignment(cost_matrix)` Solve the linear sum assignment problem.

---

`scipy.optimize.linear_sum_assignment(cost_matrix)`

Solve the linear sum assignment problem.

The linear sum assignment problem is also known as minimum weight matching in bipartite graphs. A problem instance is described by a matrix  $C$ , where each  $C[i,j]$  is the cost of matching vertex  $i$  of the first partite set (a “worker”) and vertex  $j$  of the second set (a “job”). The goal is to find a complete assignment of workers to jobs of minimal cost.

Formally, let  $X$  be a boolean matrix where  $X[i,j] = 1$  iff row  $i$  is assigned to column  $j$ . Then the optimal assignment has cost

$$\min \sum_i \sum_j C_{i,j} X_{i,j}$$

s.t. each row is assignment to at most one column, and each column to at most one row.

This function can also solve a generalization of the classic assignment problem where the cost matrix is rectangular. If it has more rows than columns, then not every row needs to be assigned to a column, and vice versa.

The method used is the Hungarian algorithm, also known as the Munkres or Kuhn-Munkres algorithm.

**Parameters** `cost_matrix` : array

**Returns** `row_ind, col_ind` : array  
The cost matrix of the bipartite graph.

An array of row indices and one of corresponding column indices giving the optimal assignment. The cost of the assignment can be computed as `cost_matrix[row_ind, col_ind].sum()`. The row indices will be sorted; in the case of a square cost matrix they will be equal to `numpy.arange(cost_matrix.shape[0])`.

## Notes

New in version 0.17.0.

## References

- 1.<http://csclab.murraystate.edu/bob.pilgrim/445/munkres.html>
- 2.Harold W. Kuhn. The Hungarian Method for the assignment problem. *Naval Research Logistics Quarterly*, 2:83-97, 1955.
- 3.Harold W. Kuhn. Variants of the Hungarian method for assignment problems. *Naval Research Logistics Quarterly*, 3: 253-258, 1956.
- 4.Munkres, J. Algorithms for the Assignment and Transportation Problems. *J. SIAM*, 5(1):32-38, March, 1957.
- 5.[https://en.wikipedia.org/wiki/Hungarian\\_algorithm](https://en.wikipedia.org/wiki/Hungarian_algorithm)

## Examples

```
>>> cost = np.array([[4, 1, 3], [2, 0, 5], [3, 2, 2]])
>>> from scipy.optimize import linear_sum_assignment
>>> row_ind, col_ind = linear_sum_assignment(cost)
>>> col_ind
array([1, 0, 2])
>>> cost[row_ind, col_ind].sum()
5
```

## 5.18.5 Utilities

<code>approx_fprime(xk, f, epsilon, *args)</code>	Finite-difference approximation of the gradient of a scalar function.
<code>bracket(func[, xa, xb, args, grow_limit, ...])</code>	Bracket the minimum of the function.
<code>check_grad(func, grad, x0, *args, **kwargs)</code>	Check the correctness of a gradient function by comparing it against a (forward) finite difference.
<code>line_search(f, myfprime, xk, pk[, gfk, ...])</code>	Find alpha that satisfies strong Wolfe conditions.
<code>show_options([solver, method, disp])</code>	Show documentation for additional options of optimization solvers.
<code>LbfgsInvHessProduct(sk, yk)</code>	Linear operator for the L-BFGS approximate inverse Hessian.

`scipy.optimize.approx_fprime(xk, f, epsilon, *args)`

Finite-difference approximation of the gradient of a scalar function.

**Parameters** `xk` : array\_like

The coordinate vector at which to determine the gradient of  $f$ .

`f` : callable

The function of which to determine the gradient (partial derivatives). Should take  $xk$  as first argument, other arguments to  $f$  can be supplied in `*args`. Should return a scalar, the value of the function at  $xk$ .

`epsilon` : array\_like

Increment to  $xk$  to use for determining the function gradient. If a scalar, uses the same finite difference delta for all partial derivatives. If an array, should contain one value per element of  $xk$ .

`*args` : args, optional

Any other arguments that are to be passed to  $f$ .

**Returns**

`grad` : ndarray

The partial derivatives of  $f$  to  $xk$ .

See also:

`check_grad` Check correctness of gradient function against `approx_fprime`.

### Notes

The function gradient is determined by the forward finite difference formula:

$$f'[i] = \frac{f(xk[i] + \text{epsilon}[i]) - f(xk[i])}{\text{epsilon}[i]}$$

The main use of `approx_fprime` is in scalar function optimizers like `fmin_bfgs`, to determine numerically the Jacobian of a function.

### Examples

```
>>> from scipy import optimize
>>> def func(x, c0, c1):
...     "Coordinate vector `x` should be an array of size two."
...     return c0 * x[0]**2 + c1*x[1]**2

>>> x = np.ones(2)
>>> c0, c1 = (1, 200)
>>> eps = np.sqrt(np.finfo(float).eps)
>>> optimize.approx_fprime(x, func, [eps, np.sqrt(200) * eps], c0, c1)
array([ 2.          , 400.00004198])
```

`scipy.optimize.bracket` (*func*, *xa=0.0*, *xb=1.0*, *args=()*, *grow\_limit=110.0*, *maxiter=1000*)

Bracket the minimum of the function.

Given a function and distinct initial points, search in the downhill direction (as defined by the initial points) and return new points *xa*, *xb*, *xc* that bracket the minimum of the function  $f(xa) > f(xb) < f(xc)$ . It doesn't always mean that obtained solution will satisfy  $xa \leq x \leq xb$

**Parameters**

- func** : callable  $f(x, *args)$   
Objective function to minimize.
- xa, xb** : float, optional  
Bracketing interval. Defaults *xa* to 0.0, and *xb* to 1.0.
- args** : tuple, optional  
Additional arguments (if present), passed to *func*.
- grow\_limit** : float, optional  
Maximum grow limit. Defaults to 110.0
- maxiter** : int, optional  
Maximum number of iterations to perform. Defaults to 1000.

**Returns**

- xa, xb, xc** : float  
Bracket.
- fa, fb, fc** : float  
Objective function values in bracket.
- funcalls** : int  
Number of function evaluations made.

`scipy.optimize.check_grad` (*func*, *grad*, *x0*, *\*args*, *\*\*kwargs*)

Check the correctness of a gradient function by comparing it against a (forward) finite-difference approximation of the gradient.

**Parameters**

- func** : callable  $func(x0, *args)$   
Function whose derivative is to be checked.
- grad** : callable  $grad(x0, *args)$   
Gradient of *func*.
- x0** : ndarray

Points to check *grad* against forward difference approximation of grad using *func*.

**args** : \*args, optional  
Extra arguments passed to *func* and *grad*.

**epsilon** : float, optional  
Step size used for the finite difference approximation. It defaults to `sqrt(numpy.finfo(float).eps)`, which is approximately 1.49e-08.

**Returns** **err** : float  
The square root of the sum of squares (i.e. the 2-norm) of the difference between `grad(x0, *args)` and the finite difference approximation of *grad* using *func* at the points *x0*.

See also:

`approx_fprime`

**Examples**

```
>>> def func(x):
...     return x[0]**2 - 0.5 * x[1]**3
>>> def grad(x):
...     return [2 * x[0], -1.5 * x[1]**2]
>>> from scipy.optimize import check_grad
>>> check_grad(func, grad, [1.5, -1.5])
2.9802322387695312e-08
```

`scipy.optimize.line_search` (*f*, *myfprime*, *xk*, *pk*, *gfk=None*, *old\_fval=None*, *old\_old\_fval=None*, *args=()*, *c1=0.0001*, *c2=0.9*, *amax=50*)

Find alpha that satisfies strong Wolfe conditions.

**Parameters**

- f** : callable *f*(*x*,\*args)  
Objective function.
- myfprime** : callable *f'*(*x*,\*args)  
Objective function gradient.
- xk** : ndarray  
Starting point.
- pk** : ndarray  
Search direction.
- gfk** : ndarray, optional  
Gradient value for *x=xk* (*xk* being the current parameter estimate). Will be recomputed if omitted.
- old\_fval** : float, optional  
Function value for *x=xk*. Will be recomputed if omitted.
- old\_old\_fval** : float, optional  
Function value for the point preceding *x=xk*
- args** : tuple, optional  
Additional arguments passed to objective function.
- c1** : float, optional  
Parameter for Armijo condition rule.
- c2** : float, optional  
Parameter for curvature condition rule.
- amax** : float, optional  
Maximum step size

**Returns** **alpha** : float or None  
Alpha for which  $x_{\text{new}} = x_0 + \text{alpha} * \text{pk}$ , or None if the line search algorithm did not converge.

**fc** : int  
Number of function evaluations made.

**gc** : int  
Number of gradient evaluations made.

**new\_fval** : float or None  
New function value  $f(x_{\text{new}}) = f(x_0 + \alpha * pk)$ , or None if the line search algorithm did not converge.

**old\_fval** : float  
Old function value  $f(x_0)$ .

**new\_slope** : float or None  
The local slope along the search direction at the new value  $\langle \text{myfprime}(x_{\text{new}}), pk \rangle$ , or None if the line search algorithm did not converge.

**Notes**

Uses the line search algorithm to enforce strong Wolfe conditions. See Wright and Nocedal, ‘Numerical Optimization’, 1999, pg. 59-60.

For the zoom phase it uses an algorithm by [...].

`scipy.optimize.show_options(solver=None, method=None, disp=True)`

Show documentation for additional options of optimization solvers.

These are method-specific options that can be supplied through the `options` dict.

**Parameters**

**solver** : str  
Type of optimization solver. One of ‘minimize’, ‘minimize\_scalar’, ‘root’, or ‘linprog’.

**method** : str, optional  
If not given, shows all methods of the specified solver. Otherwise, show only the options for the specified method. Valid values corresponds to methods’ names of respective solver (e.g. ‘BFGS’ for ‘minimize’).

**disp** : bool, optional  
Whether to print the result rather than returning it.

**Returns**

text  
Either None (for `disp=False`) or the text string (`disp=True`)

**Notes**

The solver-specific methods are:

`scipy.optimize.minimize`

- Nelder-Mead
- Powell
- CG
- BFGS
- Newton-CG
- L-BFGS-B
- TNC
- COBYLA
- SLSQP
- dogleg
- trust-ncg

`scipy.optimize.root`

- hybr
- lm
- broyden1

- broyden2*
- anderson*
- linearmixing*
- diagbroyden*
- excitingmixing*
- krylov*
- df-sane*

```
scipy.optimize.minimize_scalar
```

- brent*
- golden*
- bounded*

```
scipy.optimize.linprog
```

- simplex*

**class** `scipy.optimize.LbfgsInvHessProduct` (*sk*, *yk*)  
 Linear operator for the L-BFGS approximate inverse Hessian.

This operator computes the product of a vector with the approximate inverse of the Hessian of the objective function, using the L-BFGS limited memory approximation to the inverse Hessian, accumulated during the optimization.

Objects of this class implement the `scipy.sparse.linalg.LinearOperator` interface.

**Parameters**

- sk** : array\_like, shape=(*n\_corr*, *n*)  
 Array of *n\_corr* most recent updates to the solution vector. (See [1]).
- yk** : array\_like, shape=(*n\_corr*, *n*)  
 Array of *n\_corr* most recent updates to the gradient. (See [1]).

### References

[R143]

### Attributes

---

H	Hermitian adjoint.
T	Transpose this linear operator.

---

`LbfgsInvHessProduct.H`

Hermitian adjoint.

Returns the Hermitian adjoint of self, aka the Hermitian conjugate or Hermitian transpose. For a complex matrix, the Hermitian adjoint is equal to the conjugate transpose.

Can be abbreviated `self.H` instead of `self.adjoint()`.

**Returns**     **A\_H** : LinearOperator  
 Hermitian adjoint of self.

`LbfgsInvHessProduct.T`

Transpose this linear operator.

Returns a LinearOperator that represents the transpose of this one. Can be abbreviated `self.T` instead of `self.transpose()`.

### Methods

---

`__call__(x)`

Continued on next page

Table 5.119 – continued from previous page

<code>adjoint()</code>	Hermitian adjoint.
<code>dot(x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>matmat(X)</code>	Matrix-matrix multiplication.
<code>matvec(x)</code>	Matrix-vector multiplication.
<code>rmatvec(x)</code>	Adjoint matrix-vector multiplication.
<code>todense()</code>	Return a dense array representation of this operator.
<code>transpose()</code>	Transpose this linear operator.

`LbfgsInvHessProduct.__call__(x)`

`LbfgsInvHessProduct.adjoint()`

Hermitian adjoint.

Returns the Hermitian adjoint of self, aka the Hermitian conjugate or Hermitian transpose. For a complex matrix, the Hermitian adjoint is equal to the conjugate transpose.

Can be abbreviated self.H instead of self.adjoint().

**Returns** **A\_H** : LinearOperator  
Hermitian adjoint of self.

`LbfgsInvHessProduct.dot(x)`

Matrix-matrix or matrix-vector multiplication.

**Parameters** **x** : array\_like  
1-d or 2-d array, representing a vector or matrix.

**Returns** **Ax** : array  
1-d or 2-d array (depending on the shape of x) that represents the result of applying this linear operator on x.

`LbfgsInvHessProduct.matmat(X)`

Matrix-matrix multiplication.

Performs the operation  $y=A*X$  where A is an  $M \times N$  linear operator and X dense  $N \times K$  matrix or ndarray.

**Parameters** **X** : {matrix, ndarray}  
An array with shape (N,K).

**Returns** **Y** : {matrix, ndarray}  
A matrix or ndarray with shape (M,K) depending on the type of the X argument.

**Notes**

This matmat wraps any user-specified matmat routine or overridden `_matmat` method to ensure that y has the correct type.

`LbfgsInvHessProduct.matvec(x)`

Matrix-vector multiplication.

Performs the operation  $y=A*x$  where A is an  $M \times N$  linear operator and x is a column vector or 1-d array.

**Parameters** **x** : {matrix, ndarray}  
An array with shape (N,) or (N,1).

**Returns** **y** : {matrix, ndarray}  
A matrix or ndarray with shape (M,) or (M,1) depending on the type and shape of the x argument.



*Notes*

This matvec wraps the user-specified matvec routine or overridden `_matvec` method to ensure that `y` has the correct shape and type.

`LbfgsInvHessProduct.rmatvec(x)`  
 Adjoint matrix-vector multiplication.

Performs the operation  $y = A^H * x$  where  $A$  is an  $M \times N$  linear operator and  $x$  is a column vector or 1-d array.

**Parameters** `x` : {matrix, ndarray}

**Returns** `y` : {matrix, ndarray} An array with shape (M,) or (M,1).

A matrix or ndarray with shape (N,) or (N,1) depending on the type and shape of the `x` argument.

*Notes*

This rmatvec wraps the user-specified rmatvec routine or overridden `_rmatvec` method to ensure that `y` has the correct shape and type.

`LbfgsInvHessProduct.todense()`  
 Return a dense array representation of this operator.

**Returns** `arr` : ndarray, shape=(n, n)

An array with the same shape and containing the same data represented by this *LinearOperator*.

`LbfgsInvHessProduct.ttranspose()`  
 Transpose this linear operator.

Returns a *LinearOperator* that represents the transpose of this one. Can be abbreviated `self.T` instead of `self.transpose()`.

## 5.19 Nonlinear solvers

This is a collection of general-purpose nonlinear multidimensional solvers. These solvers find  $x$  for which  $F(x) = 0$ . Both  $x$  and  $F$  can be multidimensional.

### 5.19.1 Routines

Large-scale nonlinear solvers:

<code>newton_krylov(F, xin[, iter, rdiff, method, ...])</code>	Find a root of a function, using Krylov approximation for inverse Jacobian.
<code>anderson(F, xin[, iter, alpha, w0, M, ...])</code>	Find a root of a function, using (extended) Anderson mixing.

General nonlinear solvers:

<code>broyden1(F, xin[, iter, alpha, ...])</code>	Find a root of a function, using Broyden's first Jacobian approximation.
<code>broyden2(F, xin[, iter, alpha, ...])</code>	Find a root of a function, using Broyden's second Jacobian approximation.

Simple iterations:

<code>excitingmixing(F, xin[, iter, alpha, ...])</code>	Find a root of a function, using a tuned diagonal Jacobian approximation.
---	---

Continued on next page

Table 5.122 – continued from previous page

<code>linear mixing(F, xin[, iter, alpha, verbose, ...])</code>	Find a root of a function, using a scalar Jacobian approximation.
<code>diagbroyden(F, xin[, iter, alpha, verbose, ...])</code>	Find a root of a function, using diagonal Broyden Jacobian approximation.

## 5.19.2 Examples

### Small problem

```
>>> def F(x):
...     return np.cos(x) + x[:, :-1] - [1, 2, 3, 4]
>>> import scipy.optimize
>>> x = scipy.optimize.broyden1(F, [1, 1, 1, 1], f_tol=1e-14)
>>> x
array([ 4.04674914,  3.91158389,  2.71791677,  1.61756251])
>>> np.cos(x) + x[:, :-1]
array([ 1.,  2.,  3.,  4.])
```

### Large problem

Suppose that we needed to solve the following integrodifferential equation on the square  $[0, 1] \times [0, 1]$ :

$$\nabla^2 P = 10 \left( \int_0^1 \int_0^1 \cosh(P) dx dy \right)^2$$

with  $P(x, 1) = 1$  and  $P = 0$  elsewhere on the boundary of the square.

The solution can be found using the `newton_krylov` solver:

```
import numpy as np
from scipy.optimize import newton_krylov
from numpy import cosh, zeros_like, mgrid, zeros

# parameters
nx, ny = 75, 75
hx, hy = 1./(nx-1), 1./(ny-1)

P_left, P_right = 0, 0
P_top, P_bottom = 1, 0

def residual(P):
    d2x = zeros_like(P)
    d2y = zeros_like(P)

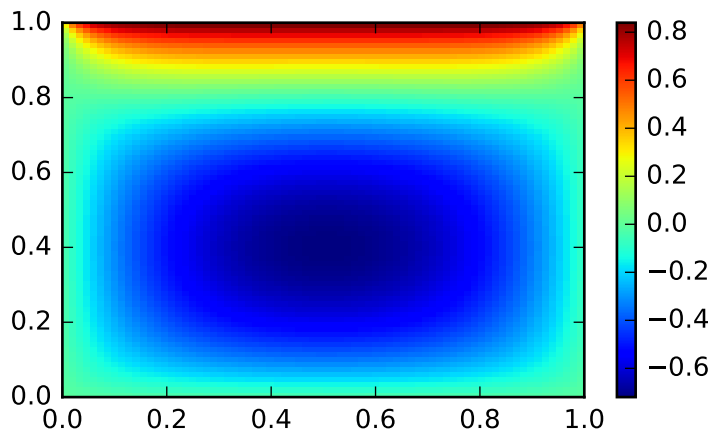
    d2x[1:-1] = (P[2:] - 2*P[1:-1] + P[:-2]) / hx/hx
    d2x[0] = (P[1] - 2*P[0] + P_left) / hx/hx
    d2x[-1] = (P_right - 2*P[-1] + P[-2]) / hx/hx

    d2y[:, 1:-1] = (P[:, 2:] - 2*P[:, 1:-1] + P[:, :-2]) / hy/hy
    d2y[:, 0] = (P[:, 1] - 2*P[:, 0] + P_bottom) / hy/hy
    d2y[:, -1] = (P_top - 2*P[:, -1] + P[:, -2]) / hy/hy

    return d2x + d2y - 10*cosh(P).mean()**2

# solve
guess = zeros((nx, ny), float)
sol = newton_krylov(residual, guess, method='lgmres', verbose=1)
print('Residual: %g' % abs(residual(sol)).max())
```

```
# visualize
import matplotlib.pyplot as plt
x, y = mgrid[0:1:(nx*1j), 0:1:(ny*1j)]
plt.pcolor(x, y, sol)
plt.colorbar()
plt.show()
```



## 5.20 Signal processing (`scipy.signal`)

### 5.20.1 Convolution

<code>convolve(in1, in2[, mode])</code>	Convolve two N-dimensional arrays.
<code>correlate(in1, in2[, mode])</code>	Cross-correlate two N-dimensional arrays.
<code>fftconvolve(in1, in2[, mode])</code>	Convolve two N-dimensional arrays using FFT.
<code>convolve2d(in1, in2[, mode, boundary, fillvalue])</code>	Convolve two 2-dimensional arrays.
<code>correlate2d(in1, in2[, mode, boundary, ...])</code>	Cross-correlate two 2-dimensional arrays.
<code>sepfir2d((input, hrow, hcol) -&gt; output)</code>	Description:

`scipy.signal.convolve` (*in1*, *in2*, *mode*='full')

Convolve two N-dimensional arrays.

Convolve *in1* and *in2*, with the output size determined by the *mode* argument.

**Parameters** **in1** : array\_like

First input.

**in2** : array\_like

Second input. Should have the same number of dimensions as *in1*. If operating in 'valid' mode, either *in1* or *in2* must be at least as large as the other in every dimension.

**mode** : str {'full', 'valid', 'same'}, optional

A string indicating the size of the output:

	<b>full</b>	The output is the full discrete linear convolution of the inputs. (Default)
	<b>valid</b>	The output consists only of those elements that do not rely on the zero-padding.
	<b>same</b>	The output is the same size as <i>in1</i> , centered with respect to the 'full' output.
<b>Returns</b>	<b>convolve</b> : array	An N-dimensional array containing a subset of the discrete linear convolution of <i>in1</i> with <i>in2</i> .

**See also:**

`numpy.polymul`

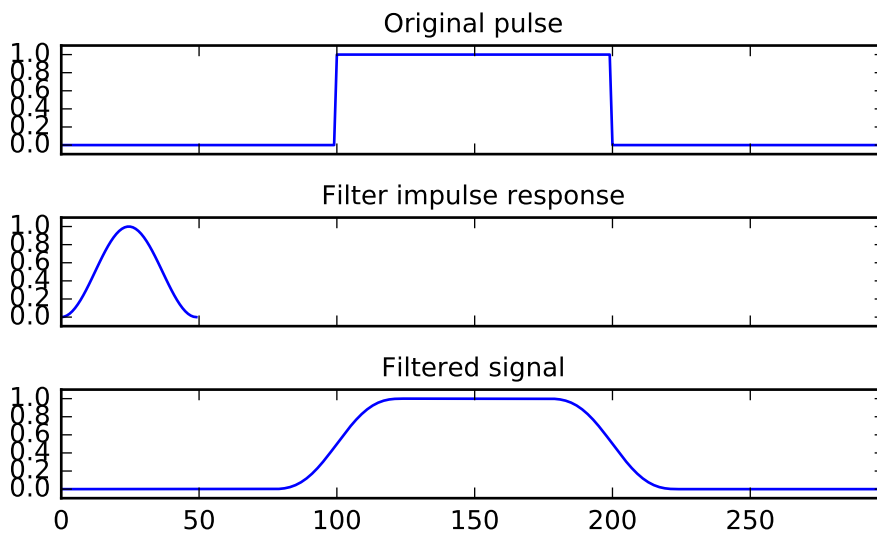
performs polynomial multiplication (same operation, but also accepts poly1d objects)

**Examples**

Smooth a square pulse using a Hann window:

```
>>> from scipy import signal
>>> sig = np.repeat([0., 1., 0.], 100)
>>> win = signal.hann(50)
>>> filtered = signal.convolve(sig, win, mode='same') / sum(win)

>>> import matplotlib.pyplot as plt
>>> fig, (ax_orig, ax_win, ax_filt) = plt.subplots(3, 1, sharex=True)
>>> ax_orig.plot(sig)
>>> ax_orig.set_title('Original pulse')
>>> ax_orig.margins(0, 0.1)
>>> ax_win.plot(win)
>>> ax_win.set_title('Filter impulse response')
>>> ax_win.margins(0, 0.1)
>>> ax_filt.plot(filtered)
>>> ax_filt.set_title('Filtered signal')
>>> ax_filt.margins(0, 0.1)
>>> fig.tight_layout()
>>> fig.show()
```



`scipy.signal.correlate` (*in1*, *in2*, *mode*='full')

Cross-correlate two N-dimensional arrays.

Cross-correlate *in1* and *in2*, with the output size determined by the *mode* argument.

**Parameters**

**in1** : array\_like  
First input.

**in2** : array\_like  
Second input. Should have the same number of dimensions as *in1*. If operating in 'valid' mode, either *in1* or *in2* must be at least as large as the other in every dimension.

**mode** : str {'full', 'valid', 'same'}, optional  
A string indicating the size of the output:

**full**      The output is the full discrete linear cross-correlation of the inputs. (Default)

**valid**      The output consists only of those elements that do not rely on the zero-padding.

**same**      The output is the same size as *in1*, centered with respect to the 'full' output.

**Returns**

**correlate** : array  
An N-dimensional array containing a subset of the discrete linear cross-correlation of *in1* with *in2*.

#### Notes

The correlation *z* of two d-dimensional arrays *x* and *y* is defined as:

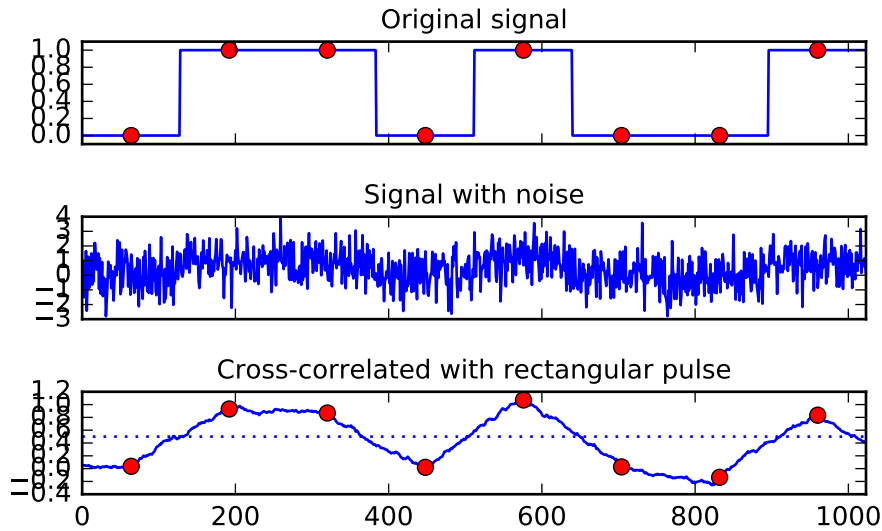
$$z[...k,...] = \text{sum}[..., i_1, ...] \\ x[..., i_1, ...] * \text{conj}(y[..., i_1 + k, ...])$$

#### Examples

Implement a matched filter using cross-correlation, to recover a signal that has passed through a noisy channel.

```
>>> from scipy import signal
>>> sig = np.repeat([0., 1., 1., 0., 1., 0., 0., 1.], 128)
>>> sig_noise = sig + np.random.randn(len(sig))
>>> corr = signal.correlate(sig_noise, np.ones(128), mode='same') / 128

>>> import matplotlib.pyplot as plt
>>> clock = np.arange(64, len(sig), 128)
>>> fig, (ax_orig, ax_noise, ax_corr) = plt.subplots(3, 1, sharex=True)
>>> ax_orig.plot(sig)
>>> ax_orig.plot(clock, sig[clock], 'ro')
>>> ax_orig.set_title('Original signal')
>>> ax_noise.plot(sig_noise)
>>> ax_noise.set_title('Signal with noise')
>>> ax_corr.plot(corr)
>>> ax_corr.plot(clock, corr[clock], 'ro')
>>> ax_corr.axhline(0.5, ls=':')
>>> ax_corr.set_title('Cross-correlated with rectangular pulse')
>>> ax_orig.margins(0, 0.1)
>>> fig.tight_layout()
>>> fig.show()
```



`scipy.signal.fftconvolve` (*in1*, *in2*, *mode*='full')

Convolve two N-dimensional arrays using FFT.

Convolve *in1* and *in2* using the fast Fourier transform method, with the output size determined by the *mode* argument.

This is generally much faster than `convolve` for large arrays ( $n > \sim 500$ ), but can be slower when only a few output values are needed, and can only output float arrays (int or object array inputs will be cast to float).

**Parameters**

- in1** : array\_like  
First input.
- in2** : array\_like  
Second input. Should have the same number of dimensions as *in1*. If operating in 'valid' mode, either *in1* or *in2* must be at least as large as the other in every dimension.
- mode** : str {'full', 'valid', 'same'}, optional  
A string indicating the size of the output:
  - full** : The output is the full discrete linear convolution of the inputs. (Default)
  - valid** : The output consists only of those elements that do not rely on the zero-padding.
  - same** : The output is the same size as *in1*, centered with respect to the 'full' output.

**Returns**

- out** : array  
An N-dimensional array containing a subset of the discrete linear convolution of *in1* with *in2*.

### Examples

Autocorrelation of white noise is an impulse. (This is at least 100 times as fast as `convolve`.)

```
>>> from scipy import signal
>>> sig = np.random.randn(1000)
>>> autocorr = signal.fftconvolve(sig, sig[::-1], mode='full')

>>> import matplotlib.pyplot as plt
>>> fig, (ax_orig, ax_mag) = plt.subplots(2, 1)
>>> ax_orig.plot(sig)
```

```

>>> ax_orig.set_title('White noise')
>>> ax_mag.plot(np.arange(-len(sig)+1,len(sig)), autocorr)
>>> ax_mag.set_title('Autocorrelation')
>>> fig.tight_layout()
>>> fig.show()

```

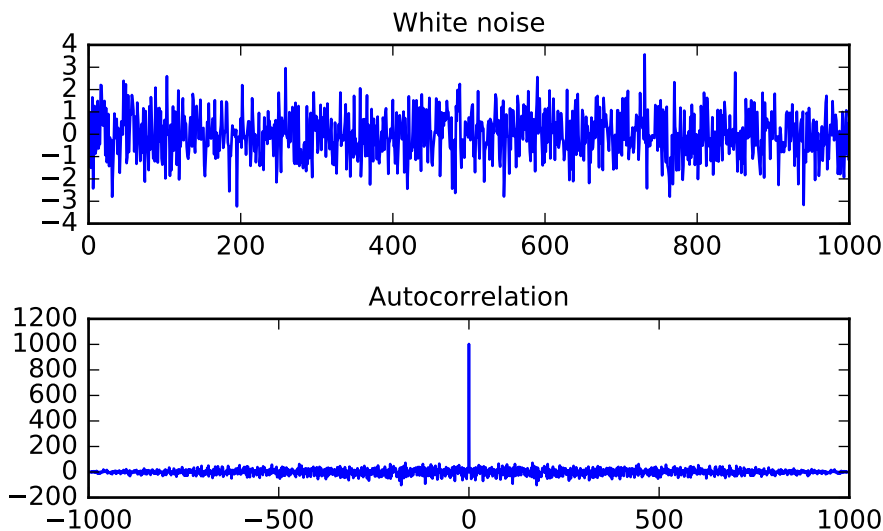
Gaussian blur implemented using FFT convolution. Notice the dark borders around the image, due to the zero-padding beyond its boundaries. The `convolve2d` function allows for other types of image boundaries, but is far slower.

```

>>> from scipy import misc
>>> face = misc.face(gray=True)
>>> kernel = np.outer(signal.gaussian(70, 8), signal.gaussian(70, 8))
>>> blurred = signal.fftconvolve(face, kernel, mode='same')

>>> fig, (ax_orig, ax_kernel, ax_blurred) = plt.subplots(3, 1,
...                                                    figsize=(6, 15))
>>> ax_orig.imshow(face, cmap='gray')
>>> ax_orig.set_title('Original')
>>> ax_orig.set_axis_off()
>>> ax_kernel.imshow(kernel, cmap='gray')
>>> ax_kernel.set_title('Gaussian kernel')
>>> ax_kernel.set_axis_off()
>>> ax_blurred.imshow(blurred, cmap='gray')
>>> ax_blurred.set_title('Blurred')
>>> ax_blurred.set_axis_off()
>>> fig.show()

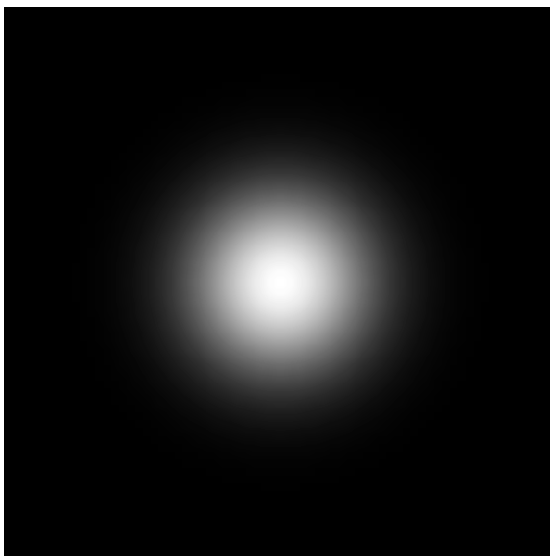
```



Original



Gaussian kernel



Blurred





`scipy.signal.convolve2d`(*in1*, *in2*, *mode*='full', *boundary*='fill', *fillvalue*=0)

Convolve two 2-dimensional arrays.

Convolve *in1* and *in2* with output size determined by *mode*, and boundary conditions determined by *boundary* and *fillvalue*.

**Parameters**

**in1** : array\_like  
First input.

**in2** : array\_like  
Second input. Should have the same number of dimensions as *in1*. If operating in 'valid' mode, either *in1* or *in2* must be at least as large as the other in every dimension.

**mode** : str {'full', 'valid', 'same'}, optional  
A string indicating the size of the output:  
**full**        The output is the full discrete linear convolution of the inputs. (Default)  
**valid**        The output consists only of those elements that do not rely on the zero-padding.  
**same**         The output is the same size as *in1*, centered with respect to the 'full' output.

**boundary** : str {'fill', 'wrap', 'symm'}, optional  
A flag indicating how to handle boundaries:  
**fill**         pad input arrays with *fillvalue*. (default)  
**wrap**         circular boundary conditions.  
**symm**         symmetrical boundary conditions.

**fillvalue** : scalar, optional  
Value to fill pad input arrays with. Default is 0.

**Returns**

**out** : ndarray  
A 2-dimensional array containing a subset of the discrete linear convolution of *in1* with *in2*.

### Examples

Compute the gradient of an image by 2D convolution with a complex Scharr operator. (Horizontal operator is real, vertical is imaginary.) Use symmetric boundary condition to avoid creating edges at the image boundaries.

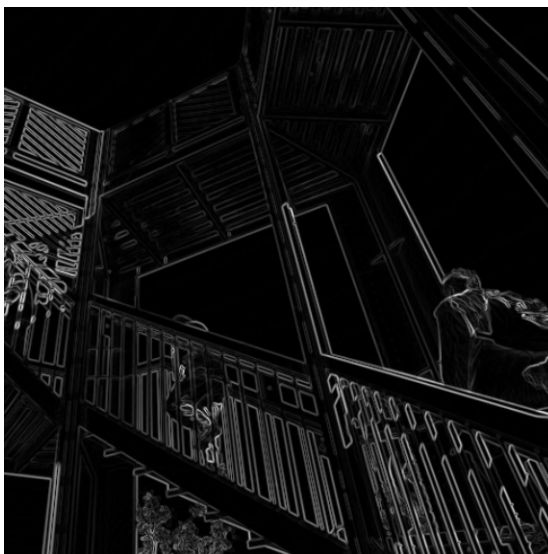
```
>>> from scipy import signal
>>> from scipy import misc
>>> ascent = misc.ascent()
>>> scharr = np.array([[ -3-3j,  0-10j,  +3 -3j],
...                   [-10+0j,  0+ 0j,  +10 +0j],
...                   [ -3+3j,  0+10j,  +3 +3j]]) # Gx + j*Gy
>>> grad = signal.convolve2d(ascent, scharr, boundary='symm', mode='same')

>>> import matplotlib.pyplot as plt
>>> fig, (ax_orig, ax_mag, ax_ang) = plt.subplots(3, 1, figsize=(6, 15))
>>> ax_orig.imshow(ascent, cmap='gray')
>>> ax_orig.set_title('Original')
>>> ax_orig.set_axis_off()
>>> ax_mag.imshow(np.absolute(grad), cmap='gray')
>>> ax_mag.set_title('Gradient magnitude')
>>> ax_mag.set_axis_off()
>>> ax_ang.imshow(np.angle(grad), cmap='hsv') # hsv is cyclic, like angles
>>> ax_ang.set_title('Gradient orientation')
>>> ax_ang.set_axis_off()
>>> fig.show()
```

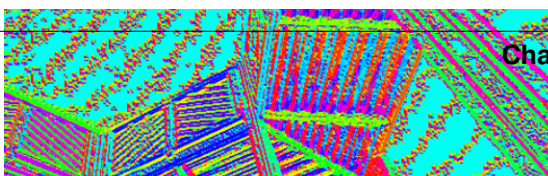
Original



Gradient magnitude



Gradient orientation



`scipy.signal.correlate2d` (*in1*, *in2*, *mode*='full', *boundary*='fill', *fillvalue*=0)

Cross-correlate two 2-dimensional arrays.

Cross correlate *in1* and *in2* with output size determined by *mode*, and boundary conditions determined by *boundary* and *fillvalue*.

**Parameters**

**in1** : array\_like  
First input.

**in2** : array\_like  
Second input. Should have the same number of dimensions as *in1*. If operating in 'valid' mode, either *in1* or *in2* must be at least as large as the other in every dimension.

**mode** : str {'full', 'valid', 'same'}, optional  
A string indicating the size of the output:  
**full**        The output is the full discrete linear cross-correlation of the inputs. (Default)  
**valid**        The output consists only of those elements that do not rely on the zero-padding.  
**same**         The output is the same size as *in1*, centered with respect to the 'full' output.

**boundary** : str {'fill', 'wrap', 'symm'}, optional  
A flag indicating how to handle boundaries:  
**fill**         pad input arrays with *fillvalue*. (default)  
**wrap**         circular boundary conditions.  
**symm**         symmetrical boundary conditions.

**fillvalue** : scalar, optional  
Value to fill pad input arrays with. Default is 0.

**Returns**

**correlate2d** : ndarray  
A 2-dimensional array containing a subset of the discrete linear cross-correlation of *in1* with *in2*.

### Examples

Use 2D cross-correlation to find the location of a template in a noisy image:

```
>>> from scipy import signal
>>> from scipy import misc
>>> face = misc.face(gray=True) - misc.face(gray=True).mean()
>>> template = np.copy(face[300:365, 670:750]) # right eye
>>> template -= template.mean()
>>> face = face + np.random.randn(*face.shape) * 50 # add noise
>>> corr = signal.correlate2d(face, template, boundary='symm', mode='same')
>>> y, x = np.unravel_index(np.argmax(corr), corr.shape) # find the match

>>> import matplotlib.pyplot as plt
>>> fig, (ax_orig, ax_template, ax_corr) = plt.subplots(3, 1,
...                                                figsize=(6, 15))
>>> ax_orig.imshow(face, cmap='gray')
>>> ax_orig.set_title('Original')
>>> ax_orig.set_axis_off()
>>> ax_template.imshow(template, cmap='gray')
>>> ax_template.set_title('Template')
>>> ax_template.set_axis_off()
>>> ax_corr.imshow(corr, cmap='gray')
>>> ax_corr.set_title('Cross-correlation')
>>> ax_corr.set_axis_off()
>>> ax_orig.plot(x, y, 'ro')
>>> fig.show()
```

Original



Template



Cross-correlation

`scipy.signal.sepfir2d(input, hrow, hcol) → output`

Description:

Convolve the rank-2 input array with the separable filter defined by the rank-1 arrays `hrow`, and `hcol`. Mirror symmetric boundary conditions are assumed. This function can be used to find an image given its B-spline representation.

## 5.20.2 B-splines

<code>bspline(x, n)</code>	B-spline basis function of order <code>n</code> .
<code>cubic(x)</code>	A cubic B-spline.
<code>quadratic(x)</code>	A quadratic B-spline.
<code>gauss_spline(x, n)</code>	Gaussian approximation to B-spline basis function of order <code>n</code> .
<code>cspline1d(signal[, lamb])</code>	Compute cubic spline coefficients for rank-1 array.
<code>qspline1d(signal[, lamb])</code>	Compute quadratic spline coefficients for rank-1 array.
<code>cspline2d((input {, lambda, precision}) -&gt; ck)</code>	Description:
<code>qspline2d((input {, lambda, precision}) -&gt; qk)</code>	Description:
<code>cspline1d_eval(cj, newx[, dx, x0])</code>	Evaluate a spline at the new set of points.
<code>qspline1d_eval(cj, newx[, dx, x0])</code>	Evaluate a quadratic spline at the new set of points.
<code>spline_filter(lin[, lmbda])</code>	Smoothing spline (cubic) filtering of a rank-2 array.

`scipy.signal.bspline(x, n)`

B-spline basis function of order `n`.

### Notes

Uses `numpy.piecewise` and automatic function-generator.

`scipy.signal.cubic(x)`

A cubic B-spline.

This is a special case of `bspline`, and equivalent to `bspline(x, 3)`.

`scipy.signal.quadratic(x)`

A quadratic B-spline.

This is a special case of `bspline`, and equivalent to `bspline(x, 2)`.

`scipy.signal.gauss_spline(x, n)`

Gaussian approximation to B-spline basis function of order `n`.

`scipy.signal.cspline1d(signal, lamb=0.0)`

Compute cubic spline coefficients for rank-1 array.

Find the cubic spline coefficients for a 1-D signal assuming mirror-symmetric boundary conditions. To obtain the signal back from the spline representation mirror-symmetric-convolve these coefficients with a length 3 FIR window `[1.0, 4.0, 1.0]/6.0`.

**Parameters**

- signal** : ndarray  
A rank-1 array representing samples of a signal.
- lamb** : float, optional  
Smoothing coefficient, default is 0.0.

**Returns**

- c** : ndarray  
Cubic spline coefficients.

`scipy.signal.qspline1d(signal, lamb=0.0)`

Compute quadratic spline coefficients for rank-1 array.

Find the quadratic spline coefficients for a 1-D signal assuming mirror-symmetric boundary conditions. To obtain the signal back from the spline representation mirror-symmetric-convolve these coefficients with a length 3 FIR window  $[1.0, 6.0, 1.0]/8.0$ .

**Parameters**

**signal** : ndarray  
A rank-1 array representing samples of a signal.

**lamb** : float, optional  
Smoothing coefficient (must be zero for now).

**Returns**

**c** : ndarray  
Cubic spline coefficients.

`scipy.signal.cspline2d(input {, lambda, precision})` → ck

Description:

Return the third-order B-spline coefficients over a regularly spaced input grid for the two-dimensional input image. The lambda argument specifies the amount of smoothing. The precision argument allows specifying the precision used when computing the infinite sum needed to apply mirror-symmetric boundary conditions.

`scipy.signal.qspline2d(input {, lambda, precision})` → qk

Description:

Return the second-order B-spline coefficients over a regularly spaced input grid for the two-dimensional input image. The lambda argument specifies the amount of smoothing. The precision argument allows specifying the precision used when computing the infinite sum needed to apply mirror-symmetric boundary conditions.

`scipy.signal.cspline1d_eval(cj, newx, dx=1.0, x0=0)`

Evaluate a spline at the new set of points.

$dx$  is the old sample-spacing while  $x0$  was the old origin. In other-words the old-sample points (knot-points) for which the  $cj$  represent spline coefficients were at equally-spaced points of:

$$\text{oldx} = x0 + j*dx \quad j=0\dots N-1, \text{ with } N=\text{len}(cj)$$

Edges are handled using mirror-symmetric boundary conditions.

`scipy.signal.qspline1d_eval(cj, newx, dx=1.0, x0=0)`

Evaluate a quadratic spline at the new set of points.

$dx$  is the old sample-spacing while  $x0$  was the old origin. In other-words the old-sample points (knot-points) for which the  $cj$  represent spline coefficients were at equally-spaced points of:

$$\text{oldx} = x0 + j*dx \quad j=0\dots N-1, \text{ with } N=\text{len}(cj)$$

Edges are handled using mirror-symmetric boundary conditions.

`scipy.signal.spline_filter(lin, lambda=5.0)`

Smoothing spline (cubic) filtering of a rank-2 array.

Filter an input data set,  $lin$ , using a (cubic) smoothing spline of fall-off  $lambda$ .

### 5.20.3 Filtering

`order_filter(a, domain, rank)`

Perform an order filter on an N-dimensional array.

`medfilt(volume[, kernel_size])`

Perform a median filter on an N-dimensional array.

`medfilt2d(input[, kernel_size])`

Median filter a 2-dimensional array.

`wiener(im[, msize, noise])`

Perform a Wiener filter on an N-dimensional array.

`symiirorder1((input, c0, z1 {, ...}))`

Implement a smoothing IIR filter with mirror-symmetric boundary conditions.

Table 5.125 – continued from previous page

<code>symiirorder2((input, r, omega {, ...)</code>	Implement a smoothing IIR filter with mirror-symmetric boundary condition
<code>lfilter(b, a, x[, axis, zi])</code>	Filter data along one-dimension with an IIR or FIR filter.
<code>lfiltic(b, a, y[, x])</code>	Construct initial conditions for lfilter.
<code>lfilter_zi(b, a)</code>	Compute an initial state <i>zi</i> for the lfilter function that corresponds to the steady state response.
<code>filtfilt(b, a, x[, axis, padtype, padlen, ...])</code>	A forward-backward filter.
<code>savgol_filter(x, window_length, polyorder[, ...])</code>	Apply a Savitzky-Golay filter to an array.
<code>deconvolve(signal, divisor)</code>	Deconvolves <i>divisor</i> out of <i>signal</i> .
<code>sosfilt(sos, x[, axis, zi])</code>	Filter data along one dimension using cascaded second-order sections
<code>sosfilt_zi(sos)</code>	Compute an initial state <i>zi</i> for the sosfilt function that corresponds to the steady state response.
<code>sosfiltfilt(sos, x[, axis, padtype, padlen])</code>	A forward-backward filter using cascaded second-order sections.
<code>hilbert(x[, N, axis])</code>	Compute the analytic signal, using the Hilbert transform.
<code>hilbert2(x[, N])</code>	Compute the ‘2-D’ analytic signal of <i>x</i>
<code>decimate(x, q[, n, ftype, axis, zero_phase])</code>	Downsample the signal after applying an anti-aliasing filter.
<code>detrend(data[, axis, type, bp])</code>	Remove linear trend along axis from data.
<code>resample(x, num[, t, axis, window])</code>	Resample <i>x</i> to <i>num</i> samples using Fourier method along the given axis.
<code>resample_poly(x, up, down[, axis, window])</code>	Resample <i>x</i> along the given axis using polyphase filtering.
<code>upfirdn(h, x[, up, down, axis])</code>	Upsample, FIR filter, and downsample

`scipy.signal.order_filter(a, domain, rank)`

Perform an order filter on an N-dimensional array.

Perform an order filter on the array *in*. The domain argument acts as a mask centered over each pixel. The non-zero elements of domain are used to select elements surrounding each input pixel which are placed in a list. The list is sorted, and the output for that pixel is the element corresponding to rank in the sorted list.

**Parameters** **a** : ndarray

The N-dimensional input array.

**domain** : array\_like

A mask array with the same number of dimensions as *a*. Each dimension should have an odd number of elements.

**rank** : int

A non-negative integer which selects the element from the sorted list (0 corresponds to the smallest element, 1 is the next smallest element, etc.).

**Returns** **out** : ndarray

The results of the order filter in an array with the same shape as *a*.

### Examples

```
>>> from scipy import signal
>>> x = np.arange(25).reshape(5, 5)
>>> domain = np.identity(3)
>>> x
array([[ 0,  1,  2,  3,  4],
       [ 5,  6,  7,  8,  9],
       [10, 11, 12, 13, 14],
       [15, 16, 17, 18, 19],
       [20, 21, 22, 23, 24]])
>>> signal.order_filter(x, domain, 0)
array([[ 0.,  0.,  0.,  0.,  0.],
       [ 0.,  0.,  1.,  2.,  0.],
       [ 0.,  5.,  6.,  7.,  0.],
       [ 0., 10., 11., 12.,  0.],
       [ 0.,  0.,  0.,  0.,  0.]])
>>> signal.order_filter(x, domain, 2)
array([[ 6.,  7.,  8.,  9.,  4.]])
```



```
[ 11., 12., 13., 14., 9.],
 [ 16., 17., 18., 19., 14.],
 [ 21., 22., 23., 24., 19.],
 [ 20., 21., 22., 23., 24.]])
```

`scipy.signal.medfilt` (*volume*, *kernel\_size=None*)

Perform a median filter on an N-dimensional array.

Apply a median filter to the input array using a local window-size given by *kernel\_size*.

**Parameters**

- volume** : array\_like  
An N-dimensional input array.
- kernel\_size** : array\_like, optional  
A scalar or an N-length list giving the size of the median filter window in each dimension. Elements of *kernel\_size* should be odd. If *kernel\_size* is a scalar, then this scalar is used as the size in each dimension. Default size is 3 for each dimension.

**Returns**

- out** : ndarray  
An array the same size as input containing the median filtered result.

`scipy.signal.medfilt2d` (*input*, *kernel\_size=3*)

Median filter a 2-dimensional array.

Apply a median filter to the *input* array using a local window-size given by *kernel\_size* (must be odd).

**Parameters**

- input** : array\_like  
A 2-dimensional input array.
- kernel\_size** : array\_like, optional  
A scalar or a list of length 2, giving the size of the median filter window in each dimension. Elements of *kernel\_size* should be odd. If *kernel\_size* is a scalar, then this scalar is used as the size in each dimension. Default is a kernel of size (3, 3).

**Returns**

- out** : ndarray  
An array the same size as input containing the median filtered result.

`scipy.signal.wiener` (*im*, *mysize=None*, *noise=None*)

Perform a Wiener filter on an N-dimensional array.

Apply a Wiener filter to the N-dimensional array *im*.

**Parameters**

- im** : ndarray  
An N-dimensional array.
- mysize** : int or array\_like, optional  
A scalar or an N-length list giving the size of the Wiener filter window in each dimension. Elements of *mysize* should be odd. If *mysize* is a scalar, then this scalar is used as the size in each dimension.
- noise** : float, optional  
The noise-power to use. If None, then noise is estimated as the average of the local variance of the input.

**Returns**

- out** : ndarray  
Wiener filtered result with the same shape as *im*.

`scipy.signal.symiirorder1` (*input*, *c0*, *z1* {, *precision*}) → output

Implement a smoothing IIR filter with mirror-symmetric boundary conditions using a cascade of first-order sections. The second section uses a reversed sequence. This implements a system with the following transfer function and mirror-symmetric boundary conditions:

$$H(z) = \frac{c0}{(1-z1/z)(1-z1z)}$$



The resulting signal will have mirror symmetric boundary conditions as well.

**Parameters**

- input** : ndarray  
The input signal.
- c0, z1** : scalar  
Parameters in the transfer function.
- precision** :  
Specifies the precision for calculating initial conditions of the recursive filter based on mirror-symmetric input.

**Returns**

- output** : ndarray  
The filtered signal.

`scipy.signal.symiirorder2(input, r, omega {, precision})` → output

Implement a smoothing IIR filter with mirror-symmetric boundary conditions using a cascade of second-order sections. The second section uses a reversed sequence. This implements the following transfer function:

$$H(z) = \frac{cs^2}{(1 - a2/z - a3/z^2) (1 - a2 z - a3 z^2)}$$

where:

$$\begin{aligned} a2 &= (2 r \cos \omega) \\ a3 &= - r^2 \\ cs &= 1 - 2 r \cos \omega + r^2 \end{aligned}$$

**Parameters**

- input** : ndarray  
The input signal.
- r, omega** : scalar  
Parameters in the transfer function.
- precision** :  
Specifies the precision for calculating initial conditions of the recursive filter based on mirror-symmetric input.

**Returns**

- output** : ndarray  
The filtered signal.

`scipy.signal.lfilter(b, a, x, axis=-1, zi=None)`

Filter data along one-dimension with an IIR or FIR filter.

Filter a data sequence, *x*, using a digital filter. This works for many fundamental data types (including Object type). The filter is a direct form II transposed implementation of the standard difference equation (see Notes).

**Parameters**

- b** : array\_like  
The numerator coefficient vector in a 1-D sequence.
- a** : array\_like  
The denominator coefficient vector in a 1-D sequence. If *a*[0] is not 1, then both *a* and *b* are normalized by *a*[0].
- x** : array\_like  
An N-dimensional input array.
- axis** : int, optional  
The axis of the input data array along which to apply the linear filter. The filter is applied to each subarray along this axis. Default is -1.
- zi** : array\_like, optional  
Initial conditions for the filter delays. It is a vector (or array of vectors for an N-dimensional input) of length  $\max(\text{len}(a), \text{len}(b)) - 1$ . If *zi* is None or is not given then initial rest is assumed. See `lfiltic` for more information.

**Returns**

- y** : array  
The output of the digital filter.
- zf** : array, optional  
If *zi* is None, this is not returned, otherwise, *zf* holds the final filter delay values.

**See also:**

**lfilteric** Construct initial conditions for `lfilter`.  
**lfilter\_zi** Compute initial state (steady state of step response) for `lfilter`.  
**filtfilt** A forward-backward filter, to obtain a filter with linear phase.  
**savgol\_filter**  
 A Savitzky-Golay filter.  
**sosfilt** Filter data using cascaded second-order sections.  
**sosfiltfilt**  
 A forward-backward filter using second-order sections.

**Notes**

The filter function is implemented as a direct II transposed structure. This means that the filter implements:

$$a[0]*y[n] = b[0]*x[n] + b[1]*x[n-1] + \dots + b[M]*x[n-M] \\ - a[1]*y[n-1] - \dots - a[N]*y[n-N]$$

where *M* is the degree of the numerator, *N* is the degree of the denominator, and *n* is the sample number. It is implemented using the following difference equations (assuming *M* = *N*):

$$a[0]*y[n] = b[0] * x[n] + d[0][n-1] \\ d[0][n] = b[1] * x[n] - a[1] * y[n] + d[1][n-1] \\ d[1][n] = b[2] * x[n] - a[2] * y[n] + d[2][n-1] \\ \dots \\ d[N-2][n] = b[N-1]*x[n] - a[N-1]*y[n] + d[N-1][n-1] \\ d[N-1][n] = b[N] * x[n] - a[N] * y[n]$$

where *d* are the state variables.

The rational transfer function describing this filter in the z-transform domain is:

$$Y(z) = \frac{b[0] + b[1]z^{-1} + \dots + b[M]z^{-M}}{a[0] + a[1]z^{-1} + \dots + a[N]z^{-N}} X(z)$$

**Examples**

Generate a noisy signal to be filtered:

```
>>> from scipy import signal
>>> import matplotlib.pyplot as plt
>>> t = np.linspace(-1, 1, 201)
>>> x = (np.sin(2*np.pi*0.75*t*(1-t)) + 2.1) + 0.1*np.sin(2*np.pi*1.25*t + 1)
...     + 0.18*np.cos(2*np.pi*3.85*t))
>>> xn = x + np.random.randn(len(t)) * 0.08
```

Create an order 3 lowpass butterworth filter:

```
>>> b, a = signal.butter(3, 0.05)
```

Apply the filter to `xn`. Use `lfilter_zi` to choose the initial condition of the filter:

```
>>> zi = signal.lfilter_zi(b, a)
>>> z, _ = signal.lfilter(b, a, xn, zi=zi*xn[0])
```

Apply the filter again, to have a result filtered at an order the same as `filtfilt`:

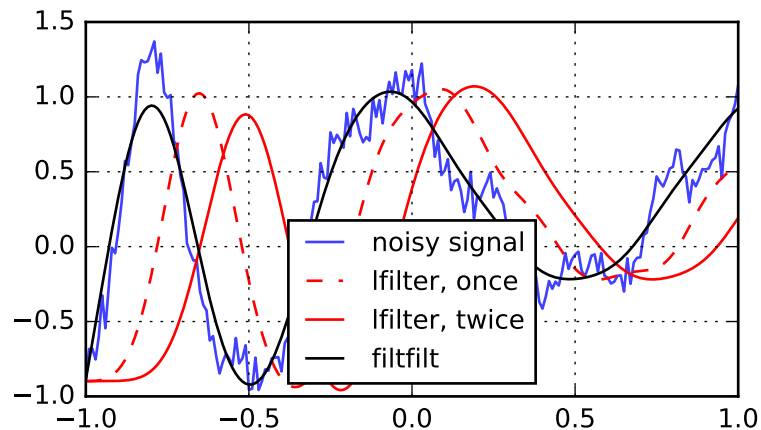
```
>>> z2, _ = signal.lfilter(b, a, z, zi=zi*z[0])
```

Use `filtfilt` to apply the filter:

```
>>> y = signal.filtfilt(b, a, xn)
```

Plot the original signal and the various filtered versions:

```
>>> plt.figure
>>> plt.plot(t, xn, 'b', alpha=0.75)
>>> plt.plot(t, z, 'r--', t, z2, 'r', t, y, 'k')
>>> plt.legend(('noisy signal', 'lfilter, once', 'lfilter, twice',
...          'filtfilt'), loc='best')
>>> plt.grid(True)
>>> plt.show()
```



```
scipy.signal.lfiltic(b, a, y, x=None)
```

Construct initial conditions for `lfilter`.

Given a linear filter (`b`, `a`) and initial conditions on the output `y` and the input `x`, return the initial conditions on the state vector `zi` which is used by `lfilter` to generate the output given the input.

**Parameters**

- b** : array\_like  
Linear filter term.
- a** : array\_like  
Linear filter term.
- y** : array\_like

Initial conditions.  
 If  $N = \text{len}(a) - 1$ , then  $y = \{y[-1], y[-2], \dots, y[-N]\}$ .  
 If  $y$  is too short, it is padded with zeros.

**x** : array\_like, optional  
 Initial conditions.  
 If  $M = \text{len}(b) - 1$ , then  $x = \{x[-1], x[-2], \dots, x[-M]\}$ .  
 If  $x$  is not given, its initial conditions are assumed zero.  
 If  $x$  is too short, it is padded with zeros.

**Returns** **zi** : ndarray  
 The state vector  $z_i = \{z_{-1}[0], z_{-1}[1], \dots, z_{-1}[K-1]\}$ , where  $K = \max(M, N)$ .

**See also:**

`lfilter`, `lfilter_zi`

`scipy.signal.lfilter_zi(b, a)`

Compute an initial state  $z_i$  for the `lfilter` function that corresponds to the steady state of the step response.

A typical use of this function is to set the initial state so that the output of the filter starts at the same value as the first element of the signal to be filtered.

**Parameters** **b, a** : array\_like (1-D)  
 The IIR filter coefficients. See `lfilter` for more information.

**Returns** **zi** : 1-D ndarray  
 The initial state for the filter.

**See also:**

`lfilter`, `lfiltic`, `filtfilt`

**Notes**

A linear filter with order  $m$  has a state space representation  $(A, B, C, D)$ , for which the output  $y$  of the filter can be expressed as:

$$\begin{aligned} z(n+1) &= A*z(n) + B*x(n) \\ y(n) &= C*z(n) + D*x(n) \end{aligned}$$

where  $z(n)$  is a vector of length  $m$ ,  $A$  has shape  $(m, m)$ ,  $B$  has shape  $(m, 1)$ ,  $C$  has shape  $(1, m)$  and  $D$  has shape  $(1, 1)$  (assuming  $x(n)$  is a scalar). `lfilter_zi` solves:

$$z_i = A*z_i + B$$

In other words, it finds the initial condition for which the response to an input of all ones is a constant.

Given the filter coefficients  $a$  and  $b$ , the state space matrices for the transposed direct form II implementation of the linear filter, which is the implementation used by `scipy.signal.lfilter`, are:

$$\begin{aligned} A &= \text{scipy.linalg.companion}(a).T \\ B &= b[1:] - a[1:]*b[0] \end{aligned}$$

assuming  $a[0]$  is 1.0; if  $a[0]$  is not 1,  $a$  and  $b$  are first divided by  $a[0]$ .

**Examples**

The following code creates a lowpass Butterworth filter. Then it applies that filter to an array whose values are all 1.0; the output is also all 1.0, as expected for a lowpass filter. If the  $z_i$  argument of `lfilter` had not been given, the output would have shown the transient signal.

```
>>> from numpy import array, ones
>>> from scipy.signal import lfilter, lfilter_zi, butter
>>> b, a = butter(5, 0.25)
>>> zi = lfilter_zi(b, a)
>>> y, zo = lfilter(b, a, ones(10), zi=zi)
>>> y
array([1., 1., 1., 1., 1., 1., 1., 1., 1., 1.])
```

Another example:

```
>>> x = array([0.5, 0.5, 0.5, 0.0, 0.0, 0.0, 0.0])
>>> y, zf = lfilter(b, a, x, zi=zi*x[0])
>>> y
array([ 0.5          ,  0.5          ,  0.5          ,  0.49836039,  0.48610528,
        0.44399389,  0.35505241])
```

Note that the *zi* argument to `lfilter` was computed using `lfilter_zi` and scaled by  $x[0]$ . Then the output *y* has no transient until the input drops from 0.5 to 0.0.

`scipy.signal.filtfilt` (*b, a, x, axis=-1, padtype='odd', padlen=None, method='pad', irlen=None*)  
A forward-backward filter.

This function applies a linear filter twice, once forward and once backwards. The combined filter has linear phase.

The function provides options for handling the edges of the signal.

When *method* is “pad”, the function pads the data along the given axis in one of three ways: odd, even or constant. The odd and even extensions have the corresponding symmetry about the end point of the data. The constant extension extends the data with the values at the end points. On both the forward and backward passes, the initial condition of the filter is found by using `lfilter_zi` and scaling it by the end point of the extended data.

When *method* is “gust”, Gustafsson’s method [R198] is used. Initial conditions are chosen for the forward and backward passes so that the forward-backward filter gives the same result as the backward-forward filter.

**Parameters**

- b** : (N,) array\_like  
The numerator coefficient vector of the filter.
- a** : (N,) array\_like  
The denominator coefficient vector of the filter. If  $a[0]$  is not 1, then both *a* and *b* are normalized by  $a[0]$ .
- x** : array\_like  
The array of data to be filtered.
- axis** : int, optional  
The axis of *x* to which the filter is applied. Default is -1.
- padtype** : str or None, optional  
Must be ‘odd’, ‘even’, ‘constant’, or None. This determines the type of extension to use for the padded signal to which the filter is applied. If *padtype* is None, no padding is used. The default is ‘odd’.
- padlen** : int or None, optional  
The number of elements by which to extend *x* at both ends of *axis* before applying the filter. This value must be less than  $x.shape[axis] - 1$ .  $padlen=0$  implies no padding. The default value is  $3 * \max(\text{len}(a), \text{len}(b))$ .
- method** : str, optional  
Determines the method for handling the edges of the signal, either “pad” or “gust”. When *method* is “pad”, the signal is padded; the type of padding is

determined by *padtype* and *padlen*, and *irlen* is ignored. When *method* is “gust”, Gustafsson’s method is used, and *padtype* and *padlen* are ignored.

**irlen** : int or None, optional

When *method* is “gust”, *irlen* specifies the length of the impulse response of the filter. If *irlen* is None, no part of the impulse response is ignored. For a long signal, specifying *irlen* can significantly improve the performance of the filter.

**Returns** **y** : ndarray

The filtered output with the same shape as *x*.

#### See also:

`sosfiltfilt`, `lfilter_zi`, `lfilter`, `lfiltic`, `savgol_filter`, `sosfilt`

#### Notes

The option to use Gustafsson’s method was added in scipy version 0.16.0.

#### References

[R198]

#### Examples

The examples will use several functions from `scipy.signal`.

```
>>> from scipy import signal
>>> import matplotlib.pyplot as plt
```

First we create a one second signal that is the sum of two pure sine waves, with frequencies 5 Hz and 250 Hz, sampled at 2000 Hz.

```
>>> t = np.linspace(0, 1.0, 2001)
>>> xlow = np.sin(2 * np.pi * 5 * t)
>>> xhigh = np.sin(2 * np.pi * 250 * t)
>>> x = xlow + xhigh
```

Now create a lowpass Butterworth filter with a cutoff of 0.125 times the Nyquist rate, or 125 Hz, and apply it to *x* with `filtfilt`. The result should be approximately *xlow*, with no phase shift.

```
>>> b, a = signal.butter(8, 0.125)
>>> y = signal.filtfilt(b, a, x, padlen=150)
>>> np.abs(y - xlow).max()
9.1086182074789912e-06
```

We get a fairly clean result for this artificial example because the odd extension is exact, and with the moderately long padding, the filter’s transients have dissipated by the time the actual data is reached. In general, transient effects at the edges are unavoidable.

The following example demonstrates the option `method="gust"`.

First, create a filter.

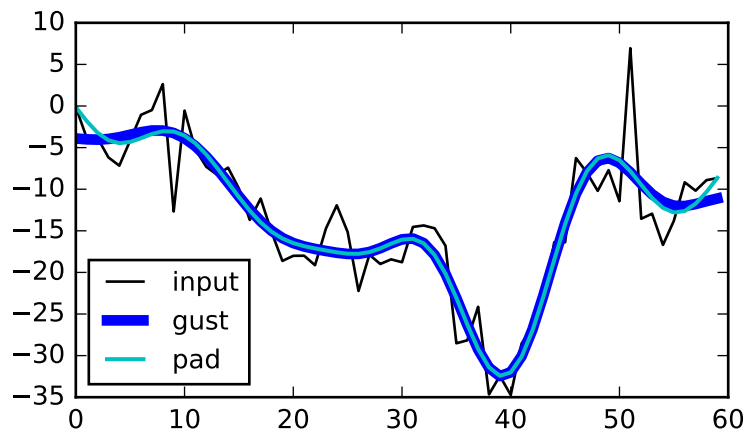
```
>>> b, a = signal.ellip(4, 0.01, 120, 0.125) # Filter to be applied.
>>> np.random.seed(123456)
```

*sig* is a random input signal to be filtered.

```
>>> n = 60
>>> sig = np.random.randn(n)**3 + 3*np.random.randn(n).cumsum()
```

Apply `filtfilt` to `sig`, once using the Gustafsson method, and once using padding, and plot the results for comparison.

```
>>> fgust = signal.filtfilt(b, a, sig, method="gust")
>>> fpad = signal.filtfilt(b, a, sig, padlen=50)
>>> plt.plot(sig, 'k-', label='input')
>>> plt.plot(fgust, 'b-', linewidth=4, label='gust')
>>> plt.plot(fpad, 'c-', linewidth=1.5, label='pad')
>>> plt.legend(loc='best')
>>> plt.show()
```



The `irlen` argument can be used to improve the performance of Gustafsson's method.

Estimate the impulse response length of the filter.

```
>>> z, p, k = signal.tf2zpk(b, a)
>>> eps = 1e-9
>>> r = np.max(np.abs(p))
>>> approx_impulse_len = int(np.ceil(np.log(eps) / np.log(r)))
>>> approx_impulse_len
137
```

Apply the filter to a longer signal, with and without the `irlen` argument. The difference between `y1` and `y2` is small. For long signals, using `irlen` gives a significant performance improvement.

```
>>> x = np.random.randn(5000)
>>> y1 = signal.filtfilt(b, a, x, method='gust')
>>> y2 = signal.filtfilt(b, a, x, method='gust', irlen=approx_impulse_len)
>>> print(np.max(np.abs(y1 - y2)))
1.80056858312e-10
```

```
scipy.signal.savgol_filter(x, window_length, polyorder, deriv=0, delta=1.0, axis=-1,
                           mode='interp', cval=0.0)
```

Apply a Savitzky-Golay filter to an array.

This is a 1-d filter. If  $x$  has dimension greater than 1, *axis* determines the axis along which the filter is applied.

**Parameters**

- x** : array\_like  
The data to be filtered. If  $x$  is not a single or double precision floating point array, it will be converted to type `numpy.float64` before filtering.
- window\_length** : int  
The length of the filter window (i.e. the number of coefficients). *window\_length* must be a positive odd integer.
- polyorder** : int  
The order of the polynomial used to fit the samples. *polyorder* must be less than *window\_length*.
- deriv** : int, optional  
The order of the derivative to compute. This must be a nonnegative integer. The default is 0, which means to filter the data without differentiating.
- delta** : float, optional  
The spacing of the samples to which the filter will be applied. This is only used if *deriv* > 0. Default is 1.0.
- axis** : int, optional  
The axis of the array  $x$  along which the filter is to be applied. Default is -1.
- mode** : str, optional  
Must be 'mirror', 'constant', 'nearest', 'wrap' or 'interp'. This determines the type of extension to use for the padded signal to which the filter is applied. When *mode* is 'constant', the padding value is given by *cval*. See the Notes for more details on 'mirror', 'constant', 'wrap', and 'nearest'. When the 'interp' mode is selected (the default), no extension is used. Instead, a degree *polyorder* polynomial is fit to the last *window\_length* values of the edges, and this polynomial is used to evaluate the last *window\_length* // 2 output values.
- cval** : scalar, optional  
Value to fill past the edges of the input if *mode* is 'constant'. Default is 0.0.

**Returns**

- y** : ndarray, same shape as  $x$   
The filtered data.

See also:

`savgol_coeffs`

**Notes**

Details on the *mode* options:

- 'mirror'**: Repeats the values at the edges in reverse order. The value closest to the edge is not included.
- 'nearest'**: The extension contains the nearest input value.
- 'constant'**: The extension contains the value given by the *cval* argument.
- 'wrap'**: The extension contains the values from the other end of the array.

For example, if the input is [1, 2, 3, 4, 5, 6, 7, 8], and *window\_length* is 7, the following shows the extended data for the various *mode* options (assuming *cval* is 0):

mode	Ext			Input								Ext		
'mirror'	4	3	2	1	2	3	4	5	6	7	8	7	6	5
'nearest'	1	1	1	1	2	3	4	5	6	7	8	8	8	8
'constant'	0	0	0	1	2	3	4	5	6	7	8	0	0	0
'wrap'	6	7	8	1	2	3	4	5	6	7	8	1	2	3

New in version 0.14.0.



### Examples

```
>>> from scipy.signal import savgol_filter
>>> np.set_printoptions(precision=2) # For compact display.
>>> x = np.array([2, 2, 5, 2, 1, 0, 1, 4, 9])
```

Filter with a window length of 5 and a degree 2 polynomial. Use the defaults for all other parameters.

```
>>> savgol_filter(x, 5, 2)
array([ 1.66,  3.17,  3.54,  2.86,  0.66,  0.17,  1.  ,  4.  ,  9.  ])
```

Note that the last five values in `x` are samples of a parabola, so when `mode='interp'` (the default) is used with `polyorder=2`, the last three values are unchanged. Compare that to, for example, `mode='nearest'`:

```
>>> savgol_filter(x, 5, 2, mode='nearest')
array([ 1.74,  3.03,  3.54,  2.86,  0.66,  0.17,  1.  ,  4.6 ,  7.97])
```

`scipy.signal.deconvolve` (*signal, divisor*)

Deconvolves divisor out of signal.

Returns the quotient and remainder such that `signal = convolve(divisor, quotient) + remainder`

<b>Parameters</b>	<b>signal</b> : array_like	Signal data, typically a recorded signal
	<b>divisor</b> : array_like	Divisor data, typically an impulse response or filter that was applied to the original signal
<b>Returns</b>	<b>quotient</b> : ndarray	Quotient, typically the recovered original signal
	<b>remainder</b> : ndarray	Remainder

See also:

`numpy.polydiv`

performs polynomial division (same operation, but also accepts `poly1d` objects)

### Examples

Deconvolve a signal that's been filtered:

```
>>> from scipy import signal
>>> original = [0, 1, 0, 0, 1, 1, 0, 0]
>>> impulse_response = [2, 1]
>>> recorded = signal.convolve(impulse_response, original)
>>> recorded
array([0, 2, 1, 0, 2, 3, 1, 0])
>>> recovered, remainder = signal.deconvolve(recorded, impulse_response)
>>> recovered
array([ 0.,  1.,  0.,  0.,  1.,  1.,  0.,  0.])
```

`scipy.signal.sosfilt` (*sos, x, axis=-1, zi=None*)

Filter data along one dimension using cascaded second-order sections

Filter a data sequence, `x`, using a digital IIR filter defined by `sos`. This is implemented by performing `lfilter` for each second-order section. See `lfilter` for details.

<b>Parameters</b>	<b>sos</b> : array_like
-------------------	-------------------------

Array of second-order filter coefficients, must have shape `(n_sections, 6)`. Each row corresponds to a second-order section, with the first three columns providing the numerator coefficients and the last three providing the denominator coefficients.

**x** : array\_like

An N-dimensional input array.

**axis** : int, optional

The axis of the input data array along which to apply the linear filter. The filter is applied to each subarray along this axis. Default is -1.

**zi** : array\_like, optional

Initial conditions for the cascaded filter delays. It is a (at least 2D) vector of shape `(n_sections, ..., 2, ...)`, where `..., 2, ...` denotes the shape of `x`, but with `x.shape[axis]` replaced by 2. If `zi` is None or is not given then initial rest (i.e. all zeros) is assumed. Note that these initial conditions are *not* the same as the initial conditions given by `lfiltic` or `lfilter_zi`.

**Returns**

**y** : ndarray

The output of the digital filter.

**zf** : ndarray, optional

If `zi` is None, this is not returned, otherwise, `zf` holds the final filter delay values.

**See also:**

`zpk2sos`, `sos2zpk`, `sosfilt_zi`, `sosfiltfilt`

**Notes**

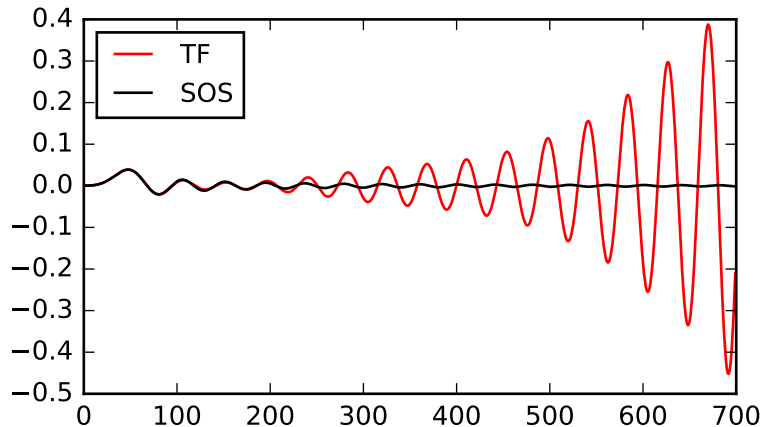
The filter function is implemented as a series of second-order filters with direct-form II transposed structure. It is designed to minimize numerical precision errors for high-order filters.

New in version 0.16.0.

**Examples**

Plot a 13th-order filter's impulse response using both `lfilter` and `sosfilt`, showing the instability that results from trying to do a 13th-order filter in a single stage (the numerical error pushes some poles outside of the unit circle):

```
>>> import matplotlib.pyplot as plt
>>> from scipy import signal
>>> b, a = signal.ellip(13, 0.009, 80, 0.05, output='ba')
>>> sos = signal.ellip(13, 0.009, 80, 0.05, output='sos')
>>> x = np.zeros(700)
>>> x[0] = 1.
>>> y_tf = signal.lfilter(b, a, x)
>>> y_sos = signal.sosfilt(sos, x)
>>> plt.plot(y_tf, 'r', label='TF')
>>> plt.plot(y_sos, 'k', label='SOS')
>>> plt.legend(loc='best')
>>> plt.show()
```



`scipy.signal.sosfilt_zi` (*sos*)

Compute an initial state *zi* for the `sosfilt` function that corresponds to the steady state of the step response.

A typical use of this function is to set the initial state so that the output of the filter starts at the same value as the first element of the signal to be filtered.

**Parameters**    **sos** : array\_like  
 Array of second-order filter coefficients, must have shape (n\_sections, 6). See `sosfilt` for the SOS filter format specification.

**Returns**        **zi** : ndarray  
 Initial conditions suitable for use with `sosfilt`, shape (n\_sections, 2).

**See also:**

`sosfilt`, `zpk2sos`

**Notes**

New in version 0.16.0.

**Examples**

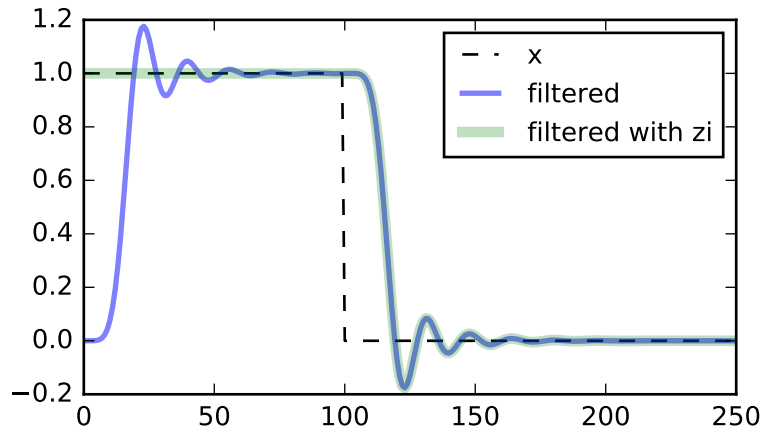
Filter a rectangular pulse that begins at time 0, with and without the use of the *zi* argument of `scipy.signal.sosfilt`.

```
>>> from scipy import signal
>>> import matplotlib.pyplot as plt

>>> sos = signal.butter(9, 0.125, output='sos')
>>> zi = signal.sosfilt_zi(sos)
>>> x = (np.arange(250) < 100).astype(int)
>>> f1 = signal.sosfilt(sos, x)
>>> f2, zo = signal.sosfilt(sos, x, zi=zi)

>>> plt.plot(x, 'k--', label='x')
>>> plt.plot(f1, 'b', alpha=0.5, linewidth=2, label='filtered')
>>> plt.plot(f2, 'g', alpha=0.25, linewidth=4, label='filtered with zi')
```

```
>>> plt.legend(loc='best')
>>> plt.show()
```



`scipy.signal.sosfiltfilt` (*sos*, *x*, *axis=-1*, *padtype='odd'*, *padlen=None*)

A forward-backward filter using cascaded second-order sections.

See `filtfilt` for more complete information about this method.

**Parameters**    *sos* : array\_like

Array of second-order filter coefficients, must have shape (n\_sections, 6). Each row corresponds to a second-order section, with the first three columns providing the numerator coefficients and the last three providing the denominator coefficients.

*x* : array\_like

The array of data to be filtered.

*axis* : int, optional

The axis of *x* to which the filter is applied. Default is -1.

*padtype* : str or None, optional

Must be 'odd', 'even', 'constant', or None. This determines the type of extension to use for the padded signal to which the filter is applied. If *padtype* is None, no padding is used. The default is 'odd'.

*padlen* : int or None, optional

The number of elements by which to extend *x* at both ends of *axis* before applying the filter. This value must be less than `x.shape[axis] - 1`. `padlen=0` implies no padding. The default value is:

```
3 * (2 * len(sos) + 1 - min((sos[:, 2] == 0).sum(),
                             (sos[:, 5] == 0).sum()))
```

The extra subtraction at the end attempts to compensate for poles and zeros at the origin (e.g. for odd-order filters) to yield equivalent estimates of *padlen* to those of `filtfilt` for second-order section filters built with `scipy.signal` functions.

**Returns**        *y* : ndarray

The filtered output with the same shape as *x*.

**See also:**

`filtfilt`, `sosfilt`, `sosfilt_zi`

**Notes**

New in version 0.18.0.

`scipy.signal.hilbert(x, N=None, axis=-1)`

Compute the analytic signal, using the Hilbert transform.

The transformation is done along the last axis by default.

**Parameters**

- x** : array\_like  
Signal data. Must be real.
- N** : int, optional  
Number of Fourier components. Default: `x.shape[axis]`
- axis** : int, optional  
Axis along which to do the transformation. Default: -1.

**Returns**

- xa** : ndarray  
Analytic signal of *x*, of each 1-D array along *axis*

**Notes**

The analytic signal  $x_a(t)$  of signal  $x(t)$  is:

$$x_a = F^{-1}(F(x)2U) = x + iy$$

where  $F$  is the Fourier transform,  $U$  the unit step function, and  $y$  the Hilbert transform of  $x$ . [R216]

In other words, the negative half of the frequency spectrum is zeroed out, turning the real-valued signal into a complex signal. The Hilbert transformed signal can be obtained from `np.imag(hilbert(x))`, and the original signal from `np.real(hilbert(x))`.

**References**

[R216], [R217], [R218]

**Examples**

In this example we use the Hilbert transform to determine the amplitude envelope and instantaneous frequency of an amplitude-modulated signal.

```
>>> import numpy as np
>>> import matplotlib.pyplot as plt
>>> from scipy.signal import hilbert, chirp

>>> duration = 1.0
>>> fs = 400.0
>>> samples = int(fs*duration)
>>> t = np.arange(samples) / fs
```

We create a chirp of which the frequency increases from 20 Hz to 100 Hz and apply an amplitude modulation.

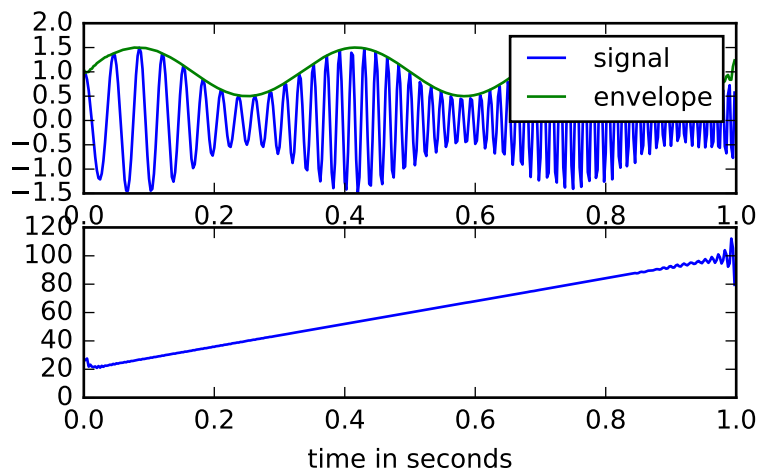
```
>>> signal = chirp(t, 20.0, t[-1], 100.0)
>>> signal *= (1.0 + 0.5 * np.sin(2.0*np.pi*3.0*t) )
```

The amplitude envelope is given by magnitude of the analytic signal. The instantaneous frequency can be obtained by differentiating the instantaneous phase in respect to time. The instantaneous phase corresponds to the phase angle of the analytic signal.

```

>>> analytic_signal = hilbert(signal)
>>> amplitude_envelope = np.abs(analytic_signal)
>>> instantaneous_phase = np.unwrap(np.angle(analytic_signal))
>>> instantaneous_frequency = np.diff(instantaneous_phase) / (2.0*np.pi) * fs

>>> fig = plt.figure()
>>> ax0 = fig.add_subplot(211)
>>> ax0.plot(t, signal, label='signal')
>>> ax0.plot(t, amplitude_envelope, label='envelope')
>>> ax0.set_xlabel("time in seconds")
>>> ax0.legend()
>>> ax1 = fig.add_subplot(212)
>>> ax1.plot(t[1:], instantaneous_frequency)
>>> ax1.set_xlabel("time in seconds")
>>> ax1.set_ylim(0.0, 120.0)
    
```



`scipy.signal.hilbert2(x, N=None)`

Compute the ‘2-D’ analytic signal of  $x$

**Parameters**

- x** : array\_like  
2-D signal data.
- N** : int or tuple of two ints, optional  
Number of Fourier components. Default is `x.shape`

**Returns**

- xa** : ndarray  
Analytic signal of  $x$  taken along axes (0,1).

### References

[R219]

`scipy.signal.decimate(x, q, n=None, ftype='iir', axis=-1, zero_phase=None)`

Downsample the signal after applying an anti-aliasing filter.

By default, an order 8 Chebyshev type I filter is used. A 30 point FIR filter with Hamming window is used if `ftype` is ‘fir’.

**Parameters**

- x** : ndarray  
The signal to be downsampled, as an N-dimensional array.
- q** : int

The downsampling factor. For downsampling factors higher than 13, it is recommended to call `decimate` multiple times.

**n** : int, optional  
The order of the filter (1 less than the length for ‘fir’). Defaults to 8 for ‘iir’ and 30 for ‘fir’.

**ftype** : str {‘iir’, ‘fir’} or `dlti` instance, optional  
If ‘iir’ or ‘fir’, specifies the type of lowpass filter. If an instance of an `dlti` object, uses that object to filter before downsampling.

**axis** : int, optional  
The axis along which to decimate.

**zero\_phase** : bool, optional  
Prevent phase shift by filtering with `filtfilt` instead of `lfilter` when using an IIR filter, and shifting the outputs back by the filter’s group delay when using an FIR filter. A value of `True` is recommended, since a phase shift is generally not desired. Using `None` defaults to `False` for backwards compatibility. This default will change to `True` in a future release, so it is best to set this argument explicitly.

**Returns** **y** : ndarray  
New in version 0.18.0.  
The down-sampled signal.

**See also:**

`resample` Resample up or down using the FFT method.

`resample_poly`

Resample using polyphase filtering and an FIR filter.

**Notes**

The `zero_phase` keyword was added in 0.18.0. The possibility to use instances of `dlti` as `ftype` was added in 0.18.0.

`scipy.signal.detrend` (*data*, *axis=-1*, *type='linear'*, *bp=0*)

Remove linear trend along axis from data.

**Parameters** **data** : array\_like

The input data.

**axis** : int, optional

The axis along which to detrend the data. By default this is the last axis (-1).

**type** : {‘linear’, ‘constant’}, optional

The type of detrending. If `type == 'linear'` (default), the result of a linear least-squares fit to *data* is subtracted from *data*. If `type == 'constant'`, only the mean of *data* is subtracted.

**bp** : array\_like of ints, optional

A sequence of break points. If given, an individual linear fit is performed for each part of *data* between two break points. Break points are specified as indices into *data*.

**Returns** **ret** : ndarray

The detrended input data.

**Examples**

```
>>> from scipy import signal
>>> randgen = np.random.RandomState(9)
>>> npoints = 1000
>>> noise = randgen.randn(npoints)
>>> x = 3 + 2*np.linspace(0, 1, npoints) + noise
```

```
>>> (signal.detrend(x) - noise).max() < 0.01
True
```

`scipy.signal.resample(x, num, t=None, axis=0, window=None)`

Resample  $x$  to  $num$  samples using Fourier method along the given axis.

The resampled signal starts at the same value as  $x$  but is sampled with a spacing of  $\text{len}(x) / num * (\text{spacing of } x)$ . Because a Fourier method is used, the signal is assumed to be periodic.

**Parameters**

- x** : array\_like  
The data to be resampled.
- num** : int  
The number of samples in the resampled signal.
- t** : array\_like, optional  
If  $t$  is given, it is assumed to be the sample positions associated with the signal data in  $x$ .
- axis** : int, optional  
The axis of  $x$  that is resampled. Default is 0.
- window** : array\_like, callable, string, float, or tuple, optional  
Specifies the window applied to the signal in the Fourier domain. See below for details.

**Returns**

resampled\_x or (resampled\_x, resampled\_t)  
Either the resampled array, or, if  $t$  was given, a tuple containing the resampled array and the corresponding resampled positions.

**See also:**

[`decimate`](#) Downsample the signal after applying an FIR or IIR filter.

[`resample\_poly`](#)

Resample using polyphase filtering and an FIR filter.

### Notes

The argument *window* controls a Fourier-domain window that tapers the Fourier spectrum before zero-padding to alleviate ringing in the resampled values for sampled signals you didn't intend to be interpreted as band-limited.

If *window* is a function, then it is called with a vector of inputs indicating the frequency bins (i.e. `fft-freq(x.shape[axis])`).

If *window* is an array of the same length as  $x.\text{shape}[\text{axis}]$  it is assumed to be the window to be applied directly in the Fourier domain (with dc and low-frequency first).

For any other type of *window*, the function `scipy.signal.get_window` is called to generate the window.

The first sample of the returned vector is the same as the first sample of the input vector. The spacing between samples is changed from  $dx$  to  $dx * \text{len}(x) / num$ .

If  $t$  is not `None`, then it represents the old sample positions, and the new sample positions will be returned as well as the new samples.

As noted, `resample` uses FFT transformations, which can be very slow if the number of input or output samples is large and prime; see `scipy.fftpack.fft`.

### Examples

Note that the end of the resampled data rises to meet the first sample of the next cycle:

```
>>> from scipy import signal
```

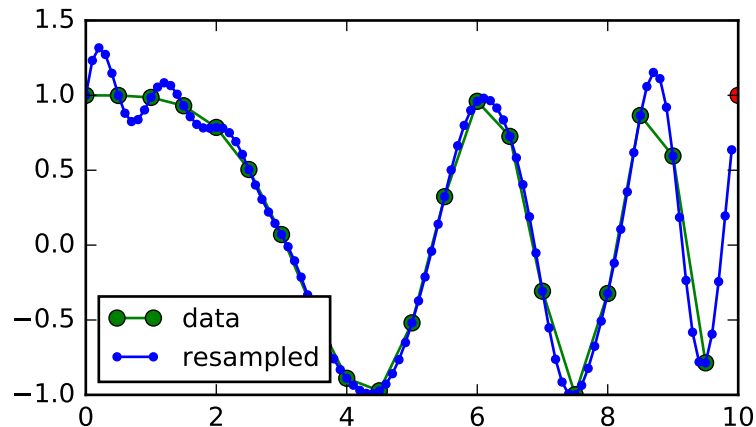


```

>>> x = np.linspace(0, 10, 20, endpoint=False)
>>> y = np.cos(-x**2/6.0)
>>> f = signal.resample(y, 100)
>>> xnew = np.linspace(0, 10, 100, endpoint=False)

>>> import matplotlib.pyplot as plt
>>> plt.plot(x, y, 'go-', xnew, f, '-.', 10, y[0], 'ro')
>>> plt.legend(['data', 'resampled'], loc='best')
>>> plt.show()

```



```
scipy.signal.resample_poly(x, up, down, axis=0, window=('kaiser', 5.0))
```

Resample  $x$  along the given axis using polyphase filtering.

The signal  $x$  is upsampled by the factor  $up$ , a zero-phase low-pass FIR filter is applied, and then it is downsampled by the factor  $down$ . The resulting sample rate is  $up / down$  times the original sample rate. Values beyond the boundary of the signal are assumed to be zero during the filtering step.

<b>Parameters</b>	<b>x</b> : array_like	The data to be resampled.
	<b>up</b> : int	The upsampling factor.
	<b>down</b> : int	The downsampling factor.
	<b>axis</b> : int, optional	The axis of $x$ that is resampled. Default is 0.
	<b>window</b> : string, tuple, or array_like, optional	Desired window to use to design the low-pass filter, or the FIR filter coefficients to employ. See below for details.
<b>Returns</b>	<b>resampled_x</b> : array	The resampled array.

See also:

**decimate** Downsample the signal after applying an FIR or IIR filter.  
**resample** Resample up or down using the FFT method.

### Notes

This polyphase method will likely be faster than the Fourier method in `scipy.signal.resample` when the number of samples is large and prime, or when the number of samples is large and *up* and *down* share a large greatest common denominator. The length of the FIR filter used will depend on  $\max(\text{up}, \text{down}) // \text{gcd}(\text{up}, \text{down})$ , and the number of operations during polyphase filtering will depend on the filter length and *down* (see `scipy.signal.upfirdn` for details).

The argument *window* specifies the FIR low-pass filter design.

If *window* is an array\_like it is assumed to be the FIR filter coefficients. Note that the FIR filter is applied after the upsampling step, so it should be designed to operate on a signal at a sampling frequency higher than the original by a factor of  $\text{up} // \text{gcd}(\text{up}, \text{down})$ . This function's output will be centered with respect to this array, so it is best to pass a symmetric filter with an odd number of samples if, as is usually the case, a zero-phase filter is desired.

For any other type of *window*, the functions `scipy.signal.get_window` and `scipy.signal.firwin` are called to generate the appropriate filter coefficients.

The first sample of the returned vector is the same as the first sample of the input vector. The spacing between samples is changed from *dx* to  $\text{dx} * \text{up} / \text{float}(\text{down})$ .

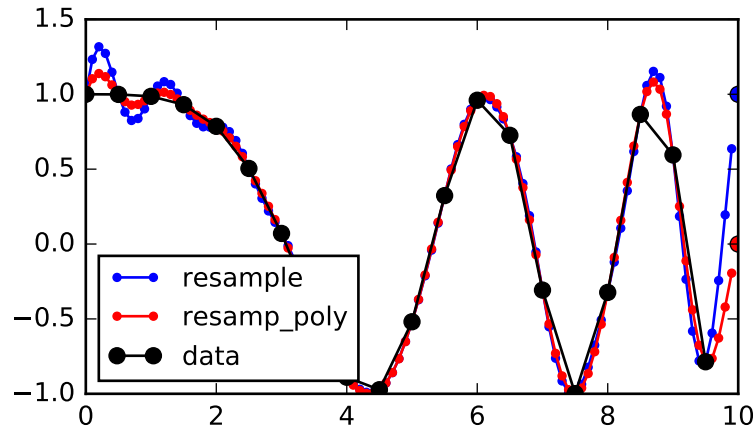
### Examples

Note that the end of the resampled data rises to meet the first sample of the next cycle for the FFT method, and gets closer to zero for the polyphase method:

```
>>> from scipy import signal

>>> x = np.linspace(0, 10, 20, endpoint=False)
>>> y = np.cos(-x**2/6.0)
>>> f_fft = signal.resample(y, 100)
>>> f_poly = signal.resample_poly(y, 100, 20)
>>> xnew = np.linspace(0, 10, 100, endpoint=False)

>>> import matplotlib.pyplot as plt
>>> plt.plot(xnew, f_fft, 'b.-', xnew, f_poly, 'r.-')
>>> plt.plot(x, y, 'ko-')
>>> plt.plot(10, y[0], 'bo', 10, 0., 'ro') # boundaries
>>> plt.legend(['resample', 'resamp_poly', 'data'], loc='best')
>>> plt.show()
```



`scipy.signal.upfirdn(h, x, up=1, down=1, axis=-1)`

Upsample, FIR filter, and downsample

**Parameters**

- h** : array\_like  
1-dimensional FIR (finite-impulse response) filter coefficients.
- x** : array\_like  
Input signal array.
- up** : int, optional  
Upsampling rate. Default is 1.
- down** : int, optional  
Downsampling rate. Default is 1.
- axis** : int, optional  
The axis of the input data array along which to apply the linear filter. The filter is applied to each subarray along this axis. Default is -1.

**Returns**

- y** : ndarray  
The output signal array. Dimensions will be the same as *x* except for along *axis*, which will change size according to the *h*, *up*, and *down* parameters.

### Notes

The algorithm is an implementation of the block diagram shown on page 129 of the Vaidyanathan text [R233] (Figure 4.3-8d).

The direct approach of upsampling by factor of *P* with zero insertion, FIR filtering of length *N*, and downsampling by factor of *Q* is  $O(N*Q)$  per output sample. The polyphase implementation used here is  $O(N/P)$ .

New in version 0.18.

### Examples

Simple operations:

```
>>> from scipy.signal import upfirdn
>>> upfirdn([1, 1, 1], [1, 1, 1]) # FIR filter
array([ 1.,  2.,  3.,  2.,  1.])
>>> upfirdn([1], [1, 2, 3], 3) # upsampling with zeros insertion
array([ 1.,  0.,  0.,  2.,  0.,  0.,  3.,  0.,  0.])
>>> upfirdn([1, 1, 1], [1, 2, 3], 3) # upsampling with sample-and-hold
array([ 1.,  1.,  1.,  2.,  2.,  2.,  3.,  3.,  3.])
```

```
>>> upfirdn([.5, 1, .5], [1, 1, 1], 2) # linear interpolation
array([ 0.5, 1. , 1. , 1. , 1. , 1. , 0.5, 0. ])
>>> upfirdn([1], np.arange(10), 1, 3) # decimation by 3
array([ 0., 3., 6., 9.])
>>> upfirdn([.5, 1, .5], np.arange(10), 2, 3) # linear interp, rate 2/3
array([ 0. , 1. , 2.5, 4. , 5.5, 7. , 8.5, 0. ])
```

Apply a single filter to multiple signals:

```
>>> x = np.reshape(np.arange(8), (4, 2))
>>> x
array([[0, 1],
       [2, 3],
       [4, 5],
       [6, 7]])
```

Apply along the last dimension of x:

```
>>> h = [1, 1]
>>> upfirdn(h, x, 2)
array([[ 0., 0., 1., 1.],
       [ 2., 2., 3., 3.],
       [ 4., 4., 5., 5.],
       [ 6., 6., 7., 7.]])
```

Apply along the 0th dimension of x:

```
>>> upfirdn(h, x, 2, axis=0)
array([[ 0., 1.],
       [ 0., 1.],
       [ 2., 3.],
       [ 2., 3.],
       [ 4., 5.],
       [ 4., 5.],
       [ 6., 7.],
       [ 6., 7.]])
```

## 5.20.4 Filter design

<code>bilinear(b, a[, fs])</code>	Return a digital filter from an analog one using a bilinear transform.
<code>findfreqs(num, den, N)</code>	Find array of frequencies for computing the response of an analog filter.
<code>firls(numtaps, bands, desired[, weight, nyq])</code>	FIR filter design using least-squares error minimization.
<code>firwin(numtaps, cutoff[, width, window, ...])</code>	FIR filter design using the window method.
<code>firwin2(numtaps, freq, gain[, nfreqs, ...])</code>	FIR filter design using the window method.
<code>freqs(b, a[, worN, plot])</code>	Compute frequency response of analog filter.
<code>freqz(b[, a, worN, whole, plot])</code>	Compute the frequency response of a digital filter.
<code>group_delay(system[, w, whole])</code>	Compute the group delay of a digital filter.
<code>iirdesign(wp, ws, gpass, gstop[, analog, ...])</code>	Complete IIR digital and analog filter design.
<code>iirfilter(N, Wn[, rp, rs, btype, analog, ...])</code>	IIR digital and analog filter design given order and critical points.
<code>kaiser_atten(numtaps, width)</code>	Compute the attenuation of a Kaiser FIR filter.
<code>kaiser_beta(a)</code>	Compute the Kaiser parameter <i>beta</i> , given the attenuation <i>a</i> .
<code>kaiserord(ripple, width)</code>	Design a Kaiser window to limit ripple and width of transition region.
<code>savgol_coeffs(window_length, polyorder[, ...])</code>	Compute the coefficients for a 1-d Savitzky-Golay FIR filter.

Continued on next page

Table 5.126 – continued from previous page

<code>remez(numtaps, bands, desired[, weight, Hz, ...])</code>	Calculate the minimax optimal filter using the Remez exchange algorithm.
<code>unique_roots(p[, tol, rtype])</code>	Determine unique roots and their multiplicities from a list of roots.
<code>residue(b, a[, tol, rtype])</code>	Compute partial-fraction expansion of $b(s) / a(s)$ .
<code>residuez(b, a[, tol, rtype])</code>	Compute partial-fraction expansion of $b(z) / a(z)$ .
<code>invres(r, p, k[, tol, rtype])</code>	Compute $b(s)$ and $a(s)$ from partial fraction expansion.
<code>invresz(r, p, k[, tol, rtype])</code>	Compute $b(z)$ and $a(z)$ from partial fraction expansion.
<code>BadCoefficients</code>	Warning about badly conditioned filter coefficients

`scipy.signal.bilinear` ( $b, a, fs=1.0$ )

Return a digital filter from an analog one using a bilinear transform.

The bilinear transform substitutes  $(z-1) / (z+1)$  for  $s$ .

`scipy.signal.findfreqs` ( $num, den, N$ )

Find array of frequencies for computing the response of an analog filter.

**Parameters** `num, den` : array\_like, 1-D

The polynomial coefficients of the numerator and denominator of the transfer function of the filter or LTI system. The coefficients are ordered from highest to lowest degree.

`N` : int

**Returns** `w` : (N,) ndarray

The length of the array to be computed.

A 1-D array of frequencies, logarithmically spaced.

### Examples

Find a set of nine frequencies that span the “interesting part” of the frequency response for the filter with the transfer function

$$H(s) = s / (s^2 + 8s + 25)$$

```
>>> from scipy import signal
>>> signal.findfreqs([1, 0], [1, 8, 25], N=9)
array([[ 1.00000000e-02,  3.16227766e-02,  1.00000000e-01,
         3.16227766e-01,  1.00000000e+00,  3.16227766e+00,
         1.00000000e+01,  3.16227766e+01,  1.00000000e+02]])
```

`scipy.signal.firls` ( $numtaps, bands, desired, weight=None, nyq=1.0$ )

FIR filter design using least-squares error minimization.

Calculate the filter coefficients for the linear-phase finite impulse response (FIR) filter which has the best approximation to the desired frequency response described by *bands* and *desired* in the least squares sense (i.e., the integral of the weighted mean-squared error within the specified bands is minimized).

**Parameters** `numtaps` : int

The number of taps in the FIR filter. *numtaps* must be odd.

`bands` : array\_like

A monotonic nondecreasing sequence containing the band edges in Hz. All elements must be non-negative and less than or equal to the Nyquist frequency given by *nyq*.

`desired` : array\_like

A sequence the same size as *bands* containing the desired gain at the start and end point of each band.

`weight` : array\_like, optional

A relative weighting to give to each band region when solving the least squares problem. *weight* has to be half the size of *bands*.

**nyq** : float, optional  
Nyquist frequency. Each frequency in *bands* must be between 0 and *nyq* (inclusive).

**Returns** **coeffs** : ndarray  
Coefficients of the optimal (in a least squares sense) FIR filter.

**See also:**

`firwin`, `firwin2`

**Notes**

This implementation follows the algorithm given in [R200]. As noted there, least squares design has multiple advantages:

1. Optimal in a least-squares sense.
2. Simple, non-iterative method.
3. The general solution can be obtained by solving a linear system of equations.
4. Allows the use of a frequency dependent weighting function.

This function constructs a Type I linear phase FIR filter, which contains an odd number of *coeffs* satisfying for  $n < numtaps$ :

$$coeffs(n) = coeffs(numtaps - 1 - n)$$

The odd number of coefficients and filter symmetry avoid boundary conditions that could otherwise occur at the Nyquist and 0 frequencies (e.g., for Type II, III, or IV variants).

New in version 0.18.

**References**

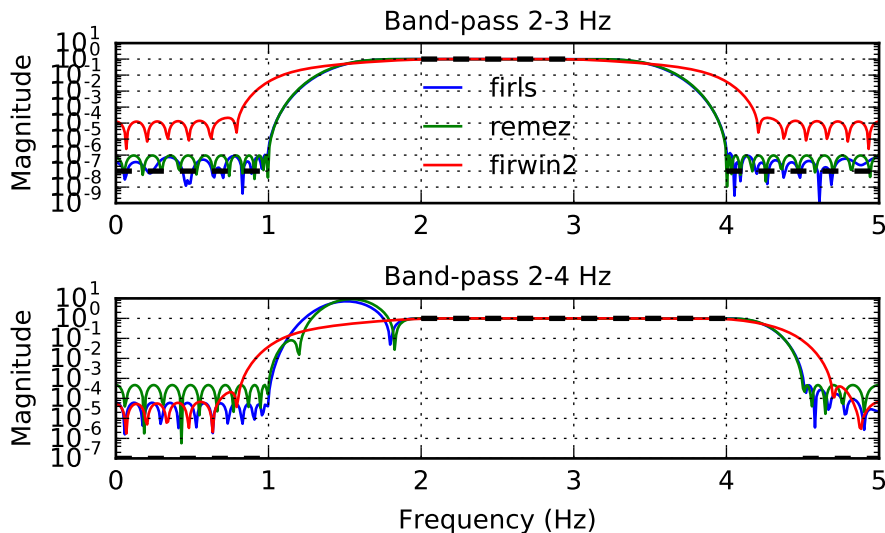
[R200]

**Examples**

We want to construct a band-pass filter. Note that the behavior in the frequency ranges between our stop bands and pass bands is unspecified, and thus may overshoot depending on the parameters of our filter:

```
>>> from scipy import signal
>>> import matplotlib.pyplot as plt
>>> fig, axs = plt.subplots(2)
>>> nyq = 5. # Hz
>>> desired = (0, 0, 1, 1, 0, 0)
>>> for bi, bands in enumerate(((0, 1, 2, 3, 4, 5), (0, 1, 2, 4, 4.5, 5))):
...     fir_firls = signal.firls(73, bands, desired, nyq=nyq)
...     fir_remez = signal.remez(73, bands, desired[::2], Hz=2 * nyq)
...     fir_firwin2 = signal.firwin2(73, bands, desired, nyq=nyq)
...     hs = list()
...     ax = axs[bi]
...     for fir in (fir_firls, fir_remez, fir_firwin2):
...         freq, response = signal.freqz(fir)
...         hs.append(ax.semilogy(nyq*freq/(np.pi), np.abs(response))[0])
...     for band, gains in zip(zip(bands[::2], bands[1::2]), zip(desired[::2], desired[1::2])):
...         ax.semilogy(band, np.maximum(gains, 1e-7), 'k--', linewidth=2)
...     if bi == 0:
...         ax.legend(hs, ('firls', 'remez', 'firwin2'), loc='lower center', frameon=False)
...     else:
...         ax.set_xlabel('Frequency (Hz)')
...     ax.grid(True)
...     ax.set(title='Band-pass %d-%d Hz' % bands[2:4], ylabel='Magnitude')
... 
```

```
>>> fig.tight_layout()
>>> plt.show()
```



```
scipy.signal.firwin(numtaps, cutoff, width=None, window='hamming', pass_zero=True, scale=True,
                    nyq=1.0)
```

FIR filter design using the window method.

This function computes the coefficients of a finite impulse response filter. The filter will have linear phase; it will be Type I if *numtaps* is odd and Type II if *numtaps* is even.

Type II filters always have zero response at the Nyquist rate, so a `ValueError` exception is raised if `firwin` is called with *numtaps* even and having a passband whose right end is at the Nyquist rate.

**Parameters** **numtaps** : int

Length of the filter (number of coefficients, i.e. the filter order + 1). *numtaps* must be even if a passband includes the Nyquist frequency.

**cutoff** : float or 1D array\_like

Cutoff frequency of filter (expressed in the same units as *nyq*) OR an array of cutoff frequencies (that is, band edges). In the latter case, the frequencies in *cutoff* should be positive and monotonically increasing between 0 and *nyq*. The values 0 and *nyq* must not be included in *cutoff*.

**width** : float or None, optional

If *width* is not None, then assume it is the approximate width of the transition region (expressed in the same units as *nyq*) for use in Kaiser FIR filter design. In this case, the *window* argument is ignored.

**window** : string or tuple of string and parameter values, optional

Desired window to use. See `scipy.signal.get_window` for a list of windows and required parameters.

**pass\_zero** : bool, optional

If True, the gain at the frequency 0 (i.e. the “DC gain”) is 1. Otherwise the DC gain is 0.

**scale** : bool, optional

Set to True to scale the coefficients so that the frequency response is exactly unity at a certain frequency. That frequency is either:

- 0 (DC) if the first passband starts at 0 (i.e. `pass_zero` is True)

•*nyq* (the Nyquist rate) if the first passband ends at *nyq* (i.e the filter is a single band highpass filter); center of first passband otherwise

**nyq** : float, optional  
 Nyquist frequency. Each frequency in *cutoff* must be between 0 and *nyq*.  
**Returns** **h** : (numtaps,) ndarray  
 Coefficients of length *numtaps* FIR filter.  
**Raises** **ValueError**  
 If any value in *cutoff* is less than or equal to 0 or greater than or equal to *nyq*, if the values in *cutoff* are not strictly monotonically increasing, or if *numtaps* is even but a passband includes the Nyquist frequency.

**See also:**

`firwin2`, `firls`, `remez`

**Examples**

Low-pass from 0 to f:

```
>>> from scipy import signal
>>> numtaps = 3
>>> f = 0.1
>>> signal.firwin(numtaps, f)
array([ 0.06799017,  0.86401967,  0.06799017])
```

Use a specific window function:

```
>>> signal.firwin(numtaps, f, window='nuttall')
array([ 3.56607041e-04,  9.99286786e-01,  3.56607041e-04])
```

High-pass ('stop' from 0 to f):

```
>>> signal.firwin(numtaps, f, pass_zero=False)
array([-0.00859313,  0.98281375, -0.00859313])
```

Band-pass:

```
>>> f1, f2 = 0.1, 0.2
>>> signal.firwin(numtaps, [f1, f2], pass_zero=False)
array([ 0.06301614,  0.88770441,  0.06301614])
```

Band-stop:

```
>>> signal.firwin(numtaps, [f1, f2])
array([-0.00801395,  1.0160279 , -0.00801395])
```

Multi-band (passbands are [0, f1], [f2, f3] and [f4, 1]):

```
>>> f3, f4 = 0.3, 0.4
>>> signal.firwin(numtaps, [f1, f2, f3, f4])
array([-0.01376344,  1.02752689, -0.01376344])
```

Multi-band (passbands are [f1, f2] and [f3,f4]):

```
>>> signal.firwin(numtaps, [f1, f2, f3, f4], pass_zero=False)
array([ 0.04890915,  0.91284326,  0.04890915])
```



`scipy.signal.firwin2` (*numtaps*, *freq*, *gain*, *nfreqs=None*, *window='hamming'*, *nyq=1.0*, *antisymmetric=False*)

FIR filter design using the window method.

From the given frequencies *freq* and corresponding gains *gain*, this function constructs an FIR filter with linear phase and (approximately) the given frequency response.

**Parameters**    **numtaps** : int

The number of taps in the FIR filter. *numtaps* must be less than *nfreqs*.

**freq** : array\_like, 1D

The frequency sampling points. Typically 0.0 to 1.0 with 1.0 being Nyquist. The Nyquist frequency can be redefined with the argument *nyq*. The values in *freq* must be nondecreasing. A value can be repeated once to implement a discontinuity. The first value in *freq* must be 0, and the last value must be *nyq*.

**gain** : array\_like

The filter gains at the frequency sampling points. Certain constraints to gain values, depending on the filter type, are applied, see Notes for details.

**nfreqs** : int, optional

The size of the interpolation mesh used to construct the filter. For most efficient behavior, this should be a power of 2 plus 1 (e.g, 129, 257, etc). The default is one more than the smallest power of 2 that is not less than *numtaps*. *nfreqs* must be greater than *numtaps*.

**window** : string or (string, float) or float, or None, optional

Window function to use. Default is “hamming”. See `scipy.signal.get_window` for the complete list of possible values. If None, no window function is applied.

**nyq** : float, optional

Nyquist frequency. Each frequency in *freq* must be between 0 and *nyq* (inclusive).

**antisymmetric** : bool, optional

Whether resulting impulse response is symmetric/antisymmetric. See Notes for more details.

**Returns**    **taps** : ndarray

The filter coefficients of the FIR filter, as a 1-D array of length *numtaps*.

**See also:**

`firls`, `firwin`, `remez`

**Notes**

From the given set of frequencies and gains, the desired response is constructed in the frequency domain. The inverse FFT is applied to the desired response to create the associated convolution kernel, and the first *numtaps* coefficients of this kernel, scaled by *window*, are returned.

The FIR filter will have linear phase. The type of filter is determined by the value of ‘numtaps’ and *antisymmetric* flag. There are four possible combinations:

- odd *numtaps*, *antisymmetric* is False, type I filter is produced
- even *numtaps*, *antisymmetric* is False, type II filter is produced
- odd *numtaps*, *antisymmetric* is True, type III filter is produced
- even *numtaps*, *antisymmetric* is True, type IV filter is produced

Magnitude response of all but type I filters are subjects to following constraints:

- type II – zero at the Nyquist frequency
- type III – zero at zero and Nyquist frequencies
- type IV – zero at zero frequency

New in version 0.9.0.

### References

[R201], [R202]

### Examples

A lowpass FIR filter with a response that is 1 on [0.0, 0.5], and that decreases linearly on [0.5, 1.0] from 1 to 0:

```

>>> from scipy import signal
>>> taps = signal.firwin2(150, [0.0, 0.5, 1.0], [1.0, 1.0, 0.0])
>>> print(taps[72:78])
[-0.02286961 -0.06362756  0.57310236  0.57310236 -0.06362756 -0.02286961]

```

`scipy.signal.freqs` (*b*, *a*, *worN=None*, *plot=None*)

Compute frequency response of analog filter.

Given the *M*-order numerator *b* and *N*-order denominator *a* of an analog filter, compute its frequency response:

$$H(\omega) = \frac{b[0]*(j\omega)^{**M} + b[1]*(j\omega)^{**(M-1)} + \dots + b[M]}{a[0]*(j\omega)^{**N} + a[1]*(j\omega)^{**(N-1)} + \dots + a[N]}$$

<b>Parameters</b>	<b>b</b> : array_like	Numerator of a linear filter.
	<b>a</b> : array_like	Denominator of a linear filter.
	<b>worN</b> : {None, int, array_like}, optional	If None, then compute at 200 frequencies around the interesting parts of the response curve (determined by pole-zero locations). If a single integer, then compute at that many frequencies. Otherwise, compute the response at the angular frequencies (e.g. rad/s) given in <i>worN</i> .
	<b>plot</b> : callable, optional	A callable that takes two arguments. If given, the return parameters <i>w</i> and <i>h</i> are passed to plot. Useful for plotting the frequency response inside <code>freqs</code> .
<b>Returns</b>	<b>w</b> : ndarray	The angular frequencies at which <i>h</i> was computed.
	<b>h</b> : ndarray	The frequency response.

### See also:

`freqz` Compute the frequency response of a digital filter.

### Notes

Using Matplotlib's "plot" function as the callable for *plot* produces unexpected results, this plots the real part of the complex transfer function, not the magnitude. Try `lambda w, h: plot(w, abs(h))`.

### Examples

```

>>> from scipy.signal import freqs, iirfilter

>>> b, a = iirfilter(4, [1, 10], 1, 60, analog=True, ftype='cheby1')

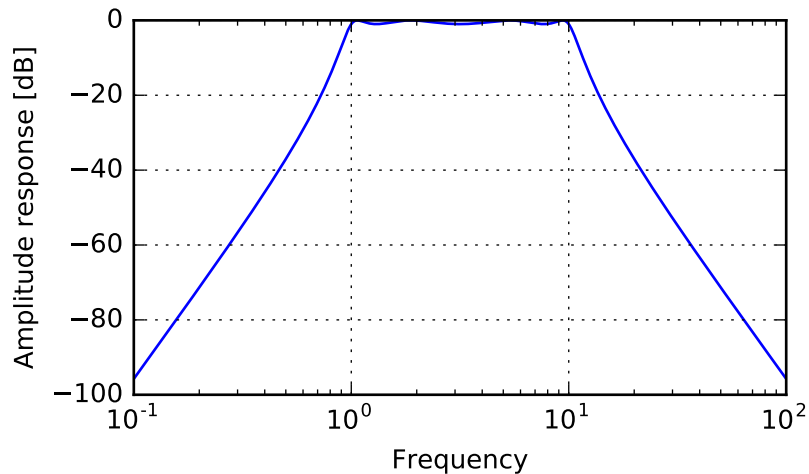
>>> w, h = freqs(b, a, worN=np.logspace(-1, 2, 1000))

```

```

>>> import matplotlib.pyplot as plt
>>> plt.semilogx(w, 20 * np.log10(abs(h)))
>>> plt.xlabel('Frequency')
>>> plt.ylabel('Amplitude response [dB]')
>>> plt.grid()
>>> plt.show()

```



`scipy.signal.freqz` (*b*, *a=1*, *worN=None*, *whole=False*, *plot=None*)  
 Compute the frequency response of a digital filter.

Given the *M*-order numerator *b* and *N*-order denominator *a* of a digital filter, compute its frequency response:

$$H(e^{j\omega}) = \frac{B(e^{j\omega})}{A(e^{j\omega})} = \frac{b[0] + b[1]e^{-j\omega} + \dots + b[M]e^{-j\omega M}}{a[0] + a[1]e^{-j\omega} + \dots + a[N]e^{-j\omega N}}$$

**Parameters**

- b** : array\_like  
numerator of a linear filter
- a** : array\_like  
denominator of a linear filter
- worN** : {None, int, array\_like}, optional  
If None (default), then compute at 512 frequencies equally spaced around the unit circle. If a single integer, then compute at that many frequencies. If an array\_like, compute the response at the frequencies given (in radians/sample).
- whole** : bool, optional  
Normally, frequencies are computed from 0 to the Nyquist frequency, pi radians/sample (upper-half of unit-circle). If *whole* is True, compute frequencies from 0 to 2\*pi radians/sample.
- plot** : callable  
A callable that takes two arguments. If given, the return parameters *w* and *h* are passed to plot. Useful for plotting the frequency response inside `freqz`.

**Returns**

- w** : ndarray  
The normalized frequencies at which *h* was computed, in radians/sample.

**h** : ndarray  
The frequency response.

### Notes

Using Matplotlib's "plot" function as the callable for *plot* produces unexpected results, this plots the real part of the complex transfer function, not the magnitude. Try `lambda w, h: plot(w, abs(h))`.

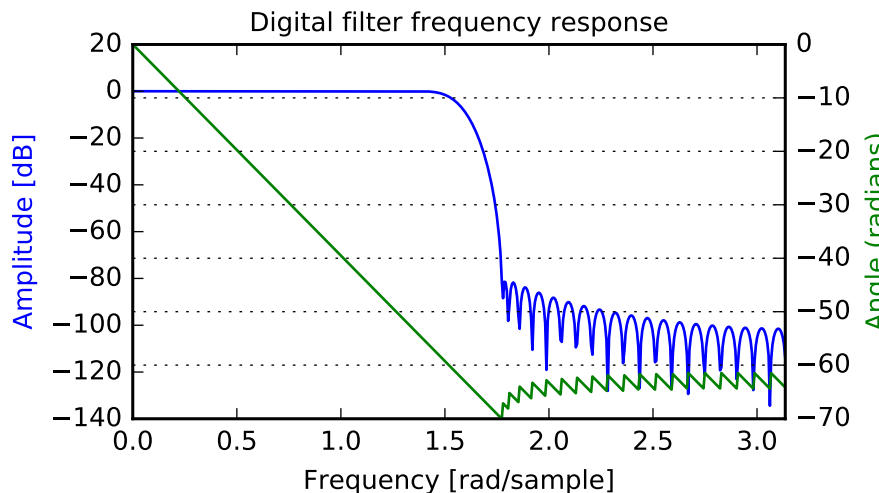
### Examples

```
>>> from scipy import signal
>>> b = signal.firwin(80, 0.5, window=('kaiser', 8))
>>> w, h = signal.freqz(b)

>>> import matplotlib.pyplot as plt
>>> fig = plt.figure()
>>> plt.title('Digital filter frequency response')
>>> ax1 = fig.add_subplot(111)

>>> plt.plot(w, 20 * np.log10(abs(h)), 'b')
>>> plt.ylabel('Amplitude [dB]', color='b')
>>> plt.xlabel('Frequency [rad/sample]')

>>> ax2 = ax1.twinx()
>>> angles = np.unwrap(np.angle(h))
>>> plt.plot(w, angles, 'g')
>>> plt.ylabel('Angle (radians)', color='g')
>>> plt.grid()
>>> plt.axis('tight')
>>> plt.show()
```



`scipy.signal.group_delay` (*system*, *w=None*, *whole=False*)

Compute the group delay of a digital filter.

The group delay measures by how many samples amplitude envelopes of various spectral components of a signal are delayed by a filter. It is formally defined as the derivative of continuous (unwrapped) phase:

$$D(\omega) = - \frac{d}{d\omega} \arg H(e^{j\omega})$$

<b>Parameters</b>	<b>system</b> : tuple of array_like (b, a) Numerator and denominator coefficients of a filter transfer function.
	<b>w</b> : {None, int, array-like}, optional If None (default), then compute at 512 frequencies equally spaced around the unit circle. If a single integer, then compute at that many frequencies. If array, compute the delay at the frequencies given (in radians/sample).
	<b>whole</b> : bool, optional Normally, frequencies are computed from 0 to the Nyquist frequency, pi radians/sample (upper-half of unit-circle). If <i>whole</i> is True, compute frequencies from 0 to 2*pi radians/sample.
<b>Returns</b>	<b>w</b> : ndarray The normalized frequencies at which the group delay was computed, in radians/sample.
	<b>gd</b> : ndarray The group delay.

See also:

[\*freqz\*](#) Frequency response of a digital filter

#### Notes

The similar function in MATLAB is called *grpdelay*.

If the transfer function  $H(z)$  has zeros or poles on the unit circle, the group delay at corresponding frequencies is undefined. When such a case arises the warning is raised and the group delay is set to 0 at those frequencies.

For the details of numerical computation of the group delay refer to [R203].

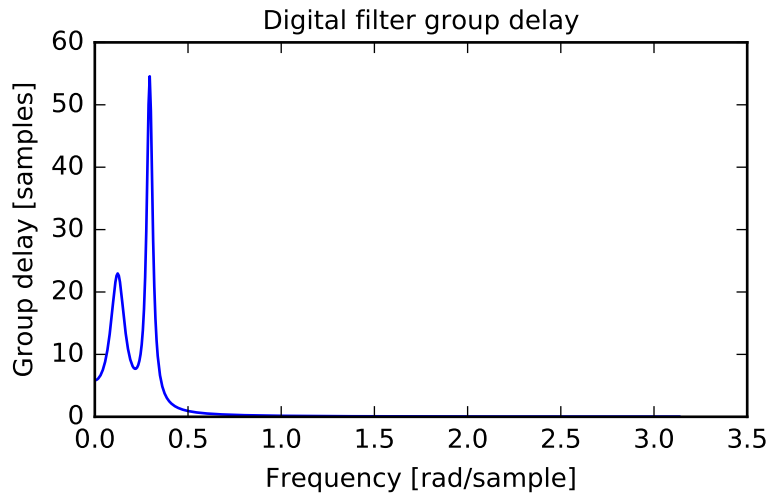
#### References

[R203]

#### Examples

```
>>> from scipy import signal
>>> b, a = signal.iirdesign(0.1, 0.3, 5, 50, ftype='cheby1')
>>> w, gd = signal.group_delay((b, a))

>>> import matplotlib.pyplot as plt
>>> plt.title('Digital filter group delay')
>>> plt.plot(w, gd)
>>> plt.ylabel('Group delay [samples]')
>>> plt.xlabel('Frequency [rad/sample]')
>>> plt.show()
```



`scipy.signal.iirdesign` (*wp*, *ws*, *gpass*, *gstop*, *analog=False*, *ftype='ellip'*, *output='ba'*)

Complete IIR digital and analog filter design.

Given passband and stopband frequencies and gains, construct an analog or digital IIR filter of minimum order for a given basic type. Return the output in numerator, denominator ('ba'), pole-zero ('zpk') or second order sections ('sos') form.

**Parameters**    **wp, ws** : float

Passband and stopband edge frequencies. For digital filters, these are normalized from 0 to 1, where 1 is the Nyquist frequency, pi radians/sample. (*wp* and *ws* are thus in half-cycles / sample.) For example:

- Lowpass: *wp* = 0.2, *ws* = 0.3
- Highpass: *wp* = 0.3, *ws* = 0.2
- Bandpass: *wp* = [0.2, 0.5], *ws* = [0.4, 0.6]
- Bandstop: *wp* = [0.1, 0.6], *ws* = [0.2, 0.5]

For analog filters, *wp* and *ws* are angular frequencies (e.g. rad/s).

**gpass** : float

The maximum loss in the passband (dB).

**gstop** : float

The minimum attenuation in the stopband (dB).

**analog** : bool, optional

When True, return an analog filter, otherwise a digital filter is returned.

**ftype** : str, optional

The type of IIR filter to design:

- Butterworth: 'butter'
- Chebyshev I: 'cheby1'
- Chebyshev II: 'cheby2'
- Cauchy/elliptic: 'ellip'
- Bessel/Thomson: 'bessel'

**output** : {'ba', 'zpk', 'sos'}, optional

Type of output: numerator/denominator ('ba'), pole-zero ('zpk'), or second-order sections ('sos'). Default is 'ba'.

**Returns**

**b, a** : ndarray, ndarray

Numerator (*b*) and denominator (*a*) polynomials of the IIR filter. Only returned if *output='ba'*.

**z, p, k** : ndarray, ndarray, float

Zeros, poles, and system gain of the IIR filter transfer function. Only returned if *output='zpk'*.

**sos** : ndarray

Second-order sections representation of the IIR filter. Only returned if `output=='sos'`.

**See also:**

**`butter`** Filter design using order and critical points

`cheby1`, `cheby2`, `ellip`, `bessel`

**`buttord`** Find order and critical points from passband and stopband spec

`cheblord`, `cheb2ord`, `ellipord`

**`iirfilter`** General filter design using order and critical frequencies

**Notes**

The `'sos'` output parameter was added in 0.16.0.

`scipy.signal.iirfilter` (*N*, *Wn*, *rp=None*, *rs=None*, *btype='band'*, *analog=False*, *fctype='butter'*, *output='ba'*)

IIR digital and analog filter design given order and critical points.

Design an *N*th-order digital or analog filter and return the filter coefficients.

**Parameters** *N* : int

The order of the filter.

***Wn*** : array\_like

A scalar or length-2 sequence giving the critical frequencies. For digital filters, *Wn* is normalized from 0 to 1, where 1 is the Nyquist frequency, pi radians/sample. (*Wn* is thus in half-cycles / sample.) For analog filters, *Wn* is an angular frequency (e.g. rad/s).

***rp*** : float, optional

For Chebyshev and elliptic filters, provides the maximum ripple in the passband. (dB)

***rs*** : float, optional

For Chebyshev and elliptic filters, provides the minimum attenuation in the stop band. (dB)

***btype*** : { 'bandpass', 'lowpass', 'highpass', 'bandstop' }, optional

The type of filter. Default is 'bandpass'.

***analog*** : bool, optional

When True, return an analog filter, otherwise a digital filter is returned.

***fctype*** : str, optional

The type of IIR filter to design:

- Butterworth: 'butter'
- Chebyshev I: 'cheby1'
- Chebyshev II: 'cheby2'
- Cauchy/elliptic: 'ellip'
- Bessel/Thomson: 'bessel'

***output*** : { 'ba', 'zpk', 'sos' }, optional

Type of output: numerator/denominator ('ba'), pole-zero ('zpk'), or second-order sections ('sos'). Default is 'ba'.

**Returns**

***b, a*** : ndarray, ndarray

Numerator (*b*) and denominator (*a*) polynomials of the IIR filter. Only returned if `output='ba'`.

***z, p, k*** : ndarray, ndarray, float

Zeros, poles, and system gain of the IIR filter transfer function. Only returned if `output='zpk'`.

***sos*** : ndarray

Second-order sections representation of the IIR filter. Only returned if `output=='sos'`.

**See also:**

**butter** Filter design using order and critical points

`cheby1`, `cheby2`, `ellip`, `bessel`

**buttord** Find order and critical points from passband and stopband spec

`cheblord`, `cheb2ord`, `ellipord`

**iirdesign** General filter design using passband and stopband spec

### Notes

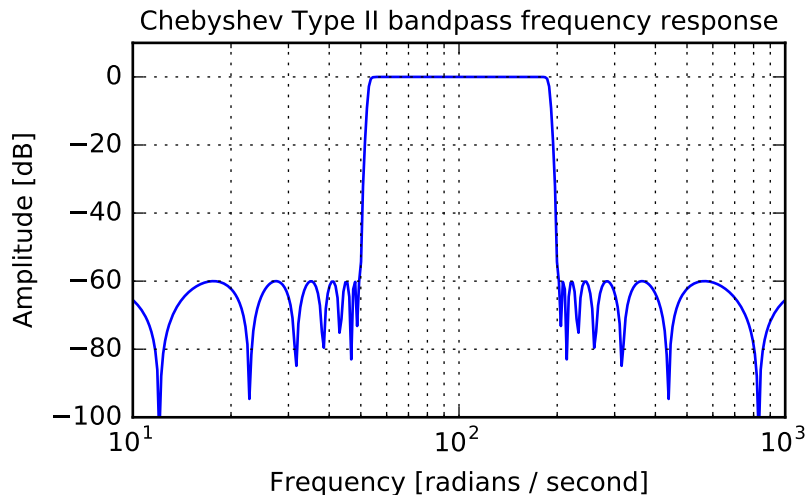
The 'sos' output parameter was added in 0.16.0.

### Examples

Generate a 17th-order Chebyshev II bandpass filter and plot the frequency response:

```
>>> from scipy import signal
>>> import matplotlib.pyplot as plt

>>> b, a = signal.iirfilter(17, [50, 200], rs=60, btype='band',
...                        analog=True, ftype='cheby2')
>>> w, h = signal.freqs(b, a, 1000)
>>> fig = plt.figure()
>>> ax = fig.add_subplot(111)
>>> ax.semilogx(w, 20 * np.log10(abs(h)))
>>> ax.set_title('Chebyshev Type II bandpass frequency response')
>>> ax.set_xlabel('Frequency [radians / second]')
>>> ax.set_ylabel('Amplitude [dB]')
>>> ax.axis((10, 1000, -100, 10))
>>> ax.grid(which='both', axis='both')
>>> plt.show()
```



`scipy.signal.kaiser_atten` (*numtaps*, *width*)

Compute the attenuation of a Kaiser FIR filter.

Given the number of taps  $N$  and the transition width  $width$ , compute the attenuation  $a$  in dB, given by Kaiser's formula:

$$a = 2.285 * (N - 1) * \pi * width + 7.95$$



**Parameters** **numtaps** : int  
The number of taps in the FIR filter.

**width** : float  
The desired width of the transition region between passband and stopband (or, in general, at any discontinuity) for the filter.

**Returns** **a** : float  
The attenuation of the ripple, in dB.

**See also:**

`kaiserord`, `kaiser_beta`

`scipy.signal.kaiser_beta` (*a*)

Compute the Kaiser parameter *beta*, given the attenuation *a*.

**Parameters** **a** : float  
The desired attenuation in the stopband and maximum ripple in the passband, in dB. This should be a *positive* number.

**Returns** **beta** : float  
The *beta* parameter to be used in the formula for a Kaiser window.

**References**

Oppenheim, Schafer, “Discrete-Time Signal Processing”, p.475-476.

`scipy.signal.kaiserord` (*ripple*, *width*)

Design a Kaiser window to limit ripple and width of transition region.

**Parameters** **ripple** : float  
Positive number specifying maximum ripple in passband (dB) and minimum ripple in stopband.

**width** : float  
Width of transition region (normalized so that 1 corresponds to pi radians / sample).

**Returns** **numtaps** : int  
The length of the kaiser window.

**beta** : float  
The beta parameter for the kaiser window.

**See also:**

`kaiser_beta`, `kaiser_atten`

**Notes**

There are several ways to obtain the Kaiser window:

- `signal.kaiser(numtaps, beta, sym=True)`
- `signal.get_window(beta, numtaps)`
- `signal.get_window('kaiser', beta), numtaps)`

The empirical equations discovered by Kaiser are used.

**References**

Oppenheim, Schafer, “Discrete-Time Signal Processing”, p.475-476.

`scipy.signal.savgol_coeffs` (*window\_length*, *polyorder*, *deriv=0*, *delta=1.0*, *pos=None*, *use='conv'*)

Compute the coefficients for a 1-d Savitzky-Golay FIR filter.

**Parameters** **window\_length** : int  
The length of the filter window (i.e. the number of coefficients). *window\_length* must be an odd positive integer.

**polyorder** : int  
 The order of the polynomial used to fit the samples. *polyorder* must be less than *window\_length*.

**deriv** : int, optional  
 The order of the derivative to compute. This must be a nonnegative integer. The default is 0, which means to filter the data without differentiating.

**delta** : float, optional  
 The spacing of the samples to which the filter will be applied. This is only used if *deriv* > 0.

**pos** : int or None, optional  
 If *pos* is not None, it specifies evaluation position within the window. The default is the middle of the window.

**use** : str, optional  
 Either 'conv' or 'dot'. This argument chooses the order of the coefficients. The default is 'conv', which means that the coefficients are ordered to be used in a convolution. With *use='dot'*, the order is reversed, so the filter is applied by dotting the coefficients with the data set.

**Returns** **coeffs** : 1-d ndarray  
 The filter coefficients.

**See also:**

`savgol_filter`

**Notes**

New in version 0.14.0.

**References**

A. Savitzky, M. J. E. Golay, Smoothing and Differentiation of Data by Simplified Least Squares Procedures. Analytical Chemistry, 1964, 36 (8), pp 1627-1639.

**Examples**

```
>>> from scipy.signal import savgol_coeffs
>>> savgol_coeffs(5, 2)
array([-0.08571429,  0.34285714,  0.48571429,  0.34285714, -0.08571429])
>>> savgol_coeffs(5, 2, deriv=1)
array([ 2.00000000e-01,  1.00000000e-01,  2.00607895e-16,
        -1.00000000e-01, -2.00000000e-01])
```

Note that *use='dot'* simply reverses the coefficients.

```
>>> savgol_coeffs(5, 2, pos=3)
array([ 0.25714286,  0.37142857,  0.34285714,  0.17142857, -0.14285714])
>>> savgol_coeffs(5, 2, pos=3, use='dot')
array([-0.14285714,  0.17142857,  0.34285714,  0.37142857,  0.25714286])
```

*x* contains data from the parabola  $x = t^2$ , sampled at  $t = -1, 0, 1, 2, 3$ . *c* holds the coefficients that will compute the derivative at the last position. When dotted with *x* the result should be 6.

```
>>> x = np.array([1, 0, 1, 4, 9])
>>> c = savgol_coeffs(5, 2, pos=4, deriv=1, use='dot')
>>> c.dot(x)
6.0000000000000018
```

```
scipy.signal.remez(numtaps, bands, desired, weight=None, Hz=1, type='bandpass', maxiter=25,
                  grid_density=16)
```

Calculate the minimax optimal filter using the Remez exchange algorithm.

Calculate the filter-coefficients for the finite impulse response (FIR) filter whose transfer function minimizes the maximum error between the desired gain and the realized gain in the specified frequency bands using the Remez exchange algorithm.

**Parameters**    **numtaps** : int

The desired number of taps in the filter. The number of taps is the number of terms in the filter, or the filter order plus one.

**bands** : array\_like

A monotonic sequence containing the band edges in Hz. All elements must be non-negative and less than half the sampling frequency as given by *Hz*.

**desired** : array\_like

A sequence half the size of *bands* containing the desired gain in each of the specified bands.

**weight** : array\_like, optional

A relative weighting to give to each band region. The length of *weight* has to be half the length of *bands*.

**Hz** : scalar, optional

The sampling frequency in Hz. Default is 1.

**type** : {'bandpass', 'differentiator', 'hilbert'}, optional

The type of filter:

- 'bandpass' : flat response in bands. This is the default.
- 'differentiator' : frequency proportional response in bands.
- 'hilbert' : [filter with odd symmetry, that is, type III] (for even order) or type IV (for odd order) linear phase filters.

**maxiter** : int, optional

Maximum number of iterations of the algorithm. Default is 25.

**grid\_density** : int, optional

Grid density. The dense grid used in `remez` is of size  $(\text{numtaps} + 1) * \text{grid\_density}$ . Default is 16.

**Returns**    **out** : ndarray

A rank-1 array containing the coefficients of the optimal (in a minimax sense) filter.

**See also:**

`freqz`, `firls`, `firwin`, `firwin2`

**References**

[R228], [R229]

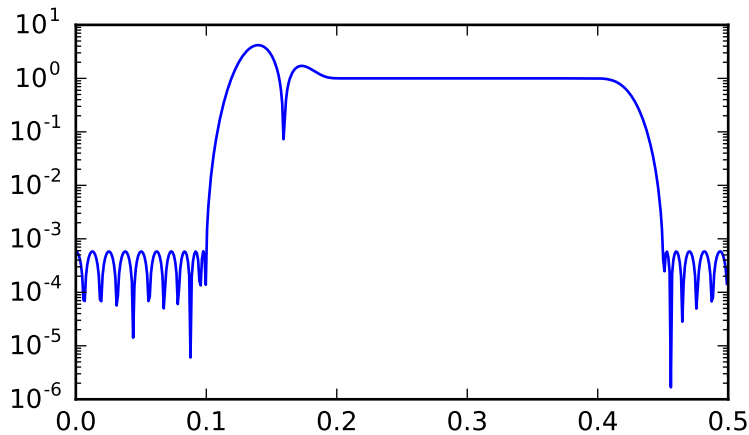
**Examples**

We want to construct a filter with a passband at 0.2-0.4 Hz, and stop bands at 0-0.1 Hz and 0.45-0.5 Hz. Note that this means that the behavior in the frequency ranges between those bands is unspecified and may overshoot.

```
>>> from scipy import signal
>>> bpass = signal.remez(72, [0, 0.1, 0.2, 0.4, 0.45, 0.5], [0, 1, 0])
>>> freq, response = signal.freqz(bpass)
>>> ampl = np.abs(response)

>>> import matplotlib.pyplot as plt
>>> fig = plt.figure()
>>> ax1 = fig.add_subplot(111)
```

```
>>> ax1.semilogy(freq/(2*np.pi), ampl, 'b-') # freq in Hz
>>> plt.show()
```



```
scipy.signal.unique_roots(p, tol=0.001, rtype='min')
```

Determine unique roots and their multiplicities from a list of roots.

**Parameters**

- p** : array\_like  
The list of roots.
- tol** : float, optional  
The tolerance for two roots to be considered equal. Default is 1e-3.
- rtype** : {'max', 'min', 'avg'}, optional  
How to determine the returned root if multiple roots are within *tol* of each other.
  - 'max': pick the maximum of those roots.
  - 'min': pick the minimum of those roots.
  - 'avg': take the average of those roots.

**Returns**

- root** : ndarray  
The list of unique roots, sorted from low to high.
- mult** : ndarray  
The multiplicity of each root.

### Notes

This utility function is not specific to roots but can be used for any sequence of values for which uniqueness and multiplicity has to be determined. For a more general routine, see `numpy.unique`.

### Examples

```
>>> from scipy import signal
>>> vals = [0, 1.3, 1.31, 2.8, 1.25, 2.2, 10.3]
>>> uniq, mult = signal.unique_roots(vals, tol=2e-2, rtype='avg')
```

Check which roots have multiplicity larger than 1:

```
>>> uniq[mult > 1]
array([ 1.305])
```

`scipy.signal.residue(b, a, tol=0.001, rtype='avg')`

Compute partial-fraction expansion of  $b(s) / a(s)$ .

If  $M$  is the degree of numerator  $b$  and  $N$  the degree of denominator  $a$ :

$$H(s) = \frac{b(s)}{a(s)} = \frac{b[0] s^{**}(M) + b[1] s^{**}(M-1) + \dots + b[M]}{a[0] s^{**}(N) + a[1] s^{**}(N-1) + \dots + a[N]}$$

then the partial-fraction expansion  $H(s)$  is defined as:

$$= \frac{r[0]}{(s-p[0])} + \frac{r[1]}{(s-p[1])} + \dots + \frac{r[-1]}{(s-p[-1])} + k(s)$$

If there are any repeated roots (closer together than  $tol$ ), then  $H(s)$  has terms like:

$$\frac{r[i]}{(s-p[i])} + \frac{r[i+1]}{(s-p[i])**2} + \dots + \frac{r[i+n-1]}{(s-p[i])**n}$$

This function is used for polynomials in positive powers of  $s$  or  $z$ , such as analog filters or digital filters in controls engineering. For negative powers of  $z$  (typical for digital filters in DSP), use `residuez`.

**Parameters**

- b** : array\_like  
Numerator polynomial coefficients.
- a** : array\_like  
Denominator polynomial coefficients.

**Returns**

- r** : ndarray  
Residues.
- p** : ndarray  
Poles.
- k** : ndarray  
Coefficients of the direct polynomial term.

**See also:**

`invres`, `residuez`, `numpy.poly`, `unique_roots`

`scipy.signal.residuez(b, a, tol=0.001, rtype='avg')`

Compute partial-fraction expansion of  $b(z) / a(z)$ .

If  $M$  is the degree of numerator  $b$  and  $N$  the degree of denominator  $a$ :

$$H(z) = \frac{b(z)}{a(z)} = \frac{b[0] + b[1] z^{**}(-1) + \dots + b[M] z^{**}(-M)}{a[0] + a[1] z^{**}(-1) + \dots + a[N] z^{**}(-N)}$$

then the partial-fraction expansion  $H(z)$  is defined as:

$$= \frac{r[0]}{(1-p[0]z^{**}(-1))} + \dots + \frac{r[-1]}{(1-p[-1]z^{**}(-1))} + k[0] + k[1]z^{**}(-1) \dots$$

If there are any repeated roots (closer than  $tol$ ), then the partial fraction expansion has terms like:

$$\frac{r[i]}{(1-p[i]z^{**}(-1))} + \frac{r[i+1]}{(1-p[i]z^{**}(-1))**2} + \dots + \frac{r[i+n-1]}{(1-p[i]z^{**}(-1))**n}$$

This function is used for polynomials in negative powers of  $z$ , such as digital filters in DSP. For positive powers, use `residue`.

**Parameters**

- b** : array\_like  
Numerator polynomial coefficients.
- a** : array\_like  
Denominator polynomial coefficients.

**Returns**

- r** : ndarray  
Residues.
- p** : ndarray  
Poles.
- k** : ndarray  
Coefficients of the direct polynomial term.

**See also:**

`invresz`, `residue`, `unique_roots`

`scipy.signal.invres(r, p, k, tol=0.001, rtype='avg')`  
 Compute  $b(s)$  and  $a(s)$  from partial fraction expansion.

If  $M$  is the degree of numerator  $b$  and  $N$  the degree of denominator  $a$ :

$$H(s) = \frac{b(s)}{a(s)} = \frac{b[0] s^{(M)} + b[1] s^{(M-1)} + \dots + b[M]}{a[0] s^{(N)} + a[1] s^{(N-1)} + \dots + a[N]}$$

then the partial-fraction expansion  $H(s)$  is defined as:

$$= \frac{r[0]}{(s-p[0])} + \frac{r[1]}{(s-p[1])} + \dots + \frac{r[-1]}{(s-p[-1])} + k(s)$$

If there are any repeated roots (closer together than `tol`), then  $H(s)$  has terms like:

$$\frac{r[i]}{(s-p[i])} + \frac{r[i+1]}{(s-p[i])**2} + \dots + \frac{r[i+n-1]}{(s-p[i])**n}$$

This function is used for polynomials in positive powers of  $s$  or  $z$ , such as analog filters or digital filters in controls engineering. For negative powers of  $z$  (typical for digital filters in DSP), use `invresz`.

**Parameters**

- r** : array\_like  
Residues.
- p** : array\_like  
Poles.
- k** : array\_like  
Coefficients of the direct polynomial term.
- tol** : float, optional  
The tolerance for two roots to be considered equal. Default is 1e-3.
- rtype** : { 'max', 'min', 'avg' }, optional  
How to determine the returned root if multiple roots are within `tol` of each other.
  - 'max': pick the maximum of those roots.
  - 'min': pick the minimum of those roots.
  - 'avg': take the average of those roots.

**Returns**

- b** : ndarray  
Numerator polynomial coefficients.
- a** : ndarray  
Denominator polynomial coefficients.

**See also:**`residue, invresz, unique_roots``scipy.signal.invresz(r, p, k, tol=0.001, rtype='avg')`Compute  $b(z)$  and  $a(z)$  from partial fraction expansion.If  $M$  is the degree of numerator  $b$  and  $N$  the degree of denominator  $a$ :

$$H(z) = \frac{b(z)}{a(z)} = \frac{b[0] + b[1]z^{*-1} + \dots + b[M]z^{*-M}}{a[0] + a[1]z^{*-1} + \dots + a[N]z^{*-N}}$$

then the partial-fraction expansion  $H(z)$  is defined as:

$$= \frac{r[0]}{(1-p[0]z^{*-1})} + \dots + \frac{r[-1]}{(1-p[-1]z^{*-1})} + k[0] + k[1]z^{*-1} \dots$$

If there are any repeated roots (closer than  $tol$ ), then the partial fraction expansion has terms like:

$$\frac{r[i]}{(1-p[i]z^{*-1})} + \frac{r[i+1]}{(1-p[i]z^{*-1})^{*2}} + \dots + \frac{r[i+n-1]}{(1-p[i]z^{*-1})^{*n}}$$

This function is used for polynomials in negative powers of  $z$ , such as digital filters in DSP. For positive powers, use `invres`.

**Parameters**

- r** : array\_like  
Residues.
- p** : array\_like  
Poles.
- k** : array\_like  
Coefficients of the direct polynomial term.
- tol** : float, optional  
The tolerance for two roots to be considered equal. Default is 1e-3.
- rtype** : { 'max', 'min', 'avg' }, optional  
How to determine the returned root if multiple roots are within  $tol$  of each other.
  - 'max': pick the maximum of those roots.
  - 'min': pick the minimum of those roots.
  - 'avg': take the average of those roots.

**Returns**

- b** : ndarray  
Numerator polynomial coefficients.
- a** : ndarray  
Denominator polynomial coefficients.

**See also:**`residuez, unique_roots, invres`**exception** `scipy.signal.BadCoefficients`

Warning about badly conditioned filter coefficients

Lower-level filter design functions:

<code>abcd_normalize([A, B, C, D])</code>	Check state-space matrices and ensure they are two-dimensional.
<code>band_stop_obj(wp, ind, passb, stopb, gpass, ...)</code>	Band Stop Objective Function for order minimization.
<code>besselap(N[, norm])</code>	Return (z,p,k) for analog prototype of an Nth-order Bessel filter.

Continued on next page

Table 5.127 – continued from previous page

<code>buttap(N)</code>	Return (z,p,k) for analog prototype of Nth-order Butterworth filter.
<code>cheb1ap(N, rp)</code>	Return (z,p,k) for Nth-order Chebyshev type I analog lowpass filter.
<code>cheb2ap(N, rs)</code>	Return (z,p,k) for Nth-order Chebyshev type I analog lowpass filter.
<code>cmplx_sort(p)</code>	Sort roots based on magnitude.
<code>ellipap(N, rp, rs)</code>	Return (z,p,k) of Nth-order elliptic analog lowpass filter.
<code>lp2bp(b, a[, wo, bw])</code>	Transform a lowpass filter prototype to a bandpass filter.
<code>lp2bs(b, a[, wo, bw])</code>	Transform a lowpass filter prototype to a bandstop filter.
<code>lp2hp(b, a[, wo])</code>	Transform a lowpass filter prototype to a highpass filter.
<code>lp2lp(b, a[, wo])</code>	Transform a lowpass filter prototype to a different frequency.
<code>normalize(b, a)</code>	Normalize polynomial representation of a transfer function.

`scipy.signal.abcd_normalize` (*A=None, B=None, C=None, D=None*)

Check state-space matrices and ensure they are two-dimensional.

If enough information on the system is provided, that is, enough properly-shaped arrays are passed to the function, the missing ones are built from this information, ensuring the correct number of rows and columns. Otherwise a `ValueError` is raised.

**Parameters** **A, B, C, D** : array\_like, optional  
State-space matrices. All of them are None (missing) by default. See `ss2tf` for format.

**Returns** **A, B, C, D** : array

**Raises** **ValueError** Properly shaped state-space matrices.  
If not enough information on the system was provided.

`scipy.signal.band_stop_obj` (*wp, ind, passb, stopb, gpass, gstop, type*)

Band Stop Objective Function for order minimization.

Returns the non-integer order for an analog band stop filter.

**Parameters** **wp** : scalar  
Edge of passband *passb*.

**ind** : int, {0, 1}  
Index specifying which *passb* edge to vary (0 or 1).

**passb** : ndarray  
Two element sequence of fixed passband edges.

**stopb** : ndarray  
Two element sequence of fixed stopband edges.

**gstop** : float  
Amount of attenuation in stopband in dB.

**gpass** : float  
Amount of ripple in the passband in dB.

**type** : {'butter', 'cheby', 'ellip'}

**Returns** **n** : scalar  
Type of filter.  
Filter order (possibly non-integer).

`scipy.signal.besselap` (*N, norm='phase'*)

Return (z,p,k) for analog prototype of an Nth-order Bessel filter.

**Parameters** **N** : int  
The order of the filter.

**norm** : {'phase', 'delay', 'mag'}, optional  
Frequency normalization:



	<b>phase</b>	The filter is normalized such that the phase response reaches its midpoint at an angular (e.g. rad/s) cutoff frequency of 1. This happens for both low-pass and high-pass filters, so this is the “phase-matched” case. [R185] The magnitude response asymptotes are the same as a Butterworth filter of the same order with a cutoff of $Wn$ . This is the default, and matches MATLAB’s implementation.
	<b>delay</b>	The filter is normalized such that the group delay in the pass-band is 1 (e.g. 1 second). This is the “natural” type obtained by solving Bessel polynomials.
	<b>mag</b>	The filter is normalized such that the gain magnitude is -3 dB at angular frequency 1. This is called “frequency normalization” by Bond. [R180]
<b>Returns</b>	<b>z</b> : ndarray	New in version 0.18.0. Zeros of the transfer function. Is always an empty array.
	<b>p</b> : ndarray	Poles of the transfer function.
	<b>k</b> : scalar	Gain of the transfer function. For phase-normalized, this is always 1.

**See also:**

**bessel** Filter design function using this prototype

**Notes**

To find the pole locations, approximate starting points are generated [R181] for the zeros of the ordinary Bessel polynomial [R182], then the Aberth-Ehrlich method [R183] [R184] is used on the  $Kv(x)$  Bessel function to calculate more accurate zeros, and these locations are then inverted about the unit circle.

**References**

[R180], [R181], [R182], [R183], [R184], [R185]

`scipy.signal.buttap` ( $N$ )

Return (z,p,k) for analog prototype of Nth-order Butterworth filter.

The filter will have an angular (e.g. rad/s) cutoff frequency of 1.

**See also:**

**butter** Filter design function using this prototype

`scipy.signal.cheb1ap` ( $N, rp$ )

Return (z,p,k) for Nth-order Chebyshev type I analog lowpass filter.

The returned filter prototype has  $rp$  decibels of ripple in the passband.

The filter’s angular (e.g. rad/s) cutoff frequency is normalized to 1, defined as the point at which the gain first drops below  $-rp$ .

**See also:**

**cheby1** Filter design function using this prototype

`scipy.signal.cheb2ap` ( $N, rs$ )

Return (z,p,k) for Nth-order Chebyshev type I analog lowpass filter.

The returned filter prototype has  $rs$  decibels of ripple in the stopband.

The filter’s angular (e.g. rad/s) cutoff frequency is normalized to 1, defined as the point at which the gain first reaches  $-rs$ .

**See also:**

`cheby2` Filter design function using this prototype

`scipy.signal.complex_sort` ( $p$ )  
Sort roots based on magnitude.

**Parameters** `p` : array\_like  
The roots to sort, as a 1-D array.

**Returns** `p_sorted` : ndarray  
Sorted roots.

`indx` : ndarray  
Array of indices needed to sort the input  $p$ .

`scipy.signal.ellipap` ( $N, rp, rs$ )  
Return (z,p,k) of Nth-order elliptic analog lowpass filter.

The filter is a normalized prototype that has  $rp$  decibels of ripple in the passband and a stopband  $rs$  decibels down.

The filter's angular (e.g. rad/s) cutoff frequency is normalized to 1, defined as the point at which the gain first drops below  $-rp$ .

**See also:**

`ellip` Filter design function using this prototype

**References**

Lutova, Tosic, and Evans, "Filter Design for Signal Processing", Chapters 5 and 12.

`scipy.signal.lp2bp` ( $b, a, wo=1.0, bw=1.0$ )  
Transform a lowpass filter prototype to a bandpass filter.

Return an analog band-pass filter with center frequency  $wo$  and bandwidth  $bw$  from an analog low-pass filter prototype with unity cutoff frequency, in transfer function ('ba') representation.

`scipy.signal.lp2bs` ( $b, a, wo=1.0, bw=1.0$ )  
Transform a lowpass filter prototype to a bandstop filter.

Return an analog band-stop filter with center frequency  $wo$  and bandwidth  $bw$  from an analog low-pass filter prototype with unity cutoff frequency, in transfer function ('ba') representation.

`scipy.signal.lp2hp` ( $b, a, wo=1.0$ )  
Transform a lowpass filter prototype to a highpass filter.

Return an analog high-pass filter with cutoff frequency  $wo$  from an analog low-pass filter prototype with unity cutoff frequency, in transfer function ('ba') representation.

`scipy.signal.lp2lp` ( $b, a, wo=1.0$ )  
Transform a lowpass filter prototype to a different frequency.

Return an analog low-pass filter with cutoff frequency  $wo$  from an analog low-pass filter prototype with unity cutoff frequency, in transfer function ('ba') representation.

`scipy.signal.normalize` ( $b, a$ )  
Normalize polynomial representation of a transfer function.

If values of  $b$  are too close to 0, they are removed. In that case, a BadCoefficients warning is emitted.

## 5.20.5 Matlab-style IIR filter design

---

<code>butter(N, Wn[, btype, analog, output])</code>	Butterworth digital and analog filter design.
<code>buttord(wp, ws, gpass, gstop[, analog])</code>	Butterworth filter order selection.
<code>cheby1(N, rp, Wn[, btype, analog, output])</code>	Chebyshev type I digital and analog filter design.
<code>cheblord(wp, ws, gpass, gstop[, analog])</code>	Chebyshev type I filter order selection.
<code>cheby2(N, rs, Wn[, btype, analog, output])</code>	Chebyshev type II digital and analog filter design.
<code>cheb2ord(wp, ws, gpass, gstop[, analog])</code>	Chebyshev type II filter order selection.
<code>ellip(N, rp, rs, Wn[, btype, analog, output])</code>	Elliptic (Cauer) digital and analog filter design.
<code>ellipord(wp, ws, gpass, gstop[, analog])</code>	Elliptic (Cauer) filter order selection.
<code>bessel(N, Wn[, btype, analog, output, norm])</code>	Bessel/Thomson digital and analog filter design.

---

`scipy.signal.butter(N, Wn, btype='low', analog=False, output='ba')`

Butterworth digital and analog filter design.

Design an Nth-order digital or analog Butterworth filter and return the filter coefficients.

**Parameters**    **N** : int

The order of the filter.

**Wn** : array\_like

A scalar or length-2 sequence giving the critical frequencies. For a Butterworth filter, this is the point at which the gain drops to  $1/\sqrt{2}$  that of the passband (the “-3 dB point”). For digital filters, *Wn* is normalized from 0 to 1, where 1 is the Nyquist frequency,  $\pi$  radians/sample. (*Wn* is thus in half-cycles / sample.) For analog filters, *Wn* is an angular frequency (e.g. rad/s).

**btype** : { ‘lowpass’, ‘highpass’, ‘bandpass’, ‘bandstop’ }, optional

The type of filter. Default is ‘lowpass’.

**analog** : bool, optional

When True, return an analog filter, otherwise a digital filter is returned.

**output** : { ‘ba’, ‘zpk’, ‘sos’ }, optional

Type of output: numerator/denominator (‘ba’), pole-zero (‘zpk’), or second-order sections (‘sos’). Default is ‘ba’.

**Returns**

**b, a** : ndarray, ndarray

Numerator (*b*) and denominator (*a*) polynomials of the IIR filter. Only returned if `output='ba'`.

**z, p, k** : ndarray, ndarray, float

Zeros, poles, and system gain of the IIR filter transfer function. Only returned if `output='zpk'`.

**sos** : ndarray

Second-order sections representation of the IIR filter. Only returned if `output=='sos'`.

**See also:**

`buttord`, `buttap`

**Notes**

The Butterworth filter has maximally flat frequency response in the passband.

The ‘`sos`’ output parameter was added in 0.16.0.

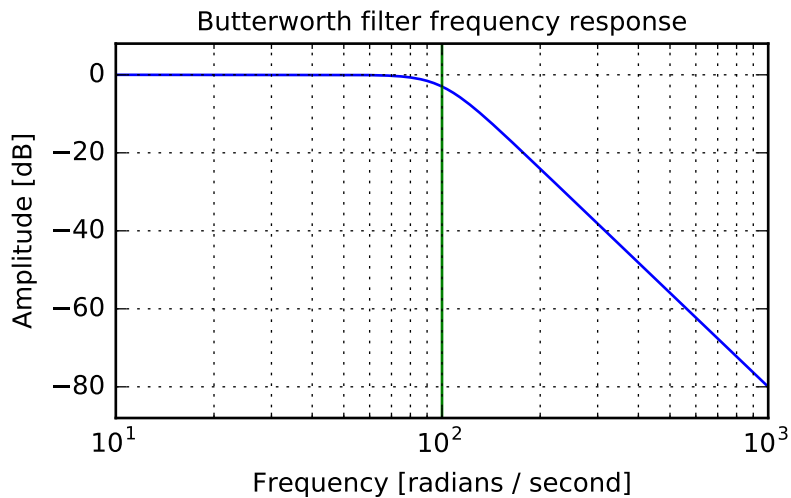
**Examples**

Plot the filter’s frequency response, showing the critical points:

```
>>> from scipy import signal
>>> import matplotlib.pyplot as plt
```

```

>>> b, a = signal.butter(4, 100, 'low', analog=True)
>>> w, h = signal.freqs(b, a)
>>> plt.semilogx(w, 20 * np.log10(abs(h)))
>>> plt.title('Butterworth filter frequency response')
>>> plt.xlabel('Frequency [radians / second]')
>>> plt.ylabel('Amplitude [dB]')
>>> plt.margins(0, 0.1)
>>> plt.grid(which='both', axis='both')
>>> plt.axvline(100, color='green') # cutoff frequency
>>> plt.show()
    
```



`scipy.signal.buttord` (*wp*, *ws*, *gpass*, *gstop*, *analog=False*)  
 Butterworth filter order selection.

Return the order of the lowest order digital or analog Butterworth filter that loses no more than *gpass* dB in the passband and has at least *gstop* dB attenuation in the stopband.

**Parameters**    **wp, ws** : float

Passband and stopband edge frequencies. For digital filters, these are normalized from 0 to 1, where 1 is the Nyquist frequency, pi radians/sample. (*wp* and *ws* are thus in half-cycles / sample.) For example:

- Lowpass: *wp* = 0.2, *ws* = 0.3
- Highpass: *wp* = 0.3, *ws* = 0.2
- Bandpass: *wp* = [0.2, 0.5], *ws* = [0.1, 0.6]
- Bandstop: *wp* = [0.1, 0.6], *ws* = [0.2, 0.5]

For analog filters, *wp* and *ws* are angular frequencies (e.g. rad/s).

**gpass** : float

The maximum loss in the passband (dB).

**gstop** : float

The minimum attenuation in the stopband (dB).

**analog** : bool, optional

When True, return an analog filter, otherwise a digital filter is returned.

**Returns**

**ord** : int

The lowest order for a Butterworth filter which meets specs.

**wn** : ndarray or float

The Butterworth natural frequency (i.e. the “3dB frequency”). Should be used with `butter` to give filter results.

See also:

**butter** Filter design using order and critical points  
**cheblord** Find order and critical points from passband and stopband spec

`cheb2ord`, `ellipord`

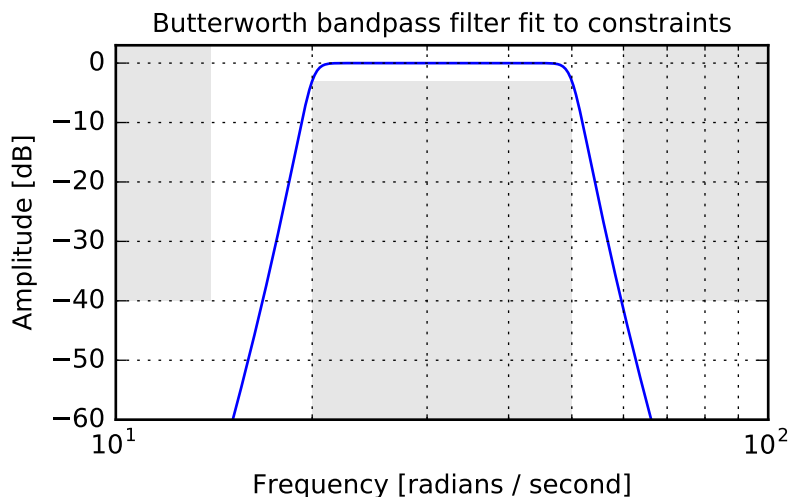
**iirfilter** General filter design using order and critical frequencies  
**iirdesign** General filter design using passband and stopband spec

### Examples

Design an analog bandpass filter with passband within 3 dB from 20 to 50 rad/s, while rejecting at least -40 dB below 14 and above 60 rad/s. Plot its frequency response, showing the passband and stopband constraints in gray.

```
>>> from scipy import signal
>>> import matplotlib.pyplot as plt

>>> N, Wn = signal.buttord([20, 50], [14, 60], 3, 40, True)
>>> b, a = signal.butter(N, Wn, 'band', True)
>>> w, h = signal.freqs(b, a, np.logspace(1, 2, 500))
>>> plt.semilogx(w, 20 * np.log10(abs(h)))
>>> plt.title('Butterworth bandpass filter fit to constraints')
>>> plt.xlabel('Frequency [radians / second]')
>>> plt.ylabel('Amplitude [dB]')
>>> plt.grid(which='both', axis='both')
>>> plt.fill([1, 14, 14, 1], [-40, -40, 99, 99], '0.9', lw=0) # stop
>>> plt.fill([20, 20, 50, 50], [-99, -3, -3, -99], '0.9', lw=0) # pass
>>> plt.fill([60, 60, 1e9, 1e9], [99, -40, -40, 99], '0.9', lw=0) # stop
>>> plt.axis([10, 100, -60, 3])
>>> plt.show()
```



`scipy.signal.cheby1` (*N*, *rp*, *Wn*, *btype*='low', *analog*=False, *output*='ba')

Chebyshev type I digital and analog filter design.

Design an Nth-order digital or analog Chebyshev type I filter and return the filter coefficients.

**Parameters**

- N** : int  
The order of the filter.
- rp** : float

The maximum ripple allowed below unity gain in the passband. Specified in decibels, as a positive number.

**Wn** : array\_like

A scalar or length-2 sequence giving the critical frequencies. For Type I filters, this is the point in the transition band at which the gain first drops below  $-rp$ . For digital filters,  $Wn$  is normalized from 0 to 1, where 1 is the Nyquist frequency,  $\pi$  radians/sample. ( $Wn$  is thus in half-cycles / sample.) For analog filters,  $Wn$  is an angular frequency (e.g. rad/s).

**btype** : { 'lowpass', 'highpass', 'bandpass', 'bandstop' }, optional

The type of filter. Default is 'lowpass'.

**analog** : bool, optional

When True, return an analog filter, otherwise a digital filter is returned.

**output** : { 'ba', 'zpk', 'sos' }, optional

Type of output: numerator/denominator ('ba'), pole-zero ('zpk'), or second-order sections ('sos'). Default is 'ba'.

#### Returns

**b, a** : ndarray, ndarray

Numerator ( $b$ ) and denominator ( $a$ ) polynomials of the IIR filter. Only returned if `output='ba'`.

**z, p, k** : ndarray, ndarray, float

Zeros, poles, and system gain of the IIR filter transfer function. Only returned if `output='zpk'`.

**sos** : ndarray

Second-order sections representation of the IIR filter. Only returned if `output=='sos'`.

#### See also:

`cheblord`, `cheblap`

#### Notes

The Chebyshev type I filter maximizes the rate of cutoff between the frequency response's passband and stopband, at the expense of ripple in the passband and increased ringing in the step response.

Type I filters roll off faster than Type II (`cheby2`), but Type II filters do not have any ripple in the passband.

The equiripple passband has  $N$  maxima or minima (for example, a 5th-order filter has 3 maxima and 2 minima). Consequently, the DC gain is unity for odd-order filters, or  $-rp$  dB for even-order filters.

The 'sos' output parameter was added in 0.16.0.

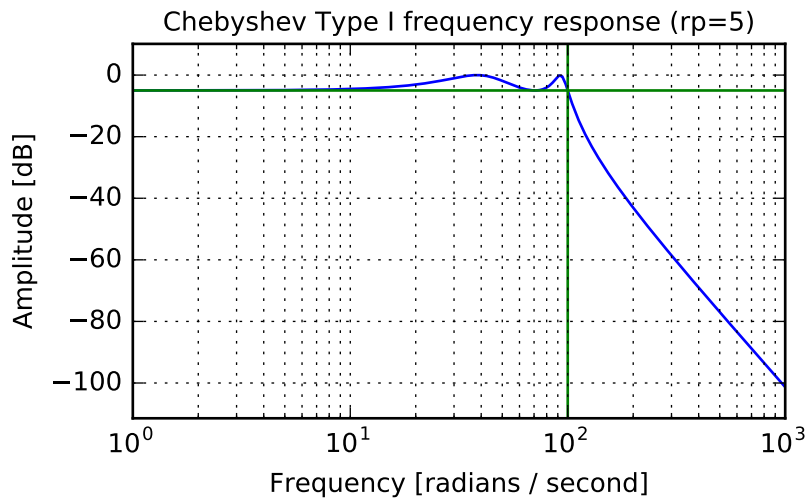
#### Examples

Plot the filter's frequency response, showing the critical points:

```
>>> from scipy import signal
>>> import matplotlib.pyplot as plt

>>> b, a = signal.cheby1(4, 5, 100, 'low', analog=True)
>>> w, h = signal.freqs(b, a)
>>> plt.semilogx(w, 20 * np.log10(abs(h)))
>>> plt.title('Chebyshev Type I frequency response (rp=5)')
>>> plt.xlabel('Frequency [radians / second]')
>>> plt.ylabel('Amplitude [dB]')
>>> plt.margins(0, 0.1)
>>> plt.grid(which='both', axis='both')
>>> plt.axvline(100, color='green') # cutoff frequency
```

```
>>> plt.axhline(-5, color='green') # rp
>>> plt.show()
```



```
scipy.signal.cheb1ord(wp, ws, gpass, gstop, analog=False)
```

Chebyshev type I filter order selection.

Return the order of the lowest order digital or analog Chebyshev Type I filter that loses no more than *gpass* dB in the passband and has at least *gstop* dB attenuation in the stopband.

**Parameters** **wp, ws** : float

Passband and stopband edge frequencies. For digital filters, these are normalized from 0 to 1, where 1 is the Nyquist frequency, pi radians/sample. (*wp* and *ws* are thus in half-cycles / sample.) For example:

- Lowpass: *wp* = 0.2, *ws* = 0.3
- Highpass: *wp* = 0.3, *ws* = 0.2
- Bandpass: *wp* = [0.2, 0.5], *ws* = [0.1, 0.6]
- Bandstop: *wp* = [0.1, 0.6], *ws* = [0.2, 0.5]

For analog filters, *wp* and *ws* are angular frequencies (e.g. rad/s).

**gpass** : float

The maximum loss in the passband (dB).

**gstop** : float

The minimum attenuation in the stopband (dB).

**analog** : bool, optional

When True, return an analog filter, otherwise a digital filter is returned.

**Returns**

**ord** : int

The lowest order for a Chebyshev type I filter that meets specs.

**wn** : ndarray or float

The Chebyshev natural frequency (the “3dB frequency”) for use with `cheby1` to give filter results.

**See also:**

`cheby1` Filter design using order and critical points

`buttord` Find order and critical points from passband and stopband spec

`cheb2ord`, `ellipord`

`iirfilter` General filter design using order and critical frequencies

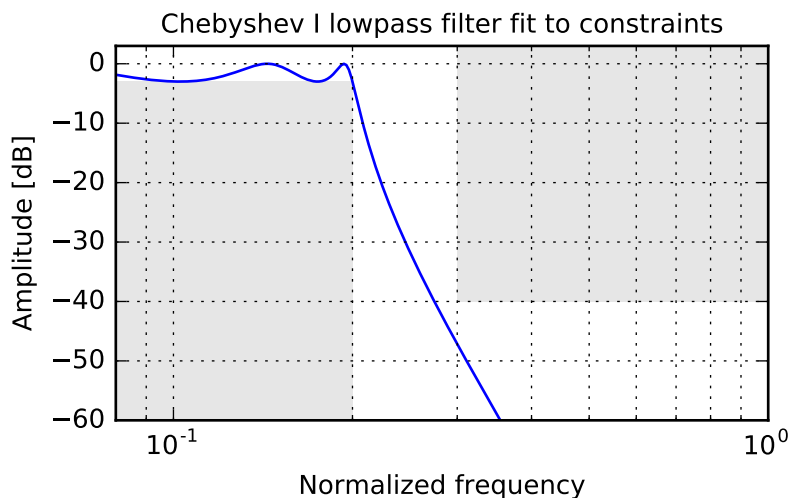
`iirdesign` General filter design using passband and stopband spec

### Examples

Design a digital lowpass filter such that the passband is within 3 dB up to  $0.2 \cdot (fs/2)$ , while rejecting at least -40 dB above  $0.3 \cdot (fs/2)$ . Plot its frequency response, showing the passband and stopband constraints in gray.

```
>>> from scipy import signal
>>> import matplotlib.pyplot as plt

>>> N, Wn = signal.cheblord(0.2, 0.3, 3, 40)
>>> b, a = signal.cheby1(N, 3, Wn, 'low')
>>> w, h = signal.freqz(b, a)
>>> plt.semilogx(w / np.pi, 20 * np.log10(abs(h)))
>>> plt.title('Chebyshev I lowpass filter fit to constraints')
>>> plt.xlabel('Normalized frequency')
>>> plt.ylabel('Amplitude [dB]')
>>> plt.grid(which='both', axis='both')
>>> plt.fill([0.01, 0.2, 0.2, .01], [-3, -3, -99, -99], '0.9', lw=0) # stop
>>> plt.fill([0.3, 0.3, 2, 2], [9, -40, -40, 9], '0.9', lw=0) # pass
>>> plt.axis([0.08, 1, -60, 3])
>>> plt.show()
```



`scipy.signal.cheby2(N, rs, Wn, btype='low', analog=False, output='ba')`

Chebyshev type II digital and analog filter design.

Design an Nth-order digital or analog Chebyshev type II filter and return the filter coefficients.

**Parameters** **N** : int

The order of the filter.

**rs** : float

The minimum attenuation required in the stop band. Specified in decibels, as a positive number.

**Wn** : array\_like

A scalar or length-2 sequence giving the critical frequencies. For Type II filters, this is the point in the transition band at which the gain first reaches  $-rs$ . For digital filters,  $Wn$  is normalized from 0 to 1, where 1 is the Nyquist frequency,  $\pi$  radians/sample. ( $Wn$  is thus in half-cycles / sample.) For analog filters,  $Wn$  is an angular frequency (e.g. rad/s).



**btype** : { 'lowpass', 'highpass', 'bandpass', 'bandstop' }, optional  
The type of filter. Default is 'lowpass'.

**analog** : bool, optional  
When True, return an analog filter, otherwise a digital filter is returned.

**output** : { 'ba', 'zpk', 'sos' }, optional  
Type of output: numerator/denominator ('ba'), pole-zero ('zpk'), or second-order sections ('sos'). Default is 'ba'.

**Returns**

**b, a** : ndarray, ndarray  
Numerator (*b*) and denominator (*a*) polynomials of the IIR filter. Only returned if `output='ba'`.

**z, p, k** : ndarray, ndarray, float  
Zeros, poles, and system gain of the IIR filter transfer function. Only returned if `output='zpk'`.

**sos** : ndarray  
Second-order sections representation of the IIR filter. Only returned if `output=='sos'`.

**See also:**`cheb2ord`, `cheb2ap`**Notes**

The Chebyshev type II filter maximizes the rate of cutoff between the frequency response's passband and stopband, at the expense of ripple in the stopband and increased ringing in the step response.

Type II filters do not roll off as fast as Type I (`cheby1`).

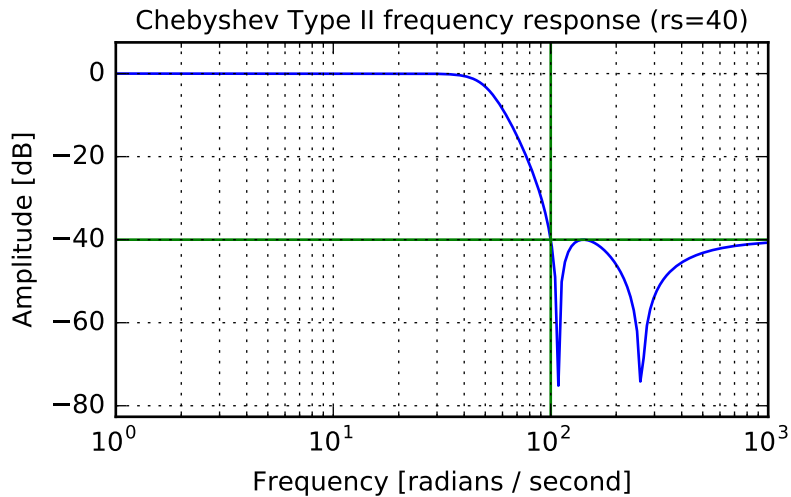
The 'sos' output parameter was added in 0.16.0.

**Examples**

Plot the filter's frequency response, showing the critical points:

```
>>> from scipy import signal
>>> import matplotlib.pyplot as plt

>>> b, a = signal.cheby2(4, 40, 100, 'low', analog=True)
>>> w, h = signal.freqs(b, a)
>>> plt.semilogx(w, 20 * np.log10(abs(h)))
>>> plt.title('Chebyshev Type II frequency response (rs=40)')
>>> plt.xlabel('Frequency [radians / second]')
>>> plt.ylabel('Amplitude [dB]')
>>> plt.margins(0, 0.1)
>>> plt.grid(which='both', axis='both')
>>> plt.axvline(100, color='green') # cutoff frequency
>>> plt.axhline(-40, color='green') # rs
>>> plt.show()
```



`scipy.signal.cheb2ord` (*wp*, *ws*, *gpass*, *gstop*, *analog=False*)

Chebyshev type II filter order selection.

Return the order of the lowest order digital or analog Chebyshev Type II filter that loses no more than *gpass* dB in the passband and has at least *gstop* dB attenuation in the stopband.

**Parameters**    **wp, ws** : float

Passband and stopband edge frequencies. For digital filters, these are normalized from 0 to 1, where 1 is the Nyquist frequency, pi radians/sample. (*wp* and *ws* are thus in half-cycles / sample.) For example:

- Lowpass: *wp* = 0.2, *ws* = 0.3
- Highpass: *wp* = 0.3, *ws* = 0.2
- Bandpass: *wp* = [0.2, 0.5], *ws* = [0.4, 0.6]
- Bandstop: *wp* = [0.1, 0.6], *ws* = [0.2, 0.5]

For analog filters, *wp* and *ws* are angular frequencies (e.g. rad/s).

**gpass** : float

The maximum loss in the passband (dB).

**gstop** : float

The minimum attenuation in the stopband (dB).

**analog** : bool, optional

When True, return an analog filter, otherwise a digital filter is returned.

**Returns**

**ord** : int

The lowest order for a Chebyshev type II filter that meets specs.

**wn** : ndarray or float

The Chebyshev natural frequency (the “3dB frequency”) for use with `cheby2` to give filter results.

**See also:**

`cheby2`    Filter design using order and critical points

`buttord`    Find order and critical points from passband and stopband spec

`cheblord`, `ellipord`

`iirfilter`    General filter design using order and critical frequencies

`iirdesign`    General filter design using passband and stopband spec

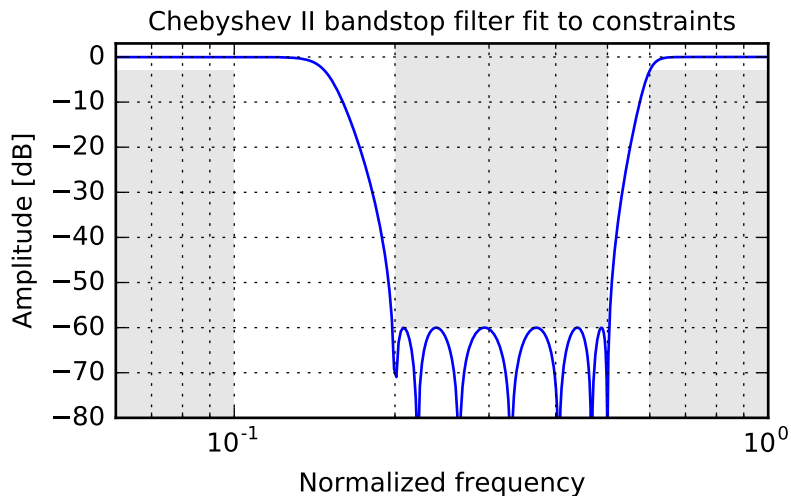
**Examples**

Design a digital bandstop filter which rejects -60 dB from  $0.2 \cdot (fs/2)$  to  $0.5 \cdot (fs/2)$ , while staying within 3 dB below  $0.1 \cdot (fs/2)$  or above  $0.6 \cdot (fs/2)$ . Plot its frequency response, showing the passband and stopband constraints

in gray.

```
>>> from scipy import signal
>>> import matplotlib.pyplot as plt

>>> N, Wn = signal.cheb2ord([0.1, 0.6], [0.2, 0.5], 3, 60)
>>> b, a = signal.cheby2(N, 60, 'stop')
>>> w, h = signal.freqz(b, a)
>>> plt.semilogx(w / np.pi, 20 * np.log10(abs(h)))
>>> plt.title('Chebyshev II bandstop filter fit to constraints')
>>> plt.xlabel('Normalized frequency')
>>> plt.ylabel('Amplitude [dB]')
>>> plt.grid(which='both', axis='both')
>>> plt.fill([.01, .1, .1, .01], [-3, -3, -99, -99], '0.9', lw=0) # stop
>>> plt.fill([.2, .2, .5, .5], [ 9, -60, -60,  9], '0.9', lw=0) # pass
>>> plt.fill([.6, .6, 2, 2], [-99, -3, -3, -99], '0.9', lw=0) # stop
>>> plt.axis([0.06, 1, -80, 3])
>>> plt.show()
```



`scipy.signal.ellip(N, rp, rs, Wn, btype='low', analog=False, output='ba')`  
Elliptic (Cauer) digital and analog filter design.

Design an Nth-order digital or analog elliptic filter and return the filter coefficients.

**Parameters**

- N** : int  
The order of the filter.
- rp** : float  
The maximum ripple allowed below unity gain in the passband. Specified in decibels, as a positive number.
- rs** : float  
The minimum attenuation required in the stop band. Specified in decibels, as a positive number.
- Wn** : array\_like  
A scalar or length-2 sequence giving the critical frequencies. For elliptic filters, this is the point in the transition band at which the gain first drops below  $-rp$ . For digital filters,  $Wn$  is normalized from 0 to 1, where 1 is the

Nyquist frequency, pi radians/sample. ( $W_n$  is thus in half-cycles / sample.)  
 For analog filters,  $W_n$  is an angular frequency (e.g. rad/s).

**btype** : { 'lowpass', 'highpass', 'bandpass', 'bandstop' }, optional  
 The type of filter. Default is 'lowpass'.

**analog** : bool, optional  
 When True, return an analog filter, otherwise a digital filter is returned.

**output** : { 'ba', 'zpk', 'sos' }, optional  
 Type of output: numerator/denominator ('ba'), pole-zero ('zpk'), or second-order sections ('sos'). Default is 'ba'.

**Returns**

**b, a** : ndarray, ndarray  
 Numerator ( $b$ ) and denominator ( $a$ ) polynomials of the IIR filter. Only returned if `output='ba'`.

**z, p, k** : ndarray, ndarray, float  
 Zeros, poles, and system gain of the IIR filter transfer function. Only returned if `output='zpk'`.

**sos** : ndarray  
 Second-order sections representation of the IIR filter. Only returned if `output=='sos'`.

**See also:**

`ellipord`, `ellipap`

**Notes**

Also known as Cauer or Zolotarev filters, the elliptical filter maximizes the rate of transition between the frequency response's passband and stopband, at the expense of ripple in both, and increased ringing in the step response.

As  $r_p$  approaches 0, the elliptical filter becomes a Chebyshev type II filter (`cheby2`). As  $r_s$  approaches 0, it becomes a Chebyshev type I filter (`cheby1`). As both approach 0, it becomes a Butterworth filter (`butter`).

The equiripple passband has  $N$  maxima or minima (for example, a 5th-order filter has 3 maxima and 2 minima). Consequently, the DC gain is unity for odd-order filters, or  $-r_p$  dB for even-order filters.

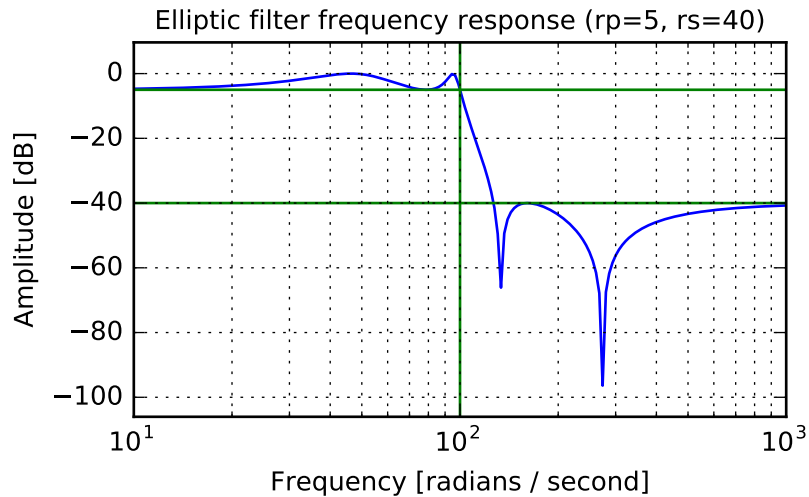
The 'sos' output parameter was added in 0.16.0.

**Examples**

Plot the filter's frequency response, showing the critical points:

```
>>> from scipy import signal
>>> import matplotlib.pyplot as plt

>>> b, a = signal.ellip(4, 5, 40, 100, 'low', analog=True)
>>> w, h = signal.freqs(b, a)
>>> plt.semilogx(w, 20 * np.log10(abs(h)))
>>> plt.title('Elliptic filter frequency response (rp=5, rs=40)')
>>> plt.xlabel('Frequency [radians / second]')
>>> plt.ylabel('Amplitude [dB]')
>>> plt.margins(0, 0.1)
>>> plt.grid(which='both', axis='both')
>>> plt.axvline(100, color='green') # cutoff frequency
>>> plt.axhline(-40, color='green') # rs
>>> plt.axhline(-5, color='green') # rp
>>> plt.show()
```



`scipy.signal.ellipord` (*wp*, *ws*, *gpass*, *gstop*, *analog=False*)  
 Elliptic (Cauer) filter order selection.

Return the order of the lowest order digital or analog elliptic filter that loses no more than *gpass* dB in the passband and has at least *gstop* dB attenuation in the stopband.

**Parameters**    **wp, ws** : float

Passband and stopband edge frequencies. For digital filters, these are normalized from 0 to 1, where 1 is the Nyquist frequency, pi radians/sample. (*wp* and *ws* are thus in half-cycles / sample.) For example:

- Lowpass: *wp* = 0.2, *ws* = 0.3
- Highpass: *wp* = 0.3, *ws* = 0.2
- Bandpass: *wp* = [0.2, 0.5], *ws* = [0.4, 0.6]
- Bandstop: *wp* = [0.1, 0.6], *ws* = [0.2, 0.5]

For analog filters, *wp* and *ws* are angular frequencies (e.g. rad/s).

**gpass** : float

The maximum loss in the passband (dB).

**gstop** : float

The minimum attenuation in the stopband (dB).

**analog** : bool, optional

When True, return an analog filter, otherwise a digital filter is returned.

**Returns**

**ord** : int

The lowest order for an Elliptic (Cauer) filter that meets specs.

**wn** : ndarray or float

The Chebyshev natural frequency (the “3dB frequency”) for use with `ellip` to give filter results.

**See also:**

`ellip`    Filter design using order and critical points

`buttord`    Find order and critical points from passband and stopband spec

`cheb1ord`, `cheb2ord`

`iirfilter`    General filter design using order and critical frequencies

`iirdesign`    General filter design using passband and stopband spec

**Examples**

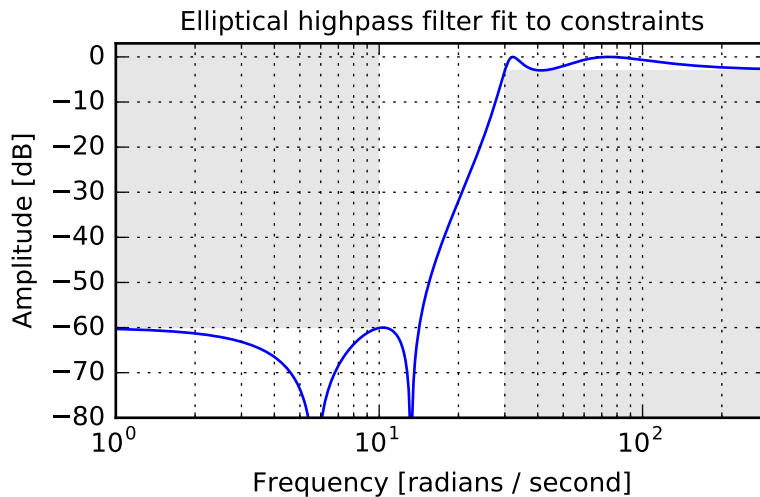
Design an analog highpass filter such that the passband is within 3 dB above 30 rad/s, while rejecting -60 dB at 10 rad/s. Plot its frequency response, showing the passband and stopband constraints in gray.

```

>>> from scipy import signal
>>> import matplotlib.pyplot as plt

>>> N, Wn = signal.ellipord(30, 10, 3, 60, True)
>>> b, a = signal.ellip(N, 3, 60, Wn, 'high', True)
>>> w, h = signal.freqs(b, a, np.logspace(0, 3, 500))
>>> plt.semilogx(w, 20 * np.log10(abs(h)))
>>> plt.title('Elliptical highpass filter fit to constraints')
>>> plt.xlabel('Frequency [radians / second]')
>>> plt.ylabel('Amplitude [dB]')
>>> plt.grid(which='both', axis='both')
>>> plt.fill([.1, 10, 10, .1], [1e4, 1e4, -60, -60], '0.9', lw=0) # stop
>>> plt.fill([30, 30, 1e9, 1e9], [-99, -3, -3, -99], '0.9', lw=0) # pass
>>> plt.axis([1, 300, -80, 3])
>>> plt.show()

```



`scipy.signal.bessel` (*N*, *Wn*, *btype*='low', *analog*=*False*, *output*='ba', *norm*='phase')

Bessel/Thomson digital and analog filter design.

Design an *N*th-order digital or analog Bessel filter and return the filter coefficients.

**Parameters**    *N* : int

The order of the filter.

**Wn** : array\_like

A scalar or length-2 sequence giving the critical frequencies (defined by the *norm* parameter). For analog filters, *Wn* is an angular frequency (e.g. rad/s). For digital filters, *Wn* is normalized from 0 to 1, where 1 is the Nyquist frequency, pi radians/sample. (*Wn* is thus in half-cycles / sample.)

**btype** : {'lowpass', 'highpass', 'bandpass', 'bandstop'}, optional

The type of filter. Default is 'lowpass'.

**analog** : bool, optional

When True, return an analog filter, otherwise a digital filter is returned. (See Notes.)

**output** : {'ba', 'zpk', 'sos'}, optional

Type of output: numerator/denominator ('ba'), pole-zero ('zpk'), or second-order sections ('sos'). Default is 'ba'.

**norm** : { 'phase', 'delay', 'mag' }, optional  
 Critical frequency normalization:

**phase** The filter is normalized such that the phase response reaches its midpoint at angular (e.g. rad/s) frequency  $Wn$ . This happens for both low-pass and high-pass filters, so this is the “phase-matched” case. The magnitude response asymptotes are the same as a Butterworth filter of the same order with a cutoff of  $Wn$ . This is the default, and matches MATLAB’s implementation.

**delay** The filter is normalized such that the group delay in the pass-band is  $1/Wn$  (e.g. seconds). This is the “natural” type obtained by solving Bessel polynomials.

**mag** The filter is normalized such that the gain magnitude is -3 dB at angular frequency  $Wn$ .

**Returns**

**b, a** : ndarray, ndarray  
 New in version 0.18.0.  
 Numerator ( $b$ ) and denominator ( $a$ ) polynomials of the IIR filter. Only returned if `output='ba'`.

**z, p, k** : ndarray, ndarray, float  
 Zeros, poles, and system gain of the IIR filter transfer function. Only returned if `output='zpk'`.

**sos** : ndarray  
 Second-order sections representation of the IIR filter. Only returned if `output=='sos'`.

### Notes

Also known as a Thomson filter, the analog Bessel filter has maximally flat group delay and maximally linear phase response, with very little ringing in the step response. [R179]

The Bessel is inherently an analog filter. This function generates digital Bessel filters using the bilinear transform, which does not preserve the phase response of the analog filter. As such, it is only approximately correct at frequencies below about  $fs/4$ . To get maximally-flat group delay at higher frequencies, the analog Bessel filter must be transformed using phase-preserving techniques.

See `besselap` for implementation details and references.

The `'sos'` output parameter was added in 0.16.0.

### References

[R179]

### Examples

Plot the phase-normalized frequency response, showing the relationship to the Butterworth’s cutoff frequency (green):

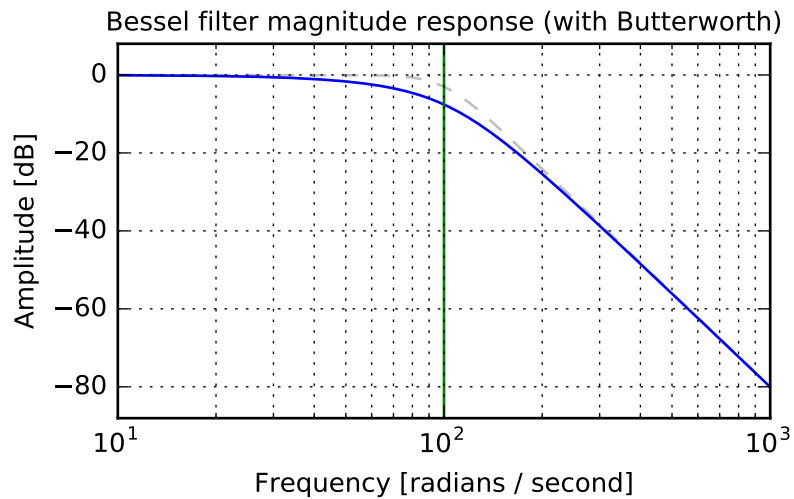
```
>>> from scipy import signal
>>> import matplotlib.pyplot as plt

>>> b, a = signal.butter(4, 100, 'low', analog=True)
>>> w, h = signal.freqs(b, a)
>>> plt.semilogx(w, 20 * np.log10(np.abs(h)), color='silver', ls='dashed')
>>> b, a = signal.bessel(4, 100, 'low', analog=True, norm='phase')
>>> w, h = signal.freqs(b, a)
>>> plt.semilogx(w, 20 * np.log10(np.abs(h)))
>>> plt.title('Bessel filter magnitude response (with Butterworth)')
```

```

>>> plt.xlabel('Frequency [radians / second]')
>>> plt.ylabel('Amplitude [dB]')
>>> plt.margins(0, 0.1)
>>> plt.grid(which='both', axis='both')
>>> plt.axvline(100, color='green') # cutoff frequency
>>> plt.show()

```



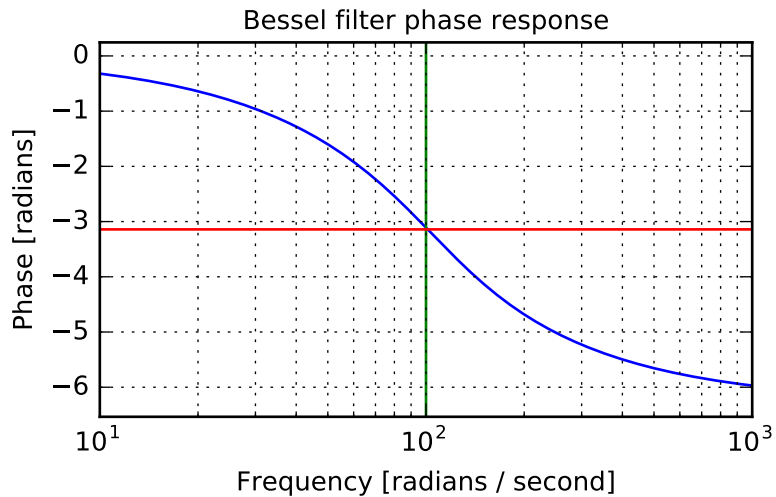
and the phase midpoint:

```

>>> plt.figure()
>>> plt.semilogx(w, np.unwrap(np.angle(h)))
>>> plt.axvline(100, color='green') # cutoff frequency
>>> plt.axhline(-np.pi, color='red') # phase midpoint
>>> plt.title('Bessel filter phase response')
>>> plt.xlabel('Frequency [radians / second]')
>>> plt.ylabel('Phase [radians]')
>>> plt.margins(0, 0.1)
>>> plt.grid(which='both', axis='both')
>>> plt.show()

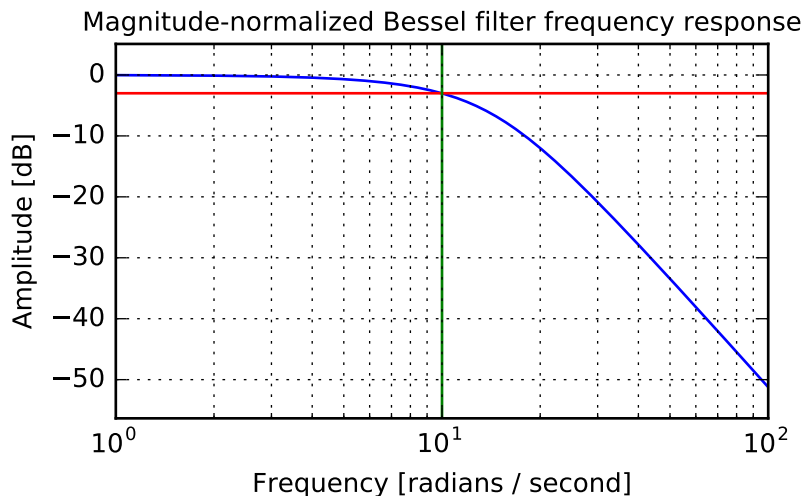
```





Plot the magnitude-normalized frequency response, showing the -3 dB cutoff:

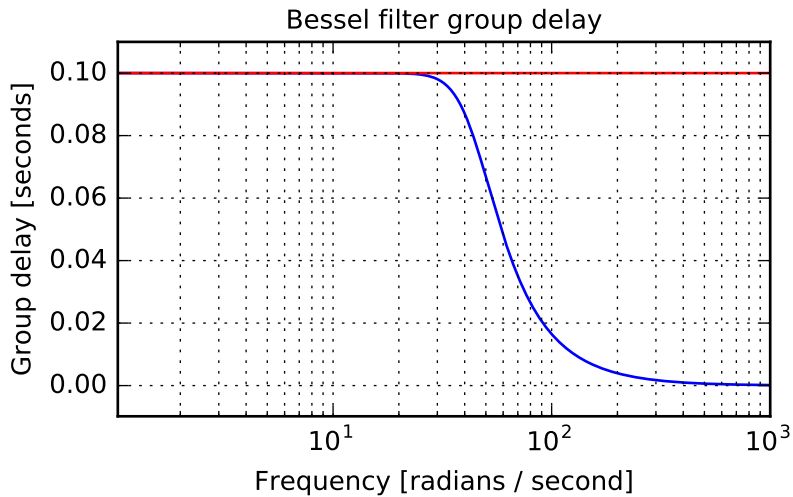
```
>>> b, a = signal.bessel(3, 10, 'low', analog=True, norm='mag')
>>> w, h = signal.freqs(b, a)
>>> plt.semilogx(w, 20 * np.log10(np.abs(h)))
>>> plt.axhline(-3, color='red') # -3 dB magnitude
>>> plt.axvline(10, color='green') # cutoff frequency
>>> plt.title('Magnitude-normalized Bessel filter frequency response')
>>> plt.xlabel('Frequency [radians / second]')
>>> plt.ylabel('Amplitude [dB]')
>>> plt.margins(0, 0.1)
>>> plt.grid(which='both', axis='both')
>>> plt.show()
```



Plot the delay-normalized filter, showing the maximally-flat group delay at 0.1 seconds:

```

>>> b, a = signal.bessel(5, 1/0.1, 'low', analog=True, norm='delay')
>>> w, h = signal.freqs(b, a)
>>> plt.figure()
>>> plt.semilogx(w[1:], -np.diff(np.unwrap(np.angle(h)))/np.diff(w))
>>> plt.axhline(0.1, color='red') # 0.1 seconds group delay
>>> plt.title('Bessel filter group delay')
>>> plt.xlabel('Frequency [radians / second]')
>>> plt.ylabel('Group delay [seconds]')
>>> plt.margins(0, 0.1)
>>> plt.grid(which='both', axis='both')
>>> plt.show()
    
```



## 5.20.6 Continuous-Time Linear Systems

<code>lti(*system)</code>	Continuous-time linear time invariant system base class.
<code>StateSpace(*system, **kwargs)</code>	Linear Time Invariant system in state-space form.
<code>TransferFunction(*system, **kwargs)</code>	Linear Time Invariant system class in transfer function form.
<code>ZerosPolesGain(*system, **kwargs)</code>	Linear Time Invariant system class in zeros, poles, gain form.
<code>lsim(system, U, T[, X0, interp])</code>	Simulate output of a continuous-time linear system.
<code>lsim2(system[, U, T, X0])</code>	Simulate output of a continuous-time linear system, by using the ODE solver <code>scipy.integrate.odeint</code> .
<code>impulse(system[, X0, T, N])</code>	Impulse response of continuous-time system.
<code>impulse2(system[, X0, T, N])</code>	Impulse response of a single-input, continuous-time linear system.
<code>step(system[, X0, T, N])</code>	Step response of continuous-time system.
<code>step2(system[, X0, T, N])</code>	Step response of continuous-time system.
<code>freqresp(system[, w, n])</code>	Calculate the frequency response of a continuous-time system.
<code>bode(system[, w, n])</code>	Calculate Bode magnitude and phase data of a continuous-time system.

**class** `scipy.signal.lti(*system)`  
 Continuous-time linear time invariant system base class.

**Parameters** `*system` : arguments

The `lti` class can be instantiated with either 2, 3 or 4 arguments. The following gives the number of arguments and the corresponding continuous-time subclass that is created:

```

•2: TransferFunction: (numerator, denominator)
•3: ZerosPolesGain: (zeros, poles, gain)
•4: StateSpace: (A, B, C, D)

```

Each argument can be an array or a sequence.

#### See also:

`ZerosPolesGain`, `StateSpace`, `TransferFunction`, `dlti`

#### Notes

`lti` instances do not exist directly. Instead, `lti` creates an instance of one of its subclasses: `StateSpace`, `TransferFunction` or `ZerosPolesGain`.

If (numerator, denominator) is passed in for `*system`, coefficients for both the numerator and denominator should be specified in descending exponent order (e.g.,  $s^2 + 3s + 5$  would be represented as `[1, 3, 5]`).

Changing the value of properties that are not directly part of the current system representation (such as the `zeros` of a `StateSpace` system) is very inefficient and may lead to numerical inaccuracies. It is better to convert to the specific system representation first. For example, call `sys = sys.to_zpk()` before accessing/changing the zeros, poles or gain.

#### Examples

```

>>> from scipy import signal

>>> signal.lti(1, 2, 3, 4)
StateSpaceContinuous(
  array([[1]]),
  array([[2]]),
  array([[3]]),
  array([[4]]),
  dt: None
)

>>> signal.lti([1, 2], [3, 4], 5)
ZerosPolesGainContinuous(
  array([1, 2]),
  array([3, 4]),
  5,
  dt: None
)

>>> signal.lti([3, 4], [1, 2])
TransferFunctionContinuous(
  array([ 3.,  4.]),
  array([ 1.,  2.]),
  dt: None
)

```

#### Attributes

A	State matrix of the <code>StateSpace</code> system.
B	Input matrix of the <code>StateSpace</code> system.
C	Output matrix of the <code>StateSpace</code> system.
D	Feedthrough matrix of the <code>StateSpace</code> system.

Continued on next page

Table 5.130 – continued from previous page

<code>den</code>	Denominator of the <code>TransferFunction</code> system.
<code>dt</code>	Return the sampling time of the system, <code>None</code> for <code>lti</code> systems.
<code>gain</code>	Gain of the <code>ZerosPolesGain</code> system.
<code>num</code>	Numerator of the <code>TransferFunction</code> system.
<code>poles</code>	Poles of the system.
<code>zeros</code>	Zeros of the system.

- `lti.A`  
State matrix of the `StateSpace` system.
- `lti.B`  
Input matrix of the `StateSpace` system.
- `lti.C`  
Output matrix of the `StateSpace` system.
- `lti.D`  
Feedthrough matrix of the `StateSpace` system.
- `lti.den`  
Denominator of the `TransferFunction` system.
- `lti.dt`  
Return the sampling time of the system, `None` for `lti` systems.
- `lti.gain`  
Gain of the `ZerosPolesGain` system.
- `lti.num`  
Numerator of the `TransferFunction` system.
- `lti.poles`  
Poles of the system.
- `lti.zeros`  
Zeros of the system.

**Methods**

<code>bode([w, n])</code>	Calculate Bode magnitude and phase data of a continuous-time system.
<code>freqresp([w, n])</code>	Calculate the frequency response of a continuous-time system.
<code>impulse([X0, T, N])</code>	Return the impulse response of a continuous-time system.
<code>output(U, T[, X0])</code>	Return the response of a continuous-time system to input <i>U</i> .
<code>step([X0, T, N])</code>	Return the step response of a continuous-time system.
<code>to_discrete(dt[, method, alpha])</code>	Return a discretized version of the current system.

`lti.bode` (*w=None, n=100*)  
 Calculate Bode magnitude and phase data of a continuous-time system.  
 Returns a 3-tuple containing arrays of frequencies [rad/s], magnitude [dB] and phase [deg]. See `bode` for details.

**Examples**

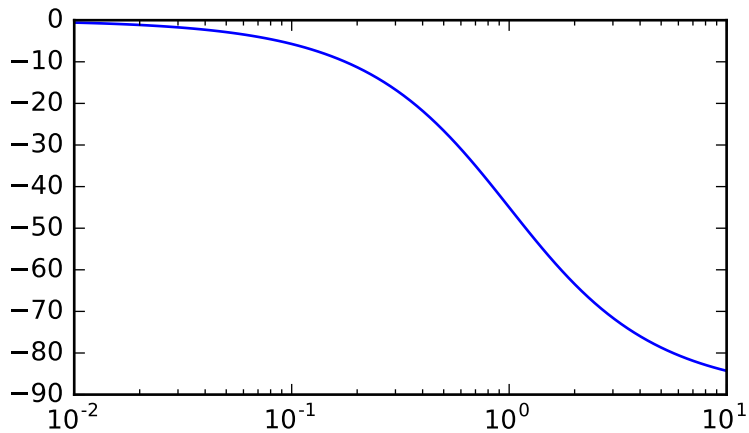
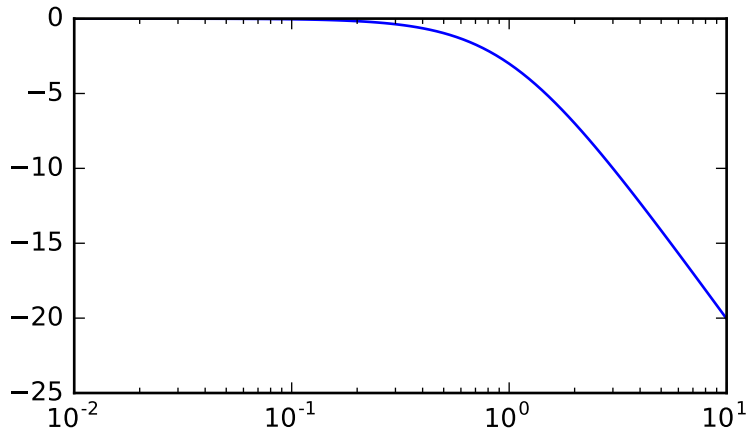
```
>>> from scipy import signal
>>> import matplotlib.pyplot as plt
```

```

>>> sys = signal.TransferFunction([1], [1, 1])
>>> w, mag, phase = sys.bode()

>>> plt.figure()
>>> plt.semilogx(w, mag)      # Bode magnitude plot
>>> plt.figure()
>>> plt.semilogx(w, phase)   # Bode phase plot
>>> plt.show()

```



`lti.freqresp` ( $w=None, n=10000$ )

Calculate the frequency response of a continuous-time system.

Returns a 2-tuple containing arrays of frequencies [rad/s] and complex magnitude. See `freqresp` for details.

`lti.impulse` ( $X0=None, T=None, N=None$ )

Return the impulse response of a continuous-time system. See `impulse` for details.

`lti.output` (*U*, *T*, *X0=None*)  
 Return the response of a continuous-time system to input *U*. See `lsim` for details.

`lti.step` (*X0=None*, *T=None*, *N=None*)  
 Return the step response of a continuous-time system. See `step` for details.

`lti.to_discrete` (*dt*, *method='zoh'*, *alpha=None*)  
 Return a discretized version of the current system.

Parameters: See `cont2discrete` for details.

**Returns** sys: instance of `dlti`

**class** `scipy.signal.StateSpace` (*\*system*, *\*\*kwargs*)

Linear Time Invariant system in state-space form.

Represents the system as the continuous-time, first order differential equation  $\dot{x} = Ax + Bu$  or the discrete-time difference equation  $x[k+1] = Ax[k] + Bu[k]$ . `StateSpace` systems inherit additional functionality from the `lti`, respectively the `dlti` classes, depending on which system representation is used.

**Parameters** **\*system: arguments**

The `StateSpace` class can be instantiated with 1 or 3 arguments. The following gives the number of input arguments and their interpretation:

- 1: `lti` or `dlti` system: (`StateSpace`, `TransferFunction` or `ZerosPolesGain`)
- 4: array\_like: (A, B, C, D)

**dt: float, optional**

Sampling time [s] of the discrete-time systems. Defaults to `None` (continuous-time). Must be specified as a keyword argument, for example, `dt=0.1`.

**See also:**

`TransferFunction`, `ZerosPolesGain`, `lti`, `dlti`, `ss2zpk`, `ss2tf`, `zpk2sos`

**Notes**

Changing the value of properties that are not part of the `StateSpace` system representation (such as `zeros` or `poles`) is very inefficient and may lead to numerical inaccuracies. It is better to convert to the specific system representation first. For example, call `sys = sys.to_zpk()` before accessing/changing the zeros, poles or gain.

**Examples**

```
>>> from scipy import signal

>>> a = np.array([[0, 1], [0, 0]])
>>> b = np.array([[0], [1]])
>>> c = np.array([[1, 0]])
>>> d = np.array([[0]])

>>> sys = signal.StateSpace(a, b, c, d)
>>> print(sys)
StateSpaceContinuous(
array([[0, 1],
       [0, 0]]),
array([[0],
       [1]]),
array([[1, 0]]),
```

```

array([[0]]),
dt: None
)

>>> sys.to_discrete(0.1)
StateSpaceDiscrete(
array([[ 1. ,  0.1],
       [ 0. ,  1. ]]),
array([[ 0.005],
       [ 0.1  ]]),
array([[1, 0]]),
array([[0]]),
dt: 0.1
)

>>> a = np.array([[1, 0.1], [0, 1]])
>>> b = np.array([[0.005], [0.1]])

>>> signal.StateSpace(a, b, c, d, dt=0.1)
StateSpaceDiscrete(
array([[ 1. ,  0.1],
       [ 0. ,  1. ]]),
array([[ 0.005],
       [ 0.1  ]]),
array([[1, 0]]),
array([[0]]),
dt: 0.1
)

```

**Attributes**


---

<code>A</code>	State matrix of the <code>StateSpace</code> system.
<code>B</code>	Input matrix of the <code>StateSpace</code> system.
<code>C</code>	Output matrix of the <code>StateSpace</code> system.
<code>D</code>	Feedthrough matrix of the <code>StateSpace</code> system.
<code>den</code>	Denominator of the <code>TransferFunction</code> system.
<code>dt</code>	Return the sampling time of the system, <i>None</i> for <code>lti</code> systems.
<code>gain</code>	Gain of the <code>ZerosPolesGain</code> system.
<code>num</code>	Numerator of the <code>TransferFunction</code> system.
<code>poles</code>	Poles of the system.
<code>zeros</code>	Zeros of the system.

---

**StateSpace.A**State matrix of the `StateSpace` system.**StateSpace.B**Input matrix of the `StateSpace` system.**StateSpace.C**Output matrix of the `StateSpace` system.**StateSpace.D**Feedthrough matrix of the `StateSpace` system.**StateSpace.den**

Denominator of the `TransferFunction` system.

`StateSpace.dt`

Return the sampling time of the system, *None* for `lti` systems.

`StateSpace.gain`

Gain of the `ZerosPolesGain` system.

`StateSpace.num`

Numerator of the `TransferFunction` system.

`StateSpace.poles`

Poles of the system.

`StateSpace.zeros`

Zeros of the system.

**Methods**

<code>to_ss()</code>	Return a copy of the current <code>StateSpace</code> system.
<code>to_tf(**kwargs)</code>	Convert system representation to <code>TransferFunction</code> .
<code>to_zpk(**kwargs)</code>	Convert system representation to <code>ZerosPolesGain</code> .

`StateSpace.to_ss()`

Return a copy of the current `StateSpace` system.

**Returns** `sys` : instance of `StateSpace`  
The current system (copy)

`StateSpace.to_tf(**kwargs)`

Convert system representation to `TransferFunction`.

**Parameters** `kwargs` : dict, optional  
Additional keywords passed to `ss2zpk`  
**Returns** `sys` : instance of `TransferFunction`  
Transfer function of the current system

`StateSpace.to_zpk(**kwargs)`

Convert system representation to `ZerosPolesGain`.

**Parameters** `kwargs` : dict, optional  
Additional keywords passed to `ss2zpk`  
**Returns** `sys` : instance of `ZerosPolesGain`  
Zeros, poles, gain representation of the current system

**class** `scipy.signal.TransferFunction(*system, **kwargs)`

Linear Time Invariant system class in transfer function form.

Represents the system as the continuous-time transfer function  $H(s) = \sum_{i=0}^N b[N-i]s^i / \sum_{j=0}^M a[M-j]s^j$  or the discrete-time transfer function  $H(z) = \sum_{i=0}^N b[N-i]z^i / \sum_{j=0}^M a[M-j]z^j$ , where *b* are elements of the numerator `num`, *a* are elements of the denominator `den`, and  $N == \text{len}(b) - 1, M == \text{len}(a) - 1$ . `TransferFunction` systems inherit additional functionality from the `lti`, respectively the `dlti` classes, depending on which system representation is used.

**Parameters** `*system: arguments`

The `TransferFunction` class can be instantiated with 1 or 2 arguments. The following gives the number of input arguments and their interpretation:

- 1: `lti` or `dlti` system: (`StateSpace`, `TransferFunction` or `ZerosPolesGain`)
- 2: `array_like`: (numerator, denominator)

**dt**: float, optional



Sampling time [s] of the discrete-time systems. Defaults to *None* (continuous-time). Must be specified as a keyword argument, for example, `dt=0.1`.

**See also:**

`ZerosPolesGain`, `StateSpace`, `lti`, `dlti`, `tf2ss`, `tf2zpk`, `tf2sos`

**Notes**

Changing the value of properties that are not part of the `TransferFunction` system representation (such as the `A`, `B`, `C`, `D` state-space matrices) is very inefficient and may lead to numerical inaccuracies. It is better to convert to the specific system representation first. For example, call `sys = sys.to_ss()` before accessing/changing the `A`, `B`, `C`, `D` system matrices.

If (numerator, denominator) is passed in for `*system`, coefficients for both the numerator and denominator should be specified in descending exponent order (e.g.  $s^2 + 3s + 5$  or  $z^2 + 3z + 5$  would be represented as `[1, 3, 5]`)

**Examples**

Construct the transfer function:

$$H(s) = \frac{s^2 + 3s + 3}{s^2 + 2s + 1}$$

```
>>> from scipy import signal
```

```
>>> num = [1, 3, 3]
```

```
>>> den = [1, 2, 1]
```

```
>>> signal.TransferFunction(num, den)
TransferFunctionContinuous(
array([ 1.,  3.,  3.]),
array([ 1.,  2.,  1.]),
dt: None
)
```

Construct the transfer function with a sampling time of 0.5 seconds:

$$H(z) = \frac{z^2 + 3z + 3}{z^2 + 2z + 1}$$

```
>>> signal.TransferFunction(num, den, dt=0.1)
TransferFunctionDiscrete(
array([ 1.,  3.,  3.]),
array([ 1.,  2.,  1.]),
dt: 0.1
)
```

Continued on next page
------------------------

Table 5.134 – continued from previous page

*Attributes*

---

<code>A</code>	State matrix of the <code>StateSpace</code> system.
<code>B</code>	Input matrix of the <code>StateSpace</code> system.
<code>C</code>	Output matrix of the <code>StateSpace</code> system.
<code>D</code>	Feedthrough matrix of the <code>StateSpace</code> system.
<code>den</code>	Denominator of the <code>TransferFunction</code> system.
<code>dt</code>	Return the sampling time of the system, <i>None</i> for <code>lti</code> systems.
<code>gain</code>	Gain of the <code>ZerosPolesGain</code> system.
<code>num</code>	Numerator of the <code>TransferFunction</code> system.
<code>poles</code>	Poles of the system.
<code>zeros</code>	Zeros of the system.

---

`TransferFunction.A`  
State matrix of the `StateSpace` system.

`TransferFunction.B`  
Input matrix of the `StateSpace` system.

`TransferFunction.C`  
Output matrix of the `StateSpace` system.

`TransferFunction.D`  
Feedthrough matrix of the `StateSpace` system.

`TransferFunction.den`  
Denominator of the `TransferFunction` system.

`TransferFunction.dt`  
Return the sampling time of the system, *None* for `lti` systems.

`TransferFunction.gain`  
Gain of the `ZerosPolesGain` system.

`TransferFunction.num`  
Numerator of the `TransferFunction` system.

`TransferFunction.poles`  
Poles of the system.

`TransferFunction.zeros`  
Zeros of the system.

*Methods*

---

<code>to_ss()</code>	Convert system representation to <code>StateSpace</code> .
<code>to_tf()</code>	Return a copy of the current <code>TransferFunction</code> system.
<code>to_zpk()</code>	Convert system representation to <code>ZerosPolesGain</code> .

---

`TransferFunction.to_ss()`  
Convert system representation to `StateSpace`.

**Returns** `sys` : instance of `StateSpace`  
State space model of the current system

`TransferFunction.to_tf()`

Return a copy of the current `TransferFunction` system.

**Returns** `sys` : instance of `TransferFunction`  
The current system (copy)

`TransferFunction.to_zpk()`

Convert system representation to `ZerosPolesGain`.

**Returns** `sys` : instance of `ZerosPolesGain`  
Zeros, poles, gain representation of the current system

**class** `scipy.signal.ZerosPolesGain(*system, **kwargs)`

Linear Time Invariant system class in zeros, poles, gain form.

Represents the system as the continuous- or discrete-time transfer function  $H(s) = k \prod_i (s - z[i]) / \prod_j (s - p[j])$ , where  $k$  is the `gain`,  $z$  are the `zeros` and  $p$  are the `poles`. `ZerosPolesGain` systems inherit additional functionality from the `lti`, respectively the `dlti` classes, depending on which system representation is used.

**Parameters** `*system` : arguments

The `ZerosPolesGain` class can be instantiated with 1 or 3 arguments. The following gives the number of input arguments and their interpretation:

- 1: `lti` or `dlti` system: (`StateSpace`, `TransferFunction` or `ZerosPolesGain`)
- 3: `array_like`: (`zeros`, `poles`, `gain`)

**dt**: float, optional

Sampling time [s] of the discrete-time systems. Defaults to `None` (continuous-time). Must be specified as a keyword argument, for example, `dt=0.1`.

**See also:**

`TransferFunction`, `StateSpace`, `lti`, `dlti`, `zpk2ss`, `zpk2tf`, `zpk2sos`

**Notes**

Changing the value of properties that are not part of the `ZerosPolesGain` system representation (such as the `A`, `B`, `C`, `D` state-space matrices) is very inefficient and may lead to numerical inaccuracies. It is better to convert to the specific system representation first. For example, call `sys = sys.to_ss()` before accessing/changing the `A`, `B`, `C`, `D` system matrices.

**Examples**

```
>>> from scipy import signal
```

Transfer function:  $H(s) = 5(s - 1)(s - 2) / (s - 3)(s - 4)$

```
>>> signal.ZerosPolesGain([1, 2], [3, 4], 5)
ZerosPolesGainContinuous(
  array([1, 2]),
  array([3, 4]),
  5,
  dt: None
)
```

Transfer function:  $H(z) = 5(z - 1)(z - 2) / (z - 3)(z - 4)$

```
>>> signal.ZerosPolesGain([1, 2], [3, 4], 5, dt=0.1)
ZerosPolesGainDiscrete(
  array([1, 2]),
  array([3, 4]),
```

```
5,
dt: 0.1
)
```

**Attributes**

---

<code>A</code>	State matrix of the <code>StateSpace</code> system.
<code>B</code>	Input matrix of the <code>StateSpace</code> system.
<code>C</code>	Output matrix of the <code>StateSpace</code> system.
<code>D</code>	Feedthrough matrix of the <code>StateSpace</code> system.
<code>den</code>	Denominator of the <code>TransferFunction</code> system.
<code>dt</code>	Return the sampling time of the system, <i>None</i> for <code>lti</code> systems.
<code>gain</code>	Gain of the <code>ZerosPolesGain</code> system.
<code>num</code>	Numerator of the <code>TransferFunction</code> system.
<code>poles</code>	Poles of the <code>ZerosPolesGain</code> system.
<code>zeros</code>	Zeros of the <code>ZerosPolesGain</code> system.

---

`ZerosPolesGain.A`  
State matrix of the `StateSpace` system.

`ZerosPolesGain.B`  
Input matrix of the `StateSpace` system.

`ZerosPolesGain.C`  
Output matrix of the `StateSpace` system.

`ZerosPolesGain.D`  
Feedthrough matrix of the `StateSpace` system.

`ZerosPolesGain.den`  
Denominator of the `TransferFunction` system.

`ZerosPolesGain.dt`  
Return the sampling time of the system, *None* for `lti` systems.

`ZerosPolesGain.gain`  
Gain of the `ZerosPolesGain` system.

`ZerosPolesGain.num`  
Numerator of the `TransferFunction` system.

`ZerosPolesGain.poles`  
Poles of the `ZerosPolesGain` system.

`ZerosPolesGain.zeros`  
Zeros of the `ZerosPolesGain` system.

**Methods**

---

<code>to_ss()</code>	Convert system representation to <code>StateSpace</code> .
<code>to_tf()</code>	Convert system representation to <code>TransferFunction</code> .
<code>to_zpk()</code>	Return a copy of the current 'ZerosPolesGain' system.

---

`ZerosPolesGain.to_ss()`  
Convert system representation to `StateSpace`.

**Returns** `sys` : instance of `StateSpace`

State space model of the current system

`ZerosPolesGain.to_tf()`

Convert system representation to `TransferFunction`.

**Returns** `sys` : instance of `TransferFunction`  
Transfer function of the current system

`ZerosPolesGain.to_zpk()`

Return a copy of the current ‘ZerosPolesGain’ system.

**Returns** `sys` : instance of `ZerosPolesGain`  
The current system (copy)

`scipy.signal.lsim(system, U, T, X0=None, interp=True)`

Simulate output of a continuous-time linear system.

**Parameters** `system` : an instance of the LTI class or a tuple describing the system.  
The following gives the number of elements in the tuple and the interpretation:

- 1: (instance of `lti`)
- 2: (num, den)
- 3: (zeros, poles, gain)
- 4: (A, B, C, D)

`U` : array\_like

An input array describing the input at each time  $T$  (interpolation is assumed between given times). If there are multiple inputs, then each column of the rank-2 array represents an input. If  $U = 0$  or `None`, a zero input is used.

`T` : array\_like

The time steps at which the input is defined and at which the output is desired. Must be nonnegative, increasing, and equally spaced.

`X0` : array\_like, optional

The initial conditions on the state vector (zero by default).

`interp` : bool, optional

Whether to use linear (`True`, the default) or zero-order-hold (`False`) interpolation for the input array.

**Returns** `T` : 1D ndarray

Time values for the output.

`yout` : 1D ndarray

System response.

`xout` : ndarray

Time evolution of the state vector.

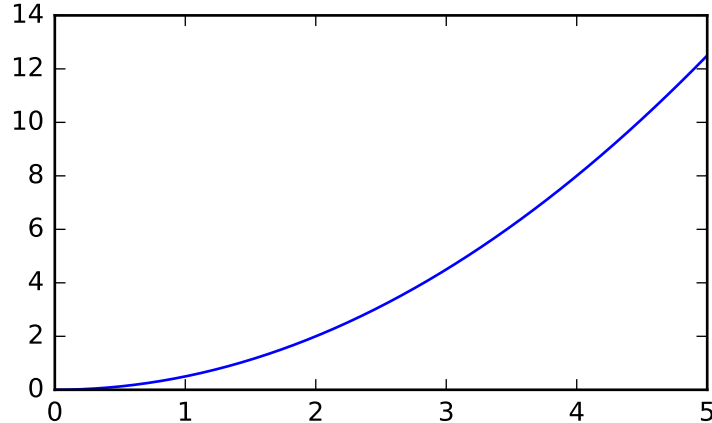
### Notes

If (num, den) is passed in for `system`, coefficients for both the numerator and denominator should be specified in descending exponent order (e.g.  $s^2 + 3s + 5$  would be represented as `[1, 3, 5]`).

### Examples

Simulate a double integrator  $y'' = u$ , with a constant input  $u = 1$

```
>>> from scipy import signal
>>> system = signal.lti([[0., 1.], [0., 0.]], [[0.], [1.]], [[1., 0.]], 0.)
>>> t = np.linspace(0, 5)
>>> u = np.ones_like(t)
>>> tout, y, x = signal.lsim(system, u, t)
>>> import matplotlib.pyplot as plt
>>> plt.plot(t, y)
```



`scipy.signal.lsim2` (*system*, *U=None*, *T=None*, *X0=None*, *\*\*kwargs*)

Simulate output of a continuous-time linear system, by using the ODE solver `scipy.integrate.odeint`.

**Parameters** **system** : an instance of the `lti` class or a tuple describing the system.  
The following gives the number of elements in the tuple and the interpretation:

- 1: (instance of `lti`)
- 2: (num, den)
- 3: (zeros, poles, gain)
- 4: (A, B, C, D)

**U** : array\_like (1D or 2D), optional

An input array describing the input at each time *T*. Linear interpolation is used between given times. If there are multiple inputs, then each column of the rank-2 array represents an input. If *U* is not given, the input is assumed to be zero.

**T** : array\_like (1D or 2D), optional

The time steps at which the input is defined and at which the output is desired. The default is 101 evenly spaced points on the interval [0,10.0].

**X0** : array\_like (1D), optional

The initial condition of the state vector. If *X0* is not given, the initial conditions are assumed to be 0.

**kwargs** : dict

Additional keyword arguments are passed on to the function `odeint`. See the notes below for more details.

**Returns** **T** : 1D ndarray

The time values for the output.

**yout** : ndarray

The response of the system.

**xout** : ndarray

The time-evolution of the state-vector.

### Notes

This function uses `scipy.integrate.odeint` to solve the system's differential equations. Additional keyword arguments given to `lsim2` are passed on to `odeint`. See the documentation for `scipy.integrate.odeint` for the full list of arguments.

If (num, den) is passed in for *system*, coefficients for both the numerator and denominator should be specified in descending exponent order (e.g.  $s^2 + 3s + 5$  would be represented as [1, 3, 5]).

`scipy.signal.impulse` (*system*, *X0=None*, *T=None*, *N=None*)

Impulse response of continuous-time system.

**Parameters**

- system** : an instance of the LTI class or a tuple of array\_like describing the system. The following gives the number of elements in the tuple and the interpretation:
  - 1 (instance of `lti`)
  - 2 (num, den)
  - 3 (zeros, poles, gain)
  - 4 (A, B, C, D)
- X0** : array\_like, optional  
Initial state-vector. Defaults to zero.
- T** : array\_like, optional  
Time points. Computed if not given.
- N** : int, optional  
The number of time points to compute (if *T* is not given).

**Returns**

- T** : ndarray  
A 1-D array of time points.
- yout** : ndarray  
A 1-D array containing the impulse response of the system (except for singularities at zero).

#### Notes

If (num, den) is passed in for *system*, coefficients for both the numerator and denominator should be specified in descending exponent order (e.g.  $s^2 + 3s + 5$  would be represented as [1, 3, 5]).

`scipy.signal.impulse2` (*system*, *X0=None*, *T=None*, *N=None*, **\*\*kwargs**)

Impulse response of a single-input, continuous-time linear system.

**Parameters**

- system** : an instance of the LTI class or a tuple of array\_like describing the system. The following gives the number of elements in the tuple and the interpretation:
  - 1 (instance of `lti`)
  - 2 (num, den)
  - 3 (zeros, poles, gain)
  - 4 (A, B, C, D)
- X0** : 1-D array\_like, optional  
The initial condition of the state vector. Default: 0 (the zero vector).
- T** : 1-D array\_like, optional  
The time steps at which the input is defined and at which the output is desired. If *T* is not given, the function will generate a set of time samples automatically.
- N** : int, optional  
Number of time points to compute. Default: 100.
- kwargs** : various types  
Additional keyword arguments are passed on to the function `scipy.signal.lsim2`, which in turn passes them on to `scipy.integrate.odeint`; see the latter's documentation for information about these arguments.

**Returns**

- T** : ndarray  
The time values for the output.
- yout** : ndarray  
The output response of the system.

See also:

`impulse`, `lsim2`, `integrate.odeint`

### Notes

The solution is generated by calling `scipy.signal.lsim2`, which uses the differential equation solver `scipy.integrate.odeint`.

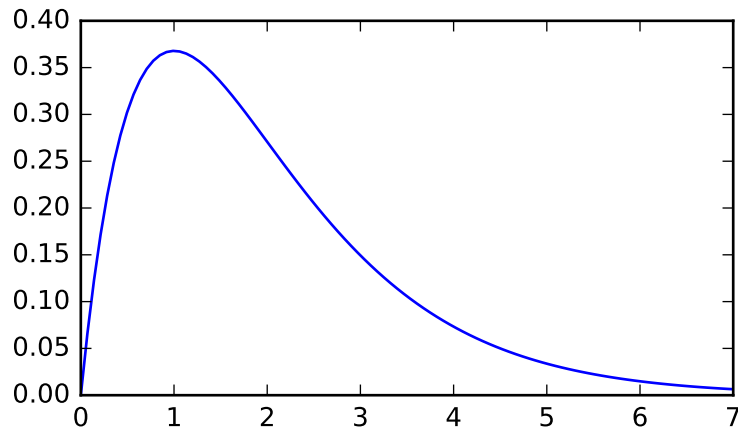
If `(num, den)` is passed in for `system`, coefficients for both the numerator and denominator should be specified in descending exponent order (e.g.  $s^2 + 3s + 5$  would be represented as `[1, 3, 5]`).

New in version 0.8.0.

### Examples

Second order system with a repeated root:  $x''(t) + 2x'(t) + x(t) = u(t)$

```
>>> from scipy import signal
>>> system = ([1.0], [1.0, 2.0, 1.0])
>>> t, y = signal.impulse2(system)
>>> import matplotlib.pyplot as plt
>>> plt.plot(t, y)
```



`scipy.signal.step` (*system*, *X0=None*, *T=None*, *N=None*)

Step response of continuous-time system.

**Parameters**

- system** : an instance of the LTI class or a tuple of array\_like describing the system. The following gives the number of elements in the tuple and the interpretation:
  - 1 (instance of `lti`)
  - 2 (num, den)
  - 3 (zeros, poles, gain)
  - 4 (A, B, C, D)

**X0** : array\_like, optional  
Initial state-vector (default is zero).

**T** : array\_like, optional  
Time points (computed if not given).

**N** : int, optional  
Number of time points to compute if *T* is not given.

**Returns**

- T** : 1D ndarray  
Output time points.

- yout** : 1D ndarray  
Step response of system.



**See also:**`scipy.signal.step2`**Notes**

If (num, den) is passed in for `system`, coefficients for both the numerator and denominator should be specified in descending exponent order (e.g.  $s^2 + 3s + 5$  would be represented as [1, 3, 5]).

`scipy.signal.step2` (*system*, *X0=None*, *T=None*, *N=None*, *\*\*kwargs*)  
 Step response of continuous-time system.

This function is functionally the same as `scipy.signal.step`, but it uses the function `scipy.signal.lsim2` to compute the step response.

**Parameters** **system** : an instance of the LTI class or a tuple of array\_like describing the system. The following gives the number of elements in the tuple and the interpretation:

- 1 (instance of `lti`)
- 2 (num, den)
- 3 (zeros, poles, gain)
- 4 (A, B, C, D)

**X0** : array\_like, optional  
 Initial state-vector (default is zero).

**T** : array\_like, optional  
 Time points (computed if not given).

**N** : int, optional  
 Number of time points to compute if *T* is not given.

**kwargs** : various types  
 Additional keyword arguments are passed on the function `scipy.signal.lsim2`, which in turn passes them on to `scipy.integrate.odeint`. See the documentation for `scipy.integrate.odeint` for information about these arguments.

**Returns** **T** : 1D ndarray  
 Output time points.

**yout** : 1D ndarray  
 Step response of system.

**See also:**`scipy.signal.step`**Notes**

If (num, den) is passed in for `system`, coefficients for both the numerator and denominator should be specified in descending exponent order (e.g.  $s^2 + 3s + 5$  would be represented as [1, 3, 5]).

New in version 0.8.0.

`scipy.signal.freqresp` (*system*, *w=None*, *n=10000*)  
 Calculate the frequency response of a continuous-time system.

**Parameters** **system** : an instance of the `lti` class or a tuple describing the system. The following gives the number of elements in the tuple and the interpretation:

- 1 (instance of `lti`)
- 2 (num, den)
- 3 (zeros, poles, gain)
- 4 (A, B, C, D)

**w** : array\_like, optional  
 Array of frequencies (in rad/s). Magnitude and phase data is calculated for every value in this array. If not given, a reasonable set will be calculated.

**n** : int, optional

Number of frequency points to compute if  $w$  is not given. The  $n$  frequencies are logarithmically spaced in an interval chosen to include the influence of the poles and zeros of the system.

**Returns**

- w** : 1D ndarray  
Frequency array [rad/s]
- H** : 1D ndarray  
Array of complex magnitude values

### Notes

If (num, den) is passed in for `system`, coefficients for both the numerator and denominator should be specified in descending exponent order (e.g.  $s^2 + 3s + 5$  would be represented as `[1, 3, 5]`).

### Examples

Generating the Nyquist plot of a transfer function

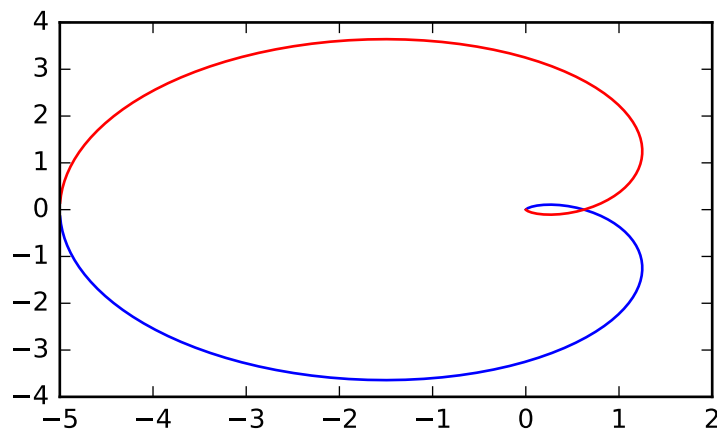
```
>>> from scipy import signal
>>> import matplotlib.pyplot as plt
```

Transfer function:  $H(s) = 5 / (s-1)^3$

```
>>> s1 = signal.ZerosPolesGain([], [1, 1, 1], [5])
```

```
>>> w, H = signal.freqresp(s1)
```

```
>>> plt.figure()
>>> plt.plot(H.real, H.imag, "b")
>>> plt.plot(H.real, -H.imag, "r")
>>> plt.show()
```



```
scipy.signal.bode(system, w=None, n=100)
```

Calculate Bode magnitude and phase data of a continuous-time system.

**Parameters**

- system** : an instance of the LTI class or a tuple describing the system.  
The following gives the number of elements in the tuple and the interpretation:

- 1 (instance of `lti`)
- 2 (num, den)
- 3 (zeros, poles, gain)
- 4 (A, B, C, D)

`w` : array\_like, optional

Array of frequencies (in rad/s). Magnitude and phase data is calculated for every value in this array. If not given a reasonable set will be calculated.

`n` : int, optional

Number of frequency points to compute if `w` is not given. The `n` frequencies are logarithmically spaced in an interval chosen to include the influence of the poles and zeros of the system.

#### Returns

`w` : 1D ndarray

Frequency array [rad/s]

`mag` : 1D ndarray

Magnitude array [dB]

`phase` : 1D ndarray

Phase array [deg]

#### Notes

If (num, den) is passed in for `system`, coefficients for both the numerator and denominator should be specified in descending exponent order (e.g.  $s^2 + 3s + 5$  would be represented as [1, 3, 5]).

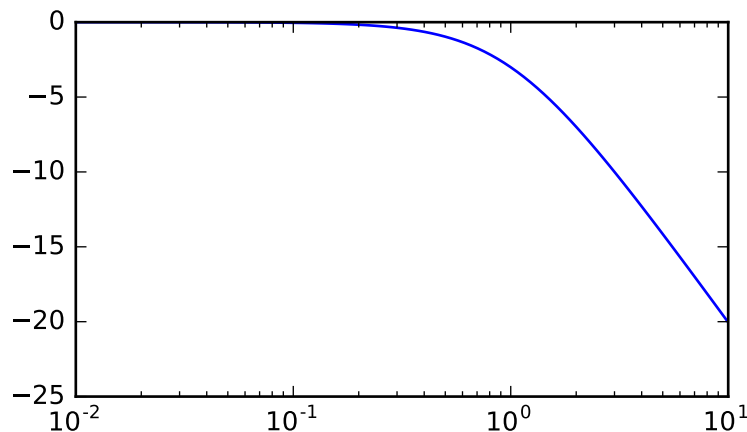
New in version 0.11.0.

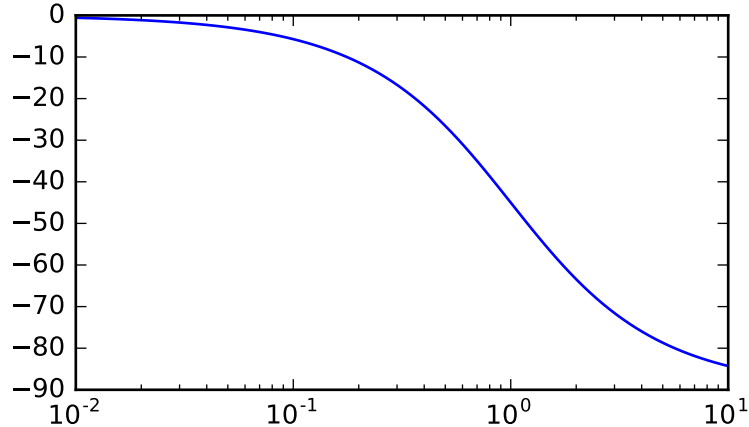
#### Examples

```
>>> from scipy import signal
>>> import matplotlib.pyplot as plt

>>> sys = signal.TransferFunction([1], [1, 1])
>>> w, mag, phase = signal.bode(sys)

>>> plt.figure()
>>> plt.semilogx(w, mag)      # Bode magnitude plot
>>> plt.figure()
>>> plt.semilogx(w, phase)   # Bode phase plot
>>> plt.show()
```





## 5.20.7 Discrete-Time Linear Systems

<code>dlti(*system, **kwargs)</code>	Discrete-time linear time invariant system base class.
<code>StateSpace(*system, **kwargs)</code>	Linear Time Invariant system in state-space form.
<code>TransferFunction(*system, **kwargs)</code>	Linear Time Invariant system class in transfer function form.
<code>ZerosPolesGain(*system, **kwargs)</code>	Linear Time Invariant system class in zeros, poles, gain form.
<code>dlsim(system, u[, t, x0])</code>	Simulate output of a discrete-time linear system.
<code>dimpulse(system[, x0, t, n])</code>	Impulse response of discrete-time system.
<code>dstep(system[, x0, t, n])</code>	Step response of discrete-time system.
<code>dfreqresp(system[, w, n, whole])</code>	Calculate the frequency response of a discrete-time system.
<code>dbode(system[, w, n])</code>	Calculate Bode magnitude and phase data of a discrete-time system.

**class** `scipy.signal.dlti` (*\*system*, *\*\*kwargs*)  
 Discrete-time linear time invariant system base class.

**Parameters** *\*system*: arguments

The `dlti` class can be instantiated with either 2, 3 or 4 arguments. The following gives the number of arguments and the corresponding discrete-time subclass that is created:

- 2: `TransferFunction`: (numerator, denominator)
- 3: `ZerosPolesGain`: (zeros, poles, gain)
- 4: `StateSpace`: (A, B, C, D)

Each argument can be an array or a sequence.

**dt**: float, optional

Sampling time [s] of the discrete-time systems. Defaults to `True` (unspecified sampling time). Must be specified as a keyword argument, for example, `dt=0.1`.

**See also:**

`ZerosPolesGain`, `StateSpace`, `TransferFunction`, `lti`

**Notes**

`dlti` instances do not exist directly. Instead, `dlti` creates an instance of one of its subclasses: `StateSpace`, `TransferFunction` or `ZerosPolesGain`.

Changing the value of properties that are not directly part of the current system representation (such as the `zeros` of a `StateSpace` system) is very inefficient and may lead to numerical inaccuracies. It is better to convert to the specific system representation first. For example, call `sys = sys.to_zpk()` before accessing/changing the zeros, poles or gain.

If (numerator, denominator) is passed in for `*system`, coefficients for both the numerator and denominator should be specified in descending exponent order (e.g.,  $z^2 + 3z + 5$  would be represented as `[1, 3, 5]`).

New in version 0.18.0.

### Examples

```
>>> from scipy import signal

>>> signal.dlti(1, 2, 3, 4)
StateSpaceDiscrete(
array([[1]]),
array([[2]]),
array([[3]]),
array([[4]]),
dt: True
)

>>> signal.dlti(1, 2, 3, 4, dt=0.1)
StateSpaceDiscrete(
array([[1]]),
array([[2]]),
array([[3]]),
array([[4]]),
dt: 0.1
)

>>> signal.dlti([1, 2], [3, 4], 5, dt=0.1)
ZerosPolesGainDiscrete(
array([1, 2]),
array([3, 4]),
5,
dt: 0.1
)

>>> signal.dlti([3, 4], [1, 2], dt=0.1)
TransferFunctionDiscrete(
array([ 3.,  4.]),
array([ 1.,  2.]),
dt: 0.1
)
```

### Attributes

A	State matrix of the <code>StateSpace</code> system.
B	Input matrix of the <code>StateSpace</code> system.
C	Output matrix of the <code>StateSpace</code> system.
D	Feedthrough matrix of the <code>StateSpace</code> system.

Continued on next page

Table 5.139 – continued from previous page

<code>den</code>	Denominator of the <code>TransferFunction</code> system.
<code>dt</code>	Return the sampling time of the system.
<code>gain</code>	Gain of the <code>ZerosPolesGain</code> system.
<code>num</code>	Numerator of the <code>TransferFunction</code> system.
<code>poles</code>	Poles of the system.
<code>zeros</code>	Zeros of the system.

**`dlti.A`**

State matrix of the `StateSpace` system.

**`dlti.B`**

Input matrix of the `StateSpace` system.

**`dlti.C`**

Output matrix of the `StateSpace` system.

**`dlti.D`**

Feedthrough matrix of the `StateSpace` system.

**`dlti.den`**

Denominator of the `TransferFunction` system.

**`dlti.dt`**

Return the sampling time of the system.

**`dlti.gain`**

Gain of the `ZerosPolesGain` system.

**`dlti.num`**

Numerator of the `TransferFunction` system.

**`dlti.poles`**

Poles of the system.

**`dlti.zeros`**

Zeros of the system.

**Methods**

<code>bode([w, n])</code>	Calculate Bode magnitude and phase data of a discrete-time system.
<code>freqresp([w, n, whole])</code>	Calculate the frequency response of a discrete-time system.
<code>impulse([x0, t, n])</code>	Return the impulse response of the discrete-time <code>dlti</code> system.
<code>output(u, t[, x0])</code>	Return the response of the discrete-time system to input $u$ .
<code>step([x0, t, n])</code>	Return the step response of the discrete-time <code>dlti</code> system.

**`dlti.bode` ( $w=None, n=100$ )**

Calculate Bode magnitude and phase data of a discrete-time system.

Returns a 3-tuple containing arrays of frequencies [rad/s], magnitude [dB] and phase [deg]. See `dbode` for details.

**Examples**

```
>>> from scipy import signal
>>> import matplotlib.pyplot as plt
```

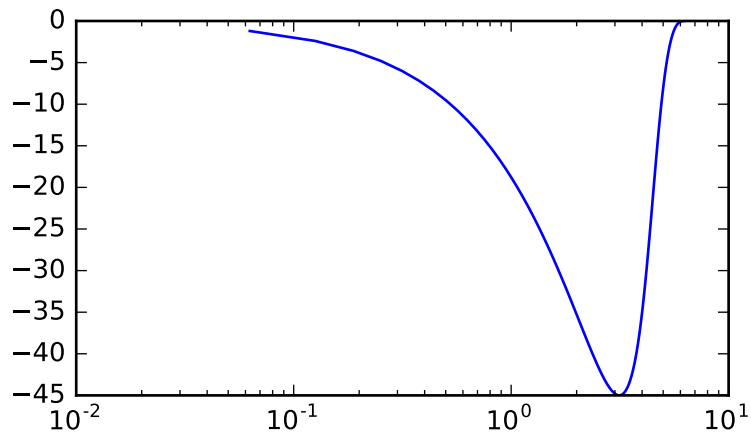
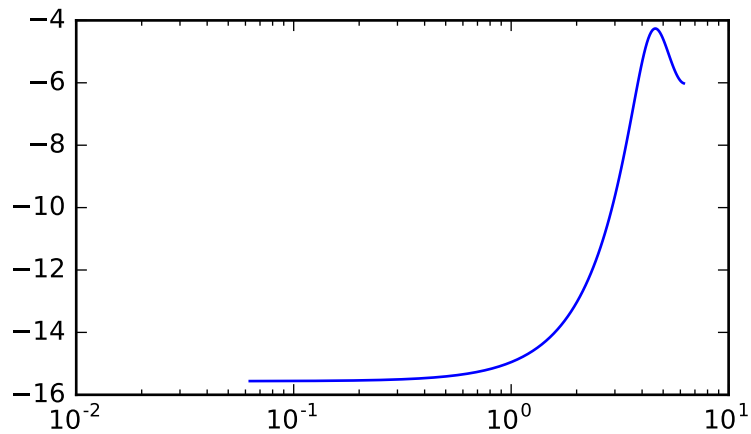
Transfer function:  $H(z) = 1 / (z^2 + 2z + 3)$  with sampling time 0.5s

```
>>> sys = signal.TransferFunction([1], [1, 2, 3], dt=0.5)
```

Equivalent: `signal.dbode(sys)`

```
>>> w, mag, phase = sys.bode()
```

```
>>> plt.figure()
>>> plt.semilogx(w, mag)      # Bode magnitude plot
>>> plt.figure()
>>> plt.semilogx(w, phase)   # Bode phase plot
>>> plt.show()
```



`dlti.freqresp(w=None, n=10000, whole=False)`

Calculate the frequency response of a discrete-time system.

Returns a 2-tuple containing arrays of frequencies [rad/s] and complex magnitude. See `dfreqresp` for details.

`dlti.impulse` ( $x0=None, t=None, n=None$ )

Return the impulse response of the discrete-time `dlti` system. See `dimpulse` for details.

`dlti.output` ( $u, t, x0=None$ )

Return the response of the discrete-time system to input  $u$ . See `dlsim` for details.

`dlti.step` ( $x0=None, t=None, n=None$ )

Return the step response of the discrete-time `dlti` system. See `dstep` for details.

**class** `scipy.signal.StateSpace` (*\*system, \*\*kwargs*)

Linear Time Invariant system in state-space form.

Represents the system as the continuous-time, first order differential equation  $\dot{x} = Ax + Bu$  or the discrete-time difference equation  $x[k+1] = Ax[k] + Bu[k]$ . `StateSpace` systems inherit additional functionality from the `lti`, respectively the `dlti` classes, depending on which system representation is used.

**Parameters** **\*system: arguments**

The `StateSpace` class can be instantiated with 1 or 3 arguments. The following gives the number of input arguments and their interpretation:

- 1: `lti` or `dlti` system: (`StateSpace`, `TransferFunction` or `ZerosPolesGain`)
- 4: `array_like`: (A, B, C, D)

**dt: float, optional**

Sampling time [s] of the discrete-time systems. Defaults to `None` (continuous-time). Must be specified as a keyword argument, for example, `dt=0.1`.

**See also:**

`TransferFunction`, `ZerosPolesGain`, `lti`, `dlti`, `ss2zpk`, `ss2tf`, `zpk2sos`

**Notes**

Changing the value of properties that are not part of the `StateSpace` system representation (such as `zeros` or `poles`) is very inefficient and may lead to numerical inaccuracies. It is better to convert to the specific system representation first. For example, call `sys = sys.to_zpk()` before accessing/changing the zeros, poles or gain.

**Examples**

```
>>> from scipy import signal

>>> a = np.array([[0, 1], [0, 0]])
>>> b = np.array([[0], [1]])
>>> c = np.array([[1, 0]])
>>> d = np.array([[0]])

>>> sys = signal.StateSpace(a, b, c, d)
>>> print(sys)
StateSpaceContinuous(
array([[0, 1],
       [0, 0]]),
array([[0],
       [1]]),
array([[1, 0]]),
array([[0]]),
dt: None
)
```



```

>>> sys.to_discrete(0.1)
StateSpaceDiscrete(
array([[ 1. ,  0.1],
       [ 0. ,  1. ]]),
array([[ 0.005],
       [ 0.1 ]]),
array([[1, 0]]),
array([[0]]),
dt: 0.1
)

>>> a = np.array([[1, 0.1], [0, 1]])
>>> b = np.array([[0.005], [0.1]])

>>> signal.StateSpace(a, b, c, d, dt=0.1)
StateSpaceDiscrete(
array([[ 1. ,  0.1],
       [ 0. ,  1. ]]),
array([[ 0.005],
       [ 0.1 ]]),
array([[1, 0]]),
array([[0]]),
dt: 0.1
)

```

**Attributes**


---

<code>A</code>	State matrix of the <code>StateSpace</code> system.
<code>B</code>	Input matrix of the <code>StateSpace</code> system.
<code>C</code>	Output matrix of the <code>StateSpace</code> system.
<code>D</code>	Feedthrough matrix of the <code>StateSpace</code> system.
<code>den</code>	Denominator of the <code>TransferFunction</code> system.
<code>dt</code>	Return the sampling time of the system, <i>None</i> for <code>lti</code> systems.
<code>gain</code>	Gain of the <code>ZerosPolesGain</code> system.
<code>num</code>	Numerator of the <code>TransferFunction</code> system.
<code>poles</code>	Poles of the system.
<code>zeros</code>	Zeros of the system.

---

**StateSpace.A**

State matrix of the `StateSpace` system.

**StateSpace.B**

Input matrix of the `StateSpace` system.

**StateSpace.C**

Output matrix of the `StateSpace` system.

**StateSpace.D**

Feedthrough matrix of the `StateSpace` system.

**StateSpace.den**

Denominator of the `TransferFunction` system.

**StateSpace.dt**

Return the sampling time of the system, *None* for `lti` systems.

`StateSpace.gain`  
Gain of the `ZerosPolesGain` system.

`StateSpace.num`  
Numerator of the `TransferFunction` system.

`StateSpace.poles`  
Poles of the system.

`StateSpace.zeros`  
Zeros of the system.

**Methods**

<code>to_ss()</code>	Return a copy of the current <code>StateSpace</code> system.
<code>to_tf(**kwargs)</code>	Convert system representation to <code>TransferFunction</code> .
<code>to_zpk(**kwargs)</code>	Convert system representation to <code>ZerosPolesGain</code> .

`StateSpace.to_ss()`  
Return a copy of the current `StateSpace` system.

**Returns** `sys` : instance of `StateSpace`  
The current system (copy)

`StateSpace.to_tf(**kwargs)`  
Convert system representation to `TransferFunction`.

**Parameters** `kwargs` : dict, optional  
Additional keywords passed to `ss2zpk`

**Returns** `sys` : instance of `TransferFunction`  
Transfer function of the current system

`StateSpace.to_zpk(**kwargs)`  
Convert system representation to `ZerosPolesGain`.

**Parameters** `kwargs` : dict, optional  
Additional keywords passed to `ss2zpk`

**Returns** `sys` : instance of `ZerosPolesGain`  
Zeros, poles, gain representation of the current system

**class** `scipy.signal.TransferFunction(*system, **kwargs)`  
Linear Time Invariant system class in transfer function form.

Represents the system as the continuous-time transfer function  $H(s) = \sum_{i=0}^N b[N-i]s^i / \sum_{j=0}^M a[M-j]s^j$  or the discrete-time transfer function  $H(z) = \sum_{i=0}^N b[N-i]z^i / \sum_{j=0}^M a[M-j]z^j$ , where `b` are elements of the numerator `num`, `a` are elements of the denominator `den`, and  $N == \text{len}(b) - 1, M == \text{len}(a) - 1$ . `TransferFunction` systems inherit additional functionality from the `lti`, respectively the `dlti` classes, depending on which system representation is used.

**Parameters** `*system: arguments`  
The `TransferFunction` class can be instantiated with 1 or 2 arguments. The following gives the number of input arguments and their interpretation:

- 1: `lti` or `dlti` system: (`StateSpace`, `TransferFunction` or `ZerosPolesGain`)
- 2: `array_like`: (numerator, denominator)

**dt: float, optional**  
Sampling time [s] of the discrete-time systems. Defaults to `None` (continuous-time). Must be specified as a keyword argument, for example, `dt=0.1`.

**See also:**

ZerosPolesGain, StateSpace, lti, dlti, tf2ss, tf2zpk, tf2sos

**Notes**

Changing the value of properties that are not part of the `TransferFunction` system representation (such as the `A`, `B`, `C`, `D` state-space matrices) is very inefficient and may lead to numerical inaccuracies. It is better to convert to the specific system representation first. For example, call `sys = sys.to_ss()` before accessing/changing the `A`, `B`, `C`, `D` system matrices.

If (numerator, denominator) is passed in for `*system`, coefficients for both the numerator and denominator should be specified in descending exponent order (e.g.  $s^2 + 3s + 5$  or  $z^2 + 3z + 5$  would be represented as `[1, 3, 5]`)

**Examples**

Construct the transfer function:

$$H(s) = \frac{s^2 + 3s + 3}{s^2 + 2s + 1}$$

```
>>> from scipy import signal

>>> num = [1, 3, 3]
>>> den = [1, 2, 1]

>>> signal.TransferFunction(num, den)
TransferFunctionContinuous(
array([ 1.,  3.,  3.]),
array([ 1.,  2.,  1.]),
dt: None
)
```

Construct the transfer function with a sampling time of 0.5 seconds:

$$H(z) = \frac{z^2 + 3z + 3}{z^2 + 2z + 1}$$

```
>>> signal.TransferFunction(num, den, dt=0.1)
TransferFunctionDiscrete(
array([ 1.,  3.,  3.]),
array([ 1.,  2.,  1.]),
dt: 0.1
)
```

**Attributes**

<code>A</code>	State matrix of the <code>StateSpace</code> system.
<code>B</code>	Input matrix of the <code>StateSpace</code> system.
<code>C</code>	Output matrix of the <code>StateSpace</code> system.
<code>D</code>	Feedthrough matrix of the <code>StateSpace</code> system.
<code>den</code>	Denominator of the <code>TransferFunction</code> system.

Continued on next page

Table 5.143 – continued from previous page

<code>dt</code>	Return the sampling time of the system, <i>None</i> for <code>lti</code> systems.
<code>gain</code>	Gain of the <code>ZerosPolesGain</code> system.
<code>num</code>	Numerator of the <code>TransferFunction</code> system.
<code>poles</code>	Poles of the system.
<code>zeros</code>	Zeros of the system.

`TransferFunction.A`

State matrix of the `StateSpace` system.

`TransferFunction.B`

Input matrix of the `StateSpace` system.

`TransferFunction.C`

Output matrix of the `StateSpace` system.

`TransferFunction.D`

Feedthrough matrix of the `StateSpace` system.

`TransferFunction.den`

Denominator of the `TransferFunction` system.

`TransferFunction.dt`

Return the sampling time of the system, *None* for `lti` systems.

`TransferFunction.gain`

Gain of the `ZerosPolesGain` system.

`TransferFunction.num`

Numerator of the `TransferFunction` system.

`TransferFunction.poles`

Poles of the system.

`TransferFunction.zeros`

Zeros of the system.

### Methods

<code>to_ss()</code>	Convert system representation to <code>StateSpace</code> .
<code>to_tf()</code>	Return a copy of the current <code>TransferFunction</code> system.
<code>to_zpk()</code>	Convert system representation to <code>ZerosPolesGain</code> .

`TransferFunction.to_ss()`

Convert system representation to `StateSpace`.

**Returns** `sys` : instance of `StateSpace`  
State space model of the current system

`TransferFunction.to_tf()`

Return a copy of the current `TransferFunction` system.

**Returns** `sys` : instance of `TransferFunction`  
The current system (copy)

`TransferFunction.to_zpk()`

Convert system representation to `ZerosPolesGain`.

**Returns** `sys` : instance of `ZerosPolesGain`

Zeros, poles, gain representation of the current system

**class** `scipy.signal.ZerosPolesGain` (\*system, \*\*kwargs)  
 Linear Time Invariant system class in zeros, poles, gain form.

Represents the system as the continuous- or discrete-time transfer function  $H(s) = k \prod_i (s - z[i]) / \prod_j (s - p[j])$ , where  $k$  is the `gain`,  $z$  are the `zeros` and  $p$  are the `poles`. `ZerosPolesGain` systems inherit additional functionality from the `lti`, respectively the `dlti` classes, depending on which system representation is used.

**Parameters** \*system : arguments

The `ZerosPolesGain` class can be instantiated with 1 or 3 arguments. The following gives the number of input arguments and their interpretation:

- 1: `lti` or `dlti` system: (`StateSpace`, `TransferFunction` or `ZerosPolesGain`)
- 3: `array_like`: (`zeros`, `poles`, `gain`)

**dt**: float, optional

Sampling time [s] of the discrete-time systems. Defaults to `None` (continuous-time). Must be specified as a keyword argument, for example, `dt=0.1`.

**See also:**

`TransferFunction`, `StateSpace`, `lti`, `dlti`, `zpk2ss`, `zpk2tf`, `zpk2sos`

**Notes**

Changing the value of properties that are not part of the `ZerosPolesGain` system representation (such as the `A`, `B`, `C`, `D` state-space matrices) is very inefficient and may lead to numerical inaccuracies. It is better to convert to the specific system representation first. For example, call `sys = sys.to_ss()` before accessing/changing the `A`, `B`, `C`, `D` system matrices.

**Examples**

```
>>> from scipy import signal
```

Transfer function:  $H(s) = 5(s - 1)(s - 2) / (s - 3)(s - 4)$

```
>>> signal.ZerosPolesGain([1, 2], [3, 4], 5)
ZerosPolesGainContinuous(
array([1, 2]),
array([3, 4]),
5,
dt: None
)
```

Transfer function:  $H(z) = 5(z - 1)(z - 2) / (z - 3)(z - 4)$

```
>>> signal.ZerosPolesGain([1, 2], [3, 4], 5, dt=0.1)
ZerosPolesGainDiscrete(
array([1, 2]),
array([3, 4]),
5,
dt: 0.1
)
```

**Attributes**

A	State matrix of the <code>StateSpace</code> system.
---	---

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Table 5.145 – continued from previous page

<code>B</code>	Input matrix of the <code>StateSpace</code> system.
<code>C</code>	Output matrix of the <code>StateSpace</code> system.
<code>D</code>	Feedthrough matrix of the <code>StateSpace</code> system.
<code>den</code>	Denominator of the <code>TransferFunction</code> system.
<code>dt</code>	Return the sampling time of the system, <code>None</code> for <code>lti</code> systems.
<code>gain</code>	Gain of the <code>ZerosPolesGain</code> system.
<code>num</code>	Numerator of the <code>TransferFunction</code> system.
<code>poles</code>	Poles of the <code>ZerosPolesGain</code> system.
<code>zeros</code>	Zeros of the <code>ZerosPolesGain</code> system.

`ZerosPolesGain.A`

State matrix of the `StateSpace` system.

`ZerosPolesGain.B`

Input matrix of the `StateSpace` system.

`ZerosPolesGain.C`

Output matrix of the `StateSpace` system.

`ZerosPolesGain.D`

Feedthrough matrix of the `StateSpace` system.

`ZerosPolesGain.den`

Denominator of the `TransferFunction` system.

`ZerosPolesGain.dt`

Return the sampling time of the system, `None` for `lti` systems.

`ZerosPolesGain.gain`

Gain of the `ZerosPolesGain` system.

`ZerosPolesGain.num`

Numerator of the `TransferFunction` system.

`ZerosPolesGain.poles`

Poles of the `ZerosPolesGain` system.

`ZerosPolesGain.zeros`

Zeros of the `ZerosPolesGain` system.

*Methods*

<code>to_ss()</code>	Convert system representation to <code>StateSpace</code> .
<code>to_tf()</code>	Convert system representation to <code>TransferFunction</code> .
<code>to_zpk()</code>	Return a copy of the current ‘ZerosPolesGain’ system.

`ZerosPolesGain.to_ss()`

Convert system representation to `StateSpace`.

**Returns** `sys` : instance of `StateSpace`  
State space model of the current system

`ZerosPolesGain.to_tf()`

Convert system representation to `TransferFunction`.

**Returns** `sys` : instance of `TransferFunction`  
Transfer function of the current system

`ZerosPolesGain.to_zpk()`

Return a copy of the current ‘ZerosPolesGain’ system.

**Returns** `sys` : instance of `ZerosPolesGain`  
The current system (copy)

`scipy.signal.dlsim(system, u, t=None, x0=None)`

Simulate output of a discrete-time linear system.

**Parameters** `system` : tuple of array\_like or instance of `dlti`  
A tuple describing the system. The following gives the number of elements in the tuple and the interpretation:

- 1: (instance of `dlti`)
- 3: (num, den, dt)
- 4: (zeros, poles, gain, dt)
- 5: (A, B, C, D, dt)

`u` : array\_like  
An input array describing the input at each time  $t$  (interpolation is assumed between given times). If there are multiple inputs, then each column of the rank-2 array represents an input.

`t` : array\_like, optional  
The time steps at which the input is defined. If  $t$  is given, it must be the same length as  $u$ , and the final value in  $t$  determines the number of steps returned in the output.

`x0` : array\_like, optional  
The initial conditions on the state vector (zero by default).

**Returns** `tout` : ndarray  
Time values for the output, as a 1-D array.

`yout` : ndarray  
System response, as a 1-D array.

`xout` : ndarray, optional  
Time-evolution of the state-vector. Only generated if the input is a `StateSpace` system.

**See also:**

`lsim`, `dstep`, `dimpulse`, `cont2discrete`

**Examples**

A simple integrator transfer function with a discrete time step of 1.0 could be implemented as:

```
>>> from scipy import signal
>>> tf = ([1.0,], [1.0, -1.0], 1.0)
>>> t_in = [0.0, 1.0, 2.0, 3.0]
>>> u = np.asarray([0.0, 0.0, 1.0, 1.0])
>>> t_out, y = signal.dlsim(tf, u, t=t_in)
>>> y.T
array([[ 0.,  0.,  0.,  1.]])
```

`scipy.signal.dimpulse(system, x0=None, t=None, n=None)`

Impulse response of discrete-time system.

**Parameters** `system` : tuple of array\_like or instance of `dlti`  
A tuple describing the system. The following gives the number of elements in the tuple and the interpretation:

- 1: (instance of `dlti`)
- 3: (num, den, dt)
- 4: (zeros, poles, gain, dt)
- 5: (A, B, C, D, dt)

`x0` : array\_like, optional  
Initial state-vector. Defaults to zero.

**Returns**

**t** : array\_like, optional  
Time points. Computed if not given.

**n** : int, optional  
The number of time points to compute (if *t* is not given).

**tout** : ndarray  
Time values for the output, as a 1-D array.

**yout** : ndarray  
Impulse response of system. Each element of the tuple represents the output of the system based on an impulse in each input.

**See also:**

`impulse`, `dstep`, `dlsim`, `cont2discrete`

`scipy.signal.dstep` (*system*, *x0=None*, *t=None*, *n=None*)

Step response of discrete-time system.

**Parameters**

**system** : tuple of array\_like  
A tuple describing the system. The following gives the number of elements in the tuple and the interpretation:

- 1: (instance of `dlti`)
- 2: (num, den, dt)
- 3: (zeros, poles, gain, dt)
- 4: (A, B, C, D, dt)

**x0** : array\_like, optional  
Initial state-vector. Defaults to zero.

**t** : array\_like, optional  
Time points. Computed if not given.

**n** : int, optional  
The number of time points to compute (if *t* is not given).

**Returns**

**tout** : ndarray  
Output time points, as a 1-D array.

**yout** : ndarray  
Step response of system. Each element of the tuple represents the output of the system based on a step response to each input.

**See also:**

`step`, `dimpulse`, `dlsim`, `cont2discrete`

`scipy.signal.dfreqresp` (*system*, *w=None*, *n=10000*, *whole=False*)

Calculate the frequency response of a discrete-time system.

**Parameters**

**system** : an instance of the `dlti` class or a tuple describing the system.  
The following gives the number of elements in the tuple and the interpretation:

- 1 (instance of `dlti`).
- 2 (numerator, denominator, dt)
- 3 (zeros, poles, gain, dt)
- 4 (A, B, C, D, dt)

**w** : array\_like, optional  
Array of frequencies (in radians/sample). Magnitude and phase data is calculated for every value in this array. If not given a reasonable set will be calculated.

**n** : int, optional  
Number of frequency points to compute if *w* is not given. The *n* frequencies are logarithmically spaced in an interval chosen to include the influence of the poles and zeros of the system.

**whole** : bool, optional  
Normally, if 'w' is not given, frequencies are computed from 0 to the Nyquist frequency, pi radians/sample (upper-half of unit-circle). If *whole* is True, compute frequencies from 0 to 2\*pi radians/sample.



**Returns** **w** : 1D ndarray  
Frequency array [radians/sample]  
**H** : 1D ndarray  
Array of complex magnitude values

**Notes**

If (num, den) is passed in for `system`, coefficients for both the numerator and denominator should be specified in descending exponent order (e.g.  $z^2 + 3z + 5$  would be represented as `[1, 3, 5]`).

New in version 0.18.0.

**Examples**

Generating the Nyquist plot of a transfer function

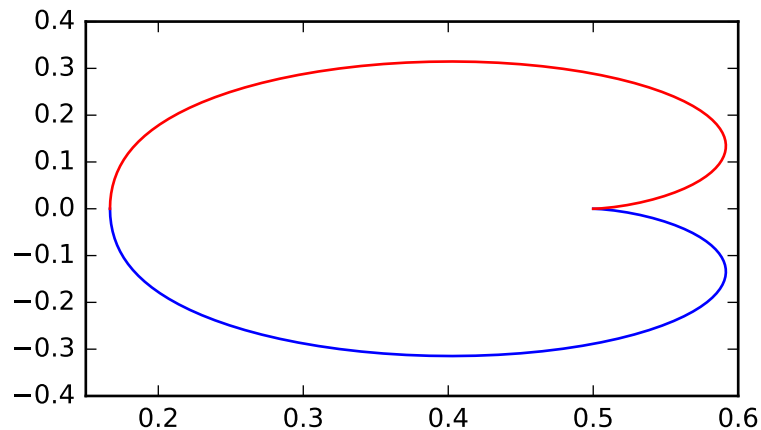
```
>>> from scipy import signal
>>> import matplotlib.pyplot as plt
```

Transfer function:  $H(z) = 1 / (z^2 + 2z + 3)$

```
>>> sys = signal.TransferFunction([1], [1, 2, 3], dt=0.05)
```

```
>>> w, H = signal.dfreqresp(sys)
```

```
>>> plt.figure()
>>> plt.plot(H.real, H.imag, "b")
>>> plt.plot(H.real, -H.imag, "r")
>>> plt.show()
```



`scipy.signal.dbode` (*system*, *w=None*, *n=100*)

Calculate Bode magnitude and phase data of a discrete-time system.

**Parameters** **system** : an instance of the LTI class or a tuple describing the system.

The following gives the number of elements in the tuple and the interpretation:

- 1 (instance of `dlti`)
- 2 (num, den, dt)

`•3 (zeros, poles, gain, dt)`  
`•4 (A, B, C, D, dt)`

**w** : array\_like, optional  
 Array of frequencies (in radians/sample). Magnitude and phase data is calculated for every value in this array. If not given a reasonable set will be calculated.

**n** : int, optional  
 Number of frequency points to compute if *w* is not given. The *n* frequencies are logarithmically spaced in an interval chosen to include the influence of the poles and zeros of the system.

**Returns**

**w** : 1D ndarray  
 Frequency array [rad/time\_unit]

**mag** : 1D ndarray  
 Magnitude array [dB]

**phase** : 1D ndarray  
 Phase array [deg]

**Notes**

If (num, den) is passed in for `system`, coefficients for both the numerator and denominator should be specified in descending exponent order (e.g.  $z^2 + 3z + 5$  would be represented as [1, 3, 5]).

New in version 0.18.0.

**Examples**

```
>>> from scipy import signal
>>> import matplotlib.pyplot as plt
```

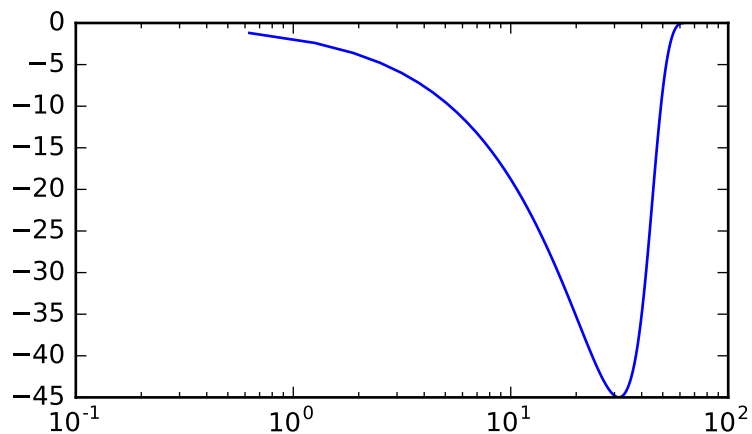
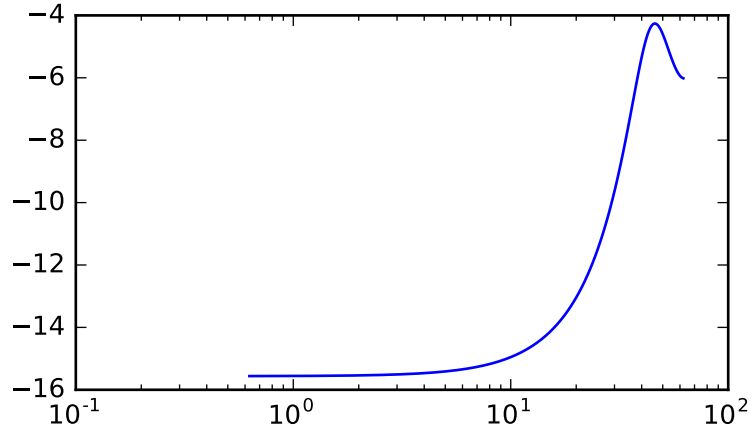
Transfer function:  $H(z) = 1 / (z^2 + 2z + 3)$

```
>>> sys = signal.TransferFunction([1], [1, 2, 3], dt=0.05)
```

Equivalent: `sys.bode()`

```
>>> w, mag, phase = signal.dbode(sys)
```

```
>>> plt.figure()
>>> plt.semilogx(w, mag)      # Bode magnitude plot
>>> plt.figure()
>>> plt.semilogx(w, phase)   # Bode phase plot
>>> plt.show()
```



### 5.20.8 LTI Representations

<code>tf2zpk(b, a)</code>	Return zero, pole, gain (z, p, k) representation from a numerator, denominator
<code>tf2sos(b, a[, pairing])</code>	Return second-order sections from transfer function representation
<code>tf2ss(num, den)</code>	Transfer function to state-space representation.
<code>zpk2tf(z, p, k)</code>	Return polynomial transfer function representation from zeros and poles
<code>zpk2sos(z, p, k[, pairing])</code>	Return second-order sections from zeros, poles, and gain of a system
<code>zpk2ss(z, p, k)</code>	Zero-pole-gain representation to state-space representation
<code>ss2tf(A, B, C, D[, input])</code>	State-space to transfer function.
<code>ss2zpk(A, B, C, D[, input])</code>	State-space representation to zero-pole-gain representation.
<code>sos2zpk(sos)</code>	Return zeros, poles, and gain of a series of second-order sections
<code>sos2tf(sos)</code>	Return a single transfer function from a series of second-order sections
<code>cont2discrete(system, dt[, method, alpha])</code>	Transform a continuous to a discrete state-space system.
<code>place_poles(A, B, poles[, method, rtol, maxiter])</code>	Compute K such that eigenvalues $(A - \text{dot}(B, K)) = \text{poles}$ .

`scipy.signal.tf2zpk(b, a)`

Return zero, pole, gain (z, p, k) representation from a numerator, denominator representation of a linear filter.

<b>Parameters</b>	<b>b</b> : array_like	Numerator polynomial coefficients.
	<b>a</b> : array_like	Denominator polynomial coefficients.
<b>Returns</b>	<b>z</b> : ndarray	Zeros of the transfer function.
	<b>p</b> : ndarray	Poles of the transfer function.
	<b>k</b> : float	System gain.

### Notes

If some values of  $b$  are too close to 0, they are removed. In that case, a `BadCoefficients` warning is emitted.

The  $b$  and  $a$  arrays are interpreted as coefficients for positive, descending powers of the transfer function variable. So the inputs  $b = [b_0, b_1, \dots, b_M]$  and  $a = [a_0, a_1, \dots, a_N]$  can represent an analog filter of the form:

$$H(s) = \frac{b_0 s^M + b_1 s^{(M-1)} + \dots + b_M}{a_0 s^N + a_1 s^{(N-1)} + \dots + a_N}$$

or a discrete-time filter of the form:

$$H(z) = \frac{b_0 z^M + b_1 z^{(M-1)} + \dots + b_M}{a_0 z^N + a_1 z^{(N-1)} + \dots + a_N}$$

This “positive powers” form is found more commonly in controls engineering. If  $M$  and  $N$  are equal (which is true for all filters generated by the bilinear transform), then this happens to be equivalent to the “negative powers” discrete-time form preferred in DSP:

$$H(z) = \frac{b_0 + b_1 z^{-1} + \dots + b_M z^{-M}}{a_0 + a_1 z^{-1} + \dots + a_N z^{-N}}$$

Although this is true for common filters, remember that this is not true in the general case. If  $M$  and  $N$  are not equal, the discrete-time transfer function coefficients must first be converted to the “positive powers” form before finding the poles and zeros.

`scipy.signal.tf2sos(b, a, pairing='nearest')`

Return second-order sections from transfer function representation

<b>Parameters</b>	<b>b</b> : array_like	Numerator polynomial coefficients.
	<b>a</b> : array_like	Denominator polynomial coefficients.
	<b>pairing</b> : {'nearest', 'keep_odd'}, optional	The method to use to combine pairs of poles and zeros into sections. See <a href="#">zpk2sos</a> .
<b>Returns</b>	<b>sos</b> : ndarray	Array of second-order filter coefficients, with shape <code>(n_sections, 6)</code> . See <a href="#">sosfilt</a> for the SOS filter format specification.

See also:

[zpk2sos](#), [sosfilt](#)

**Notes**

It is generally discouraged to convert from TF to SOS format, since doing so usually will not improve numerical precision errors. Instead, consider designing filters in ZPK format and converting directly to SOS. TF is converted to SOS by first converting to ZPK format, then converting ZPK to SOS.

New in version 0.16.0.

`scipy.signal.tf2ss(num, den)`

Transfer function to state-space representation.

**Parameters** **num, den** : array\_like  
Sequences representing the coefficients of the numerator and denominator polynomials, in order of descending degree. The denominator needs to be at least as long as the numerator.

**Returns** **A, B, C, D** : ndarray  
State space representation of the system, in controller canonical form.

**Examples**

Convert the transfer function:

$$H(s) = \frac{s^2 + 3s + 3}{s^2 + 2s + 1}$$

```
>>> num = [1, 3, 3]
>>> den = [1, 2, 1]
```

to the state-space representation:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \mathbf{u}(t)$$

$$\mathbf{y}(t) = [1 \quad 2] \mathbf{x}(t) + [1] \mathbf{u}(t)$$

```
>>> from scipy.signal import tf2ss
>>> A, B, C, D = tf2ss(num, den)
>>> A
array([[ -2.,  -1.],
       [  1.,   0.]])
>>> B
array([[ 1.],
       [ 0.]])
>>> C
array([[ 1.,  2.]])
>>> D
array([[ 1.]])
```

`scipy.signal.zpk2tf(z, p, k)`

Return polynomial transfer function representation from zeros and poles

**Parameters** **z** : array\_like  
Zeros of the transfer function.

**p** : array\_like  
Poles of the transfer function.

**k** : float  
System gain.

**Returns** **b** : ndarray

**a** : ndarray  
 Numerator polynomial coefficients.  
**b** : ndarray  
 Denominator polynomial coefficients.

`scipy.signal.zpk2sos(z, p, k, pairing='nearest')`

Return second-order sections from zeros, poles, and gain of a system

**Parameters**

- z** : array\_like  
Zeros of the transfer function.
- p** : array\_like  
Poles of the transfer function.
- k** : float  
System gain.
- pairing** : {'nearest', 'keep\_odd'}, optional  
The method to use to combine pairs of poles and zeros into sections. See Notes below.

**Returns**

- sos** : ndarray  
Array of second-order filter coefficients, with shape `(n_sections, 6)`. See `sosfilt` for the SOS filter format specification.

**See also:**

`sosfilt`

**Notes**

The algorithm used to convert ZPK to SOS format is designed to minimize errors due to numerical precision issues. The pairing algorithm attempts to minimize the peak gain of each biquadratic section. This is done by pairing poles with the nearest zeros, starting with the poles closest to the unit circle.

**Algorithms**

The current algorithms are designed specifically for use with digital filters. (The output coefficients are not correct for analog filters.)

The steps in the `pairing='nearest'` and `pairing='keep_odd'` algorithms are mostly shared. The `nearest` algorithm attempts to minimize the peak gain, while `'keep_odd'` minimizes peak gain under the constraint that odd-order systems should retain one section as first order. The algorithm steps and are as follows:

As a pre-processing step, add poles or zeros to the origin as necessary to obtain the same number of poles and zeros for pairing. If `pairing == 'nearest'` and there are an odd number of poles, add an additional pole and a zero at the origin.

The following steps are then iterated over until no more poles or zeros remain:

1. Take the (next remaining) pole (complex or real) closest to the unit circle to begin a new filter section.
2. If the pole is real and there are no other remaining real poles<sup>4</sup>, add the closest real zero to the section and leave it as a first order section. Note that after this step we are guaranteed to be left with an even number of real poles, complex poles, real zeros, and complex zeros for subsequent pairing iterations.
3. Else:
  - (a) If the pole is complex and the zero is the only remaining real zero\*, then pair the pole with the *next* closest zero (guaranteed to be complex). This is necessary to ensure that there will be a real zero remaining to eventually create a first-order section (thus keeping the odd order).
  - (b) Else pair the pole with the closest remaining zero (complex or real).
  - (c) Proceed to complete the second-order section by adding another pole and zero to the current pole and zero in the section:
    - i. If the current pole and zero are both complex, add their conjugates.
    - ii. Else if the pole is complex and the zero is real, add the conjugate pole and the next closest real zero.

---

<sup>4</sup> This conditional can only be met for specific odd-order inputs with the `pairing == 'keep_odd'` method.

- iii. Else if the pole is real and the zero is complex, add the conjugate zero and the real pole closest to those zeros.
- iv. Else (we must have a real pole and real zero) add the next real pole closest to the unit circle, and then add the real zero closest to that pole.

New in version 0.16.0.

### Examples

Design a 6th order low-pass elliptic digital filter for a system with a sampling rate of 8000 Hz that has a pass-band corner frequency of 1000 Hz. The ripple in the pass-band should not exceed 0.087 dB, and the attenuation in the stop-band should be at least 90 dB.

In the following call to `signal.ellip`, we could use `output='sos'`, but for this example, we'll use `output='zpk'`, and then convert to SOS format with `zpk2sos`:

```
>>> from scipy import signal
>>> z, p, k = signal.ellip(6, 0.087, 90, 1000/(0.5*8000), output='zpk')
```

Now convert to SOS format.

```
>>> sos = signal.zpk2sos(z, p, k)
```

The coefficients of the numerators of the sections:

```
>>> sos[:, :3]
array([[ 0.0014154 ,  0.00248707,  0.0014154 ],
       [ 1.          ,  0.72965193,  1.          ],
       [ 1.          ,  0.17594966,  1.          ]])
```

The symmetry in the coefficients occurs because all the zeros are on the unit circle.

The coefficients of the denominators of the sections:

```
>>> sos[:, 3:]
array([[ 1.          , -1.32543251,  0.46989499],
       [ 1.          , -1.26117915,  0.6262586 ],
       [ 1.          , -1.25707217,  0.86199667]])
```

The next example shows the effect of the `pairing` option. We have a system with three poles and three zeros, so the SOS array will have shape (2, 6). The means there is, in effect, an extra pole and an extra zero at the origin in the SOS representation.

```
>>> z1 = np.array([-1, -0.5-0.5j, -0.5+0.5j])
>>> p1 = np.array([0.75, 0.8+0.1j, 0.8-0.1j])
```

With `pairing='nearest'` (the default), we obtain

```
>>> signal.zpk2sos(z1, p1, 1)
array([[ 1. ,  1. ,  0.5 ,  1. , -0.75,  0. ],
       [ 1. ,  1. ,  0. ,  1. , -1.6 ,  0.65]])
```

The first section has the zeros  $\{-0.5-0.05j, -0.5+0.5j\}$  and the poles  $\{0, 0.75\}$ , and the second section has the zeros  $\{-1, 0\}$  and poles  $\{0.8+0.1j, 0.8-0.1j\}$ . Note that the extra pole and zero at the origin have been assigned to different sections.

With `pairing='keep_odd'`, we obtain:

```
>>> signal.zpk2sos(z1, p1, 1, pairing='keep_odd')
array([[ 1. ,  1. ,  0. ,  1. , -0.75,  0. ],
       [ 1. ,  1. ,  0.5,  1. , -1.6 ,  0.65]])
```

The extra pole and zero at the origin are in the same section. The first section is, in effect, a first-order section.

`scipy.signal.zpk2ss(z, p, k)`

Zero-pole-gain representation to state-space representation

**Parameters** **z, p** : sequence  
Zeros and poles.

**k** : float  
System gain.

**Returns** **A, B, C, D** : ndarray  
State space representation of the system, in controller canonical form.

`scipy.signal.ss2tf(A, B, C, D, input=0)`

State-space to transfer function.

A, B, C, D defines a linear state-space system with  $p$  inputs,  $q$  outputs, and  $n$  state variables.

**Parameters** **A** : array\_like  
State (or system) matrix of shape  $(n, n)$

**B** : array\_like  
Input matrix of shape  $(n, p)$

**C** : array\_like  
Output matrix of shape  $(q, n)$

**D** : array\_like  
Feedthrough (or feedforward) matrix of shape  $(q, p)$

**input** : int, optional  
For multiple-input systems, the index of the input to use.

**Returns** **num** : 2-D ndarray  
Numerator(s) of the resulting transfer function(s). *num* has one row for each of the system's outputs. Each row is a sequence representation of the numerator polynomial.

**den** : 1-D ndarray  
Denominator of the resulting transfer function(s). *den* is a sequence representation of the denominator polynomial.

### Examples

Convert the state-space representation:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \mathbf{u}(t)$$

$$\mathbf{y}(t) = \begin{bmatrix} 1 & 2 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 \end{bmatrix} \mathbf{u}(t)$$

```
>>> A = [[-2, -1], [1, 0]]
>>> B = [[1], [0]] # 2-dimensional column vector
>>> C = [[1, 2]] # 2-dimensional row vector
>>> D = 1
```

to the transfer function:

$$H(s) = \frac{s^2 + 3s + 3}{s^2 + 2s + 1}$$



```
>>> from scipy.signal import ss2tf
>>> ss2tf(A, B, C, D)
(array([[1, 3, 3]]), array([ 1.,  2.,  1.]))
```

`scipy.signal.ss2zpk(A, B, C, D, input=0)`  
State-space representation to zero-pole-gain representation.

A, B, C, D defines a linear state-space system with  $p$  inputs,  $q$  outputs, and  $n$  state variables.

**Parameters**

- A** : array\_like  
State (or system) matrix of shape  $(n, n)$
- B** : array\_like  
Input matrix of shape  $(n, p)$
- C** : array\_like  
Output matrix of shape  $(q, n)$
- D** : array\_like  
Feedthrough (or feedforward) matrix of shape  $(q, p)$
- input** : int, optional  
For multiple-input systems, the index of the input to use.

**Returns**

- z, p** : sequence  
Zeros and poles.
- k** : float  
System gain.

`scipy.signal.sos2zpk(sos)`  
Return zeros, poles, and gain of a series of second-order sections

**Parameters**

- sos** : array\_like  
Array of second-order filter coefficients, must have shape  $(n\_sections, 6)$ . See `sosfilt` for the SOS filter format specification.

**Returns**

- z** : ndarray  
Zeros of the transfer function.
- p** : ndarray  
Poles of the transfer function.
- k** : float  
System gain.

#### Notes

New in version 0.16.0.

`scipy.signal.sos2tf(sos)`  
Return a single transfer function from a series of second-order sections

**Parameters**

- sos** : array\_like  
Array of second-order filter coefficients, must have shape  $(n\_sections, 6)$ . See `sosfilt` for the SOS filter format specification.

**Returns**

- b** : ndarray  
Numerator polynomial coefficients.
- a** : ndarray  
Denominator polynomial coefficients.

#### Notes

New in version 0.16.0.

`scipy.signal.cont2discrete(system, dt, method='zoh', alpha=None)`  
Transform a continuous to a discrete state-space system.

**Parameters**

- system** : a tuple describing the system or an instance of `lti`

The following gives the number of elements in the tuple and the interpretation:

- 1: (instance of `lti`)
- 2: (num, den)
- 3: (zeros, poles, gain)
- 4: (A, B, C, D)

**dt** : float

The discretization time step.

**method** : {"gbt", "bilinear", "euler", "backward\_diff", "zoh"}, optional

Which method to use:

- gbt: generalized bilinear transformation
- bilinear: Tustin's approximation ("gbt" with `alpha=0.5`)
- euler: Euler (or forward differencing) method ("gbt" with `alpha=0`)
- backward\_diff: Backwards differencing ("gbt" with `alpha=1.0`)
- zoh: zero-order hold (default)

**alpha** : float within [0, 1], optional

The generalized bilinear transformation weighting parameter, which should only be specified with `method="gbt"`, and is ignored otherwise

**Returns**

**sysd** : tuple containing the discrete system

Based on the input type, the output will be of the form

- (num, den, dt) for transfer function input.
- (zeros, poles, gain, dt) for zeros-poles-gain input
- (A, B, C, D, dt) for state-space input

**Notes**

By default, the routine uses a Zero-Order Hold (zoh) method to perform the transformation. Alternatively, a generalized bilinear transformation may be used, which includes the common Tustin's bilinear approximation, an Euler's method technique, or a backwards differencing technique.

The Zero-Order Hold (zoh) method is based on [R193], the generalized bilinear approximation is based on [R194] and [R195].

**References**

[R193], [R194], [R195]

`scipy.signal.place_poles(A, B, poles, method='YT', rtol=0.001, maxiter=30)`

Compute K such that eigenvalues  $(A - \text{dot}(B, K)) = \text{poles}$ .

K is the gain matrix such as the plant described by the linear system  $AX+BU$  will have its closed-loop poles, i.e the eigenvalues  $A - B*K$ , as close as possible to those asked for in poles.

SISO, MISO and MIMO systems are supported.

**Parameters** **A, B** : ndarray

State-space representation of linear system  $AX + BU$ .

**poles** : array\_like

Desired real poles and/or complex conjugates poles. Complex poles are only supported with `method="YT"` (default).

**method**: {'YT', 'KNV0'}, optional

Which method to choose to find the gain matrix K. One of:

- 'YT': Yang Tits
- 'KNV0': Kautsky, Nichols, Van Dooren update method 0

See References and Notes for details on the algorithms.

**rtol**: float, optional

After each iteration the determinant of the eigenvectors of  $A - B*K$  is compared to its previous value, when the relative error between these two values becomes lower than `rtol` the algorithm stops. Default is `1e-3`.

**maxiter**: int, optional

Maximum number of iterations to compute the gain matrix. Default is 30.

**Returns**

**full\_state\_feedback** : Bunch object

*full\_state\_feedback* is composed of:

<b>gain_matrix</b>	[1-D ndarray] The closed loop matrix $K$ such as the eigenvalues of $A-BK$ are as close as possible to the requested poles.
<b>computed_poles</b>	[1-D ndarray] The poles corresponding to $A-BK$ sorted as first the real poles in increasing order, then the complex conjugates in lexicographic order.
<b>requested_poles</b>	[1-D ndarray] The poles the algorithm was asked to place sorted as above, they may differ from what was achieved.
<b>X</b>	[2-D ndarray] The transfer matrix such as $X * \text{diag}(\text{poles}) = (A - B*K) * X$ (see Notes)
<b>rtol</b>	[float] The relative tolerance achieved on $\det(X)$ (see Notes). <i>rtol</i> will be NaN if it is possible to solve the system $\text{diag}(\text{poles}) = (A - B*K)$ , or 0 when the optimization algorithms can't do anything i.e when $B.\text{shape}[1] == 1$ .
<b>nb_iter</b>	[int] The number of iterations performed before converging. <i>nb_iter</i> will be NaN if it is possible to solve the system $\text{diag}(\text{poles}) = (A - B*K)$ , or 0 when the optimization algorithms can't do anything i.e when $B.\text{shape}[1] == 1$ .

### Notes

The Tits and Yang (YT), [R227] paper is an update of the original Kautsky et al. (KNV) paper [R226]. KNV relies on rank-1 updates to find the transfer matrix  $X$  such that  $X * \text{diag}(\text{poles}) = (A - B*K) * X$ , whereas YT uses rank-2 updates. This yields on average more robust solutions (see [R227] pp 21-22), furthermore the YT algorithm supports complex poles whereas KNV does not in its original version. Only update method 0 proposed by KNV has been implemented here, hence the name 'KNV0'.

KNV extended to complex poles is used in Matlab's `place` function, YT is distributed under a non-free licence by Slicot under the name `robpole`. It is unclear and undocumented how KNV0 has been extended to complex poles (Tits and Yang claim on page 14 of their paper that their method can not be used to extend KNV to complex poles), therefore only YT supports them in this implementation.

As the solution to the problem of pole placement is not unique for MIMO systems, both methods start with a tentative transfer matrix which is altered in various way to increase its determinant. Both methods have been proven to converge to a stable solution, however depending on the way the initial transfer matrix is chosen they will converge to different solutions and therefore there is absolutely no guarantee that using 'KNV0' will yield results similar to Matlab's or any other implementation of these algorithms.

Using the default method 'YT' should be fine in most cases; 'KNV0' is only provided because it is needed by 'YT' in some specific cases. Furthermore 'YT' gives on average more robust results than 'KNV0' when  $\text{abs}(\det(X))$  is used as a robustness indicator.

[R227] is available as a technical report on the following URL: <http://drum.lib.umd.edu/handle/1903/5598>

## References

[R226], [R227]

## Examples

A simple example demonstrating real pole placement using both KNV and YT algorithms. This is example number 1 from section 4 of the reference KNV publication ([R226]):

```

>>> from scipy import signal
>>> import matplotlib.pyplot as plt

>>> A = np.array([[ 1.380,  -0.2077,  6.715,  -5.676  ],
...             [-0.5814, -4.290,   0,      0.6750 ],
...             [ 1.067,   4.273, -6.654,  5.893  ],
...             [ 0.0480,  4.273,  1.343, -2.104  ]])
>>> B = np.array([[ 0,    5.679 ],
...             [ 1.136, 1.136 ],
...             [ 0,    0,    ],
...             [-3.146, 0    ]])
>>> P = np.array([-0.2, -0.5, -5.0566, -8.6659])

```

Now compute  $K$  with KNV method 0, with the default YT method and with the YT method while forcing 100 iterations of the algorithm and print some results after each call.

```

>>> fsf1 = signal.place_poles(A, B, P, method='KNV0')
>>> fsf1.gain_matrix
array([[ 0.20071427, -0.96665799,  0.24066128, -0.10279785],
       [ 0.50587268,  0.57779091,  0.51795763, -0.41991442]])

>>> fsf2 = signal.place_poles(A, B, P) # uses YT method
>>> fsf2.computed_poles
array([-8.6659, -5.0566, -0.5    , -0.2    ])

>>> fsf3 = signal.place_poles(A, B, P, rtol=-1, maxiter=100)
>>> fsf3.X
array([[ 0.52072442+0.j, -0.08409372+0.j, -0.56847937+0.j,  0.74823657+0.j],
       [-0.04977751+0.j, -0.80872954+0.j,  0.13566234+0.j, -0.29322906+0.j],
       [-0.82266932+0.j, -0.19168026+0.j, -0.56348322+0.j, -0.43815060+0.j],
       [ 0.22267347+0.j,  0.54967577+0.j, -0.58387806+0.j, -0.40271926+0.j]])

```

The absolute value of the determinant of  $X$  is a good indicator to check the robustness of the results, both 'KNV0' and 'YT' aim at maximizing it. Below a comparison of the robustness of the results above:

```

>>> abs(np.linalg.det(fsfl.X)) < abs(np.linalg.det(fsf2.X))
True
>>> abs(np.linalg.det(fsf2.X)) < abs(np.linalg.det(fsf3.X))
True

```

Now a simple example for complex poles:

```

>>> A = np.array([[ 0,  7/3.,  0,  0  ],
...             [ 0,  0,  0,  7/9. ],
...             [ 0,  0,  0,  0  ],
...             [ 0,  0,  0,  0  ]])
>>> B = np.array([[ 0,  0 ]],

```

```

...         [ 0,  0 ],
...         [ 1,  0 ],
...         [ 0,  1 ]])
>>> P = np.array([-3, -1, -2-1j, -2+1j]) / 3.
>>> fsf = signal.place_poles(A, B, P, method='YT')

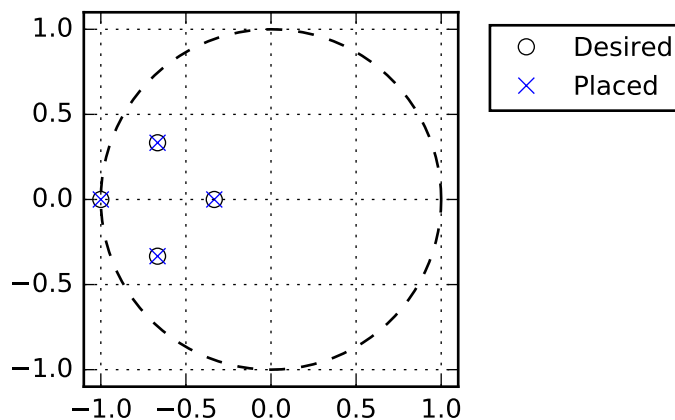
```

We can plot the desired and computed poles in the complex plane:

```

>>> t = np.linspace(0, 2*np.pi, 401)
>>> plt.plot(np.cos(t), np.sin(t), 'k--') # unit circle
>>> plt.plot(fsf.requested_poles.real, fsf.requested_poles.imag,
...         'wo', label='Desired')
>>> plt.plot(fsf.computed_poles.real, fsf.computed_poles.imag, 'bx',
...         label='Placed')
>>> plt.grid()
>>> plt.axis('image')
>>> plt.axis([-1.1, 1.1, -1.1, 1.1])
>>> plt.legend(bbox_to_anchor=(1.05, 1), loc=2, numpoints=1)

```



## 5.20.9 Waveforms

<code>chirp(t, f0, t1, f1[, method, phi, vertex_zero])</code>	Frequency-swept cosine generator.
<code>gausspulse(t[, fc, bw, bwr, tpr, retquad, ...])</code>	Return a Gaussian modulated sinusoid:
<code>max_len_seq(nbbits[, state, length, taps])</code>	Maximum length sequence (MLS) generator.
<code>sawtooth(t[, width])</code>	Return a periodic sawtooth or triangle waveform.
<code>square(t[, duty])</code>	Return a periodic square-wave waveform.
<code>sweep_poly(t, poly[, phi])</code>	Frequency-swept cosine generator, with a time-dependent frequency.

`scipy.signal.chirp(t, f0, t1, f1, method='linear', phi=0, vertex_zero=True)`  
 Frequency-swept cosine generator.

In the following, ‘Hz’ should be interpreted as ‘cycles per unit’; there is no requirement here that the unit is one second. The important distinction is that the units of rotation are cycles, not radians. Likewise,  $t$  could be a measurement of space instead of time.

<b>Parameters</b>	<b>t</b> : array_like	Times at which to evaluate the waveform.	
	<b>f0</b> : float	Frequency (e.g. Hz) at time t=0.	
	<b>t1</b> : float	Time at which <i>f1</i> is specified.	
	<b>f1</b> : float	Frequency (e.g. Hz) of the waveform at time <i>t1</i> .	
	<b>method</b> : { 'linear', 'quadratic', 'logarithmic', 'hyperbolic' }, optional	Kind of frequency sweep. If not given, <i>linear</i> is assumed. See Notes below for more details.	
	<b>phi</b> : float, optional	Phase offset, in degrees. Default is 0.	
	<b>vertex_zero</b> : bool, optional	This parameter is only used when <i>method</i> is 'quadratic'. It determines whether the vertex of the parabola that is the graph of the frequency is at t=0 or t=t1.	
	<b>Returns</b>	<b>y</b> : ndarray	A numpy array containing the signal evaluated at <i>t</i> with the requested time-varying frequency. More precisely, the function returns $\cos(\text{phase} + (\pi/180) * \text{phi})$ where <i>phase</i> is the integral (from 0 to <i>t</i> ) of $2 * \pi * f(t)$ . <i>f(t)</i> is defined below.

**See also:**

[sweep\\_poly](#)

**Notes**

There are four options for the *method*. The following formulas give the instantaneous frequency (in Hz) of the signal generated by *chirp()*. For convenience, the shorter names shown below may also be used.

linear, lin, li:

$$f(t) = f0 + (f1 - f0) * t / t1$$

quadratic, quad, q:

The graph of the frequency *f(t)* is a parabola through (0, *f0*) and (*t1*, *f1*). By default, the vertex of the parabola is at (0, *f0*). If *vertex\_zero* is False, then the vertex is at (*t1*, *f1*). The formula is:

if *vertex\_zero* is True:

$$f(t) = f0 + (f1 - f0) * t**2 / t1**2$$

else:

$$f(t) = f1 - (f1 - f0) * (t1 - t)**2 / t1**2$$

To use a more general quadratic function, or an arbitrary polynomial, use the function `scipy.signal.waveforms.sweep_poly`.

logarithmic, log, lo:

$$f(t) = f0 * (f1/f0)**(t/t1)$$

*f0* and *f1* must be nonzero and have the same sign.

This signal is also known as a geometric or exponential chirp.

hyperbolic, hyp:

$$f(t) = f0*f1*t1 / ((f0 - f1)*t + f1*t1)$$

*f0* and *f1* must be nonzero.

`scipy.signal.gausspulse` (*t, fc=1000, bw=0.5, bwr=-6, tpr=-60, retquad=False, retenv=False*)

Return a Gaussian modulated sinusoid:

$$\exp(-a t^2) \exp(1j*2*\pi*fc*t).$$

If *retquad* is True, then return the real and imaginary parts (in-phase and quadrature). If *retenv* is True, then return the envelope (unmodulated signal). Otherwise, return the real part of the modulated sinusoid.

**Parameters**

- t** : ndarray or the string 'cutoff'  
Input array.
- fc** : int, optional  
Center frequency (e.g. Hz). Default is 1000.
- bw** : float, optional  
Fractional bandwidth in frequency domain of pulse (e.g. Hz). Default is 0.5.
- bwr** : float, optional  
Reference level at which fractional bandwidth is calculated (dB). Default is -6.
- tpr** : float, optional  
If *t* is 'cutoff', then the function returns the cutoff time for when the pulse amplitude falls below *tpr* (in dB). Default is -60.
- retquad** : bool, optional  
If True, return the quadrature (imaginary) as well as the real part of the signal. Default is False.
- retenv** : bool, optional  
If True, return the envelope of the signal. Default is False.

**Returns**

- yI** : ndarray  
Real part of signal. Always returned.
- yQ** : ndarray  
Imaginary part of signal. Only returned if *retquad* is True.
- yenv** : ndarray  
Envelope of signal. Only returned if *retenv* is True.

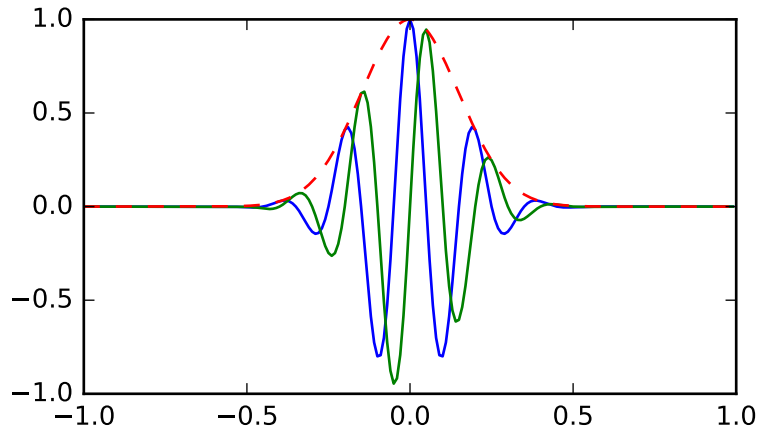
**See also:**

`scipy.signal.morlet`

### Examples

Plot real component, imaginary component, and envelope for a 5 Hz pulse, sampled at 100 Hz for 2 seconds:

```
>>> from scipy import signal
>>> import matplotlib.pyplot as plt
>>> t = np.linspace(-1, 1, 2 * 100, endpoint=False)
>>> i, q, e = signal.gausspulse(t, fc=5, retquad=True, retenv=True)
>>> plt.plot(t, i, t, q, t, e, '--')
```



`scipy.signal.max_len_seq` (*nbits*, *state=None*, *length=None*, *taps=None*)

Maximum length sequence (MLS) generator.

**Parameters** `nbits` : int

Number of bits to use. Length of the resulting sequence will be  $(2^{**nbits}) - 1$ . Note that generating long sequences (e.g., greater than `nbits == 16`) can take a long time.

`state` : array\_like, optional

If array, must be of length `nbits`, and will be cast to binary (bool) representation. If `None`, a seed of ones will be used, producing a repeatable representation. If `state` is all zeros, an error is raised as this is invalid. Default: `None`.

`length` : int, optional

Number of samples to compute. If `None`, the entire length  $(2^{**nbits}) - 1$  is computed.

`taps` : array\_like, optional

Polynomial taps to use (e.g., `[7, 6, 1]` for an 8-bit sequence). If `None`, taps will be automatically selected (for up to `nbits == 32`).

**Returns**

`seq` : array

Resulting MLS sequence of 0's and 1's.

`state` : array

The final state of the shift register.

### Notes

The algorithm for MLS generation is generically described in:

[https://en.wikipedia.org/wiki/Maximum\\_length\\_sequence](https://en.wikipedia.org/wiki/Maximum_length_sequence)

The default values for `taps` are specifically taken from the first option listed for each value of `nbits` in:

[http://www.newwaveinstruments.com/resources/articles/m\\_sequence\\_linear\\_feedback\\_shift\\_register\\_lfsr.htm](http://www.newwaveinstruments.com/resources/articles/m_sequence_linear_feedback_shift_register_lfsr.htm)

New in version 0.15.0.

### Examples

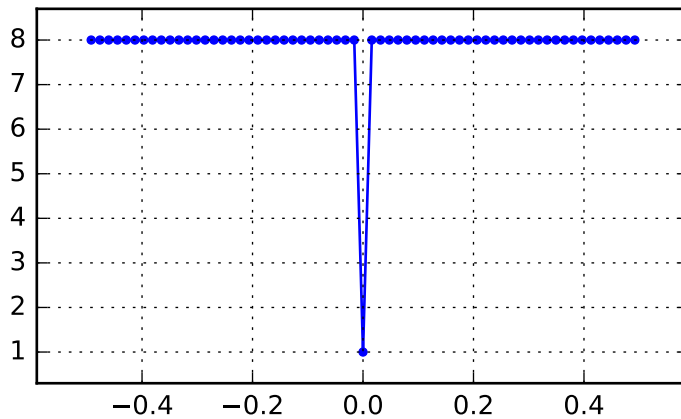
MLS uses binary convention:



```
>>> from scipy.signal import max_len_seq
>>> max_len_seq(4)[0]
array([1, 1, 1, 1, 0, 1, 0, 1, 1, 0, 0, 1, 0, 0, 0], dtype=int8)
```

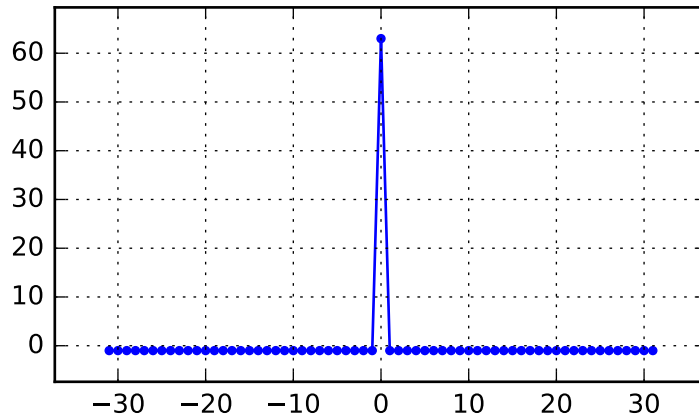
MLS has a white spectrum (except for DC):

```
>>> import matplotlib.pyplot as plt
>>> from numpy.fft import fft, ifft, fftshift, fftfreq
>>> seq = max_len_seq(6)[0]*2-1 # +1 and -1
>>> spec = fft(seq)
>>> N = len(seq)
>>> plt.plot(fftshift(fftfreq(N)), fftshift(np.abs(spec)), '.-')
>>> plt.margins(0.1, 0.1)
>>> plt.grid(True)
>>> plt.show()
```



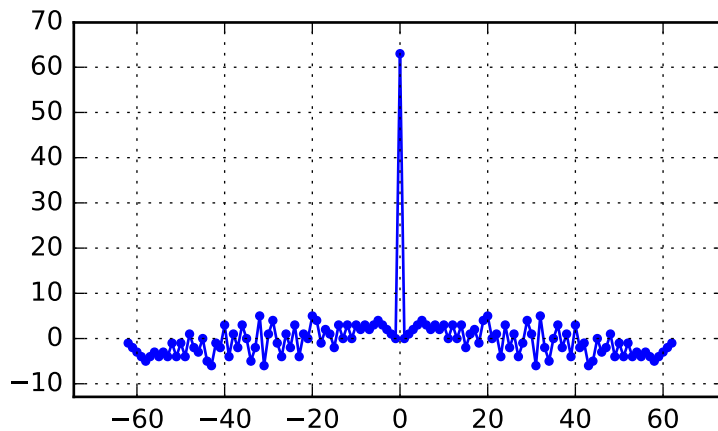
Circular autocorrelation of MLS is an impulse:

```
>>> acorrirc = ifft(spec * np.conj(spec)).real
>>> plt.figure()
>>> plt.plot(np.arange(-N/2+1, N/2+1), fftshift(acorrirc), '.-')
>>> plt.margins(0.1, 0.1)
>>> plt.grid(True)
>>> plt.show()
```



Linear autocorrelation of MLS is approximately an impulse:

```
>>> acorr = np.correlate(seq, seq, 'full')
>>> plt.figure()
>>> plt.plot(np.arange(-N+1, N), acorr, '-.-')
>>> plt.margins(0.1, 0.1)
>>> plt.grid(True)
>>> plt.show()
```



`scipy.signal.sawtooth(t, width=1)`

Return a periodic sawtooth or triangle waveform.

The sawtooth waveform has a period  $2\pi$ , rises from -1 to 1 on the interval 0 to  $\text{width} \cdot 2\pi$ , then drops from 1 to -1 on the interval  $\text{width} \cdot 2\pi$  to  $2\pi$ . *width* must be in the interval [0, 1].

Note that this is not band-limited. It produces an infinite number of harmonics, which are aliased back and forth across the frequency spectrum.

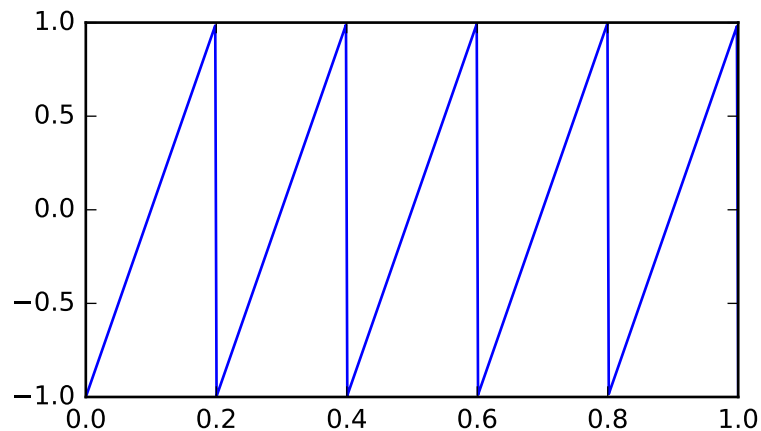
**Parameters** `t`: array\_like

Time.  
**width** : array\_like, optional  
 Width of the rising ramp as a proportion of the total cycle. Default is 1, producing a rising ramp, while 0 produces a falling ramp. *width* = 0.5 produces a triangle wave. If an array, causes wave shape to change over time, and must be the same length as *t*.  
**Returns** **y** : ndarray  
 Output array containing the sawtooth waveform.

### Examples

A 5 Hz waveform sampled at 500 Hz for 1 second:

```
>>> from scipy import signal
>>> import matplotlib.pyplot as plt
>>> t = np.linspace(0, 1, 500)
>>> plt.plot(t, signal.sawtooth(2 * np.pi * 5 * t))
```



`scipy.signal.square(t, duty=0.5)`

Return a periodic square-wave waveform.

The square wave has a period  $2\pi$ , has value +1 from 0 to  $2\pi \cdot \text{duty}$  and -1 from  $2\pi \cdot \text{duty}$  to  $2\pi$ . *duty* must be in the interval [0,1].

Note that this is not band-limited. It produces an infinite number of harmonics, which are aliased back and forth across the frequency spectrum.

**Parameters** **t** : array\_like  
 The input time array.  
**duty** : array\_like, optional  
 Duty cycle. Default is 0.5 (50% duty cycle). If an array, causes wave shape to change over time, and must be the same length as *t*.  
**Returns** **y** : ndarray  
 Output array containing the square waveform.

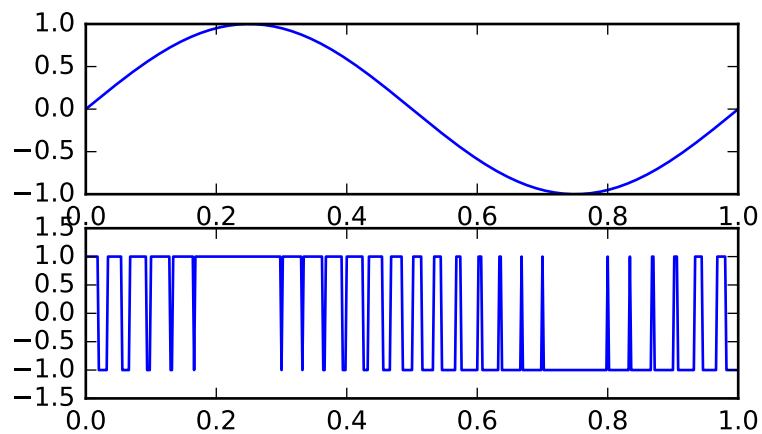
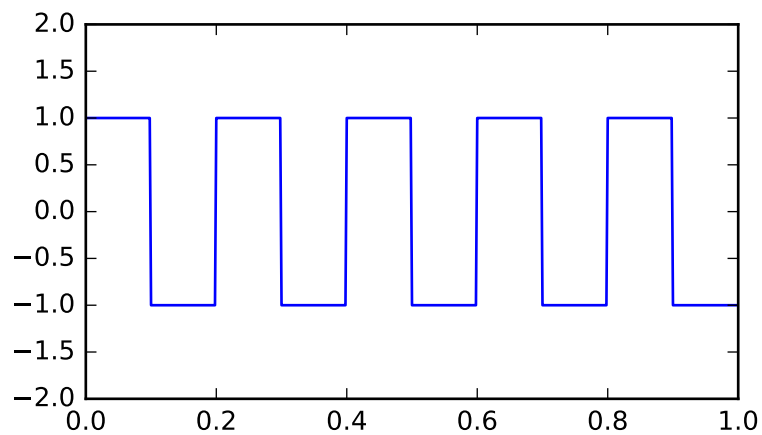
### Examples

A 5 Hz waveform sampled at 500 Hz for 1 second:

```
>>> from scipy import signal
>>> import matplotlib.pyplot as plt
>>> t = np.linspace(0, 1, 500, endpoint=False)
>>> plt.plot(t, signal.square(2 * np.pi * 5 * t))
>>> plt.ylim(-2, 2)
```

A pulse-width modulated sine wave:

```
>>> plt.figure()
>>> sig = np.sin(2 * np.pi * t)
>>> pwm = signal.square(2 * np.pi * 30 * t, duty=(sig + 1)/2)
>>> plt.subplot(2, 1, 1)
>>> plt.plot(t, sig)
>>> plt.subplot(2, 1, 2)
>>> plt.plot(t, pwm)
>>> plt.ylim(-1.5, 1.5)
```



`scipy.signal.sweep_poly(t, poly, phi=0)`

Frequency-swept cosine generator, with a time-dependent frequency.

This function generates a sinusoidal function whose instantaneous frequency varies with time. The frequency at time  $t$  is given by the polynomial  $poly$ .

**Parameters** `t` : ndarray

Times at which to evaluate the waveform.

**poly** : 1-D array\_like or instance of `numpy.poly1d`

The desired frequency expressed as a polynomial. If  $poly$  is a list or ndarray of length  $n$ , then the elements of  $poly$  are the coefficients of the polynomial, and the instantaneous frequency is

$$f(t) = poly[0]*t^{n-1} + poly[1]*t^{n-2} + \dots + poly[n-1]$$

If  $poly$  is an instance of `numpy.poly1d`, then the instantaneous frequency is

$$f(t) = poly(t)$$

**phi** : float, optional

Phase offset, in degrees, Default: 0.

**Returns**

**sweep\_poly** : ndarray

A numpy array containing the signal evaluated at  $t$  with the requested time-varying frequency. More precisely, the function returns  $\cos(\text{phase} + (\pi/180)*\text{phi})$ , where  $phase$  is the integral (from 0 to  $t$ ) of  $2 * \pi * f(t)$ ;  $f(t)$  is defined above.

**See also:**

[chirp](#)

**Notes**

New in version 0.8.0.

If  $poly$  is a list or ndarray of length  $n$ , then the elements of  $poly$  are the coefficients of the polynomial, and the instantaneous frequency is:

$$f(t) = poly[0]*t^{n-1} + poly[1]*t^{n-2} + \dots + poly[n-1]$$

If  $poly$  is an instance of `numpy.poly1d`, then the instantaneous frequency is:

$$f(t) = poly(t)$$

Finally, the output  $s$  is:

$$\cos(\text{phase} + (\pi/180)*\text{phi})$$

where  $phase$  is the integral from 0 to  $t$  of  $2 * \pi * f(t)$ ,  $f(t)$  as defined above.

## 5.20.10 Window functions

<code>get_window(window, Nx[, fftbins])</code>	Return a window.
<code>barthann(M[, sym])</code>	Return a modified Bartlett-Hann window.
<code>bartlett(M[, sym])</code>	Return a Bartlett window.
<code>blackman(M[, sym])</code>	Return a Blackman window.
<code>blackmanharris(M[, sym])</code>	Return a minimum 4-term Blackman-Harris window.
<code>bohman(M[, sym])</code>	Return a Bohman window.
<code>boxcar(M[, sym])</code>	Return a boxcar or rectangular window.
<code>chebwin(M, at[, sym])</code>	Return a Dolph-Chebyshev window.
<code>cosine(M[, sym])</code>	Return a window with a simple cosine shape.

Continued on next page

Table 5.149 – continued from previous page

<code>exponential(M[, center, tau, sym])</code>	Return an exponential (or Poisson) window.
<code>flattop(M[, sym])</code>	Return a flat top window.
<code>gaussian(M, std[, sym])</code>	Return a Gaussian window.
<code>general_gaussian(M, p, sig[, sym])</code>	Return a window with a generalized Gaussian shape.
<code>hamming(M[, sym])</code>	Return a Hamming window.
<code>hann(M[, sym])</code>	Return a Hann window.
<code>hanning(M[, sym])</code>	Return a Hann window.
<code>kaiser(M, beta[, sym])</code>	Return a Kaiser window.
<code>nuttall(M[, sym])</code>	Return a minimum 4-term Blackman-Harris window according to Nuttall.
<code>parzen(M[, sym])</code>	Return a Parzen window.
<code>slepian(M, width[, sym])</code>	Return a digital Slepian (DPSS) window.
<code>triang(M[, sym])</code>	Return a triangular window.
<code>tukey(M[, alpha, sym])</code>	Return a Tukey window, also known as a tapered cosine window.

`scipy.signal.get_window(window, Nx, fftbins=True)`

Return a window.

**Parameters**

- window** : string, float, or tuple  
The type of window to create. See below for more details.
- Nx** : int  
The number of samples in the window.
- fftbins** : bool, optional  
If True (default), create a “periodic” window, ready to use with `fftshift` and be multiplied by the result of an FFT (see also `fftpack.fftfreq`). If False, create a “symmetric” window, for use in filter design.

**Returns**

- get\_window** : ndarray  
Returns a window of length `Nx` and type `window`

### Notes

Window types:

`boxcar`, `triang`, `blackman`, `hamming`, `hann`, `bartlett`, `flattop`, `parzen`, `bohman`, `blackmanharris`, `nuttall`, `barthann`, `kaiser` (needs `beta`), `gaussian` (needs standard deviation), `general_gaussian` (needs power, width), `slepian` (needs width), `chebwin` (needs attenuation), `exponential` (needs decay scale), `tukey` (needs taper fraction)

If the window requires no parameters, then `window` can be a string.

If the window requires parameters, then `window` must be a tuple with the first argument the string name of the window, and the next arguments the needed parameters.

If `window` is a floating point number, it is interpreted as the `beta` parameter of the `kaiser` window.

Each of the window types listed above is also the name of a function that can be called directly to create a window of that type.

### Examples

```
>>> from scipy import signal
>>> signal.get_window('triang', 7)
array([ 0.25,  0.5 ,  0.75,  1.   ,  0.75,  0.5 ,  0.25])
>>> signal.get_window(('kaiser', 4.0), 9)
array([ 0.08848053,  0.32578323,  0.63343178,  0.89640418,  1.         ,
        0.89640418,  0.63343178,  0.32578323,  0.08848053])
>>> signal.get_window(4.0, 9)
```

```
array([ 0.08848053,  0.32578323,  0.63343178,  0.89640418,  1.          ,
        0.89640418,  0.63343178,  0.32578323,  0.08848053])
```

`scipy.signal.barthann` ( $M$ ,  $sym=True$ )

Return a modified Bartlett-Hann window.

**Parameters**     $M$  : int

Number of points in the output window. If zero or less, an empty array is returned.

$sym$  : bool, optional

When True (default), generates a symmetric window, for use in filter design. When False, generates a periodic window, for use in spectral analysis.

**Returns**         $w$  : ndarray

The window, with the maximum value normalized to 1 (though the value 1 does not appear if  $M$  is even and  $sym$  is True).

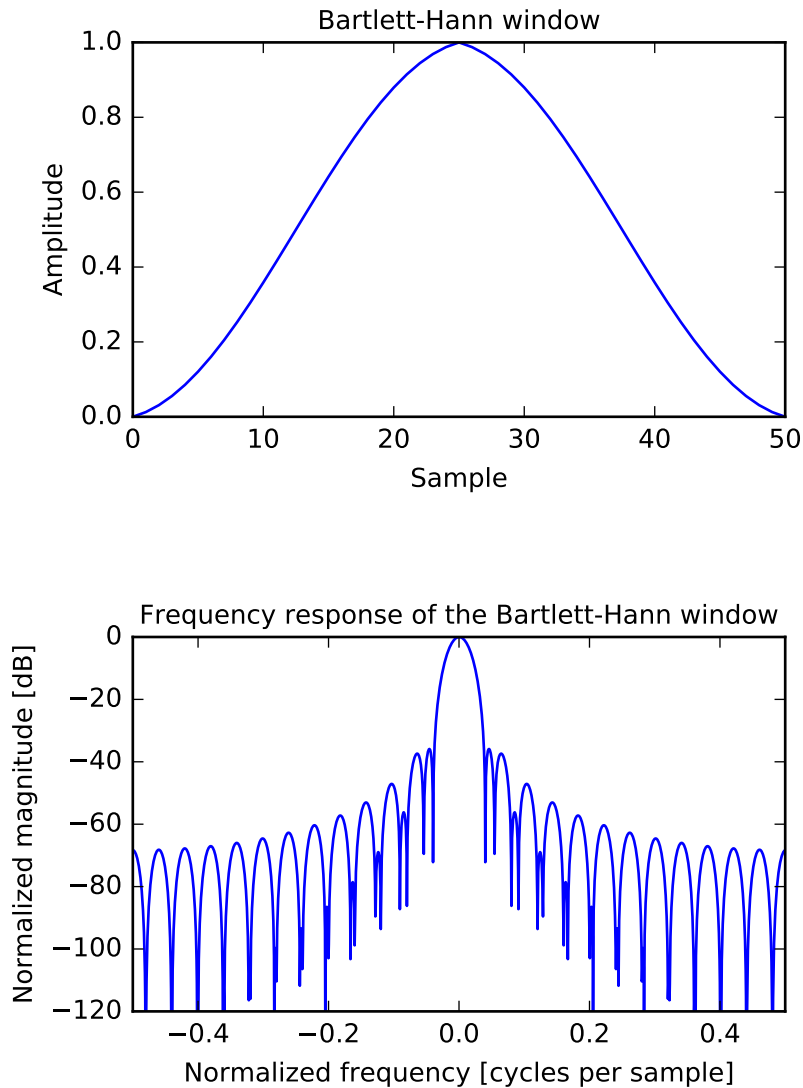
### Examples

Plot the window and its frequency response:

```
>>> from scipy import signal
>>> from scipy.fftpack import fft, fftshift
>>> import matplotlib.pyplot as plt

>>> window = signal.barthann(51)
>>> plt.plot(window)
>>> plt.title("Bartlett-Hann window")
>>> plt.ylabel("Amplitude")
>>> plt.xlabel("Sample")

>>> plt.figure()
>>> A = fft(window, 2048) / (len(window)/2.0)
>>> freq = np.linspace(-0.5, 0.5, len(A))
>>> response = 20 * np.log10(np.abs(fftshift(A / abs(A).max())))
>>> plt.plot(freq, response)
>>> plt.axis([-0.5, 0.5, -120, 0])
>>> plt.title("Frequency response of the Bartlett-Hann window")
>>> plt.ylabel("Normalized magnitude [dB]")
>>> plt.xlabel("Normalized frequency [cycles per sample]")
```



`scipy.signal.bartlett` ( $M$ ,  $sym=True$ )

Return a Bartlett window.

The Bartlett window is very similar to a triangular window, except that the end points are at zero. It is often used in signal processing for tapering a signal, without generating too much ripple in the frequency domain.

**Parameters**  $M$  : int

Number of points in the output window. If zero or less, an empty array is returned.

$sym$  : bool, optional

When True (default), generates a symmetric window, for use in filter design. When False, generates a periodic window, for use in spectral analysis.

**Returns**  $w$  : ndarray

The triangular window, with the first and last samples equal to zero and the maximum value normalized to 1 (though the value 1 does not appear if  $M$  is even and  $sym$  is True).



### Notes

The Bartlett window is defined as

$$w(n) = \frac{2}{M-1} \left( \frac{M-1}{2} - \left| n - \frac{M-1}{2} \right| \right)$$

Most references to the Bartlett window come from the signal processing literature, where it is used as one of many windowing functions for smoothing values. Note that convolution with this window produces linear interpolation. It is also known as an apodization (which means "removing the foot", i.e. smoothing discontinuities at the beginning and end of the sampled signal) or tapering function. The Fourier transform of the Bartlett is the product of two sinc functions. Note the excellent discussion in Kanasewich.

### References

[R174], [R175], [R176], [R177], [R178]

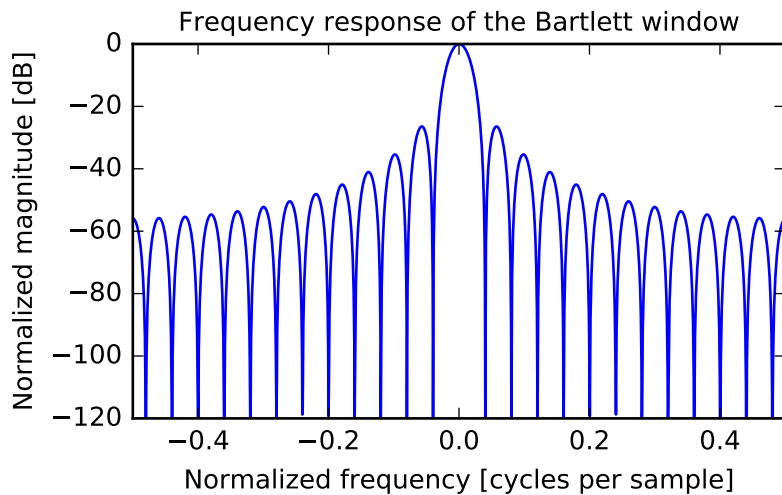
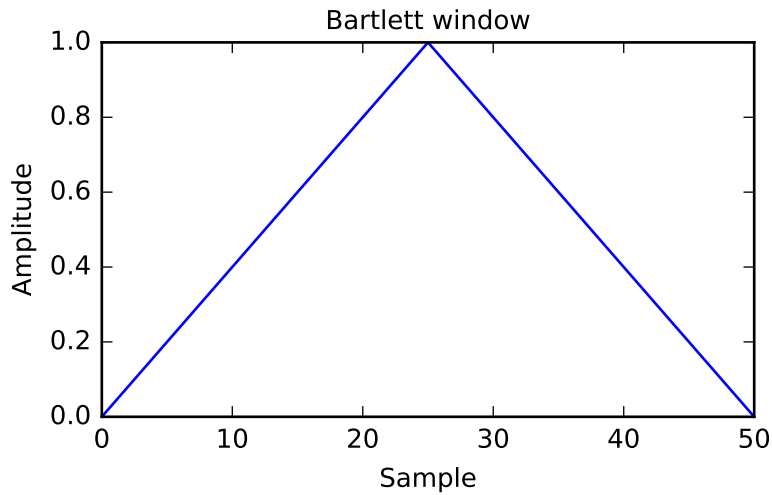
### Examples

Plot the window and its frequency response:

```
>>> from scipy import signal
>>> from scipy.fftpack import fft, fftshift
>>> import matplotlib.pyplot as plt

>>> window = signal.bartlett(51)
>>> plt.plot(window)
>>> plt.title("Bartlett window")
>>> plt.ylabel("Amplitude")
>>> plt.xlabel("Sample")

>>> plt.figure()
>>> A = fft(window, 2048) / (len(window)/2.0)
>>> freq = np.linspace(-0.5, 0.5, len(A))
>>> response = 20 * np.log10(np.abs(fftshift(A / abs(A).max())))
>>> plt.plot(freq, response)
>>> plt.axis([-0.5, 0.5, -120, 0])
>>> plt.title("Frequency response of the Bartlett window")
>>> plt.ylabel("Normalized magnitude [dB]")
>>> plt.xlabel("Normalized frequency [cycles per sample]")
```



`scipy.signal.blackman` ( $M$ ,  $sym=True$ )

Return a Blackman window.

The Blackman window is a taper formed by using the first three terms of a summation of cosines. It was designed to have close to the minimal leakage possible. It is close to optimal, only slightly worse than a Kaiser window.

**Parameters**     $M$  : int

Number of points in the output window. If zero or less, an empty array is returned.

$sym$  : bool, optional

When True (default), generates a symmetric window, for use in filter design. When False, generates a periodic window, for use in spectral analysis.

**Returns**         $w$  : ndarray

The window, with the maximum value normalized to 1 (though the value 1 does not appear if  $M$  is even and  $sym$  is True).

### Notes

The Blackman window is defined as

$$w(n) = 0.42 - 0.5 \cos(2\pi n/M) + 0.08 \cos(4\pi n/M)$$

Most references to the Blackman window come from the signal processing literature, where it is used as one of many windowing functions for smoothing values. It is also known as an apodization (which means “removing the foot”, i.e. smoothing discontinuities at the beginning and end of the sampled signal) or tapering function. It is known as a “near optimal” tapering function, almost as good (by some measures) as the Kaiser window.

### References

[R186], [R187]

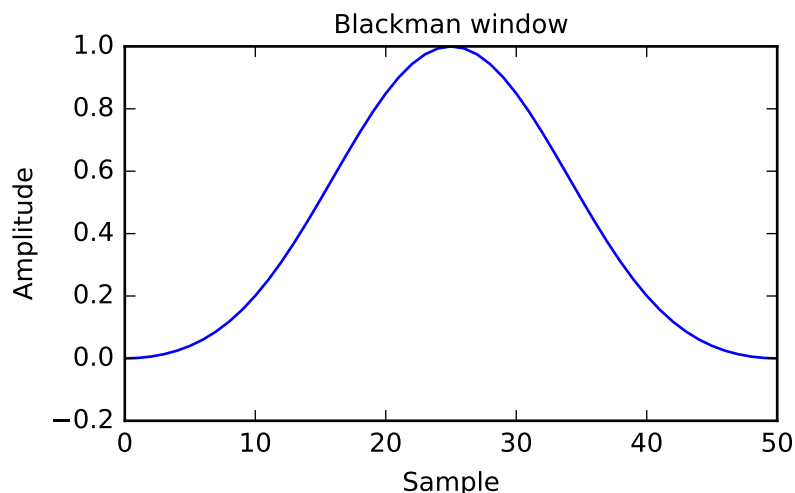
### Examples

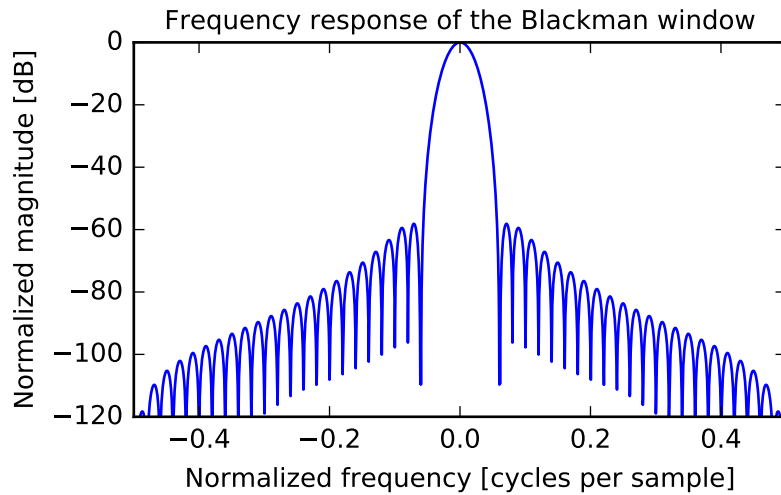
Plot the window and its frequency response:

```
>>> from scipy import signal
>>> from scipy.fftpack import fft, fftshift
>>> import matplotlib.pyplot as plt

>>> window = signal.blackman(51)
>>> plt.plot(window)
>>> plt.title("Blackman window")
>>> plt.ylabel("Amplitude")
>>> plt.xlabel("Sample")

>>> plt.figure()
>>> A = fft(window, 2048) / (len(window)/2.0)
>>> freq = np.linspace(-0.5, 0.5, len(A))
>>> response = 20 * np.log10(np.abs(fftshift(A / abs(A).max())))
>>> plt.plot(freq, response)
>>> plt.axis([-0.5, 0.5, -120, 0])
>>> plt.title("Frequency response of the Blackman window")
>>> plt.ylabel("Normalized magnitude [dB]")
>>> plt.xlabel("Normalized frequency [cycles per sample]")
```





`scipy.signal.blackmanharris` ( $M$ ,  $sym=True$ )  
Return a minimum 4-term Blackman-Harris window.

**Parameters**

- M** : int  
Number of points in the output window. If zero or less, an empty array is returned.
- sym** : bool, optional  
When True (default), generates a symmetric window, for use in filter design.  
When False, generates a periodic window, for use in spectral analysis.

**Returns**

- w** : ndarray  
The window, with the maximum value normalized to 1 (though the value 1 does not appear if  $M$  is even and  $sym$  is True).

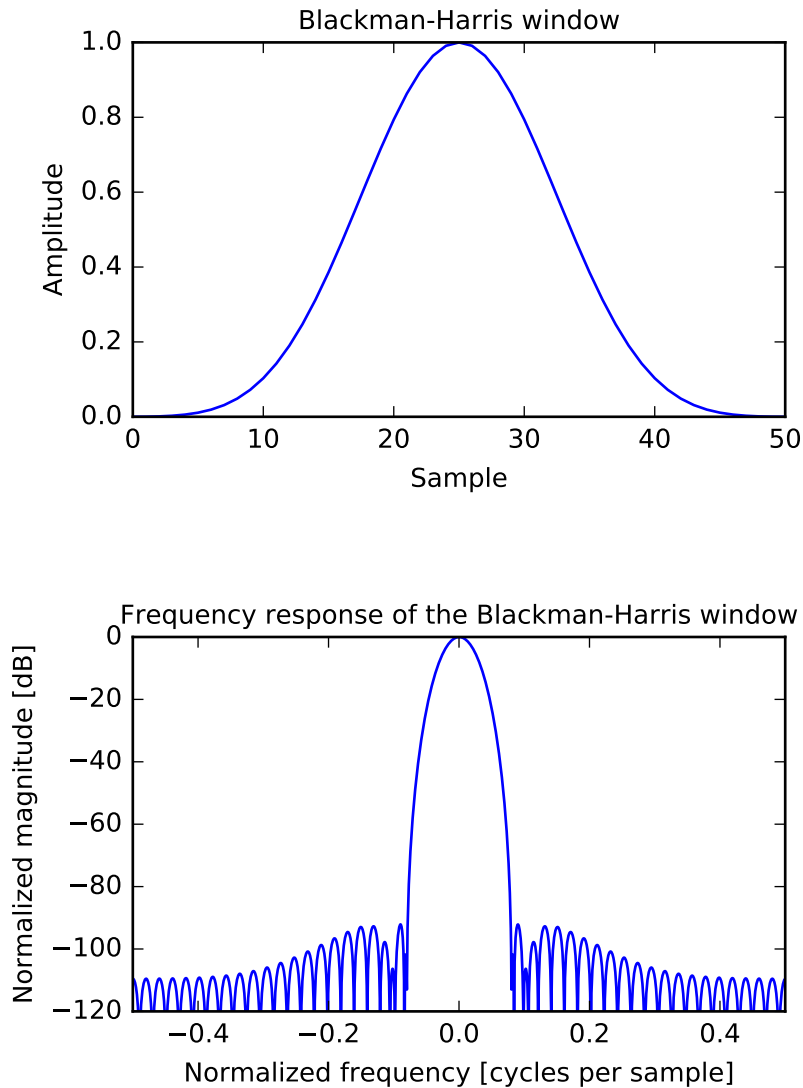
### Examples

Plot the window and its frequency response:

```
>>> from scipy import signal
>>> from scipy.fftpack import fft, fftshift
>>> import matplotlib.pyplot as plt

>>> window = signal.blackmanharris(51)
>>> plt.plot(window)
>>> plt.title("Blackman-Harris window")
>>> plt.ylabel("Amplitude")
>>> plt.xlabel("Sample")

>>> plt.figure()
>>> A = fft(window, 2048) / (len(window)/2.0)
>>> freq = np.linspace(-0.5, 0.5, len(A))
>>> response = 20 * np.log10(np.abs(fftshift(A / abs(A).max())))
>>> plt.plot(freq, response)
>>> plt.axis([-0.5, 0.5, -120, 0])
>>> plt.title("Frequency response of the Blackman-Harris window")
>>> plt.ylabel("Normalized magnitude [dB]")
>>> plt.xlabel("Normalized frequency [cycles per sample]")
```



`scipy.signal.bohman` ( $M$ ,  $sym=True$ )

Return a Bohman window.

**Parameters**  $M$  : int

Number of points in the output window. If zero or less, an empty array is returned.

$sym$  : bool, optional

When True (default), generates a symmetric window, for use in filter design. When False, generates a periodic window, for use in spectral analysis.

**Returns**  $w$  : ndarray

The window, with the maximum value normalized to 1 (though the value 1 does not appear if  $M$  is even and  $sym$  is True).

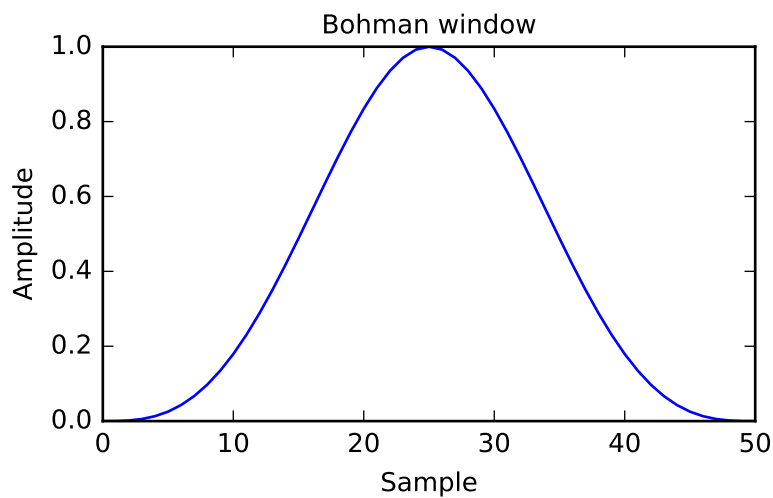
### Examples

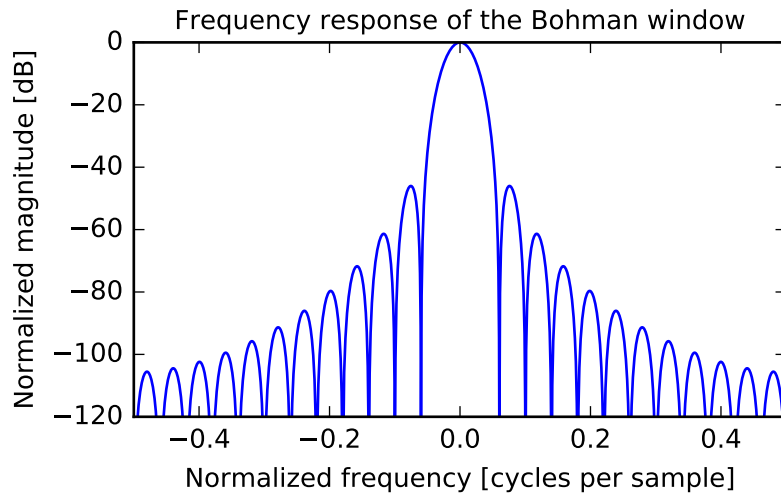
Plot the window and its frequency response:

```
>>> from scipy import signal
>>> from scipy.fftpack import fft, fftshift
>>> import matplotlib.pyplot as plt

>>> window = signal.bohman(51)
>>> plt.plot(window)
>>> plt.title("Bohman window")
>>> plt.ylabel("Amplitude")
>>> plt.xlabel("Sample")

>>> plt.figure()
>>> A = fft(window, 2048) / (len(window)/2.0)
>>> freq = np.linspace(-0.5, 0.5, len(A))
>>> response = 20 * np.log10(np.abs(fftshift(A / abs(A).max())))
>>> plt.plot(freq, response)
>>> plt.axis([-0.5, 0.5, -120, 0])
>>> plt.title("Frequency response of the Bohman window")
>>> plt.ylabel("Normalized magnitude [dB]")
>>> plt.xlabel("Normalized frequency [cycles per sample]")
```





`scipy.signal.boxcar` ( $M$ ,  $\text{sym}=\text{True}$ )

Return a boxcar or rectangular window.

Included for completeness, this is equivalent to no window at all.

**Parameters**     $M$  : int

Number of points in the output window. If zero or less, an empty array is returned.

$\text{sym}$  : bool, optional

Whether the window is symmetric. (Has no effect for boxcar.)

**Returns**         $w$  : ndarray

The window, with the maximum value normalized to 1.

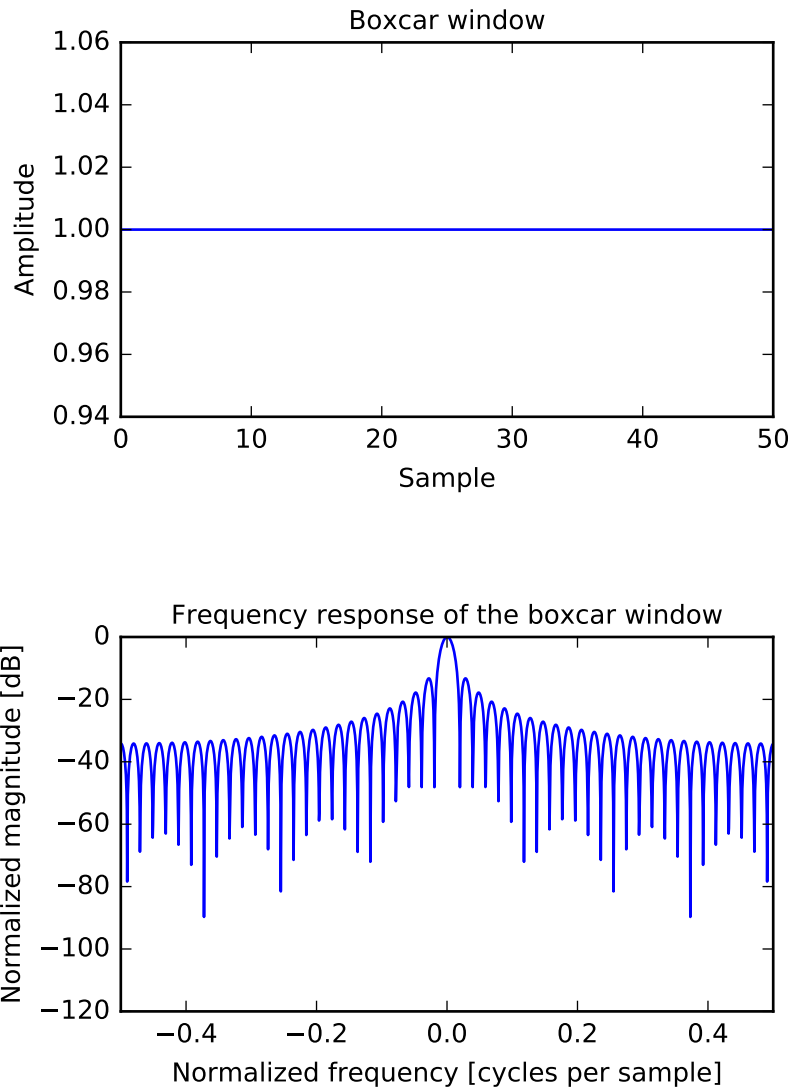
### Examples

Plot the window and its frequency response:

```
>>> from scipy import signal
>>> from scipy.fftpack import fft, fftshift
>>> import matplotlib.pyplot as plt

>>> window = signal.boxcar(51)
>>> plt.plot(window)
>>> plt.title("Boxcar window")
>>> plt.ylabel("Amplitude")
>>> plt.xlabel("Sample")

>>> plt.figure()
>>> A = fft(window, 2048) / (len(window)/2.0)
>>> freq = np.linspace(-0.5, 0.5, len(A))
>>> response = 20 * np.log10(np.abs(fftshift(A / abs(A).max())))
>>> plt.plot(freq, response)
>>> plt.axis([-0.5, 0.5, -120, 0])
>>> plt.title("Frequency response of the boxcar window")
>>> plt.ylabel("Normalized magnitude [dB]")
>>> plt.xlabel("Normalized frequency [cycles per sample])"
```



`scipy.signal.chebwin` ( $M$ ,  $at$ ,  $sym=True$ )

Return a Dolph-Chebyshev window.

**Parameters**

- M** : int  
Number of points in the output window. If zero or less, an empty array is returned.
- at** : float  
Attenuation (in dB).
- sym** : bool, optional  
When True (default), generates a symmetric window, for use in filter design.  
When False, generates a periodic window, for use in spectral analysis.

**Returns**

- w** : ndarray  
The window, with the maximum value always normalized to 1

**Notes**

This window optimizes for the narrowest main lobe width for a given order  $M$  and sidelobe equiripple attenuation  $at$ , using Chebyshev polynomials. It was originally developed by Dolph to optimize the directionality of radio



antenna arrays.

Unlike most windows, the Dolph-Chebyshev is defined in terms of its frequency response:

$$W(k) = \frac{\cos\{M \cos^{-1}[\beta \cos(\frac{\pi k}{M})]\}}{\cosh[M \cosh^{-1}(\beta)]}$$

where

$$\beta = \cosh \left[ \frac{1}{M} \cosh^{-1}(10^{\frac{A}{20}}) \right]$$

and  $0 \leq \text{abs}(k) \leq M-1$ .  $A$  is the attenuation in decibels ( $at$ ).

The time domain window is then generated using the IFFT, so power-of-two  $M$  are the fastest to generate, and prime number  $M$  are the slowest.

The equiripple condition in the frequency domain creates impulses in the time domain, which appear at the ends of the window.

### References

[R188], [R189], [R190]

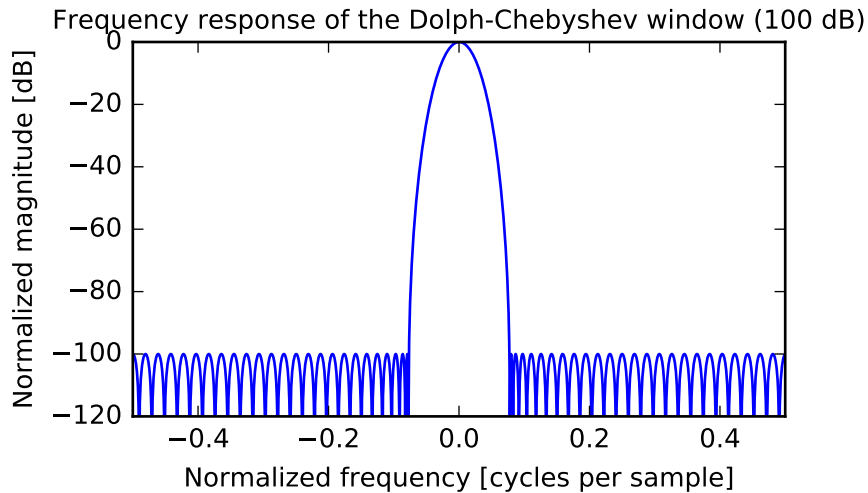
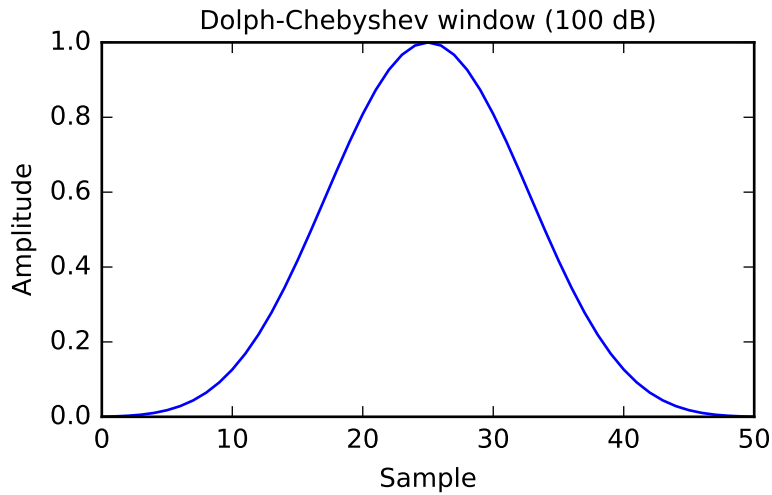
### Examples

Plot the window and its frequency response:

```
>>> from scipy import signal
>>> from scipy.fftpack import fft, fftshift
>>> import matplotlib.pyplot as plt

>>> window = signal.chebwin(51, at=100)
>>> plt.plot(window)
>>> plt.title("Dolph-Chebyshev window (100 dB)")
>>> plt.ylabel("Amplitude")
>>> plt.xlabel("Sample")

>>> plt.figure()
>>> A = fft(window, 2048) / (len(window)/2.0)
>>> freq = np.linspace(-0.5, 0.5, len(A))
>>> response = 20 * np.log10(np.abs(fftshift(A / abs(A).max())))
>>> plt.plot(freq, response)
>>> plt.axis([-0.5, 0.5, -120, 0])
>>> plt.title("Frequency response of the Dolph-Chebyshev window (100 dB)")
>>> plt.ylabel("Normalized magnitude [dB]")
>>> plt.xlabel("Normalized frequency [cycles per sample]")
```



`scipy.signal.cosine` ( $M$ ,  $sym=True$ )

Return a window with a simple cosine shape.

**Parameters**     $M$  : int

Number of points in the output window. If zero or less, an empty array is returned.

$sym$  : bool, optional

When True (default), generates a symmetric window, for use in filter design. When False, generates a periodic window, for use in spectral analysis.

**Returns**         $w$  : ndarray

The window, with the maximum value normalized to 1 (though the value 1 does not appear if  $M$  is even and  $sym$  is True).

**Notes**

New in version 0.13.0.

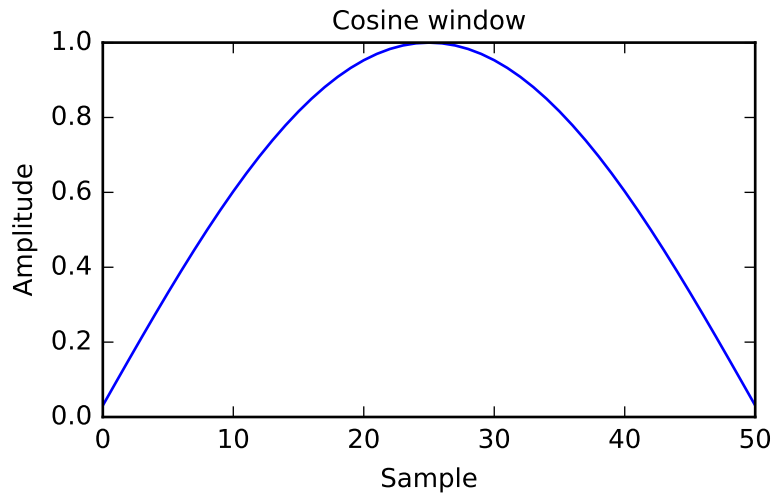
### Examples

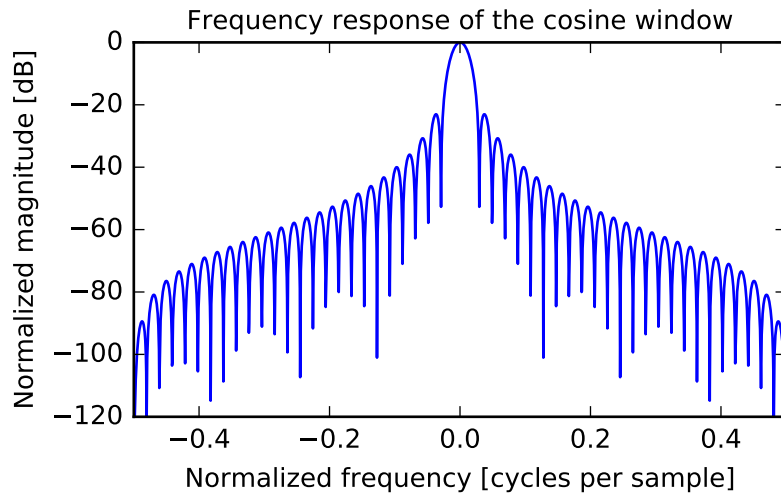
Plot the window and its frequency response:

```
>>> from scipy import signal
>>> from scipy.fftpack import fft, fftshift
>>> import matplotlib.pyplot as plt

>>> window = signal.cosine(51)
>>> plt.plot(window)
>>> plt.title("Cosine window")
>>> plt.ylabel("Amplitude")
>>> plt.xlabel("Sample")

>>> plt.figure()
>>> A = fft(window, 2048) / (len(window)/2.0)
>>> freq = np.linspace(-0.5, 0.5, len(A))
>>> response = 20 * np.log10(np.abs(fftshift(A / abs(A).max())))
>>> plt.plot(freq, response)
>>> plt.axis([-0.5, 0.5, -120, 0])
>>> plt.title("Frequency response of the cosine window")
>>> plt.ylabel("Normalized magnitude [dB]")
>>> plt.xlabel("Normalized frequency [cycles per sample]")
>>> plt.show()
```





`scipy.signal.exponential` (*M*, *center=None*, *tau=1.0*, *sym=True*)

Return an exponential (or Poisson) window.

**Parameters** *M* : int

Number of points in the output window. If zero or less, an empty array is returned.

**center** : float, optional

Parameter defining the center location of the window function. The default value if not given is  $center = (M-1) / 2$ . This parameter must take its default value for symmetric windows.

**tau** : float, optional

Parameter defining the decay. For  $center = 0$  use  $tau = -(M-1) / \ln(x)$  if  $x$  is the fraction of the window remaining at the end.

**sym** : bool, optional

When True (default), generates a symmetric window, for use in filter design. When False, generates a periodic window, for use in spectral analysis.

**Returns** *w* : ndarray

The window, with the maximum value normalized to 1 (though the value 1 does not appear if *M* is even and *sym* is True).

### Notes

The Exponential window is defined as

$$w(n) = e^{-|n-center|/\tau}$$

### References

S. Gade and H. Herlufsen, "Windows to FFT analysis (Part I)", Technical Review 3, Bruel & Kjaer, 1987.

### Examples

Plot the symmetric window and its frequency response:

```
>>> from scipy import signal
>>> from scipy.fftpack import fft, fftshift
>>> import matplotlib.pyplot as plt
```

```

>>> M = 51
>>> tau = 3.0
>>> window = signal.exponential(M, tau=tau)
>>> plt.plot(window)
>>> plt.title("Exponential Window (tau=3.0)")
>>> plt.ylabel("Amplitude")
>>> plt.xlabel("Sample")

>>> plt.figure()
>>> A = fft(window, 2048) / (len(window)/2.0)
>>> freq = np.linspace(-0.5, 0.5, len(A))
>>> response = 20 * np.log10(np.abs(fftshift(A / abs(A).max())))
>>> plt.plot(freq, response)
>>> plt.axis([-0.5, 0.5, -35, 0])
>>> plt.title("Frequency response of the Exponential window (tau=3.0)")
>>> plt.ylabel("Normalized magnitude [dB]")
>>> plt.xlabel("Normalized frequency [cycles per sample]")

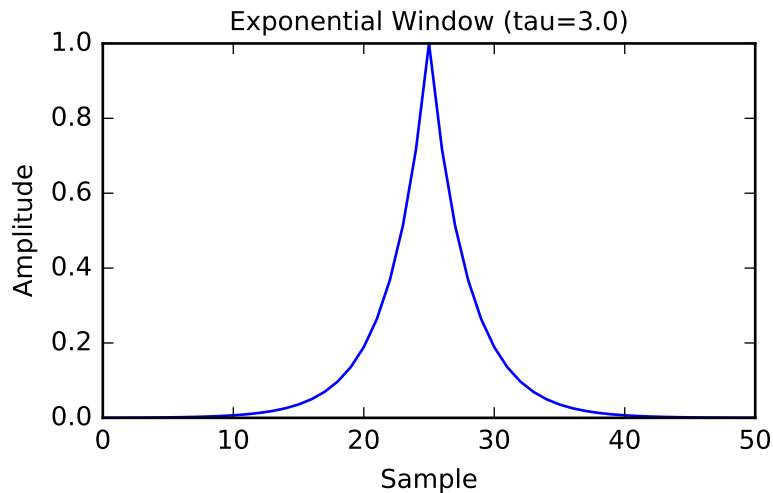
```

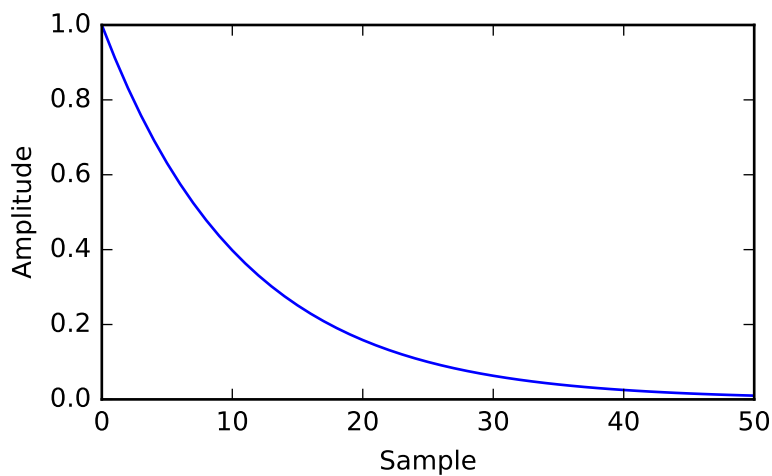
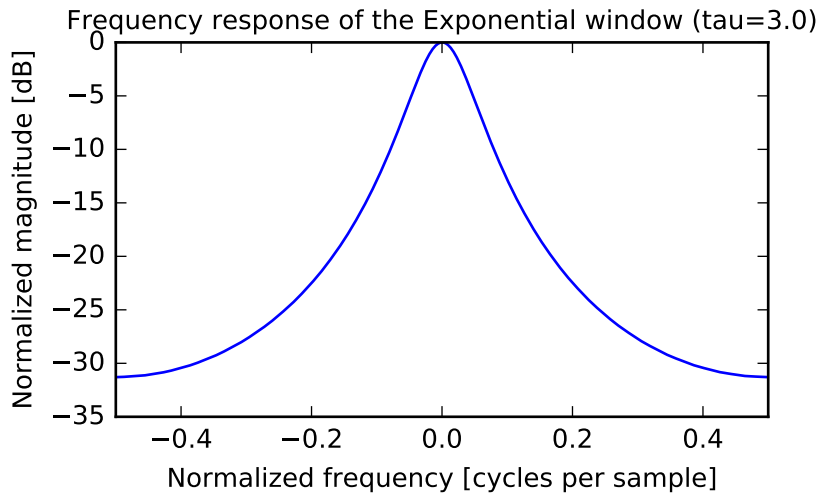
This function can also generate non-symmetric windows:

```

>>> tau2 = -(M-1) / np.log(0.01)
>>> window2 = signal.exponential(M, 0, tau2, False)
>>> plt.figure()
>>> plt.plot(window2)
>>> plt.ylabel("Amplitude")
>>> plt.xlabel("Sample")

```





`scipy.signal.flattop` ( $M, sym=True$ )

Return a flat top window.

**Parameters**  $M$  : int

Number of points in the output window. If zero or less, an empty array is returned.

$sym$  : bool, optional

When True (default), generates a symmetric window, for use in filter design. When False, generates a periodic window, for use in spectral analysis.

**Returns**  $w$  : ndarray

The window, with the maximum value normalized to 1 (though the value 1 does not appear if  $M$  is even and  $sym$  is True).

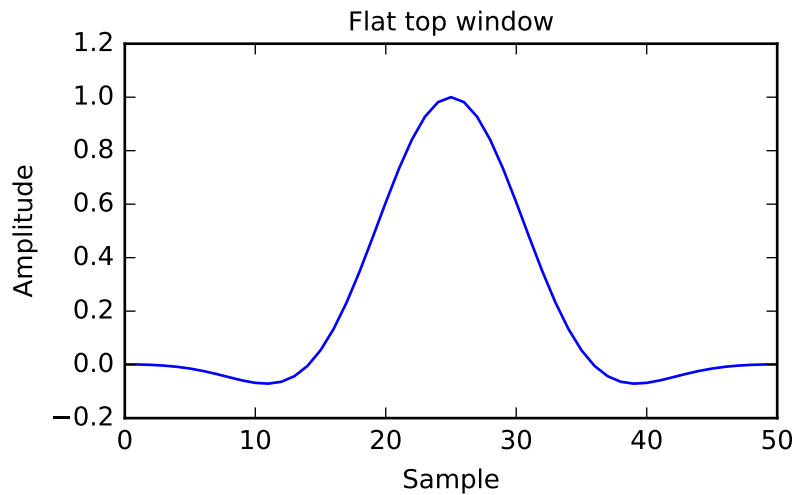
### Examples

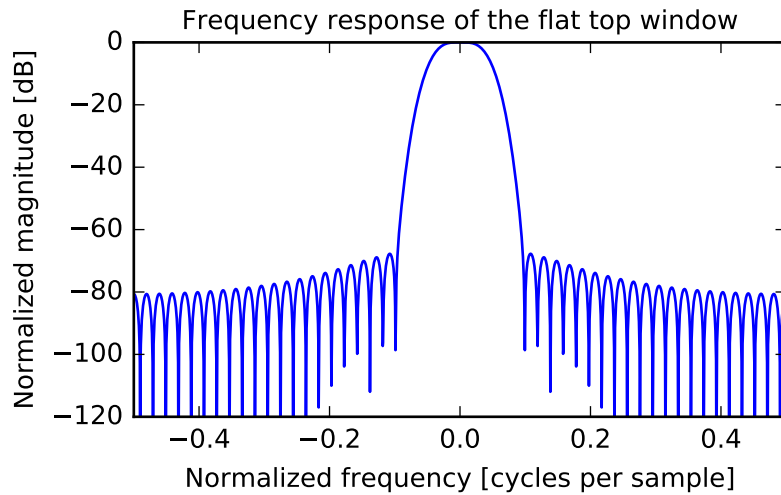
Plot the window and its frequency response:

```
>>> from scipy import signal
>>> from scipy.fftpack import fft, fftshift
>>> import matplotlib.pyplot as plt

>>> window = signal.flatop(51)
>>> plt.plot(window)
>>> plt.title("Flat top window")
>>> plt.ylabel("Amplitude")
>>> plt.xlabel("Sample")

>>> plt.figure()
>>> A = fft(window, 2048) / (len(window)/2.0)
>>> freq = np.linspace(-0.5, 0.5, len(A))
>>> response = 20 * np.log10(np.abs(fftshift(A / abs(A).max())))
>>> plt.plot(freq, response)
>>> plt.axis([-0.5, 0.5, -120, 0])
>>> plt.title("Frequency response of the flat top window")
>>> plt.ylabel("Normalized magnitude [dB]")
>>> plt.xlabel("Normalized frequency [cycles per sample]")
```





`scipy.signal.gaussian` ( $M$ ,  $std$ ,  $sym=True$ )

Return a Gaussian window.

**Parameters**  $M$  : int

Number of points in the output window. If zero or less, an empty array is returned.

$std$  : float

The standard deviation, sigma.

$sym$  : bool, optional

When True (default), generates a symmetric window, for use in filter design. When False, generates a periodic window, for use in spectral analysis.

**Returns**  $w$  : ndarray

The window, with the maximum value normalized to 1 (though the value 1 does not appear if  $M$  is even and  $sym$  is True).

### Notes

The Gaussian window is defined as

$$w(n) = e^{-\frac{1}{2}\left(\frac{n}{\sigma}\right)^2}$$

### Examples

Plot the window and its frequency response:

```
>>> from scipy import signal
>>> from scipy.fftpack import fft, fftshift
>>> import matplotlib.pyplot as plt

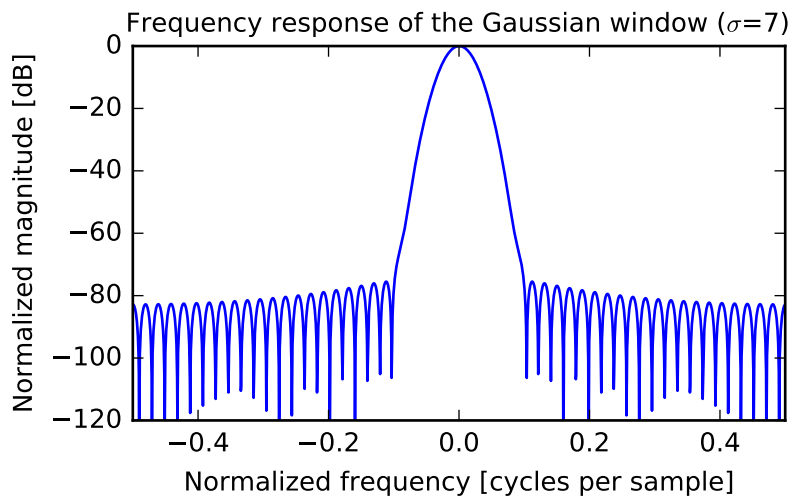
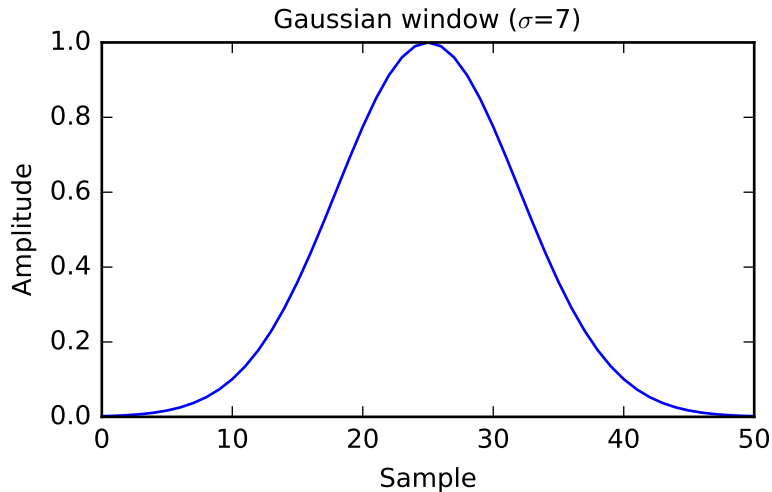
>>> window = signal.gaussian(51, std=7)
>>> plt.plot(window)
>>> plt.title(r"Gaussian window (\sigma=7)")
>>> plt.ylabel("Amplitude")
>>> plt.xlabel("Sample")
```



```

>>> plt.figure()
>>> A = fft(window, 2048) / (len(window)/2.0)
>>> freq = np.linspace(-0.5, 0.5, len(A))
>>> response = 20 * np.log10(np.abs(fftshift(A / abs(A).max())))
>>> plt.plot(freq, response)
>>> plt.axis([-0.5, 0.5, -120, 0])
>>> plt.title(r"Frequency response of the Gaussian window ( $\sigma=7$ )")
>>> plt.ylabel("Normalized magnitude [dB]")
>>> plt.xlabel("Normalized frequency [cycles per sample]")

```



`scipy.signal.general_gaussian` ( $M, p, sig, sym=True$ )

Return a window with a generalized Gaussian shape.

**Parameters**  $M$ : int

Number of points in the output window. If zero or less, an empty array is returned.

$p$ : float

		Shape parameter. $p = 1$ is identical to <code>gaussian</code> , $p = 0.5$ is the same shape as the Laplace distribution.
	<b>sig</b> : float	The standard deviation, <code>sigma</code> .
	<b>sym</b> : bool, optional	When True (default), generates a symmetric window, for use in filter design. When False, generates a periodic window, for use in spectral analysis.
<b>Returns</b>	<b>w</b> : ndarray	The window, with the maximum value normalized to 1 (though the value 1 does not appear if $M$ is even and <code>sym</code> is True).

### Notes

The generalized Gaussian window is defined as

$$w(n) = e^{-\frac{1}{2} \left| \frac{n}{\sigma} \right|^{2p}}$$

the half-power point is at

$$(2 \log(2))^{1/(2p)} \sigma$$

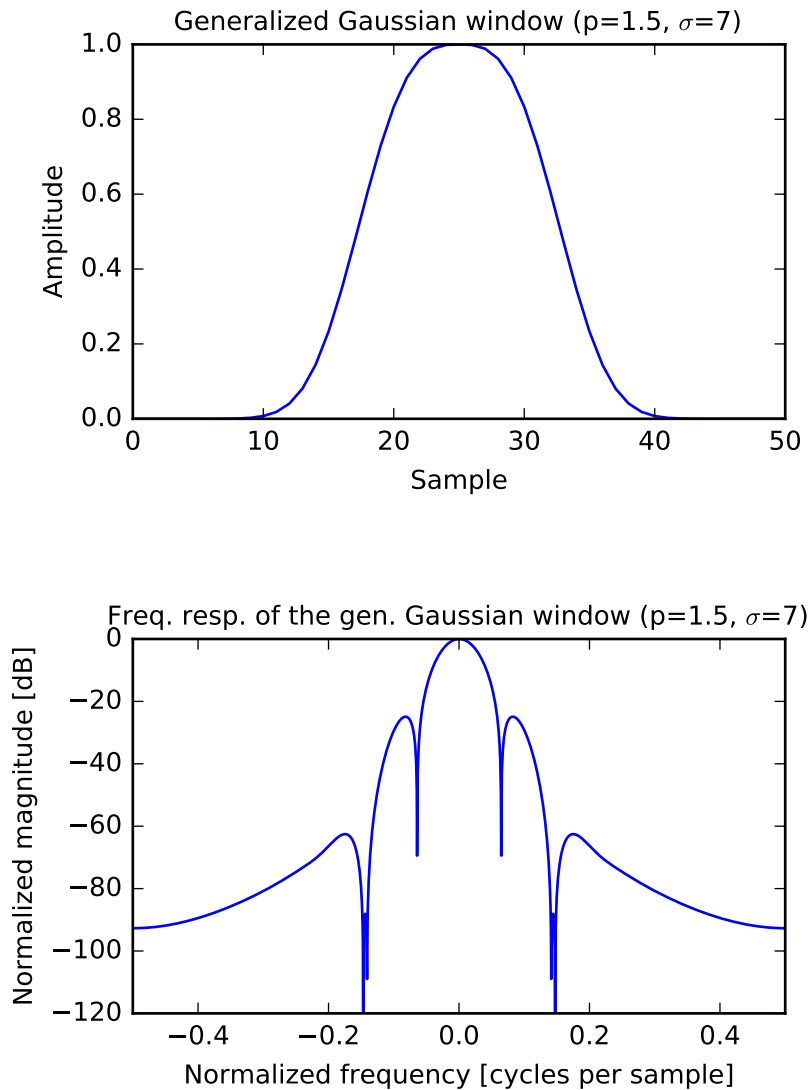
### Examples

Plot the window and its frequency response:

```
>>> from scipy import signal
>>> from scipy.fftpack import fft, fftshift
>>> import matplotlib.pyplot as plt

>>> window = signal.general_gaussian(51, p=1.5, sig=7)
>>> plt.plot(window)
>>> plt.title(r"Generalized Gaussian window (p=1.5, $\sigma$=7)")
>>> plt.ylabel("Amplitude")
>>> plt.xlabel("Sample")

>>> plt.figure()
>>> A = fft(window, 2048) / (len(window)/2.0)
>>> freq = np.linspace(-0.5, 0.5, len(A))
>>> response = 20 * np.log10(np.abs(fftshift(A / abs(A).max())))
>>> plt.plot(freq, response)
>>> plt.axis([-0.5, 0.5, -120, 0])
>>> plt.title(r"Freq. resp. of the gen. Gaussian window (p=1.5, $\sigma$=7)")
>>> plt.ylabel("Normalized magnitude [dB]")
>>> plt.xlabel("Normalized frequency [cycles per sample]")
```



`scipy.signal.hamming` ( $M$ ,  $sym=True$ )

Return a Hamming window.

The Hamming window is a taper formed by using a raised cosine with non-zero endpoints, optimized to minimize the nearest side lobe.

**Parameters**  $M$  : int

Number of points in the output window. If zero or less, an empty array is returned.

$sym$  : bool, optional

When True (default), generates a symmetric window, for use in filter design. When False, generates a periodic window, for use in spectral analysis.

**Returns**  $w$  : ndarray

The window, with the maximum value normalized to 1 (though the value 1 does not appear if  $M$  is even and  $sym$  is True).

### Notes

The Hamming window is defined as

$$w(n) = 0.54 - 0.46 \cos\left(\frac{2\pi n}{M-1}\right) \quad 0 \leq n \leq M-1$$

The Hamming was named for R. W. Hamming, an associate of J. W. Tukey and is described in Blackman and Tukey. It was recommended for smoothing the truncated autocovariance function in the time domain. Most references to the Hamming window come from the signal processing literature, where it is used as one of many windowing functions for smoothing values. It is also known as an apodization (which means “removing the foot”, i.e. smoothing discontinuities at the beginning and end of the sampled signal) or tapering function.

### References

[R204], [R205], [R206], [R207]

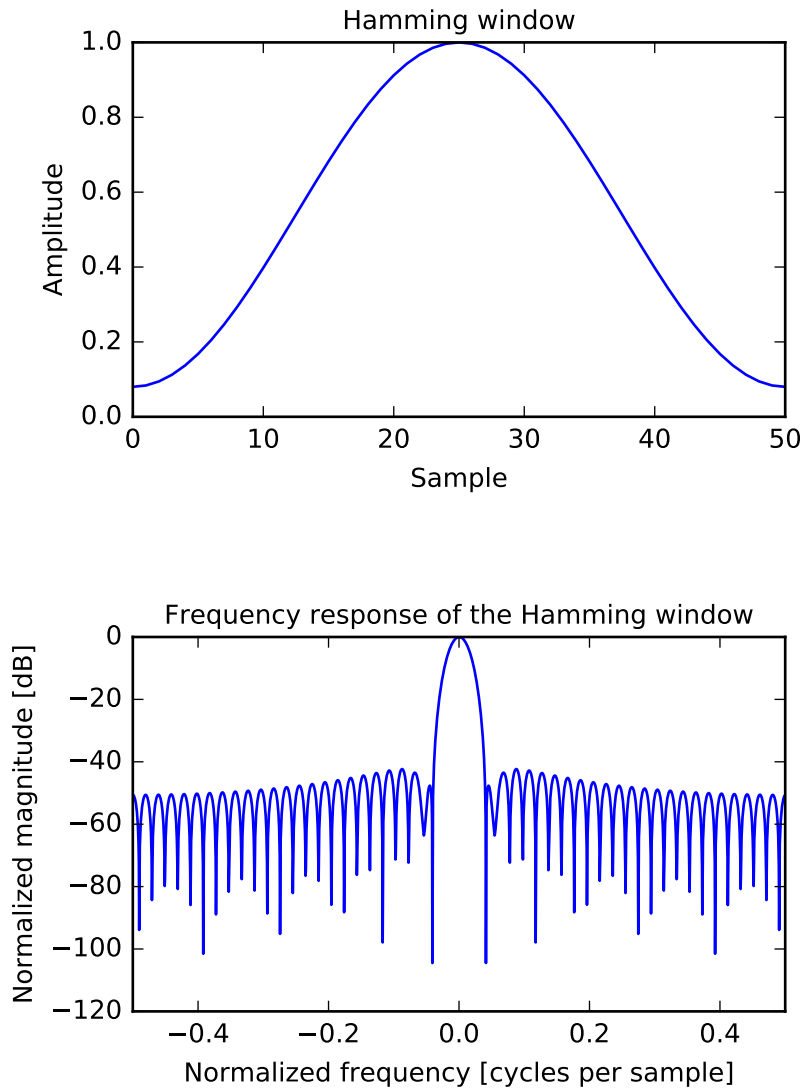
### Examples

Plot the window and its frequency response:

```
>>> from scipy import signal
>>> from scipy.fftpack import fft, fftshift
>>> import matplotlib.pyplot as plt

>>> window = signal.hamming(51)
>>> plt.plot(window)
>>> plt.title("Hamming window")
>>> plt.ylabel("Amplitude")
>>> plt.xlabel("Sample")

>>> plt.figure()
>>> A = fft(window, 2048) / (len(window)/2.0)
>>> freq = np.linspace(-0.5, 0.5, len(A))
>>> response = 20 * np.log10(np.abs(fftshift(A / abs(A).max())))
>>> plt.plot(freq, response)
>>> plt.axis([-0.5, 0.5, -120, 0])
>>> plt.title("Frequency response of the Hamming window")
>>> plt.ylabel("Normalized magnitude [dB]")
>>> plt.xlabel("Normalized frequency [cycles per sample]")
```



`scipy.signal.hann` ( $M$ ,  $sym=True$ )

Return a Hann window.

The Hann window is a taper formed by using a raised cosine or sine-squared with ends that touch zero.

**Parameters**  $M$  : int

Number of points in the output window. If zero or less, an empty array is returned.

$sym$  : bool, optional

When True (default), generates a symmetric window, for use in filter design. When False, generates a periodic window, for use in spectral analysis.

**Returns**  $w$  : ndarray

The window, with the maximum value normalized to 1 (though the value 1 does not appear if  $M$  is even and  $sym$  is True).

### Notes

The Hann window is defined as

$$w(n) = 0.5 - 0.5 \cos\left(\frac{2\pi n}{M-1}\right) \quad 0 \leq n \leq M-1$$

The window was named for Julius von Hann, an Austrian meteorologist. It is also known as the Cosine Bell. It is sometimes erroneously referred to as the “Hanning” window, from the use of “hann” as a verb in the original paper and confusion with the very similar Hamming window.

Most references to the Hann window come from the signal processing literature, where it is used as one of many windowing functions for smoothing values. It is also known as an apodization (which means “removing the foot”, i.e. smoothing discontinuities at the beginning and end of the sampled signal) or tapering function.

### References

[R208], [R209], [R210], [R211]

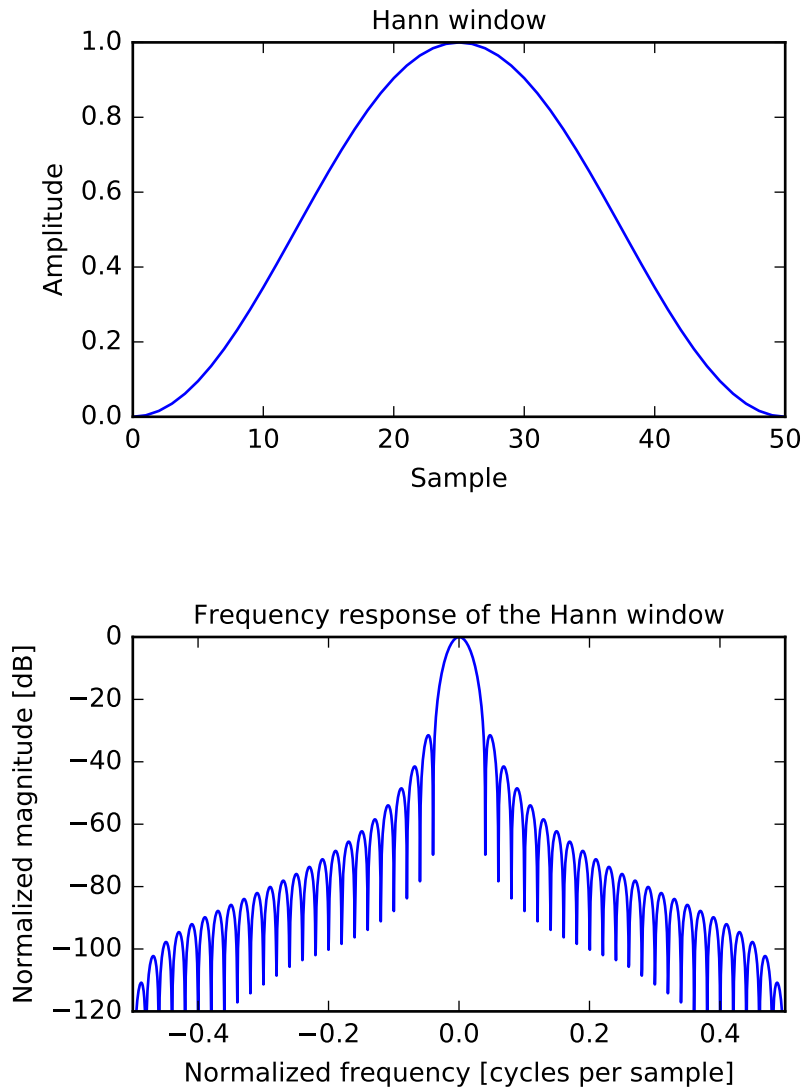
### Examples

Plot the window and its frequency response:

```
>>> from scipy import signal
>>> from scipy.fftpack import fft, fftshift
>>> import matplotlib.pyplot as plt

>>> window = signal.hann(51)
>>> plt.plot(window)
>>> plt.title("Hann window")
>>> plt.ylabel("Amplitude")
>>> plt.xlabel("Sample")

>>> plt.figure()
>>> A = fft(window, 2048) / (len(window)/2.0)
>>> freq = np.linspace(-0.5, 0.5, len(A))
>>> response = 20 * np.log10(np.abs(fftshift(A / abs(A).max())))
>>> plt.plot(freq, response)
>>> plt.axis([-0.5, 0.5, -120, 0])
>>> plt.title("Frequency response of the Hann window")
>>> plt.ylabel("Normalized magnitude [dB]")
>>> plt.xlabel("Normalized frequency [cycles per sample]")
```



`scipy.signal.hanning` ( $M$ ,  $sym=True$ )

Return a Hann window.

The Hann window is a taper formed by using a raised cosine or sine-squared with ends that touch zero.

**Parameters**  $M$  : int

Number of points in the output window. If zero or less, an empty array is returned.

$sym$  : bool, optional

When True (default), generates a symmetric window, for use in filter design. When False, generates a periodic window, for use in spectral analysis.

**Returns**  $w$  : ndarray

The window, with the maximum value normalized to 1 (though the value 1 does not appear if  $M$  is even and  $sym$  is True).

### Notes

The Hann window is defined as

$$w(n) = 0.5 - 0.5 \cos\left(\frac{2\pi n}{M-1}\right) \quad 0 \leq n \leq M-1$$

The window was named for Julius von Hann, an Austrian meteorologist. It is also known as the Cosine Bell. It is sometimes erroneously referred to as the “Hanning” window, from the use of “hann” as a verb in the original paper and confusion with the very similar Hamming window.

Most references to the Hann window come from the signal processing literature, where it is used as one of many windowing functions for smoothing values. It is also known as an apodization (which means “removing the foot”, i.e. smoothing discontinuities at the beginning and end of the sampled signal) or tapering function.

### References

[R212], [R213], [R214], [R215]

### Examples

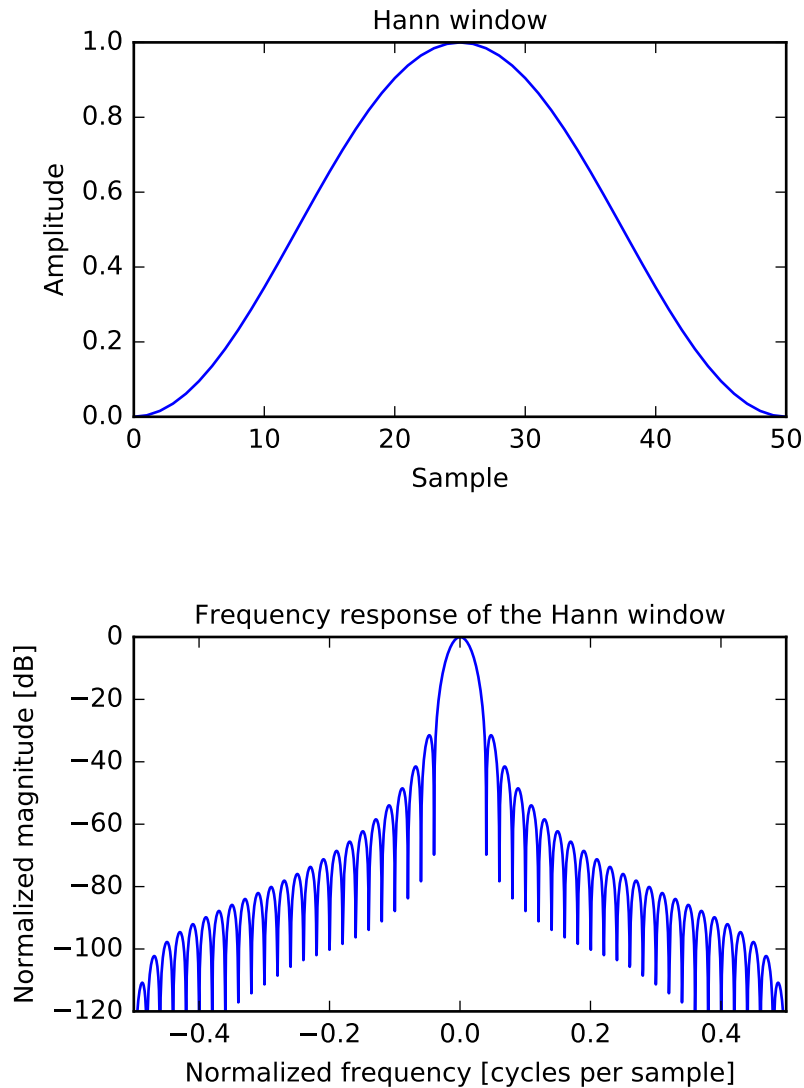
Plot the window and its frequency response:

```
>>> from scipy import signal
>>> from scipy.fftpack import fft, fftshift
>>> import matplotlib.pyplot as plt

>>> window = signal.hann(51)
>>> plt.plot(window)
>>> plt.title("Hann window")
>>> plt.ylabel("Amplitude")
>>> plt.xlabel("Sample")

>>> plt.figure()
>>> A = fft(window, 2048) / (len(window)/2.0)
>>> freq = np.linspace(-0.5, 0.5, len(A))
>>> response = 20 * np.log10(np.abs(fftshift(A / abs(A).max())))
>>> plt.plot(freq, response)
>>> plt.axis([-0.5, 0.5, -120, 0])
>>> plt.title("Frequency response of the Hann window")
>>> plt.ylabel("Normalized magnitude [dB]")
>>> plt.xlabel("Normalized frequency [cycles per sample]")
```





`scipy.signal.kaiser` ( $M$ ,  $\beta$ ,  $\text{sym}=\text{True}$ )

Return a Kaiser window.

The Kaiser window is a taper formed by using a Bessel function.

**Parameters**     $M$  : int

Number of points in the output window. If zero or less, an empty array is returned.

**beta** : float

Shape parameter, determines trade-off between main-lobe width and side lobe level. As beta gets large, the window narrows.

**sym** : bool, optional

When True (default), generates a symmetric window, for use in filter design. When False, generates a periodic window, for use in spectral analysis.

**Returns**         $w$  : ndarray

The window, with the maximum value normalized to 1 (though the value 1 does not appear if  $M$  is even and  $\text{sym}$  is True).

### Notes

The Kaiser window is defined as

$$w(n) = I_0 \left( \beta \sqrt{1 - \frac{4n^2}{(M-1)^2}} \right) / I_0(\beta)$$

with

$$-\frac{M-1}{2} \leq n \leq \frac{M-1}{2},$$

where  $I_0$  is the modified zeroth-order Bessel function.

The Kaiser was named for Jim Kaiser, who discovered a simple approximation to the DPSS window based on Bessel functions. The Kaiser window is a very good approximation to the Digital Prolate Spheroidal Sequence, or Slepian window, which is the transform which maximizes the energy in the main lobe of the window relative to total energy.

The Kaiser can approximate many other windows by varying the beta parameter.

beta	Window shape
0	Rectangular
5	Similar to a Hamming
6	Similar to a Hann
8.6	Similar to a Blackman

A beta value of 14 is probably a good starting point. Note that as beta gets large, the window narrows, and so the number of samples needs to be large enough to sample the increasingly narrow spike, otherwise NaNs will get returned.

Most references to the Kaiser window come from the signal processing literature, where it is used as one of many windowing functions for smoothing values. It is also known as an apodization (which means “removing the foot”, i.e. smoothing discontinuities at the beginning and end of the sampled signal) or tapering function.

### References

[R220], [R221], [R222]

### Examples

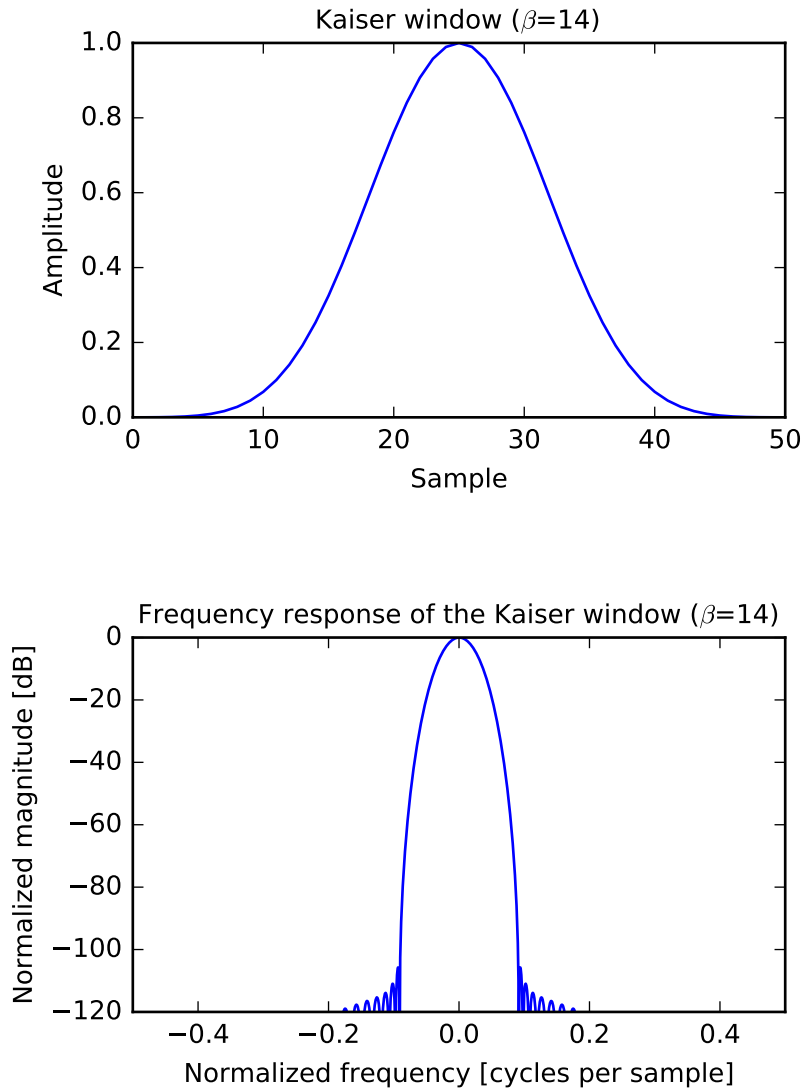
Plot the window and its frequency response:

```
>>> from scipy import signal
>>> from scipy.fftpack import fft, fftshift
>>> import matplotlib.pyplot as plt

>>> window = signal.kaiser(51, beta=14)
>>> plt.plot(window)
>>> plt.title(r"Kaiser window (\beta=14)")
>>> plt.ylabel("Amplitude")
>>> plt.xlabel("Sample")

>>> plt.figure()
>>> A = fft(window, 2048) / (len(window)/2.0)
>>> freq = np.linspace(-0.5, 0.5, len(A))
>>> response = 20 * np.log10(np.abs(fftshift(A / abs(A).max())))
>>> plt.plot(freq, response)
>>> plt.axis([-0.5, 0.5, -120, 0])
```

```
>>> plt.title(r"Frequency response of the Kaiser window ( $\beta=14$ )")
>>> plt.ylabel("Normalized magnitude [dB]")
>>> plt.xlabel("Normalized frequency [cycles per sample]")
```



`scipy.signal.nuttall` ( $M$ ,  $sym=True$ )

Return a minimum 4-term Blackman-Harris window according to Nuttall.

**Parameters**     $M$  : int

Number of points in the output window. If zero or less, an empty array is returned.

$sym$  : bool, optional

When True (default), generates a symmetric window, for use in filter design. When False, generates a periodic window, for use in spectral analysis.

**Returns**         $w$  : ndarray

The window, with the maximum value normalized to 1 (though the value 1 does not appear if  $M$  is even and  $sym$  is True).

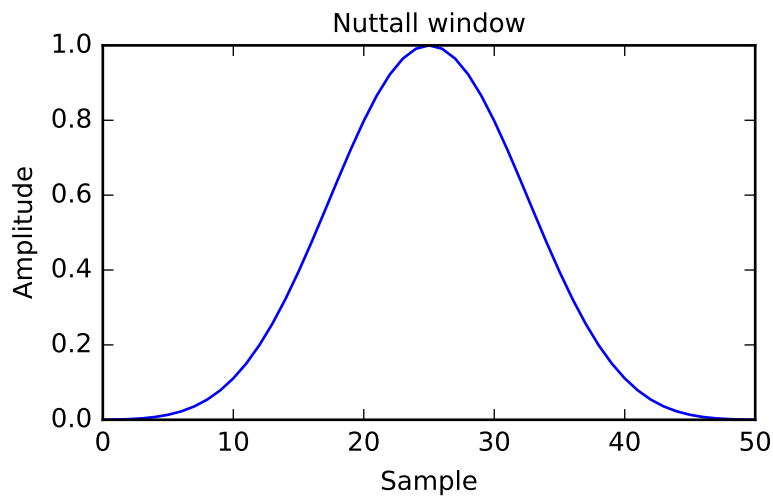
### Examples

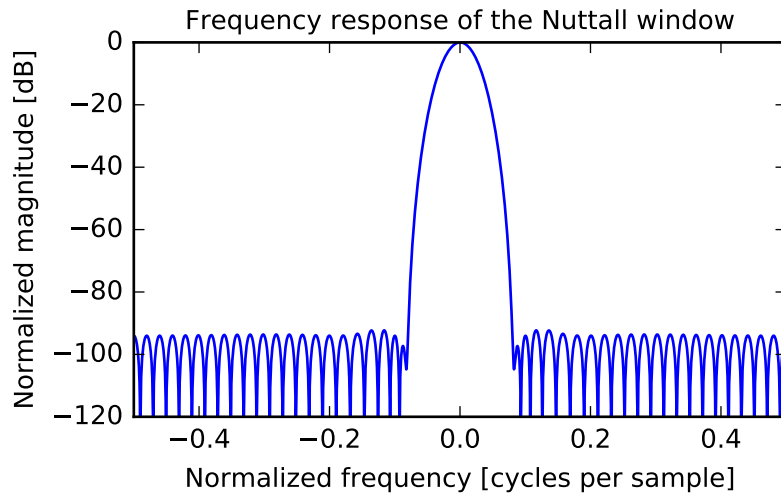
Plot the window and its frequency response:

```
>>> from scipy import signal
>>> from scipy.fftpack import fft, fftshift
>>> import matplotlib.pyplot as plt

>>> window = signal.nuttall(51)
>>> plt.plot(window)
>>> plt.title("Nuttall window")
>>> plt.ylabel("Amplitude")
>>> plt.xlabel("Sample")

>>> plt.figure()
>>> A = fft(window, 2048) / (len(window)/2.0)
>>> freq = np.linspace(-0.5, 0.5, len(A))
>>> response = 20 * np.log10(np.abs(fftshift(A / abs(A).max()))))
>>> plt.plot(freq, response)
>>> plt.axis([-0.5, 0.5, -120, 0])
>>> plt.title("Frequency response of the Nuttall window")
>>> plt.ylabel("Normalized magnitude [dB]")
>>> plt.xlabel("Normalized frequency [cycles per sample]")
```





`scipy.signal.parzen` ( $M$ ,  $sym=True$ )

Return a Parzen window.

**Parameters**  $M$  : int

Number of points in the output window. If zero or less, an empty array is returned.

$sym$  : bool, optional

When True (default), generates a symmetric window, for use in filter design. When False, generates a periodic window, for use in spectral analysis.

**Returns**  $w$  : ndarray

The window, with the maximum value normalized to 1 (though the value 1 does not appear if  $M$  is even and  $sym$  is True).

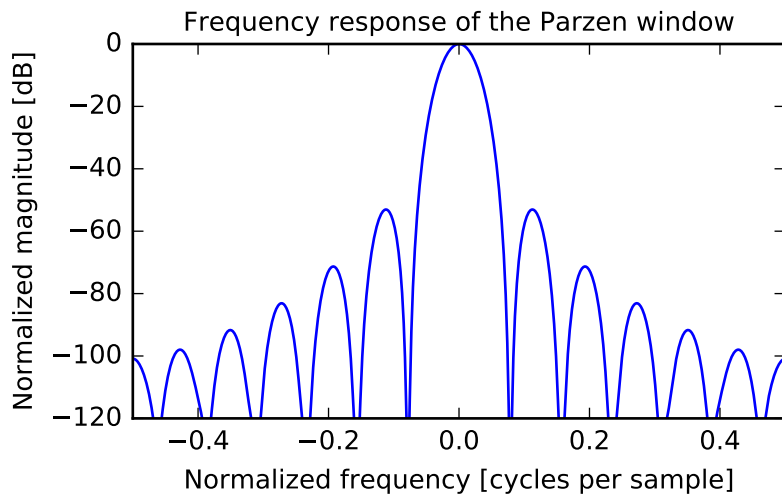
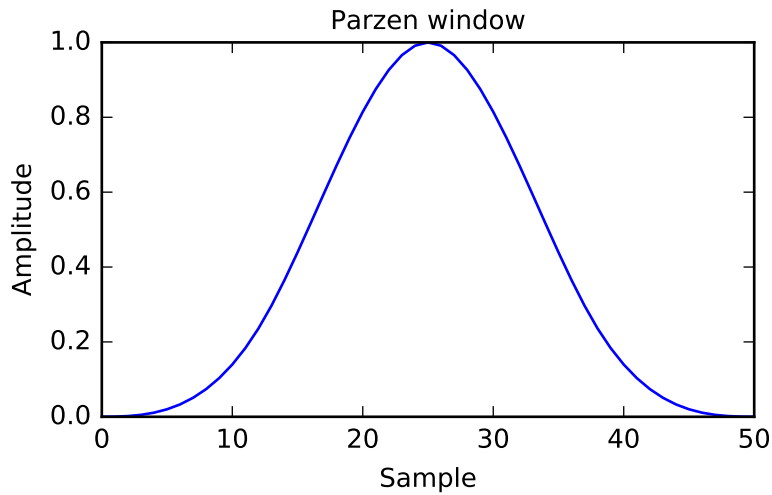
### Examples

Plot the window and its frequency response:

```
>>> from scipy import signal
>>> from scipy.fftpack import fft, fftshift
>>> import matplotlib.pyplot as plt

>>> window = signal.parzen(51)
>>> plt.plot(window)
>>> plt.title("Parzen window")
>>> plt.ylabel("Amplitude")
>>> plt.xlabel("Sample")

>>> plt.figure()
>>> A = fft(window, 2048) / (len(window)/2.0)
>>> freq = np.linspace(-0.5, 0.5, len(A))
>>> response = 20 * np.log10(np.abs(fftshift(A / abs(A).max())))
>>> plt.plot(freq, response)
>>> plt.axis([-0.5, 0.5, -120, 0])
>>> plt.title("Frequency response of the Parzen window")
>>> plt.ylabel("Normalized magnitude [dB]")
>>> plt.xlabel("Normalized frequency [cycles per sample]")
```



`scipy.signal.slepian` ( $M$ ,  $width$ ,  $sym=True$ )

Return a digital Slepian (DPSS) window.

Used to maximize the energy concentration in the main lobe. Also called the digital prolate spheroidal sequence (DPSS).

**Parameters**  $M$  : int

Number of points in the output window. If zero or less, an empty array is returned.

$width$  : float

Bandwidth

$sym$  : bool, optional

When True (default), generates a symmetric window, for use in filter design.

When False, generates a periodic window, for use in spectral analysis.

**Returns**  $w$  : ndarray

The window, with the maximum value always normalized to 1

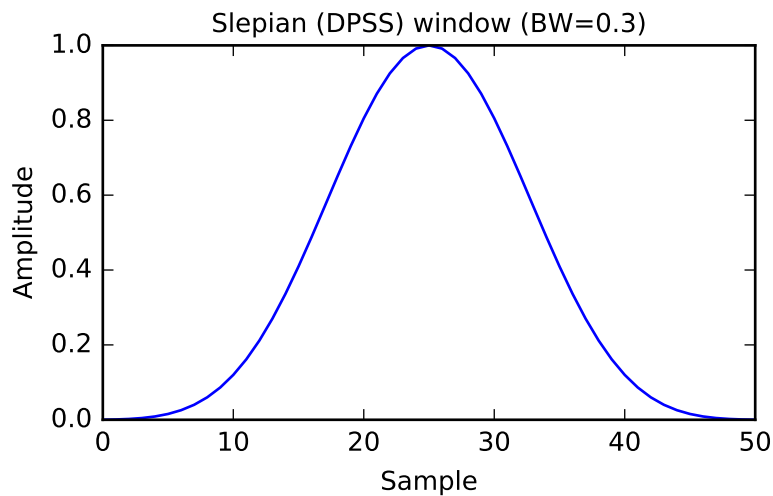
*Examples*

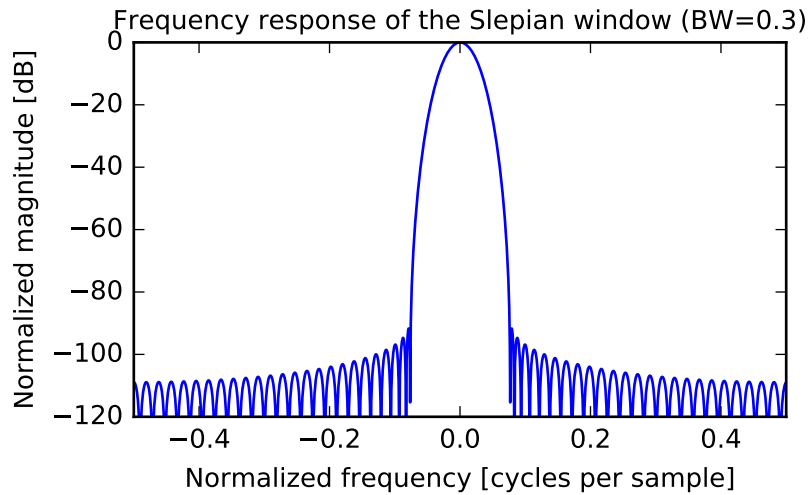
Plot the window and its frequency response:

```
>>> from scipy import signal
>>> from scipy.fftpack import fft, fftshift
>>> import matplotlib.pyplot as plt

>>> window = signal.slepian(51, width=0.3)
>>> plt.plot(window)
>>> plt.title("Slepian (DPSS) window (BW=0.3)")
>>> plt.ylabel("Amplitude")
>>> plt.xlabel("Sample")

>>> plt.figure()
>>> A = fft(window, 2048) / (len(window)/2.0)
>>> freq = np.linspace(-0.5, 0.5, len(A))
>>> response = 20 * np.log10(np.abs(fftshift(A / abs(A).max())))
>>> plt.plot(freq, response)
>>> plt.axis([-0.5, 0.5, -120, 0])
>>> plt.title("Frequency response of the Slepian window (BW=0.3)")
>>> plt.ylabel("Normalized magnitude [dB]")
>>> plt.xlabel("Normalized frequency [cycles per sample]")
```





`scipy.signal.triang` ( $M$ ,  $sym=True$ )

Return a triangular window.

**Parameters**  $M$  : int

Number of points in the output window. If zero or less, an empty array is returned.

$sym$  : bool, optional

When True (default), generates a symmetric window, for use in filter design. When False, generates a periodic window, for use in spectral analysis.

**Returns**  $w$  : ndarray

The window, with the maximum value normalized to 1 (though the value 1 does not appear if  $M$  is even and  $sym$  is True).

### Examples

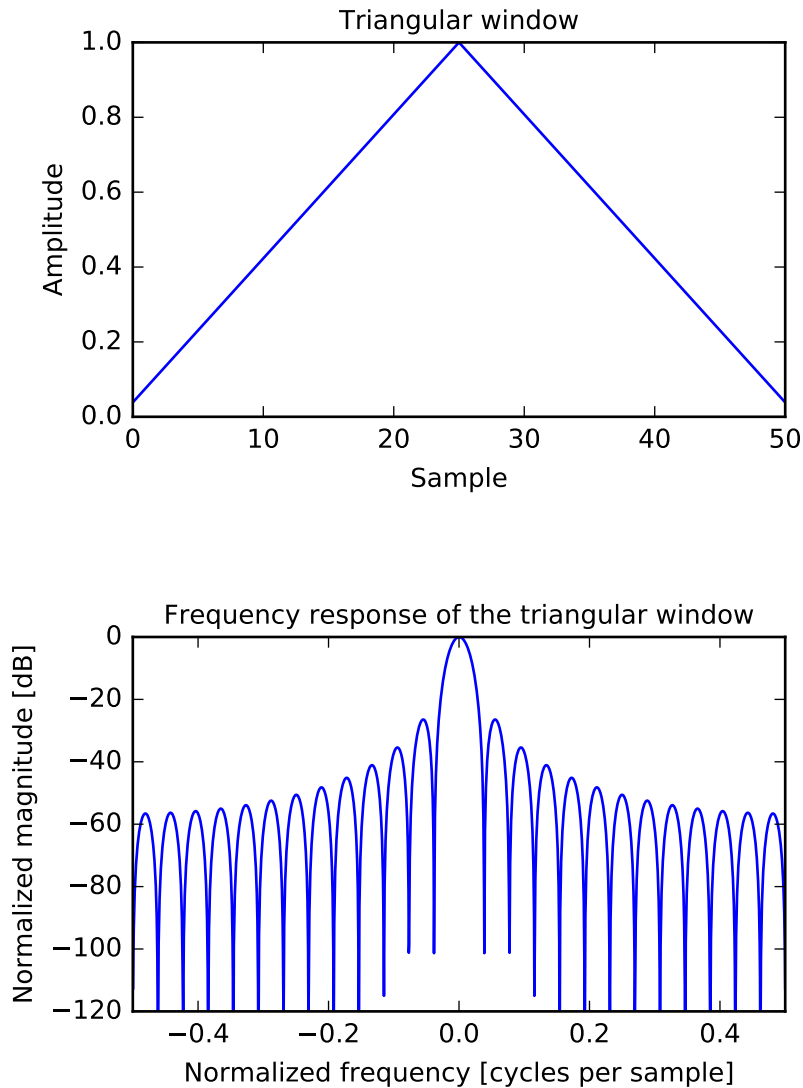
Plot the window and its frequency response:

```
>>> from scipy import signal
>>> from scipy.fftpack import fft, fftshift
>>> import matplotlib.pyplot as plt

>>> window = signal.triang(51)
>>> plt.plot(window)
>>> plt.title("Triangular window")
>>> plt.ylabel("Amplitude")
>>> plt.xlabel("Sample")

>>> plt.figure()
>>> A = fft(window, 2048) / (len(window)/2.0)
>>> freq = np.linspace(-0.5, 0.5, len(A))
>>> response = 20 * np.log10(np.abs(fftshift(A / abs(A).max())))
>>> plt.plot(freq, response)
>>> plt.axis([-0.5, 0.5, -120, 0])
>>> plt.title("Frequency response of the triangular window")
>>> plt.ylabel("Normalized magnitude [dB]")
>>> plt.xlabel("Normalized frequency [cycles per sample]")
```





`scipy.signal.tukey` ( $M$ ,  $\alpha=0.5$ ,  $\text{sym}=\text{True}$ )

Return a Tukey window, also known as a tapered cosine window.

**Parameters**  $M$  : int

Number of points in the output window. If zero or less, an empty array is returned.

**alpha** : float, optional

Shape parameter of the Tukey window, representing the fraction of the window inside the cosine tapered region. If zero, the Tukey window is equivalent to a rectangular window. If one, the Tukey window is equivalent to a Hann window.

**sym** : bool, optional

When True (default), generates a symmetric window, for use in filter design. When False, generates a periodic window, for use in spectral analysis.

**Returns**  $w$  : ndarray

The window, with the maximum value normalized to 1 (though the value 1 does not appear if  $M$  is even and  $\text{sym}$  is True).

## References

[R231], [R232]

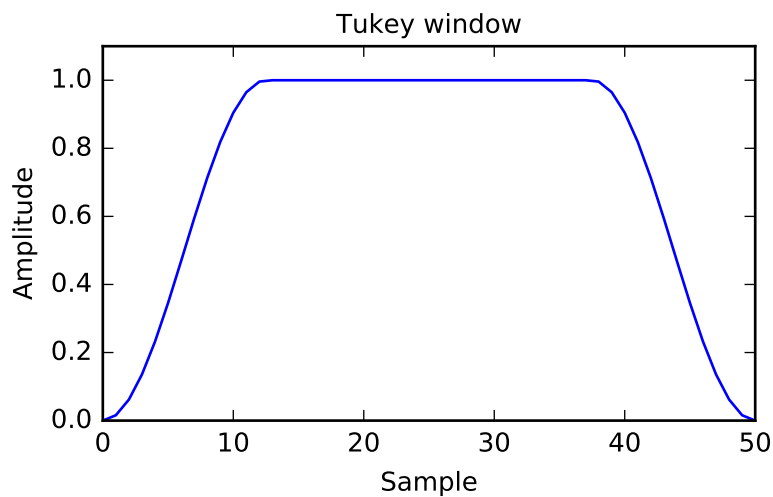
## Examples

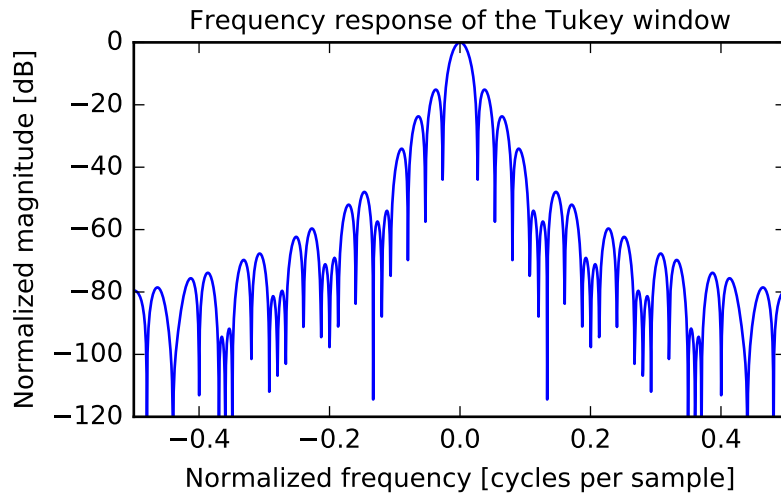
Plot the window and its frequency response:

```
>>> from scipy import signal
>>> from scipy.fftpack import fft, fftshift
>>> import matplotlib.pyplot as plt

>>> window = signal.tukey(51)
>>> plt.plot(window)
>>> plt.title("Tukey window")
>>> plt.ylabel("Amplitude")
>>> plt.xlabel("Sample")
>>> plt.ylim([0, 1.1])

>>> plt.figure()
>>> A = fft(window, 2048) / (len(window)/2.0)
>>> freq = np.linspace(-0.5, 0.5, len(A))
>>> response = 20 * np.log10(np.abs(fftshift(A / abs(A).max())))
>>> plt.plot(freq, response)
>>> plt.axis([-0.5, 0.5, -120, 0])
>>> plt.title("Frequency response of the Tukey window")
>>> plt.ylabel("Normalized magnitude [dB]")
>>> plt.xlabel("Normalized frequency [cycles per sample]")
```





### 5.20.11 Wavelets

---

<code>cascade(hk[, J])</code>	Return (x, phi, psi) at dyadic points $K/2^{**J}$ from filter coefficients.
<code>daub(p)</code>	The coefficients for the FIR low-pass filter producing Daubechies wavelets.
<code>morlet(M[, w, s, complete])</code>	Complex Morlet wavelet.
<code>qmf(hk)</code>	Return high-pass qmf filter from low-pass
<code>ricker(points, a)</code>	Return a Ricker wavelet, also known as the “Mexican hat wavelet”.
<code>cwt(data, wavelet, widths)</code>	Continuous wavelet transform.

---

`scipy.signal.cascade(hk, J=7)`

Return (x, phi, psi) at dyadic points  $K/2^{**J}$  from filter coefficients.

<b>Parameters</b>	<b>hk</b> : array_like Coefficients of low-pass filter.
	<b>J</b> : int, optional Values will be computed at grid points $K/2^{**J}$ . Default is 7.
<b>Returns</b>	<b>x</b> : ndarray The dyadic points $K/2^{**J}$ for $K=0 \dots N * (2^{**J}) - 1$ where $\text{len}(hk) = \text{len}(gk) = N + 1$ .
	<b>phi</b> : ndarray The scaling function $\phi(x)$ at $x$ : $\phi(x) = \sum(hk * \phi(2x - k))$ , where $k$ is from 0 to $N$ .
	<b>psi</b> : ndarray, optional The wavelet function $\psi(x)$ at $x$ : $\psi(x) = \sum(gk * \phi(2x - k))$ , where $k$ is from 0 to $N$ . $\psi$ is only returned if $gk$ is not None.

#### Notes

The algorithm uses the vector cascade algorithm described by Strang and Nguyen in “Wavelets and Filter Banks”. It builds a dictionary of values and slices for quick reuse. Then inserts vectors into final vector at the end.

`scipy.signal.daub(p)`

The coefficients for the FIR low-pass filter producing Daubechies wavelets.

$p \geq 1$  gives the order of the zero at  $f=1/2$ . There are  $2p$  filter coefficients.

**Parameters** **p** : int  
Order of the zero at  $f=1/2$ , can have values from 1 to 34.

**Returns** **daub** : ndarray  
Return

`scipy.signal.morlet` (*M*, *w*=5.0, *s*=1.0, *complete*=True)

Complex Morlet wavelet.

**Parameters** **M** : int  
Length of the wavelet.

**w** : float, optional  
Omega0. Default is 5

**s** : float, optional  
Scaling factor, windowed from  $-s*2*\pi$  to  $+s*2*\pi$ . Default is 1.

**complete** : bool, optional  
Whether to use the complete or the standard version.

**Returns** **morlet** : (M,) ndarray

**See also:**

`scipy.signal.gausspulse`

**Notes**

The standard version:

```
pi**-0.25 * exp(1j*w*x) * exp(-0.5*(x**2))
```

This commonly used wavelet is often referred to simply as the Morlet wavelet. Note that this simplified version can cause admissibility problems at low values of  $w$ .

The complete version:

```
pi**-0.25 * (exp(1j*w*x) - exp(-0.5*(w**2))) * exp(-0.5*(x**2))
```

This version has a correction term to improve admissibility. For  $w$  greater than 5, the correction term is negligible.

Note that the energy of the return wavelet is not normalised according to  $s$ .

The fundamental frequency of this wavelet in Hz is given by  $f = 2*s*w*r / M$  where  $r$  is the sampling rate.

Note: This function was created before `cwt` and is not compatible with it.

`scipy.signal.qmf` (*hk*)

Return high-pass qmf filter from low-pass

**Parameters** **hk** : array\_like  
Coefficients of high-pass filter.

`scipy.signal.ricker` (*points*, *a*)

Return a Ricker wavelet, also known as the ‘‘Mexican hat wavelet’’.

It models the function:

$$A (1 - x^2/a^2) \exp(-x^2/2 a^2),$$

where  $A = 2/\sqrt{3a} \pi^{1/4}$ .

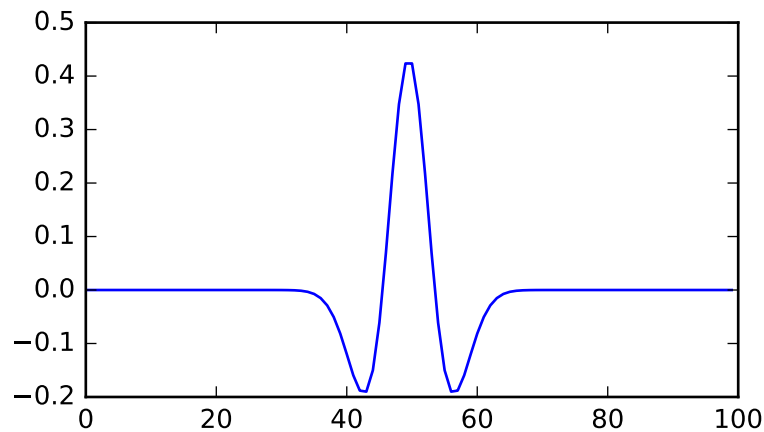
**Parameters** **points** : int  
Number of points in *vector*. Will be centered around 0.

**Returns** **a** : scalar  
**vector** : (N,) ndarray  
 Width parameter of the wavelet.  
 Array of length *points* in shape of ricker curve.

**Examples**

```
>>> from scipy import signal
>>> import matplotlib.pyplot as plt

>>> points = 100
>>> a = 4.0
>>> vec2 = signal.ricker(points, a)
>>> print(len(vec2))
100
>>> plt.plot(vec2)
>>> plt.show()
```



`scipy.signal.cwt` (*data*, *wavelet*, *widths*)

Continuous wavelet transform.

Performs a continuous wavelet transform on *data*, using the *wavelet* function. A CWT performs a convolution with *data* using the *wavelet* function, which is characterized by a width parameter and length parameter.

**Parameters** **data** : (N,) ndarray  
 data on which to perform the transform.

**wavelet** : function  
 Wavelet function, which should take 2 arguments. The first argument is the number of points that the returned vector will have (`len(wavelet(length,width)) == length`). The second is a width parameter, defining the size of the wavelet (e.g. standard deviation of a gaussian). See [ricker](#), which satisfies these requirements.

**widths** : (M,) sequence  
 Widths to use for transform.

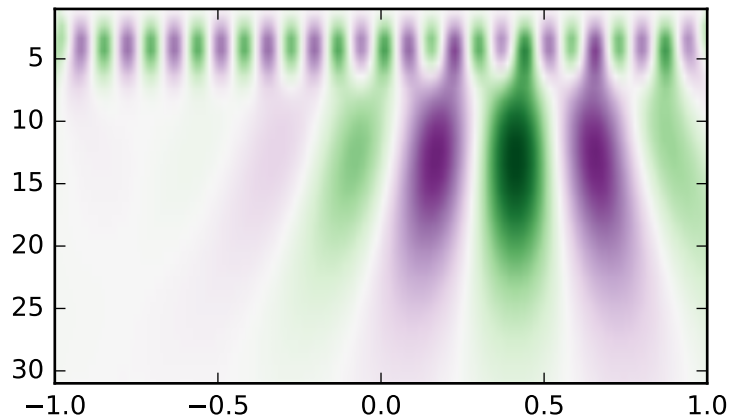
**Returns** **cwt**: (M, N) ndarray  
 Will have shape of `(len(widths), len(data))`.

### Notes

```
length = min(10 * width[ii], len(data))
cwt[ii,:] = signal.convolve(data, wavelet(length,
                                         width[ii]), mode='same')
```

### Examples

```
>>> from scipy import signal
>>> import matplotlib.pyplot as plt
>>> t = np.linspace(-1, 1, 200, endpoint=False)
>>> sig = np.cos(2 * np.pi * 7 * t) + signal.gausspulse(t - 0.4, fc=2)
>>> widths = np.arange(1, 31)
>>> cwtmatr = signal.cwt(sig, signal.ricker, widths)
>>> plt.imshow(cwtmatr, extent=[-1, 1, 31, 1], cmap='PRGn', aspect='auto',
...           vmax=abs(cwtmatr).max(), vmin=-abs(cwtmatr).max())
>>> plt.show()
```



## 5.20.12 Peak finding

<code>find_peaks_cwt(vector, widths[, wavelet, ...])</code>	Attempt to find the peaks in a 1-D array.
<code>argrelmin(data[, axis, order, mode])</code>	Calculate the relative minima of <i>data</i> .
<code>argrelmax(data[, axis, order, mode])</code>	Calculate the relative maxima of <i>data</i> .
<code>argrelextrema(data, comparator[, axis, ...])</code>	Calculate the relative extrema of <i>data</i> .

`scipy.signal.find_peaks_cwt` (*vector*, *widths*, *wavelet=None*, *max\_distances=None*, *gap\_thresh=None*, *min\_length=None*, *min\_snr=1*, *noise\_perc=10*)  
 Attempt to find the peaks in a 1-D array.

The general approach is to smooth *vector* by convolving it with *wavelet(width)* for each width in *widths*. Relative maxima which appear at enough length scales, and with sufficiently high SNR, are accepted.

**Parameters** **vector** : ndarray  
 1-D array in which to find the peaks.

**widths** : sequence

1-D array of widths to use for calculating the CWT matrix. In general, this range should cover the expected width of peaks of interest.

**wavelet** : callable, optional

Should take two parameters and return a 1-D array to convolve with *vector*. The first parameter determines the number of points of the returned wavelet array, the second parameter is the scale (*width*) of the wavelet. Should be normalized and symmetric. Default is the ricker wavelet.

**max\_distances** : ndarray, optional

At each row, a ridge line is only connected if the relative max at row[n] is within `max_distances[n]` from the relative max at row[n+1]. Default value is `widths/4`.

**gap\_thresh** : float, optional

If a relative maximum is not found within *max\_distances*, there will be a gap. A ridge line is discontinued if there are more than *gap\_thresh* points without connecting a new relative maximum. Default is 2.

**min\_length** : int, optional

Minimum length a ridge line needs to be acceptable. Default is `cwt.shape[0] / 4`, ie 1/4-th the number of widths.

**min\_snr** : float, optional

Minimum SNR ratio. Default 1. The signal is the value of the cwt matrix at the shortest length scale (`cwt[0, loc]`), the noise is the *noise\_perc*'th percentile of datapoints contained within a window of *window\_size* around `cwt[0, loc]`.

**noise\_perc** : float, optional

When calculating the noise floor, percentile of data points examined below which to consider noise. Calculated using *stats.scoreatpercentile*. Default

**Returns** **peaks\_indices** : list

Indices of the locations in the *vector* where peaks were found. The list is sorted.

**See also:**

`cwt`

**Notes**

This approach was designed for finding sharp peaks among noisy data, however with proper parameter selection it should function well for different peak shapes.

*The algorithm is as follows:*

1. Perform a continuous wavelet transform on *vector*, for the supplied *widths*. This is a convolution of *vector* with *wavelet(width)* for each width in *widths*. See `cwt`
2. Identify “ridge lines” in the cwt matrix. These are relative maxima at each row, connected across adjacent rows. See `identify_ridge_lines`
3. Filter the `ridge_lines` using `filter_ridge_lines`.

New in version 0.11.0.

**References**

[R199]

**Examples**

```
>>> from scipy import signal
>>> xs = np.arange(0, np.pi, 0.05)
```

```
>>> data = np.sin(xs)
>>> peakind = signal.find_peaks_cwt(data, np.arange(1,10))
>>> peakind, xs[peakind], data[peakind]
([32], array([ 1.6]), array([ 0.9995736]))
```

`scipy.signal.argrelmin` (*data*, *axis=0*, *order=1*, *mode='clip'*)

Calculate the relative minima of *data*.

<b>Parameters</b>	<p><b>data</b> : ndarray Array in which to find the relative minima.</p> <p><b>axis</b> : int, optional Axis over which to select from <i>data</i>. Default is 0.</p> <p><b>order</b> : int, optional How many points on each side to use for the comparison to consider <code>comparator(n, n+x)</code> to be True.</p> <p><b>mode</b> : str, optional How the edges of the vector are treated. Available options are 'wrap' (wrap around) or 'clip' (treat overflow as the same as the last (or first) element).</p>
<b>Returns</b>	<p><b>extrema</b> : tuple of ndarrays Default 'clip'. See <code>numpy.take</code>. Indices of the minima in arrays of integers. <code>extrema[k]</code> is the array of indices of axis <i>k</i> of <i>data</i>. Note that the return value is a tuple even when <i>data</i> is one-dimensional.</p>

See also:

`argrelextrema`, `argrelmax`

#### Notes

This function uses `argrelextrema` with `np.less` as comparator.

New in version 0.11.0.

#### Examples

```
>>> from scipy.signal import argrelmin
>>> x = np.array([2, 1, 2, 3, 2, 0, 1, 0])
>>> argrelmin(x)
(array([1, 5]),)
>>> y = np.array([[1, 2, 1, 2],
...               [2, 2, 0, 0],
...               [5, 3, 4, 4]])
...
>>> argrelmin(y, axis=1)
(array([0, 2]), array([2, 1]))
```

`scipy.signal.argrelmax` (*data*, *axis=0*, *order=1*, *mode='clip'*)

Calculate the relative maxima of *data*.

<b>Parameters</b>	<p><b>data</b> : ndarray Array in which to find the relative maxima.</p> <p><b>axis</b> : int, optional Axis over which to select from <i>data</i>. Default is 0.</p> <p><b>order</b> : int, optional How many points on each side to use for the comparison to consider <code>comparator(n, n+x)</code> to be True.</p> <p><b>mode</b> : str, optional</p>
-------------------	---



**Returns** **extrema** : tuple of ndarrays

How the edges of the vector are treated. Available options are ‘wrap’ (wrap around) or ‘clip’ (treat overflow as the same as the last (or first) element). Default ‘clip’. See `numpy.take`.

Indices of the maxima in arrays of integers. `extrema[k]` is the array of indices of axis  $k$  of `data`. Note that the return value is a tuple even when `data` is one-dimensional.

**See also:**

`argrelextrema`, `argrelmin`

**Notes**

This function uses `argrelextrema` with `np.greater` as comparator.

New in version 0.11.0.

**Examples**

```
>>> from scipy.signal import argrelmax
>>> x = np.array([2, 1, 2, 3, 2, 0, 1, 0])
>>> argrelmax(x)
(array([3, 6]),)
>>> y = np.array([[1, 2, 1, 2],
...               [2, 2, 0, 0],
...               [5, 3, 4, 4]])
...
>>> argrelmax(y, axis=1)
(array([0]), array([1]))
```

`scipy.signal.argrelextrema` (`data`, `comparator`, `axis=0`, `order=1`, `mode='clip'`)  
Calculate the relative extrema of `data`.

**Parameters**

- data** : ndarray  
Array in which to find the relative extrema.
- comparator** : callable  
Function to use to compare two data points. Should take 2 numbers as arguments.
- axis** : int, optional  
Axis over which to select from `data`. Default is 0.
- order** : int, optional  
How many points on each side to use for the comparison to consider `comparator(n, n+x)` to be True.
- mode** : str, optional  
How the edges of the vector are treated. ‘wrap’ (wrap around) or ‘clip’ (treat overflow as the same as the last (or first) element). Default is ‘clip’.

**Returns** **extrema** : tuple of ndarrays

See `numpy.take`.

Indices of the maxima in arrays of integers. `extrema[k]` is the array of indices of axis  $k$  of `data`. Note that the return value is a tuple even when `data` is one-dimensional.

**See also:**

`argrelmin`, `argrelmax`

**Notes**

New in version 0.11.0.

### Examples

```
>>> from scipy.signal import argrelextrema
>>> x = np.array([2, 1, 2, 3, 2, 0, 1, 0])
>>> argrelextrema(x, np.greater)
(array([3, 6]),)
>>> y = np.array([[1, 2, 1, 2],
...              [2, 2, 0, 0],
...              [5, 3, 4, 4]])
...
>>> argrelextrema(y, np.less, axis=1)
(array([0, 2]), array([2, 1]))
```

## 5.20.13 Spectral Analysis

<code>periodogram(x[, fs, window, nfft, detrend, ...])</code>	Estimate power spectral density using a periodogram.
<code>welch(x[, fs, window, nperseg, noverlap, ...])</code>	Estimate power spectral density using Welch's method.
<code>csd(x, y[, fs, window, nperseg, noverlap, ...])</code>	Estimate the cross power spectral density, $P_{xy}$ , using Welch's method.
<code>coherence(x, y[, fs, window, nperseg, ...])</code>	Estimate the magnitude squared coherence estimate, $C_{xy}$ , of discrete-time signals.
<code>spectrogram(x[, fs, window, nperseg, ...])</code>	Compute a spectrogram with consecutive Fourier transforms.
<code>lombscargle(x, y, freqs)</code>	Computes the Lomb-Scargle periodogram.
<code>vectorstrength(events, period)</code>	Determine the vector strength of the events corresponding to the given period.

`scipy.signal.periodogram(x, fs=1.0, window=None, nfft=None, detrend='constant', return_onesided=True, scaling='density', axis=-1)`  
 Estimate power spectral density using a periodogram.

**Parameters** `x` : array\_like

Time series of measurement values

`fs` : float, optional

Sampling frequency of the  $x$  time series. Defaults to 1.0.

`window` : str or tuple or array\_like, optional

Desired window to use. See `get_window` for a list of windows and required parameters. If `window` is an array it will be used directly as the window. Defaults to None; equivalent to 'boxcar'.

`nfft` : int, optional

Length of the FFT used. If None the length of  $x$  will be used.

`detrend` : str or function or False, optional

Specifies how to detrend  $x$  prior to computing the spectrum. If `detrend` is a string, it is passed as the `type` argument to `detrend`. If it is a function, it should return a detrended array. If `detrend` is False, no detrending is done. Defaults to 'constant'.

`return_onesided` : bool, optional

If True, return a one-sided spectrum for real data. If False return a two-sided spectrum. Note that for complex data, a two-sided spectrum is always returned.

`scaling` : { 'density', 'spectrum' }, optional

Selects between computing the power spectral density ('density') where  $P_{xx}$  has units of  $V^2/Hz$  and computing the power spectrum ('spectrum') where  $P_{xx}$  has units of  $V^2$ , if  $x$  is measured in V and  $fs$  is measured in Hz. Defaults to 'density'

`axis` : int, optional

**Returns**

**f**: ndarray Axis along which the periodogram is computed; the default is over the last axis (i.e. `axis=-1`).

Array of sample frequencies.

**Pxx**: ndarray Power spectral density or power spectrum of *x*.

**See also:**

`welch` Estimate power spectral density using Welch's method

`lombscargle` Lomb-Scargle periodogram for unevenly sampled data

**Notes**

New in version 0.12.0.

**Examples**

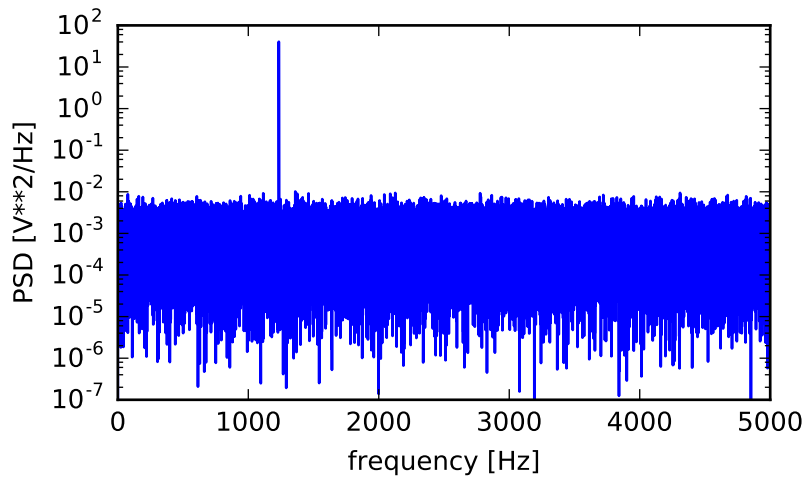
```
>>> from scipy import signal
>>> import matplotlib.pyplot as plt
>>> np.random.seed(1234)
```

Generate a test signal, a 2 Vrms sine wave at 1234 Hz, corrupted by 0.001 V\*\*2/Hz of white noise sampled at 10 kHz.

```
>>> fs = 10e3
>>> N = 1e5
>>> amp = 2*np.sqrt(2)
>>> freq = 1234.0
>>> noise_power = 0.001 * fs / 2
>>> time = np.arange(N) / fs
>>> x = amp*np.sin(2*np.pi*freq*time)
>>> x += np.random.normal(scale=np.sqrt(noise_power), size=time.shape)
```

Compute and plot the power spectral density.

```
>>> f, Pxx_den = signal.periodogram(x, fs)
>>> plt.semilogy(f, Pxx_den)
>>> plt.ylim([1e-7, 1e2])
>>> plt.xlabel('frequency [Hz]')
>>> plt.ylabel('PSD [V**2/Hz]')
>>> plt.show()
```

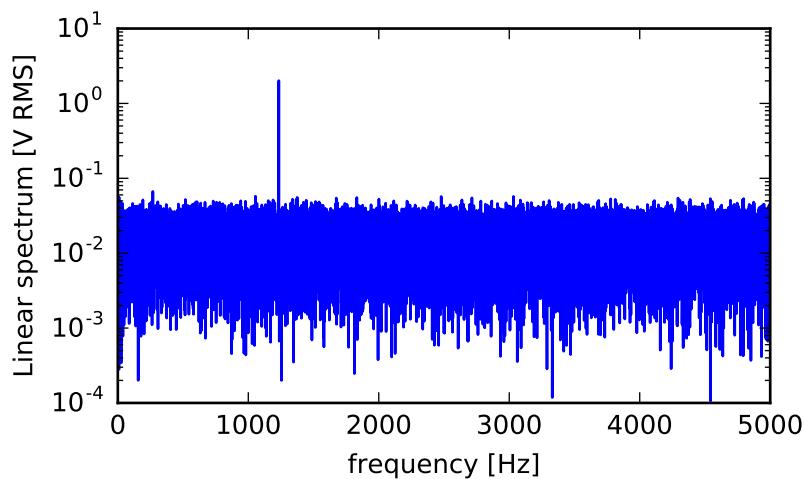


If we average the last half of the spectral density, to exclude the peak, we can recover the noise power on the signal.

```
>>> np.mean(Pxx_den[256:])
0.0018156616014838548
```

Now compute and plot the power spectrum.

```
>>> f, Pxx_spec = signal.periodogram(x, fs, 'flattop', scaling='spectrum')
>>> plt.figure()
>>> plt.semilogy(f, np.sqrt(Pxx_spec))
>>> plt.ylim([1e-4, 1e1])
>>> plt.xlabel('frequency [Hz]')
>>> plt.ylabel('Linear spectrum [V RMS]')
>>> plt.show()
```



The peak height in the power spectrum is an estimate of the RMS amplitude.

```
>>> np.sqrt(Pxx_spec.max())
2.0077340678640727
```

`scipy.signal.welch(x, fs=1.0, window='hann', nperseg=256, noverlap=None, nfft=None, detrend='constant', return_onesided=True, scaling='density', axis=-1)`  
Estimate power spectral density using Welch's method.

Welch's method [R234] computes an estimate of the power spectral density by dividing the data into overlapping segments, computing a modified periodogram for each segment and averaging the periodograms.

**Parameters**

- x** : array\_like  
Time series of measurement values
- fs** : float, optional  
Sampling frequency of the *x* time series. Defaults to 1.0.
- window** : str or tuple or array\_like, optional  
Desired window to use. See `get_window` for a list of windows and required parameters. If *window* is array\_like it will be used directly as the window and its length will be used for *nperseg*. Defaults to 'hann'.
- nperseg** : int, optional  
Length of each segment. Defaults to 256.
- noverlap** : int, optional  
Number of points to overlap between segments. If None, `noverlap = nperseg // 2`. Defaults to None.
- nfft** : int, optional  
Length of the FFT used, if a zero padded FFT is desired. If None, the FFT length is *nperseg*. Defaults to None.
- detrend** : str or function or False, optional  
Specifies how to detrend each segment. If *detrend* is a string, it is passed as the `type` argument to `detrend`. If it is a function, it takes a segment and returns a detrended segment. If *detrend* is False, no detrending is done. Defaults to 'constant'.
- return\_onesided** : bool, optional  
If True, return a one-sided spectrum for real data. If False return a two-sided spectrum. Note that for complex data, a two-sided spectrum is always returned.
- scaling** : { 'density', 'spectrum' }, optional  
Selects between computing the power spectral density ('density') where *Pxx* has units of  $V^{**2}/Hz$  and computing the power spectrum ('spectrum') where *Pxx* has units of  $V^{**2}$ , if *x* is measured in V and fs is measured in Hz. Defaults to 'density'
- axis** : int, optional  
Axis along which the periodogram is computed; the default is over the last axis (i.e. `axis=-1`).

**Returns**

- f** : ndarray  
Array of sample frequencies.
- Pxx** : ndarray  
Power spectral density or power spectrum of *x*.

See also:

[`periodogram`](#)

Simple, optionally modified periodogram

[`lombscargle`](#)

Lomb-Scargle periodogram for unevenly sampled data

### Notes

An appropriate amount of overlap will depend on the choice of window and on your requirements. For the default 'hann' window an overlap of 50% is a reasonable trade off between accurately estimating the signal power, while not over counting any of the data. Narrower windows may require a larger overlap.

If *noverlap* is 0, this method is equivalent to Bartlett's method [R235].

New in version 0.12.0.

### References

[R234], [R235]

### Examples

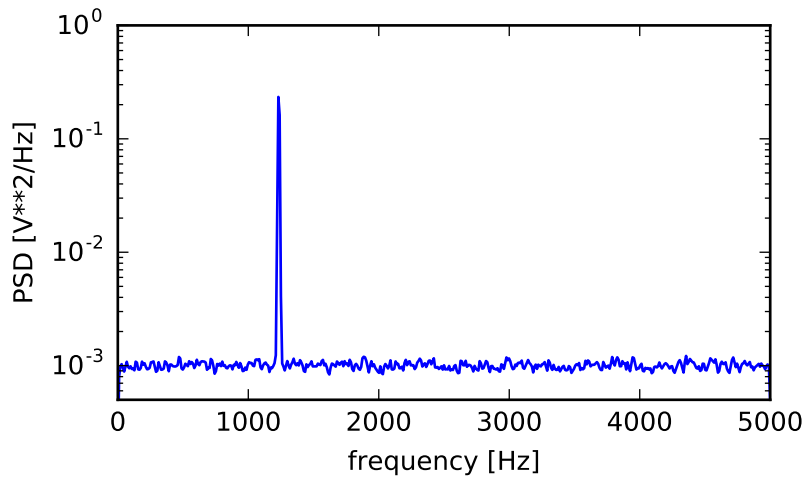
```
>>> from scipy import signal
>>> import matplotlib.pyplot as plt
>>> np.random.seed(1234)
```

Generate a test signal, a 2 Vrms sine wave at 1234 Hz, corrupted by 0.001 V\*\*2/Hz of white noise sampled at 10 kHz.

```
>>> fs = 10e3
>>> N = 1e5
>>> amp = 2*np.sqrt(2)
>>> freq = 1234.0
>>> noise_power = 0.001 * fs / 2
>>> time = np.arange(N) / fs
>>> x = amp*np.sin(2*np.pi*freq*time)
>>> x += np.random.normal(scale=np.sqrt(noise_power), size=time.shape)
```

Compute and plot the power spectral density.

```
>>> f, Pxx_den = signal.welch(x, fs, nperseg=1024)
>>> plt.semilogy(f, Pxx_den)
>>> plt.ylim([0.5e-3, 1])
>>> plt.xlabel('frequency [Hz]')
>>> plt.ylabel('PSD [V**2/Hz]')
>>> plt.show()
```

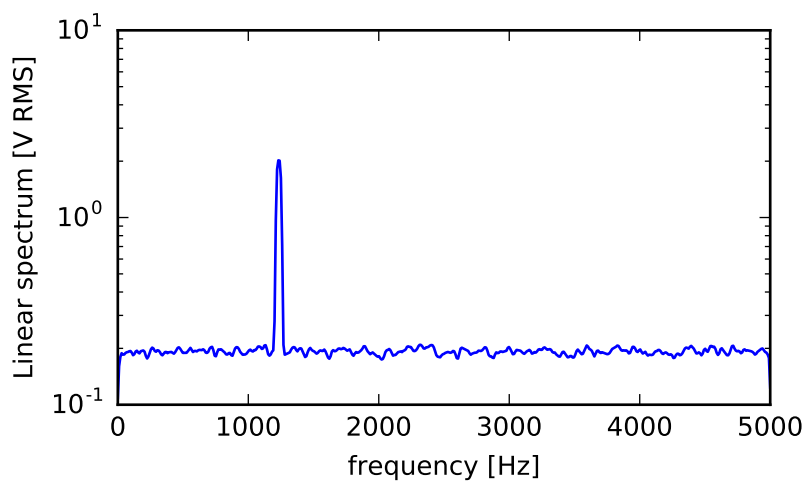


If we average the last half of the spectral density, to exclude the peak, we can recover the noise power on the signal.

```
>>> np.mean(Pxx_den[256:])
0.0009924865443739191
```

Now compute and plot the power spectrum.

```
>>> f, Pxx_spec = signal.welch(x, fs, 'flattop', 1024, scaling='spectrum')
>>> plt.figure()
>>> plt.semilogy(f, np.sqrt(Pxx_spec))
>>> plt.xlabel('frequency [Hz]')
>>> plt.ylabel('Linear spectrum [V RMS]')
>>> plt.show()
```



The peak height in the power spectrum is an estimate of the RMS amplitude.

```
>>> np.sqrt(Pxx_spec.max())
2.0077340678640727
```

`scipy.signal.csd(x, y, fs=1.0, window='hann', nperseg=256, noverlap=None, nfft=None, detrend='constant', return_onesided=True, scaling='density', axis=-1)`  
 Estimate the cross power spectral density,  $P_{xy}$ , using Welch's method.

**Parameters**

- x** : array\_like  
Time series of measurement values
- y** : array\_like  
Time series of measurement values
- fs** : float, optional  
Sampling frequency of the  $x$  and  $y$  time series. Defaults to 1.0.
- window** : str or tuple or array\_like, optional  
Desired window to use. See `get_window` for a list of windows and required parameters. If `window` is array\_like it will be used directly as the window and its length will be used for `nperseg`. Defaults to 'hann'.
- nperseg** : int, optional  
Length of each segment. Defaults to 256.
- noverlap** : int, optional  
Number of points to overlap between segments. If None, `noverlap = nperseg // 2`. Defaults to None.
- nfft** : int, optional  
Length of the FFT used, if a zero padded FFT is desired. If None, the FFT length is `nperseg`. Defaults to None.
- detrend** : str or function or False, optional  
Specifies how to detrend each segment. If `detrend` is a string, it is passed as the `type` argument to `detrend`. If it is a function, it takes a segment and returns a detrended segment. If `detrend` is False, no detrending is done. Defaults to 'constant'.
- return\_onesided** : bool, optional  
If True, return a one-sided spectrum for real data. If False return a two-sided spectrum. Note that for complex data, a two-sided spectrum is always returned.
- scaling** : { 'density', 'spectrum' }, optional  
Selects between computing the cross spectral density ('density') where  $P_{xy}$  has units of  $V^2/Hz$  and computing the cross spectrum ('spectrum') where  $P_{xy}$  has units of  $V^2$ , if  $x$  and  $y$  are measured in  $V$  and  $fs$  is measured in  $Hz$ . Defaults to 'density'
- axis** : int, optional  
Axis along which the CSD is computed for both inputs; the default is over the last axis (i.e. `axis=-1`).

**Returns**

- f** : ndarray  
Array of sample frequencies.
- Pxy** : ndarray  
Cross spectral density or cross power spectrum of  $x, y$ .

**See also:**

[`periodogram`](#)

Simple, optionally modified periodogram

[`lombscargle`](#)

Lomb-Scargle periodogram for unevenly sampled data

[`welch`](#)

Power spectral density by Welch's method. [Equivalent to `csd(x,x)`]

[`coherence`](#)

Magnitude squared coherence by Welch's method.



**Notes**

By convention,  $P_{xy}$  is computed with the conjugate FFT of  $X$  multiplied by the FFT of  $Y$ .

If the input series differ in length, the shorter series will be zero-padded to match.

An appropriate amount of overlap will depend on the choice of window and on your requirements. For the default ‘hann’ window an overlap of 50% is a reasonable trade off between accurately estimating the signal power, while not over counting any of the data. Narrower windows may require a larger overlap.

New in version 0.16.0.

**References**

[R196], [R197]

**Examples**

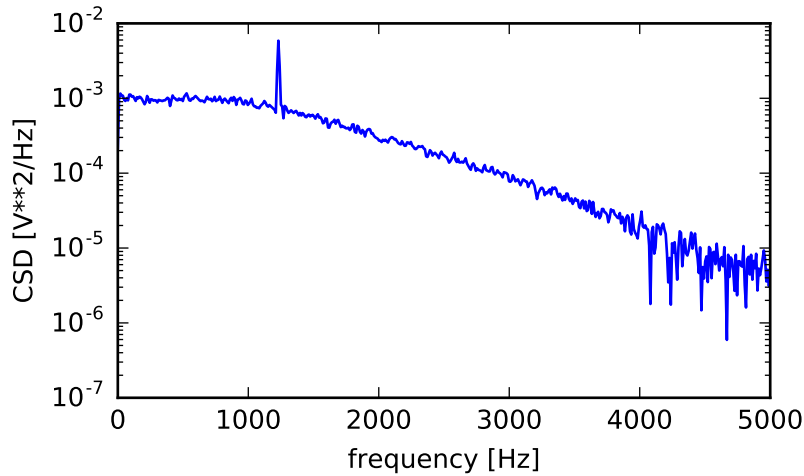
```
>>> from scipy import signal
>>> import matplotlib.pyplot as plt
```

Generate two test signals with some common features.

```
>>> fs = 10e3
>>> N = 1e5
>>> amp = 20
>>> freq = 1234.0
>>> noise_power = 0.001 * fs / 2
>>> time = np.arange(N) / fs
>>> b, a = signal.butter(2, 0.25, 'low')
>>> x = np.random.normal(scale=np.sqrt(noise_power), size=time.shape)
>>> y = signal.lfilter(b, a, x)
>>> x += amp*np.sin(2*np.pi*freq*time)
>>> y += np.random.normal(scale=0.1*np.sqrt(noise_power), size=time.shape)
```

Compute and plot the magnitude of the cross spectral density.

```
>>> f, Pxy = signal.csd(x, y, fs, nperseg=1024)
>>> plt.semilogy(f, np.abs(Pxy))
>>> plt.xlabel('frequency [Hz]')
>>> plt.ylabel('CSD [V**2/Hz]')
>>> plt.show()
```



`scipy.signal.coherence` ( $x$ ,  $y$ ,  $fs=1.0$ ,  $window='hann'$ ,  $nperseg=256$ ,  $noverlap=None$ ,  $nfft=None$ ,  $detrend='constant'$ ,  $axis=-1$ )

Estimate the magnitude squared coherence estimate,  $C_{xy}$ , of discrete-time signals  $X$  and  $Y$  using Welch's method.

$C_{xy} = \text{abs}(P_{xy})^{**2} / (P_{xx} * P_{yy})$ , where  $P_{xx}$  and  $P_{yy}$  are power spectral density estimates of  $X$  and  $Y$ , and  $P_{xy}$  is the cross spectral density estimate of  $X$  and  $Y$ .

**Parameters**

- x** : array\_like  
Time series of measurement values
- y** : array\_like  
Time series of measurement values
- fs** : float, optional  
Sampling frequency of the  $x$  and  $y$  time series. Defaults to 1.0.
- window** : str or tuple or array\_like, optional  
Desired window to use. See `get_window` for a list of windows and required parameters. If  $window$  is array\_like it will be used directly as the window and its length will be used for  $nperseg$ . Defaults to 'hann'.
- nperseg** : int, optional  
Length of each segment. Defaults to 256.
- noverlap** : int, optional  
Number of points to overlap between segments. If `None`,  $noverlap = nperseg // 2$ . Defaults to `None`.
- nfft** : int, optional  
Length of the FFT used, if a zero padded FFT is desired. If `None`, the FFT length is  $nperseg$ . Defaults to `None`.
- detrend** : str or function or `False`, optional  
Specifies how to detrend each segment. If `detrend` is a string, it is passed as the `type` argument to `detrend`. If it is a function, it takes a segment and returns a detrended segment. If `detrend` is `False`, no detrending is done. Defaults to 'constant'.
- axis** : int, optional  
Axis along which the coherence is computed for both inputs; the default is over the last axis (i.e.  $axis=-1$ ).

**Returns**

- f** : ndarray  
Array of sample frequencies.
- Cxy** : ndarray

Magnitude squared coherence of  $x$  and  $y$ .

**See also:**

*periodogram*

Simple, optionally modified periodogram

*lombscargle*

Lomb-Scargle periodogram for unevenly sampled data

*welch*

Power spectral density by Welch's method.

*csd*

Cross spectral density by Welch's method.

**Notes**

An appropriate amount of overlap will depend on the choice of window and on your requirements. For the default 'hann' window an overlap of 50% is a reasonable trade off between accurately estimating the signal power, while not over counting any of the data. Narrower windows may require a larger overlap.

New in version 0.16.0.

**References**

[R191], [R192]

**Examples**

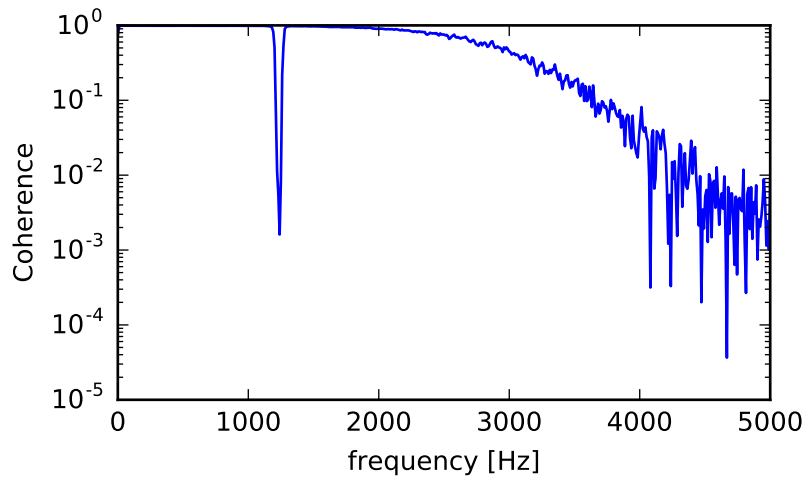
```
>>> from scipy import signal
>>> import matplotlib.pyplot as plt
```

Generate two test signals with some common features.

```
>>> fs = 10e3
>>> N = 1e5
>>> amp = 20
>>> freq = 1234.0
>>> noise_power = 0.001 * fs / 2
>>> time = np.arange(N) / fs
>>> b, a = signal.butter(2, 0.25, 'low')
>>> x = np.random.normal(scale=np.sqrt(noise_power), size=time.shape)
>>> y = signal.lfilter(b, a, x)
>>> x += amp*np.sin(2*np.pi*freq*time)
>>> y += np.random.normal(scale=0.1*np.sqrt(noise_power), size=time.shape)
```

Compute and plot the coherence.

```
>>> f, Cxy = signal.coherence(x, y, fs, nperseg=1024)
>>> plt.semilogy(f, Cxy)
>>> plt.xlabel('frequency [Hz]')
>>> plt.ylabel('Coherence')
>>> plt.show()
```



```
scipy.signal.spectrogram(x, fs=1.0, window=('tukey', 0.25), nperseg=256, noverlap=None,
                        nfft=None, detrend='constant', return_onesided=True, scaling='density', axis=-1, mode='psd')
```

Compute a spectrogram with consecutive Fourier transforms.

Spectrograms can be used as a way of visualizing the change of a nonstationary signal's frequency content over time.

**Parameters**

- x** : array\_like  
Time series of measurement values
- fs** : float, optional  
Sampling frequency of the *x* time series. Defaults to 1.0.
- window** : str or tuple or array\_like, optional  
Desired window to use. See `get_window` for a list of windows and required parameters. If *window* is array\_like it will be used directly as the window and its length will be used for *nperseg*. Defaults to a Tukey window with shape parameter of 0.25.
- nperseg** : int, optional  
Length of each segment. Defaults to 256.
- noverlap** : int, optional  
Number of points to overlap between segments. If None, `noverlap = nperseg // 8`. Defaults to None.
- nfft** : int, optional  
Length of the FFT used, if a zero padded FFT is desired. If None, the FFT length is *nperseg*. Defaults to None.
- detrend** : str or function or False, optional  
Specifies how to detrend each segment. If *detrend* is a string, it is passed as the *type* argument to `detrend`. If it is a function, it takes a segment and returns a detrended segment. If *detrend* is False, no detrending is done. Defaults to 'constant'.
- return\_onesided** : bool, optional  
If True, return a one-sided spectrum for real data. If False return a two-sided spectrum. Note that for complex data, a two-sided spectrum is always returned.
- scaling** : { 'density', 'spectrum' }, optional

Selects between computing the power spectral density ('density') where  $P_{xx}$  has units of  $V^{**2}/Hz$  and computing the power spectrum ('spectrum') where  $P_{xx}$  has units of  $V^{**2}$ , if  $x$  is measured in  $V$  and  $fs$  is measured in  $Hz$ . Defaults to 'density'

**axis** : int, optional

Axis along which the spectrogram is computed; the default is over the last axis (i.e. `axis=-1`).

**mode** : str, optional

Defines what kind of return values are expected. Options are ['psd', 'complex', 'magnitude', 'angle', 'phase'].

#### Returns

**f** : ndarray

Array of sample frequencies.

**t** : ndarray

Array of segment times.

**Sxx** : ndarray

Spectrogram of  $x$ . By default, the last axis of  $S_{xx}$  corresponds to the segment times.

#### See also:

##### *periodogram*

Simple, optionally modified periodogram

##### *lombscargle*

Lomb-Scargle periodogram for unevenly sampled data

##### *welch*

Power spectral density by Welch's method.

##### *csd*

Cross spectral density by Welch's method.

#### Notes

An appropriate amount of overlap will depend on the choice of window and on your requirements. In contrast to `welch`'s method, where the entire data stream is averaged over, one may wish to use a smaller overlap (or perhaps none at all) when computing a spectrogram, to maintain some statistical independence between individual segments.

New in version 0.16.0.

#### References

[R230]

#### Examples

```
>>> from scipy import signal
>>> import matplotlib.pyplot as plt
```

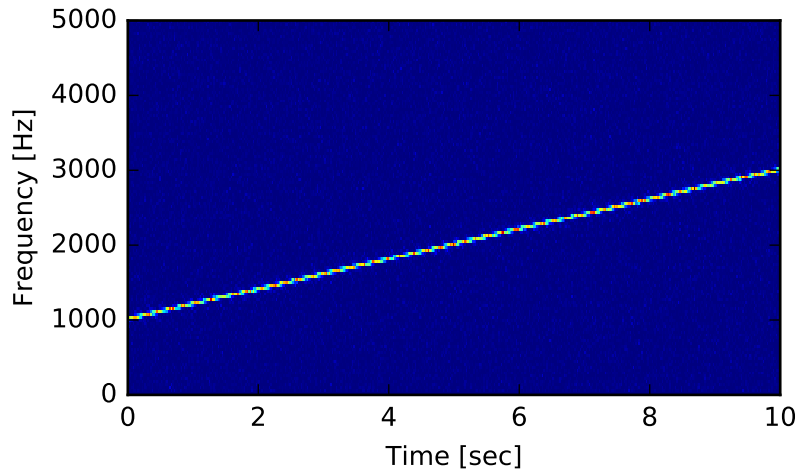
Generate a test signal, a 2 Vrms sine wave whose frequency linearly changes with time from 1kHz to 2kHz, corrupted by 0.001  $V^{**2}/Hz$  of white noise sampled at 10 kHz.

```
>>> fs = 10e3
>>> N = 1e5
>>> amp = 2 * np.sqrt(2)
>>> noise_power = 0.001 * fs / 2
>>> time = np.arange(N) / fs
>>> freq = np.linspace(1e3, 2e3, N)
>>> x = amp * np.sin(2*np.pi*freq*time)
>>> x += np.random.normal(scale=np.sqrt(noise_power), size=time.shape)
```

Compute and plot the spectrogram.

```

>>> f, t, Sxx = signal.spectrogram(x, fs)
>>> plt.pcolormesh(t, f, Sxx)
>>> plt.ylabel('Frequency [Hz]')
>>> plt.xlabel('Time [sec]')
>>> plt.show()
    
```



`scipy.signal.lombscargle(x, y, freqs)`

Computes the Lomb-Scargle periodogram.

The Lomb-Scargle periodogram was developed by Lomb [R223] and further extended by Scargle [R224] to find, and test the significance of weak periodic signals with uneven temporal sampling.

The computed periodogram is unnormalized, it takes the value  $(A**2) * N/4$  for a harmonic signal with amplitude  $A$  for sufficiently large  $N$ .

**Parameters**

- x** : array\_like  
Sample times.
- y** : array\_like  
Measurement values.
- freqs** : array\_like  
Angular frequencies for output periodogram.

**Returns**

- pgram** : array\_like  
Lomb-Scargle periodogram.

**Raises**

- ValueError**  
If the input arrays  $x$  and  $y$  do not have the same shape.

#### Notes

This subroutine calculates the periodogram using a slightly modified algorithm due to Townsend [R225] which allows the periodogram to be calculated using only a single pass through the input arrays for each frequency.

The algorithm running time scales roughly as  $O(x * \text{freqs})$  or  $O(N^2)$  for a large number of samples and frequencies.

#### References

[R223], [R224], [R225]

**Examples**

```
>>> import scipy.signal
>>> import matplotlib.pyplot as plt
```

First define some input parameters for the signal:

```
>>> A = 2.
>>> w = 1.
>>> phi = 0.5 * np.pi
>>> nin = 1000
>>> nout = 100000
>>> frac_points = 0.9 # Fraction of points to select
```

Randomly select a fraction of an array with timesteps:

```
>>> r = np.random.rand(nin)
>>> x = np.linspace(0.01, 10*np.pi, nin)
>>> x = x[r >= frac_points]
>>> normval = x.shape[0] # For normalization of the periodogram
```

Plot a sine wave for the selected times:

```
>>> y = A * np.sin(w*x+phi)
```

Define the array of frequencies for which to compute the periodogram:

```
>>> f = np.linspace(0.01, 10, nout)
```

Calculate Lomb-Scargle periodogram:

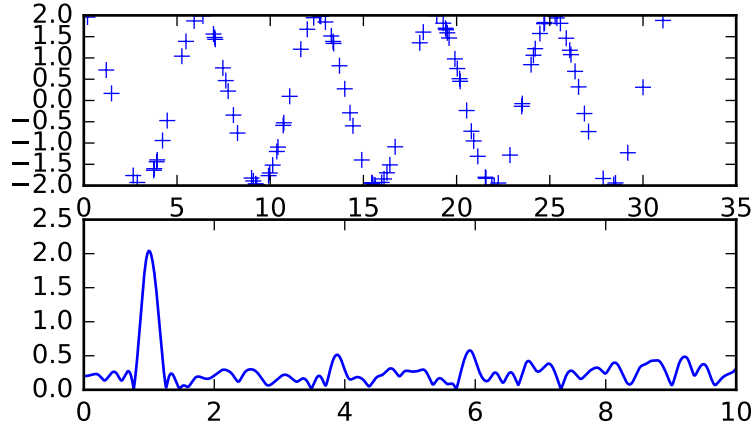
```
>>> import scipy.signal as signal
>>> pgram = signal.lombscargle(x, y, f)
```

Now make a plot of the input data:

```
>>> plt.subplot(2, 1, 1)
<matplotlib.axes.AxesSubplot object at 0x102154f50>
>>> plt.plot(x, y, 'b+')
[<matplotlib.lines.Line2D object at 0x102154a10>]
```

Then plot the normalized periodogram:

```
>>> plt.subplot(2, 1, 2)
<matplotlib.axes.AxesSubplot object at 0x104b0a990>
>>> plt.plot(f, np.sqrt(4*(pgram/normval)))
[<matplotlib.lines.Line2D object at 0x104b2f910>]
>>> plt.show()
```



`scipy.signal.vectorstrength(events, period)`

Determine the vector strength of the events corresponding to the given period.

The vector strength is a measure of phase synchrony, how well the timing of the events is synchronized to a single period of a periodic signal.

If multiple periods are used, calculate the vector strength of each. This is called the “resonating vector strength”.

**Parameters**    **events** : 1D array\_like

An array of time points containing the timing of the events.

**period** : float or array\_like

The period of the signal that the events should synchronize to. The period is in the same units as *events*. It can also be an array of periods, in which case the outputs are arrays of the same length.

**Returns**        **strength** : float or 1D array

The strength of the synchronization. 1.0 is perfect synchronization and 0.0 is no synchronization. If *period* is an array, this is also an array with each element containing the vector strength at the corresponding period.

**phase** : float or array

The phase that the events are most strongly synchronized to in radians. If *period* is an array, this is also an array with each element containing the phase for the corresponding period.

### References

*van Hemmen, JL, Longtin, A, and Vollmayr, AN. Testing resonating vector*

*strength: Auditory system, electric fish, and noise.* Chaos 21, 047508 (2011); doi: 10.1063/1.3670512

*van Hemmen, JL. Vector strength after Goldberg, Brown, and von Mises:*

*biological and mathematical perspectives.* Biol Cybern. 2013 Aug;107(4):385-96. doi: 10.1007/s00422-013-0561-7.

*van Hemmen, JL and Vollmayr, AN. Resonating vector strength: what happens*

*when we vary the “probing” frequency while keeping the spike times fixed.* Biol Cybern. 2013 Aug;107(4):491-94. doi: 10.1007/s00422-013-0560-8



## 5.21 Sparse matrices (`scipy.sparse`)

SciPy 2-D sparse matrix package for numeric data.

### 5.21.1 Contents

#### Sparse matrix classes

---

<code>bsr_matrix(arg1[, shape, dtype, copy, blocksize])</code>	Block Sparse Row matrix
<code>coo_matrix(arg1[, shape, dtype, copy])</code>	A sparse matrix in COOrdinate format.
<code>csc_matrix(arg1[, shape, dtype, copy])</code>	Compressed Sparse Column matrix
<code>csr_matrix(arg1[, shape, dtype, copy])</code>	Compressed Sparse Row matrix
<code>dia_matrix(arg1[, shape, dtype, copy])</code>	Sparse matrix with DIAgonal storage
<code>dok_matrix(arg1[, shape, dtype, copy])</code>	Dictionary Of Keys based sparse matrix.
<code>lil_matrix(arg1[, shape, dtype, copy])</code>	Row-based linked list sparse matrix
<code>spmatrix([maxprint])</code>	This class provides a base class for all sparse matrices.

---

**class** `scipy.sparse.bsr_matrix` (*arg1*, *shape=None*, *dtype=None*, *copy=False*, *blocksize=None*)  
 Block Sparse Row matrix

*This can be instantiated in several ways:*

*`bsr_matrix(D, [blocksize=(R,C)])`*

where D is a dense matrix or 2-D ndarray.

*`bsr_matrix(S, [blocksize=(R,C)])`*

with another sparse matrix S (equivalent to `S.tobsr()`)

*`bsr_matrix((M, N), [blocksize=(R,C), dtype])`*

to construct an empty matrix with shape (M, N) dtype is optional, defaulting to `dtype='d'`.

*`bsr_matrix((data, ij), [blocksize=(R,C), shape=(M, N)])`*

where data and ij satisfy `a[ij[0, k], ij[1, k]] = data[k]`

*`bsr_matrix((data, indices, indptr), [shape=(M, N)])`*

is the standard BSR representation where the block column indices for row i are stored in `indices[indptr[i]:indptr[i+1]]` and their corresponding block values are stored in `data[indptr[i]: indptr[i+1]]`. If the shape parameter is not supplied, the matrix dimensions are inferred from the index arrays.

#### Notes

Sparse matrices can be used in arithmetic operations: they support addition, subtraction, multiplication, division, and matrix power.

#### Summary of BSR format

The Block Compressed Row (BSR) format is very similar to the Compressed Sparse Row (CSR) format. BSR is appropriate for sparse matrices with dense sub matrices like the last example below. Block matrices often arise in vector-valued finite element discretizations. In such cases, BSR is considerably more efficient than CSR and CSC for many sparse arithmetic operations.

#### Blocksize

The blocksize (R,C) must evenly divide the shape of the matrix (M,N). That is, R and C must satisfy the relationship  $M \% R = 0$  and  $N \% C = 0$ .

If no blocksize is specified, a simple heuristic is applied to determine an appropriate blocksize.

**Examples**

```
>>> from scipy.sparse import bsr_matrix
>>> bsr_matrix((3, 4), dtype=np.int8).toarray()
array([[0, 0, 0, 0],
       [0, 0, 0, 0],
       [0, 0, 0, 0]], dtype=int8)

>>> row = np.array([0, 0, 1, 2, 2, 2])
>>> col = np.array([0, 2, 2, 0, 1, 2])
>>> data = np.array([1, 2, 3, 4, 5, 6])
>>> bsr_matrix((data, (row, col)), shape=(3, 3)).toarray()
array([[1, 0, 2],
       [0, 0, 3],
       [4, 5, 6]])

>>> indptr = np.array([0, 2, 3, 6])
>>> indices = np.array([0, 2, 2, 0, 1, 2])
>>> data = np.array([1, 2, 3, 4, 5, 6]).repeat(4).reshape(6, 2, 2)
>>> bsr_matrix((data, indices, indptr), shape=(6, 6)).toarray()
array([[1, 1, 0, 0, 2, 2],
       [1, 1, 0, 0, 2, 2],
       [0, 0, 0, 0, 3, 3],
       [0, 0, 0, 0, 3, 3],
       [4, 4, 5, 5, 6, 6],
       [4, 4, 5, 5, 6, 6]])
```

**Attributes**

---

<code>nnz</code>	Number of stored values, including explicit zeros.
<code>has_sorted_indices</code>	Determine whether the matrix has sorted indices

---

`bsr_matrix.nnz`  
 Number of stored values, including explicit zeros.

**See also:**

`count_nonzero`  
 Number of non-zero entries

`bsr_matrix.has_sorted_indices`  
 Determine whether the matrix has sorted indices

**Returns**

- True: if the indices of the matrix are in sorted order
- False: otherwise

<code>dtype</code>	(dtype) Data type of the matrix
<code>shape</code>	(2-tuple) Shape of the matrix
<code>ndim</code>	(int) Number of dimensions (this is always 2)
<code>data</code>	Data array of the matrix
<code>indices</code>	BSR format index array
<code>indptr</code>	BSR format index pointer array
<code>blocksize</code>	Block size of the matrix

**Methods**

<code>arcsin()</code>	Element-wise arcsin.
<code>arcsinh()</code>	Element-wise arcsinh.
<code>arctan()</code>	Element-wise arctan.
<code>arctanh()</code>	Element-wise arctanh.
<code>asformat(format)</code>	Return this matrix in a given sparse format
<code>asfptype()</code>	Upcast matrix to a floating point format (if necessary)
<code>astype(t)</code>	
<code>ceil()</code>	Element-wise ceil.
<code>check_format([full_check])</code>	check whether the matrix format is valid
<code>conj()</code>	
<code>conjugate()</code>	
<code>copy()</code>	
<code>count_nonzero()</code>	Number of non-zero entries, equivalent to
<code>deg2rad()</code>	Element-wise deg2rad.
<code>diagonal()</code>	Returns the main diagonal of the matrix
<code>dot(other)</code>	Ordinary dot product
<code>eliminate_zeros()</code>	
<code>expm1()</code>	Element-wise expm1.
<code>floor()</code>	Element-wise floor.
<code>getH()</code>	
<code>get_shape()</code>	
<code>getcol(j)</code>	Returns a copy of column j of the matrix, as an (m x 1) sparse matrix (column vector).
<code>getdata(ind)</code>	
<code>getformat()</code>	
<code>getmaxprint()</code>	
<code>getnnz([axis])</code>	Number of stored values, including explicit zeros.
<code>getrow(i)</code>	Returns a copy of row i of the matrix, as a (1 x n) sparse matrix (row vector).
<code>log1p()</code>	Element-wise log1p.
<code>matmat(other)</code>	
<code>matvec(other)</code>	
<code>max([axis, out])</code>	Return the maximum of the matrix or maximum along an axis.
<code>maximum(other)</code>	
<code>mean([axis, dtype, out])</code>	Compute the arithmetic mean along the specified axis.
<code>min([axis, out])</code>	Return the minimum of the matrix or maximum along an axis.
<code>minimum(other)</code>	
<code>multiply(other)</code>	Point-wise multiplication by another matrix, vector, or scalar.
<code>nonzero()</code>	nonzero indices
<code>power(n[, dtype])</code>	This function performs element-wise power.
<code>prune()</code>	Remove empty space after all non-zero elements.
<code>rad2deg()</code>	Element-wise rad2deg.
<code>reshape(shape[, order])</code>	Gives a new shape to a sparse matrix without changing its data.
<code>rint()</code>	Element-wise rint.
<code>set_shape(shape)</code>	
<code>setdiag(values[, k])</code>	Set diagonal or off-diagonal elements of the array.
<code>sign()</code>	Element-wise sign.
<code>sin()</code>	Element-wise sin.
<code>sinh()</code>	Element-wise sinh.
<code>sort_indices()</code>	Sort the indices of this matrix <i>in place</i>
<code>sorted_indices()</code>	Return a copy of this matrix with sorted indices
<code>sqrt()</code>	Element-wise sqrt.
<code>sum([axis, dtype, out])</code>	Sum the matrix elements over a given axis.

Continued on next page

Table 5.155 – continued from previous page

<code>sum_duplicates()</code>	Eliminate duplicate matrix entries by adding them together
<code>tan()</code>	Element-wise tan.
<code>tanh()</code>	Element-wise tanh.
<code>toarray([order, out])</code>	See the docstring for <code>spmatrix.toarray</code> .
<code>tobsr([blocksize, copy])</code>	Convert this matrix into Block Sparse Row Format.
<code>tocoo([copy])</code>	Convert this matrix to COOrdinate format.
<code>tocsc([copy])</code>	Convert this matrix to Compressed Sparse Column format.
<code>tocsr([copy])</code>	Convert this matrix to Compressed Sparse Row format.
<code>todense([order, out])</code>	Return a dense matrix representation of this matrix.
<code>todia([copy])</code>	Convert this matrix to sparse DIAgonal format.
<code>todok([copy])</code>	Convert this matrix to Dictionary Of Keys format.
<code>tolil([copy])</code>	Convert this matrix to LInked List format.
<code>transpose([axes, copy])</code>	Reverses the dimensions of the sparse matrix.
<code>trunc()</code>	Element-wise trunc.

`bsr_matrix.arcsin()`  
Element-wise arcsin.

See `numpy.arcsin` for more information.

`bsr_matrix.arcsinh()`  
Element-wise arcsinh.

See `numpy.arcsinh` for more information.

`bsr_matrix.arctan()`  
Element-wise arctan.

See `numpy.arctan` for more information.

`bsr_matrix.arctanh()`  
Element-wise arctanh.

See `numpy.arctanh` for more information.

`bsr_matrix.asformat(format)`  
Return this matrix in a given sparse format

**Parameters** `format` : {string, None}  
*desired sparse matrix format*

- None for no format conversion
- “csr” for `csr_matrix` format
- “csc” for `csc_matrix` format
- “lil” for `lil_matrix` format
- “dok” for `dok_matrix` format and so on

`bsr_matrix.asfptype()`  
Upcast matrix to a floating point format (if necessary)

`bsr_matrix.astype(t)`

`bsr_matrix.ceil()`  
Element-wise ceil.

See `numpy.ceil` for more information.

`bsr_matrix.check_format` (*full\_check=True*)

check whether the matrix format is valid

**Parameters:**

*full\_check*: True - rigorous check, O(N) operations : default False - basic check, O(1) operations

`bsr_matrix.conj` ()

`bsr_matrix.conjugate` ()

`bsr_matrix.copy` ()

`bsr_matrix.count_nonzero` ()

Number of non-zero entries, equivalent to

`np.count_nonzero(a.toarray())`

Unlike `getnnz`() and the `nnz` property, which return the number of stored entries (the length of the data attribute), this method counts the actual number of non-zero entries in data.

`bsr_matrix.deg2rad` ()

Element-wise `deg2rad`.

See `numpy.deg2rad` for more information.

`bsr_matrix.diagonal` ()

Returns the main diagonal of the matrix

`bsr_matrix.dot` (*other*)

Ordinary dot product

**Examples**

```
>>> import numpy as np
>>> from scipy.sparse import csr_matrix
>>> A = csr_matrix([[1, 2, 0], [0, 0, 3], [4, 0, 5]])
>>> v = np.array([1, 0, -1])
>>> A.dot(v)
array([ 1, -3, -1], dtype=int64)
```

`bsr_matrix.eliminate_zeros` ()

`bsr_matrix.expm1` ()

Element-wise `expm1`.

See `numpy.expm1` for more information.

`bsr_matrix.floor` ()

Element-wise floor.

See `numpy.floor` for more information.

`bsr_matrix.getH` ()

`bsr_matrix.get_shape` ()

`bsr_matrix.getcol(j)`

Returns a copy of column *j* of the matrix, as an (*m* x 1) sparse matrix (column vector).

`bsr_matrix.getdata(ind)`

`bsr_matrix.getformat()`

`bsr_matrix.getmaxprint()`

`bsr_matrix.getnnz(axis=None)`

Number of stored values, including explicit zeros.

**Parameters** `axis` : None, 0, or 1

Select between the number of values across the whole matrix, in each column, or in each row.

**See also:**

`count_nonzero`

Number of non-zero entries

`bsr_matrix.getrow(i)`

Returns a copy of row *i* of the matrix, as a (1 x *n*) sparse matrix (row vector).

`bsr_matrix.log1p()`

Element-wise log1p.

See `numpy.log1p` for more information.

`bsr_matrix.matmat(other)`

`bsr_matrix.matvec(other)`

`bsr_matrix.max(axis=None, out=None)`

Return the maximum of the matrix or maximum along an axis. This takes all elements into account, not just the non-zero ones.

**Parameters** `axis` : {-2, -1, 0, 1, None} optional

Axis along which the sum is computed. The default is to compute the maximum over all the matrix elements, returning a scalar (i.e. `axis = None`).

`out` : None, optional

This argument is in the signature *solely* for NumPy compatibility reasons. Do not pass in anything except for the default value, as this argument is not used.

**Returns** `amax` : `coo_matrix` or scalar

Maximum of *a*. If `axis` is None, the result is a scalar value. If `axis` is given, the result is a `sparse.coo_matrix` of dimension `a.ndim - 1`.

**See also:**

`min` The minimum value of a sparse matrix along a given axis.

`np.matrix.max`

NumPy's implementation of 'max' for matrices

`bsr_matrix.maximum(other)`

`bsr_matrix.mean` (*axis=None, dtype=None, out=None*)

Compute the arithmetic mean along the specified axis.

Returns the average of the matrix elements. The average is taken over all elements in the matrix by default, otherwise over the specified axis. *float64* intermediate and return values are used for integer inputs.

**Parameters**

- axis** : {-2, -1, 0, 1, None} optional  
Axis along which the mean is computed. The default is to compute the mean of all elements in the matrix (i.e. *axis = None*).
- dtype** : data-type, optional  
Type to use in computing the mean. For integer inputs, the default is *float64*; for floating point inputs, it is the same as the input dtype.
- out** : np.matrix, optional  
Alternative output matrix in which to place the result. It must have the same shape as the expected output, but the type of the output values will be cast if necessary.

**Returns** **m** : np.matrix

**See also:**

**`np.matrix.mean`**

NumPy's implementation of 'mean' for matrices

`bsr_matrix.min` (*axis=None, out=None*)

Return the minimum of the matrix or maximum along an axis. This takes all elements into account, not just the non-zero ones.

**Parameters**

- axis** : {-2, -1, 0, 1, None} optional  
Axis along which the sum is computed. The default is to compute the minimum over all the matrix elements, returning a scalar (i.e. *axis = None*).
- out** : None, optional  
This argument is in the signature *solely* for NumPy compatibility reasons. Do not pass in anything except for the default value, as this argument is not used.

**Returns** **amin** : `coo_matrix` or scalar  
Minimum of *a*. If *axis* is None, the result is a scalar value. If *axis* is given, the result is a `sparse.coo_matrix` of dimension `a.ndim - 1`.

**See also:**

**`max`** The maximum value of a sparse matrix along a given axis.

**`np.matrix.min`**

NumPy's implementation of 'min' for matrices

`bsr_matrix.minimum` (*other*)

`bsr_matrix.multiply` (*other*)

Point-wise multiplication by another matrix, vector, or scalar.

`bsr_matrix.nonzero` ()

nonzero indices

Returns a tuple of arrays (row,col) containing the indices of the non-zero elements of the matrix.

**Examples**

```
>>> from scipy.sparse import csr_matrix
>>> A = csr_matrix([[1, 2, 0], [0, 0, 3], [4, 0, 5]])
```

```
>>> A.nonzero()
(array([0, 0, 1, 2, 2]), array([0, 1, 2, 0, 2]))
```

`bsr_matrix.power` (*n*, *dtype=None*)  
 This function performs element-wise power.

**Parameters** **n** : *n* is a scalar  
**dtype** : If *dtype* is not specified, the current *dtype* will be preserved.

`bsr_matrix.prune` ()  
 Remove empty space after all non-zero elements.

`bsr_matrix.rad2deg` ()  
 Element-wise `rad2deg`.  
 See `numpy.rad2deg` for more information.

`bsr_matrix.reshape` (*shape*, *order='C'*)  
 Gives a new shape to a sparse matrix without changing its data.

**Parameters** **shape** : length-2 tuple of ints  
 The new shape should be compatible with the original shape.  
**order** : 'C', optional  
 This argument is in the signature *solely* for NumPy compatibility reasons. Do not pass in anything except for the default value, as this argument is not used.  
**Returns** **reshaped\_matrix** : *self* with the new dimensions of *shape*

**See also:**

**`np.matrix.reshape`**  
 NumPy's implementation of 'reshape' for matrices

`bsr_matrix.rint` ()  
 Element-wise `rint`.  
 See `numpy.rint` for more information.

`bsr_matrix.set_shape` (*shape*)

`bsr_matrix.setdiag` (*values*, *k=0*)  
 Set diagonal or off-diagonal elements of the array.

**Parameters** **values** : *array\_like*  
 New values of the diagonal elements.  
 Values may have any length. If the diagonal is longer than *values*, then the remaining diagonal entries will not be set. If *values* is longer than the diagonal, then the remaining values are ignored.  
 If a scalar value is given, all of the diagonal is set to it.  
**k** : int, optional  
 Which off-diagonal to set, corresponding to elements `a[i,i+k]`. Default: 0 (the main diagonal).

`bsr_matrix.sign` ()  
 Element-wise `sign`.  
 See `numpy.sign` for more information.

`bsr_matrix.sin` ()  
 Element-wise `sin`.  
 See `numpy.sin` for more information.



`bsr_matrix.sinh()`  
Element-wise sinh.

See `numpy.sinh` for more information.

`bsr_matrix.sort_indices()`  
Sort the indices of this matrix *in place*

`bsr_matrix.sorted_indices()`  
Return a copy of this matrix with sorted indices

`bsr_matrix.sqrt()`  
Element-wise sqrt.

See `numpy.sqrt` for more information.

`bsr_matrix.sum(axis=None, dtype=None, out=None)`  
Sum the matrix elements over a given axis.

**Parameters** **axis** : {-2, -1, 0, 1, None} optional

Axis along which the sum is computed. The default is to compute the sum of all the matrix elements, returning a scalar (i.e. *axis = None*).

**dtype** : dtype, optional

The type of the returned matrix and of the accumulator in which the elements are summed. The dtype of *a* is used by default unless *a* has an integer dtype of less precision than the default platform integer. In that case, if *a* is signed then the platform integer is used while if *a* is unsigned then an unsigned integer of the same precision as the platform integer is used.

**out** : np.matrix, optional

Alternative output matrix in which to place the result. It must have the same shape as the expected output, but the type of the output values will be cast if necessary.

**Returns** **sum\_along\_axis** : np.matrix

A matrix with the same shape as *self*, with the specified axis removed.

**See also:**

**`np.matrix.sum`**

NumPy's implementation of 'sum' for matrices

`bsr_matrix.sum_duplicates()`  
Eliminate duplicate matrix entries by adding them together

The is an *in place* operation

`bsr_matrix.tan()`  
Element-wise tan.

See `numpy.tan` for more information.

`bsr_matrix.tanh()`  
Element-wise tanh.

See `numpy.tanh` for more information.

`bsr_matrix.toarray(order=None, out=None)`  
See the docstring for `spmatrix.toarray`.

`bsr_matrix.tobsr(blocksize=None, copy=False)`  
Convert this matrix into Block Sparse Row Format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `bsr_matrix`.

If `blocksize=(R, C)` is provided, it will be used for determining block size of the `bsr_matrix`.

`bsr_matrix.tocoo` (*copy=True*)

Convert this matrix to COOrdinate format.

When `copy=False` the data array will be shared between this matrix and the resultant `coo_matrix`.

`bsr_matrix.tocsf` (*copy=False*)

Convert this matrix to Compressed Sparse Column format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `csc_matrix`.

`bsr_matrix.tocsr` (*copy=False*)

Convert this matrix to Compressed Sparse Row format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `csr_matrix`.

`bsr_matrix.todense` (*order=None, out=None*)

Return a dense matrix representation of this matrix.

**Parameters**    **order** : {'C', 'F'}, optional

Whether to store multi-dimensional data in C (row-major) or Fortran (column-major) order in memory. The default is 'None', indicating the NumPy default of C-ordered. Cannot be specified in conjunction with the *out* argument.

**out** : ndarray, 2-dimensional, optional

If specified, uses this array (or `numpy.matrix`) as the output buffer instead of allocating a new array to return. The provided array must have the same shape and dtype as the sparse matrix on which you are calling the method.

**Returns**        **arr** : `numpy.matrix`, 2-dimensional

A NumPy matrix object with the same shape and containing the same data represented by the sparse matrix, with the requested memory order. If *out* was passed and was an array (rather than a `numpy.matrix`), it will be filled with the appropriate values and returned wrapped in a `numpy.matrix` object that shares the same memory.

`bsr_matrix.todia` (*copy=False*)

Convert this matrix to sparse DIAgonal format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `dia_matrix`.

`bsr_matrix.todok` (*copy=False*)

Convert this matrix to Dictionary Of Keys format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `dok_matrix`.

`bsr_matrix.tolil` (*copy=False*)

Convert this matrix to LInked List format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `lil_matrix`.

`bsr_matrix.ttranspose` (*axes=None, copy=False*)

Reverses the dimensions of the sparse matrix.

**Parameters**    **axes** : None, optional

This argument is in the signature *solely* for NumPy compatibility reasons. Do not pass in anything except for the default value.

**copy** : bool, optional

Indicates whether or not attributes of *self* should be copied whenever possible. The degree to which attributes are copied varies depending on the type of sparse matrix being used.

**Returns** `p` : *self* with the dimensions reversed.

**See also:**

`np.matrix.transpose`

NumPy's implementation of 'transpose' for matrices

`bsr_matrix.trunc()`

Element-wise trunc.

See `numpy.trunc` for more information.

**class** `scipy.sparse.coo_matrix` (*arg1*, *shape=None*, *dtype=None*, *copy=False*)

A sparse matrix in COOrdinate format.

Also known as the 'ijv' or 'triplet' format.

*This can be instantiated in several ways:*

`coo_matrix(D)`

with a dense matrix D

`coo_matrix(S)` with another sparse matrix S (equivalent to `S.tocoo()`)

`coo_matrix((M, N), [dtype])`

to construct an empty matrix with shape (M, N) dtype is optional, defaulting to dtype='d'.

`coo_matrix((data, (i, j)), [shape=(M, N)])`

*to construct from three arrays:*

1.`data[:]` the entries of the matrix, in any order

2.`i[:]` the row indices of the matrix entries

3.`j[:]` the column indices of the matrix entries

Where `A[i[k], j[k]] = data[k]`. When shape is not specified, it is inferred from the index arrays

**Notes**

Sparse matrices can be used in arithmetic operations: they support addition, subtraction, multiplication, division, and matrix power.

**Advantages of the COO format**

- facilitates fast conversion among sparse formats
- permits duplicate entries (see example)
- very fast conversion to and from CSR/CSC formats

**Disadvantages of the COO format**

• *does not directly support:*

– arithmetic operations

– slicing

**Intended Usage**

- COO is a fast format for constructing sparse matrices
- Once a matrix has been constructed, convert to CSR or CSC format for fast arithmetic and matrix vector operations

- By default when converting to CSR or CSC format, duplicate (i,j) entries will be summed together. This facilitates efficient construction of finite element matrices and the like. (see example)

**Examples**

```
>>> from scipy.sparse import coo_matrix
>>> coo_matrix((3, 4), dtype=np.int8).toarray()
array([[0, 0, 0, 0],
       [0, 0, 0, 0],
       [0, 0, 0, 0]], dtype=int8)

>>> row = np.array([0, 3, 1, 0])
>>> col = np.array([0, 3, 1, 2])
>>> data = np.array([4, 5, 7, 9])
>>> coo_matrix((data, (row, col)), shape=(4, 4)).toarray()
array([[4, 0, 9, 0],
       [0, 7, 0, 0],
       [0, 0, 0, 0],
       [0, 0, 0, 5]])

>>> # example with duplicates
>>> row = np.array([0, 0, 1, 3, 1, 0, 0])
>>> col = np.array([0, 2, 1, 3, 1, 0, 0])
>>> data = np.array([1, 1, 1, 1, 1, 1, 1])
>>> coo_matrix((data, (row, col)), shape=(4, 4)).toarray()
array([[3, 0, 1, 0],
       [0, 2, 0, 0],
       [0, 0, 0, 0],
       [0, 0, 0, 1]])
```

**Attributes**

nnz Number of stored values, including explicit zeros.

`coo_matrix.nnz`  
Number of stored values, including explicit zeros.

**See also:**

`count_nonzero`  
Number of non-zero entries

<code>dtype</code>	(dtype) Data type of the matrix
<code>shape</code>	(2-tuple) Shape of the matrix
<code>ndim</code>	(int) Number of dimensions (this is always 2)
<code>data</code>	COO format data array of the matrix
<code>row</code>	COO format row index array of the matrix
<code>col</code>	COO format column index array of the matrix

**Methods**

---

<code>arcsin()</code>	Element-wise arcsin.
<code>arcsinh()</code>	Element-wise arcsinh.

Continued on next page

Table 5.157 – continued from previous page

<code>arctan()</code>	Element-wise arctan.
<code>arctanh()</code>	Element-wise arctanh.
<code>asformat(format)</code>	Return this matrix in a given sparse format
<code>asfptype()</code>	Upcast matrix to a floating point format (if necessary)
<code>astype(t)</code>	
<code>ceil()</code>	Element-wise ceil.
<code>conj()</code>	
<code>conjugate()</code>	
<code>copy()</code>	
<code>count_nonzero()</code>	Number of non-zero entries, equivalent to
<code>deg2rad()</code>	Element-wise deg2rad.
<code>diagonal()</code>	Returns the main diagonal of the matrix
<code>dot(other)</code>	Ordinary dot product
<code>eliminate_zeros()</code>	Remove zero entries from the matrix
<code>expm1()</code>	Element-wise expm1.
<code>floor()</code>	Element-wise floor.
<code>getH()</code>	
<code>get_shape()</code>	
<code>getcol(j)</code>	Returns a copy of column j of the matrix, as an (m x 1) sparse matrix (column vector).
<code>getformat()</code>	
<code>getmaxprint()</code>	
<code>getnnz([axis])</code>	Number of stored values, including explicit zeros.
<code>getrow(i)</code>	Returns a copy of row i of the matrix, as a (1 x n) sparse matrix (row vector).
<code>log1p()</code>	Element-wise log1p.
<code>max([axis, out])</code>	Return the maximum of the matrix or maximum along an axis.
<code>maximum(other)</code>	
<code>mean([axis, dtype, out])</code>	Compute the arithmetic mean along the specified axis.
<code>min([axis, out])</code>	Return the minimum of the matrix or maximum along an axis.
<code>minimum(other)</code>	
<code>multiply(other)</code>	Point-wise multiplication by another matrix
<code>nonzero()</code>	nonzero indices
<code>power(n[, dtype])</code>	This function performs element-wise power.
<code>rad2deg()</code>	Element-wise rad2deg.
<code>reshape(shape[, order])</code>	Gives a new shape to a sparse matrix without changing its data.
<code>rint()</code>	Element-wise rint.
<code>set_shape(shape)</code>	
<code>setdiag(values[, k])</code>	Set diagonal or off-diagonal elements of the array.
<code>sign()</code>	Element-wise sign.
<code>sin()</code>	Element-wise sin.
<code>sinh()</code>	Element-wise sinh.
<code>sqrt()</code>	Element-wise sqrt.
<code>sum([axis, dtype, out])</code>	Sum the matrix elements over a given axis.
<code>sum_duplicates()</code>	Eliminate duplicate matrix entries by adding them together
<code>tan()</code>	Element-wise tan.
<code>tanh()</code>	Element-wise tanh.
<code>toarray([order, out])</code>	See the docstring for <code>spmatrix.toarray</code> .
<code>tobsr([blocksize, copy])</code>	Convert this matrix to Block Sparse Row format.
<code>tocoo([copy])</code>	Convert this matrix to COOrdinate format.
<code>tocsc([copy])</code>	Convert this matrix to Compressed Sparse Column format
<code>tocsr([copy])</code>	Convert this matrix to Compressed Sparse Row format
<code>todense([order, out])</code>	Return a dense matrix representation of this matrix.
<code>todia([copy])</code>	Convert this matrix to sparse DIAgonal format.

Continued on next page

Table 5.157 – continued from previous page

<code>todok([copy])</code>	Convert this matrix to Dictionary Of Keys format.
<code>tolil([copy])</code>	Convert this matrix to LInked List format.
<code>transpose([axes, copy])</code>	Reverses the dimensions of the sparse matrix.
<code>trunc()</code>	Element-wise trunc.

`coo_matrix.arcsin()`  
Element-wise arcsin.

See `numpy.arcsin` for more information.

`coo_matrix.arcsinh()`  
Element-wise arcsinh.

See `numpy.arcsinh` for more information.

`coo_matrix.arctan()`  
Element-wise arctan.

See `numpy.arctan` for more information.

`coo_matrix.arctanh()`  
Element-wise arctanh.

See `numpy.arctanh` for more information.

`coo_matrix.asformat(format)`  
Return this matrix in a given sparse format

**Parameters** `format` : {string, None}  
*desired sparse matrix format*

- None for no format conversion
- “csr” for `csr_matrix` format
- “csc” for `csc_matrix` format
- “lil” for `lil_matrix` format
- “dok” for `dok_matrix` format and so on

`coo_matrix.asfptype()`  
Upcast matrix to a floating point format (if necessary)

`coo_matrix.astype(t)`

`coo_matrix.ceil()`  
Element-wise ceil.

See `numpy.ceil` for more information.

`coo_matrix.conj()`

`coo_matrix.conjugate()`

`coo_matrix.copy()`

`coo_matrix.count_nonzero()`  
Number of non-zero entries, equivalent to  
`np.count_nonzero(a.toarray())`

Unlike `getnnz()` and the `nnz` property, which return the number of stored entries (the length of the data attribute), this method counts the actual number of non-zero entries in data.

`coo_matrix.deg2rad()`

Element-wise `deg2rad`.

See `numpy.deg2rad` for more information.

`coo_matrix.diagonal()`

Returns the main diagonal of the matrix

`coo_matrix.dot(other)`

Ordinary dot product

### Examples

```
>>> import numpy as np
>>> from scipy.sparse import csr_matrix
>>> A = csr_matrix([[1, 2, 0], [0, 0, 3], [4, 0, 5]])
>>> v = np.array([1, 0, -1])
>>> A.dot(v)
array([ 1, -3, -1], dtype=int64)
```

`coo_matrix.eliminate_zeros()`

Remove zero entries from the matrix

This is an *in place* operation

`coo_matrix.expm1()`

Element-wise `expm1`.

See `numpy.expm1` for more information.

`coo_matrix.floor()`

Element-wise `floor`.

See `numpy.floor` for more information.

`coo_matrix.getH()`

`coo_matrix.get_shape()`

`coo_matrix.getcol(j)`

Returns a copy of column `j` of the matrix, as an  $(m \times 1)$  sparse matrix (column vector).

`coo_matrix.getformat()`

`coo_matrix.getmaxprint()`

`coo_matrix.getnnz(axis=None)`

Number of stored values, including explicit zeros.

**Parameters** `axis` : None, 0, or 1

Select between the number of values across the whole matrix, in each column, or in each row.

**See also:**

`count_nonzero`

Number of non-zero entries

`coo_matrix.getrow(i)`

Returns a copy of row *i* of the matrix, as a (1 x *n*) sparse matrix (row vector).

`coo_matrix.log1p()`

Element-wise `log1p`.

See `numpy.log1p` for more information.

`coo_matrix.max(axis=None, out=None)`

Return the maximum of the matrix or maximum along an axis. This takes all elements into account, not just the non-zero ones.

**Parameters** **axis** : {-2, -1, 0, 1, None} optional

Axis along which the sum is computed. The default is to compute the maximum over all the matrix elements, returning a scalar (i.e. *axis* = *None*).

**out** : None, optional

This argument is in the signature *solely* for NumPy compatibility reasons. Do not pass in anything except for the default value, as this argument is not used.

**Returns** **amax** : `coo_matrix` or scalar

Maximum of *a*. If *axis* is *None*, the result is a scalar value. If *axis* is given, the result is a sparse `coo_matrix` of dimension `a.ndim - 1`.

**See also:**

`min` The minimum value of a sparse matrix along a given axis.

`np.matrix.max`

NumPy's implementation of 'max' for matrices

`coo_matrix.maximum(other)`

`coo_matrix.mean(axis=None, dtype=None, out=None)`

Compute the arithmetic mean along the specified axis.

Returns the average of the matrix elements. The average is taken over all elements in the matrix by default, otherwise over the specified axis. *float64* intermediate and return values are used for integer inputs.

**Parameters** **axis** : {-2, -1, 0, 1, None} optional

Axis along which the mean is computed. The default is to compute the mean of all elements in the matrix (i.e. *axis* = *None*).

**dtype** : data-type, optional

Type to use in computing the mean. For integer inputs, the default is *float64*; for floating point inputs, it is the same as the input dtype.

**out** : `np.matrix`, optional

Alternative output matrix in which to place the result. It must have the same shape as the expected output, but the type of the output values will be cast if necessary.

**Returns** **m** : `np.matrix`

**See also:**

`np.matrix.mean`

NumPy's implementation of 'mean' for matrices

`coo_matrix.min(axis=None, out=None)`

Return the minimum of the matrix or maximum along an axis. This takes all elements into account, not just the non-zero ones.

**Parameters** **axis** : {-2, -1, 0, 1, None} optional



Axis along which the sum is computed. The default is to compute the minimum over all the matrix elements, returning a scalar (i.e. `axis = None`).

**out** : None, optional

This argument is in the signature *solely* for NumPy compatibility reasons. Do not pass in anything except for the default value, as this argument is not used.

**Returns**

**amin** : `coo_matrix` or scalar

Minimum of *a*. If *axis* is None, the result is a scalar value. If *axis* is given, the result is a `sparse.coo_matrix` of dimension `a.ndim - 1`.

**See also:**

**`max`** The maximum value of a sparse matrix along a given axis.

**`np.matrix.min`**

NumPy's implementation of 'min' for matrices

`coo_matrix.minimum` (*other*)

`coo_matrix.multiply` (*other*)

Point-wise multiplication by another matrix

`coo_matrix.nonzero` ()

nonzero indices

Returns a tuple of arrays (row,col) containing the indices of the non-zero elements of the matrix.

**Examples**

```
>>> from scipy.sparse import csr_matrix
>>> A = csr_matrix([[1, 2, 0], [0, 0, 3], [4, 0, 5]])
>>> A.nonzero()
(array([0, 0, 1, 2, 2]), array([0, 1, 2, 0, 2]))
```

`coo_matrix.power` (*n*, *dtype=None*)

This function performs element-wise power.

**Parameters** **n** : *n* is a scalar

**dtype** : If dtype is not specified, the current dtype will be preserved.

`coo_matrix.rad2deg` ()

Element-wise rad2deg.

See `numpy.rad2deg` for more information.

`coo_matrix.reshape` (*shape*, *order='C'*)

Gives a new shape to a sparse matrix without changing its data.

**Parameters** **shape** : length-2 tuple of ints

The new shape should be compatible with the original shape.

**order** : 'C', optional

This argument is in the signature *solely* for NumPy compatibility reasons. Do not pass in anything except for the default value, as this argument is not used.

**Returns**

**reshaped\_matrix** : *self* with the new dimensions of shape

**See also:**

**`np.matrix.reshape`**

NumPy's implementation of 'reshape' for matrices

`coo_matrix.rint()`  
 Element-wise rint.

See `numpy rint` for more information.

`coo_matrix.set_shape(shape)`

`coo_matrix.setdiag(values, k=0)`  
 Set diagonal or off-diagonal elements of the array.

**Parameters**

- values** : array\_like  
 New values of the diagonal elements.  
 Values may have any length. If the diagonal is longer than values, then the remaining diagonal entries will not be set. If values if longer than the diagonal, then the remaining values are ignored.  
 If a scalar value is given, all of the diagonal is set to it.
- k** : int, optional  
 Which off-diagonal to set, corresponding to elements `a[i,i+k]`. Default: 0 (the main diagonal).

`coo_matrix.sign()`  
 Element-wise sign.

See `numpy.sign` for more information.

`coo_matrix.sin()`  
 Element-wise sin.

See `numpy.sin` for more information.

`coo_matrix.sinh()`  
 Element-wise sinh.

See `numpy.sinh` for more information.

`coo_matrix.sqrt()`  
 Element-wise sqrt.

See `numpy.sqrt` for more information.

`coo_matrix.sum(axis=None, dtype=None, out=None)`  
 Sum the matrix elements over a given axis.

**Parameters**

- axis** : {-2, -1, 0, 1, None} optional  
 Axis along which the sum is computed. The default is to compute the sum of all the matrix elements, returning a scalar (i.e. `axis = None`).
- dtype** : dtype, optional  
 The type of the returned matrix and of the accumulator in which the elements are summed. The dtype of `a` is used by default unless `a` has an integer dtype of less precision than the default platform integer. In that case, if `a` is signed then the platform integer is used while if `a` is unsigned then an unsigned integer of the same precision as the platform integer is used.
- out** : np.matrix, optional  
 Alternative output matrix in which to place the result. It must have the same shape as the expected output, but the type of the output values will be cast if necessary.

**Returns**

- sum\_along\_axis** : np.matrix  
 A matrix with the same shape as `self`, with the specified axis removed.

See also:

**`np.matrix.sum`**

NumPy's implementation of 'sum' for matrices

**`coo_matrix.sum_duplicates()`**

Eliminate duplicate matrix entries by adding them together

This is an *in place* operation

**`coo_matrix.tan()`**

Element-wise tan.

See `numpy.tan` for more information.

**`coo_matrix.tanh()`**

Element-wise tanh.

See `numpy.tanh` for more information.

**`coo_matrix.toarray`** (*order=None, out=None*)

See the docstring for `spmatrix.toarray`.

**`coo_matrix.tobsr`** (*blocksize=None, copy=False*)

Convert this matrix to Block Sparse Row format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `bsr_matrix`.

When `blocksize=(R, C)` is provided, it will be used for construction of the `bsr_matrix`.

**`coo_matrix.tocoo`** (*copy=False*)

Convert this matrix to COOrdinate format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `coo_matrix`.

**`coo_matrix.tocsc`** (*copy=False*)

Convert this matrix to Compressed Sparse Column format

Duplicate entries will be summed together.

***Examples***

```
>>> from numpy import array
>>> from scipy.sparse import coo_matrix
>>> row = array([0, 0, 1, 3, 1, 0, 0])
>>> col = array([0, 2, 1, 3, 1, 0, 0])
>>> data = array([1, 1, 1, 1, 1, 1, 1])
>>> A = coo_matrix((data, (row, col)), shape=(4, 4)).tocsc()
>>> A.toarray()
array([[3, 0, 1, 0],
       [0, 2, 0, 0],
       [0, 0, 0, 0],
       [0, 0, 0, 1]])
```

**`coo_matrix.tocsr`** (*copy=False*)

Convert this matrix to Compressed Sparse Row format

Duplicate entries will be summed together.

***Examples***

```
>>> from numpy import array
>>> from scipy.sparse import coo_matrix
>>> row = array([0, 0, 1, 3, 1, 0, 0])
>>> col = array([0, 2, 1, 3, 1, 0, 0])
```

```
>>> data = array([1, 1, 1, 1, 1, 1, 1])
>>> A = coo_matrix((data, (row, col)), shape=(4, 4)).tocsr()
>>> A.toarray()
array([[3, 0, 1, 0],
       [0, 2, 0, 0],
       [0, 0, 0, 0],
       [0, 0, 0, 1]])
```

`coo_matrix.todense` (*order=None, out=None*)

Return a dense matrix representation of this matrix.

**Parameters**    **order** : {'C', 'F'}, optional

Whether to store multi-dimensional data in C (row-major) or Fortran (column-major) order in memory. The default is 'None', indicating the NumPy default of C-ordered. Cannot be specified in conjunction with the *out* argument.

**out** : ndarray, 2-dimensional, optional

If specified, uses this array (or `numpy.matrix`) as the output buffer instead of allocating a new array to return. The provided array must have the same shape and dtype as the sparse matrix on which you are calling the method.

**Returns**        **arr** : `numpy.matrix`, 2-dimensional

A NumPy matrix object with the same shape and containing the same data represented by the sparse matrix, with the requested memory order. If *out* was passed and was an array (rather than a `numpy.matrix`), it will be filled with the appropriate values and returned wrapped in a `numpy.matrix` object that shares the same memory.

`coo_matrix.todia` (*copy=False*)

Convert this matrix to sparse DIAgonal format.

With *copy=False*, the data/indices may be shared between this matrix and the resultant `dia_matrix`.

`coo_matrix.todok` (*copy=False*)

Convert this matrix to Dictionary Of Keys format.

With *copy=False*, the data/indices may be shared between this matrix and the resultant `dok_matrix`.

`coo_matrix.tolil` (*copy=False*)

Convert this matrix to LInked List format.

With *copy=False*, the data/indices may be shared between this matrix and the resultant `lil_matrix`.

`coo_matrix.transpose` (*axes=None, copy=False*)

Reverses the dimensions of the sparse matrix.

**Parameters**    **axes** : None, optional

This argument is in the signature *solely* for NumPy compatibility reasons. Do not pass in anything except for the default value.

**copy** : bool, optional

Indicates whether or not attributes of *self* should be copied whenever possible. The degree to which attributes are copied varies depending on the type of sparse matrix being used.

**Returns**        **p** : *self* with the dimensions reversed.

**See also:**

`np.matrix.transpose`

NumPy's implementation of 'transpose' for matrices

`coo_matrix.trunc()`  
Element-wise trunc.

See `numpy.trunc` for more information.

**class** `scipy.sparse.csc_matrix` (*arg1*, *shape=None*, *dtype=None*, *copy=False*)  
Compressed Sparse Column matrix

This can be instantiated in several ways:

`csc_matrix(D)` with a dense matrix or rank-2 ndarray *D*  
`csc_matrix(S)` with another sparse matrix *S* (equivalent to `S.tocsc()`)  
`csc_matrix((M, N), [dtype])`  
 to construct an empty matrix with shape (M, N) dtype is optional, defaulting to dtype='d'.  
`csc_matrix((data, (row_ind, col_ind)), [shape=(M, N)])`  
 where *data*, *row\_ind* and *col\_ind* satisfy the relationship `a[row_ind[k], col_ind[k]] = data[k]`  
`csc_matrix((data, indices, indptr), [shape=(M, N)])`  
 is the standard CSC representation where the row indices for column *i* are stored in `indices[indptr[i]:indptr[i+1]]` and their corresponding values are stored in `data[indptr[i]:indptr[i+1]]`. If the *shape* parameter is not supplied, the matrix dimensions are inferred from the index arrays.

### Notes

Sparse matrices can be used in arithmetic operations: they support addition, subtraction, multiplication, division, and matrix power.

#### Advantages of the CSC format

- efficient arithmetic operations CSC + CSC, CSC \* CSC, etc.
- efficient column slicing
- fast matrix vector products (CSR, BSR may be faster)

#### Disadvantages of the CSC format

- slow row slicing operations (consider CSR)
- changes to the sparsity structure are expensive (consider LIL or DOK)

### Examples

```
>>> import numpy as np
>>> from scipy.sparse import csc_matrix
>>> csc_matrix((3, 4), dtype=np.int8).toarray()
array([[0, 0, 0, 0],
       [0, 0, 0, 0],
       [0, 0, 0, 0]], dtype=int8)

>>> row = np.array([0, 2, 2, 0, 1, 2])
>>> col = np.array([0, 0, 1, 2, 2, 2])
>>> data = np.array([1, 2, 3, 4, 5, 6])
>>> csc_matrix((data, (row, col)), shape=(3, 3)).toarray()
array([[1, 0, 4],
       [0, 0, 5],
       [2, 3, 6]])

>>> indptr = np.array([0, 2, 3, 6])
>>> indices = np.array([0, 2, 2, 0, 1, 2])
>>> data = np.array([1, 2, 3, 4, 5, 6])
>>> csc_matrix((data, indices, indptr), shape=(3, 3)).toarray()
array([[1, 0, 4],
```

```
[0, 0, 5],
 [2, 3, 6]])
```

**Attributes**

<code>nnz</code>	Number of stored values, including explicit zeros.
<code>has_sorted_indices</code>	Determine whether the matrix has sorted indices

`csc_matrix.nnz`

Number of stored values, including explicit zeros.

**See also:**

`count_nonzero`

Number of non-zero entries

`csc_matrix.has_sorted_indices`

Determine whether the matrix has sorted indices

**Returns**

- True: if the indices of the matrix are in sorted order
- False: otherwise

<code>dtype</code>	(dtype) Data type of the matrix
<code>shape</code>	(2-tuple) Shape of the matrix
<code>ndim</code>	(int) Number of dimensions (this is always 2)
<code>data</code>	Data array of the matrix
<code>indices</code>	CSC format index array
<code>indptr</code>	CSC format index pointer array

**Methods**

---

<code>arcsin()</code>	Element-wise arcsin.
<code>arcsinh()</code>	Element-wise arcsinh.
<code>arctan()</code>	Element-wise arctan.
<code>arctanh()</code>	Element-wise arctanh.
<code>asformat(format)</code>	Return this matrix in a given sparse format
<code>asfptype()</code>	Upcast matrix to a floating point format (if necessary)
<code>astype(t)</code>	
<code>ceil()</code>	Element-wise ceil.
<code>check_format([full_check])</code>	check whether the matrix format is valid
<code>conj()</code>	
<code>conjugate()</code>	
<code>copy()</code>	
<code>count_nonzero()</code>	Number of non-zero entries, equivalent to
<code>deg2rad()</code>	Element-wise deg2rad.
<code>diagonal()</code>	Returns the main diagonal of the matrix
<code>dot(other)</code>	Ordinary dot product
<code>eliminate_zeros()</code>	Remove zero entries from the matrix
<code>expm1()</code>	Element-wise expm1.
<code>floor()</code>	Element-wise floor.
<code>getH()</code>	
<code>get_shape()</code>	
<code>getcol(i)</code>	Returns a copy of column i of the matrix, as a (m x 1) CSC matrix (column vector).
<code>getformat()</code>	

Continued on next page

Table 5.159 – continued from previous page

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<code>getmaxprint()</code>	
<code>getnnz([axis])</code>	Number of stored values, including explicit zeros.
<code>getrow(i)</code>	Returns a copy of row <i>i</i> of the matrix, as a (1 x <i>n</i> ) CSR matrix (row vector).
<code>log1p()</code>	Element-wise <code>log1p</code> .
<code>max([axis, out])</code>	Return the maximum of the matrix or maximum along an axis.
<code>maximum(other)</code>	
<code>mean([axis, dtype, out])</code>	Compute the arithmetic mean along the specified axis.
<code>min([axis, out])</code>	Return the minimum of the matrix or maximum along an axis.
<code>minimum(other)</code>	
<code>multiply(other)</code>	Point-wise multiplication by another matrix, vector, or scalar.
<code>nonzero()</code>	nonzero indices
<code>power(n[, dtype])</code>	This function performs element-wise power.
<code>prune()</code>	Remove empty space after all non-zero elements.
<code>rad2deg()</code>	Element-wise <code>rad2deg</code> .
<code>reshape(shape[, order])</code>	Gives a new shape to a sparse matrix without changing its data.
<code>rint()</code>	Element-wise <code>rint</code> .
<code>set_shape(shape)</code>	
<code>setdiag(values[, k])</code>	Set diagonal or off-diagonal elements of the array.
<code>sign()</code>	Element-wise <code>sign</code> .
<code>sin()</code>	Element-wise <code>sin</code> .
<code>sinh()</code>	Element-wise <code>sinh</code> .
<code>sort_indices()</code>	Sort the indices of this matrix <i>in place</i>
<code>sorted_indices()</code>	Return a copy of this matrix with sorted indices
<code>sqrt()</code>	Element-wise <code>sqrt</code> .
<code>sum([axis, dtype, out])</code>	Sum the matrix elements over a given axis.
<code>sum_duplicates()</code>	Eliminate duplicate matrix entries by adding them together
<code>tan()</code>	Element-wise <code>tan</code> .
<code>tanh()</code>	Element-wise <code>tanh</code> .
<code>toarray([order, out])</code>	See the docstring for <code>spmatrix.toarray</code> .
<code>tobsr([blocksize, copy])</code>	Convert this matrix to Block Sparse Row format.
<code>tocoo([copy])</code>	Convert this matrix to COOrdinate format.
<code>tocsc([copy])</code>	Convert this matrix to Compressed Sparse Column format.
<code>tocsr([copy])</code>	Convert this matrix to Compressed Sparse Row format.
<code>todense([order, out])</code>	Return a dense matrix representation of this matrix.
<code>todia([copy])</code>	Convert this matrix to sparse DIAgonal format.
<code>todok([copy])</code>	Convert this matrix to Dictionary Of Keys format.
<code>tolil([copy])</code>	Convert this matrix to LInked List format.
<code>transpose([axes, copy])</code>	Reverses the dimensions of the sparse matrix.
<code>trunc()</code>	Element-wise <code>trunc</code> .

---

`csc_matrix.arcsin()`

Element-wise `arcsin`.

See `numpy.arcsin` for more information.

`csc_matrix.arcsinh()`

Element-wise `arcsinh`.

See `numpy.arcsinh` for more information.

`csc_matrix.arctan()`

Element-wise `arctan`.

See `numpy.arctan` for more information.

`csc_matrix.arctanh()`

Element-wise arctanh.

See `numpy.arctanh` for more information.

`csc_matrix.asformat(format)`

Return this matrix in a given sparse format

**Parameters** `format` : {string, None}

*desired sparse matrix format*

- None for no format conversion
- “csr” for `csr_matrix` format
- “csc” for `csc_matrix` format
- “lil” for `lil_matrix` format
- “dok” for `dok_matrix` format and so on

`csc_matrix.asfptype()`

Upcast matrix to a floating point format (if necessary)

`csc_matrix.astype(t)`

`csc_matrix.ceil()`

Element-wise ceil.

See `numpy.ceil` for more information.

`csc_matrix.check_format(full_check=True)`

check whether the matrix format is valid

**Parameters** `full_check` : bool, optional

If *True*, rigorous check, O(N) operations. Otherwise basic check, O(1) operations (default *True*).

`csc_matrix.conj()`

`csc_matrix.conjugate()`

`csc_matrix.copy()`

`csc_matrix.count_nonzero()`

Number of non-zero entries, equivalent to

`np.count_nonzero(a.toarray())`

Unlike `getnnz()` and the `nnz` property, which return the number of stored entries (the length of the data attribute), this method counts the actual number of non-zero entries in data.

`csc_matrix.deg2rad()`

Element-wise `deg2rad`.

See `numpy.deg2rad` for more information.

`csc_matrix.diagonal()`

Returns the main diagonal of the matrix

`csc_matrix.dot(other)`

Ordinary dot product



*Examples*

```
>>> import numpy as np
>>> from scipy.sparse import csr_matrix
>>> A = csr_matrix([[1, 2, 0], [0, 0, 3], [4, 0, 5]])
>>> v = np.array([1, 0, -1])
>>> A.dot(v)
array([ 1, -3, -1], dtype=int64)
```

`csc_matrix.eliminate_zeros()`  
Remove zero entries from the matrix

This is an *in place* operation

`csc_matrix.expm1()`  
Element-wise `expm1`.

See `numpy.expm1` for more information.

`csc_matrix.floor()`  
Element-wise floor.

See `numpy.floor` for more information.

`csc_matrix.getH()`

`csc_matrix.get_shape()`

`csc_matrix.getcol(i)`  
Returns a copy of column `i` of the matrix, as a  $(m \times 1)$  CSC matrix (column vector).

`csc_matrix.getformat()`

`csc_matrix.getmaxprint()`

`csc_matrix.getnnz(axis=None)`  
Number of stored values, including explicit zeros.

**Parameters** `axis` : None, 0, or 1

Select between the number of values across the whole matrix, in each column, or in each row.

**See also:**

`count_nonzero`  
Number of non-zero entries

`csc_matrix.getrow(i)`  
Returns a copy of row `i` of the matrix, as a  $(1 \times n)$  CSR matrix (row vector).

`csc_matrix.log1p()`  
Element-wise `log1p`.

See `numpy.log1p` for more information.

`csc_matrix.max(axis=None, out=None)`  
Return the maximum of the matrix or maximum along an axis. This takes all elements into account, not just the non-zero ones.

**Parameters** `axis` : {-2, -1, 0, 1, None} optional

Axis along which the sum is computed. The default is to compute the maximum over all the matrix elements, returning a scalar (i.e. *axis* = *None*).

**out** : None, optional

This argument is in the signature *solely* for NumPy compatibility reasons. Do not pass in anything except for the default value, as this argument is not used.

**Returns**

**amax** : `coo_matrix` or scalar

Maximum of *a*. If *axis* is *None*, the result is a scalar value. If *axis* is given, the result is a `sparse.coo_matrix` of dimension `a.ndim - 1`.

**See also:**

`min` The minimum value of a sparse matrix along a given axis.

`np.matrix.max`

NumPy's implementation of 'max' for matrices

`csc_matrix.maximum` (*other*)

`csc_matrix.mean` (*axis=None, dtype=None, out=None*)

Compute the arithmetic mean along the specified axis.

Returns the average of the matrix elements. The average is taken over all elements in the matrix by default, otherwise over the specified axis. *float64* intermediate and return values are used for integer inputs.

**Parameters** **axis** : {-2, -1, 0, 1, None} optional

Axis along which the mean is computed. The default is to compute the mean of all elements in the matrix (i.e. *axis* = *None*).

**dtype** : data-type, optional

Type to use in computing the mean. For integer inputs, the default is *float64*; for floating point inputs, it is the same as the input dtype.

**out** : `np.matrix`, optional

Alternative output matrix in which to place the result. It must have the same shape as the expected output, but the type of the output values will be cast if necessary.

**Returns**

**m** : `np.matrix`

**See also:**

`np.matrix.mean`

NumPy's implementation of 'mean' for matrices

`csc_matrix.min` (*axis=None, out=None*)

Return the minimum of the matrix or maximum along an axis. This takes all elements into account, not just the non-zero ones.

**Parameters** **axis** : {-2, -1, 0, 1, None} optional

Axis along which the sum is computed. The default is to compute the minimum over all the matrix elements, returning a scalar (i.e. *axis* = *None*).

**out** : None, optional

This argument is in the signature *solely* for NumPy compatibility reasons. Do not pass in anything except for the default value, as this argument is not used.

**Returns**

**amin** : `coo_matrix` or scalar

Minimum of *a*. If *axis* is *None*, the result is a scalar value. If *axis* is given, the result is a `sparse.coo_matrix` of dimension `a.ndim - 1`.

**See also:**

**max** The maximum value of a sparse matrix along a given axis.  
**np.matrix.min**  
 NumPy's implementation of 'min' for matrices

`csc_matrix.minimum` (*other*)

`csc_matrix.multiply` (*other*)

Point-wise multiplication by another matrix, vector, or scalar.

`csc_matrix.nonzero` ()

nonzero indices

Returns a tuple of arrays (row,col) containing the indices of the non-zero elements of the matrix.

### Examples

```
>>> from scipy.sparse import csr_matrix
>>> A = csr_matrix([[1,2,0],[0,0,3],[4,0,5]])
>>> A.nonzero()
(array([0, 0, 1, 2, 2]), array([0, 1, 2, 0, 2]))
```

`csc_matrix.power` (*n*, *dtype=None*)

This function performs element-wise power.

**Parameters** **n** : n is a scalar  
**dtype** : If dtype is not specified, the current dtype will be preserved.

`csc_matrix.prune` ()

Remove empty space after all non-zero elements.

`csc_matrix.rad2deg` ()

Element-wise rad2deg.

See `numpy.rad2deg` for more information.

`csc_matrix.reshape` (*shape*, *order='C'*)

Gives a new shape to a sparse matrix without changing its data.

**Parameters** **shape** : length-2 tuple of ints  
 The new shape should be compatible with the original shape.  
**order** : 'C', optional  
 This argument is in the signature *solely* for NumPy compatibility reasons. Do not pass in anything except for the default value, as this argument is not used.

**Returns** **reshaped\_matrix** : *self* with the new dimensions of shape

See also:

**np.matrix.reshape**

NumPy's implementation of 'reshape' for matrices

`csc_matrix.rint` ()

Element-wise rint.

See `numpy.rint` for more information.

`csc_matrix.set_shape` (*shape*)

`csc_matrix.setdiag` (*values*, *k=0*)

Set diagonal or off-diagonal elements of the array.

**Parameters**

- values** : array\_like  
New values of the diagonal elements.  
Values may have any length. If the diagonal is longer than values, then the remaining diagonal entries will not be set. If values is longer than the diagonal, then the remaining values are ignored.  
If a scalar value is given, all of the diagonal is set to it.
- k** : int, optional  
Which off-diagonal to set, corresponding to elements  $a[i,i+k]$ . Default: 0 (the main diagonal).

`csc_matrix.sign()`  
Element-wise sign.

See `numpy.sign` for more information.

`csc_matrix.sin()`  
Element-wise sin.

See `numpy.sin` for more information.

`csc_matrix.sinh()`  
Element-wise sinh.

See `numpy.sinh` for more information.

`csc_matrix.sort_indices()`  
Sort the indices of this matrix *in place*

`csc_matrix.sorted_indices()`  
Return a copy of this matrix with sorted indices

`csc_matrix.sqrt()`  
Element-wise sqrt.

See `numpy.sqrt` for more information.

`csc_matrix.sum(axis=None, dtype=None, out=None)`  
Sum the matrix elements over a given axis.

**Parameters**

- axis** : {-2, -1, 0, 1, None} optional  
Axis along which the sum is computed. The default is to compute the sum of all the matrix elements, returning a scalar (i.e. *axis = None*).
- dtype** : dtype, optional  
The type of the returned matrix and of the accumulator in which the elements are summed. The dtype of *a* is used by default unless *a* has an integer dtype of less precision than the default platform integer. In that case, if *a* is signed then the platform integer is used while if *a* is unsigned then an unsigned integer of the same precision as the platform integer is used.
- out** : np.matrix, optional  
Alternative output matrix in which to place the result. It must have the same shape as the expected output, but the type of the output values will be cast if necessary.

**Returns**

- sum\_along\_axis** : np.matrix  
A matrix with the same shape as *self*, with the specified axis removed.

See also:

`np.matrix.sum`  
NumPy's implementation of 'sum' for matrices

`csc_matrix.sum_duplicates()`  
Eliminate duplicate matrix entries by adding them together

The is an *in place* operation

`csc_matrix.tan()`  
Element-wise tan.

See `numpy.tan` for more information.

`csc_matrix.tanh()`  
Element-wise tanh.

See `numpy.tanh` for more information.

`csc_matrix.toarray (order=None, out=None)`  
See the docstring for `spmatrix.toarray`.

`csc_matrix.tobsr (blocksize=None, copy=False)`  
Convert this matrix to Block Sparse Row format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `bsr_matrix`.

When `blocksize=(R, C)` is provided, it will be used for construction of the `bsr_matrix`.

`csc_matrix.tocoo (copy=True)`  
Convert this matrix to COOrdinate format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `coo_matrix`.

`csc_matrix.tocsc (copy=False)`  
Convert this matrix to Compressed Sparse Column format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `csc_matrix`.

`csc_matrix.tocsr (copy=False)`  
Convert this matrix to Compressed Sparse Row format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `csr_matrix`.

`csc_matrix.todense (order=None, out=None)`  
Return a dense matrix representation of this matrix.

**Parameters**

- order** : {'C', 'F'}, optional  
Whether to store multi-dimensional data in C (row-major) or Fortran (column-major) order in memory. The default is 'None', indicating the NumPy default of C-ordered. Cannot be specified in conjunction with the `out` argument.
- out** : ndarray, 2-dimensional, optional  
If specified, uses this array (or `numpy.matrix`) as the output buffer instead of allocating a new array to return. The provided array must have the same shape and dtype as the sparse matrix on which you are calling the method.

**Returns**

- arr** : `numpy.matrix`, 2-dimensional  
A NumPy matrix object with the same shape and containing the same data represented by the sparse matrix, with the requested memory order. If `out` was passed and was an array (rather than a `numpy.matrix`), it will be filled with the appropriate values and returned wrapped in a `numpy.matrix` object that shares the same memory.

`csc_matrix.todia (copy=False)`  
Convert this matrix to sparse DIAgonal format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `dia_matrix`.

`csc_matrix.todok` (*copy=False*)  
 Convert this matrix to Dictionary Of Keys format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `dok_matrix`.

`csc_matrix.tolil` (*copy=False*)  
 Convert this matrix to LInked List format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `lil_matrix`.

`csc_matrix.transpose` (*axes=None, copy=False*)  
 Reverses the dimensions of the sparse matrix.

**Parameters**

- axes** : None, optional  
 This argument is in the signature *solely* for NumPy compatibility reasons. Do not pass in anything except for the default value.
- copy** : bool, optional  
 Indicates whether or not attributes of *self* should be copied whenever possible. The degree to which attributes are copied varies depending on the type of sparse matrix being used.

**Returns**

- p** : *self* with the dimensions reversed.

**See also:**

**`np.matrix.transpose`**  
 NumPy's implementation of 'transpose' for matrices

`csc_matrix.trunc` ()  
 Element-wise trunc.

See `numpy.trunc` for more information.

**class** `scipy.sparse.csr_matrix` (*arg1, shape=None, dtype=None, copy=False*)  
 Compressed Sparse Row matrix

*This can be instantiated in several ways:*

- `csr_matrix(D)` with a dense matrix or rank-2 ndarray D
- `csr_matrix(S)` with another sparse matrix S (equivalent to `S.tocsr()`)
- `csr_matrix((M, N), [dtype])`  
 to construct an empty matrix with shape (M, N) dtype is optional, defaulting to `dtype='d'`.
- `csr_matrix((data, (row_ind, col_ind)), [shape=(M, N)])`  
 where `data`, `row_ind` and `col_ind` satisfy the relationship `a[row_ind[k], col_ind[k]] = data[k]`.
- `csr_matrix((data, indices, indptr), [shape=(M, N)])`  
 is the standard CSR representation where the column indices for row `i` are stored in `indices[indptr[i]:indptr[i+1]]` and their corresponding values are stored in `data[indptr[i]:indptr[i+1]]`. If the shape parameter is not supplied, the matrix dimensions are inferred from the index arrays.

**Notes**

Sparse matrices can be used in arithmetic operations: they support addition, subtraction, multiplication, division, and matrix power.

**Advantages of the CSR format**

- efficient arithmetic operations `CSR + CSR`, `CSR * CSR`, etc.
- efficient row slicing

- fast matrix vector products

### *Disadvantages of the CSR format*

- slow column slicing operations (consider CSC)
- changes to the sparsity structure are expensive (consider LIL or DOK)

### *Examples*

```
>>> import numpy as np
>>> from scipy.sparse import csr_matrix
>>> csr_matrix((3, 4), dtype=np.int8).toarray()
array([[0, 0, 0, 0],
       [0, 0, 0, 0],
       [0, 0, 0, 0]], dtype=int8)

>>> row = np.array([0, 0, 1, 2, 2, 2])
>>> col = np.array([0, 2, 2, 0, 1, 2])
>>> data = np.array([1, 2, 3, 4, 5, 6])
>>> csr_matrix((data, (row, col)), shape=(3, 3)).toarray()
array([[1, 0, 2],
       [0, 0, 3],
       [4, 5, 6]])

>>> indptr = np.array([0, 2, 3, 6])
>>> indices = np.array([0, 2, 2, 0, 1, 2])
>>> data = np.array([1, 2, 3, 4, 5, 6])
>>> csr_matrix((data, indices, indptr), shape=(3, 3)).toarray()
array([[1, 0, 2],
       [0, 0, 3],
       [4, 5, 6]])
```

As an example of how to construct a CSR matrix incrementally, the following snippet builds a term-document matrix from texts:

```
>>> docs = [["hello", "world", "hello"], ["goodbye", "cruel", "world"]]
>>> indptr = [0]
>>> indices = []
>>> data = []
>>> vocabulary = {}
>>> for d in docs:
...     for term in d:
...         index = vocabulary.setdefault(term, len(vocabulary))
...         indices.append(index)
...         data.append(1)
...     indptr.append(len(indices))
...
>>> csr_matrix((data, indices, indptr), dtype=int).toarray()
array([[2, 1, 0, 0],
       [0, 1, 1, 1]])
```

### *Attributes*

<code>nnz</code>	Number of stored values, including explicit zeros.
<code>has_sorted_indices</code>	Determine whether the matrix has sorted indices

`csr_matrix.nnz`  
 Number of stored values, including explicit zeros.

**See also:**

`count_nonzero`  
 Number of non-zero entries

`csr_matrix.has_sorted_indices`  
 Determine whether the matrix has sorted indices

**Returns**

- True: if the indices of the matrix are in sorted order
- False: otherwise

<code>dtype</code>	(dtype) Data type of the matrix
<code>shape</code>	(2-tuple) Shape of the matrix
<code>ndim</code>	(int) Number of dimensions (this is always 2)
<code>data</code>	CSR format data array of the matrix
<code>indices</code>	CSR format index array of the matrix
<code>indptr</code>	CSR format index pointer array of the matrix

**Methods**

---

<code>arcsin()</code>	Element-wise arcsin.
<code>arcsinh()</code>	Element-wise arcsinh.
<code>arctan()</code>	Element-wise arctan.
<code>arctanh()</code>	Element-wise arctanh.
<code>asformat(format)</code>	Return this matrix in a given sparse format
<code>asfptype()</code>	Upcast matrix to a floating point format (if necessary)
<code>astype(t)</code>	
<code>ceil()</code>	Element-wise ceil.
<code>check_format([full_check])</code>	check whether the matrix format is valid
<code>conj()</code>	
<code>conjugate()</code>	
<code>copy()</code>	
<code>count_nonzero()</code>	Number of non-zero entries, equivalent to
<code>deg2rad()</code>	Element-wise deg2rad.
<code>diagonal()</code>	Returns the main diagonal of the matrix
<code>dot(other)</code>	Ordinary dot product
<code>eliminate_zeros()</code>	Remove zero entries from the matrix
<code>expm1()</code>	Element-wise expm1.
<code>floor()</code>	Element-wise floor.
<code>getH()</code>	
<code>get_shape()</code>	
<code>getcol(i)</code>	Returns a copy of column i of the matrix, as a (m x 1) CSR matrix (column vector).
<code>getformat()</code>	
<code>getmaxprint()</code>	
<code>getnnz([axis])</code>	Number of stored values, including explicit zeros.
<code>getrow(i)</code>	Returns a copy of row i of the matrix, as a (1 x n) CSR matrix (row vector).
<code>log1p()</code>	Element-wise log1p.
<code>max([axis, out])</code>	Return the maximum of the matrix or maximum along an axis.
<code>maximum(other)</code>	
<code>mean([axis, dtype, out])</code>	Compute the arithmetic mean along the specified axis.
<code>min([axis, out])</code>	Return the minimum of the matrix or maximum along an axis.
<code>minimum(other)</code>	

Continued on next page



Table 5.161 – continued from previous page

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<code>multiply(other)</code>	Point-wise multiplication by another matrix, vector, or scalar.
<code>nonzero()</code>	nonzero indices
<code>power(n[, dtype])</code>	This function performs element-wise power.
<code>prune()</code>	Remove empty space after all non-zero elements.
<code>rad2deg()</code>	Element-wise rad2deg.
<code>reshape(shape[, order])</code>	Gives a new shape to a sparse matrix without changing its data.
<code>rint()</code>	Element-wise rint.
<code>set_shape(shape)</code>	
<code>setdiag(values[, k])</code>	Set diagonal or off-diagonal elements of the array.
<code>sign()</code>	Element-wise sign.
<code>sin()</code>	Element-wise sin.
<code>sinh()</code>	Element-wise sinh.
<code>sort_indices()</code>	Sort the indices of this matrix <i>in place</i>
<code>sorted_indices()</code>	Return a copy of this matrix with sorted indices
<code>sqrt()</code>	Element-wise sqrt.
<code>sum([axis, dtype, out])</code>	Sum the matrix elements over a given axis.
<code>sum_duplicates()</code>	Eliminate duplicate matrix entries by adding them together
<code>tan()</code>	Element-wise tan.
<code>tanh()</code>	Element-wise tanh.
<code>toarray([order, out])</code>	See the docstring for <code>spmatrix.toarray</code> .
<code>tobsr([blocksize, copy])</code>	Convert this matrix to Block Sparse Row format.
<code>tocoo([copy])</code>	Convert this matrix to COOrdinate format.
<code>tocsc([copy])</code>	
<code>tocsr([copy])</code>	Convert this matrix to Compressed Sparse Row format.
<code>todense([order, out])</code>	Return a dense matrix representation of this matrix.
<code>todia([copy])</code>	Convert this matrix to sparse DIAgonal format.
<code>todok([copy])</code>	Convert this matrix to Dictionary Of Keys format.
<code>tolil([copy])</code>	Convert this matrix to LInked List format.
<code>transpose([axes, copy])</code>	Reverses the dimensions of the sparse matrix.
<code>trunc()</code>	Element-wise trunc.

---

`csr_matrix.arcsin()`  
Element-wise arcsin.

See `numpy.arcsin` for more information.

`csr_matrix.arcsinh()`  
Element-wise arcsinh.

See `numpy.arcsinh` for more information.

`csr_matrix.arctan()`  
Element-wise arctan.

See `numpy.arctan` for more information.

`csr_matrix.arctanh()`  
Element-wise arctanh.

See `numpy.arctanh` for more information.

`csr_matrix.asformat(format)`

Return this matrix in a given sparse format

**Parameters** `format` : {string, None}  
*desired sparse matrix format*

- None for no format conversion
- “csr” for csr\_matrix format
- “csc” for csc\_matrix format
- “lil” for lil\_matrix format
- “dok” for dok\_matrix format and so on

`csr_matrix.astype()`

Upcast matrix to a floating point format (if necessary)

`csr_matrix.astype(t)`

`csr_matrix.ceil()`

Element-wise ceil.

See `numpy.ceil` for more information.

`csr_matrix.check_format(full_check=True)`

check whether the matrix format is valid

**Parameters** `full_check` : bool, optional

If *True*, rigorous check, O(N) operations. Otherwise basic check, O(1) operations (default *True*).

`csr_matrix.conj()`

`csr_matrix.conjugate()`

`csr_matrix.copy()`

`csr_matrix.count_nonzero()`

Number of non-zero entries, equivalent to

`np.count_nonzero(a.toarray())`

Unlike `getnnz()` and the `nnz` property, which return the number of stored entries (the length of the data attribute), this method counts the actual number of non-zero entries in data.

`csr_matrix.deg2rad()`

Element-wise `deg2rad`.

See `numpy.deg2rad` for more information.

`csr_matrix.diagonal()`

Returns the main diagonal of the matrix

`csr_matrix.dot(other)`

Ordinary dot product

### Examples

```
>>> import numpy as np
>>> from scipy.sparse import csr_matrix
>>> A = csr_matrix([[1, 2, 0], [0, 0, 3], [4, 0, 5]])
>>> v = np.array([1, 0, -1])
>>> A.dot(v)
array([ 1, -3, -1], dtype=int64)
```

`csr_matrix.eliminate_zeros()`

Remove zero entries from the matrix

This is an *in place* operation

`csr_matrix.expm1()`

Element-wise `expm1`.

See `numpy.expm1` for more information.

`csr_matrix.floor()`

Element-wise floor.

See `numpy.floor` for more information.

`csr_matrix.getH()`

`csr_matrix.get_shape()`

`csr_matrix.getcol(i)`

Returns a copy of column *i* of the matrix, as a (m x 1) CSR matrix (column vector).

`csr_matrix.getformat()`

`csr_matrix.getmaxprint()`

`csr_matrix.getnnz(axis=None)`

Number of stored values, including explicit zeros.

**Parameters** **axis** : None, 0, or 1

Select between the number of values across the whole matrix, in each column, or in each row.

**See also:**

[`count\_nonzero`](#)

Number of non-zero entries

`csr_matrix.getrow(i)`

Returns a copy of row *i* of the matrix, as a (1 x n) CSR matrix (row vector).

`csr_matrix.log1p()`

Element-wise `log1p`.

See `numpy.log1p` for more information.

`csr_matrix.max(axis=None, out=None)`

Return the maximum of the matrix or maximum along an axis. This takes all elements into account, not just the non-zero ones.

**Parameters** **axis** : {-2, -1, 0, 1, None} optional

Axis along which the sum is computed. The default is to compute the maximum over all the matrix elements, returning a scalar (i.e. *axis* = *None*).

**out** : None, optional

This argument is in the signature *solely* for NumPy compatibility reasons. Do not pass in anything except for the default value, as this argument is not used.

**Returns** **amax** : `coo_matrix` or scalar

Maximum of  $a$ . If  $axis$  is `None`, the result is a scalar value. If  $axis$  is given, the result is a `sparse.coo_matrix` of dimension `a.ndim - 1`.

**See also:**

`min` The minimum value of a sparse matrix along a given axis.

`np.matrix.max`  
NumPy's implementation of 'max' for matrices

`csr_matrix.maximum` (*other*)

`csr_matrix.mean` ( $axis=None$ ,  $dtype=None$ ,  $out=None$ )

Compute the arithmetic mean along the specified axis.

Returns the average of the matrix elements. The average is taken over all elements in the matrix by default, otherwise over the specified axis. `float64` intermediate and return values are used for integer inputs.

**Parameters**

- axis** : {-2, -1, 0, 1, None} optional  
Axis along which the mean is computed. The default is to compute the mean of all elements in the matrix (i.e.  $axis = None$ ).
- dtype** : data-type, optional  
Type to use in computing the mean. For integer inputs, the default is `float64`; for floating point inputs, it is the same as the input dtype.
- out** : `np.matrix`, optional  
Alternative output matrix in which to place the result. It must have the same shape as the expected output, but the type of the output values will be cast if necessary.

**Returns** **m** : `np.matrix`

**See also:**

`np.matrix.mean`  
NumPy's implementation of 'mean' for matrices

`csr_matrix.min` ( $axis=None$ ,  $out=None$ )

Return the minimum of the matrix or maximum along an axis. This takes all elements into account, not just the non-zero ones.

**Parameters**

- axis** : {-2, -1, 0, 1, None} optional  
Axis along which the sum is computed. The default is to compute the minimum over all the matrix elements, returning a scalar (i.e.  $axis = None$ ).
- out** : None, optional  
This argument is in the signature *solely* for NumPy compatibility reasons. Do not pass in anything except for the default value, as this argument is not used.

**Returns** **amin** : `coo_matrix` or scalar  
Minimum of  $a$ . If  $axis$  is `None`, the result is a scalar value. If  $axis$  is given, the result is a `sparse.coo_matrix` of dimension `a.ndim - 1`.

**See also:**

`max` The maximum value of a sparse matrix along a given axis.

`np.matrix.min`  
NumPy's implementation of 'min' for matrices

`csr_matrix.minimum` (*other*)

`csr_matrix.multiply` (*other*)

Point-wise multiplication by another matrix, vector, or scalar.

`csr_matrix.nonzero()`  
nonzero indices

Returns a tuple of arrays (row,col) containing the indices of the non-zero elements of the matrix.

#### Examples

```
>>> from scipy.sparse import csr_matrix
>>> A = csr_matrix([[1,2,0],[0,0,3],[4,0,5]])
>>> A.nonzero()
(array([0, 0, 1, 2, 2]), array([0, 1, 2, 0, 2]))
```

`csr_matrix.power(n, dtype=None)`

This function performs element-wise power.

**Parameters** **n** : n is a scalar  
**dtype** : If dtype is not specified, the current dtype will be preserved.

`csr_matrix.prune()`

Remove empty space after all non-zero elements.

`csr_matrix.rad2deg()`

Element-wise rad2deg.

See `numpy.rad2deg` for more information.

`csr_matrix.reshape(shape, order='C')`

Gives a new shape to a sparse matrix without changing its data.

**Parameters** **shape** : length-2 tuple of ints  
The new shape should be compatible with the original shape.  
**order** : 'C', optional  
This argument is in the signature *solely* for NumPy compatibility reasons. Do not pass in anything except for the default value, as this argument is not used.

**Returns** **reshaped\_matrix** : *self* with the new dimensions of shape

See also:

**`np.matrix.reshape`**

NumPy's implementation of 'reshape' for matrices

`csr_matrix.rint()`

Element-wise rint.

See `numpy.rint` for more information.

`csr_matrix.set_shape(shape)`

`csr_matrix.setdiag(values, k=0)`

Set diagonal or off-diagonal elements of the array.

**Parameters** **values** : array\_like  
New values of the diagonal elements.  
Values may have any length. If the diagonal is longer than values, then the remaining diagonal entries will not be set. If values is longer than the diagonal, then the remaining values are ignored.  
If a scalar value is given, all of the diagonal is set to it.  
**k** : int, optional  
Which off-diagonal to set, corresponding to elements `a[i,i+k]`. Default: 0 (the main diagonal).

`csr_matrix.sign()`  
Element-wise sign.

See `numpy.sign` for more information.

`csr_matrix.sin()`  
Element-wise sin.

See `numpy.sin` for more information.

`csr_matrix.sinh()`  
Element-wise sinh.

See `numpy.sinh` for more information.

`csr_matrix.sort_indices()`  
Sort the indices of this matrix *in place*

`csr_matrix.sorted_indices()`  
Return a copy of this matrix with sorted indices

`csr_matrix.sqrt()`  
Element-wise sqrt.

See `numpy.sqrt` for more information.

`csr_matrix.sum(axis=None, dtype=None, out=None)`  
Sum the matrix elements over a given axis.

**Parameters** **axis** : {-2, -1, 0, 1, None} optional

Axis along which the sum is computed. The default is to compute the sum of all the matrix elements, returning a scalar (i.e. *axis = None*).

**dtype** : dtype, optional

The type of the returned matrix and of the accumulator in which the elements are summed. The dtype of *a* is used by default unless *a* has an integer dtype of less precision than the default platform integer. In that case, if *a* is signed then the platform integer is used while if *a* is unsigned then an unsigned integer of the same precision as the platform integer is used.

**out** : np.matrix, optional

Alternative output matrix in which to place the result. It must have the same shape as the expected output, but the type of the output values will be cast if necessary.

**Returns** **sum\_along\_axis** : np.matrix

A matrix with the same shape as *self*, with the specified axis removed.

**See also:**

**`np.matrix.sum`**

NumPy's implementation of 'sum' for matrices

`csr_matrix.sum_duplicates()`  
Eliminate duplicate matrix entries by adding them together

The is an *in place* operation

`csr_matrix.tan()`  
Element-wise tan.

See `numpy.tan` for more information.

`csr_matrix.tanh()`  
Element-wise tanh.

See `numpy.tanh` for more information.

`csr_matrix.toarray` (*order=None, out=None*)

See the docstring for `spmatrix.toarray`.

`csr_matrix.tobsr` (*blocksize=None, copy=True*)

Convert this matrix to Block Sparse Row format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `bsr_matrix`.

When `blocksize=(R, C)` is provided, it will be used for construction of the `bsr_matrix`.

`csr_matrix.tocoo` (*copy=True*)

Convert this matrix to COOrdinate format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `coo_matrix`.

`csr_matrix.tocsc` (*copy=False*)

`csr_matrix.tocsr` (*copy=False*)

Convert this matrix to Compressed Sparse Row format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `csr_matrix`.

`csr_matrix.todense` (*order=None, out=None*)

Return a dense matrix representation of this matrix.

**Parameters**    **order** : {'C', 'F'}, optional

Whether to store multi-dimensional data in C (row-major) or Fortran (column-major) order in memory. The default is 'None', indicating the NumPy default of C-ordered. Cannot be specified in conjunction with the `out` argument.

**out** : ndarray, 2-dimensional, optional

If specified, uses this array (or `numpy.matrix`) as the output buffer instead of allocating a new array to return. The provided array must have the same shape and dtype as the sparse matrix on which you are calling the method.

**Returns**        **arr** : `numpy.matrix`, 2-dimensional

A NumPy matrix object with the same shape and containing the same data represented by the sparse matrix, with the requested memory order. If `out` was passed and was an array (rather than a `numpy.matrix`), it will be filled with the appropriate values and returned wrapped in a `numpy.matrix` object that shares the same memory.

`csr_matrix.todia` (*copy=False*)

Convert this matrix to sparse DIAgonal format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `dia_matrix`.

`csr_matrix.todok` (*copy=False*)

Convert this matrix to Dictionary Of Keys format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `dok_matrix`.

`csr_matrix.tolil` (*copy=False*)

Convert this matrix to LInked List format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `lil_matrix`.

`csr_matrix.transpose` (*axes=None, copy=False*)

Reverses the dimensions of the sparse matrix.

**Parameters** `axes` : None, optional  
 This argument is in the signature *solely* for NumPy compatibility reasons. Do not pass in anything except for the default value.

`copy` : bool, optional  
 Indicates whether or not attributes of *self* should be copied whenever possible. The degree to which attributes are copied varies depending on the type of sparse matrix being used.

**Returns** `p` : *self* with the dimensions reversed.

**See also:**

`np.matrix.transpose`

NumPy's implementation of 'transpose' for matrices

`csr_matrix.trunc()`

Element-wise trunc.

See `numpy.trunc` for more information.

**class** `scipy.sparse.dia_matrix` (*arg1*, *shape=None*, *dtype=None*, *copy=False*)

Sparse matrix with DIAGONAL storage

*This can be instantiated in several ways:*

`dia_matrix(D)` with a dense matrix

`dia_matrix(S)` with another sparse matrix *S* (equivalent to *S.todia()*)

`dia_matrix((M, N), [dtype])`

to construct an empty matrix with shape (M, N), *dtype* is optional, defaulting to *dtype='d'*.

`dia_matrix((data, offsets), shape=(M, N))`

where the `data[k, :]` stores the diagonal entries for diagonal `offsets[k]`  
 (See example below)

### Notes

Sparse matrices can be used in arithmetic operations: they support addition, subtraction, multiplication, division, and matrix power.

### Examples

```
>>> import numpy as np
>>> from scipy.sparse import dia_matrix
>>> dia_matrix((3, 4), dtype=np.int8).toarray()
array([[0, 0, 0, 0],
       [0, 0, 0, 0],
       [0, 0, 0, 0]], dtype=int8)

>>> data = np.array([[1, 2, 3, 4]].repeat(3, axis=0)
>>> offsets = np.array([0, -1, 2])
>>> dia_matrix((data, offsets), shape=(4, 4)).toarray()
array([[1, 0, 3, 0],
       [1, 2, 0, 4],
       [0, 2, 3, 0],
       [0, 0, 3, 4]])
```

### Attributes

---

`nnz` Number of stored values, including explicit zeros.

---



`dia_matrix.nnz`

Number of stored values, including explicit zeros.

**See also:**`count_nonzero`

Number of non-zero entries

<code>dtype</code>	(dtype) Data type of the matrix
<code>shape</code>	(2-tuple) Shape of the matrix
<code>ndim</code>	(int) Number of dimensions (this is always 2)
<code>data</code>	DIA format data array of the matrix
<code>offsets</code>	DIA format offset array of the matrix

**Methods**


---

<code>arcsin()</code>	Element-wise arcsin.
<code>arcsinh()</code>	Element-wise arcsinh.
<code>arctan()</code>	Element-wise arctan.
<code>arctanh()</code>	Element-wise arctanh.
<code>asformat(format)</code>	Return this matrix in a given sparse format
<code>asfptype()</code>	Upcast matrix to a floating point format (if necessary)
<code>astype(t)</code>	
<code>ceil()</code>	Element-wise ceil.
<code>conj()</code>	
<code>conjugate()</code>	
<code>copy()</code>	
<code>count_nonzero()</code>	Number of non-zero entries, equivalent to
<code>deg2rad()</code>	Element-wise deg2rad.
<code>diagonal()</code>	Returns the main diagonal of the matrix
<code>dot(other)</code>	Ordinary dot product
<code>expm1()</code>	Element-wise expm1.
<code>floor()</code>	Element-wise floor.
<code>getH()</code>	
<code>get_shape()</code>	
<code>getcol(j)</code>	Returns a copy of column <i>j</i> of the matrix, as an ( <i>m</i> x 1) sparse matrix (column vector).
<code>getformat()</code>	
<code>getmaxprint()</code>	
<code>getnnz([axis])</code>	Number of stored values, including explicit zeros.
<code>getrow(i)</code>	Returns a copy of row <i>i</i> of the matrix, as a (1 x <i>n</i> ) sparse matrix (row vector).
<code>log1p()</code>	Element-wise log1p.
<code>maximum(other)</code>	
<code>mean([axis, dtype, out])</code>	Compute the arithmetic mean along the specified axis.
<code>minimum(other)</code>	
<code>multiply(other)</code>	Point-wise multiplication by another matrix
<code>nonzero()</code>	nonzero indices
<code>power(n[, dtype])</code>	This function performs element-wise power.
<code>rad2deg()</code>	Element-wise rad2deg.
<code>reshape(shape[, order])</code>	Gives a new shape to a sparse matrix without changing its data.
<code>rint()</code>	Element-wise rint.
<code>set_shape(shape)</code>	
<code>setdiag(values[, k])</code>	Set diagonal or off-diagonal elements of the array.
<code>sign()</code>	Element-wise sign.
<code>sin()</code>	Element-wise sin.
<code>sinh()</code>	Element-wise sinh.

Continued on next page

Table 5.163 – continued from previous page

<code>sqrt()</code>	Element-wise sqrt.
<code>sum([axis, dtype, out])</code>	Sum the matrix elements over a given axis.
<code>tan()</code>	Element-wise tan.
<code>tanh()</code>	Element-wise tanh.
<code>toarray([order, out])</code>	Return a dense ndarray representation of this matrix.
<code>tobsr([blocksize, copy])</code>	Convert this matrix to Block Sparse Row format.
<code>tocoo([copy])</code>	Convert this matrix to COOrdinate format.
<code>tocsc([copy])</code>	Convert this matrix to Compressed Sparse Column format.
<code>tocsr([copy])</code>	Convert this matrix to Compressed Sparse Row format.
<code>todense([order, out])</code>	Return a dense matrix representation of this matrix.
<code>todia([copy])</code>	Convert this matrix to sparse DIAgonal format.
<code>todok([copy])</code>	Convert this matrix to Dictionary Of Keys format.
<code>tolil([copy])</code>	Convert this matrix to LInked List format.
<code>transpose([axes, copy])</code>	Reverses the dimensions of the sparse matrix.
<code>trunc()</code>	Element-wise trunc.

`dia_matrix.arcsin()`

Element-wise arcsin.

See `numpy.arcsin` for more information.

`dia_matrix.arcsinh()`

Element-wise arcsinh.

See `numpy.arcsinh` for more information.

`dia_matrix.arctan()`

Element-wise arctan.

See `numpy.arctan` for more information.

`dia_matrix.arctanh()`

Element-wise arctanh.

See `numpy.arctanh` for more information.

`dia_matrix.asformat(format)`

Return this matrix in a given sparse format

**Parameters** `format` : {string, None}

*desired sparse matrix format*

- None for no format conversion
- “csr” for `csr_matrix` format
- “csc” for `csc_matrix` format
- “lil” for `lil_matrix` format
- “dok” for `dok_matrix` format and so on

`dia_matrix.asfptype()`

Upcast matrix to a floating point format (if necessary)

`dia_matrix.astype(t)`

`dia_matrix.ceil()`

Element-wise ceil.

See `numpy.ceil` for more information.

`dia_matrix.conj()`

`dia_matrix.conjugate()`

`dia_matrix.copy()`

`dia_matrix.count_nonzero()`

Number of non-zero entries, equivalent to

`np.count_nonzero(a.toarray())`

Unlike `getnnz()` and the `nnz` property, which return the number of stored entries (the length of the data attribute), this method counts the actual number of non-zero entries in data.

`dia_matrix.deg2rad()`

Element-wise `deg2rad`.

See `numpy.deg2rad` for more information.

`dia_matrix.diagonal()`

Returns the main diagonal of the matrix

`dia_matrix.dot(other)`

Ordinary dot product

### *Examples*

```
>>> import numpy as np
>>> from scipy.sparse import csr_matrix
>>> A = csr_matrix([[1, 2, 0], [0, 0, 3], [4, 0, 5]])
>>> v = np.array([1, 0, -1])
>>> A.dot(v)
array([ 1, -3, -1], dtype=int64)
```

`dia_matrix.expm1()`

Element-wise `expm1`.

See `numpy.expm1` for more information.

`dia_matrix.floor()`

Element-wise `floor`.

See `numpy.floor` for more information.

`dia_matrix.getH()`

`dia_matrix.get_shape()`

`dia_matrix.getcol(j)`

Returns a copy of column `j` of the matrix, as an  $(m \times 1)$  sparse matrix (column vector).

`dia_matrix.getformat()`

`dia_matrix.getmaxprint()`

`dia_matrix.getnnz(axis=None)`

Number of stored values, including explicit zeros.

**Parameters** **axis** : None, 0, or 1

Select between the number of values across the whole matrix, in each column, or in each row.

**See also:**

`count_nonzero`

Number of non-zero entries

`dia_matrix.getrow(i)`

Returns a copy of row *i* of the matrix, as a (1 x *n*) sparse matrix (row vector).

`dia_matrix.log1p()`

Element-wise log1p.

See `numpy.log1p` for more information.

`dia_matrix.maximum(other)`

`dia_matrix.mean(axis=None, dtype=None, out=None)`

Compute the arithmetic mean along the specified axis.

Returns the average of the matrix elements. The average is taken over all elements in the matrix by default, otherwise over the specified axis. *float64* intermediate and return values are used for integer inputs.

**Parameters** **axis** : {-2, -1, 0, 1, None} optional

Axis along which the mean is computed. The default is to compute the mean of all elements in the matrix (i.e. *axis = None*).

**dtype** : data-type, optional

Type to use in computing the mean. For integer inputs, the default is *float64*; for floating point inputs, it is the same as the input dtype.

**out** : np.matrix, optional

Alternative output matrix in which to place the result. It must have the same shape as the expected output, but the type of the output values will be cast if necessary.

**Returns** **m** : np.matrix

**See also:**

`np.matrix.mean`

NumPy's implementation of 'mean' for matrices

`dia_matrix.minimum(other)`

`dia_matrix.multiply(other)`

Point-wise multiplication by another matrix

`dia_matrix.nonzero()`

nonzero indices

Returns a tuple of arrays (row,col) containing the indices of the non-zero elements of the matrix.

**Examples**

```
>>> from scipy.sparse import csr_matrix
>>> A = csr_matrix([[1, 2, 0], [0, 0, 3], [4, 0, 5]])
>>> A.nonzero()
(array([0, 0, 1, 2, 2]), array([0, 1, 2, 0, 2]))
```

`dia_matrix.power` ( $n$ ,  $dtype=None$ )

This function performs element-wise power.

**Parameters** **n** :  $n$  is a scalar  
**dtype** : If dtype is not specified, the current dtype will be preserved.

`dia_matrix.rad2deg` ()

Element-wise rad2deg.

See `numpy.rad2deg` for more information.

`dia_matrix.reshape` ( $shape$ ,  $order='C'$ )

Gives a new shape to a sparse matrix without changing its data.

**Parameters** **shape** : length-2 tuple of ints  
The new shape should be compatible with the original shape.  
**order** : 'C', optional  
This argument is in the signature *solely* for NumPy compatibility reasons. Do not pass in anything except for the default value, as this argument is not used.

**Returns** **reshaped\_matrix** : *self* with the new dimensions of shape

**See also:**

**`np.matrix.reshape`**

NumPy's implementation of 'reshape' for matrices

`dia_matrix.rint` ()

Element-wise rint.

See `numpy.rint` for more information.

`dia_matrix.set_shape` ( $shape$ )

`dia_matrix.setdiag` ( $values$ ,  $k=0$ )

Set diagonal or off-diagonal elements of the array.

**Parameters** **values** : array\_like  
New values of the diagonal elements.  
Values may have any length. If the diagonal is longer than values, then the remaining diagonal entries will not be set. If values is longer than the diagonal, then the remaining values are ignored.  
If a scalar value is given, all of the diagonal is set to it.

**k** : int, optional

Which off-diagonal to set, corresponding to elements `a[i,i+k]`. Default: 0 (the main diagonal).

`dia_matrix.sign` ()

Element-wise sign.

See `numpy.sign` for more information.

`dia_matrix.sin` ()

Element-wise sin.

See `numpy.sin` for more information.

`dia_matrix.sinh` ()

Element-wise sinh.

See `numpy.sinh` for more information.

`dia_matrix.sqrt()`  
Element-wise sqrt.

See `numpy.sqrt` for more information.

`dia_matrix.sum(axis=None, dtype=None, out=None)`  
Sum the matrix elements over a given axis.

**Parameters**

- axis** : {-2, -1, 0, 1, None} optional  
Axis along which the sum is computed. The default is to compute the sum of all the matrix elements, returning a scalar (i.e. *axis = None*).
- dtype** : dtype, optional  
The type of the returned matrix and of the accumulator in which the elements are summed. The dtype of *a* is used by default unless *a* has an integer dtype of less precision than the default platform integer. In that case, if *a* is signed then the platform integer is used while if *a* is unsigned then an unsigned integer of the same precision as the platform integer is used.
- out** : np.matrix, optional  
Alternative output matrix in which to place the result. It must have the same shape as the expected output, but the type of the output values will be cast if necessary.

**Returns**

- sum\_along\_axis** : np.matrix  
A matrix with the same shape as *self*, with the specified axis removed.

**See also:**

**`np.matrix.sum`**  
NumPy's implementation of 'sum' for matrices

`dia_matrix.tan()`  
Element-wise tan.

See `numpy.tan` for more information.

`dia_matrix.tanh()`  
Element-wise tanh.

See `numpy.tanh` for more information.

`dia_matrix.toarray(order=None, out=None)`  
Return a dense ndarray representation of this matrix.

**Parameters**

- order** : {'C', 'F'}, optional  
Whether to store multi-dimensional data in C (row-major) or Fortran (column-major) order in memory. The default is 'None', indicating the NumPy default of C-ordered. Cannot be specified in conjunction with the *out* argument.
- out** : ndarray, 2-dimensional, optional  
If specified, uses this array as the output buffer instead of allocating a new array to return. The provided array must have the same shape and dtype as the sparse matrix on which you are calling the method. For most sparse types, *out* is required to be memory contiguous (either C or Fortran ordered).

**Returns**

- arr** : ndarray, 2-dimensional  
An array with the same shape and containing the same data represented by the sparse matrix, with the requested memory order. If *out* was passed, the same object is returned after being modified in-place to contain the appropriate values.

`dia_matrix.tobsr` (*blocksize=None, copy=False*)

Convert this matrix to Block Sparse Row format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `bsr_matrix`.

When `blocksize=(R, C)` is provided, it will be used for construction of the `bsr_matrix`.

`dia_matrix.tocoo` (*copy=False*)

Convert this matrix to COOrdinate format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `coo_matrix`.

`dia_matrix.tocsc` (*copy=False*)

Convert this matrix to Compressed Sparse Column format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `csc_matrix`.

`dia_matrix.tocsr` (*copy=False*)

Convert this matrix to Compressed Sparse Row format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `csr_matrix`.

`dia_matrix.todense` (*order=None, out=None*)

Return a dense matrix representation of this matrix.

**Parameters**    **order** : {'C', 'F'}, optional

Whether to store multi-dimensional data in C (row-major) or Fortran (column-major) order in memory. The default is 'None', indicating the NumPy default of C-ordered. Cannot be specified in conjunction with the `out` argument.

**out** : ndarray, 2-dimensional, optional

If specified, uses this array (or `numpy.matrix`) as the output buffer instead of allocating a new array to return. The provided array must have the same shape and dtype as the sparse matrix on which you are calling the method.

**Returns**        **arr** : `numpy.matrix`, 2-dimensional

A NumPy matrix object with the same shape and containing the same data represented by the sparse matrix, with the requested memory order. If `out` was passed and was an array (rather than a `numpy.matrix`), it will be filled with the appropriate values and returned wrapped in a `numpy.matrix` object that shares the same memory.

`dia_matrix.todia` (*copy=False*)

Convert this matrix to sparse DIAgonal format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `dia_matrix`.

`dia_matrix.todok` (*copy=False*)

Convert this matrix to Dictionary Of Keys format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `dok_matrix`.

`dia_matrix.tolil` (*copy=False*)

Convert this matrix to LInked List format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `lil_matrix`.

`dia_matrix.ttranspose` (*axes=None, copy=False*)

Reverses the dimensions of the sparse matrix.

**Parameters**    **axes** : None, optional

This argument is in the signature *solely* for NumPy compatibility reasons. Do not pass in anything except for the default value.

**copy** : bool, optional

Indicates whether or not attributes of *self* should be copied whenever possible. The degree to which attributes are copied varies depending on the type of sparse matrix being used.

**Returns** **p** : *self* with the dimensions reversed.

**See also:**

`np.matrix.transpose`

NumPy's implementation of 'transpose' for matrices

`dia_matrix.trunc()`

Element-wise trunc.

See `numpy.trunc` for more information.

**class** `scipy.sparse.dok_matrix` (*arg1, shape=None, dtype=None, copy=False*)

Dictionary Of Keys based sparse matrix.

This is an efficient structure for constructing sparse matrices incrementally.

*This can be instantiated in several ways:*

`dok_matrix(D)`

with a dense matrix, D

`dok_matrix(S)` with a sparse matrix, S

`dok_matrix((M,N), [dtype])`

create the matrix with initial shape (M,N) dtype is optional, defaulting to dtype='d'

**Notes**

Sparse matrices can be used in arithmetic operations: they support addition, subtraction, multiplication, division, and matrix power.

Allows for efficient O(1) access of individual elements. Duplicates are not allowed. Can be efficiently converted to a `coo_matrix` once constructed.

**Examples**

```
>>> import numpy as np
>>> from scipy.sparse import dok_matrix
>>> S = dok_matrix((5, 5), dtype=np.float32)
>>> for i in range(5):
...     for j in range(5):
...         S[i, j] = i + j      # Update element
```

**Attributes**

---

**nnz** Number of stored values, including explicit zeros.

---

`dok_matrix.nnz`

Number of stored values, including explicit zeros.

**See also:**

`count_nonzero`

Number of non-zero entries



<code>dtype</code>	(dtype) Data type of the matrix
<code>shape</code>	(2-tuple) Shape of the matrix
<code>ndim</code>	(int) Number of dimensions (this is always 2)

**Methods**

<code>asformat(format)</code>	Return this matrix in a given sparse format
<code>asfptype()</code>	Upcast matrix to a floating point format (if necessary)
<code>astype(t)</code>	
<code>clear()</code> -> None. Remove all items from D.)	
<code>conj()</code>	
<code>conjtransp()</code>	Return the conjugate transpose
<code>conjugate()</code>	
<code>copy()</code>	Returns a copy of this matrix.
<code>count_nonzero()</code>	Number of non-zero entries, equivalent to
<code>diagonal()</code>	Returns the main diagonal of the matrix
<code>dot(other)</code>	Ordinary dot product
<code>fromkeys(...)</code>	v defaults to None.
<code>get(key[, default])</code>	This overrides the dict.get method, providing type checking but otherwise equivalent
<code>getH()</code>	
<code>get_shape()</code>	
<code>getcol(j)</code>	Returns a copy of column j of the matrix as a (m x 1) DOK matrix.
<code>getformat()</code>	
<code>getmaxprint()</code>	
<code>getnnz([axis])</code>	Number of stored values, including explicit zeros.
<code>getrow(i)</code>	Returns a copy of row i of the matrix as a (1 x n) DOK matrix.
<code>has_key(k)</code> -> True if D has a key k, else False)	
<code>items()</code> -> list of D's (key, value) pairs, ...)	
<code>iteritems()</code> -> an iterator over the (key, ...)	
<code>iterkeys()</code> -> an iterator over the keys of D)	
<code>itervalues(...)</code>	
<code>keys()</code> -> list of D's keys)	
<code>maximum(other)</code>	
<code>mean([axis, dtype, out])</code>	Compute the arithmetic mean along the specified axis.
<code>minimum(other)</code>	
<code>multiply(other)</code>	Point-wise multiplication by another matrix
<code>nonzero()</code>	nonzero indices
<code>pop((k[,d]) -&gt; v, ...)</code>	If key is not found, d is returned if given, otherwise KeyError is raised
<code>popitem()</code> -> (k, v), ...)	2-tuple; but raise KeyError if D is empty.
<code>power(n[, dtype])</code>	
<code>reshape(shape[, order])</code>	Gives a new shape to a sparse matrix without changing its data.
<code>resize(shape)</code>	Resize the matrix in-place to dimensions given by 'shape'.
<code>set_shape(shape)</code>	
<code>setdefault((k[,d]) -&gt; D.get(k,d), ...)</code>	
<code>setdiag(values[, k])</code>	Set diagonal or off-diagonal elements of the array.
<code>sum([axis, dtype, out])</code>	Sum the matrix elements over a given axis.
<code>toarray([order, out])</code>	Return a dense ndarray representation of this matrix.
<code>tobsr([blocksize, copy])</code>	Convert this matrix to Block Sparse Row format.
<code>tocoo([copy])</code>	Convert this matrix to COOrdinate format.
<code>tocsc([copy])</code>	Convert this matrix to Compressed Sparse Column format.
<code>tocsr([copy])</code>	Convert this matrix to Compressed Sparse Row format.
<code>todense([order, out])</code>	Return a dense matrix representation of this matrix.
<code>todia([copy])</code>	Convert this matrix to sparse DIAgonal format.

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Table 5.165 – continued from previous page

<code>todok([copy])</code>	Convert this matrix to Dictionary Of Keys format.
<code>tolil([copy])</code>	Convert this matrix to LInked List format.
<code>transpose([axes, copy])</code>	Reverses the dimensions of the sparse matrix.
<code>update([E, ...])</code>	If E present and has a <code>.keys()</code> method, does: for k in E: $D[k] = E[k]$
<code>values()</code> -> list of D's values)	
<code>viewitems(...)</code>	
<code>viewkeys(...)</code>	
<code>viewvalues(...)</code>	

`dok_matrix.asformat` (*format*)

Return this matrix in a given sparse format

**Parameters** `format` : {string, None}  
*desired sparse matrix format*

- None for no format conversion
- “csr” for `csr_matrix` format
- “csc” for `csc_matrix` format
- “li” for `lil_matrix` format
- “dok” for `dok_matrix` format and so on

`dok_matrix.asfptype` ()

Upcast matrix to a floating point format (if necessary)

`dok_matrix.astype` (*t*)

`dok_matrix.clear` () → None. Remove all items from D.

`dok_matrix.conj` ()

`dok_matrix.conjtransp` ()

Return the conjugate transpose

`dok_matrix.conjugate` ()

`dok_matrix.copy` ()

Returns a copy of this matrix.

No data/indices will be shared between the returned value and current matrix.

`dok_matrix.count_nonzero` ()

Number of non-zero entries, equivalent to

`np.count_nonzero(a.toarray())`

Unlike `getnnz()` and the `nnz` property, which return the number of stored entries (the length of the data attribute), this method counts the actual number of non-zero entries in data.

`dok_matrix.diagonal` ()

Returns the main diagonal of the matrix

`dok_matrix.dot` (*other*)

Ordinary dot product

*Examples*

```
>>> import numpy as np
>>> from scipy.sparse import csr_matrix
>>> A = csr_matrix([[1, 2, 0], [0, 0, 3], [4, 0, 5]])
>>> v = np.array([1, 0, -1])
>>> A.dot(v)
array([ 1, -3, -1], dtype=int64)
```

**static** `dok_matrix.fromkeys(S[, v])` → New dict with keys from S and values equal to v.  
v defaults to None.

`dok_matrix.get(key, default=0.0)`

This overrides the dict.get method, providing type checking but otherwise equivalent functionality.

`dok_matrix.getH()`

`dok_matrix.get_shape()`

`dok_matrix.getcol(j)`

Returns a copy of column j of the matrix as a (m x 1) DOK matrix.

`dok_matrix.getformat()`

`dok_matrix.getmaxprint()`

`dok_matrix.getnnz(axis=None)`

Number of stored values, including explicit zeros.

**Parameters** axis : None, 0, or 1

Select between the number of values across the whole matrix, in each column, or in each row.

**See also:**

[\*count\\_nonzero\*](#)

Number of non-zero entries

`dok_matrix.getrow(i)`

Returns a copy of row i of the matrix as a (1 x n) DOK matrix.

`dok_matrix.has_key(k)` → True if D has a key k, else False

`dok_matrix.items()` → list of D's (key, value) pairs, as 2-tuples

`dok_matrix.iteritems()` → an iterator over the (key, value) items of D

`dok_matrix.iterkeys()` → an iterator over the keys of D

`dok_matrix.itervalues()` → an iterator over the values of D

`dok_matrix.keys()` → list of D's keys

`dok_matrix.maximum` (*other*)

`dok_matrix.mean` (*axis=None, dtype=None, out=None*)

Compute the arithmetic mean along the specified axis.

Returns the average of the matrix elements. The average is taken over all elements in the matrix by default, otherwise over the specified axis. *float64* intermediate and return values are used for integer inputs.

**Parameters**

- axis** : {-2, -1, 0, 1, None} optional  
Axis along which the mean is computed. The default is to compute the mean of all elements in the matrix (i.e. *axis = None*).
- dtype** : data-type, optional  
Type to use in computing the mean. For integer inputs, the default is *float64*; for floating point inputs, it is the same as the input dtype.
- out** : np.matrix, optional  
Alternative output matrix in which to place the result. It must have the same shape as the expected output, but the type of the output values will be cast if necessary.

**Returns** **m** : np.matrix

**See also:**

**`np.matrix.mean`**

NumPy's implementation of 'mean' for matrices

`dok_matrix.minimum` (*other*)

`dok_matrix.multiply` (*other*)

Point-wise multiplication by another matrix

`dok_matrix.nonzero` ()

nonzero indices

Returns a tuple of arrays (row,col) containing the indices of the non-zero elements of the matrix.

**Examples**

```
>>> from scipy.sparse import csr_matrix
>>> A = csr_matrix([[1, 2, 0], [0, 0, 3], [4, 0, 5]])
>>> A.nonzero()
(array([0, 0, 1, 2, 2]), array([0, 1, 2, 0, 2]))
```

`dok_matrix.pop` (*k*[, *d*]) → *v*, remove specified key and return the corresponding value.

If key is not found, *d* is returned if given, otherwise `KeyError` is raised

`dok_matrix.popitem` () → (*k*, *v*), remove and return some (key, value) pair as a

2-tuple; but raise `KeyError` if *D* is empty.

`dok_matrix.power` (*n, dtype=None*)

`dok_matrix.reshape` (*shape, order='C'*)

Gives a new shape to a sparse matrix without changing its data.

**Parameters**

- shape** : length-2 tuple of ints  
The new shape should be compatible with the original shape.
- order** : 'C', optional

This argument is in the signature *solely* for NumPy compatibility reasons. Do not pass in anything except for the default value, as this argument is not used.

**Returns** `reshaped_matrix`: *self* with the new dimensions of shape

**See also:**

`np.matrix.reshape`

NumPy's implementation of 'reshape' for matrices

`dok_matrix.resize` (*shape*)

Resize the matrix in-place to dimensions given by 'shape'.

Any non-zero elements that lie outside the new shape are removed.

`dok_matrix.set_shape` (*shape*)

`dok_matrix.setdefault` (*k*[, *d*]) → *D.get(k,d)*, also set *D[k]=d* if *k* not in *D*

`dok_matrix.setdiag` (*values*, *k=0*)

Set diagonal or off-diagonal elements of the array.

**Parameters** `values`: array\_like

New values of the diagonal elements.

Values may have any length. If the diagonal is longer than *values*, then the remaining diagonal entries will not be set. If *values* is longer than the diagonal, then the remaining *values* are ignored.

If a scalar value is given, all of the diagonal is set to it.

`k`: int, optional

Which off-diagonal to set, corresponding to elements *a*[*i*,*i*+*k*]. Default: 0 (the main diagonal).

`dok_matrix.sum` (*axis=None*, *dtype=None*, *out=None*)

Sum the matrix elements over a given axis.

**Parameters** `axis`: {-2, -1, 0, 1, None} optional

Axis along which the sum is computed. The default is to compute the sum of all the matrix elements, returning a scalar (i.e. *axis = None*).

`dtype`: dtype, optional

The type of the returned matrix and of the accumulator in which the elements are summed. The dtype of *a* is used by default unless *a* has an integer dtype of less precision than the default platform integer. In that case, if *a* is signed then the platform integer is used while if *a* is unsigned then an unsigned integer of the same precision as the platform integer is used.

`out`: np.matrix, optional

Alternative output matrix in which to place the result. It must have the same shape as the expected output, but the type of the output values will be cast if necessary.

**Returns** `sum_along_axis`: np.matrix

A matrix with the same shape as *self*, with the specified axis removed.

**See also:**

`np.matrix.sum`

NumPy's implementation of 'sum' for matrices

`dok_matrix.toarray` (*order=None*, *out=None*)

Return a dense ndarray representation of this matrix.

**Parameters** **order** : {'C', 'F'}, optional  
 Whether to store multi-dimensional data in C (row-major) or Fortran (column-major) order in memory. The default is 'None', indicating the NumPy default of C-ordered. Cannot be specified in conjunction with the *out* argument.

**out** : ndarray, 2-dimensional, optional  
 If specified, uses this array as the output buffer instead of allocating a new array to return. The provided array must have the same shape and dtype as the sparse matrix on which you are calling the method. For most sparse types, *out* is required to be memory contiguous (either C or Fortran ordered).

**Returns** **arr** : ndarray, 2-dimensional  
 An array with the same shape and containing the same data represented by the sparse matrix, with the requested memory order. If *out* was passed, the same object is returned after being modified in-place to contain the appropriate values.

`dok_matrix.tobsr` (*blocksize=None, copy=False*)

Convert this matrix to Block Sparse Row format.

With *copy=False*, the data/indices may be shared between this matrix and the resultant `bsr_matrix`.

When *blocksize=(R, C)* is provided, it will be used for construction of the `bsr_matrix`.

`dok_matrix.tocoo` (*copy=False*)

Convert this matrix to COOrdinate format.

With *copy=False*, the data/indices may be shared between this matrix and the resultant `coo_matrix`.

`dok_matrix.tocsc` (*copy=False*)

Convert this matrix to Compressed Sparse Column format.

With *copy=False*, the data/indices may be shared between this matrix and the resultant `csc_matrix`.

`dok_matrix.tocsr` (*copy=False*)

Convert this matrix to Compressed Sparse Row format.

With *copy=False*, the data/indices may be shared between this matrix and the resultant `csr_matrix`.

`dok_matrix.todense` (*order=None, out=None*)

Return a dense matrix representation of this matrix.

**Parameters** **order** : {'C', 'F'}, optional  
 Whether to store multi-dimensional data in C (row-major) or Fortran (column-major) order in memory. The default is 'None', indicating the NumPy default of C-ordered. Cannot be specified in conjunction with the *out* argument.

**out** : ndarray, 2-dimensional, optional  
 If specified, uses this array (or `numpy.matrix`) as the output buffer instead of allocating a new array to return. The provided array must have the same shape and dtype as the sparse matrix on which you are calling the method.

**Returns** **arr** : `numpy.matrix`, 2-dimensional  
 A NumPy matrix object with the same shape and containing the same data represented by the sparse matrix, with the requested memory order. If *out* was passed and was an array (rather than a `numpy.matrix`), it will be filled with the appropriate values and returned wrapped in a `numpy.matrix` object that shares the same memory.

`dok_matrix.todia (copy=False)`

Convert this matrix to sparse DIAgonal format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `dia_matrix`.

`dok_matrix.todok (copy=False)`

Convert this matrix to Dictionary Of Keys format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `dok_matrix`.

`dok_matrix.tolil (copy=False)`

Convert this matrix to LInked List format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `lil_matrix`.

`dok_matrix.transpose (axes=None, copy=False)`

Reverses the dimensions of the sparse matrix.

**Parameters** `axes` : None, optional

This argument is in the signature *solely* for NumPy compatibility reasons. Do not pass in anything except for the default value.

`copy` : bool, optional

Indicates whether or not attributes of *self* should be copied whenever possible. The degree to which attributes are copied varies depending on the type of sparse matrix being used.

**Returns** `p` : *self* with the dimensions reversed.

**See also:**

`np.matrix.transpose`

NumPy's implementation of 'transpose' for matrices

`dok_matrix.update ([E], **F) → None`. Update D from dict/iterable E and F.

If E present and has a `.keys()` method, does: for k in E: D[k] = E[k] If E present and lacks `.keys()` method, does: for (k, v) in E: D[k] = v In either case, this is followed by: for k in F: D[k] = F[k]

`dok_matrix.values () → list of D's values`

`dok_matrix.viewitems () → a set-like object providing a view on D's items`

`dok_matrix.viewkeys () → a set-like object providing a view on D's keys`

`dok_matrix.viewvalues () → an object providing a view on D's values`

**class** `scipy.sparse.lil_matrix (arg1, shape=None, dtype=None, copy=False)`

Row-based linked list sparse matrix

This is a structure for constructing sparse matrices incrementally. Note that inserting a single item can take linear time in the worst case; to construct a matrix efficiently, make sure the items are pre-sorted by index, per row.

*This can be instantiated in several ways:*

`lil_matrix(D)` with a dense matrix or rank-2 ndarray D

`lil_matrix(S)` with another sparse matrix S (equivalent to `S.tolil()`)

`lil_matrix((M, N), [dtype])`

to construct an empty matrix with shape (M, N) dtype is optional, defaulting to `dtype='d'`.

*Notes*

Sparse matrices can be used in arithmetic operations: they support addition, subtraction, multiplication, division, and matrix power.

*Advantages of the LIL format*

- supports flexible slicing
- changes to the matrix sparsity structure are efficient

*Disadvantages of the LIL format*

- arithmetic operations LIL + LIL are slow (consider CSR or CSC)
- slow column slicing (consider CSC)
- slow matrix vector products (consider CSR or CSC)

*Intended Usage*

- LIL is a convenient format for constructing sparse matrices
- once a matrix has been constructed, convert to CSR or CSC format for fast arithmetic and matrix vector operations
- consider using the COO format when constructing large matrices

*Data Structure*

- An array (`self.rows`) of rows, each of which is a sorted list of column indices of non-zero elements.
- The corresponding nonzero values are stored in similar fashion in `self.data`.

*Attributes*

---

`nnz` Number of stored values, including explicit zeros.

---

`lil_matrix.nnz`

Number of stored values, including explicit zeros.

**See also:**

`count_nonzero`

Number of non-zero entries

<code>dtype</code>	(dtype) Data type of the matrix
<code>shape</code>	(2-tuple) Shape of the matrix
<code>ndim</code>	(int) Number of dimensions (this is always 2)
<code>data</code>	LIL format data array of the matrix
<code>rows</code>	LIL format row index array of the matrix

*Methods*

---

<code>asformat(format)</code>	Return this matrix in a given sparse format
<code>asfptype()</code>	Upcast matrix to a floating point format (if necessary)
<code>astype(t)</code>	
<code>conj()</code>	
<code>conjugate()</code>	
<code>copy()</code>	Returns a copy of this matrix.
<code>count_nonzero()</code>	Number of non-zero entries, equivalent to
<code>diagonal()</code>	Returns the main diagonal of the matrix
<code>dot(other)</code>	Ordinary dot product

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Table 5.167 – continued from previous page

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<code>getH()</code>	
<code>get_shape()</code>	
<code>getcol(j)</code>	Returns a copy of column <i>j</i> of the matrix, as an (m x 1) sparse matrix (column vector).
<code>getformat()</code>	
<code>getmaxprint()</code>	
<code>getnnz([axis])</code>	Number of stored values, including explicit zeros.
<code>getrow(i)</code>	Returns a copy of the ‘i’th row.
<code>getrowview(i)</code>	Returns a view of the ‘i’th row (without copying).
<code>maximum(other)</code>	
<code>mean([axis, dtype, out])</code>	Compute the arithmetic mean along the specified axis.
<code>minimum(other)</code>	
<code>multiply(other)</code>	Point-wise multiplication by another matrix
<code>nonzero()</code>	nonzero indices
<code>power(n[, dtype])</code>	
<code>reshape(shape[, order])</code>	Gives a new shape to a sparse matrix without changing its data.
<code>set_shape(shape)</code>	
<code>setdiag(values[, k])</code>	Set diagonal or off-diagonal elements of the array.
<code>sum([axis, dtype, out])</code>	Sum the matrix elements over a given axis.
<code>toarray([order, out])</code>	See the docstring for <code>spmatrix.toarray</code> .
<code>tobsr([blocksize, copy])</code>	Convert this matrix to Block Sparse Row format.
<code>tocoo([copy])</code>	Convert this matrix to COOrdinate format.
<code>tocsc([copy])</code>	Convert this matrix to Compressed Sparse Column format.
<code>tocsr([copy])</code>	Convert this matrix to Compressed Sparse Row format.
<code>todense([order, out])</code>	Return a dense matrix representation of this matrix.
<code>todia([copy])</code>	Convert this matrix to sparse DIAgonal format.
<code>todok([copy])</code>	Convert this matrix to Dictionary Of Keys format.
<code>tolil([copy])</code>	Convert this matrix to LInked List format.
<code>transpose([axes, copy])</code>	

---

`lil_matrix.asformat` (*format*)

Return this matrix in a given sparse format

**Parameters** **format** : {string, None}  
*desired sparse matrix format*

- None for no format conversion
- “csr” for `csr_matrix` format
- “csc” for `csc_matrix` format
- “lil” for `lil_matrix` format
- “dok” for `dok_matrix` format and so on

`lil_matrix.asfptype` ()

Upcast matrix to a floating point format (if necessary)

`lil_matrix.astype` (*t*)

`lil_matrix.conj` ()

`lil_matrix.conjugate` ()

`lil_matrix.copy` ()

Returns a copy of this matrix.

No data/indices will be shared between the returned value and current matrix.

`lil_matrix.count_nonzero()`  
 Number of non-zero entries, equivalent to  
`np.count_nonzero(a.toarray())`

Unlike `getnnz()` and the `nnz` property, which return the number of stored entries (the length of the data attribute), this method counts the actual number of non-zero entries in data.

`lil_matrix.diagonal()`  
 Returns the main diagonal of the matrix

`lil_matrix.dot(other)`  
 Ordinary dot product

**Examples**

```
>>> import numpy as np
>>> from scipy.sparse import csr_matrix
>>> A = csr_matrix([[1, 2, 0], [0, 0, 3], [4, 0, 5]])
>>> v = np.array([1, 0, -1])
>>> A.dot(v)
array([ 1, -3, -1], dtype=int64)
```

`lil_matrix.getH()`

`lil_matrix.get_shape()`

`lil_matrix.getcol(j)`  
 Returns a copy of column `j` of the matrix, as an  $(m \times 1)$  sparse matrix (column vector).

`lil_matrix.getformat()`

`lil_matrix.getmaxprint()`

`lil_matrix.getnnz(axis=None)`  
 Number of stored values, including explicit zeros.

**Parameters** `axis`: None, 0, or 1

Select between the number of values across the whole matrix, in each column, or in each row.

**See also:**

`count_nonzero`  
 Number of non-zero entries

`lil_matrix.getrow(i)`  
 Returns a copy of the 'i'th row.

`lil_matrix.getrowview(i)`  
 Returns a view of the 'i'th row (without copying).

`lil_matrix.maximum(other)`

`lil_matrix.mean(axis=None, dtype=None, out=None)`  
 Compute the arithmetic mean along the specified axis.

Returns the average of the matrix elements. The average is taken over all elements in the matrix by default, otherwise over the specified axis. *float64* intermediate and return values are used for integer inputs.

**Parameters**

- axis** : {-2, -1, 0, 1, None} optional  
Axis along which the mean is computed. The default is to compute the mean of all elements in the matrix (i.e. *axis = None*).
- dtype** : data-type, optional  
Type to use in computing the mean. For integer inputs, the default is *float64*; for floating point inputs, it is the same as the input dtype.
- out** : np.matrix, optional  
Alternative output matrix in which to place the result. It must have the same shape as the expected output, but the type of the output values will be cast if necessary.

**Returns** **m** : np.matrix

**See also:**

**np.matrix.mean**

NumPy's implementation of 'mean' for matrices

`lil_matrix.minimum` (*other*)

`lil_matrix.multiply` (*other*)

Point-wise multiplication by another matrix

`lil_matrix.nonzero` ()

nonzero indices

Returns a tuple of arrays (row,col) containing the indices of the non-zero elements of the matrix.

**Examples**

```
>>> from scipy.sparse import csr_matrix
>>> A = csr_matrix([[1, 2, 0], [0, 0, 3], [4, 0, 5]])
>>> A.nonzero()
(array([0, 0, 1, 2, 2]), array([0, 1, 2, 0, 2]))
```

`lil_matrix.power` (*n*, *dtype=None*)

`lil_matrix.reshape` (*shape*, *order='C'*)

Gives a new shape to a sparse matrix without changing its data.

**Parameters**

- shape** : length-2 tuple of ints  
The new shape should be compatible with the original shape.

- order** : 'C', optional  
This argument is in the signature *solely* for NumPy compatibility reasons. Do not pass in anything except for the default value, as this argument is not used.

**Returns** **reshaped\_matrix** : *self* with the new dimensions of shape

**See also:**

**np.matrix.reshape**

NumPy's implementation of 'reshape' for matrices

`lil_matrix.set_shape` (*shape*)

`lil_matrix.setdiag` (*values*, *k=0*)

Set diagonal or off-diagonal elements of the array.

**Parameters** **values** : array\_like  
 New values of the diagonal elements.  
 Values may have any length. If the diagonal is longer than values, then the remaining diagonal entries will not be set. If values is longer than the diagonal, then the remaining values are ignored.  
 If a scalar value is given, all of the diagonal is set to it.

**k** : int, optional  
 Which off-diagonal to set, corresponding to elements  $a[i,i+k]$ . Default: 0 (the main diagonal).

`lil_matrix.sum` (*axis=None, dtype=None, out=None*)  
 Sum the matrix elements over a given axis.

**Parameters** **axis** : {-2, -1, 0, 1, None} optional  
 Axis along which the sum is computed. The default is to compute the sum of all the matrix elements, returning a scalar (i.e. *axis = None*).

**dtype** : dtype, optional  
 The type of the returned matrix and of the accumulator in which the elements are summed. The dtype of *a* is used by default unless *a* has an integer dtype of less precision than the default platform integer. In that case, if *a* is signed then the platform integer is used while if *a* is unsigned then an unsigned integer of the same precision as the platform integer is used.

**out** : np.matrix, optional  
 Alternative output matrix in which to place the result. It must have the same shape as the expected output, but the type of the output values will be cast if necessary.

**Returns** **sum\_along\_axis** : np.matrix  
 A matrix with the same shape as *self*, with the specified axis removed.

**See also:**

**`np.matrix.sum`**  
 NumPy's implementation of 'sum' for matrices

`lil_matrix.toarray` (*order=None, out=None*)  
 See the docstring for `spmatrix.toarray`.

`lil_matrix.tobsr` (*blocksize=None, copy=False*)  
 Convert this matrix to Block Sparse Row format.  
 With `copy=False`, the data/indices may be shared between this matrix and the resultant `bsr_matrix`.  
 When `blocksize=(R, C)` is provided, it will be used for construction of the `bsr_matrix`.

`lil_matrix.tocoo` (*copy=False*)  
 Convert this matrix to COOrdinate format.  
 With `copy=False`, the data/indices may be shared between this matrix and the resultant `coo_matrix`.

`lil_matrix.tocsc` (*copy=False*)  
 Convert this matrix to Compressed Sparse Column format.  
 With `copy=False`, the data/indices may be shared between this matrix and the resultant `csc_matrix`.

`lil_matrix.tocsr` (*copy=False*)  
 Convert this matrix to Compressed Sparse Row format.  
 With `copy=False`, the data/indices may be shared between this matrix and the resultant `csr_matrix`.

`lil_matrix.todense` (*order=None, out=None*)  
 Return a dense matrix representation of this matrix.

**Parameters**

**order** : {'C', 'F'}, optional  
 Whether to store multi-dimensional data in C (row-major) or Fortran (column-major) order in memory. The default is 'None', indicating the NumPy default of C-ordered. Cannot be specified in conjunction with the *out* argument.

**out** : ndarray, 2-dimensional, optional  
 If specified, uses this array (or `numpy.matrix`) as the output buffer instead of allocating a new array to return. The provided array must have the same shape and dtype as the sparse matrix on which you are calling the method.

**Returns**

**arr** : `numpy.matrix`, 2-dimensional  
 A NumPy matrix object with the same shape and containing the same data represented by the sparse matrix, with the requested memory order. If *out* was passed and was an array (rather than a `numpy.matrix`), it will be filled with the appropriate values and returned wrapped in a `numpy.matrix` object that shares the same memory.

`lil_matrix.todia (copy=False)`

Convert this matrix to sparse DIAgonal format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `dia_matrix`.

`lil_matrix.todok (copy=False)`

Convert this matrix to Dictionary Of Keys format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `dok_matrix`.

`lil_matrix.tolil (copy=False)`

Convert this matrix to LInked List format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `lil_matrix`.

`lil_matrix.ttranspose (axes=None, copy=False)`

**class** `scipy.sparse.spmatrix (maxprint=50)`

This class provides a base class for all sparse matrices. It cannot be instantiated. Most of the work is provided by subclasses.

**Attributes**

<code>nnz</code>	Number of stored values, including explicit zeros.
<code>shape</code>	

`spmatrix.nnz`

Number of stored values, including explicit zeros.

**See also:**

`count_nonzero`

Number of non-zero entries

`spmatrix.shape`

**Methods**

<code>asformat(format)</code>	Return this matrix in a given sparse format
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Table 5.169 – continued from previous page

<code>asfptype()</code>	Upcast matrix to a floating point format (if necessary)
<code>astype(t)</code>	
<code>conj()</code>	
<code>conjugate()</code>	
<code>copy()</code>	Returns a copy of this matrix.
<code>count_nonzero()</code>	Number of non-zero entries, equivalent to
<code>diagonal()</code>	Returns the main diagonal of the matrix
<code>dot(other)</code>	Ordinary dot product
<code>getH()</code>	
<code>get_shape()</code>	
<code>getcol(j)</code>	Returns a copy of column j of the matrix, as an (m x 1) sparse matrix (column vector).
<code>getformat()</code>	
<code>getmaxprint()</code>	
<code>getnnz([axis])</code>	Number of stored values, including explicit zeros.
<code>getrow(i)</code>	Returns a copy of row i of the matrix, as a (1 x n) sparse matrix (row vector).
<code>maximum(other)</code>	
<code>mean([axis, dtype, out])</code>	Compute the arithmetic mean along the specified axis.
<code>minimum(other)</code>	
<code>multiply(other)</code>	Point-wise multiplication by another matrix
<code>nonzero()</code>	nonzero indices
<code>power(n[, dtype])</code>	
<code>reshape(shape[, order])</code>	Gives a new shape to a sparse matrix without changing its data.
<code>set_shape(shape)</code>	
<code>setdiag(values[, k])</code>	Set diagonal or off-diagonal elements of the array.
<code>sum([axis, dtype, out])</code>	Sum the matrix elements over a given axis.
<code>toarray([order, out])</code>	Return a dense ndarray representation of this matrix.
<code>tobsr([blocksize, copy])</code>	Convert this matrix to Block Sparse Row format.
<code>tocoo([copy])</code>	Convert this matrix to COOrdinate format.
<code>tocsc([copy])</code>	Convert this matrix to Compressed Sparse Column format.
<code>tocsr([copy])</code>	Convert this matrix to Compressed Sparse Row format.
<code>todense([order, out])</code>	Return a dense matrix representation of this matrix.
<code>todia([copy])</code>	Convert this matrix to sparse DIAgonal format.
<code>todok([copy])</code>	Convert this matrix to Dictionary Of Keys format.
<code>tolil([copy])</code>	Convert this matrix to LInked List format.
<code>transpose([axes, copy])</code>	Reverses the dimensions of the sparse matrix.

`spmatrix.asformat` (*format*)

Return this matrix in a given sparse format

**Parameters** **format** : {string, None}  
*desired sparse matrix format*

- None for no format conversion
- “csr” for csr\_matrix format
- “csc” for csc\_matrix format
- “lil” for lil\_matrix format
- “dok” for dok\_matrix format and so on

`spmatrix.asfptype` ()

Upcast matrix to a floating point format (if necessary)

`spmatrix.astype` (*t*)

`spmatrix.conj()`

`spmatrix.conjugate()`

`spmatrix.copy()`

Returns a copy of this matrix.

No data/indices will be shared between the returned value and current matrix.

`spmatrix.count_nonzero()`

Number of non-zero entries, equivalent to

`np.count_nonzero(a.toarray())`

Unlike `getnnz()` and the `nnz` property, which return the number of stored entries (the length of the data attribute), this method counts the actual number of non-zero entries in data.

`spmatrix.diagonal()`

Returns the main diagonal of the matrix

`spmatrix.dot(other)`

Ordinary dot product

### Examples

```
>>> import numpy as np
>>> from scipy.sparse import csr_matrix
>>> A = csr_matrix([[1, 2, 0], [0, 0, 3], [4, 0, 5]])
>>> v = np.array([1, 0, -1])
>>> A.dot(v)
array([ 1, -3, -1], dtype=int64)
```

`spmatrix.getH()`

`spmatrix.get_shape()`

`spmatrix.getcol(j)`

Returns a copy of column `j` of the matrix, as an  $(m \times 1)$  sparse matrix (column vector).

`spmatrix.getformat()`

`spmatrix.getmaxprint()`

`spmatrix.getnnz(axis=None)`

Number of stored values, including explicit zeros.

**Parameters** `axis`: None, 0, or 1

Select between the number of values across the whole matrix, in each column, or in each row.

### See also:

`count_nonzero`

Number of non-zero entries

`spmatrix.getrow(i)`

Returns a copy of row `i` of the matrix, as a  $(1 \times n)$  sparse matrix (row vector).

`spmatrix.maximum` (*other*)

`spmatrix.mean` (*axis=None, dtype=None, out=None*)

Compute the arithmetic mean along the specified axis.

Returns the average of the matrix elements. The average is taken over all elements in the matrix by default, otherwise over the specified axis. *float64* intermediate and return values are used for integer inputs.

**Parameters**

- axis** : {-2, -1, 0, 1, None} optional  
Axis along which the mean is computed. The default is to compute the mean of all elements in the matrix (i.e. *axis = None*).
- dtype** : data-type, optional  
Type to use in computing the mean. For integer inputs, the default is *float64*; for floating point inputs, it is the same as the input dtype.
- out** : np.matrix, optional  
Alternative output matrix in which to place the result. It must have the same shape as the expected output, but the type of the output values will be cast if necessary.

**Returns** **m** : np.matrix

**See also:**

**`np.matrix.mean`**

NumPy's implementation of 'mean' for matrices

`spmatrix.minimum` (*other*)

`spmatrix.multiply` (*other*)

Point-wise multiplication by another matrix

`spmatrix.nonzero` ()

nonzero indices

Returns a tuple of arrays (row,col) containing the indices of the non-zero elements of the matrix.

**Examples**

```
>>> from scipy.sparse import csr_matrix
>>> A = csr_matrix([[1, 2, 0], [0, 0, 3], [4, 0, 5]])
>>> A.nonzero()
(array([0, 0, 1, 2, 2]), array([0, 1, 2, 0, 2]))
```

`spmatrix.power` (*n, dtype=None*)

`spmatrix.reshape` (*shape, order='C'*)

Gives a new shape to a sparse matrix without changing its data.

**Parameters**

- shape** : length-2 tuple of ints  
The new shape should be compatible with the original shape.
- order** : 'C', optional  
This argument is in the signature *solely* for NumPy compatibility reasons. Do not pass in anything except for the default value, as this argument is not used.

**Returns** **reshaped\_matrix** : *self* with the new dimensions of `shape`

**See also:**

**`np.matrix.reshape`**

NumPy's implementation of 'reshape' for matrices



`spmatrix.set_shape` (*shape*)

`spmatrix.setdiag` (*values*, *k=0*)

Set diagonal or off-diagonal elements of the array.

**Parameters** **values** : array\_like

New values of the diagonal elements.

Values may have any length. If the diagonal is longer than values, then the remaining diagonal entries will not be set. If values is longer than the diagonal, then the remaining values are ignored.

If a scalar value is given, all of the diagonal is set to it.

**k** : int, optional

Which off-diagonal to set, corresponding to elements  $a[i,i+k]$ . Default: 0 (the main diagonal).

`spmatrix.sum` (*axis=None*, *dtype=None*, *out=None*)

Sum the matrix elements over a given axis.

**Parameters** **axis** : {-2, -1, 0, 1, None} optional

Axis along which the sum is computed. The default is to compute the sum of all the matrix elements, returning a scalar (i.e. *axis = None*).

**dtype** : dtype, optional

The type of the returned matrix and of the accumulator in which the elements are summed. The dtype of *a* is used by default unless *a* has an integer dtype of less precision than the default platform integer. In that case, if *a* is signed then the platform integer is used while if *a* is unsigned then an unsigned integer of the same precision as the platform integer is used.

**out** : np.matrix, optional

Alternative output matrix in which to place the result. It must have the same shape as the expected output, but the type of the output values will be cast if necessary.

**Returns** **sum\_along\_axis** : np.matrix

A matrix with the same shape as *self*, with the specified axis removed.

See also:

`np.matrix.sum`

NumPy's implementation of 'sum' for matrices

`spmatrix.toarray` (*order=None*, *out=None*)

Return a dense ndarray representation of this matrix.

**Parameters** **order** : {'C', 'F'}, optional

Whether to store multi-dimensional data in C (row-major) or Fortran (column-major) order in memory. The default is 'None', indicating the NumPy default of C-ordered. Cannot be specified in conjunction with the *out* argument.

**out** : ndarray, 2-dimensional, optional

If specified, uses this array as the output buffer instead of allocating a new array to return. The provided array must have the same shape and dtype as the sparse matrix on which you are calling the method. For most sparse types, *out* is required to be memory contiguous (either C or Fortran ordered).

**Returns** **arr** : ndarray, 2-dimensional

An array with the same shape and containing the same data represented by the sparse matrix, with the requested memory order. If *out*

was passed, the same object is returned after being modified in-place to contain the appropriate values.

`spmatrix.tobsr` (*blocksize=None, copy=False*)

Convert this matrix to Block Sparse Row format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `bsr_matrix`.

When `blocksize=(R, C)` is provided, it will be used for construction of the `bsr_matrix`.

`spmatrix.tocoo` (*copy=False*)

Convert this matrix to COOrdinate format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `coo_matrix`.

`spmatrix.tocsc` (*copy=False*)

Convert this matrix to Compressed Sparse Column format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `csc_matrix`.

`spmatrix.tocsr` (*copy=False*)

Convert this matrix to Compressed Sparse Row format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `csr_matrix`.

`spmatrix.todense` (*order=None, out=None*)

Return a dense matrix representation of this matrix.

**Parameters**    **order** : {'C', 'F'}, optional

Whether to store multi-dimensional data in C (row-major) or Fortran (column-major) order in memory. The default is 'None', indicating the NumPy default of C-ordered. Cannot be specified in conjunction with the `out` argument.

**out** : ndarray, 2-dimensional, optional

If specified, uses this array (or `numpy.matrix`) as the output buffer instead of allocating a new array to return. The provided array must have the same shape and dtype as the sparse matrix on which you are calling the method.

**Returns**        **arr** : `numpy.matrix`, 2-dimensional

A NumPy matrix object with the same shape and containing the same data represented by the sparse matrix, with the requested memory order. If `out` was passed and was an array (rather than a `numpy.matrix`), it will be filled with the appropriate values and returned wrapped in a `numpy.matrix` object that shares the same memory.

`spmatrix.todia` (*copy=False*)

Convert this matrix to sparse DIAgonal format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `dia_matrix`.

`spmatrix.todok` (*copy=False*)

Convert this matrix to Dictionary Of Keys format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `dok_matrix`.

`spmatrix.tolil` (*copy=False*)

Convert this matrix to LInked List format.

With `copy=False`, the data/indices may be shared between this matrix and the resultant `lil_matrix`.

`spmatrix.ttranspose` (*axes=None, copy=False*)

Reverses the dimensions of the sparse matrix.

**Parameters** **axes** : None, optional  
 This argument is in the signature *solely* for NumPy compatibility reasons. Do not pass in anything except for the default value.

**copy** : bool, optional  
 Indicates whether or not attributes of *self* should be copied whenever possible. The degree to which attributes are copied varies depending on the type of sparse matrix being used.

**Returns** **p** : *self* with the dimensions reversed.

**See also:**

**`np.matrix.transpose`**

NumPy's implementation of 'transpose' for matrices

## Functions

Building sparse matrices:

<code>eye(m[, n, k, dtype, format])</code>	Sparse matrix with ones on diagonal
<code>identity(n[, dtype, format])</code>	Identity matrix in sparse format
<code>kron(A, B[, format])</code>	kroncker product of sparse matrices A and B
<code>kronsum(A, B[, format])</code>	kroncker sum of sparse matrices A and B
<code>diags(diagonals[, offsets, shape, format, dtype])</code>	Construct a sparse matrix from diagonals.
<code>spdiags(data, diags, m, n[, format])</code>	Return a sparse matrix from diagonals.
<code>block_diag(mats[, format, dtype])</code>	Build a block diagonal sparse matrix from provided matrices.
<code>tril(A[, k, format])</code>	Return the lower triangular portion of a matrix in sparse format
<code>triu(A[, k, format])</code>	Return the upper triangular portion of a matrix in sparse format
<code>bmat(blocks[, format, dtype])</code>	Build a sparse matrix from sparse sub-blocks
<code>hstack(blocks[, format, dtype])</code>	Stack sparse matrices horizontally (column wise)
<code>vstack(blocks[, format, dtype])</code>	Stack sparse matrices vertically (row wise)
<code>rand(m, n[, density, format, dtype, ...])</code>	Generate a sparse matrix of the given shape and density with uniformly distributed values
<code>random(m, n[, density, format, dtype, ...])</code>	Generate a sparse matrix of the given shape and density with randomly distributed values

`scipy.sparse.eye` (*m*, *n=None*, *k=0*, *dtype=<type 'float'>*, *format=None*)

Sparse matrix with ones on diagonal

Returns a sparse (*m* x *n*) matrix where the *k*-th diagonal is all ones and everything else is zeros.

**Parameters** **m** : int  
 Number of rows in the matrix.

**n** : int, optional  
 Number of columns. Default: *m*.

**k** : int, optional  
 Diagonal to place ones on. Default: 0 (main diagonal).

**dtype** : dtype, optional  
 Data type of the matrix.

**format** : str, optional  
 Sparse format of the result, e.g. `format="csr"`, etc.

### Examples

```
>>> from scipy import sparse
>>> sparse.eye(3).toarray()
array([[ 1.,  0.,  0.],
       [ 0.,  1.,  0.]])
```

```
[ 0.,  0.,  1.])
>>> sparse.eye(3, dtype=np.int8)
<3x3 sparse matrix of type '<type 'numpy.int8'>'
  with 3 stored elements (1 diagonals) in DIAgonal format>
```

`scipy.sparse.identity(n, dtype='d', format=None)`

Identity matrix in sparse format

Returns an identity matrix with shape (n,n) using a given sparse format and dtype.

**Parameters**

- n** : int  
Shape of the identity matrix.
- dtype** : dtype, optional  
Data type of the matrix
- format** : str, optional  
Sparse format of the result, e.g. format="csr", etc.

**Examples**

```
>>> from scipy.sparse import identity
>>> identity(3).toarray()
array([[ 1.,  0.,  0.],
       [ 0.,  1.,  0.],
       [ 0.,  0.,  1.]])
>>> identity(3, dtype='int8', format='dia')
<3x3 sparse matrix of type '<type 'numpy.int8'>'
  with 3 stored elements (1 diagonals) in DIAgonal format>
```

`scipy.sparse.kron(A, B, format=None)`

kroncker product of sparse matrices A and B

**Parameters**

- A** : sparse or dense matrix  
first matrix of the product
- B** : sparse or dense matrix  
second matrix of the product
- format** : str, optional  
format of the result (e.g. "csr")

**Returns** kroncker product in a sparse matrix format

**Examples**

```
>>> from scipy import sparse
>>> A = sparse.csr_matrix(np.array([[0, 2], [5, 0]]))
>>> B = sparse.csr_matrix(np.array([[1, 2], [3, 4]]))
>>> sparse.kron(A, B).toarray()
array([[ 0,  0,  2,  4],
       [ 0,  0,  6,  8],
       [ 5, 10,  0,  0],
       [15, 20,  0,  0]])

>>> sparse.kron(A, [[1, 2], [3, 4]]).toarray()
array([[ 0,  0,  2,  4],
       [ 0,  0,  6,  8],
       [ 5, 10,  0,  0],
       [15, 20,  0,  0]])
```

`scipy.sparse.kronsum(A, B, format=None)`

kroncker sum of sparse matrices A and B

Kronecker sum of two sparse matrices is a sum of two Kronecker products  $\text{kron}(I_n, A) + \text{kron}(B, I_m)$  where  $A$  has shape  $(m, m)$  and  $B$  has shape  $(n, n)$  and  $I_m$  and  $I_n$  are identity matrices of shape  $(m, m)$  and  $(n, n)$  respectively.

**Parameters**

- A** square matrix
- B** square matrix
- format** : str format of the result, (e.g. "csr")

**Returns** kronecker sum in a sparse matrix format

`scipy.sparse.diags` (*diagonals, offsets=0, shape=None, format=None, dtype=None*)

Construct a sparse matrix from diagonals.

**Parameters**

- diagonals** : sequence of array\_like  
Sequence of arrays containing the matrix diagonals, corresponding to *offsets*.
- offsets** : sequence of int or an int, optional  
**Diagonals to set:**
  - $k = 0$  the main diagonal (default)
  - $k > 0$  the  $k$ -th upper diagonal
  - $k < 0$  the  $k$ -th lower diagonal
- shape** : tuple of int, optional  
Shape of the result. If omitted, a square matrix large enough to contain the diagonals is returned.
- format** : {"dia", "csr", "csc", "lil", ...}, optional  
Matrix format of the result. By default (`format=None`) an appropriate sparse matrix format is returned. This choice is subject to change.
- dtype** : dtype, optional  
Data type of the matrix.

See also:

`spdiags` construct matrix from diagonals

**Notes**

This function differs from `spdiags` in the way it handles off-diagonals.

The result from `diags` is the sparse equivalent of:

```
np.diag(diagonals[0], offsets[0])
+ ...
+ np.diag(diagonals[k], offsets[k])
```

Repeated diagonal offsets are disallowed.

New in version 0.11.

**Examples**

```
>>> from scipy.sparse import diags
>>> diagonals = [[1, 2, 3, 4], [1, 2, 3], [1, 2]]
>>> diags(diagonals, [0, -1, 2]).toarray()
array([[1, 0, 1, 0],
       [1, 2, 0, 2],
```

```
[0, 2, 3, 0],
 [0, 0, 3, 4]])
```

Broadcasting of scalars is supported (but shape needs to be specified):

```
>>> diags([1, -2, 1], [-1, 0, 1], shape=(4, 4)).toarray()
array([[ -2.,  1.,  0.,  0.],
       [  1., -2.,  1.,  0.],
       [  0.,  1., -2.,  1.],
       [  0.,  0.,  1., -2.]])
```

If only one diagonal is wanted (as in `numpy.diag`), the following works as well:

```
>>> diags([1, 2, 3], 1).toarray()
array([[ 0.,  1.,  0.,  0.],
       [ 0.,  0.,  2.,  0.],
       [ 0.,  0.,  0.,  3.],
       [ 0.,  0.,  0.,  0.]])
```

`scipy.sparse.spdiags` (*data, diags, m, n, format=None*)

Return a sparse matrix from diagonals.

**Parameters** **data** : array\_like

matrix diagonals stored row-wise

**diags** : diagonals to set

- $k = 0$  the main diagonal
- $k > 0$  the  $k$ -th upper diagonal
- $k < 0$  the  $k$ -th lower diagonal

**m, n** : int

shape of the result

**format** : str, optional

Format of the result. By default (`format=None`) an appropriate sparse matrix format is returned. This choice is subject to change.

See also:

[`diags`](#) more convenient form of this function

[`dia\_matrix`](#) the sparse DIAGONAL format.

**Examples**

```
>>> from scipy.sparse import spdiags
>>> data = np.array([[1, 2, 3, 4], [1, 2, 3, 4], [1, 2, 3, 4]])
>>> diags = np.array([0, -1, 2])
>>> spdiags(data, diags, 4, 4).toarray()
array([[1, 0, 3, 0],
       [1, 2, 0, 4],
       [0, 2, 3, 0],
       [0, 0, 3, 4]])
```

`scipy.sparse.block_diag` (*mats, format=None, dtype=None*)

Build a block diagonal sparse matrix from provided matrices.

**Parameters** **mats** : sequence of matrices

Input matrices.

**format** : str, optional

The sparse format of the result (e.g. “csr”). If not given, the matrix is returned in “coo” format.

**dtype** : dtype specifier, optional

**Returns** `res` : sparse matrix  
The data-type of the output matrix. If not given, the dtype is determined from that of *blocks*.

**See also:**

`bmat`, `diags`

**Notes**

New in version 0.11.0.

**Examples**

```
>>> from scipy.sparse import coo_matrix, block_diag
>>> A = coo_matrix([[1, 2], [3, 4]])
>>> B = coo_matrix([[5], [6]])
>>> C = coo_matrix([[7]])
>>> block_diag((A, B, C)).toarray()
array([[1, 2, 0, 0],
       [3, 4, 0, 0],
       [0, 0, 5, 0],
       [0, 0, 6, 0],
       [0, 0, 0, 7]])
```

`scipy.sparse.tril(A, k=0, format=None)`

Return the lower triangular portion of a matrix in sparse format

**Returns the elements on or below the *k*-th diagonal of the matrix *A*.**

- `k = 0` corresponds to the main diagonal
- `k > 0` is above the main diagonal
- `k < 0` is below the main diagonal

**Parameters** `A` : dense or sparse matrix  
Matrix whose lower triangular portion is desired.

`k` : integer  
The top-most diagonal of the lower triangle.

**format** : string  
Sparse format of the result, e.g. `format="csr"`, etc.

**Returns** `L` : sparse matrix  
Lower triangular portion of `A` in sparse format.

**See also:**

`triu` upper triangle in sparse format

**Examples**

```
>>> from scipy.sparse import csr_matrix, tril
>>> A = csr_matrix([[1, 2, 0, 0, 3], [4, 5, 0, 6, 7], [0, 0, 8, 9, 0]],
...               dtype='int32')
>>> A.toarray()
array([[1, 2, 0, 0, 3],
       [4, 5, 0, 6, 7],
       [0, 0, 8, 9, 0]])
>>> tril(A).toarray()
array([[1, 0, 0, 0, 0],
       [4, 5, 0, 0, 0],
       [0, 0, 8, 0, 0]])
```

```

>>> tril(A).nnz
4
>>> tril(A, k=1).toarray()
array([[1, 2, 0, 0, 0],
       [4, 5, 0, 0, 0],
       [0, 0, 8, 9, 0]])
>>> tril(A, k=-1).toarray()
array([[0, 0, 0, 0, 0],
       [4, 0, 0, 0, 0],
       [0, 0, 0, 0, 0]])
>>> tril(A, format='csc')
<3x5 sparse matrix of type '<type 'numpy.int32'>'
  with 4 stored elements in Compressed Sparse Column format>
    
```

`scipy.sparse.triu(A, k=0, format=None)`

Return the upper triangular portion of a matrix in sparse format

*Returns the elements on or above the k-th diagonal of the matrix A.*

- `k = 0` corresponds to the main diagonal
- `k > 0` is above the main diagonal
- `k < 0` is below the main diagonal

**Parameters**

- A** : dense or sparse matrix  
Matrix whose upper triangular portion is desired.
- k** : integer  
The bottom-most diagonal of the upper triangle.
- format** : string  
Sparse format of the result, e.g. `format="csr"`, etc.

**Returns**

- L** : sparse matrix  
Upper triangular portion of A in sparse format.

**See also:**

`tril` lower triangle in sparse format

**Examples**

```

>>> from scipy.sparse import csr_matrix, triu
>>> A = csr_matrix([[1, 2, 0, 0, 3], [4, 5, 0, 6, 7], [0, 0, 8, 9, 0]],
...                dtype='int32')
>>> A.toarray()
array([[1, 2, 0, 0, 3],
       [4, 5, 0, 6, 7],
       [0, 0, 8, 9, 0]])
>>> triu(A).toarray()
array([[1, 2, 0, 0, 3],
       [0, 5, 0, 6, 7],
       [0, 0, 8, 9, 0]])
>>> triu(A).nnz
8
>>> triu(A, k=1).toarray()
array([[0, 2, 0, 0, 3],
       [0, 0, 0, 6, 7],
       [0, 0, 0, 9, 0]])
>>> triu(A, k=-1).toarray()
array([[1, 2, 0, 0, 3],
       [4, 5, 0, 6, 7],
    
```



```
[0, 0, 8, 9, 0]])
>>> triu(A, format='csc')
<3x5 sparse matrix of type '<type 'numpy.int32'>'
with 8 stored elements in Compressed Sparse Column format>
```

`scipy.sparse.bmat` (*blocks*, *format=None*, *dtype=None*)

Build a sparse matrix from sparse sub-blocks

**Parameters**

- blocks** : array\_like  
Grid of sparse matrices with compatible shapes. An entry of None implies an all-zero matrix.
- format** : {'bsr', 'coo', 'csc', 'csr', 'dia', 'dok', 'lil'}, optional  
The sparse format of the result (e.g. "csr"). By default an appropriate sparse matrix format is returned. This choice is subject to change.
- dtype** : dtype, optional  
The data-type of the output matrix. If not given, the dtype is determined from that of *blocks*.

**Returns**

- bmat** : sparse matrix

**See also:**

`block_diag`, `diags`

**Examples**

```
>>> from scipy.sparse import coo_matrix, bmat
>>> A = coo_matrix([[1, 2], [3, 4]])
>>> B = coo_matrix([[5], [6]])
>>> C = coo_matrix([[7]])
>>> bmat([[A, B], [None, C]]).toarray()
array([[1, 2, 5],
       [3, 4, 6],
       [0, 0, 7]])

>>> bmat([[A, None], [None, C]]).toarray()
array([[1, 2, 0],
       [3, 4, 0],
       [0, 0, 7]])
```

`scipy.sparse.hstack` (*blocks*, *format=None*, *dtype=None*)

Stack sparse matrices horizontally (column wise)

**Parameters**

- blocks**  
sequence of sparse matrices with compatible shapes
- format** : str  
sparse format of the result (e.g. "csr") by default an appropriate sparse matrix format is returned. This choice is subject to change.
- dtype** : dtype, optional  
The data-type of the output matrix. If not given, the dtype is determined from that of *blocks*.

**See also:**

`vstack` stack sparse matrices vertically (row wise)

*Examples*

```
>>> from scipy.sparse import coo_matrix, hstack
>>> A = coo_matrix([[1, 2], [3, 4]])
>>> B = coo_matrix([[5], [6]])
>>> hstack([A,B]).toarray()
array([[1, 2, 5],
       [3, 4, 6]])
```

`scipy.sparse.vstack` (*blocks, format=None, dtype=None*)

Stack sparse matrices vertically (row wise)

**Parameters** **blocks**

sequence of sparse matrices with compatible shapes

**format** : str, optional

sparse format of the result (e.g. “csr”) by default an appropriate sparse matrix format is returned. This choice is subject to change.

**dtype** : dtype, optional

The data-type of the output matrix. If not given, the dtype is determined from that of *blocks*.

**See also:**

[\*hstack\*](#) stack sparse matrices horizontally (column wise)

*Examples*

```
>>> from scipy.sparse import coo_matrix, vstack
>>> A = coo_matrix([[1, 2], [3, 4]])
>>> B = coo_matrix([[5, 6]])
>>> vstack([A, B]).toarray()
array([[1, 2],
       [3, 4],
       [5, 6]])
```

`scipy.sparse.rand` (*m, n, density=0.01, format='coo', dtype=None, random\_state=None*)

Generate a sparse matrix of the given shape and density with uniformly distributed values.

**Parameters** **m, n** : int

shape of the matrix

**density** : real, optional

density of the generated matrix: density equal to one means a full matrix, density of 0 means a matrix with no non-zero items.

**format** : str, optional

sparse matrix format.

**dtype** : dtype, optional

type of the returned matrix values.

**random\_state** : {numpy.random.RandomState, int}, optional

Random number generator or random seed. If not given, the singleton `numpy.random` will be used.

*Notes*

Only float types are supported for now.

`scipy.sparse.random` (*m, n, density=0.01, format='coo', dtype=None, random\_state=None, data\_rvs=None*)

Generate a sparse matrix of the given shape and density with randomly distributed values.

**Parameters**

- m, n** : int  
shape of the matrix
- density** : real, optional  
density of the generated matrix: density equal to one means a full matrix, density of 0 means a matrix with no non-zero items.
- format** : str, optional  
sparse matrix format.
- dtype** : dtype, optional  
type of the returned matrix values.
- random\_state** : {numpy.random.RandomState, int}, optional  
Random number generator or random seed. If not given, the singleton numpy.random will be used. This random state will be used for sampling the sparsity structure, but not necessarily for sampling the values of the structurally nonzero entries of the matrix.
- data\_rvs** : callable, optional  
Samples a requested number of random values. This function should take a single argument specifying the length of the ndarray that it will return. The structurally nonzero entries of the sparse random matrix will be taken from the array sampled by this function. By default, uniform [0, 1) random values will be sampled using the same random state as is used for sampling the sparsity structure.

**Notes**

Only float types are supported for now.

**Examples**

```
>>> from scipy.sparse import random
>>> from scipy import stats
>>> class CustomRandomState(object):
...     def randint(self, k):
...         i = np.random.randint(k)
...         return i - i % 2
>>> rs = CustomRandomState()
>>> rvs = stats.poisson(25, loc=10).rvs
>>> S = random(3, 4, density=0.25, random_state=rs, data_rvs=rvs)
>>> S.A
array([[ 36.,   0.,  33.,   0.], # random
       [  0.,   0.,   0.,   0.],
       [  0.,   0.,  36.,   0.]])
```

Sparse matrix tools:

---

`find(A)` Return the indices and values of the nonzero elements of a matrix

---

`scipy.sparse.find(A)`

Return the indices and values of the nonzero elements of a matrix

**Parameters** **A** : dense or sparse matrix

**Returns** **(I,J,V)** : tuple of arrays  
Matrix whose nonzero elements are desired.

I, J, and V contain the row indices, column indices, and values of the nonzero matrix entries.

### Examples

```
>>> from scipy.sparse import csr_matrix, find
>>> A = csr_matrix([[7.0, 8.0, 0],[0, 0, 9.0]])
>>> find(A)
(array([0, 0, 1], dtype=int32), array([0, 1, 2], dtype=int32), array([ 7.,  8.,  9.]))
```

Identifying sparse matrices:

---

```
issparse(x)
isspmatrix(x)
isspmatrix_csc(x)
isspmatrix_csr(x)
isspmatrix_bsr(x)
isspmatrix_lil(x)
isspmatrix_dok(x)
isspmatrix_coo(x)
isspmatrix_dia(x)
```

---

`scipy.sparse.issparse(x)`

`scipy.sparse.isspmatrix(x)`

`scipy.sparse.isspmatrix_csc(x)`

`scipy.sparse.isspmatrix_csr(x)`

`scipy.sparse.isspmatrix_bsr(x)`

`scipy.sparse.isspmatrix_lil(x)`

`scipy.sparse.isspmatrix_dok(x)`

`scipy.sparse.isspmatrix_coo(x)`

`scipy.sparse.isspmatrix_dia(x)`

### Submodules

---

```
csgraph
linalg
```

---

**Compressed Sparse Graph Routines (`scipy.sparse.csgraph`)** Fast graph algorithms based on sparse matrix representations.

---

<code>connected_components(csgraph[, directed, ...])</code>	Analyze the connected components of a sparse graph
<code>laplacian(csgraph[, normed, return_diag, ...])</code>	Return the Laplacian matrix of a directed graph.
<code>shortest_path(csgraph[, method, directed, ...])</code>	Perform a shortest-path graph search on a positive directed or undirected graph
<code>dijkstra(csgraph[, directed, indices, ...])</code>	Dijkstra algorithm using Fibonacci Heaps
<code>floyd_warshall(csgraph[, directed, ...])</code>	Compute the shortest path lengths using the Floyd-Warshall algorithm
<code>bellman_ford(csgraph[, directed, indices, ...])</code>	Compute the shortest path lengths using the Bellman-Ford algorithm.
<code>johnson(csgraph[, directed, indices, ...])</code>	Compute the shortest path lengths using Johnson's algorithm.
<code>breadth_first_order(csgraph, i_start[, ...])</code>	Return a breadth-first ordering starting with specified node.
<code>depth_first_order(csgraph, i_start[, ...])</code>	Return a depth-first ordering starting with specified node.
<code>breadth_first_tree(csgraph, i_start[, directed])</code>	Return the tree generated by a breadth-first search
<code>depth_first_tree(csgraph, i_start[, directed])</code>	Return a tree generated by a depth-first search.
<code>minimum_spanning_tree(csgraph[, overwrite])</code>	Return a minimum spanning tree of an undirected graph
<code>reverse_cuthill_mckee</code>	Returns the permutation array that orders a sparse CSR or CSC matrix in Reverse Cuthill-McKee order
<code>maximum_bipartite_matching</code>	Returns an array of row or column permutations that makes the diagonal of a sparse matrix non-zero
<code>NegativeCycleError</code>	

---

## Contents

`scipy.sparse.csgraph.connected_components` (*csgraph*, *directed=True*, *connection='weak'*, *return\_labels=True*)

Analyze the connected components of a sparse graph

New in version 0.11.0.

**Parameters**

**csgraph** : array\_like or sparse matrix  
 The N x N matrix representing the compressed sparse graph. The input csgraph will be converted to csr format for the calculation.

**directed** : bool, optional  
 If True (default), then operate on a directed graph: only move from point i to point j along paths csgraph[i, j]. If False, then find the shortest path on an undirected graph: the algorithm can progress from point i to j along csgraph[i, j] or csgraph[j, i].

**connection** : str, optional  
 ['weak'|'strong']. For directed graphs, the type of connection to use. Nodes i and j are strongly connected if a path exists both from i to j and from j to i. Nodes i and j are weakly connected if only one of these paths exists. If directed == False, this keyword is not referenced.

**return\_labels** : bool, optional  
 If True (default), then return the labels for each of the connected components.

**Returns**

n\_components: int  
 The number of connected components.

labels: ndarray  
 The length-N array of labels of the connected components.

## References

[R14]

`scipy.sparse.csgraph.laplacian` (*csgraph*, *normed=False*, *return\_diag=False*, *use\_out\_degree=False*)

Return the Laplacian matrix of a directed graph.

**Parameters**

**csgraph** : array\_like or sparse matrix, 2 dimensions  
 compressed-sparse graph, with shape (N, N).

**normed** : bool, optional  
 If True, then compute normalized Laplacian.

**return\_diag** : bool, optional  
 If True, then also return an array related to vertex degrees.

**use\_out\_degree** : bool, optional  
 If True, then use out-degree instead of in-degree. This distinction matters only if the graph is asymmetric. Default: False.

**Returns** **lap** : ndarray or sparse matrix  
 The N x N laplacian matrix of csgraph. It will be a numpy array (dense) if the input was dense, or a sparse matrix otherwise.

**diag** : ndarray, optional  
 The length-N diagonal of the Laplacian matrix. For the normalized Laplacian, this is the array of square roots of vertex degrees or 1 if the degree is zero.

### Notes

The Laplacian matrix of a graph is sometimes referred to as the “Kirchoff matrix” or the “admittance matrix”, and is useful in many parts of spectral graph theory. In particular, the eigen-decomposition of the laplacian matrix can give insight into many properties of the graph.

### Examples

```

>>> from scipy.sparse import csgraph
>>> G = np.arange(5) * np.arange(5)[:, np.newaxis]
>>> G
array([[ 0,  0,  0,  0,  0],
       [ 0,  1,  2,  3,  4],
       [ 0,  2,  4,  6,  8],
       [ 0,  3,  6,  9, 12],
       [ 0,  4,  8, 12, 16]])
>>> csgraph.laplacian(G, normed=False)
array([[ 0,  0,  0,  0,  0],
       [ 0,  9, -2, -3, -4],
       [ 0, -2, 16, -6, -8],
       [ 0, -3, -6, 21, -12],
       [ 0, -4, -8, -12, 24]])
    
```

`scipy.sparse.csgraph.shortest_path(csgraph, method='auto', directed=True, return_predecessors=False, unweighted=False, overwrite=False, indices=None)`

Perform a shortest-path graph search on a positive directed or undirected graph.

New in version 0.11.0.

**Parameters** **csgraph** : array, matrix, or sparse matrix, 2 dimensions  
 The N x N array of distances representing the input graph.

**method** : string ['auto'|'FW'|'D'], optional  
 Algorithm to use for shortest paths. Options are:

- 'auto'** – (default) select the best among 'FW', 'D', 'BF', or 'J' based on the input data.
- 'FW'** – Floyd-Warshall algorithm. Computational cost is approximately  $O[N^3]$ . The input csgraph will be converted to a dense representation.
- 'D'** – Dijkstra's algorithm with Fibonacci heaps. Computational cost is approximately  $O[N(N*k + N*\log(N))]$ , where k is the average number of connected edges per node. The

input csgraph will be converted to a csr representation.

**‘BF’ – Bellman-Ford algorithm.** *This algorithm can be used when*

*weights are negative. If a negative cycle is encountered, an error will be raised. Computational cost is approximately  $O[N(N^2 k)]$ , where  $k$  is the average number of connected edges per node. The input csgraph will be converted to a csr representation.*

**‘J’ – Johnson’s algorithm.** *Like the Bellman-Ford algorithm,*

*Johnson’s algorithm is designed for use when the weights are negative. It combines the Bellman-Ford algorithm with Dijkstra’s algorithm for faster computation.*

**directed** : bool, optional

If True (default), then find the shortest path on a directed graph: only move from point  $i$  to point  $j$  along paths  $csgraph[i, j]$ . If False, then find the shortest path on an undirected graph: the algorithm can progress from point  $i$  to  $j$  along  $csgraph[i, j]$  or  $csgraph[j, i]$

**return\_predecessors** : bool, optional

If True, return the size  $(N, N)$  predecessor matrix

**unweighted** : bool, optional

If True, then find unweighted distances. That is, rather than finding the path between each point such that the sum of weights is minimized, find the path such that the number of edges is minimized.

**overwrite** : bool, optional

If True, overwrite csgraph with the result. This applies only if method == ‘FW’ and csgraph is a dense,  $c$ -ordered array with dtype=float64.

**indices** : array\_like or int, optional

If specified, only compute the paths for the points at the given indices. Incompatible with method == ‘FW’.

**Returns**

**dist\_matrix** : ndarray

The  $N \times N$  matrix of distances between graph nodes.  $dist\_matrix[i,j]$  gives the shortest distance from point  $i$  to point  $j$  along the graph.

**predecessors** : ndarray

Returned only if return\_predecessors == True. The  $N \times N$  matrix of predecessors, which can be used to reconstruct the shortest paths. Row  $i$  of the predecessor matrix contains information on the shortest paths from point  $i$ : each entry predecessors[ $i, j$ ] gives the index of the previous node in the path from point  $i$  to point  $j$ . If no path exists between point  $i$  and  $j$ , then predecessors[ $i, j$ ] = -9999

**Raises**

**NegativeCycleError:**

if there are negative cycles in the graph

**Notes**

As currently implemented, Dijkstra’s algorithm and Johnson’s algorithm do not work for graphs with direction-dependent distances when directed == False. i.e., if  $csgraph[i,j]$  and  $csgraph[j,i]$  are non-equal edges, method=‘D’ may yield an incorrect result.

`scipy.sparse.csgraph.dijkstra(csgraph, directed=True, indices=None, return_predecessors=False, unweighted=False, limit=np.inf)`

Dijkstra algorithm using Fibonacci Heaps

New in version 0.11.0.

**Parameters**

- csgraph** : array, matrix, or sparse matrix, 2 dimensions  
The N x N array of non-negative distances representing the input graph.
- directed** : bool, optional  
If True (default), then find the shortest path on a directed graph: only move from point i to point j along paths csgraph[i, j]. If False, then find the shortest path on an undirected graph: the algorithm can progress from point i to j along csgraph[i, j] or csgraph[j, i]
- indices** : array\_like or int, optional  
if specified, only compute the paths for the points at the given indices.
- return\_predecessors** : bool, optional  
If True, return the size (N, N) predecessor matrix
- unweighted** : bool, optional  
If True, then find unweighted distances. That is, rather than finding the path between each point such that the sum of weights is minimized, find the path such that the number of edges is minimized.
- limit** : float, optional  
The maximum distance to calculate, must be  $\geq 0$ . Using a smaller limit will decrease computation time by aborting calculations between pairs that are separated by a distance  $> \text{limit}$ . For such pairs, the distance will be equal to `np.inf` (i.e., not connected).

**Returns**

- dist\_matrix** : ndarray  
New in version 0.14.0.  
The matrix of distances between graph nodes. `dist_matrix[i,j]` gives the shortest distance from point i to point j along the graph.
- predecessors** : ndarray  
Returned only if `return_predecessors == True`. The matrix of predecessors, which can be used to reconstruct the shortest paths. Row i of the predecessor matrix contains information on the shortest paths from point i: each entry `predecessors[i, j]` gives the index of the previous node in the path from point i to point j. If no path exists between point i and j, then `predecessors[i, j] = -9999`

**Notes**

As currently implemented, Dijkstra’s algorithm does not work for graphs with direction-dependent distances when `directed == False`. i.e., if `csgraph[i,j]` and `csgraph[j,i]` are not equal and both are nonzero, setting `directed=False` will not yield the correct result.

Also, this routine does not work for graphs with negative distances. Negative distances can lead to infinite cycles that must be handled by specialized algorithms such as Bellman-Ford’s algorithm or Johnson’s algorithm.

`scipy.sparse.csgraph.floyd_warshall` (*csgraph*, *directed=True*, *return\_predecessors=False*, *unweighted=False*, *overwrite=False*)

Compute the shortest path lengths using the Floyd-Warshall algorithm

New in version 0.11.0.

**Parameters**

- csgraph** : array, matrix, or sparse matrix, 2 dimensions  
The N x N array of distances representing the input graph.
- directed** : bool, optional  
If True (default), then find the shortest path on a directed graph: only move from point i to point j along paths csgraph[i, j]. If False, then find the shortest path on an undirected graph: the algorithm can progress from point i to j along csgraph[i, j] or csgraph[j, i]
- return\_predecessors** : bool, optional  
If True, return the size (N, N) predecessor matrix
- unweighted** : bool, optional



If True, then find unweighted distances. That is, rather than finding the path between each point such that the sum of weights is minimized, find the path such that the number of edges is minimized.

**overwrite** : bool, optional  
 If True, overwrite csgraph with the result. This applies only if csgraph is a dense, c-ordered array with dtype=float64.

**Returns** **dist\_matrix** : ndarray  
 The N x N matrix of distances between graph nodes. dist\_matrix[i,j] gives the shortest distance from point i to point j along the graph.

**predecessors** : ndarray  
 Returned only if return\_predecessors == True. The N x N matrix of predecessors, which can be used to reconstruct the shortest paths. Row i of the predecessor matrix contains information on the shortest paths from point i: each entry predecessors[i, j] gives the index of the previous node in the path from point i to point j. If no path exists between point i and j, then predecessors[i, j] = -9999

**Raises** **NegativeCycleError**:  
 if there are negative cycles in the graph

`scipy.sparse.csgraph.bellman_ford(csgraph, directed=True, indices=None, return_predecessors=False, unweighted=False)`

Compute the shortest path lengths using the Bellman-Ford algorithm.

The Bellman-ford algorithm can robustly deal with graphs with negative weights. If a negative cycle is detected, an error is raised. For graphs without negative edge weights, dijkstra's algorithm may be faster.

New in version 0.11.0.

**Parameters** **csgraph** : array, matrix, or sparse matrix, 2 dimensions  
 The N x N array of distances representing the input graph.

**directed** : bool, optional  
 If True (default), then find the shortest path on a directed graph: only move from point i to point j along paths csgraph[i, j]. If False, then find the shortest path on an undirected graph: the algorithm can progress from point i to j along csgraph[i, j] or csgraph[j, i]

**indices** : array\_like or int, optional  
 if specified, only compute the paths for the points at the given indices.

**return\_predecessors** : bool, optional  
 If True, return the size (N, N) predecessor matrix

**unweighted** : bool, optional  
 If True, then find unweighted distances. That is, rather than finding the path between each point such that the sum of weights is minimized, find the path such that the number of edges is minimized.

**Returns** **dist\_matrix** : ndarray  
 The N x N matrix of distances between graph nodes. dist\_matrix[i,j] gives the shortest distance from point i to point j along the graph.

**predecessors** : ndarray  
 Returned only if return\_predecessors == True. The N x N matrix of predecessors, which can be used to reconstruct the shortest paths. Row i of the predecessor matrix contains information on the shortest paths from point i: each entry predecessors[i, j] gives the index of the previous node in the path from point i to point j. If no path exists between point i and j, then predecessors[i, j] = -9999

**Raises** **NegativeCycleError**:  
 if there are negative cycles in the graph

*Notes*

This routine is specially designed for graphs with negative edge weights. If all edge weights are positive, then Dijkstra's algorithm is a better choice.

`scipy.sparse.csgraph.johnson` (*csgraph*, *directed=True*, *indices=None*, *return\_predecessors=False*, *unweighted=False*)

Compute the shortest path lengths using Johnson's algorithm.

Johnson's algorithm combines the Bellman-Ford algorithm and Dijkstra's algorithm to quickly find shortest paths in a way that is robust to the presence of negative cycles. If a negative cycle is detected, an error is raised. For graphs without negative edge weights, `dijkstra()` may be faster.

New in version 0.11.0.

- Parameters**
- csgraph** : array, matrix, or sparse matrix, 2 dimensions  
The N x N array of distances representing the input graph.
  - directed** : bool, optional  
If True (default), then find the shortest path on a directed graph: only move from point i to point j along paths `csgraph[i, j]`. If False, then find the shortest path on an undirected graph: the algorithm can progress from point i to j along `csgraph[i, j]` or `csgraph[j, i]`
  - indices** : array\_like or int, optional  
if specified, only compute the paths for the points at the given indices.
  - return\_predecessors** : bool, optional  
If True, return the size (N, N) predecessor matrix
  - unweighted** : bool, optional  
If True, then find unweighted distances. That is, rather than finding the path between each point such that the sum of weights is minimized, find the path such that the number of edges is minimized.
- Returns**
- dist\_matrix** : ndarray  
The N x N matrix of distances between graph nodes. `dist_matrix[i,j]` gives the shortest distance from point i to point j along the graph.
  - predecessors** : ndarray  
Returned only if `return_predecessors == True`. The N x N matrix of predecessors, which can be used to reconstruct the shortest paths. Row i of the predecessor matrix contains information on the shortest paths from point i: each entry `predecessors[i, j]` gives the index of the previous node in the path from point i to point j. If no path exists between point i and j, then `predecessors[i, j] = -9999`
- Raises**
- NegativeCycleError**:  
if there are negative cycles in the graph

*Notes*

This routine is specially designed for graphs with negative edge weights. If all edge weights are positive, then Dijkstra's algorithm is a better choice.

`scipy.sparse.csgraph.breadth_first_order` (*csgraph*, *i\_start*, *directed=True*, *return\_predecessors=True*)

Return a breadth-first ordering starting with specified node.

Note that a breadth-first order is not unique, but the tree which it generates is unique.

New in version 0.11.0.

- Parameters**
- csgraph** : array\_like or sparse matrix  
The N x N compressed sparse graph. The input `csgraph` will be converted to csr format for the calculation.
  - i\_start** : int

The index of starting node.

**directed** : bool, optional  
If True (default), then operate on a directed graph: only move from point *i* to point *j* along paths `csgraph[i, j]`. If False, then find the shortest path on an undirected graph: the algorithm can progress from point *i* to *j* along `csgraph[i, j]` or `csgraph[j, i]`.

**return\_predecessors** : bool, optional  
If True (default), then return the predecessor array (see below).

**Returns** **node\_array** : ndarray, one dimension  
The breadth-first list of nodes, starting with specified node. The length of `node_array` is the number of nodes reachable from the specified node.

**predecessors** : ndarray, one dimension  
Returned only if `return_predecessors` is True. The length-*N* list of predecessors of each node in a breadth-first tree. If node *i* is in the tree, then its parent is given by `predecessors[i]`. If node *i* is not in the tree (and for the parent node) then `predecessors[i] = -9999`.

`scipy.sparse.csgraph.depth_first_order` (*csgraph*, *i\_start*, *directed=True*, *return\_predecessors=True*)

Return a depth-first ordering starting with specified node.

Note that a depth-first order is not unique. Furthermore, for graphs with cycles, the tree generated by a depth-first search is not unique either.

New in version 0.11.0.

**Parameters** **csgraph** : array\_like or sparse matrix  
The *N* x *N* compressed sparse graph. The input `csgraph` will be converted to csr format for the calculation.

**i\_start** : int  
The index of starting node.

**directed** : bool, optional  
If True (default), then operate on a directed graph: only move from point *i* to point *j* along paths `csgraph[i, j]`. If False, then find the shortest path on an undirected graph: the algorithm can progress from point *i* to *j* along `csgraph[i, j]` or `csgraph[j, i]`.

**return\_predecessors** : bool, optional  
If True (default), then return the predecessor array (see below).

**Returns** **node\_array** : ndarray, one dimension  
The breadth-first list of nodes, starting with specified node. The length of `node_array` is the number of nodes reachable from the specified node.

**predecessors** : ndarray, one dimension  
Returned only if `return_predecessors` is True. The length-*N* list of predecessors of each node in a breadth-first tree. If node *i* is in the tree, then its parent is given by `predecessors[i]`. If node *i* is not in the tree (and for the parent node) then `predecessors[i] = -9999`.

`scipy.sparse.csgraph.breadth_first_tree` (*csgraph*, *i\_start*, *directed=True*)

Return the tree generated by a breadth-first search

Note that a breadth-first tree from a specified node is unique.

New in version 0.11.0.

**Parameters** **csgraph** : array\_like or sparse matrix  
The *N* x *N* matrix representing the compressed sparse graph. The input `csgraph` will be converted to csr format for the calculation.

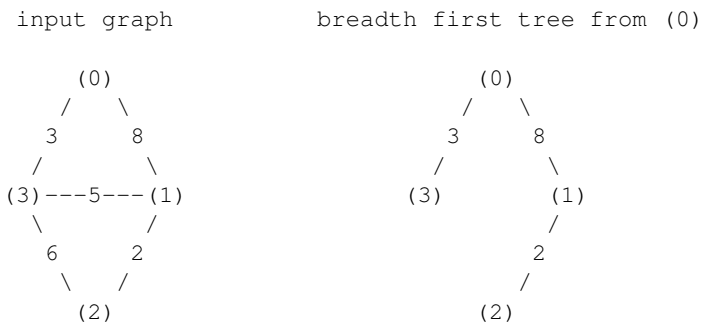
**i\_start** : int  
The index of starting node.

**directed** : bool, optional  
 If True (default), then operate on a directed graph: only move from point *i* to point *j* along paths `csgraph[i, j]`. If False, then find the shortest path on an undirected graph: the algorithm can progress from point *i* to *j* along `csgraph[i, j]` or `csgraph[j, i]`.

**Returns** **cstree** : csr matrix  
 The N x N directed compressed-sparse representation of the breadth- first tree drawn from `csgraph`, starting at the specified node.

**Examples**

The following example shows the computation of a depth-first tree over a simple four-component graph, starting at node 0:



In compressed sparse representation, the solution looks like this:

```
>>> from scipy.sparse import csr_matrix
>>> from scipy.sparse.csgraph import breadth_first_tree
>>> X = csr_matrix([[0, 8, 0, 3],
...                [0, 0, 2, 5],
...                [0, 0, 0, 6],
...                [0, 0, 0, 0]])
>>> Tcsr = breadth_first_tree(X, 0, directed=False)
>>> Tcsr.toarray().astype(int)
array([[0, 8, 0, 3],
       [0, 0, 2, 0],
       [0, 0, 0, 0],
       [0, 0, 0, 0]])
```

Note that the resulting graph is a Directed Acyclic Graph which spans the graph. A breadth-first tree from a given node is unique.

`scipy.sparse.csgraph.depth_first_tree(csgraph, i_start, directed=True)`

Return a tree generated by a depth-first search.

Note that a tree generated by a depth-first search is not unique: it depends on the order that the children of each node are searched.

New in version 0.11.0.

**Parameters** **csgraph** : array\_like or sparse matrix  
 The N x N matrix representing the compressed sparse graph. The input `csgraph` will be converted to `csr` format for the calculation.

**i\_start** : int  
 The index of starting node.

**directed** : bool, optional

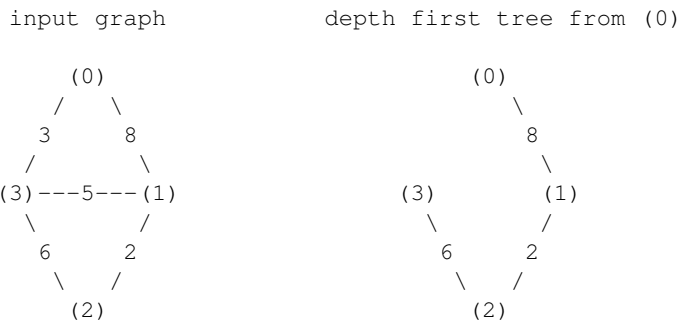
**Returns** **cstree** : csr matrix

If True (default), then operate on a directed graph: only move from point  $i$  to point  $j$  along paths  $\text{csgraph}[i, j]$ . If False, then find the shortest path on an undirected graph: the algorithm can progress from point  $i$  to  $j$  along  $\text{csgraph}[i, j]$  or  $\text{csgraph}[j, i]$ .

The  $N \times N$  directed compressed-sparse representation of the depth-first tree drawn from  $\text{csgraph}$ , starting at the specified node.

### Examples

The following example shows the computation of a depth-first tree over a simple four-component graph, starting at node 0:



In compressed sparse representation, the solution looks like this:

```
>>> from scipy.sparse import csr_matrix
>>> from scipy.sparse.csgraph import depth_first_tree
>>> X = csr_matrix([[0, 8, 0, 3],
...                 [0, 0, 2, 5],
...                 [0, 0, 0, 6],
...                 [0, 0, 0, 0]])
>>> Tcsr = depth_first_tree(X, 0, directed=False)
>>> Tcsr.toarray().astype(int)
array([[0, 8, 0, 0],
       [0, 0, 2, 0],
       [0, 0, 0, 6],
       [0, 0, 0, 0]])
```

Note that the resulting graph is a Directed Acyclic Graph which spans the graph. Unlike a breadth-first tree, a depth-first tree of a given graph is not unique if the graph contains cycles. If the above solution had begun with the edge connecting nodes 0 and 3, the result would have been different.

`scipy.sparse.csgraph.minimum_spanning_tree` (*csgraph*, *overwrite=False*)

Return a minimum spanning tree of an undirected graph

A minimum spanning tree is a graph consisting of the subset of edges which together connect all connected nodes, while minimizing the total sum of weights on the edges. This is computed using the Kruskal algorithm.

New in version 0.11.0.

**Parameters** **csgraph** : array\_like or sparse matrix, 2 dimensions  
 The  $N \times N$  matrix representing an undirected graph over  $N$  nodes (see notes below).

**overwrite** : bool, optional  
 if true, then parts of the input graph will be overwritten for efficiency.

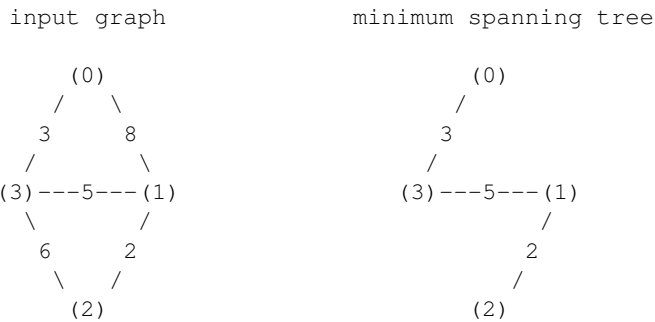
**Returns** **span\_tree** : csr matrix  
 The  $N \times N$  compressed-sparse representation of the undirected minimum spanning tree over the input (see notes below).

### Notes

This routine uses undirected graphs as input and output. That is, if `graph[i, j]` and `graph[j, i]` are both zero, then nodes `i` and `j` do not have an edge connecting them. If either is nonzero, then the two are connected by the minimum nonzero value of the two.

### Examples

The following example shows the computation of a minimum spanning tree over a simple four-component graph:



It is easy to see from inspection that the minimum spanning tree involves removing the edges with weights 8 and 6. In compressed sparse representation, the solution looks like this:

```
>>> from scipy.sparse import csr_matrix
>>> from scipy.sparse.csgraph import minimum_spanning_tree
>>> X = csr_matrix([[0, 8, 0, 3],
...                [0, 0, 2, 5],
...                [0, 0, 0, 6],
...                [0, 0, 0, 0]])
>>> Tcsr = minimum_spanning_tree(X)
>>> Tcsr.toarray().astype(int)
array([[0, 0, 0, 3],
       [0, 0, 2, 5],
       [0, 0, 0, 0],
       [0, 0, 0, 0]])
```

`scipy.sparse.csgraph.reverse_cuthill_mckee()`

Returns the permutation array that orders a sparse CSR or CSC matrix in Reverse-Cuthill McKee ordering.

It is assumed by default, `symmetric_mode=False`, that the input matrix is not symmetric and works on the matrix `A+A.T`. If you are guaranteed that the matrix is symmetric in structure (values of matrix elements do not matter) then set `symmetric_mode=True`.

**Parameters**

- graph** : sparse matrix  
Input sparse in CSC or CSR sparse matrix format.
- symmetric\_mode** : bool, optional  
Is input matrix guaranteed to be symmetric.

**Returns**

- perm** : ndarray  
Array of permuted row and column indices.

### Notes

New in version 0.15.0.

**References**

E. Cuthill and J. McKee, “Reducing the Bandwidth of Sparse Symmetric Matrices”, ACM ‘69 Proceedings of the 1969 24th national conference, (1969).

`scipy.sparse.csgraph.maximum_bipartite_matching()`

Returns an array of row or column permutations that makes the diagonal of a nonsingular square CSC sparse matrix zero free.

Such a permutation is always possible provided that the matrix is nonsingular. This function looks at the structure of the matrix only. The input matrix will be converted to CSC matrix format if necessary.

**Parameters**

- graph** : sparse matrix  
Input sparse in CSC format
- perm\_type** : str, {'row', 'column'}  
Type of permutation to generate.

**Returns**

- perm** : ndarray  
Array of row or column permutations.

**Notes**

This function relies on a maximum cardinality bipartite matching algorithm based on a breadth-first search (BFS) of the underlying graph.

New in version 0.15.0.

**References**

I. S. Duff, K. Kaya, and B. Ucar, “Design, Implementation, and Analysis of Maximum Transversal Algorithms”, ACM Trans. Math. Softw. 38, no. 2, (2011).

**exception** `scipy.sparse.csgraph.NegativeCycleError`

---

<code>construct_dist_matrix(graph, predecessors[, ...])</code>	Construct distance matrix from a predecessor matrix
<code>csgraph_from_dense(graph[, null_value, ...])</code>	Construct a CSR-format sparse graph from a dense matrix.
<code>csgraph_from_masked(graph)</code>	Construct a CSR-format graph from a masked array.
<code>csgraph_masked_from_dense(graph[, ...])</code>	Construct a masked array graph representation from a dense matrix.
<code>csgraph_to_dense(csgraph[, null_value])</code>	Convert a sparse graph representation to a dense representation
<code>csgraph_to_masked(csgraph)</code>	Convert a sparse graph representation to a masked array representation
<code>reconstruct_path(csgraph, predecessors[, ...])</code>	Construct a tree from a graph and a predecessor list.

---

`scipy.sparse.csgraph.construct_dist_matrix(graph, predecessors, directed=True, null_value=np.inf)`

Construct distance matrix from a predecessor matrix

New in version 0.11.0.

**Parameters**

- graph** : array\_like or sparse  
The N x N matrix representation of a directed or undirected graph. If dense, then non-edges are indicated by zeros or infinities.
- predecessors** : array\_like  
The N x N matrix of predecessors of each node (see Notes below).
- directed** : bool, optional  
If True (default), then operate on a directed graph: only move from point i to point j along paths `csgraph[i, j]`. If False, then operate on an undirected graph: the algorithm can progress from point i to j along `csgraph[i, j]` or `csgraph[j, i]`.
- null\_value** : bool, optional

**Returns** **dist\_matrix** : ndarray  
 value to use for distances between unconnected nodes. Default is np.inf  
 The N x N matrix of distances between nodes along the path specified by the predecessor matrix. If no path exists, the distance is zero.

**Notes**

The predecessor matrix is of the form returned by `graph_shortest_path`. Row *i* of the predecessor matrix contains information on the shortest paths from point *i*: each entry `predecessors[i, j]` gives the index of the previous node in the path from point *i* to point *j*. If no path exists between point *i* and *j*, then `predecessors[i, j] = -9999`

`scipy.sparse.csgraph.csgraph_from_dense` (*graph*, *null\_value=0*, *nan\_null=True*, *infinity\_null=True*)

Construct a CSR-format sparse graph from a dense matrix.

New in version 0.11.0.

**Parameters** **graph** : array\_like  
 Input graph. Shape should be (n\_nodes, n\_nodes).  
**null\_value** : float or None (optional)  
 Value that denotes non-edges in the graph. Default is zero.  
**infinity\_null** : bool  
 If True (default), then infinite entries (both positive and negative) are treated as null edges.  
**nan\_null** : bool  
 If True (default), then NaN entries are treated as non-edges

**Returns** **csgraph** : csr\_matrix  
 Compressed sparse representation of graph,

`scipy.sparse.csgraph.csgraph_from_masked` (*graph*)

Construct a CSR-format graph from a masked array.

New in version 0.11.0.

**Parameters** **graph** : MaskedArray  
 Input graph. Shape should be (n\_nodes, n\_nodes).

**Returns** **csgraph** : csr\_matrix  
 Compressed sparse representation of graph,

`scipy.sparse.csgraph.csgraph_masked_from_dense` (*graph*, *null\_value=0*, *nan\_null=True*, *infinity\_null=True*, *copy=True*)

Construct a masked array graph representation from a dense matrix.

New in version 0.11.0.

**Parameters** **graph** : array\_like  
 Input graph. Shape should be (n\_nodes, n\_nodes).  
**null\_value** : float or None (optional)  
 Value that denotes non-edges in the graph. Default is zero.  
**infinity\_null** : bool  
 If True (default), then infinite entries (both positive and negative) are treated as null edges.  
**nan\_null** : bool  
 If True (default), then NaN entries are treated as non-edges

**Returns** **csgraph** : MaskedArray  
 masked array representation of graph

`scipy.sparse.csgraph.csgraph_to_dense` (*csgraph*, *null\_value=0*)

Convert a sparse graph representation to a dense representation

New in version 0.11.0.



**Parameters** **csgraph** : csr\_matrix, csc\_matrix, or lil\_matrix  
Sparse representation of a graph.  
**null\_value** : float, optional  
The value used to indicate null edges in the dense representation. Default is 0.

**Returns** **graph** : ndarray  
The dense representation of the sparse graph.

### Notes

For normal sparse graph representations, calling `csgraph_to_dense` with `null_value=0` produces an equivalent result to using dense format conversions in the main sparse package. When the sparse representations have repeated values, however, the results will differ. The tools in `scipy.sparse` will add repeating values to obtain a final value. This function will select the minimum among repeating values to obtain a final value. For example, here we'll create a two-node directed sparse graph with multiple edges from node 0 to node 1, of weights 2 and 3. This illustrates the difference in behavior:

```
>>> from scipy.sparse import csr_matrix, csgraph
>>> data = np.array([2, 3])
>>> indices = np.array([1, 1])
>>> indptr = np.array([0, 2, 2])
>>> M = csr_matrix((data, indices, indptr), shape=(2, 2))
>>> M.toarray()
array([[0, 5],
       [0, 0]])
>>> csgraph.csgraph_to_dense(M)
array([[0., 2.],
       [0., 0.]])
```

The reason for this difference is to allow a compressed sparse graph to represent multiple edges between any two nodes. As most sparse graph algorithms are concerned with the single lowest-cost edge between any two nodes, the default `scipy.sparse` behavior of summing multiple weights does not make sense in this context.

The other reason for using this routine is to allow for graphs with zero-weight edges. Let's look at the example of a two-node directed graph, connected by an edge of weight zero:

```
>>> from scipy.sparse import csr_matrix, csgraph
>>> data = np.array([0.0])
>>> indices = np.array([1])
>>> indptr = np.array([0, 1, 1])
>>> M = csr_matrix((data, indices, indptr), shape=(2, 2))
>>> M.toarray()
array([[0, 0],
       [0, 0]])
>>> csgraph.csgraph_to_dense(M, np.inf)
array([[ inf,  0.],
       [ inf,  inf]])
```

In the first case, the zero-weight edge gets lost in the dense representation. In the second case, we can choose a different null value and see the true form of the graph.

`scipy.sparse.csgraph.csgraph_to_masked(csgraph)`

Convert a sparse graph representation to a masked array representation

New in version 0.11.0.

**Parameters** **csgraph** : csr\_matrix, csc\_matrix, or lil\_matrix  
Sparse representation of a graph.  
**Returns** **graph** : MaskedArray

The masked dense representation of the sparse graph.

`scipy.sparse.csgraph.reconstruct_path` (*csgraph*, *predecessors*, *directed=True*)  
 Construct a tree from a graph and a predecessor list.

New in version 0.11.0.

**Parameters**

- csgraph** : array\_like or sparse matrix  
 The N x N matrix representing the directed or undirected graph from which the predecessors are drawn.
- predecessors** : array\_like, one dimension  
 The length-N array of indices of predecessors for the tree. The index of the parent of node i is given by predecessors[i].
- directed** : bool, optional  
 If True (default), then operate on a directed graph: only move from point i to point j along paths csgraph[i, j]. If False, then operate on an undirected graph: the algorithm can progress from point i to j along csgraph[i, j] or csgraph[j, i].

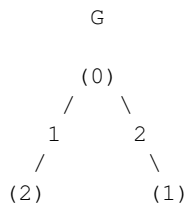
**Returns**

- cstree** : csr matrix  
 The N x N directed compressed-sparse representation of the tree drawn from csgraph which is encoded by the predecessor list.

**Graph Representations** This module uses graphs which are stored in a matrix format. A graph with N nodes can be represented by an (N x N) adjacency matrix G. If there is a connection from node i to node j, then  $G[i, j] = w$ , where w is the weight of the connection. For nodes i and j which are not connected, the value depends on the representation:

- for dense array representations, non-edges are represented by  $G[i, j] = 0$ , infinity, or NaN.
- for dense masked representations (of type `np.ma.MaskedArray`), non-edges are represented by masked values. This can be useful when graphs with zero-weight edges are desired.
- for sparse array representations, non-edges are represented by non-entries in the matrix. This sort of sparse representation also allows for edges with zero weights.

As a concrete example, imagine that you would like to represent the following undirected graph:

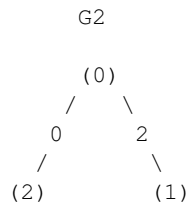


This graph has three nodes, where node 0 and 1 are connected by an edge of weight 2, and nodes 0 and 2 are connected by an edge of weight 1. We can construct the dense, masked, and sparse representations as follows, keeping in mind that an undirected graph is represented by a symmetric matrix:

```

>>> G_dense = np.array([[0, 2, 1],
...                    [2, 0, 0],
...                    [1, 0, 0]])
>>> G_masked = np.ma.masked_values(G_dense, 0)
>>> from scipy.sparse import csr_matrix
>>> G_sparse = csr_matrix(G_dense)
    
```

This becomes more difficult when zero edges are significant. For example, consider the situation when we slightly modify the above graph:



This is identical to the previous graph, except nodes 0 and 2 are connected by an edge of zero weight. In this case, the dense representation above leads to ambiguities: how can non-edges be represented if zero is a meaningful value? In this case, either a masked or sparse representation must be used to eliminate the ambiguity:

```

>>> G2_data = np.array([[np.inf, 2,      0      ],
...                    [2,      np.inf, np.inf],
...                    [0,      np.inf, np.inf]])
>>> G2_masked = np.ma.masked_invalid(G2_data)
>>> from scipy.sparse.csgraph import csgraph_from_dense
>>> # G2_sparse = csr_matrix(G2_data) would give the wrong result
>>> G2_sparse = csgraph_from_dense(G2_data, null_value=np.inf)
>>> G2_sparse.data
array([ 2.,  0.,  2.,  0.])

```

Here we have used a utility routine from the `csgraph` submodule in order to convert the dense representation to a sparse representation which can be understood by the algorithms in submodule. By viewing the data array, we can see that the zero values are explicitly encoded in the graph.

**Directed vs. Undirected** Matrices may represent either directed or undirected graphs. This is specified throughout the `csgraph` module by a boolean keyword. Graphs are assumed to be directed by default. In a directed graph, traversal from node  $i$  to node  $j$  can be accomplished over the edge  $G[i, j]$ , but not the edge  $G[j, i]$ . In a non-directed graph, traversal from node  $i$  to node  $j$  can be accomplished over either  $G[i, j]$  or  $G[j, i]$ . If both edges are not null, and the two have unequal weights, then the smaller of the two is used. Note that a symmetric matrix will represent an undirected graph, regardless of whether the ‘directed’ keyword is set to True or False. In this case, using `directed=True` generally leads to more efficient computation.

The routines in this module accept as input either `scipy.sparse` representations (`csr`, `csc`, or `lil` format), masked representations, or dense representations with non-edges indicated by zeros, infinities, and NaN entries.

### Functions

<code>bellman_ford(csgraph[, directed, indices, ...])</code>	Compute the shortest path lengths using the Bellman-Ford algorithm.
<code>breadth_first_order(csgraph, i_start[, ...])</code>	Return a breadth-first ordering starting with specified node.
<code>breadth_first_tree(csgraph, i_start[, directed])</code>	Return the tree generated by a breadth-first search
<code>connected_components(csgraph[, directed, ...])</code>	Analyze the connected components of a sparse graph
<code>construct_dist_matrix(graph, predecessors[, ...])</code>	Construct distance matrix from a predecessor matrix
<code>cs_graph_components(*args, **kwds)</code>	<code>cs_graph_components</code> is deprecated!
<code>csgraph_from_dense(graph[, null_value, ...])</code>	Construct a CSR-format sparse graph from a dense matrix.
<code>csgraph_from_masked(graph)</code>	Construct a CSR-format graph from a masked array.
<code>csgraph_masked_from_dense(graph[, ...])</code>	Construct a masked array graph representation from a dense matrix.
<code>csgraph_to_dense(csgraph[, null_value])</code>	Convert a sparse graph representation to a dense representation
<code>csgraph_to_masked(csgraph)</code>	Convert a sparse graph representation to a masked array representation
<code>depth_first_order(csgraph, i_start[, ...])</code>	Return a depth-first ordering starting with specified node.
<code>depth_first_tree(csgraph, i_start[, directed])</code>	Return a tree generated by a depth-first search.
<code>dijkstra(csgraph[, directed, indices, ...])</code>	Dijkstra algorithm using Fibonacci Heaps
<code>floyd_warshall(csgraph[, directed, ...])</code>	Compute the shortest path lengths using the Floyd-Warshall algorithm

Table 5.176 – continued from previous page

<code>johnson(csgraph[, directed, indices, ...])</code>	Compute the shortest path lengths using Johnson’s algorithm.
<code>laplacian(csgraph[, normed, return_diag, ...])</code>	Return the Laplacian matrix of a directed graph.
<code>maximum_bipartite_matching</code>	Returns an array of row or column permutations that makes the diagonal
<code>minimum_spanning_tree(csgraph[, overwrite])</code>	Return a minimum spanning tree of an undirected graph
<code>reconstruct_path(csgraph, predecessors[, ...])</code>	Construct a tree from a graph and a predecessor list.
<code>reverse_cuthill_mckee</code>	Returns the permutation array that orders a sparse CSR or CSC matrix in
<code>shortest_path(csgraph[, method, directed, ...])</code>	Perform a shortest-path graph search on a positive directed or undirected

**Classes**

<code>Tester</code>	alias of <code>NoseTester</code>
---------------------	----------------------------------

**Exceptions**

<code>NegativeCycleError</code>
---------------------------------

**Sparse linear algebra (`scipy.sparse.linalg`)**

<code>LinearOperator(dtype, shape)</code>	Common interface for performing matrix vector products
<code>aslinearoperator(A)</code>	Return A as a <code>LinearOperator</code> .

**Abstract linear operators**

**class** `scipy.sparse.linalg.LinearOperator` (*dtype, shape*)

Common interface for performing matrix vector products

Many iterative methods (e.g. `cg`, `gmres`) do not need to know the individual entries of a matrix to solve a linear system  $A*x=b$ . Such solvers only require the computation of matrix vector products,  $A*v$  where  $v$  is a dense vector. This class serves as an abstract interface between iterative solvers and matrix-like objects.

To construct a concrete `LinearOperator`, either pass appropriate callables to the constructor of this class, or subclass it.

A subclass must implement either one of the methods `_matvec` and `_matmat`, and the attributes/properties `shape` (pair of integers) and `dtype` (may be `None`). It may call the `__init__` on this class to have these attributes validated. Implementing `_matvec` automatically implements `_matmat` (using a naive algorithm) and vice-versa.

Optionally, a subclass may implement `_rmatvec` or `_adjoint` to implement the Hermitian adjoint (conjugate transpose). As with `_matvec` and `_matmat`, implementing either `_rmatvec` or `_adjoint` implements the other automatically. Implementing `_adjoint` is preferable; `_rmatvec` is mostly there for backwards compatibility.

**Parameters**

- shape** : tuple  
Matrix dimensions (M,N).
- matvec** : callable f(v)  
Returns returns  $A * v$ .
- rmatvec** : callable f(v)  
Returns  $A^H * v$ , where  $A^H$  is the conjugate transpose of A.
- matmat** : callable f(V)  
Returns  $A * V$ , where V is a dense matrix with dimensions (N,K).
- dtype** : dtype

Data type of the matrix.

**See also:**

[\*aslinearoperator\*](#)

Construct LinearOperators

**Notes**

The user-defined `matvec()` function must properly handle the case where `v` has shape `(N,)` as well as the `(N,1)` case. The shape of the return type is handled internally by `LinearOperator`.

`LinearOperator` instances can also be multiplied, added with each other and exponentiated, all lazily: the result of these operations is always a new, composite `LinearOperator`, that defers linear operations to the original operators and combines the results.

**Examples**

```
>>> import numpy as np
>>> from scipy.sparse.linalg import LinearOperator
>>> def mv(v):
...     return np.array([2*v[0], 3*v[1]])
...
>>> A = LinearOperator((2,2), matvec=mv)
>>> A
<2x2 _CustomLinearOperator with dtype=float64>
>>> A.matvec(np.ones(2))
array([ 2.,  3.])
>>> A * np.ones(2)
array([ 2.,  3.])
```

**Attributes**

args	(tuple) For linear operators describing products etc. of other linear operators, the operands of the binary operation.
------	--

**Methods**

<code>__call__(x)</code>	
<code>adjoint()</code>	Hermitian adjoint.
<code>dot(x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>matmat(X)</code>	Matrix-matrix multiplication.
<code>matvec(x)</code>	Matrix-vector multiplication.
<code>rmatvec(x)</code>	Adjoint matrix-vector multiplication.
<code>transpose()</code>	Transpose this linear operator.

`LinearOperator.__call__(x)`

`LinearOperator.adjoint()`

Hermitian adjoint.

Returns the Hermitian adjoint of self, aka the Hermitian conjugate or Hermitian transpose. For a complex matrix, the Hermitian adjoint is equal to the conjugate transpose.

Can be abbreviated `self.H` instead of `self.adjoint()`.

**Returns** `A_H`: `LinearOperator`

Hermitian adjoint of self.

`LinearOperator.dot(x)`

Matrix-matrix or matrix-vector multiplication.

**Parameters** `x` : array\_like

**Returns** `Ax` : array

1-d or 2-d array, representing a vector or matrix.

1-d or 2-d array (depending on the shape of `x`) that represents the result of applying this linear operator on `x`.

`LinearOperator.matmat(X)`

Matrix-matrix multiplication.

Performs the operation  $y=A*X$  where `A` is an  $M \times N$  linear operator and `X` dense  $N \times K$  matrix or ndarray.

**Parameters** `X` : {matrix, ndarray}

**Returns** `Y` : {matrix, ndarray} An array with shape (N,K).

A matrix or ndarray with shape (M,K) depending on the type of the `X` argument.

**Notes**

This `matmat` wraps any user-specified `matmat` routine or overridden `_matmat` method to ensure that `y` has the correct type.

`LinearOperator.matvec(x)`

Matrix-vector multiplication.

Performs the operation  $y=A*x$  where `A` is an  $M \times N$  linear operator and `x` is a column vector or 1-d array.

**Parameters** `x` : {matrix, ndarray}

**Returns** `y` : {matrix, ndarray} An array with shape (N,) or (N,1).

A matrix or ndarray with shape (M,) or (M,1) depending on the type and shape of the `x` argument.

**Notes**

This `matvec` wraps the user-specified `matvec` routine or overridden `_matvec` method to ensure that `y` has the correct shape and type.

`LinearOperator.rmatvec(x)`

Adjoint matrix-vector multiplication.

Performs the operation  $y = A^H * x$  where `A` is an  $M \times N$  linear operator and `x` is a column vector or 1-d array.

**Parameters** `x` : {matrix, ndarray}

**Returns** `y` : {matrix, ndarray} An array with shape (M,) or (M,1).

A matrix or ndarray with shape (N,) or (N,1) depending on the type and shape of the `x` argument.

**Notes**

This `rmatvec` wraps the user-specified `rmatvec` routine or overridden `_rmatvec` method to ensure that `y` has the correct shape and type.

`LinearOperator.transpose()`

Transpose this linear operator.

Returns a LinearOperator that represents the transpose of this one. Can be abbreviated self.T instead of self.transpose().

`scipy.sparse.linalg.aslinearoperator(A)`

Return A as a LinearOperator.

'A' may be any of the following types:

- ndarray
- matrix
- sparse matrix (e.g. csr\_matrix, lil\_matrix, etc.)
- LinearOperator
- An object with .shape and .matvec attributes

See the LinearOperator documentation for additional information.

### Examples

```
>>> from scipy.sparse.linalg import aslinearoperator
>>> M = np.array([[1, 2, 3], [4, 5, 6]], dtype=np.int32)
>>> aslinearoperator(M)
<2x3 MatrixLinearOperator with dtype=int32>
```

---

<code>inv(A)</code>	Compute the inverse of a sparse matrix
<code>expm(A)</code>	Compute the matrix exponential using Pade approximation.
<code>expm_multiply(A, B[, start, stop, num, endpoint])</code>	Compute the action of the matrix exponential of A on B.

---

## Matrix Operations

`scipy.sparse.linalg.inv(A)`

Compute the inverse of a sparse matrix

**Parameters** **A** : (M,M) ndarray or sparse matrix  
**Returns** **Ainv** : (M,M) ndarray or sparse matrix  
 square matrix to be inverted  
 inverse of A

### Notes

This computes the sparse inverse of A. If the inverse of A is expected to be non-sparse, it will likely be faster to convert A to dense and use `scipy.linalg.inv`.

New in version 0.12.0.

`scipy.sparse.linalg.expm(A)`

Compute the matrix exponential using Pade approximation.

**Parameters** **A** : (M,M) array\_like or sparse matrix  
**Returns** **expA** : (M,M) ndarray  
 2D Array or Matrix (sparse or dense) to be exponentiated  
 Matrix exponential of A

### Notes

This is algorithm (6.1) which is a simplification of algorithm (5.1).

New in version 0.12.0.

### References

[R19]

`scipy.sparse.linalg.expm_multiply` (*A*, *B*, *start=None*, *stop=None*, *num=None*, *endpoint=None*)  
 Compute the action of the matrix exponential of *A* on *B*.

**Parameters**

- A** : transposable linear operator  
The operator whose exponential is of interest.
- B** : ndarray  
The matrix or vector to be multiplied by the matrix exponential of *A*.
- start** : scalar, optional  
The starting time point of the sequence.
- stop** : scalar, optional  
The end time point of the sequence, unless *endpoint* is set to `False`. In that case, the sequence consists of all but the last of `num + 1` evenly spaced time points, so that *stop* is excluded. Note that the step size changes when *endpoint* is `False`.
- num** : int, optional  
Number of time points to use.
- endpoint** : bool, optional  
If `True`, *stop* is the last time point. Otherwise, it is not included.

**Returns**

- expm\_A\_B** : ndarray  
The result of the action  $e^{t_k A} B$ .

**Notes**

The optional arguments defining the sequence of evenly spaced time points are compatible with the arguments of `numpy.linspace`.

The output ndarray shape is somewhat complicated so I explain it here. The `ndim` of the output could be either 1, 2, or 3. It would be 1 if you are computing the `expm` action on a single vector at a single time point. It would be 2 if you are computing the `expm` action on a vector at multiple time points, or if you are computing the `expm` action on a matrix at a single time point. It would be 3 if you want the action on a matrix with multiple columns at multiple time points. If multiple time points are requested, `expm_A_B[0]` will always be the action of the `expm` at the first time point, regardless of whether the action is on a vector or a matrix.

**References**

[R20], [R21]

---

<code>norm(x[, ord, axis])</code>	Norm of a sparse matrix
<code>onenormest(A[, t, itmax, compute_v, compute_w])</code>	Compute a lower bound of the 1-norm of a sparse matrix.

---

**Matrix norms**

`scipy.sparse.linalg.norm` (*x*, *ord=None*, *axis=None*)  
 Norm of a sparse matrix

This function is able to return one of seven different matrix norms, depending on the value of the `ord` parameter.

**Parameters**

- x** : a sparse matrix  
Input sparse matrix.
- ord** : {non-zero int, inf, -inf, 'fro'}, optional  
Order of the norm (see table under `Notes`). `inf` means numpy's *inf* object.
- axis** : {int, 2-tuple of ints, None}, optional  
If *axis* is an integer, it specifies the axis of *x* along which to compute the vector norms. If *axis* is a 2-tuple, it specifies the axes that hold 2-D matrices, and the matrix norms of these matrices are computed. If *axis* is `None` then either a vector norm (when *x* is 1-D) or a matrix norm (when *x* is 2-D) is returned.



**Returns** `n` : float or ndarray

### Notes

Some of the ord are not implemented because some associated functions like, `_multi_svd_norm`, are not yet available for sparse matrix.

This docstring is modified based on `numpy.linalg.norm`. <https://github.com/numpy/numpy/blob/master/numpy/linalg/linalg.py>

The following norms can be calculated:

ord	norm for sparse matrices
None	Frobenius norm
'fro'	Frobenius norm
inf	<code>max(sum(abs(x), axis=1))</code>
-inf	<code>min(sum(abs(x), axis=1))</code>
0	<code>abs(x).sum(axis=axis)</code>
1	<code>max(sum(abs(x), axis=0))</code>
-1	<code>min(sum(abs(x), axis=0))</code>
2	Not implemented
-2	Not implemented
other	Not implemented

The Frobenius norm is given by [R32]:

$$\|A\|_F = [\sum_{i,j} abs(a_{i,j})^2]^{1/2}$$

### References

[R32]

### Examples

```
>>> from scipy.sparse import *
>>> import numpy as np
>>> from scipy.sparse.linalg import norm
>>> a = np.arange(9) - 4
>>> a
array([-4, -3, -2, -1, 0, 1, 2, 3, 4])
>>> b = a.reshape((3, 3))
>>> b
array([[ -4,  -3,  -2],
       [ -1,   0,   1],
       [  2,   3,   4]])

>>> b = csr_matrix(b)
>>> norm(b)
7.745966692414834
>>> norm(b, 'fro')
7.745966692414834
>>> norm(b, np.inf)
9
>>> norm(b, -np.inf)
2
>>> norm(b, 1)
7
>>> norm(b, -1)
6
```

`scipy.sparse.linalg.onenormest` (*A*, *t*=2, *itmax*=5, *compute\_v*=False, *compute\_w*=False)

Compute a lower bound of the 1-norm of a sparse matrix.

**Parameters**

- A** : ndarray or other linear operator  
A linear operator that can be transposed and that can produce matrix products.
- t** : int, optional  
A positive parameter controlling the tradeoff between accuracy versus time and memory usage. Larger values take longer and use more memory but give more accurate output.
- itmax** : int, optional  
Use at most this many iterations.
- compute\_v** : bool, optional  
Request a norm-maximizing linear operator input vector if True.
- compute\_w** : bool, optional  
Request a norm-maximizing linear operator output vector if True.

**Returns**

- est** : float  
An underestimate of the 1-norm of the sparse matrix.
- v** : ndarray, optional  
The vector such that  $\|Av\|_1 == est*\|v\|_1$ . It can be thought of as an input to the linear operator that gives an output with particularly large norm.
- w** : ndarray, optional  
The vector  $Av$  which has relatively large 1-norm. It can be thought of as an output of the linear operator that is relatively large in norm compared to the input.

**Notes**

This is algorithm 2.4 of [1].

In [2] it is described as follows. “This algorithm typically requires the evaluation of about 4t matrix-vector products and almost invariably produces a norm estimate (which is, in fact, a lower bound on the norm) correct to within a factor 3.”

New in version 0.13.0.

**References**

[R33], [R34]

**Solving linear problems** Direct methods for linear equation systems:

---

<code>spsolve(A, b[, permc_spec, use_umfpack])</code>	Solve the sparse linear system $Ax=b$ , where <i>b</i> may be a vector or a matrix.
<code>factorized(A)</code>	Return a function for solving a sparse linear system, with <i>A</i> pre-factorized.
<code>MatrixRankWarning</code>	
<code>use_solver(**kwargs)</code>	Select default sparse direct solver to be used.

---

`scipy.sparse.linalg.spsolve` (*A*, *b*, *permc\_spec*=None, *use\_umfpack*=True)

Solve the sparse linear system  $Ax=b$ , where *b* may be a vector or a matrix.

**Parameters**

- A** : ndarray or sparse matrix  
The square matrix *A* will be converted into CSC or CSR form
- b** : ndarray or sparse matrix  
The matrix or vector representing the right hand side of the equation. If a vector, *b.shape* must be (n,) or (n, 1).
- permc\_spec** : str, optional

How to permute the columns of the matrix for sparsity preservation. (default: 'COLAMD')

- **NATURAL**: natural ordering.
- **MMD\_ATA**: minimum degree ordering on the structure of  $A^T A$ .
- **MMD\_AT\_PLUS\_A**: minimum degree ordering on the structure of  $A^T A + A$ .
- **COLAMD**: approximate minimum degree column ordering

**use\_umfpack** : bool, optional

if True (default) then use umfpack for the solution. This is only referenced if **b** is a vector and `scikit-umfpack` is installed.

**Returns**

**x** : ndarray or sparse matrix

the solution of the sparse linear equation. If **b** is a vector, then **x** is a vector of size `A.shape[1]`. If **b** is a matrix, then **x** is a matrix of size `(A.shape[1], b.shape[1])`.

### Notes

For solving the matrix expression  $AX = B$ , this solver assumes the resulting matrix **X** is sparse, as is often the case for very sparse inputs. If the resulting **X** is dense, the construction of this sparse result will be relatively expensive. In that case, consider converting **A** to a dense matrix and using `scipy.linalg.solve` or its variants.

`scipy.sparse.linalg.factorized(A)`

Return a function for solving a sparse linear system, with **A** pre-factorized.

**Parameters** **A** : (N, N) array\_like

Input.

**Returns**

**solve** : callable

To solve the linear system of equations given in **A**, the *solve* callable should be passed an ndarray of shape (N,).

### Examples

```
>>> from scipy.sparse.linalg import factorized
>>> A = np.array([[ 3. ,  2. , -1. ],
...              [ 2. , -2. ,  4. ],
...              [-1. ,  0.5, -1. ]])
>>> solve = factorized(A) # Makes LU decomposition.
>>> rhs1 = np.array([1, -2, 0])
>>> solve(rhs1) # Uses the LU factors.
array([ 1., -2., -2.]
```

**exception** `scipy.sparse.linalg.MatrixRankWarning`

`scipy.sparse.linalg.use_solver(**kwargs)`

Select default sparse direct solver to be used.

**Parameters** **useUmfpack** : bool, optional

Use UMFPACK over SuperLU. Has effect only if `scikits.umfpack` is installed. Default: True

### Notes

The default sparse solver is umfpack when available (`scikits.umfpack` is installed). This can be changed by passing `useUmfpack = False`, which then causes the always present SuperLU based solver to be used.

Umfpack requires a CSR/CSC matrix to have sorted column/row indices. If sure that the matrix fulfills this, pass `assumeSortedIndices=True` to gain some speed.

Iterative methods for linear equation systems:

<code>bicg(A, b[, x0, tol, maxiter, xtype, M, ...])</code>	Use BIConjugate Gradient iteration to solve $Ax = b$
<code>bicgstab(A, b[, x0, tol, maxiter, xtype, M, ...])</code>	Use BIConjugate Gradient STABILized iteration to solve $Ax = b$
<code>cg(A, b[, x0, tol, maxiter, xtype, M, callback])</code>	Use Conjugate Gradient iteration to solve $Ax = b$
<code>cgs(A, b[, x0, tol, maxiter, xtype, M, callback])</code>	Use Conjugate Gradient Squared iteration to solve $Ax = b$
<code>gmres(A, b[, x0, tol, restart, maxiter, ...])</code>	Use Generalized Minimal RESidual iteration to solve $Ax = b$ .
<code>lgmres(A, b[, x0, tol, maxiter, M, ...])</code>	Solve a matrix equation using the LGMRES algorithm.
<code>minres(A, b[, x0, shift, tol, maxiter, ...])</code>	Use MINimum RESidual iteration to solve $Ax=b$
<code>qmr(A, b[, x0, tol, maxiter, xtype, M1, M2, ...])</code>	Use Quasi-Minimal Residual iteration to solve $Ax = b$

`scipy.sparse.linalg.bicg(A, b, x0=None, tol=1e-05, maxiter=None, xtype=None, M=None, callback=None)`  
 Use BIConjugate Gradient iteration to solve  $Ax = b$

**Parameters** **A** : {sparse matrix, dense matrix, LinearOperator}  
 The real or complex N-by-N matrix of the linear system It is required that the linear operator can produce  $Ax$  and  $A^T x$ .

**b** : {array, matrix}  
 Right hand side of the linear system. Has shape (N,) or (N,1).

**Returns** **x** : {array, matrix}  
 The converged solution.

**info** : integer  
**Provides convergence information:**  
 0 : successful exit >0 : convergence to tolerance not achieved, number of iterations <0 : illegal input or breakdown

**Other Parameters**

**x0** : {array, matrix}  
 Starting guess for the solution.

**tol** : float  
 Tolerance to achieve. The algorithm terminates when either the relative or the absolute residual is below *tol*.

**maxiter** : integer  
 Maximum number of iterations. Iteration will stop after maxiter steps even if the specified tolerance has not been achieved.

**M** : {sparse matrix, dense matrix, LinearOperator}  
 Preconditioner for A. The preconditioner should approximate the inverse of A. Effective preconditioning dramatically improves the rate of convergence, which implies that fewer iterations are needed to reach a given error tolerance.

**callback** : function  
 User-supplied function to call after each iteration. It is called as `callback(xk)`, where `xk` is the current solution vector.

**xtype** : {'f','d','F','D'}  
 This parameter is deprecated – avoid using it.  
 The type of the result. If None, then it will be determined from `A.dtype.char` and `b`. If A does not have a `typecode` method then it will compute `A.matvec(x0)` to get a `typecode`. To save the extra computation when A does not have a `typecode` attribute use `xtype=0` for the same type as `b` or use `xtype='f','d','F',`or `'D'`. This parameter has been superseded by `LinearOperator`.

`scipy.sparse.linalg.bicgstab(A, b, x0=None, tol=1e-05, maxiter=None, xtype=None, M=None, callback=None)`  
 Use BIConjugate Gradient STABILized iteration to solve  $Ax = b$

**Parameters** **A** : {sparse matrix, dense matrix, LinearOperator}  
 The real or complex N-by-N matrix of the linear system A must represent a hermitian, positive definite matrix

**b** : {array, matrix}

**Returns** **x** : {array, matrix} Right hand side of the linear system. Has shape (N,) or (N,1).  
 The converged solution.

**info** : integer

**Provides convergence information:**  
 0 : successful exit >0 : convergence to tolerance not achieved, number of iterations <0 : illegal input or breakdown

**Other Parameters**

**x0** : {array, matrix}  
 Starting guess for the solution.

**tol** : float  
 Tolerance to achieve. The algorithm terminates when either the relative or the absolute residual is below *tol*.

**maxiter** : integer  
 Maximum number of iterations. Iteration will stop after maxiter steps even if the specified tolerance has not been achieved.

**M** : {sparse matrix, dense matrix, LinearOperator}  
 Preconditioner for A. The preconditioner should approximate the inverse of A. Effective preconditioning dramatically improves the rate of convergence, which implies that fewer iterations are needed to reach a given error tolerance.

**callback** : function  
 User-supplied function to call after each iteration. It is called as `callback(xk)`, where `xk` is the current solution vector.

**xtype** : {'f','d','F','D'}  
 This parameter is deprecated – avoid using it.  
 The type of the result. If None, then it will be determined from `A.dtype.char` and `b`. If A does not have a `typecode` method then it will compute `A.matvec(x0)` to get a `typecode`. To save the extra computation when A does not have a `typecode` attribute use `xtype=0` for the same type as `b` or use `xtype='f','d','F',` or `'D'`. This parameter has been superseded by `LinearOperator`.

`scipy.sparse.linalg.cg(A, b, x0=None, tol=1e-05, maxiter=None, xtype=None, M=None, callback=None)`

Use Conjugate Gradient iteration to solve  $Ax = b$

**Parameters** **A** : {sparse matrix, dense matrix, LinearOperator}  
 The real or complex N-by-N matrix of the linear system A must represent a hermitian, positive definite matrix

**b** : {array, matrix}

**Returns** **x** : {array, matrix} Right hand side of the linear system. Has shape (N,) or (N,1).  
 The converged solution.

**info** : integer

**Provides convergence information:**  
 0 : successful exit >0 : convergence to tolerance not achieved, number of iterations <0 : illegal input or breakdown

**Other Parameters**

**x0** : {array, matrix}  
 Starting guess for the solution.

**tol** : float  
Tolerance to achieve. The algorithm terminates when either the relative or the absolute residual is below *tol*.

**maxiter** : integer  
Maximum number of iterations. Iteration will stop after maxiter steps even if the specified tolerance has not been achieved.

**M** : {sparse matrix, dense matrix, LinearOperator}  
Preconditioner for A. The preconditioner should approximate the inverse of A. Effective preconditioning dramatically improves the rate of convergence, which implies that fewer iterations are needed to reach a given error tolerance.

**callback** : function  
User-supplied function to call after each iteration. It is called as `callback(xk)`, where `xk` is the current solution vector.

**xtype** : {'f','d','F','D'}  
This parameter is deprecated – avoid using it.  
The type of the result. If None, then it will be determined from `A.dtype.char` and `b`. If A does not have a `typecode` method then it will compute `A.matvec(x0)` to get a `typecode`. To save the extra computation when A does not have a `typecode` attribute use `xtype=0` for the same type as `b` or use `xtype='f','d','F',` or `'D'`. This parameter has been superseded by `LinearOperator`.

`scipy.sparse.linalg.cgs(A, b, x0=None, tol=1e-05, maxiter=None, xtype=None, M=None, callback=None)`

Use Conjugate Gradient Squared iteration to solve  $Ax = b$

**Parameters** **A** : {sparse matrix, dense matrix, LinearOperator}  
The real-valued N-by-N matrix of the linear system

**b** : {array, matrix}

**Returns** **x** : {array, matrix}  
Right hand side of the linear system. Has shape (N,) or (N,1).  
The converged solution.

**info** : integer  
**Provides convergence information:**  
0 : successful exit >0 : convergence to tolerance not achieved, number of iterations <0 : illegal input or breakdown

**Other Parameters**

**x0** : {array, matrix}  
Starting guess for the solution.

**tol** : float  
Tolerance to achieve. The algorithm terminates when either the relative or the absolute residual is below *tol*.

**maxiter** : integer  
Maximum number of iterations. Iteration will stop after maxiter steps even if the specified tolerance has not been achieved.

**M** : {sparse matrix, dense matrix, LinearOperator}  
Preconditioner for A. The preconditioner should approximate the inverse of A. Effective preconditioning dramatically improves the rate of convergence, which implies that fewer iterations are needed to reach a given error tolerance.

**callback** : function  
User-supplied function to call after each iteration. It is called as `callback(xk)`, where `xk` is the current solution vector.

**xtype** : {'f','d','F','D'}

This parameter is deprecated – avoid using it.

The type of the result. If `None`, then it will be determined from `A.dtype.char` and `b`. If `A` does not have a `typecode` method then it will compute `A.matvec(x0)` to get a `typecode`. To save the extra computation when `A` does not have a `typecode` attribute use `xtype=0` for the same type as `b` or use `xtype='f','d','F',or 'D'`. This parameter has been superseded by `LinearOperator`.

`scipy.sparse.linalg.gmres` (`A`, `b`, `x0=None`, `tol=1e-05`, `restart=None`, `maxiter=None`, `xtype=None`, `M=None`, `callback=None`, `restrt=None`)

Use Generalized Minimal RESidual iteration to solve  $Ax = b$ .

**Parameters** `A` : {sparse matrix, dense matrix, `LinearOperator`}

The real or complex N-by-N matrix of the linear system.

`b` : {array, matrix}

**Returns** `x` : {array, matrix} Right hand side of the linear system. Has shape (N,) or (N,1).

The converged solution.

`info` : int

**Provides convergence information:**

- 0 : successful exit
- >0 : convergence to tolerance not achieved, number of iterations
- <0 : illegal input or breakdown

**Other Parameters**

`x0` : {array, matrix}

Starting guess for the solution (a vector of zeros by default).

`tol` : float

Tolerance to achieve. The algorithm terminates when either the relative or the absolute residual is below `tol`.

`restart` : int, optional

Number of iterations between restarts. Larger values increase iteration cost, but may be necessary for convergence. Default is 20.

`maxiter` : int, optional

Maximum number of iterations (restart cycles). Iteration will stop after `maxiter` steps even if the specified tolerance has not been achieved.

`xtype` : {'f','d','F','D'}

This parameter is DEPRECATED — avoid using it.

The type of the result. If `None`, then it will be determined from `A.dtype.char` and `b`. If `A` does not have a `typecode` method then it will compute `A.matvec(x0)` to get a `typecode`. To save the extra computation when `A` does not have a `typecode` attribute use `xtype=0` for the same type as `b` or use `xtype='f','d','F',or 'D'`. This parameter has been superseded by `LinearOperator`.

`M` : {sparse matrix, dense matrix, `LinearOperator`}

Inverse of the preconditioner of `A`. `M` should approximate the inverse of `A` and be easy to solve for (see Notes). Effective preconditioning dramatically improves the rate of convergence, which implies that fewer iterations are needed to reach a given error tolerance. By default, no preconditioner is used.

`callback` : function

User-supplied function to call after each iteration. It is called as `callback(rk)`, where `rk` is the current residual vector.

`restrt` : int, optional

DEPRECATED - use `restart` instead.

See also:

[LinearOperator](#)

### Notes

A preconditioner,  $P$ , is chosen such that  $P$  is close to  $A$  but easy to solve for. The preconditioner parameter required by this routine is  $M = P^{-1}$ . The inverse should preferably not be calculated explicitly. Rather, use the following template to produce  $M$ :

```
# Construct a linear operator that computes  $P^{-1} * x$ .
import scipy.sparse.linalg as spla
M_x = lambda x: spla.spsolve(P, x)
M = spla.LinearOperator((n, n), M_x)
```

```
scipy.sparse.linalg.lgmres(A, b, x0=None, tol=1e-05, maxiter=1000, M=None, callback=None,
                           inner_m=30, outer_k=3, outer_v=None, store_outer_Av=True)
```

Solve a matrix equation using the LGMRES algorithm.

The LGMRES algorithm [R22] [R23] is designed to avoid some problems in the convergence in restarted GMRES, and often converges in fewer iterations.

**Parameters**

- A** : {sparse matrix, dense matrix, LinearOperator}
  - The real or complex  $N$ -by- $N$  matrix of the linear system.
- b** : {array, matrix}
  - Right hand side of the linear system. Has shape  $(N,)$  or  $(N,1)$ .
- x0** : {array, matrix}
  - Starting guess for the solution.
- tol** : float, optional
  - Tolerance to achieve. The algorithm terminates when either the relative or the absolute residual is below *tol*.
- maxiter** : int, optional
  - Maximum number of iterations. Iteration will stop after maxiter steps even if the specified tolerance has not been achieved.
- M** : {sparse matrix, dense matrix, LinearOperator}, optional
  - Preconditioner for  $A$ . The preconditioner should approximate the inverse of  $A$ . Effective preconditioning dramatically improves the rate of convergence, which implies that fewer iterations are needed to reach a given error tolerance.
- callback** : function, optional
  - User-supplied function to call after each iteration. It is called as `callback(xk)`, where `xk` is the current solution vector.
- inner\_m** : int, optional
  - Number of inner GMRES iterations per each outer iteration.
- outer\_k** : int, optional
  - Number of vectors to carry between inner GMRES iterations. According to [R22], good values are in the range of 1...3. However, note that if you want to use the additional vectors to accelerate solving multiple similar problems, larger values may be beneficial.
- outer\_v** : list of tuples, optional
  - List containing tuples  $(v, Av)$  of vectors and corresponding matrix-vector products, used to augment the Krylov subspace, and carried between inner GMRES iterations. The element  $Av$  can be *None* if the matrix-vector product should be re-evaluated. This parameter is modified in-place by `lgmres`, and can be used to pass “guess” vectors in and out of the algorithm when solving similar problems.
- store\_outer\_Av** : bool, optional



Whether LGMRES should store also  $A*v$  in addition to vectors  $v$  in the *outer\_v* list. Default is True.

**Returns** **x** : array or matrix  
The converged solution.

**info** : int  
Provides convergence information:

- 0 : successful exit
- >0 : convergence to tolerance not achieved, number of iterations
- <0 : illegal input or breakdown

**Notes**

The LGMRES algorithm [R22] [R23] is designed to avoid the slowing of convergence in restarted GMRES, due to alternating residual vectors. Typically, it often outperforms GMRES(m) of comparable memory requirements by some measure, or at least is not much worse.

Another advantage in this algorithm is that you can supply it with ‘guess’ vectors in the *outer\_v* argument that augment the Krylov subspace. If the solution lies close to the span of these vectors, the algorithm converges faster. This can be useful if several very similar matrices need to be inverted one after another, such as in Newton-Krylov iteration where the Jacobian matrix often changes little in the nonlinear steps.

**References**

[R22], [R23]

`scipy.sparse.linalg.minres` (*A*, *b*, *x0=None*, *shift=0.0*, *tol=1e-05*, *maxiter=None*, *xtype=None*, *M=None*, *callback=None*, *show=False*, *check=False*)

Use MINimum RESidual iteration to solve  $Ax=b$

MINRES minimizes  $\text{norm}(A*x - b)$  for a real symmetric matrix  $A$ . Unlike the Conjugate Gradient method,  $A$  can be indefinite or singular.

If  $\text{shift} \neq 0$  then the method solves  $(A - \text{shift}*I)x = b$

**Parameters** **A** : {sparse matrix, dense matrix, LinearOperator}  
The real symmetric N-by-N matrix of the linear system

**b** : {array, matrix}

**Returns** **x** : {array, matrix} Right hand side of the linear system. Has shape (N,) or (N,1).  
The converged solution.

**info** : integer  
**Provides convergence information:**

- 0 : successful exit
- >0 : convergence to tolerance not achieved, number of iterations
- <0 : illegal input or breakdown

**Other Parameters**

**x0** : {array, matrix}  
Starting guess for the solution.

**tol** : float  
Tolerance to achieve. The algorithm terminates when either the relative or the absolute residual is below *tol*.

**maxiter** : integer  
Maximum number of iterations. Iteration will stop after maxiter steps even if the specified tolerance has not been achieved.

**M** : {sparse matrix, dense matrix, LinearOperator}  
Preconditioner for  $A$ . The preconditioner should approximate the inverse of  $A$ . Effective preconditioning dramatically improves the rate of convergence, which implies that fewer iterations are needed to reach a given error tolerance.

**callback** : function

User-supplied function to call after each iteration. It is called as `callback(xk)`, where `xk` is the current solution vector.

**xtype** : {'f','d','F','D'}

This parameter is deprecated – avoid using it.

The type of the result. If `None`, then it will be determined from `A.dtype.char` and `b`. If `A` does not have a `typecode` method then it will compute `A.matvec(x0)` to get a `typecode`. To save the extra computation when `A` does not have a `typecode` attribute use `xtype=0` for the same type as `b` or use `xtype='f','d','F',or 'D'`. This parameter has been superseded by `LinearOperator`.

### Notes

THIS FUNCTION IS EXPERIMENTAL AND SUBJECT TO CHANGE!

### References

#### *Solution of sparse indefinite systems of linear equations,*

C. C. Paige and M. A. Saunders (1975), *SIAM J. Numer. Anal.* 12(4), pp. 617-629.  
<http://www.stanford.edu/group/SOL/software/minres.html>

#### *This file is a translation of the following MATLAB implementation:*

<http://www.stanford.edu/group/SOL/software/minres/matlab/>

`scipy.sparse.linalg.qmr(A, b, x0=None, tol=1e-05, maxiter=None, xtype=None, M1=None, M2=None, callback=None)`

Use Quasi-Minimal Residual iteration to solve  $Ax = b$

**Parameters** **A** : {sparse matrix, dense matrix, LinearOperator}

The real-valued N-by-N matrix of the linear system. It is required that the linear operator can produce  $Ax$  and  $A^T x$ .

**b** : {array, matrix}

Right hand side of the linear system. Has shape (N,) or (N,1).

**Returns** **x** : {array, matrix}

The converged solution.

**info** : integer

#### *Provides convergence information:*

0 : successful exit >0 : convergence to tolerance not achieved, number of iterations <0 : illegal input or breakdown

### *Other Parameters*

**x0** : {array, matrix}

Starting guess for the solution.

**tol** : float

Tolerance to achieve. The algorithm terminates when either the relative or the absolute residual is below `tol`.

**maxiter** : integer

Maximum number of iterations. Iteration will stop after `maxiter` steps even if the specified tolerance has not been achieved.

**M1** : {sparse matrix, dense matrix, LinearOperator}

Left preconditioner for `A`.

**M2** : {sparse matrix, dense matrix, LinearOperator}

Right preconditioner for `A`. Used together with the left preconditioner `M1`. The matrix `M1*A*M2` should have better conditioned than `A` alone.

**callback** : function

User-supplied function to call after each iteration. It is called as `callback(xk)`, where `xk` is the current solution vector.

**xtype** : {'f','d','F','D'}

This parameter is DEPRECATED – avoid using it.

The type of the result. If None, then it will be determined from `A.dtype.char` and `b`. If `A` does not have a `typecode` method then it will compute `A.matvec(x0)` to get a `typecode`. To save the extra computation when `A` does not have a `typecode` attribute use `xtype=0` for the same type as `b` or use `xtype='f','d','F',` or `'D'`. This parameter has been superseded by `LinearOperator`.

**See also:**

`LinearOperator`

Iterative methods for least-squares problems:

---

<code>lsqr(A, b[, damp, atol, btol, conlim, ...])</code>	Find the least-squares solution to a large, sparse, linear system of equations.
<code>lsmr(A, b[, damp, atol, btol, conlim, ...])</code>	Iterative solver for least-squares problems.

---

`scipy.sparse.linalg.lsqr(A, b, damp=0.0, atol=1e-08, btol=1e-08, conlim=100000000.0, iter_lim=None, show=False, calc_var=False)`

Find the least-squares solution to a large, sparse, linear system of equations.

The function solves  $Ax = b$  or  $\min ||b - Ax||^2$  or  $\min ||Ax - b||^2 + d^2 ||x||^2$ .

The matrix `A` may be square or rectangular (over-determined or under-determined), and may have any rank.

1. Unsymmetric equations -- solve  $Ax = b$
2. Linear least squares -- solve  $Ax = b$   
in the least-squares sense
3. Damped least squares -- solve  $\begin{pmatrix} A \\ \text{damp} \cdot I \end{pmatrix} x = \begin{pmatrix} b \\ 0 \end{pmatrix}$   
in the least-squares sense

**Parameters** `A` : {sparse matrix, ndarray, LinearOperator}

Representation of an `m`-by-`n` matrix. It is required that the linear operator can produce  $Ax$  and  $A^T x$ .

`b` : array\_like, shape (`m`,)

Right-hand side vector `b`.

`damp` : float

Damping coefficient.

`atol`, `btol` : float, optional

Stopping tolerances. If both are  $1.0e-9$  (say), the final residual norm should be accurate to about 9 digits. (The final `x` will usually have fewer correct digits, depending on `cond(A)` and the size of `damp`.)

`conlim` : float, optional

Another stopping tolerance. `lsqr` terminates if an estimate of `cond(A)` exceeds `conlim`. For compatible systems  $Ax = b$ , `conlim` could be as large as  $1.0e+12$  (say). For least-squares problems, `conlim` should be less than  $1.0e+8$ . Maximum precision can be obtained by setting `atol = btol = conlim = zero`, but the number of iterations may then be excessive.

`iter_lim` : int, optional

Explicit limitation on number of iterations (for safety).

`show` : bool, optional

Display an iteration log.

**Returns**

**calc\_var** : bool, optional  
Whether to estimate diagonals of  $(A'A + \text{damp}^2 * I)^{-1}$ .

**x** : ndarray of float  
The final solution.

**istop** : int  
Gives the reason for termination. 1 means x is an approximate solution to  $Ax = b$ . 2 means x approximately solves the least-squares problem.

**itn** : int  
Iteration number upon termination.

**r1norm** : float  
 $\text{norm}(r)$ , where  $r = b - Ax$ .

**r2norm** : float  
 $\sqrt{\text{norm}(r)^2 + \text{damp}^2 * \text{norm}(x)^2}$ . Equal to *r1norm* if  $\text{damp} == 0$ .

**anorm** : float  
Estimate of Frobenius norm of  $Abar = \begin{bmatrix} A \\ \text{damp} * I \end{bmatrix}$ .

**acond** : float  
Estimate of  $\text{cond}(Abar)$ .

**arnorm** : float  
Estimate of  $\text{norm}(A' * r - \text{damp}^2 * x)$ .

**xnorm** : float  
 $\text{norm}(x)$

**var** : ndarray of float  
If **calc\_var** is True, estimates all diagonals of  $(A'A)^{-1}$  (if  $\text{damp} == 0$ ) or more generally  $(A'A + \text{damp}^2 * I)^{-1}$ . This is well defined if A has full column rank or  $\text{damp} > 0$ . (Not sure what var means if  $\text{rank}(A) < n$  and  $\text{damp} = 0$ .)

### Notes

LSQR uses an iterative method to approximate the solution. The number of iterations required to reach a certain accuracy depends strongly on the scaling of the problem. Poor scaling of the rows or columns of A should therefore be avoided where possible.

For example, in problem 1 the solution is unaltered by row-scaling. If a row of A is very small or large compared to the other rows of A, the corresponding row of  $(A \ b)$  should be scaled up or down.

In problems 1 and 2, the solution x is easily recovered following column-scaling. Unless better information is known, the nonzero columns of A should be scaled so that they all have the same Euclidean norm (e.g., 1.0).

In problem 3, there is no freedom to re-scale if damp is nonzero. However, the value of damp should be assigned only after attention has been paid to the scaling of A.

The parameter damp is intended to help regularize ill-conditioned systems, by preventing the true solution from being very large. Another aid to regularization is provided by the parameter acond, which may be used to terminate iterations before the computed solution becomes very large.

If some initial estimate  $x_0$  is known and if  $\text{damp} == 0$ , one could proceed as follows:

1. Compute a residual vector  $r_0 = b - A * x_0$ .
2. Use LSQR to solve the system  $A * dx = r_0$ .
3. Add the correction dx to obtain a final solution  $x = x_0 + dx$ .

This requires that  $x_0$  be available before and after the call to LSQR. To judge the benefits, suppose LSQR takes  $k_1$  iterations to solve  $A * x = b$  and  $k_2$  iterations to solve  $A * dx = r_0$ . If  $x_0$  is “good”,  $\text{norm}(r_0)$  will be smaller than  $\text{norm}(b)$ . If the same stopping tolerances atol and btol are used for each system,  $k_1$  and  $k_2$  will be similar, but the final solution  $x_0 + dx$  should be more accurate. The only way to reduce the total work is to use a larger stopping tolerance for the second system. If some value btol is suitable for  $A * x = b$ , the larger value  $\text{btol} * \text{norm}(b) / \text{norm}(r_0)$  should be suitable for  $A * dx = r_0$ .

Preconditioning is another way to reduce the number of iterations. If it is possible to solve a related system  $Mx = b$  efficiently, where  $M$  approximates  $A$  in some helpful way (e.g.  $M - A$  has low rank or its elements are small relative to those of  $A$ ), LSQR may converge more rapidly on the system  $A * M(\text{inverse}) * z = b$ , after which  $x$  can be recovered by solving  $M * x = z$ .

If  $A$  is symmetric, LSQR should not be used!

Alternatives are the symmetric conjugate-gradient method (`cg`) and/or `SYMMLQ`. `SYMMLQ` is an implementation of symmetric `cg` that applies to any symmetric  $A$  and will converge more rapidly than LSQR. If  $A$  is positive definite, there are other implementations of symmetric `cg` that require slightly less work per iteration than `SYMMLQ` (but will take the same number of iterations).

### References

[R29], [R30], [R31]

`scipy.sparse.linalg.lsmr`(*A*, *b*, *damp*=0.0, *atol*=1e-06, *btol*=1e-06, *conlim*=100000000.0, *maxiter*=None, *show*=False)

Iterative solver for least-squares problems.

`lsmr` solves the system of linear equations  $Ax = b$ . If the system is inconsistent, it solves the least-squares problem  $\min \|b - Ax\|_2$ .  $A$  is a rectangular matrix of dimension  $m$ -by- $n$ , where all cases are allowed:  $m = n$ ,  $m > n$ , or  $m < n$ .  $B$  is a vector of length  $m$ . The matrix  $A$  may be dense or sparse (usually sparse).

**Parameters** **A** : {matrix, sparse matrix, ndarray, LinearOperator}  
Matrix  $A$  in the linear system.

**b** : array\_like, shape (m,)   
Vector  $b$  in the linear system.

**damp** : float   
Damping factor for regularized least-squares. `lsmr` solves the regularized least-squares problem:

$$\min \| (b - (A) x) \|_2 + \| (0) (damp * I) x \|_2$$

where `damp` is a scalar. If `damp` is `None` or `0`, the system is solved without regularization.

**atol, btol** : float, optional   
Stopping tolerances. `lsmr` continues iterations until a certain backward error estimate is smaller than some quantity depending on `atol` and `btol`. Let  $r = b - Ax$  be the residual vector for the current approximate solution  $x$ . If  $Ax = b$  seems to be consistent, `lsmr` terminates when  $\text{norm}(r) \leq \text{atol} * \text{norm}(A) * \text{norm}(x) + \text{btol} * \text{norm}(b)$ . Otherwise, `lsmr` terminates when  $\text{norm}(A^T r) \leq \text{atol} * \text{norm}(A) * \text{norm}(r)$ . If both tolerances are  $1.0e-6$  (say), the final  $\text{norm}(r)$  should be accurate to about 6 digits. (The final  $x$  will usually have fewer correct digits, depending on  $\text{cond}(A)$  and the size of `LAMBDA`.) If `atol` or `btol` is `None`, a default value of  $1.0e-6$  will be used. Ideally, they should be estimates of the relative error in the entries of  $A$  and  $B$  respectively. For example, if the entries of  $A$  have 7 correct digits, set `atol` =  $1e-7$ . This prevents the algorithm from doing unnecessary work beyond the uncertainty of the input data.

**conlim** : float, optional   
`lsmr` terminates if an estimate of  $\text{cond}(A)$  exceeds `conlim`. For compatible systems  $Ax = b$ , `conlim` could be as large as  $1.0e+12$  (say). For least-squares problems, `conlim` should be less than  $1.0e+8$ . If `conlim` is `None`, the default value is  $1e+8$ . Maximum precision can be obtained by setting

`atol = btol = conlim = 0`, but the number of iterations may then be excessive.

**maxiter** : int, optional  
`lsqr` terminates if the number of iterations reaches *maxiter*. The default is `maxiter = min(m, n)`. For ill-conditioned systems, a larger value of *maxiter* may be needed.

**show** : bool, optional  
 Print iterations logs if `show=True`.

**Returns** **x** : ndarray of float  
 Least-square solution returned.

**istop** : int  
 istop gives the reason for stopping:

- `istop = 0` means `x=0` is a solution.
- `= 1` means `x` is an approximate solution to `A*x = B`, according to `atol` and `btol`.
- `= 2` means `x` approximately solves the least-squares problem according to `atol`.
- `= 3` means `COND(A)` seems to be greater than `CONLIM`.
- `= 4` is the same as 1 with `atol = btol = eps` (machine precision)
- `= 5` is the same as 2 with `atol = eps`.
- `= 6` is the same as 3 with `CONLIM = 1/eps`.
- `= 7` means ITN reached `maxiter` before the other stopping conditions were satisfied.

**itn** : int  
 Number of iterations used.

**normr** : float  
`norm(b-Ax)`

**normar** : float  
`norm(A^T (b - Ax))`

**norma** : float  
`norm(A)`

**conda** : float  
 Condition number of A.

**normx** : float  
`norm(x)`

**Notes**

New in version 0.11.0.

**References**

[R27], [R28]

**Matrix factorizations** Eigenvalue problems:

---

<code>eigs(A[, k, M, sigma, which, v0, ncv, ...])</code>	Find <i>k</i> eigenvalues and eigenvectors of the square matrix <i>A</i> .
<code>eigsh(A[, k, M, sigma, which, v0, ncv, ...])</code>	Find <i>k</i> eigenvalues and eigenvectors of the real symmetric square matrix or complex hermitian matrix <i>A</i> .
<code>lobpcg(A, X[, B, M, Y, tol, maxiter, ...])</code>	Locally Optimal Block Preconditioned Conjugate Gradient Method (LOBPCG)

---

`scipy.sparse.linalg.eigs` (*A*, *k*=6, *M*=None, *sigma*=None, *which*='LM', *v0*=None, *ncv*=None, *maxiter*=None, *tol*=0, *return\_eigenvectors*=True, *Minv*=None, *OPinv*=None, *OPpart*=None)

Find  $k$  eigenvalues and eigenvectors of the square matrix  $A$ .

Solves  $A * x[i] = w[i] * x[i]$ , the standard eigenvalue problem for  $w[i]$  eigenvalues with corresponding eigenvectors  $x[i]$ .

If  $M$  is specified, solves  $A * x[i] = w[i] * M * x[i]$ , the generalized eigenvalue problem for  $w[i]$  eigenvalues with corresponding eigenvectors  $x[i]$

**Parameters** **A** : ndarray, sparse matrix or LinearOperator

An array, sparse matrix, or LinearOperator representing the operation  $A * x$ , where  $A$  is a real or complex square matrix.

**k** : int, optional

The number of eigenvalues and eigenvectors desired.  $k$  must be smaller than  $N$ . It is not possible to compute all eigenvectors of a matrix.

**M** : ndarray, sparse matrix or LinearOperator, optional

An array, sparse matrix, or LinearOperator representing the operation  $M*x$  for the generalized eigenvalue problem

$$A * x = w * M * x.$$

$M$  must represent a real, symmetric matrix if  $A$  is real, and must represent a complex, hermitian matrix if  $A$  is complex. For best results, the data type of  $M$  should be the same as that of  $A$ . Additionally:

If  $\sigma$  is None,  $M$  is positive definite

If  $\sigma$  is specified,  $M$  is positive semi-definite

If  $\sigma$  is None, `eigs` requires an operator to compute the solution of the linear equation  $M * x = b$ . This is done internally via a (sparse) LU decomposition for an explicit matrix  $M$ , or via an iterative solver for a general linear operator. Alternatively, the user can supply the matrix or operator `Minv`, which gives  $x = Minv * b = M^{-1} * b$ .

**sigma** : real or complex, optional

Find eigenvalues near  $\sigma$  using shift-invert mode. This requires an operator to compute the solution of the linear system  $[A - \sigma * M] * x = b$ , where  $M$  is the identity matrix if unspecified. This is computed internally via a (sparse) LU decomposition for explicit matrices  $A$  &  $M$ , or via an iterative solver if either  $A$  or  $M$  is a general linear operator. Alternatively, the user can supply the matrix or operator `OPinv`, which gives  $x = OPinv * b = [A - \sigma * M]^{-1} * b$ . For a real matrix  $A$ , shift-invert can either be done in imaginary mode or real mode, specified by the parameter `OPpart` ('r' or 'i'). Note that when  $\sigma$  is specified, the keyword 'which' (below) refers to the shifted eigenvalues  $w' [i]$  where:

*If  $A$  is real and `OPpart == 'r'` (default),*

$$w' [i] = 1/2 * [1/(w[i]-\sigma) + 1/(w[i]-\text{conj}(\sigma))].$$

*If  $A$  is real and `OPpart == 'i'`,*

$$w' [i] = 1/2i * [1/(w[i]-\sigma) - 1/(w[i]-\text{conj}(\sigma))].$$

*If  $A$  is complex,  $w' [i] = 1/(w[i]-\sigma)$ .*

**v0** : ndarray, optional

Starting vector for iteration. Default: random

**ncv** : int, optional

The number of Lanczos vectors generated  $ncv$  must be greater than  $k$ ; it is recommended that  $ncv > 2*k$ . Default:  $\min(n, \max(2*k + 1, 20))$

**which** : str, ['LM' | 'SM' | 'LR' | 'SR' | 'LI' | 'SI'], optional

Which  $k$  eigenvectors and eigenvalues to find:

‘LM’ : largest magnitude  
 ‘SM’ : smallest magnitude  
 ‘LR’ : largest real part  
 ‘SR’ : smallest real part  
 ‘LI’ : largest imaginary part  
 ‘SI’ : smallest imaginary part

When `sigma != None`, ‘which’ refers to the shifted eigenvalues `w[i]` (see discussion in ‘sigma’, above). ARPACK is generally better at finding large values than small values. If small eigenvalues are desired, consider using shift-invert mode for better performance.

**maxiter** : int, optional

Maximum number of Arnoldi update iterations allowed Default: `n*10`

**tol** : float, optional

Relative accuracy for eigenvalues (stopping criterion) The default value of 0 implies machine precision.

**return\_eigenvectors** : bool, optional

Return eigenvectors (True) in addition to eigenvalues

**Minv** : ndarray, sparse matrix or LinearOperator, optional

See notes in `M`, above.

**OPinv** : ndarray, sparse matrix or LinearOperator, optional

See notes in `sigma`, above.

**OPpart** : {‘r’ or ‘i’}, optional

See notes in `sigma`, above

#### Returns

**w** : ndarray

Array of `k` eigenvalues.

**v** : ndarray

An array of `k` eigenvectors. `v[:, i]` is the eigenvector corresponding to the eigenvalue `w[i]`.

#### Raises

**ArpackNoConvergence**

When the requested convergence is not obtained. The currently converged eigenvalues and eigenvectors can be found as `eigenvalues` and `eigenvectors` attributes of the exception object.

#### See also:

[`eigsh`](#) eigenvalues and eigenvectors for symmetric matrix `A`  
[`svds`](#) singular value decomposition for a matrix `A`

#### Notes

This function is a wrapper to the ARPACK [R15] SNEUPD, DNEUPD, CNEUPD, ZNEUPD, functions which use the Implicitly Restarted Arnoldi Method to find the eigenvalues and eigenvectors [R16].

#### References

[R15], [R16]

#### Examples

Find 6 eigenvectors of the identity matrix:

```

>>> import scipy.sparse as sparse
>>> id = np.eye(13)
>>> vals, vecs = sparse.linalg.eigs(id, k=6)
>>> vals
array([ 1.+0.j,  1.+0.j,  1.+0.j,  1.+0.j,  1.+0.j,  1.+0.j])
>>> vecs.shape
(13, 6)
    
```



`scipy.sparse.linalg.eigsh` ( $A$ ,  $k=6$ ,  $M=None$ ,  $sigma=None$ ,  $which='LM'$ ,  $v0=None$ ,  $ncv=None$ ,  $maxiter=None$ ,  $tol=0$ ,  $return_eigenvectors=True$ ,  $Minv=None$ ,  $OPinv=None$ ,  $mode='normal'$ )

Find  $k$  eigenvalues and eigenvectors of the real symmetric square matrix or complex hermitian matrix  $A$ .

Solves  $A * x[i] = w[i] * x[i]$ , the standard eigenvalue problem for  $w[i]$  eigenvalues with corresponding eigenvectors  $x[i]$ .

If  $M$  is specified, solves  $A * x[i] = w[i] * M * x[i]$ , the generalized eigenvalue problem for  $w[i]$  eigenvalues with corresponding eigenvectors  $x[i]$

**Parameters**  $A$  : An  $N \times N$  matrix, array, sparse matrix, or LinearOperator representing the operation  $A * x$ , where  $A$  is a real symmetric matrix For buckling mode (see below)  $A$  must additionally be positive-definite

$k$  : int, optional

The number of eigenvalues and eigenvectors desired.  $k$  must be smaller than  $N$ . It is not possible to compute all eigenvectors of a matrix.

**Returns**  $w$  : array

Array of  $k$  eigenvalues

$v$  : array

An array representing the  $k$  eigenvectors. The column  $v[:, i]$  is the eigenvector corresponding to the eigenvalue  $w[i]$ .

**Other Parameters**

$M$  : An  $N \times N$  matrix, array, sparse matrix, or linear operator representing the operation  $M * x$  for the generalized eigenvalue problem

$$A * x = w * M * x.$$

$M$  must represent a real, symmetric matrix if  $A$  is real, and must represent a complex, hermitian matrix if  $A$  is complex. For best results, the data type of  $M$  should be the same as that of  $A$ . Additionally:

If  $sigma$  is `None`,  $M$  is symmetric positive definite

If  $sigma$  is specified,  $M$  is symmetric positive semi-definite

In buckling mode,  $M$  is symmetric indefinite.

If  $sigma$  is `None`, `eigsh` requires an operator to compute the solution of the linear equation  $M * x = b$ . This is done internally via a (sparse) LU decomposition for an explicit matrix  $M$ , or via an iterative solver for a general linear operator. Alternatively, the user can supply the matrix or operator `Minv`, which gives  $x = Minv * b = M^{-1} * b$ .

$sigma$  : real

Find eigenvalues near  $sigma$  using shift-invert mode. This requires an operator to compute the solution of the linear system  $[A - sigma * M] x = b$ , where  $M$  is the identity matrix if unspecified. This is computed internally via a (sparse) LU decomposition for explicit matrices  $A$  &  $M$ , or via an iterative solver if either  $A$  or  $M$  is a general linear operator. Alternatively, the user can supply the matrix or operator `OPinv`, which gives  $x = OPinv * b = [A - sigma * M]^{-1} * b$ . Note that when  $sigma$  is specified, the keyword 'which' refers to the shifted eigenvalues  $w' [i]$  where:

if `mode == 'normal'`,  $w' [i] = 1 / (w[i] - sigma)$ .

if `mode == 'cayley'`,  $w' [i] = (w[i] + sigma) / (w[i] - sigma)$ .

if `mode == 'buckling'`,  $w' [i] = w[i] / (w[i] - sigma)$ .

(see further discussion in 'mode' below)

$v0$  : ndarray, optional

Starting vector for iteration. Default: random

$ncv$  : int, optional

The number of Lanczos vectors generated `ncv` must be greater than `k` and smaller than `n`; it is recommended that `ncv > 2*k`. Default: `min(n, max(2*k + 1, 20))`

**which** : str ['LM' | 'SM' | 'LA' | 'SA' | 'BE']

If `A` is a complex hermitian matrix, 'BE' is invalid. Which `k` eigenvectors and eigenvalues to find:

- 'LM' : Largest (in magnitude) eigenvalues
- 'SM' : Smallest (in magnitude) eigenvalues
- 'LA' : Largest (algebraic) eigenvalues
- 'SA' : Smallest (algebraic) eigenvalues
- 'BE' : Half (`k/2`) from each end of the spectrum

When `k` is odd, return one more (`k/2+1`) from the high end. When `sigma != None`, 'which' refers to the shifted eigenvalues `w' [i]` (see discussion in 'sigma', above). ARPACK is generally better at finding large values than small values. If small eigenvalues are desired, consider using shift-invert mode for better performance.

**maxiter** : int, optional

Maximum number of Arnoldi update iterations allowed Default: `n*10`

**tol** : float

Relative accuracy for eigenvalues (stopping criterion). The default value of 0 implies machine precision.

**Minv** : N x N matrix, array, sparse matrix, or LinearOperator

See notes in `M`, above

**OPinv** : N x N matrix, array, sparse matrix, or LinearOperator

See notes in `sigma`, above.

**return\_eigenvectors** : bool

Return eigenvectors (True) in addition to eigenvalues

**mode** : string ['normal' | 'buckling' | 'cayley']

Specify strategy to use for shift-invert mode. This argument applies only for real-valued `A` and `sigma != None`. For shift-invert mode, ARPACK internally solves the eigenvalue problem  $OP * x' [i] = w' [i] * B * x' [i]$  and transforms the resulting Ritz vectors `x'[i]` and Ritz values `w'[i]` into the desired eigenvectors and eigenvalues of the problem  $A * x [i] = w [i] * M * x [i]$ . The modes are as follows:

- 'normal' :  $OP = [A - \text{sigma} * M]^{-1} * M, B = M, w'[i] = 1 / (w[i] - \text{sigma})$
- 'buckling' :  $OP = [A - \text{sigma} * M]^{-1} * A, B = A, w'[i] = w[i] / (w[i] - \text{sigma})$
- 'cayley' :  $OP = [A - \text{sigma} * M]^{-1} * [A + \text{sigma} * M], B = M, w'[i] = (w[i] + \text{sigma}) / (w[i] - \text{sigma})$

The choice of mode will affect which eigenvalues are selected by the keyword 'which', and can also impact the stability of convergence (see [2] for a discussion)

**Raises**

**ArpackNoConvergence**

When the requested convergence is not obtained.

The currently converged eigenvalues and eigenvectors can be found as `eigenvalues` and `eigenvectors` attributes of the exception object.

**See also:**

- `eigs` eigenvalues and eigenvectors for a general (nonsymmetric) matrix `A`
- `svds` singular value decomposition for a matrix `A`

**Notes**

This function is a wrapper to the ARPACK [R17] SSEUPD and DSEUPD functions which use the Implicitly Restarted Lanczos Method to find the eigenvalues and eigenvectors [R18].

**References**

[R17], [R18]

**Examples**

```
>>> import scipy.sparse as sparse
>>> id = np.eye(13)
>>> vals, vecs = sparse.linalg.eigsh(id, k=6)
>>> vals
array([ 1.+0.j,  1.+0.j,  1.+0.j,  1.+0.j,  1.+0.j,  1.+0.j])
>>> vecs.shape
(13, 6)
```

```
scipy.sparse.linalg.lobpcg(A, X, B=None, M=None, Y=None, tol=None, maxiter=20,
                           largest=True, verbosityLevel=0, retLambdaHistory=False, re-
                           tResidualNormsHistory=False)
```

Locally Optimal Block Preconditioned Conjugate Gradient Method (LOBPCG)

LOBPCG is a preconditioned eigensolver for large symmetric positive definite (SPD) generalized eigenproblems.

**Parameters**

- A** : {sparse matrix, dense matrix, LinearOperator}
 

The symmetric linear operator of the problem, usually a sparse matrix. Often called the “stiffness matrix”.
- X** : array\_like
 

Initial approximation to the k eigenvectors. If A has shape=(n,n) then X should have shape shape=(n,k).
- B** : {dense matrix, sparse matrix, LinearOperator}, optional
 

the right hand side operator in a generalized eigenproblem. by default, B = Identity often called the “mass matrix”
- M** : {dense matrix, sparse matrix, LinearOperator}, optional
 

preconditioner to A; by default M = Identity M should approximate the inverse of A
- Y** : array\_like, optional
 

n-by-sizeY matrix of constraints, sizeY < n The iterations will be performed in the B-orthogonal complement of the column-space of Y. Y must be full rank.

**Returns**

- w** : array
 

Array of k eigenvalues
- v** : array
 

An array of k eigenvectors. V has the same shape as X.

**Other Parameters**

- tol** : scalar, optional
 

Solver tolerance (stopping criterion) by default: tol=n\*sqrt(eps)
- maxiter** : integer, optional
 

maximum number of iterations by default: maxiter=min(n,20)
- largest** : bool, optional
 

when True, solve for the largest eigenvalues, otherwise the smallest
- verbosityLevel** : integer, optional
 

controls solver output. default: verbosityLevel = 0.
- retLambdaHistory** : boolean, optional
 

whether to return eigenvalue history

**retResidualNormsHistory** : boolean, optional  
whether to return history of residual norms

### Notes

If both `retLambdaHistory` and `retResidualNormsHistory` are `True`, the return tuple has the following format (lambda, V, lambda history, residual norms history).

In the following  $n$  denotes the matrix size and  $m$  the number of required eigenvalues (smallest or largest).

The LOBPCG code internally solves eigenproblems of the size  $3 \cdot m$  on every iteration by calling the “standard” dense eigensolver, so if  $m$  is not small enough compared to  $n$ , it does not make sense to call the LOBPCG code, but rather one should use the “standard” eigensolver, e.g. `numpy` or `scipy` function in this case. If one calls the LOBPCG algorithm for  $5 \cdot m > n$ , it will most likely break internally, so the code tries to call the standard function instead.

It is not that  $n$  should be large for the LOBPCG to work, but rather the ratio  $n/m$  should be large. If you call the LOBPCG code with  $m=1$  and  $n=10$ , it should work, though  $n$  is small. The method is intended for extremely large  $n/m$ , see e.g., reference [28] in <http://arxiv.org/abs/0705.2626>

The convergence speed depends basically on two factors:

1. How well relatively separated the seeking eigenvalues are from the rest of the eigenvalues. One can try to vary  $m$  to make this better.
2. How well conditioned the problem is. This can be changed by using proper preconditioning. For example, a rod vibration test problem (under tests directory) is ill-conditioned for large  $n$ , so convergence will be slow, unless efficient preconditioning is used. For this specific problem, a good simple preconditioner function would be a linear solve for  $A$ , which is easy to code since  $A$  is tridiagonal.

### Acknowledgements

`lobpcg.py` code was written by Robert Cimrman. Many thanks belong to Andrew Knyazev, the author of the algorithm, for lots of advice and support.

### References

[R24], [R25], [R26]

### Examples

Solve  $Ax = \lambda Bx$  with constraints and preconditioning.

```
>>> from scipy.sparse import spdiags, issparse
>>> from scipy.sparse.linalg import lobpcg, LinearOperator
>>> n = 100
>>> vals = [np.arange(n, dtype=np.float64) + 1]
>>> A = spdiags(vals, 0, n, n)
>>> A.toarray()
array([[ 1.,  0.,  0., ...,  0.,  0.,  0.],
       [ 0.,  2.,  0., ...,  0.,  0.,  0.],
       [ 0.,  0.,  3., ...,  0.,  0.,  0.],
       ...,
       [ 0.,  0.,  0., ..., 98.,  0.,  0.],
       [ 0.,  0.,  0., ...,  0., 99.,  0.],
       [ 0.,  0.,  0., ...,  0.,  0., 100.]])
```

Constraints.

```
>>> Y = np.eye(n, 3)
```

Initial guess for eigenvectors, should have linearly independent columns. Column dimension = number of requested eigenvalues.

```
>>> X = np.random.rand(n, 3)
```

Preconditioner – inverse of A (as an abstract linear operator).

```
>>> invA = spdiags([1./vals[0]], 0, n, n)
>>> def precondition(x):
...     return invA * x
>>> M = LinearOperator(matvec=precond, shape=(n, n), dtype=float)
```

Here, `invA` could of course have been used directly as a preconditioner. Let us then solve the problem:

```
>>> eigs, vecs = lobpcg(A, X, Y=Y, M=M, tol=1e-4, maxiter=40, largest=False)
>>> eigs
array([ 4.,  5.,  6.])
```

Note that the vectors passed in `Y` are the eigenvectors of the 3 smallest eigenvalues. The results returned are orthogonal to those.

Singular values problems:

---

`svds(A[, k, ncv, tol, which, v0, maxiter, ...])` Compute the largest k singular values/vectors for a sparse matrix.

---

`scipy.sparse.linalg.svds(A, k=6, ncv=None, tol=0, which='LM', v0=None, maxiter=None, return_singular_vectors=True)`

Compute the largest k singular values/vectors for a sparse matrix.

**Parameters** `A` : {sparse matrix, LinearOperator}

Array to compute the SVD on, of shape (M, N)

`k` : int, optional

Number of singular values and vectors to compute. Must be  $1 \leq k < \min(A.shape)$ .

`ncv` : int, optional

The number of Lanczos vectors generated `ncv` must be greater than  $k+1$  and smaller than  $n$ ; it is recommended that  $ncv > 2*k$  Default:  $\min(n, \max(2*k + 1, 20))$

`tol` : float, optional

Tolerance for singular values. Zero (default) means machine precision.

`which` : str, ['LM' | 'SM'], optional

Which  $k$  singular values to find:

- 'LM' : largest singular values
- 'SM' : smallest singular values

New in version 0.12.0.

`v0` : ndarray, optional

Starting vector for iteration, of length  $\min(A.shape)$ . Should be an (approximate) left singular vector if  $N > M$  and a right singular vector otherwise. Default: random

New in version 0.12.0.

`maxiter` : int, optional

Maximum number of iterations.

New in version 0.12.0.

`return_singular_vectors` : bool or str, optional

- True: return singular vectors (True) in addition to singular values.

New in version 0.12.0.

- “u”: only return the u matrix, without computing vh (if  $N > M$ ).
- “vh”: only return the vh matrix, without computing u (if  $N \leq M$ ).

**Returns**

**u** : ndarray, shape=(M, k)  
 New in version 0.16.0.  
 Unitary matrix having left singular vectors as columns. If *return\_singular\_vectors* is “vh”, this variable is not computed, and None is returned instead.

**s** : ndarray, shape=(k,)  
 The singular values.

**vt** : ndarray, shape=(k, N)  
 Unitary matrix having right singular vectors as rows. If *return\_singular\_vectors* is “u”, this variable is not computed, and None is returned instead.

**Notes**

This is a naive implementation using ARPACK as an eigensolver on  $A.H * A$  or  $A * A.H$ , depending on which one is more efficient.

Complete or incomplete LU factorizations

<code>splu(A[, permc_spec, diag_pivot_thresh, ...])</code>	Compute the LU decomposition of a sparse, square matrix.
<code>spilu(A[, drop_tol, fill_factor, drop_rule, ...])</code>	Compute an incomplete LU decomposition for a sparse, square matrix.
SuperLU	LU factorization of a sparse matrix.

`scipy.sparse.linalg.splu(A, permc_spec=None, diag_pivot_thresh=None, drop_tol=None, relax=None, panel_size=None, options={})`

Compute the LU decomposition of a sparse, square matrix.

**Parameters**

**A** : sparse matrix  
 Sparse matrix to factorize. Should be in CSR or CSC format.

**permc\_spec** : str, optional  
 How to permute the columns of the matrix for sparsity preservation. (default: ‘COLAMD’)

- NATURAL: natural ordering.
- MMD\_ATA: minimum degree ordering on the structure of  $A^T A$ .
- MMD\_AT\_PLUS\_A: minimum degree ordering on the structure of  $A^T + A$ .
- COLAMD: approximate minimum degree column ordering

**diag\_pivot\_thresh** : float, optional  
 Threshold used for a diagonal entry to be an acceptable pivot. See SuperLU user’s guide for details [R37]

**drop\_tol** : float, optional  
 (deprecated) No effect.

**relax** : int, optional  
 Expert option for customizing the degree of relaxing supernodes. See SuperLU user’s guide for details [R37]

**panel\_size** : int, optional  
 Expert option for customizing the panel size. See SuperLU user’s guide for details [R37]

**options** : dict, optional  
 Dictionary containing additional expert options to SuperLU. See SuperLU user guide [R37] (section 2.4 on the ‘Options’ argument) for more details. For example, you can specify `options=dict(Equil=False,`

**Returns** `invA` : `scipy.sparse.linalg.SuperLU` Object, which has a `solve` method.

**See also:**

`spilu` incomplete LU decomposition

**Notes**

This function uses the SuperLU library.

**References**

[R37]

```
scipy.sparse.linalg.spilu(A, drop_tol=None, fill_factor=None, drop_rule=None,
                          permc_spec=None, diag_pivot_thresh=None, relax=None,
                          panel_size=None, options=None)
```

Compute an incomplete LU decomposition for a sparse, square matrix.

The resulting object is an approximation to the inverse of A.

**Parameters** `A` : (N, N) array\_like  
Sparse matrix to factorize

**drop\_tol** : float, optional  
Drop tolerance ( $0 \leq \text{tol} \leq 1$ ) for an incomplete LU decomposition. (default:  $1e-4$ )

**fill\_factor** : float, optional  
Specifies the fill ratio upper bound ( $\geq 1.0$ ) for ILU. (default: 10)

**drop\_rule** : str, optional  
Comma-separated string of drop rules to use. Available rules: `basic`, `prows`, `column`, `area`, `secondary`, `dynamic`, `interp`. (Default: `basic, area`)  
See SuperLU documentation for details.

**Remaining other options**

**Returns** `invA_approx` : `scipy.sparse.linalg.SuperLU` Object, which has a `solve` method.

**See also:**

`splu` complete LU decomposition

**Notes**

To improve the better approximation to the inverse, you may need to increase `fill_factor` AND decrease `drop_tol`.

This function uses the SuperLU library.

**class** `scipy.sparse.linalg.SuperLU`  
LU factorization of a sparse matrix.

Factorization is represented as:

$$P_r * A * P_c = L * U$$

To construct these SuperLU objects, call the `splu` and `spilu` functions.

**Notes**

New in version 0.14.0.

### Examples

The LU decomposition can be used to solve matrix equations. Consider:

```
>>> import numpy as np
>>> from scipy.sparse import csc_matrix, linalg as sla
>>> A = csc_matrix([[1,2,0,4],[1,0,0,1],[1,0,2,1],[2,2,1,0.]])
```

This can be solved for a given right-hand side:

```
>>> lu = sla.splu(A)
>>> b = np.array([1, 2, 3, 4])
>>> x = lu.solve(b)
>>> A.dot(x)
array([ 1.,  2.,  3.,  4.]
```

The `lu` object also contains an explicit representation of the decomposition. The permutations are represented as mappings of indices:

```
>>> lu.perm_r
array([0, 2, 1, 3], dtype=int32)
>>> lu.perm_c
array([2, 0, 1, 3], dtype=int32)
```

The L and U factors are sparse matrices in CSC format:

```
>>> lu.L.A
array([[ 1. ,  0. ,  0. ,  0. ],
       [ 0. ,  1. ,  0. ,  0. ],
       [ 0. ,  0. ,  1. ,  0. ],
       [ 1. ,  0.5,  0.5,  1. ]])
>>> lu.U.A
array([[ 2.,  0.,  1.,  4.],
       [ 0.,  2.,  1.,  1.],
       [ 0.,  0.,  1.,  1.],
       [ 0.,  0.,  0., -5.]])
```

The permutation matrices can be constructed:

```
>>> Pr = csc_matrix((4, 4))
>>> Pr[lu.perm_r, np.arange(4)] = 1
>>> Pc = csc_matrix((4, 4))
>>> Pc[np.arange(4), lu.perm_c] = 1
```

We can reassemble the original matrix:

```
>>> (Pr.T * (lu.L * lu.U) * Pc.T).A
array([[ 1.,  2.,  0.,  4.],
       [ 1.,  0.,  0.,  1.],
       [ 1.,  0.,  2.,  1.],
       [ 2.,  2.,  1.,  0.]])
```

Continued on next page



Table 5.189 – continued from previous page

**Attributes**


---

<code>shape</code>	Shape of the original matrix as a tuple of ints.
<code>nnz</code>	Number of nonzero elements in the matrix.
<code>perm_c</code>	Permutation Pc represented as an array of indices.
<code>perm_r</code>	Permutation Pr represented as an array of indices.
<code>L</code>	Lower triangular factor with unit diagonal as a <code>scipy.sparse.csc_matrix</code> .
<code>U</code>	Upper triangular factor as a <code>scipy.sparse.csc_matrix</code> .

---

**SuperLU.shape**

Shape of the original matrix as a tuple of ints.

**SuperLU.nnz**

Number of nonzero elements in the matrix.

**SuperLU.perm\_c**

Permutation Pc represented as an array of indices.

The column permutation matrix can be reconstructed via:

```
>>> Pc = np.zeros((n, n))
>>> Pc[np.arange(n), perm_c] = 1
```

**SuperLU.perm\_r**

Permutation Pr represented as an array of indices.

The row permutation matrix can be reconstructed via:

```
>>> Pr = np.zeros((n, n))
>>> Pr[perm_r, np.arange(n)] = 1
```

**SuperLU.L**

Lower triangular factor with unit diagonal as a `scipy.sparse.csc_matrix`.

New in version 0.14.0.

**SuperLU.U**

Upper triangular factor as a `scipy.sparse.csc_matrix`.

New in version 0.14.0.

**Methods**


---

`solve(rhs[, trans])` Solves linear system of equations with one or several right-hand sides.

---

**SuperLU.solve(rhs[, trans])**

Solves linear system of equations with one or several right-hand sides.

**Parameters** **rhs** : ndarray, shape (n,) or (n, k)  
 Right hand side(s) of equation  
**trans** : { 'N', 'T', 'H' }, optional  
 Type of system to solve:

```
'N':  A   * x == rhs  (default)
'T':  A^T * x == rhs
```

'H':  $A^H * x == rhs$   
*Returns* **x** : ndarray, shape  $rhs.shape$  i.e., normal, transposed, and hermitian conjugate.  
 Solution vector(s)

---

<code>ArpackNoConvergence(msg, eigenvalues, ...)</code>	ARPACK iteration did not converge
<code>ArpackError(info[, infodict])</code>	ARPACK error

---

**Exceptions**

**exception** `scipy.sparse.linalg.ArpackNoConvergence` (*msg, eigenvalues, eigenvectors*)  
 ARPACK iteration did not converge

*Attributes*

eigenvalues	(ndarray) Partial result. Converged eigenvalues.
eigenvectors	(ndarray) Partial result. Converged eigenvectors.

**exception** `scipy.sparse.linalg.ArpackError` (*info*, *infodict*={*c*: {0: 'Normal exit.', 1: 'Maximum number of iterations taken. All possible eigenvalues of OP has been found. IPARAM(5) returns the number of wanted converged Ritz values.', 2: 'No longer an informational error. Deprecated starting with release 2 of ARPACK.', 3: 'No shifts could be applied during a cycle of the Implicitly restarted Arnoldi iteration. One possibility is to increase the size of NCV relative to NEV.', -9999: 'Could not build an Arnoldi factorization. IPARAM(5) returns the size of the current Arnoldi factorization. The user is advised to check that enough workspace and array storage has been allocated.', -13: "NEV and WHICH = 'BE' are incompatible.", -12: 'IPARAM(1) must be equal to 0 or 1.', -1: 'N must be positive.', -10: 'IPARAM(7) must be 1, 2, 3.', -9: 'Starting vector is zero.', -8: 'Error return from LAPACK eigenvalue calculation;', -7: 'Length of private work array WORKL is not sufficient.', -6: "BMAT must be one of 'I' or 'G'.", -5: " WHICH must be one of 'LM', 'SM', 'LR', 'SR', 'LI', 'SI'", -4: 'The maximum number of Arnoldi update iterations allowed must be greater than zero.', -3: 'NCV-NEV >= 2 and less than or equal to N.', -2: 'NEV must be positive.', -11: "IPARAM(7) = 1 and BMAT = 'G' are incompatible."}, *s*: {0: 'Normal exit.', 1: 'Maximum number of iterations taken. All possible eigenvalues of OP has been found. IPARAM(5) returns the number of wanted converged Ritz values.', 2: 'No longer an informational error. Deprecated starting with release 2 of ARPACK.', 3: 'No shifts could be applied during a cycle of the Implicitly restarted Arnoldi iteration. One possibility is to increase the size of NCV relative to NEV.', -9999: 'Could not build an Arnoldi factorization. IPARAM(5) returns the size of the current Arnoldi factorization. The user is advised to check that enough workspace and array storage has been allocated.', -13: "NEV and WHICH = 'BE' are incompatible.", -12: 'IPARAM(1) must be equal to 0 or 1.', -2: 'NEV must be positive.', -10: 'IPARAM(7) must be 1, 2, 3, 4.', -9: 'Starting vector is zero.', -8: 'Error return from LAPACK eigenvalue calculation;', -7: 'Length of private work array WORKL is not sufficient.', -6: "BMAT must be one of 'I' or 'G'.", -5: " WHICH must be one of 'LM', 'SM', 'LR', 'SR', 'LI', 'SI'", -4: 'The maximum number of Arnoldi update iterations allowed must be greater than zero.', -3: 'NCV-NEV >= 2 and less than or equal to N.', -1: 'N must be positive.', -11: "IPARAM(7) = 1 and BMAT = 'G' are incompatible."}, *z*: {0: 'Normal exit.', 1: 'Maximum number of iterations taken. All possible eigenvalues of OP has been found. IPARAM(5) returns the number of wanted converged Ritz values.', 2: 'No longer an informational error. Deprecated starting with release 2 of ARPACK.', 3: 'No shifts could be applied during a cycle of the Implicitly restarted Arnoldi iteration. One possibility is to increase the size of NCV relative to NEV.', -9999: 'Could not build an Arnoldi factorization. IPARAM(5) returns the size

ARPACK error

**Functions**

<code>aslinearoperator(A)</code>	Return A as a LinearOperator.
<code>bicg(A, b[, x0, tol, maxiter, xtype, M, ...])</code>	Use BIConjugate Gradient iteration to solve $Ax = b$
<code>bicgstab(A, b[, x0, tol, maxiter, xtype, M, ...])</code>	Use BIConjugate Gradient STABILized iteration to solve $Ax = b$
<code>cg(A, b[, x0, tol, maxiter, xtype, M, callback])</code>	Use Conjugate Gradient iteration to solve $Ax = b$
<code>cgs(A, b[, x0, tol, maxiter, xtype, M, callback])</code>	Use Conjugate Gradient Squared iteration to solve $Ax = b$
<code>eigs(A[, k, M, sigma, which, v0, ncv, ...])</code>	Find k eigenvalues and eigenvectors of the square matrix A.
<code>eigsh(A[, k, M, sigma, which, v0, ncv, ...])</code>	Find k eigenvalues and eigenvectors of the real symmetric square matrix or c
<code>expm(A)</code>	Compute the matrix exponential using Pade approximation.
<code>expm_multiply(A, B[, start, stop, num, endpoint])</code>	Compute the action of the matrix exponential of A on B.
<code>factorized(A)</code>	Return a fuction for solving a sparse linear system, with A pre-factorized.
<code>gmres(A, b[, x0, tol, restart, maxiter, ...])</code>	Use Generalized Minimal RESidual iteration to solve $Ax = b$ .
<code>inv(A)</code>	Compute the inverse of a sparse matrix
<code>lgmres(A, b[, x0, tol, maxiter, M, ...])</code>	Solve a matrix equation using the LGMRES algorithm.
<code>lobpcg(A, X[, B, M, Y, tol, maxiter, ...])</code>	Locally Optimal Block Preconditioned Conjugate Gradient Method (LOBPC
<code>lsmr(A, b[, damp, atol, btol, conlim, ...])</code>	Iterative solver for least-squares problems.
<code>lsqr(A, b[, damp, atol, btol, conlim, ...])</code>	Find the least-squares solution to a large, sparse, linear system of equations.
<code>minres(A, b[, x0, shift, tol, maxiter, ...])</code>	Use MINimum RESidual iteration to solve $Ax=b$
<code>norm(x[, ord, axis])</code>	Norm of a sparse matrix
<code>onenormest(A[, t, itmax, compute_v, compute_w])</code>	Compute a lower bound of the 1-norm of a sparse matrix.
<code>qmr(A, b[, x0, tol, maxiter, xtype, M1, M2, ...])</code>	Use Quasi-Minimal Residual iteration to solve $Ax = b$
<code>spilu(A[, drop_tol, fill_factor, drop_rule, ...])</code>	Compute an incomplete LU decomposition for a sparse, square matrix.
<code>splu(A[, permc_spec, diag_pivot_thresh, ...])</code>	Compute the LU decomposition of a sparse, square matrix.
<code>spsolve(A, b[, permc_spec, use_umfpack])</code>	Solve the sparse linear system $Ax=b$ , where b may be a vector or a matrix.
<code>svds(A[, k, ncv, tol, which, v0, maxiter, ...])</code>	Compute the largest k singular values/vectors for a sparse matrix.
<code>use_solver(**kwargs)</code>	Select default sparse direct solver to be used.

**Classes**

<code>LinearOperator(dtype, shape)</code>	Common interface for performing matrix vector products
<code>SuperLU</code>	LU factorization of a sparse matrix.
<code>Tester</code>	alias of NoseTester

**Exceptions**

<code>ArpackError(info[, infodict])</code>	ARPACK error
<code>ArpackNoConvergence(msg, eigenvalues, ...)</code>	ARPACK iteration did not converge
<code>MatrixRankWarning</code>	

**Exceptions**

<code>SparseEfficiencyWarning</code>
<code>SparseWarning</code>

**exception** `scipy.sparse.SparseEfficiencyWarning`

**exception** `scipy.sparse.SparseWarning`

## 5.21.2 Usage information

There are seven available sparse matrix types:

1. `csc_matrix`: Compressed Sparse Column format
2. `csr_matrix`: Compressed Sparse Row format
3. `bsr_matrix`: Block Sparse Row format
4. `lil_matrix`: List of Lists format
5. `dok_matrix`: Dictionary of Keys format
6. `coo_matrix`: COOrdinate format (aka IJV, triplet format)
7. `dia_matrix`: DIAgonal format

To construct a matrix efficiently, use either `dok_matrix` or `lil_matrix`. The `lil_matrix` class supports basic slicing and fancy indexing with a similar syntax to NumPy arrays. As illustrated below, the COO format may also be used to efficiently construct matrices. Despite their similarity to NumPy arrays, it is **strongly discouraged** to use NumPy functions directly on these matrices because NumPy may not properly convert them for computations, leading to unexpected (and incorrect) results. If you do want to apply a NumPy function to these matrices, first check if SciPy has its own implementation for the given sparse matrix class, or **convert the sparse matrix to a NumPy array** (e.g. using the `toarray()` method of the class) first before applying the method.

To perform manipulations such as multiplication or inversion, first convert the matrix to either CSC or CSR format. The `lil_matrix` format is row-based, so conversion to CSR is efficient, whereas conversion to CSC is less so.

All conversions among the CSR, CSC, and COO formats are efficient, linear-time operations.

### Matrix vector product

To do a vector product between a sparse matrix and a vector simply use the matrix `dot` method, as described in its docstring:

```
>>> import numpy as np
>>> from scipy.sparse import csr_matrix
>>> A = csr_matrix([[1, 2, 0], [0, 0, 3], [4, 0, 5]])
>>> v = np.array([1, 0, -1])
>>> A.dot(v)
array([ 1, -3, -1], dtype=int64)
```

**Warning:** As of NumPy 1.7, `np.dot` is not aware of sparse matrices, therefore using it will result on unexpected results or errors. The corresponding dense array should be obtained first instead:

```
>>> np.dot(A.toarray(), v)
array([ 1, -3, -1], dtype=int64)
```

but then all the performance advantages would be lost.

The CSR format is specially suitable for fast matrix vector products.

## Example 1

Construct a 1000x1000 `lil_matrix` and add some values to it:

```
>>> from scipy.sparse import lil_matrix
>>> from scipy.sparse.linalg import spsolve
>>> from numpy.linalg import solve, norm
>>> from numpy.random import rand

>>> A = lil_matrix((1000, 1000))
>>> A[0, :100] = rand(100)
>>> A[1, 100:200] = A[0, :100]
>>> A.setdiag(rand(1000))
```

Now convert it to CSR format and solve  $Ax = b$  for  $x$ :

```
>>> A = A.tocsr()
>>> b = rand(1000)
>>> x = spsolve(A, b)
```

Convert it to a dense matrix and solve, and check that the result is the same:

```
>>> x_ = solve(A.toarray(), b)
```

Now we can compute norm of the error with:

```
>>> err = norm(x-x_)
>>> err < 1e-10
True
```

It should be small :)

## Example 2

Construct a matrix in COO format:

```
>>> from scipy import sparse
>>> from numpy import array
>>> I = array([0, 3, 1, 0])
>>> J = array([0, 3, 1, 2])
>>> V = array([4, 5, 7, 9])
>>> A = sparse.coo_matrix((V, (I, J)), shape=(4, 4))
```

Notice that the indices do not need to be sorted.

Duplicate  $(i,j)$  entries are summed when converting to CSR or CSC.

```
>>> I = array([0, 0, 1, 3, 1, 0, 0])
>>> J = array([0, 2, 1, 3, 1, 0, 0])
>>> V = array([1, 1, 1, 1, 1, 1, 1])
>>> B = sparse.coo_matrix((V, (I, J)), shape=(4, 4)).tocsr()
```

This is useful for constructing finite-element stiffness and mass matrices.

## Further Details

CSR column indices are not necessarily sorted. Likewise for CSC row indices. Use the `.sorted_indices()` and `.sort_indices()` methods when sorted indices are required (e.g. when passing data to other libraries).

## 5.22 Sparse linear algebra (`scipy.sparse.linalg`)

### 5.22.1 Abstract linear operators

<code>LinearOperator(dtype, shape)</code>	Common interface for performing matrix vector products
<code>aslinearoperator(A)</code>	Return A as a LinearOperator.

**class** `scipy.sparse.linalg.LinearOperator` (*dtype, shape*)

Common interface for performing matrix vector products

Many iterative methods (e.g. `cg`, `gmres`) do not need to know the individual entries of a matrix to solve a linear system  $A*x=b$ . Such solvers only require the computation of matrix vector products,  $A*v$  where  $v$  is a dense vector. This class serves as an abstract interface between iterative solvers and matrix-like objects.

To construct a concrete `LinearOperator`, either pass appropriate callables to the constructor of this class, or subclass it.

A subclass must implement either one of the methods `_matvec` and `_matmat`, and the attributes/properties `shape` (pair of integers) and `dtype` (may be `None`). It may call the `__init__` on this class to have these attributes validated. Implementing `_matvec` automatically implements `_matmat` (using a naive algorithm) and vice-versa.

Optionally, a subclass may implement `_rmatvec` or `_adjoint` to implement the Hermitian adjoint (conjugate transpose). As with `_matvec` and `_matmat`, implementing either `_rmatvec` or `_adjoint` implements the other automatically. Implementing `_adjoint` is preferable; `_rmatvec` is mostly there for backwards compatibility.

**Parameters**

- shape** : tuple  
Matrix dimensions (M,N).
- matvec** : callable  $f(v)$   
Returns  $A * v$ .
- rmatvec** : callable  $f(v)$   
Returns  $A^H * v$ , where  $A^H$  is the conjugate transpose of  $A$ .
- matmat** : callable  $f(V)$   
Returns  $A * V$ , where  $V$  is a dense matrix with dimensions (N,K).
- dtype** : dtype  
Data type of the matrix.

**See also:**

[`aslinearoperator`](#)

Construct `LinearOperator`s

**Notes**

The user-defined `matvec()` function must properly handle the case where  $v$  has shape  $(N,)$  as well as the  $(N,1)$  case. The shape of the return type is handled internally by `LinearOperator`.

LinearOperator instances can also be multiplied, added with each other and exponentiated, all lazily: the result of these operations is always a new, composite LinearOperator, that defers linear operations to the original operators and combines the results.

**Examples**

```
>>> import numpy as np
>>> from scipy.sparse.linalg import LinearOperator
>>> def mv(v):
...     return np.array([2*v[0], 3*v[1]])
...
>>> A = LinearOperator((2,2), matvec=mv)
>>> A
<2x2 _CustomLinearOperator with dtype=float64>
>>> A.matvec(np.ones(2))
array([ 2.,  3.])
>>> A * np.ones(2)
array([ 2.,  3.])
```

**Attributes**

args	(tuple) For linear operators describing products etc. of other linear operators, the operands of the binary operation.
------	--

**Methods**

<code>__call__(x)</code>	
<code>adjoint()</code>	Hermitian adjoint.
<code>dot(x)</code>	Matrix-matrix or matrix-vector multiplication.
<code>matmat(X)</code>	Matrix-matrix multiplication.
<code>matvec(x)</code>	Matrix-vector multiplication.
<code>rmatvec(x)</code>	Adjoint matrix-vector multiplication.
<code>transpose()</code>	Transpose this linear operator.

LinearOperator.**\_\_call\_\_**(x)

LinearOperator.**adjoint**()

Hermitian adjoint.

Returns the Hermitian adjoint of self, aka the Hermitian conjugate or Hermitian transpose. For a complex matrix, the Hermitian adjoint is equal to the conjugate transpose.

Can be abbreviated self.H instead of self.adjoint().

**Returns**     **A\_H** : LinearOperator  
Hermitian adjoint of self.

LinearOperator.**dot**(x)

Matrix-matrix or matrix-vector multiplication.

**Parameters**   **x** : array\_like  
1-d or 2-d array, representing a vector or matrix.

**Returns**     **Ax** : array  
1-d or 2-d array (depending on the shape of x) that represents the result of applying this linear operator on x.



`LinearOperator.matmat(X)`

Matrix-matrix multiplication.

Performs the operation  $y=A*X$  where  $A$  is an  $M \times N$  linear operator and  $X$  dense  $N \times K$  matrix or ndarray.

**Parameters**  $X$  : {matrix, ndarray}

**Returns**  $Y$  : {matrix, ndarray} An array with shape  $(N,K)$ .

A matrix or ndarray with shape  $(M,K)$  depending on the type of the  $X$  argument.

#### Notes

This `matmat` wraps any user-specified `matmat` routine or overridden `_matmat` method to ensure that  $y$  has the correct type.

`LinearOperator.matvec(x)`

Matrix-vector multiplication.

Performs the operation  $y=A*x$  where  $A$  is an  $M \times N$  linear operator and  $x$  is a column vector or 1-d array.

**Parameters**  $x$  : {matrix, ndarray}

**Returns**  $y$  : {matrix, ndarray} An array with shape  $(N,)$  or  $(N,1)$ .

A matrix or ndarray with shape  $(M,)$  or  $(M,1)$  depending on the type and shape of the  $x$  argument.

#### Notes

This `matvec` wraps the user-specified `matvec` routine or overridden `_matvec` method to ensure that  $y$  has the correct shape and type.

`LinearOperator.rmatvec(x)`

Adjoint matrix-vector multiplication.

Performs the operation  $y = A^H * x$  where  $A$  is an  $M \times N$  linear operator and  $x$  is a column vector or 1-d array.

**Parameters**  $x$  : {matrix, ndarray}

**Returns**  $y$  : {matrix, ndarray} An array with shape  $(M,)$  or  $(M,1)$ .

A matrix or ndarray with shape  $(N,)$  or  $(N,1)$  depending on the type and shape of the  $x$  argument.

#### Notes

This `rmatvec` wraps the user-specified `rmatvec` routine or overridden `_rmatvec` method to ensure that  $y$  has the correct shape and type.

`LinearOperator.transpose()`

Transpose this linear operator.

Returns a `LinearOperator` that represents the transpose of this one. Can be abbreviated `self.T` instead of `self.transpose()`.

`scipy.sparse.linalg.aslinearoperator(A)`

Return  $A$  as a `LinearOperator`.

*'A' may be any of the following types:*

- ndarray
- matrix
- sparse matrix (e.g. `csr_matrix`, `lil_matrix`, etc.)
- `LinearOperator`

- An object with `.shape` and `.matvec` attributes

See the `LinearOperator` documentation for additional information.

*Examples*

```
>>> from scipy.sparse.linalg import aslinearoperator
>>> M = np.array([[1,2,3],[4,5,6]], dtype=np.int32)
>>> aslinearoperator(M)
<2x3 MatrixLinearOperator with dtype=int32>
```

## 5.22.2 Matrix Operations

---

<code>inv(A)</code>	Compute the inverse of a sparse matrix
<code>expm(A)</code>	Compute the matrix exponential using Pade approximation.
<code>expm_multiply(A, B[, start, stop, num, endpoint])</code>	Compute the action of the matrix exponential of A on B.

---

`scipy.sparse.linalg.inv(A)`  
 Compute the inverse of a sparse matrix

**Parameters** `A` : (M,M) ndarray or sparse matrix  
**Returns** `Ainv` : (M,M) ndarray or sparse matrix  
 square matrix to be inverted  
 inverse of `A`

*Notes*

This computes the sparse inverse of `A`. If the inverse of `A` is expected to be non-sparse, it will likely be faster to convert `A` to dense and use `scipy.linalg.inv`.

New in version 0.12.0.

`scipy.sparse.linalg.expm(A)`  
 Compute the matrix exponential using Pade approximation.

**Parameters** `A` : (M,M) array\_like or sparse matrix  
**Returns** `expA` : (M,M) ndarray  
 2D Array or Matrix (sparse or dense) to be exponentiated  
 Matrix exponential of `A`

*Notes*

This is algorithm (6.1) which is a simplification of algorithm (5.1).

New in version 0.12.0.

*References*

[R289]

`scipy.sparse.linalg.expm_multiply(A, B, start=None, stop=None, num=None, endpoint=None)`  
 Compute the action of the matrix exponential of A on B.

**Parameters** `A` : transposable linear operator  
 The operator whose exponential is of interest.  
`B` : ndarray  
 The matrix or vector to be multiplied by the matrix exponential of `A`.  
`start` : scalar, optional  
 The starting time point of the sequence.

**stop** : scalar, optional  
The end time point of the sequence, unless *endpoint* is set to False. In that case, the sequence consists of all but the last of `num + 1` evenly spaced time points, so that *stop* is excluded. Note that the step size changes when *endpoint* is False.

**num** : int, optional  
Number of time points to use.

**endpoint** : bool, optional  
If True, *stop* is the last time point. Otherwise, it is not included.

**Returns** **expm\_A\_B** : ndarray  
The result of the action  $e^{t_k A} B$ .

### Notes

The optional arguments defining the sequence of evenly spaced time points are compatible with the arguments of `numpy.linspace`.

The output ndarray shape is somewhat complicated so I explain it here. The ndim of the output could be either 1, 2, or 3. It would be 1 if you are computing the expm action on a single vector at a single time point. It would be 2 if you are computing the expm action on a vector at multiple time points, or if you are computing the expm action on a matrix at a single time point. It would be 3 if you want the action on a matrix with multiple columns at multiple time points. If multiple time points are requested, `expm_A_B[0]` will always be the action of the expm at the first time point, regardless of whether the action is on a vector or a matrix.

### References

[R290], [R291]

## 5.22.3 Matrix norms

---

<code>norm(x[, ord, axis])</code>	Norm of a sparse matrix
<code>onenormest(A[, t, itmax, compute_v, compute_w])</code>	Compute a lower bound of the 1-norm of a sparse matrix.

---

`scipy.sparse.linalg.norm(x, ord=None, axis=None)`

Norm of a sparse matrix

This function is able to return one of seven different matrix norms, depending on the value of the `ord` parameter.

**Parameters**

- x** : a sparse matrix  
Input sparse matrix.
- ord** : {non-zero int, inf, -inf, 'fro'}, optional  
Order of the norm (see table under *Notes*). `inf` means numpy's *inf* object.
- axis** : {int, 2-tuple of ints, None}, optional  
If *axis* is an integer, it specifies the axis of *x* along which to compute the vector norms. If *axis* is a 2-tuple, it specifies the axes that hold 2-D matrices, and the matrix norms of these matrices are computed. If *axis* is None then either a vector norm (when *x* is 1-D) or a matrix norm (when *x* is 2-D) is returned.

**Returns** **n** : float or ndarray

### Notes

Some of the `ord` are not implemented because some associated functions like, `_multi_svd_norm`, are not yet available for sparse matrix.

This docstring is modified based on `numpy.linalg.norm`. <https://github.com/numpy/numpy/blob/master/numpy/linalg/linalg.py>

The following norms can be calculated:

ord	norm for sparse matrices
None	Frobenius norm
'fro'	Frobenius norm
inf	max(sum(abs(x), axis=1))
-inf	min(sum(abs(x), axis=1))
0	abs(x).sum(axis=axis)
1	max(sum(abs(x), axis=0))
-1	min(sum(abs(x), axis=0))
2	Not implemented
-2	Not implemented
other	Not implemented

The Frobenius norm is given by [R302]:

$$\|A\|_F = [\sum_{i,j} \text{abs}(a_{i,j})^2]^{1/2}$$

### References

[R302]

### Examples

```
>>> from scipy.sparse import *
>>> import numpy as np
>>> from scipy.sparse.linalg import norm
>>> a = np.arange(9) - 4
>>> a
array([-4, -3, -2, -1, 0, 1, 2, 3, 4])
>>> b = a.reshape((3, 3))
>>> b
array([[ -4,  -3,  -2],
       [ -1,   0,   1],
       [  2,   3,   4]])

>>> b = csr_matrix(b)
>>> norm(b)
7.745966692414834
>>> norm(b, 'fro')
7.745966692414834
>>> norm(b, np.inf)
9
>>> norm(b, -np.inf)
2
>>> norm(b, 1)
7
>>> norm(b, -1)
6
```

`scipy.sparse.linalg.onenormest` (*A*, *t*=2, *itmax*=5, *compute\_v*=False, *compute\_w*=False)

Compute a lower bound of the 1-norm of a sparse matrix.

**Parameters**

- A** : ndarray or other linear operator  
A linear operator that can be transposed and that can produce matrix products.
- t** : int, optional

A positive parameter controlling the tradeoff between accuracy versus time and memory usage. Larger values take longer and use more memory but give more accurate output.

**itmax** : int, optional

Use at most this many iterations.

**compute\_v** : bool, optional

Request a norm-maximizing linear operator input vector if True.

**compute\_w** : bool, optional

Request a norm-maximizing linear operator output vector if True.

#### Returns

**est** : float

An underestimate of the 1-norm of the sparse matrix.

**v** : ndarray, optional

The vector such that  $\|Av\|_1 == est * \|v\|_1$ . It can be thought of as an input to the linear operator that gives an output with particularly large norm.

**w** : ndarray, optional

The vector  $Av$  which has relatively large 1-norm. It can be thought of as an output of the linear operator that is relatively large in norm compared to the input.

#### Notes

This is algorithm 2.4 of [1].

In [2] it is described as follows. “This algorithm typically requires the evaluation of about  $4t$  matrix-vector products and almost invariably produces a norm estimate (which is, in fact, a lower bound on the norm) correct to within a factor 3.”

New in version 0.13.0.

#### References

[R303], [R304]

## 5.22.4 Solving linear problems

Direct methods for linear equation systems:

<code>spsolve(A, b[, permc_spec, use_umfpack])</code>	Solve the sparse linear system $Ax=b$ , where $b$ may be a vector or a matrix.
<code>factorized(A)</code>	Return a function for solving a sparse linear system, with $A$ pre-factorized.
<code>MatrixRankWarning</code>	
<code>use_solver(**kwargs)</code>	Select default sparse direct solver to be used.

`scipy.sparse.linalg.spsolve(A, b, permc_spec=None, use_umfpack=True)`

Solve the sparse linear system  $Ax=b$ , where  $b$  may be a vector or a matrix.

**Parameters** **A** : ndarray or sparse matrix

The square matrix  $A$  will be converted into CSC or CSR form

**b** : ndarray or sparse matrix

The matrix or vector representing the right hand side of the equation. If a vector,  $b.shape$  must be  $(n,)$  or  $(n, 1)$ .

**permc\_spec** : str, optional

How to permute the columns of the matrix for sparsity preservation. (default: ‘COLAMD’)

- **NATURAL**: natural ordering.
- **MMD\_ATA**: minimum degree ordering on the structure of  $A^T A$ .

- MMD\_AT\_PLUS\_A: minimum degree ordering on the structure of  $A^T+A$ .
- COLAMD: approximate minimum degree column ordering

**use\_umfpack** : bool, optional  
 if True (default) then use umfpack for the solution. This is only referenced if b is a vector and scikit-umfpack is installed.

**Returns** **x** : ndarray or sparse matrix  
 the solution of the sparse linear equation. If b is a vector, then x is a vector of size A.shape[1] If b is a matrix, then x is a matrix of size (A.shape[1], b.shape[1])

**Notes**

For solving the matrix expression  $AX = B$ , this solver assumes the resulting matrix X is sparse, as is often the case for very sparse inputs. If the resulting X is dense, the construction of this sparse result will be relatively expensive. In that case, consider converting A to a dense matrix and using `scipy.linalg.solve` or its variants.

`scipy.sparse.linalg.factorized(A)`

Return a fuction for solving a sparse linear system, with A pre-factorized.

**Parameters** **A** : (N, N) array\_like

**Returns** **solve** : callable  
 Input.

To solve the linear system of equations given in A, the *solve* callable should be passed an ndarray of shape (N,).

**Examples**

```
>>> from scipy.sparse.linalg import factorized
>>> A = np.array([[ 3. ,  2. , -1. ],
...              [ 2. , -2. ,  4. ],
...              [-1. ,  0.5, -1. ]])
>>> solve = factorized(A) # Makes LU decomposition.
>>> rhs1 = np.array([1, -2, 0])
>>> solve(rhs1) # Uses the LU factors.
array([ 1., -2., -2.]
```

**exception** `scipy.sparse.linalg.MatrixRankWarning`

`scipy.sparse.linalg.use_solver(**kwargs)`

Select default sparse direct solver to be used.

**Parameters** **useUmfpack** : bool, optional

Use UMFPACK over SuperLU. Has effect only if scikits.umfpack is installed. Default: True

**Notes**

The default sparse solver is umfpack when available (scikits.umfpack is installed). This can be changed by passing `useUmfpack = False`, which then causes the always present SuperLU based solver to be used.

Umfpack requires a CSR/CSC matrix to have sorted column/row indices. If sure that the matrix fulfills this, pass `assumeSortedIndices=True` to gain some speed.

Iterative methods for linear equation systems:

<code>bicg(A, b[, x0, tol, maxiter, xtype, M, ...])</code>	Use BIConjugate Gradient iteration to solve $Ax = b$
<code>bicgstab(A, b[, x0, tol, maxiter, xtype, M, ...])</code>	Use BIConjugate Gradient STABILized iteration to solve $Ax = b$
<code>cg(A, b[, x0, tol, maxiter, xtype, M, callback])</code>	Use Conjugate Gradient iteration to solve $Ax = b$

Continued on next page

Table 5.201 – continued from previous page

<code>cgs(A, b[, x0, tol, maxiter, xtype, M, callback])</code>	Use Conjugate Gradient Squared iteration to solve $A x = b$
<code>gmres(A, b[, x0, tol, restart, maxiter, ...])</code>	Use Generalized Minimal RESidual iteration to solve $A x = b$ .
<code>lgmres(A, b[, x0, tol, maxiter, M, ...])</code>	Solve a matrix equation using the LGMRES algorithm.
<code>minres(A, b[, x0, shift, tol, maxiter, ...])</code>	Use MINimum RESidual iteration to solve $Ax=b$
<code>qmr(A, b[, x0, tol, maxiter, xtype, M1, M2, ...])</code>	Use Quasi-Minimal Residual iteration to solve $A x = b$

`scipy.sparse.linalg.bicg(A, b, x0=None, tol=1e-05, maxiter=None, xtype=None, M=None, callback=None)`

Use BiConjugate Gradient iteration to solve  $A x = b$

**Parameters** **A** : {sparse matrix, dense matrix, LinearOperator}  
The real or complex N-by-N matrix of the linear system It is required that the linear operator can produce  $Ax$  and  $A^T x$ .

**b** : {array, matrix}

**Returns** **x** : {array, matrix} Right hand side of the linear system. Has shape (N,) or (N,1).

The converged solution.

**info** : integer

**Provides convergence information:**

0 : successful exit >0 : convergence to tolerance not achieved, number of iterations <0 : illegal input or breakdown

**Other Parameters**

**x0** : {array, matrix}

Starting guess for the solution.

**tol** : float

Tolerance to achieve. The algorithm terminates when either the relative or the absolute residual is below *tol*.

**maxiter** : integer

Maximum number of iterations. Iteration will stop after maxiter steps even if the specified tolerance has not been achieved.

**M** : {sparse matrix, dense matrix, LinearOperator}

Preconditioner for A. The preconditioner should approximate the inverse of A. Effective preconditioning dramatically improves the rate of convergence, which implies that fewer iterations are needed to reach a given error tolerance.

**callback** : function

User-supplied function to call after each iteration. It is called as `callback(xk)`, where `xk` is the current solution vector.

**xtype** : {'f','d','F','D'}

This parameter is deprecated – avoid using it.

The type of the result. If None, then it will be determined from `A.dtype.char` and `b`. If A does not have a `typecode` method then it will compute `A.matvec(x0)` to get a `typecode`. To save the extra computation when A does not have a `typecode` attribute use `xtype=0` for the same type as `b` or use `xtype='f','d','F',` or `'D'`. This parameter has been superseded by `LinearOperator`.

`scipy.sparse.linalg.bicgstab(A, b, x0=None, tol=1e-05, maxiter=None, xtype=None, M=None, callback=None)`

Use BiConjugate Gradient STABILized iteration to solve  $A x = b$

**Parameters** **A** : {sparse matrix, dense matrix, LinearOperator}

The real or complex N-by-N matrix of the linear system A must represent a hermitian, positive definite matrix

**Returns**

- b** : {array, matrix} Right hand side of the linear system. Has shape (N,) or (N,1).
- x** : {array, matrix} The converged solution.
- info** : integer *Provides convergence information:*
  - 0 : successful exit >0 : convergence to tolerance not achieved, number of iterations <0 : illegal input or breakdown

**Other Parameters**

- x0** : {array, matrix} Starting guess for the solution.
- tol** : float Tolerance to achieve. The algorithm terminates when either the relative or the absolute residual is below *tol*.
- maxiter** : integer Maximum number of iterations. Iteration will stop after maxiter steps even if the specified tolerance has not been achieved.
- M** : {sparse matrix, dense matrix, LinearOperator} Preconditioner for A. The preconditioner should approximate the inverse of A. Effective preconditioning dramatically improves the rate of convergence, which implies that fewer iterations are needed to reach a given error tolerance.
- callback** : function User-supplied function to call after each iteration. It is called as `callback(xk)`, where `xk` is the current solution vector.
- xtype** : {'f','d','F','D'} This parameter is deprecated – avoid using it. The type of the result. If None, then it will be determined from `A.dtype.char` and `b`. If A does not have a `typecode` method then it will compute `A.matvec(x0)` to get a typecode. To save the extra computation when A does not have a `typecode` attribute use `xtype=0` for the same type as `b` or use `xtype='f','d','F',` or `'D'`. This parameter has been superseded by `LinearOperator`.

`scipy.sparse.linalg.cg(A, b, x0=None, tol=1e-05, maxiter=None, xtype=None, M=None, callback=None)`

Use Conjugate Gradient iteration to solve  $Ax = b$

**Parameters**

- A** : {sparse matrix, dense matrix, LinearOperator} The real or complex N-by-N matrix of the linear system A must represent a hermitian, positive definite matrix
- b** : {array, matrix}

**Returns**

- x** : {array, matrix} Right hand side of the linear system. Has shape (N,) or (N,1).
- info** : integer The converged solution.
- info** : integer *Provides convergence information:*
  - 0 : successful exit >0 : convergence to tolerance not achieved, number of iterations <0 : illegal input or breakdown

**Other Parameters**

- x0** : {array, matrix} Starting guess for the solution.
- tol** : float Tolerance to achieve. The algorithm terminates when either the relative or the absolute residual is below *tol*.



**maxiter** : integer

Maximum number of iterations. Iteration will stop after maxiter steps even if the specified tolerance has not been achieved.

**M** : {sparse matrix, dense matrix, LinearOperator}

Preconditioner for A. The preconditioner should approximate the inverse of A. Effective preconditioning dramatically improves the rate of convergence, which implies that fewer iterations are needed to reach a given error tolerance.

**callback** : function

User-supplied function to call after each iteration. It is called as callback(xk), where xk is the current solution vector.

**xtype** : {'f','d','F','D'}

This parameter is deprecated – avoid using it.

The type of the result. If None, then it will be determined from A.dtype.char and b. If A does not have a typecode method then it will compute A.matvec(x0) to get a typecode. To save the extra computation when A does not have a typecode attribute use xtype=0 for the same type as b or use xtype='f','d','F',or 'D'. This parameter has been superseded by LinearOperator.

`scipy.sparse.linalg.cg(A, b, x0=None, tol=1e-05, maxiter=None, xtype=None, M=None, callback=None)`

Use Conjugate Gradient Squared iteration to solve  $Ax = b$

**Parameters** **A** : {sparse matrix, dense matrix, LinearOperator}

The real-valued N-by-N matrix of the linear system

**b** : {array, matrix}

Right hand side of the linear system. Has shape (N,) or (N,1).

**Returns**

**x** : {array, matrix}

The converged solution.

**info** : integer

**Provides convergence information:**

0 : successful exit >0 : convergence to tolerance not achieved, number of iterations <0 : illegal input or breakdown

**Other Parameters**

**x0** : {array, matrix}

Starting guess for the solution.

**tol** : float

Tolerance to achieve. The algorithm terminates when either the relative or the absolute residual is below *tol*.

**maxiter** : integer

Maximum number of iterations. Iteration will stop after maxiter steps even if the specified tolerance has not been achieved.

**M** : {sparse matrix, dense matrix, LinearOperator}

Preconditioner for A. The preconditioner should approximate the inverse of A. Effective preconditioning dramatically improves the rate of convergence, which implies that fewer iterations are needed to reach a given error tolerance.

**callback** : function

User-supplied function to call after each iteration. It is called as callback(xk), where xk is the current solution vector.

**xtype** : {'f','d','F','D'}

This parameter is deprecated – avoid using it.

The type of the result. If None, then it will be determined from A.dtype.char and b. If A does not have a typecode method then it will compute

`A.matvec(x0)` to get a typecode. To save the extra computation when `A` does not have a typecode attribute use `xtype=0` for the same type as `b` or use `xtype='f','d','F',` or `'D'`. This parameter has been superseded by `LinearOperator`.

`scipy.sparse.linalg.gmres` (*A*, *b*, *x0=None*, *tol=1e-05*, *restart=None*, *maxiter=None*, *xtype=None*, *M=None*, *callback=None*, *restrt=None*)  
 Use Generalized Minimal RESidual iteration to solve  $Ax = b$ .

**Parameters** **A** : {sparse matrix, dense matrix, LinearOperator}  
 The real or complex N-by-N matrix of the linear system.

**b** : {array, matrix}

**Returns** **x** : {array, matrix} Right hand side of the linear system. Has shape (N,) or (N,1).

The converged solution.

**info** : int

*Provides convergence information:*

- 0 : successful exit
- >0 : convergence to tolerance not achieved, number of iterations
- <0 : illegal input or breakdown

**Other Parameters**

**x0** : {array, matrix}

Starting guess for the solution (a vector of zeros by default).

**tol** : float

Tolerance to achieve. The algorithm terminates when either the relative or the absolute residual is below *tol*.

**restart** : int, optional

Number of iterations between restarts. Larger values increase iteration cost, but may be necessary for convergence. Default is 20.

**maxiter** : int, optional

Maximum number of iterations (restart cycles). Iteration will stop after *maxiter* steps even if the specified tolerance has not been achieved.

**xtype** : {'f','d','F','D'}

This parameter is DEPRECATED — avoid using it.

The type of the result. If `None`, then it will be determined from `A.dtype.char` and `b`. If `A` does not have a typecode method then it will compute `A.matvec(x0)` to get a typecode. To save the extra computation when `A` does not have a typecode attribute use `xtype=0` for the same type as `b` or use `xtype='f','d','F',` or `'D'`. This parameter has been superseded by `LinearOperator`.

**M** : {sparse matrix, dense matrix, LinearOperator}

Inverse of the preconditioner of `A`. `M` should approximate the inverse of `A` and be easy to solve for (see Notes). Effective preconditioning dramatically improves the rate of convergence, which implies that fewer iterations are needed to reach a given error tolerance. By default, no preconditioner is used.

**callback** : function

User-supplied function to call after each iteration. It is called as `callback(rk)`, where `rk` is the current residual vector.

**restrt** : int, optional

DEPRECATED - use *restart* instead.

See also:

[LinearOperator](#)

**Notes**

A preconditioner,  $P$ , is chosen such that  $P$  is close to  $A$  but easy to solve for. The preconditioner parameter required by this routine is  $M = P^{-1}$ . The inverse should preferably not be calculated explicitly. Rather, use the following template to produce  $M$ :

```
# Construct a linear operator that computes  $P^{-1} * x$ .
import scipy.sparse.linalg as spla
M_x = lambda x: spla.spsolve(P, x)
M = spla.LinearOperator((n, n), M_x)
```

```
scipy.sparse.linalg.lgmres(A, b, x0=None, tol=1e-05, maxiter=1000, M=None, callback=None,
                           inner_m=30, outer_k=3, outer_v=None, store_outer_Av=True)
```

Solve a matrix equation using the LGMRES algorithm.

The LGMRES algorithm [R292] [R293] is designed to avoid some problems in the convergence in restarted GMRES, and often converges in fewer iterations.

**Parameters**

- A** : {sparse matrix, dense matrix, LinearOperator}  
The real or complex N-by-N matrix of the linear system.
- b** : {array, matrix}  
Right hand side of the linear system. Has shape (N,) or (N,1).
- x0** : {array, matrix}  
Starting guess for the solution.
- tol** : float, optional  
Tolerance to achieve. The algorithm terminates when either the relative or the absolute residual is below *tol*.
- maxiter** : int, optional  
Maximum number of iterations. Iteration will stop after maxiter steps even if the specified tolerance has not been achieved.
- M** : {sparse matrix, dense matrix, LinearOperator}, optional  
Preconditioner for A. The preconditioner should approximate the inverse of A. Effective preconditioning dramatically improves the rate of convergence, which implies that fewer iterations are needed to reach a given error tolerance.
- callback** : function, optional  
User-supplied function to call after each iteration. It is called as `callback(xk)`, where `xk` is the current solution vector.
- inner\_m** : int, optional  
Number of inner GMRES iterations per each outer iteration.
- outer\_k** : int, optional  
Number of vectors to carry between inner GMRES iterations. According to [R292], good values are in the range of 1...3. However, note that if you want to use the additional vectors to accelerate solving multiple similar problems, larger values may be beneficial.
- outer\_v** : list of tuples, optional  
List containing tuples  $(v, Av)$  of vectors and corresponding matrix-vector products, used to augment the Krylov subspace, and carried between inner GMRES iterations. The element  $Av$  can be *None* if the matrix-vector product should be re-evaluated. This parameter is modified in-place by `lgmres`, and can be used to pass “guess” vectors in and out of the algorithm when solving similar problems.
- store\_outer\_Av** : bool, optional  
Whether LGMRES should store also  $A*v$  in addition to vectors  $v$  in the *outer\_v* list. Default is True.

**Returns**

- x** : array or matrix

**info** : int

The converged solution.

Provides convergence information:

- 0 : successful exit
- >0 : convergence to tolerance not achieved, number of iterations
- <0 : illegal input or breakdown

**Notes**

The LGMRES algorithm [R292] [R293] is designed to avoid the slowing of convergence in restarted GMRES, due to alternating residual vectors. Typically, it often outperforms GMRES(m) of comparable memory requirements by some measure, or at least is not much worse.

Another advantage in this algorithm is that you can supply it with ‘guess’ vectors in the *outer\_v* argument that augment the Krylov subspace. If the solution lies close to the span of these vectors, the algorithm converges faster. This can be useful if several very similar matrices need to be inverted one after another, such as in Newton-Krylov iteration where the Jacobian matrix often changes little in the nonlinear steps.

**References**

[R292], [R293]

`scipy.sparse.linalg.minres` (*A*, *b*, *x0=None*, *shift=0.0*, *tol=1e-05*, *maxiter=None*, *xtype=None*, *M=None*, *callback=None*, *show=False*, *check=False*)

Use MINimum RESidual iteration to solve  $Ax=b$

MINRES minimizes  $\text{norm}(A*x - b)$  for a real symmetric matrix *A*. Unlike the Conjugate Gradient method, *A* can be indefinite or singular.

If *shift* != 0 then the method solves  $(A - \text{shift}*I)x = b$

**Parameters** **A** : {sparse matrix, dense matrix, LinearOperator}  
 The real symmetric N-by-N matrix of the linear system

**b** : {array, matrix}  
 Right hand side of the linear system. Has shape (N,) or (N,1).

**Returns** **x** : {array, matrix}  
 The converged solution.

**info** : integer

**Provides convergence information:**

- 0 : successful exit
- >0 : convergence to tolerance not achieved, number of iterations
- <0 : illegal input or breakdown

**Other Parameters**

**x0** : {array, matrix}  
 Starting guess for the solution.

**tol** : float  
 Tolerance to achieve. The algorithm terminates when either the relative or the absolute residual is below *tol*.

**maxiter** : integer  
 Maximum number of iterations. Iteration will stop after *maxiter* steps even if the specified tolerance has not been achieved.

**M** : {sparse matrix, dense matrix, LinearOperator}  
 Preconditioner for *A*. The preconditioner should approximate the inverse of *A*. Effective preconditioning dramatically improves the rate of convergence, which implies that fewer iterations are needed to reach a given error tolerance.

**callback** : function  
 User-supplied function to call after each iteration. It is called as `callback(xk)`, where *xk* is the current solution vector.

**xtype** : {'f','d','F','D'}

This parameter is deprecated – avoid using it.

The type of the result. If None, then it will be determined from `A.dtype.char` and `b`. If `A` does not have a `typecode` method then it will compute `A.matvec(x0)` to get a `typecode`. To save the extra computation when `A` does not have a `typecode` attribute use `xtype=0` for the same type as `b` or use `xtype='f','d','F',or 'D'`. This parameter has been superseded by `LinearOperator`.

#### Notes

THIS FUNCTION IS EXPERIMENTAL AND SUBJECT TO CHANGE!

#### References

##### *Solution of sparse indefinite systems of linear equations,*

C. C. Paige and M. A. Saunders (1975), SIAM J. Numer. Anal. 12(4), pp. 617-629.  
<http://www.stanford.edu/group/SOL/software/minres.html>

##### *This file is a translation of the following MATLAB implementation:*

<http://www.stanford.edu/group/SOL/software/minres/matlab/>

`scipy.sparse.linalg.qmr(A, b, x0=None, tol=1e-05, maxiter=None, xtype=None, M1=None, M2=None, callback=None)`

Use Quasi-Minimal Residual iteration to solve  $Ax = b$

**Parameters** **A** : {sparse matrix, dense matrix, LinearOperator}

The real-valued N-by-N matrix of the linear system. It is required that the linear operator can produce  $Ax$  and  $A^T x$ .

**b** : {array, matrix}

**Returns** **x** : {array, matrix} Right hand side of the linear system. Has shape (N,) or (N,1).

The converged solution.

**info** : integer

##### *Provides convergence information:*

0 : successful exit >0 : convergence to tolerance not achieved, number of iterations <0 : illegal input or breakdown

#### *Other Parameters*

**x0** : {array, matrix}

Starting guess for the solution.

**tol** : float

Tolerance to achieve. The algorithm terminates when either the relative or the absolute residual is below `tol`.

**maxiter** : integer

Maximum number of iterations. Iteration will stop after `maxiter` steps even if the specified tolerance has not been achieved.

**M1** : {sparse matrix, dense matrix, LinearOperator}

Left preconditioner for `A`.

**M2** : {sparse matrix, dense matrix, LinearOperator}

Right preconditioner for `A`. Used together with the left preconditioner `M1`. The matrix `M1*A*M2` should have better conditioned than `A` alone.

**callback** : function

User-supplied function to call after each iteration. It is called as `callback(xk)`, where `xk` is the current solution vector.

**xtype** : {'f','d','F','D'}

This parameter is DEPRECATED – avoid using it.

The type of the result. If None, then it will be determined from `A.dtype.char` and `b`. If `A` does not have a `typecode` method then it will compute `A.matvec(x0)` to get a `typecode`. To save the extra computation when `A` does not have a `typecode` attribute use `xtype=0` for the same type as `b` or use `xtype='f','d','F',` or `'D'`. This parameter has been superseded by `LinearOperator`.

**See also:**

`LinearOperator`

Iterative methods for least-squares problems:

---

<code>lsqr(A, b[, damp, atol, btol, conlim, ...])</code>	Find the least-squares solution to a large, sparse, linear system of equations.
<code>lsmr(A, b[, damp, atol, btol, conlim, ...])</code>	Iterative solver for least-squares problems.

---

`scipy.sparse.linalg.lsqr(A, b, damp=0.0, atol=1e-08, btol=1e-08, conlim=100000000.0, iter_lim=None, show=False, calc_var=False)`

Find the least-squares solution to a large, sparse, linear system of equations.

The function solves  $Ax = b$  or  $\min ||b - Ax||^2$  or  $\min ||Ax - b||^2 + d^2 ||x||^2$ .

The matrix `A` may be square or rectangular (over-determined or under-determined), and may have any rank.

1. Unsymmetric equations -- solve  $Ax = b$
2. Linear least squares -- solve  $Ax = b$   
in the least-squares sense
3. Damped least squares -- solve  $\begin{pmatrix} A \\ \text{damp} \times I \end{pmatrix} x = \begin{pmatrix} b \\ 0 \end{pmatrix}$   
in the least-squares sense

**Parameters** `A` : {sparse matrix, ndarray, LinearOperator}

Representation of an m-by-n matrix. It is required that the linear operator can produce  $Ax$  and  $A^T x$ .

`b` : array\_like, shape (m,)
   
Right-hand side vector `b`.

`damp` : float
   
Damping coefficient.

`atol, btol` : float, optional
   
Stopping tolerances. If both are 1.0e-9 (say), the final residual norm should be accurate to about 9 digits. (The final `x` will usually have fewer correct digits, depending on `cond(A)` and the size of `damp`.)

`conlim` : float, optional
   
Another stopping tolerance. `lsqr` terminates if an estimate of `cond(A)` exceeds `conlim`. For compatible systems  $Ax = b$ , `conlim` could be as large as 1.0e+12 (say). For least-squares problems, `conlim` should be less than 1.0e+8. Maximum precision can be obtained by setting `atol = btol = conlim = zero`, but the number of iterations may then be excessive.

`iter_lim` : int, optional
   
Explicit limitation on number of iterations (for safety).

`show` : bool, optional
   
Display an iteration log.

`calc_var` : bool, optional
   
Whether to estimate diagonals of  $(A^T A + \text{damp}^2 \times I)^{-1}$ .

**Returns**

- x** : ndarray of float  
The final solution.
- istop** : int  
Gives the reason for termination. 1 means x is an approximate solution to  $Ax = b$ . 2 means x approximately solves the least-squares problem.
- itn** : int  
Iteration number upon termination.
- r1norm** : float  
 $\text{norm}(r)$ , where  $r = b - Ax$ .
- r2norm** : float  
 $\text{sqrt}(\text{norm}(r)^2 + \text{damp}^2 * \text{norm}(x)^2)$ . Equal to *r1norm* if  $\text{damp} == 0$ .
- anorm** : float  
Estimate of Frobenius norm of  $A_{\text{bar}} = [[A]; [\text{damp}*I]]$ .
- acond** : float  
Estimate of  $\text{cond}(A_{\text{bar}})$ .
- arnorm** : float  
Estimate of  $\text{norm}(A' * r - \text{damp}^2 * x)$ .
- xnorm** : float  
 $\text{norm}(x)$
- var** : ndarray of float  
If `calc_var` is True, estimates all diagonals of  $(A' A)^{-1}$  (if  $\text{damp} == 0$ ) or more generally  $(A' A + \text{damp}^2 * I)^{-1}$ . This is well defined if A has full column rank or  $\text{damp} > 0$ . (Not sure what var means if  $\text{rank}(A) < n$  and  $\text{damp} = 0$ .)

### Notes

LSQR uses an iterative method to approximate the solution. The number of iterations required to reach a certain accuracy depends strongly on the scaling of the problem. Poor scaling of the rows or columns of A should therefore be avoided where possible.

For example, in problem 1 the solution is unaltered by row-scaling. If a row of A is very small or large compared to the other rows of A, the corresponding row of  $(A b)$  should be scaled up or down.

In problems 1 and 2, the solution x is easily recovered following column-scaling. Unless better information is known, the nonzero columns of A should be scaled so that they all have the same Euclidean norm (e.g., 1.0).

In problem 3, there is no freedom to re-scale if damp is nonzero. However, the value of damp should be assigned only after attention has been paid to the scaling of A.

The parameter damp is intended to help regularize ill-conditioned systems, by preventing the true solution from being very large. Another aid to regularization is provided by the parameter acond, which may be used to terminate iterations before the computed solution becomes very large.

If some initial estimate  $x_0$  is known and if  $\text{damp} == 0$ , one could proceed as follows:

1. Compute a residual vector  $r_0 = b - A * x_0$ .
2. Use LSQR to solve the system  $A * dx = r_0$ .
3. Add the correction dx to obtain a final solution  $x = x_0 + dx$ .

This requires that  $x_0$  be available before and after the call to LSQR. To judge the benefits, suppose LSQR takes  $k_1$  iterations to solve  $A * x = b$  and  $k_2$  iterations to solve  $A * dx = r_0$ . If  $x_0$  is “good”,  $\text{norm}(r_0)$  will be smaller than  $\text{norm}(b)$ . If the same stopping tolerances atol and btol are used for each system,  $k_1$  and  $k_2$  will be similar, but the final solution  $x_0 + dx$  should be more accurate. The only way to reduce the total work is to use a larger stopping tolerance for the second system. If some value btol is suitable for  $A * x = b$ , the larger value  $\text{btol} * \text{norm}(b) / \text{norm}(r_0)$  should be suitable for  $A * dx = r_0$ .

Preconditioning is another way to reduce the number of iterations. If it is possible to solve a related system  $M*x = b$  efficiently, where  $M$  approximates  $A$  in some helpful way (e.g.  $M - A$  has low rank or its elements are small relative to those of  $A$ ), LSQR may converge more rapidly on the system  $A*M(\text{inverse})*z = b$ , after which  $x$  can be recovered by solving  $M*x = z$ .

If  $A$  is symmetric, LSQR should not be used!

Alternatives are the symmetric conjugate-gradient method (`cg`) and/or SYMMLQ. SYMMLQ is an implementation of symmetric `cg` that applies to any symmetric  $A$  and will converge more rapidly than LSQR. If  $A$  is positive definite, there are other implementations of symmetric `cg` that require slightly less work per iteration than SYMMLQ (but will take the same number of iterations).

### References

[R299], [R300], [R301]

`scipy.sparse.linalg.lsmr`(*A*, *b*, *damp*=0.0, *atol*=1e-06, *btol*=1e-06, *conlim*=100000000.0, *max-iter*=None, *show*=False)

Iterative solver for least-squares problems.

`lsmr` solves the system of linear equations  $Ax = b$ . If the system is inconsistent, it solves the least-squares problem  $\min ||b - Ax||_2$ .  $A$  is a rectangular matrix of dimension  $m$ -by- $n$ , where all cases are allowed:  $m = n$ ,  $m > n$ , or  $m < n$ .  $B$  is a vector of length  $m$ . The matrix  $A$  may be dense or sparse (usually sparse).

**Parameters** **A** : {matrix, sparse matrix, ndarray, LinearOperator}  
Matrix  $A$  in the linear system.

**b** : array\_like, shape (m,)  
Vector  $b$  in the linear system.

**damp** : float  
Damping factor for regularized least-squares. `lsmr` solves the regularized least-squares problem:

$$\min ||(b - (A) x)||_2 + \text{damp} ||(0) (damp * I) x||_2$$

where `damp` is a scalar. If `damp` is `None` or `0`, the system is solved without regularization.

**atol, btol** : float, optional  
Stopping tolerances. `lsmr` continues iterations until a certain backward error estimate is smaller than some quantity depending on `atol` and `btol`. Let  $r = b - Ax$  be the residual vector for the current approximate solution  $x$ . If  $Ax = b$  seems to be consistent, `lsmr` terminates when  $\text{norm}(r) \leq \text{atol} * \text{norm}(A) * \text{norm}(x) + \text{btol} * \text{norm}(b)$ . Otherwise, `lsmr` terminates when  $\text{norm}(A^T r) \leq \text{atol} * \text{norm}(A) * \text{norm}(r)$ . If both tolerances are  $1.0e-6$  (say), the final  $\text{norm}(r)$  should be accurate to about 6 digits. (The final  $x$  will usually have fewer correct digits, depending on  $\text{cond}(A)$  and the size of `LAMBDA`.) If `atol` or `btol` is `None`, a default value of  $1.0e-6$  will be used. Ideally, they should be estimates of the relative error in the entries of  $A$  and  $B$  respectively. For example, if the entries of  $A$  have 7 correct digits, set `atol` =  $1e-7$ . This prevents the algorithm from doing unnecessary work beyond the uncertainty of the input data.

**conlim** : float, optional  
`lsmr` terminates if an estimate of  $\text{cond}(A)$  exceeds `conlim`. For compatible systems  $Ax = b$ , `conlim` could be as large as  $1.0e+12$  (say). For least-squares problems, `conlim` should be less than  $1.0e+8$ . If `conlim` is `None`, the default value is  $1e+8$ . Maximum precision can be obtained by setting



`atol = btol = conlim = 0`, but the number of iterations may then be excessive.

**maxiter** : int, optional  
`lsqr` terminates if the number of iterations reaches *maxiter*. The default is `maxiter = min(m, n)`. For ill-conditioned systems, a larger value of *maxiter* may be needed.

**show** : bool, optional  
 Print iterations logs if `show=True`.

**Returns**  
**x** : ndarray of float  
 Least-square solution returned.

**istop** : int  
 istop gives the reason for stopping:

istop = 0 means `x=0` is a solution.  
 = 1 means `x` is an approximate solution to `A*x = B`, according to `atol` and `btol`.  
 = 2 means `x` approximately solves the least-squares problem according to `atol`.  
 = 3 means `COND(A)` seems to be greater than `CONLIM`.  
 = 4 is the same as 1 with `atol = btol = eps` (machine precision)  
 = 5 is the same as 2 with `atol = eps`.  
 = 6 is the same as 3 with `CONLIM = 1/eps`.  
 = 7 means ITN reached `maxiter` before the other stopping conditions were satisfied.

**itn** : int  
 Number of iterations used.

**normr** : float  
`norm(b-Ax)`

**normar** : float  
`norm(A^T (b - Ax))`

**norma** : float  
`norm(A)`

**conda** : float  
 Condition number of A.

**normx** : float  
`norm(x)`

**Notes**

New in version 0.11.0.

**References**

[R297], [R298]

### 5.22.5 Matrix factorizations

Eigenvalue problems:

<code>eigs(A[, k, M, sigma, which, v0, ncv, ...])</code>	Find <code>k</code> eigenvalues and eigenvectors of the square matrix <code>A</code> .
<code>eigsh(A[, k, M, sigma, which, v0, ncv, ...])</code>	Find <code>k</code> eigenvalues and eigenvectors of the real symmetric square matrix or complex hermitian square matrix <code>A</code> .
<code>lobpcg(A, X[, B, M, Y, tol, maxiter, ...])</code>	Locally Optimal Block Preconditioned Conjugate Gradient Method (LOBPCG)

`scipy.sparse.linalg.eigs` (*A*, *k=6*, *M=None*, *sigma=None*, *which='LM'*, *v0=None*, *ncv=None*, *max-iter=None*, *tol=0*, *return\_eigenvectors=True*, *Minv=None*, *OPinv=None*, *OPpart=None*)

Find *k* eigenvalues and eigenvectors of the square matrix *A*.

Solves  $A * x[i] = w[i] * x[i]$ , the standard eigenvalue problem for *w*[*i*] eigenvalues with corresponding eigenvectors *x*[*i*].

If *M* is specified, solves  $A * x[i] = w[i] * M * x[i]$ , the generalized eigenvalue problem for *w*[*i*] eigenvalues with corresponding eigenvectors *x*[*i*]

**Parameters** *A* : ndarray, sparse matrix or LinearOperator

An array, sparse matrix, or LinearOperator representing the operation  $A * x$ , where *A* is a real or complex square matrix.

*k* : int, optional

The number of eigenvalues and eigenvectors desired. *k* must be smaller than *N*. It is not possible to compute all eigenvectors of a matrix.

*M* : ndarray, sparse matrix or LinearOperator, optional

An array, sparse matrix, or LinearOperator representing the operation  $M*x$  for the generalized eigenvalue problem

$$A * x = w * M * x.$$

*M* must represent a real, symmetric matrix if *A* is real, and must represent a complex, hermitian matrix if *A* is complex. For best results, the data type of *M* should be the same as that of *A*. Additionally:

If *sigma* is None, *M* is positive definite

If *sigma* is specified, *M* is positive semi-definite

If *sigma* is None, `eigs` requires an operator to compute the solution of the linear equation  $M * x = b$ . This is done internally via a (sparse) LU decomposition for an explicit matrix *M*, or via an iterative solver for a general linear operator. Alternatively, the user can supply the matrix or operator *Minv*, which gives  $x = Minv * b = M^{-1} * b$ .

*sigma* : real or complex, optional

Find eigenvalues near *sigma* using shift-invert mode. This requires an operator to compute the solution of the linear system  $[A - sigma * M] * x = b$ , where *M* is the identity matrix if unspecified. This is computed internally via a (sparse) LU decomposition for explicit matrices *A* & *M*, or via an iterative solver if either *A* or *M* is a general linear operator. Alternatively, the user can supply the matrix or operator *OPinv*, which gives  $x = OPinv * b = [A - sigma * M]^{-1} * b$ . For a real matrix *A*, shift-invert can either be done in imaginary mode or real mode, specified by the parameter *OPpart* ('r' or 'i'). Note that when *sigma* is specified, the keyword 'which' (below) refers to the shifted eigenvalues *w'* [*i*] where:

**If *A* is real and *OPpart* == 'r' (default),**

$$w' [i] = 1/2 * [1/(w[i]-sigma) + 1/(w[i]-conj(sigma))].$$

**If *A* is real and *OPpart* == 'i',**

$$w' [i] = 1/2i * [1/(w[i]-sigma) - 1/(w[i]-conj(sigma))].$$

**If *A* is complex, *w'* [*i*] = 1/(*w*[*i*]-*sigma*).**

*v0* : ndarray, optional

Starting vector for iteration. Default: random

*ncv* : int, optional

The number of Lanczos vectors generated *ncv* must be greater than *k*; it is recommended that  $ncv > 2*k$ . Default:  $\min(n, \max(2*k + 1, 20))$

**which** : str, ['LM' | 'SM' | 'LR' | 'SR' | 'LI' | 'SI'], optional  
 Which  $k$  eigenvectors and eigenvalues to find:  
     'LM' : largest magnitude  
     'SM' : smallest magnitude  
     'LR' : largest real part  
     'SR' : smallest real part  
     'LI' : largest imaginary part  
     'SI' : smallest imaginary part  
 When  $\sigma \neq \text{None}$ , 'which' refers to the shifted eigenvalues  $w[i]$  (see discussion in 'sigma', above). ARPACK is generally better at finding large values than small values. If small eigenvalues are desired, consider using shift-invert mode for better performance.

**maxiter** : int, optional  
 Maximum number of Arnoldi update iterations allowed Default:  $n \times 10$

**tol** : float, optional  
 Relative accuracy for eigenvalues (stopping criterion) The default value of 0 implies machine precision.

**return\_eigenvectors** : bool, optional  
 Return eigenvectors (True) in addition to eigenvalues

**Minv** : ndarray, sparse matrix or LinearOperator, optional  
 See notes in *M*, above.

**OPinv** : ndarray, sparse matrix or LinearOperator, optional  
 See notes in *sigma*, above.

**OPpart** : {'r' or 'i'}, optional  
 See notes in *sigma*, above

**Returns**

**w** : ndarray  
 Array of  $k$  eigenvalues.

**v** : ndarray  
 An array of  $k$  eigenvectors.  $v[:, i]$  is the eigenvector corresponding to the eigenvalue  $w[i]$ .

**Raises**

**ArpackNoConvergence**  
 When the requested convergence is not obtained. The currently converged eigenvalues and eigenvectors can be found as *eigenvalues* and *eigenvectors* attributes of the exception object.

**See also:**

[\*eigsh\*](#) eigenvalues and eigenvectors for symmetric matrix *A*  
[\*svds\*](#) singular value decomposition for a matrix *A*

**Notes**

This function is a wrapper to the ARPACK [R285] SNEUPD, DNEUPD, CNEUPD, ZNEUPD, functions which use the Implicitly Restarted Arnoldi Method to find the eigenvalues and eigenvectors [R286].

**References**

[R285], [R286]

**Examples**

Find 6 eigenvectors of the identity matrix:

```
>>> import scipy.sparse as sparse
>>> id = np.eye(13)
>>> vals, vecs = sparse.linalg.eigs(id, k=6)
>>> vals
array([ 1.+0.j,  1.+0.j,  1.+0.j,  1.+0.j,  1.+0.j,  1.+0.j])
```

```
>>> vecs.shape
(13, 6)
```

```
scipy.sparse.linalg.eigsh(A, k=6, M=None, sigma=None, which='LM', v0=None, ncv=None,
                          maxiter=None, tol=0, return_eigenvectors=True, Minv=None,
                          OPinv=None, mode='normal')
```

Find  $k$  eigenvalues and eigenvectors of the real symmetric square matrix or complex hermitian matrix  $A$ .

Solves  $A * x[i] = w[i] * x[i]$ , the standard eigenvalue problem for  $w[i]$  eigenvalues with corresponding eigenvectors  $x[i]$ .

If  $M$  is specified, solves  $A * x[i] = w[i] * M * x[i]$ , the generalized eigenvalue problem for  $w[i]$  eigenvalues with corresponding eigenvectors  $x[i]$

**Parameters** **A** : An  $N \times N$  matrix, array, sparse matrix, or LinearOperator representing the operation  $A * x$ , where  $A$  is a real symmetric matrix For buckling mode (see below)  $A$  must additionally be positive-definite

**k** : int, optional  
The number of eigenvalues and eigenvectors desired.  $k$  must be smaller than  $N$ . It is not possible to compute all eigenvectors of a matrix.

**Returns** **w** : array  
Array of  $k$  eigenvalues

**v** : array  
An array representing the  $k$  eigenvectors. The column  $v[:, i]$  is the eigenvector corresponding to the eigenvalue  $w[i]$ .

**Other Parameters**

**M** : An  $N \times N$  matrix, array, sparse matrix, or linear operator representing the operation  $M * x$  for the generalized eigenvalue problem  
 $A * x = w * M * x$ .

$M$  must represent a real, symmetric matrix if  $A$  is real, and must represent a complex, hermitian matrix if  $A$  is complex. For best results, the data type of  $M$  should be the same as that of  $A$ . Additionally:

- If  $\sigma$  is `None`,  $M$  is symmetric positive definite
- If  $\sigma$  is specified,  $M$  is symmetric positive semi-definite
- In buckling mode,  $M$  is symmetric indefinite.

If  $\sigma$  is `None`, `eigsh` requires an operator to compute the solution of the linear equation  $M * x = b$ . This is done internally via a (sparse) LU decomposition for an explicit matrix  $M$ , or via an iterative solver for a general linear operator. Alternatively, the user can supply the matrix or operator `Minv`, which gives  $x = \text{Minv} * b = M^{-1} * b$ .

**sigma** : real

Find eigenvalues near  $\sigma$  using shift-invert mode. This requires an operator to compute the solution of the linear system  $[A - \sigma * M] x = b$ , where  $M$  is the identity matrix if unspecified. This is computed internally via a (sparse) LU decomposition for explicit matrices  $A$  &  $M$ , or via an iterative solver if either  $A$  or  $M$  is a general linear operator. Alternatively, the user can supply the matrix or operator `OPinv`, which gives  $x = \text{OPinv} * b = [A - \sigma * M]^{-1} * b$ . Note that when  $\sigma$  is specified, the keyword 'which' refers to the shifted eigenvalues  $w'[i]$  where:

- if `mode == 'normal'`,  $w'[i] = 1 / (w[i] - \sigma)$ .
- if `mode == 'cayley'`,  $w'[i] = (w[i] + \sigma) / (w[i] - \sigma)$ .
- if `mode == 'buckling'`,  $w'[i] = w[i] / (w[i] - \sigma)$ .

(see further discussion in 'mode' below)

**v0** : ndarray, optional

Starting vector for iteration. Default: random

**ncv** : int, optional

The number of Lanczos vectors generated `ncv` must be greater than `k` and smaller than `n`; it is recommended that `ncv > 2*k`. Default: `min(n, max(2*k + 1, 20))`

**which** : str ['LM' | 'SM' | 'LA' | 'SA' | 'BE']

If `A` is a complex hermitian matrix, 'BE' is invalid. Which `k` eigenvectors and eigenvalues to find:

'LM' : Largest (in magnitude) eigenvalues

'SM' : Smallest (in magnitude) eigenvalues

'LA' : Largest (algebraic) eigenvalues

'SA' : Smallest (algebraic) eigenvalues

'BE' : Half (`k/2`) from each end of the spectrum

When `k` is odd, return one more (`k/2+1`) from the high end. When `sigma != None`, 'which' refers to the shifted eigenvalues `w' [i]` (see discussion in 'sigma', above). ARPACK is generally better at finding large values than small values. If small eigenvalues are desired, consider using shift-invert mode for better performance.

**maxiter** : int, optional

Maximum number of Arnoldi update iterations allowed Default: `n*10`

**tol** : float

Relative accuracy for eigenvalues (stopping criterion). The default value of 0 implies machine precision.

**Minv** : N x N matrix, array, sparse matrix, or LinearOperator

See notes in `M`, above

**OPinv** : N x N matrix, array, sparse matrix, or LinearOperator

See notes in `sigma`, above.

**return\_eigenvectors** : bool

Return eigenvectors (True) in addition to eigenvalues

**mode** : string ['normal' | 'buckling' | 'cayley']

Specify strategy to use for shift-invert mode. This argument applies only for real-valued `A` and `sigma != None`. For shift-invert mode, ARPACK internally solves the eigenvalue problem  $OP * x' [i] = w' [i] * B * x' [i]$  and transforms the resulting Ritz vectors `x'[i]` and Ritz values `w'[i]` into the desired eigenvectors and eigenvalues of the problem  $A * x [i] = w [i] * M * x [i]$ . The modes are as follows:

'normal' :  $OP = [A - \text{sigma} * M]^{-1} * M$ ,  $B = M$ ,  $w'[i] = 1 / (w[i] - \text{sigma})$

'buckling' :  $OP = [A - \text{sigma} * M]^{-1} * A$ ,  $B = A$ ,  $w'[i] = w[i] / (w[i] - \text{sigma})$

'cayley' :  $OP = [A - \text{sigma} * M]^{-1} * [A + \text{sigma} * M]$ ,  $B = M$ ,  $w'[i] = (w[i] + \text{sigma}) / (w[i] - \text{sigma})$

The choice of mode will affect which eigenvalues are selected by the keyword 'which', and can also impact the stability of convergence (see [2] for a discussion)

**Raises**

**ArpackNoConvergence**

When the requested convergence is not obtained.

The currently converged eigenvalues and eigenvectors can be found as `eigenvalues` and `eigenvectors` attributes of the exception object.

See also:

`eigs` eigenvalues and eigenvectors for a general (nonsymmetric) matrix `A`  
`svds` singular value decomposition for a matrix `A`

*Notes*

This function is a wrapper to the ARPACK [R287] SSEUPD and DSEUPD functions which use the Implicitly Restarted Lanczos Method to find the eigenvalues and eigenvectors [R288].

*References*

[R287], [R288]

*Examples*

```
>>> import scipy.sparse as sparse
>>> id = np.eye(13)
>>> vals, vecs = sparse.linalg.eigsh(id, k=6)
>>> vals
array([ 1.+0.j,  1.+0.j,  1.+0.j,  1.+0.j,  1.+0.j,  1.+0.j])
>>> vecs.shape
(13, 6)
```

```
scipy.sparse.linalg.lobpcg(A, X, B=None, M=None, Y=None, tol=None, maxiter=20,
                           largest=True, verbosityLevel=0, retLambdaHistory=False, re-
                           tResidualNormsHistory=False)
```

Locally Optimal Block Preconditioned Conjugate Gradient Method (LOBPCG)

LOBPCG is a preconditioned eigensolver for large symmetric positive definite (SPD) generalized eigenproblems.

- Parameters**
  - A** : {sparse matrix, dense matrix, LinearOperator}
 

The symmetric linear operator of the problem, usually a sparse matrix. Often called the “stiffness matrix”.
  - X** : array\_like
 

Initial approximation to the k eigenvectors. If A has shape=(n,n) then X should have shape shape=(n,k).
  - B** : {dense matrix, sparse matrix, LinearOperator}, optional
 

the right hand side operator in a generalized eigenproblem. by default, B = Identity often called the “mass matrix”
  - M** : {dense matrix, sparse matrix, LinearOperator}, optional
 

preconditioner to A; by default M = Identity M should approximate the inverse of A
  - Y** : array\_like, optional
 

n-by-sizeY matrix of constraints, sizeY < n The iterations will be performed in the B-orthogonal complement of the column-space of Y. Y must be full rank.
- Returns**
  - w** : array
 

Array of k eigenvalues
  - v** : array
 

An array of k eigenvectors. V has the same shape as X.
- Other Parameters**
  - tol** : scalar, optional
 

Solver tolerance (stopping criterion) by default: tol=n\*sqrt(eps)
  - maxiter** : integer, optional
 

maximum number of iterations by default: maxiter=min(n,20)
  - largest** : bool, optional
 

when True, solve for the largest eigenvalues, otherwise the smallest
  - verbosityLevel** : integer, optional
 

controls solver output. default: verbosityLevel = 0.
  - retLambdaHistory** : boolean, optional
 

whether to return eigenvalue history

**retResidualNormsHistory** : boolean, optional  
whether to return history of residual norms

### Notes

If both `retLambdaHistory` and `retResidualNormsHistory` are `True`, the return tuple has the following format (lambda, V, lambda history, residual norms history).

In the following  $n$  denotes the matrix size and  $m$  the number of required eigenvalues (smallest or largest).

The LOBPCG code internally solves eigenproblems of the size  $3 \times m$  on every iteration by calling the “standard” dense eigensolver, so if  $m$  is not small enough compared to  $n$ , it does not make sense to call the LOBPCG code, but rather one should use the “standard” eigensolver, e.g. `numpy` or `scipy` function in this case. If one calls the LOBPCG algorithm for  $5 \times m > n$ , it will most likely break internally, so the code tries to call the standard function instead.

It is not that  $n$  should be large for the LOBPCG to work, but rather the ratio  $n/m$  should be large. If you call the LOBPCG code with  $m=1$  and  $n=10$ , it should work, though  $n$  is small. The method is intended for extremely large  $n/m$ , see e.g., reference [28] in <http://arxiv.org/abs/0705.2626>

The convergence speed depends basically on two factors:

1. How well relatively separated the seeking eigenvalues are from the rest of the eigenvalues. One can try to vary  $m$  to make this better.
2. How well conditioned the problem is. This can be changed by using proper preconditioning. For example, a rod vibration test problem (under tests directory) is ill-conditioned for large  $n$ , so convergence will be slow, unless efficient preconditioning is used. For this specific problem, a good simple preconditioner function would be a linear solve for  $A$ , which is easy to code since  $A$  is tridiagonal.

### Acknowledgements

`lobpcg.py` code was written by Robert Cimrman. Many thanks belong to Andrew Knyazev, the author of the algorithm, for lots of advice and support.

### References

[R294], [R295], [R296]

### Examples

Solve  $Ax = \lambda Bx$  with constraints and preconditioning.

```
>>> from scipy.sparse import spdiags, issparse
>>> from scipy.sparse.linalg import lobpcg, LinearOperator
>>> n = 100
>>> vals = [np.arange(n, dtype=np.float64) + 1]
>>> A = spdiags(vals, 0, n, n)
>>> A.toarray()
array([[ 1.,  0.,  0., ...,  0.,  0.,  0.],
       [ 0.,  2.,  0., ...,  0.,  0.,  0.],
       [ 0.,  0.,  3., ...,  0.,  0.,  0.],
       ...,
       [ 0.,  0.,  0., ..., 98.,  0.,  0.],
       [ 0.,  0.,  0., ...,  0., 99.,  0.],
       [ 0.,  0.,  0., ...,  0.,  0., 100.]])
```

Constraints.

```
>>> Y = np.eye(n, 3)
```

Initial guess for eigenvectors, should have linearly independent columns. Column dimension = number of requested eigenvalues.

```
>>> X = np.random.rand(n, 3)
```

Preconditioner – inverse of A (as an abstract linear operator).

```
>>> invA = spdiags([1./vals[0]], 0, n, n)
>>> def precondition(x):
...     return invA * x
>>> M = LinearOperator(matvec=precond, shape=(n, n), dtype=float)
```

Here, `invA` could of course have been used directly as a preconditioner. Let us then solve the problem:

```
>>> eigs, vecs = lobpcg(A, X, Y=Y, M=M, tol=1e-4, maxiter=40, largest=False)
>>> eigs
array([ 4.,  5.,  6.])
```

Note that the vectors passed in `Y` are the eigenvectors of the 3 smallest eigenvalues. The results returned are orthogonal to those.

Singular values problems:

---

`svds(A[, k, ncv, tol, which, v0, maxiter, ...])` Compute the largest  $k$  singular values/vectors for a sparse matrix.

---

`scipy.sparse.linalg.svds(A, k=6, ncv=None, tol=0, which='LM', v0=None, maxiter=None, return_singular_vectors=True)`

Compute the largest  $k$  singular values/vectors for a sparse matrix.

**Parameters** `A` : {sparse matrix, LinearOperator}

Array to compute the SVD on, of shape (M, N)

`k` : int, optional

Number of singular values and vectors to compute. Must be  $1 \leq k < \min(A.shape)$ .

`ncv` : int, optional

The number of Lanczos vectors generated `ncv` must be greater than  $k+1$  and smaller than  $n$ ; it is recommended that  $ncv > 2*k$  Default:  $\min(n, \max(2*k + 1, 20))$

`tol` : float, optional

Tolerance for singular values. Zero (default) means machine precision.

`which` : str, ['LM' | 'SM'], optional

Which  $k$  singular values to find:

- 'LM' : largest singular values
- 'SM' : smallest singular values

New in version 0.12.0.

`v0` : ndarray, optional

Starting vector for iteration, of length  $\min(A.shape)$ . Should be an (approximate) left singular vector if  $N > M$  and a right singular vector otherwise. Default: random

New in version 0.12.0.

`maxiter` : int, optional

Maximum number of iterations.

New in version 0.12.0.

`return_singular_vectors` : bool or str, optional

- True: return singular vectors (True) in addition to singular values.



New in version 0.12.0.

- “u”: only return the u matrix, without computing vh (if  $N > M$ ).
- “vh”: only return the vh matrix, without computing u (if  $N \leq M$ ).

**Returns**

**u** : ndarray, shape=(M, k)  
 New in version 0.16.0.  
 Unitary matrix having left singular vectors as columns. If *return\_singular\_vectors* is “vh”, this variable is not computed, and None is returned instead.

**s** : ndarray, shape=(k,)  
 The singular values.

**vt** : ndarray, shape=(k, N)  
 Unitary matrix having right singular vectors as rows. If *return\_singular\_vectors* is “u”, this variable is not computed, and None is returned instead.

**Notes**

This is a naive implementation using ARPACK as an eigensolver on  $A.H * A$  or  $A * A.H$ , depending on which one is more efficient.

Complete or incomplete LU factorizations

<code>splu(A[, permc_spec, diag_pivot_thresh, ...])</code>	Compute the LU decomposition of a sparse, square matrix.
<code>spilu(A[, drop_tol, fill_factor, drop_rule, ...])</code>	Compute an incomplete LU decomposition for a sparse, square matrix.
SuperLU	LU factorization of a sparse matrix.

`scipy.sparse.linalg.splu(A, permc_spec=None, diag_pivot_thresh=None, drop_tol=None, relax=None, panel_size=None, options={})`

Compute the LU decomposition of a sparse, square matrix.

**Parameters**

**A** : sparse matrix  
 Sparse matrix to factorize. Should be in CSR or CSC format.

**permc\_spec** : str, optional  
 How to permute the columns of the matrix for sparsity preservation. (default: ‘COLAMD’)

- NATURAL: natural ordering.
- MMD\_ATA: minimum degree ordering on the structure of  $A^T A$ .
- MMD\_AT\_PLUS\_A: minimum degree ordering on the structure of  $A^T + A$ .
- COLAMD: approximate minimum degree column ordering

**diag\_pivot\_thresh** : float, optional  
 Threshold used for a diagonal entry to be an acceptable pivot. See SuperLU user’s guide for details [R307]

**drop\_tol** : float, optional  
 (deprecated) No effect.

**relax** : int, optional  
 Expert option for customizing the degree of relaxing supernodes. See SuperLU user’s guide for details [R307]

**panel\_size** : int, optional  
 Expert option for customizing the panel size. See SuperLU user’s guide for details [R307]

**options** : dict, optional  
 Dictionary containing additional expert options to SuperLU. See SuperLU user guide [R307] (section 2.4 on the ‘Options’ argument) for more details. For example, you can specify `options=dict(Equil=False,`

**Returns** `invA` : `scipy.sparse.linalg.SuperLU` object, which has a `solve` method.

**See also:**

`spilu` incomplete LU decomposition

**Notes**

This function uses the SuperLU library.

**References**

[R307]

```
scipy.sparse.linalg.spilu(A, drop_tol=None, fill_factor=None, drop_rule=None,
                          permc_spec=None, diag_pivot_thresh=None, relax=None,
                          panel_size=None, options=None)
```

Compute an incomplete LU decomposition for a sparse, square matrix.

The resulting object is an approximation to the inverse of A.

**Parameters**

- A** : (N, N) array\_like  
Sparse matrix to factorize
- drop\_tol** : float, optional  
Drop tolerance (0 <= tol <= 1) for an incomplete LU decomposition. (default: 1e-4)
- fill\_factor** : float, optional  
Specifies the fill ratio upper bound (>= 1.0) for ILU. (default: 10)
- drop\_rule** : str, optional  
Comma-separated string of drop rules to use. Available rules: `basic`, `prows`, `column`, `area`, `secondary`, `dynamic`, `interp`. (Default: `basic, area`)  
See SuperLU documentation for details.

**Remaining other options**

**Returns** `invA_approx` : `scipy.sparse.linalg.SuperLU` object, which has a `solve` method.

**See also:**

`splu` complete LU decomposition

**Notes**

To improve the better approximation to the inverse, you may need to increase `fill_factor` AND decrease `drop_tol`.

This function uses the SuperLU library.

**class** `scipy.sparse.linalg.SuperLU`  
LU factorization of a sparse matrix.

Factorization is represented as:

$$P_r * A * P_c = L * U$$

To construct these SuperLU objects, call the `splu` and `spilu` functions.

**Notes**

New in version 0.14.0.

### Examples

The LU decomposition can be used to solve matrix equations. Consider:

```
>>> import numpy as np
>>> from scipy.sparse import csc_matrix, linalg as sla
>>> A = csc_matrix([[1,2,0,4],[1,0,0,1],[1,0,2,1],[2,2,1,0.]])
```

This can be solved for a given right-hand side:

```
>>> lu = sla.splu(A)
>>> b = np.array([1, 2, 3, 4])
>>> x = lu.solve(b)
>>> A.dot(x)
array([ 1.,  2.,  3.,  4.]
```

The `lu` object also contains an explicit representation of the decomposition. The permutations are represented as mappings of indices:

```
>>> lu.perm_r
array([0, 2, 1, 3], dtype=int32)
>>> lu.perm_c
array([2, 0, 1, 3], dtype=int32)
```

The L and U factors are sparse matrices in CSC format:

```
>>> lu.L.A
array([[ 1. ,  0. ,  0. ,  0. ],
       [ 0. ,  1. ,  0. ,  0. ],
       [ 0. ,  0. ,  1. ,  0. ],
       [ 1. ,  0.5,  0.5,  1. ]])
>>> lu.U.A
array([[ 2.,  0.,  1.,  4.],
       [ 0.,  2.,  1.,  1.],
       [ 0.,  0.,  1.,  1.],
       [ 0.,  0.,  0., -5.]])
```

The permutation matrices can be constructed:

```
>>> Pr = csc_matrix((4, 4))
>>> Pr[lu.perm_r, np.arange(4)] = 1
>>> Pc = csc_matrix((4, 4))
>>> Pc[np.arange(4), lu.perm_c] = 1
```

We can reassemble the original matrix:

```
>>> (Pr.T * (lu.L * lu.U) * Pc.T).A
array([[ 1.,  2.,  0.,  4.],
       [ 1.,  0.,  0.,  1.],
       [ 1.,  0.,  2.,  1.],
       [ 2.,  2.,  1.,  0.]])
```

Continued on next page

Table 5.206 – continued from previous page

*Attributes*

---

<code>shape</code>	Shape of the original matrix as a tuple of ints.
<code>nnz</code>	Number of nonzero elements in the matrix.
<code>perm_c</code>	Permutation Pc represented as an array of indices.
<code>perm_r</code>	Permutation Pr represented as an array of indices.
<code>L</code>	Lower triangular factor with unit diagonal as a <code>scipy.sparse.csc_matrix</code> .
<code>U</code>	Upper triangular factor as a <code>scipy.sparse.csc_matrix</code> .

---

`SuperLU.shape`

Shape of the original matrix as a tuple of ints.

`SuperLU.nnz`

Number of nonzero elements in the matrix.

`SuperLU.perm_c`

Permutation Pc represented as an array of indices.

The column permutation matrix can be reconstructed via:

```
>>> Pc = np.zeros((n, n))
>>> Pc[np.arange(n), perm_c] = 1
```

`SuperLU.perm_r`

Permutation Pr represented as an array of indices.

The row permutation matrix can be reconstructed via:

```
>>> Pr = np.zeros((n, n))
>>> Pr[perm_r, np.arange(n)] = 1
```

`SuperLU.L`

Lower triangular factor with unit diagonal as a `scipy.sparse.csc_matrix`.

New in version 0.14.0.

`SuperLU.U`

Upper triangular factor as a `scipy.sparse.csc_matrix`.

New in version 0.14.0.

*Methods*

---

<code>solve(rhs[, trans])</code>	Solves linear system of equations with one or several right-hand sides.
----------------------------------	---

---

`SuperLU.solve(rhs[, trans])`

Solves linear system of equations with one or several right-hand sides.

**Parameters** **rhs** : ndarray, shape (n,) or (n, k)  
 Right hand side(s) of equation  
**trans** : { 'N', 'T', 'H' }, optional  
 Type of system to solve:

```
'N':  A  * x == rhs  (default)
'T':  A^T * x == rhs
```

**Returns** `x`: ndarray, shape  $\text{rhs.shape}$  *i.e.*, normal, transposed, and hermitian conjugate.  
 Solution vector(s)

### 5.22.6 Exceptions

<code>ArpackNoConvergence(msg, eigenvalues, ...)</code>	ARPACK iteration did not converge
<code>ArpackError(info[, infodict])</code>	ARPACK error

**exception** `scipy.sparse.linalg.ArpackNoConvergence` (*msg, eigenvalues, eigenvectors*)  
 ARPACK iteration did not converge

**Attributes**

<code>eigenvalues</code>	(ndarray) Partial result. Converged eigenvalues.
<code>eigenvectors</code>	(ndarray) Partial result. Converged eigenvectors.

**exception** `scipy.sparse.linalg.ArpackError` (*info*, *infodict*={*c*: {0: 'Normal exit.', 1: 'Maximum number of iterations taken. All possible eigenvalues of OP has been found. IPARAM(5) returns the number of wanted converged Ritz values.', 2: 'No longer an informational error. Deprecated starting with release 2 of ARPACK.', 3: 'No shifts could be applied during a cycle of the Implicitly restarted Arnoldi iteration. One possibility is to increase the size of NCV relative to NEV. ', -9999: 'Could not build an Arnoldi factorization. IPARAM(5) returns the size of the current Arnoldi factorization. The user is advised to check that enough workspace and array storage has been allocated.', -13: "NEV and WHICH = 'BE' are incompatible.", -12: 'IPARAM(1) must be equal to 0 or 1.', -1: 'N must be positive.', -10: 'IPARAM(7) must be 1, 2, 3.', -9: 'Starting vector is zero.', -8: 'Error return from LAPACK eigenvalue calculation;', -7: 'Length of private work array WORKL is not sufficient.', -6: "BMAT must be one of 'I' or 'G'."}, -5: " WHICH must be one of 'LM', 'SM', 'LR', 'SR', 'LI', 'SI'", -4: 'The maximum number of Arnoldi update iterations allowed must be greater than zero.', -3: 'NCV-NEV >= 2 and less than or equal to N.', -2: 'NEV must be positive.', -11: "IPARAM(7) = 1 and BMAT = 'G' are incompatible."}, *s*: {0: 'Normal exit.', 1: 'Maximum number of iterations taken. All possible eigenvalues of OP has been found. IPARAM(5) returns the number of wanted converged Ritz values.', 2: 'No longer an informational error. Deprecated starting with release 2 of ARPACK.', 3: 'No shifts could be applied during a cycle of the Implicitly restarted Arnoldi iteration. One possibility is to increase the size of NCV relative to NEV. ', -9999: 'Could not build an Arnoldi factorization. IPARAM(5) returns the size of the current Arnoldi factorization. The user is advised to check that enough workspace and array storage has been allocated.', -13: "NEV and WHICH = 'BE' are incompatible.", -12: 'IPARAM(1) must be equal to 0 or 1.', -2: 'NEV must be positive.', -10: 'IPARAM(7) must be 1, 2, 3, 4.', -9: 'Starting vector is zero.', -8: 'Error return from LAPACK eigenvalue calculation;', -7: 'Length of private work array WORKL is not sufficient.', -6: "BMAT must be one of 'I' or 'G'."}, -5: " WHICH must be one of 'LM', 'SM', 'LR', 'SR', 'LI', 'SI'", -4: 'The maximum number of Arnoldi update iterations allowed must be greater than zero.', -3: 'NCV-NEV >= 2 and less than or equal to N.', -1: 'N must be positive.', -11: "IPARAM(7) = 1 and BMAT = 'G' are incompatible."}, *z*: {0: 'Normal exit.', 1: 'Maximum number of iterations taken. All possible eigenvalues of OP has been found. IPARAM(5) returns the number of wanted converged Ritz values.', 2: 'No longer an informational error. Deprecated starting with release 2 of ARPACK.', 3: 'No shifts could be applied during a cycle of the Implicitly restarted Arnoldi iteration. One possibility is to increase the size of NCV relative to NEV. ', -9999: 'Could not build an Arnoldi factorization. IPARAM(5) returns the size

ARPACK error

## 5.23 Compressed Sparse Graph Routines (`scipy.sparse.csgraph`)

Fast graph algorithms based on sparse matrix representations.

### 5.23.1 Contents

---

<code>connected_components(csgraph[, directed, ...])</code>	Analyze the connected components of a sparse graph
<code>laplacian(csgraph[, normed, return_diag, ...])</code>	Return the Laplacian matrix of a directed graph.
<code>shortest_path(csgraph[, method, directed, ...])</code>	Perform a shortest-path graph search on a positive directed or undirected graph
<code>dijkstra(csgraph[, directed, indices, ...])</code>	Dijkstra algorithm using Fibonacci Heaps
<code>floyd_warshall(csgraph[, directed, ...])</code>	Compute the shortest path lengths using the Floyd-Warshall algorithm
<code>bellman_ford(csgraph[, directed, indices, ...])</code>	Compute the shortest path lengths using the Bellman-Ford algorithm.
<code>johnson(csgraph[, directed, indices, ...])</code>	Compute the shortest path lengths using Johnson's algorithm.
<code>breadth_first_order(csgraph, i_start[, ...])</code>	Return a breadth-first ordering starting with specified node.
<code>depth_first_order(csgraph, i_start[, ...])</code>	Return a depth-first ordering starting with specified node.
<code>breadth_first_tree(csgraph, i_start[, directed])</code>	Return the tree generated by a breadth-first search
<code>depth_first_tree(csgraph, i_start[, directed])</code>	Return a tree generated by a depth-first search.
<code>minimum_spanning_tree(csgraph[, overwrite])</code>	Return a minimum spanning tree of an undirected graph
<code>reverse_cuthill_mckee</code>	Returns the permutation array that orders a sparse CSR or CSC matrix in Reverse Cuthill-McKee order
<code>maximum_bipartite_matching</code>	Returns an array of row or column permutations that makes the diagonal of a bipartite graph
<code>NegativeCycleError</code>	

---

`scipy.sparse.csgraph.connected_components` (*csgraph*, *directed=True*, *connection='weak'*, *return\_labels=True*)

Analyze the connected components of a sparse graph

New in version 0.11.0.

**Parameters** `csgraph` : array\_like or sparse matrix

The N x N matrix representing the compressed sparse graph. The input `csgraph` will be converted to `csr` format for the calculation.

**directed** : bool, optional

If `True` (default), then operate on a directed graph: only move from point `i` to point `j` along paths `csgraph[i, j]`. If `False`, then find the shortest path on an undirected graph: the algorithm can progress from point `i` to `j` along `csgraph[i, j]` or `csgraph[j, i]`.

**connection** : str, optional

['weak'|'strong']. For directed graphs, the type of connection to use. Nodes `i` and `j` are strongly connected if a path exists both from `i` to `j` and from `j` to `i`. Nodes `i` and `j` are weakly connected if only one of these paths exists. If `directed == False`, this keyword is not referenced.

**return\_labels** : bool, optional

If `True` (default), then return the labels for each of the connected components.

**Returns**

`n_components`: int

The number of connected components.

`labels`: ndarray

The length-N array of labels of the connected components.

*References*

[R238]

`scipy.sparse.csgraph.laplacian` (*csgraph*, *normed=False*, *return\_diag=False*, *use\_out\_degree=False*)

Return the Laplacian matrix of a directed graph.

**Parameters**

- csgraph** : array\_like or sparse matrix, 2 dimensions  
compressed-sparse graph, with shape (N, N).
- normed** : bool, optional  
If True, then compute normalized Laplacian.
- return\_diag** : bool, optional  
If True, then also return an array related to vertex degrees.
- use\_out\_degree** : bool, optional  
If True, then use out-degree instead of in-degree. This distinction matters only if the graph is asymmetric. Default: False.

**Returns**

- lap** : ndarray or sparse matrix  
The N x N laplacian matrix of *csgraph*. It will be a numpy array (dense) if the input was dense, or a sparse matrix otherwise.
- diag** : ndarray, optional  
The length-N diagonal of the Laplacian matrix. For the normalized Laplacian, this is the array of square roots of vertex degrees or 1 if the degree is zero.

*Notes*

The Laplacian matrix of a graph is sometimes referred to as the “Kirchoff matrix” or the “admittance matrix”, and is useful in many parts of spectral graph theory. In particular, the eigen-decomposition of the laplacian matrix can give insight into many properties of the graph.

*Examples*

```
>>> from scipy.sparse import csgraph
>>> G = np.arange(5) * np.arange(5)[:, np.newaxis]
>>> G
array([[ 0,  0,  0,  0,  0],
       [ 0,  1,  2,  3,  4],
       [ 0,  2,  4,  6,  8],
       [ 0,  3,  6,  9, 12],
       [ 0,  4,  8, 12, 16]])
>>> csgraph.laplacian(G, normed=False)
array([[ 0,  0,  0,  0,  0],
       [ 0,  9, -2, -3, -4],
       [ 0, -2, 16, -6, -8],
       [ 0, -3, -6, 21, -12],
       [ 0, -4, -8, -12, 24]])
```

`scipy.sparse.csgraph.shortest_path` (*csgraph*, *method='auto'*, *directed=True*, *return\_predecessors=False*, *unweighted=False*, *overwrite=False*, *indices=None*)

Perform a shortest-path graph search on a positive directed or undirected graph.

New in version 0.11.0.

**Parameters**

- csgraph** : array, matrix, or sparse matrix, 2 dimensions  
The N x N array of distances representing the input graph.
- method** : string ['auto'|'FW'|'D'], optional  
Algorithm to use for shortest paths. Options are:



*‘auto’ – (default) select the best among ‘FW’, ‘D’, ‘BF’, or ‘J’*

based on the input data.

*‘FW’ – Floyd-Warshall algorithm. Computational cost is approximately  $O[N^3]$ . The input csgraph will be converted to a dense representation.*

*‘D’ – Dijkstra’s algorithm with Fibonacci heaps. Computational*

*cost is approximately  $O[N(N*k + N*\log(N))]$ , where  $k$  is the average number of connected edges per node. The input csgraph will be converted to a csr representation.*

*‘BF’ – Bellman-Ford algorithm. This algorithm can be used when*

*weights are negative. If a negative cycle is encountered, an error will be raised. Computational cost is approximately  $O[N(N^2*k)]$ , where  $k$  is the average number of connected edges per node. The input csgraph will be converted to a csr representation.*

*‘J’ – Johnson’s algorithm. Like the Bellman-Ford algorithm,*

*Johnson’s algorithm is designed for use when the weights are negative. It combines the Bellman-Ford algorithm with Dijkstra’s algorithm for faster computation.*

**directed** : bool, optional

If True (default), then find the shortest path on a directed graph: only move from point  $i$  to point  $j$  along paths  $csgraph[i, j]$ . If False, then find the shortest path on an undirected graph: the algorithm can progress from point  $i$  to  $j$  along  $csgraph[i, j]$  or  $csgraph[j, i]$

**return\_predecessors** : bool, optional

If True, return the size  $(N, N)$  predecessor matrix

**unweighted** : bool, optional

If True, then find unweighted distances. That is, rather than finding the path between each point such that the sum of weights is minimized, find the path such that the number of edges is minimized.

**overwrite** : bool, optional

If True, overwrite csgraph with the result. This applies only if method == ‘FW’ and csgraph is a dense, c-ordered array with dtype=float64.

**indices** : array\_like or int, optional

If specified, only compute the paths for the points at the given indices. Incompatible with method == ‘FW’.

**Returns**

**dist\_matrix** : ndarray

The  $N \times N$  matrix of distances between graph nodes.  $dist\_matrix[i,j]$  gives the shortest distance from point  $i$  to point  $j$  along the graph.

**predecessors** : ndarray

Returned only if return\_predecessors == True. The  $N \times N$  matrix of predecessors, which can be used to reconstruct the shortest paths. Row  $i$  of the predecessor matrix contains information on the shortest paths from point  $i$ : each entry predecessors[ $i, j$ ] gives the index of the previous node in the path from point  $i$  to point  $j$ . If no path exists between point  $i$  and  $j$ , then predecessors[ $i, j$ ] = -9999

**Raises**

**NegativeCycleError:**

if there are negative cycles in the graph

**Notes**

As currently implemented, Dijkstra's algorithm and Johnson's algorithm do not work for graphs with direction-dependent distances when `directed == False`. i.e., if `csgraph[i,j]` and `csgraph[j,i]` are non-equal edges, `method='D'` may yield an incorrect result.

```
scipy.sparse.csgraph.dijkstra(csgraph, directed=True, indices=None, return_predecessors=False, unweighted=False, limit=np.inf)
```

Dijkstra algorithm using Fibonacci Heaps

New in version 0.11.0.

- Parameters**
- csgraph** : array, matrix, or sparse matrix, 2 dimensions  
The N x N array of non-negative distances representing the input graph.
  - directed** : bool, optional  
If True (default), then find the shortest path on a directed graph: only move from point i to point j along paths `csgraph[i, j]`. If False, then find the shortest path on an undirected graph: the algorithm can progress from point i to j along `csgraph[i, j]` or `csgraph[j, i]`
  - indices** : array\_like or int, optional  
if specified, only compute the paths for the points at the given indices.
  - return\_predecessors** : bool, optional  
If True, return the size (N, N) predecessor matrix
  - unweighted** : bool, optional  
If True, then find unweighted distances. That is, rather than finding the path between each point such that the sum of weights is minimized, find the path such that the number of edges is minimized.
  - limit** : float, optional  
The maximum distance to calculate, must be  $\geq 0$ . Using a smaller limit will decrease computation time by aborting calculations between pairs that are separated by a distance  $> \text{limit}$ . For such pairs, the distance will be equal to `np.inf` (i.e., not connected).
- Returns**
- dist\_matrix** : ndarray  
New in version 0.14.0.  
The matrix of distances between graph nodes. `dist_matrix[i,j]` gives the shortest distance from point i to point j along the graph.
  - predecessors** : ndarray  
Returned only if `return_predecessors == True`. The matrix of predecessors, which can be used to reconstruct the shortest paths. Row i of the predecessor matrix contains information on the shortest paths from point i: each entry `predecessors[i, j]` gives the index of the previous node in the path from point i to point j. If no path exists between point i and j, then `predecessors[i, j] = -9999`

**Notes**

As currently implemented, Dijkstra's algorithm does not work for graphs with direction-dependent distances when `directed == False`. i.e., if `csgraph[i,j]` and `csgraph[j,i]` are not equal and both are nonzero, setting `directed=False` will not yield the correct result.

Also, this routine does not work for graphs with negative distances. Negative distances can lead to infinite cycles that must be handled by specialized algorithms such as Bellman-Ford's algorithm or Johnson's algorithm.

```
scipy.sparse.csgraph.floyd_warshall(csgraph, directed=True, return_predecessors=False, unweighted=False, overwrite=False)
```

Compute the shortest path lengths using the Floyd-Warshall algorithm

New in version 0.11.0.

- Parameters**
- csgraph** : array, matrix, or sparse matrix, 2 dimensions  
The N x N array of distances representing the input graph.
  - directed** : bool, optional  
If True (default), then find the shortest path on a directed graph: only move from point i to point j along paths csgraph[i, j]. If False, then find the shortest path on an undirected graph: the algorithm can progress from point i to j along csgraph[i, j] or csgraph[j, i]
  - return\_predecessors** : bool, optional  
If True, return the size (N, N) predecessor matrix
  - unweighted** : bool, optional  
If True, then find unweighted distances. That is, rather than finding the path between each point such that the sum of weights is minimized, find the path such that the number of edges is minimized.
  - overwrite** : bool, optional  
If True, overwrite csgraph with the result. This applies only if csgraph is a dense, c-ordered array with dtype=float64.
- Returns**
- dist\_matrix** : ndarray  
The N x N matrix of distances between graph nodes. dist\_matrix[i,j] gives the shortest distance from point i to point j along the graph.
  - predecessors** : ndarray  
Returned only if return\_predecessors == True. The N x N matrix of predecessors, which can be used to reconstruct the shortest paths. Row i of the predecessor matrix contains information on the shortest paths from point i: each entry predecessors[i, j] gives the index of the previous node in the path from point i to point j. If no path exists between point i and j, then predecessors[i, j] = -9999
- Raises**
- NegativeCycleError**:  
if there are negative cycles in the graph

```
scipy.sparse.csgraph.bellman_ford(csgraph, directed=True, indices=None,
                                return_predecessors=False, unweighted=False)
```

Compute the shortest path lengths using the Bellman-Ford algorithm.

The Bellman-ford algorithm can robustly deal with graphs with negative weights. If a negative cycle is detected, an error is raised. For graphs without negative edge weights, dijkstra's algorithm may be faster.

New in version 0.11.0.

- Parameters**
- csgraph** : array, matrix, or sparse matrix, 2 dimensions  
The N x N array of distances representing the input graph.
  - directed** : bool, optional  
If True (default), then find the shortest path on a directed graph: only move from point i to point j along paths csgraph[i, j]. If False, then find the shortest path on an undirected graph: the algorithm can progress from point i to j along csgraph[i, j] or csgraph[j, i]
  - indices** : array\_like or int, optional  
if specified, only compute the paths for the points at the given indices.
  - return\_predecessors** : bool, optional  
If True, return the size (N, N) predecessor matrix
  - unweighted** : bool, optional  
If True, then find unweighted distances. That is, rather than finding the path between each point such that the sum of weights is minimized, find the path such that the number of edges is minimized.
- Returns**
- dist\_matrix** : ndarray  
The N x N matrix of distances between graph nodes. dist\_matrix[i,j] gives the shortest distance from point i to point j along the graph.

**predecessors** : ndarray  
 Returned only if `return_predecessors == True`. The  $N \times N$  matrix of predecessors, which can be used to reconstruct the shortest paths. Row  $i$  of the predecessor matrix contains information on the shortest paths from point  $i$ : each entry `predecessors[i, j]` gives the index of the previous node in the path from point  $i$  to point  $j$ . If no path exists between point  $i$  and  $j$ , then `predecessors[i, j] = -9999`.

**Raises** **NegativeCycleError**:  
 if there are negative cycles in the graph

*Notes*

This routine is specially designed for graphs with negative edge weights. If all edge weights are positive, then Dijkstra's algorithm is a better choice.

`scipy.sparse.csgraph.johnson` (*csgraph*, *directed=True*, *indices=None*, *return\_predecessors=False*, *unweighted=False*)

Compute the shortest path lengths using Johnson's algorithm.

Johnson's algorithm combines the Bellman-Ford algorithm and Dijkstra's algorithm to quickly find shortest paths in a way that is robust to the presence of negative cycles. If a negative cycle is detected, an error is raised. For graphs without negative edge weights, `dijkstra()` may be faster.

New in version 0.11.0.

**Parameters**

- csgraph** : array, matrix, or sparse matrix, 2 dimensions  
 The  $N \times N$  array of distances representing the input graph.
- directed** : bool, optional  
 If True (default), then find the shortest path on a directed graph: only move from point  $i$  to point  $j$  along paths `csgraph[i, j]`. If False, then find the shortest path on an undirected graph: the algorithm can progress from point  $i$  to  $j$  along `csgraph[i, j]` or `csgraph[j, i]`
- indices** : array\_like or int, optional  
 if specified, only compute the paths for the points at the given indices.
- return\_predecessors** : bool, optional  
 If True, return the size  $(N, N)$  predecessor matrix
- unweighted** : bool, optional  
 If True, then find unweighted distances. That is, rather than finding the path between each point such that the sum of weights is minimized, find the path such that the number of edges is minimized.

**Returns**

- dist\_matrix** : ndarray  
 The  $N \times N$  matrix of distances between graph nodes. `dist_matrix[i,j]` gives the shortest distance from point  $i$  to point  $j$  along the graph.
- predecessors** : ndarray  
 Returned only if `return_predecessors == True`. The  $N \times N$  matrix of predecessors, which can be used to reconstruct the shortest paths. Row  $i$  of the predecessor matrix contains information on the shortest paths from point  $i$ : each entry `predecessors[i, j]` gives the index of the previous node in the path from point  $i$  to point  $j$ . If no path exists between point  $i$  and  $j$ , then `predecessors[i, j] = -9999`.

**Raises** **NegativeCycleError**:  
 if there are negative cycles in the graph

*Notes*

This routine is specially designed for graphs with negative edge weights. If all edge weights are positive, then Dijkstra's algorithm is a better choice.

`scipy.sparse.csgraph.breadth_first_order` (*csgraph*, *i\_start*, *directed=True*, *return\_predecessors=True*)

Return a breadth-first ordering starting with specified node.

Note that a breadth-first order is not unique, but the tree which it generates is unique.

New in version 0.11.0.

**Parameters**

- csgraph** : array\_like or sparse matrix  
The N x N compressed sparse graph. The input csgraph will be converted to csr format for the calculation.
- i\_start** : int  
The index of starting node.
- directed** : bool, optional  
If True (default), then operate on a directed graph: only move from point i to point j along paths csgraph[i, j]. If False, then find the shortest path on an undirected graph: the algorithm can progress from point i to j along csgraph[i, j] or csgraph[j, i].
- return\_predecessors** : bool, optional  
If True (default), then return the predecessor array (see below).

**Returns**

- node\_array** : ndarray, one dimension  
The breadth-first list of nodes, starting with specified node. The length of node\_array is the number of nodes reachable from the specified node.
- predecessors** : ndarray, one dimension  
Returned only if return\_predecessors is True. The length-N list of predecessors of each node in a breadth-first tree. If node i is in the tree, then its parent is given by predecessors[i]. If node i is not in the tree (and for the parent node) then predecessors[i] = -9999.

`scipy.sparse.csgraph.depth_first_order` (*csgraph*, *i\_start*, *directed=True*, *return\_predecessors=True*)

Return a depth-first ordering starting with specified node.

Note that a depth-first order is not unique. Furthermore, for graphs with cycles, the tree generated by a depth-first search is not unique either.

New in version 0.11.0.

**Parameters**

- csgraph** : array\_like or sparse matrix  
The N x N compressed sparse graph. The input csgraph will be converted to csr format for the calculation.
- i\_start** : int  
The index of starting node.
- directed** : bool, optional  
If True (default), then operate on a directed graph: only move from point i to point j along paths csgraph[i, j]. If False, then find the shortest path on an undirected graph: the algorithm can progress from point i to j along csgraph[i, j] or csgraph[j, i].
- return\_predecessors** : bool, optional  
If True (default), then return the predecessor array (see below).

**Returns**

- node\_array** : ndarray, one dimension  
The breadth-first list of nodes, starting with specified node. The length of node\_array is the number of nodes reachable from the specified node.
- predecessors** : ndarray, one dimension  
Returned only if return\_predecessors is True. The length-N list of predecessors of each node in a breadth-first tree. If node i is in the tree, then its parent is given by predecessors[i]. If node i is not in the tree (and for the parent node) then predecessors[i] = -9999.

`scipy.sparse.csgraph.breadth_first_tree` (*csgraph*, *i\_start*, *directed=True*)

Return the tree generated by a breadth-first search

Note that a breadth-first tree from a specified node is unique.

New in version 0.11.0.

**Parameters**

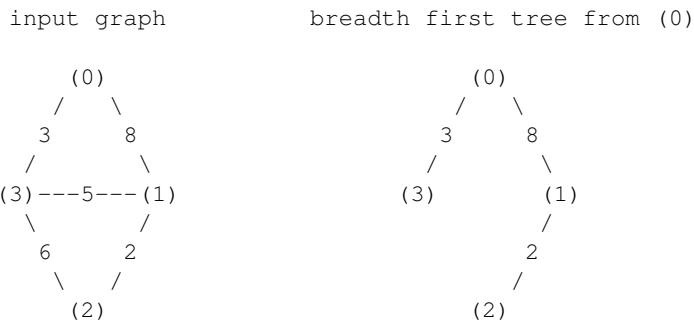
- csgraph** : array\_like or sparse matrix  
The N x N matrix representing the compressed sparse graph. The input csgraph will be converted to csr format for the calculation.
- i\_start** : int  
The index of starting node.
- directed** : bool, optional  
If True (default), then operate on a directed graph: only move from point i to point j along paths `csgraph[i, j]`. If False, then find the shortest path on an undirected graph: the algorithm can progress from point i to j along `csgraph[i, j]` or `csgraph[j, i]`.

**Returns**

- cstree** : csr matrix  
The N x N directed compressed-sparse representation of the breadth- first tree drawn from `csgraph`, starting at the specified node.

### Examples

The following example shows the computation of a depth-first tree over a simple four-component graph, starting at node 0:



In compressed sparse representation, the solution looks like this:

```
>>> from scipy.sparse import csr_matrix
>>> from scipy.sparse.csgraph import breadth_first_tree
>>> X = csr_matrix([[0, 8, 0, 3],
...                [0, 0, 2, 5],
...                [0, 0, 0, 6],
...                [0, 0, 0, 0]])
>>> Tcsr = breadth_first_tree(X, 0, directed=False)
>>> Tcsr.toarray().astype(int)
array([[0, 8, 0, 3],
       [0, 0, 2, 0],
       [0, 0, 0, 0],
       [0, 0, 0, 0]])
```

Note that the resulting graph is a Directed Acyclic Graph which spans the graph. A breadth-first tree from a given node is unique.

`scipy.sparse.csgraph.depth_first_tree` (*csgraph*, *i\_start*, *directed=True*)

Return a tree generated by a depth-first search.

Note that a tree generated by a depth-first search is not unique: it depends on the order that the children of each node are searched.

New in version 0.11.0.

**Parameters**

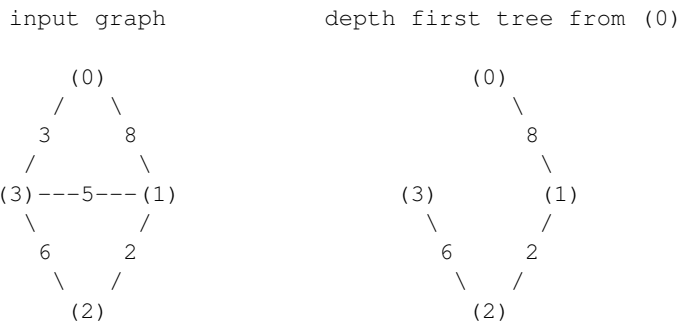
- csgraph** : array\_like or sparse matrix  
The N x N matrix representing the compressed sparse graph. The input csgraph will be converted to csr format for the calculation.
- i\_start** : int  
The index of starting node.
- directed** : bool, optional  
If True (default), then operate on a directed graph: only move from point i to point j along paths csgraph[i, j]. If False, then find the shortest path on an undirected graph: the algorithm can progress from point i to j along csgraph[i, j] or csgraph[j, i].

**Returns**

- csmtree** : csr matrix  
The N x N directed compressed-sparse representation of the depth- first tree drawn from csgraph, starting at the specified node.

### Examples

The following example shows the computation of a depth-first tree over a simple four-component graph, starting at node 0:



In compressed sparse representation, the solution looks like this:

```

>>> from scipy.sparse import csr_matrix
>>> from scipy.sparse.csgraph import depth_first_tree
>>> X = csr_matrix([[0, 8, 0, 3],
...                [0, 0, 2, 5],
...                [0, 0, 0, 6],
...                [0, 0, 0, 0]])
>>> Tcsr = depth_first_tree(X, 0, directed=False)
>>> Tcsr.toarray().astype(int)
array([[0, 8, 0, 0],
       [0, 0, 2, 0],
       [0, 0, 0, 6],
       [0, 0, 0, 0]])

```

Note that the resulting graph is a Directed Acyclic Graph which spans the graph. Unlike a breadth-first tree, a depth-first tree of a given graph is not unique if the graph contains cycles. If the above solution had begun with the edge connecting nodes 0 and 3, the result would have been different.

`scipy.sparse.csgraph.minimum_spanning_tree(csgraph, overwrite=False)`  
Return a minimum spanning tree of an undirected graph

A minimum spanning tree is a graph consisting of the subset of edges which together connect all connected nodes, while minimizing the total sum of weights on the edges. This is computed using the Kruskal algorithm.

New in version 0.11.0.

**Parameters**

- csgraph** : array\_like or sparse matrix, 2 dimensions  
The N x N matrix representing an undirected graph over N nodes (see notes below).
- overwrite** : bool, optional  
if true, then parts of the input graph will be overwritten for efficiency.

**Returns**

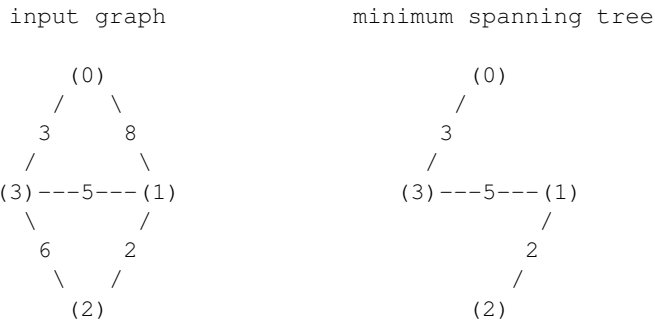
- span\_tree** : csr matrix  
The N x N compressed-sparse representation of the undirected minimum spanning tree over the input (see notes below).

**Notes**

This routine uses undirected graphs as input and output. That is, if graph[i, j] and graph[j, i] are both zero, then nodes i and j do not have an edge connecting them. If either is nonzero, then the two are connected by the minimum nonzero value of the two.

**Examples**

The following example shows the computation of a minimum spanning tree over a simple four-component graph:



It is easy to see from inspection that the minimum spanning tree involves removing the edges with weights 8 and 6. In compressed sparse representation, the solution looks like this:

```

>>> from scipy.sparse import csr_matrix
>>> from scipy.sparse.csgraph import minimum_spanning_tree
>>> X = csr_matrix([[0, 8, 0, 3],
...                [0, 0, 2, 5],
...                [0, 0, 0, 6],
...                [0, 0, 0, 0]])
>>> Tcsr = minimum_spanning_tree(X)
>>> Tcsr.toarray().astype(int)
array([[0, 0, 0, 3],
       [0, 0, 2, 5],
       [0, 0, 0, 0],
       [0, 0, 0, 0]])
    
```

`scipy.sparse.csgraph.reverse_cuthill_mckee()`

Returns the permutation array that orders a sparse CSR or CSC matrix in Reverse-Cuthill McKee ordering.

It is assumed by default, `symmetric_mode=False`, that the input matrix is not symmetric and works on the matrix  $A+A.T$ . If you are guaranteed that the matrix is symmetric in structure (values of matrix elements do not matter) then set `symmetric_mode=True`.



**Parameters** **graph** : sparse matrix  
Input sparse in CSC or CSR sparse matrix format.  
**symmetric\_mode** : bool, optional  
Is input matrix guaranteed to be symmetric.

**Returns** **perm** : ndarray  
Array of permuted row and column indices.

**Notes**

New in version 0.15.0.

**References**

E. Cuthill and J. McKee, “Reducing the Bandwidth of Sparse Symmetric Matrices”, ACM ‘69 Proceedings of the 1969 24th national conference, (1969).

`scipy.sparse.csgraph.maximum_bipartite_matching()`

Returns an array of row or column permutations that makes the diagonal of a nonsingular square CSC sparse matrix zero free.

Such a permutation is always possible provided that the matrix is nonsingular. This function looks at the structure of the matrix only. The input matrix will be converted to CSC matrix format if necessary.

**Parameters** **graph** : sparse matrix  
Input sparse in CSC format  
**perm\_type** : str, {‘row’, ‘column’}  
Type of permutation to generate.

**Returns** **perm** : ndarray  
Array of row or column permutations.

**Notes**

This function relies on a maximum cardinality bipartite matching algorithm based on a breadth-first search (BFS) of the underlying graph.

New in version 0.15.0.

**References**

I. S. Duff, K. Kaya, and B. Ucar, “Design, Implementation, and Analysis of Maximum Transversal Algorithms”, ACM Trans. Math. Softw. 38, no. 2, (2011).

**exception** `scipy.sparse.csgraph.NegativeCycleError`

---

<code>construct_dist_matrix(graph, predecessors[, ...])</code>	Construct distance matrix from a predecessor matrix
<code>csgraph_from_dense(graph[, null_value, ...])</code>	Construct a CSR-format sparse graph from a dense matrix.
<code>csgraph_from_masked(graph)</code>	Construct a CSR-format graph from a masked array.
<code>csgraph_masked_from_dense(graph[, ...])</code>	Construct a masked array graph representation from a dense matrix.
<code>csgraph_to_dense(csgraph[, null_value])</code>	Convert a sparse graph representation to a dense representation
<code>csgraph_to_masked(csgraph)</code>	Convert a sparse graph representation to a masked array representation
<code>reconstruct_path(csgraph, predecessors[, ...])</code>	Construct a tree from a graph and a predecessor list.

---

`scipy.sparse.csgraph.construct_dist_matrix(graph, predecessors, directed=True, null_value=np.inf)`

Construct distance matrix from a predecessor matrix

New in version 0.11.0.

**Parameters** **graph** : array\_like or sparse

The N x N matrix representation of a directed or undirected graph. If dense, then non-edges are indicated by zeros or infinities.

**predecessors** : array\_like

The N x N matrix of predecessors of each node (see Notes below).

**directed** : bool, optional

If True (default), then operate on a directed graph: only move from point i to point j along paths `csgraph[i, j]`. If False, then operate on an undirected graph: the algorithm can progress from point i to j along `csgraph[i, j]` or `csgraph[j, i]`.

**null\_value** : bool, optional

**Returns** **dist\_matrix** : ndarray  
value to use for distances between unconnected nodes. Default is `np.inf`

The N x N matrix of distances between nodes along the path specified by the predecessor matrix. If no path exists, the distance is zero.

**Notes**

The predecessor matrix is of the form returned by `graph_shortest_path`. Row i of the predecessor matrix contains information on the shortest paths from point i: each entry `predecessors[i, j]` gives the index of the previous node in the path from point i to point j. If no path exists between point i and j, then `predecessors[i, j] = -9999`

`scipy.sparse.csgraph.csgraph_from_dense` (*graph*, *null\_value=0*, *nan\_null=True*, *infinity\_null=True*)

Construct a CSR-format sparse graph from a dense matrix.

New in version 0.11.0.

**Parameters** **graph** : array\_like

Input graph. Shape should be (n\_nodes, n\_nodes).

**null\_value** : float or None (optional)

Value that denotes non-edges in the graph. Default is zero.

**infinity\_null** : bool

If True (default), then infinite entries (both positive and negative) are treated as null edges.

**nan\_null** : bool

**Returns** **csgraph** : csr\_matrix  
If True (default), then NaN entries are treated as non-edges

Compressed sparse representation of graph,

`scipy.sparse.csgraph.csgraph_from_masked` (*graph*)

Construct a CSR-format graph from a masked array.

New in version 0.11.0.

**Parameters** **graph** : MaskedArray

Input graph. Shape should be (n\_nodes, n\_nodes).

**Returns** **csgraph** : csr\_matrix

Compressed sparse representation of graph,

`scipy.sparse.csgraph.csgraph_masked_from_dense` (*graph*, *null\_value=0*, *nan\_null=True*, *infinity\_null=True*, *copy=True*)

Construct a masked array graph representation from a dense matrix.

New in version 0.11.0.

**Parameters** **graph** : array\_like

Input graph. Shape should be (n\_nodes, n\_nodes).

**null\_value** : float or None (optional)

Value that denotes non-edges in the graph. Default is zero.

**infinity\_null** : bool

If True (default), then infinite entries (both positive and negative) are treated as null edges.

**nan\_null** : bool

If True (default), then NaN entries are treated as non-edges

**Returns** **csgraph** : MaskedArray  
masked array representation of graph

`scipy.sparse.csgraph.csgraph_to_dense` (*csgraph*, *null\_value=0*)  
Convert a sparse graph representation to a dense representation

New in version 0.11.0.

**Parameters** **csgraph** : csr\_matrix, csc\_matrix, or lil\_matrix  
Sparse representation of a graph.

**null\_value** : float, optional  
The value used to indicate null edges in the dense representation. Default is 0.

**Returns** **graph** : ndarray  
The dense representation of the sparse graph.

### Notes

For normal sparse graph representations, calling `csgraph_to_dense` with `null_value=0` produces an equivalent result to using dense format conversions in the main sparse package. When the sparse representations have repeated values, however, the results will differ. The tools in `scipy.sparse` will add repeating values to obtain a final value. This function will select the minimum among repeating values to obtain a final value. For example, here we'll create a two-node directed sparse graph with multiple edges from node 0 to node 1, of weights 2 and 3. This illustrates the difference in behavior:

```
>>> from scipy.sparse import csr_matrix, csgraph
>>> data = np.array([2, 3])
>>> indices = np.array([1, 1])
>>> indptr = np.array([0, 2, 2])
>>> M = csr_matrix((data, indices, indptr), shape=(2, 2))
>>> M.toarray()
array([[0, 5],
       [0, 0]])
>>> csgraph.csgraph_to_dense(M)
array([[0., 2.],
       [0., 0.]])
```

The reason for this difference is to allow a compressed sparse graph to represent multiple edges between any two nodes. As most sparse graph algorithms are concerned with the single lowest-cost edge between any two nodes, the default `scipy.sparse` behavior of summing multiple weights does not make sense in this context.

The other reason for using this routine is to allow for graphs with zero-weight edges. Let's look at the example of a two-node directed graph, connected by an edge of weight zero:

```
>>> from scipy.sparse import csr_matrix, csgraph
>>> data = np.array([0.0])
>>> indices = np.array([1])
>>> indptr = np.array([0, 1, 1])
>>> M = csr_matrix((data, indices, indptr), shape=(2, 2))
>>> M.toarray()
array([[0, 0],
       [0, 0]])
>>> csgraph.csgraph_to_dense(M, np.inf)
array([[ inf,  0.],
       [ inf,  inf]])
```

In the first case, the zero-weight edge gets lost in the dense representation. In the second case, we can choose a different null value and see the true form of the graph.

`scipy.sparse.csgraph.csgraph_to_masked(csgraph)`  
 Convert a sparse graph representation to a masked array representation

New in version 0.11.0.

**Parameters** **csgraph** : csr\_matrix, csc\_matrix, or lil\_matrix  
 Sparse representation of a graph.  
**Returns** **graph** : MaskedArray  
 The masked dense representation of the sparse graph.

`scipy.sparse.csgraph.reconstruct_path(csgraph, predecessors, directed=True)`  
 Construct a tree from a graph and a predecessor list.

New in version 0.11.0.

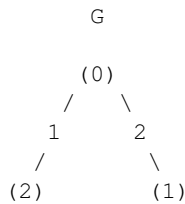
**Parameters** **csgraph** : array\_like or sparse matrix  
 The N x N matrix representing the directed or undirected graph from which the predecessors are drawn.  
**predecessors** : array\_like, one dimension  
 The length-N array of indices of predecessors for the tree. The index of the parent of node i is given by predecessors[i].  
**directed** : bool, optional  
 If True (default), then operate on a directed graph: only move from point i to point j along paths csgraph[i, j]. If False, then operate on an undirected graph: the algorithm can progress from point i to j along csgraph[i, j] or csgraph[j, i].  
**Returns** **cstree** : csr matrix  
 The N x N directed compressed-sparse representation of the tree drawn from csgraph which is encoded by the predecessor list.

### 5.23.2 Graph Representations

This module uses graphs which are stored in a matrix format. A graph with N nodes can be represented by an (N x N) adjacency matrix G. If there is a connection from node i to node j, then G[i, j] = w, where w is the weight of the connection. For nodes i and j which are not connected, the value depends on the representation:

- for dense array representations, non-edges are represented by G[i, j] = 0, infinity, or NaN.
- for dense masked representations (of type np.ma.MaskedArray), non-edges are represented by masked values. This can be useful when graphs with zero-weight edges are desired.
- for sparse array representations, non-edges are represented by non-entries in the matrix. This sort of sparse representation also allows for edges with zero weights.

As a concrete example, imagine that you would like to represent the following undirected graph:



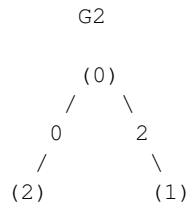
This graph has three nodes, where node 0 and 1 are connected by an edge of weight 2, and nodes 0 and 2 are connected by an edge of weight 1. We can construct the dense, masked, and sparse representations as follows, keeping in mind that an undirected graph is represented by a symmetric matrix:

```

>>> G_dense = np.array([[0, 2, 1],
...                     [2, 0, 0],
...                     [1, 0, 0]])
>>> G_masked = np.ma.masked_values(G_dense, 0)
>>> from scipy.sparse import csr_matrix
>>> G_sparse = csr_matrix(G_dense)

```

This becomes more difficult when zero edges are significant. For example, consider the situation when we slightly modify the above graph:



This is identical to the previous graph, except nodes 0 and 2 are connected by an edge of zero weight. In this case, the dense representation above leads to ambiguities: how can non-edges be represented if zero is a meaningful value? In this case, either a masked or sparse representation must be used to eliminate the ambiguity:

```

>>> G2_data = np.array([[np.inf, 2, 0],
...                    [2, np.inf, np.inf],
...                    [0, np.inf, np.inf]])
>>> G2_masked = np.ma.masked_invalid(G2_data)
>>> from scipy.sparse.csgraph import csgraph_from_dense
>>> # G2_sparse = csr_matrix(G2_data) would give the wrong result
>>> G2_sparse = csgraph_from_dense(G2_data, null_value=np.inf)
>>> G2_sparse.data
array([ 2.,  0.,  2.,  0.])

```

Here we have used a utility routine from the `csgraph` submodule in order to convert the dense representation to a sparse representation which can be understood by the algorithms in submodule. By viewing the data array, we can see that the zero values are explicitly encoded in the graph.

## Directed vs. Undirected

Matrices may represent either directed or undirected graphs. This is specified throughout the `csgraph` module by a boolean keyword. Graphs are assumed to be directed by default. In a directed graph, traversal from node  $i$  to node  $j$  can be accomplished over the edge  $G[i, j]$ , but not the edge  $G[j, i]$ . In a non-directed graph, traversal from node  $i$  to node  $j$  can be accomplished over either  $G[i, j]$  or  $G[j, i]$ . If both edges are not null, and the two have unequal weights, then the smaller of the two is used. Note that a symmetric matrix will represent an undirected graph, regardless of whether the ‘directed’ keyword is set to `True` or `False`. In this case, using `directed=True` generally leads to more efficient computation.

The routines in this module accept as input either `scipy.sparse` representations (`csr`, `csc`, or `lil` format), masked representations, or dense representations with non-edges indicated by zeros, infinities, and NaN entries.

## 5.24 Spatial algorithms and data structures (`scipy.spatial`)

### 5.24.1 Nearest-neighbor Queries

<code>KDTree(data[, leafsize])</code>	kd-tree for quick nearest-neighbor lookup
<code>cKDTree</code>	kd-tree for quick nearest-neighbor lookup
<code>distance</code>	
<code>Rectangle(maxes, mins)</code>	Hyperrectangle class.

**class** `scipy.spatial.KDTree` (*data*, *leafsize=10*)  
kd-tree for quick nearest-neighbor lookup

This class provides an index into a set of k-dimensional points which can be used to rapidly look up the nearest neighbors of any point.

**Parameters**    **data** : (N,K) array\_like  
The data points to be indexed. This array is not copied, and so modifying this data will result in bogus results.

**leafsize** : int, optional  
The number of points at which the algorithm switches over to brute-force. Has to be positive.

**Raises**         **RuntimeError**  
The maximum recursion limit can be exceeded for large data sets. If this happens, either increase the value for the *leafsize* parameter or increase the recursion limit by:

```
>>> import sys
>>> sys.setrecursionlimit(10000)
```

**See also:**

`cKDTree`    Implementation of `KDTree` in Cython

**Notes**

The algorithm used is described in Maneewongvatana and Mount 1999. The general idea is that the kd-tree is a binary tree, each of whose nodes represents an axis-aligned hyperrectangle. Each node specifies an axis and splits the set of points based on whether their coordinate along that axis is greater than or less than a particular value.

During construction, the axis and splitting point are chosen by the “sliding midpoint” rule, which ensures that the cells do not all become long and thin.

The tree can be queried for the *r* closest neighbors of any given point (optionally returning only those within some maximum distance of the point). It can also be queried, with a substantial gain in efficiency, for the *r* approximate closest neighbors.

For large dimensions (20 is already large) do not expect this to run significantly faster than brute force. High-dimensional nearest-neighbor queries are a substantial open problem in computer science.

The tree also supports all-neighbors queries, both with arrays of points and with other kd-trees. These do use a reasonably efficient algorithm, but the kd-tree is not necessarily the best data structure for this sort of calculation.

**Methods**

<code>count_neighbors(other, r[, p])</code>	Count how many nearby pairs can be formed.
<code>innernode(split_dim, split, less, greater)</code>	
<code>leafnode(idx)</code>	
<code>node</code>	
<code>query(x[, k, eps, p, distance_upper_bound])</code>	Query the kd-tree for nearest neighbors

Continued on next page

Table 5.212 – continued from previous page

<code>query_ball_point(x, r[, p, eps])</code>	Find all points within distance $r$ of point(s) $x$ .
<code>query_ball_tree(other, r[, p, eps])</code>	Find all pairs of points whose distance is at most $r$
<code>query_pairs(r[, p, eps])</code>	Find all pairs of points within a distance.
<code>sparse_distance_matrix(other, max_distance)</code>	Compute a sparse distance matrix

`KDTree.count_neighbors` (*other*,  $r$ ,  $p=2.0$ )

Count how many nearby pairs can be formed.

Count the number of pairs ( $x_1, x_2$ ) can be formed, with  $x_1$  drawn from self and  $x_2$  drawn from *other*, and where  $\text{distance}(x_1, x_2, p) \leq r$ . This is the “two-point correlation” described in Gray and Moore 2000, “N-body problems in statistical learning”, and the code here is based on their algorithm.

**Parameters**

- other** : KDTree instance  
The other tree to draw points from.
- r** : float or one-dimensional array of floats  
The radius to produce a count for. Multiple radii are searched with a single tree traversal.
- p** : float,  $1 \leq p \leq \text{infinity}$ , optional  
Which Minkowski  $p$ -norm to use

**Returns**

- result** : int or 1-D array of ints  
The number of pairs. Note that this is internally stored in a numpy int, and so may overflow if very large ( $2e9$ ).

`KDTree.query` ( $x$ ,  $k=1$ ,  $\text{eps}=0$ ,  $p=2$ ,  $\text{distance\_upper\_bound}=\text{inf}$ )

Query the kd-tree for nearest neighbors

**Parameters**

- x** : array\_like, last dimension self.m  
An array of points to query.
- k** : int, optional  
The number of nearest neighbors to return.
- eps** : nonnegative float, optional  
Return approximate nearest neighbors; the  $k$ th returned value is guaranteed to be no further than  $(1+\text{eps})$  times the distance to the real  $k$ th nearest neighbor.
- p** : float,  $1 \leq p \leq \text{infinity}$ , optional  
Which Minkowski  $p$ -norm to use. 1 is the sum-of-absolute-values “Manhattan” distance 2 is the usual Euclidean distance infinity is the maximum-coordinate-difference distance
- distance\_upper\_bound** : nonnegative float, optional  
Return only neighbors within this distance. This is used to prune tree searches, so if you are doing a series of nearest-neighbor queries, it may help to supply the distance to the nearest neighbor of the most recent point.

**Returns**

- d** : float or array of floats  
The distances to the nearest neighbors. If  $x$  has shape  $\text{tuple}+(\text{self.m},)$ , then  $d$  has shape  $\text{tuple}$  if  $k$  is one, or  $\text{tuple}+(k,)$  if  $k$  is larger than one. Missing neighbors (e.g. when  $k > n$  or  $\text{distance\_upper\_bound}$  is given) are indicated with infinite distances. If  $k$  is `None`, then  $d$  is an object array of shape  $\text{tuple}$ , containing lists of distances. In either case the hits are sorted by distance (nearest first).
- i** : integer or array of integers  
The locations of the neighbors in `self.data`.  $i$  is the same shape as  $d$ .

*Examples*

```
>>> from scipy import spatial
>>> x, y = np.mgrid[0:5, 2:8]
>>> tree = spatial.KDTree(list(zip(x.ravel(), y.ravel())))
>>> tree.data
array([[0, 2],
       [0, 3],
       [0, 4],
       [0, 5],
       [0, 6],
       [0, 7],
       [1, 2],
       [1, 3],
       [1, 4],
       [1, 5],
       [1, 6],
       [1, 7],
       [2, 2],
       [2, 3],
       [2, 4],
       [2, 5],
       [2, 6],
       [2, 7],
       [3, 2],
       [3, 3],
       [3, 4],
       [3, 5],
       [3, 6],
       [3, 7],
       [4, 2],
       [4, 3],
       [4, 4],
       [4, 5],
       [4, 6],
       [4, 7]])
>>> pts = np.array([[0, 0], [2.1, 2.9]])
>>> tree.query(pts)
(array([ 2.         ,  0.14142136]), array([ 0, 13]))
>>> tree.query(pts[0])
(2.0, 0)
```

`KDTree.query_ball_point` ( $x, r, p=2.0, eps=0$ )

Find all points within distance  $r$  of point(s)  $x$ .

**Parameters**

- x** : array\_like, shape tuple + (self.m,)
  - The point or points to search for neighbors of.
- r** : positive float
  - The radius of points to return.
- p** : float, optional
  - Which Minkowski  $p$ -norm to use. Should be in the range  $[1, \text{inf}]$ .
- eps** : nonnegative float, optional
  - Approximate search. Branches of the tree are not explored if their nearest points are further than  $r / (1 + \text{eps})$ , and branches are added in bulk if their furthest points are nearer than  $r * (1 + \text{eps})$ .

**Returns**

- results** : list or array of lists
  - If  $x$  is a single point, returns a list of the indices of the neighbors of



*x*. If *x* is an array of points, returns an object array of shape tuple containing lists of neighbors.

### Notes

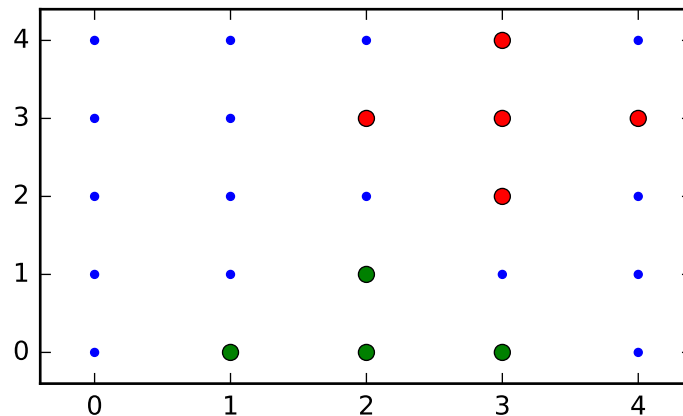
If you have many points whose neighbors you want to find, you may save substantial amounts of time by putting them in a KDTree and using `query_ball_tree`.

### Examples

```
>>> from scipy import spatial
>>> x, y = np.mgrid[0:5, 0:5]
>>> points = zip(x.ravel(), y.ravel())
>>> tree = spatial.KDTree(points)
>>> tree.query_ball_point([2, 0], 1)
[5, 10, 11, 15]
```

Query multiple points and plot the results:

```
>>> import matplotlib.pyplot as plt
>>> points = np.asarray(points)
>>> plt.plot(points[:,0], points[:,1], '.')
>>> for results in tree.query_ball_point([2, 0], [3, 3], 1):
...     nearby_points = points[results]
...     plt.plot(nearby_points[:,0], nearby_points[:,1], 'o')
>>> plt.margins(0.1, 0.1)
>>> plt.show()
```



`KDTree.query_ball_tree` (*other*, *r*, *p*=2.0, *eps*=0)

Find all pairs of points whose distance is at most *r*

**Parameters**

- other** : KDTree instance  
The tree containing points to search against.
- r** : float  
The maximum distance, has to be positive.
- p** : float, optional  
Which Minkowski norm to use. *p* has to meet the condition  $1 \leq p \leq \infty$ .

**eps** : float, optional

Approximate search. Branches of the tree are not explored if their nearest points are further than  $r / (1+eps)$ , and branches are added in bulk if their furthest points are nearer than  $r * (1+eps)$ . *eps* has to be non-negative.

**Returns** **results** : list of lists

For each element `self.data[i]` of this tree, `results[i]` is a list of the indices of its neighbors in `other.data`.

`KDTree.query_pairs` (*r*, *p*=2.0, *eps*=0)

Find all pairs of points within a distance.

**Parameters** **r** : positive float

The maximum distance.

**p** : float, optional

Which Minkowski norm to use. *p* has to meet the condition  $1 \leq p \leq \text{infinity}$ .

**eps** : float, optional

Approximate search. Branches of the tree are not explored if their nearest points are further than  $r / (1+eps)$ , and branches are added in bulk if their furthest points are nearer than  $r * (1+eps)$ . *eps* has to be non-negative.

**Returns** **results** : set

Set of pairs  $(i, j)$ , with  $i < j$ , for which the corresponding positions are close.

`KDTree.sparse_distance_matrix` (*other*, *max\_distance*, *p*=2.0)

Compute a sparse distance matrix

Computes a distance matrix between two KDTrees, leaving as zero any distance greater than *max\_distance*.

**Parameters** **other** : KDTree

**max\_distance** : positive float

**Returns** **result** : dok\_matrix

Sparse matrix representing the results in “dictionary of keys” format.

**class** `scipy.spatial.cKDTree`

kd-tree for quick nearest-neighbor lookup

This class provides an index into a set of k-dimensional points which can be used to rapidly look up the nearest neighbors of any point.

The algorithm used is described in Maneewongvatana and Mount 1999. The general idea is that the kd-tree is a binary trie, each of whose nodes represents an axis-aligned hyperrectangle. Each node specifies an axis and splits the set of points based on whether their coordinate along that axis is greater than or less than a particular value.

During construction, the axis and splitting point are chosen by the “sliding midpoint” rule, which ensures that the cells do not all become long and thin.

The tree can be queried for the *r* closest neighbors of any given point (optionally returning only those within some maximum distance of the point). It can also be queried, with a substantial gain in efficiency, for the *r* approximate closest neighbors.

For large dimensions (20 is already large) do not expect this to run significantly faster than brute force. High-dimensional nearest-neighbor queries are a substantial open problem in computer science.

**Parameters** **data** : array\_like, shape (n,m)

The  $n$  data points of dimension  $m$  to be indexed. This array is not copied unless this is necessary to produce a contiguous array of doubles, and so modifying this data will result in bogus results. The data are also copied if the kd-tree is built with `copy_data=True`.

**leafsize** : positive int, optional

The number of points at which the algorithm switches over to brute-force. Default: 16.

**compact\_nodes** : bool, optional

If True, the kd-tree is built to shrink the hyperrectangles to the actual data range. This usually gives a more compact tree and faster queries at the expense of longer build time. Default: True.

**copy\_data** : bool, optional

If True the data is always copied to protect the kd-tree against data corruption. Default: False.

**balanced\_tree** : bool, optional

If True, the median is used to split the hyperrectangles instead of the midpoint. This usually gives a more compact tree and faster queries at the expense of longer build time. Default: True.

**boxsize** : array\_like or scalar, optional

Apply a  $m$ -d toroidal topology to the KDTree.. The topology is generated by  $x_i + n_i L_i$  where  $n_i$  are integers and  $L_i$  is the boxsize along  $i$ -th dimension. The input data shall be wrapped into  $[0, L_i)$ . A `ValueError` is raised if any of the data is outside of this bound.

#### See also:

[KDTree](#) Implementation of `cKDTree` in pure Python

#### Attributes

---

```

boxsize
data
indices
leafsize
m
maxes
mins
n
tree

```

---

`cKDTree.boxesize`

`cKDTree.data`

`cKDTree.indices`

`cKDTree.leafsize`

`cKDTree.m`

`cKDTree.maxes`

`cKDTree.mins`

`cKDTree.n`

`cKDTree.tree`

*Methods*

---

<code>count_neighbors(self, other, r[, p])</code>	Count how many nearby pairs can be formed.
<code>query(self, x[, k, eps, p, ...])</code>	Query the kd-tree for nearest neighbors
<code>query_ball_point(self, x, r[, p, eps])</code>	Find all points within distance r of point(s) x.
<code>query_ball_tree(self, other, r[, p, eps])</code>	Find all pairs of points whose distance is at most r
<code>query_pairs(self, r[, p, eps])</code>	Find all pairs of points whose distance is at most r.
<code>sparse_distance_matrix(self, other, max_distance)</code>	Compute a sparse distance matrix

---

`cKDTree.count_neighbors` (*self, other, r, p=2.*)

Count how many nearby pairs can be formed.

Count the number of pairs (x1,x2) can be formed, with x1 drawn from self and x2 drawn from *other*, and where `distance(x1, x2, p) <= r`. This is the “two-point correlation” described in Gray and Moore 2000, “N-body problems in statistical learning”, and the code here is based on their algorithm.

**Parameters** **other** : cKDTree instance

The other tree to draw points from.

**r** : float or one-dimensional array of floats

The radius to produce a count for. Multiple radii are searched with a single tree traversal.

**p** : float, optional

**Returns** **result** : int or 1-D array of ints  
 $1 \leq p \leq \text{infinity}$ , default 2.0 Which Minkowski p-norm to use

The number of pairs.

`cKDTree.query` (*self, x, k=1, eps=0, p=2, distance\_upper\_bound=np.inf, n\_jobs=1*)

Query the kd-tree for nearest neighbors

**Parameters** **x** : array\_like, last dimension self.m

An array of points to query.

**k** : list of integer or integer

The list of k-th nearest neighbors to return. If k is an integer it is treated as a list of [1, ... k] (range(1, k+1)). Note that the counting starts from 1.

**eps** : non-negative float

Return approximate nearest neighbors; the k-th returned value is guaranteed to be no further than (1+eps) times the distance to the real k-th nearest neighbor.

**p** : float,  $1 \leq p \leq \text{infinity}$

Which Minkowski p-norm to use. 1 is the sum-of-absolute-values “Manhattan” distance 2 is the usual Euclidean distance infinity is the maximum-coordinate-difference distance

**distance\_upper\_bound** : nonnegative float

Return only neighbors within this distance. This is used to prune tree searches, so if you are doing a series of nearest-neighbor queries, it may help to supply the distance to the nearest neighbor of the most recent point.

**Returns**

**n\_jobs** : int, optional  
Number of jobs to schedule for parallel processing. If -1 is given all processors are used. Default: 1.

**d** : array of floats  
The distances to the nearest neighbors. If  $x$  has shape  $\text{tuple}+(\text{self.m},)$ , then  $d$  has shape  $\text{tuple}+(\text{len}(k),)$ . When  $k == 1$ , the last dimension of the output is squeezed. Missing neighbors are indicated with infinite distances.

**i** : ndarray of ints  
The locations of the neighbors in  $\text{self.data}$ . If  $x$  has shape  $\text{tuple}+(\text{self.m},)$ , then  $i$  has shape  $\text{tuple}+(\text{len}(k),)$ . When  $k == 1$ , the last dimension of the output is squeezed. Missing neighbors are indicated with  $\text{self.n}$ .

### Notes

If the KD-Tree is periodic, the position `py:code:'x'` is wrapped into the box.

When the input  $k$  is a list, a query for  $\text{arange}(\max(k))$  is performed, but only columns that store the requested values of  $k$  are preserved. This is implemented in a manner that reduces memory usage.

### Examples

```
>>> tree = cKDTree(data)
```

To query the nearest neighbours and return squeezed result, use

```
>>> dd, ii = tree.query(x, k=1)
```

To query the nearest neighbours and return unsqueezed result, use

```
>>> dd, ii = tree.query(x, k=[1])
```

To query the second nearest neighbours and return unsqueezed result, use

```
>>> dd, ii = tree.query(x, k=[2])
```

To query the first and second nearest neighbours, use

```
>>> dd, ii = tree.query(x, k=2)
```

or, be more specific

```
>>> dd, ii = tree.query(x, k=[1, 2])
```

`cKDTree.query_ball_point` (*self*,  $x$ ,  $r$ ,  $p=2$ ,  $\text{eps}=0$ )

Find all points within distance  $r$  of point(s)  $x$ .

**Parameters**

**x** : array\_like, shape  $\text{tuple} + (\text{self.m},)$   
The point or points to search for neighbors of.

**r** : positive float  
The radius of points to return.

**p** : float, optional  
Which Minkowski  $p$ -norm to use. Should be in the range  $[1, \text{inf}]$ .

**eps** : nonnegative float, optional

Approximate search. Branches of the tree are not explored if their nearest points are further than  $r / (1 + \text{eps})$ , and branches are added in bulk if their furthest points are nearer than  $r * (1 + \text{eps})$ .

**n\_jobs** : int, optional

Number of jobs to schedule for parallel processing. If -1 is given all processors are used. Default: 1.

**Returns** **results** : list or array of lists

If  $x$  is a single point, returns a list of the indices of the neighbors of  $x$ . If  $x$  is an array of points, returns an object array of shape tuple containing lists of neighbors.

**Notes**

If you have many points whose neighbors you want to find, you may save substantial amounts of time by putting them in a `cKDTree` and using `query_ball_tree`.

**Examples**

```
>>> from scipy import spatial
>>> x, y = np.mgrid[0:4, 0:4]
>>> points = zip(x.ravel(), y.ravel())
>>> tree = spatial.cKDTree(points)
>>> tree.query_ball_point([2, 0], 1)
[4, 8, 9, 12]
```

`cKDTree.query_ball_tree` (*self*, *other*, *r*, *p=2.*, *eps=0*)

Find all pairs of points whose distance is at most *r*

**Parameters**

**other** : cKDTree instance

The tree containing points to search against.

**r** : float

The maximum distance, has to be positive.

**p** : float, optional

Which Minkowski norm to use. *p* has to meet the condition  $1 \leq p \leq \text{infinity}$ .

**eps** : float, optional

Approximate search. Branches of the tree are not explored if their nearest points are further than  $r / (1 + \text{eps})$ , and branches are added in bulk if their furthest points are nearer than  $r * (1 + \text{eps})$ . *eps* has to be non-negative.

**Returns** **results** : list of lists

For each element `self.data[i]` of this tree, `results[i]` is a list of the indices of its neighbors in `other.data`.

`cKDTree.query_pairs` (*self*, *r*, *p=2.*, *eps=0*)

Find all pairs of points whose distance is at most *r*.

**Parameters**

**r** : positive float

The maximum distance.

**p** : float, optional

Which Minkowski norm to use. *p* has to meet the condition  $1 \leq p \leq \text{infinity}$ .

**eps** : float, optional

Approximate search. Branches of the tree are not explored if their nearest points are further than  $r / (1 + \text{eps})$ , and branches are added in bulk if their furthest points are nearer than  $r * (1 + \text{eps})$ . *eps* has to be non-negative.

**output\_type** : string, optional  
**Returns** **results** : set or ndarray  
 Choose the output container, 'set' or 'ndarray'. Default: 'set'  
 Set of pairs (i, j), with i < j, for which the corresponding positions are close. If output\_type is 'ndarray', an ndarray is returned instead of a set.

cKDTree.**sparse\_distance\_matrix**(self, other, max\_distance, p=2.)  
 Compute a sparse distance matrix

Computes a distance matrix between two cKDTrees, leaving as zero any distance greater than max\_distance.

**Parameters** **other** : cKDTree  
**max\_distance** : positive float  
**p** : float, 1<=p<=infinity  
 Which Minkowski p-norm to use.  
**output\_type** : string, optional  
 Which container to use for output data. Options: 'dok\_matrix', 'coo\_matrix', 'dict' or 'ndarray'. Default: 'dok\_matrix'.  
**Returns** **result** : dok\_matrix, coo\_matrix, dict or ndarray  
 Sparse matrix representing the results in "dictionary of keys" format. If a dict is returned the keys are (i,j) tuples of indices. If output\_type is 'ndarray' a record array with fields 'i', 'j', and 'k' is returned,

*Distance computations (scipy.spatial.distance)*

**Function Reference** Distance matrix computation from a collection of raw observation vectors stored in a rectangular array.

---

pdist(X[, metric, p, w, V, VI])	Pairwise distances between observations in n-dimensional space.
cdist(XA, XB[, metric, p, V, VI, w])	Computes distance between each pair of the two collections of inputs.
squareform(X[, force, checks])	Converts a vector-form distance vector to a square-form distance matrix, and vice-versa.

---

scipy.spatial.distance.**pdist**(X, metric='euclidean', p=2, w=None, V=None, VI=None)  
 Pairwise distances between observations in n-dimensional space.

The following are common calling conventions.

1.Y = pdist(X, 'euclidean')

Computes the distance between m points using Euclidean distance (2-norm) as the distance metric between the points. The points are arranged as m n-dimensional row vectors in the matrix X.

2.Y = pdist(X, 'minkowski', p)

Computes the distances using the Minkowski distance  $\|u - v\|_p$  (p-norm) where  $p \geq 1$ .

3.Y = pdist(X, 'cityblock')

Computes the city block or Manhattan distance between the points.

4.Y = pdist(X, 'seuclidean', V=None)

Computes the standardized Euclidean distance. The standardized Euclidean distance between two n-vectors u and v is

$$\sqrt{\sum (u_i - v_i)^2 / V[x_i]}$$

V is the variance vector; V[i] is the variance computed over all the i'th components of the points. If not passed, it is automatically computed.

5.Y = pdist(X, 'sqeuclidean')

Computes the squared Euclidean distance  $\|u - v\|_2^2$  between the vectors.

6.Y = pdist(X, 'cosine')

Computes the cosine distance between vectors  $u$  and  $v$ ,

$$1 - \frac{u \cdot v}{\|u\|_2 \|v\|_2}$$

where  $\|*\|_2$  is the 2-norm of its argument  $*$ , and  $u \cdot v$  is the dot product of  $u$  and  $v$ .

7.Y = pdist(X, 'correlation')

Computes the correlation distance between vectors  $u$  and  $v$ . This is

$$1 - \frac{(u - \bar{u}) \cdot (v - \bar{v})}{\|(u - \bar{u})\|_2 \|(v - \bar{v})\|_2}$$

where  $\bar{v}$  is the mean of the elements of vector  $v$ , and  $x \cdot y$  is the dot product of  $x$  and  $y$ .

8.Y = pdist(X, 'hamming')

Computes the normalized Hamming distance, or the proportion of those vector elements between two  $n$ -vectors  $u$  and  $v$  which disagree. To save memory, the matrix  $X$  can be of type boolean.

9.Y = pdist(X, 'jaccard')

Computes the Jaccard distance between the points. Given two vectors,  $u$  and  $v$ , the Jaccard distance is the proportion of those elements  $u[i]$  and  $v[i]$  that disagree.

10.Y = pdist(X, 'chebyshev')

Computes the Chebyshev distance between the points. The Chebyshev distance between two  $n$ -vectors  $u$  and  $v$  is the maximum norm-1 distance between their respective elements. More precisely, the distance is given by

$$d(u, v) = \max_i |u_i - v_i|$$

11.Y = pdist(X, 'canberra')

Computes the Canberra distance between the points. The Canberra distance between two points  $u$  and  $v$  is

$$d(u, v) = \sum_i \frac{|u_i - v_i|}{|u_i| + |v_i|}$$

12.Y = pdist(X, 'braycurtis')

Computes the Bray-Curtis distance between the points. The Bray-Curtis distance between two points  $u$  and  $v$  is

$$d(u, v) = \frac{\sum_i u_i - v_i}{\sum_i u_i + v_i}$$

13.Y = pdist(X, 'mahalanobis', VI=None)

Computes the Mahalanobis distance between the points. The Mahalanobis distance between two points  $u$  and  $v$  is  $(u - v)(1/V)(u - v)^T$  where  $(1/V)$  (the VI variable) is the inverse covariance. If VI is not None, VI will be used as the inverse covariance matrix.

14.Y = pdist(X, 'yule')

Computes the Yule distance between each pair of boolean vectors. (see yule function documentation)

15.Y = pdist(X, 'matching')

Synonym for 'hamming'.



```
16.Y = pdist(X, 'dice')
```

Computes the Dice distance between each pair of boolean vectors. (see dice function documentation)

```
17.Y = pdist(X, 'kulsinski')
```

Computes the Kulsinski distance between each pair of boolean vectors. (see kulsinski function documentation)

```
18.Y = pdist(X, 'rogerstanimoto')
```

Computes the Rogers-Tanimoto distance between each pair of boolean vectors. (see rogerstanimoto function documentation)

```
19.Y = pdist(X, 'russellrao')
```

Computes the Russell-Rao distance between each pair of boolean vectors. (see russellrao function documentation)

```
20.Y = pdist(X, 'sokalmichener')
```

Computes the Sokal-Michener distance between each pair of boolean vectors. (see sokalmichener function documentation)

```
21.Y = pdist(X, 'sokalsneath')
```

Computes the Sokal-Sneath distance between each pair of boolean vectors. (see sokalsneath function documentation)

```
22.Y = pdist(X, 'wminkowski')
```

Computes the weighted Minkowski distance between each pair of vectors. (see wminkowski function documentation)

```
23.Y = pdist(X, f)
```

Computes the distance between all pairs of vectors in X using the user supplied 2-arity function f. For example, Euclidean distance between the vectors could be computed as follows:

```
dm = pdist(X, lambda u, v: np.sqrt(((u-v)**2).sum()))
```

Note that you should avoid passing a reference to one of the distance functions defined in this library. For example,:

```
dm = pdist(X, sokalsneath)
```

would calculate the pair-wise distances between the vectors in X using the Python function sokalsneath. This would result in sokalsneath being called  $\binom{n}{2}$  times, which is inefficient. Instead, the optimized C version is more efficient, and we call it using the following syntax.:

```
dm = pdist(X, 'sokalsneath')
```

**Parameters** **X**: ndarray

An m by n array of m original observations in an n-dimensional space.

**metric**: str or function, optional

The distance metric to use. The distance function can be 'braycurtis', 'canberra', 'chebyshev', 'cityblock', 'correlation', 'cosine', 'dice', 'euclidean', 'hamming', 'jaccard', 'kulsinski', 'mahalanobis', 'matching', 'minkowski', 'rogerstanimoto', 'russellrao', 'seuclidean', 'sokalmichener', 'sokalsneath', 'sqeuclidean', 'yule'.

**w** : ndarray, optional  
The weight vector (for weighted Minkowski).

**p** : double, optional  
The p-norm to apply (for Minkowski, weighted and unweighted)

**V** : ndarray, optional  
The variance vector (for standardized Euclidean).

**VI** : ndarray, optional  
The inverse of the covariance matrix (for Mahalanobis).

**Returns** **Y** : ndarray  
Returns a condensed distance matrix Y. For each  $i$  and  $j$  (where  $i < j < n$ ), the metric `dist(u=X[i], v=X[j])` is computed and stored in entry  $ij$ .

**See also:**

[`squareform`](#) converts between condensed distance matrices and square distance matrices.

**Notes**

See `squareform` for information on how to calculate the index of this entry or to convert the condensed distance matrix to a redundant square matrix.

`scipy.spatial.distance.cdist(XA, XB, metric='euclidean', p=2, V=None, VI=None, w=None)`  
Computes distance between each pair of the two collections of inputs.

The following are common calling conventions:

1. `Y = cdist(XA, XB, 'euclidean')`

Computes the distance between  $m$  points using Euclidean distance (2-norm) as the distance metric between the points. The points are arranged as  $m$   $n$ -dimensional row vectors in the matrix  $X$ .

2. `Y = cdist(XA, XB, 'minkowski', p)`

Computes the distances using the Minkowski distance  $\|u - v\|_p$  ( $p$ -norm) where  $p \geq 1$ .

3. `Y = cdist(XA, XB, 'cityblock')`

Computes the city block or Manhattan distance between the points.

4. `Y = cdist(XA, XB, 'seuclidean', V=None)`

Computes the standardized Euclidean distance. The standardized Euclidean distance between two  $n$ -vectors  $u$  and  $v$  is

$$\sqrt{\sum (u_i - v_i)^2 / V[x_i]}$$

$V$  is the variance vector;  $V[i]$  is the variance computed over all the  $i$ 'th components of the points. If not passed, it is automatically computed.

5. `Y = cdist(XA, XB, 'sqeuclidean')`

Computes the squared Euclidean distance  $\|u - v\|_2^2$  between the vectors.

6. `Y = cdist(XA, XB, 'cosine')`

Computes the cosine distance between vectors  $u$  and  $v$ ,

$$1 - \frac{u \cdot v}{\|u\|_2 \|v\|_2}$$

where  $\|* \|_2$  is the 2-norm of its argument  $*$ , and  $u \cdot v$  is the dot product of  $u$  and  $v$ .

7. `Y = cdist(XA, XB, 'correlation')`

Computes the correlation distance between vectors  $u$  and  $v$ . This is

$$1 - \frac{(u - \bar{u}) \cdot (v - \bar{v})}{\|(u - \bar{u})\|_2 \|(v - \bar{v})\|_2}$$

where  $\bar{v}$  is the mean of the elements of vector  $v$ , and  $x \cdot y$  is the dot product of  $x$  and  $y$ .

8. `Y = cdist(XA, XB, 'hamming')`

Computes the normalized Hamming distance, or the proportion of those vector elements between two  $n$ -vectors  $u$  and  $v$  which disagree. To save memory, the matrix  $X$  can be of type boolean.

9. `Y = cdist(XA, XB, 'jaccard')`

Computes the Jaccard distance between the points. Given two vectors,  $u$  and  $v$ , the Jaccard distance is the proportion of those elements  $u[i]$  and  $v[i]$  that disagree where at least one of them is non-zero.

10. `Y = cdist(XA, XB, 'chebyshev')`

Computes the Chebyshev distance between the points. The Chebyshev distance between two  $n$ -vectors  $u$  and  $v$  is the maximum norm-1 distance between their respective elements. More precisely, the distance is given by

$$d(u, v) = \max_i |u_i - v_i|.$$

11. `Y = cdist(XA, XB, 'canberra')`

Computes the Canberra distance between the points. The Canberra distance between two points  $u$  and  $v$  is

$$d(u, v) = \sum_i \frac{|u_i - v_i|}{|u_i| + |v_i|}.$$

12. `Y = cdist(XA, XB, 'braycurtis')`

Computes the Bray-Curtis distance between the points. The Bray-Curtis distance between two points  $u$  and  $v$  is

$$d(u, v) = \frac{\sum_i (u_i - v_i)}{\sum_i (u_i + v_i)}$$

13. `Y = cdist(XA, XB, 'mahalanobis', VI=None)`

Computes the Mahalanobis distance between the points. The Mahalanobis distance between two points  $u$  and  $v$  is  $\sqrt{(u - v)(1/V)(u - v)^T}$  where  $(1/V)$  (the `VI` variable) is the inverse covariance. If `VI` is not `None`, `VI` will be used as the inverse covariance matrix.

14. `Y = cdist(XA, XB, 'yule')`

Computes the Yule distance between the boolean vectors. (see `yule` function documentation)

15. `Y = cdist(XA, XB, 'matching')`

Synonym for 'hamming'.

16. `Y = cdist(XA, XB, 'dice')`

Computes the Dice distance between the boolean vectors. (see `dice` function documentation)

17. `Y = cdist(XA, XB, 'kulsinski')`

Computes the Kulsinski distance between the boolean vectors. (see `kulsinski` function documentation)

18. `Y = cdist(XA, XB, 'rogerstanimoto')`

Computes the Rogers-Tanimoto distance between the boolean vectors. (see `rogerstanimoto` function documentation)

```
19.Y = cdist(XA, XB, 'russellrao')
```

Computes the Russell-Rao distance between the boolean vectors. (see `russellrao` function documentation)

```
20.Y = cdist(XA, XB, 'sokalmichener')
```

Computes the Sokal-Michener distance between the boolean vectors. (see `sokalmichener` function documentation)

```
21.Y = cdist(XA, XB, 'sokalsneath')
```

Computes the Sokal-Sneath distance between the vectors. (see `sokalsneath` function documentation)

```
22.Y = cdist(XA, XB, 'wminkowski')
```

Computes the weighted Minkowski distance between the vectors. (see `wminkowski` function documentation)

```
23.Y = cdist(XA, XB, f)
```

Computes the distance between all pairs of vectors in X using the user supplied 2-arity function f. For example, Euclidean distance between the vectors could be computed as follows:

```
dm = cdist(XA, XB, lambda u, v: np.sqrt(((u-v)**2).sum()))
```

Note that you should avoid passing a reference to one of the distance functions defined in this library. For example,:

```
dm = cdist(XA, XB, sokalsneath)
```

would calculate the pair-wise distances between the vectors in X using the Python function `sokalsneath`. This would result in `sokalsneath` being called  $\binom{n}{2}$  times, which is inefficient. Instead, the optimized C version is more efficient, and we call it using the following syntax:

```
dm = cdist(XA, XB, 'sokalsneath')
```

**Parameters** **XA** : ndarray

An  $m_A$  by  $n$  array of  $m_A$  original observations in an  $n$ -dimensional space. Inputs are converted to float type.

**XB** : ndarray

An  $m_B$  by  $n$  array of  $m_B$  original observations in an  $n$ -dimensional space. Inputs are converted to float type.

**metric** : str or callable, optional

The distance metric to use. If a string, the distance function can be 'braycurtis', 'canberra', 'chebyshev', 'cityblock', 'correlation', 'cosine', 'dice', 'euclidean', 'hamming', 'jaccard', 'kulsinski', 'mahalanobis', 'matching', 'minkowski', 'rogerstanimoto', 'russellrao', 'seuclidean', 'sokalmichener', 'sokalsneath', 'sqeuclidean', 'wminkowski', 'yule'.

**w** : ndarray, optional

The weight vector (for weighted Minkowski).

**p** : scalar, optional

The p-norm to apply (for Minkowski, weighted and unweighted)

**V** : ndarray, optional

The variance vector (for standardized Euclidean).

**VI** : ndarray, optional

The inverse of the covariance matrix (for Mahalanobis).

**Returns**

**Y** : ndarray

**Raises** **ValueError** A  $m_A$  by  $m_B$  distance matrix is returned. For each  $i$  and  $j$ , the metric `dist(u=XA[i], v=XB[j])` is computed and stored in the  $ij$  th entry. An exception is thrown if  $XA$  and  $XB$  do not have the same number of columns.

**Examples**

Find the Euclidean distances between four 2-D coordinates:

```
>>> from scipy.spatial import distance
>>> coords = [(35.0456, -85.2672),
...           (35.1174, -89.9711),
...           (35.9728, -83.9422),
...           (36.1667, -86.7833)]
>>> distance.cdist(coords, coords, 'euclidean')
array([[ 0.         ,  4.7044,  1.6172,  1.8856],
       [ 4.7044,  0.         ,  6.0893,  3.3561],
       [ 1.6172,  6.0893,  0.         ,  2.8477],
       [ 1.8856,  3.3561,  2.8477,  0.         ]])
```

Find the Manhattan distance from a 3-D point to the corners of the unit cube:

```
>>> a = np.array([[0, 0, 0],
...               [0, 0, 1],
...               [0, 1, 0],
...               [0, 1, 1],
...               [1, 0, 0],
...               [1, 0, 1],
...               [1, 1, 0],
...               [1, 1, 1]])
>>> b = np.array([[0.1, 0.2, 0.4]])
>>> distance.cdist(a, b, 'cityblock')
array([[ 0.7],
       [ 0.9],
       [ 1.3],
       [ 1.5],
       [ 1.5],
       [ 1.7],
       [ 2.1],
       [ 2.3]])
```

`scipy.spatial.distance.squareform(X, force='no', checks=True)`

Converts a vector-form distance vector to a square-form distance matrix, and vice-versa.

**Parameters** **X** : ndarray

Either a condensed or redundant distance matrix.

**force** : str, optional

As with MATLAB(TM), if `force` is equal to 'tovector' or 'tomatrix', the input will be treated as a distance matrix or distance vector respectively.

**checks** : bool, optional

If `checks` is set to False, no checks will be made for matrix symmetry nor zero diagonals. This is useful if it is known that  $X - X.T$  is small and  $\text{diag}(X)$  is close to zero. These values are ignored any way so they do not disrupt the squareform transformation.

**Returns** **Y** : ndarray

If a condensed distance matrix is passed, a redundant one is returned, or if a redundant one is passed, a condensed distance matrix is returned.

*Notes*

1. `v = squareform(X)`

Given a square  $d$ -by- $d$  symmetric distance matrix  $X$ , `v=squareform(X)` returns a  $d * (d-1) / 2$  (or  $\binom{n}{2}$ ) sized vector  $v$ .

$v[\binom{2}{n}-\binom{2}{n-i} + (j-i-1)]$  is the distance between points  $i$  and  $j$ . If  $X$  is non-square or asymmetric, an error is returned.

2. `X = squareform(v)`

Given a  $d*(d-1)/2$  sized  $v$  for some integer  $d \geq 2$  encoding distances as described, `X=squareform(v)` returns a  $d$  by  $d$  distance matrix  $X$ . The  $X[i, j]$  and  $X[j, i]$  values are set to  $v[\binom{2}{n}-\binom{2}{n-i} + (j-i-1)]$  and all diagonal elements are zero.

Predicates for checking the validity of distance matrices, both condensed and redundant. Also contained in this module are functions for computing the number of observations in a distance matrix.

---

<code>is_valid_dm(D[, tol, throw, name, warning])</code>	Returns True if input array is a valid distance matrix.
<code>is_valid_y(y[, warning, throw, name])</code>	Returns True if the input array is a valid condensed distance matrix.
<code>num_obs_dm(d)</code>	Returns the number of original observations that correspond to a square, redundant distance matrix.
<code>num_obs_y(Y)</code>	Returns the number of original observations that correspond to a condensed distance matrix.

---

`scipy.spatial.distance.is_valid_dm(D, tol=0.0, throw=False, name='D', warning=False)`

Returns True if input array is a valid distance matrix.

Distance matrices must be 2-dimensional numpy arrays containing doubles. They must have a zero-diagonal, and they must be symmetric.

**Parameters** **D** : ndarray

The candidate object to test for validity.

**tol** : float, optional

The distance matrix should be symmetric. *tol* is the maximum difference between entries  $i, j$  and  $j, i$  for the distance metric to be considered symmetric.

**throw** : bool, optional

An exception is thrown if the distance matrix passed is not valid.

**name** : str, optional

The name of the variable to checked. This is useful if *throw* is set to True so the offending variable can be identified in the exception message when an exception is thrown.

**warning** : bool, optional

**Returns** **valid** : bool Instead of throwing an exception, a warning message is raised.

True if the variable *D* passed is a valid distance matrix.

*Notes*

Small numerical differences in *D* and *D.T* and non-zerosness of the diagonal are ignored if they are within the tolerance specified by *tol*.

`scipy.spatial.distance.is_valid_y(y, warning=False, throw=False, name=None)`

Returns True if the input array is a valid condensed distance matrix.

Condensed distance matrices must be 1-dimensional numpy arrays containing doubles. Their length must be a binomial coefficient  $\binom{n}{2}$  for some positive integer  $n$ .

**Parameters** **y** : ndarray  
The condensed distance matrix.

**warning** : bool, optional  
Invokes a warning if the variable passed is not a valid condensed distance matrix. The warning message explains why the distance matrix is not valid. `name` is used when referencing the offending variable.

**throw** : bool, optional  
Throws an exception if the variable passed is not a valid condensed distance matrix.

**name** : bool, optional  
Used when referencing the offending variable in the warning or exception message.

`scipy.spatial.distance.num_obs_dm(d)`

Returns the number of original observations that correspond to a square, redundant distance matrix.

**Parameters** **d** : ndarray  
The target distance matrix.

**Returns** **num\_obs\_dm** : int  
The number of observations in the redundant distance matrix.

`scipy.spatial.distance.num_obs_y(Y)`

Returns the number of original observations that correspond to a condensed distance matrix.

**Parameters** **Y** : ndarray  
Condensed distance matrix.

**Returns** **n** : int  
The number of observations in the condensed distance matrix *Y*.

Distance functions between two numeric vectors *u* and *v*. Computing distances over a large collection of vectors is inefficient for these functions. Use `pdist` for this purpose.

---

<code>braycurtis(u, v)</code>	Computes the Bray-Curtis distance between two 1-D arrays.
<code>canberra(u, v)</code>	Computes the Canberra distance between two 1-D arrays.
<code>chebyshev(u, v)</code>	Computes the Chebyshev distance.
<code>cityblock(u, v)</code>	Computes the City Block (Manhattan) distance.
<code>correlation(u, v)</code>	Computes the correlation distance between two 1-D arrays.
<code>cosine(u, v)</code>	Computes the Cosine distance between 1-D arrays.
<code>euclidean(u, v)</code>	Computes the Euclidean distance between two 1-D arrays.
<code>mahalanobis(u, v, VI)</code>	Computes the Mahalanobis distance between two 1-D arrays.
<code>minkowski(u, v, p)</code>	Computes the Minkowski distance between two 1-D arrays.
<code>seuclidean(u, v, V)</code>	Returns the standardized Euclidean distance between two 1-D arrays.
<code>squeuclidean(u, v)</code>	Computes the squared Euclidean distance between two 1-D arrays.
<code>wminkowski(u, v, p, w)</code>	Computes the weighted Minkowski distance between two 1-D arrays.

---

`scipy.spatial.distance.braycurtis(u, v)`

Computes the Bray-Curtis distance between two 1-D arrays.

Bray-Curtis distance is defined as

$$\sum |u_i - v_i| / \sum |u_i + v_i|$$

The Bray-Curtis distance is in the range [0, 1] if all coordinates are positive, and is undefined if the inputs are of length zero.

**Parameters** **u** : (N,) array\_like  
Input array.

**v** : (N,) array\_like

**Returns** **braycurtis** : double  
Input array.  
 The Bray-Curtis distance between 1-D arrays  $u$  and  $v$ .

`scipy.spatial.distance.canberra` ( $u, v$ )  
 Computes the Canberra distance between two 1-D arrays.

The Canberra distance is defined as

$$d(u, v) = \sum_i \frac{|u_i - v_i|}{|u_i| + |v_i|}.$$

**Parameters** **u** : (N,) array\_like  
Input array.  
**v** : (N,) array\_like

**Returns** **canberra** : double  
Input array.  
 The Canberra distance between vectors  $u$  and  $v$ .

**Notes**

When  $u[i]$  and  $v[i]$  are 0 for given  $i$ , then the fraction  $0/0 = 0$  is used in the calculation.

`scipy.spatial.distance.chebyshev` ( $u, v$ )  
 Computes the Chebyshev distance.

Computes the Chebyshev distance between two 1-D arrays  $u$  and  $v$ , which is defined as

$$\max_i |u_i - v_i|.$$

**Parameters** **u** : (N,) array\_like  
Input vector.  
**v** : (N,) array\_like

**Returns** **chebyshev** : double  
Input vector.  
 The Chebyshev distance between vectors  $u$  and  $v$ .

`scipy.spatial.distance.cityblock` ( $u, v$ )  
 Computes the City Block (Manhattan) distance.

Computes the Manhattan distance between two 1-D arrays  $u$  and  $v$ , which is defined as

$$\sum_i |u_i - v_i|.$$

**Parameters** **u** : (N,) array\_like  
Input array.  
**v** : (N,) array\_like

**Returns** **cityblock** : double  
Input array.  
 The City Block (Manhattan) distance between vectors  $u$  and  $v$ .

`scipy.spatial.distance.correlation` ( $u, v$ )  
 Computes the correlation distance between two 1-D arrays.

The correlation distance between  $u$  and  $v$ , is defined as

$$1 - \frac{(u - \bar{u}) \cdot (v - \bar{v})}{\|(u - \bar{u})\|_2 \|(v - \bar{v})\|_2}$$

where  $\bar{u}$  is the mean of the elements of  $u$  and  $x \cdot y$  is the dot product of  $x$  and  $y$ .

**Parameters** **u** : (N,) array\_like



**Returns** **correlation** : double  
 Input array.  
 $v$  : (N,) array\_like  
 Input array.  
 The correlation distance between 1-D array  $u$  and  $v$ .

`scipy.spatial.distance.cosine` ( $u, v$ )  
 Computes the Cosine distance between 1-D arrays.

The Cosine distance between  $u$  and  $v$ , is defined as

$$1 - \frac{u \cdot v}{\|u\|_2 \|v\|_2}.$$

where  $u \cdot v$  is the dot product of  $u$  and  $v$ .

**Parameters**  $u$  : (N,) array\_like  
 Input array.  
 $v$  : (N,) array\_like  
 Input array.  
**Returns** **cosine** : double  
 Input array.  
 The Cosine distance between vectors  $u$  and  $v$ .

`scipy.spatial.distance.euclidean` ( $u, v$ )  
 Computes the Euclidean distance between two 1-D arrays.

The Euclidean distance between 1-D arrays  $u$  and  $v$ , is defined as

$$\|u - v\|_2$$

**Parameters**  $u$  : (N,) array\_like  
 Input array.  
 $v$  : (N,) array\_like  
 Input array.  
**Returns** **euclidean** : double  
 Input array.  
 The Euclidean distance between vectors  $u$  and  $v$ .

`scipy.spatial.distance.mahalanobis` ( $u, v, VI$ )  
 Computes the Mahalanobis distance between two 1-D arrays.

The Mahalanobis distance between 1-D arrays  $u$  and  $v$ , is defined as

$$\sqrt{(u - v)V^{-1}(u - v)^T}$$

where  $V$  is the covariance matrix. Note that the argument  $VI$  is the inverse of  $V$ .

**Parameters**  $u$  : (N,) array\_like  
 Input array.  
 $v$  : (N,) array\_like  
 Input array.  
 $VI$  : ndarray  
 The inverse of the covariance matrix.  
**Returns** **mahalanobis** : double  
 The Mahalanobis distance between vectors  $u$  and  $v$ .

`scipy.spatial.distance.minkowski` ( $u, v, p$ )  
 Computes the Minkowski distance between two 1-D arrays.

The Minkowski distance between 1-D arrays  $u$  and  $v$ , is defined as

$$\|u - v\|_p = \left( \sum |u_i - v_i|^p \right)^{1/p}.$$

**Parameters**  $u$  : (N,) array\_like

**Parameters**  
**v** : (N,) array\_like  
 Input array.  
**p** : int  
 Input array.  
**Returns**  
**d** : double  
 The order of the norm of the difference  $\|u - v\|_p$ .  
 The Minkowski distance between vectors  $u$  and  $v$ .

`scipy.spatial.distance.seuclidean(u, v, V)`  
 Returns the standardized Euclidean distance between two 1-D arrays.  
 The standardized Euclidean distance between  $u$  and  $v$ .

**Parameters**  
**u** : (N,) array\_like  
 Input array.  
**v** : (N,) array\_like  
 Input array.  
**V** : (N,) array\_like  
 $V$  is an 1-D array of component variances. It is usually computed among a larger collection vectors.  
**Returns**  
**seuclidean** : double  
 The standardized Euclidean distance between vectors  $u$  and  $v$ .

`scipy.spatial.distance.sqeuclidean(u, v)`  
 Computes the squared Euclidean distance between two 1-D arrays.  
 The squared Euclidean distance between  $u$  and  $v$  is defined as

$$\|u - v\|_2^2.$$

**Parameters**  
**u** : (N,) array\_like  
 Input array.  
**v** : (N,) array\_like  
 Input array.  
**Returns**  
**sqeuclidean** : double  
 The squared Euclidean distance between vectors  $u$  and  $v$ .

`scipy.spatial.distance.wminkowski(u, v, p, w)`  
 Computes the weighted Minkowski distance between two 1-D arrays.  
 The weighted Minkowski distance between  $u$  and  $v$ , defined as

$$\left( \sum (|w_i(u_i - v_i)|^p) \right)^{1/p}.$$

**Parameters**  
**u** : (N,) array\_like  
 Input array.  
**v** : (N,) array\_like  
 Input array.  
**p** : int  
 The order of the norm of the difference  $\|u - v\|_p$ .  
**w** : (N,) array\_like  
 The weight vector.  
**Returns**  
**wminkowski** : double  
 The weighted Minkowski distance between vectors  $u$  and  $v$ .

Distance functions between two boolean vectors (representing sets)  $u$  and  $v$ . As in the case of numerical vectors, `pdist` is more efficient for computing the distances between all pairs.

<code>dice(u, v)</code>	Computes the Dice dissimilarity between two boolean 1-D arrays.
Continued on next page	

Table 5.218 – continued from previous page

<code>hamming(u, v)</code>	Computes the Hamming distance between two 1-D arrays.
<code>jaccard(u, v)</code>	Computes the Jaccard-Needham dissimilarity between two boolean 1-D arrays.
<code>kulsinski(u, v)</code>	Computes the Kulsinski dissimilarity between two boolean 1-D arrays.
<code>matching(u, v)</code>	Computes the Hamming distance between two boolean 1-D arrays.
<code>rogerstanimoto(u, v)</code>	Computes the Rogers-Tanimoto dissimilarity between two boolean 1-D arrays.
<code>russellrao(u, v)</code>	Computes the Russell-Rao dissimilarity between two boolean 1-D arrays.
<code>sokalmichener(u, v)</code>	Computes the Sokal-Michener dissimilarity between two boolean 1-D arrays.
<code>sokalsneath(u, v)</code>	Computes the Sokal-Sneath dissimilarity between two boolean 1-D arrays.
<code>yule(u, v)</code>	Computes the Yule dissimilarity between two boolean 1-D arrays.

`scipy.spatial.distance.dice(u, v)`

Computes the Dice dissimilarity between two boolean 1-D arrays.

The Dice dissimilarity between  $u$  and  $v$ , is

$$\frac{c_{TF} + c_{FT}}{2c_{TT} + c_{FT} + c_{TF}}$$

where  $c_{ij}$  is the number of occurrences of  $u[k] = i$  and  $v[k] = j$  for  $k < n$ .

**Parameters** **u** : (N,) ndarray, bool  
Input 1-D array.  
**v** : (N,) ndarray, bool  
Input 1-D array.

**Returns** **dice** : double  
The Dice dissimilarity between 1-D arrays  $u$  and  $v$ .

`scipy.spatial.distance.hamming(u, v)`

Computes the Hamming distance between two 1-D arrays.

The Hamming distance between 1-D arrays  $u$  and  $v$ , is simply the proportion of disagreeing components in  $u$  and  $v$ . If  $u$  and  $v$  are boolean vectors, the Hamming distance is

$$\frac{c_{01} + c_{10}}{n}$$

where  $c_{ij}$  is the number of occurrences of  $u[k] = i$  and  $v[k] = j$  for  $k < n$ .

**Parameters** **u** : (N,) array\_like  
Input array.  
**v** : (N,) array\_like  
Input array.

**Returns** **hamming** : double  
The Hamming distance between vectors  $u$  and  $v$ .

`scipy.spatial.distance.jaccard(u, v)`

Computes the Jaccard-Needham dissimilarity between two boolean 1-D arrays.

The Jaccard-Needham dissimilarity between 1-D boolean arrays  $u$  and  $v$ , is defined as

$$\frac{c_{TF} + c_{FT}}{c_{TT} + c_{FT} + c_{TF}}$$

where  $c_{ij}$  is the number of occurrences of  $u[k] = i$  and  $v[k] = j$  for  $k < n$ .

**Parameters** **u** : (N,) array\_like, bool  
Input array.  
**v** : (N,) array\_like, bool  
Input array.

**Returns** **jaccard** : double

The Jaccard distance between vectors  $u$  and  $v$ .

`scipy.spatial.distance.kulsinski` ( $u, v$ )

Computes the Kulsinski dissimilarity between two boolean 1-D arrays.

The Kulsinski dissimilarity between two boolean 1-D arrays  $u$  and  $v$ , is defined as

$$\frac{c_{TF} + c_{FT} - c_{TT} + n}{c_{FT} + c_{TF} + n}$$

where  $c_{ij}$  is the number of occurrences of  $u[k] = i$  and  $v[k] = j$  for  $k < n$ .

**Parameters** **u** : (N,) array\_like, bool  
Input array.

**v** : (N,) array\_like, bool

**Returns** **kulsinski** : double

The Kulsinski distance between vectors  $u$  and  $v$ .

`scipy.spatial.distance.matching` ( $u, v$ )

Computes the Hamming distance between two boolean 1-D arrays.

This is a deprecated synonym for `hamming`.

`scipy.spatial.distance.rogerstanimoto` ( $u, v$ )

Computes the Rogers-Tanimoto dissimilarity between two boolean 1-D arrays.

The Rogers-Tanimoto dissimilarity between two boolean 1-D arrays  $u$  and  $v$ , is defined as

$$\frac{R}{c_{TT} + c_{FF} + R}$$

where  $c_{ij}$  is the number of occurrences of  $u[k] = i$  and  $v[k] = j$  for  $k < n$  and  $R = 2(c_{TF} + c_{FT})$ .

**Parameters** **u** : (N,) array\_like, bool  
Input array.

**v** : (N,) array\_like, bool

**Returns** **rogerstanimoto** : double

The Rogers-Tanimoto dissimilarity between vectors  $u$  and  $v$ .

`scipy.spatial.distance.russellrao` ( $u, v$ )

Computes the Russell-Rao dissimilarity between two boolean 1-D arrays.

The Russell-Rao dissimilarity between two boolean 1-D arrays,  $u$  and  $v$ , is defined as

$$\frac{n - c_{TT}}{n}$$

where  $c_{ij}$  is the number of occurrences of  $u[k] = i$  and  $v[k] = j$  for  $k < n$ .

**Parameters** **u** : (N,) array\_like, bool  
Input array.

**v** : (N,) array\_like, bool

**Returns** **russellrao** : double

The Russell-Rao dissimilarity between vectors  $u$  and  $v$ .

`scipy.spatial.distance.sokalmichener` ( $u, v$ )

Computes the Sokal-Michener dissimilarity between two boolean 1-D arrays.

The Sokal-Michener dissimilarity between boolean 1-D arrays  $u$  and  $v$ , is defined as

$$\frac{R}{S + R}$$

where  $c_{ij}$  is the number of occurrences of  $u[k] = i$  and  $v[k] = j$  for  $k < n$ ,  $R = 2 * (c_{TF} + c_{FT})$  and  $S = c_{FF} + c_{TT}$ .

**Parameters** **u** : (N,) array\_like, bool  
                   Input array.  
**v** : (N,) array\_like, bool  
                   Input array.  
**Returns** **sokalmichener** : double  
                   The Sokal-Michener dissimilarity between vectors  $u$  and  $v$ .

`scipy.spatial.distance.sokalsneath(u, v)`  
 Computes the Sokal-Sneath dissimilarity between two boolean 1-D arrays.  
 The Sokal-Sneath dissimilarity between  $u$  and  $v$ ,

$$\frac{R}{c_{TT} + R}$$

where  $c_{ij}$  is the number of occurrences of  $u[k] = i$  and  $v[k] = j$  for  $k < n$  and  $R = 2(c_{TF} + c_{FT})$ .

**Parameters** **u** : (N,) array\_like, bool  
                   Input array.  
**v** : (N,) array\_like, bool  
                   Input array.  
**Returns** **sokalsneath** : double  
                   The Sokal-Sneath dissimilarity between vectors  $u$  and  $v$ .

`scipy.spatial.distance.yule(u, v)`  
 Computes the Yule dissimilarity between two boolean 1-D arrays.  
 The Yule dissimilarity is defined as

$$\frac{R}{c_{TT} * c_{FF} + \frac{R}{2}}$$

where  $c_{ij}$  is the number of occurrences of  $u[k] = i$  and  $v[k] = j$  for  $k < n$  and  $R = 2.0 * c_{TF} * c_{FT}$ .

**Parameters** **u** : (N,) array\_like, bool  
                   Input array.  
**v** : (N,) array\_like, bool  
                   Input array.  
**Returns** **yule** : double  
                   The Yule dissimilarity between vectors  $u$  and  $v$ .

`hamming` also operates over discrete numerical vectors.

**Functions**

<code>braycurtis(u, v)</code>	Computes the Bray-Curtis distance between two 1-D arrays.
<code>callable(object) -&gt; bool</code>	Return whether the object is callable (i.e., some kind of function).
<code>canberra(u, v)</code>	Computes the Canberra distance between two 1-D arrays.
<code>cdist(XA, XB[, metric, p, V, VI, w])</code>	Computes distance between each pair of the two collections of inputs.
<code>cdist_fn</code>	
<code>chebyshev(u, v)</code>	Computes the Chebyshev distance.
<code>cityblock(u, v)</code>	Computes the City Block (Manhattan) distance.
<code>correlation(u, v)</code>	Computes the correlation distance between two 1-D arrays.
<code>cosine(u, v)</code>	Computes the Cosine distance between 1-D arrays.
<code>dice(u, v)</code>	Computes the Dice dissimilarity between two boolean 1-D arrays.
<code>euclidean(u, v)</code>	Computes the Euclidean distance between two 1-D arrays.
<code>hamming(u, v)</code>	Computes the Hamming distance between two 1-D arrays.

Continue

Table 5.219 – continued from previous page

<code>is_valid_dm(D[, tol, throw, name, warning])</code>	Returns True if input array is a valid distance matrix.
<code>is_valid_y(y[, warning, throw, name])</code>	Returns True if the input array is a valid condensed distance matrix.
<code>jaccard(u, v)</code>	Computes the Jaccard-Needham dissimilarity between two boolean 1-D arrays.
<code>kulsinski(u, v)</code>	Computes the Kulsinski dissimilarity between two boolean 1-D arrays.
<code>mahalanobis(u, v, VI)</code>	Computes the Mahalanobis distance between two 1-D arrays.
<code>matching(u, v)</code>	Computes the Hamming distance between two boolean 1-D arrays.
<code>minkowski(u, v, p)</code>	Computes the Minkowski distance between two 1-D arrays.
<code>norm(a[, ord, axis, keepdims])</code>	Matrix or vector norm.
<code>num_obs_dm(d)</code>	Returns the number of original observations that correspond to a square, redundant distance matrix.
<code>num_obs_y(Y)</code>	Returns the number of original observations that correspond to a condensed distance matrix.
<code>pdist(X[, metric, p, w, V, VI])</code>	Pairwise distances between observations in n-dimensional space.
<code>pdist_fn</code>	
<code>rogerstanimoto(u, v)</code>	Computes the Rogers-Tanimoto dissimilarity between two boolean 1-D arrays.
<code>russellrao(u, v)</code>	Computes the Russell-Rao dissimilarity between two boolean 1-D arrays.
<code>seuclidean(u, v, V)</code>	Returns the standardized Euclidean distance between two 1-D arrays.
<code>sokalmichener(u, v)</code>	Computes the Sokal-Michener dissimilarity between two boolean 1-D arrays.
<code>sokalsneath(u, v)</code>	Computes the Sokal-Sneath dissimilarity between two boolean 1-D arrays.
<code>squeuclidean(u, v)</code>	Computes the squared Euclidean distance between two 1-D arrays.
<code>squareform(X[, force, checks])</code>	Converts a vector-form distance vector to a square-form distance matrix, and vice-versa.
<code>wminkowski(u, v, p, w)</code>	Computes the weighted Minkowski distance between two 1-D arrays.
<code>yule(u, v)</code>	Computes the Yule dissimilarity between two boolean 1-D arrays.

### Classes

---

`xrange`    `xrange(stop)` -> xrange object

---

**class** `scipy.spatial.Rectangle` (*maxes, mins*)

Hyperrectangle class.

Represents a Cartesian product of intervals.

#### Methods

<code>max_distance_point(x[, p])</code>	Return the maximum distance between input and points in the hyperrectangle.
<code>max_distance_rectangle(other[, p])</code>	Compute the maximum distance between points in the two hyperrectangles.
<code>min_distance_point(x[, p])</code>	Return the minimum distance between input and points in the hyperrectangle.
<code>min_distance_rectangle(other[, p])</code>	Compute the minimum distance between points in the two hyperrectangles.
<code>split(d, split)</code>	Produce two hyperrectangles by splitting.
<code>volume()</code>	Total volume.

`Rectangle.max_distance_point` (*x, p=2.0*)

Return the maximum distance between input and points in the hyperrectangle.

**Parameters**

- x** : array\_like  
Input array.
- p** : float, optional  
Input.

`Rectangle.max_distance_rectangle` (*other, p=2.0*)

Compute the maximum distance between points in the two hyperrectangles.

**Parameters**

- other** : hyperrectangle  
Input.

**p** : float, optional  
Input.

Rectangle.**min\_distance\_point** (*x*, *p=2.0*)

Return the minimum distance between input and points in the hyperrectangle.

**Parameters** **x** : array\_like  
Input.  
**p** : float, optional  
Input.

Rectangle.**min\_distance\_rectangle** (*other*, *p=2.0*)

Compute the minimum distance between points in the two hyperrectangles.

**Parameters** **other** : hyperrectangle  
Input.  
**p** : float  
Input.

Rectangle.**split** (*d*, *split*)

Produce two hyperrectangles by splitting.

In general, if you need to compute maximum and minimum distances to the children, it can be done more efficiently by updating the maximum and minimum distances to the parent.

**Parameters** **d** : int  
Axis to split hyperrectangle along.  
**split** : float  
Position along axis *d* to split at.

Rectangle.**volume** ()  
Total volume.

## 5.24.2 Delaunay Triangulation, Convex Hulls and Voronoi Diagrams

<code>Delaunay(points[, furthest_site, ...])</code>	Delaunay tessellation in N dimensions.
<code>ConvexHull(points[, incremental, qhull_options])</code>	Convex hulls in N dimensions.
<code>Voronoi(points[, furthest_site, ...])</code>	Voronoi diagrams in N dimensions.
<code>SphericalVoronoi(points[, radius, center])</code>	Voronoi diagrams on the surface of a sphere.

**class** `scipy.spatial.Delaunay` (*points*, *furthest\_site=False*, *incremental=False*, *qhull\_options=None*)  
Delaunay tessellation in N dimensions.

New in version 0.9.

**Parameters** **points** : ndarray of floats, shape (npoints, ndim)  
Coordinates of points to triangulate  
**furthest\_site** : bool, optional  
Whether to compute a furthest-site Delaunay triangulation. Default: False  
New in version 0.12.0.  
**incremental** : bool, optional  
Allow adding new points incrementally. This takes up some additional resources.  
**qhull\_options** : str, optional  
Additional options to pass to Qhull. See Qhull manual for details. Option “Qt” is always enabled. Default: “Qbb Qc Qz Qx Q12” for ndim > 4 and “Qbb Qc Qz Q12” otherwise. Incremental mode omits “Qz”.

<i>Raises</i>	<b>QhullError</b>	New in version 0.12.0. Raised when Qhull encounters an error condition, such as geometrical degeneracy when options to resolve are not enabled.
	<b>ValueError</b>	Raised if an incompatible array is given as input.

*Notes*

The tessellation is computed using the Qhull library [Qhull library](#).

---

**Note:** Unless you pass in the Qhull option “QJ”, Qhull does not guarantee that each input point appears as a vertex in the Delaunay triangulation. Omitted points are listed in the *coplanar* attribute.

---

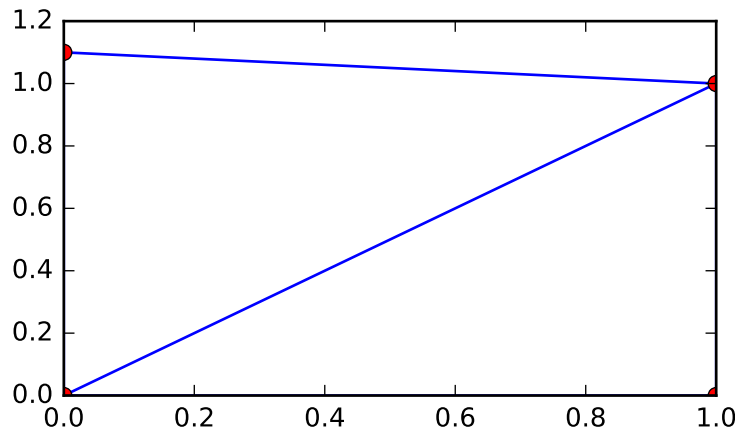
*Examples*

Triangulation of a set of points:

```
>>> points = np.array([[0, 0], [0, 1.1], [1, 0], [1, 1]])
>>> from scipy.spatial import Delaunay
>>> tri = Delaunay(points)
```

We can plot it:

```
>>> import matplotlib.pyplot as plt
>>> plt.triplot(points[:,0], points[:,1], tri.simplices.copy())
>>> plt.plot(points[:,0], points[:,1], 'o')
>>> plt.show()
```



Point indices and coordinates for the two triangles forming the triangulation:

```
>>> tri.simplices
array([[2, 3, 0],          # may vary
       [3, 1, 0]], dtype=int32)
```

Note that depending on how rounding errors go, the simplices may be in a different order than above.



```
>>> points[tri.simplices]
array([[ 1. ,  0. ],          # may vary
       [ 1. ,  1. ],
       [ 0. ,  0. ]],
      [[ 1. ,  1. ],
       [ 0. ,  1.1],
       [ 0. ,  0. ]]])
```

Triangle 0 is the only neighbor of triangle 1, and it's opposite to vertex 1 of triangle 1:

```
>>> tri.neighbors[1]
array([-1,  0, -1], dtype=int32)
>>> points[tri.simplices[1,1]]
array([ 0. ,  1.1])
```

We can find out which triangle points are in:

```
>>> p = np.array([(0.1, 0.2), (1.5, 0.5)])
>>> tri.find_simplex(p)
array([ 1, -1], dtype=int32)
```

We can also compute barycentric coordinates in triangle 1 for these points:

```
>>> b = tri.transform[1,:2].dot(p - tri.transform[1,2])
>>> np.c_[b, 1 - b.sum(axis=1)]
array([[ 0.1          ,  0.2          ,  0.7          ],
       [ 1.27272727,  0.27272727, -0.54545455]])
```

The coordinates for the first point are all positive, meaning it is indeed inside the triangle.

### Attributes

<code>transform</code>	Affine transform from $x$ to the barycentric coordinates $c$ .
<code>vertex_to_simplex</code>	Lookup array, from a vertex, to some simplex which it is a part of.
<code>convex_hull</code>	Vertices of facets forming the convex hull of the point set.
<code>vertex_neighbor_vertices</code>	Neighboring vertices of vertices.

### Delaunay.`transform`

Affine transform from  $x$  to the barycentric coordinates  $c$ .

**Type** ndarray of double, shape (nsimplex, ndim+1, ndim)

This is defined by:

$$T c = x - r$$

At vertex  $j$ ,  $c_j = 1$  and the other coordinates zero.

For simplex  $i$ , `transform[i,:ndim,:ndim]` contains inverse of the matrix  $T$ , and `transform[i,ndim,:]` contains the vector  $r$ .

### Delaunay.`vertex_to_simplex`

Lookup array, from a vertex, to some simplex which it is a part of.

**Type** ndarray of int, shape (npoints,)

Delaunay.**convex\_hull**

Vertices of facets forming the convex hull of the point set.

**Type** ndarray of int, shape (nfaces, ndim)

The array contains the indices of the points belonging to the (N-1)-dimensional facets that form the convex hull of the triangulation.

---

**Note:** Computing convex hulls via the Delaunay triangulation is inefficient and subject to increased numerical instability. Use [ConvexHull](#) instead.

---

Delaunay.**vertex\_neighbor\_vertices**

Neighboring vertices of vertices.

Tuple of two ndarrays of int: (indices, indptr). The indices of neighboring vertices of vertex *k* are `indptr[indices[k]:indices[k+1]]`.

points	(ndarray of double, shape (npoints, ndim)) Coordinates of input points.
simplices	(ndarray of ints, shape (nsimplex, ndim+1)) Indices of the points forming the simplices in the triangulation. For 2-D, the points are oriented counterclockwise.
neighbors	(ndarray of ints, shape (nsimplex, ndim+1)) Indices of neighbor simplices for each simplex. The <i>k</i> th neighbor is opposite to the <i>k</i> th vertex. For simplices at the boundary, -1 denotes no neighbor.
equations	(ndarray of double, shape (nsimplex, ndim+2)) [normal, offset] forming the hyperplane equation of the facet on the paraboloid (see <a href="#">Qhull documentation</a> for more).
paraboloid_scale, paraboloid_shift	(float) Scale and shift for the extra paraboloid dimension (see <a href="#">Qhull documentation</a> for more).
coplanar	(ndarray of int, shape (ncoplanar, 3)) Indices of coplanar points and the corresponding indices of the nearest facet and the nearest vertex. Coplanar points are input points which were <i>not</i> included in the triangulation due to numerical precision issues. If option “Qc” is not specified, this list is not computed. .. versionadded:: 0.12.0
vertices	Same as <i>simplices</i> , but deprecated.

**Methods**

---

<code>add_points(points[, restart])</code>	Process a set of additional new points.
<code>close()</code>	Finish incremental processing.
<code>find_simplex(self, xi[, bruteforce, tol])</code>	Find the simplices containing the given points.
<code>lift_points(self, x)</code>	Lift points to the Qhull paraboloid.
<code>plane_distance(self, xi)</code>	Compute hyperplane distances to the point <i>xi</i> from all simplices.

---

Delaunay.**add\_points** (*points*, *restart=False*)

Process a set of additional new points.

**Parameters**

- points** : ndarray  
New points to add. The dimensionality should match that of the initial points.
- restart** : bool, optional  
Whether to restart processing from scratch, rather than adding points incrementally.

**Raises**

- QhullError**  
Raised when Qhull encounters an error condition, such as geometrical degeneracy when options to resolve are not enabled.

**See also:**

`close`

*Notes*

You need to specify `incremental=True` when constructing the object to be able to add points incrementally. Incremental addition of points is also not possible after `close` has been called.

`Delaunay.close()`

Finish incremental processing.

Call this to free resources taken up by Qhull, when using the incremental mode. After calling this, adding more points is no longer possible.

`Delaunay.find_simplex(self, xi, bruteforce=False, tol=None)`

Find the simplices containing the given points.

**Parameters**

- tri** : `DelaunayInfo`  
Delaunay triangulation
- xi** : `ndarray` of `double`, shape `(..., ndim)`  
Points to locate
- bruteforce** : `bool`, optional  
Whether to only perform a brute-force search
- tol** : `float`, optional  
Tolerance allowed in the inside-triangle check. Default is `100*eps`.

**Returns**

- i** : `ndarray` of `int`, same shape as `xi`  
Indices of simplices containing each point. Points outside the triangulation get the value -1.

*Notes*

This uses an algorithm adapted from Qhull's `qh_findbestfacet`, which makes use of the connection between a convex hull and a Delaunay triangulation. After finding the simplex closest to the point in N+1 dimensions, the algorithm falls back to directed search in N dimensions.

`Delaunay.lift_points(self, x)`

Lift points to the Qhull paraboloid.

`Delaunay.plane_distance(self, xi)`

Compute hyperplane distances to the point `xi` from all simplices.

**class** `scipy.spatial.ConvexHull(points, incremental=False, qhull_options=None)`

Convex hulls in N dimensions.

New in version 0.12.0.

**Parameters**

- points** : `ndarray` of `floats`, shape `(npoints, ndim)`  
Coordinates of points to construct a convex hull from
- incremental** : `bool`, optional  
Allow adding new points incrementally. This takes up some additional resources.
- qhull\_options** : `str`, optional  
Additional options to pass to Qhull. See Qhull manual for details. (Default: "Qx" for `ndim > 4` and "" otherwise) Option "Qt" is always enabled.

**Raises**

- QhullError**  
Raised when Qhull encounters an error condition, such as geometrical degeneracy when options to resolve are not enabled.
- ValueError**  
Raised if an incompatible array is given as input.

*Notes*

The convex hull is computed using the [Qhull library](#).

## References

[Qhull]

## Examples

Convex hull of a random set of points:

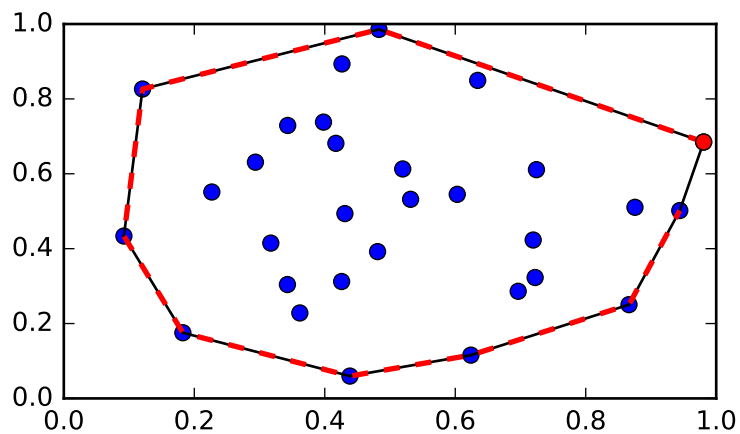
```
>>> from scipy.spatial import ConvexHull
>>> points = np.random.rand(30, 2) # 30 random points in 2-D
>>> hull = ConvexHull(points)
```

Plot it:

```
>>> import matplotlib.pyplot as plt
>>> plt.plot(points[:,0], points[:,1], 'o')
>>> for simplex in hull.simplices:
...     plt.plot(points[simplex, 0], points[simplex, 1], 'k-')
```

We could also have directly used the vertices of the hull, which for 2-D are guaranteed to be in counterclockwise order:

```
>>> plt.plot(points[hull.vertices,0], points[hull.vertices,1], 'r--', lw=2)
>>> plt.plot(points[hull.vertices[0],0], points[hull.vertices[0],1], 'ro')
>>> plt.show()
```



**Attributes**

points	(ndarray of double, shape (npoints, ndim)) Coordinates of input points.
vertices	(ndarray of ints, shape (nvertices,)) Indices of points forming the vertices of the convex hull. For 2-D convex hulls, the vertices are in counterclockwise order. For other dimensions, they are in input order.
simplices	(ndarray of ints, shape (nfacet, ndim)) Indices of points forming the simplicial facets of the convex hull.
neighbors	(ndarray of ints, shape (nfacet, ndim)) Indices of neighbor facets for each facet. The kth neighbor is opposite to the kth vertex. -1 denotes no neighbor.
equations	(ndarray of double, shape (nfacet, ndim+1)) [normal, offset] forming the hyperplane equation of the facet (see <a href="#">Qhull documentation</a> for more).
coplanar	(ndarray of int, shape (ncoplanar, 3)) Indices of coplanar points and the corresponding indices of the nearest facets and nearest vertex indices. Coplanar points are input points which were <i>not</i> included in the triangulation due to numerical precision issues. If option “Qc” is not specified, this list is not computed.
area	(float) Area of the convex hull
volume	(float) Volume of the convex hull

**Methods**

<code>add_points(points[, restart])</code>	Process a set of additional new points.
<code>close()</code>	Finish incremental processing.

`ConvexHull.add_points(points, restart=False)`

Process a set of additional new points.

**Parameters** **points** : ndarray

New points to add. The dimensionality should match that of the initial points.

**restart** : bool, optional

Whether to restart processing from scratch, rather than adding points incrementally.

**Raises** **QhullError**

Raised when Qhull encounters an error condition, such as geometrical degeneracy when options to resolve are not enabled.

**See also:**

`close`

**Notes**

You need to specify `incremental=True` when constructing the object to be able to add points incrementally. Incremental addition of points is also not possible after `close` has been called.

`ConvexHull.close()`

Finish incremental processing.

Call this to free resources taken up by Qhull, when using the incremental mode. After calling this, adding more points is no longer possible.

**class** `scipy.spatial.Voronoi(points, furthest_site=False, incremental=False, qhull_options=None)`  
 Voronoi diagrams in N dimensions.

New in version 0.12.0.

**Parameters** **points** : ndarray of floats, shape (npoints, ndim)

		Coordinates of points to construct a convex hull from
	<b>furthest_site</b> : bool, optional	Whether to compute a furthest-site Voronoi diagram. Default: False
	<b>incremental</b> : bool, optional	Allow adding new points incrementally. This takes up some additional resources.
	<b>qhull_options</b> : str, optional	Additional options to pass to Qhull. See Qhull manual for details. (Default: “Qbb Qc Qz Qx” for ndim > 4 and “Qbb Qc Qz” otherwise. Incremental mode omits “Qz”.)
<b>Raises</b>	<b>QhullError</b>	Raised when Qhull encounters an error condition, such as geometrical degeneracy when options to resolve are not enabled.
	<b>ValueError</b>	Raised if an incompatible array is given as input.

### Notes

The Voronoi diagram is computed using the [Qhull library](#).

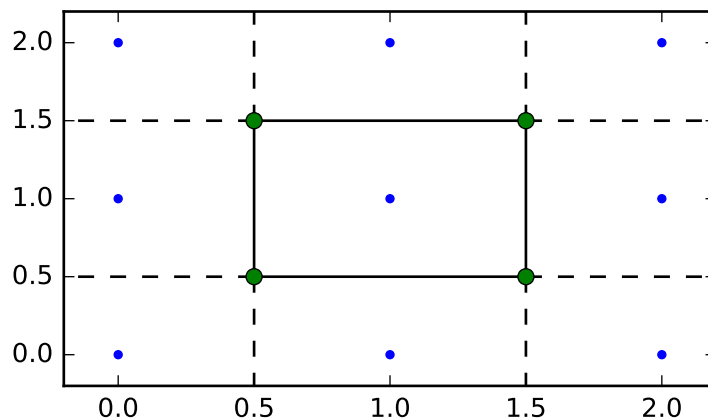
### Examples

Voronoi diagram for a set of point:

```
>>> points = np.array([[0, 0], [0, 1], [0, 2], [1, 0], [1, 1], [1, 2],
...                    [2, 0], [2, 1], [2, 2]])
>>> from scipy.spatial import Voronoi, voronoi_plot_2d
>>> vor = Voronoi(points)
```

Plot it:

```
>>> import matplotlib.pyplot as plt
>>> voronoi_plot_2d(vor)
>>> plt.show()
```



The Voronoi vertices:

```
>>> vor.vertices
array([[ 0.5,  0.5],
       [ 1.5,  0.5],
       [ 0.5,  1.5],
       [ 1.5,  1.5]])
```

There is a single finite Voronoi region, and four finite Voronoi ridges:

```
>>> vor.regions
[[], [-1, 0], [-1, 1], [1, -1, 0], [3, -1, 2], [-1, 3], [-1, 2], [3, 2, 0, 1], [2, -1, 0], [3, -
>>> vor.ridge_vertices
[[-1, 0], [-1, 0], [-1, 1], [-1, 1], [0, 1], [-1, 3], [-1, 2], [2, 3], [-1, 3], [-1, 2], [0, 2],
```

The ridges are perpendicular between lines drawn between the following input points:

```
>>> vor.ridge_points
array([[0, 1],
       [0, 3],
       [6, 3],
       [6, 7],
       [3, 4],
       [5, 8],
       [5, 2],
       [5, 4],
       [8, 7],
       [2, 1],
       [4, 1],
       [4, 7]], dtype=int32)
```

### Attributes

points	(ndarray of double, shape (npoints, ndim)) Coordinates of input points.
vertices	(ndarray of double, shape (nvertices, ndim)) Coordinates of the Voronoi vertices.
ridge_points	(ndarray of ints, shape (nridges, 2)) Indices of the points between which each Voronoi ridge lies.
ridge_vertices	(list of list of ints, shape (nridges, *)) Indices of the Voronoi vertices forming each Voronoi ridge.
regions	(list of list of ints, shape (nregions, *)) Indices of the Voronoi vertices forming each Voronoi region. -1 indicates vertex outside the Voronoi diagram.
point_region	(list of ints, shape (npoints)) Index of the Voronoi region for each input point. If qhull option "Qc" was not specified, the list will contain -1 for points that are not associated with a Voronoi region.

### Methods

<code>add_points(points[, restart])</code>	Process a set of additional new points.
<code>close()</code>	Finish incremental processing.

`Voronoi.add_points` (*points*, *restart=False*)

Process a set of additional new points.

**Parameters**    **points** : ndarray

New points to add. The dimensionality should match that of the initial points.

**restart** : bool, optional

*Raises*

**QhullError**

Whether to restart processing from scratch, rather than adding points incrementally.

Raised when Qhull encounters an error condition, such as geometrical degeneracy when options to resolve are not enabled.

**See also:**

`close`

*Notes*

You need to specify `incremental=True` when constructing the object to be able to add points incrementally. Incremental addition of points is also not possible after `close` has been called.

`Voronoi.close()`

Finish incremental processing.

Call this to free resources taken up by Qhull, when using the incremental mode. After calling this, adding more points is no longer possible.

**class** `scipy.spatial.SphericalVoronoi` (*points*, *radius=None*, *center=None*)

Voronoi diagrams on the surface of a sphere.

New in version 0.18.0.

*Parameters*

**points** : ndarray of floats, shape (npoints, 3)

Coordinates of points to construct a spherical Voronoi diagram from

**radius** : float, optional

Radius of the sphere (Default: 1)

**center** : ndarray of floats, shape (3,)

Center of sphere (Default: origin)

**See also:**

[Voronoi](#) Conventional Voronoi diagrams in N dimensions.

*Notes*

The spherical Voronoi diagram algorithm proceeds as follows. The Convex Hull of the input points (generators) is calculated, and is equivalent to their Delaunay triangulation on the surface of the sphere [Caroli]. A 3D Delaunay tetrahedralization is obtained by including the origin of the coordinate system as the fourth vertex of each simplex of the Convex Hull. The circumcenters of all tetrahedra in the system are calculated and projected to the surface of the sphere, producing the Voronoi vertices. The Delaunay tetrahedralization neighbour information is then used to order the Voronoi region vertices around each generator. The latter approach is substantially less sensitive to floating point issues than angle-based methods of Voronoi region vertex sorting.

The surface area of spherical polygons is calculated by decomposing them into triangles and using L'Huilier's Theorem to calculate the spherical excess of each triangle [Weisstein]. The sum of the spherical excesses is multiplied by the square of the sphere radius to obtain the surface area of the spherical polygon. For nearly-degenerate spherical polygons an area of approximately 0 is returned by default, rather than attempting the unstable calculation.

Empirical assessment of spherical Voronoi algorithm performance suggests quadratic time complexity (loglinear is optimal, but algorithms are more challenging to implement). The reconstitution of the surface area of the sphere, measured as the sum of the surface areas of all Voronoi regions, is closest to 100 % for larger (>> 10) numbers of generators.

*References*

[Caroli], [Weisstein]

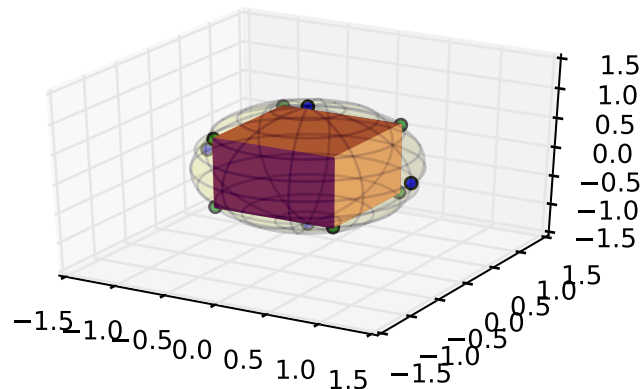


*Examples*

```

>>> from matplotlib import colors
>>> from mpl_toolkits.mplot3d.art3d import Poly3DCollection
>>> import matplotlib.pyplot as plt
>>> from scipy.spatial import SphericalVoronoi
>>> from mpl_toolkits.mplot3d import proj3d
>>> # set input data
>>> points = np.array([[0, 0, 1], [0, 0, -1], [1, 0, 0],
...                   [0, 1, 0], [0, -1, 0], [-1, 0, 0], ])
>>> center = np.array([0, 0, 0])
>>> radius = 1
>>> # calculate spherical Voronoi diagram
>>> sv = SphericalVoronoi(points, radius, center)
>>> # sort vertices (optional, helpful for plotting)
>>> sv.sort_vertices_of_regions()
>>> # generate plot
>>> fig = plt.figure()
>>> ax = fig.add_subplot(111, projection='3d')
>>> # plot the unit sphere for reference (optional)
>>> u = np.linspace(0, 2 * np.pi, 100)
>>> v = np.linspace(0, np.pi, 100)
>>> x = np.outer(np.cos(u), np.sin(v))
>>> y = np.outer(np.sin(u), np.sin(v))
>>> z = np.outer(np.ones(np.size(u)), np.cos(v))
>>> ax.plot_surface(x, y, z, color='y', alpha=0.1)
>>> # plot generator points
>>> ax.scatter(points[:, 0], points[:, 1], points[:, 2], c='b')
>>> # plot Voronoi vertices
>>> ax.scatter(sv.vertices[:, 0], sv.vertices[:, 1], sv.vertices[:, 2],
...          c='g')
>>> # indicate Voronoi regions (as Euclidean polygons)
>>> for region in sv.regions:
...     random_color = colors.rgb2hex(np.random.rand(3))
...     polygon = Poly3DCollection([sv.vertices[region]], alpha=1.0)
...     polygon.set_color(random_color)
...     ax.add_collection3d(polygon)
>>> plt.show()

```



*Attributes*

points	(double array of shape (npoints, 3)) the points in 3D to generate the Voronoi diagram from
radius	(double) radius of the sphere Default: None (forces estimation, which is less precise)
center	(double array of shape (3,)) center of the sphere Default: None (assumes sphere is centered at origin)
vertices	(double array of shape (nvertices, 3)) Voronoi vertices corresponding to points
regions	(list of list of integers of shape (npoints, _)) the n-th entry is a list consisting of the indices of the vertices belonging to the n-th point in points

*Methods*

---

`sort_vertices_of_regions()` For each region in regions, it sorts the indices of the Voronoi vertices such that the resulting p

---

`SphericalVoronoi.sort_vertices_of_regions()`

For each region in regions, it sorts the indices of the Voronoi vertices such that the resulting points are in a clockwise or counterclockwise order around the generator point.

This is done as follows: Recall that the n-th region in regions surrounds the n-th generator in points and that the k-th Voronoi vertex in vertices is the projected circumcenter of the tetrahedron obtained by the k-th triangle in `_tri.simplices` (and the origin). For each region n, we choose the first triangle (=Voronoi vertex) in `_tri.simplices` and a vertex of that triangle not equal to the center n. These determine a unique neighbor of that triangle, which is then chosen as the second triangle. The second triangle will have a unique vertex not equal to the current vertex or the center. This determines a unique neighbor of the second triangle, which is then chosen as the third triangle and so forth. We proceed through all the triangles (=Voronoi vertices) belonging to the generator in points and obtain a sorted version of the vertices of its surrounding region.

### 5.24.3 Plotting Helpers

---

<code>del aunay_plot_2d(tri[, ax])</code>	Plot the given Delaunay triangulation in 2-D
<code>convex_hull_plot_2d(hull[, ax])</code>	Plot the given convex hull diagram in 2-D
<code>voronoi_plot_2d(vor[, ax])</code>	Plot the given Voronoi diagram in 2-D

---

`scipy.spatial.delaunay_plot_2d(tri, ax=None)`

Plot the given Delaunay triangulation in 2-D

**Parameters** **tri** : `scipy.spatial.Delaunay` instance  
 Triangulation to plot  
**ax** : `matplotlib.axes.Axes` instance, optional

**Returns** **fig** : `matplotlib.figure.Figure` instance  
 Axes to plot on  
 Figure for the plot

**See also:**

`Delaunay`, `matplotlib.pyplot.triplot`

**Notes**

Requires Matplotlib.

```
scipy.spatial.convex_hull_plot_2d(hull, ax=None)
```

Plot the given convex hull diagram in 2-D

**Parameters**

- hull** : scipy.spatial.ConvexHull instance  
Convex hull to plot
- ax** : matplotlib.axes.Axes instance, optional  
Axes to plot on

**Returns**

- fig** : matplotlib.figure.Figure instance  
Figure for the plot

**See also:**

[ConvexHull](#)

**Notes**

Requires Matplotlib.

```
scipy.spatial.voronoi_plot_2d(vor, ax=None, **kw)
```

Plot the given Voronoi diagram in 2-D

**Parameters**

- vor** : scipy.spatial.Voronoi instance  
Diagram to plot
- ax** : matplotlib.axes.Axes instance, optional  
Axes to plot on
- show\_points**: **bool, optional**  
Add the Voronoi points to the plot.
- show\_vertices** : bool, optional  
Add the Voronoi vertices to the plot.
- line\_colors** : string, optional  
Specifies the line color for polygon boundaries
- line\_width** : float, optional  
Specifies the line width for polygon boundaries
- line\_alpha**: **float, optional**  
Specifies the line alpha for polygon boundaries

**Returns**

- fig** : matplotlib.figure.Figure instance  
Figure for the plot

**See also:**

[Voronoi](#)

**Notes**

Requires Matplotlib.

**See also:**

[Tutorial](#)

## 5.24.4 Simplex representation

The simplices (triangles, tetrahedra, ...) appearing in the Delaunay tessellation (N-dim simplices), convex hull facets, and Voronoi ridges (N-1 dim simplices) are represented in the following scheme:

```
tess = Delaunay(points)
hull = ConvexHull(points)
voro = Voronoi(points)

# coordinates of the j-th vertex of the i-th simplex
```

```
tess.points[tess.simplices[i, j], :]      # tessellation element
hull.points[hull.simplices[i, j], :]     # convex hull facet
vor.vortices[vor.ridge_vertices[i, j], :] # ridge between Voronoi cells
```

For Delaunay triangulations and convex hulls, the neighborhood structure of the simplices satisfies the condition:

`tess.neighbors[i, j]` is the neighboring simplex of the *i*-th simplex, opposite to the *j*-vertex. It is -1 in case of no neighbor.

Convex hull facets also define a hyperplane equation:

```
(hull.equations[i, :-1] * coord).sum() + hull.equations[i, -1] == 0
```

Similar hyperplane equations for the Delaunay triangulation correspond to the convex hull facets on the corresponding *N*+1 dimensional paraboloid.

The Delaunay triangulation objects offer a method for locating the simplex containing a given point, and barycentric coordinate computations.

## Functions

---

<code>tsearch(tri, xi)</code>	Find simplices containing the given points.
<code>distance_matrix(x, y[, p, threshold])</code>	Compute the distance matrix.
<code>minkowski_distance(x, y[, p])</code>	Compute the $L^{**p}$ distance between two arrays.
<code>minkowski_distance_p(x, y[, p])</code>	Compute the <i>p</i> -th power of the $L^{**p}$ distance between two arrays.
<code>procrustes(data1, data2)</code>	Procrustes analysis, a similarity test for two data sets.

---

`scipy.spatial.tsearch` (*tri, xi*)

Find simplices containing the given points. This function does the same thing as `Delaunay.find_simplex`.

New in version 0.9.

**See also:**

`Delaunay.find_simplex`

`scipy.spatial.distance_matrix` (*x, y, p=2, threshold=1000000*)

Compute the distance matrix.

Returns the matrix of all pair-wise distances.

**Parameters**

- x** : (M, K) array\_like  
TODO: description needed
- y** : (N, K) array\_like  
TODO: description needed
- p** : float, 1 <= p <= infinity  
Which Minkowski *p*-norm to use.
- threshold** : positive int  
If  $M * N * K > threshold$ , algorithm uses a Python loop instead of large temporary arrays.

**Returns**

**result** : (M, N) ndarray  
Distance matrix.

### Examples

```
>>> from scipy.spatial import distance_matrix
>>> distance_matrix([[0,0],[0,1]], [[1,0],[1,1]])
```

```
array([[ 1.          ,  1.41421356],
       [ 1.41421356,  1.          ]])
```

`scipy.spatial.minkowski_distance(x, y, p=2)`

Compute the L\*\*p distance between two arrays.

**Parameters**

- x** : (M, K) array\_like  
Input array.
- y** : (N, K) array\_like  
Input array.
- p** : float, 1 <= p <= infinity  
Which Minkowski p-norm to use.

### Examples

```
>>> from scipy.spatial import minkowski_distance
>>> minkowski_distance([[0,0],[0,0]], [[1,1],[0,1]])
array([ 1.41421356,  1.          ])
```

`scipy.spatial.minkowski_distance_p(x, y, p=2)`

Compute the p-th power of the L\*\*p distance between two arrays.

For efficiency, this function computes the L\*\*p distance but does not extract the pth root. If *p* is 1 or infinity, this is equal to the actual L\*\*p distance.

**Parameters**

- x** : (M, K) array\_like  
Input array.
- y** : (N, K) array\_like  
Input array.
- p** : float, 1 <= p <= infinity  
Which Minkowski p-norm to use.

### Examples

```
>>> from scipy.spatial import minkowski_distance_p
>>> minkowski_distance_p([[0,0],[0,0]], [[1,1],[0,1]])
array([2, 1])
```

`scipy.spatial.procrustes(data1, data2)`

Procrustes analysis, a similarity test for two data sets.

Each input matrix is a set of points or vectors (the rows of the matrix). The dimension of the space is the number of columns of each matrix. Given two identically sized matrices, procrustes standardizes both such that:

- $tr(AA^T) = 1$ .
- Both sets of points are centered around the origin.

Procrustes ([R309], [R310]) then applies the optimal transform to the second matrix (including scaling/dilation, rotations, and reflections) to minimize  $M^2 = \sum (data1 - data2)^2$ , or the sum of the squares of the pointwise differences between the two input datasets.

This function was not designed to handle datasets with different numbers of datapoints (rows). If two data sets have different dimensionality (different number of columns), simply add columns of zeros the smaller of the two.

**Parameters**

- data1** : array\_like  
Matrix, n rows represent points in k (columns) space *data1* is the reference data, after it is standardised, the data from *data2* will be transformed to fit the pattern in *data1* (must have >1 unique points).
- data2** : array\_like

**Returns**

**mtx1** : array\_like  
 n rows of data in k space to be fit to *data1*. Must be the same shape (numrows, numcols) as data1 (must have >1 unique points).  
 A standardized version of *data1*.

**mtx2** : array\_like  
 The orientation of *data2* that best fits *data1*. Centered, but not necessarily  $tr(AA^T) = 1$ .

**disparity** : float  
 $M^2$  as defined above.

**Raises**

**ValueError**  
 If the input arrays are not two-dimensional. If the shape of the input arrays is different. If the input arrays have zero columns or zero rows.

See also:

`scipy.linalg.orthogonal_procrustes`

Notes

- The disparity should not depend on the order of the input matrices, but the output matrices will, as only the first output matrix is guaranteed to be scaled such that  $tr(AA^T) = 1$ .
- Duplicate data points are generally ok, duplicating a data point will increase its effect on the procrustes fit.
- The disparity scales as the number of points per input matrix.

References

[R309], [R310]

Examples

```
>>> from scipy.spatial import procrustes
```

The matrix b is a rotated, shifted, scaled and mirrored version of a here:

```
>>> a = np.array([[1, 3], [1, 2], [1, 1], [2, 1]], 'd')
>>> b = np.array([[4, -2], [4, -4], [4, -6], [2, -6]], 'd')
>>> mtx1, mtx2, disparity = procrustes(a, b)
>>> round(disparity)
0.0
```

## 5.25 Distance computations (`scipy.spatial.distance`)

### 5.25.1 Function Reference

Distance matrix computation from a collection of raw observation vectors stored in a rectangular array.

---

<code>pdist(X[, metric, p, w, V, VI])</code>	Pairwise distances between observations in n-dimensional space.
<code>cdist(XA, XB[, metric, p, V, VI, w])</code>	Computes distance between each pair of the two collections of inputs.
<code>squareform(X[, force, checks])</code>	Converts a vector-form distance vector to a square-form distance matrix, and vice-versa.

---

`scipy.spatial.distance.pdist` (*X*, *metric*='euclidean', *p*=2, *w*=None, *V*=None, *VI*=None)

Pairwise distances between observations in n-dimensional space.

The following are common calling conventions.

1. `Y = pdist(X, 'euclidean')`

Computes the distance between  $m$  points using Euclidean distance (2-norm) as the distance metric between the points. The points are arranged as  $m$   $n$ -dimensional row vectors in the matrix  $X$ .

2. `Y = pdist(X, 'minkowski', p)`

Computes the distances using the Minkowski distance  $\|u - v\|_p$  ( $p$ -norm) where  $p \geq 1$ .

3. `Y = pdist(X, 'cityblock')`

Computes the city block or Manhattan distance between the points.

4. `Y = pdist(X, 'seuclidean', V=None)`

Computes the standardized Euclidean distance. The standardized Euclidean distance between two  $n$ -vectors  $u$  and  $v$  is

$$\sqrt{\sum (u_i - v_i)^2 / V[x_i]}$$

$V$  is the variance vector;  $V[i]$  is the variance computed over all the  $i$ 'th components of the points. If not passed, it is automatically computed.

5. `Y = pdist(X, 'sqeuclidean')`

Computes the squared Euclidean distance  $\|u - v\|_2^2$  between the vectors.

6. `Y = pdist(X, 'cosine')`

Computes the cosine distance between vectors  $u$  and  $v$ ,

$$1 - \frac{u \cdot v}{\|u\|_2 \|v\|_2}$$

where  $\|* \|_2$  is the 2-norm of its argument  $*$ , and  $u \cdot v$  is the dot product of  $u$  and  $v$ .

7. `Y = pdist(X, 'correlation')`

Computes the correlation distance between vectors  $u$  and  $v$ . This is

$$1 - \frac{(u - \bar{u}) \cdot (v - \bar{v})}{\|(u - \bar{u})\|_2 \|(v - \bar{v})\|_2}$$

where  $\bar{v}$  is the mean of the elements of vector  $v$ , and  $x \cdot y$  is the dot product of  $x$  and  $y$ .

8. `Y = pdist(X, 'hamming')`

Computes the normalized Hamming distance, or the proportion of those vector elements between two  $n$ -vectors  $u$  and  $v$  which disagree. To save memory, the matrix  $X$  can be of type boolean.

9. `Y = pdist(X, 'jaccard')`

Computes the Jaccard distance between the points. Given two vectors,  $u$  and  $v$ , the Jaccard distance is the proportion of those elements  $u[i]$  and  $v[i]$  that disagree.

10. `Y = pdist(X, 'chebyshev')`

Computes the Chebyshev distance between the points. The Chebyshev distance between two  $n$ -vectors  $u$  and  $v$  is the maximum norm-1 distance between their respective elements. More precisely, the distance is given by

$$d(u, v) = \max_i |u_i - v_i|$$

11. `Y = pdist(X, 'canberra')`

Computes the Canberra distance between the points. The Canberra distance between two points  $u$  and  $v$  is

$$d(u, v) = \sum_i \frac{|u_i - v_i|}{|u_i| + |v_i|}$$

12.Y = pdist(X, 'braycurtis')

Computes the Bray-Curtis distance between the points. The Bray-Curtis distance between two points  $u$  and  $v$  is

$$d(u, v) = \frac{\sum_i u_i - v_i}{\sum_i u_i + v_i}$$

13.Y = pdist(X, 'mahalanobis', VI=None)

Computes the Mahalanobis distance between the points. The Mahalanobis distance between two points  $u$  and  $v$  is  $(u - v)(1/V)(u - v)^T$  where  $(1/V)$  (the VI variable) is the inverse covariance. If VI is not None, VI will be used as the inverse covariance matrix.

14.Y = pdist(X, 'yule')

Computes the Yule distance between each pair of boolean vectors. (see yule function documentation)

15.Y = pdist(X, 'matching')

Synonym for 'hamming'.

16.Y = pdist(X, 'dice')

Computes the Dice distance between each pair of boolean vectors. (see dice function documentation)

17.Y = pdist(X, 'kulsinski')

Computes the Kulsinski distance between each pair of boolean vectors. (see kulsinski function documentation)

18.Y = pdist(X, 'rogerstanimoto')

Computes the Rogers-Tanimoto distance between each pair of boolean vectors. (see rogerstanimoto function documentation)

19.Y = pdist(X, 'russellrao')

Computes the Russell-Rao distance between each pair of boolean vectors. (see russellrao function documentation)

20.Y = pdist(X, 'sokalmichener')

Computes the Sokal-Michener distance between each pair of boolean vectors. (see sokalmichener function documentation)

21.Y = pdist(X, 'sokalsneath')

Computes the Sokal-Sneath distance between each pair of boolean vectors. (see sokalsneath function documentation)

22.Y = pdist(X, 'wminkowski')

Computes the weighted Minkowski distance between each pair of vectors. (see wminkowski function documentation)

23.Y = pdist(X, f)

Computes the distance between all pairs of vectors in X using the user supplied 2-arity function f. For example, Euclidean distance between the vectors could be computed as follows:



```
dm = pdist(X, lambda u, v: np.sqrt(((u-v)**2).sum()))
```

Note that you should avoid passing a reference to one of the distance functions defined in this library. For example,:

```
dm = pdist(X, sokalsneath)
```

would calculate the pair-wise distances between the vectors in X using the Python function sokalsneath. This would result in sokalsneath being called  $\binom{n}{2}$  times, which is inefficient. Instead, the optimized C version is more efficient, and we call it using the following syntax.:

```
dm = pdist(X, 'sokalsneath')
```

**Parameters** **X** : ndarray

An m by n array of m original observations in an n-dimensional space.

**metric** : str or function, optional

The distance metric to use. The distance function can be 'braycurtis', 'canberra', 'chebyshev', 'cityblock', 'correlation', 'cosine', 'dice', 'euclidean', 'hamming', 'jaccard', 'kulsinski', 'mahalanobis', 'matching', 'minkowski', 'rogerstanimoto', 'russellrao', 'seuclidean', 'sokalmichener', 'sokalsneath', 'sqeuclidean', 'yule'.

**w** : ndarray, optional

The weight vector (for weighted Minkowski).

**p** : double, optional

The p-norm to apply (for Minkowski, weighted and unweighted)

**V** : ndarray, optional

The variance vector (for standardized Euclidean).

**VI** : ndarray, optional

The inverse of the covariance matrix (for Mahalanobis).

**Returns**

**Y** : ndarray

Returns a condensed distance matrix Y. For each  $i$  and  $j$  (where  $i < j < n$ ), the metric `dist(u=X[i], v=X[j])` is computed and stored in entry  $ij$ .

**See also:**

[\*squareform\*](#) converts between condensed distance matrices and square distance matrices.

**Notes**

See [squareform](#) for information on how to calculate the index of this entry or to convert the condensed distance matrix to a redundant square matrix.

`scipy.spatial.distance.cdist(XA, XB, metric='euclidean', p=2, V=None, VI=None, w=None)`  
Computes distance between each pair of the two collections of inputs.

The following are common calling conventions:

1. `Y = cdist(XA, XB, 'euclidean')`

Computes the distance between  $m$  points using Euclidean distance (2-norm) as the distance metric between the points. The points are arranged as  $m$   $n$ -dimensional row vectors in the matrix X.

2. `Y = cdist(XA, XB, 'minkowski', p)`

Computes the distances using the Minkowski distance  $\|u - v\|_p$  ( $p$ -norm) where  $p \geq 1$ .

3. `Y = cdist(XA, XB, 'cityblock')`

Computes the city block or Manhattan distance between the points.

4. `Y = cdist(XA, XB, 'seuclidean', V=None)`

Computes the standardized Euclidean distance. The standardized Euclidean distance between two n-vectors  $u$  and  $v$  is

$$\sqrt{\sum (u_i - v_i)^2 / V[x_i]}.$$

$V$  is the variance vector;  $V[i]$  is the variance computed over all the  $i$ 'th components of the points. If not passed, it is automatically computed.

5.Y = cdist(XA, XB, 'sqeuclidean')

Computes the squared Euclidean distance  $\|u - v\|_2^2$  between the vectors.

6.Y = cdist(XA, XB, 'cosine')

Computes the cosine distance between vectors  $u$  and  $v$ ,

$$1 - \frac{u \cdot v}{\|u\|_2 \|v\|_2}$$

where  $\|*\|_2$  is the 2-norm of its argument  $*$ , and  $u \cdot v$  is the dot product of  $u$  and  $v$ .

7.Y = cdist(XA, XB, 'correlation')

Computes the correlation distance between vectors  $u$  and  $v$ . This is

$$1 - \frac{(u - \bar{u}) \cdot (v - \bar{v})}{\|(u - \bar{u})\|_2 \|(v - \bar{v})\|_2}$$

where  $\bar{v}$  is the mean of the elements of vector  $v$ , and  $x \cdot y$  is the dot product of  $x$  and  $y$ .

8.Y = cdist(XA, XB, 'hamming')

Computes the normalized Hamming distance, or the proportion of those vector elements between two n-vectors  $u$  and  $v$  which disagree. To save memory, the matrix  $X$  can be of type boolean.

9.Y = cdist(XA, XB, 'jaccard')

Computes the Jaccard distance between the points. Given two vectors,  $u$  and  $v$ , the Jaccard distance is the proportion of those elements  $u[i]$  and  $v[i]$  that disagree where at least one of them is non-zero.

10.Y = cdist(XA, XB, 'chebyshev')

Computes the Chebyshev distance between the points. The Chebyshev distance between two n-vectors  $u$  and  $v$  is the maximum norm-1 distance between their respective elements. More precisely, the distance is given by

$$d(u, v) = \max_i |u_i - v_i|.$$

11.Y = cdist(XA, XB, 'canberra')

Computes the Canberra distance between the points. The Canberra distance between two points  $u$  and  $v$  is

$$d(u, v) = \sum_i \frac{|u_i - v_i|}{|u_i| + |v_i|}.$$

12.Y = cdist(XA, XB, 'braycurtis')

Computes the Bray-Curtis distance between the points. The Bray-Curtis distance between two points  $u$  and  $v$  is

$$d(u, v) = \frac{\sum_i (u_i - v_i)}{\sum_i (u_i + v_i)}$$

13.Y = cdist(XA, XB, 'mahalanobis', VI=None)

Computes the Mahalanobis distance between the points. The Mahalanobis distance between two points  $u$  and  $v$  is  $\sqrt{(u-v)(1/V)(u-v)^T}$  where  $(1/V)$  (the `VI` variable) is the inverse covariance. If `VI` is not `None`, `VI` will be used as the inverse covariance matrix.

```
14.Y = cdist(XA, XB, 'yule')
```

Computes the Yule distance between the boolean vectors. (see `yule` function documentation)

```
15.Y = cdist(XA, XB, 'matching')
```

Synonym for `'hamming'`.

```
16.Y = cdist(XA, XB, 'dice')
```

Computes the Dice distance between the boolean vectors. (see `dice` function documentation)

```
17.Y = cdist(XA, XB, 'kulsinski')
```

Computes the Kulsinski distance between the boolean vectors. (see `kulsinski` function documentation)

```
18.Y = cdist(XA, XB, 'rogerstanimoto')
```

Computes the Rogers-Tanimoto distance between the boolean vectors. (see `rogerstanimoto` function documentation)

```
19.Y = cdist(XA, XB, 'russellrao')
```

Computes the Russell-Rao distance between the boolean vectors. (see `russellrao` function documentation)

```
20.Y = cdist(XA, XB, 'sokalmichener')
```

Computes the Sokal-Michener distance between the boolean vectors. (see `sokalmichener` function documentation)

```
21.Y = cdist(XA, XB, 'sokalsneath')
```

Computes the Sokal-Sneath distance between the vectors. (see `sokalsneath` function documentation)

```
22.Y = cdist(XA, XB, 'wminkowski')
```

Computes the weighted Minkowski distance between the vectors. (see `wminkowski` function documentation)

```
23.Y = cdist(XA, XB, f)
```

Computes the distance between all pairs of vectors in `X` using the user supplied 2-arity function `f`. For example, Euclidean distance between the vectors could be computed as follows:

```
dm = cdist(XA, XB, lambda u, v: np.sqrt(((u-v)**2).sum()))
```

Note that you should avoid passing a reference to one of the distance functions defined in this library. For example,:

```
dm = cdist(XA, XB, sokalsneath)
```

would calculate the pair-wise distances between the vectors in `X` using the Python function `sokalsneath`. This would result in `sokalsneath` being called  $\binom{n}{2}$  times, which is inefficient. Instead, the optimized C version is more efficient, and we call it using the following syntax:

```
dm = cdist(XA, XB, 'sokalsneath')
```

<b>Parameters</b>	<b>XA</b> : ndarray	An $m_A$ by $n$ array of $m_A$ original observations in an $n$ -dimensional space. Inputs are converted to float type.
	<b>XB</b> : ndarray	An $m_B$ by $n$ array of $m_B$ original observations in an $n$ -dimensional space. Inputs are converted to float type.
	<b>metric</b> : str or callable, optional	The distance metric to use. If a string, the distance function can be 'bray-curtis', 'canberra', 'chebyshev', 'cityblock', 'correlation', 'cosine', 'dice', 'euclidean', 'hamming', 'jaccard', 'kulsinski', 'mahalanobis', 'matching', 'minkowski', 'rogerstanimoto', 'russellrao', 'seuclidean', 'sokalmichener', 'sokalsneath', 'sqeuclidean', 'wminkowski', 'yule'.
	<b>w</b> : ndarray, optional	The weight vector (for weighted Minkowski).
	<b>p</b> : scalar, optional	The p-norm to apply (for Minkowski, weighted and unweighted)
	<b>V</b> : ndarray, optional	The variance vector (for standardized Euclidean).
	<b>VI</b> : ndarray, optional	The inverse of the covariance matrix (for Mahalanobis).
<b>Returns</b>	<b>Y</b> : ndarray	A $m_A$ by $m_B$ distance matrix is returned. For each $i$ and $j$ , the metric <code>dist(u=XA[i], v=XB[j])</code> is computed and stored in the $ij$ th entry.
<b>Raises</b>	<b>ValueError</b>	An exception is thrown if $XA$ and $XB$ do not have the same number of columns.

### Examples

Find the Euclidean distances between four 2-D coordinates:

```
>>> from scipy.spatial import distance
>>> coords = [(35.0456, -85.2672),
...           (35.1174, -89.9711),
...           (35.9728, -83.9422),
...           (36.1667, -86.7833)]
>>> distance.cdist(coords, coords, 'euclidean')
array([[ 0.         ,  4.7044,  1.6172,  1.8856],
       [ 4.7044,  0.         ,  6.0893,  3.3561],
       [ 1.6172,  6.0893,  0.         ,  2.8477],
       [ 1.8856,  3.3561,  2.8477,  0.         ]])
```

Find the Manhattan distance from a 3-D point to the corners of the unit cube:

```
>>> a = np.array([[0, 0, 0],
...               [0, 0, 1],
...               [0, 1, 0],
...               [0, 1, 1],
...               [1, 0, 0],
...               [1, 0, 1],
...               [1, 1, 0],
...               [1, 1, 1]])
>>> b = np.array([[0.1, 0.2, 0.4]])
>>> distance.cdist(a, b, 'cityblock')
```

```
array([[ 0.7],
       [ 0.9],
       [ 1.3],
       [ 1.5],
       [ 1.5],
       [ 1.7],
       [ 2.1],
       [ 2.3]])
```

`scipy.spatial.distance.squareform(X, force='no', checks=True)`

Converts a vector-form distance vector to a square-form distance matrix, and vice-versa.

**Parameters** **X** : ndarray  
 Either a condensed or redundant distance matrix.

**force** : str, optional  
 As with MATLAB(TM), if force is equal to 'tovector' or 'tomatrix', the input will be treated as a distance matrix or distance vector respectively.

**checks** : bool, optional  
 If checks is set to False, no checks will be made for matrix symmetry nor zero diagonals. This is useful if it is known that  $X - X.T$  is small and  $\text{diag}(X)$  is close to zero. These values are ignored any way so they do not disrupt the squareform transformation.

**Returns** **Y** : ndarray  
 If a condensed distance matrix is passed, a redundant one is returned, or if a redundant one is passed, a condensed distance matrix is returned.

**Notes**

1.  $v = \text{squareform}(X)$

Given a square  $d$ -by- $d$  symmetric distance matrix  $X$ ,  $v = \text{squareform}(X)$  returns a  $d * (d-1) / 2$  (or  $\binom{n}{2}$ ) sized vector  $v$ .

$v[\binom{n}{2} - \binom{n-i}{2} + (j-i-1)]$  is the distance between points  $i$  and  $j$ . If  $X$  is non-square or asymmetric, an error is returned.

2.  $X = \text{squareform}(v)$

Given a  $d*(d-1)/2$  sized  $v$  for some integer  $d \geq 2$  encoding distances as described,  $X = \text{squareform}(v)$  returns a  $d$  by  $d$  distance matrix  $X$ . The  $X[i, j]$  and  $X[j, i]$  values are set to  $v[\binom{n}{2} - \binom{n-i}{2} + (j-i-1)]$  and all diagonal elements are zero.

Predicates for checking the validity of distance matrices, both condensed and redundant. Also contained in this module are functions for computing the number of observations in a distance matrix.

---

<code>is_valid_dm(D[, tol, throw, name, warning])</code>	Returns True if input array is a valid distance matrix.
<code>is_valid_y(y[, warning, throw, name])</code>	Returns True if the input array is a valid condensed distance matrix.
<code>num_obs_dm(d)</code>	Returns the number of original observations that correspond to a square, redundant
<code>num_obs_y(Y)</code>	Returns the number of original observations that correspond to a condensed distance

---

`scipy.spatial.distance.is_valid_dm(D, tol=0.0, throw=False, name='D', warning=False)`

Returns True if input array is a valid distance matrix.

Distance matrices must be 2-dimensional numpy arrays containing doubles. They must have a zero-diagonal, and they must be symmetric.

**Parameters** **D** : ndarray

The candidate object to test for validity.

**tol** : float, optional  
The distance matrix should be symmetric. *tol* is the maximum difference between entries  $i_j$  and  $j_i$  for the distance metric to be considered symmetric.

**throw** : bool, optional  
An exception is thrown if the distance matrix passed is not valid.

**name** : str, optional  
The name of the variable to checked. This is useful if *throw* is set to True so the offending variable can be identified in the exception message when an exception is thrown.

**warning** : bool, optional  
Instead of throwing an exception, a warning message is raised.

**Returns** **valid** : bool  
True if the variable *D* passed is a valid distance matrix.

**Notes**

Small numerical differences in *D* and *D.T* and non-zeroneess of the diagonal are ignored if they are within the tolerance specified by *tol*.

`scipy.spatial.distance.is_valid_y` (*y*, *warning=False*, *throw=False*, *name=None*)  
Returns True if the input array is a valid condensed distance matrix.

Condensed distance matrices must be 1-dimensional numpy arrays containing doubles. Their length must be a binomial coefficient  $\binom{n}{2}$  for some positive integer *n*.

**Parameters** **y** : ndarray  
The condensed distance matrix.

**warning** : bool, optional  
Invokes a warning if the variable passed is not a valid condensed distance matrix. The warning message explains why the distance matrix is not valid. *name* is used when referencing the offending variable.

**throw** : bool, optional  
Throws an exception if the variable passed is not a valid condensed distance matrix.

**name** : bool, optional  
Used when referencing the offending variable in the warning or exception message.

`scipy.spatial.distance.num_obs_dm` (*d*)  
Returns the number of original observations that correspond to a square, redundant distance matrix.

**Parameters** **d** : ndarray  
The target distance matrix.

**Returns** **num\_obs\_dm** : int  
The number of observations in the redundant distance matrix.

`scipy.spatial.distance.num_obs_y` (*Y*)  
Returns the number of original observations that correspond to a condensed distance matrix.

**Parameters** **Y** : ndarray  
Condensed distance matrix.

**Returns** **n** : int  
The number of observations in the condensed distance matrix *Y*.

Distance functions between two numeric vectors *u* and *v*. Computing distances over a large collection of vectors is inefficient for these functions. Use `pdist` for this purpose.

Continued on next page

Table 5.232 – continued from previous page

<code>braycurtis(u, v)</code>	Computes the Bray-Curtis distance between two 1-D arrays.
<code>canberra(u, v)</code>	Computes the Canberra distance between two 1-D arrays.
<code>chebyshev(u, v)</code>	Computes the Chebyshev distance.
<code>cityblock(u, v)</code>	Computes the City Block (Manhattan) distance.
<code>correlation(u, v)</code>	Computes the correlation distance between two 1-D arrays.
<code>cosine(u, v)</code>	Computes the Cosine distance between 1-D arrays.
<code>euclidean(u, v)</code>	Computes the Euclidean distance between two 1-D arrays.
<code>mahalanobis(u, v, VI)</code>	Computes the Mahalanobis distance between two 1-D arrays.
<code>minkowski(u, v, p)</code>	Computes the Minkowski distance between two 1-D arrays.
<code>seuclidean(u, v, V)</code>	Returns the standardized Euclidean distance between two 1-D arrays.
<code>squclidean(u, v)</code>	Computes the squared Euclidean distance between two 1-D arrays.
<code>wminkowski(u, v, p, w)</code>	Computes the weighted Minkowski distance between two 1-D arrays.

`scipy.spatial.distance.braycurtis(u, v)`

Computes the Bray-Curtis distance between two 1-D arrays.

Bray-Curtis distance is defined as

$$\sum |u_i - v_i| / \sum |u_i + v_i|$$

The Bray-Curtis distance is in the range [0, 1] if all coordinates are positive, and is undefined if the inputs are of length zero.

**Parameters** **u** : (N,) array\_like  
Input array.

**v** : (N,) array\_like

**Returns** **braycurtis** : double  
Input array.

The Bray-Curtis distance between 1-D arrays *u* and *v*.

`scipy.spatial.distance.canberra(u, v)`

Computes the Canberra distance between two 1-D arrays.

The Canberra distance is defined as

$$d(u, v) = \sum_i \frac{|u_i - v_i|}{|u_i| + |v_i|}$$

**Parameters** **u** : (N,) array\_like  
Input array.

**v** : (N,) array\_like

**Returns** **canberra** : double  
Input array.

The Canberra distance between vectors *u* and *v*.

#### Notes

When  $u[i]$  and  $v[i]$  are 0 for given *i*, then the fraction  $0/0 = 0$  is used in the calculation.

`scipy.spatial.distance.chebyshev(u, v)`

Computes the Chebyshev distance.

Computes the Chebyshev distance between two 1-D arrays *u* and *v*, which is defined as

$$\max_i |u_i - v_i|$$

**Parameters** **u** : (N,) array\_like

**Returns** **chebyshev** : double  
 Input vector.  
 $v$  : (N,) array\_like  
 Input vector.  
 The Chebyshev distance between vectors  $u$  and  $v$ .

`scipy.spatial.distance.cityblock` ( $u$ ,  $v$ )

Computes the City Block (Manhattan) distance.

Computes the Manhattan distance between two 1-D arrays  $u$  and  $v$ , which is defined as

$$\sum_i |u_i - v_i|.$$

**Parameters** **u** : (N,) array\_like  
 Input array.  
**v** : (N,) array\_like  
 Input array.  
**Returns** **cityblock** : double  
 The City Block (Manhattan) distance between vectors  $u$  and  $v$ .

`scipy.spatial.distance.correlation` ( $u$ ,  $v$ )

Computes the correlation distance between two 1-D arrays.

The correlation distance between  $u$  and  $v$ , is defined as

$$1 - \frac{(u - \bar{u}) \cdot (v - \bar{v})}{\| (u - \bar{u}) \|_2 \| (v - \bar{v}) \|_2}$$

where  $\bar{u}$  is the mean of the elements of  $u$  and  $x \cdot y$  is the dot product of  $x$  and  $y$ .

**Parameters** **u** : (N,) array\_like  
 Input array.  
**v** : (N,) array\_like  
 Input array.  
**Returns** **correlation** : double  
 The correlation distance between 1-D array  $u$  and  $v$ .

`scipy.spatial.distance.cosine` ( $u$ ,  $v$ )

Computes the Cosine distance between 1-D arrays.

The Cosine distance between  $u$  and  $v$ , is defined as

$$1 - \frac{u \cdot v}{\|u\|_2 \|v\|_2}.$$

where  $u \cdot v$  is the dot product of  $u$  and  $v$ .

**Parameters** **u** : (N,) array\_like  
 Input array.  
**v** : (N,) array\_like  
 Input array.  
**Returns** **cosine** : double  
 The Cosine distance between vectors  $u$  and  $v$ .

`scipy.spatial.distance.euclidean` ( $u$ ,  $v$ )

Computes the Euclidean distance between two 1-D arrays.

The Euclidean distance between 1-D arrays  $u$  and  $v$ , is defined as

$$\|u - v\|_2$$

**Parameters** **u** : (N,) array\_like



**Returns** **euclidean** : double  
 Input array.  
 $v$  : (N,) array\_like  
 Input array.  
 The Euclidean distance between vectors  $u$  and  $v$ .

`scipy.spatial.distance.mahalanobis` ( $u, v, VI$ )

Computes the Mahalanobis distance between two 1-D arrays.

The Mahalanobis distance between 1-D arrays  $u$  and  $v$ , is defined as

$$\sqrt{(u - v)V^{-1}(u - v)^T}$$

where  $V$  is the covariance matrix. Note that the argument  $VI$  is the inverse of  $V$ .

**Parameters**  $u$  : (N,) array\_like  
 Input array.  
 $v$  : (N,) array\_like  
 Input array.  
 $VI$  : ndarray  
 The inverse of the covariance matrix.  
**Returns** **mahalanobis** : double  
 The Mahalanobis distance between vectors  $u$  and  $v$ .

`scipy.spatial.distance.minkowski` ( $u, v, p$ )

Computes the Minkowski distance between two 1-D arrays.

The Minkowski distance between 1-D arrays  $u$  and  $v$ , is defined as

$$\|u - v\|_p = \left( \sum |u_i - v_i|^p \right)^{1/p}.$$

**Parameters**  $u$  : (N,) array\_like  
 Input array.  
 $v$  : (N,) array\_like  
 Input array.  
 $p$  : int  
 The order of the norm of the difference  $\|u - v\|_p$ .  
**Returns** **d** : double  
 The Minkowski distance between vectors  $u$  and  $v$ .

`scipy.spatial.distance.seuclidean` ( $u, v, V$ )

Returns the standardized Euclidean distance between two 1-D arrays.

The standardized Euclidean distance between  $u$  and  $v$ .

**Parameters**  $u$  : (N,) array\_like  
 Input array.  
 $v$  : (N,) array\_like  
 Input array.  
 $V$  : (N,) array\_like  
 $V$  is an 1-D array of component variances. It is usually computed among a larger collection vectors.  
**Returns** **seuclidean** : double  
 The standardized Euclidean distance between vectors  $u$  and  $v$ .

`scipy.spatial.distance.sqeuclidean` ( $u, v$ )

Computes the squared Euclidean distance between two 1-D arrays.

The squared Euclidean distance between  $u$  and  $v$  is defined as

$$\|u - v\|_2^2.$$

**Parameters** **u** : (N,) array\_like  
 Input array.  
**v** : (N,) array\_like  
 Input array.  
**Returns** **squeclidean** : double  
 The squared Euclidean distance between vectors  $u$  and  $v$ .

`scipy.spatial.distance.wminkowski(u, v, p, w)`  
 Computes the weighted Minkowski distance between two 1-D arrays.

The weighted Minkowski distance between  $u$  and  $v$ , defined as

$$\left( \sum (|w_i(u_i - v_i)|^p) \right)^{1/p}.$$

**Parameters** **u** : (N,) array\_like  
 Input array.  
**v** : (N,) array\_like  
 Input array.  
**p** : int  
 The order of the norm of the difference  $\|u - v\|_p$ .  
**w** : (N,) array\_like  
 The weight vector.  
**Returns** **wminkowski** : double  
 The weighted Minkowski distance between vectors  $u$  and  $v$ .

Distance functions between two boolean vectors (representing sets)  $u$  and  $v$ . As in the case of numerical vectors, `pdist` is more efficient for computing the distances between all pairs.

---

<code>dice(u, v)</code>	Computes the Dice dissimilarity between two boolean 1-D arrays.
<code>hamming(u, v)</code>	Computes the Hamming distance between two 1-D arrays.
<code>jaccard(u, v)</code>	Computes the Jaccard-Needham dissimilarity between two boolean 1-D arrays.
<code>kulsinski(u, v)</code>	Computes the Kulsinski dissimilarity between two boolean 1-D arrays.
<code>matching(u, v)</code>	Computes the Hamming distance between two boolean 1-D arrays.
<code>rogerstanimoto(u, v)</code>	Computes the Rogers-Tanimoto dissimilarity between two boolean 1-D arrays.
<code>russellrao(u, v)</code>	Computes the Russell-Rao dissimilarity between two boolean 1-D arrays.
<code>sokalmichener(u, v)</code>	Computes the Sokal-Michener dissimilarity between two boolean 1-D arrays.
<code>sokalsneath(u, v)</code>	Computes the Sokal-Sneath dissimilarity between two boolean 1-D arrays.
<code>yule(u, v)</code>	Computes the Yule dissimilarity between two boolean 1-D arrays.

---

`scipy.spatial.distance.dice(u, v)`  
 Computes the Dice dissimilarity between two boolean 1-D arrays.

The Dice dissimilarity between  $u$  and  $v$ , is

$$\frac{c_{TF} + c_{FT}}{2c_{TT} + c_{FT} + c_{TF}}$$

where  $c_{ij}$  is the number of occurrences of  $u[k] = i$  and  $v[k] = j$  for  $k < n$ .

**Parameters** **u** : (N,) ndarray, bool  
 Input 1-D array.  
**v** : (N,) ndarray, bool  
 Input 1-D array.  
**Returns** **dice** : double  
 The Dice dissimilarity between 1-D arrays  $u$  and  $v$ .

`scipy.spatial.distance.hamming(u, v)`  
 Computes the Hamming distance between two 1-D arrays.

The Hamming distance between 1-D arrays  $u$  and  $v$ , is simply the proportion of disagreeing components in  $u$  and  $v$ . If  $u$  and  $v$  are boolean vectors, the Hamming distance is

$$\frac{c_{01} + c_{10}}{n}$$

where  $c_{ij}$  is the number of occurrences of  $u[k] = i$  and  $v[k] = j$  for  $k < n$ .

**Parameters** **u** : (N,) array\_like  
Input array.  
**v** : (N,) array\_like  
Input array.  
**Returns** **hamming** : double  
The Hamming distance between vectors  $u$  and  $v$ .

`scipy.spatial.distance.jaccard` ( $u, v$ )

Computes the Jaccard-Needham dissimilarity between two boolean 1-D arrays.

The Jaccard-Needham dissimilarity between 1-D boolean arrays  $u$  and  $v$ , is defined as

$$\frac{c_{TF} + c_{FT}}{c_{TT} + c_{FT} + c_{TF}}$$

where  $c_{ij}$  is the number of occurrences of  $u[k] = i$  and  $v[k] = j$  for  $k < n$ .

**Parameters** **u** : (N,) array\_like, bool  
Input array.  
**v** : (N,) array\_like, bool  
Input array.  
**Returns** **jaccard** : double  
The Jaccard distance between vectors  $u$  and  $v$ .

`scipy.spatial.distance.kulsinski` ( $u, v$ )

Computes the Kulsinski dissimilarity between two boolean 1-D arrays.

The Kulsinski dissimilarity between two boolean 1-D arrays  $u$  and  $v$ , is defined as

$$\frac{c_{TF} + c_{FT} - c_{TT} + n}{c_{FT} + c_{TF} + n}$$

where  $c_{ij}$  is the number of occurrences of  $u[k] = i$  and  $v[k] = j$  for  $k < n$ .

**Parameters** **u** : (N,) array\_like, bool  
Input array.  
**v** : (N,) array\_like, bool  
Input array.  
**Returns** **kulsinski** : double  
The Kulsinski distance between vectors  $u$  and  $v$ .

`scipy.spatial.distance.matching` ( $u, v$ )

Computes the Hamming distance between two boolean 1-D arrays.

This is a deprecated synonym for `hamming`.

`scipy.spatial.distance.rogerstanimoto` ( $u, v$ )

Computes the Rogers-Tanimoto dissimilarity between two boolean 1-D arrays.

The Rogers-Tanimoto dissimilarity between two boolean 1-D arrays  $u$  and  $v$ , is defined as

$$\frac{R}{c_{TT} + c_{FF} + R}$$

where  $c_{ij}$  is the number of occurrences of  $u[k] = i$  and  $v[k] = j$  for  $k < n$  and  $R = 2(c_{TF} + c_{FT})$ .

**Parameters** **u** : (N,) array\_like, bool

**Returns** **rogerstanimoto** : double  
 Input array.  
**v** : (N,) array\_like, bool  
 Input array.  
 The Rogers-Tanimoto dissimilarity between vectors  $u$  and  $v$ .

`scipy.spatial.distance.russellrao` ( $u, v$ )  
 Computes the Russell-Rao dissimilarity between two boolean 1-D arrays.

The Russell-Rao dissimilarity between two boolean 1-D arrays,  $u$  and  $v$ , is defined as

$$\frac{n - c_{TT}}{n}$$

where  $c_{ij}$  is the number of occurrences of  $u[k] = i$  and  $v[k] = j$  for  $k < n$ .

**Parameters** **u** : (N,) array\_like, bool  
 Input array.  
**v** : (N,) array\_like, bool  
**Returns** **russellrao** : double  
 Input array.  
 The Russell-Rao dissimilarity between vectors  $u$  and  $v$ .

`scipy.spatial.distance.sokalmichener` ( $u, v$ )  
 Computes the Sokal-Michener dissimilarity between two boolean 1-D arrays.

The Sokal-Michener dissimilarity between boolean 1-D arrays  $u$  and  $v$ , is defined as

$$\frac{R}{S + R}$$

where  $c_{ij}$  is the number of occurrences of  $u[k] = i$  and  $v[k] = j$  for  $k < n$ ,  $R = 2 * (c_{TF} + c_{FT})$  and  $S = c_{FF} + c_{TT}$ .

**Parameters** **u** : (N,) array\_like, bool  
 Input array.  
**v** : (N,) array\_like, bool  
**Returns** **sokalmichener** : double  
 Input array.  
 The Sokal-Michener dissimilarity between vectors  $u$  and  $v$ .

`scipy.spatial.distance.sokalsneath` ( $u, v$ )  
 Computes the Sokal-Sneath dissimilarity between two boolean 1-D arrays.

The Sokal-Sneath dissimilarity between  $u$  and  $v$ ,

$$\frac{R}{c_{TT} + R}$$

where  $c_{ij}$  is the number of occurrences of  $u[k] = i$  and  $v[k] = j$  for  $k < n$  and  $R = 2(c_{TF} + c_{FT})$ .

**Parameters** **u** : (N,) array\_like, bool  
 Input array.  
**v** : (N,) array\_like, bool  
**Returns** **sokalsneath** : double  
 Input array.  
 The Sokal-Sneath dissimilarity between vectors  $u$  and  $v$ .

`scipy.spatial.distance.yule` ( $u, v$ )  
 Computes the Yule dissimilarity between two boolean 1-D arrays.

The Yule dissimilarity is defined as

$$\frac{R}{c_{TT} * c_{FF} + \frac{R}{2}}$$

where  $c_{ij}$  is the number of occurrences of  $u[k] = i$  and  $v[k] = j$  for  $k < n$  and  $R = 2.0 * c_{TF} * c_{FT}$ .

**Parameters** **u** : (N,) array\_like, bool  
Input array.  
**v** : (N,) array\_like, bool  
Input array.

**Returns** **yule** : double  
The Yule dissimilarity between vectors  $u$  and  $v$ .

`hamming` also operates over discrete numerical vectors.

## 5.26 Special functions (`scipy.special`)

Nearly all of the functions below are universal functions and follow broadcasting and automatic array-looping rules. Exceptions are noted.

### 5.26.1 Error handling

Errors are handled by returning nans, or other appropriate values. Some of the special function routines will emit warnings when an error occurs. By default this is disabled. To enable such messages use `errprint(1)`, and to disable such messages use `errprint(0)`.

Example:

```
>>> print scipy.special.bdtr(-1,10,0.3)
>>> scipy.special.errprint(1)
>>> print scipy.special.bdtr(-1,10,0.3)
```

<code>errprint([inflag])</code>	Sets or returns the error printing flag for special functions.
<code>SpecialFunctionWarning</code>	Warning that can be issued with <code>errprint(True)</code>

`scipy.special.errprint` (*inflag=None*)

Sets or returns the error printing flag for special functions.

**Parameters** **inflag** : bool, optional  
Whether warnings concerning evaluation of special functions in `scipy.special` are shown. If omitted, no change is made to the current setting.

**Returns** **old\_flag**  
Previous value of the error flag

**exception** `scipy.special.SpecialFunctionWarning`

Warning that can be issued with `errprint(True)`

### 5.26.2 Available functions

#### Airy functions

<code>airy(z)</code>	Airy functions and their derivatives.
<code>airyex(z)</code>	Exponentially scaled Airy functions and their derivatives.
<code>ai_zeros(nt)</code>	Compute $nt$ zeros and values of the Airy function $A_i$ and its derivative.
<code>bi_zeros(nt)</code>	Compute $nt$ zeros and values of the Airy function $B_i$ and its derivative.
<code>itairy(x)</code>	Integrals of Airy functions

`scipy.special.airy(z) = <ufunc 'airy'>`

Airy functions and their derivatives.

**Parameters** `z` : array\_like

**Returns** `Ai, Aip, Bi, Bip` : ndarrays  
Real or complex argument.

Airy functions Ai and Bi, and their derivatives Aip and Bip.

**See also:**

`airy` exponentially scaled Airy functions.

**Notes**

The Airy functions Ai and Bi are two independent solutions of

$$y''(x) = xy(x).$$

For real  $z$  in  $[-10, 10]$ , the computation is carried out by calling the Cephes [R312] `airy` routine, which uses power series summation for small  $z$  and rational minimax approximations for large  $z$ .

Outside this range, the AMOS [R313] `zairy` and `zbiry` routines are employed. They are computed using power series for  $|z| < 1$  and the following relations to modified Bessel functions for larger  $z$  (where  $t \equiv 2z^{3/2}/3$ ):

$$\begin{aligned} Ai(z) &= \frac{1}{\pi\sqrt{3}}K_{1/3}(t) \\ Ai'(z) &= -\frac{z}{\pi\sqrt{3}}K_{2/3}(t) \\ Bi(z) &= \sqrt{\frac{z}{3}}(I_{-1/3}(t) + I_{1/3}(t)) \\ Bi'(z) &= \frac{z}{\sqrt{3}}(I_{-2/3}(t) + I_{2/3}(t)) \end{aligned}$$

**References**

[R312], [R313]

`scipy.special.airye(z) = <ufunc 'airye'>`

Exponentially scaled Airy functions and their derivatives.

Scaling:

```
eAi = Ai * exp(2.0/3.0*z*sqrt(z))
eAip = Aip * exp(2.0/3.0*z*sqrt(z))
eBi = Bi * exp(-abs((2.0/3.0*z*sqrt(z)).real))
eBip = Bip * exp(-abs((2.0/3.0*z*sqrt(z)).real))
```

**Parameters** `z` : array\_like

**Returns** `eAi, eAip, eBi, eBip` : array\_like  
Real or complex argument.

Airy functions Ai and Bi, and their derivatives Aip and Bip

**See also:**

`airy`

**Notes**

Wrapper for the AMOS [R314] routines `zairy` and `zbiry`.

**References**

[R314]

`scipy.special.ai_zeros` (*nt*)Compute *nt* zeros and values of the Airy function  $Ai$  and its derivative.Computes the first *nt* zeros, *a*, of the Airy function  $Ai(x)$ ; first *nt* zeros, *ap*, of the derivative of the Airy function  $Ai'(x)$ ; the corresponding values  $Ai(a')$ ; and the corresponding values  $Ai'(a)$ .

<b>Parameters</b>	<b>nt</b> : int	Number of zeros to compute
<b>Returns</b>	<b>a</b> : ndarray	First <i>nt</i> zeros of $Ai(x)$
	<b>ap</b> : ndarray	First <i>nt</i> zeros of $Ai'(x)$
	<b>ai</b> : ndarray	Values of $Ai(x)$ evaluated at first <i>nt</i> zeros of $Ai'(x)$
	<b>aip</b> : ndarray	Values of $Ai'(x)$ evaluated at first <i>nt</i> zeros of $Ai(x)$

**References**

[R311]

`scipy.special.bi_zeros` (*nt*)Compute *nt* zeros and values of the Airy function  $Bi$  and its derivative.Computes the first *nt* zeros, *b*, of the Airy function  $Bi(x)$ ; first *nt* zeros, *b'*, of the derivative of the Airy function  $Bi'(x)$ ; the corresponding values  $Bi(b')$ ; and the corresponding values  $Bi'(b)$ .

<b>Parameters</b>	<b>nt</b> : int	Number of zeros to compute
<b>Returns</b>	<b>b</b> : ndarray	First <i>nt</i> zeros of $Bi(x)$
	<b>bp</b> : ndarray	First <i>nt</i> zeros of $Bi'(x)$
	<b>bi</b> : ndarray	Values of $Bi(x)$ evaluated at first <i>nt</i> zeros of $Bi'(x)$
	<b>bip</b> : ndarray	Values of $Bi'(x)$ evaluated at first <i>nt</i> zeros of $Bi(x)$

**References**

[R327]

`scipy.special.itairy` (*x*) = <ufunc 'itairy'>

Integrals of Airy functions

Calculates the integrals of Airy functions from 0 to *x*.

<b>Parameters</b>	<b>x</b> : array_like	Upper limit of integration (float).
<b>Returns</b>	Apt	Integral of $Ai(t)$ from 0 to <i>x</i> .
	Bpt	Integral of $Bi(t)$ from 0 to <i>x</i> .
	Ant	Integral of $Ai(-t)$ from 0 to <i>x</i> .
	Bnt	Integral of $Bi(-t)$ from 0 to <i>x</i> .

*Notes*

Wrapper for a Fortran routine created by Shanjie Zhang and Jianming Jin [R384].

*References*

[R384]

**Elliptic Functions and Integrals**

<code>ellipj(u, m)</code>	Jacobian elliptic functions
<code>ellipk(m)</code>	Complete elliptic integral of the first kind.
<code>ellipkm1(p)</code>	Complete elliptic integral of the first kind around $m = 1$
<code>ellipkinc(phi, m)</code>	Incomplete elliptic integral of the first kind
<code>ellipe(m)</code>	Complete elliptic integral of the second kind
<code>ellipeinc(phi, m)</code>	Incomplete elliptic integral of the second kind

`scipy.special.ellipj(u, m) = <ufunc 'ellipj'>`

Jacobian elliptic functions

Calculates the Jacobian elliptic functions of parameter  $m$  between 0 and 1, and real argument  $u$ .

**Parameters** `m` : array\_like

Parameter.

`u` : array\_like

**Returns** `sn, cn, dn, ph` : ndarrays

Argument.  
The returned functions:

`sn(u|m)`, `cn(u|m)`, `dn(u|m)`

The value  $ph$  is such that if  $u = \text{ellipk}(ph, m)$ , then  $sn(u|m) = \sin(ph)$  and  $cn(u|m) = \cos(ph)$ .

**See also:**

`ellipk` Complete elliptic integral of the first kind.

*Notes*

Wrapper for the Cephes [R346] routine `ellpj`.

These functions are periodic, with quarter-period on the real axis equal to the complete elliptic integral `ellipk(m)`.

Relation to incomplete elliptic integral: If  $u = \text{ellipk}(phi, m)$ , then  $sn(ulm) = \sin(phi)$ , and  $cn(ulm) = \cos(phi)$ . The  $phi$  is called the amplitude of  $u$ .

Computation is by means of the arithmetic-geometric mean algorithm, except when  $m$  is within 1e-9 of 0 or 1. In the latter case with  $m$  close to 1, the approximation applies only for  $phi < pi/2$ .

*References*

[R346]

`scipy.special.ellipk(m)`

Complete elliptic integral of the first kind.

This function is defined as

$$K(m) = \int_0^{\pi/2} [1 - m \sin(t)^2]^{-1/2} dt$$



**Parameters** **m** : array\_like  
**Returns** **K** : array\_like The parameter of the elliptic integral.  
Value of the elliptic integral.

**See also:**

**ellipkm1** Complete elliptic integral of the first kind around  $m = 1$   
**ellipkinc** Incomplete elliptic integral of the first kind  
**ellipe** Complete elliptic integral of the second kind  
**ellipeinc** Incomplete elliptic integral of the second kind

**Notes**

For more precision around point  $m = 1$ , use `ellipkm1`, which this function calls.

`scipy.special.ellipkm1(p) = <ufunc 'ellipkm1'>`  
Complete elliptic integral of the first kind around  $m = 1$

This function is defined as

$$K(p) = \int_0^{\pi/2} [1 - m \sin(t)^2]^{-1/2} dt$$

where  $m = 1 - p$ .

**Parameters** **p** : array\_like  
**Returns** **K** : ndarray Defines the parameter of the elliptic integral as  $m = 1 - p$ .  
Value of the elliptic integral.

**See also:**

**ellipk** Complete elliptic integral of the first kind  
**ellipkinc** Incomplete elliptic integral of the first kind  
**ellipe** Complete elliptic integral of the second kind  
**ellipeinc** Incomplete elliptic integral of the second kind

**Notes**

Wrapper for the Cephes [R348] routine `ellpk`.

For  $p \leq 1$ , computation uses the approximation,

$$K(p) \approx P(p) - \log(p)Q(p),$$

where  $P$  and  $Q$  are tenth-order polynomials. The argument  $p$  is used internally rather than  $m$  so that the logarithmic singularity at  $m = 1$  will be shifted to the origin; this preserves maximum accuracy. For  $p > 1$ , the identity

$$K(p) = K(1/p)/\sqrt{p}$$

is used.

**References**

[R348]

`scipy.special.ellipkinc(phi, m) = <ufunc 'ellipkinc'>`  
Incomplete elliptic integral of the first kind

This function is defined as

$$K(\phi, m) = \int_0^{\phi} [1 - m \sin(t)^2]^{-1/2} dt$$

This function is also called  $F(\phi, m)$ .

**Parameters** **phi** : array\_like  
 amplitude of the elliptic integral  
**m** : array\_like  
 parameter of the elliptic integral  
**Returns** **K** : ndarray  
 Value of the elliptic integral

**See also:**

**ellipkm1** Complete elliptic integral of the first kind, near  $m = 1$   
**ellipk** Complete elliptic integral of the first kind  
**ellipe** Complete elliptic integral of the second kind  
**ellipeinc** Incomplete elliptic integral of the second kind

**Notes**

Wrapper for the Cephes [R347] routine *ellik*. The computation is carried out using the arithmetic-geometric mean algorithm.

**References**

[R347]

`scipy.special.ellipe(m) = <ufunc 'ellipe'>`  
 Complete elliptic integral of the second kind

This function is defined as

$$E(m) = \int_0^{\pi/2} [1 - m \sin(t)^2]^{1/2} dt$$

**Parameters** **m** : array\_like  
 Defines the parameter of the elliptic integral.  
**Returns** **E** : ndarray  
 Value of the elliptic integral.

**See also:**

**ellipkm1** Complete elliptic integral of the first kind, near  $m = 1$   
**ellipk** Complete elliptic integral of the first kind  
**ellipkinc** Incomplete elliptic integral of the first kind  
**ellipeinc** Incomplete elliptic integral of the second kind

**Notes**

Wrapper for the Cephes [R344] routine *ellpe*.

For  $m > 0$  the computation uses the approximation,

$$E(m) \approx P(1 - m) - (1 - m) \log(1 - m)Q(1 - m),$$

where  $P$  and  $Q$  are tenth-order polynomials. For  $m < 0$ , the relation

$$E(m) = E(m/(m - 1))\sqrt{1 - m}$$

is used.

**References**

[R344]

`scipy.special.ellipeinc(phi, m) = <ufunc 'ellipeinc'>`

Incomplete elliptic integral of the second kind

This function is defined as

$$E(\phi, m) = \int_0^\phi [1 - m \sin(t)^2]^{1/2} dt$$

**Parameters**

**phi** : array\_like  
amplitude of the elliptic integral.

**m** : array\_like  
parameter of the elliptic integral.

**Returns**

**E** : ndarray  
Value of the elliptic integral.

**See also:**

**ellipkm1** Complete elliptic integral of the first kind, near  $m = 1$

**ellipk** Complete elliptic integral of the first kind

**ellipkinc** Incomplete elliptic integral of the first kind

**ellipe** Complete elliptic integral of the second kind

**Notes**Wrapper for the Cephes [R345] routine *ellie*.

Computation uses arithmetic-geometric means algorithm.

**References**

[R345]

**Bessel Functions**


---

<code>jv(v, z)</code>	Bessel function of the first kind of real order and complex argument.
<code>jn(v, z)</code>	Bessel function of the first kind of real order and complex argument.
<code>jve(v, z)</code>	Exponentially scaled Bessel function of order $v$ .
<code>yn(n, x)</code>	Bessel function of the second kind of integer order and real argument.
<code>yv(v, z)</code>	Bessel function of the second kind of real order and complex argument.
<code>yve(v, z)</code>	Exponentially scaled Bessel function of the second kind of real order.
<code>kn(n, x)</code>	Modified Bessel function of the second kind of integer order $n$
<code>kv(v, z)</code>	Modified Bessel function of the second kind of real order $v$
<code>kve(v, z)</code>	Exponentially scaled modified Bessel function of the second kind.
<code>iv(v, z)</code>	Modified Bessel function of the first kind of real order.
<code>ive(v, z)</code>	Exponentially scaled modified Bessel function of the first kind
<code>hankel1(v, z)</code>	Hankel function of the first kind
<code>hankel1e(v, z)</code>	Exponentially scaled Hankel function of the first kind
<code>hankel2(v, z)</code>	Hankel function of the second kind
<code>hankel2e(v, z)</code>	Exponentially scaled Hankel function of the second kind

---

`scipy.special.jv(v, z) = <ufunc 'jv'>`

Bessel function of the first kind of real order and complex argument.

**Parameters** **v** : array\_like

**Returns**
**z** : array\_like
Order (float).  
**J** : ndarray
Argument (float or complex).  

Value of the Bessel function,  $J_\nu(z)$ .

**See also:**

[jve](#)  $J_\nu$  with leading exponential behavior stripped off.

**Notes**

For positive  $\nu$  values, the computation is carried out using the AMOS [R399] *zbesj* routine, which exploits the connection to the modified Bessel function  $I_\nu$ ,

$$\begin{aligned}
 J_\nu(z) &= \exp(n\pi i/2)I_\nu(-iz) & (\Im z > 0) \\
 J_\nu(z) &= \exp(-n\pi i/2)I_\nu(iz) & (\Im z < 0)
 \end{aligned}$$

For negative  $\nu$  values the formula,

$$J_{-\nu}(z) = J_\nu(z) \cos(\pi\nu) - Y_\nu(z) \sin(\pi\nu)$$

is used, where  $Y_\nu(z)$  is the Bessel function of the second kind, computed using the AMOS routine *zbesy*. Note that the second term is exactly zero for integer  $\nu$ ; to improve accuracy the second term is explicitly omitted for  $\nu$  values such that  $\nu = \text{floor}(\nu)$ .

**References**

[R399]

`scipy.special.jn` ( $\nu, z$ ) = <ufunc 'jv'>

Bessel function of the first kind of real order and complex argument.

**Parameters**
**v** : array\_like
Order (float).  
**z** : array\_like
Argument (float or complex).  
**Returns**
**J** : ndarray
Value of the Bessel function,  $J_\nu(z)$ .

**See also:**

[jve](#)  $J_\nu$  with leading exponential behavior stripped off.

**Notes**

For positive  $\nu$  values, the computation is carried out using the AMOS [R394] *zbesj* routine, which exploits the connection to the modified Bessel function  $I_\nu$ ,

$$\begin{aligned}
 J_\nu(z) &= \exp(n\pi i/2)I_\nu(-iz) & (\Im z > 0) \\
 J_\nu(z) &= \exp(-n\pi i/2)I_\nu(iz) & (\Im z < 0)
 \end{aligned}$$

For negative  $\nu$  values the formula,

$$J_{-\nu}(z) = J_\nu(z) \cos(\pi\nu) - Y_\nu(z) \sin(\pi\nu)$$

is used, where  $Y_\nu(z)$  is the Bessel function of the second kind, computed using the AMOS routine *zbesy*. Note that the second term is exactly zero for integer  $\nu$ ; to improve accuracy the second term is explicitly omitted for  $\nu$  values such that  $\nu = \text{floor}(\nu)$ .

**References**

[R394]

`scipy.special.jve`( $\nu, z$ ) = <ufunc 'jve'>  
Exponentially scaled Bessel function of order  $\nu$ .

Defined as:

$$\text{jve}(\nu, z) = \text{jv}(\nu, z) * \exp(-\text{abs}(z.\text{imag}))$$

**Parameters**

- $\nu$**  : array\_like  
Order (float).
- $z$**  : array\_like  
Argument (float or complex).

**Returns**

- $J$**  : ndarray  
Value of the exponentially scaled Bessel function.

**Notes**

For positive  $\nu$  values, the computation is carried out using the AMOS [R400] `zbesj` routine, which exploits the connection to the modified Bessel function  $I_\nu$ ,

$$\begin{aligned} J_\nu(z) &= \exp(n\pi i/2) I_\nu(-iz) & (\Im z > 0) \\ J_\nu(z) &= \exp(-n\pi i/2) I_\nu(iz) & (\Im z < 0) \end{aligned}$$

For negative  $\nu$  values the formula,

$$J_{-\nu}(z) = J_\nu(z) \cos(\pi\nu) - Y_\nu(z) \sin(\pi\nu)$$

is used, where  $Y_\nu(z)$  is the Bessel function of the second kind, computed using the AMOS routine `zbesy`. Note that the second term is exactly zero for integer  $\nu$ ; to improve accuracy the second term is explicitly omitted for  $\nu$  values such that  $\nu = \text{floor}(\nu)$ .

**References**

[R400]

`scipy.special.yn`( $n, x$ ) = <ufunc 'yn'>  
Bessel function of the second kind of integer order and real argument.

**Parameters**

- $n$**  : array\_like  
Order (integer).
- $z$**  : array\_like  
Argument (float).

**Returns**

- $Y$**  : ndarray  
Value of the Bessel function,  $Y_n(x)$ .

**See also:**

**`yv`** For real order and real or complex argument.

**Notes**

Wrapper for the Cephes [R479] routine `yn`.

The function is evaluated by forward recurrence on  $n$ , starting with values computed by the Cephes routines `y0` and `y1`. If  $n = 0$  or 1, the routine for `y0` or `y1` is called directly.

**References**

[R479]

`scipy.special.yv(v, z) = <ufunc 'yv'>`

Bessel function of the second kind of real order and complex argument.

**Parameters** `v` : array\_like  
Order (float).

`z` : array\_like  
Argument (float or complex).

**Returns** `Y` : ndarray  
Value of the Bessel function of the second kind,  $Y_v(x)$ .

**See also:**`yve`  $Y_v$  with leading exponential behavior stripped off.**Notes**

For positive  $\nu$  values, the computation is carried out using the AMOS [R482] `zbesy` routine, which exploits the connection to the Hankel Bessel functions  $H_v^{(1)}$  and  $H_v^{(2)}$ ,

$$Y_\nu(z) = \frac{1}{2i}(H_\nu^{(1)} - H_\nu^{(2)}).$$

For negative  $\nu$  values the formula,

$$Y_{-\nu}(z) = Y_\nu(z) \cos(\pi\nu) + J_\nu(z) \sin(\pi\nu)$$

is used, where  $J_\nu(z)$  is the Bessel function of the first kind, computed using the AMOS routine `zbesj`. Note that the second term is exactly zero for integer  $\nu$ ; to improve accuracy the second term is explicitly omitted for  $\nu$  values such that  $\nu = \text{floor}(\nu)$ .

**References**

[R482]

`scipy.special.yve(v, z) = <ufunc 'yve'>`

Exponentially scaled Bessel function of the second kind of real order.

Returns the exponentially scaled Bessel function of the second kind of real order  $\nu$  at complex  $z$ :
$$\text{yve}(\nu, z) = \text{yv}(\nu, z) * \exp(-\text{abs}(z.\text{imag}))$$

**Parameters** `v` : array\_like  
Order (float).

`z` : array\_like  
Argument (float or complex).

**Returns** `Y` : ndarray  
Value of the exponentially scaled Bessel function.

**Notes**

For positive  $\nu$  values, the computation is carried out using the AMOS [R483] `zbesy` routine, which exploits the connection to the Hankel Bessel functions  $H_v^{(1)}$  and  $H_v^{(2)}$ ,

$$Y_\nu(z) = \frac{1}{2i}(H_\nu^{(1)} - H_\nu^{(2)}).$$

For negative  $\nu$  values the formula,

$$Y_{-\nu}(z) = Y_\nu(z) \cos(\pi\nu) + J_\nu(z) \sin(\pi\nu)$$

is used, where  $J_\nu(z)$  is the Bessel function of the first kind, computed using the AMOS routine `zbesj`. Note that the second term is exactly zero for integer  $\nu$ ; to improve accuracy the second term is explicitly omitted for  $\nu$  values such that  $\nu = \text{floor}(\nu)$ .

### References

[R483]

`scipy.special.kn(n, x) = <ufunc 'kn'>`

Modified Bessel function of the second kind of integer order  $n$

Returns the modified Bessel function of the second kind for integer order  $n$  at real  $z$ .

These are also sometimes called functions of the third kind, Basset functions, or Macdonald functions.

<b>Parameters</b>	<b>n</b> : array_like of int	Order of Bessel functions (floats will truncate with a warning)
	<b>z</b> : array_like of float	Argument at which to evaluate the Bessel functions
<b>Returns</b>	<b>out</b> : ndarray	The results

### See also:

**kv** Same function, but accepts real order and complex argument

**kvp** Derivative of this function

### Notes

Wrapper for AMOS [R411] routine `zbesk`. For a discussion of the algorithm used, see [R412] and the references therein.

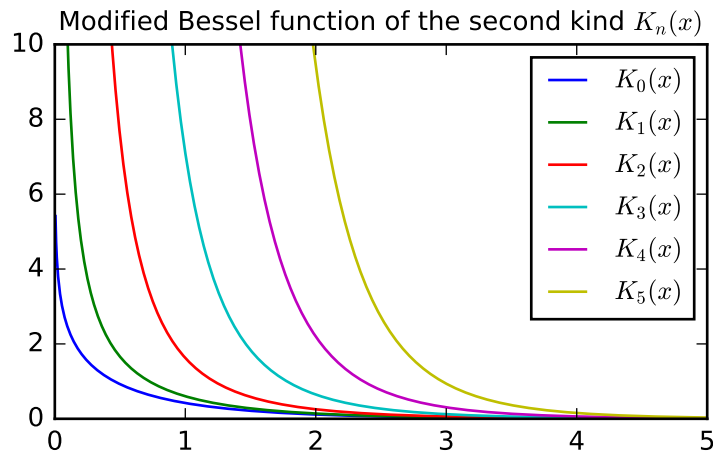
### References

[R411], [R412]

### Examples

Plot the function of several orders for real input:

```
>>> from scipy.special import kn
>>> import matplotlib.pyplot as plt
>>> x = np.linspace(0, 5, 1000)
>>> for N in range(6):
...     plt.plot(x, kn(N, x), label='$K_{%d}(x)$'.format(N))
>>> plt.ylim(0, 10)
>>> plt.legend()
>>> plt.title(r'Modified Bessel function of the second kind $K_n(x)$')
>>> plt.show()
```



Calculate for a single value at multiple orders:

```
>>> kn([4, 5, 6], 1)
array([ 44.23241585,  360.9605896 , 3653.83831186])
```

`scipy.special.kv`( $\nu, z$ ) = <ufunc 'kv'>

Modified Bessel function of the second kind of real order  $\nu$

Returns the modified Bessel function of the second kind for real order  $\nu$  at complex  $z$ .

These are also sometimes called functions of the third kind, Basset functions, or Macdonald functions. They are defined as those solutions of the modified Bessel equation for which,

$$K_\nu(x) \sim \sqrt{\pi/(2x)} \exp(-x)$$

as  $x \rightarrow \infty$  [R415].

**Parameters**

- v** : array\_like of float  
Order of Bessel functions
- z** : array\_like of complex  
Argument at which to evaluate the Bessel functions

**Returns**

- out** : ndarray  
The results. Note that input must be of complex type to get complex output, e.g. `kv(3, -2+0j)` instead of `kv(3, -2)`.

**See also:**

***kve*** This function with leading exponential behavior stripped off.  
***kvp*** Derivative of this function

**Notes**

Wrapper for AMOS [R413] routine `zbesk`. For a discussion of the algorithm used, see [R414] and the references therein.

**References**

[R413], [R414], [R415]



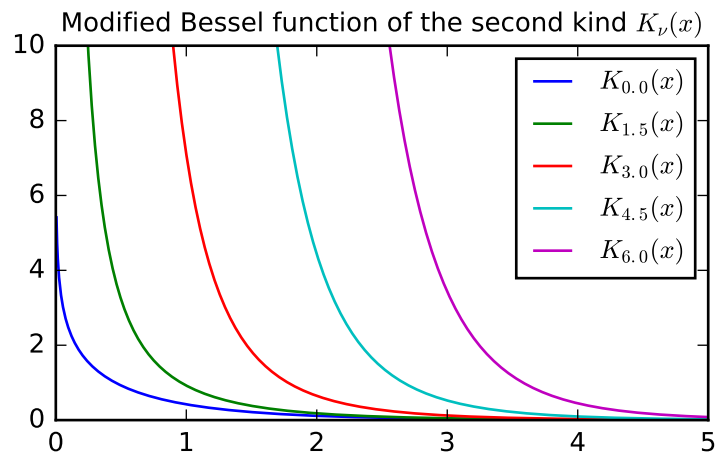
### Examples

Plot the function of several orders for real input:

```

>>> from scipy.special import kv
>>> import matplotlib.pyplot as plt
>>> x = np.linspace(0, 5, 1000)
>>> for N in np.linspace(0, 6, 5):
...     plt.plot(x, kv(N, x), label='$K_{\{\}\}(x)$'.format(N))
>>> plt.ylim(0, 10)
>>> plt.legend()
>>> plt.title(r'Modified Bessel function of the second kind $K_{\nu}(x)$')
>>> plt.show()

```



Calculate for a single value at multiple orders:

```

>>> kv([4, 4.5, 5], 1+2j)
array([ 0.1992+2.3892j,  2.3493+3.6j      ,  7.2827+3.8104j])

```

`scipy.special.kve` ( $v, z$ ) = <ufunc 'kve'>

Exponentially scaled modified Bessel function of the second kind.

Returns the exponentially scaled, modified Bessel function of the second kind (sometimes called the third kind) for real order  $v$  at complex  $z$ :

$kve(v, z) = kv(v, z) * \exp(z)$

**Parameters**

- v**: array\_like of float  
Order of Bessel functions
- z**: array\_like of complex  
Argument at which to evaluate the Bessel functions

**Returns**

- out**: ndarray  
The exponentially scaled modified Bessel function of the second kind.

### Notes

Wrapper for AMOS [R416] routine `zbesk`. For a discussion of the algorithm used, see [R417] and the references therein.

**References**

[R416], [R417]

`scipy.special.iv(v, z) = <ufunc 'iv'>`

Modified Bessel function of the first kind of real order.

**Parameters** **v** : array\_like  
 Order. If *z* is of real type and negative, *v* must be integer valued.  
**z** : array\_like of float or complex  
 Argument.  
**Returns** **out** : ndarray  
 Values of the modified Bessel function.

**See also:**

[kve](#) This function with leading exponential behavior stripped off.

**Notes**

For real *z* and *v* ∈ [−50, 50], the evaluation is carried out using Temme’s method [R387]. For larger orders, uniform asymptotic expansions are applied.

For complex *z* and positive *v*, the AMOS [R388] *zbesi* routine is called. It uses a power series for small *z*, the asymptotic expansion for large *abs(z)*, the Miller algorithm normalized by the Wronskian and a Neumann series for intermediate magnitudes, and the uniform asymptotic expansions for *I<sub>v</sub>(z)* and *J<sub>v</sub>(z)* for large orders. Backward recurrence is used to generate sequences or reduce orders when necessary.

The calculations above are done in the right half plane and continued into the left half plane by the formula,

$$I_v(z \exp(\pm i\pi)) = \exp(\pm \pi v) I_v(z)$$

(valid when the real part of *z* is positive). For negative *v*, the formula

$$I_{-v}(z) = I_v(z) + \frac{2}{\pi} \sin(\pi v) K_v(z)$$

is used, where *K<sub>v</sub>(z)* is the modified Bessel function of the second kind, evaluated using the AMOS routine *zbesk*.

**References**

[R387], [R388]

`scipy.special.ive(v, z) = <ufunc 'ive'>`

Exponentially scaled modified Bessel function of the first kind

Defined as:

```
ive(v, z) = iv(v, z) * exp(-abs(z.real))
```

**Parameters** **v** : array\_like of float  
 Order.  
**z** : array\_like of float or complex  
 Argument.  
**Returns** **out** : ndarray  
 Values of the exponentially scaled modified Bessel function.

**Notes**

For positive  $\nu$ , the AMOS [R389] *zbesi* routine is called. It uses a power series for small  $z$ , the asymptotic expansion for large  $\text{abs}(z)$ , the Miller algorithm normalized by the Wronskian and a Neumann series for intermediate magnitudes, and the uniform asymptotic expansions for  $I_\nu(z)$  and  $J_\nu(z)$  for large orders. Backward recurrence is used to generate sequences or reduce orders when necessary.

The calculations above are done in the right half plane and continued into the left half plane by the formula,

$$I_\nu(z \exp(\pm i\pi)) = \exp(\pm \pi\nu) I_\nu(z)$$

(valid when the real part of  $z$  is positive). For negative  $\nu$ , the formula

$$I_{-\nu}(z) = I_\nu(z) + \frac{2}{\pi} \sin(\pi\nu) K_\nu(z)$$

is used, where  $K_\nu(z)$  is the modified Bessel function of the second kind, evaluated using the AMOS routine *zbesk*.

**References**

[R389]

`scipy.special.hankel1` ( $\nu, z$ ) = <ufunc 'hankel1'>

Hankel function of the first kind

**Parameters** **v** : array\_like  
Order (float).  
**z** : array\_like  
Argument (float or complex).  
**Returns** **out** : Values of the Hankel function of the first kind.

**See also:**

**`hankelle`** this function with leading exponential behavior stripped off.

**Notes**

A wrapper for the AMOS [R375] routine *zbesh*, which carries out the computation using the relation,

$$H_\nu^{(1)}(z) = \frac{2}{i\pi} \exp(-i\pi\nu/2) K_\nu(z \exp(-i\pi/2))$$

where  $K_\nu$  is the modified Bessel function of the second kind. For negative orders, the relation

$$H_{-\nu}^{(1)}(z) = H_\nu^{(1)}(z) \exp(i\pi\nu)$$

is used.

**References**

[R375]

`scipy.special.hankelle` ( $\nu, z$ ) = <ufunc 'hankelle'>

Exponentially scaled Hankel function of the first kind

Defined as:

`hankelle(v, z) = hankel1(v, z) * exp(-1j * z)`

**Parameters** **v** : array\_like  
Order (float).  
**z** : array\_like  
Argument (float or complex).  
**Returns** **out** : Values of the exponentially scaled Hankel function.

**Notes**

A wrapper for the AMOS [R376] routine *zbesb*, which carries out the computation using the relation,

$$H_v^{(1)}(z) = \frac{2}{i\pi} \exp(-i\pi v/2) K_v(z \exp(-i\pi/2))$$

where  $K_v$  is the modified Bessel function of the second kind. For negative orders, the relation

$$H_{-v}^{(1)}(z) = H_v^{(1)}(z) \exp(i\pi v)$$

is used.

**References**

[R376]

`scipy.special.hankel12` (*v*, *z*) = <ufunc 'hankel2'>

Hankel function of the second kind

**Parameters** *v* : array\_like  
 Order (float).  
*z* : array\_like  
 Argument (float or complex).  
**Returns** **out** : Values of the Hankel function of the second kind.

**See also:**

**`hankel12e`** this function with leading exponential behavior stripped off.

**Notes**

A wrapper for the AMOS [R377] routine *zbesb*, which carries out the computation using the relation,

$$H_v^{(2)}(z) = -\frac{2}{i\pi} \exp(i\pi v/2) K_v(z \exp(i\pi/2))$$

where  $K_v$  is the modified Bessel function of the second kind. For negative orders, the relation

$$H_{-v}^{(2)}(z) = H_v^{(2)}(z) \exp(-i\pi v)$$

is used.

**References**

[R377]

`scipy.special.hankel12e` (*v*, *z*) = <ufunc 'hankel2e'>

Exponentially scaled Hankel function of the second kind

Defined as:

`hankel12e(v, z) = hankel12(v, z) * exp(1j * z)`

**Parameters** *v* : array\_like  
 Order (float).  
*z* : array\_like  
 Argument (float or complex).  
**Returns** **out** : Values of the exponentially scaled Hankel function of the second kind.

**Notes**

A wrapper for the AMOS [R378] routine *zbesb*, which carries out the computation using the relation,

$$H_v^{(2)}(z) = -\frac{2}{i\pi} \exp\left(\frac{i\pi v}{2}\right) K_v\left(z \exp\left(\frac{i\pi}{2}\right)\right)$$

where  $K_v$  is the modified Bessel function of the second kind. For negative orders, the relation

$$H_{-v}^{(2)}(z) = H_v^{(2)}(z) \exp(-i\pi v)$$

is used.

**References**

[R378]

The following is not an universal function:

---

`lmbda(v, x)` Jahnke-Emden Lambda function, `Lambdav(x)`.

---

`scipy.special.lmbda(v, x)`

Jahnke-Emden Lambda function, `Lambdav(x)`.

This function is defined as [R423],

$$\Lambda_v(x) = \Gamma(v+1) \frac{J_v(x)}{(x/2)^v},$$

where  $\Gamma$  is the gamma function and  $J_v$  is the Bessel function of the first kind.

<b>Parameters</b>	<b>v</b> : float	Order of the Lambda function
	<b>x</b> : float	Value at which to evaluate the function and derivatives
<b>Returns</b>	<b>vl</b> : ndarray	Values of <code>Lambda_vi(x)</code> , for $v_i=v-\text{int}(v)$ , $v_i=1+v-\text{int}(v)$ , ..., $v_i=v$ .
	<b>dl</b> : ndarray	Derivatives <code>Lambda_vi'(x)</code> , for $v_i=v-\text{int}(v)$ , $v_i=1+v-\text{int}(v)$ , ..., $v_i=v$ .

**References**

[R422], [R423]

**Zeros of Bessel Functions**

These are not universal functions:

---

<code>jnjnp_zeros(nt)</code>	Compute zeros of integer-order Bessel functions $J_n$ and $J_n'$ .
<code>jny_n_zeros(n, nt)</code>	Compute <i>nt</i> zeros of Bessel functions $J_n(x)$ , $J_n'(x)$ , $Y_n(x)$ , and $Y_n'(x)$ .
<code>j_n_zeros(n, nt)</code>	Compute zeros of integer-order Bessel function $J_n(x)$ .
<code>jnp_zeros(n, nt)</code>	Compute zeros of integer-order Bessel function derivative $J_n'(x)$ .
<code>yn_zeros(n, nt)</code>	Compute zeros of integer-order Bessel function $Y_n(x)$ .
<code>ynp_zeros(n, nt)</code>	Compute zeros of integer-order Bessel function derivative $Y_n'(x)$ .
<code>y0_zeros(nt[, complex])</code>	Compute <i>nt</i> zeros of Bessel function $Y_0(z)$ , and derivative at each zero.
<code>y1_zeros(nt[, complex])</code>	Compute <i>nt</i> zeros of Bessel function $Y_1(z)$ , and derivative at each zero.
<code>y1p_zeros(nt[, complex])</code>	Compute <i>nt</i> zeros of Bessel derivative $Y_1'(z)$ , and value at each zero.

---

`scipy.special.jnjnp_zeros(nt)`

Compute zeros of integer-order Bessel functions  $J_n$  and  $J_n'$ .

Results are arranged in order of the magnitudes of the zeros.

**Parameters** **nt** : int  
 Number ( $\leq 1200$ ) of zeros to compute

**Returns** **zo[l-1]** : ndarray  
 Value of the  $l$ th zero of  $J_n(x)$  and  $J_n'(x)$ . Of length  $nt$ .

**n[l-1]** : ndarray  
 Order of the  $J_n(x)$  or  $J_n'(x)$  associated with  $l$ th zero. Of length  $nt$ .

**m[l-1]** : ndarray  
 Serial number of the zeros of  $J_n(x)$  or  $J_n'(x)$  associated with  $l$ th zero. Of length  $nt$ .

**t[l-1]** : ndarray  
 0 if  $l$ th zero in  $zo$  is zero of  $J_n(x)$ , 1 if it is a zero of  $J_n'(x)$ . Of length  $nt$ .

**See also:**

`jn_zeros`, `jnp_zeros`

**References**

[R396]

`scipy.special.jnyn_zeros` ( $n, nt$ )

Compute  $nt$  zeros of Bessel functions  $J_n(x)$ ,  $J_n'(x)$ ,  $Y_n(x)$ , and  $Y_n'(x)$ .

Returns 4 arrays of length  $nt$ , corresponding to the first  $nt$  zeros of  $J_n(x)$ ,  $J_n'(x)$ ,  $Y_n(x)$ , and  $Y_n'(x)$ , respectively.

**Parameters** **n** : int  
 Order of the Bessel functions

**nt** : int  
 Number ( $\leq 1200$ ) of zeros to compute

**See `jn_zeros`, `jnp_zeros`, `yn_zeros`, `ynp_zeros` to get separate arrays.**

**References**

[R398]

`scipy.special.jn_zeros` ( $n, nt$ )

Compute zeros of integer-order Bessel function  $J_n(x)$ .

**Parameters** **n** : int  
 Order of Bessel function

**nt** : int  
 Number of zeros to return

**References**

[R395]

`scipy.special.jnp_zeros` ( $n, nt$ )

Compute zeros of integer-order Bessel function derivative  $J_n'(x)$ .

**Parameters** **n** : int  
 Order of Bessel function

**nt** : int  
 Number of zeros to return

**References**[\[R397\]](#)`scipy.special.yn_zeros(n, nt)`Compute zeros of integer-order Bessel function  $Y_n(x)$ .

**Parameters**

**n** : int  
Order of Bessel function

**nt** : int  
Number of zeros to return

**References**[\[R480\]](#)`scipy.special.ynp_zeros(n, nt)`Compute zeros of integer-order Bessel function derivative  $Y_n'(x)$ .

**Parameters**

**n** : int  
Order of Bessel function

**nt** : int  
Number of zeros to return

**References**[\[R481\]](#)`scipy.special.y0_zeros(nt, complex=False)`Compute  $nt$  zeros of Bessel function  $Y_0(z)$ , and derivative at each zero.The derivatives are given by  $Y_0'(z_0) = -Y_1(z_0)$  at each zero  $z_0$ .

**Parameters**

**nt** : int  
Number of zeros to return

**complex** : bool, default False  
Set to False to return only the real zeros; set to True to return only the complex zeros with negative real part and positive imaginary part. Note that the complex conjugates of the latter are also zeros of the function, but are not returned by this routine.

**Returns**

**z0n** : ndarray  
Location of  $n$ th zero of  $Y_0(z)$

**y0pz0n** : ndarray  
Value of derivative  $Y_0'(z_0)$  for  $n$ th zero

**References**[\[R475\]](#)`scipy.special.y1_zeros(nt, complex=False)`Compute  $nt$  zeros of Bessel function  $Y_1(z)$ , and derivative at each zero.The derivatives are given by  $Y_1'(z_1) = Y_0(z_1)$  at each zero  $z_1$ .

**Parameters**

**nt** : int  
Number of zeros to return

**complex** : bool, default False  
Set to False to return only the real zeros; set to True to return only the complex zeros with negative real part and positive imaginary part. Note that the complex conjugates of the latter are also zeros of the function, but are not returned by this routine.

**Returns**

**z1n** : ndarray

Location of nth zero of  $Y_1(z)$   
**y1pz1n** : ndarray  
 Value of derivative  $Y_1'(z_1)$  for nth zero

### References

[R477]

`scipy.special.y1p_zeros(nt, complex=False)`  
 Compute nt zeros of Bessel derivative  $Y_1'(z)$ , and value at each zero.

The values are given by  $Y_1(z_1)$  at each  $z_1$  where  $Y_1'(z_1)=0$ .

**Parameters**    **nt** : int  
                         Number of zeros to return  
                         **complex** : bool, default False  
                         Set to False to return only the real zeros; set to True to return only the  
                         complex zeros with negative real part and positive imaginary part. Note  
                         that the complex conjugates of the latter are also zeros of the function, but  
                         are not returned by this routine.

**Returns**        **z1pn** : ndarray  
                         Location of nth zero of  $Y_1'(z)$   
                         **y1z1pn** : ndarray  
                         Value of derivative  $Y_1'(z_1)$  for nth zero

### References

[R478]

### Faster versions of common Bessel Functions

$j_0(x)$	Bessel function of the first kind of order 0.
$j_1(x)$	Bessel function of the first kind of order 1.
$y_0(x)$	Bessel function of the second kind of order 0.
$y_1(x)$	Bessel function of the second kind of order 1.
$i_0(x)$	Modified Bessel function of order 0.
$i_0e(x)$	Exponentially scaled modified Bessel function of order 0.
$i_1(x)$	Modified Bessel function of order 1.
$i_1e(x)$	Exponentially scaled modified Bessel function of order 1.
$k_0(x)$	Modified Bessel function of the second kind of order 0, $K_0$ .
$k_0e(x)$	Exponentially scaled modified Bessel function K of order 0
$k_1(x)$	Modified Bessel function of the second kind of order 1, $K_1(x)$ .
$k_1e(x)$	Exponentially scaled modified Bessel function K of order 1

`scipy.special.j0(x) = <ufunc 'j0'>`  
 Bessel function of the first kind of order 0.

**Parameters**    **x** : array\_like  
                         Argument (float).  
**Returns**        **J** : ndarray  
                         Value of the Bessel function of the first kind of order 0 at x.

### See also:

**jv**                Bessel function of real order and complex argument.



**Notes**

The domain is divided into the intervals  $[0, 5]$  and  $(5, \text{infinity})$ . In the first interval the following rational approximation is used:

$$J_0(x) \approx (w - r_1^2)(w - r_2^2) \frac{P_3(w)}{Q_8(w)},$$

where  $w = x^2$  and  $r_1, r_2$  are the zeros of  $J_0$ , and  $P_3$  and  $Q_8$  are polynomials of degrees 3 and 8, respectively.

In the second interval, the Hankel asymptotic expansion is employed with two rational functions of degree 6/6 and 7/7.

This function is a wrapper for the Cephes [R392] routine `j0`.

**References**

[R392]

`scipy.special.j1(x) = <ufunc 'j1'>`

Bessel function of the first kind of order 1.

**Parameters** `x` : array\_like  
Argument (float).

**Returns** `J` : ndarray  
Value of the Bessel function of the first kind of order 1 at  $x$ .

**See also:**

`jv`

**Notes**

The domain is divided into the intervals  $[0, 8]$  and  $(8, \text{infinity})$ . In the first interval a 24 term Chebyshev expansion is used. In the second, the asymptotic trigonometric representation is employed using two rational functions of degree 5/5.

This function is a wrapper for the Cephes [R393] routine `j1`.

**References**

[R393]

`scipy.special.y0(x) = <ufunc 'y0'>`

Bessel function of the second kind of order 0.

**Parameters** `x` : array\_like  
Argument (float).

**Returns** `Y` : ndarray  
Value of the Bessel function of the second kind of order 0 at  $x$ .

**See also:**

`j0`, `yv`

**Notes**

The domain is divided into the intervals  $[0, 5]$  and  $(5, \text{infinity})$ . In the first interval a rational approximation  $R(x)$  is employed to compute,

$$Y_0(x) = R(x) + \frac{2 \log(x) J_0(x)}{\pi},$$

where  $J_0$  is the Bessel function of the first kind of order 0.

In the second interval, the Hankel asymptotic expansion is employed with two rational functions of degree 6/6 and 7/7.

This function is a wrapper for the Cephes [R474] routine `y0`.

**References**

[R474]

`scipy.special.y1(x) = <ufunc 'y1'>`  
 Bessel function of the second kind of order 1.

**Parameters** `x` : array\_like  
 Argument (float).  
**Returns** `Y` : ndarray  
 Value of the Bessel function of the second kind of order 1 at `x`.

**See also:**

`j1`, `yn`, `yv`

**Notes**

The domain is divided into the intervals [0, 8] and (8, infinity). In the first interval a 25 term Chebyshev expansion is used, and computing  $J_1$  (the Bessel function of the first kind) is required. In the second, the asymptotic trigonometric representation is employed using two rational functions of degree 5/5.

This function is a wrapper for the Cephes [R476] routine `y1`.

**References**

[R476]

`scipy.special.i0(x) = <ufunc 'i0'>`  
 Modified Bessel function of order 0.

Defined as,

$$I_0(x) = \sum_{k=0}^{\infty} \frac{(x^2/4)^k}{(k!)^2} = J_0(ix),$$

where  $J_0$  is the Bessel function of the first kind of order 0.

**Parameters** `x` : array\_like  
 Argument (float).  
**Returns** `I` : ndarray  
 Value of the modified Bessel function of order 0 at `x`.

**See also:**

`iv`, `i0e`

**Notes**

The range is partitioned into the two intervals [0, 8] and (8, infinity). Chebyshev polynomial expansions are employed in each interval.

This function is a wrapper for the Cephes [R379] routine `i0`.

**References**

[R379]

`scipy.special.i0e(x) = <ufunc 'i0e'>`

Exponentially scaled modified Bessel function of order 0.

Defined as:

$$i0e(x) = \exp(-\text{abs}(x)) * i0(x).$$

**Parameters** `x`: array\_like

**Returns** `I`: ndarray Argument (float)

Value of the exponentially scaled modified Bessel function of order 0 at  $x$ .

**See also:**

`iv`, `i0`

**Notes**

The range is partitioned into the two intervals [0, 8] and (8, infinity). Chebyshev polynomial expansions are employed in each interval. The polynomial expansions used are the same as those in `i0`, but they are not multiplied by the dominant exponential factor.

This function is a wrapper for the Cephes [R380] routine `i0e`.

**References**

[R380]

`scipy.special.i1(x) = <ufunc 'i1'>`

Modified Bessel function of order 1.

Defined as,

$$I_1(x) = \frac{1}{2}x \sum_{k=0}^{\infty} \frac{(x^2/4)^k}{k!(k+1)!} = -\iota J_1(\iota x),$$

where  $J_1$  is the Bessel function of the first kind of order 1.

**Parameters** `x`: array\_like

**Returns** `I`: ndarray Argument (float)

Value of the modified Bessel function of order 1 at  $x$ .

**See also:**

`iv`, `ile`

**Notes**

The range is partitioned into the two intervals [0, 8] and (8, infinity). Chebyshev polynomial expansions are employed in each interval.

This function is a wrapper for the Cephes [R381] routine `i1`.

**References**

[R381]

`scipy.special.ile(x) = <ufunc 'ile'>`

Exponentially scaled modified Bessel function of order 1.

Defined as:

$i1e(x) = \exp(-\text{abs}(x)) * i1(x)$

**Parameters** `x` : array\_like  
**Returns** `I` : ndarray      Argument (float)  
Value of the exponentially scaled modified Bessel function of order 1 at  $x$ .

**See also:**

`iv`, `i1`

**Notes**

The range is partitioned into the two intervals [0, 8] and (8, infinity). Chebyshev polynomial expansions are employed in each interval. The polynomial expansions used are the same as those in `i1`, but they are not multiplied by the dominant exponential factor.

This function is a wrapper for the Cephys [R382] routine `i1e`.

**References**

[R382]

`scipy.special.k0(x) = <ufunc 'k0'>`

Modified Bessel function of the second kind of order 0,  $K_0$ .

This function is also sometimes referred to as the modified Bessel function of the third kind of order 0.

**Parameters** `x` : array\_like  
**Returns** `K` : ndarray      Argument (float).  
Value of the modified Bessel function  $K_0$  at  $x$ .

**See also:**

`kv`, `k0e`

**Notes**

The range is partitioned into the two intervals [0, 2] and (2, infinity). Chebyshev polynomial expansions are employed in each interval.

This function is a wrapper for the Cephys [R403] routine `k0`.

**References**

[R403]

`scipy.special.k0e(x) = <ufunc 'k0e'>`

Exponentially scaled modified Bessel function K of order 0

Defined as:

$k0e(x) = \exp(x) * k0(x)$ .

**Parameters** `x` : array\_like  
**Returns** `K` : ndarray      Argument (float)  
Value of the exponentially scaled modified Bessel function K of order 0 at  $x$ .

**See also:**`kv, k0`**Notes**

The range is partitioned into the two intervals  $[0, 2]$  and  $(2, \text{infinity})$ . Chebyshev polynomial expansions are employed in each interval.

This function is a wrapper for the Cephes [R404] routine `k0e`.

**References**[\[R404\]](#)

`scipy.special.k1` ( $x$ ) = <ufunc 'k1'>

Modified Bessel function of the second kind of order 1,  $K_1(x)$ .

**Parameters** `x` : array\_like

**Returns** `K` : ndarray Argument (float)

Value of the modified Bessel function  $K$  of order 1 at  $x$ .

**See also:**`kv, k1e`**Notes**

The range is partitioned into the two intervals  $[0, 2]$  and  $(2, \text{infinity})$ . Chebyshev polynomial expansions are employed in each interval.

This function is a wrapper for the Cephes [R405] routine `k1`.

**References**[\[R405\]](#)

`scipy.special.k1e` ( $x$ ) = <ufunc 'k1e'>

Exponentially scaled modified Bessel function  $K$  of order 1

Defined as:

$$k1e(x) = \exp(x) * k1(x)$$

**Parameters** `x` : array\_like

**Returns** `K` : ndarray Argument (float)

Value of the exponentially scaled modified Bessel function  $K$  of order 1 at  $x$ .

**See also:**`kv, k1`**Notes**

The range is partitioned into the two intervals  $[0, 2]$  and  $(2, \text{infinity})$ . Chebyshev polynomial expansions are employed in each interval.

This function is a wrapper for the Cephes [R406] routine `k1e`.

**References**

[R406]

**Integrals of Bessel Functions**

---

<code>itj0y0(x)</code>	Integrals of Bessel functions of order 0
<code>it2j0y0(x)</code>	Integrals related to Bessel functions of order 0
<code>iti0k0(x)</code>	Integrals of modified Bessel functions of order 0
<code>it2i0k0(x)</code>	Integrals related to modified Bessel functions of order 0
<code>besselpoly(a, lmb, nu)</code>	Weighted integral of a Bessel function.

---

`scipy.special.itj0y0(x) = <ufunc 'itj0y0'>`

Integrals of Bessel functions of order 0

Returns simple integrals from 0 to  $x$  of the zeroth order Bessel functions  $j_0$  and  $y_0$ .

**Returns**  $ij_0, iy_0$

`scipy.special.it2j0y0(x) = <ufunc 'it2j0y0'>`

Integrals related to Bessel functions of order 0

**Returns**  $ij_0$

`integral((1-j0(t))/t, t=0..x)`

$iy_0$

`integral(y0(t)/t, t=x..inf)`

`scipy.special.iti0k0(x) = <ufunc 'iti0k0'>`

Integrals of modified Bessel functions of order 0

Returns simple integrals from 0 to  $x$  of the zeroth order modified Bessel functions  $i_0$  and  $k_0$ .

**Returns**  $ii_0, ik_0$

`scipy.special.it2i0k0(x) = <ufunc 'it2i0k0'>`

Integrals related to modified Bessel functions of order 0

**Returns**  $ii_0$

`integral((i0(t)-1)/t, t=0..x)`

$ik_0$

`int(k0(t)/t, t=x..inf)`

`scipy.special.besselpoly(a, lmb, nu) = <ufunc 'besselpoly'>`

Weighted integral of a Bessel function.

$$\int_0^1 x^\lambda J_\nu(2ax) dx$$

where  $J_\nu$  is a Bessel function and  $\lambda = lmb, \nu = nu$ .

**Derivatives of Bessel Functions**

---

<code>jvp(v, z[, n])</code>	Compute nth derivative of Bessel function $J_\nu(z)$ with respect to $z$ .
<code>yvp(v, z[, n])</code>	Compute nth derivative of Bessel function $Y_\nu(z)$ with respect to $z$ .
<code>kvp(v, z[, n])</code>	Compute nth derivative of real-order modified Bessel function $K_\nu(z)$
<code>ivp(v, z[, n])</code>	Compute nth derivative of modified Bessel function $I_\nu(z)$ with respect to $z$ .
<code>h1vp(v, z[, n])</code>	Compute nth derivative of Hankel function $H_1(z)$ with respect to $z$ .
<code>h2vp(v, z[, n])</code>	Compute nth derivative of Hankel function $H_2(z)$ with respect to $z$ .

---

`scipy.special.jvp(v, z, n=1)`

Compute nth derivative of Bessel function  $J_v(z)$  with respect to  $z$ .

**Parameters**

- v** : float  
Order of Bessel function
- z** : complex  
Argument at which to evaluate the derivative
- n** : int, default 1  
Order of derivative

**Notes**

The derivative is computed using the relation DLFM 10.6.7 [R402].

**References**

[R401], [R402]

`scipy.special.yvp(v, z, n=1)`

Compute nth derivative of Bessel function  $Y_v(z)$  with respect to  $z$ .

**Parameters**

- v** : float  
Order of Bessel function
- z** : complex  
Argument at which to evaluate the derivative
- n** : int, default 1  
Order of derivative

**Notes**

The derivative is computed using the relation DLFM 10.6.7 [R485].

**References**

[R484], [R485]

`scipy.special.kvp(v, z, n=1)`

Compute nth derivative of real-order modified Bessel function  $K_v(z)$

$K_v(z)$  is the modified Bessel function of the second kind. Derivative is calculated with respect to  $z$ .

**Parameters**

- v** : array\_like of float  
Order of Bessel function
- z** : array\_like of complex  
Argument at which to evaluate the derivative
- n** : int  
Order of derivative. Default is first derivative.

**Returns**

- out** : ndarray  
The results

**Notes**

The derivative is computed using the relation DLFM 10.29.5 [R419].

**References**

[R418], [R419]

### Examples

Calculate multiple values at order 5:

```
>>> from scipy.special import kvp
>>> kvp(5, (1, 2, 3+5j))
array([-1849.0354+0.j      , -25.7735+0.j      , -0.0307+0.0875jj])
```

Calculate for a single value at multiple orders:

```
>>> kvp((4, 4.5, 5), 1)
array([-184.0309, -568.9585, -1849.0354])
```

`scipy.special.ivp(v, z, n=1)`

Compute nth derivative of modified Bessel function  $I_\nu(z)$  with respect to  $z$ .

**Parameters**

- v** : array\_like of float  
Order of Bessel function
- z** : array\_like of complex  
Argument at which to evaluate the derivative
- n** : int, default 1  
Order of derivative

### Notes

The derivative is computed using the relation DLFM 10.29.5 [R391].

### References

[R390], [R391]

`scipy.special.h1vp(v, z, n=1)`

Compute nth derivative of Hankel function  $H_1^\nu(z)$  with respect to  $z$ .

**Parameters**

- v** : float  
Order of Hankel function
- z** : complex  
Argument at which to evaluate the derivative
- n** : int, default 1  
Order of derivative

### Notes

The derivative is computed using the relation DLFM 10.6.7 [R372].

### References

[R371], [R372]

`scipy.special.h2vp(v, z, n=1)`

Compute nth derivative of Hankel function  $H_2^\nu(z)$  with respect to  $z$ .

**Parameters**

- v** : float  
Order of Hankel function
- z** : complex  
Argument at which to evaluate the derivative
- n** : int, default 1  
Order of derivative



**Notes**

The derivative is computed using the relation DLFM 10.6.7 [R374].

**References**

[R373], [R374]

**Spherical Bessel Functions**


---

<code>spherical_jn(n, z[, derivative])</code>	Spherical Bessel function of the first kind or its derivative.
<code>spherical_yn(n, z[, derivative])</code>	Spherical Bessel function of the second kind or its derivative.
<code>spherical_in(n, z[, derivative])</code>	Modified spherical Bessel function of the first kind or its derivative.
<code>spherical_kn(n, z[, derivative])</code>	Modified spherical Bessel function of the second kind or its derivative.

---

`scipy.special.spherical_jn` ( $n, z, derivative=False$ )

Spherical Bessel function of the first kind or its derivative.

Defined as [R464],

$$j_n(z) = \sqrt{\frac{\pi}{2z}} J_{n+1/2}(z),$$

where  $J_n$  is the Bessel function of the first kind.

**Parameters**    **n** : int, array\_like

Order of the Bessel function ( $n \geq 0$ ).

**z** : complex or float, array\_like

Argument of the Bessel function.

**derivative** : bool, optional

If True, the value of the derivative (rather than the function itself) is returned.

**Returns**        **jn** : ndarray

**Notes**

For real arguments greater than the order, the function is computed using the ascending recurrence [R465]. For small real or complex arguments, the definitional relation to the cylindrical Bessel function of the first kind is used.

The derivative is computed using the relations [R466],

$$j'_n = j_{n-1} - \frac{n+1}{2} j_n.$$

$$j'_0 = -j_1$$

New in version 0.18.0.

**References**

[R464], [R465], [R466]

`scipy.special.spherical_yn` ( $n, z, derivative=False$ )

Spherical Bessel function of the second kind or its derivative.

Defined as [R469],

$$y_n(z) = \sqrt{\frac{\pi}{2z}} Y_{n+1/2}(z),$$

where  $Y_n$  is the Bessel function of the second kind.

**Parameters** **n** : int, array\_like  
Order of the Bessel function ( $n \geq 0$ ).  
**z** : complex or float, array\_like  
Argument of the Bessel function.  
**derivative** : bool, optional  
If True, the value of the derivative (rather than the function itself) is returned.

**Returns** **yn** : ndarray

**Notes**

For real arguments, the function is computed using the ascending recurrence [R470]. For complex arguments, the definitional relation to the cylindrical Bessel function of the second kind is used.

The derivative is computed using the relations [R471],

$$y'_n = y_{n-1} - \frac{n+1}{2} y_n.$$

$$y'_0 = -y_1$$

New in version 0.18.0.

**References**

[R469], [R470], [R471]

`scipy.special.spherical_in` ( $n, z, derivative=False$ )  
Modified spherical Bessel function of the first kind or its derivative.

Defined as [R462],

$$i_n(z) = \sqrt{\frac{\pi}{2z}} I_{n+1/2}(z),$$

where  $I_n$  is the modified Bessel function of the first kind.

**Parameters** **n** : int, array\_like  
Order of the Bessel function ( $n \geq 0$ ).  
**z** : complex or float, array\_like  
Argument of the Bessel function.  
**derivative** : bool, optional  
If True, the value of the derivative (rather than the function itself) is returned.

**Returns** **in** : ndarray

**Notes**

The function is computed using its definitional relation to the modified cylindrical Bessel function of the first kind.

The derivative is computed using the relations [R463],

$$i'_n = i_{n-1} - \frac{n+1}{2} i_n.$$

$$i'_1 = i_0$$

New in version 0.18.0.

**References**

[R462], [R463]

`scipy.special.spherical_kn` (*n*, *z*, *derivative=False*)

Modified spherical Bessel function of the second kind or its derivative.

Defined as [R467],

$$k_n(z) = \sqrt{\frac{\pi}{2z}} K_{n+1/2}(z),$$

where  $K_n$  is the modified Bessel function of the second kind.**Parameters** **n** : int, array\_likeOrder of the Bessel function ( $n \geq 0$ ).**z** : complex or float, array\_like

Argument of the Bessel function.

**derivative** : bool, optional

If True, the value of the derivative (rather than the function itself) is returned.

**Returns** **kn** : ndarray**Notes**

The function is computed using its definitional relation to the modified cylindrical Bessel function of the second kind.

The derivative is computed using the relations [R468],

$$k'_n = -k_{n-1} - \frac{n+1}{2}k_n.$$

$$k'_0 = -k_1$$

New in version 0.18.0.

**References**

[R467], [R468]

These are not universal functions:

<code>sph_jn(*args, **kws)</code>	<code>sph_jn</code> is deprecated!
<code>sph_yn(*args, **kws)</code>	<code>sph_yn</code> is deprecated!
<code>sph_jnyn(*args, **kws)</code>	<code>sph_jnyn</code> is deprecated!
<code>sph_in(*args, **kws)</code>	<code>sph_in</code> is deprecated!
<code>sph_kn(*args, **kws)</code>	<code>sph_kn</code> is deprecated!
<code>sph_inkn(*args, **kws)</code>	<code>sph_inkn</code> is deprecated!

`scipy.special.sph_jn` (*\*args*, *\*\*kws*)`sph_jn` is deprecated! `scipy.special.sph_jn` is deprecated in SciPy 0.18.0. Use `scipy.special.spherical_jn` instead. Note that the new function has a different signature.Compute spherical Bessel function  $j_n(z)$  and derivative.This function computes the value and first derivative of  $j_n(z)$  for all orders up to and including  $n$ .**Parameters** **n** : intMaximum order of  $j_n$  to compute

<b>Returns</b>	<b>jn</b> : ndarray	<b>z</b>	[complex] Argument at which to evaluate
			Value of $j_0(z)$ , ..., $j_n(z)$
		<b>jnp</b>	[ndarray] First derivative $j_0'(z)$ , ..., $j_n'(z)$

`scipy.special.sph_yn(*args, **kwargs)`

`sph_yn` is deprecated! `scipy.special.sph_yn` is deprecated in scipy 0.18.0. Use `scipy.special.spherical_yn` instead. Note that the new function has a different signature.

Compute spherical Bessel function  $yn(z)$  and derivative.

This function computes the value and first derivative of  $yn(z)$  for all orders up to and including  $n$ .

<b>Parameters</b>	<b>n</b> : int		Maximum order of $yn$ to compute
<b>Returns</b>	<b>yn</b> : ndarray	<b>z</b>	[complex] Argument at which to evaluate
			Value of $y_0(z)$ , ..., $y_n(z)$
		<b>ynp</b>	[ndarray] First derivative $y_0'(z)$ , ..., $y_n'(z)$

`scipy.special.sph_jnyn(*args, **kwargs)`

`sph_jnyn` is deprecated! `scipy.special.sph_jnyn` is deprecated in scipy 0.18.0. Use `scipy.special.spherical_jn` and `scipy.special.spherical_yn` instead. Note that the new function has a different signature.

Compute spherical Bessel functions  $jn(z)$  and  $yn(z)$  and derivatives.

This function computes the value and first derivative of  $jn(z)$  and  $yn(z)$  for all orders up to and including  $n$ .

<b>Parameters</b>	<b>n</b> : int		Maximum order of $jn$ and $yn$ to compute
<b>Returns</b>	<b>jn</b> : ndarray	<b>z</b>	[complex] Argument at which to evaluate
			Value of $j_0(z)$ , ..., $j_n(z)$
		<b>jnp</b>	[ndarray] First derivative $j_0'(z)$ , ..., $j_n'(z)$
		<b>yn</b>	[ndarray] Value of $y_0(z)$ , ..., $y_n(z)$
		<b>ynp</b>	[ndarray] First derivative $y_0'(z)$ , ..., $y_n'(z)$

`scipy.special.sph_in(*args, **kwargs)`

`sph_in` is deprecated! `scipy.special.sph_in` is deprecated in scipy 0.18.0. Use `scipy.special.spherical_in` instead. Note that the new function has a different signature.

Compute spherical Bessel function  $in(z)$  and derivative.

This function computes the value and first derivative of  $in(z)$  for all orders up to and including  $n$ .

<b>Parameters</b>	<b>n</b> : int		Maximum order of $in$ to compute
<b>Returns</b>	<b>in</b> : ndarray	<b>z</b>	[complex] Argument at which to evaluate
			Value of $i_0(z)$ , ..., $i_n(z)$
		<b>inp</b>	[ndarray] First derivative $i_0'(z)$ , ..., $i_n'(z)$

`scipy.special.sph_kn(*args, **kwargs)`

`sph_kn` is deprecated! `scipy.special.sph_kn` is deprecated in scipy 0.18.0. Use `scipy.special.spherical_kn` instead. Note that the new function has a different signature.

Compute spherical Bessel function  $kn(z)$  and derivative.

This function computes the value and first derivative of  $kn(z)$  for all orders up to and including  $n$ .

<b>Parameters</b>	<b>n</b> : int	Maximum order of $kn$ to compute
	<b>z</b>	[complex] Argument at which to evaluate
<b>Returns</b>	<b>kn</b> : ndarray	Value of $k_0(z), \dots, kn(z)$
	<b>knp</b>	[ndarray] First derivative $k_0'(z), \dots, kn'(z)$

`scipy.special.sph_inkn(*args, **kwargs)`

`sph_inkn` is deprecated! `scipy.special.sph_inkn` is deprecated in scipy 0.18.0. Use `scipy.special.spherical_in` and `scipy.special.spherical_kn` instead. Note that the new function has a different signature.

Compute spherical Bessel functions  $in(z)$ ,  $kn(z)$ , and derivatives.

This function computes the value and first derivative of  $in(z)$  and  $kn(z)$  for all orders up to and including  $n$ .

<b>Parameters</b>	<b>n</b> : int	Maximum order of $in$ and $kn$ to compute
	<b>z</b>	[complex] Argument at which to evaluate
<b>Returns</b>	<b>in</b> : ndarray	Value of $i_0(z), \dots, in(z)$
	<b>inp</b>	[ndarray] First derivative $i_0'(z), \dots, in'(z)$
	<b>kn</b>	[ndarray] Value of $k_0(z), \dots, kn(z)$
	<b>knp</b>	[ndarray] First derivative $k_0'(z), \dots, kn'(z)$

### Riccati-Bessel Functions

These are not universal functions:

---

<code>riccati_jn(n, x)</code>	Compute Riccati-Bessel function of the first kind and its derivative.
<code>riccati_yn(n, x)</code>	Compute Riccati-Bessel function of the second kind and its derivative.

---

`scipy.special.riccati_jn(n, x)`

Compute Riccati-Bessel function of the first kind and its derivative.

The Riccati-Bessel function of the first kind is defined as  $xj_n(x)$ , where  $j_n$  is the spherical Bessel function of the first kind of order  $n$ .

This function computes the value and first derivative of the Riccati-Bessel function for all orders up to and including  $n$ .

<b>Parameters</b>	<b>n</b> : int	Maximum order of function to compute
	<b>x</b> : float	Argument at which to evaluate
<b>Returns</b>	<b>jn</b> : ndarray	Value of $j_0(x), \dots, j_n(x)$
	<b>jnp</b> : ndarray	First derivative $j_0'(x), \dots, j_n'(x)$

**Notes**

The computation is carried out via backward recurrence, using the relation DLMF 10.51.1 [R450].  
 Wrapper for a Fortran routine created by Shanjie Zhang and Jianming Jin [R449].

**References**

[R449], [R450]

`scipy.special.riccati_yn` (*n*, *x*)

Compute Riccati-Bessel function of the second kind and its derivative.

The Riccati-Bessel function of the second kind is defined as  $xy_n(x)$ , where  $y_n$  is the spherical Bessel function of the second kind of order  $n$ .

This function computes the value and first derivative of the function for all orders up to and including  $n$ .

<b>Parameters</b>	<b>n</b> : int	Maximum order of function to compute
	<b>x</b> : float	Argument at which to evaluate
<b>Returns</b>	<b>yn</b> : ndarray	Value of $y_0(x), \dots, y_n(x)$
	<b>ynp</b> : ndarray	First derivative $y_0'(x), \dots, y_n'(x)$

**Notes**

The computation is carried out via ascending recurrence, using the relation DLMF 10.51.1 [R452].  
 Wrapper for a Fortran routine created by Shanjie Zhang and Jianming Jin [R451].

**References**

[R451], [R452]

**Struve Functions**

---

<code>struve</code> ( <i>v</i> , <i>x</i> )	Struve function.
<code>modstruve</code> ( <i>v</i> , <i>x</i> )	Modified Struve function.
<code>itstruve0</code> ( <i>x</i> )	Integral of the Struve function of order 0.
<code>it2struve0</code> ( <i>x</i> )	Integral related to the Struve function of order 0.
<code>itmodstruve0</code> ( <i>x</i> )	Integral of the modified Struve function of order 0.

---

`scipy.special.struve` (*v*, *x*) = <ufunc 'struve'>

Struve function.

Return the value of the Struve function of order  $v$  at  $x$ . The Struve function is defined as,

$$H_v(x) = (z/2)^{v+1} \sum_{n=0}^{\infty} \frac{(-1)^n (z/2)^{2n}}{\Gamma(n + \frac{3}{2})\Gamma(n + v + \frac{3}{2})},$$

where  $\Gamma$  is the gamma function.

<b>Parameters</b>	<b>v</b> : array_like	Order of the Struve function (float).
	<b>x</b> : array_like	

**Returns** **H** : ndarray  
 Argument of the Struve function (float; must be positive unless  $\nu$  is an integer).  
 Value of the Struve function of order  $\nu$  at  $x$ .

**See also:**`modstruve`**Notes**

Three methods discussed in [R472] are used to evaluate the Struve function:

- power series
- expansion in Bessel functions (if  $|z| < |v| + 20$ )
- asymptotic large- $z$  expansion (if  $z \geq 0.7v + 12$ )

Rounding errors are estimated based on the largest terms in the sums, and the result associated with the smallest error is returned.

**References**

[R472]

`scipy.special.modstruve` ( $\nu, x$ ) = <ufunc 'modstruve'>

Modified Struve function.

Return the value of the modified Struve function of order  $\nu$  at  $x$ . The modified Struve function is defined as,

$$L_\nu(x) = -\nu \exp(-\pi\nu/2)H_\nu(x),$$

where  $H_\nu$  is the Struve function.

**Parameters** **v** : array\_like  
 Order of the modified Struve function (float).  
**x** : array\_like  
 Argument of the Struve function (float; must be positive unless  $\nu$  is an integer).  
**Returns** **L** : ndarray  
 Value of the modified Struve function of order  $\nu$  at  $x$ .

**See also:**`struve`**Notes**

Three methods discussed in [R432] are used to evaluate the function:

- power series
- expansion in Bessel functions (if  $|z| < |v| + 20$ )
- asymptotic large- $z$  expansion (if  $z \geq 0.7v + 12$ )

Rounding errors are estimated based on the largest terms in the sums, and the result associated with the smallest error is returned.

**References**

[R432]

`scipy.special.itstruve0` ( $x$ ) = <ufunc 'itstruve0'>

Integral of the Struve function of order 0.

$$I = \int_0^x H_0(t) dt$$

**Parameters** **x** : array\_like

**Returns** **I** : ndarray Upper limit of integration (float).  
The integral of  $H_0$  from 0 to  $x$ .

**See also:**

`struve`

**Notes**

Wrapper for a Fortran routine created by Shanjie Zhang and Jianming Jin [R386].

**References**

[R386]

`scipy.special.it2struve0(x) = <ufunc 'it2struve0'>`  
Integral related to the Struve function of order 0.

Returns the integral,

$$\int_x^\infty \frac{H_0(t)}{t} dt$$

where  $H_0$  is the Struve function of order 0.

**Parameters** **x** : array\_like

**Returns** **I** : ndarray Lower limit of integration.  
The value of the integral.

**See also:**

`struve`

**Notes**

Wrapper for a Fortran routine created by Shanjie Zhang and Jianming Jin [R383].

**References**

[R383]

`scipy.special.itmodstruve0(x) = <ufunc 'itmodstruve0'>`  
Integral of the modified Struve function of order 0.

$$I = \int_0^x L_0(t) dt$$

**Parameters** **x** : array\_like

**Returns** **I** : ndarray Upper limit of integration (float).  
The integral of  $L_0$  from 0 to  $x$ .

**Notes**

Wrapper for a Fortran routine created by Shanjie Zhang and Jianming Jin [R385].

**References**

[R385]



## Raw Statistical Functions

### See also:

`scipy.stats`: Friendly versions of these functions.

<code>bdtr(k, n, p)</code>	Binomial distribution cumulative distribution function.
<code>bdtrc(k, n, p)</code>	Binomial distribution survival function.
<code>bdtri(k, n, y)</code>	Inverse function to <code>bdtr</code> with respect to $p$ .
<code>bdtrik(y, n, p)</code>	Inverse function to <code>bdtr</code> with respect to $k$ .
<code>bdtrin(k, y, p)</code>	Inverse function to <code>bdtr</code> with respect to $n$ .
<code>btddtr(a, b, x)</code>	Cumulative density function of the beta distribution.
<code>btddtri(a, b, p)</code>	The $p$ -th quantile of the beta distribution.
<code>btddtria(p, b, x)</code>	Inverse of <code>btddtr</code> with respect to $a$ .
<code>btddtrib(a, p, x)</code>	Inverse of <code>btddtr</code> with respect to $b$ .
<code>fdtr(dfn, dfd, x)</code>	F cumulative distribution function.
<code>fdtrc(dfn, dfd, x)</code>	F survival function.
<code>fdtri(dfn, dfd, p)</code>	The $p$ -th quantile of the F-distribution.
<code>fdtridfd(dfn, p, x)</code>	Inverse to <code>fdtr</code> vs <code>dfd</code>
<code>gdtr(a, b, x)</code>	Gamma distribution cumulative distribution function.
<code>gdtrc(a, b, x)</code>	Gamma distribution survival function.
<code>gdtria(p, b, x[, out])</code>	Inverse of <code>gdtr</code> vs $a$ .
<code>gdtrib(a, p, x[, out])</code>	Inverse of <code>gdtr</code> vs $b$ .
<code>gdtrix(a, b, p[, out])</code>	Inverse of <code>gdtr</code> vs $x$ .
<code>nbdtr(k, n, p)</code>	Negative binomial cumulative distribution function.
<code>nbdtrc(k, n, p)</code>	Negative binomial survival function.
<code>nbdtri(k, n, y)</code>	Inverse of <code>nbdtr</code> vs $p$ .
<code>nbdtrik(y, n, p)</code>	Inverse of <code>nbdtr</code> vs $k$ .
<code>nbdtrin(k, y, p)</code>	Inverse of <code>nbdtr</code> vs $n$ .
<code>ncfdtr(dfn, dfd, nc, f)</code>	Cumulative distribution function of the non-central F distribution.
<code>ncfdtridfd(p, f, dfn, nc)</code>	Calculate degrees of freedom (denominator) for the noncentral F-distribution.
<code>ncfdtridfn(p, f, dfd, nc)</code>	Calculate degrees of freedom (numerator) for the noncentral F-distribution.
<code>ncfdtri(p, dfn, dfd, nc)</code>	Inverse cumulative distribution function of the non-central F distribution.
<code>ncfdtrinc(p, f, dfn, dfd)</code>	Calculate non-centrality parameter for non-central F distribution.
<code>nctdtr(df, nc, t)</code>	Cumulative distribution function of the non-central $t$ distribution.
<code>nctdtridf(p, nc, t)</code>	Calculate degrees of freedom for non-central $t$ distribution.
<code>nctdtrit(df, nc, p)</code>	Inverse cumulative distribution function of the non-central $t$ distribution.
<code>nctdtrinc(df, p, t)</code>	Calculate non-centrality parameter for non-central $t$ distribution.
<code>nrdrtrimn(p, x, std)</code>	Calculate mean of normal distribution given other params.
<code>nrdrtrisd(p, x, mn)</code>	Calculate standard deviation of normal distribution given other params.
<code>pdtr(k, m)</code>	Poisson cumulative distribution function
<code>pdtrc(k, m)</code>	Poisson survival function
<code>pdtri(k, y)</code>	Inverse to <code>pdtr</code> vs $m$
<code>pdtrik(p, m)</code>	Inverse to <code>pdtr</code> vs $k$
<code>stdtr(df, t)</code>	Student $t$ distribution cumulative density function
<code>stdtridf(p, t)</code>	Inverse of <code>stdtr</code> vs <code>df</code>
<code>stdtrit(df, p)</code>	Inverse of <code>stdtr</code> vs $t$
<code>chdtr(v, x)</code>	Chi square cumulative distribution function
<code>chdtrc(v, x)</code>	Chi square survival function
<code>chdtri(v, p)</code>	Inverse to <code>chdtrc</code>
<code>chdtriv(p, x)</code>	Inverse to <code>chdtr</code> vs $v$
<code>ndtr(x)</code>	Gaussian cumulative distribution function.
<code>log_ndtr(x)</code>	Logarithm of Gaussian cumulative distribution function.
<code>ndtri(y)</code>	Inverse of <code>ndtr</code> vs $x$

Continued on next page

Table 5.247 – continued from previous page

<code>chndtr(x, df, nc)</code>	Non-central chi square cumulative distribution function
<code>chndtridf(x, p, nc)</code>	Inverse to <code>chndtr</code> vs <code>df</code>
<code>chndtrinc(x, df, p)</code>	Inverse to <code>chndtr</code> vs <code>nc</code>
<code>chndtrix(p, df, nc)</code>	Inverse to <code>chndtr</code> vs <code>x</code>
<code>smirnov(n, e)</code>	Kolmogorov-Smirnov complementary cumulative distribution function
<code>smirnovi(n, y)</code>	Inverse to <code>smirnov</code>
<code>kolmogorov(y)</code>	Complementary cumulative distribution function of Kolmogorov distribution
<code>kolmogi(p)</code>	Inverse function to <code>kolmogorov</code>
<code>tklmbda(x, lmbda)</code>	Tukey-Lambda cumulative distribution function
<code>logit(x)</code>	Logit ufunc for ndarrays.
<code>expit(x)</code>	Expit ufunc for ndarrays.
<code>boxcox(x, lmbda)</code>	Compute the Box-Cox transformation.
<code>boxcox1p(x, lmbda)</code>	Compute the Box-Cox transformation of $1 + x$ .
<code>inv_boxcox(y, lmbda)</code>	Compute the inverse of the Box-Cox transformation.
<code>inv_boxcox1p(y, lmbda)</code>	Compute the inverse of the Box-Cox transformation.

`scipy.special.bdtr(k, n, p) = <ufunc 'bdtr'>`

Binomial distribution cumulative distribution function.

Sum of the terms 0 through  $k$  of the Binomial probability density.

$$\text{bdtr}(k, n, p) = \sum_{j=0}^k \binom{n}{j} p^j (1-p)^{n-j}$$

**Parameters** **k** : array\_like

Number of successes (int).

**n** : array\_like

Number of events (int).

**p** : array\_like

Probability of success in a single event (float).

**Returns** **y** : ndarray

Probability of  $k$  or fewer successes in  $n$  independent events with success probabilities of  $p$ .

**Notes**

The terms are not summed directly; instead the regularized incomplete beta function is employed, according to the formula,

$$\text{bdtr}(k, n, p) = I_{1-p}(n - k, k + 1).$$

Wrapper for the Cephes [R315] routine `bdtr`.

**References**

[R315]

`scipy.special.bdtrc(k, n, p) = <ufunc 'bdtrc'>`

Binomial distribution survival function.

Sum of the terms  $k + 1$  through  $n$  of the binomial probability density,

$$\text{bdtrc}(k, n, p) = \sum_{j=k+1}^n \binom{n}{j} p^j (1-p)^{n-j}$$

**Parameters** **k** : array\_like

	<code>n</code> : array_like	Number of successes (int).
	<code>p</code> : array_like	Number of events (int)
<b>Returns</b>	<code>y</code> : ndarray	Probability of success in a single event.
		Probability of $k + 1$ or more successes in $n$ independent events with success probabilities of $p$ .

**See also:**`bdtr`, `betainc`**Notes**

The terms are not summed directly; instead the regularized incomplete beta function is employed, according to the formula,

$$\text{bdtrc}(k, n, p) = I_p(k + 1, n - k).$$

Wrapper for the Cephes [R316] routine `bdtrc`.

**References**

[R316]

`scipy.special.bdtri`( $k, n, y$ ) = <ufunc 'bdtri'>

Inverse function to `bdtr` with respect to  $p$ .

Finds the event probability  $p$  such that the sum of the terms 0 through  $k$  of the binomial probability density is equal to the given cumulative probability  $y$ .

<b>Parameters</b>	<code>k</code> : array_like	Number of successes (float).
	<code>n</code> : array_like	Number of events (float)
	<code>y</code> : array_like	Cumulative probability (probability of $k$ or fewer successes in $n$ events).
<b>Returns</b>	<code>p</code> : ndarray	The event probability such that $\text{bdtr}(k, n, p) = y$ .

**See also:**`bdtr`, `betaincinv`**Notes**

The computation is carried out using the inverse beta integral function and the relation,:

$$1 - p = \text{betaincinv}(n - k, k + 1, y).$$

Wrapper for the Cephes [R317] routine `bdtri`.

**References**

[R317]

`scipy.special.bdtrik`( $y, n, p$ ) = <ufunc 'bdtrik'>

Inverse function to `bdtr` with respect to  $k$ .

Finds the number of successes  $k$  such that the sum of the terms 0 through  $k$  of the Binomial probability density for  $n$  events with probability  $p$  is equal to the given cumulative probability  $y$ .

**Parameters** **y** : array\_like  
 Cumulative probability (probability of  $k$  or fewer successes in  $n$  events).  
**n** : array\_like  
 Number of events (float).  
**p** : array\_like  
 Success probability (float).  
**Returns** **k** : ndarray  
 The number of successes  $k$  such that  $bdtr(k, n, p) = y$ .

**See also:**

`bdtr`

**Notes**

Formula 26.5.24 of [R318] is used to reduce the binomial distribution to the cumulative incomplete beta distribution.

Computation of  $k$  involves a search for a value that produces the desired value of  $y$ . The search relies on the monotonicity of  $y$  with  $k$ .

Wrapper for the CDFLIB [R319] Fortran routine *cdfbin*.

**References**

[R318], [R319]

`scipy.special.bdtrin(k, y, p) = <ufunc 'bdtrin'>`

Inverse function to `bdtr` with respect to  $n$ .

Finds the number of events  $n$  such that the sum of the terms 0 through  $k$  of the Binomial probability density for events with probability  $p$  is equal to the given cumulative probability  $y$ .

**Parameters** **k** : array\_like  
 Number of successes (float).  
**y** : array\_like  
 Cumulative probability (probability of  $k$  or fewer successes in  $n$  events).  
**p** : array\_like  
 Success probability (float).  
**Returns** **n** : ndarray  
 The number of events  $n$  such that  $bdtr(k, n, p) = y$ .

**See also:**

`bdtr`

**Notes**

Formula 26.5.24 of [R320] is used to reduce the binomial distribution to the cumulative incomplete beta distribution.

Computation of  $n$  involves a search for a value that produces the desired value of  $y$ . The search relies on the monotonicity of  $y$  with  $n$ .

Wrapper for the CDFLIB [R321] Fortran routine *cdfbin*.

**References**

[R320], [R321]

`scipy.special.btdtr(a, b, x) = <ufunc 'btdtr'>`

Cumulative density function of the beta distribution.

Returns the integral from zero to  $x$  of the beta probability density function,

$$I = \int_0^x \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} t^{a-1} (1-t)^{b-1} dt$$

where  $\Gamma$  is the gamma function.

**Parameters**

- a** : array\_like  
Shape parameter ( $a > 0$ ).
- b** : array\_like  
Shape parameter ( $b > 0$ ).
- x** : array\_like  
Upper limit of integration, in  $[0, 1]$ .

**Returns**

- I** : ndarray  
Cumulative density function of the beta distribution with parameters  $a$  and  $b$  at  $x$ .

**See also:**

`betainc`

**Notes**

This function is identical to the incomplete beta integral function `betainc`.

Wrapper for the Cephes [R328] routine `btdtr`.

**References**

[R328]

`scipy.special.btdtri` ( $a, b, p$ ) = <ufunc 'btdtri'>

The  $p$ -th quantile of the beta distribution.

This function is the inverse of the beta cumulative distribution function, `btdtr`, returning the value of  $x$  for which  $btdtr(a, b, x) = p$ , or

$$p = \int_0^x \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} t^{a-1} (1-t)^{b-1} dt$$

**Parameters**

- a** : array\_like  
Shape parameter ( $a > 0$ ).
- b** : array\_like  
Shape parameter ( $b > 0$ ).
- p** : array\_like  
Cumulative probability, in  $[0, 1]$ .

**Returns**

- x** : ndarray  
The quantile corresponding to  $p$ .

**See also:**

`betaincinv`, `btdtr`

**Notes**

The value of  $x$  is found by interval halving or Newton iterations.

Wrapper for the Cephes [R329] routine `incbi`, which solves the equivalent problem of finding the inverse of the incomplete beta integral.

**References**

[R329]

`scipy.special.btdtria` ( $p, b, x$ ) = <ufunc 'btdtria'>

Inverse of `btdtr` with respect to  $a$ .

This is the inverse of the beta cumulative distribution function, `btdtr`, considered as a function of  $a$ , returning the value of  $a$  for which  $btdtr(a, b, x) = p$ , or

$$p = \int_0^x \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} t^{a-1} (1-t)^{b-1} dt$$

**Parameters**

- p** : array\_like  
Cumulative probability, in [0, 1].
- b** : array\_like  
Shape parameter ( $b > 0$ ).
- x** : array\_like  
The quantile, in [0, 1].

**Returns**

- a** : ndarray  
The value of the shape parameter  $a$  such that  $btdtr(a, b, x) = p$ .

**See also:**

`btdtr` Cumulative density function of the beta distribution.  
`btdtri` Inverse with respect to  $x$ .  
`btdtrib` Inverse with respect to  $b$ .

**Notes**

Wrapper for the CDFLIB [R330] Fortran routine `cdfbet`.

The cumulative distribution function  $p$  is computed using a routine by DiDinato and Morris [R331]. Computation of  $a$  involves a search for a value that produces the desired value of  $p$ . The search relies on the monotonicity of  $p$  with  $a$ .

**References**

[R330], [R331]

`scipy.special.btdtrib` ( $a, p, x$ ) = <ufunc 'btdtrib'>

Inverse of `btdtr` with respect to  $b$ .

This is the inverse of the beta cumulative distribution function, `btdtr`, considered as a function of  $b$ , returning the value of  $b$  for which  $btdtr(a, b, x) = p$ , or

$$p = \int_0^x \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} t^{a-1} (1-t)^{b-1} dt$$

**Parameters**

- a** : array\_like  
Shape parameter ( $a > 0$ ).
- p** : array\_like  
Cumulative probability, in [0, 1].
- x** : array\_like  
The quantile, in [0, 1].

**Returns**

- b** : ndarray  
The value of the shape parameter  $b$  such that  $btdtr(a, b, x) = p$ .

**See also:**

`btdtr` Cumulative density function of the beta distribution.  
`btdtri` Inverse with respect to  $x$ .  
`btdtria` Inverse with respect to  $a$ .

**Notes**

Wrapper for the CDFLIB [R332] Fortran routine `cdfbet`.

The cumulative distribution function  $p$  is computed using a routine by DiDinato and Morris [R333]. Computation of  $b$  involves a search for a value that produces the desired value of  $p$ . The search relies on the monotonicity of  $p$  with  $b$ .

**References**

[R332], [R333]

`scipy.special.fdtf` ( $dfn, dfd, x$ ) = <ufunc 'fdtr'>

F cumulative distribution function.

Returns the value of the cumulative density function of the F-distribution, also known as Snedecor's F-distribution or the Fisher-Snedecor distribution.

The F-distribution with parameters  $d_n$  and  $d_d$  is the distribution of the random variable,

$$X = \frac{U_n/d_n}{U_d/d_d},$$

where  $U_n$  and  $U_d$  are random variables distributed  $\chi^2$ , with  $d_n$  and  $d_d$  degrees of freedom, respectively.

**Parameters**

- dfn** : array\_like  
First parameter (positive float).
- dfd** : array\_like  
Second parameter (positive float).
- x** : array\_like  
Argument (nonnegative float).

**Returns**

- y** : ndarray  
The CDF of the F-distribution with parameters  $dfn$  and  $dfd$  at  $x$ .

**Notes**

The regularized incomplete beta function is used, according to the formula,

$$F(d_n, d_d; x) = I_{x d_n / (d_d + x d_n)}(d_n/2, d_d/2).$$

Wrapper for the Cephes [R357] routine `fdtr`.

**References**

[R357]

`scipy.special.fdtfc` ( $dfn, dfd, x$ ) = <ufunc 'fdtrc'>

F survival function.

Returns the complemented F-distribution function (the integral of the density from  $x$  to infinity).

**Parameters**

- dfn** : array\_like  
First parameter (positive float).
- dfd** : array\_like  
Second parameter (positive float).
- x** : array\_like  
Argument (nonnegative float).

**Returns**

- y** : ndarray  
The complemented F-distribution function with parameters  $dfn$  and  $dfd$  at  $x$ .

**See also:**

`fdtr`

**Notes**

The regularized incomplete beta function is used, according to the formula,

$$F(d_n, d_d; x) = I_{d_d/(d_d+x d_n)}(d_d/2, d_n/2).$$

Wrapper for the Cephes [R358] routine `fdtrc`.

**References**

[R358]

`scipy.special.fdttri(dfn, dfd, p) = <ufunc 'fdtri'>`

The  $p$ -th quantile of the F-distribution.

This function is the inverse of the F-distribution CDF, `fdtr`, returning the  $x$  such that `fdtr(dfn, dfd, x) = p`.

<b>Parameters</b>	<b>dfn</b> : array_like First parameter (positive float).
	<b>dfd</b> : array_like Second parameter (positive float).
	<b>p</b> : array_like Cumulative probability, in [0, 1].
<b>Returns</b>	<b>x</b> : ndarray The quantile corresponding to $p$ .

**Notes**

The computation is carried out using the relation to the inverse regularized beta function,  $I_x^{-1}(a, b)$ . Let  $z = I_p^{-1}(d_d/2, d_n/2)$ . Then,

$$x = \frac{d_d(1-z)}{d_n z}.$$

If  $p$  is such that  $x < 0.5$ , the following relation is used instead for improved stability: let  $z' = I_{1-p}^{-1}(d_n/2, d_d/2)$ . Then,

$$x = \frac{d_d z'}{d_n(1-z')}.$$

Wrapper for the Cephes [R359] routine `fdtri`.

**References**

[R359]

`scipy.special.fdttridfd(dfn, p, x) = <ufunc 'fdtridfd'>`

Inverse to `fdtr` vs `dfd`

Finds the F density argument `dfd` such that `fdtr(dfn, dfd, x) == p`.

`scipy.special.gdtr(a, b, x) = <ufunc 'gdtr'>`

Gamma distribution cumulative density function.

Returns the integral from zero to  $x$  of the gamma probability density function,

$$F = \int_0^x \frac{a^b}{\Gamma(b)} t^{b-1} e^{-at} dt,$$

where  $\Gamma$  is the gamma function.

**Parameters** **a** : array\_like



The rate parameter of the gamma distribution, sometimes denoted  $\beta$  (float). It is also the reciprocal of the scale parameter  $\theta$ .

**b** : array\_like  
The shape parameter of the gamma distribution, sometimes denoted  $\alpha$  (float).

**x** : array\_like  
The quantile (upper limit of integration; float).

**Returns** **F** : ndarray  
The CDF of the gamma distribution with parameters  $a$  and  $b$  evaluated at  $x$ .

**See also:**

`gdtrc` 1 - CDF of the gamma distribution.

**Notes**

The evaluation is carried out using the relation to the incomplete gamma integral (regularized gamma function). Wrapper for the Cephes [R363] routine `gdt r`.

**References**

[R363]

`scipy.special.gdtrc` ( $a, b, x$ ) = <ufunc 'gdtrc'>  
Gamma distribution survival function.

Integral from  $x$  to infinity of the gamma probability density function,

$$F = \int_x^\infty \frac{a^b}{\Gamma(b)} t^{b-1} e^{-at} dt,$$

where  $\Gamma$  is the gamma function.

**Parameters** **a** : array\_like  
The rate parameter of the gamma distribution, sometimes denoted  $\beta$  (float). It is also the reciprocal of the scale parameter  $\theta$ .

**b** : array\_like  
The shape parameter of the gamma distribution, sometimes denoted  $\alpha$  (float).

**x** : array\_like  
The quantile (lower limit of integration; float).

**Returns** **F** : ndarray  
The survival function of the gamma distribution with parameters  $a$  and  $b$  evaluated at  $x$ .

**See also:**

`gdt r`, `gdtri`

**Notes**

The evaluation is carried out using the relation to the incomplete gamma integral (regularized gamma function). Wrapper for the Cephes [R364] routine `gdt rc`.

**References**

[R364]

`scipy.special.gdttria` ( $p, b, x, out=None$ ) = <ufunc 'gdttria'>  
Inverse of `gdt r` vs  $a$ .

Returns the inverse with respect to the parameter  $a$  of  $p = \text{gdtr}(a, b, x)$ , the cumulative distribution function of the gamma distribution.

**Parameters**

- p** : array\_like  
Probability values.
- b** : array\_like  
 $b$  parameter values of  $\text{gdtr}(a, b, x)$ .  $b$  is the “shape” parameter of the gamma distribution.
- x** : array\_like  
Nonnegative real values, from the domain of the gamma distribution.
- out** : ndarray, optional  
If a fourth argument is given, it must be a `numpy.ndarray` whose size matches the broadcast result of  $a$ ,  $b$  and  $x$ . *out* is then the array returned by the function.

**Returns**

- a** : ndarray  
Values of the  $a$  parameter such that  $p = \text{gdtr}(a, b, x)$ .  $1/a$  is the “scale” parameter of the gamma distribution.

**See also:**

`gdtr` CDF of the gamma distribution.  
`gdtrib` Inverse with respect to  $b$  of  $\text{gdtr}(a, b, x)$ .  
`gdtrix` Inverse with respect to  $x$  of  $\text{gdtr}(a, b, x)$ .

**Notes**

Wrapper for the CDFLIB [R365] Fortran routine *cdfgam*.

The cumulative distribution function  $p$  is computed using a routine by DiDinato and Morris [R366]. Computation of  $a$  involves a search for a value that produces the desired value of  $p$ . The search relies on the monotonicity of  $p$  with  $a$ .

**References**

[R365], [R366]

**Examples**

First evaluate `gdtr`.

```
>>> from scipy.special import gdtr, gdtria
>>> p = gdtr(1.2, 3.4, 5.6)
>>> print(p)
0.94378087442
```

Verify the inverse.

```
>>> gdtria(p, 3.4, 5.6)
1.2
```

`scipy.special.gdtrib(a, p, x, out=None) = <ufunc 'gdtrib'>`

Inverse of `gdtr` vs  $b$ .

Returns the inverse with respect to the parameter  $b$  of  $p = \text{gdtr}(a, b, x)$ , the cumulative distribution function of the gamma distribution.

**Parameters**

- a** : array\_like  
 $a$  parameter values of  $\text{gdtr}(a, b, x)$ .  $1/a$  is the “scale” parameter of the gamma distribution.
- p** : array\_like

**x** : array\_like  
 Probability values.

**out** : ndarray, optional  
 Nonnegative real values, from the domain of the gamma distribution.

**Returns** **b** : ndarray  
 If a fourth argument is given, it must be a `numpy.ndarray` whose size matches the broadcast result of `a`, `b` and `x`. `out` is then the array returned by the function.

Values of the `b` parameter such that  $p = \text{gdftr}(a, b, x)$ . `b` is the “shape” parameter of the gamma distribution.

**See also:**

**`gdftr`** CDF of the gamma distribution.  
**`gdftria`** Inverse with respect to `a` of  $\text{gdftr}(a, b, x)$ .  
**`gdftrix`** Inverse with respect to `x` of  $\text{gdftr}(a, b, x)$ .

**Notes**

Wrapper for the CDFLIB [R367] Fortran routine `cdfgam`.

The cumulative distribution function  $p$  is computed using a routine by DiDinato and Morris [R368]. Computation of  $b$  involves a search for a value that produces the desired value of  $p$ . The search relies on the monotonicity of  $p$  with  $b$ .

**References**

[R367], [R368]

**Examples**

First evaluate `gdftr`.

```
>>> from scipy.special import gdftr, gdftrib
>>> p = gdftr(1.2, 3.4, 5.6)
>>> print(p)
0.94378087442
```

Verify the inverse.

```
>>> gdftrib(1.2, p, 5.6)
3.3999999999723882
```

`scipy.special.gdftrix(a, b, p, out=None) = <ufunc 'gdftrix'>`

Inverse of `gdftr` vs `x`.

Returns the inverse with respect to the parameter `x` of  $p = \text{gdftr}(a, b, x)$ , the cumulative distribution function of the gamma distribution. This is also known as the  $p$ 'th quantile of the distribution.

**Parameters** **a** : array\_like  
`a` parameter values of  $\text{gdftr}(a, b, x)$ .  $1/a$  is the “scale” parameter of the gamma distribution.

**b** : array\_like  
`b` parameter values of  $\text{gdftr}(a, b, x)$ . `b` is the “shape” parameter of the gamma distribution.

**p** : array\_like  
 Probability values.

**out** : ndarray, optional

**Returns** `x` : ndarray  
 If a fourth argument is given, it must be a `numpy.ndarray` whose size matches the broadcast result of `a`, `b` and `x`. `out` is then the array returned by the function.  
 Values of the `x` parameter such that  $p = \text{gdftr}(a, b, x)$ .

**See also:**

`gdftr` CDF of the gamma distribution.  
`gdftria` Inverse with respect to `a` of `gdftr(a, b, x)`.  
`gdftrib` Inverse with respect to `b` of `gdftr(a, b, x)`.

**Notes**

Wrapper for the CDFLIB [R369] Fortran routine `cdfgam`.

The cumulative distribution function `p` is computed using a routine by DiDinato and Morris [R370]. Computation of `x` involves a search for a value that produces the desired value of `p`. The search relies on the monotonicity of `p` with `x`.

**References**

[R369], [R370]

**Examples**

First evaluate `gdftr`.

```
>>> from scipy.special import gdftr, gdftria
>>> p = gdftr(1.2, 3.4, 5.6)
>>> print(p)
0.94378087442
```

Verify the inverse.

```
>>> gdftria(1.2, 3.4, p)
5.5999999999999996
```

`scipy.special.nbdtr(k, n, p) = <ufunc 'nbdtr'>`  
 Negative binomial cumulative distribution function.

Returns the sum of the terms 0 through `k` of the negative binomial distribution probability mass function,

$$F = \sum_{j=0}^k \binom{n+j-1}{j} p^n (1-p)^j.$$

In a sequence of Bernoulli trials with individual success probabilities `p`, this is the probability that `k` or fewer failures precede the `n`th success.

**Parameters** `k` : array\_like  
 The maximum number of allowed failures (nonnegative int).  
`n` : array\_like  
 The target number of successes (positive int).  
`p` : array\_like  
 Probability of success in a single event (float).  
**Returns** `F` : ndarray  
 The probability of `k` or fewer failures before `n` successes in a sequence of events with individual success probability `p`.

**See also:**

`nbdtrc`

**Notes**

If floating point values are passed for  $k$  or  $n$ , they will be truncated to integers.

The terms are not summed directly; instead the regularized incomplete beta function is employed, according to the formula,

$$\text{nbdt r}(k, n, p) = I_p(n, k + 1).$$

Wrapper for the Cephes [R433] routine `nbdt r`.

**References**

[R433]

`scipy.special.nbdtrc(k, n, p) = <ufunc 'nbdtrc'>`

Negative binomial survival function.

Returns the sum of the terms  $k + 1$  to infinity of the negative binomial distribution probability mass function,

$$F = \sum_{j=k+1}^{\infty} \binom{n+j-1}{j} p^n (1-p)^j.$$

In a sequence of Bernoulli trials with individual success probabilities  $p$ , this is the probability that more than  $k$  failures precede the  $n$ th success.

<b>Parameters</b>	<b>k</b> : array_like	The maximum number of allowed failures (nonnegative int).
	<b>n</b> : array_like	The target number of successes (positive int).
	<b>p</b> : array_like	Probability of success in a single event (float).
<b>Returns</b>	<b>F</b> : ndarray	The probability of $k + 1$ or more failures before $n$ successes in a sequence of events with individual success probability $p$ .

**Notes**

If floating point values are passed for  $k$  or  $n$ , they will be truncated to integers.

The terms are not summed directly; instead the regularized incomplete beta function is employed, according to the formula,

$$\text{nbdt rc}(k, n, p) = I_{1-p}(k + 1, n).$$

Wrapper for the Cephes [R434] routine `nbdt rc`.

**References**

[R434]

`scipy.special.nbdtri(k, n, y) = <ufunc 'nbdtri'>`

Inverse of `nbdt r` vs  $p$ .

Returns the inverse with respect to the parameter  $p$  of  $y = \text{nbdt r}(k, n, p)$ , the negative binomial cumulative distribution function.

<b>Parameters</b>	<b>k</b> : array_like	The maximum number of allowed failures (nonnegative int).
	<b>n</b> : array_like	The target number of successes (positive int).
	<b>y</b> : array_like	

**Returns** **p** : ndarray The probability of  $k$  or fewer failures before  $n$  successes (float).  
Probability of success in a single event (float) such that  $nbdr(k, n, p) = y$ .

**See also:**

[nbdr](#) Cumulative distribution function of the negative binomial.  
[nbdrtrik](#) Inverse with respect to  $k$  of  $nbdr(k, n, p)$ .  
[nbdrtrin](#) Inverse with respect to  $n$  of  $nbdr(k, n, p)$ .

**Notes**

Wrapper for the Cephes [R435] routine `nbdrtri`.

**References**

[R435]

`scipy.special.nbdrtrik`( $y, n, p$ ) = <ufunc 'nbdrtrik'>  
Inverse of `nbdr` vs  $k$ .

Returns the inverse with respect to the parameter  $k$  of  $y = nbdr(k, n, p)$ , the negative binomial cumulative distribution function.

**Parameters** **y** : array\_like The probability of  $k$  or fewer failures before  $n$  successes (float).  
**n** : array\_like The target number of successes (positive int).  
**p** : array\_like Probability of success in a single event (float).  
**Returns** **k** : ndarray The maximum number of allowed failures such that  $nbdr(k, n, p) = y$ .

**See also:**

[nbdr](#) Cumulative distribution function of the negative binomial.  
[nbdrtri](#) Inverse with respect to  $p$  of  $nbdr(k, n, p)$ .  
[nbdrtrin](#) Inverse with respect to  $n$  of  $nbdr(k, n, p)$ .

**Notes**

Wrapper for the CDFLIB [R436] Fortran routine `cdfnbn`.

Formula 26.5.26 of [R437],

$$\sum_{j=k+1}^{\infty} \binom{n+j-1}{j} p^n (1-p)^j = I_{1-p}(k+1, n),$$

is used to reduce calculation of the cumulative distribution function to that of a regularized incomplete beta  $I$ .

Computation of  $k$  involves a search for a value that produces the desired value of  $y$ . The search relies on the monotonicity of  $y$  with  $k$ .

**References**

[R436], [R437]

`scipy.special.nbdrtrin`( $k, y, p$ ) = <ufunc 'nbdrtrin'>  
Inverse of `nbdr` vs  $n$ .

Returns the inverse with respect to the parameter  $n$  of  $y = nbdr(k, n, p)$ , the negative binomial cumulative distribution function.

**Parameters** **k** : array\_like

	<b>y</b> : array_like	The maximum number of allowed failures (nonnegative int).
	<b>p</b> : array_like	The probability of $k$ or fewer failures before $n$ successes (float).
<b>Returns</b>	<b>n</b> : ndarray	Probability of success in a single event (float).
		The number of successes $n$ such that $nbdftr(k, n, p) = y$ .

**See also:**

<b>nbdftr</b>	Cumulative distribution function of the negative binomial.
<b>nbdftri</b>	Inverse with respect to $p$ of $nbdftr(k, n, p)$ .
<b>nbdftrik</b>	Inverse with respect to $k$ of $nbdftr(k, n, p)$ .

**Notes**

Wrapper for the CDFLIB [R438] Fortran routine *cdfnbn*.

Formula 26.5.26 of [R439],

$$\sum_{j=k+1}^{\infty} \binom{n+j-1}{j} p^n (1-p)^j = I_{1-p}(k+1, n),$$

is used to reduce calculation of the cumulative distribution function to that of a regularized incomplete beta  $I$ .

Computation of  $n$  involves a search for a value that produces the desired value of  $y$ . The search relies on the monotonicity of  $y$  with  $n$ .

**References**

[R438], [R439]

`scipy.special.ncfdtr` (*dfn, dfd, nc, f*) = <ufunc 'ncfdtr'>  
Cumulative distribution function of the non-central F distribution.

The non-central F describes the distribution of,

$$Z = \frac{X/d_n}{Y/d_d}$$

where  $X$  and  $Y$  are independently distributed, with  $X$  distributed non-central  $\chi^2$  with noncentrality parameter  $nc$  and  $d_n$  degrees of freedom, and  $Y$  distributed  $\chi^2$  with  $d_d$  degrees of freedom.

<b>Parameters</b>	<b>dfn</b> : array_like	Degrees of freedom of the numerator sum of squares. Range (0, inf).
	<b>dfd</b> : array_like	Degrees of freedom of the denominator sum of squares. Range (0, inf).
	<b>nc</b> : array_like	Noncentrality parameter. Should be in range (0, 1e4).
	<b>f</b> : array_like	Quantiles, i.e. the upper limit of integration.
<b>Returns</b>	<b>cdf</b> : float or ndarray	The calculated CDF. If all inputs are scalar, the return will be a float. Otherwise it will be an array.

**See also:**

<b>ncdfdtri</b>	Inverse CDF (iCDF) of the non-central F distribution.
<b>ncdfdtridfd</b>	Calculate dfd, given CDF and iCDF values.
<b>ncdfdtridfn</b>	Calculate dfn, given CDF and iCDF values.

**`ncfdtrinc`** Calculate noncentrality parameter, given CDF, iCDF, dfn, dfd.

#### Notes

Wrapper for the CDFLIB [R440] Fortran routine `cdffnc`.

The cumulative distribution function is computed using Formula 26.6.20 of [R441]:

$$F(d_n, d_d, n_c, f) = \sum_{j=0}^{\infty} e^{-n_c/2} \frac{(n_c/2)^j}{j!} I_x\left(\frac{d_n}{2} + j, \frac{d_d}{2}\right),$$

where  $I$  is the regularized incomplete beta function, and  $x = fd_n/(fd_n + d_d)$ .

The computation time required for this routine is proportional to the noncentrality parameter  $nc$ . Very large values of this parameter can consume immense computer resources. This is why the search range is bounded by 10,000.

#### References

[R440], [R441]

#### Examples

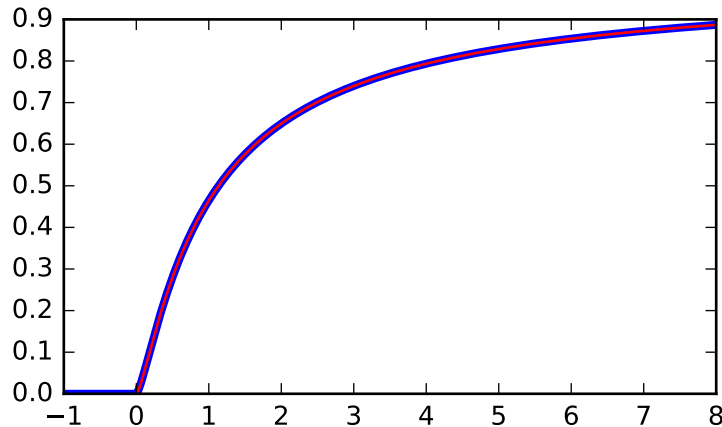
```
>>> from scipy import special
>>> from scipy import stats
>>> import matplotlib.pyplot as plt
```

Plot the CDF of the non-central F distribution, for  $nc=0$ . Compare with the F-distribution from `scipy.stats`:

```
>>> x = np.linspace(-1, 8, num=500)
>>> dfn = 3
>>> dfd = 2
>>> ncf_stats = stats.f.cdf(x, dfn, dfd)
>>> ncf_special = special.ncfdtr(dfn, dfd, 0, x)

>>> fig = plt.figure()
>>> ax = fig.add_subplot(111)
>>> ax.plot(x, ncf_stats, 'b-', lw=3)
>>> ax.plot(x, ncf_special, 'r-')
>>> plt.show()
```





`scipy.special.ncfdtridfd(p, f, dfn, nc) = <ufunc 'ncfdtridfd'>`  
 Calculate degrees of freedom (denominator) for the noncentral F-distribution.

See `ncfdtr` for more details.

#### Notes

The value of the cumulative noncentral F distribution is not necessarily monotone in either degrees of freedom. There thus may be two values that provide a given CDF value. This routine assumes monotonicity and will find an arbitrary one of the two values.

`scipy.special.ncfdtridfn(p, f, dfd, nc) = <ufunc 'ncfdtridfn'>`  
 Calculate degrees of freedom (numerator) for the noncentral F-distribution.

See `ncfdtr` for more details.

#### Notes

The value of the cumulative noncentral F distribution is not necessarily monotone in either degrees of freedom. There thus may be two values that provide a given CDF value. This routine assumes monotonicity and will find an arbitrary one of the two values.

`scipy.special.ncfdtri(p, dfn, dfd, nc) = <ufunc 'ncfdtri'>`  
 Inverse cumulative distribution function of the non-central F distribution.

See `ncfdtr` for more details.

`scipy.special.ncfdtrinc(p, f, dfn, dfd) = <ufunc 'ncfdtrinc'>`  
 Calculate non-centrality parameter for non-central F distribution.

See `ncfdtr` for more details.

`scipy.special.nctdtr(df, nc, t) = <ufunc 'nctdtr'>`  
 Cumulative distribution function of the non-central  $t$  distribution.

**Parameters**

- df**: array\_like  
Degrees of freedom of the distribution. Should be in range (0, inf).
- nc**: array\_like  
Noncentrality parameter. Should be in range (-1e6, 1e6).
- t**: array\_like  
Quantiles, i.e. the upper limit of integration.

**Returns** `cdf` : float or ndarray

The calculated CDF. If all inputs are scalar, the return will be a float. Otherwise it will be an array.

**See also:**

`nctdtrit` Inverse CDF (iCDF) of the non-central t distribution.

`nctdtridf` Calculate degrees of freedom, given CDF and iCDF values.

`nctdtrinc` Calculate non-centrality parameter, given CDF iCDF values.

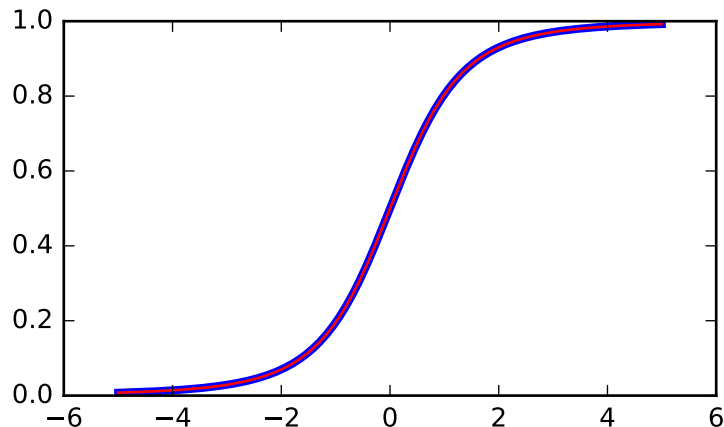
**Examples**

```
>>> from scipy import special
>>> from scipy import stats
>>> import matplotlib.pyplot as plt
```

Plot the CDF of the non-central t distribution, for `nc=0`. Compare with the t-distribution from `scipy.stats`:

```
>>> x = np.linspace(-5, 5, num=500)
>>> df = 3
>>> nct_stats = stats.t.cdf(x, df)
>>> nct_special = special.nctdtr(df, 0, x)

>>> fig = plt.figure()
>>> ax = fig.add_subplot(111)
>>> ax.plot(x, nct_stats, 'b-', lw=3)
>>> ax.plot(x, nct_special, 'r-')
>>> plt.show()
```



`scipy.special.nctdtridf` ( $p, nc, t$ ) = <ufunc 'nctdtridf'>

Calculate degrees of freedom for non-central t distribution.

See `nctdtr` for more details.

**Parameters** `p` : array\_like

CDF values, in range (0, 1].

`nc` : array\_like

Noncentrality parameter. Should be in range (-1e6, 1e6).

**t** : array\_like  
Quantiles, i.e. the upper limit of integration.

`scipy.special.nctdtrit(df, nc, p) = <ufunc 'nctdtrit'>`  
Inverse cumulative distribution function of the non-central t distribution.  
See `nctdtr` for more details.

**Parameters**

- df** : array\_like  
Degrees of freedom of the distribution. Should be in range (0, inf).
- nc** : array\_like  
Noncentrality parameter. Should be in range (-1e6, 1e6).
- p** : array\_like  
CDF values, in range (0, 1].

`scipy.special.nctdtrinc(df, p, t) = <ufunc 'nctdtrinc'>`  
Calculate non-centrality parameter for non-central t distribution.  
See `nctdtr` for more details.

**Parameters**

- df** : array\_like  
Degrees of freedom of the distribution. Should be in range (0, inf).
- p** : array\_like  
CDF values, in range (0, 1].
- t** : array\_like  
Quantiles, i.e. the upper limit of integration.

`scipy.special.nrdtrimn(p, x, std) = <ufunc 'nrdtrimn'>`  
Calculate mean of normal distribution given other params.

**Parameters**

- p** : array\_like  
CDF values, in range (0, 1].
- x** : array\_like  
Quantiles, i.e. the upper limit of integration.
- std** : array\_like  
Standard deviation.

**Returns**

- mn** : float or ndarray  
The mean of the normal distribution.

**See also:**

`nrdtrimn`, `ndtr`

`scipy.special.nrdtrisd(p, x, mn) = <ufunc 'nrdtrisd'>`  
Calculate standard deviation of normal distribution given other params.

**Parameters**

- p** : array\_like  
CDF values, in range (0, 1].
- x** : array\_like  
Quantiles, i.e. the upper limit of integration.
- mn** : float or ndarray  
The mean of the normal distribution.

**Returns**

- std** : array\_like  
Standard deviation.

**See also:**

`nrdtrisd`, `ndtr`

`scipy.special.pdtr(k, m) = <ufunc 'pdtr'>`  
Poisson cumulative distribution function

Returns the sum of the first  $k$  terms of the Poisson distribution:  $\sum(\exp(-m) * m^{**j} / j!, j=0..k) = \text{gammaincc}(k+1, m)$ . Arguments must both be positive and  $k$  an integer.

`scipy.special.pdtrc(k, m) = <ufunc 'pdtrc'>`

Poisson survival function

Returns the sum of the terms from  $k+1$  to infinity of the Poisson distribution:  $\sum(\exp(-m) * m^{**j} / j!, j=k+1..inf) = \text{gammainc}(k+1, m)$ . Arguments must both be positive and  $k$  an integer.

`scipy.special.pdtri(k, y) = <ufunc 'pdtri'>`

Inverse to `pdtr` vs  $m$

Returns the Poisson variable  $m$  such that the sum from 0 to  $k$  of the Poisson density is equal to the given probability  $y$ : calculated by `gammaincinv(k+1, y)`.  $k$  must be a nonnegative integer and  $y$  between 0 and 1.

`scipy.special.pdtrik(p, m) = <ufunc 'pdtrik'>`

Inverse to `pdtr` vs  $k$

Returns the quantile  $k$  such that `pdtr(k, m) = p`

`scipy.special.stdtr(df, t) = <ufunc 'stdtr'>`

Student t distribution cumulative density function

Returns the integral from minus infinity to  $t$  of the Student t distribution with  $df > 0$  degrees of freedom:

```
gamma((df+1)/2) / (sqrt(df*pi) * gamma(df/2)) *
integral((1+x**2/df)**(-df/2-1/2), x=-inf..t)
```

`scipy.special.stdtridf(p, t) = <ufunc 'stdtridf'>`

Inverse of `stdtr` vs  $df$

Returns the argument  $df$  such that `stdtr(df, t)` is equal to  $p$ .

`scipy.special.stdtrit(df, p) = <ufunc 'stdtrit'>`

Inverse of `stdtr` vs  $t$

Returns the argument  $t$  such that `stdtr(df, t)` is equal to  $p$ .

`scipy.special.chdtr(v, x) = <ufunc 'chdtr'>`

Chi square cumulative distribution function

Returns the area under the left hand tail (from 0 to  $x$ ) of the Chi square probability density function with  $v$  degrees of freedom:

```
1 / (2**(v/2) * gamma(v/2)) * integral(t**(v/2-1) * exp(-t/2), t=0..x)
```

`scipy.special.chdtrc(v, x) = <ufunc 'chdtrc'>`

Chi square survival function

Returns the area under the right hand tail (from  $x$  to infinity) of the Chi square probability density function with  $v$  degrees of freedom:

```
1 / (2**(v/2) * gamma(v/2)) * integral(t**(v/2-1) * exp(-t/2), t=x..inf)
```

`scipy.special.chdtri(v, p) = <ufunc 'chdtri'>`

Inverse to `chdtrc`

Returns the argument  $x$  such that `chdtrc(v, x) == p`.

`scipy.special.chdtriv(p, x) = <ufunc 'chdtriv'>`

Inverse to `chdtr` vs  $v$

Returns the argument  $v$  such that `chdtr(v, x) == p`.

`scipy.special.ndtr(x) = <ufunc 'ndtr'>`

Gaussian cumulative distribution function.

Returns the area under the standard Gaussian probability density function, integrated from minus infinity to  $x$

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^x \exp(-t^2/2) dt$$

**Parameters** `x` : array\_like, real or complex

**Returns** ndarray  
Argument

The value of the normal CDF evaluated at  $x$

**See also:**

`erf`, `erfc`, `scipy.stats.norm`, `log_ndtr`

`scipy.special.log_ndtr(x) = <ufunc 'log_ndtr'>`

Logarithm of Gaussian cumulative distribution function.

Returns the log of the area under the standard Gaussian probability density function, integrated from minus infinity to  $x$ :

```
log(1/sqrt(2*pi) * integral(exp(-t**2 / 2), t=-inf..x))
```

**Parameters** `x` : array\_like, real or complex

**Returns** ndarray  
Argument

The value of the log of the normal CDF evaluated at  $x$

**See also:**

`erf`, `erfc`, `scipy.stats.norm`, `ndtr`

`scipy.special.ndtri(y) = <ufunc 'ndtri'>`

Inverse of `ndtr` vs  $x$

Returns the argument  $x$  for which the area under the Gaussian probability density function (integrated from minus infinity to  $x$ ) is equal to  $y$ .

`scipy.special.chndtr(x, df, nc) = <ufunc 'chndtr'>`

Non-central chi square cumulative distribution function

`scipy.special.chndtridf(x, p, nc) = <ufunc 'chndtridf'>`

Inverse to `chndtr` vs  $df$

`scipy.special.chndtrinc(x, df, p) = <ufunc 'chndtrinc'>`

Inverse to `chndtr` vs  $nc$

`scipy.special.chndtrix(p, df, nc) = <ufunc 'chndtrix'>`

Inverse to `chndtr` vs  $x$

`scipy.special.smirnov(n, e) = <ufunc 'smirnov'>`

Kolmogorov-Smirnov complementary cumulative distribution function

Returns the exact Kolmogorov-Smirnov complementary cumulative distribution function (Dn+ or Dn-) for a one-sided test of equality between an empirical and a theoretical distribution. It is equal to the probability that the maximum difference between a theoretical distribution and an empirical one based on  $n$  samples is greater than  $e$ .

`scipy.special.smirnovi(n, y) = <ufunc 'smirnovi'>`

Inverse to `smirnov`

Returns  $e$  such that `smirnov(n, e) = y`.

`scipy.special.kolmogorov` (*y*) = <ufunc 'kolmogorov'>

Complementary cumulative distribution function of Kolmogorov distribution

Returns the complementary cumulative distribution function of Kolmogorov's limiting distribution ( $K_n^*$  for large  $n$ ) of a two-sided test for equality between an empirical and a theoretical distribution. It is equal to the (limit as  $n \rightarrow \infty$  of the) probability that  $\sqrt{n} * \max \text{absolute deviation} > y$ .

`scipy.special.kolmogi` (*p*) = <ufunc 'kolmogi'>

Inverse function to kolmogorov

Returns *y* such that `kolmogorov(y) == p`.

`scipy.special.tklmbda` (*x*, *lmbda*) = <ufunc 'tklmbda'>

Tukey-Lambda cumulative distribution function

`scipy.special.logit` (*x*) = <ufunc 'logit'>

Logit ufunc for ndarrays.

The logit function is defined as  $\text{logit}(p) = \log(p/(1-p))$ . Note that  $\text{logit}(0) = -\infty$ ,  $\text{logit}(1) = \infty$ , and  $\text{logit}(p)$  for  $p < 0$  or  $p > 1$  yields nan.

**Parameters**    **x** : ndarray

The ndarray to apply logit to element-wise.

**Returns**        **out** : ndarray

An ndarray of the same shape as *x*. Its entries are logit of the corresponding entry of *x*.

**Notes**

As a ufunc logit takes a number of optional keyword arguments. For more information see [ufuncs](#)

New in version 0.10.0.

`scipy.special.expit` (*x*) = <ufunc 'expit'>

Expit ufunc for ndarrays.

The expit function, also known as the logistic function, is defined as  $\text{expit}(x) = 1/(1+\exp(-x))$ . It is the inverse of the logit function.

**Parameters**    **x** : ndarray

The ndarray to apply expit to element-wise.

**Returns**        **out** : ndarray

An ndarray of the same shape as *x*. Its entries are expit of the corresponding entry of *x*.

**Notes**

As a ufunc expit takes a number of optional keyword arguments. For more information see [ufuncs](#)

New in version 0.10.0.

`scipy.special.boxcox` (*x*, *lmbda*) = <ufunc 'boxcox'>

Compute the Box-Cox transformation.

The Box-Cox transformation is:

$$y = \begin{cases} (x^{**lmbda} - 1) / lmbda & \text{if } lmbda \neq 0 \\ \log(x) & \text{if } lmbda == 0 \end{cases}$$

Returns *nan* if  $x < 0$ . Returns *-inf* if  $x == 0$  and  $lmbda < 0$ .

**Parameters**    **x** : array\_like

Data to be transformed.

**lmbda** : array\_like

**Returns** **y** : array  
Power parameter of the Box-Cox transform.  
Transformed data.

**Notes**

New in version 0.14.0.

**Examples**

```
>>> from scipy.special import boxcox
>>> boxcox([1, 4, 10], 2.5)
array([ 0.          , 12.4          , 126.09110641])
>>> boxcox(2, [0, 1, 2])
array([ 0.69314718, 1.          , 1.5          ])
```

`scipy.special.boxcox1p(x, lambda) = <ufunc 'boxcox1p'>`  
Compute the Box-Cox transformation of  $1 + x$ .

The Box-Cox transformation computed by `boxcox1p` is:

$$y = \begin{cases} ((1+x)**\lambda - 1) / \lambda & \text{if } \lambda \neq 0 \\ \log(1+x) & \text{if } \lambda == 0 \end{cases}$$

Returns *nan* if  $x < -1$ . Returns *-inf* if  $x == -1$  and  $\lambda < 0$ .

**Parameters** **x** : array\_like  
Data to be transformed.  
**lambda** : array\_like  
Power parameter of the Box-Cox transform.  
**Returns** **y** : array  
Transformed data.

**Notes**

New in version 0.14.0.

**Examples**

```
>>> from scipy.special import boxcox1p
>>> boxcox1p(1e-4, [0, 0.5, 1])
array([ 9.99950003e-05,  9.99975001e-05,  1.00000000e-04])
>>> boxcox1p([0.01, 0.1], 0.25)
array([ 0.00996272,  0.09645476])
```

`scipy.special.inv_boxcox(y, lambda) = <ufunc 'inv_boxcox'>`  
Compute the inverse of the Box-Cox transformation.

Find  $x$  such that:

$$y = \begin{cases} (x**\lambda - 1) / \lambda & \text{if } \lambda \neq 0 \\ \log(x) & \text{if } \lambda == 0 \end{cases}$$

**Parameters** **y** : array\_like  
Data to be transformed.  
**lambda** : array\_like  
Power parameter of the Box-Cox transform.  
**Returns** **x** : array  
Transformed data.

*Notes*

New in version 0.16.0.

*Examples*

```
>>> from scipy.special import boxcox, inv_boxcox
>>> y = boxcox([1, 4, 10], 2.5)
>>> inv_boxcox(y, 2.5)
array([1., 4., 10.]
```

`scipy.special.inv_boxcox1p(y, lambda) = <ufunc 'inv_boxcox1p'>`  
 Compute the inverse of the Box-Cox transformation.

Find  $x$  such that:

$$y = \begin{cases} ((1+x)**\lambda - 1) / \lambda & \text{if } \lambda \neq 0 \\ \log(1+x) & \text{if } \lambda == 0 \end{cases}$$

**Parameters** `y`: array\_like  
 Data to be transformed.  
`lambda`: array\_like  
 Power parameter of the Box-Cox transform.  
**Returns** `x`: array  
 Transformed data.

*Notes*

New in version 0.16.0.

*Examples*

```
>>> from scipy.special import boxcox1p, inv_boxcox1p
>>> y = boxcox1p([1, 4, 10], 2.5)
>>> inv_boxcox1p(y, 2.5)
array([1., 4., 10.]
```

## Information Theory Functions

---

<code>entr(x)</code>	Elementwise function for computing entropy.
<code>rel_entr(x, y)</code>	Elementwise function for computing relative entropy.
<code>kl_div(x, y)</code>	Elementwise function for computing Kullback-Leibler divergence.
<code>huber(delta, r)</code>	Huber loss function.
<code>pseudo_huber(delta, r)</code>	Pseudo-Huber loss function.

---

`scipy.special.entr(x) = <ufunc 'entr'>`  
 Elementwise function for computing entropy.

$$\text{entr}(x) = \begin{cases} -x \log(x) & x > 0 \\ 0 & x = 0 \\ -\infty & \text{otherwise} \end{cases}$$

**Parameters** `x`: ndarray  
 Input array.



**Returns** `res` : ndarray

The value of the elementwise entropy function at the given points  $x$ .

**See also:**

`kl_div`, `rel_entr`

**Notes**

This function is concave.

New in version 0.15.0.

`scipy.special.rel_entr(x, y) = <ufunc 'rel_entr'>`

Elementwise function for computing relative entropy.

$$\text{rel\_entr}(x, y) = \begin{cases} x \log(x/y) & x > 0, y > 0 \\ 0 & x = 0, y \geq 0 \\ \infty & \text{otherwise} \end{cases}$$

**Parameters** `x` : ndarray

First input array.

`y` : ndarray

Second input array.

**Returns** `res` : ndarray

Output array.

**See also:**

`entr`, `kl_div`

**Notes**

This function is jointly convex in  $x$  and  $y$ .

New in version 0.15.0.

`scipy.special.kl_div(x, y) = <ufunc 'kl_div'>`

Elementwise function for computing Kullback-Leibler divergence.

$$\text{kl\_div}(x, y) = \begin{cases} x \log(x/y) - x + y & x > 0, y > 0 \\ y & x = 0, y \geq 0 \\ \infty & \text{otherwise} \end{cases}$$

**Parameters** `x` : ndarray

First input array.

`y` : ndarray

Second input array.

**Returns** `res` : ndarray

Output array.

**See also:**

`entr`, `rel_entr`

**Notes**

This function is non-negative and is jointly convex in  $x$  and  $y$ .

New in version 0.15.0.

`scipy.special.huber` (*delta*, *r*) = <ufunc 'huber'>  
 Huber loss function.

$$\text{huber}(\delta, r) = \begin{cases} \infty & \delta < 0 \\ \frac{1}{2}r^2 & 0 \leq \delta, |r| \leq \delta \\ \delta(|r| - \frac{1}{2}\delta) & \text{otherwise} \end{cases}$$

**Parameters**

- delta** : ndarray  
Input array, indicating the quadratic vs. linear loss changepoint.
- r** : ndarray  
Input array, possibly representing residuals.

**Returns**

- res** : ndarray  
The computed Huber loss function values.

**Notes**

This function is convex in *r*.  
 New in version 0.15.0.

`scipy.special.pseudo_huber` (*delta*, *r*) = <ufunc 'pseudo\_huber'>  
 Pseudo-Huber loss function.

$$\text{pseudo\_huber}(\delta, r) = \delta^2 \left( \sqrt{1 + \left(\frac{r}{\delta}\right)^2} - 1 \right)$$

**Parameters**

- delta** : ndarray  
Input array, indicating the soft quadratic vs. linear loss changepoint.
- r** : ndarray  
Input array, possibly representing residuals.

**Returns**

- res** : ndarray  
The computed Pseudo-Huber loss function values.

**Notes**

This function is convex in *r*.  
 New in version 0.15.0.

**Gamma and Related Functions**

---

<code>gamma(z)</code>	Gamma function.
<code>gammaaln(x)</code>	Logarithm of the absolute value of the Gamma function for real inputs.
<code>loggamma(z[, out])</code>	Principal branch of the logarithm of the Gamma function.
<code>gammasgn(x)</code>	Sign of the gamma function.
<code>gammainc(a, x)</code>	Incomplete gamma function
<code>gammaincinv(a, y)</code>	Inverse to <code>gammainc</code>
<code>gammaincc(a, x)</code>	Complemented incomplete gamma integral
<code>gammainccinv(a, y)</code>	Inverse to <code>gammaincc</code>
<code>beta(a, b)</code>	Beta function.
<code>betaln(a, b)</code>	Natural logarithm of absolute value of beta function.
<code>betainc(a, b, x)</code>	Incomplete beta integral.
<code>betaincinv(a, b, y)</code>	Inverse function to beta integral.
<code>psi(z[, out])</code>	The digamma function.
<code>rgamma(z)</code>	Gamma function inverted
<code>polygamma(n, x)</code>	Polygamma function <i>n</i> .

Continued on next page

Table 5.249 – continued from previous page

<code>multigammaln(a, d)</code>	Returns the log of multivariate gamma, also sometimes called the generalized gamma.
<code>digamma(z[, out])</code>	The digamma function.
<code>poch(z, m)</code>	Rising factorial $(z)_m$

`scipy.special.gamma(z) = <ufunc 'gamma'>`

Gamma function.

The gamma function is often referred to as the generalized factorial since  $z * \text{gamma}(z) = \text{gamma}(z+1)$  and  $\text{gamma}(n+1) = n!$  for natural number  $n$ .

`scipy.special.gammaln(x)`

Logarithm of the absolute value of the Gamma function for real inputs.

**Parameters** `x`: array-like

**Returns** `gammaln`: ndarray  
Values on the real line at which to compute `gammaln`  
Values of `gammaln` at `x`.

**See also:**

`gamma` sign of the gamma function

`loggamma` principal branch of the logarithm of the gamma function

**Notes**

When used in conjunction with `gamma`, this function is useful for working in logspace on the real axis without having to deal with complex numbers, via the relation  $\exp(\text{gammaln}(x)) = \text{gamma}(x) * \text{gamma}(x)$ .

Note that `gammaln` currently accepts complex-valued inputs, but it is not the same function as for real-valued inputs, and the branch is not well-defined — using `gammaln` with complex is deprecated and will be disallowed in future SciPy versions.

For complex-valued log-gamma, use `loggamma` instead of `gammaln`.

`scipy.special.loggamma(z, out=None) = <ufunc 'loggamma'>`

Principal branch of the logarithm of the Gamma function. It is defined to be  $\log(\Gamma(x))$  for  $x > 0$  and extended to the complex plane by analytic continuation. The implementation here is based on [hare1997].

The function has a single branch cut on the negative real axis and is taken to be continuous when approaching the axis from above. Note that it is not generally true that  $\log \Gamma(z) = \log(\Gamma(z))$ , though the real parts of the functions do agree. The benefit of not defining `loggamma` as  $\log(\Gamma(z))$  is that the latter function has a complicated branch cut structure whereas `loggamma` is analytic except for on the negative real axis.

The identities

$$\exp(\log \Gamma(z)) = \Gamma(z)$$

$$\log \Gamma(z + 1) = \log(z) + \log \Gamma(z)$$

make `loggamma` useful for working in complex logspace. However, `loggamma` necessarily returns complex outputs for real inputs, so if you want to work only with real numbers use `gammaln`. On the real line the two functions are related by  $\exp(\text{loggamma}(x)) = \text{gamma}(x) * \exp(\text{gammaln}(x))$ , though in practice rounding errors will introduce small spurious imaginary components in  $\exp(\text{loggamma}(x))$ .

New in version 0.18.0.

**Parameters** `z`: array-like

Values in the complex plain at which to compute `loggamma`

**out**: ndarray, optional

Output array for computed values of `loggamma`

**Returns** `loggamma` : ndarray  
 Values of loggamma at z.

**See also:**

`gamma ln` logarithm of the absolute value of the Gamma function  
`gamma sgn` sign of the gamma function

**References**

[hare1997]

`scipy.special.gammalsgn` (*x*) = <ufunc 'gammalsgn'>  
 Sign of the gamma function.

**See also:**

`gammaln`, `loggamma`

`scipy.special.gammainc` (*a*, *x*) = <ufunc 'gammainc'>  
 Incomplete gamma function

Defined as:

$$1 / \text{gamma}(a) * \text{integral}(\exp(-t) * t^{(a-1)}, t=0..x)$$

*a* must be positive and *x* must be >= 0.

`scipy.special.gammaincinv` (*a*, *y*) = <ufunc 'gammaincinv'>  
 Inverse to `gammainc`

Returns *x* such that `gammainc(a, x) = y`.

`scipy.special.gammaincc` (*a*, *x*) = <ufunc 'gammaincc'>  
 Complemented incomplete gamma integral

Defined as:

$$1 / \text{gamma}(a) * \text{integral}(\exp(-t) * t^{(a-1)}, t=x..inf) = 1 - \text{gammainc}(a, x)$$

*a* must be positive and *x* must be >= 0.

`scipy.special.gammainccinv` (*a*, *y*) = <ufunc 'gammainccinv'>  
 Inverse to `gammaincc`

Returns *x* such that `gammaincc(a, x) == y`.

`scipy.special.beta` (*a*, *b*) = <ufunc 'beta'>  
 Beta function.

$$\text{beta}(a, b) = \text{gamma}(a) * \text{gamma}(b) / \text{gamma}(a+b)$$

`scipy.special.betaln` (*a*, *b*) = <ufunc 'betaln'>  
 Natural logarithm of absolute value of beta function.

Computes `ln(abs(beta(a, b)))`.

`scipy.special.betainc` (*a*, *b*, *x*) = <ufunc 'betainc'>  
 Incomplete beta integral.

Compute the incomplete beta integral of the arguments, evaluated from zero to *x*:

$$\text{gamma}(a+b) / (\text{gamma}(a) * \text{gamma}(b)) * \text{integral}(t^{(a-1)} (1-t)^{(b-1)}, t=0..x).$$

**Notes**

The incomplete beta is also sometimes defined without the terms in gamma, in which case the above definition is the so-called regularized incomplete beta. Under this definition, you can get the incomplete beta by multiplying the result of the scipy function by `beta(a, b)`.

`scipy.special.betaincinv(a, b, y) = <ufunc 'betaincinv'>`

Inverse function to beta integral.

Compute  $x$  such that `betainc(a, b, x) = y`.

`scipy.special.psi(z, out=None) = <ufunc 'psi'>`

The digamma function.

The logarithmic derivative of the gamma function evaluated at  $z$ .

<b>Parameters</b>	<b>z</b> : array_like	Real or complex argument.
	<b>out</b> : ndarray, optional	
<b>Returns</b>	<b>digamma</b> : ndarray	Array for the computed values of <code>psi</code> . Computed values of <code>psi</code> .

**Notes**

For large values not close to the negative real axis `psi` is computed using the asymptotic series (5.11.2) from [R447]. For small arguments not close to the negative real axis the recurrence relation (5.5.2) from [R447] is used until the argument is large enough to use the asymptotic series. For values close to the negative real axis the reflection formula (5.5.4) from [R447] is used first. Note that `psi` has a family of zeros on the negative real axis which occur between the poles at nonpositive integers. Around the zeros the reflection formula suffers from cancellation and the implementation loses precision. The sole positive zero and the first negative zero, however, are handled separately by precomputing series expansions using [R448], so the function should maintain full accuracy around the origin.

**References**

[R447], [R448]

`scipy.special.rgamma(z) = <ufunc 'rgamma'>`

Gamma function inverted

Returns  $1/\text{gamma}(x)$

`scipy.special.polygamma(n, x)`

Polygamma function  $n$ .

This is the  $n$ th derivative of the digamma (`psi`) function.

<b>Parameters</b>	<b>n</b> : array_like of int	The order of the derivative of <code>psi</code> .
	<b>x</b> : array_like	
<b>Returns</b>	<b>polygamma</b> : ndarray	Where to evaluate the polygamma function. The result.

**Examples**

```
>>> from scipy import special
>>> x = [2, 3, 25.5]
>>> special.polygamma(1, x)
array([ 0.64493407,  0.39493407,  0.03999467])
```

```
>>> special.polygamma(0, x) == special.psi(x)
array([ True,  True,  True], dtype=bool)
```

`scipy.special.multigammaln(a, d)`

Returns the log of multivariate gamma, also sometimes called the generalized gamma.

**Parameters**

- a** : ndarray  
The multivariate gamma is computed for each item of *a*.
- d** : int  
The dimension of the space of integration.

**Returns**

- res** : ndarray  
The values of the log multivariate gamma at the given points *a*.

### Notes

The formal definition of the multivariate gamma of dimension *d* for a real *a* is

$$\Gamma_d(a) = \int_{A>0} e^{-tr(A)} |A|^{a-(d+1)/2} dA$$

with the condition  $a > (d - 1)/2$ , and  $A > 0$  being the set of all the positive definite matrices of dimension *d*. Note that *a* is a scalar: the integrand only is multivariate, the argument is not (the function is defined over a subset of the real set).

This can be proven to be equal to the much friendlier equation

$$\Gamma_d(a) = \pi^{d(d-1)/4} \prod_{i=1}^d \Gamma(a - (i - 1)/2).$$

### References

R. J. Muirhead, Aspects of multivariate statistical theory (Wiley Series in probability and mathematical statistics).

`scipy.special.digamma(z, out=None) = <ufunc 'psi'>`

The digamma function.

The logarithmic derivative of the gamma function evaluated at *z*.

**Parameters**

- z** : array\_like  
Real or complex argument.
- out** : ndarray, optional  
Array for the computed values of `psi`.

**Returns**

- digamma** : ndarray  
Computed values of `psi`.

### Notes

For large values not close to the negative real axis `psi` is computed using the asymptotic series (5.11.2) from [R338]. For small arguments not close to the negative real axis the recurrence relation (5.5.2) from [R338] is used until the argument is large enough to use the asymptotic series. For values close to the negative real axis the reflection formula (5.5.4) from [R338] is used first. Note that `psi` has a family of zeros on the negative real axis which occur between the poles at nonpositive integers. Around the zeros the reflection formula suffers from cancellation and the implementation loses precision. The sole positive zero and the first negative zero, however, are handled separately by precomputing series expansions using [R339], so the function should maintain full accuracy around the origin.

**References**

[R338], [R339]

`scipy.special.poch(z, m) = <ufunc 'poch'>`Rising factorial (z)<sub>m</sub>

The Pochhammer symbol (rising factorial), is defined as:

$$(z)_m = \text{gamma}(z + m) / \text{gamma}(z)$$

For positive integer *m* it reads:

$$(z)_m = z * (z + 1) * \dots * (z + m - 1)$$

**Error Function and Fresnel Integrals**


---

<code>erf(z)</code>	Returns the error function of complex argument.
<code>erfc(x)</code>	Complementary error function, $1 - \text{erf}(x)$ .
<code>erfcx(x)</code>	Scaled complementary error function, $\exp(x^2) * \text{erfc}(x)$ .
<code>erfi(z)</code>	Imaginary error function, $-i \text{erf}(i z)$ .
<code>erfinv(y)</code>	Inverse function for erf.
<code>erfcinv(y)</code>	Inverse function for erfc.
<code>wofz(z)</code>	Faddeeva function
<code>dawson(x)</code>	Dawson's integral.
<code>fresnel(z)</code>	Fresnel sin and cos integrals
<code>fresnel_zeros(nt)</code>	Compute nt complex zeros of sine and cosine Fresnel integrals S(z) and C(z).
<code>modfresnelp(x)</code>	Modified Fresnel positive integrals
<code>modfresnelm(x)</code>	Modified Fresnel negative integrals

---

`scipy.special.erf(z) = <ufunc 'erf'>`

Returns the error function of complex argument.

It is defined as  $2/\sqrt{\pi} * \int_0^z \exp(-t^2) dt$ .

**Parameters** `x`: ndarray  
**Returns** `res`: ndarray

Input array.

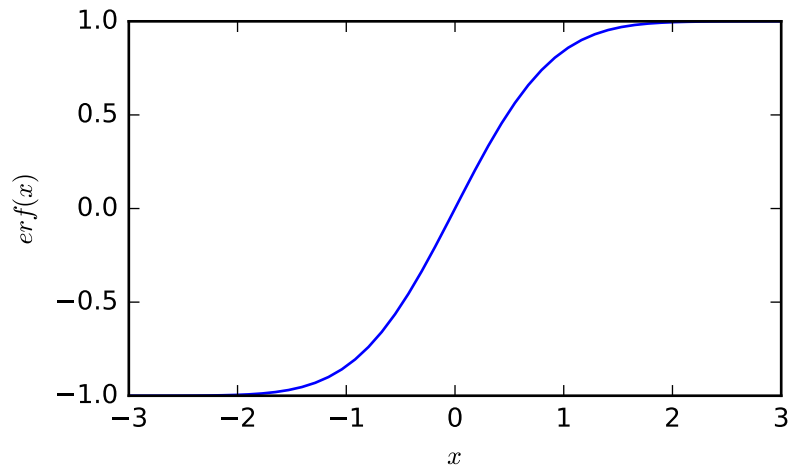
The values of the error function at the given points *x*.**See also:**`erfc, erfinv, erfcinv, wofz, erfcx, erfi`**Notes**The cumulative of the unit normal distribution is given by  $\Phi(z) = 1/2[1 + \text{erf}(z/\sqrt{2})]$ .**References**

[R349], [R350], [R351]

**Examples**

```
>>> from scipy import special
>>> import matplotlib.pyplot as plt
>>> x = np.linspace(-3, 3)
```

```
>>> plt.plot(x, special.erf(x))
>>> plt.xlabel('$x$')
>>> plt.ylabel('$\text{erf}(x)$')
>>> plt.show()
```



`scipy.special.erfc(x)` = <ufunc 'erfc'>  
Complementary error function,  $1 - \text{erf}(x)$ .

**See also:**

`erf`, `erfi`, `erfcx`, `dawsn`, `wofz`

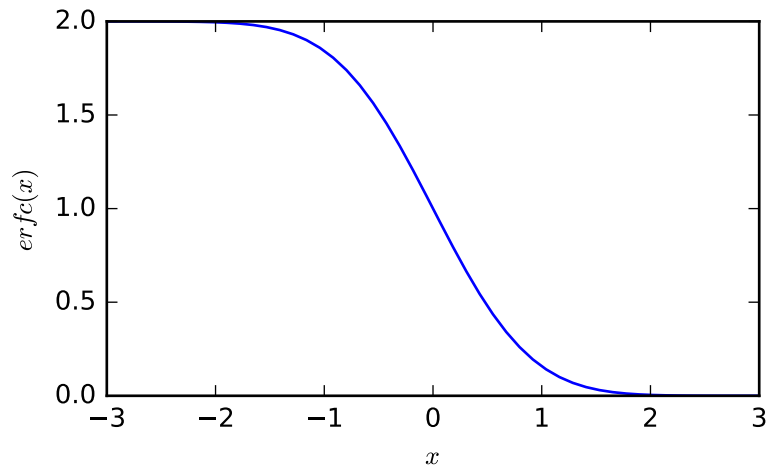
**References**

[R353]

**Examples**

```
>>> from scipy import special
>>> import matplotlib.pyplot as plt
>>> x = np.linspace(-3, 3)
>>> plt.plot(x, special.erfc(x))
>>> plt.xlabel('$x$')
>>> plt.ylabel('$\text{erfc}(x)$')
>>> plt.show()
```





`scipy.special.erfcx(x) = <ufunc 'erfcx'>`  
 Scaled complementary error function,  $\exp(x^2) * \text{erfc}(x)$ .

**See also:**

`erf`, `erfc`, `erfi`, `dawsn`, `wofz`

**Notes**

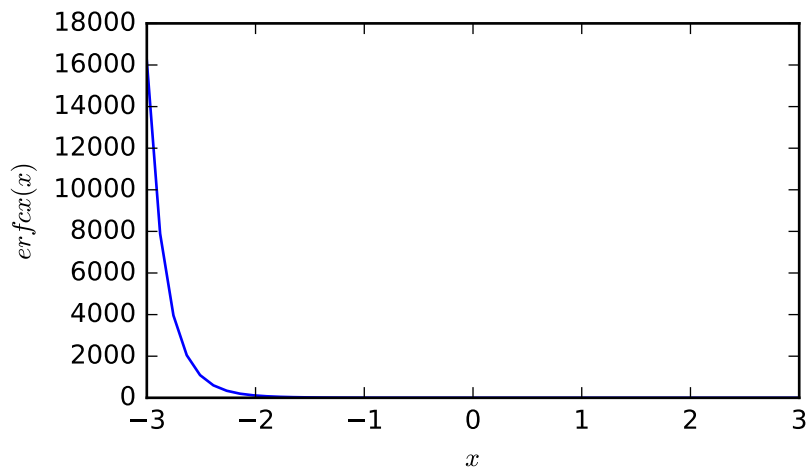
New in version 0.12.0.

**References**

[R354]

**Examples**

```
>>> from scipy import special
>>> import matplotlib.pyplot as plt
>>> x = np.linspace(-3, 3)
>>> plt.plot(x, special.erfcx(x))
>>> plt.xlabel('$x$')
>>> plt.ylabel('$\text{erfcx}(x)$')
>>> plt.show()
```



`scipy.special.erfi(z) = <ufunc 'erfi'>`  
Imaginary error function,  $-i \operatorname{erf}(i z)$ .

**See also:**

`erf`, `erfc`, `erfcx`, `dawsn`, `wofz`

**Notes**

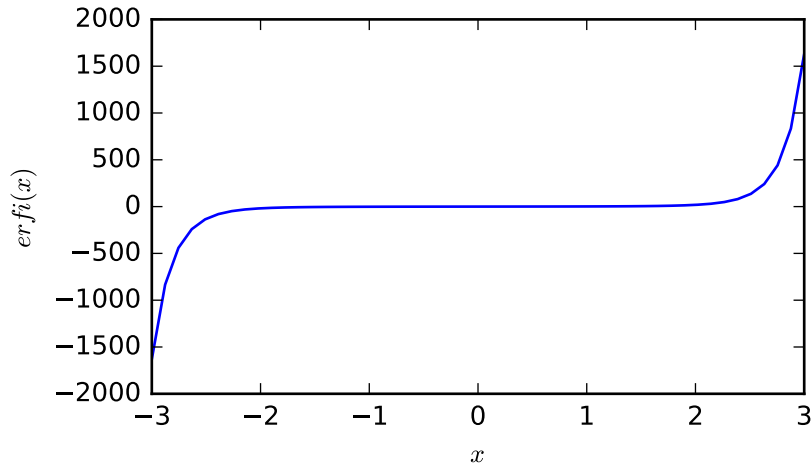
New in version 0.12.0.

**References**

[R355]

**Examples**

```
>>> from scipy import special
>>> import matplotlib.pyplot as plt
>>> x = np.linspace(-3, 3)
>>> plt.plot(x, special.erfi(x))
>>> plt.xlabel('$x$')
>>> plt.ylabel('$erfi(x)$')
>>> plt.show()
```



`scipy.special.erfinv`(y)  
Inverse function for erf.

`scipy.special.erfcinv`(y)  
Inverse function for erfc.

`scipy.special.wofz`(z) = <ufunc 'wofz'>  
Faddeeva function

Returns the value of the Faddeeva function for complex argument:

```
exp(-z**2) * erfc(-i*z)
```

**See also:**

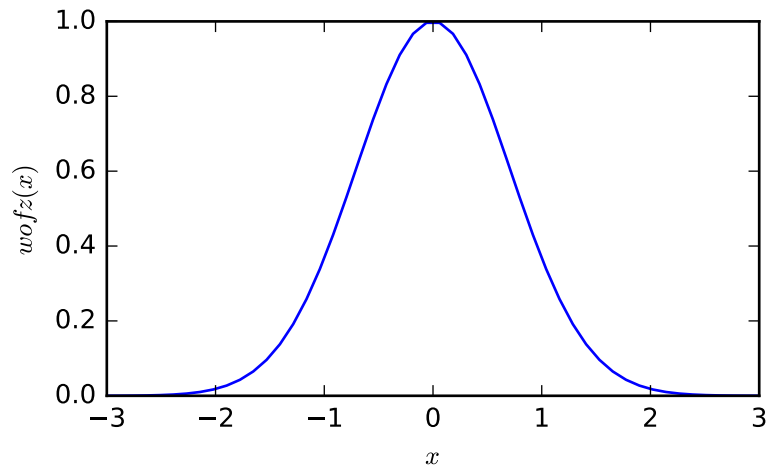
`dawsn`, `erf`, `erfc`, `erfcx`, `erfi`

### References

[R473]

### Examples

```
>>> from scipy import special
>>> import matplotlib.pyplot as plt
>>> x = np.linspace(-3, 3)
>>> plt.plot(x, special.wofz(x))
>>> plt.xlabel('$x$')
>>> plt.ylabel('$wofz(x)$')
>>> plt.show()
```



`scipy.special.dawsn(x) = <ufunc 'dawsn'>`

Dawson's integral.

Computes:

```
exp(-x**2) * integral(exp(t**2), t=0..x).
```

**See also:**

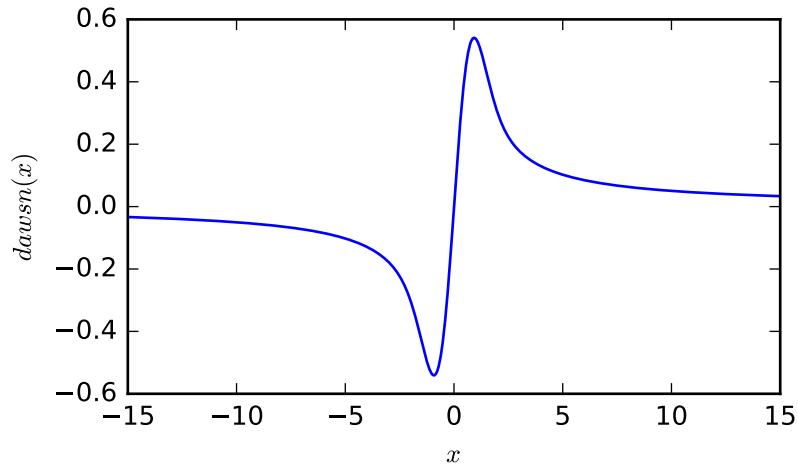
`wofz`, `erf`, `erfc`, `erfcx`, `erfi`

**References**

[R337]

**Examples**

```
>>> from scipy import special
>>> import matplotlib.pyplot as plt
>>> x = np.linspace(-15, 15, num=1000)
>>> plt.plot(x, special.dawsn(x))
>>> plt.xlabel('$x$')
>>> plt.ylabel('$dawsn(x)$')
>>> plt.show()
```



`scipy.special.fresnel(z) = <ufunc 'fresnel'>`

Fresnel sin and cos integrals

Defined as:

```
ssa = integral(sin(pi/2 * t**2), t=0..z)
csa = integral(cos(pi/2 * t**2), t=0..z)
```

**Parameters** `z`: float or complex array\_like  
Argument

**Returns** `ssa, csa`  
Fresnel sin and cos integral values

`scipy.special.fresnel_zeros(nt)`

Compute `nt` complex zeros of sine and cosine Fresnel integrals  $S(z)$  and  $C(z)$ .

**References**

[R360]

`scipy.special.modfresnelp(x) = <ufunc 'modfresnelp'>`

Modified Fresnel positive integrals

**Returns** `fp`  
Integral  $F_+(x)$ :  $\text{integral}(\exp(1j*t*t), t=x..\text{inf})$

`kp`  
Integral  $K_+(x)$ :  $1/\text{sqrt}(\text{pi}) * \exp(-1j*(x*x+\text{pi}/4)) * \text{fp}$

`scipy.special.modfresnelm(x) = <ufunc 'modfresnelm'>`

Modified Fresnel negative integrals

**Returns** `fm`  
Integral  $F_-(x)$ :  $\text{integral}(\exp(-1j*t*t), t=x..\text{inf})$

`km`  
Integral  $K_-(x)$ :  $1/\text{sqrt}(\text{pi}) * \exp(1j*(x*x+\text{pi}/4)) * \text{fp}$

These are not universal functions:

Continued on next page

Table 5.251 – continued from previous page

<code>erf_zeros(nt)</code>	Compute $nt$ complex zeros of error function $\text{erf}(z)$ .
<code>fresnelc_zeros(nt)</code>	Compute $nt$ complex zeros of cosine Fresnel integral $C(z)$ .
<code>fresnels_zeros(nt)</code>	Compute $nt$ complex zeros of sine Fresnel integral $S(z)$ .

`scipy.special.erf_zeros` ( $nt$ )  
 Compute  $nt$  complex zeros of error function  $\text{erf}(z)$ .

**References**

[R352]

`scipy.special.fresnelc_zeros` ( $nt$ )  
 Compute  $nt$  complex zeros of cosine Fresnel integral  $C(z)$ .

**References**

[R361]

`scipy.special.fresnels_zeros` ( $nt$ )  
 Compute  $nt$  complex zeros of sine Fresnel integral  $S(z)$ .

**References**

[R362]

**Legendre Functions**

<code>lpmv(m, v, x)</code>	Associated legendre function of integer order.
<code>sph_harm(m, n, theta, phi)</code>	Compute spherical harmonics.

`scipy.special.lpmv` ( $m, v, x$ ) = <ufunc ‘lpmv’>  
 Associated legendre function of integer order.

**Parameters**

- m** : int  
Order
- v** : float  
Degree.
- x** : float  
Argument. Must be  $|x| \leq 1$ .

**Returns**

- res** : float  
The value of the function.

**See also:**

`lpmn` Similar, but computes values for all orders  $0..m$  and degrees  $0..n$ .  
`clpmn` Similar to `lpmn` but allows a complex argument.

**Notes**

It is possible to extend the domain of this function to all complex  $m, v, x$ , but this is not yet implemented.

`scipy.special.sph_harm` ( $m, n, theta, phi$ ) = <ufunc ‘sph\_harm’>

Compute spherical harmonics.

$$Y_n^m(\theta, \phi) = \sqrt{\frac{2n+1}{4\pi} \frac{(n-m)!}{(n+m)!}} e^{im\theta} P_n^m(\cos(\phi))$$

<b>Parameters</b>	<b>m</b> : int	$ m  \leq n$ ; the order of the harmonic.
	<b>n</b> : int	where $n \geq 0$ ; the degree of the harmonic. This is often called $l$ (lower case L) in descriptions of spherical harmonics.
	<b>theta</b> : float	$[0, 2\pi]$ ; the azimuthal (longitudinal) coordinate.
	<b>phi</b> : float	$[0, \pi]$ ; the polar (colatitudinal) coordinate.
<b>Returns</b>	<b>y_mn</b> : complex float	The harmonic $Y_n^m$ sampled at <i>theta</i> and <i>phi</i>

#### Notes

There are different conventions for the meaning of input arguments *theta* and *phi*. We take *theta* to be the azimuthal angle and *phi* to be the polar angle. It is common to see the opposite convention - that is *theta* as the polar angle and *phi* as the azimuthal angle.

#### References

[R455]

These are not universal functions:

<code>clpmn(m, n, z[, type])</code>	Associated Legendre function of the first kind, $P_{mn}(z)$ .
<code>lpn(n, z)</code>	Legendre functions of the first kind, $P_n(z)$ .
<code>lqn(n, z)</code>	Legendre functions of the second kind, $Q_n(z)$ .
<code>lpmn(m, n, z)</code>	Associated Legendre function of the first kind, $P_{mn}(z)$ .
<code>lqmn(m, n, z)</code>	Associated Legendre function of the second kind, $Q_{mn}(z)$ .

`scipy.special.clpmn(m, n, z, type=3)`

Associated Legendre function of the first kind,  $P_{mn}(z)$ .

Computes the associated Legendre function of the first kind of order  $m$  and degree  $n$ ,  $P_{mn}(z) = P_n^m(z)$ , and its derivative,  $P_{mn}'(z)$ . Returns two arrays of size  $(m+1, n+1)$  containing  $P_{mn}(z)$  and  $P_{mn}'(z)$  for all orders from  $0 \dots m$  and degrees from  $0 \dots n$ .

<b>Parameters</b>	<b>m</b> : int	$ m  \leq n$ ; the order of the Legendre function.
	<b>n</b> : int	where $n \geq 0$ ; the degree of the Legendre function. Often called $l$ (lower case L) in descriptions of the associated Legendre function
	<b>z</b> : float or complex	Input value.
	<b>type</b> : int, optional	takes values 2 or 3: 2: cut on the real axis $ x  > 1$ 3: cut on the real axis $-1 \leq x \leq 1$ (default)
<b>Returns</b>	<b>Pmn_z</b> : $(m+1, n+1)$ array	Values for all orders $0 \dots m$ and degrees $0 \dots n$
	<b>Pmn_d_z</b> : $(m+1, n+1)$ array	Derivatives for all orders $0 \dots m$ and degrees $0 \dots n$

See also:

`lpmn` associated Legendre functions of the first kind for real  $z$

**Notes**

By default, i.e. for `type=3`, phase conventions are chosen according to [R334] such that the function is analytic. The cut lies on the interval  $(-1, 1)$ . Approaching the cut from above or below in general yields a phase factor with respect to Ferrer's function of the first kind (cf. `lpmn`).

For `type=2` a cut at  $|x| > 1$  is chosen. Approaching the real values on the interval  $(-1, 1)$  in the complex plane yields Ferrer's function of the first kind.

**References**

[R334], [R335]

`scipy.special.lpn` ( $n, z$ )

Legendre functions of the first kind,  $P_n(z)$ .

Compute sequence of Legendre functions of the first kind (polynomials),  $P_n(z)$  and derivatives for all degrees from 0 to  $n$  (inclusive).

See also `special.legendre` for polynomial class.

**References**

[R426]

`scipy.special.lqn` ( $n, z$ )

Legendre functions of the second kind,  $Q_n(z)$ .

Compute sequence of Legendre functions of the second kind,  $Q_n(z)$  and derivatives for all degrees from 0 to  $n$  (inclusive).

**References**

[R428]

`scipy.special.lpmn` ( $m, n, z$ )

Associated Legendre function of the first kind,  $P_{mn}(z)$ .

Computes the associated Legendre function of the first kind of order  $m$  and degree  $n$ ,  $P_{mn}(z) = P_n^m(z)$ , and its derivative,  $P_{mn}'(z)$ . Returns two arrays of size  $(m+1, n+1)$  containing  $P_{mn}(z)$  and  $P_{mn}'(z)$  for all orders from  $0..m$  and degrees from  $0..n$ .

This function takes a real argument  $z$ . For complex arguments  $z$  use `clpmn` instead.

<b>Parameters</b>	<b>m</b> : int	$ m  \leq n$ ; the order of the Legendre function.
	<b>n</b> : int	where $n \geq 0$ ; the degree of the Legendre function. Often called $l$ (lower case L) in descriptions of the associated Legendre function
	<b>z</b> : float	
<b>Returns</b>	<b>Pmn_z</b> : $(m+1, n+1)$ array	Input value. Values for all orders $0..m$ and degrees $0..n$
	<b>Pmn_d_z</b> : $(m+1, n+1)$ array	Derivatives for all orders $0..m$ and degrees $0..n$

**See also:**

`clpmn` associated Legendre functions of the first kind for complex  $z$



**Notes**

In the interval  $(-1, 1)$ , Ferrer's function of the first kind is returned. The phase convention used for the intervals  $(1, \text{inf})$  and  $(-\text{inf}, -1)$  is such that the result is always real.

**References**

[R424], [R425]

`scipy.special.lqmn` ( $m, n, z$ )

Associated Legendre function of the second kind,  $Q_{mn}(z)$ .

Computes the associated Legendre function of the second kind of order  $m$  and degree  $n$ ,  $Q_{mn}(z) = Q_n^m(z)$ , and its derivative,  $Q_{mn}'(z)$ . Returns two arrays of size  $(m+1, n+1)$  containing  $Q_{mn}(z)$  and  $Q_{mn}'(z)$  for all orders from  $0..m$  and degrees from  $0..n$ .

**Parameters** **m** : int

$|m| \leq n$ ; the order of the Legendre function.

**n** : int

where  $n \geq 0$ ; the degree of the Legendre function. Often called  $l$  (lower case L) in descriptions of the associated Legendre function

**z** : complex

**Returns** **Qmn\_z** :  $(m+1, n+1)$  array

Input value.  
Values for all orders  $0..m$  and degrees  $0..n$

**Qmn\_d\_z** :  $(m+1, n+1)$  array

Derivatives for all orders  $0..m$  and degrees  $0..n$

**References**

[R427]

**Ellipsoidal Harmonics**


---

<code>ellip_harm</code> ( $h2, k2, n, p, s[, \text{signm}, \text{signn}]$ )	Ellipsoidal harmonic functions $E^p_n(l)$
<code>ellip_harm_2</code> ( $h2, k2, n, p, s$ )	Ellipsoidal harmonic functions $F^p_n(l)$
<code>ellip_normal</code> ( $h2, k2, n, p$ )	Ellipsoidal harmonic normalization constants $\gamma^p_n$

---

`scipy.special.ellip_harm` ( $h2, k2, n, p, s, \text{signm}=1, \text{signn}=1$ )

Ellipsoidal harmonic functions  $E^p_n(l)$

These are also known as Lamé functions of the first kind, and are solutions to the Lamé equation:

$$(s^2 - h^2)(s^2 - k^2)E''(s) + s(2s^2 - h^2 - k^2)E'(s) + (a - qs^2)E(s) = 0$$

where  $q = (n+1)n$  and  $a$  is the eigenvalue (not returned) corresponding to the solutions.

**Parameters** **h2** : float

$h^2$

**k2** : float

$k^2$ ; should be larger than  $h^2$

**n** : int

Degree

**s** : float

Coordinate

**p** : int

Order, can range between  $[1, 2n+1]$

**signm** : {1, -1}, optional  
 Sign of prefactor of functions. Can be +/-1. See Notes.  
**signn** : {1, -1}, optional  
 Sign of prefactor of functions. Can be +/-1. See Notes.  
**Returns** **E** : float  
 the harmonic  $E_n^p(s)$

**See also:**

`ellip_harm_2`, `ellip_normal`

**Notes**

The geometric interpretation of the ellipsoidal functions is explained in [R341], [R342], [R343]. The *signm* and *signn* arguments control the sign of prefactors for functions according to their type:

K : +1  
 L : signm  
 M : signn  
 N : signm\*signn

New in version 0.15.0.

**References**

[R340], [R341], [R342], [R343]

**Examples**

```
>>> from scipy.special import ellip_harm
>>> w = ellip_harm(5, 8, 1, 1, 2.5)
>>> w
2.5
```

Check that the functions indeed are solutions to the Lamé equation:

```
>>> from scipy.interpolate import UnivariateSpline
>>> def eigenvalue(f, df, ddf):
...     r = ((s**2 - h**2)*(s**2 - k**2)*ddf + s*(2*s**2 - h**2 - k**2)*df - n*(n+1)*s**2*f)/f
...     return -r.mean(), r.std()
>>> s = np.linspace(0.1, 10, 200)
>>> k, h, n, p = 8.0, 2.2, 3, 2
>>> E = ellip_harm(h**2, k**2, n, p, s)
>>> E_spl = UnivariateSpline(s, E)
>>> a, a_err = eigenvalue(E_spl(s), E_spl(s, 1), E_spl(s, 2))
>>> a, a_err
(583.44366156701483, 6.4580890640310646e-11)
```

`scipy.special.ellip_harm_2(h2, k2, n, p, s)`  
 Ellipsoidal harmonic functions  $F_n^p(l)$

These are also known as Lamé functions of the second kind, and are solutions to the Lamé equation:

$$(s^2 - h^2)(s^2 - k^2)F''(s) + s(2s^2 - h^2 - k^2)F'(s) + (a - qs^2)F(s) = 0$$

where  $q = (n + 1)n$  and  $a$  is the eigenvalue (not returned) corresponding to the solutions.

**Parameters** **h2** : float  
                   h\*\*2  
**k2** : float

	<b>n</b> : int	$k^{**2}$ ; should be larger than $h^{**2}$
		Degree.
	<b>p</b> : int	Order, can range between $[1, 2n+1]$ .
	<b>s</b> : float	Coordinate
<b>Returns</b>	<b>F</b> : float	The harmonic $F_n^p(s)$

**See also:**

`ellip_harm`, `ellip_normal`

**Notes**

Lame functions of the second kind are related to the functions of the first kind:

$$F_n^p(s) = (2n + 1)E_n^p(s) \int_0^{1/s} \frac{du}{(E_n^p(1/u))^2 \sqrt{(1 - u^2 k^2)(1 - u^2 h^2)}}$$

New in version 0.15.0.

**Examples**

```
>>> from scipy.special import ellip_harm_2
>>> w = ellip_harm_2(5, 8, 2, 1, 10)
>>> w
0.00108056853382
```

`scipy.special.ellip_normal(h2, k2, n, p)`

Ellipsoidal harmonic normalization constants  $\gamma_n^p$

The normalization constant is defined as

$$\gamma_n^p = 8 \int_0^h dx \int_h^k dy \frac{(y^2 - x^2)(E_n^p(y)E_n^p(x))^2}{\sqrt{((k^2 - y^2)(y^2 - h^2)(h^2 - x^2)(k^2 - x^2))}}$$

<b>Parameters</b>	<b>h2</b> : float	$h^{**2}$
	<b>k2</b> : float	$k^{**2}$ ; should be larger than $h^{**2}$
	<b>n</b> : int	Degree.
	<b>p</b> : int	Order, can range between $[1, 2n+1]$ .
<b>Returns</b>	<b>gamma</b> : float	The normalization constant $\gamma_n^p$

**See also:**

`ellip_harm`, `ellip_harm_2`

**Notes**

New in version 0.15.0.

**Examples**

```
>>> from scipy.special import ellip_normal
>>> w = ellip_normal(5, 8, 3, 7)
>>> w
1723.38796997
```

## Orthogonal polynomials

The following functions evaluate values of orthogonal polynomials:

<code>assoc_laguerre(x, n[, k])</code>	Compute the generalized (associated) Laguerre polynomial of degree $n$ and order $k$ .
<code>eval_legendre(n, x[, out])</code>	Evaluate Legendre polynomial at a point.
<code>eval_chebyt(n, x[, out])</code>	Evaluate Chebyshev T polynomial at a point.
<code>eval_chebyu(n, x[, out])</code>	Evaluate Chebyshev U polynomial at a point.
<code>eval_chebysc(n, x[, out])</code>	Evaluate Chebyshev C polynomial at a point.
<code>eval_chebys(n, x[, out])</code>	Evaluate Chebyshev S polynomial at a point.
<code>eval_jacobi(n, alpha, beta, x[, out])</code>	Evaluate Jacobi polynomial at a point.
<code>eval_laguerre(n, x[, out])</code>	Evaluate Laguerre polynomial at a point.
<code>eval_genlaguerre(n, alpha, x[, out])</code>	Evaluate generalized Laguerre polynomial at a point.
<code>eval_hermite(n, x[, out])</code>	Evaluate Hermite polynomial at a point.
<code>eval_hermitenorm(n, x[, out])</code>	Evaluate normalized Hermite polynomial at a point.
<code>eval_gegenbauer(n, alpha, x[, out])</code>	Evaluate Gegenbauer polynomial at a point.
<code>eval_sh_legendre(n, x[, out])</code>	Evaluate shifted Legendre polynomial at a point.
<code>eval_sh_chebyt(n, x[, out])</code>	Evaluate shifted Chebyshev T polynomial at a point.
<code>eval_sh_chebyu(n, x[, out])</code>	Evaluate shifted Chebyshev U polynomial at a point.
<code>eval_sh_jacobi(n, p, q, x[, out])</code>	Evaluate shifted Jacobi polynomial at a point.

`scipy.special.assoc_laguerre(x, n, k=0.0)`

Compute the generalized (associated) Laguerre polynomial of degree  $n$  and order  $k$ .

The polynomial  $L_n^{(k)}(x)$  is orthogonal over  $[0, \infty)$ , with weighting function  $\exp(-x) * x^{**k}$  with  $k > -1$ .

### Notes

`assoc_laguerre` is a simple wrapper around `eval_genlaguerre`, with reversed argument order  $(x, n, k=0.0) \rightarrow (n, k, x)$ .

`scipy.special.eval_legendre(n, x, out=None) = <ufunc 'eval_legendre'>`

Evaluate Legendre polynomial at a point.

`scipy.special.eval_chebyt(n, x, out=None) = <ufunc 'eval_chebyt'>`

Evaluate Chebyshev T polynomial at a point.

This routine is numerically stable for  $x$  in  $[-1, 1]$  at least up to order 10000.

`scipy.special.eval_chebyu(n, x, out=None) = <ufunc 'eval_chebyu'>`

Evaluate Chebyshev U polynomial at a point.

`scipy.special.eval_chebysc(n, x, out=None) = <ufunc 'eval_chebysc'>`

Evaluate Chebyshev C polynomial at a point.

`scipy.special.eval_chebys(n, x, out=None) = <ufunc 'eval_chebys'>`

Evaluate Chebyshev S polynomial at a point.

`scipy.special.eval_jacobi(n, alpha, beta, x, out=None) = <ufunc 'eval_jacobi'>`

Evaluate Jacobi polynomial at a point.

`scipy.special.eval_laguerre` ( $n, x, out=None$ ) = <ufunc 'eval\_laguerre'>  
Evaluate Laguerre polynomial at a point.

`scipy.special.eval_genlaguerre` ( $n, alpha, x, out=None$ ) = <ufunc 'eval\_genlaguerre'>  
Evaluate generalized Laguerre polynomial at a point.

`scipy.special.eval_hermite` ( $n, x, out=None$ ) = <ufunc 'eval\_hermite'>  
Evaluate Hermite polynomial at a point.

`scipy.special.eval_hermitenorm` ( $n, x, out=None$ ) = <ufunc 'eval\_hermitenorm'>  
Evaluate normalized Hermite polynomial at a point.

`scipy.special.eval_gegenbauer` ( $n, alpha, x, out=None$ ) = <ufunc 'eval\_gegenbauer'>  
Evaluate Gegenbauer polynomial at a point.

`scipy.special.eval_sh_legendre` ( $n, x, out=None$ ) = <ufunc 'eval\_sh\_legendre'>  
Evaluate shifted Legendre polynomial at a point.

`scipy.special.eval_sh_chebyt` ( $n, x, out=None$ ) = <ufunc 'eval\_sh\_chebyt'>  
Evaluate shifted Chebyshev T polynomial at a point.

`scipy.special.eval_sh_chebyu` ( $n, x, out=None$ ) = <ufunc 'eval\_sh\_chebyu'>  
Evaluate shifted Chebyshev U polynomial at a point.

`scipy.special.eval_sh_jacobi` ( $n, p, q, x, out=None$ ) = <ufunc 'eval\_sh\_jacobi'>  
Evaluate shifted Jacobi polynomial at a point.

The functions below, in turn, return the polynomial coefficients in `orthopoly1d` objects, which function similarly as `numpy.poly1d`. The `orthopoly1d` class also has an attribute `weights` which returns the roots, weights, and total weights for the appropriate form of Gaussian quadrature. These are returned in an  $n \times 3$  array with roots in the first column, weights in the second column, and total weights in the final column. Note that `orthopoly1d` objects are converted to `poly1d` when doing arithmetic, and lose information of the original orthogonal polynomial.

<code>legendre</code> ( $n$ [, monic])	Legendre polynomial coefficients
<code>chebyt</code> ( $n$ [, monic])	Return $n$ th order Chebyshev polynomial of first kind, $T_n(x)$ .
<code>chebyu</code> ( $n$ [, monic])	Return $n$ th order Chebyshev polynomial of second kind, $U_n(x)$ .
<code>chebyc</code> ( $n$ [, monic])	Return $n$ -th order Chebyshev polynomial of first kind, $C_n(x)$ .
<code>chebys</code> ( $n$ [, monic])	Return $n$ th order Chebyshev polynomial of second kind, $S_n(x)$ .
<code>jacobi</code> ( $n, alpha, beta$ [, monic])	Returns the $n$ th order Jacobi polynomial, $P^{(alpha,beta)}_n(x)$ orthogonal over $[-1,1]$ with weight
<code>laguerre</code> ( $n$ [, monic])	Return the $n$ th order Laguerre polynomial, $L_n(x)$ , orthogonal over
<code>genlaguerre</code> ( $n, alpha$ [, monic])	Returns the $n$ th order generalized (associated) Laguerre polynomial,
<code>hermite</code> ( $n$ [, monic])	Return the $n$ th order Hermite polynomial, $H_n(x)$ , orthogonal over
<code>hermitenorm</code> ( $n$ [, monic])	Return the $n$ th order normalized Hermite polynomial, $He_n(x)$ , orthogonal
<code>gegenbauer</code> ( $n, alpha$ [, monic])	Return the $n$ th order Gegenbauer (ultraspherical) polynomial,
<code>sh_legendre</code> ( $n$ [, monic])	Returns the $n$ th order shifted Legendre polynomial, $P_n^*(x)$ , orthogonal over $[0,1]$ with weight
<code>sh_chebyt</code> ( $n$ [, monic])	Return $n$ th order shifted Chebyshev polynomial of first kind, $T_n(x)$ .
<code>sh_chebyu</code> ( $n$ [, monic])	Return $n$ th order shifted Chebyshev polynomial of second kind, $U_n(x)$ .
<code>sh_jacobi</code> ( $n, p, q$ [, monic])	Returns the $n$ th order Jacobi polynomial, $G_n(p,q,x)$ orthogonal over $[0,1]$ with weighting funct

`scipy.special.legendre` ( $n, monic=False$ )

Legendre polynomial coefficients

Returns the  $n$ th-order Legendre polynomial,  $P_n(x)$ , orthogonal over  $[-1, 1]$  with weight function 1.

**Parameters**    **n**

Order of the polynomial

**monic** : bool, optional

If True, output is a monic polynomial (normalized so the leading coefficient is 1). Default is False.

**Returns** `P`: orthopoly1d  
The Legendre polynomial object

**Examples**

Generate the 3rd-order Legendre polynomial  $1/2*(5x^3 + 0x^2 - 3x + 0)$ :

```
>>> from scipy.special import legendre
>>> legendre(3)
poly1d([ 2.5,  0. , -1.5,  0. ])
```

`scipy.special.chebyt` (*n*, *monic=False*)

Return *n*th order Chebyshev polynomial of first kind,  $T_n(x)$ . Orthogonal over  $[-1,1]$  with weight function  $(1-x^{**2})^{**(-1/2)}$ .

`scipy.special.chebyu` (*n*, *monic=False*)

Return *n*th order Chebyshev polynomial of second kind,  $U_n(x)$ . Orthogonal over  $[-1,1]$  with weight function  $(1-x^{**2})^{**(1/2)}$ .

`scipy.special.chebyc` (*n*, *monic=False*)

Return *n*-th order Chebyshev polynomial of first kind,  $C_n(x)$ . Orthogonal over  $[-2, 2]$  with weight function  $f(x) = 1/\sqrt{1 - (x/2)^2}$

`scipy.special.chebys` (*n*, *monic=False*)

Return *n*th order Chebyshev polynomial of second kind,  $S_n(x)$ . Orthogonal over  $[-2, 2]$  with weight function  $f(x) = \sqrt{1 - (x/2)^2}$ .

`scipy.special.jacobi` (*n*, *alpha*, *beta*, *monic=False*)

Returns the *n*th order Jacobi polynomial,  $P^{(\alpha,\beta)}_n(x)$  orthogonal over  $[-1,1]$  with weighting function  $(1-x)^{**\alpha} (1+x)^{**\beta}$  with  $\alpha,\beta > -1$ .

`scipy.special.laguerre` (*n*, *monic=False*)

Return the *n*th order Laguerre polynomial,  $L_n(x)$ , orthogonal over  $[0,\infty)$  with weighting function  $\exp(-x)$

`scipy.special.genlaguerre` (*n*, *alpha*, *monic=False*)

Returns the *n*th order generalized (associated) Laguerre polynomial,  $L^{(\alpha)}_n(x)$ , orthogonal over  $[0,\infty)$  with weighting function  $\exp(-x) x^{**\alpha}$  with  $\alpha > -1$

`scipy.special.hermite` (*n*, *monic=False*)

Return the *n*th order Hermite polynomial,  $H_n(x)$ , orthogonal over  $(-\infty,\infty)$  with weighting function  $\exp(-x^{**2})$

`scipy.special.hermite`norm (*n*, *monic=False*)

Return the *n*th order normalized Hermite polynomial,  $He_n(x)$ , orthogonal over  $(-\infty,\infty)$  with weighting function  $\exp(-(x/2)^{**2})$

`scipy.special.gegenbauer` (*n*, *alpha*, *monic=False*)

Return the *n*th order Gegenbauer (ultraspherical) polynomial,  $C^{(\alpha)}_n(x)$ , orthogonal over  $[-1,1]$  with weighting function  $(1-x^{**2})^{**(\alpha-1/2)}$  with  $\alpha > -1/2$

`scipy.special.sh_legendre` (*n*, *monic=False*)

Returns the *n*th order shifted Legendre polynomial,  $P^{*}_n(x)$ , orthogonal over  $[0,1]$  with weighting function 1.

`scipy.special.sh_chebyt` (*n*, *monic=False*)

Return *n*th order shifted Chebyshev polynomial of first kind,  $T_n(x)$ . Orthogonal over  $[0,1]$  with weight function  $(x-x^{**2})^{**(-1/2)}$ .

`scipy.special.sh_chebyu` (*n*, *monic=False*)

Return *n*th order shifted Chebyshev polynomial of second kind,  $U_n(x)$ . Orthogonal over  $[0,1]$  with weight function  $(x-x^{**2})^{**(1/2)}$ .

`scipy.special.sh_jacobi` ( $n, p, q, \text{monic}=\text{False}$ )

Returns the  $n$ th order Jacobi polynomial,  $G_n(p,q,x)$  orthogonal over  $[0,1]$  with weighting function  $(1-x)^{p-q}(x)^{q-1}$  with  $p>q-1$  and  $q > 0$ .

**Warning:** Computing values of high-order polynomials (around `order > 20`) using polynomial coefficients is numerically unstable. To evaluate polynomial values, the `eval_*` functions should be used instead.

Roots and weights for orthogonal polynomials

<code>c_roots</code> ( $n[, \text{mu}]$ )	Gauss-Chebyshev (first kind) quadrature.
<code>cg_roots</code> ( $n, \text{alpha}[, \text{mu}]$ )	Gauss-Gegenbauer quadrature.
<code>h_roots</code> ( $n[, \text{mu}]$ )	Gauss-Hermite (physicist's) quadrature.
<code>he_roots</code> ( $n[, \text{mu}]$ )	Gauss-Hermite (statistician's) quadrature.
<code>j_roots</code> ( $n, \text{alpha}, \text{beta}[, \text{mu}]$ )	Gauss-Jacobi quadrature.
<code>js_roots</code> ( $n, p1, q1[, \text{mu}]$ )	Gauss-Jacobi (shifted) quadrature.
<code>l_roots</code> ( $n[, \text{mu}]$ )	Gauss-Laguerre quadrature.
<code>la_roots</code> ( $n, \text{alpha}[, \text{mu}]$ )	Gauss-generalized Laguerre quadrature.
<code>p_roots</code> ( $n[, \text{mu}]$ )	Gauss-Legendre quadrature.
<code>ps_roots</code> ( $n[, \text{mu}]$ )	Gauss-Legendre (shifted) quadrature.
<code>s_roots</code> ( $n[, \text{mu}]$ )	Gauss-Chebyshev (second kind) quadrature.
<code>t_roots</code> ( $n[, \text{mu}]$ )	Gauss-Chebyshev (first kind) quadrature.
<code>ts_roots</code> ( $n[, \text{mu}]$ )	Gauss-Chebyshev (first kind, shifted) quadrature.
<code>u_roots</code> ( $n[, \text{mu}]$ )	Gauss-Chebyshev (second kind) quadrature.
<code>us_roots</code> ( $n[, \text{mu}]$ )	Gauss-Chebyshev (second kind, shifted) quadrature.

`scipy.special.c_roots` ( $n, \text{mu}=\text{False}$ )

Gauss-Chebyshev (first kind) quadrature.

Computes the sample points and weights for Gauss-Chebyshev quadrature. The sample points are the roots of the  $n$ -th degree Chebyshev polynomial of the first kind,  $C_n(x)$ . These sample points and weights correctly integrate polynomials of degree  $2n - 1$  or less over the interval  $[-2, 2]$  with weight function  $f(x) = 1/\sqrt{1 - (x/2)^2}$ .

**Parameters**

- n** : int  
quadrature order
- mu** : bool, optional  
If True, return the sum of the weights, optional.

**Returns**

- x** : ndarray  
Sample points
- w** : ndarray  
Weights
- mu** : float  
Sum of the weights

**See also:**

`scipy.integrate.quadrature`, `scipy.integrate.fixed_quad`

`scipy.special.cg_roots` ( $n, \text{alpha}, \text{mu}=\text{False}$ )

Gauss-Gegenbauer quadrature.

Computes the sample points and weights for Gauss-Gegenbauer quadrature. The sample points are the roots of the  $n$ -th degree Gegenbauer polynomial,  $C_n^\alpha(x)$ . These sample points and weights correctly integrate polynomials of degree  $2n - 1$  or less over the interval  $[-1, 1]$  with weight function  $f(x) = (1 - x^2)^{\alpha-1/2}$ .

**Parameters**

- n** : int  
quadrature order
- alpha** : float

**Returns**  
**mu** : bool, optional  
     alpha must be > -0.5  
     If True, return the sum of the weights, optional.  
**x** : ndarray  
     Sample points  
**w** : ndarray  
     Weights  
**mu** : float  
     Sum of the weights

**See also:**

`scipy.integrate.quadrature`, `scipy.integrate.fixed_quad`

`scipy.special.h_roots` (*n*, *mu=False*)

Gauss-Hermite (physicst's) quadrature.

Computes the sample points and weights for Gauss-Hermite quadrature. The sample points are the roots of the *n*-th degree Hermite polynomial,  $H_n(x)$ . These sample points and weights correctly integrate polynomials of degree  $2n - 1$  or less over the interval  $[-\infty, \infty]$  with weight function  $f(x) = e^{-x^2}$ .

**Parameters**  
**n** : int  
     quadrature order  
**mu** : bool, optional  
     If True, return the sum of the weights, optional.  
**Returns**  
**x** : ndarray  
     Sample points  
**w** : ndarray  
     Weights  
**mu** : float  
     Sum of the weights

**See also:**

`scipy.integrate.quadrature`, `scipy.integrate.fixed_quad`,  
`numpy.polynomial.hermite.hermgauss`, `he_roots`

**Notes**

For small *n* up to 150 a modified version of the Golub-Welsch algorithm is used. Nodes are computed from the eigenvalue problem and improved by one step of a Newton iteration. The weights are computed from the well-known analytical formula.

For *n* larger than 150 an optimal asymptotic algorithm is applied which computes nodes and weights in a numerically stable manner. The algorithm has linear runtime making computation for very large *n* (several thousand or more) feasible.

**References**

[townsend.trogdon.olver-2014], [townsend.trogdon.olver-2015]

`scipy.special.he_roots` (*n*, *mu=False*)

Gauss-Hermite (statistician's) quadrature.

Computes the sample points and weights for Gauss-Hermite quadrature. The sample points are the roots of the *n*-th degree Hermite polynomial,  $He_n(x)$ . These sample points and weights correctly integrate polynomials of degree  $2n - 1$  or less over the interval  $[-\infty, \infty]$  with weight function  $f(x) = e^{-(x/2)^2}$ .

**Parameters**  
**n** : int  
     quadrature order  
**mu** : bool, optional



**Returns**

<b>x</b> : ndarray	If True, return the sum of the weights, optional.
<b>w</b> : ndarray	Sample points
<b>mu</b> : float	Weights
	Sum of the weights

**See also:**

`scipy.integrate.quadrature`, `scipy.integrate.fixed_quad`,  
`numpy.polynomial.hermite_e.hermegauss`

**Notes**

For small  $n$  up to 150 a modified version of the Golub-Welsch algorithm is used. Nodes are computed from the eigenvalue problem and improved by one step of a Newton iteration. The weights are computed from the well-known analytical formula.

For  $n$  larger than 150 an optimal asymptotic algorithm is used which computes nodes and weights in a numerical stable manner. The algorithm has linear runtime making computation for very large  $n$  (several thousand or more) feasible.

`scipy.special.j_roots` ( $n$ ,  $alpha$ ,  $beta$ ,  $mu=False$ )  
 Gauss-Jacobi quadrature.

Computes the sample points and weights for Gauss-Jacobi quadrature. The sample points are the roots of the  $n$ -th degree Jacobi polynomial,  $P_n^{\alpha,\beta}(x)$ . These sample points and weights correctly integrate polynomials of degree  $2n - 1$  or less over the interval  $[-1, 1]$  with weight function  $f(x) = (1 - x)^\alpha(1 + x)^\beta$ .

**Parameters**

<b>n</b> : int	quadrature order
<b>alpha</b> : float	alpha must be > -1
<b>beta</b> : float	beta must be > 0
<b>mu</b> : bool, optional	

**Returns**

<b>x</b> : ndarray	If True, return the sum of the weights, optional.
<b>w</b> : ndarray	Sample points
<b>mu</b> : float	Weights
	Sum of the weights

**See also:**

`scipy.integrate.quadrature`, `scipy.integrate.fixed_quad`

`scipy.special.js_roots` ( $n$ ,  $p1$ ,  $q1$ ,  $mu=False$ )  
 Gauss-Jacobi (shifted) quadrature.

Computes the sample points and weights for Gauss-Jacobi (shifted) quadrature. The sample points are the roots of the  $n$ -th degree shifted Jacobi polynomial,  $G_n^{p,q}(x)$ . These sample points and weights correctly integrate polynomials of degree  $2n - 1$  or less over the interval  $[0, 1]$  with weight function  $f(x) = (1 - x)^{p-1}x^{q-1}$

**Parameters**

<b>n</b> : int	quadrature order
<b>p1</b> : float	(p1 - q1) must be > -1
<b>q1</b> : float	

**Returns**

- mu** : bool, optional  
q1 must be > 0
- x** : ndarray  
If True, return the sum of the weights, optional.
- w** : ndarray  
Sample points
- mu** : float  
Weights
- mu** : float  
Sum of the weights

**See also:**

`scipy.integrate.quadrature`, `scipy.integrate.fixed_quad`

`scipy.special.l_roots` (*n*, *mu=False*)

Gauss-Laguerre quadrature.

Computes the sample points and weights for Gauss-Laguerre quadrature. The sample points are the roots of the *n*-th degree Laguerre polynomial,  $L_n(x)$ . These sample points and weights correctly integrate polynomials of degree  $2n - 1$  or less over the interval  $[0, \infty]$  with weight function  $f(x) = e^{-x}$ .

**Parameters**

- n** : int  
quadrature order
- mu** : bool, optional  
If True, return the sum of the weights, optional.

**Returns**

- x** : ndarray  
Sample points
- w** : ndarray  
Weights
- mu** : float  
Sum of the weights

**See also:**

`scipy.integrate.quadrature`, `scipy.integrate.fixed_quad`,  
`numpy.polynomial.laguerre.laggauss`

`scipy.special.la_roots` (*n*, *alpha*, *mu=False*)

Gauss-generalized Laguerre quadrature.

Computes the sample points and weights for Gauss-generalized Laguerre quadrature. The sample points are the roots of the *n*-th degree generalized Laguerre polynomial,  $L_n^\alpha(x)$ . These sample points and weights correctly integrate polynomials of degree  $2n - 1$  or less over the interval  $[0, \infty]$  with weight function  $f(x) = x^\alpha e^{-x}$ .

**Parameters**

- n** : int  
quadrature order
- alpha** : float  
alpha must be > -1
- mu** : bool, optional  
If True, return the sum of the weights, optional.

**Returns**

- x** : ndarray  
Sample points
- w** : ndarray  
Weights
- mu** : float  
Sum of the weights

**See also:**

`scipy.integrate.quadrature`, `scipy.integrate.fixed_quad`

`scipy.special.p_roots(n, mu=False)`

Gauss-Legendre quadrature.

Computes the sample points and weights for Gauss-Legendre quadrature. The sample points are the roots of the  $n$ -th degree Legendre polynomial  $P_n(x)$ . These sample points and weights correctly integrate polynomials of degree  $2n - 1$  or less over the interval  $[-1, 1]$  with weight function  $f(x) = 1.0$ .

<b>Parameters</b>	<b>n</b> : int	quadrature order
	<b>mu</b> : bool, optional	If True, return the sum of the weights, optional.
<b>Returns</b>	<b>x</b> : ndarray	Sample points
	<b>w</b> : ndarray	Weights
	<b>mu</b> : float	Sum of the weights

**See also:**

`scipy.integrate.quadrature`, `scipy.integrate.fixed_quad`,  
`numpy.polynomial.legendre.leggauss`

`scipy.special.ps_roots(n, mu=False)`

Gauss-Legendre (shifted) quadrature.

Computes the sample points and weights for Gauss-Legendre quadrature. The sample points are the roots of the  $n$ -th degree shifted Legendre polynomial  $P_n^*(x)$ . These sample points and weights correctly integrate polynomials of degree  $2n - 1$  or less over the interval  $[0, 1]$  with weight function  $f(x) = 1.0$ .

<b>Parameters</b>	<b>n</b> : int	quadrature order
	<b>mu</b> : bool, optional	If True, return the sum of the weights, optional.
<b>Returns</b>	<b>x</b> : ndarray	Sample points
	<b>w</b> : ndarray	Weights
	<b>mu</b> : float	Sum of the weights

**See also:**

`scipy.integrate.quadrature`, `scipy.integrate.fixed_quad`

`scipy.special.s_roots(n, mu=False)`

Gauss-Chebyshev (second kind) quadrature.

Computes the sample points and weights for Gauss-Chebyshev quadrature. The sample points are the roots of the  $n$ -th degree Chebyshev polynomial of the second kind,  $S_n(x)$ . These sample points and weights correctly integrate polynomials of degree  $2n - 1$  or less over the interval  $[-2, 2]$  with weight function  $f(x) = \sqrt{1 - (x/2)^2}$ .

<b>Parameters</b>	<b>n</b> : int	quadrature order
	<b>mu</b> : bool, optional	If True, return the sum of the weights, optional.
<b>Returns</b>	<b>x</b> : ndarray	Sample points
	<b>w</b> : ndarray	Weights
	<b>mu</b> : float	

Sum of the weights

**See also:**

`scipy.integrate.quadrature`, `scipy.integrate.fixed_quad`

`scipy.special.t_roots` (*n*, *mu=False*)  
 Gauss-Chebyshev (first kind) quadrature.

Computes the sample points and weights for Gauss-Chebyshev quadrature. The sample points are the roots of the *n*-th degree Chebyshev polynomial of the first kind,  $T_n(x)$ . These sample points and weights correctly integrate polynomials of degree  $2n - 1$  or less over the interval  $[-1, 1]$  with weight function  $f(x) = 1/\sqrt{1 - x^2}$ .

<b>Parameters</b>	<b>n</b> : int	quadrature order
	<b>mu</b> : bool, optional	If True, return the sum of the weights, optional.
<b>Returns</b>	<b>x</b> : ndarray	Sample points
	<b>w</b> : ndarray	Weights
	<b>mu</b> : float	Sum of the weights

**See also:**

`scipy.integrate.quadrature`, `scipy.integrate.fixed_quad`,  
`numpy.polynomial.chebyshev.chebgauss`

`scipy.special.ts_roots` (*n*, *mu=False*)  
 Gauss-Chebyshev (first kind, shifted) quadrature.

Computes the sample points and weights for Gauss-Chebyshev quadrature. The sample points are the roots of the *n*-th degree shifted Chebyshev polynomial of the first kind,  $T_n(x)$ . These sample points and weights correctly integrate polynomials of degree  $2n - 1$  or less over the interval  $[0, 1]$  with weight function  $f(x) = 1/\sqrt{x - x^2}$ .

<b>Parameters</b>	<b>n</b> : int	quadrature order
	<b>mu</b> : bool, optional	If True, return the sum of the weights, optional.
<b>Returns</b>	<b>x</b> : ndarray	Sample points
	<b>w</b> : ndarray	Weights
	<b>mu</b> : float	Sum of the weights

**See also:**

`scipy.integrate.quadrature`, `scipy.integrate.fixed_quad`

`scipy.special.u_roots` (*n*, *mu=False*)  
 Gauss-Chebyshev (second kind) quadrature.

Computes the sample points and weights for Gauss-Chebyshev quadrature. The sample points are the roots of the *n*-th degree Chebyshev polynomial of the second kind,  $U_n(x)$ . These sample points and weights correctly integrate polynomials of degree  $2n - 1$  or less over the interval  $[-1, 1]$  with weight function  $f(x) = \sqrt{1 - x^2}$ .

<b>Parameters</b>	<b>n</b> : int	quadrature order
	<b>mu</b> : bool, optional	If True, return the sum of the weights, optional.
<b>Returns</b>	<b>x</b> : ndarray	

**w** : ndarray      Sample points  
                     Weights  
**mu** : float        Sum of the weights

**See also:**

`scipy.integrate.quadrature`, `scipy.integrate.fixed_quad`

`scipy.special.us_roots` (*n*, *mu=False*)

Gauss-Chebyshev (second kind, shifted) quadrature.

Computes the sample points and weights for Gauss-Chebyshev quadrature. The sample points are the roots of the *n*-th degree shifted Chebyshev polynomial of the second kind,  $U_n(x)$ . These sample points and weights correctly integrate polynomials of degree  $2n - 1$  or less over the interval  $[0, 1]$  with weight function  $f(x) = \sqrt{x - x^2}$ .

**Parameters**    **n** : int                      quadrature order  
                     **mu** : bool, optional  
**Returns**        **x** : ndarray                      If True, return the sum of the weights, optional.  
                     Sample points  
                     **w** : ndarray                      Weights  
                     **mu** : float                      Sum of the weights

**See also:**

`scipy.integrate.quadrature`, `scipy.integrate.fixed_quad`

**Hypergeometric Functions**


---

<code>hyp2f1(a, b, c, z)</code>	Gauss hypergeometric function ${}_2F_1(a, b; c; z)$ .
<code>hyp1f1(a, b, x)</code>	Confluent hypergeometric function ${}_1F_1(a, b; x)$
<code>hyperu(a, b, x)</code>	Confluent hypergeometric function $U(a, b, x)$ of the second kind
<code>hyp0f1(v, x)</code>	Confluent hypergeometric limit function ${}_0F_1$ .
<code>hyp2f0(a, b, x, type)</code>	Hypergeometric function ${}_2F_0$ in <i>y</i> and an error estimate
<code>hyp1f2(a, b, c, x)</code>	Hypergeometric function ${}_1F_2$ and error estimate
<code>hyp3f0(a, b, c, x)</code>	Hypergeometric function ${}_3F_0$ in <i>y</i> and an error estimate

---

`scipy.special.hyp2f1` (*a*, *b*, *c*, *z*) = <ufunc 'hyp2f1'>

Gauss hypergeometric function  ${}_2F_1(a, b; c; z)$ .

`scipy.special.hyp1f1` (*a*, *b*, *x*) = <ufunc 'hyp1f1'>

Confluent hypergeometric function  ${}_1F_1(a, b; x)$

`scipy.special.hyperu` (*a*, *b*, *x*) = <ufunc 'hyperu'>

Confluent hypergeometric function  $U(a, b, x)$  of the second kind

`scipy.special.hyp0f1` (*v*, *x*) = <ufunc 'hyp0f1'>

Confluent hypergeometric limit function  ${}_0F_1$ .

**Parameters**    **v**, **z** : array\_like

**Returns**        **hyp0f1** : ndarray                      Input values.  
                     The confluent hypergeometric limit function.

*Notes*

This function is defined as:

$${}_0F_1(v, z) = \sum_{k=0}^{\infty} \frac{z^k}{(v)_k k!}.$$

It's also the limit as  $q \rightarrow \infty$  of  ${}_1F_1(q; v; z/q)$ , and satisfies the differential equation  $f''(z) + v f'(z) = f(z)$ .

`scipy.special.hyp2f0(a, b, x, type) = <ufunc 'hyp2f0'>`  
 Hypergeometric function 2F0 in y and an error estimate

The parameter *type* determines a convergence factor and can be either 1 or 2.

**Returns**     y                                     Value of the function  
                   err                                 Error estimate

`scipy.special.hyp1f2(a, b, c, x) = <ufunc 'hyp1f2'>`  
 Hypergeometric function 1F2 and error estimate

**Returns**     y                                     Value of the function  
                   err                                 Error estimate

`scipy.special.hyp3f0(a, b, c, x) = <ufunc 'hyp3f0'>`  
 Hypergeometric function 3F0 in y and an error estimate

**Returns**     y                                     Value of the function  
                   err                                 Error estimate

**Parabolic Cylinder Functions**

---

<code>pbdv(v, x)</code>	Parabolic cylinder function D
<code>pbvv(v, x)</code>	Parabolic cylinder function V
<code>pbwa(a, x)</code>	Parabolic cylinder function W

---

`scipy.special.pbdv(v, x) = <ufunc 'pbdv'>`  
 Parabolic cylinder function D

Returns (d, dp) the parabolic cylinder function Dv(x) in d and the derivative, Dv'(x) in dp.

**Returns**     d                                     Value of the function  
                   dp                                 Value of the derivative vs x

`scipy.special.pbvv(v, x) = <ufunc 'pbvv'>`  
 Parabolic cylinder function V

Returns the parabolic cylinder function Vv(x) in v and the derivative, Vv'(x) in vp.

**Returns**     v                                     Value of the function

`vp`  
Value of the derivative vs  $x$

`scipy.special.pbwa(a, x) = <ufunc 'pbwa'>`

Parabolic cylinder function  $W$

Returns the parabolic cylinder function  $W(a, x)$  in  $w$  and the derivative,  $W'(a, x)$  in  $w_p$ .

**Warning:** May not be accurate for large ( $>5$ ) arguments in  $a$  and/or  $x$ .

**Returns** `w`  
Value of the function

`w_p`  
Value of the derivative vs  $x$

These are not universal functions:

<code>pbdv_seq(v, x)</code>	Parabolic cylinder functions $Dv(x)$ and derivatives.
<code>pbvv_seq(v, x)</code>	Parabolic cylinder functions $Vv(x)$ and derivatives.
<code>pbdn_seq(n, z)</code>	Parabolic cylinder functions $Dn(z)$ and derivatives.

`scipy.special.pbdv_seq(v, x)`

Parabolic cylinder functions  $Dv(x)$  and derivatives.

**Parameters** `v` : float  
Order of the parabolic cylinder function

`x` : float  
Value at which to evaluate the function and derivatives

**Returns** `dv` : ndarray  
Values of  $D_{vi}(x)$ , for  $vi=v-int(v)$ ,  $vi=1+v-int(v)$ , ...,  $vi=v$ .

`dp` : ndarray  
Derivatives  $D_{vi}'(x)$ , for  $vi=v-int(v)$ ,  $vi=1+v-int(v)$ , ...,  $vi=v$ .

#### References

[R444]

`scipy.special.pbvv_seq(v, x)`

Parabolic cylinder functions  $Vv(x)$  and derivatives.

**Parameters** `v` : float  
Order of the parabolic cylinder function

`x` : float  
Value at which to evaluate the function and derivatives

**Returns** `dv` : ndarray  
Values of  $V_{vi}(x)$ , for  $vi=v-int(v)$ ,  $vi=1+v-int(v)$ , ...,  $vi=v$ .

`dp` : ndarray  
Derivatives  $V_{vi}'(x)$ , for  $vi=v-int(v)$ ,  $vi=1+v-int(v)$ , ...,  $vi=v$ .

#### References

[R445]

`scipy.special.pbdn_seq(n, z)`

Parabolic cylinder functions  $Dn(z)$  and derivatives.

**Parameters** `n` : int  
Order of the parabolic cylinder function

`z` : complex  
Value at which to evaluate the function and derivatives

**Returns**     **dv** : ndarray     Values of  $D_i(z)$ , for  $i=0, \dots, i=n$ .  
                   **dp** : ndarray     Derivatives  $D_i'(z)$ , for  $i=0, \dots, i=n$ .

**References**

[R443]

**Mathieu and Related Functions**

---

<code>mathieu_a(m, q)</code>	Characteristic value of even Mathieu functions
<code>mathieu_b(m, q)</code>	Characteristic value of odd Mathieu functions

---

`scipy.special.mathieu_a(m, q) = <ufunc 'mathieu_a'>`  
 Characteristic value of even Mathieu functions

Returns the characteristic value for the even solution,  $ce_m(z, q)$ , of Mathieu's equation.

`scipy.special.mathieu_b(m, q) = <ufunc 'mathieu_b'>`  
 Characteristic value of odd Mathieu functions

Returns the characteristic value for the odd solution,  $se_m(z, q)$ , of Mathieu's equation.

These are not universal functions:

---

<code>mathieu_even_coef(m, q)</code>	Fourier coefficients for even Mathieu and modified Mathieu functions.
<code>mathieu_odd_coef(m, q)</code>	Fourier coefficients for even Mathieu and modified Mathieu functions.

---

`scipy.special.mathieu_even_coef(m, q)`  
 Fourier coefficients for even Mathieu and modified Mathieu functions.

The Fourier series of the even solutions of the Mathieu differential equation are of the form

$$ce_{2n}(z, q) = \sum_{k=0}^{\infty} A_{(2n)}^{(2k)} \cos 2kz$$

$$ce_{2n+1}(z, q) = \sum_{k=0}^{\infty} A_{(2n+1)}^{(2k+1)} \cos(2k+1)z$$

This function returns the coefficients  $A_{(2n)}^{(2k)}$  for even input  $m=2n$ , and the coefficients  $A_{(2n+1)}^{(2k+1)}$  for odd input  $m=2n+1$ .

**Parameters**    **m** : int     Order of Mathieu functions. Must be non-negative.  
                     **q** : float (>=0)     Parameter of Mathieu functions. Must be non-negative.  
**Returns**        **Ak** : ndarray     Even or odd Fourier coefficients, corresponding to even or odd m.

**References**

[R429], [R430]

`scipy.special.mathieu_odd_coef(m, q)`  
 Fourier coefficients for even Mathieu and modified Mathieu functions.



The Fourier series of the odd solutions of the Mathieu differential equation are of the form

$$se_{2n+1}(z, q) = \sum_{k=0}^{\infty} B_{(2n+1)}^{(2k+1)} \sin(2k+1)z$$

$$se_{2n+2}(z, q) = \sum_{k=0}^{\infty} B_{(2n+2)}^{(2k+2)} \sin(2k+2)z$$

This function returns the coefficients  $B_{(2n+2)}^{(2k+2)}$  for even input  $m=2n+2$ , and the coefficients  $B_{(2n+1)}^{(2k+1)}$  for odd input  $m=2n+1$ .

**Parameters**

- m** : int  
Order of Mathieu functions. Must be non-negative.
- q** : float (>=0)  
Parameter of Mathieu functions. Must be non-negative.

**Returns**

- Bk** : ndarray  
Even or odd Fourier coefficients, corresponding to even or odd m.

### References

[R431]

The following return both function and first derivative:

---

<code>mathieu_cem(m, q, x)</code>	Even Mathieu function and its derivative
<code>mathieu_sem(m, q, x)</code>	Odd Mathieu function and its derivative
<code>mathieu_modcem1(m, q, x)</code>	Even modified Mathieu function of the first kind and its derivative
<code>mathieu_modcem2(m, q, x)</code>	Even modified Mathieu function of the second kind and its derivative
<code>mathieu_modsem1(m, q, x)</code>	Odd modified Mathieu function of the first kind and its derivative
<code>mathieu_modsem2(m, q, x)</code>	Odd modified Mathieu function of the second kind and its derivative

---

`scipy.special.mathieu_cem(m, q, x) = <ufunc 'mathieu_cem'>`

Even Mathieu function and its derivative

Returns the even Mathieu function,  $ce_m(x, q)$ , of order  $m$  and parameter  $q$  evaluated at  $x$  (given in degrees). Also returns the derivative with respect to  $x$  of  $ce_m(x, q)$ .

**Parameters**

- m**  
Order of the function
- q**  
Parameter of the function
- x**  
Argument of the function, *given in degrees, not radians*

**Returns**

- y**  
Value of the function
- yp**  
Value of the derivative vs  $x$

`scipy.special.mathieu_sem(m, q, x) = <ufunc 'mathieu_sem'>`

Odd Mathieu function and its derivative

Returns the odd Mathieu function,  $se_m(x, q)$ , of order  $m$  and parameter  $q$  evaluated at  $x$  (given in degrees). Also returns the derivative with respect to  $x$  of  $se_m(x, q)$ .

**Parameters**

- m**  
Order of the function
- q**

	<code>x</code>	Parameter of the function
<b>Returns</b>	<code>y</code>	Argument of the function, <i>given in degrees, not radians.</i>
		Value of the function
	<code>YP</code>	Value of the derivative vs <code>x</code>

`scipy.special.mathieu_modcem1(m, q, x) = <ufunc 'mathieu_modcem1'>`

Even modified Mathieu function of the first kind and its derivative

Evaluates the even modified Mathieu function of the first kind,  $Mc1m(x, q)$ , and its derivative at  $x$  for order  $m$  and parameter  $q$ .

<b>Returns</b>	<code>y</code>	Value of the function
	<code>YP</code>	Value of the derivative vs <code>x</code>

`scipy.special.mathieu_modcem2(m, q, x) = <ufunc 'mathieu_modcem2'>`

Even modified Mathieu function of the second kind and its derivative

Evaluates the even modified Mathieu function of the second kind,  $Mc2m(x, q)$ , and its derivative at  $x$  (given in degrees) for order  $m$  and parameter  $q$ .

<b>Returns</b>	<code>y</code>	Value of the function
	<code>YP</code>	Value of the derivative vs <code>x</code>

`scipy.special.mathieu_modsem1(m, q, x) = <ufunc 'mathieu_modsem1'>`

Odd modified Mathieu function of the first kind and its derivative

Evaluates the odd modified Mathieu function of the first kind,  $Ms1m(x, q)$ , and its derivative at  $x$  (given in degrees) for order  $m$  and parameter  $q$ .

<b>Returns</b>	<code>y</code>	Value of the function
	<code>YP</code>	Value of the derivative vs <code>x</code>

`scipy.special.mathieu_modsem2(m, q, x) = <ufunc 'mathieu_modsem2'>`

Odd modified Mathieu function of the second kind and its derivative

Evaluates the odd modified Mathieu function of the second kind,  $Ms2m(x, q)$ , and its derivative at  $x$  (given in degrees) for order  $m$  and parameter  $q$ .

<b>Returns</b>	<code>y</code>	Value of the function
	<code>YP</code>	Value of the derivative vs <code>x</code>

## Spheroidal Wave Functions

<code>pro_ang1(m, n, c, x)</code>	Prolate spheroidal angular function of the first kind and its derivative
<code>pro_rad1(m, n, c, x)</code>	Prolate spheroidal radial function of the first kind and its derivative
<code>pro_rad2(m, n, c, x)</code>	Prolate spheroidal radial function of the second kind and its derivative
<code>obl_ang1(m, n, c, x)</code>	Oblate spheroidal angular function of the first kind and its derivative

Continued on next page

Table 5.264 – continued from previous page

<code>obl_rad1(m, n, c, x)</code>	Oblate spheroidal radial function of the first kind and its derivative
<code>obl_rad2(m, n, c, x)</code>	Oblate spheroidal radial function of the second kind and its derivative.
<code>pro_cv(m, n, c)</code>	Characteristic value of prolate spheroidal function
<code>obl_cv(m, n, c)</code>	Characteristic value of oblate spheroidal function
<code>pro_cv_seq(m, n, c)</code>	Characteristic values for prolate spheroidal wave functions.
<code>obl_cv_seq(m, n, c)</code>	Characteristic values for oblate spheroidal wave functions.

`scipy.special.pro_ang1(m, n, c, x) = <ufunc 'pro_ang1'>`

Prolate spheroidal angular function of the first kind and its derivative

Computes the prolate spheroidal angular function of the first kind and its derivative (with respect to  $x$ ) for mode parameters  $m \geq 0$  and  $n \geq m$ , spheroidal parameter  $c$  and  $|x| < 1.0$ .

**Returns**

s	Value of the function
sp	Value of the derivative vs x

`scipy.special.pro_rad1(m, n, c, x) = <ufunc 'pro_rad1'>`

Prolate spheroidal radial function of the first kind and its derivative

Computes the prolate spheroidal radial function of the first kind and its derivative (with respect to  $x$ ) for mode parameters  $m \geq 0$  and  $n \geq m$ , spheroidal parameter  $c$  and  $|x| < 1.0$ .

**Returns**

s	Value of the function
sp	Value of the derivative vs x

`scipy.special.pro_rad2(m, n, c, x) = <ufunc 'pro_rad2'>`

Prolate spheroidal radial function of the second kind and its derivative

Computes the prolate spheroidal radial function of the second kind and its derivative (with respect to  $x$ ) for mode parameters  $m \geq 0$  and  $n \geq m$ , spheroidal parameter  $c$  and  $|x| < 1.0$ .

**Returns**

s	Value of the function
sp	Value of the derivative vs x

`scipy.special.obl_ang1(m, n, c, x) = <ufunc 'obl_ang1'>`

Oblate spheroidal angular function of the first kind and its derivative

Computes the oblate spheroidal angular function of the first kind and its derivative (with respect to  $x$ ) for mode parameters  $m \geq 0$  and  $n \geq m$ , spheroidal parameter  $c$  and  $|x| < 1.0$ .

**Returns**

s	Value of the function
sp	Value of the derivative vs x

`scipy.special.obl_rad1(m, n, c, x) = <ufunc 'obl_rad1'>`

Oblate spheroidal radial function of the first kind and its derivative

Computes the oblate spheroidal radial function of the first kind and its derivative (with respect to  $x$ ) for mode parameters  $m \geq 0$  and  $n \geq m$ , spheroidal parameter  $c$  and  $|x| < 1.0$ .

**Returns**

s	Value of the function
---	-----------------------

sp Value of the derivative vs x

`scipy.special.obl_rad2(m, n, c, x) = <ufunc 'obl_rad2'>`  
 Oblate spheroidal radial function of the second kind and its derivative.

Computes the oblate spheroidal radial function of the second kind and its derivative (with respect to  $x$ ) for mode parameters  $m \geq 0$  and  $n \geq m$ , spheroidal parameter  $c$  and  $|x| < 1.0$ .

**Returns** s Value of the function  
 sp Value of the derivative vs x

`scipy.special.pro_cv(m, n, c) = <ufunc 'pro_cv'>`  
 Characteristic value of prolate spheroidal function

Computes the characteristic value of prolate spheroidal wave functions of order  $m, n$  ( $n \geq m$ ) and spheroidal parameter  $c$ .

`scipy.special.obl_cv(m, n, c) = <ufunc 'obl_cv'>`  
 Characteristic value of oblate spheroidal function

Computes the characteristic value of oblate spheroidal wave functions of order  $m, n$  ( $n \geq m$ ) and spheroidal parameter  $c$ .

`scipy.special.pro_cv_seq(m, n, c)`  
 Characteristic values for prolate spheroidal wave functions.

Compute a sequence of characteristic values for the prolate spheroidal wave functions for mode  $m$  and  $n' = m..n$  and spheroidal parameter  $c$ .

**References**

[R446]

`scipy.special.obl_cv_seq(m, n, c)`  
 Characteristic values for oblate spheroidal wave functions.

Compute a sequence of characteristic values for the oblate spheroidal wave functions for mode  $m$  and  $n' = m..n$  and spheroidal parameter  $c$ .

**References**

[R442]

The following functions require pre-computed characteristic value:

---

<code>pro_ang1_cv(m, n, c, cv, x)</code>	Prolate spheroidal angular function <code>pro_ang1</code> for precomputed characteristic value
<code>pro_rad1_cv(m, n, c, cv, x)</code>	Prolate spheroidal radial function <code>pro_rad1</code> for precomputed characteristic value
<code>pro_rad2_cv(m, n, c, cv, x)</code>	Prolate spheroidal radial function <code>pro_rad2</code> for precomputed characteristic value
<code>obl_ang1_cv(m, n, c, cv, x)</code>	Oblate spheroidal angular function <code>obl_ang1</code> for precomputed characteristic value
<code>obl_rad1_cv(m, n, c, cv, x)</code>	Oblate spheroidal radial function <code>obl_rad1</code> for precomputed characteristic value
<code>obl_rad2_cv(m, n, c, cv, x)</code>	Oblate spheroidal radial function <code>obl_rad2</code> for precomputed characteristic value

---

`scipy.special.pro_ang1_cv(m, n, c, cv, x) = <ufunc 'pro_ang1_cv'>`  
 Prolate spheroidal angular function `pro_ang1` for precomputed characteristic value

Computes the prolate spheroidal angular function of the first kind and its derivative (with respect to  $x$ ) for mode parameters  $m \geq 0$  and  $n \geq m$ , spheroidal parameter  $c$  and  $|x| < 1.0$ . Requires pre-computed characteristic value.

**Returns**

s	Value of the function
sp	Value of the derivative vs x

`scipy.special.pro_rad1_cv(m, n, c, cv, x) = <ufunc 'pro_rad1_cv'>`  
 Prolate spheroidal radial function pro\_rad1 for precomputed characteristic value

Computes the prolate spheroidal radial function of the first kind and its derivative (with respect to  $x$ ) for mode parameters  $m \geq 0$  and  $n \geq m$ , spheroidal parameter  $c$  and  $|x| < 1.0$ . Requires pre-computed characteristic value.

**Returns**

s	Value of the function
sp	Value of the derivative vs x

`scipy.special.pro_rad2_cv(m, n, c, cv, x) = <ufunc 'pro_rad2_cv'>`  
 Prolate spheroidal radial function pro\_rad2 for precomputed characteristic value

Computes the prolate spheroidal radial function of the second kind and its derivative (with respect to  $x$ ) for mode parameters  $m \geq 0$  and  $n \geq m$ , spheroidal parameter  $c$  and  $|x| < 1.0$ . Requires pre-computed characteristic value.

**Returns**

s	Value of the function
sp	Value of the derivative vs x

`scipy.special.obl_ang1_cv(m, n, c, cv, x) = <ufunc 'obl_ang1_cv'>`  
 Oblate spheroidal angular function obl\_ang1 for precomputed characteristic value

Computes the oblate spheroidal angular function of the first kind and its derivative (with respect to  $x$ ) for mode parameters  $m \geq 0$  and  $n \geq m$ , spheroidal parameter  $c$  and  $|x| < 1.0$ . Requires pre-computed characteristic value.

**Returns**

s	Value of the function
sp	Value of the derivative vs x

`scipy.special.obl_rad1_cv(m, n, c, cv, x) = <ufunc 'obl_rad1_cv'>`  
 Oblate spheroidal radial function obl\_rad1 for precomputed characteristic value

Computes the oblate spheroidal radial function of the first kind and its derivative (with respect to  $x$ ) for mode parameters  $m \geq 0$  and  $n \geq m$ , spheroidal parameter  $c$  and  $|x| < 1.0$ . Requires pre-computed characteristic value.

**Returns**

s	Value of the function
sp	Value of the derivative vs x

`scipy.special.obl_rad2_cv(m, n, c, cv, x) = <ufunc 'obl_rad2_cv'>`  
 Oblate spheroidal radial function obl\_rad2 for precomputed characteristic value

Computes the oblate spheroidal radial function of the second kind and its derivative (with respect to  $x$ ) for mode parameters  $m \geq 0$  and  $n \geq m$ , spheroidal parameter  $c$  and  $|x| < 1.0$ . Requires pre-computed characteristic value.

**Returns**

s	
---	--

Value of the function  
 sp  
 Value of the derivative vs x

## Kelvin Functions

<code>kelvin(x)</code>	Kelvin functions as complex numbers
<code>kelvin_zeros(nt)</code>	Compute nt zeros of all Kelvin functions.
<code>ber(x)</code>	Kelvin function ber.
<code>bei(x)</code>	Kelvin function bei
<code>berp(x)</code>	Derivative of the Kelvin function ber
<code>beip(x)</code>	Derivative of the Kelvin function bei
<code>ker(x)</code>	Kelvin function ker
<code>kei(x)</code>	Kelvin function ker
<code>kerp(x)</code>	Derivative of the Kelvin function ker
<code>keip(x)</code>	Derivative of the Kelvin function kei

`scipy.special.kelvin(x) = <ufunc 'kelvin'>`  
 Kelvin functions as complex numbers

**Returns** Be, Ke, Bep, Kep

The tuple (Be, Ke, Bep, Kep) contains complex numbers representing the real and imaginary Kelvin functions and their derivatives evaluated at  $x$ . For example, `kelvin(x)[0].real = ber x` and `kelvin(x)[0].imag = bei x` with similar relationships for ker and kei.

`scipy.special.kelvin_zeros(nt)`  
 Compute nt zeros of all Kelvin functions.

Returned in a length-8 tuple of arrays of length nt. The tuple contains the arrays of zeros of (ber, bei, ker, kei, ber', bei', ker', kei').

### References

[R408]

`scipy.special.ber(x) = <ufunc 'ber'>`  
 Kelvin function ber.

`scipy.special.bei(x) = <ufunc 'bei'>`  
 Kelvin function bei

`scipy.special.berp(x) = <ufunc 'berp'>`  
 Derivative of the Kelvin function ber

`scipy.special.beip(x) = <ufunc 'beip'>`  
 Derivative of the Kelvin function bei

`scipy.special.ker(x) = <ufunc 'ker'>`  
 Kelvin function ker

`scipy.special.kei(x) = <ufunc 'kei'>`  
 Kelvin function ker

`scipy.special.kerp(x) = <ufunc 'kerp'>`  
 Derivative of the Kelvin function ker

`scipy.special.keip(x) = <ufunc 'keip'>`  
 Derivative of the Kelvin function `kei`

These are not universal functions:

<code>ber_zeros(nt)</code>	Compute <code>nt</code> zeros of the Kelvin function <code>ber(x)</code> .
<code>bei_zeros(nt)</code>	Compute <code>nt</code> zeros of the Kelvin function <code>bei(x)</code> .
<code>berp_zeros(nt)</code>	Compute <code>nt</code> zeros of the Kelvin function <code>ber'(x)</code> .
<code>beip_zeros(nt)</code>	Compute <code>nt</code> zeros of the Kelvin function <code>bei'(x)</code> .
<code>ker_zeros(nt)</code>	Compute <code>nt</code> zeros of the Kelvin function <code>ker(x)</code> .
<code>kei_zeros(nt)</code>	Compute <code>nt</code> zeros of the Kelvin function <code>kei(x)</code> .
<code>kerp_zeros(nt)</code>	Compute <code>nt</code> zeros of the Kelvin function <code>ker'(x)</code> .
<code>keip_zeros(nt)</code>	Compute <code>nt</code> zeros of the Kelvin function <code>kei'(x)</code> .

`scipy.special.ber_zeros(nt)`  
 Compute `nt` zeros of the Kelvin function `ber(x)`.

**References**

[R324]

`scipy.special.bei_zeros(nt)`  
 Compute `nt` zeros of the Kelvin function `bei(x)`.

**References**

[R322]

`scipy.special.berp_zeros(nt)`  
 Compute `nt` zeros of the Kelvin function `ber'(x)`.

**References**

[R326]

`scipy.special.beip_zeros(nt)`  
 Compute `nt` zeros of the Kelvin function `bei'(x)`.

**References**

[R323]

`scipy.special.ker_zeros(nt)`  
 Compute `nt` zeros of the Kelvin function `ker(x)`.

**References**

[R409]

`scipy.special.kei_zeros(nt)`  
 Compute `nt` zeros of the Kelvin function `kei(x)`.

`scipy.special.kerp_zeros(nt)`  
 Compute `nt` zeros of the Kelvin function `ker'(x)`.

**References**

[R410]

`scipy.special.keip_zeros(nt)`  
 Compute `nt` zeros of the Kelvin function `kei'(x)`.

*References*

[R407]

**Combinatorics**

<code>comb(N, k[, exact, repetition])</code>	The number of combinations of N things taken k at a time.
<code>perm(N, k[, exact])</code>	Permutations of N things taken k at a time, i.e., k-permutations of N.

`scipy.special.comb(N, k, exact=False, repetition=False)`  
 The number of combinations of N things taken k at a time.

This is often expressed as “N choose k”.

<b>Parameters</b>	<b>N</b> : int, ndarray Number of things.
	<b>k</b> : int, ndarray Number of elements taken.
	<b>exact</b> : bool, optional If <i>exact</i> is False, then floating point precision is used, otherwise exact long integer is computed.
	<b>repetition</b> : bool, optional If <i>repetition</i> is True, then the number of combinations with repetition is computed.
<b>Returns</b>	<b>val</b> : int, ndarray The total number of combinations.

*Notes*

- Array arguments accepted only for exact=False case.
- If  $k > N$ ,  $N < 0$ , or  $k < 0$ , then a 0 is returned.

*Examples*

```
>>> from scipy.special import comb
>>> k = np.array([3, 4])
>>> n = np.array([10, 10])
>>> comb(n, k, exact=False)
array([ 120.,  210.])
>>> comb(10, 3, exact=True)
120L
>>> comb(10, 3, exact=True, repetition=True)
220L
```

`scipy.special.perm(N, k, exact=False)`  
 Permutations of N things taken k at a time, i.e., k-permutations of N.

It’s also known as “partial permutations”.

<b>Parameters</b>	<b>N</b> : int, ndarray Number of things.
	<b>k</b> : int, ndarray Number of elements taken.
	<b>exact</b> : bool, optional If <i>exact</i> is False, then floating point precision is used, otherwise exact long integer is computed.
<b>Returns</b>	<b>val</b> : int, ndarray



The number of k-permutations of N.

### Notes

- Array arguments accepted only for exact=False case.
- If  $k > N$ ,  $N < 0$ , or  $k < 0$ , then a 0 is returned.

### Examples

```
>>> from scipy.special import perm
>>> k = np.array([3, 4])
>>> n = np.array([10, 10])
>>> perm(n, k)
array([ 720., 5040.])
>>> perm(10, 3, exact=True)
720
```

## Other Special Functions

<code>agm(a, b)</code>	Arithmetic, Geometric Mean.
<code>bernoulli(n)</code>	Bernoulli numbers $B_0..B_n$ (inclusive).
<code>binom(n, k)</code>	Binomial coefficient
<code>diric(x, n)</code>	Periodic sinc function, also called the Dirichlet function.
<code>euler(n)</code>	Euler numbers $E_0..E_n$ (inclusive).
<code>expn(n, x)</code>	Exponential integral $E_n$
<code>expl(z)</code>	Exponential integral $E_1$ of complex argument $z$
<code>expi(x)</code>	Exponential integral $E_i$
<code>factorial(n[, exact])</code>	The factorial of a number or array of numbers.
<code>factorial2(n[, exact])</code>	Double factorial.
<code>factorialk(n, k[, exact])</code>	Multifactorial of $n$ of order $k$ , $n(!...!).$
<code>shichi(x)</code>	Hyperbolic sine and cosine integrals
<code>sici(x)</code>	Sine and cosine integrals
<code>spence(z)</code>	Spence's function, also known as the dilogarithm.
<code>lambertw(z[, k, tol])</code>	Lambert W function [R994].
<code>zeta(x[, q, out])</code>	Riemann zeta function.
<code>zetac(x)</code>	Riemann zeta function minus 1.

`scipy.special.agm(a, b)`

Arithmetic, Geometric Mean.

Start with  $a_0=a$  and  $b_0=b$  and iteratively compute

$$a_{n+1} = (a_n + b_n) / 2 \quad b_{n+1} = \sqrt{a_n * b_n}$$

until  $a_n = b_n$ . The result is `agm(a, b)`

$$\text{agm}(a, b) = \text{agm}(b, a) \quad \text{agm}(a, a) = a \quad \min(a, b) < \text{agm}(a, b) < \max(a, b)$$

`scipy.special.bernoulli(n)`

Bernoulli numbers  $B_0..B_n$  (inclusive).

### References

[R325]

`scipy.special.binom(n, k) = <ufunc 'binom'>`

Binomial coefficient

`scipy.special.diric(x, n)`

Periodic sinc function, also called the Dirichlet function.

The Dirichlet function is defined as:

$$\text{diric}(x) = \sin(x * n/2) / (n * \sin(x / 2)),$$

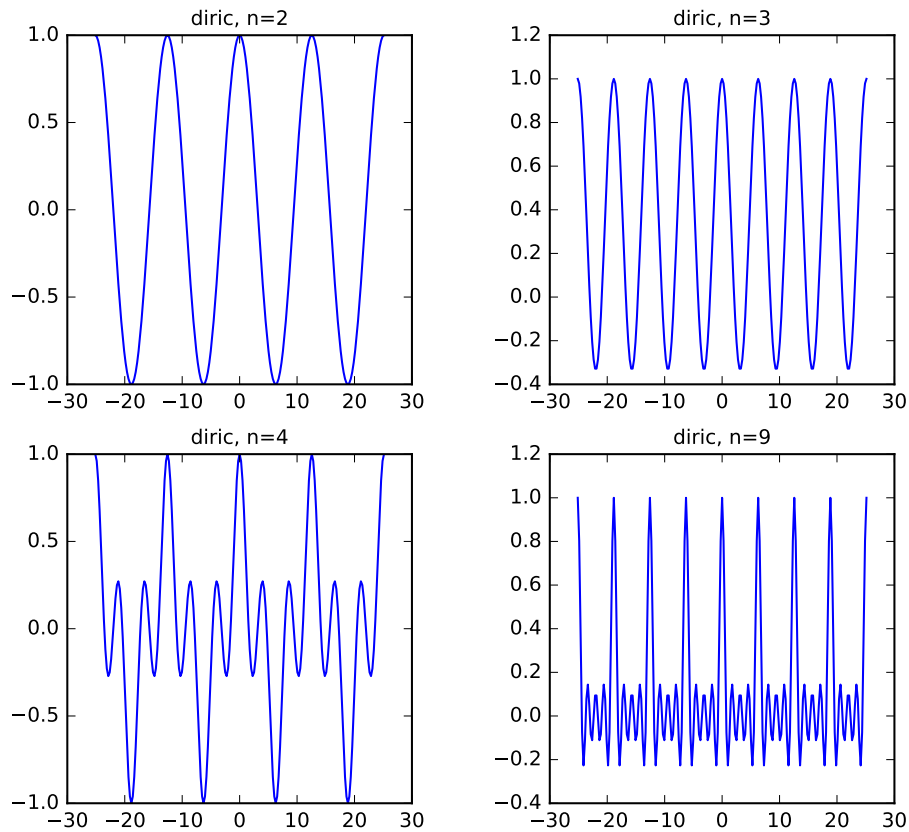
where  $n$  is a positive integer.

<b>Parameters</b>	<b>x</b> : array_like	Input data
	<b>n</b> : int	Integer defining the periodicity.
<b>Returns</b>	<b>diric</b> : ndarray	

### Examples

```
>>> from scipy import special
>>> import matplotlib.pyplot as plt

>>> x = np.linspace(-8*np.pi, 8*np.pi, num=201)
>>> plt.figure(figsize=(8, 8));
>>> for idx, n in enumerate([2, 3, 4, 9]):
...     plt.subplot(2, 2, idx+1)
...     plt.plot(x, special.diric(x, n))
...     plt.title('diric, n={}'.format(n))
>>> plt.show()
```



The following example demonstrates that `diric` gives the magnitudes (modulo the sign and scaling) of the Fourier coefficients of a rectangular pulse.

Suppress output of values that are effectively 0:

```
>>> np.set_printoptions(suppress=True)
```

Create a signal  $x$  of length  $m$  with  $k$  ones:

```
>>> m = 8
>>> k = 3
>>> x = np.zeros(m)
>>> x[:k] = 1
```

Use the FFT to compute the Fourier transform of  $x$ , and inspect the magnitudes of the coefficients:

```
>>> np.abs(np.fft.fft(x))
array([ 3.          ,  2.41421356,  1.          ,  0.41421356,  1.          ,
        0.41421356,  1.          ,  2.41421356])
```

Now find the same values (up to sign) using `diric`. We multiply by  $k$  to account for the different scaling conventions of `numpy.fft.fft` and `diric`:

```
>>> theta = np.linspace(0, 2*np.pi, m, endpoint=False)
>>> k * special.diric(theta, k)
array([ 3.          ,  2.41421356,  1.          , -0.41421356, -1.          ,
       -0.41421356,  1.          ,  2.41421356])
```

`scipy.special.euler` ( $n$ )  
Euler numbers  $E_0..E_n$  (inclusive).

### References

[R356]

`scipy.special.expn` ( $n, x$ ) = <ufunc 'expn'>  
Exponential integral  $E_n$

Returns the exponential integral for integer  $n$  and non-negative  $x$  and  $n$ :

```
integral(exp(-x*t) / t**n, t=1..inf).
```

`scipy.special.exp1` ( $z$ ) = <ufunc 'exp1'>  
Exponential integral  $E_1$  of complex argument  $z$

```
integral(exp(-z*t)/t, t=1..inf).
```

`scipy.special.expi` ( $x$ ) = <ufunc 'expi'>  
Exponential integral  $E_i$

Defined as:

```
integral(exp(t)/t, t=-inf..x)
```

See `expn` for a different exponential integral.

`scipy.special.factorial` ( $n$ , *exact*=False)  
The factorial of a number or array of numbers.

The factorial of non-negative integer  $n$  is the product of all positive integers less than or equal to  $n$ :

```
n! = n * (n - 1) * (n - 2) * ... * 1
```

<b>Parameters</b>	<p><b>n</b> : int or array_like of ints Input values. If <math>n &lt; 0</math>, the return value is 0.</p> <p><b>exact</b> : bool, optional If True, calculate the answer exactly using long integer arithmetic. If False, result is approximated in floating point rapidly using the <code>gamma</code> function.</p>
<b>Returns</b>	<p><b>nf</b> : float or int or ndarray Default is False. Factorial of <math>n</math>, as integer or float depending on <i>exact</i>.</p>

**Notes**

For arrays with `exact=True`, the factorial is computed only once, for the largest input, with each other result computed in the process. The output dtype is increased to `int64` or `object` if necessary.

With `exact=False` the factorial is approximated using the gamma function:

$$n! = \Gamma(n + 1)$$

**Examples**

```
>>> from scipy.special import factorial
>>> arr = np.array([3, 4, 5])
>>> factorial(arr, exact=False)
array([ 6., 24., 120.])
>>> factorial(arr, exact=True)
array([ 6, 24, 120])
>>> factorial(5, exact=True)
120L
```

`scipy.special.factorial2` (*n*, *exact=False*)

Double factorial.

This is the factorial with every second value skipped. E.g.,  $7!! = 7 * 5 * 3 * 1$ . It can be approximated numerically as:

```
n!! = special.gamma(n/2+1)*2**((m+1)/2)/sqrt(pi)  n odd
      = 2**(n/2) * (n/2)!                          n even
```

**Parameters** `n` : int or array\_like

Calculate  $n!!$ . Arrays are only supported with *exact* set to `False`. If  $n < 0$ , the return value is 0.

**exact** : bool, optional

The result can be approximated rapidly using the gamma-formula above (default). If *exact* is set to `True`, calculate the answer exactly using integer arithmetic.

**Returns** `nff` : float or int

Double factorial of *n*, as an int or a float depending on *exact*.

**Examples**

```
>>> from scipy.special import factorial2
>>> factorial2(7, exact=False)
array(105.00000000000001)
>>> factorial2(7, exact=True)
105L
```

`scipy.special.factorialk` (*n*, *k*, *exact=True*)

Multifactorial of *n* of order *k*,  $n(!\dots!)$ .

This is the multifactorial of *n* skipping *k* values. For example,

$$\text{factorialk}(17, 4) = 17!!!! = 17 * 13 * 9 * 5 * 1$$

In particular, for any integer *n*, we have

$$\begin{aligned} \text{factorialk}(n, 1) &= \text{factorial}(n) \\ \text{factorialk}(n, 2) &= \text{factorial2}(n) \end{aligned}$$

**Parameters**

- n** : int  
Calculate multifactorial. If  $n < 0$ , the return value is 0.
- k** : int  
Order of multifactorial.
- exact** : bool, optional  
If exact is set to True, calculate the answer exactly using integer arithmetic.

**Returns**

- val** : int  
Multifactorial of  $n$ .

**Raises**

- NotImplementedError**  
Raises when exact is False

**Examples**

```
>>> from scipy.special import factorialk
>>> factorialk(5, 1, exact=True)
120L
>>> factorialk(5, 3, exact=True)
10L
```

`scipy.special.shichi(x) = <ufunc 'shichi'>`  
Hyperbolic sine and cosine integrals

**Returns**

- shi**  
integral( $\sinh(t)/t$ ,  $t=0..x$ )
- chi**  
 $eul + \ln x + \text{integral}((\cosh(t)-1)/t, t=0..x)$  where  $eul$  is Euler's constant.

`scipy.special.sici(x) = <ufunc 'sici'>`  
Sine and cosine integrals

**Returns**

- si**  
integral( $\sin(t)/t$ ,  $t=0..x$ )
- ci**  
 $eul + \ln x + \text{integral}((\cos(t) - 1)/t, t=0..x)$  where  $eul$  is Euler's constant.

`scipy.special.spence(z) = <ufunc 'spence'>`  
Spence's function, also known as the dilogarithm. It is defined to be

$$\int_0^z \frac{\log(t)}{1-t} dt$$

for complex  $z$ , where the contour of integration is taken to avoid the branch cut of the logarithm. Spence's function is analytic everywhere except the negative real axis where it has a branch cut.

Note that there is a different convention which defines Spence's function by the integral

$$-\int_0^z \frac{\log(1-t)}{t} dt;$$

this is our `spence(1 - z)`.

`scipy.special.lambertw(z, k=0, tol=1e-8)`  
Lambert W function [R420].

The Lambert W function  $W(z)$  is defined as the inverse function of  $w * \exp(w)$ . In other words, the value of  $W(z)$  is such that  $z = W(z) * \exp(W(z))$  for any complex number  $z$ .

The Lambert W function is a multivalued function with infinitely many branches. Each branch gives a separate solution of the equation  $z = w \exp(w)$ . Here, the branches are indexed by the integer  $k$ .

**Parameters**

- z** : array\_like  
Input argument.
- k** : int, optional  
Branch index.
- tol** : float, optional  
Evaluation tolerance.

**Returns**

- w** : array  
 $w$  will have the same shape as  $z$ .

**Notes**

All branches are supported by `lambertw`:

- `lambertw(z)` gives the principal solution (branch 0)
- `lambertw(z, k)` gives the solution on branch  $k$

The Lambert W function has two partially real branches: the principal branch ( $k = 0$ ) is real for real  $z > -1/e$ , and the  $k = -1$  branch is real for  $-1/e < z < 0$ . All branches except  $k = 0$  have a logarithmic singularity at  $z = 0$ .

**Possible issues**

The evaluation can become inaccurate very close to the branch point at  $-1/e$ . In some corner cases, `lambertw` might currently fail to converge, or can end up on the wrong branch.

**Algorithm**

Halley's iteration is used to invert  $w * \exp(w)$ , using a first-order asymptotic approximation ( $O(\log(w))$  or  $O(w)$ ) as the initial estimate.

The definition, implementation and choice of branches is based on [R421].

**References**

[R420], [R421]

**Examples**

The Lambert W function is the inverse of  $w * \exp(w)$ :

```
>>> from scipy.special import lambertw
>>> w = lambertw(1)
>>> w
(0.56714329040978384+0j)
>>> w * np.exp(w)
(1.0+0j)
```

Any branch gives a valid inverse:

```
>>> w = lambertw(1, k=3)
>>> w
(-2.8535817554090377+17.113535539412148j)
>>> w*np.exp(w)
(1.0000000000000002+1.609823385706477e-15j)
```

**Applications to equation-solving**

The Lambert W function may be used to solve various kinds of equations, such as finding the value of the infinite power tower  $z^{z^{z^{\dots}}}$ :

```
>>> def tower(z, n):
...     if n == 0:
...         return z
...     return z ** tower(z, n-1)
...
>>> tower(0.5, 100)
0.641185744504986
>>> -lambertw(-np.log(0.5)) / np.log(0.5)
(0.64118574450498589+0j)
```

`scipy.special.zeta` (*x*, *q=None*, *out=None*)  
 Riemann zeta function.

The two-argument version is the Hurwitz zeta function:

$$\zeta(x, q) = \sum_{k=0}^{\infty} \frac{1}{(k+q)^x},$$

Riemann zeta function corresponds to  $q = 1$ .

**See also:**

`zetac`

`scipy.special.zetac` (*x*) = <ufunc 'zetac'>  
 Riemann zeta function minus 1.

This function is defined as

$$\zeta(x) = \sum_{k=2}^{\infty} 1/k^x,$$

where  $x > 1$ .

**See also:**

`zeta`

## Convenience Functions

---

<code>cbrt(x)</code>	Cube root of <i>x</i>
<code>expl0(x)</code>	$10^{**x}$
<code>exp2(x)</code>	$2^{**x}$
<code>radian(d, m, s)</code>	Convert from degrees to radians
<code>cosdg(x)</code>	Cosine of the angle <i>x</i> given in degrees.
<code>sindg(x)</code>	Sine of angle given in degrees
<code>tandg(x)</code>	Tangent of angle <i>x</i> given in degrees.
<code>cotdg(x)</code>	Cotangent of the angle <i>x</i> given in degrees.
<code>log1p(x)</code>	Calculates $\log(1+x)$ for use when <i>x</i> is near zero
<code>expml(x)</code>	$\exp(x) - 1$ for use when <i>x</i> is near zero.
<code>cosml(x)</code>	$\cos(x) - 1$ for use when <i>x</i> is near zero.
<code>round(x)</code>	Round to nearest integer
<code>xlogy(x, y)</code>	Compute $x * \log(y)$ so that the result is 0 if $x = 0$ .
<code>xlog1py(x, y)</code>	Compute $x * \log1p(y)$ so that the result is 0 if $x = 0$ .
<code>exprel(x)</code>	Relative error exponential, $(\exp(x)-1)/x$ , for use when <i>x</i> is near zero.
<code>sinc(x)</code>	Return the sinc function.

---



`scipy.special.cbrt(x)` = <ufunc 'cbrt'>  
Cube root of  $x$

`scipy.special.exp10(x)` = <ufunc 'exp10'>  
 $10^{**x}$

`scipy.special.exp2(x)` = <ufunc 'exp2'>  
 $2^{**x}$

`scipy.special.radians(d, m, s)` = <ufunc 'radians'>  
Convert from degrees to radians

Returns the angle given in (d)egrees, (m)inutes, and (s)econds in radians.

`scipy.special.cosdg(x)` = <ufunc 'cosdg'>  
Cosine of the angle  $x$  given in degrees.

`scipy.special.sindg(x)` = <ufunc 'sindg'>  
Sine of angle given in degrees

`scipy.special.tandg(x)` = <ufunc 'tandg'>  
Tangent of angle  $x$  given in degrees.

`scipy.special.cotdg(x)` = <ufunc 'cotdg'>  
Cotangent of the angle  $x$  given in degrees.

`scipy.special.log1p(x)` = <ufunc 'log1p'>  
Calculates  $\log(1+x)$  for use when  $x$  is near zero

`scipy.special.expm1(x)` = <ufunc 'expm1'>  
 $\exp(x) - 1$  for use when  $x$  is near zero.

`scipy.special.cosm1(x)` = <ufunc 'cosm1'>  
 $\cos(x) - 1$  for use when  $x$  is near zero.

`scipy.special.round(x)` = <ufunc 'round'>  
Round to nearest integer

Returns the nearest integer to  $x$  as a double precision floating point result. If  $x$  ends in 0.5 exactly, the nearest even integer is chosen.

`scipy.special.xlogy(x, y)` = <ufunc 'xlogy'>  
Compute  $x \cdot \log(y)$  so that the result is 0 if  $x = 0$ .

**Parameters**

- x**: array\_like  
Multiplier
- y**: array\_like  
Argument

**Returns**

- z**: array\_like  
Computed  $x \cdot \log(y)$

#### Notes

New in version 0.13.0.

`scipy.special.xlog1py(x, y)` = <ufunc 'xlog1py'>  
Compute  $x \cdot \log_1 p(y)$  so that the result is 0 if  $x = 0$ .

**Parameters**

- x**: array\_like  
Multiplier
- y**: array\_like  
Argument

**Returns**

- z**: array\_like  
Computed  $x \cdot \log_1 p(y)$

**Notes**

New in version 0.13.0.

`scipy.special.exprel(x) = <ufunc 'exprel'>`

Relative error exponential,  $(\exp(x)-1)/x$ , for use when  $x$  is near zero.

**Parameters** `x` : ndarray  
Input array.

**Returns** `res` : ndarray  
Output array.

**See also:**

`expm1`,

`scipy.special.sinc(x)`

Return the sinc function.

The sinc function is  $\sin(\pi x)/(\pi x)$ .

**Parameters** `x` : ndarray  
Array (possibly multi-dimensional) of values for which to calculate `sinc(x)`.

**Returns** `out` : ndarray  
`sinc(x)`, which has the same shape as the input.

**Notes**

`sinc(0)` is the limit value 1.

The name sinc is short for “sine cardinal” or “sinus cardinalis”.

The sinc function is used in various signal processing applications, including in anti-aliasing, in the construction of a Lanczos resampling filter, and in interpolation.

For bandlimited interpolation of discrete-time signals, the ideal interpolation kernel is proportional to the sinc function.

**References**

[R453], [R454]

**Examples**

```
>>> x = np.linspace(-4, 4, 41)
>>> np.sinc(x)
array([-3.89804309e-17, -4.92362781e-02, -8.40918587e-02,
       -8.90384387e-02, -5.84680802e-02,  3.89804309e-17,
        6.68206631e-02,  1.16434881e-01,  1.26137788e-01,
        8.50444803e-02, -3.89804309e-17, -1.03943254e-01,
       -1.89206682e-01, -2.16236208e-01, -1.55914881e-01,
        3.89804309e-17,  2.33872321e-01,  5.04551152e-01,
        7.56826729e-01,  9.35489284e-01,  1.00000000e+00,
        9.35489284e-01,  7.56826729e-01,  5.04551152e-01,
        2.33872321e-01,  3.89804309e-17, -1.55914881e-01,
       -2.16236208e-01, -1.89206682e-01, -1.03943254e-01,
       -3.89804309e-17,  8.50444803e-02,  1.26137788e-01,
        1.16434881e-01,  6.68206631e-02,  3.89804309e-17,
       -5.84680802e-02, -8.90384387e-02, -8.40918587e-02,
       -4.92362781e-02, -3.89804309e-17])
```

```
>>> plt.plot(x, np.sinc(x))
[<matplotlib.lines.Line2D object at 0x...>]
>>> plt.title("Sinc Function")
<matplotlib.text.Text object at 0x...>
>>> plt.ylabel("Amplitude")
<matplotlib.text.Text object at 0x...>
>>> plt.xlabel("X")
<matplotlib.text.Text object at 0x...>
>>> plt.show()
```

It works in 2-D as well:

```
>>> x = np.linspace(-4, 4, 401)
>>> xx = np.outer(x, x)
>>> plt.imshow(np.sinc(xx))
<matplotlib.image.AxesImage object at 0x...>
```

## 5.27 Statistical functions (`scipy.stats`)

This module contains a large number of probability distributions as well as a growing library of statistical functions.

Each univariate distribution is an instance of a subclass of `rv_continuous` (`rv_discrete` for discrete distributions):

---

<code>rv_continuous</code> ([ <i>momtype</i> , <i>a</i> , <i>b</i> , <i>xtol</i> , ...])	A generic continuous random variable class meant for subclassing.
<code>rv_discrete</code> ([ <i>a</i> , <i>b</i> , <i>name</i> , <i>badvalue</i> , ...])	A generic discrete random variable class meant for subclassing.

---

**class** `scipy.stats.rv_continuous` (*momtype*=1, *a*=None, *b*=None, *xtol*=1e-14, *badvalue*=None, *name*=None, *longname*=None, *shapes*=None, *extradoc*=None, *seed*=None)

A generic continuous random variable class meant for subclassing.

`rv_continuous` is a base class to construct specific distribution classes and instances for continuous random variables. It cannot be used directly as a distribution.

**Parameters** **momtype** : int, optional

The type of generic moment calculation to use: 0 for pdf, 1 (default) for ppf.

**a** : float, optional

Lower bound of the support of the distribution, default is minus infinity.

**b** : float, optional

Upper bound of the support of the distribution, default is plus infinity.

**xtol** : float, optional

The tolerance for fixed point calculation for generic ppf.

**badvalue** : float, optional

The value in a result arrays that indicates a value that for which some argument restriction is violated, default is `np.nan`.

**name** : str, optional

The name of the instance. This string is used to construct the default example for distributions.

**longname** : str, optional

This string is used as part of the first line of the docstring returned when a subclass has no docstring of its own. Note: *longname* exists for backwards compatibility, do not use for new subclasses.

**shapes** : str, optional

The shape of the distribution. For example "m, n" for a distribution that takes two integers as the two shape arguments for all its methods. If not provided, shape parameters will be inferred from the signature of the private methods, `_pdf` and `_cdf` of the instance.

**extradoc** : str, optional, deprecated

This string is used as the last part of the docstring returned when a subclass has no docstring of its own. Note: *extradoc* exists for backwards compatibility, do not use for new subclasses.

**seed** : None or int or `numpy.random.RandomState` instance, optional

This parameter defines the `RandomState` object to use for drawing random variates. If None (or `np.random`), the global `np.random` state is used. If integer, it is used to seed the local `RandomState` instance. Default is None.

### Notes

Public methods of an instance of a distribution class (e.g., `pdf`, `cdf`) check their arguments and pass valid arguments to private, computational methods (`_pdf`, `_cdf`). For `pdf(x)`, `x` is valid if it is within the support of a distribution, `self.a <= x <= self.b`. Whether a shape parameter is valid is decided by an `_argcheck` method (which defaults to checking that its arguments are strictly positive.)

### Subclassing

New random variables can be defined by subclassing the `rv_continuous` class and re-defining at least the `_pdf` or the `_cdf` method (normalized to location 0 and scale 1).

If positive argument checking is not correct for your RV then you will also need to re-define the `_argcheck` method.

Correct, but potentially slow defaults exist for the remaining methods but for speed and/or accuracy you can over-ride:

```
_logpdf, _cdf, _logcdf, _ppf, _rvs, _isf, _sf, _logsf
```

Rarely would you override `_isf`, `_sf` or `_logsf`, but you could.

### Methods that can be overwritten by subclasses

```
_rvs
_pdf
_cdf
_sf
_ppf
_isf
_stats
_munp
_entropy
_argcheck
```

There are additional (internal and private) generic methods that can be useful for cross-checking and for debugging, but might work in all cases when directly called.

A note on `shapes`: subclasses need not specify them explicitly. In this case, `shapes` will be automatically deduced from the signatures of the overridden methods (`pdf`, `cdf` etc). If, for some reason, you prefer to avoid relying on introspection, you can specify `shapes` explicitly as an argument to the instance constructor.

## Frozen Distributions

Normally, you must provide shape parameters (and, optionally, location and scale parameters to each call of a method of a distribution.

Alternatively, the object may be called (as a function) to fix the shape, location, and scale parameters returning a “frozen” continuous RV object:

```
rv = generic(<shape(s)>, loc=0, scale=1)
```

frozen RV object with the same methods but holding the given shape, location, and scale fixed

## Statistics

Statistics are computed using numerical integration by default. For speed you can redefine this using `_stats`:

- take shape parameters and return mu, mu2, g1, g2
- If you can’t compute one of these, return it as None
- Can also be defined with a keyword argument `moments`, which is a string composed of “m”, “v”, “s”, and/or “k”. Only the components appearing in string should be computed and returned in the order “m”, “v”, “s”, or “k” with missing values returned as None.

Alternatively, you can override `_munp`, which takes `n` and shape parameters and returns the `n`-th non-central moment of the distribution.

## Examples

To create a new Gaussian distribution, we would do the following:

```
>>> from scipy.stats import rv_continuous
>>> class gaussian_gen(rv_continuous):
...     "Gaussian distribution"
...     def _pdf(self, x):
...         return np.exp(-x**2 / 2.) / np.sqrt(2.0 * np.pi)
>>> gaussian = gaussian_gen(name='gaussian')
```

`scipy.stats` distributions are *instances*, so here we subclass `rv_continuous` and create an instance. With this, we now have a fully functional distribution with all relevant methods automatically generated by the framework.

Note that above we defined a standard normal distribution, with zero mean and unit variance. Shifting and scaling of the distribution can be done by using `loc` and `scale` parameters: `gaussian.pdf(x, loc, scale)` essentially computes  $y = (x - \text{loc}) / \text{scale}$  and `gaussian._pdf(y) / scale`.

## Attributes

---

`random_state` Get or set the RandomState object for generating random variates.

---

`rv_continuous.random_state`

Get or set the RandomState object for generating random variates.

This can be either None or an existing RandomState object.

If None (or `np.random`), use the RandomState singleton used by `np.random`. If already a RandomState instance, use it. If an int, use a new RandomState instance seeded with `seed`.

## Methods

---

<code>rvs(*args, **kwargs)</code>	Random variates of given type.
<code>pdf(x, *args, **kwargs)</code>	Probability density function at <code>x</code> of the given RV.
<code>logpdf(x, *args, **kwargs)</code>	Log of the probability density function at <code>x</code> of the given RV.

---

Continued on next page

Table 5.273 – continued from previous page

<code>cdf(x, *args, **kwargs)</code>	Cumulative distribution function of the given RV.
<code>logcdf(x, *args, **kwargs)</code>	Log of the cumulative distribution function at x of the given RV.
<code>sf(x, *args, **kwargs)</code>	Survival function (1 - <code>cdf</code> ) at x of the given RV.
<code>logsf(x, *args, **kwargs)</code>	Log of the survival function of the given RV.
<code>ppf(q, *args, **kwargs)</code>	Percent point function (inverse of <code>cdf</code> ) at q of the given RV.
<code>isf(q, *args, **kwargs)</code>	Inverse survival function (inverse of <code>sf</code> ) at q of the given RV.
<code>moment(n, *args, **kwargs)</code>	n-th order non-central moment of distribution.
<code>stats(*args, **kwargs)</code>	Some statistics of the given RV.
<code>entropy(*args, **kwargs)</code>	Differential entropy of the RV.
<code>expect([func, args, loc, scale, lb, ub, ...])</code>	Calculate expected value of a function with respect to the distribution.
<code>median(*args, **kwargs)</code>	Median of the distribution.
<code>mean(*args, **kwargs)</code>	Mean of the distribution.
<code>std(*args, **kwargs)</code>	Standard deviation of the distribution.
<code>var(*args, **kwargs)</code>	Variance of the distribution.
<code>interval(alpha, *args, **kwargs)</code>	Confidence interval with equal areas around the median.
<code>__call__(*args, **kwargs)</code>	Freeze the distribution for the given arguments.
<code>fit(data, *args, **kwargs)</code>	Return MLEs for shape, location, and scale parameters from data.
<code>fit_loc_scale(data, *args)</code>	Estimate loc and scale parameters from data using 1st and 2nd moments.
<code>nnlf(theta, x)</code>	Return negative loglikelihood function.

`rv_continuous.rvs(*args, **kwargs)`

Random variates of given type.

**Parameters** `arg1, arg2, arg3,...` : array\_like

The shape parameter(s) for the distribution (see docstring of the instance object for more information).

**loc** : array\_like, optional

Location parameter (default=0).

**scale** : array\_like, optional

Scale parameter (default=1).

**size** : int or tuple of ints, optional

Defining number of random variates (default is 1).

**random\_state** : None or int or `np.random.RandomState` instance, optional

If int or `RandomState`, use it for drawing the random variates. If

None, rely on `self.random_state`. Default is None.

**Returns** `rvs` : ndarray or scalar

Random variates of given *size*.

`rv_continuous.pdf(x, *args, **kwargs)`

Probability density function at x of the given RV.

**Parameters** `x` : array\_like

quantiles

**arg1, arg2, arg3,...** : array\_like

The shape parameter(s) for the distribution (see docstring of the instance object for more information)

**loc** : array\_like, optional

location parameter (default=0)

**scale** : array\_like, optional

scale parameter (default=1)

**Returns** `pdf` : ndarray

Probability density function evaluated at x

`rv_continuous.logpdf(x, *args, **kwargs)`

Log of the probability density function at x of the given RV.

This uses a more numerically accurate calculation if available.

**Parameters** **x** : array\_like  
 quantiles  
**arg1, arg2, arg3,...** : array\_like  
 The shape parameter(s) for the distribution (see docstring of the instance object for more information)  
**loc** : array\_like, optional  
 location parameter (default=0)  
**scale** : array\_like, optional  
 scale parameter (default=1)  
**Returns** **logpdf** : array\_like  
 Log of the probability density function evaluated at x

`rv_continuous.cdf(x, *args, **kws)`

Cumulative distribution function of the given RV.

**Parameters** **x** : array\_like  
 quantiles  
**arg1, arg2, arg3,...** : array\_like  
 The shape parameter(s) for the distribution (see docstring of the instance object for more information)  
**loc** : array\_like, optional  
 location parameter (default=0)  
**scale** : array\_like, optional  
 scale parameter (default=1)  
**Returns** **cdf** : ndarray  
 Cumulative distribution function evaluated at x

`rv_continuous.logcdf(x, *args, **kws)`

Log of the cumulative distribution function at x of the given RV.

**Parameters** **x** : array\_like  
 quantiles  
**arg1, arg2, arg3,...** : array\_like  
 The shape parameter(s) for the distribution (see docstring of the instance object for more information)  
**loc** : array\_like, optional  
 location parameter (default=0)  
**scale** : array\_like, optional  
 scale parameter (default=1)  
**Returns** **logcdf** : array\_like  
 Log of the cumulative distribution function evaluated at x

`rv_continuous.sf(x, *args, **kws)`

Survival function (1 - cdf) at x of the given RV.

**Parameters** **x** : array\_like  
 quantiles  
**arg1, arg2, arg3,...** : array\_like  
 The shape parameter(s) for the distribution (see docstring of the instance object for more information)  
**loc** : array\_like, optional  
 location parameter (default=0)  
**scale** : array\_like, optional  
 scale parameter (default=1)  
**Returns** **sf** : array\_like  
 Survival function evaluated at x

`rv_continuous.logsf(x, *args, **kwargs)`

Log of the survival function of the given RV.

Returns the log of the “survival function,” defined as  $(1 - \text{cdf})$ , evaluated at  $x$ .

**Parameters**

- x** : array\_like  
quantiles
- arg1, arg2, arg3,...** : array\_like  
The shape parameter(s) for the distribution (see docstring of the instance object for more information)
- loc** : array\_like, optional  
location parameter (default=0)
- scale** : array\_like, optional  
scale parameter (default=1)

**Returns**

- logsf** : ndarray  
Log of the survival function evaluated at  $x$ .

`rv_continuous.ppf(q, *args, **kwargs)`

Percent point function (inverse of `cdf`) at  $q$  of the given RV.

**Parameters**

- q** : array\_like  
lower tail probability
- arg1, arg2, arg3,...** : array\_like  
The shape parameter(s) for the distribution (see docstring of the instance object for more information)
- loc** : array\_like, optional  
location parameter (default=0)
- scale** : array\_like, optional  
scale parameter (default=1)

**Returns**

- x** : array\_like  
quantile corresponding to the lower tail probability  $q$ .

`rv_continuous.isf(q, *args, **kwargs)`

Inverse survival function (inverse of `sf`) at  $q$  of the given RV.

**Parameters**

- q** : array\_like  
upper tail probability
- arg1, arg2, arg3,...** : array\_like  
The shape parameter(s) for the distribution (see docstring of the instance object for more information)
- loc** : array\_like, optional  
location parameter (default=0)
- scale** : array\_like, optional  
scale parameter (default=1)

**Returns**

- x** : ndarray or scalar  
Quantile corresponding to the upper tail probability  $q$ .

`rv_continuous.moment(n, *args, **kwargs)`

$n$ -th order non-central moment of distribution.

**Parameters**

- n** : int,  $n \geq 1$   
Order of moment.
- arg1, arg2, arg3,...** : float  
The shape parameter(s) for the distribution (see docstring of the instance object for more information).
- loc** : array\_like, optional  
location parameter (default=0)
- scale** : array\_like, optional  
scale parameter (default=1)



`rv_continuous.stats` (\*args, \*\*kws)

Some statistics of the given RV.

**Parameters** **arg1, arg2, arg3,...** : array\_like  
The shape parameter(s) for the distribution (see docstring of the instance object for more information)

**loc** : array\_like, optional  
location parameter (default=0)

**scale** : array\_like, optional (continuous RVs only)  
scale parameter (default=1)

**moments** : str, optional  
composed of letters ['mvsk'] defining which moments to compute: 'm' = mean, 'v' = variance, 's' = (Fisher's) skew, 'k' = (Fisher's) kurtosis. (default is 'mv')

**Returns** **stats** : sequence  
of requested moments.

`rv_continuous.entropy` (\*args, \*\*kws)

Differential entropy of the RV.

**Parameters** **arg1, arg2, arg3,...** : array\_like  
The shape parameter(s) for the distribution (see docstring of the instance object for more information).

**loc** : array\_like, optional  
Location parameter (default=0).

**scale** : array\_like, optional (continuous distributions only).  
Scale parameter (default=1).

### Notes

Entropy is defined base  $e$ :

```
>>> drv = rv_discrete(values=((0, 1), (0.5, 0.5)))
>>> np.allclose(drv.entropy(), np.log(2.0))
True
```

`rv_continuous.expect` (func=None, args=(), loc=0, scale=1, lb=None, ub=None, conditional=False, \*\*kws)

Calculate expected value of a function with respect to the distribution.

The expected value of a function  $f(x)$  with respect to a distribution `dist` is defined as:

$$E[x] = \int_{lb}^{ub} f(x) * dist.pdf(x)$$

**Parameters** **func** : callable, optional  
Function for which integral is calculated. Takes only one argument. The default is the identity mapping  $f(x) = x$ .

**args** : tuple, optional  
Shape parameters of the distribution.

**loc** : float, optional  
Location parameter (default=0).

**scale** : float, optional  
Scale parameter (default=1).

**lb, ub** : scalar, optional  
Lower and upper bound for integration. Default is set to the support of the distribution.

**conditional** : bool, optional

If True, the integral is corrected by the conditional probability of the integration interval. The return value is the expectation of the function, conditional on being in the given interval. Default is False.

**Returns** **Additional keyword arguments are passed to the integration routine.**  
**expect** : float

The calculated expected value.

**Notes**

The integration behavior of this function is inherited from *integrate.quad*.

`rv_continuous.median(*args, **kws)`

Median of the distribution.

**Parameters** **arg1, arg2, arg3,...** : array\_like

The shape parameter(s) for the distribution (see docstring of the instance object for more information)

**loc** : array\_like, optional

Location parameter, Default is 0.

**scale** : array\_like, optional

Scale parameter, Default is 1.

**Returns** **median** : float

The median of the distribution.

**See also:**

***stats.distributions.rv\_discrete.ppf***

Inverse of the CDF

`rv_continuous.mean(*args, **kws)`

Mean of the distribution.

**Parameters** **arg1, arg2, arg3,...** : array\_like

The shape parameter(s) for the distribution (see docstring of the instance object for more information)

**loc** : array\_like, optional

location parameter (default=0)

**scale** : array\_like, optional

scale parameter (default=1)

**Returns** **mean** : float

the mean of the distribution

`rv_continuous.std(*args, **kws)`

Standard deviation of the distribution.

**Parameters** **arg1, arg2, arg3,...** : array\_like

The shape parameter(s) for the distribution (see docstring of the instance object for more information)

**loc** : array\_like, optional

location parameter (default=0)

**scale** : array\_like, optional

scale parameter (default=1)

**Returns** **std** : float

standard deviation of the distribution

`rv_continuous.var(*args, **kws)`

Variance of the distribution.

**Parameters** **arg1, arg2, arg3,...** : array\_like

The shape parameter(s) for the distribution (see docstring of the instance object for more information)

**loc** : array\_like, optional  
location parameter (default=0)

**scale** : array\_like, optional  
scale parameter (default=1)

**Returns** **var** : float  
the variance of the distribution

`rv_continuous.interval(alpha, *args, **kwargs)`  
Confidence interval with equal areas around the median.

**Parameters** **alpha** : array\_like of float  
Probability that an rv will be drawn from the returned range. Each value should be in the range [0, 1].

**arg1, arg2, ...** : array\_like  
The shape parameter(s) for the distribution (see docstring of the instance object for more information).

**loc** : array\_like, optional  
location parameter, Default is 0.

**scale** : array\_like, optional  
scale parameter, Default is 1.

**Returns** **a, b** : ndarray of float  
end-points of range that contain  $100 * \text{alpha} \%$  of the rv's possible values.

`rv_continuous.__call__(*args, **kwargs)`  
Freeze the distribution for the given arguments.

**Parameters** **arg1, arg2, arg3, ...** : array\_like  
The shape parameter(s) for the distribution. Should include all the non-optional arguments, may include `loc` and `scale`.

**Returns** **rv\_frozen** : rv\_frozen instance  
The frozen distribution.

`rv_continuous.fit(data, *args, **kwargs)`  
Return MLEs for shape, location, and scale parameters from data.

MLE stands for Maximum Likelihood Estimate. Starting estimates for the fit are given by input arguments; for any arguments not provided with starting estimates, `self._fitstart(data)` is called to generate such.

One can hold some parameters fixed to specific values by passing in keyword arguments `f0`, `f1`, ..., `fn` (for shape parameters) and `floc` and `fscale` (for location and scale parameters, respectively).

**Parameters** **data** : array\_like  
Data to use in calculating the MLEs.

**args** : floats, optional  
Starting value(s) for any shape-characterizing arguments (those not provided will be determined by a call to `_fitstart(data)`). No default value.

**kwargs** : floats, optional  
Starting values for the location and scale parameters; no default. Special keyword arguments are recognized as holding certain parameters fixed:

- `f0...fn` : hold respective shape parameters fixed. Alternatively, shape parameters to fix can be specified by name. For example, if `self.shapes == "a, b"`, `fix_a` and `fix_b` are equivalent to `f0`, and `fb` and `fix_b` are equivalent to `f1`.

- `floc` : hold location parameter fixed to specified value.
- `fscale` : hold scale parameter fixed to specified value.
- `optimizer` : The optimizer to use. The optimizer must take `func`, and starting position as the first two arguments, plus `args` (for extra arguments to pass to the function to be optimized) and `disp=0` to suppress output as keyword arguments.

**Returns** `shape, loc, scale` : tuple of floats  
MLEs for any shape statistics, followed by those for location and scale.

**Notes**

This fit is computed by maximizing a log-likelihood function, with penalty applied for samples outside of range of the distribution. The returned answer is not guaranteed to be the globally optimal MLE, it may only be locally optimal, or the optimization may fail altogether.

**Examples**

Generate some data to fit: draw random variates from the `beta` distribution

```
>>> from scipy.stats import beta
>>> a, b = 1., 2.
>>> x = beta.rvs(a, b, size=1000)
```

Now we can fit all four parameters (`a`, `b`, `loc` and `scale`):

```
>>> a1, b1, loc1, scale1 = beta.fit(x)
```

We can also use some prior knowledge about the dataset: let's keep `loc` and `scale` fixed:

```
>>> a1, b1, loc1, scale1 = beta.fit(x, floc=0, fscale=1)
>>> loc1, scale1
(0, 1)
```

We can also keep shape parameters fixed by using `f`-keywords. To keep the zero-th shape parameter `a` equal 1, use `f0=1` or, equivalently, `fa=1`:

```
>>> a1, b1, loc1, scale1 = beta.fit(x, fa=1, floc=0, fscale=1)
>>> a1
1
```

`rv_continuous.fit_loc_scale` (*data*, \*args)

Estimate `loc` and `scale` parameters from data using 1st and 2nd moments.

**Parameters** `data` : array\_like  
Data to fit.  
`arg1, arg2, arg3,...` : array\_like  
The shape parameter(s) for the distribution (see docstring of the instance object for more information).

**Returns** `Lhat` : float  
Estimated location parameter for the data.  
`Shat` : float  
Estimated scale parameter for the data.

`rv_continuous.nllf` (*theta*, *x*)  
Return negative loglikelihood function.

**Notes**

This is  $-\text{sum}(\log \text{pdf}(x, \text{theta}), \text{axis}=0)$  where *theta* are the parameters (including loc and scale).

**class** `scipy.stats.rv_discrete` (*a=0, b=inf, name=None, badvalue=None, moment\_tol=1e-08, values=None, inc=1, longname=None, shapes=None, extradoc=None, seed=None*)

A generic discrete random variable class meant for subclassing.

`rv_discrete` is a base class to construct specific distribution classes and instances for discrete random variables. It can also be used to construct an arbitrary distribution defined by a list of support points and corresponding probabilities.

**Parameters**

- a** : float, optional  
Lower bound of the support of the distribution, default: 0
- b** : float, optional  
Upper bound of the support of the distribution, default: plus infinity
- moment\_tol** : float, optional  
The tolerance for the generic calculation of moments.
- values** : tuple of two array\_like, optional  
(*xk, pk*) where *xk* are integers with non-zero probabilities *pk* with  $\text{sum}(pk) = 1$ .
- inc** : integer, optional  
Increment for the support of the distribution. Default is 1. (other values have not been tested)
- badvalue** : float, optional  
The value in a result arrays that indicates a value that for which some argument restriction is violated, default is `np.nan`.
- name** : str, optional  
The name of the instance. This string is used to construct the default example for distributions.
- longname** : str, optional  
This string is used as part of the first line of the docstring returned when a subclass has no docstring of its own. Note: *longname* exists for backwards compatibility, do not use for new subclasses.
- shapes** : str, optional  
The shape of the distribution. For example “m, n” for a distribution that takes two integers as the two shape arguments for all its methods. If not provided, shape parameters will be inferred from the signatures of the private methods, `_pmf` and `_cdf` of the instance.
- extradoc** : str, optional  
This string is used as the last part of the docstring returned when a subclass has no docstring of its own. Note: *extradoc* exists for backwards compatibility, do not use for new subclasses.
- seed** : None or int or `numpy.random.RandomState` instance, optional  
This parameter defines the `RandomState` object to use for drawing random variates. If None, the global `np.random` state is used. If integer, it is used to seed the local `RandomState` instance. Default is None.

**Notes**

This class is similar to `rv_continuous`, the main differences being:

- the support of the distribution is a set of integers
- instead of the probability density function, `pdf` (and the corresponding private `_pdf`), this class defines the *probability mass function*, `pmf` (and the corresponding private `_pmf`.)
- scale parameter is not defined.

To create a new discrete distribution, we would do the following:

```
>>> from scipy.stats import rv_discrete
>>> class poisson_gen(rv_discrete):
...     "Poisson distribution"
...     def _pmf(self, k, mu):
...         return exp(-mu) * mu**k / factorial(k)
```

and create an instance:

```
>>> poisson = poisson_gen(name="poisson")
```

Note that above we defined the Poisson distribution in the standard form. Shifting the distribution can be done by providing the `loc` parameter to the methods of the instance. For example, `poisson.pmf(x, mu, loc)` delegates the work to `poisson._pmf(x-loc, mu)`.

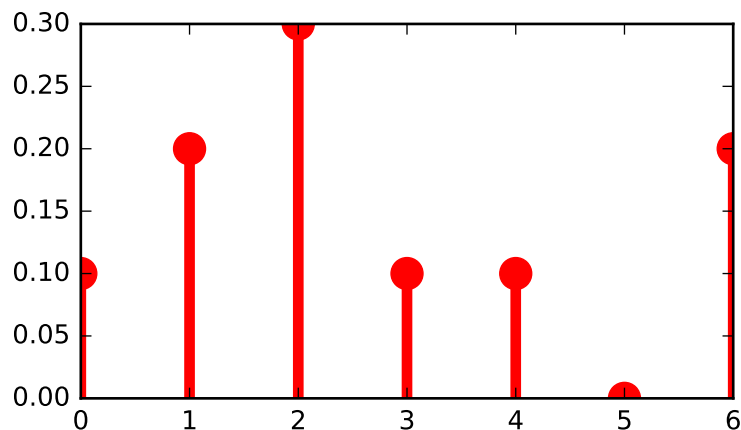
### Discrete distributions from a list of probabilities

Alternatively, you can construct an arbitrary discrete rv defined on a finite set of values  $x_k$  with  $\text{Prob}\{X=x_k\} = p_k$  by using the `values` keyword argument to the `rv_discrete` constructor.

### Examples

Custom made discrete distribution:

```
>>> from scipy import stats
>>> xk = np.arange(7)
>>> pk = (0.1, 0.2, 0.3, 0.1, 0.1, 0.0, 0.2)
>>> custm = stats.rv_discrete(name='custm', values=(xk, pk))
>>>
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
>>> ax.plot(xk, custm.pmf(xk), 'ro', ms=12, mec='r')
>>> ax.vlines(xk, 0, custm.pmf(xk), colors='r', lw=4)
>>> plt.show()
```



Random number generation:

```
>>> R = custm.rvs(size=100)
```

### Attributes

---

<code>random_state</code>	Get or set the RandomState object for generating random variates.
<code>return_integers</code>	<code>return_integers</code> is deprecated!

---

#### `rv_discrete.random_state`

Get or set the RandomState object for generating random variates.

This can be either None or an existing RandomState object.

If None (or `np.random`), use the RandomState singleton used by `np.random`. If already a RandomState instance, use it. If an int, use a new RandomState instance seeded with `seed`.

#### `rv_discrete.return_integers`

`return_integers` is deprecated! `return_integers` attribute is not used anywhere any longer and is deprecated in scipy 0.18.

### Methods

---

<code>rvs(*args, **kwargs)</code>	Random variates of given type.
<code>pmf(k, *args, **kwargs)</code>	Probability mass function at <code>k</code> of the given RV.
<code>logpmf(k, *args, **kwargs)</code>	Log of the probability mass function at <code>k</code> of the given RV.
<code>cdf(k, *args, **kwargs)</code>	Cumulative distribution function of the given RV.
<code>logcdf(k, *args, **kwargs)</code>	Log of the cumulative distribution function at <code>k</code> of the given RV.
<code>sf(k, *args, **kwargs)</code>	Survival function ( $1 - \text{cdf}$ ) at <code>k</code> of the given RV.
<code>logsf(k, *args, **kwargs)</code>	Log of the survival function of the given RV.
<code>ppf(q, *args, **kwargs)</code>	Percent point function (inverse of <code>cdf</code> ) at <code>q</code> of the given RV.
<code>isf(q, *args, **kwargs)</code>	Inverse survival function (inverse of <code>sf</code> ) at <code>q</code> of the given RV.
<code>moment(n, *args, **kwargs)</code>	$n$ -th order non-central moment of distribution.
<code>stats(*args, **kwargs)</code>	Some statistics of the given RV.
<code>entropy(*args, **kwargs)</code>	Differential entropy of the RV.
<code>expect([func, args, loc, lb, ub, ...])</code>	Calculate expected value of a function with respect to the distribution for discrete distribution.
<code>median(*args, **kwargs)</code>	Median of the distribution.
<code>mean(*args, **kwargs)</code>	Mean of the distribution.
<code>std(*args, **kwargs)</code>	Standard deviation of the distribution.
<code>var(*args, **kwargs)</code>	Variance of the distribution.
<code>interval(alpha, *args, **kwargs)</code>	Confidence interval with equal areas around the median.
<code>__call__(*args, **kwargs)</code>	Freeze the distribution for the given arguments.

---

#### `rv_discrete.rvs(*args, **kwargs)`

Random variates of given type.

**Parameters** `arg1, arg2, arg3,...` : array\_like

The shape parameter(s) for the distribution (see docstring of the instance object for more information).

**loc** : array\_like, optional

Location parameter (default=0).

**size** : int or tuple of ints, optional

Defining number of random variates (Default is 1). Note that *size* has to be given as keyword, not as positional argument.

**random\_state** : None or int or `np.random.RandomState` instance, optional

**Returns** **rvs** : ndarray or scalar If int or RandomState, use it for drawing the random variates. If None, rely on `self.random_state`. Default is None.  
 Random variates of given *size*.

`rv_discrete.pmf(k, *args, **kws)`  
 Probability mass function at *k* of the given RV.

**Parameters** **k** : array\_like  
 Quantiles.  
**arg1, arg2, arg3,...** : array\_like  
 The shape parameter(s) for the distribution (see docstring of the instance object for more information)

**loc** : array\_like, optional  
 Location parameter (default=0).  
**Returns** **pmf** : array\_like  
 Probability mass function evaluated at *k*

`rv_discrete.logpmf(k, *args, **kws)`  
 Log of the probability mass function at *k* of the given RV.

**Parameters** **k** : array\_like  
 Quantiles.  
**arg1, arg2, arg3,...** : array\_like  
 The shape parameter(s) for the distribution (see docstring of the instance object for more information).

**loc** : array\_like, optional  
 Location parameter. Default is 0.  
**Returns** **logpmf** : array\_like  
 Log of the probability mass function evaluated at *k*.

`rv_discrete.cdf(k, *args, **kws)`  
 Cumulative distribution function of the given RV.

**Parameters** **k** : array\_like, int  
 Quantiles.  
**arg1, arg2, arg3,...** : array\_like  
 The shape parameter(s) for the distribution (see docstring of the instance object for more information).

**loc** : array\_like, optional  
 Location parameter (default=0).  
**Returns** **cdf** : ndarray  
 Cumulative distribution function evaluated at *k*.

`rv_discrete.logcdf(k, *args, **kws)`  
 Log of the cumulative distribution function at *k* of the given RV.

**Parameters** **k** : array\_like, int  
 Quantiles.  
**arg1, arg2, arg3,...** : array\_like  
 The shape parameter(s) for the distribution (see docstring of the instance object for more information).

**loc** : array\_like, optional  
 Location parameter (default=0).  
**Returns** **logcdf** : array\_like  
 Log of the cumulative distribution function evaluated at *k*.

`rv_discrete.sf(k, *args, **kws)`  
 Survival function (1 - `cdf`) at *k* of the given RV.

**Parameters** **k** : array\_like



Quantiles.  
**arg1, arg2, arg3,...** : array\_like  
 The shape parameter(s) for the distribution (see docstring of the instance object for more information).  
**loc** : array\_like, optional  
 Location parameter (default=0).  
**Returns** **sf** : array\_like  
 Survival function evaluated at *k*.

`rv_discrete.logsf(k, *args, **kws)`  
 Log of the survival function of the given RV.  
 Returns the log of the “survival function,” defined as  $1 - \text{cdf}$ , evaluated at *k*.

**Parameters** **k** : array\_like  
 Quantiles.  
**arg1, arg2, arg3,...** : array\_like  
 The shape parameter(s) for the distribution (see docstring of the instance object for more information).  
**loc** : array\_like, optional  
 Location parameter (default=0).  
**Returns** **logsf** : ndarray  
 Log of the survival function evaluated at *k*.

`rv_discrete.ppf(q, *args, **kws)`  
 Percent point function (inverse of `cdf`) at *q* of the given RV.

**Parameters** **q** : array\_like  
 Lower tail probability.  
**arg1, arg2, arg3,...** : array\_like  
 The shape parameter(s) for the distribution (see docstring of the instance object for more information).  
**loc** : array\_like, optional  
 Location parameter (default=0).  
**Returns** **k** : array\_like  
 Quantile corresponding to the lower tail probability, *q*.

`rv_discrete.isf(q, *args, **kws)`  
 Inverse survival function (inverse of `sf`) at *q* of the given RV.

**Parameters** **q** : array\_like  
 Upper tail probability.  
**arg1, arg2, arg3,...** : array\_like  
 The shape parameter(s) for the distribution (see docstring of the instance object for more information).  
**loc** : array\_like, optional  
 Location parameter (default=0).  
**Returns** **k** : ndarray or scalar  
 Quantile corresponding to the upper tail probability, *q*.

`rv_discrete.moment(n, *args, **kws)`  
*n*-th order non-central moment of distribution.

**Parameters** **n** : int,  $n \geq 1$   
 Order of moment.  
**arg1, arg2, arg3,...** : float  
 The shape parameter(s) for the distribution (see docstring of the instance object for more information).  
**loc** : array\_like, optional  
 location parameter (default=0)

**scale** : array\_like, optional  
scale parameter (default=1)

`rv_discrete.stats` (\*args, \*\*kws)

Some statistics of the given RV.

**Parameters** **arg1, arg2, arg3,...** : array\_like  
The shape parameter(s) for the distribution (see docstring of the instance object for more information)

**loc** : array\_like, optional  
location parameter (default=0)

**scale** : array\_like, optional (continuous RVs only)  
scale parameter (default=1)

**moments** : str, optional  
composed of letters ['mvsk'] defining which moments to compute: 'm' = mean, 'v' = variance, 's' = (Fisher's) skew, 'k' = (Fisher's) kurtosis. (default is 'mv')

**Returns** **stats** : sequence  
of requested moments.

`rv_discrete.entropy` (\*args, \*\*kws)

Differential entropy of the RV.

**Parameters** **arg1, arg2, arg3,...** : array\_like  
The shape parameter(s) for the distribution (see docstring of the instance object for more information).

**loc** : array\_like, optional  
Location parameter (default=0).

**scale** : array\_like, optional (continuous distributions only).  
Scale parameter (default=1).

### Notes

Entropy is defined base  $e$ :

```
>>> drv = rv_discrete(values=((0, 1), (0.5, 0.5)))
>>> np.allclose(drv.entropy(), np.log(2.0))
True
```

`rv_discrete.expect` (func=None, args=(), loc=0, lb=None, ub=None, conditional=False, max-count=1000, tolerance=1e-10, chunksize=32)

Calculate expected value of a function with respect to the distribution for discrete distribution.

**Parameters** **func** : callable, optional  
Function for which the expectation value is calculated. Takes only one argument. The default is the identity mapping  $f(k) = k$ .

**args** : tuple, optional  
Shape parameters of the distribution.

**loc** : float, optional  
Location parameter. Default is 0.

**lb, ub** : int, optional  
Lower and upper bound for the summation, default is set to the support of the distribution, inclusive ( $ub \leq k \leq lb$ ).

**conditional** : bool, optional  
If true then the expectation is corrected by the conditional probability of the summation interval. The return value is the expectation of the function, *func*, conditional on being in the given interval ( $k$  such that  $ub \leq k \leq lb$ ). Default is False.

**maxcount** : int, optional  
Maximal number of terms to evaluate (to avoid an endless loop for an infinite sum). Default is 1000.

**tolerance** : float, optional  
Absolute tolerance for the summation. Default is 1e-10.

**chunksize** : int, optional  
Iterate over the support of a distributions in chunks of this size. Default is 32.

**Returns** **expect** : float  
Expected value.

**Notes**

For heavy-tailed distributions, the expected value may or may not exist, depending on the function, *func*. If it does exist, but the sum converges slowly, the accuracy of the result may be rather low. For instance, for `zipf(4)`, accuracy for mean, variance in example is only 1e-5. increasing *maxcount* and/or *chunksize* may improve the result, but may also make zipf very slow.

The function is not vectorized.

`rv_discrete.median(*args, **kws)`

Median of the distribution.

**Parameters** **arg1, arg2, arg3,...** : array\_like  
The shape parameter(s) for the distribution (see docstring of the instance object for more information)

**loc** : array\_like, optional  
Location parameter, Default is 0.

**scale** : array\_like, optional  
Scale parameter, Default is 1.

**Returns** **median** : float  
The median of the distribution.

**See also:**

**`stats.distributions.rv_discrete.ppf`**

Inverse of the CDF

`rv_discrete.mean(*args, **kws)`

Mean of the distribution.

**Parameters** **arg1, arg2, arg3,...** : array\_like  
The shape parameter(s) for the distribution (see docstring of the instance object for more information)

**loc** : array\_like, optional  
location parameter (default=0)

**scale** : array\_like, optional  
scale parameter (default=1)

**Returns** **mean** : float  
the mean of the distribution

`rv_discrete.std(*args, **kws)`

Standard deviation of the distribution.

**Parameters** **arg1, arg2, arg3,...** : array\_like  
The shape parameter(s) for the distribution (see docstring of the instance object for more information)

**loc** : array\_like, optional  
location parameter (default=0)

**scale** : array\_like, optional

**Returns** **std** : float scale parameter (default=1)  
standard deviation of the distribution

`rv_discrete.var (*args, **kws)`  
Variance of the distribution.

**Parameters** **arg1, arg2, arg3,...** : array\_like  
The shape parameter(s) for the distribution (see docstring of the instance object for more information)  
**loc** : array\_like, optional  
location parameter (default=0)  
**scale** : array\_like, optional

**Returns** **var** : float scale parameter (default=1)  
the variance of the distribution

`rv_discrete.interval (alpha, *args, **kws)`  
Confidence interval with equal areas around the median.

**Parameters** **alpha** : array\_like of float  
Probability that an rv will be drawn from the returned range. Each value should be in the range [0, 1].  
**arg1, arg2, ...** : array\_like  
The shape parameter(s) for the distribution (see docstring of the instance object for more information).  
**loc** : array\_like, optional  
location parameter, Default is 0.  
**scale** : array\_like, optional  
scale parameter, Default is 1.

**Returns** **a, b** : ndarray of float  
end-points of range that contain  $100 * \alpha$  % of the rv's possible values.

`rv_discrete.__call__ (*args, **kws)`  
Freeze the distribution for the given arguments.

**Parameters** **arg1, arg2, arg3,...** : array\_like  
The shape parameter(s) for the distribution. Should include all the non-optional arguments, may include `loc` and `scale`.

**Returns** **rv\_frozen** : `rv_frozen` instance  
The frozen distribution.

### 5.27.1 Continuous distributions

<code>alpha</code>	An alpha continuous random variable.
<code>anglit</code>	An anglit continuous random variable.
<code>arcsine</code>	An arcsine continuous random variable.
<code>beta</code>	A beta continuous random variable.
<code>betaprime</code>	A beta prime continuous random variable.
<code>bradford</code>	A Bradford continuous random variable.
<code>burr</code>	A Burr (Type III) continuous random variable.
<code>burr12</code>	A Burr (Type XII) continuous random variable.
<code>cauchy</code>	A Cauchy continuous random variable.
<code>chi</code>	A chi continuous random variable.
<code>chi2</code>	A chi-squared continuous random variable.

Continued on next page

Table 5.276 – continued from previous page

cosine	A cosine continuous random variable.
dgamma	A double gamma continuous random variable.
dweibull	A double Weibull continuous random variable.
erlang	An Erlang continuous random variable.
expon	An exponential continuous random variable.
exponnorm	An exponentially modified Normal continuous random variable.
exponweib	An exponentiated Weibull continuous random variable.
exponpow	An exponential power continuous random variable.
f	An F continuous random variable.
fatiguelife	A fatigue-life (Birnbau-Saunders) continuous random variable.
fisk	A Fisk continuous random variable.
foldcauchy	A folded Cauchy continuous random variable.
foldnorm	A folded normal continuous random variable.
frechet_r	A Frechet right (or Weibull minimum) continuous random variable.
frechet_l	A Frechet left (or Weibull maximum) continuous random variable.
genlogistic	A generalized logistic continuous random variable.
gennorm	A generalized normal continuous random variable.
genpareto	A generalized Pareto continuous random variable.
genexpon	A generalized exponential continuous random variable.
genextreme	A generalized extreme value continuous random variable.
gausshyper	A Gauss hypergeometric continuous random variable.
gamma	A gamma continuous random variable.
gengamma	A generalized gamma continuous random variable.
genhalflogistic	A generalized half-logistic continuous random variable.
gilbrat	A Gilbrat continuous random variable.
gompertz	A Gompertz (or truncated Gumbel) continuous random variable.
gumbel_r	A right-skewed Gumbel continuous random variable.
gumbel_l	A left-skewed Gumbel continuous random variable.
halfcauchy	A Half-Cauchy continuous random variable.
halflogistic	A half-logistic continuous random variable.
halfnorm	A half-normal continuous random variable.
halfgennorm	The upper half of a generalized normal continuous random variable.
hypsecant	A hyperbolic secant continuous random variable.
invgamma	An inverted gamma continuous random variable.
invgauss	An inverse Gaussian continuous random variable.
invweibull	An inverted Weibull continuous random variable.
johnsonsb	A Johnson SB continuous random variable.
johnsonsu	A Johnson SU continuous random variable.
kappa4	Kappa 4 parameter distribution.
kappa3	Kappa 3 parameter distribution.
ksone	General Kolmogorov-Smirnov one-sided test.
kstwobign	Kolmogorov-Smirnov two-sided test for large N.
laplace	A Laplace continuous random variable.
levy	A Levy continuous random variable.
levy_l	A left-skewed Levy continuous random variable.
levy_stable	A Levy-stable continuous random variable.
logistic	A logistic (or Sech-squared) continuous random variable.
loggamma	A log gamma continuous random variable.
loglaplace	A log-Laplace continuous random variable.
lognorm	A lognormal continuous random variable.
lomax	A Lomax (Pareto of the second kind) continuous random variable.
maxwell	A Maxwell continuous random variable.

Continued on next page

Table 5.276 – continued from previous page

<code>mielke</code>	A Mielke’s Beta-Kappa continuous random variable.
<code>nakagami</code>	A Nakagami continuous random variable.
<code>ncx2</code>	A non-central chi-squared continuous random variable.
<code>ncf</code>	A non-central F distribution continuous random variable.
<code>nct</code>	A non-central Student’s T continuous random variable.
<code>norm</code>	A normal continuous random variable.
<code>pareto</code>	A Pareto continuous random variable.
<code>pearson3</code>	A pearson type III continuous random variable.
<code>powerlaw</code>	A power-function continuous random variable.
<code>powerlognorm</code>	A power log-normal continuous random variable.
<code>powernorm</code>	A power normal continuous random variable.
<code>rdist</code>	An R-distributed continuous random variable.
<code>reciprocal</code>	A reciprocal continuous random variable.
<code>rayleigh</code>	A Rayleigh continuous random variable.
<code>rice</code>	A Rice continuous random variable.
<code>recipinvgauss</code>	A reciprocal inverse Gaussian continuous random variable.
<code>semicircular</code>	A semicircular continuous random variable.
<code>skewnorm</code>	A skew-normal random variable.
<code>t</code>	A Student’s T continuous random variable.
<code>trapz</code>	A trapezoidal continuous random variable.
<code>triang</code>	A triangular continuous random variable.
<code>truncexpon</code>	A truncated exponential continuous random variable.
<code>truncnorm</code>	A truncated normal continuous random variable.
<code>tukeylambda</code>	A Tukey-Lambda continuous random variable.
<code>uniform</code>	A uniform continuous random variable.
<code>vonmises</code>	A Von Mises continuous random variable.
<code>vonmises_line</code>	A Von Mises continuous random variable.
<code>wald</code>	A Wald continuous random variable.
<code>weibull_min</code>	A Frechet right (or Weibull minimum) continuous random variable.
<code>weibull_max</code>	A Frechet left (or Weibull maximum) continuous random variable.
<code>wrapcauchy</code>	A wrapped Cauchy continuous random variable.

`scipy.stats.alpha = <scipy.stats._continuous_distns.alpha_gen object at 0x2b909baabe10>`

An alpha continuous random variable.

As an instance of the `rv_continuous` class, `alpha` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**Notes**

The probability density function for `alpha` is:

$$\text{alpha.pdf}(x, a) = 1/(x**2*\text{Phi}(a)*\text{sqrt}(2*\text{pi})) * \exp(-1/2 * (a-1/x)**2),$$

where `Phi(alpha)` is the normal CDF,  $x > 0$ , and  $a > 0$ .

`alpha` takes `a` as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `alpha.pdf(x, a, loc, scale)` is identically equivalent to `alpha.pdf(y, a) / scale` with  $y = (x - loc) / scale$ .

*Examples*

```
>>> from scipy.stats import alpha
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> a = 3.57
>>> mean, var, skew, kurt = alpha.stats(a, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(alpha.ppf(0.01, a),
...                 alpha.ppf(0.99, a), 100)
>>> ax.plot(x, alpha.pdf(x, a),
...        'r-', lw=5, alpha=0.6, label='alpha pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = alpha(a)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

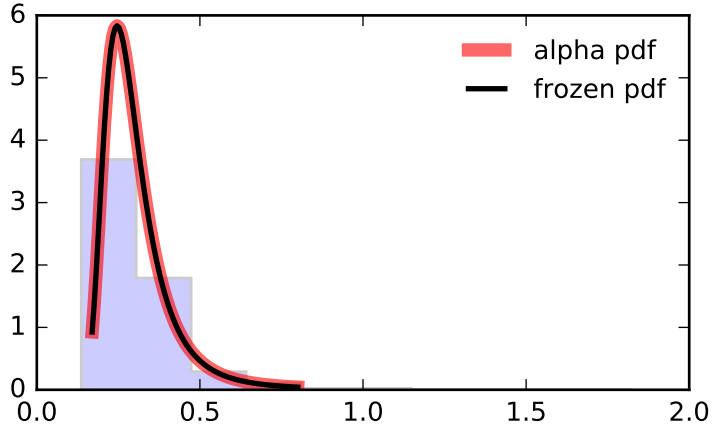
```
>>> vals = alpha.ppf([0.001, 0.5, 0.999], a)
>>> np.allclose([0.001, 0.5, 0.999], alpha.cdf(vals, a))
True
```

Generate random numbers:

```
>>> r = alpha.rvs(a, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre> rvs(a, loc=0, scale=1, size=1,     random_state=None) pdf(x, a, loc=0, scale=1) logpdf(x, a, loc=0, scale=1) cdf(x, a, loc=0, scale=1) logcdf(x, a, loc=0, scale=1)  sf(x, a, loc=0, scale=1)  logsf(x, a, loc=0, scale=1) ppf(q, a, loc=0, scale=1)  isf(q, a, loc=0, scale=1) moment(n, a, loc=0, scale=1) stats(a, loc=0, scale=1, moments='mv')  entropy(a, loc=0, scale=1) fit(data, a, loc=0, scale=1) expect(func, args=(a,), loc=0, scale=1,         lb=None, ub=None, conditional=False,         **kwds) median(a, loc=0, scale=1) mean(a, loc=0, scale=1) var(a, loc=0, scale=1) std(a, loc=0, scale=1) interval(alpha, a, loc=0, scale=1)                     </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

`scipy.stats.anglit` = <scipy.stats.continuous\_distns.anglit\_gen object at 0x2b909bab90d0>

An `anglit` continuous random variable.

As an instance of the `rv_continuous` class, `anglit` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.



### Notes

The probability density function for `anglit` is:

```
anglit.pdf(x) = sin(2*x + pi/2) = cos(2*x),
```

for  $-\pi/4 \leq x \leq \pi/4$ .

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `anglit.pdf(x, loc, scale)` is identically equivalent to `anglit.pdf(y) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

### Examples

```
>>> from scipy.stats import anglit
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> mean, var, skew, kurt = anglit.stats(moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(anglit.ppf(0.01),
...                 anglit.ppf(0.99), 100)
>>> ax.plot(x, anglit.pdf(x),
...        'r-', lw=5, alpha=0.6, label='anglit pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = anglit()
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

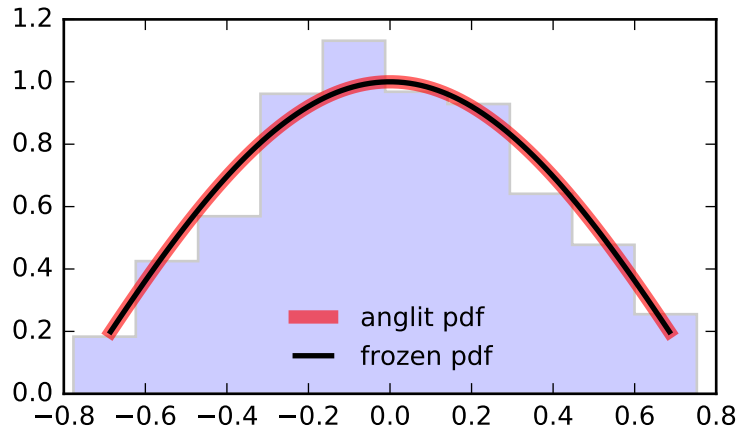
```
>>> vals = anglit.ppf([0.001, 0.5, 0.999])
>>> np.allclose([0.001, 0.5, 0.999], anglit.cdf(vals))
True
```

Generate random numbers:

```
>>> r = anglit.rvs(size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre> rvs(loc=0, scale=1, size=1,     random_state=None) pdf(x, loc=0, scale=1) logpdf(x, loc=0, scale=1) cdf(x, loc=0, scale=1) logcdf(x, loc=0, scale=1)  sf(x, loc=0, scale=1)  logsf(x, loc=0, scale=1) ppf(q, loc=0, scale=1)  isf(q, loc=0, scale=1) moment(n, loc=0, scale=1) stats(loc=0, scale=1, moments='mv')  entropy(loc=0, scale=1) fit(data, loc=0, scale=1) expect(func, args=(), loc=0, scale=1,         lb=None, ub=None, conditional=False,         **kwargs) median(loc=0, scale=1) mean(loc=0, scale=1) var(loc=0, scale=1) std(loc=0, scale=1) interval(alpha, loc=0, scale=1)                     </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <math>\text{sf}</math> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <math>\text{cdf}</math> — percentiles).</p> <p>Inverse survival function (inverse of <math>\text{sf}</math>).</p> <p>Non-central moment of order <math>n</math></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	---

`scipy.stats.arcsine` = <scipy.stats.continuous\_distns.arcsine\_gen object at 0x2b909bab92d0>

An arcsine continuous random variable.

As an instance of the `rv_continuous` class, `arcsine` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**Notes**

The probability density function for `arcsine` is:

$$\text{arcsine.pdf}(x) = 1/(\pi \cdot \sqrt{x \cdot (1-x)})$$

for  $0 < x < 1$ .

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `arcsine.pdf(x, loc, scale)` is identically equivalent to `arcsine.pdf(y) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

**Examples**

```
>>> from scipy.stats import arcsine
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> mean, var, skew, kurt = arcsine.stats(moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(arcsine.ppf(0.01),
...                 arcsine.ppf(0.99), 100)
>>> ax.plot(x, arcsine.pdf(x),
...         'r-', lw=5, alpha=0.6, label='arcsine pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = arcsine()
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

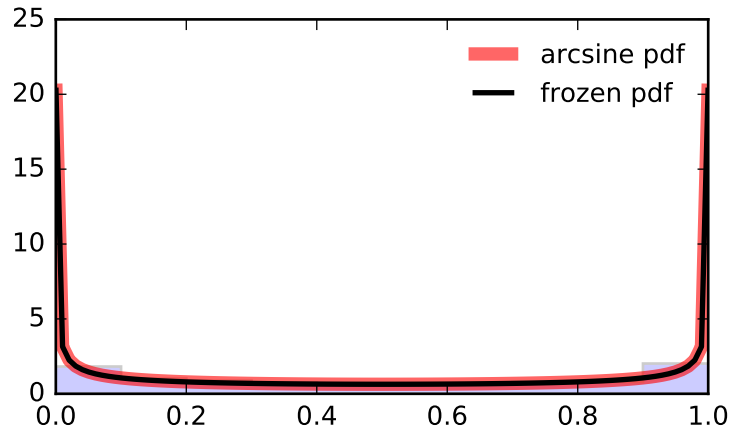
```
>>> vals = arcsine.ppf([0.001, 0.5, 0.999])
>>> np.allclose([0.001, 0.5, 0.999], arcsine.cdf(vals))
True
```

Generate random numbers:

```
>>> r = arcsine.rvs(size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre> rvs(loc=0, scale=1, size=1,     random_state=None) pdf(x, loc=0, scale=1) logpdf(x, loc=0, scale=1) cdf(x, loc=0, scale=1) logcdf(x, loc=0, scale=1)  sf(x, loc=0, scale=1)  logsf(x, loc=0, scale=1) ppf(q, loc=0, scale=1)  isf(q, loc=0, scale=1) moment(n, loc=0, scale=1) stats(loc=0, scale=1, moments='mv')  entropy(loc=0, scale=1) fit(data, loc=0, scale=1) expect(func, args=(), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kwds) median(loc=0, scale=1) mean(loc=0, scale=1) var(loc=0, scale=1) std(loc=0, scale=1) interval(alpha, loc=0, scale=1)                     </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <math>\text{sf}</math> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <math>\text{cdf}</math> — percentiles).</p> <p>Inverse survival function (inverse of <math>\text{sf}</math>).</p> <p>Non-central moment of order <math>n</math></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	---

`scipy.stats.beta = <scipy.stats.continuous_distns.beta_gen object at 0x2b909bab9550>`

A beta continuous random variable.

As an instance of the `rv_continuous` class, `beta` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**Notes**

The probability density function for `beta` is:

$$\text{beta.pdf}(x, a, b) = \frac{\text{gamma}(a+b) * x^{(a-1)} * (1-x)^{(b-1)}}{\text{gamma}(a) * \text{gamma}(b)}$$

for  $0 < x < 1, a > 0, b > 0$ , where `gamma(z)` is the gamma function (`scipy.special.gamma`).

`beta` takes `a` and `b` as shape parameters.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `beta.pdf(x, a, b, loc, scale)` is identically equivalent to `beta.pdf(y, a, b) / scale` with `y = (x - loc) / scale`.

**Examples**

```
>>> from scipy.stats import beta
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> a, b = 2.31, 0.627
>>> mean, var, skew, kurt = beta.stats(a, b, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(beta.ppf(0.01, a, b),
...                 beta.ppf(0.99, a, b), 100)
>>> ax.plot(x, beta.pdf(x, a, b),
...         'r-', lw=5, alpha=0.6, label='beta pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = beta(a, b)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

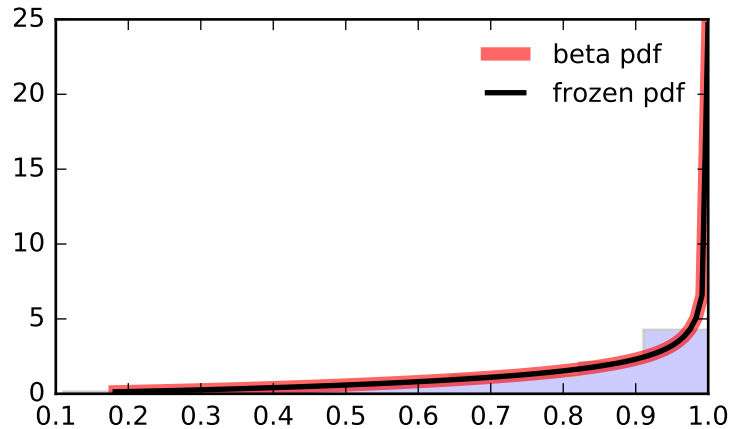
```
>>> vals = beta.ppf([0.001, 0.5, 0.999], a, b)
>>> np.allclose([0.001, 0.5, 0.999], beta.cdf(vals, a, b))
True
```

Generate random numbers:

```
>>> r = beta.rvs(a, b, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<code>rvs(a, b, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, a, b, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, a, b, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, a, b, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, a, b, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, a, b, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, a, b, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, a, b, loc=0, scale=1)</code>	Percent point function (inverse of <i>cdf</i> — percentiles).
<code>isf(q, a, b, loc=0, scale=1)</code>	Inverse survival function (inverse of <i>sf</i> ).
<code>moment(n, a, b, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(a, b, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(a, b, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, a, b, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(a, b), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(a, b, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(a, b, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(a, b, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(a, b, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, a, b, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.betaprime = <scipy.stats.continuous_distns.betaprime_gen object at 0x2b909bab97d0>`  
 A beta prime continuous random variable.

As an instance of the `rv_continuous` class, `betaprime` object inherits from it a collection of generic

methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `betaprime` is:

```
betaprime.pdf(x, a, b) = x**(a-1) * (1+x)**(-a-b) / beta(a, b)
```

for  $x > 0, a > 0, b > 0$ , where `beta(a, b)` is the beta function (see `scipy.special.beta`).

`betaprime` takes `a` and `b` as shape parameters.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `betaprime.pdf(x, a, b, loc, scale)` is identically equivalent to `betaprime.pdf(y, a, b) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

### Examples

```
>>> from scipy.stats import betaprime
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> a, b = 5, 6
>>> mean, var, skew, kurt = betaprime.stats(a, b, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(betaprime.ppf(0.01, a, b),
...                 betaprime.ppf(0.99, a, b), 100)
>>> ax.plot(x, betaprime.pdf(x, a, b),
...         'r-', lw=5, alpha=0.6, label='betaprime pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = betaprime(a, b)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

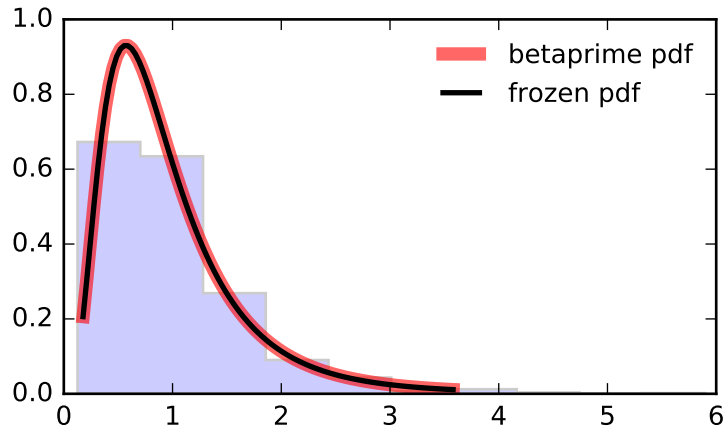
```
>>> vals = betaprime.ppf([0.001, 0.5, 0.999], a, b)
>>> np.allclose([0.001, 0.5, 0.999], betaprime.cdf(vals, a, b))
True
```

Generate random numbers:

```
>>> r = betaprime.rvs(a, b, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<code>rvs(a, b, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, a, b, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, a, b, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, a, b, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, a, b, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, a, b, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, a, b, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, a, b, loc=0, scale=1)</code>	Percent point function (inverse of <i>cdf</i> — percentiles).
<code>isf(q, a, b, loc=0, scale=1)</code>	Inverse survival function (inverse of <i>sf</i> ).
<code>moment(n, a, b, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(a, b, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(a, b, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, a, b, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(a, b), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(a, b, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(a, b, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(a, b, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(a, b, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, a, b, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.bradford` = <scipy.stats.continuous\_distns.bradford\_gen object at 0x2b909bab9a50>  
 A Bradford continuous random variable.

As an instance of the `rv_continuous` class, `bradford` object inherits from it a collection of generic meth-



ods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `bradford` is:

```
bradford.pdf(x, c) = c / (k * (1+c*x)),
```

for  $0 < x < 1, c > 0$  and  $k = \log(1+c)$ .

`bradford` takes `c` as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `bradford.pdf(x, c, loc, scale)` is identically equivalent to `bradford.pdf(y, c) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

### Examples

```
>>> from scipy.stats import bradford
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> c = 0.299
>>> mean, var, skew, kurt = bradford.stats(c, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(bradford.ppf(0.01, c),
...                 bradford.ppf(0.99, c), 100)
>>> ax.plot(x, bradford.pdf(x, c),
...        'r-', lw=5, alpha=0.6, label='bradford pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = bradford(c)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

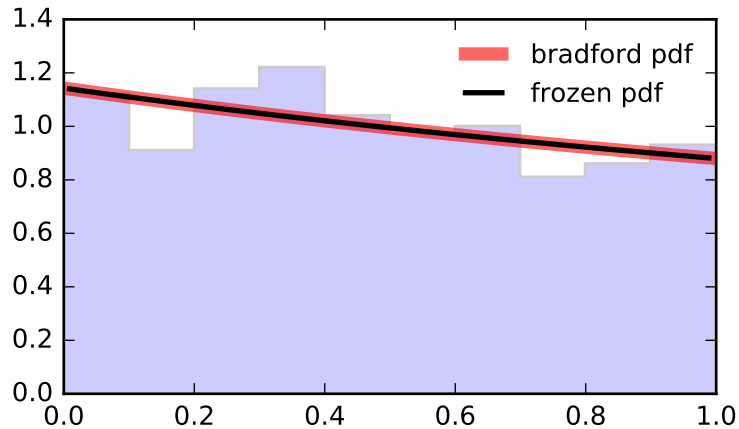
```
>>> vals = bradford.ppf([0.001, 0.5, 0.999], c)
>>> np.allclose([0.001, 0.5, 0.999], bradford.cdf(vals, c))
True
```

Generate random numbers:

```
>>> r = bradford.rvs(c, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre> rvs(c, loc=0, scale=1, size=1,     random_state=None) pdf(x, c, loc=0, scale=1) logpdf(x, c, loc=0, scale=1) cdf(x, c, loc=0, scale=1) logcdf(x, c, loc=0, scale=1)  sf(x, c, loc=0, scale=1)  logsf(x, c, loc=0, scale=1) ppf(q, c, loc=0, scale=1)  isf(q, c, loc=0, scale=1) moment(n, c, loc=0, scale=1) stats(c, loc=0, scale=1, moments='mv')  entropy(c, loc=0, scale=1) fit(data, c, loc=0, scale=1) expect(func, args=(c,), loc=0, scale=1,         lb=None, ub=None, conditional=False,         **kwds) median(c, loc=0, scale=1) mean(c, loc=0, scale=1) var(c, loc=0, scale=1) std(c, loc=0, scale=1) interval(alpha, c, loc=0, scale=1)                     </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

`scipy.stats.burr` = <scipy.stats.continuous\_distns.burr\_gen object at 0x2b909bab9d10>

A Burr (Type III) continuous random variable.

As an instance of the `rv_continuous` class, `burr` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**See also:**

**`fisk`** a special case of either `burr` or `burr12` with `d = 1`  
**`burr12`** Burr Type XII distribution

### Notes

The probability density function for `burr` is:

```
burr.pdf(x, c, d) = c * d * x**(-c-1) * (1+x**(-c))**(-d-1)
```

for  $x > 0$ .

`burr` takes `c` and `d` as shape parameters.

This is the PDF corresponding to the third CDF given in Burr’s list; specifically, it is equation (11) in Burr’s paper [R499].

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `burr.pdf(x, c, d, loc, scale)` is identically equivalent to `burr.pdf(y, c, d) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

### References

[R499]

### Examples

```
>>> from scipy.stats import burr
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> c, d = 10.5, 4.3
>>> mean, var, skew, kurt = burr.stats(c, d, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(burr.ppf(0.01, c, d),
...                 burr.ppf(0.99, c, d), 100)
>>> ax.plot(x, burr.pdf(x, c, d),
...        'r-', lw=5, alpha=0.6, label='burr pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = burr(c, d)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

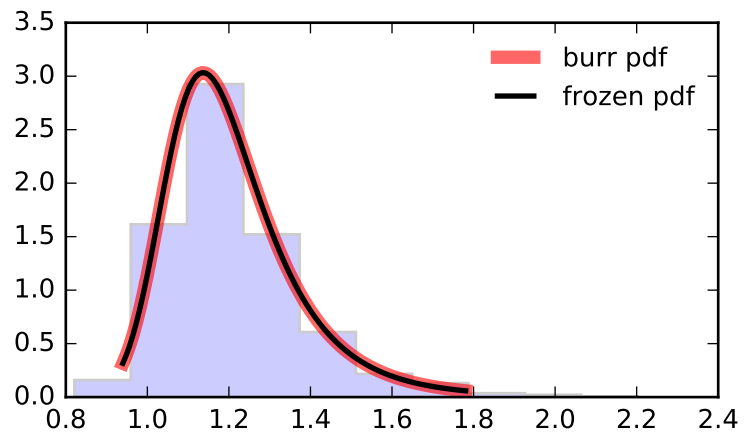
```
>>> vals = burr.ppf([0.001, 0.5, 0.999], c, d)
>>> np.allclose([0.001, 0.5, 0.999], burr.cdf(vals, c, d))
True
```

Generate random numbers:

```
>>> r = burr.rvs(c, d, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



**Methods**

<code>rvs(c, d, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, c, d, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, c, d, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, c, d, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, c, d, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, c, d, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, c, d, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, c, d, loc=0, scale=1)</code>	Percent point function (inverse of <i>cdf</i> — percentiles).
<code>isf(q, c, d, loc=0, scale=1)</code>	Inverse survival function (inverse of <i>sf</i> ).
<code>moment(n, c, d, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(c, d, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(c, d, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, c, d, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(c, d), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(c, d, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(c, d, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(c, d, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(c, d, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, c, d, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.burr12 = <scipy.stats._continuous_distns.burr12_gen object at 0x2b909bab9f90>`

A Burr (Type XII) continuous random variable.

As an instance of the `rv_continuous` class, `burr12` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**See also:**

**`fisk`** a special case of either `burr` or `burr12` with  $d = 1$   
**`burr`** Burr Type III distribution

**Notes**

The probability density function for `burr` is:

$$\text{burr12.pdf}(x, c, d) = c * d * x^{(c-1)} * (1+x^{(c)})^{-(d-1)}$$

for  $x > 0$ .

`burr12` takes *c* and *d* as shape parameters.

This is the PDF corresponding to the twelfth CDF given in Burr's list; specifically, it is equation (20) in Burr's paper [R500].

The probability density above is defined in the "standardized" form. To shift and/or scale the distribution use the *loc* and *scale* parameters. Specifically, `burr12.pdf(x, c, d, loc, scale)` is identically equivalent to `burr12.pdf(y, c, d) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

The Burr type 12 distribution is also sometimes referred to as the Singh-Maddala distribution from NIST [R501].

### References

[R500], [R501]

### Examples

```
>>> from scipy.stats import burr12
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> c, d = 10, 4
>>> mean, var, skew, kurt = burr12.stats(c, d, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(burr12.ppf(0.01, c, d),
...                 burr12.ppf(0.99, c, d), 100)
>>> ax.plot(x, burr12.pdf(x, c, d),
...        'r-', lw=5, alpha=0.6, label='burr12 pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = burr12(c, d)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

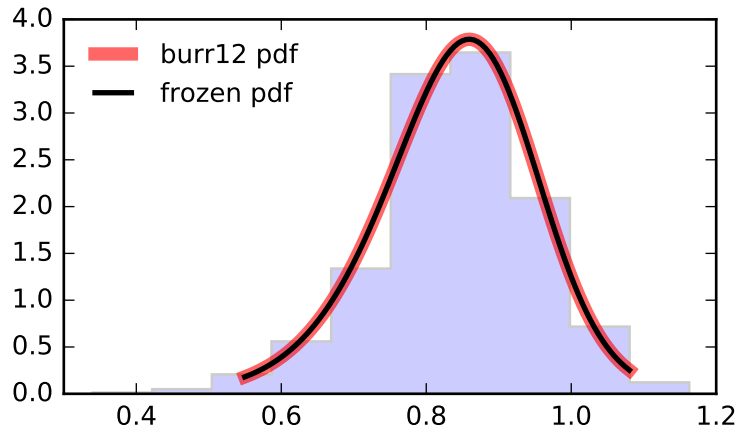
```
>>> vals = burr12.ppf([0.001, 0.5, 0.999], c, d)
>>> np.allclose([0.001, 0.5, 0.999], burr12.cdf(vals, c, d))
True
```

Generate random numbers:

```
>>> r = burr12.rvs(c, d, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<code>rvs(c, d, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, c, d, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, c, d, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, c, d, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, c, d, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, c, d, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, c, d, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, c, d, loc=0, scale=1)</code>	Percent point function (inverse of <i>cdf</i> — percentiles).
<code>isf(q, c, d, loc=0, scale=1)</code>	Inverse survival function (inverse of <i>sf</i> ).
<code>moment(n, c, d, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(c, d, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(c, d, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, c, d, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(c, d), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(c, d, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(c, d, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(c, d, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(c, d, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, c, d, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.cauchy = <scipy.stats.continuous_distns.cauchy_gen object at 0x2b909bac74d0>`

A Cauchy continuous random variable.

As an instance of the `rv_continuous` class, `cauchy` object inherits from it a collection of generic methods

(see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `cauchy` is:

```
cauchy.pdf(x) = 1 / (pi * (1 + x**2))
```

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `cauchy.pdf(x, loc, scale)` is identically equivalent to `cauchy.pdf(y) / scale` with `y = (x - loc) / scale`.

### Examples

```
>>> from scipy.stats import cauchy
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> mean, var, skew, kurt = cauchy.stats(moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(cauchy.ppf(0.01),
...                 cauchy.ppf(0.99), 100)
>>> ax.plot(x, cauchy.pdf(x),
...         'r-', lw=5, alpha=0.6, label='cauchy pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = cauchy()
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

```
>>> vals = cauchy.ppf([0.001, 0.5, 0.999])
>>> np.allclose([0.001, 0.5, 0.999], cauchy.cdf(vals))
True
```

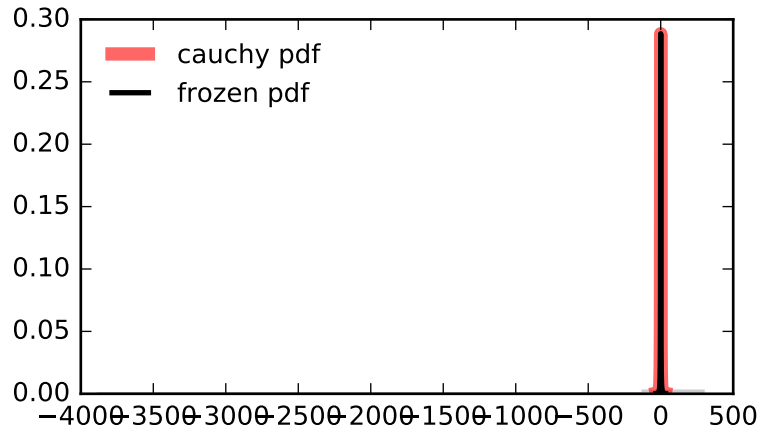
Generate random numbers:

```
>>> r = cauchy.rvs(size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```





### Methods

<pre> rvs(loc=0, scale=1, size=1,     random_state=None) pdf(x, loc=0, scale=1) logpdf(x, loc=0, scale=1) cdf(x, loc=0, scale=1) logcdf(x, loc=0, scale=1)  sf(x, loc=0, scale=1)  logsf(x, loc=0, scale=1) ppf(q, loc=0, scale=1)  isf(q, loc=0, scale=1) moment(n, loc=0, scale=1) stats(loc=0, scale=1, moments='mv')  entropy(loc=0, scale=1) fit(data, loc=0, scale=1) expect(func, args=(), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kwds) median(loc=0, scale=1) mean(loc=0, scale=1) var(loc=0, scale=1) std(loc=0, scale=1) interval(alpha, loc=0, scale=1) </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

`scipy.stats.chi` = <scipy.stats.continuous\_distns.chi\_gen object at 0x2b909bac7710>

A chi continuous random variable.

As an instance of the `rv_continuous` class, `chi` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `chi` is:

```
chi.pdf(x, df) = x**(df-1) * exp(-x**2/2) / (2**(df/2-1) * gamma(df/2))
```

for  $x > 0$ .

Special cases of `chi` are:

- `chi(1, loc, scale)` is equivalent to `halfnorm`
- `chi(2, 0, scale)` is equivalent to `rayleigh`
- `chi(3, 0, scale)` is equivalent to `maxwell`

`chi` takes `df` as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `chi.pdf(x, df, loc, scale)` is identically equivalent to `chi.pdf(y, df) / scale` with  $y = (x - loc) / scale$ .

### Examples

```
>>> from scipy.stats import chi
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> df = 78
>>> mean, var, skew, kurt = chi.stats(df, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(chi.ppf(0.01, df),
...                 chi.ppf(0.99, df), 100)
>>> ax.plot(x, chi.pdf(x, df),
...        'r-', lw=5, alpha=0.6, label='chi pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = chi(df)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

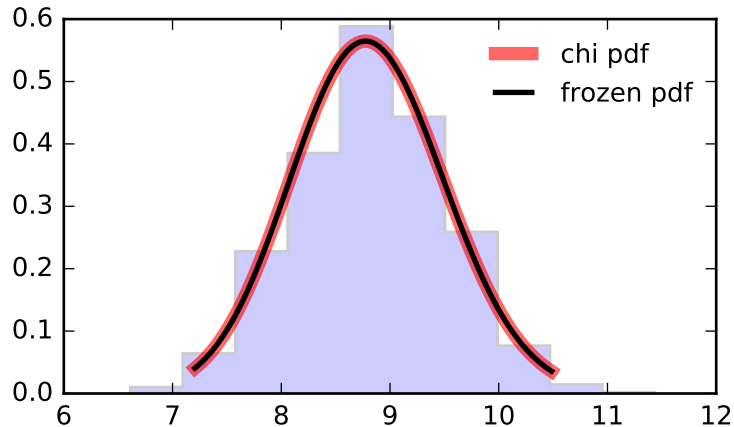
```
>>> vals = chi.ppf([0.001, 0.5, 0.999], df)
>>> np.allclose([0.001, 0.5, 0.999], chi.cdf(vals, df))
True
```

Generate random numbers:

```
>>> r = chi.rvs(df, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<code>rvs(df, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, df, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, df, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, df, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, df, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, df, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, df, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, df, loc=0, scale=1)</code>	Percent point function (inverse of <i>cdf</i> — percentiles).
<code>isf(q, df, loc=0, scale=1)</code>	Inverse survival function (inverse of <i>sf</i> ).
<code>moment(n, df, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(df, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(df, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, df, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(df,), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(df, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(df, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(df, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(df, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, df, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.chi2 = <scipy.stats.continuous_distns.chi2_gen object at 0x2b909bac7950>`

A chi-squared continuous random variable.

As an instance of the `rv_continuous` class, `chi2` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `chi2` is:

```
chi2.pdf(x, df) = 1 / (2*gamma(df/2)) * (x/2)**(df/2-1) * exp(-x/2)
```

`chi2` takes `df` as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `chi2.pdf(x, df, loc, scale)` is identically equivalent to `chi2.pdf(y, df) / scale` with `y = (x - loc) / scale`.

### Examples

```
>>> from scipy.stats import chi2
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> df = 55
>>> mean, var, skew, kurt = chi2.stats(df, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(chi2.ppf(0.01, df),
...                 chi2.ppf(0.99, df), 100)
>>> ax.plot(x, chi2.pdf(x, df),
...        'r-', lw=5, alpha=0.6, label='chi2 pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = chi2(df)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

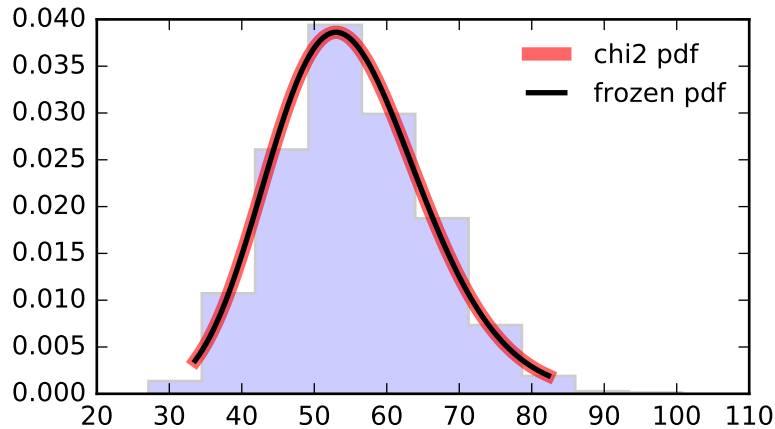
```
>>> vals = chi2.ppf([0.001, 0.5, 0.999], df)
>>> np.allclose([0.001, 0.5, 0.999], chi2.cdf(vals, df))
True
```

Generate random numbers:

```
>>> r = chi2.rvs(df, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre> rvs(df, loc=0, scale=1, size=1,     random_state=None) pdf(x, df, loc=0, scale=1) logpdf(x, df, loc=0, scale=1) cdf(x, df, loc=0, scale=1) logcdf(x, df, loc=0, scale=1)  sf(x, df, loc=0, scale=1)  logsf(x, df, loc=0, scale=1) ppf(q, df, loc=0, scale=1)  isf(q, df, loc=0, scale=1) moment(n, df, loc=0, scale=1) stats(df, loc=0, scale=1, moments='mv')  entropy(df, loc=0, scale=1) fit(data, df, loc=0, scale=1) expect(func, args=(df,), loc=0, scale=1,         lb=None, ub=None, conditional=False,         **kwds) median(df, loc=0, scale=1) mean(df, loc=0, scale=1) var(df, loc=0, scale=1) std(df, loc=0, scale=1) interval(alpha, df, loc=0, scale=1) </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
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`scipy.stats.cosine = <scipy.stats.continuous_distns.cosine_gen object at 0x2b909bac7bd0>`  
 A cosine continuous random variable.

As an instance of the `rv_continuous` class, `cosine` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The cosine distribution is an approximation to the normal distribution. The probability density function for `cosine` is:

```
cosine.pdf(x) = 1/(2*pi) * (1+cos(x))
```

for  $-\pi \leq x \leq \pi$ .

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `cosine.pdf(x, loc, scale)` is identically equivalent to `cosine.pdf(y) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

### Examples

```
>>> from scipy.stats import cosine
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> mean, var, skew, kurt = cosine.stats(moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(cosine.ppf(0.01),
...                 cosine.ppf(0.99), 100)
>>> ax.plot(x, cosine.pdf(x),
...        'r-', lw=5, alpha=0.6, label='cosine pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = cosine()
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

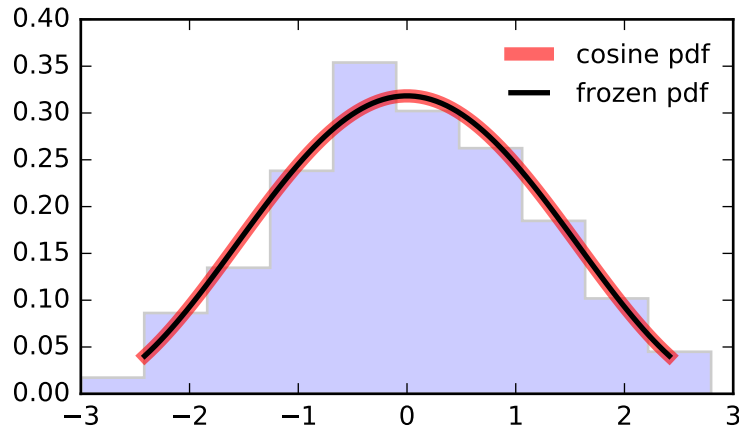
```
>>> vals = cosine.ppf([0.001, 0.5, 0.999])
>>> np.allclose([0.001, 0.5, 0.999], cosine.cdf(vals))
True
```

Generate random numbers:

```
>>> r = cosine.rvs(size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre> rvs(loc=0, scale=1, size=1,     random_state=None) pdf(x, loc=0, scale=1) logpdf(x, loc=0, scale=1) cdf(x, loc=0, scale=1) logcdf(x, loc=0, scale=1)  sf(x, loc=0, scale=1)  logsf(x, loc=0, scale=1) ppf(q, loc=0, scale=1)  isf(q, loc=0, scale=1) moment(n, loc=0, scale=1) stats(loc=0, scale=1, moments='mv')  entropy(loc=0, scale=1) fit(data, loc=0, scale=1) expect(func, args=(), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kwds) median(loc=0, scale=1) mean(loc=0, scale=1) var(loc=0, scale=1) std(loc=0, scale=1) interval(alpha, loc=0, scale=1) </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <math>\text{sf}</math> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <math>\text{cdf}</math> — percentiles).</p> <p>Inverse survival function (inverse of <math>\text{sf}</math>).</p> <p>Non-central moment of order <math>n</math></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
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`scipy.stats.dgamma = <scipy.stats.continuous_distns.dgamma_gen object at 0x2b909bac7dd0>`

A double gamma continuous random variable.

As an instance of the `rv_continuous` class, `dgamma` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `dgamma` is:

```
dgamma.pdf(x, a) = 1 / (2*gamma(a)) * abs(x)**(a-1) * exp(-abs(x))
```

for  $a > 0$ .

`dgamma` takes `a` as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `dgamma.pdf(x, a, loc, scale)` is identically equivalent to `dgamma.pdf(y, a) / scale` with  $y = (x - loc) / scale$ .

### Examples

```
>>> from scipy.stats import dgamma
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> a = 1.1
>>> mean, var, skew, kurt = dgamma.stats(a, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(dgamma.ppf(0.01, a),
...                 dgamma.ppf(0.99, a), 100)
>>> ax.plot(x, dgamma.pdf(x, a),
...         'r-', lw=5, alpha=0.6, label='dgamma pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = dgamma(a)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

```
>>> vals = dgamma.ppf([0.001, 0.5, 0.999], a)
>>> np.allclose([0.001, 0.5, 0.999], dgamma.cdf(vals, a))
True
```

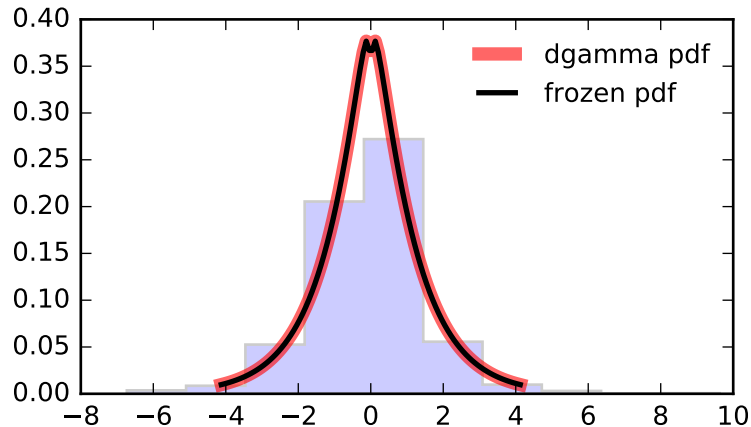
Generate random numbers:

```
>>> r = dgamma.rvs(a, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```





### Methods

<pre> rvs(a, loc=0, scale=1, size=1,     random_state=None) pdf(x, a, loc=0, scale=1) logpdf(x, a, loc=0, scale=1) cdf(x, a, loc=0, scale=1) logcdf(x, a, loc=0, scale=1)  sf(x, a, loc=0, scale=1)  logsf(x, a, loc=0, scale=1) ppf(q, a, loc=0, scale=1)  isf(q, a, loc=0, scale=1) moment(n, a, loc=0, scale=1) stats(a, loc=0, scale=1, moments='mv')  entropy(a, loc=0, scale=1) fit(data, a, loc=0, scale=1) expect(func, args=(a,), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kwds) median(a, loc=0, scale=1) mean(a, loc=0, scale=1) var(a, loc=0, scale=1) std(a, loc=0, scale=1) interval(alpha, a, loc=0, scale=1) </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
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`scipy.stats.dweibull` = <scipy.stats.continuous\_distns.dweibull\_gen object at 0x2b909bada090>  
 A double Weibull continuous random variable.

As an instance of the `rv_continuous` class, `dweibull` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `dweibull` is:

```
dweibull.pdf(x, c) = c / 2 * abs(x)**(c-1) * exp(-abs(x)**c)
```

`dweibull` takes `d` as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `dweibull.pdf(x, c, loc, scale)` is identically equivalent to `dweibull.pdf(y, c) / scale` with `y = (x - loc) / scale`.

### Examples

```
>>> from scipy.stats import dweibull
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> c = 2.07
>>> mean, var, skew, kurt = dweibull.stats(c, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(dweibull.ppf(0.01, c),
...                 dweibull.ppf(0.99, c), 100)
>>> ax.plot(x, dweibull.pdf(x, c),
...        'r-', lw=5, alpha=0.6, label='dweibull pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = dweibull(c)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

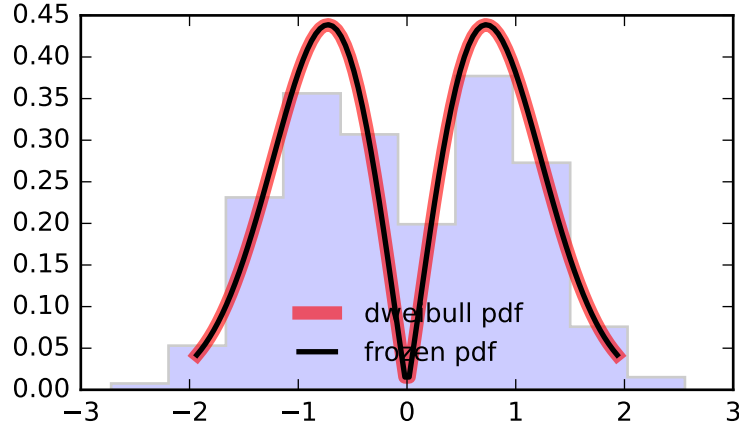
```
>>> vals = dweibull.ppf([0.001, 0.5, 0.999], c)
>>> np.allclose([0.001, 0.5, 0.999], dweibull.cdf(vals, c))
True
```

Generate random numbers:

```
>>> r = dweibull.rvs(c, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre> rvs(c, loc=0, scale=1, size=1,     random_state=None) pdf(x, c, loc=0, scale=1) logpdf(x, c, loc=0, scale=1) cdf(x, c, loc=0, scale=1) logcdf(x, c, loc=0, scale=1)  sf(x, c, loc=0, scale=1)  logsf(x, c, loc=0, scale=1) ppf(q, c, loc=0, scale=1)  isf(q, c, loc=0, scale=1) moment(n, c, loc=0, scale=1) stats(c, loc=0, scale=1, moments='mv')  entropy(c, loc=0, scale=1) fit(data, c, loc=0, scale=1) expect(func, args=(c,), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kwds) median(c, loc=0, scale=1) mean(c, loc=0, scale=1) var(c, loc=0, scale=1) std(c, loc=0, scale=1) interval(alpha, c, loc=0, scale=1) </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
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`scipy.stats.erlang` = <scipy.stats.continuous\_distns.erlang\_gen object at 0x2b909bd28d50>  
 An Erlang continuous random variable.

As an instance of the `rv_continuous` class, `erlang` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

See also:

gamma

**Notes**

The Erlang distribution is a special case of the Gamma distribution, with the shape parameter *a* an integer. Note that this restriction is not enforced by `erlang`. It will, however, generate a warning the first time a non-integer value is used for the shape parameter.

Refer to `gamma` for examples.

**Methods**

<pre> rvs(a, loc=0, scale=1, size=1,     random_state=None) pdf(x, a, loc=0, scale=1) logpdf(x, a, loc=0, scale=1) cdf(x, a, loc=0, scale=1) logcdf(x, a, loc=0, scale=1)  sf(x, a, loc=0, scale=1)  logsf(x, a, loc=0, scale=1) ppf(q, a, loc=0, scale=1)  isf(q, a, loc=0, scale=1) moment(n, a, loc=0, scale=1) stats(a, loc=0, scale=1, moments='mv')  entropy(a, loc=0, scale=1) fit(data, a, loc=0, scale=1) expect(func, args=(a,), loc=0, scale=1,         lb=None, ub=None, conditional=False,         **kwds) median(a, loc=0, scale=1) mean(a, loc=0, scale=1) var(a, loc=0, scale=1) std(a, loc=0, scale=1) interval(alpha, a, loc=0, scale=1) </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <code>cdf</code> — percentiles).</p> <p>Inverse survival function (inverse of <code>sf</code>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

`scipy.stats.expon = <scipy.stats.continuous_distns.expon_gen object at 0x2b909bada310>`

An exponential continuous random variable.

As an instance of the `rv_continuous` class, `expon` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**Notes**

The probability density function for `expon` is:

$$\text{expon.pdf}(x) = \exp(-x)$$

for  $x \geq 0$ .

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `expon.pdf(x, loc, scale)` is identically equivalent to `expon.pdf(y) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

A common parameterization for `expon` is in terms of the rate parameter `lambda`, such that  $\text{pdf} = \text{lambda} * \exp(-\text{lambda} * x)$ . This parameterization corresponds to using `scale = 1 / lambda`.

### Examples

```
>>> from scipy.stats import expon
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> mean, var, skew, kurt = expon.stats(moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(expon.ppf(0.01),
...                 expon.ppf(0.99), 100)
>>> ax.plot(x, expon.pdf(x),
...         'r-', lw=5, alpha=0.6, label='expon pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = expon()
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

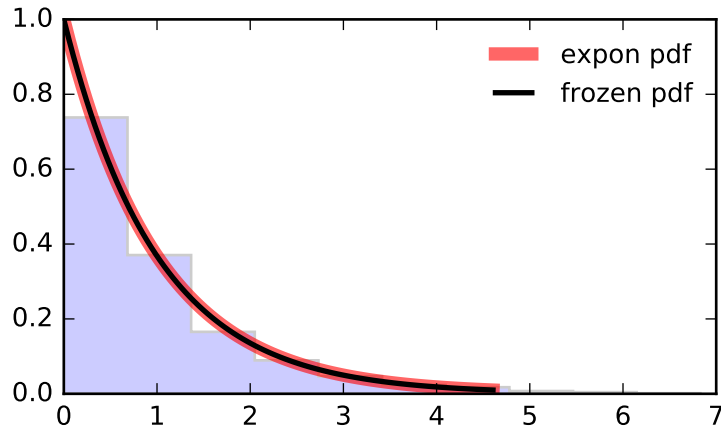
```
>>> vals = expon.ppf([0.001, 0.5, 0.999])
>>> np.allclose([0.001, 0.5, 0.999], expon.cdf(vals))
True
```

Generate random numbers:

```
>>> r = expon.rvs(size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre> rvs(loc=0, scale=1, size=1,     random_state=None) pdf(x, loc=0, scale=1) logpdf(x, loc=0, scale=1) cdf(x, loc=0, scale=1) logcdf(x, loc=0, scale=1)  sf(x, loc=0, scale=1)  logsf(x, loc=0, scale=1) ppf(q, loc=0, scale=1)  isf(q, loc=0, scale=1) moment(n, loc=0, scale=1) stats(loc=0, scale=1, moments='mv')  entropy(loc=0, scale=1) fit(data, loc=0, scale=1) expect(func, args=(), loc=0, scale=1,         lb=None, ub=None, conditional=False,         **kwds) median(loc=0, scale=1) mean(loc=0, scale=1) var(loc=0, scale=1) std(loc=0, scale=1) interval(alpha, loc=0, scale=1)                     </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

`scipy.stats.exponnorm` = <scipy.stats.continuous\_distns.exponnorm\_gen object at 0x2b909bada510>  
 An exponentially modified Normal continuous random variable.

As an instance of the `rv_continuous` class, `exponnorm` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**Notes**

The probability density function for `exponnorm` is:

$$\text{exponnorm.pdf}(x, K) = 1/(2*K) \exp(1/(2 * K**2)) \exp(-x / K) * \text{erfc}(-(x - 1/K) / \text{sqrt}(2))$$

where the shape parameter  $K > 0$ .

It can be thought of as the sum of a normally distributed random value with mean `loc` and sigma `scale` and an exponentially distributed random number with a pdf proportional to  $\exp(-\text{lambda} * x)$  where  $\text{lambda} = (K * \text{scale}) ** (-1)$ .

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `exponnorm.pdf(x, K, loc, scale)` is identically equivalent to `exponnorm.pdf(y, K) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

An alternative parameterization of this distribution (for example, in [Wikipedia](#)) involves three parameters,  $\mu$ ,  $\lambda$  and  $\sigma$ . In the present parameterization this corresponds to having `loc` and `scale` equal to  $\mu$  and  $\sigma$ , respectively, and shape parameter  $K = 1/\sigma\lambda$ .

New in version 0.16.0.

**Examples**

```
>>> from scipy.stats import exponnorm
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> K = 1.5
>>> mean, var, skew, kurt = exponnorm.stats(K, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(exponnorm.ppf(0.01, K),
...                 exponnorm.ppf(0.99, K), 100)
>>> ax.plot(x, exponnorm.pdf(x, K),
...         'r-', lw=5, alpha=0.6, label='exponnorm pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = exponnorm(K)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

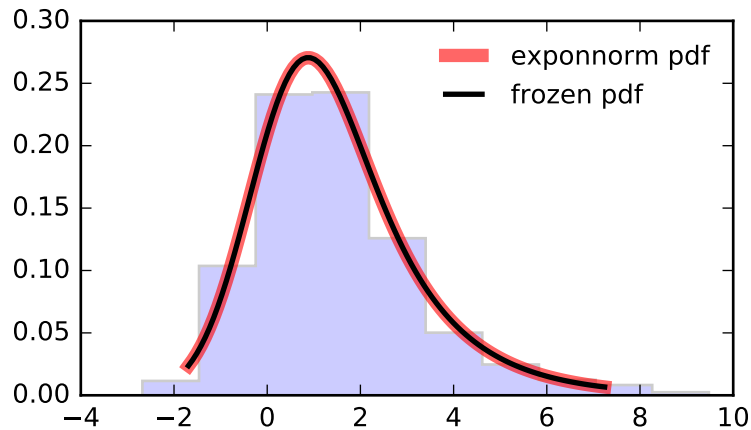
```
>>> vals = exponnorm.ppf([0.001, 0.5, 0.999], K)
>>> np.allclose([0.001, 0.5, 0.999], exponnorm.cdf(vals, K))
True
```

Generate random numbers:

```
>>> r = exponnorm.rvs(K, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre>rvs(K, loc=0, scale=1, size=1,     random_state=None) pdf(x, K, loc=0, scale=1) logpdf(x, K, loc=0, scale=1) cdf(x, K, loc=0, scale=1) logcdf(x, K, loc=0, scale=1)  sf(x, K, loc=0, scale=1)  logsf(x, K, loc=0, scale=1) ppf(q, K, loc=0, scale=1)  isf(q, K, loc=0, scale=1) moment(n, K, loc=0, scale=1) stats(K, loc=0, scale=1, moments='mv')  entropy(K, loc=0, scale=1) fit(data, K, loc=0, scale=1) expect(func, args=(K,), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kwds) median(K, loc=0, scale=1) mean(K, loc=0, scale=1) var(K, loc=0, scale=1) std(K, loc=0, scale=1) interval(alpha, K, loc=0, scale=1)</pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--



`scipy.stats.exponweib` = <scipy.stats.\_continuous\_distns.exponweib\_gen object at 0x2b909bada790>  
An exponentiated Weibull continuous random variable.

As an instance of the `rv_continuous` class, `exponweib` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `exponweib` is:

```
exponweib.pdf(x, a, c) =
    a * c * (1-exp(-x**c))**(a-1) * exp(-x**c)*x**(c-1)
```

for  $x > 0, a > 0, c > 0$ .

`exponweib` takes `a` and `c` as shape parameters.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `exponweib.pdf(x, a, c, loc, scale)` is identically equivalent to `exponweib.pdf(y, a, c) / scale` with  $y = (x - loc) / scale$ .

### Examples

```
>>> from scipy.stats import exponweib
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> a, c = 2.89, 1.95
>>> mean, var, skew, kurt = exponweib.stats(a, c, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(exponweib.ppf(0.01, a, c),
...                 exponweib.ppf(0.99, a, c), 100)
>>> ax.plot(x, exponweib.pdf(x, a, c),
...         'r-', lw=5, alpha=0.6, label='exponweib pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = exponweib(a, c)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

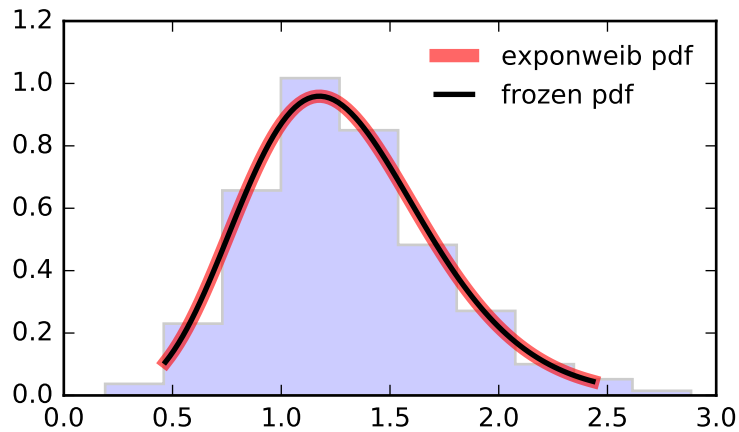
```
>>> vals = exponweib.ppf([0.001, 0.5, 0.999], a, c)
>>> np.allclose([0.001, 0.5, 0.999], exponweib.cdf(vals, a, c))
True
```

Generate random numbers:

```
>>> r = exponweib.rvs(a, c, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



**Methods**

<pre>rvs(a, c, loc=0, scale=1, size=1,     random_state=None) pdf(x, a, c, loc=0, scale=1) logpdf(x, a, c, loc=0, scale=1) cdf(x, a, c, loc=0, scale=1) logcdf(x, a, c, loc=0, scale=1)  sf(x, a, c, loc=0, scale=1)  logsf(x, a, c, loc=0, scale=1) ppf(q, a, c, loc=0, scale=1) isf(q, a, c, loc=0, scale=1)  moment(n, a, c, loc=0, scale=1) stats(a, c, loc=0, scale=1, moments='mv')  entropy(a, c, loc=0, scale=1) fit(data, a, c, loc=0, scale=1) expect(func, args=(a, c), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kwds) median(a, c, loc=0, scale=1) mean(a, c, loc=0, scale=1) var(a, c, loc=0, scale=1) std(a, c, loc=0, scale=1) interval(alpha, a, c, loc=0, scale=1)</pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

`scipy.stats.exponpow` = <scipy.stats.\_continuous\_distns.exponpow\_gen object at 0x2b909badaa10>  
An exponential power continuous random variable.

As an instance of the `rv_continuous` class, `exponpow` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `exponpow` is:

$$\text{exponpow.pdf}(x, b) = b * x^{(b-1)} * \exp(1 + x^{*b} - \exp(x^{*b}))$$

for  $x \geq 0, b > 0$ . Note that this is a different distribution from the exponential power distribution that is also known under the names “generalized normal” or “generalized Gaussian”.

`exponpow` takes `b` as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `exponpow.pdf(x, b, loc, scale)` is identically equivalent to `exponpow.pdf(y, b) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

### References

<http://www.math.wm.edu/~leemis/chart/UDR/PDFs/Exponentialpower.pdf>

### Examples

```
>>> from scipy.stats import exponpow
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> b = 2.7
>>> mean, var, skew, kurt = exponpow.stats(b, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(exponpow.ppf(0.01, b),
...                 exponpow.ppf(0.99, b), 100)
>>> ax.plot(x, exponpow.pdf(x, b),
...         'r-', lw=5, alpha=0.6, label='exponpow pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = exponpow(b)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

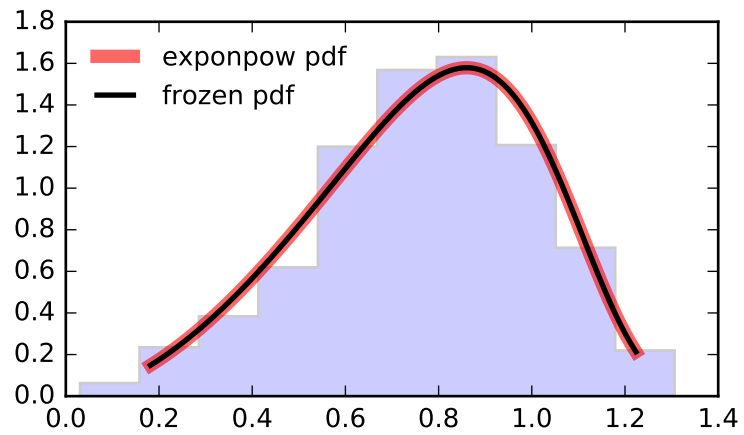
```
>>> vals = exponpow.ppf([0.001, 0.5, 0.999], b)
>>> np.allclose([0.001, 0.5, 0.999], exponpow.cdf(vals, b))
True
```

Generate random numbers:

```
>>> r = exponpow.rvs(b, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



**Methods**

<code>rvs(b, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, b, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, b, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, b, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, b, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, b, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, b, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, b, loc=0, scale=1)</code>	Percent point function (inverse of <i>cdf</i> — percentiles).
<code>isf(q, b, loc=0, scale=1)</code>	Inverse survival function (inverse of <i>sf</i> ).
<code>moment(n, b, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(b, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(b, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, b, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(b,), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(b, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(b, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(b, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(b, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, b, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.f` = <scipy.stats.continuous\_distns.f\_gen object at 0x2b909bae41d0>

An F continuous random variable.

As an instance of the `rv_continuous` class, `f` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**Notes**

The probability density function for `f` is:

$$F.\text{pdf}(x, \text{df1}, \text{df2}) = \frac{\text{df2}^{df2/2} * \text{df1}^{df1/2} * x^{df1/2-1}}{(\text{df2} + \text{df1} * x)^{df1/2} * B(\text{df1}/2, \text{df2}/2)}$$

for  $x > 0$ .

`f` takes `dfn` and `dfd` as shape parameters.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `f.pdf(x, dfn, dfd, loc, scale)` is identically equivalent to `f.pdf(y, dfn, dfd) / scale` with `y = (x - loc) / scale`.

**Examples**

```
>>> from scipy.stats import f
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> dfn, dfd = 29, 18
>>> mean, var, skew, kurt = f.stats(dfn, dfd, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(f.ppf(0.01, dfn, dfd),
...                 f.ppf(0.99, dfn, dfd), 100)
>>> ax.plot(x, f.pdf(x, dfn, dfd),
...         'r-', lw=5, alpha=0.6, label='f pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = f(dfn, dfd)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

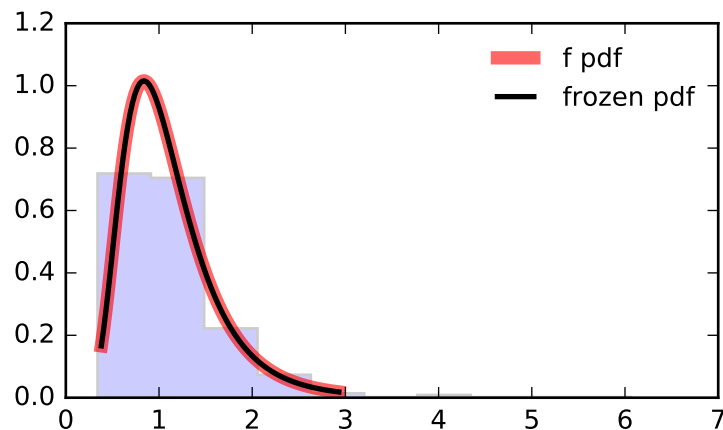
```
>>> vals = f.ppf([0.001, 0.5, 0.999], dfn, dfd)
>>> np.allclose([0.001, 0.5, 0.999], f.cdf(vals, dfn, dfd))
True
```

Generate random numbers:

```
>>> r = f.rvs(dfn, dfd, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



**Methods**

<code>rvs(dfn, dfd, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, dfn, dfd, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, dfn, dfd, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, dfn, dfd, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, dfn, dfd, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, dfn, dfd, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, dfn, dfd, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, dfn, dfd, loc=0, scale=1)</code>	Percent point function (inverse of <code>cdf</code> — percentiles).
<code>isf(q, dfn, dfd, loc=0, scale=1)</code>	Inverse survival function (inverse of <code>sf</code> ).
<code>moment(n, dfn, dfd, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(dfn, dfd, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(dfn, dfd, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, dfn, dfd, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(dfn, dfd), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(dfn, dfd, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(dfn, dfd, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(dfn, dfd, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(dfn, dfd, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, dfn, dfd, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.fatiguelife = <scipy.stats._continuous_distns.fatiguelife_gen object at 0x2b909badac90>`  
 A fatigue-life (Birnbbaum-Saunders) continuous random variable.

As an instance of the `rv_continuous` class, `fatiguelife` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**Notes**

The probability density function for `fatiguelife` is:

$$\text{fatiguelife.pdf}(x, c) = \frac{(x+1)}{(2*c*\sqrt{2*\pi*x**3})} * \exp(-(x-1)**2/(2*x*c**2))$$

for  $x > 0$ .

`fatiguelife` takes *c* as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `fatiguelife.pdf(x, c, loc, scale)` is identically equivalent to `fatiguelife.pdf(y, c) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

**References**

[R515]

### Examples

```
>>> from scipy.stats import fatiguelife
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> c = 29
>>> mean, var, skew, kurt = fatiguelife.stats(c, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(fatiguelife.ppf(0.01, c),
...                 fatiguelife.ppf(0.99, c), 100)
>>> ax.plot(x, fatiguelife.pdf(x, c),
...         'r-', lw=5, alpha=0.6, label='fatiguelife pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = fatiguelife(c)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

```
>>> vals = fatiguelife.ppf([0.001, 0.5, 0.999], c)
>>> np.allclose([0.001, 0.5, 0.999], fatiguelife.cdf(vals, c))
True
```

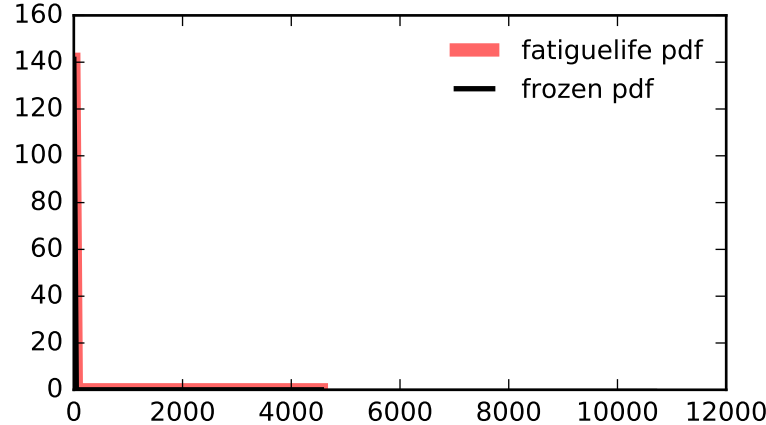
Generate random numbers:

```
>>> r = fatiguelife.rvs(c, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```





### Methods

<code>rvs(c, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, c, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, c, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, c, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, c, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, c, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, c, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, c, loc=0, scale=1)</code>	Percent point function (inverse of <i>cdf</i> — percentiles).
<code>isf(q, c, loc=0, scale=1)</code>	Inverse survival function (inverse of <i>sf</i> ).
<code>moment(n, c, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(c, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(c, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, c, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(c,), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(c, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(c, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(c, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(c, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, c, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.fisk = <scipy.stats.continuous_distns.fisk_gen object at 0x2b909bac7250>`

A Fisk continuous random variable.

The Fisk distribution is also known as the log-logistic distribution, and equals the Burr distribution with  $d == 1$ .

`fisk` takes `c` as a shape parameter.

As an instance of the `rv_continuous` class, `fisk` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**See also:**

`burr`

**Notes**

The probability density function for `fisk` is:

```
fisk.pdf(x, c) = c * x**(-c-1) * (1 + x**(-c))**(-2)
```

for  $x > 0$ .

`fisk` takes `c` as a shape parameters.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `fisk.pdf(x, c, loc, scale)` is identically equivalent to `fisk.pdf(y, c) / scale` with  $y = (x - loc) / scale$ .

**Examples**

```
>>> from scipy.stats import fisk
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> c = 3.09
>>> mean, var, skew, kurt = fisk.stats(c, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(fisk.ppf(0.01, c),
...                 fisk.ppf(0.99, c), 100)
>>> ax.plot(x, fisk.pdf(x, c),
...        'r-', lw=5, alpha=0.6, label='fisk pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = fisk(c)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

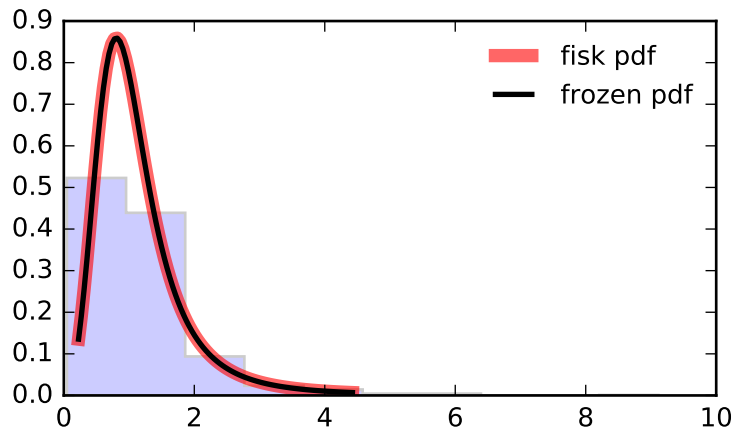
```
>>> vals = fisk.ppf([0.001, 0.5, 0.999], c)
>>> np.allclose([0.001, 0.5, 0.999], fisk.cdf(vals, c))
True
```

Generate random numbers:

```
>>> r = fisk.rvs(c, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre>rvs(c, loc=0, scale=1, size=1,     random_state=None) pdf(x, c, loc=0, scale=1) logpdf(x, c, loc=0, scale=1) cdf(x, c, loc=0, scale=1) logcdf(x, c, loc=0, scale=1)  sf(x, c, loc=0, scale=1)  logsf(x, c, loc=0, scale=1) ppf(q, c, loc=0, scale=1)  isf(q, c, loc=0, scale=1) moment(n, c, loc=0, scale=1) stats(c, loc=0, scale=1, moments='mv')  entropy(c, loc=0, scale=1) fit(data, c, loc=0, scale=1) expect(func, args=(c,), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kwargs) median(c, loc=0, scale=1) mean(c, loc=0, scale=1) var(c, loc=0, scale=1) std(c, loc=0, scale=1) interval(alpha, c, loc=0, scale=1)</pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

```
scipy.stats.foldcauchy = <scipy.stats._continuous_distns.foldcauchy_gen object at 0x2b909badaf10>
```

A folded Cauchy continuous random variable.

As an instance of the `rv_continuous` class, `foldcauchy` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `foldcauchy` is:

```
foldcauchy.pdf(x, c) = 1/(pi*(1+(x-c)**2)) + 1/(pi*(1+(x+c)**2))
```

for  $x \geq 0$ .

`foldcauchy` takes `c` as a shape parameter.

### Examples

```
>>> from scipy.stats import foldcauchy
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> c = 4.72
>>> mean, var, skew, kurt = foldcauchy.stats(c, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(foldcauchy.ppf(0.01, c),
...                 foldcauchy.ppf(0.99, c), 100)
>>> ax.plot(x, foldcauchy.pdf(x, c),
...         'r-', lw=5, alpha=0.6, label='foldcauchy pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = foldcauchy(c)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

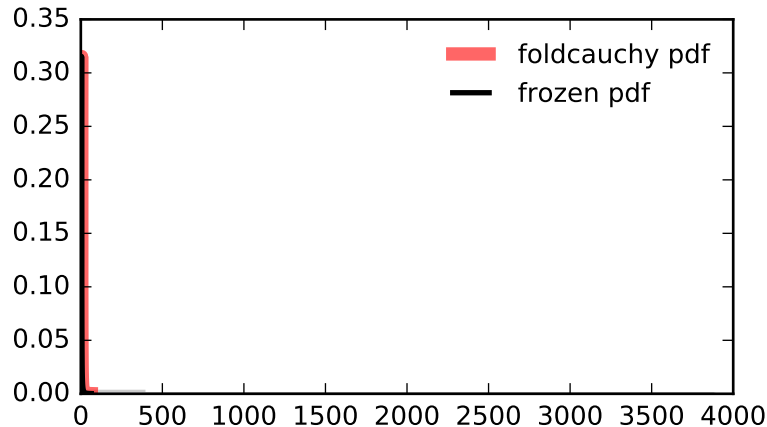
```
>>> vals = foldcauchy.ppf([0.001, 0.5, 0.999], c)
>>> np.allclose([0.001, 0.5, 0.999], foldcauchy.cdf(vals, c))
True
```

Generate random numbers:

```
>>> r = foldcauchy.rvs(c, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<code>rvs(c, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, c, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, c, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, c, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, c, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, c, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, c, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, c, loc=0, scale=1)</code>	Percent point function (inverse of <i>cdf</i> — percentiles).
<code>isf(q, c, loc=0, scale=1)</code>	Inverse survival function (inverse of <i>sf</i> ).
<code>moment(n, c, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(c, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(c, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, c, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(c,), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(c, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(c, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(c, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(c, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, c, loc=0, scale=1)</code>	Endpoints of the range that contains <i>alpha</i> percent of the distribution

`scipy.stats.foldnorm = <scipy.stats.continuous_distns.foldnorm_gen object at 0x2b909bae4450>`  
 A folded normal continuous random variable.

As an instance of the `rv_continuous` class, `foldnorm` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `foldnorm` is:

```
foldnormal.pdf(x, c) = sqrt(2/pi) * cosh(c*x) * exp(-(x**2+c**2)/2)
```

for  $c \geq 0$ .

`foldnorm` takes  $c$  as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `foldnorm.pdf(x, c, loc, scale)` is identically equivalent to `foldnorm.pdf(y, c) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

### Examples

```
>>> from scipy.stats import foldnorm
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> c = 1.95
>>> mean, var, skew, kurt = foldnorm.stats(c, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(foldnorm.ppf(0.01, c),
...                 foldnorm.ppf(0.99, c), 100)
>>> ax.plot(x, foldnorm.pdf(x, c),
...        'r-', lw=5, alpha=0.6, label='foldnorm pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = foldnorm(c)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

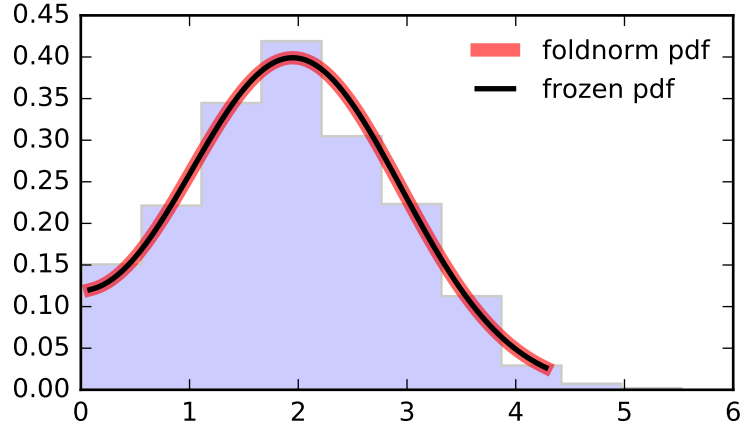
```
>>> vals = foldnorm.ppf([0.001, 0.5, 0.999], c)
>>> np.allclose([0.001, 0.5, 0.999], foldnorm.cdf(vals, c))
True
```

Generate random numbers:

```
>>> r = foldnorm.rvs(c, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<code>rvs(c, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, c, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, c, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, c, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, c, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, c, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, c, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, c, loc=0, scale=1)</code>	Percent point function (inverse of <i>cdf</i> — percentiles).
<code>isf(q, c, loc=0, scale=1)</code>	Inverse survival function (inverse of <i>sf</i> ).
<code>moment(n, c, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(c, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(c, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, c, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(c,), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(c, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(c, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(c, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(c, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, c, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.frechet_r = <scipy.stats.continuous_distns.frechet_r_gen object at 0x2b909bae46d0>`  
 A Frchet right (or Weibull minimum) continuous random variable.

As an instance of the `rv_continuous` class, `frechet_r` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

See also:

**`weibull_min`**

The same distribution as `frechet_r`.

`frechet_l`, `weibull_max`

**Notes**

The probability density function for `frechet_r` is:

```
frechet_r.pdf(x, c) = c * x**(c-1) * exp(-x**c)
```

for  $x > 0, c > 0$ .

`frechet_r` takes `c` as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `frechet_r.pdf(x, c, loc, scale)` is identically equivalent to `frechet_r.pdf(y, c) / scale` with  $y = (x - loc) / scale$ .

**Examples**

```
>>> from scipy.stats import frechet_r
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> c = 1.89
>>> mean, var, skew, kurt = frechet_r.stats(c, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(frechet_r.ppf(0.01, c),
...                 frechet_r.ppf(0.99, c), 100)
>>> ax.plot(x, frechet_r.pdf(x, c),
...         'r-', lw=5, alpha=0.6, label='frechet_r pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = frechet_r(c)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

```
>>> vals = frechet_r.ppf([0.001, 0.5, 0.999], c)
>>> np.allclose([0.001, 0.5, 0.999], frechet_r.cdf(vals, c))
True
```

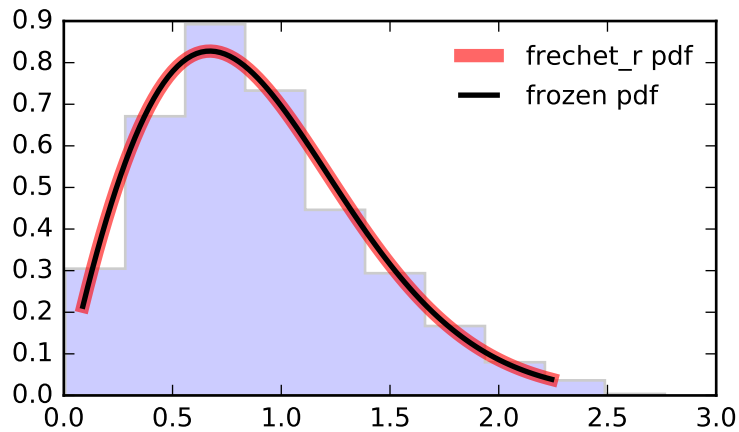
Generate random numbers:

```
>>> r = frechet_r.rvs(c, size=1000)
```

And compare the histogram:



```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre>rvs(c, loc=0, scale=1, size=1,     random_state=None) pdf(x, c, loc=0, scale=1) logpdf(x, c, loc=0, scale=1) cdf(x, c, loc=0, scale=1) logcdf(x, c, loc=0, scale=1)  sf(x, c, loc=0, scale=1)  logsf(x, c, loc=0, scale=1) ppf(q, c, loc=0, scale=1)  isf(q, c, loc=0, scale=1) moment(n, c, loc=0, scale=1) stats(c, loc=0, scale=1, moments='mv')  entropy(c, loc=0, scale=1) fit(data, c, loc=0, scale=1) expect(func, args=(c,), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kwargs) median(c, loc=0, scale=1) mean(c, loc=0, scale=1) var(c, loc=0, scale=1) std(c, loc=0, scale=1) interval(alpha, c, loc=0, scale=1)</pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

`scipy.stats.frechet_1` = <scipy.stats.\_continuous\_distns.frechet\_1\_gen object at 0x2b909bae4b90>

A Frechet left (or Weibull maximum) continuous random variable.

As an instance of the `rv_continuous` class, `frechet_l` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**See also:**

`weibull_max`

The same distribution as `frechet_l`.

`frechet_r`, `weibull_min`

**Notes**

The probability density function for `frechet_l` is:

```
frechet_l.pdf(x, c) = c * (-x)**(c-1) * exp(-(-x)**c)
```

for  $x < 0, c > 0$ .

`frechet_l` takes `c` as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `frechet_l.pdf(x, c, loc, scale)` is identically equivalent to `frechet_l.pdf(y, c) / scale` with  $y = (x - loc) / scale$ .

**Examples**

```
>>> from scipy.stats import frechet_l
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> c = 3.63
>>> mean, var, skew, kurt = frechet_l.stats(c, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(frechet_l.ppf(0.01, c),
...                 frechet_l.ppf(0.99, c), 100)
>>> ax.plot(x, frechet_l.pdf(x, c),
...        'r-', lw=5, alpha=0.6, label='frechet_l pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = frechet_l(c)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

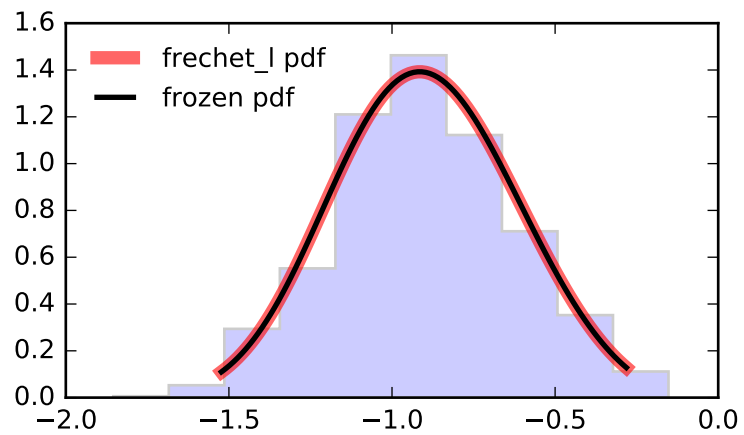
```
>>> vals = frechet_l.ppf([0.001, 0.5, 0.999], c)
>>> np.allclose([0.001, 0.5, 0.999], frechet_l.cdf(vals, c))
True
```

Generate random numbers:

```
>>> r = frechet_l.rvs(c, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<code>rvs(c, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, c, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, c, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, c, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, c, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, c, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, c, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, c, loc=0, scale=1)</code>	Percent point function (inverse of <i>cdf</i> — percentiles).
<code>isf(q, c, loc=0, scale=1)</code>	Inverse survival function (inverse of <i>sf</i> ).
<code>moment(n, c, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(c, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(c, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, c, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(c,), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(c, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(c, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(c, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(c, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, c, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.genlogistic = <scipy.stats.continuous_distns.genlogistic_gen object at 0x2b909bd28090>`  
 A generalized logistic continuous random variable.

As an instance of the `rv_continuous` class, `genlogistic` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `genlogistic` is:

$$\text{genlogistic.pdf}(x, c) = c * \exp(-x) / (1 + \exp(-x))^{c+1}$$

for  $x > 0, c > 0$ .

`genlogistic` takes *c* as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the *loc* and *scale* parameters. Specifically, `genlogistic.pdf(x, c, loc, scale)` is identically equivalent to `genlogistic.pdf(y, c) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

### Examples

```
>>> from scipy.stats import genlogistic
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> c = 0.412
>>> mean, var, skew, kurt = genlogistic.stats(c, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(genlogistic.ppf(0.01, c),
...                 genlogistic.ppf(0.99, c), 100)
>>> ax.plot(x, genlogistic.pdf(x, c),
...        'r-', lw=5, alpha=0.6, label='genlogistic pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = genlogistic(c)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

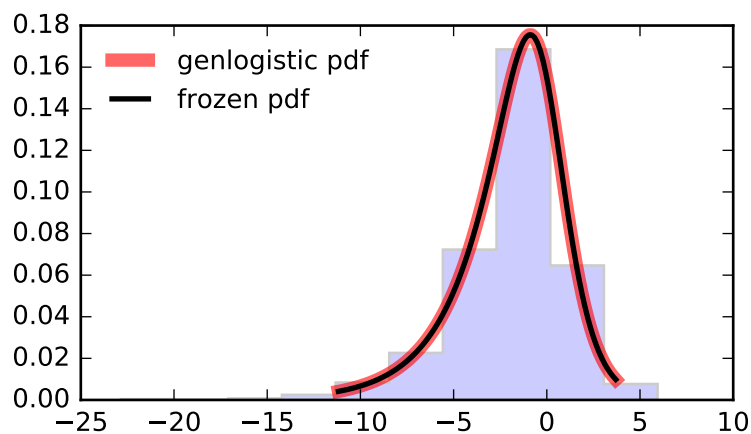
```
>>> vals = genlogistic.ppf([0.001, 0.5, 0.999], c)
>>> np.allclose([0.001, 0.5, 0.999], genlogistic.cdf(vals, c))
True
```

Generate random numbers:

```
>>> r = genlogistic.rvs(c, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



**Methods**

<pre> rvs(c, loc=0, scale=1, size=1,     random_state=None) pdf(x, c, loc=0, scale=1) logpdf(x, c, loc=0, scale=1) cdf(x, c, loc=0, scale=1) logcdf(x, c, loc=0, scale=1)  sf(x, c, loc=0, scale=1)  logsf(x, c, loc=0, scale=1) ppf(q, c, loc=0, scale=1)  isf(q, c, loc=0, scale=1) moment(n, c, loc=0, scale=1) stats(c, loc=0, scale=1, moments='mv')  entropy(c, loc=0, scale=1) fit(data, c, loc=0, scale=1) expect(func, args=(c,), loc=0, scale=1,         lb=None, ub=None, conditional=False,         **kwds) median(c, loc=0, scale=1) mean(c, loc=0, scale=1) var(c, loc=0, scale=1) std(c, loc=0, scale=1) interval(alpha, c, loc=0, scale=1) </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

`scipy.stats.gennorm` = <scipy.stats.continuous\_distns.gennorm\_gen object at 0x2b909bd9a6d0>  
 A generalized normal continuous random variable.

As an instance of the `rv_continuous` class, `gennorm` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**See also:**

- `laplace` Laplace distribution
- `norm` normal distribution

**Notes**

The probability density function for `gennorm` is [R525]:

$$\text{gennorm.pdf}(x, \text{beta}) = \frac{\text{beta}}{2 \cdot \text{gamma}(1/\text{beta})} \exp(-|x|**\text{beta})$$

`gennorm` takes `beta` as a shape parameter. For `beta = 1`, it is identical to a Laplace distribution. For `beta = 2`, it is identical to a normal distribution (with `scale=1/sqrt(2)`).

**References**

[R525]

*Examples*

```
>>> from scipy.stats import gennorm
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> beta = 1.3
>>> mean, var, skew, kurt = gennorm.stats(beta, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(gennorm.ppf(0.01, beta),
...                 gennorm.ppf(0.99, beta), 100)
>>> ax.plot(x, gennorm.pdf(x, beta),
...         'r-', lw=5, alpha=0.6, label='gennorm pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = gennorm(beta)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

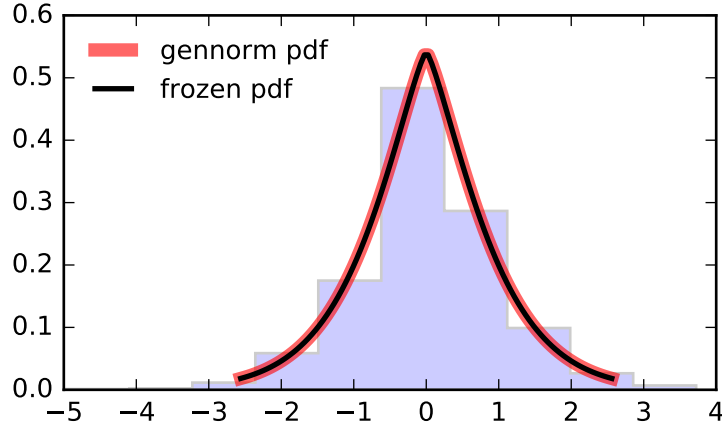
```
>>> vals = gennorm.ppf([0.001, 0.5, 0.999], beta)
>>> np.allclose([0.001, 0.5, 0.999], gennorm.cdf(vals, beta))
True
```

Generate random numbers:

```
>>> r = gennorm.rvs(beta, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre> rvs(beta, loc=0, scale=1, size=1,     random_state=None) pdf(x, beta, loc=0, scale=1) logpdf(x, beta, loc=0, scale=1) cdf(x, beta, loc=0, scale=1) logcdf(x, beta, loc=0, scale=1)  sf(x, beta, loc=0, scale=1)  logsf(x, beta, loc=0, scale=1) ppf(q, beta, loc=0, scale=1)  isf(q, beta, loc=0, scale=1)  moment(n, beta, loc=0, scale=1) stats(beta, loc=0, scale=1, moments='mv')  entropy(beta, loc=0, scale=1) fit(data, beta, loc=0, scale=1) expect(func, args=(beta,), loc=0, scale=1,     lb=None, ub=None, conditional=False,     **kwds) median(beta, loc=0, scale=1) mean(beta, loc=0, scale=1) var(beta, loc=0, scale=1) std(beta, loc=0, scale=1) interval(alpha, beta, loc=0, scale=1)                 </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <math>\text{sf}</math> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <math>\text{cdf}</math> — percentiles).</p> <p>Inverse survival function (inverse of <math>\text{sf}</math>).</p> <p>Non-central moment of order <math>n</math></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains <math>\alpha</math> percent of the distribution</p>
---	---

`scipy.stats.genpareto` = <scipy.stats.continuous\_distns.genpareto\_gen object at 0x2b909bd28310>  
 A generalized Pareto continuous random variable.

As an instance of the `rv_continuous` class, `genpareto` object inherits from it a collection of generic



methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `genpareto` is:

$$\text{genpareto.pdf}(x, c) = (1 + c * x)^{-1 - 1/c}$$

defined for  $x \geq 0$  if  $c \geq 0$ , and for  $0 \leq x \leq -1/c$  if  $c < 0$ .

`genpareto` takes  $c$  as a shape parameter.

For  $c == 0$ , `genpareto` reduces to the exponential distribution, `expon`:

$$\text{genpareto.pdf}(x, c=0) = \exp(-x)$$

For  $c == -1$ , `genpareto` is uniform on  $[0, 1]$ :

$$\text{genpareto.cdf}(x, c=-1) = x$$

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `genpareto.pdf(x, c, loc, scale)` is identically equivalent to `genpareto.pdf(y, c) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

### Examples

```
>>> from scipy.stats import genpareto
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> c = 0.1
>>> mean, var, skew, kurt = genpareto.stats(c, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(genpareto.ppf(0.01, c),
...                 genpareto.ppf(0.99, c), 100)
>>> ax.plot(x, genpareto.pdf(x, c),
...        'r-', lw=5, alpha=0.6, label='genpareto pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = genpareto(c)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

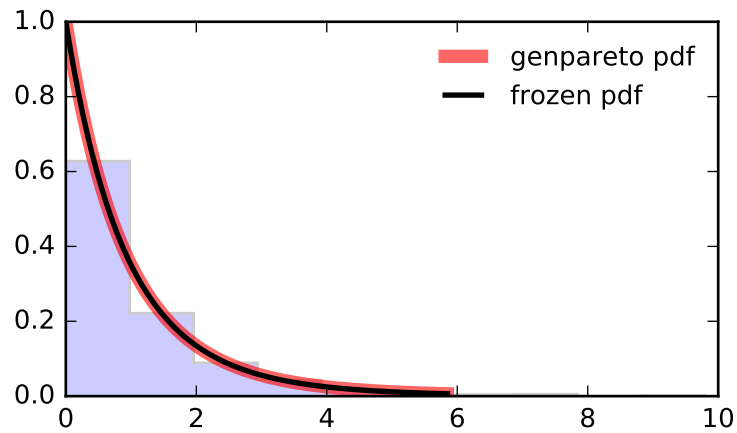
```
>>> vals = genpareto.ppf([0.001, 0.5, 0.999], c)
>>> np.allclose([0.001, 0.5, 0.999], genpareto.cdf(vals, c))
True
```

Generate random numbers:

```
>>> r = genpareto.rvs(c, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



**Methods**

<pre> rvs(c, loc=0, scale=1, size=1,     random_state=None) pdf(x, c, loc=0, scale=1) logpdf(x, c, loc=0, scale=1) cdf(x, c, loc=0, scale=1) logcdf(x, c, loc=0, scale=1)  sf(x, c, loc=0, scale=1)  logsf(x, c, loc=0, scale=1) ppf(q, c, loc=0, scale=1)  isf(q, c, loc=0, scale=1) moment(n, c, loc=0, scale=1) stats(c, loc=0, scale=1, moments='mv')  entropy(c, loc=0, scale=1) fit(data, c, loc=0, scale=1) expect(func, args=(c,), loc=0, scale=1,         lb=None, ub=None, conditional=False,         **kwds) median(c, loc=0, scale=1) mean(c, loc=0, scale=1) var(c, loc=0, scale=1) std(c, loc=0, scale=1) interval(alpha, c, loc=0, scale=1) </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

`scipy.stats.genexpon = <scipy.stats.continuous_distns.genexpon_gen object at 0x2b909bd28590>`  
 A generalized exponential continuous random variable.

As an instance of the `rv_continuous` class, `genexpon` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**Notes**

The probability density function for `genexpon` is:

$$\text{genexpon.pdf}(x, a, b, c) = (a + b * (1 - \exp(-c*x))) * \exp(-a - b * \exp(-c*x))$$

for  $x \geq 0, a, b, c > 0$ .

`genexpon` takes *a*, *b* and *c* as shape parameters.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `genexpon.pdf(x, a, b, c, loc, scale)` is identically equivalent to `genexpon.pdf(y, a, b, c) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

**References**

H.K. Ryu, “An Extension of Marshall and Olkin’s Bivariate Exponential Distribution”, Journal of the American Statistical Association, 1993.

N. Balakrishnan, “The Exponential Distribution: Theory, Methods and Applications”, Asit P. Basu.

### Examples

```
>>> from scipy.stats import genexpon
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> a, b, c = 9.13, 16.2, 3.28
>>> mean, var, skew, kurt = genexpon.stats(a, b, c, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(genexpon.ppf(0.01, a, b, c),
...                 genexpon.ppf(0.99, a, b, c), 100)
>>> ax.plot(x, genexpon.pdf(x, a, b, c),
...        'r-', lw=5, alpha=0.6, label='genexpon pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = genexpon(a, b, c)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

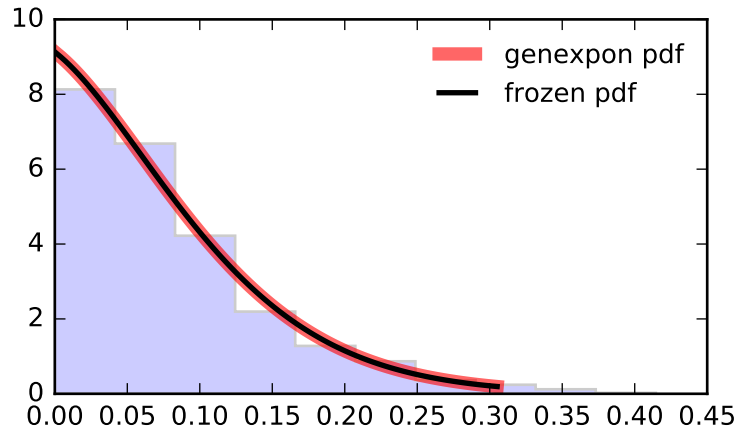
```
>>> vals = genexpon.ppf([0.001, 0.5, 0.999], a, b, c)
>>> np.allclose([0.001, 0.5, 0.999], genexpon.cdf(vals, a, b, c))
True
```

Generate random numbers:

```
>>> r = genexpon.rvs(a, b, c, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<code>rvs(a, b, c, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, a, b, c, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, a, b, c, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, a, b, c, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, a, b, c, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, a, b, c, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, a, b, c, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, a, b, c, loc=0, scale=1)</code>	Percent point function (inverse of <i>cdf</i> — percentiles).
<code>isf(q, a, b, c, loc=0, scale=1)</code>	Inverse survival function (inverse of <i>sf</i> ).
<code>moment(n, a, b, c, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(a, b, c, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(a, b, c, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, a, b, c, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(a, b, c), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(a, b, c, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(a, b, c, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(a, b, c, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(a, b, c, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, a, b, c, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.genextreme = <scipy.stats.continuous_distns.genextreme_gen object at 0x2b909bd28850>`  
 A generalized extreme value continuous random variable.

As an instance of the `rv_continuous` class, `genextreme` object inherits from it a collection of generic

methods (see below for the full list), and completes them with details specific for this particular distribution.

**See also:**

`gumbel_r`

**Notes**

For  $c=0$ , `genextreme` is equal to `gumbel_r`. The probability density function for `genextreme` is:

$$\text{genextreme.pdf}(x, c) = \begin{cases} \exp(-\exp(-x)) * \exp(-x), & \text{for } c=0 \\ \exp(-(1-c*x)**(1/c)) * (1-c*x)**(1/c-1), & \text{for } x \leq 1/c, c > 0 \end{cases}$$

Note that several sources and software packages use the opposite convention for the sign of the shape parameter  $c$ .

`genextreme` takes  $c$  as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `genextreme.pdf(x, c, loc, scale)` is identically equivalent to `genextreme.pdf(y, c) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

**Examples**

```
>>> from scipy.stats import genextreme
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> c = -0.1
>>> mean, var, skew, kurt = genextreme.stats(c, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(genextreme.ppf(0.01, c),
...                 genextreme.ppf(0.99, c), 100)
>>> ax.plot(x, genextreme.pdf(x, c),
...         'r-', lw=5, alpha=0.6, label='genextreme pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = genextreme(c)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

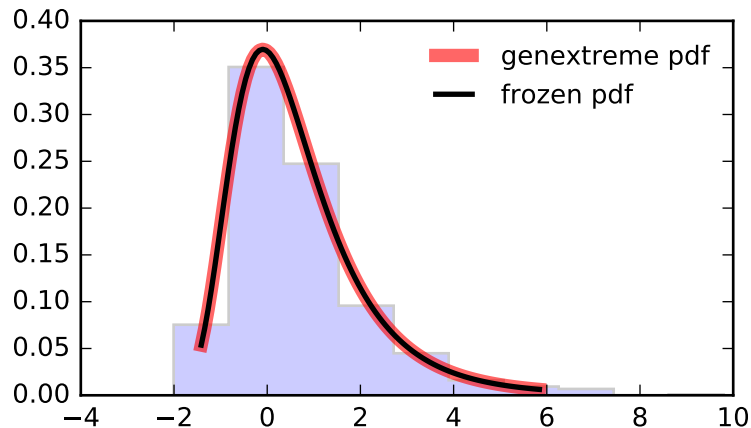
```
>>> vals = genextreme.ppf([0.001, 0.5, 0.999], c)
>>> np.allclose([0.001, 0.5, 0.999], genextreme.cdf(vals, c))
True
```

Generate random numbers:

```
>>> r = genextreme.rvs(c, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

```
rvs(c, loc=0, scale=1, size=1,
    random_state=None)
pdf(x, c, loc=0, scale=1)
logpdf(x, c, loc=0, scale=1)
cdf(x, c, loc=0, scale=1)
logcdf(x, c, loc=0, scale=1)

sf(x, c, loc=0, scale=1)

logsf(x, c, loc=0, scale=1)
ppf(q, c, loc=0, scale=1)

isf(q, c, loc=0, scale=1)
moment(n, c, loc=0, scale=1)
stats(c, loc=0, scale=1, moments='mv')

entropy(c, loc=0, scale=1)
fit(data, c, loc=0, scale=1)
expect(func, args=(c,), loc=0, scale=1,
       lb=None, ub=None, conditional=False,
       **kwds)
median(c, loc=0, scale=1)
mean(c, loc=0, scale=1)
var(c, loc=0, scale=1)
std(c, loc=0, scale=1)
interval(alpha, c, loc=0, scale=1)
```

Random variates.

Probability density function.

Log of the probability density function.

Cumulative distribution function.

Log of the cumulative distribution function.

Survival function (also defined as  $1 - \text{cdf}$ , but *sf* is sometimes more accurate).

Log of the survival function.

Percent point function (inverse of *cdf* — percentiles).

Inverse survival function (inverse of *sf*).

Non-central moment of order *n*

Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').

(Differential) entropy of the RV.

Parameter estimates for generic data.

Expected value of a function (of one argument) with respect to the distribution.

Median of the distribution.

Mean of the distribution.

Variance of the distribution.

Standard deviation of the distribution.

Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.gausshyper` = <scipy.stats.\_continuous\_distns.gausshyper\_gen object at 0x2b909bd413d0>  
 A Gauss hypergeometric continuous random variable.

As an instance of the `rv_continuous` class, `gausshyper` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `gausshyper` is:

```
gausshyper.pdf(x, a, b, c, z) =
    C * x**(a-1) * (1-x)**(b-1) * (1+z*x)**(-c)
```

for  $0 \leq x \leq 1, a > 0, b > 0,$  and  $C = 1 / (B(a, b) F[2, 1](c, a; a+b; -z))$

`gausshyper` takes `a, b, c` and `z` as shape parameters.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `gausshyper.pdf(x, a, b, c, z, loc, scale)` is identically equivalent to `gausshyper.pdf(y, a, b, c, z) / scale` with  $y = (x - loc) / scale$ .

### Examples

```
>>> from scipy.stats import gausshyper
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> a, b, c, z = 13.8, 3.12, 2.51, 5.18
>>> mean, var, skew, kurt = gausshyper.stats(a, b, c, z, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(gausshyper.ppf(0.01, a, b, c, z),
...                 gausshyper.ppf(0.99, a, b, c, z), 100)
>>> ax.plot(x, gausshyper.pdf(x, a, b, c, z),
...         'r-', lw=5, alpha=0.6, label='gausshyper pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = gausshyper(a, b, c, z)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

```
>>> vals = gausshyper.ppf([0.001, 0.5, 0.999], a, b, c, z)
>>> np.allclose([0.001, 0.5, 0.999], gausshyper.cdf(vals, a, b, c, z))
True
```

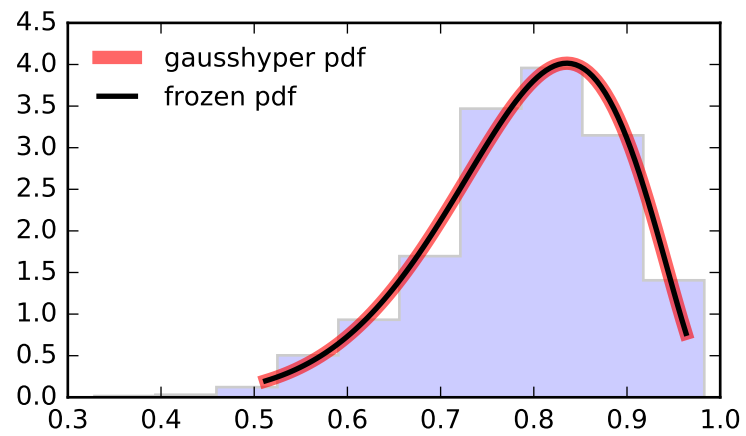
Generate random numbers:

```
>>> r = gausshyper.rvs(a, b, c, z, size=1000)
```



And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



**Methods**

<pre> rvs(a, b, c, z, loc=0, scale=1, size=1,     random_state=None) pdf(x, a, b, c, z, loc=0, scale=1) logpdf(x, a, b, c, z, loc=0, scale=1)  cdf(x, a, b, c, z, loc=0, scale=1) logcdf(x, a, b, c, z, loc=0, scale=1)  sf(x, a, b, c, z, loc=0, scale=1)  logsf(x, a, b, c, z, loc=0, scale=1) ppf(q, a, b, c, z, loc=0, scale=1)  isf(q, a, b, c, z, loc=0, scale=1)  moment(n, a, b, c, z, loc=0, scale=1) stats(a, b, c, z, loc=0, scale=1,       moments='mv') entropy(a, b, c, z, loc=0, scale=1) fit(data, a, b, c, z, loc=0, scale=1) expect(func, args=(a, b, c, z), loc=0,        scale=1, lb=None, ub=None,        conditional=False, **kwds) median(a, b, c, z, loc=0, scale=1) mean(a, b, c, z, loc=0, scale=1) var(a, b, c, z, loc=0, scale=1) std(a, b, c, z, loc=0, scale=1) interval(alpha, a, b, c, z, loc=0, scale=1) </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains <i>alpha</i> percent of the distribution</p>
--	---

`scipy.stats.gamma` = <scipy.stats.\_continuous\_distns.gamma\_gen object at 0x2b909bd28ad0>

A gamma continuous random variable.

As an instance of the `rv_continuous` class, `gamma` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**See also:**

`erlang`, `expon`

**Notes**

The probability density function for `gamma` is:

$$\text{gamma.pdf}(x, a) = x^{a-1} * \exp(-x) / \text{gamma}(a)$$

for  $x \geq 0, a > 0$ . Here `gamma(a)` refers to the gamma function.

`gamma` has a shape parameter *a* which needs to be set explicitly.

When *a* is an integer, `gamma` reduces to the Erlang distribution, and when *a*=1 to the exponential distribution.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `gamma.pdf(x, a, loc, scale)` is identically equivalent to `gamma.pdf(y, a) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

*Examples*

```
>>> from scipy.stats import gamma
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> a = 1.99
>>> mean, var, skew, kurt = gamma.stats(a, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(gamma.ppf(0.01, a),
...                 gamma.ppf(0.99, a), 100)
>>> ax.plot(x, gamma.pdf(x, a),
...         'r-', lw=5, alpha=0.6, label='gamma pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = gamma(a)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

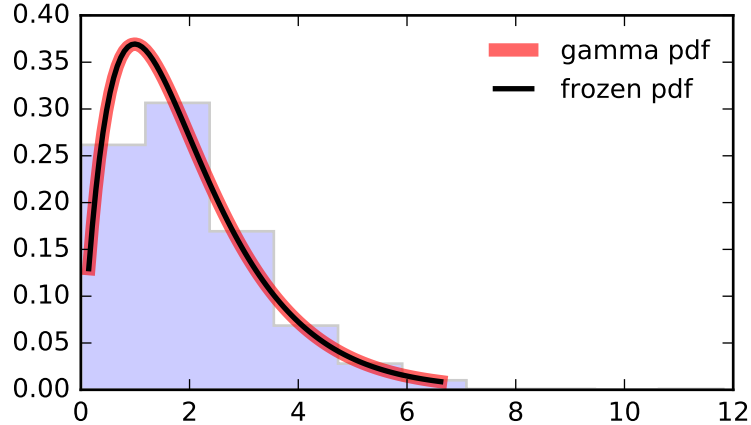
```
>>> vals = gamma.ppf([0.001, 0.5, 0.999], a)
>>> np.allclose([0.001, 0.5, 0.999], gamma.cdf(vals, a))
True
```

Generate random numbers:

```
>>> r = gamma.rvs(a, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre> rvs(a, loc=0, scale=1, size=1,     random_state=None) pdf(x, a, loc=0, scale=1) logpdf(x, a, loc=0, scale=1) cdf(x, a, loc=0, scale=1) logcdf(x, a, loc=0, scale=1)  sf(x, a, loc=0, scale=1)  logsf(x, a, loc=0, scale=1) ppf(q, a, loc=0, scale=1)  isf(q, a, loc=0, scale=1) moment(n, a, loc=0, scale=1) stats(a, loc=0, scale=1, moments='mv')  entropy(a, loc=0, scale=1) fit(data, a, loc=0, scale=1) expect(func, args=(a,), loc=0, scale=1,         lb=None, ub=None, conditional=False,         **kwds) median(a, loc=0, scale=1) mean(a, loc=0, scale=1) var(a, loc=0, scale=1) std(a, loc=0, scale=1) interval(alpha, a, loc=0, scale=1)                     </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

`scipy.stats.gengamma = <scipy.stats.continuous_distns.gengamma_gen object at 0x2b909bd28fd0>`

A generalized gamma continuous random variable.

As an instance of the `rv_continuous` class, `gengamma` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `gengamma` is:

```
gengamma.pdf(x, a, c) = abs(c) * x**(c*a-1) * exp(-x**c) / gamma(a)
```

for  $x \geq 0$ ,  $a > 0$ , and  $c \neq 0$ .

`gengamma` takes `a` and `c` as shape parameters.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `gengamma.pdf(x, a, c, loc, scale)` is identically equivalent to `gengamma.pdf(y, a, c) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

### Examples

```
>>> from scipy.stats import gengamma
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> a, c = 4.42, -3.12
>>> mean, var, skew, kurt = gengamma.stats(a, c, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(gengamma.ppf(0.01, a, c),
...                 gengamma.ppf(0.99, a, c), 100)
>>> ax.plot(x, gengamma.pdf(x, a, c),
...        'r-', lw=5, alpha=0.6, label='gengamma pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = gengamma(a, c)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

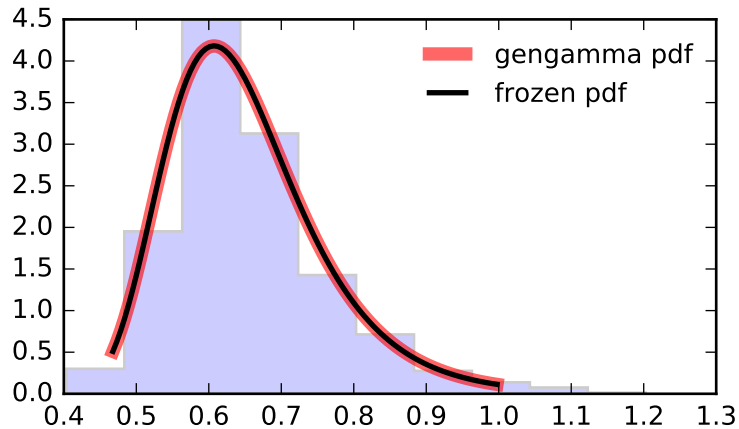
```
>>> vals = gengamma.ppf([0.001, 0.5, 0.999], a, c)
>>> np.allclose([0.001, 0.5, 0.999], gengamma.cdf(vals, a, c))
True
```

Generate random numbers:

```
>>> r = gengamma.rvs(a, c, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<code>rvs(a, c, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, a, c, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, a, c, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, a, c, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, a, c, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, a, c, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, a, c, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, a, c, loc=0, scale=1)</code>	Percent point function (inverse of <i>cdf</i> — percentiles).
<code>isf(q, a, c, loc=0, scale=1)</code>	Inverse survival function (inverse of <i>sf</i> ).
<code>moment(n, a, c, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(a, c, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(a, c, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, a, c, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(a, c), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(a, c, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(a, c, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(a, c, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(a, c, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, a, c, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.genhalflogistic` = <scipy.stats.continuous\_distns.genhalflogistic\_gen object at 0x2b909bd37290>  
 A generalized half-logistic continuous random variable.

As an instance of the `rv_continuous` class, `genhalflogistic` object inherits from it a collection of

generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `genhalflogistic` is:

$$\text{genhalflogistic.pdf}(x, c) = 2 * (1-c*x)**(1/c-1) / (1+(1-c*x)**(1/c))**2$$

for  $0 \leq x \leq 1/c$ , and  $c > 0$ .

`genhalflogistic` takes  $c$  as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `genhalflogistic.pdf(x, c, loc, scale)` is identically equivalent to `genhalflogistic.pdf(y, c) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

### Examples

```
>>> from scipy.stats import genhalflogistic
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> c = 0.773
>>> mean, var, skew, kurt = genhalflogistic.stats(c, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(genhalflogistic.ppf(0.01, c),
...                 genhalflogistic.ppf(0.99, c), 100)
>>> ax.plot(x, genhalflogistic.pdf(x, c),
...         'r-', lw=5, alpha=0.6, label='genhalflogistic pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = genhalflogistic(c)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

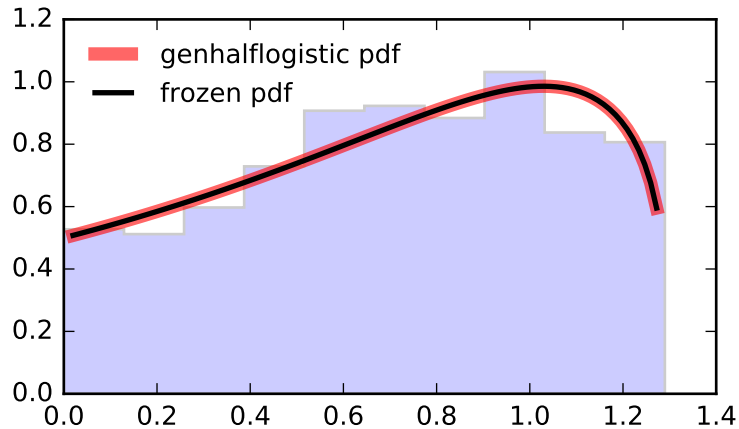
```
>>> vals = genhalflogistic.ppf([0.001, 0.5, 0.999], c)
>>> np.allclose([0.001, 0.5, 0.999], genhalflogistic.cdf(vals, c))
True
```

Generate random numbers:

```
>>> r = genhalflogistic.rvs(c, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre> rvs(c, loc=0, scale=1, size=1,     random_state=None) pdf(x, c, loc=0, scale=1) logpdf(x, c, loc=0, scale=1) cdf(x, c, loc=0, scale=1) logcdf(x, c, loc=0, scale=1)  sf(x, c, loc=0, scale=1)  logsf(x, c, loc=0, scale=1) ppf(q, c, loc=0, scale=1)  isf(q, c, loc=0, scale=1) moment(n, c, loc=0, scale=1) stats(c, loc=0, scale=1, moments='mv')  entropy(c, loc=0, scale=1) fit(data, c, loc=0, scale=1) expect(func, args=(c,), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kwds) median(c, loc=0, scale=1) mean(c, loc=0, scale=1) var(c, loc=0, scale=1) std(c, loc=0, scale=1) interval(alpha, c, loc=0, scale=1)                     </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

`scipy.stats.gilbrat` = <scipy.stats.continuous\_distns.gilbrat\_gen object at 0x2b909bd59550>

A Gilbrat continuous random variable.

As an instance of the `rv_continuous` class, `gilbrat` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.



### Notes

The probability density function for `gilbrat` is:

```
gilbrat.pdf(x) = 1/(x*sqrt(2*pi)) * exp(-1/2*(log(x))**2)
```

`gilbrat` is a special case of `lognorm` with `s = 1`.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `gilbrat.pdf(x, loc, scale)` is identically equivalent to `gilbrat.pdf(y) / scale` with `y = (x - loc) / scale`.

### Examples

```
>>> from scipy.stats import gilbrat
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> mean, var, skew, kurt = gilbrat.stats(moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(gilbrat.ppf(0.01),
...                 gilbrat.ppf(0.99), 100)
>>> ax.plot(x, gilbrat.pdf(x),
...         'r-', lw=5, alpha=0.6, label='gilbrat pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = gilbrat()
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

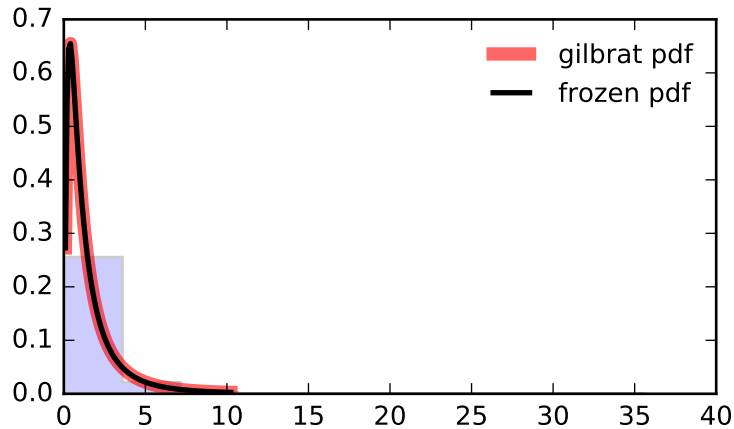
```
>>> vals = gilbrat.ppf([0.001, 0.5, 0.999])
>>> np.allclose([0.001, 0.5, 0.999], gilbrat.cdf(vals))
True
```

Generate random numbers:

```
>>> r = gilbrat.rvs(size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre> rvs(loc=0, scale=1, size=1,     random_state=None) pdf(x, loc=0, scale=1) logpdf(x, loc=0, scale=1) cdf(x, loc=0, scale=1) logcdf(x, loc=0, scale=1)  sf(x, loc=0, scale=1)  logsf(x, loc=0, scale=1) ppf(q, loc=0, scale=1)  isf(q, loc=0, scale=1) moment(n, loc=0, scale=1) stats(loc=0, scale=1, moments='mv')  entropy(loc=0, scale=1) fit(data, loc=0, scale=1) expect(func, args=(), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kwds) median(loc=0, scale=1) mean(loc=0, scale=1) var(loc=0, scale=1) std(loc=0, scale=1) interval(alpha, loc=0, scale=1)                     </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <math>\text{sf}</math> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <math>\text{cdf}</math> — percentiles).</p> <p>Inverse survival function (inverse of <math>\text{sf}</math>).</p> <p>Non-central moment of order <math>n</math></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	---

`scipy.stats.gompertz` = <scipy.stats.\_continuous\_distns.gompertz\_gen object at 0x2b909bd37510>

A Gompertz (or truncated Gumbel) continuous random variable.

As an instance of the `rv_continuous` class, `gompertz` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `gompertz` is:

```
gompertz.pdf(x, c) = c * exp(x) * exp(-c*(exp(x)-1))
```

for  $x \geq 0, c > 0$ .

`gompertz` takes `c` as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `gompertz.pdf(x, c, loc, scale)` is identically equivalent to `gompertz.pdf(y, c) / scale` with  $y = (x - loc) / scale$ .

### Examples

```
>>> from scipy.stats import gompertz
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> c = 0.947
>>> mean, var, skew, kurt = gompertz.stats(c, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(gompertz.ppf(0.01, c),
...                 gompertz.ppf(0.99, c), 100)
>>> ax.plot(x, gompertz.pdf(x, c),
...        'r-', lw=5, alpha=0.6, label='gompertz pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = gompertz(c)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

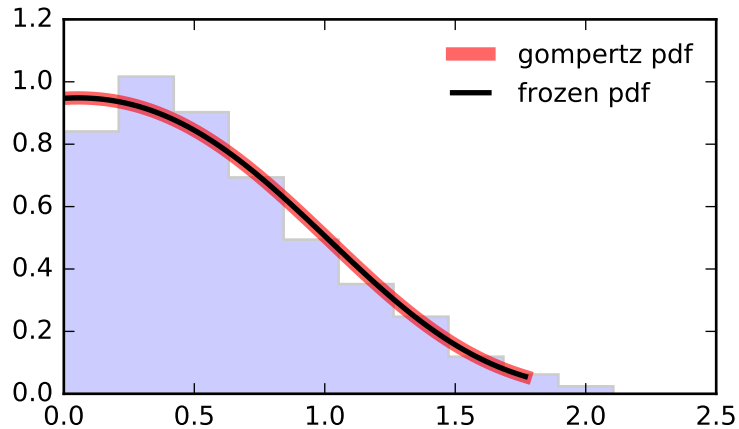
```
>>> vals = gompertz.ppf([0.001, 0.5, 0.999], c)
>>> np.allclose([0.001, 0.5, 0.999], gompertz.cdf(vals, c))
True
```

Generate random numbers:

```
>>> r = gompertz.rvs(c, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre> rvs(c, loc=0, scale=1, size=1,     random_state=None) pdf(x, c, loc=0, scale=1) logpdf(x, c, loc=0, scale=1) cdf(x, c, loc=0, scale=1) logcdf(x, c, loc=0, scale=1)  sf(x, c, loc=0, scale=1)  logsf(x, c, loc=0, scale=1) ppf(q, c, loc=0, scale=1)  isf(q, c, loc=0, scale=1) moment(n, c, loc=0, scale=1) stats(c, loc=0, scale=1, moments='mv')  entropy(c, loc=0, scale=1) fit(data, c, loc=0, scale=1) expect(func, args=(c,), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kwds) median(c, loc=0, scale=1) mean(c, loc=0, scale=1) var(c, loc=0, scale=1) std(c, loc=0, scale=1) interval(alpha, c, loc=0, scale=1)                     </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

`scipy.stats.gumbel_r` = <scipy.stats.continuous\_distns.gumbel\_r\_gen object at 0x2b909bd37790>

A right-skewed Gumbel continuous random variable.

As an instance of the `rv_continuous` class, `gumbel_r` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

See also:

```
gumbel_l, gompertz, genextreme
```

### Notes

The probability density function for `gumbel_r` is:

```
gumbel_r.pdf(x) = exp(-(x + exp(-x)))
```

The Gumbel distribution is sometimes referred to as a type I Fisher-Tippett distribution. It is also related to the extreme value distribution, log-Weibull and Gompertz distributions.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `gumbel_r.pdf(x, loc, scale)` is identically equivalent to `gumbel_r.pdf(y) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

### Examples

```
>>> from scipy.stats import gumbel_r
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> mean, var, skew, kurt = gumbel_r.stats(moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(gumbel_r.ppf(0.01),
...                 gumbel_r.ppf(0.99), 100)
>>> ax.plot(x, gumbel_r.pdf(x),
...         'r-', lw=5, alpha=0.6, label='gumbel_r pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = gumbel_r()
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

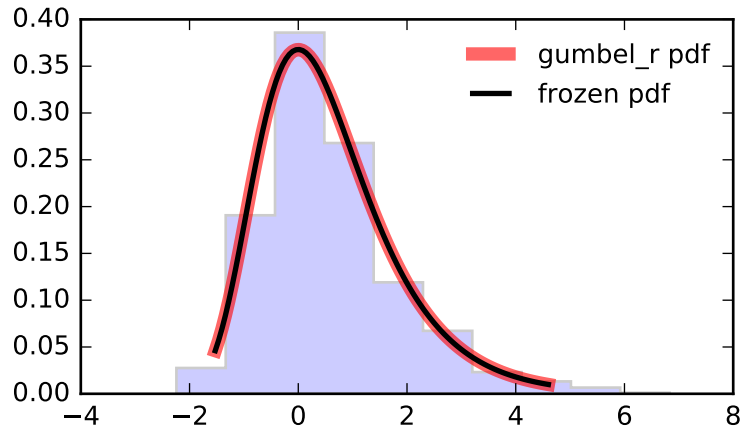
```
>>> vals = gumbel_r.ppf([0.001, 0.5, 0.999])
>>> np.allclose([0.001, 0.5, 0.999], gumbel_r.cdf(vals))
True
```

Generate random numbers:

```
>>> r = gumbel_r.rvs(size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre> rvs(loc=0, scale=1, size=1,     random_state=None) pdf(x, loc=0, scale=1) logpdf(x, loc=0, scale=1) cdf(x, loc=0, scale=1) logcdf(x, loc=0, scale=1)  sf(x, loc=0, scale=1)  logsf(x, loc=0, scale=1) ppf(q, loc=0, scale=1)  isf(q, loc=0, scale=1) moment(n, loc=0, scale=1) stats(loc=0, scale=1, moments='mv')  entropy(loc=0, scale=1) fit(data, loc=0, scale=1) expect(func, args=(), loc=0, scale=1,         lb=None, ub=None, conditional=False,         **kwds) median(loc=0, scale=1) mean(loc=0, scale=1) var(loc=0, scale=1) std(loc=0, scale=1) interval(alpha, loc=0, scale=1)                     </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

`scipy.stats.gumbel_1` = <scipy.stats.continuous\_distns.gumbel\_1\_gen object at 0x2b909bd37990>

A left-skewed Gumbel continuous random variable.

As an instance of the `rv_continuous` class, `gumbel_1` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**See also:**

`gumbel_r`, `gompertz`, `genextreme`

### Notes

The probability density function for `gumbel_1` is:

$$\text{gumbel\_1.pdf}(x) = \exp(x - \exp(x))$$

The Gumbel distribution is sometimes referred to as a type I Fisher-Tippett distribution. It is also related to the extreme value distribution, log-Weibull and Gompertz distributions.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `gumbel_1.pdf(x, loc, scale)` is identically equivalent to `gumbel_1.pdf(y) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

### Examples

```
>>> from scipy.stats import gumbel_1
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> mean, var, skew, kurt = gumbel_1.stats(moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(gumbel_1.ppf(0.01),
...                 gumbel_1.ppf(0.99), 100)
>>> ax.plot(x, gumbel_1.pdf(x),
...         'r-', lw=5, alpha=0.6, label='gumbel_1 pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = gumbel_1()
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

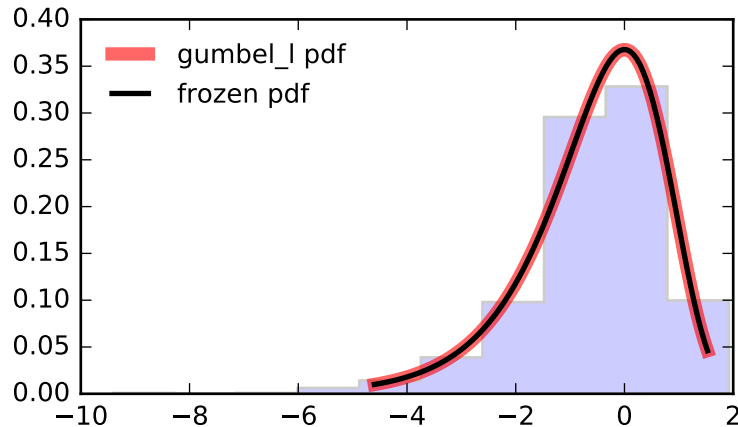
```
>>> vals = gumbel_1.ppf([0.001, 0.5, 0.999])
>>> np.allclose([0.001, 0.5, 0.999], gumbel_1.cdf(vals))
True
```

Generate random numbers:

```
>>> r = gumbel_1.rvs(size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre> rvs(loc=0, scale=1, size=1,     random_state=None) pdf(x, loc=0, scale=1) logpdf(x, loc=0, scale=1) cdf(x, loc=0, scale=1) logcdf(x, loc=0, scale=1)  sf(x, loc=0, scale=1)  logsf(x, loc=0, scale=1) ppf(q, loc=0, scale=1)  isf(q, loc=0, scale=1) moment(n, loc=0, scale=1) stats(loc=0, scale=1, moments='mv')  entropy(loc=0, scale=1) fit(data, loc=0, scale=1) expect(func, args=(), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kwds) median(loc=0, scale=1) mean(loc=0, scale=1) var(loc=0, scale=1) std(loc=0, scale=1) interval(alpha, loc=0, scale=1)                     </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

`scipy.stats.halfcauchy` = <scipy.stats.continuous\_distns.halfcauchy\_gen object at 0x2b909bd37b90>  
 A Half-Cauchy continuous random variable.

As an instance of the `rv_continuous` class, `halfcauchy` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.



**Notes**

The probability density function for `halfcauchy` is:

$$\text{halfcauchy.pdf}(x) = 2 / (\pi * (1 + x**2))$$

for  $x \geq 0$ .

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `halfcauchy.pdf(x, loc, scale)` is identically equivalent to `halfcauchy.pdf(y) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

**Examples**

```
>>> from scipy.stats import halfcauchy
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> mean, var, skew, kurt = halfcauchy.stats(moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(halfcauchy.ppf(0.01),
...                 halfcauchy.ppf(0.99), 100)
>>> ax.plot(x, halfcauchy.pdf(x),
...         'r-', lw=5, alpha=0.6, label='halfcauchy pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = halfcauchy()
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

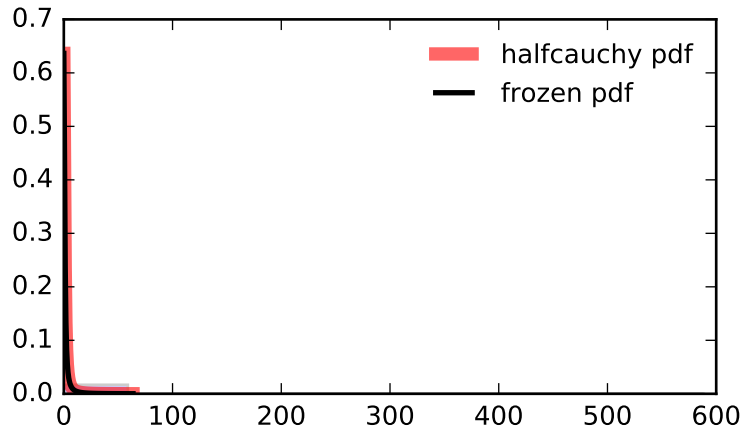
```
>>> vals = halfcauchy.ppf([0.001, 0.5, 0.999])
>>> np.allclose([0.001, 0.5, 0.999], halfcauchy.cdf(vals))
True
```

Generate random numbers:

```
>>> r = halfcauchy.rvs(size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre> rvs(loc=0, scale=1, size=1,     random_state=None) pdf(x, loc=0, scale=1) logpdf(x, loc=0, scale=1) cdf(x, loc=0, scale=1) logcdf(x, loc=0, scale=1)  sf(x, loc=0, scale=1)  logsf(x, loc=0, scale=1) ppf(q, loc=0, scale=1)  isf(q, loc=0, scale=1) moment(n, loc=0, scale=1) stats(loc=0, scale=1, moments='mv')  entropy(loc=0, scale=1) fit(data, loc=0, scale=1) expect(func, args=(), loc=0, scale=1,         lb=None, ub=None, conditional=False,         **kwds) median(loc=0, scale=1) mean(loc=0, scale=1) var(loc=0, scale=1) std(loc=0, scale=1) interval(alpha, loc=0, scale=1)                     </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

`scipy.stats.halflogistic = <scipy.stats._continuous_distns.halflogistic_gen object at 0x2b909bd37d90>`  
 A half-logistic continuous random variable.

As an instance of the `rv_continuous` class, `halflogistic` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `halflogistic` is:

$$\text{halflogistic.pdf}(x) = 2 * \exp(-x) / (1+\exp(-x))^{**2} = 1/2 * \text{sech}(x/2)^{**2}$$

for  $x \geq 0$ .

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `halflogistic.pdf(x, loc, scale)` is identically equivalent to `halflogistic.pdf(y) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

### Examples

```
>>> from scipy.stats import halflogistic
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> mean, var, skew, kurt = halflogistic.stats(moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(halflogistic.ppf(0.01),
...                 halflogistic.ppf(0.99), 100)
>>> ax.plot(x, halflogistic.pdf(x),
...         'r-', lw=5, alpha=0.6, label='halflogistic pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = halflogistic()
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

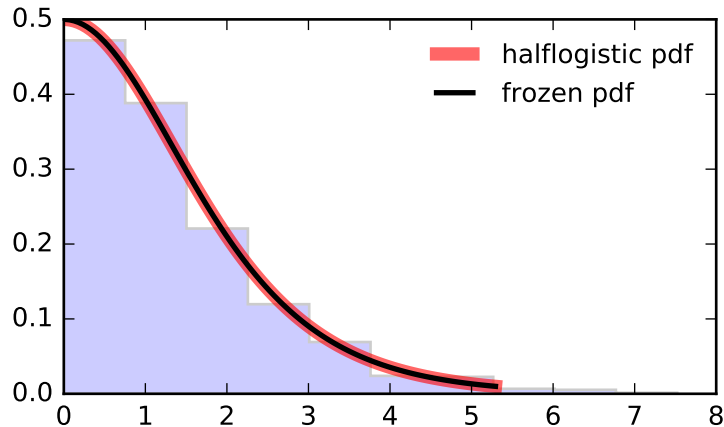
```
>>> vals = halflogistic.ppf([0.001, 0.5, 0.999])
>>> np.allclose([0.001, 0.5, 0.999], halflogistic.cdf(vals))
True
```

Generate random numbers:

```
>>> r = halflogistic.rvs(size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre> rvs(loc=0, scale=1, size=1,     random_state=None) pdf(x, loc=0, scale=1) logpdf(x, loc=0, scale=1) cdf(x, loc=0, scale=1) logcdf(x, loc=0, scale=1)  sf(x, loc=0, scale=1)  logsf(x, loc=0, scale=1) ppf(q, loc=0, scale=1)  isf(q, loc=0, scale=1) moment(n, loc=0, scale=1) stats(loc=0, scale=1, moments='mv')  entropy(loc=0, scale=1) fit(data, loc=0, scale=1) expect(func, args=(), loc=0, scale=1,         lb=None, ub=None, conditional=False,         **kwds) median(loc=0, scale=1) mean(loc=0, scale=1) var(loc=0, scale=1) std(loc=0, scale=1) interval(alpha, loc=0, scale=1)                     </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

`scipy.stats.halfnorm = <scipy.stats.continuous_distns.halfnorm_gen object at 0x2b909bd37f90>`  
 A half-normal continuous random variable.

As an instance of the `rv_continuous` class, `halfnorm` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**Notes**

The probability density function for `halfnorm` is:

```
halfnorm.pdf(x) = sqrt(2/pi) * exp(-x**2/2)
```

for  $x > 0$ .

`halfnorm` is a special case of `chi` with `df == 1`.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `halfnorm.pdf(x, loc, scale)` is identically equivalent to `halfnorm.pdf(y) / scale` with  $y = (x - loc) / scale$ .

**Examples**

```
>>> from scipy.stats import halfnorm
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> mean, var, skew, kurt = halfnorm.stats(moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(halfnorm.ppf(0.01),
...                 halfnorm.ppf(0.99), 100)
>>> ax.plot(x, halfnorm.pdf(x),
...         'r-', lw=5, alpha=0.6, label='halfnorm pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = halfnorm()
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

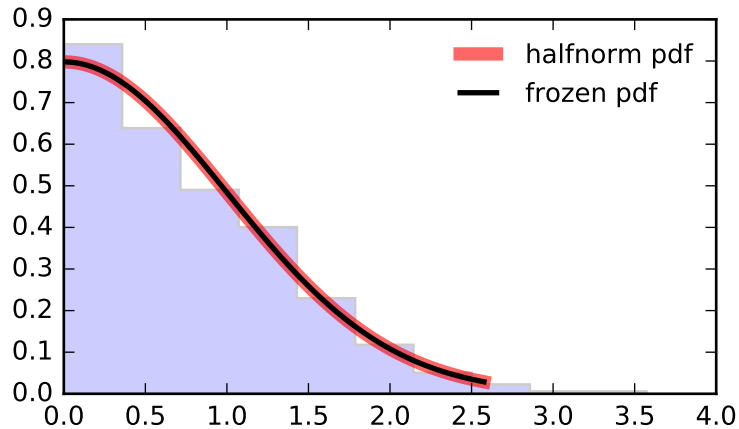
```
>>> vals = halfnorm.ppf([0.001, 0.5, 0.999])
>>> np.allclose([0.001, 0.5, 0.999], halfnorm.cdf(vals))
True
```

Generate random numbers:

```
>>> r = halfnorm.rvs(size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre> rvs(loc=0, scale=1, size=1,     random_state=None) pdf(x, loc=0, scale=1) logpdf(x, loc=0, scale=1) cdf(x, loc=0, scale=1) logcdf(x, loc=0, scale=1)  sf(x, loc=0, scale=1)  logsf(x, loc=0, scale=1) ppf(q, loc=0, scale=1)  isf(q, loc=0, scale=1) moment(n, loc=0, scale=1) stats(loc=0, scale=1, moments='mv')  entropy(loc=0, scale=1) fit(data, loc=0, scale=1) expect(func, args=(), loc=0, scale=1,         lb=None, ub=None, conditional=False,         **kwds) median(loc=0, scale=1) mean(loc=0, scale=1) var(loc=0, scale=1) std(loc=0, scale=1) interval(alpha, loc=0, scale=1)                     </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

`scipy.stats.halfgennorm` = <scipy.stats.continuous\_distns.halfgennorm\_gen object at 0x2b909bd9a950>  
 The upper half of a generalized normal continuous random variable.

As an instance of the `rv_continuous` class, `halfgennorm` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

See also:

**gennorm** generalized normal distribution  
**expon** exponential distribution  
**halfnorm** half normal distribution

### Notes

The probability density function for `halfgennorm` is:

$$\text{halfgennorm.pdf}(x, \text{beta}) = \frac{\text{beta}}{\text{gamma}(1/\text{beta})} \exp(-|x|**\text{beta})$$

`gennorm` takes `beta` as a shape parameter. For `beta = 1`, it is identical to an exponential distribution. For `beta = 2`, it is identical to a half normal distribution (with `scale=1/sqrt(2)`).

### References

[R526]

### Examples

```
>>> from scipy.stats import halfgennorm
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> beta = 0.675
>>> mean, var, skew, kurt = halfgennorm.stats(beta, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(halfgennorm.ppf(0.01, beta),
...                 halfgennorm.ppf(0.99, beta), 100)
>>> ax.plot(x, halfgennorm.pdf(x, beta),
...         'r-', lw=5, alpha=0.6, label='halfgennorm pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = halfgennorm(beta)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

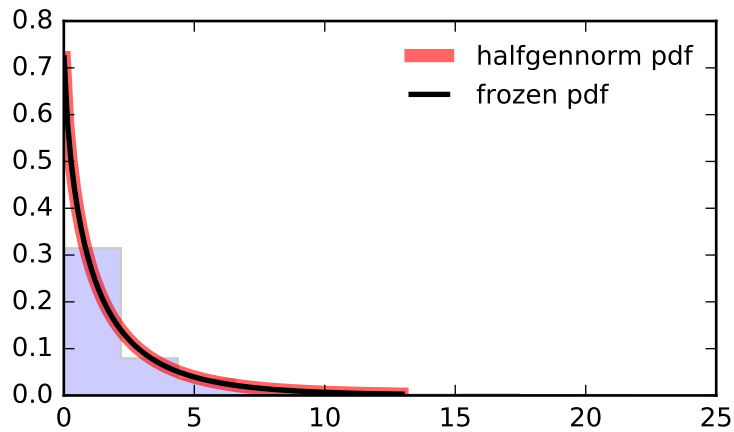
```
>>> vals = halfgennorm.ppf([0.001, 0.5, 0.999], beta)
>>> np.allclose([0.001, 0.5, 0.999], halfgennorm.cdf(vals, beta))
True
```

Generate random numbers:

```
>>> r = halfgennorm.rvs(beta, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre>rvs(beta, loc=0, scale=1, size=1,     random_state=None) pdf(x, beta, loc=0, scale=1) logpdf(x, beta, loc=0, scale=1) cdf(x, beta, loc=0, scale=1) logcdf(x, beta, loc=0, scale=1)  sf(x, beta, loc=0, scale=1)  logsf(x, beta, loc=0, scale=1) ppf(q, beta, loc=0, scale=1) isf(q, beta, loc=0, scale=1)  moment(n, beta, loc=0, scale=1) stats(beta, loc=0, scale=1, moments='mv')  entropy(beta, loc=0, scale=1) fit(data, beta, loc=0, scale=1) expect(func, args=(beta,), loc=0, scale=1,     lb=None, ub=None, conditional=False,     **kwds) median(beta, loc=0, scale=1) mean(beta, loc=0, scale=1) var(beta, loc=0, scale=1) std(beta, loc=0, scale=1) interval(alpha, beta, loc=0, scale=1)</pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <math>\text{sf}</math> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <math>\text{cdf}</math> — percentiles).</p> <p>Inverse survival function (inverse of <math>\text{sf}</math>).</p> <p>Non-central moment of order <math>n</math></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains <math>\alpha</math> percent of the distribution</p>
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`scipy.stats.hypsecant` = <scipy.stats.\_continuous\_distns.hypsecant\_gen object at 0x2b909bd411d0>  
 A hyperbolic secant continuous random variable.

As an instance of the `rv_continuous` class, `hypsecant` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `hypsecant` is:

$$\text{hypsecant.pdf}(x) = 1/\pi * \text{sech}(x)$$

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `hypsecant.pdf(x, loc, scale)` is identically equivalent to `hypsecant.pdf(y) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

### Examples

```
>>> from scipy.stats import hypsecant
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> mean, var, skew, kurt = hypsecant.stats(moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(hypsecant.ppf(0.01),
...                 hypsecant.ppf(0.99), 100)
>>> ax.plot(x, hypsecant.pdf(x),
...         'r-', lw=5, alpha=0.6, label='hypsecant pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = hypsecant()
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

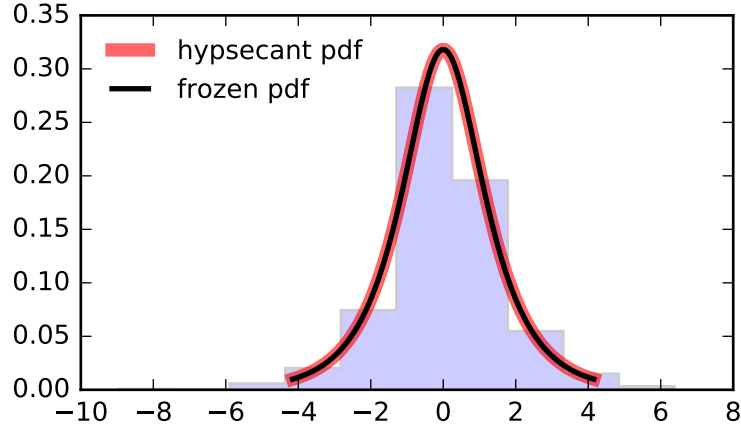
```
>>> vals = hypsecant.ppf([0.001, 0.5, 0.999])
>>> np.allclose([0.001, 0.5, 0.999], hypsecant.cdf(vals))
True
```

Generate random numbers:

```
>>> r = hypsecant.rvs(size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre> rvs(loc=0, scale=1, size=1,     random_state=None) pdf(x, loc=0, scale=1) logpdf(x, loc=0, scale=1) cdf(x, loc=0, scale=1) logcdf(x, loc=0, scale=1)  sf(x, loc=0, scale=1)  logsf(x, loc=0, scale=1) ppf(q, loc=0, scale=1)  isf(q, loc=0, scale=1) moment(n, loc=0, scale=1) stats(loc=0, scale=1, moments='mv')  entropy(loc=0, scale=1) fit(data, loc=0, scale=1) expect(func, args=(), loc=0, scale=1,     lb=None, ub=None, conditional=False,     **kwargs) median(loc=0, scale=1) mean(loc=0, scale=1) var(loc=0, scale=1) std(loc=0, scale=1) interval(alpha, loc=0, scale=1)                 </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

`scipy.stats.invgamma` = <scipy.stats.continuous\_distns.invgamma\_gen object at 0x2b909bd41650>  
 An inverted gamma continuous random variable.

As an instance of the `rv_continuous` class, `invgamma` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**Notes**

The probability density function for `invgamma` is:

```
invgamma.pdf(x, a) = x**(-a-1) / gamma(a) * exp(-1/x)
```

for  $x > 0, a > 0$ .

`invgamma` takes `a` as a shape parameter.

`invgamma` is a special case of `gengamma` with `c == -1`.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `invgamma.pdf(x, a, loc, scale)` is identically equivalent to `invgamma.pdf(y, a) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

**Examples**

```
>>> from scipy.stats import invgamma
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> a = 4.07
>>> mean, var, skew, kurt = invgamma.stats(a, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(invgamma.ppf(0.01, a),
...                 invgamma.ppf(0.99, a), 100)
>>> ax.plot(x, invgamma.pdf(x, a),
...        'r-', lw=5, alpha=0.6, label='invgamma pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = invgamma(a)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

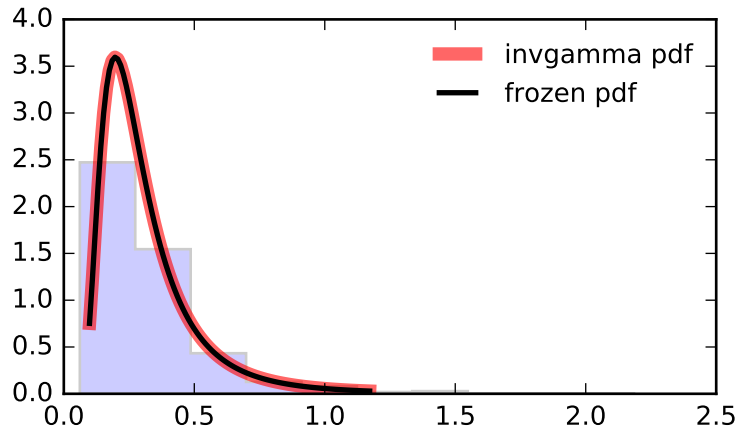
```
>>> vals = invgamma.ppf([0.001, 0.5, 0.999], a)
>>> np.allclose([0.001, 0.5, 0.999], invgamma.cdf(vals, a))
True
```

Generate random numbers:

```
>>> r = invgamma.rvs(a, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre> rvs(a, loc=0, scale=1, size=1,     random_state=None) pdf(x, a, loc=0, scale=1) logpdf(x, a, loc=0, scale=1) cdf(x, a, loc=0, scale=1) logcdf(x, a, loc=0, scale=1)  sf(x, a, loc=0, scale=1)  logsf(x, a, loc=0, scale=1) ppf(q, a, loc=0, scale=1)  isf(q, a, loc=0, scale=1) moment(n, a, loc=0, scale=1) stats(a, loc=0, scale=1, moments='mv')  entropy(a, loc=0, scale=1) fit(data, a, loc=0, scale=1) expect(func, args=(a,), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds) median(a, loc=0, scale=1) mean(a, loc=0, scale=1) var(a, loc=0, scale=1) std(a, loc=0, scale=1) interval(alpha, a, loc=0, scale=1)                     </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

`scipy.stats.invgauss` = <scipy.stats.continuous\_distns.invgauss\_gen object at 0x2b909bd418d0>  
 An inverse Gaussian continuous random variable.

As an instance of the `rv_continuous` class, `invgauss` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**Notes**

The probability density function for `invgauss` is:

$$\text{invgauss.pdf}(x, \mu) = 1 / \sqrt{2\pi x^3} * \exp(-(x-\mu)^2 / (2x\mu^2))$$

for  $x > 0$ .

`invgauss` takes `mu` as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `invgauss.pdf(x, mu, loc, scale)` is identically equivalent to `invgauss.pdf(y, mu) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

When `mu` is too small, evaluating the cumulative distribution function will be inaccurate due to `cdf(mu -> 0) = inf * 0`. NaNs are returned for `mu <= 0.0028`.

**Examples**

```
>>> from scipy.stats import invgauss
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> mu = 0.145
>>> mean, var, skew, kurt = invgauss.stats(mu, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(invgauss.ppf(0.01, mu),
...                 invgauss.ppf(0.99, mu), 100)
>>> ax.plot(x, invgauss.pdf(x, mu),
...         'r-', lw=5, alpha=0.6, label='invgauss pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = invgauss(mu)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

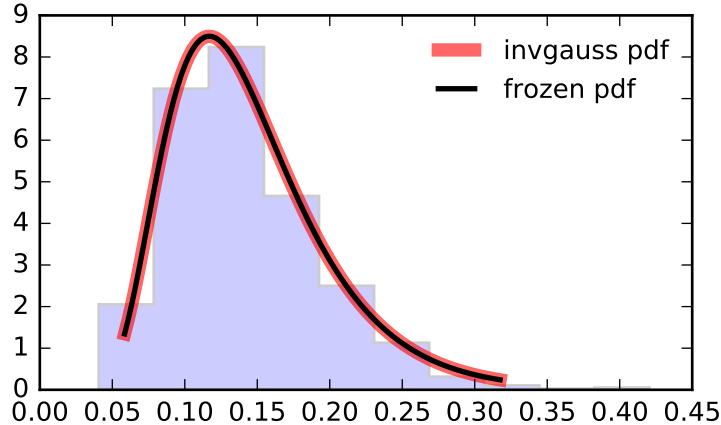
```
>>> vals = invgauss.ppf([0.001, 0.5, 0.999], mu)
>>> np.allclose([0.001, 0.5, 0.999], invgauss.cdf(vals, mu))
True
```

Generate random numbers:

```
>>> r = invgauss.rvs(mu, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre> rvs(mu, loc=0, scale=1, size=1,     random_state=None) pdf(x, mu, loc=0, scale=1) logpdf(x, mu, loc=0, scale=1) cdf(x, mu, loc=0, scale=1) logcdf(x, mu, loc=0, scale=1)  sf(x, mu, loc=0, scale=1)  logsf(x, mu, loc=0, scale=1) ppf(q, mu, loc=0, scale=1)  isf(q, mu, loc=0, scale=1) moment(n, mu, loc=0, scale=1) stats(mu, loc=0, scale=1, moments='mv')  entropy(mu, loc=0, scale=1) fit(data, mu, loc=0, scale=1) expect(func, args=(mu,), loc=0, scale=1,         lb=None, ub=None, conditional=False,         **kwds) median(mu, loc=0, scale=1) mean(mu, loc=0, scale=1) var(mu, loc=0, scale=1) std(mu, loc=0, scale=1) interval(alpha, mu, loc=0, scale=1)                     </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
--	--

`scipy.stats.invweibull` = <scipy.stats.continuous\_distns.invweibull\_gen object at 0x2b909bd41b50>  
 An inverted Weibull continuous random variable.

As an instance of the `rv_continuous` class, `invweibull` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**Notes**

The probability density function for `invweibull` is:

```
invweibull.pdf(x, c) = c * x**(-c-1) * exp(-x**(-c))
```

for  $x > 0, c > 0$ .

`invweibull` takes `c` as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `invweibull.pdf(x, c, loc, scale)` is identically equivalent to `invweibull.pdf(y, c) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

**References**

F.R.S. de Gusmao, E.M.M Ortega and G.M. Cordeiro, “The generalized inverse Weibull distribution”, Stat. Papers, vol. 52, pp. 591-619, 2011.

**Examples**

```
>>> from scipy.stats import invweibull
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> c = 10.6
>>> mean, var, skew, kurt = invweibull.stats(c, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(invweibull.ppf(0.01, c),
...                 invweibull.ppf(0.99, c), 100)
>>> ax.plot(x, invweibull.pdf(x, c),
...         'r-', lw=5, alpha=0.6, label='invweibull pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = invweibull(c)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

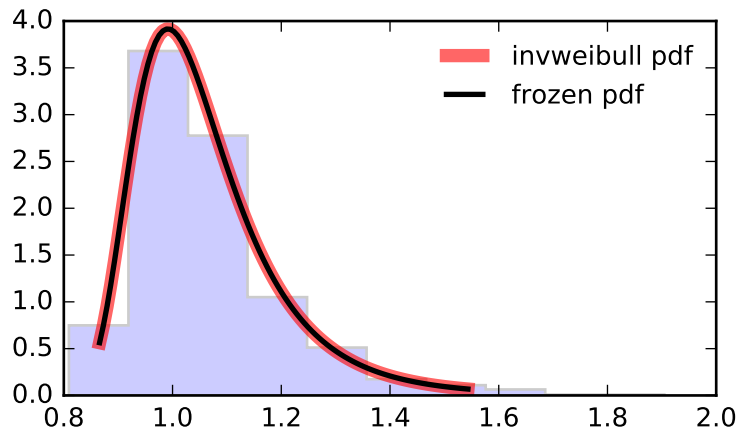
```
>>> vals = invweibull.ppf([0.001, 0.5, 0.999], c)
>>> np.allclose([0.001, 0.5, 0.999], invweibull.cdf(vals, c))
True
```

Generate random numbers:

```
>>> r = invweibull.rvs(c, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre>rvs(c, loc=0, scale=1, size=1,     random_state=None) pdf(x, c, loc=0, scale=1) logpdf(x, c, loc=0, scale=1) cdf(x, c, loc=0, scale=1) logcdf(x, c, loc=0, scale=1)  sf(x, c, loc=0, scale=1)  logsf(x, c, loc=0, scale=1) ppf(q, c, loc=0, scale=1)  isf(q, c, loc=0, scale=1) moment(n, c, loc=0, scale=1) stats(c, loc=0, scale=1, moments='mv')  entropy(c, loc=0, scale=1) fit(data, c, loc=0, scale=1) expect(func, args=(c,), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kwargs) median(c, loc=0, scale=1) mean(c, loc=0, scale=1) var(c, loc=0, scale=1) std(c, loc=0, scale=1) interval(alpha, c, loc=0, scale=1)</pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

```
scipy.stats.johnsonsb = <scipy.stats._continuous_distns.johnsonsb_gen object at 0x2b909bd41dd0>
```



A Johnson SB continuous random variable.

As an instance of the `rv_continuous` class, `johnsonsb` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**See also:**

`johnsonsu`

**Notes**

The probability density function for `johnsonsb` is:

$$\text{johnsonsb.pdf}(x, a, b) = b / (x*(1-x)) * \text{phi}(a + b * \log(x/(1-x)))$$

for  $0 < x < 1$  and  $a, b > 0$ , and `phi` is the normal pdf.

`johnsonsb` takes `a` and `b` as shape parameters.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `johnsonsb.pdf(x, a, b, loc, scale)` is identically equivalent to `johnsonsb.pdf(y, a, b) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

**Examples**

```
>>> from scipy.stats import johnsonsb
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> a, b = 4.32, 3.18
>>> mean, var, skew, kurt = johnsonsb.stats(a, b, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(johnsonsb.ppf(0.01, a, b),
...                 johnsonsb.ppf(0.99, a, b), 100)
>>> ax.plot(x, johnsonsb.pdf(x, a, b),
...         'r-', lw=5, alpha=0.6, label='johnsonsb pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = johnsonsb(a, b)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

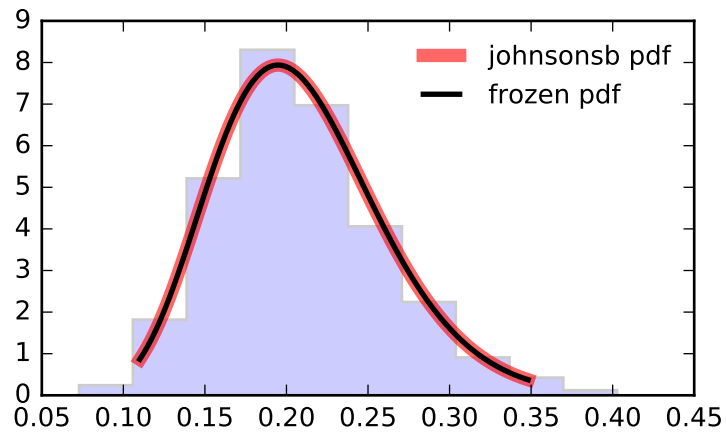
```
>>> vals = johnsonsb.ppf([0.001, 0.5, 0.999], a, b)
>>> np.allclose([0.001, 0.5, 0.999], johnsonsb.cdf(vals, a, b))
True
```

Generate random numbers:

```
>>> r = johnsonsb.rvs(a, b, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



**Methods**

<code>rvs(a, b, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, a, b, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, a, b, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, a, b, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, a, b, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, a, b, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, a, b, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, a, b, loc=0, scale=1)</code>	Percent point function (inverse of <i>cdf</i> — percentiles).
<code>isf(q, a, b, loc=0, scale=1)</code>	Inverse survival function (inverse of <i>sf</i> ).
<code>moment(n, a, b, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(a, b, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(a, b, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, a, b, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(a, b), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(a, b, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(a, b, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(a, b, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(a, b, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, a, b, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.johnsonsu = <scipy.stats._continuous_distns.johnsonsu_gen object at 0x2b909bd4f090>`  
 A Johnson SU continuous random variable.

As an instance of the `rv_continuous` class, `johnsonsu` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**See also:**

`johnsonsb`

**Notes**

The probability density function for `johnsonsu` is:

$$\text{johnsonsu.pdf}(x, a, b) = b / \sqrt{x^2 + 1} * \text{phi}(a + b * \log(x + \sqrt{x^2 + 1}))$$

for all  $x$ ,  $a, b > 0$ , and *phi* is the normal pdf.

`johnsonsu` takes *a* and *b* as shape parameters.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the *loc* and *scale* parameters. Specifically, `johnsonsu.pdf(x, a, b, loc, scale)` is identically equivalent to `johnsonsu.pdf(y, a, b) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

### Examples

```
>>> from scipy.stats import johnsonsu
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> a, b = 2.55, 2.25
>>> mean, var, skew, kurt = johnsonsu.stats(a, b, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(johnsonsu.ppf(0.01, a, b),
...                 johnsonsu.ppf(0.99, a, b), 100)
>>> ax.plot(x, johnsonsu.pdf(x, a, b),
...         'r-', lw=5, alpha=0.6, label='johnsonsu pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = johnsonsu(a, b)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

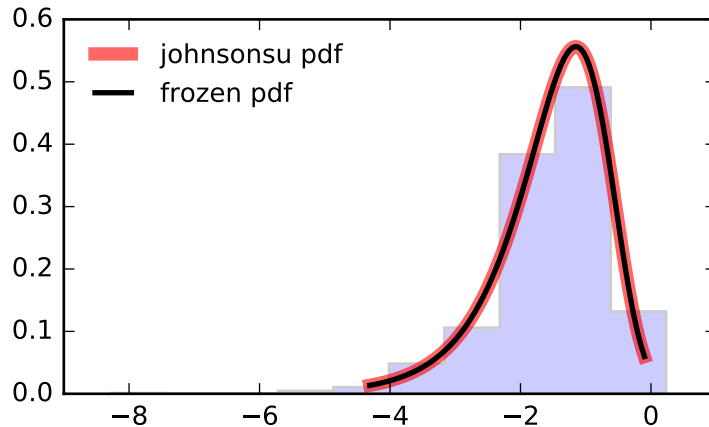
```
>>> vals = johnsonsu.ppf([0.001, 0.5, 0.999], a, b)
>>> np.allclose([0.001, 0.5, 0.999], johnsonsu.cdf(vals, a, b))
True
```

Generate random numbers:

```
>>> r = johnsonsu.rvs(a, b, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<code>rvs(a, b, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, a, b, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, a, b, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, a, b, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, a, b, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, a, b, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, a, b, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, a, b, loc=0, scale=1)</code>	Percent point function (inverse of <i>cdf</i> — percentiles).
<code>isf(q, a, b, loc=0, scale=1)</code>	Inverse survival function (inverse of <i>sf</i> ).
<code>moment(n, a, b, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(a, b, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(a, b, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, a, b, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(a, b), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(a, b, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(a, b, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(a, b, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(a, b, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, a, b, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.kappa4 = <scipy.stats.continuous_distns.kappa4_gen object at 0x2b909bd59bd0>`  
 Kappa 4 parameter distribution.

As an instance of the `rv_continuous` class, `kappa4` object inherits from it a collection of generic methods

(see below for the full list), and completes them with details specific for this particular distribution.

**Notes**

The probability density function for kappa4 is:

$$\text{kappa4.pdf}(x, h, k) = (1.0 - k*x)**(1.0/k - 1.0) * (1.0 - h*(1.0 - k*x)**(1.0/k))**(1.0/h-1)$$

if h and k are not equal to 0.

If h or k are zero then the pdf can be simplified:

h = 0 and k != 0:

$$\text{kappa4.pdf}(x, h, k) = (1.0 - k*x)**(1.0/k - 1.0) * \exp(-(1.0 - k*x)**(1.0/k))$$

h != 0 and k = 0:

$$\text{kappa4.pdf}(x, h, k) = \exp(-x) * (1.0 - h*\exp(-x))**(1.0/h - 1.0)$$

h = 0 and k = 0:

$$\text{kappa4.pdf}(x, h, k) = \exp(-x) * \exp(-\exp(-x))$$

kappa4 takes h and k as shape parameters.

The kappa4 distribution returns other distributions when certain h and k values are used.

h	k=0.0	k=1.0	-inf<=k<=inf
-1.0	Logistic logistic(x)		Generalized Logistic(1)
0.0	Gumbel gumbel_r(x)	Reverse Exponential(2)	Generalized Extreme Value genextreme(x, k)
1.0	Exponential expon(x)	Uniform uniform(x)	Generalized Pareto genpareto(x, -k)

1. There are at least five generalized logistic distributions. Four are described here: [https://en.wikipedia.org/wiki/Generalized\\_logistic\\_distribution](https://en.wikipedia.org/wiki/Generalized_logistic_distribution) The “fifth” one is the one kappa4 should match which currently isn’t implemented in scipy: [https://en.wikipedia.org/wiki/Talk:Generalized\\_logistic\\_distribution](https://en.wikipedia.org/wiki/Talk:Generalized_logistic_distribution) [http://www.mathwave.com/help/easyfit/html/analyses/distributions/gen\\_logistic.html](http://www.mathwave.com/help/easyfit/html/analyses/distributions/gen_logistic.html)

2. This distribution is currently not in scipy.

**References**

J.C. Finney, “Optimization of a Skewed Logistic Distribution With Respect to the Kolmogorov-Smirnov Test”, A Dissertation Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College, (August, 2004), [http://etd.lsu.edu/docs/available/etd-05182004-144851/unrestricted/Finney\\_dis.pdf](http://etd.lsu.edu/docs/available/etd-05182004-144851/unrestricted/Finney_dis.pdf)

J.R.M. Hosking, “The four-parameter kappa distribution”. IBM J. Res. Develop. 38 (3), 25 1-258 (1994).

B. Kumphon, A. Kaew-Man, P. Seenoi, “A Rainfall Distribution for the Lampao Site in the Chi River Basin, Thailand”, Journal of Water Resource and Protection, vol. 4, 866-869, (2012). [http://file.scirp.org/pdf/JWARP20121000009\\_14676002.pdf](http://file.scirp.org/pdf/JWARP20121000009_14676002.pdf)

C. Winchester, “On Estimation of the Four-Parameter Kappa Distribution”, A Thesis Submitted to Dalhousie University, Halifax, Nova Scotia, (March 2000). <http://www.nlc-bnc.ca/obj/s4/f2/dsk2/ftp01/MQ57336.pdf>

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `kappa4.pdf(x, h, k, loc, scale)` is identically equivalent to `kappa4.pdf(y, h, k) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

### Examples

```
>>> from scipy.stats import kappa4
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> h, k = 0.1, 0
>>> mean, var, skew, kurt = kappa4.stats(h, k, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(kappa4.ppf(0.01, h, k),
...                 kappa4.ppf(0.99, h, k), 100)
>>> ax.plot(x, kappa4.pdf(x, h, k),
...         'r-', lw=5, alpha=0.6, label='kappa4 pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = kappa4(h, k)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

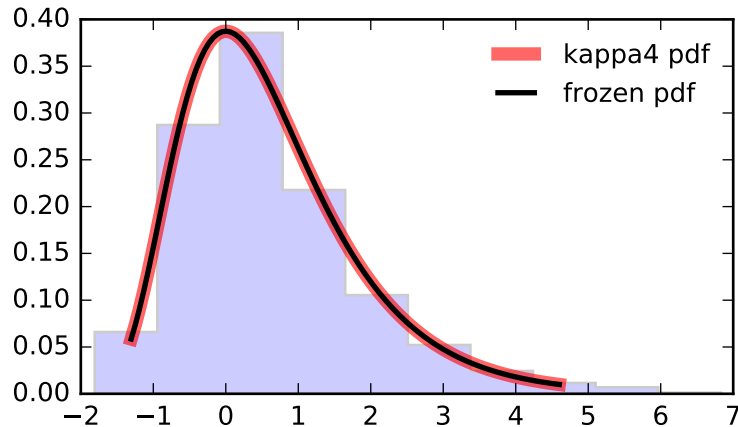
```
>>> vals = kappa4.ppf([0.001, 0.5, 0.999], h, k)
>>> np.allclose([0.001, 0.5, 0.999], kappa4.cdf(vals, h, k))
True
```

Generate random numbers:

```
>>> r = kappa4.rvs(h, k, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<code>rvs(h, k, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, h, k, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, h, k, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, h, k, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, h, k, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, h, k, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, h, k, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, h, k, loc=0, scale=1)</code>	Percent point function (inverse of <i>cdf</i> — percentiles).
<code>isf(q, h, k, loc=0, scale=1)</code>	Inverse survival function (inverse of <i>sf</i> ).
<code>moment(n, h, k, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(h, k, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(h, k, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, h, k, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(h, k), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(h, k, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(h, k, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(h, k, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(h, k, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, h, k, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.kappa3` = <scipy.stats.continuous\_distns.kappa3\_gen object at 0x2b909bd59e50>  
 Kappa 3 parameter distribution.

As an instance of the `rv_continuous` class, `kappa3` object inherits from it a collection of generic methods



(see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for *kappa* is:

```
kappa3.pdf(x, a) =
    a*[a + x**a]**(-(a + 1)/a),    for ``x > 0``
    0.0,                            for ``x <= 0``
```

`kappa3` takes `a` as a shape parameter and `a > 0`.

### References

P.W. Mielke and E.S. Johnson, “Three-Parameter Kappa Distribution Maximum Likelihood and Likelihood Ratio Tests”, *Methods in Weather Research*, 701-707, (September, 1973), <http://docs.lib.noaa.gov/rescue/mwr/101/mwr-101-09-0701.pdf>

B. Kumphon, “Maximum Entropy and Maximum Likelihood Estimation for the Three-Parameter Kappa Distribution”, *Open Journal of Statistics*, vol 2, 415-419 (2012) [http://file.scirp.org/pdf/OJS20120400011\\_95789012.pdf](http://file.scirp.org/pdf/OJS20120400011_95789012.pdf)

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `kappa3.pdf(x, a, loc, scale)` is identically equivalent to `kappa3.pdf(y, a) / scale` with `y = (x - loc) / scale`.

### Examples

```
>>> from scipy.stats import kappa3
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> a = 1
>>> mean, var, skew, kurt = kappa3.stats(a, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(kappa3.ppf(0.01, a),
...                 kappa3.ppf(0.99, a), 100)
>>> ax.plot(x, kappa3.pdf(x, a),
...        'r-', lw=5, alpha=0.6, label='kappa3 pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = kappa3(a)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

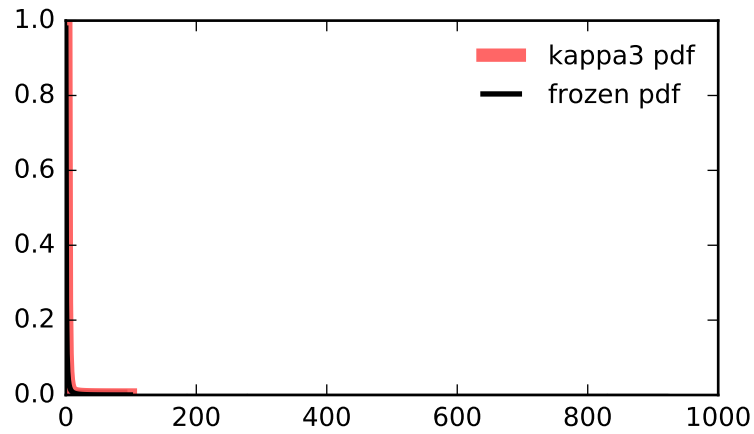
```
>>> vals = kappa3.ppf([0.001, 0.5, 0.999], a)
>>> np.allclose([0.001, 0.5, 0.999], kappa3.cdf(vals, a))
True
```

Generate random numbers:

```
>>> r = kappa3.rvs(a, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



**Methods**

<code>rvs(a, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, a, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, a, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, a, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, a, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, a, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, a, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, a, loc=0, scale=1)</code>	Percent point function (inverse of <i>cdf</i> — percentiles).
<code>isf(q, a, loc=0, scale=1)</code>	Inverse survival function (inverse of <i>sf</i> ).
<code>moment(n, a, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(a, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(a, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, a, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(a,), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(a, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(a, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(a, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(a, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, a, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.ksone = <scipy.stats.continuous_distns.ksone_gen object at 0x2b909baab810>`

General Kolmogorov-Smirnov one-sided test.

As an instance of the `rv_continuous` class, `ksone` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**Examples**

```
>>> from scipy.stats import ksone
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> n = 1e+03
>>> mean, var, skew, kurt = ksone.stats(n, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(ksone.ppf(0.01, n),
...                 ksone.ppf(0.99, n), 100)
>>> ax.plot(x, ksone.pdf(x, n),
...         'r-', lw=5, alpha=0.6, label='ksone pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = ksone(n)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

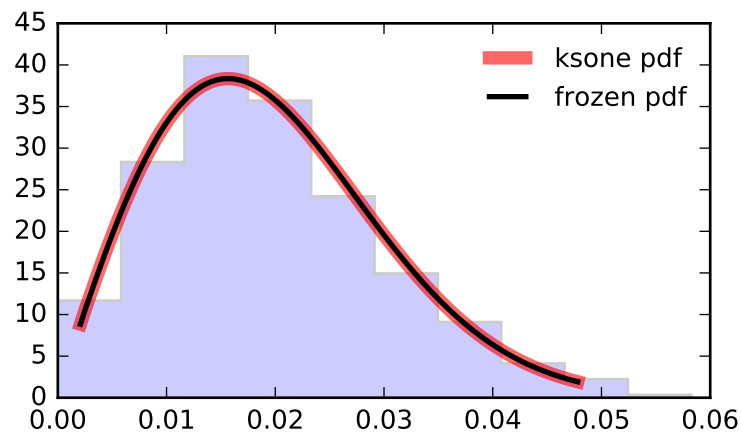
```
>>> vals = ksone.ppf([0.001, 0.5, 0.999], n)
>>> np.allclose([0.001, 0.5, 0.999], ksone.cdf(vals, n))
True
```

Generate random numbers:

```
>>> r = ksone.rvs(n, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



**Methods**

<code>rvs(n, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, n, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, n, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, n, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, n, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, n, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, n, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, n, loc=0, scale=1)</code>	Percent point function (inverse of <i>cdf</i> — percentiles).
<code>isf(q, n, loc=0, scale=1)</code>	Inverse survival function (inverse of <i>sf</i> ).
<code>moment(n, n, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(n, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(n, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, n, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(n,), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(n, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(n, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(n, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(n, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, n, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.kstwobign = <scipy.stats.continuous_distns.kstwobign_gen object at 0x2b909baaba10>`  
Kolmogorov-Smirnov two-sided test for large *N*.

As an instance of the `rv_continuous` class, `kstwobign` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**Examples**

```
>>> from scipy.stats import kstwobign
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> mean, var, skew, kurt = kstwobign.stats(moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(kstwobign.ppf(0.01),
...                 kstwobign.ppf(0.99), 100)
>>> ax.plot(x, kstwobign.pdf(x),
...         'r-', lw=5, alpha=0.6, label='kstwobign pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = kstwobign()  
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

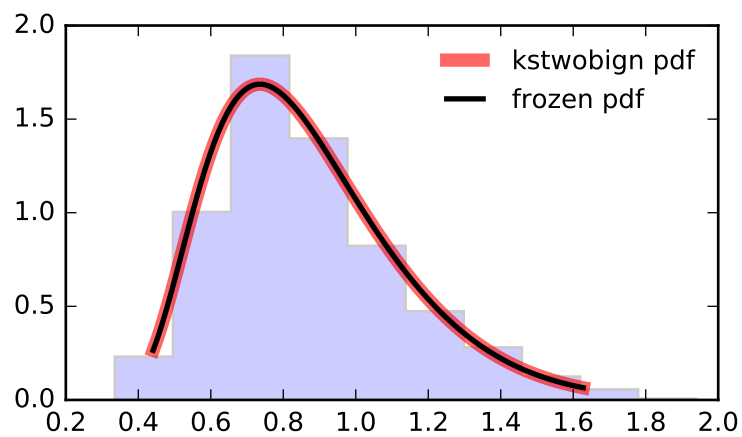
```
>>> vals = kstwobign.ppf([0.001, 0.5, 0.999])  
>>> np.allclose([0.001, 0.5, 0.999], kstwobign.cdf(vals))  
True
```

Generate random numbers:

```
>>> r = kstwobign.rvs(size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)  
>>> ax.legend(loc='best', frameon=False)  
>>> plt.show()
```



**Methods**

<pre> rvs(loc=0, scale=1, size=1,     random_state=None) pdf(x, loc=0, scale=1) logpdf(x, loc=0, scale=1) cdf(x, loc=0, scale=1) logcdf(x, loc=0, scale=1)  sf(x, loc=0, scale=1)  logsf(x, loc=0, scale=1) ppf(q, loc=0, scale=1)  isf(q, loc=0, scale=1) moment(n, loc=0, scale=1) stats(loc=0, scale=1, moments='mv')  entropy(loc=0, scale=1) fit(data, loc=0, scale=1) expect(func, args=(), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kwds) median(loc=0, scale=1) mean(loc=0, scale=1) var(loc=0, scale=1) std(loc=0, scale=1) interval(alpha, loc=0, scale=1) </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

`scipy.stats.laplace` = <scipy.stats.continuous\_distns.laplace\_gen object at 0x2b909bd4f310>  
 A Laplace continuous random variable.

As an instance of the `rv_continuous` class, `laplace` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**Notes**

The probability density function for `laplace` is:

$$\text{laplace.pdf}(x) = 1/2 * \exp(-\text{abs}(x))$$

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `laplace.pdf(x, loc, scale)` is identically equivalent to `laplace.pdf(y) / scale` with `y = (x - loc) / scale`.

**Examples**

```

>>> from scipy.stats import laplace
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)

```

Calculate a few first moments:

```

>>> mean, var, skew, kurt = laplace.stats(moments='mvsk')

```

Display the probability density function (pdf):

```
>>> x = np.linspace(laplace.ppf(0.01),
...                 laplace.ppf(0.99), 100)
>>> ax.plot(x, laplace.pdf(x),
...         'r-', lw=5, alpha=0.6, label='laplace pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = laplace()
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

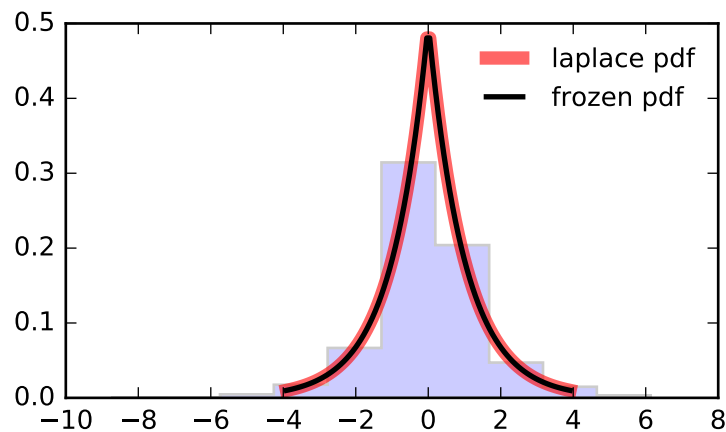
```
>>> vals = laplace.ppf([0.001, 0.5, 0.999])
>>> np.allclose([0.001, 0.5, 0.999], laplace.cdf(vals))
True
```

Generate random numbers:

```
>>> r = laplace.rvs(size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```





**Methods**

<pre> rvs(loc=0, scale=1, size=1,     random_state=None) pdf(x, loc=0, scale=1) logpdf(x, loc=0, scale=1) cdf(x, loc=0, scale=1) logcdf(x, loc=0, scale=1)  sf(x, loc=0, scale=1)  logsf(x, loc=0, scale=1) ppf(q, loc=0, scale=1)  isf(q, loc=0, scale=1) moment(n, loc=0, scale=1) stats(loc=0, scale=1, moments='mv')  entropy(loc=0, scale=1) fit(data, loc=0, scale=1) expect(func, args=(), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kwds) median(loc=0, scale=1) mean(loc=0, scale=1) var(loc=0, scale=1) std(loc=0, scale=1) interval(alpha, loc=0, scale=1) </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <math>\text{sf}</math> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <math>\text{cdf}</math> — percentiles).</p> <p>Inverse survival function (inverse of <math>\text{sf}</math>).</p> <p>Non-central moment of order <math>n</math></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	---

`scipy.stats.levy = <scipy.stats._continuous_distns.levy_gen object at 0x2b909bd4f410>`

A Levy continuous random variable.

As an instance of the `rv_continuous` class, `levy` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**See also:**

`levy_stable`, `levy_l`

**Notes**

The probability density function for `levy` is:

$$\text{levy.pdf}(x) = 1 / (x * \sqrt{2 * \pi * x}) * \exp(-1 / (2 * x))$$

for  $x > 0$ .

This is the same as the Levy-stable distribution with  $a=1/2$  and  $b=1$ .

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `levy.pdf(x, loc, scale)` is identically equivalent to `levy.pdf(y) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

**Examples**

```
>>> from scipy.stats import levy
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> mean, var, skew, kurt = levy.stats(moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(levy.ppf(0.01),
...                 levy.ppf(0.99), 100)
>>> ax.plot(x, levy.pdf(x),
...         'r-', lw=5, alpha=0.6, label='levy pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = levy()
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

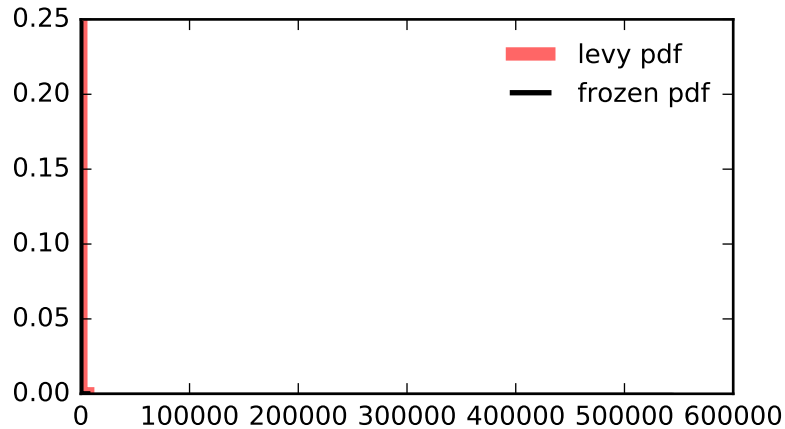
```
>>> vals = levy.ppf([0.001, 0.5, 0.999])
>>> np.allclose([0.001, 0.5, 0.999], levy.cdf(vals))
True
```

Generate random numbers:

```
>>> r = levy.rvs(size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre> rvs(loc=0, scale=1, size=1,     random_state=None) pdf(x, loc=0, scale=1) logpdf(x, loc=0, scale=1) cdf(x, loc=0, scale=1) logcdf(x, loc=0, scale=1)  sf(x, loc=0, scale=1)  logsf(x, loc=0, scale=1) ppf(q, loc=0, scale=1)  isf(q, loc=0, scale=1) moment(n, loc=0, scale=1) stats(loc=0, scale=1, moments='mv')  entropy(loc=0, scale=1) fit(data, loc=0, scale=1) expect(func, args=(), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kwds) median(loc=0, scale=1) mean(loc=0, scale=1) var(loc=0, scale=1) std(loc=0, scale=1) interval(alpha, loc=0, scale=1) </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

`scipy.stats.levy_1` = <scipy.stats.continuous\_distns.levy\_1\_gen object at 0x2b909bd4f710>

A left-skewed Levy continuous random variable.

As an instance of the `rv_continuous` class, `levy_1` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

See also:

```
levy, levy_stable
```

### Notes

The probability density function for `levy_l` is:

```
levy_l.pdf(x) = 1 / (abs(x) * sqrt(2*pi*abs(x))) * exp(-1/(2*abs(x)))
```

for  $x < 0$ .

This is the same as the Levy-stable distribution with  $a=1/2$  and  $b=-1$ .

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `levy_l.pdf(x, loc, scale)` is identically equivalent to `levy_l.pdf(y) / scale` with  $y = (x - loc) / scale$ .

### Examples

```
>>> from scipy.stats import levy_l
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> mean, var, skew, kurt = levy_l.stats(moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(levy_l.ppf(0.01),
...                 levy_l.ppf(0.99), 100)
>>> ax.plot(x, levy_l.pdf(x),
...        'r-', lw=5, alpha=0.6, label='levy_l pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = levy_l()
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

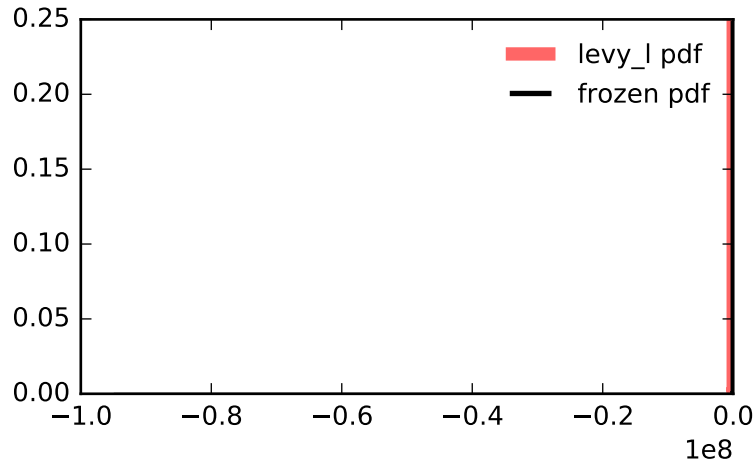
```
>>> vals = levy_l.ppf([0.001, 0.5, 0.999])
>>> np.allclose([0.001, 0.5, 0.999], levy_l.cdf(vals))
True
```

Generate random numbers:

```
>>> r = levy_l.rvs(size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre> rvs(loc=0, scale=1, size=1,     random_state=None) pdf(x, loc=0, scale=1) logpdf(x, loc=0, scale=1) cdf(x, loc=0, scale=1) logcdf(x, loc=0, scale=1)  sf(x, loc=0, scale=1)  logsf(x, loc=0, scale=1) ppf(q, loc=0, scale=1)  isf(q, loc=0, scale=1) moment(n, loc=0, scale=1) stats(loc=0, scale=1, moments='mv')  entropy(loc=0, scale=1) fit(data, loc=0, scale=1) expect(func, args=(), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kwds) median(loc=0, scale=1) mean(loc=0, scale=1) var(loc=0, scale=1) std(loc=0, scale=1) interval(alpha, loc=0, scale=1) </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

`scipy.stats.levy_stable` = <scipy.stats.continuous\_distns.levy\_stable\_gen object at 0x2b909bd4f910>  
 A Levy-stable continuous random variable.

As an instance of the `rv_continuous` class, `levy_stable` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

See also:

```
levy, levy_l
```

### Notes

Levy-stable distribution (only random variates available – ignore other docs)

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `levy_stable.pdf(x, alpha, beta, loc, scale)` is identically equivalent to `levy_stable.pdf(y, alpha, beta) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

### Examples

```
>>> from scipy.stats import levy_stable
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> alpha, beta = 0.357, -0.675
>>> mean, var, skew, kurt = levy_stable.stats(alpha, beta, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(levy_stable.ppf(0.01, alpha, beta),
...                 levy_stable.ppf(0.99, alpha, beta), 100)
>>> ax.plot(x, levy_stable.pdf(x, alpha, beta),
...        'r-', lw=5, alpha=0.6, label='levy_stable pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = levy_stable(alpha, beta)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

```
>>> vals = levy_stable.ppf([0.001, 0.5, 0.999], alpha, beta)
>>> np.allclose([0.001, 0.5, 0.999], levy_stable.cdf(vals, alpha, beta))
True
```

Generate random numbers:

```
>>> r = levy_stable.rvs(alpha, beta, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```

**Methods**

<code>rvs(alpha, beta, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, alpha, beta, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, alpha, beta, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, alpha, beta, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, alpha, beta, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, alpha, beta, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, alpha, beta, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, alpha, beta, loc=0, scale=1)</code>	Percent point function (inverse of <i>cdf</i> — percentiles).
<code>isf(q, alpha, beta, loc=0, scale=1)</code>	Inverse survival function (inverse of <i>sf</i> ).
<code>moment(n, alpha, beta, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(alpha, beta, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(alpha, beta, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, alpha, beta, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(alpha, beta), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(alpha, beta, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(alpha, beta, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(alpha, beta, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(alpha, beta, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, alpha, beta, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.logistic = <scipy.stats._continuous_distns.logistic_gen object at 0x2b909bd4fb90>`

A logistic (or Sech-squared) continuous random variable.

As an instance of the `rv_continuous` class, `logistic` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**Notes**

The probability density function for `logistic` is:

$$\text{logistic.pdf}(x) = \exp(-x) / (1 + \exp(-x))^{*2}$$

`logistic` is a special case of `genlogistic` with `c == 1`.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `logistic.pdf(x, loc, scale)` is identically equivalent to `logistic.pdf(y) / scale` with `y = (x - loc) / scale`.

**Examples**

```
>>> from scipy.stats import logistic
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> mean, var, skew, kurt = logistic.stats(moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(logistic.ppf(0.01),
...                 logistic.ppf(0.99), 100)
>>> ax.plot(x, logistic.pdf(x),
...         'r-', lw=5, alpha=0.6, label='logistic pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = logistic()
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

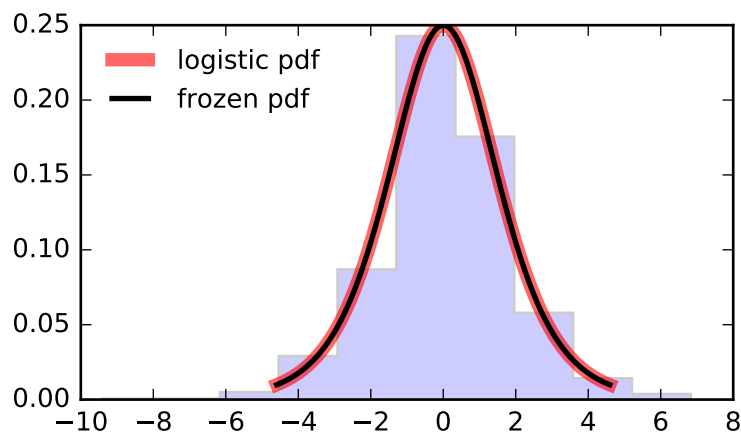
```
>>> vals = logistic.ppf([0.001, 0.5, 0.999])
>>> np.allclose([0.001, 0.5, 0.999], logistic.cdf(vals))
True
```

Generate random numbers:

```
>>> r = logistic.rvs(size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```





**Methods**

<pre> rvs(loc=0, scale=1, size=1,     random_state=None) pdf(x, loc=0, scale=1) logpdf(x, loc=0, scale=1) cdf(x, loc=0, scale=1) logcdf(x, loc=0, scale=1)  sf(x, loc=0, scale=1)  logsf(x, loc=0, scale=1) ppf(q, loc=0, scale=1)  isf(q, loc=0, scale=1) moment(n, loc=0, scale=1) stats(loc=0, scale=1, moments='mv')  entropy(loc=0, scale=1) fit(data, loc=0, scale=1) expect(func, args=(), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kwds) median(loc=0, scale=1) mean(loc=0, scale=1) var(loc=0, scale=1) std(loc=0, scale=1) interval(alpha, loc=0, scale=1) </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

`scipy.stats.loggamma = <scipy.stats.continuous_distns.loggamma_gen object at 0x2b909bd4fd90>`  
 A log gamma continuous random variable.

As an instance of the `rv_continuous` class, `loggamma` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**Notes**

The probability density function for `loggamma` is:

$$\text{loggamma.pdf}(x, c) = \exp(c \cdot x - \exp(x)) / \text{gamma}(c)$$

for all  $x, c > 0$ .

`loggamma` takes *c* as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the *loc* and *scale* parameters. Specifically, `loggamma.pdf(x, c, loc, scale)` is identically equivalent to `loggamma.pdf(y, c) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

**Examples**

```

>>> from scipy.stats import loggamma
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)

```

Calculate a few first moments:

```
>>> c = 0.414
>>> mean, var, skew, kurt = loggamma.stats(c, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(loggamma.ppf(0.01, c),
...                 loggamma.ppf(0.99, c), 100)
>>> ax.plot(x, loggamma.pdf(x, c),
...        'r-', lw=5, alpha=0.6, label='loggamma pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = loggamma(c)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

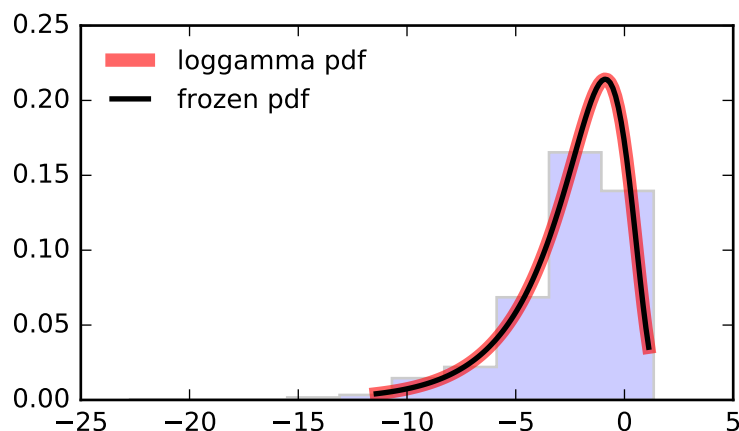
```
>>> vals = loggamma.ppf([0.001, 0.5, 0.999], c)
>>> np.allclose([0.001, 0.5, 0.999], loggamma.cdf(vals, c))
True
```

Generate random numbers:

```
>>> r = loggamma.rvs(c, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



**Methods**

<code>rvs(c, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, c, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, c, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, c, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, c, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, c, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, c, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, c, loc=0, scale=1)</code>	Percent point function (inverse of <i>cdf</i> — percentiles).
<code>isf(q, c, loc=0, scale=1)</code>	Inverse survival function (inverse of <i>sf</i> ).
<code>moment(n, c, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(c, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(c, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, c, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(c,), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(c, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(c, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(c, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(c, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, c, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.loglaplace = <scipy.stats.continuous_distns.loglaplace_gen object at 0x2b909bd59050>`  
 A log-Laplace continuous random variable.

As an instance of the `rv_continuous` class, `loglaplace` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**Notes**

The probability density function for `loglaplace` is:

$$\begin{aligned} \text{loglaplace.pdf}(x, c) &= c / 2 * x^{c-1}, & \text{for } 0 < x < 1 \\ &= c / 2 * x^{-c-1}, & \text{for } x \geq 1 \end{aligned}$$

for  $c > 0$ .

`loglaplace` takes *c* as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the *loc* and *scale* parameters. Specifically, `loglaplace.pdf(x, c, loc, scale)` is identically equivalent to `loglaplace.pdf(y, c) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

**References**

T.J. Kozubowski and K. Podgorski, “A log-Laplace growth rate model”, *The Mathematical Scientist*, vol. 28, pp. 49-60, 2003.

### Examples

```
>>> from scipy.stats import loglaplace
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> c = 3.25
>>> mean, var, skew, kurt = loglaplace.stats(c, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(loglaplace.ppf(0.01, c),
...                 loglaplace.ppf(0.99, c), 100)
>>> ax.plot(x, loglaplace.pdf(x, c),
...        'r-', lw=5, alpha=0.6, label='loglaplace pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = loglaplace(c)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

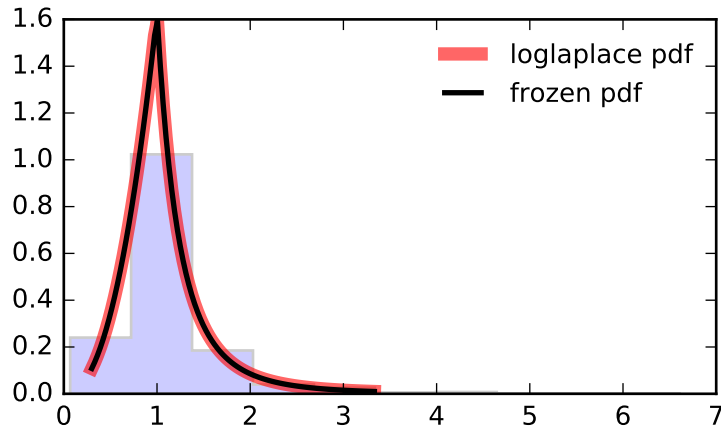
```
>>> vals = loglaplace.ppf([0.001, 0.5, 0.999], c)
>>> np.allclose([0.001, 0.5, 0.999], loglaplace.cdf(vals, c))
True
```

Generate random numbers:

```
>>> r = loglaplace.rvs(c, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<code>rvs(c, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, c, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, c, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, c, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, c, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, c, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, c, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, c, loc=0, scale=1)</code>	Percent point function (inverse of <i>cdf</i> — percentiles).
<code>isf(q, c, loc=0, scale=1)</code>	Inverse survival function (inverse of <i>sf</i> ).
<code>moment(n, c, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(c, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(c, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, c, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(c,), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(c, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(c, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(c, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(c, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, c, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.lognorm = <scipy.stats.continuous_distns.lognorm_gen object at 0x2b909bd592d0>`

A lognormal continuous random variable.

As an instance of the `rv_continuous` class, `lognorm` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `lognorm` is:

```
lognorm.pdf(x, s) = 1 / (s*x*sqrt(2*pi)) * exp(-1/2*(log(x)/s)**2)
```

for  $x > 0, s > 0$ .

`lognorm` takes `s` as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `lognorm.pdf(x, s, loc, scale)` is identically equivalent to `lognorm.pdf(y, s) / scale` with  $y = (x - loc) / scale$ .

A common parametrization for a lognormal random variable  $Y$  is in terms of the mean, `mu`, and standard deviation, `sigma`, of the unique normally distributed random variable  $X$  such that  $\exp(X) = Y$ . This parametrization corresponds to setting `s = sigma` and `scale = exp(mu)`.

### Examples

```
>>> from scipy.stats import lognorm
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> s = 0.954
>>> mean, var, skew, kurt = lognorm.stats(s, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(lognorm.ppf(0.01, s),
...                 lognorm.ppf(0.99, s), 100)
>>> ax.plot(x, lognorm.pdf(x, s),
...        'r-', lw=5, alpha=0.6, label='lognorm pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = lognorm(s)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

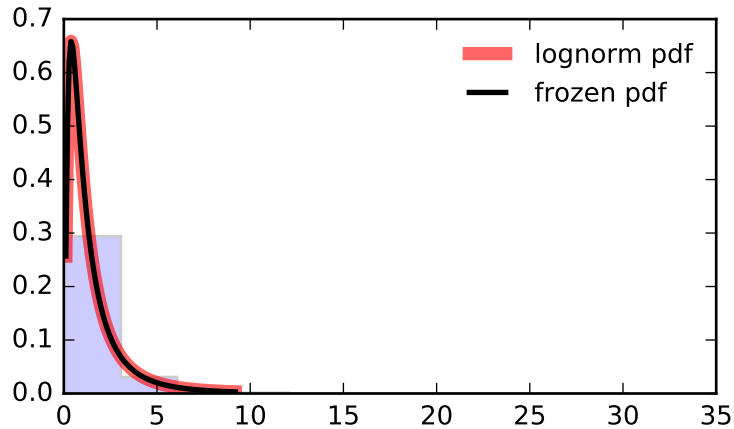
```
>>> vals = lognorm.ppf([0.001, 0.5, 0.999], s)
>>> np.allclose([0.001, 0.5, 0.999], lognorm.cdf(vals, s))
True
```

Generate random numbers:

```
>>> r = lognorm.rvs(s, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre>rvs(s, loc=0, scale=1, size=1,     random_state=None) pdf(x, s, loc=0, scale=1) logpdf(x, s, loc=0, scale=1) cdf(x, s, loc=0, scale=1) logcdf(x, s, loc=0, scale=1)  sf(x, s, loc=0, scale=1)  logsf(x, s, loc=0, scale=1) ppf(q, s, loc=0, scale=1)  isf(q, s, loc=0, scale=1) moment(n, s, loc=0, scale=1) stats(s, loc=0, scale=1, moments='mv')  entropy(s, loc=0, scale=1) fit(data, s, loc=0, scale=1) expect(func, args=(s,), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kwds) median(s, loc=0, scale=1) mean(s, loc=0, scale=1) var(s, loc=0, scale=1) std(s, loc=0, scale=1) interval(alpha, s, loc=0, scale=1)</pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains <i>alpha</i> percent of the distribution</p>
---	---

`scipy.stats.lomax` = <scipy.stats.continuous\_distns.lomax\_gen object at 0x2b909bd78110>

A Lomax (Pareto of the second kind) continuous random variable.

As an instance of the `rv_continuous` class, `lomax` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The Lomax distribution is a special case of the Pareto distribution, with (`loc=-1.0`).

The probability density function for `lomax` is:

```
lomax.pdf(x, c) = c / (1+x)**(c+1)
```

for  $x \geq 0, c > 0$ .

`lomax` takes `c` as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `lomax.pdf(x, c, loc, scale)` is identically equivalent to `lomax.pdf(y, c) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

### Examples

```
>>> from scipy.stats import lomax
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> c = 1.88
>>> mean, var, skew, kurt = lomax.stats(c, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(lomax.ppf(0.01, c),
...                 lomax.ppf(0.99, c), 100)
>>> ax.plot(x, lomax.pdf(x, c),
...        'r-', lw=5, alpha=0.6, label='lomax pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = lomax(c)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

```
>>> vals = lomax.ppf([0.001, 0.5, 0.999], c)
>>> np.allclose([0.001, 0.5, 0.999], lomax.cdf(vals, c))
True
```

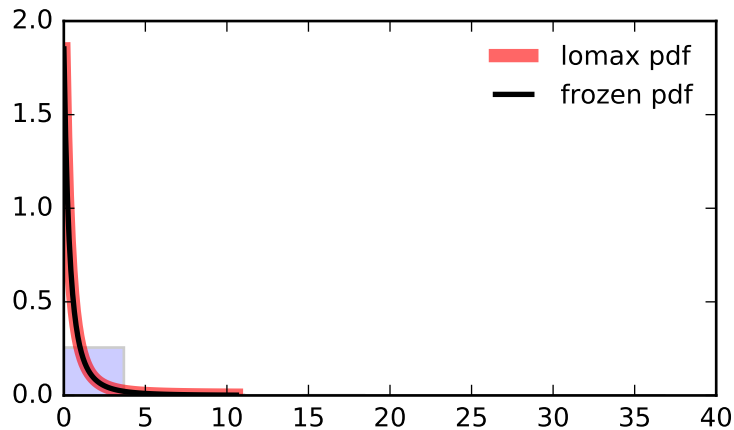
Generate random numbers:

```
>>> r = lomax.rvs(c, size=1000)
```

And compare the histogram:



```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre>rvs(c, loc=0, scale=1, size=1,     random_state=None) pdf(x, c, loc=0, scale=1) logpdf(x, c, loc=0, scale=1) cdf(x, c, loc=0, scale=1) logcdf(x, c, loc=0, scale=1)  sf(x, c, loc=0, scale=1)  logsf(x, c, loc=0, scale=1) ppf(q, c, loc=0, scale=1)  isf(q, c, loc=0, scale=1) moment(n, c, loc=0, scale=1) stats(c, loc=0, scale=1, moments='mv')  entropy(c, loc=0, scale=1) fit(data, c, loc=0, scale=1) expect(func, args=(c,), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kwargs) median(c, loc=0, scale=1) mean(c, loc=0, scale=1) var(c, loc=0, scale=1) std(c, loc=0, scale=1) interval(alpha, c, loc=0, scale=1)</pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

```
scipy.stats.maxwell = <scipy.stats._continuous_distns.maxwell_gen object at 0x2b909bd59750>
```

A Maxwell continuous random variable.

As an instance of the `rv_continuous` class, `maxwell` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

A special case of a `chi` distribution, with `df = 3`, `loc = 0.0`, and given `scale = a`, where `a` is the parameter used in the Mathworld description [R540].

The probability density function for `maxwell` is:

$$\text{maxwell.pdf}(x) = \sqrt{2/\pi} x^{*2} * \exp(-x^{*2}/2)$$

for  $x > 0$ .

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `maxwell.pdf(x, loc, scale)` is identically equivalent to `maxwell.pdf(y) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

### References

[R540]

### Examples

```
>>> from scipy.stats import maxwell
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> mean, var, skew, kurt = maxwell.stats(moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(maxwell.ppf(0.01),
...                 maxwell.ppf(0.99), 100)
>>> ax.plot(x, maxwell.pdf(x),
...         'r-', lw=5, alpha=0.6, label='maxwell pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = maxwell()
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

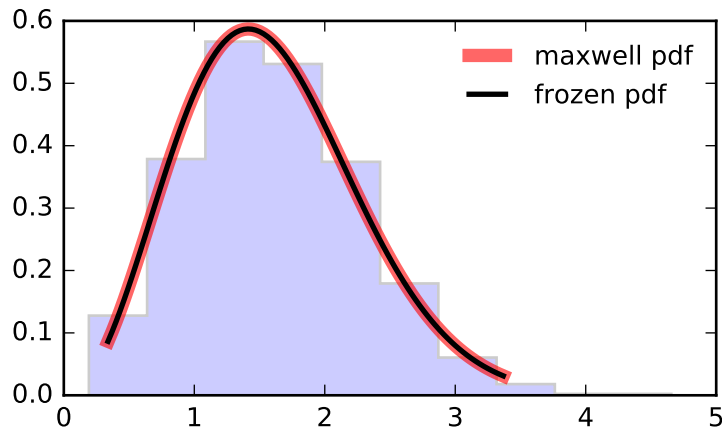
```
>>> vals = maxwell.ppf([0.001, 0.5, 0.999])
>>> np.allclose([0.001, 0.5, 0.999], maxwell.cdf(vals))
True
```

Generate random numbers:

```
>>> r = maxwell.rvs(size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre>rvs(loc=0, scale=1, size=1,     random_state=None) pdf(x, loc=0, scale=1) logpdf(x, loc=0, scale=1) cdf(x, loc=0, scale=1) logcdf(x, loc=0, scale=1)  sf(x, loc=0, scale=1)  logsf(x, loc=0, scale=1) ppf(q, loc=0, scale=1)  isf(q, loc=0, scale=1) moment(n, loc=0, scale=1) stats(loc=0, scale=1, moments='mv')  entropy(loc=0, scale=1) fit(data, loc=0, scale=1) expect(func, args=(), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kws) median(loc=0, scale=1) mean(loc=0, scale=1) var(loc=0, scale=1) std(loc=0, scale=1) interval(alpha, loc=0, scale=1)</pre>	<p>Random variates.</p> <p>Probability density function. Log of the probability density function. Cumulative distribution function. Log of the cumulative distribution function. Survival function (also defined as <math>1 - \text{cdf}</math>, but <math>\text{sf}</math> is sometimes more accurate). Log of the survival function. Percent point function (inverse of <math>\text{cdf}</math> — percentiles). Inverse survival function (inverse of <math>\text{sf}</math>). Non-central moment of order <math>n</math> Mean('m'), variance('v'), skew('s'), and/or kurtosis('k'). (Differential) entropy of the RV. Parameter estimates for generic data. Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution. Mean of the distribution. Variance of the distribution. Standard deviation of the distribution. Endpoints of the range that contains alpha percent of the distribution</p>
--	---

`scipy.stats.mielke` = <scipy.stats.\_continuous\_distns.mielke\_gen object at 0x2b909bd59950>

A Mielke’s Beta-Kappa continuous random variable.

As an instance of the `rv_continuous` class, `mielke` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**Notes**

The probability density function for `mielke` is:

$$\text{mielke.pdf}(x, k, s) = k * x^{k-1} / (1+x*s)^{k+1/s}$$

for  $x > 0$ .

`mielke` takes `k` and `s` as shape parameters.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `mielke.pdf(x, k, s, loc, scale)` is identically equivalent to `mielke.pdf(y, k, s) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

**Examples**

```
>>> from scipy.stats import mielke
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> k, s = 10.4, 3.6
>>> mean, var, skew, kurt = mielke.stats(k, s, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(mielke.ppf(0.01, k, s),
...                 mielke.ppf(0.99, k, s), 100)
>>> ax.plot(x, mielke.pdf(x, k, s),
...         'r-', lw=5, alpha=0.6, label='mielke pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = mielke(k, s)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

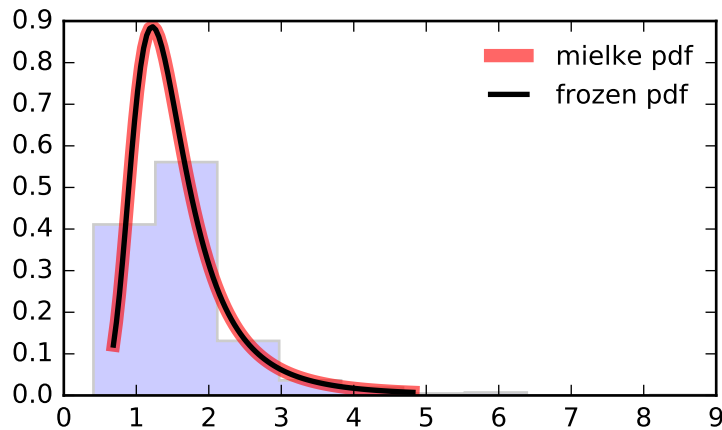
```
>>> vals = mielke.ppf([0.001, 0.5, 0.999], k, s)
>>> np.allclose([0.001, 0.5, 0.999], mielke.cdf(vals, k, s))
True
```

Generate random numbers:

```
>>> r = mielke.rvs(k, s, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre>rvs(k, s, loc=0, scale=1, size=1,     random_state=None) pdf(x, k, s, loc=0, scale=1) logpdf(x, k, s, loc=0, scale=1) cdf(x, k, s, loc=0, scale=1) logcdf(x, k, s, loc=0, scale=1)  sf(x, k, s, loc=0, scale=1)  logsf(x, k, s, loc=0, scale=1) ppf(q, k, s, loc=0, scale=1)  isf(q, k, s, loc=0, scale=1)  moment(n, k, s, loc=0, scale=1) stats(k, s, loc=0, scale=1, moments='mv')  entropy(k, s, loc=0, scale=1) fit(data, k, s, loc=0, scale=1) expect(func, args=(k, s), loc=0, scale=1,         lb=None, ub=None, conditional=False,         **kwds) median(k, s, loc=0, scale=1) mean(k, s, loc=0, scale=1) var(k, s, loc=0, scale=1) std(k, s, loc=0, scale=1) interval(alpha, k, s, loc=0, scale=1)</pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
--	--

`scipy.stats.nakagami` = <scipy.stats.\_continuous\_distns.nakagami\_gen object at 0x2b909bd67110>

A Nakagami continuous random variable.

As an instance of the `rv_continuous` class, `nakagami` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `nakagami` is:

$$\text{nakagami.pdf}(x, \text{nu}) = 2 * \text{nu} ** \text{nu} / \text{gamma}(\text{nu}) * x ** (2 * \text{nu} - 1) * \exp(-\text{nu} * x ** 2)$$

for  $x > 0, \text{nu} > 0$ .

`nakagami` takes `nu` as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `nakagami.pdf(x, nu, loc, scale)` is identically equivalent to `nakagami.pdf(y, nu) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

### Examples

```
>>> from scipy.stats import nakagami
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> nu = 4.97
>>> mean, var, skew, kurt = nakagami.stats(nu, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(nakagami.ppf(0.01, nu),
...                 nakagami.ppf(0.99, nu), 100)
>>> ax.plot(x, nakagami.pdf(x, nu),
...        'r-', lw=5, alpha=0.6, label='nakagami pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = nakagami(nu)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

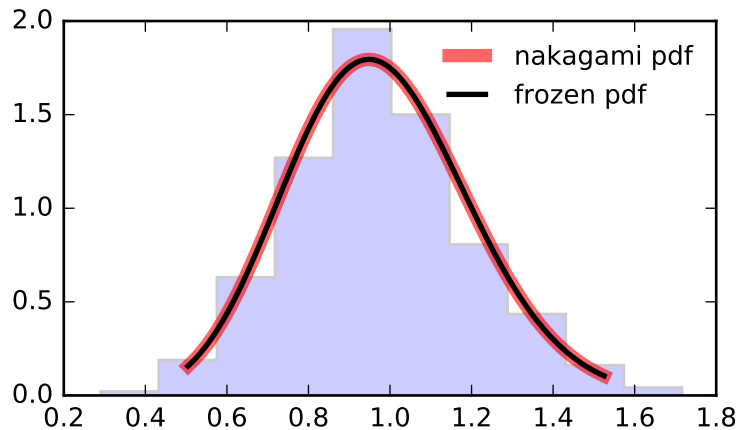
```
>>> vals = nakagami.ppf([0.001, 0.5, 0.999], nu)
>>> np.allclose([0.001, 0.5, 0.999], nakagami.cdf(vals, nu))
True
```

Generate random numbers:

```
>>> r = nakagami.rvs(nu, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre>rvs(nu, loc=0, scale=1, size=1,     random_state=None) pdf(x, nu, loc=0, scale=1) logpdf(x, nu, loc=0, scale=1) cdf(x, nu, loc=0, scale=1) logcdf(x, nu, loc=0, scale=1)  sf(x, nu, loc=0, scale=1)  logsf(x, nu, loc=0, scale=1) ppf(q, nu, loc=0, scale=1)  isf(q, nu, loc=0, scale=1) moment(n, nu, loc=0, scale=1) stats(nu, loc=0, scale=1, moments='mv')  entropy(nu, loc=0, scale=1) fit(data, nu, loc=0, scale=1) expect(func, args=(nu,), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kwds) median(nu, loc=0, scale=1) mean(nu, loc=0, scale=1) var(nu, loc=0, scale=1) std(nu, loc=0, scale=1) interval(alpha, nu, loc=0, scale=1)</pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
--	--

`scipy.stats.ncx2` = <scipy.stats.\_continuous\_distns.ncx2\_gen object at 0x2b909bd67390>

A non-central chi-squared continuous random variable.

As an instance of the `rv_continuous` class, `ncx2` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `ncx2` is:

$$\text{ncx2.pdf}(x, \text{df}, \text{nc}) = \exp(-(nc+x)/2) * 1/2 * (x/nc)**((df-2)/4) * I[(df-2)/2](\text{sqrt}(nc*x))$$

for  $x > 0$ .

`ncx2` takes `df` and `nc` as shape parameters.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `ncx2.pdf(x, df, nc, loc, scale)` is identically equivalent to `ncx2.pdf(y, df, nc) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

### Examples

```
>>> from scipy.stats import ncx2
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> df, nc = 21, 1.06
>>> mean, var, skew, kurt = ncx2.stats(df, nc, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(ncx2.ppf(0.01, df, nc),
...                 ncx2.ppf(0.99, df, nc), 100)
>>> ax.plot(x, ncx2.pdf(x, df, nc),
...        'r-', lw=5, alpha=0.6, label='ncx2 pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = ncx2(df, nc)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

```
>>> vals = ncx2.ppf([0.001, 0.5, 0.999], df, nc)
>>> np.allclose([0.001, 0.5, 0.999], ncx2.cdf(vals, df, nc))
True
```

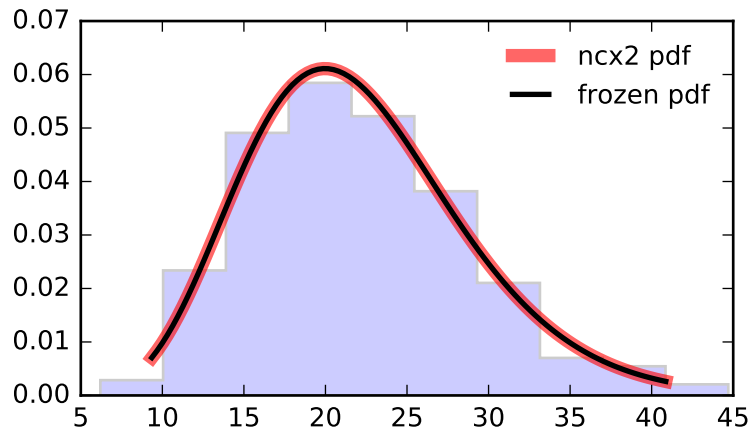
Generate random numbers:

```
>>> r = ncx2.rvs(df, nc, size=1000)
```

And compare the histogram:



```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre>rvs(df, nc, loc=0, scale=1, size=1,     random_state=None) pdf(x, df, nc, loc=0, scale=1) logpdf(x, df, nc, loc=0, scale=1) cdf(x, df, nc, loc=0, scale=1) logcdf(x, df, nc, loc=0, scale=1)  sf(x, df, nc, loc=0, scale=1)  logsf(x, df, nc, loc=0, scale=1) ppf(q, df, nc, loc=0, scale=1)  isf(q, df, nc, loc=0, scale=1)  moment(n, df, nc, loc=0, scale=1) stats(df, nc, loc=0, scale=1, moments='mv')  entropy(df, nc, loc=0, scale=1) fit(data, df, nc, loc=0, scale=1) expect(func, args=(df, nc), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kwds) median(df, nc, loc=0, scale=1) mean(df, nc, loc=0, scale=1) var(df, nc, loc=0, scale=1) std(df, nc, loc=0, scale=1) interval(alpha, df, nc, loc=0, scale=1)</pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <math>\text{sf}</math> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <math>\text{cdf}</math> — percentiles).</p> <p>Inverse survival function (inverse of <math>\text{sf}</math>).</p> <p>Non-central moment of order <math>n</math></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
--	---

`scipy.stats.ncf` = <scipy.stats.\_continuous\_distns.ncf\_gen object at 0x2b909bd67610>

A non-central F distribution continuous random variable.

As an instance of the `rv_continuous` class, `ncf` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `ncf` is:

$$\begin{aligned} \text{ncf.pdf}(x, \text{df1}, \text{df2}, \text{nc}) = & \exp(\text{nc}/2 + \text{nc} \cdot \text{df1} \cdot x / (2 \cdot (\text{df1} \cdot x + \text{df2}))) \cdot \\ & \text{df1} \cdot (\text{df1}/2) \cdot \text{df2} \cdot (\text{df2}/2) \cdot x \cdot (\text{df1}/2 - 1) \cdot \\ & (\text{df2} + \text{df1} \cdot x) \cdot (- (\text{df1} + \text{df2}) / 2) \cdot \\ & \text{gamma}(\text{df1}/2) \cdot \text{gamma}(1 + \text{df2}/2) \cdot \\ & L^{\{\text{v1}/2 - 1\}} \cdot \text{v2}/2 \cdot (-\text{nc} \cdot \text{v1} \cdot x / (2 \cdot (\text{v1} \cdot x + \text{v2}))) / \\ & (B(\text{v1}/2, \text{v2}/2) \cdot \text{gamma}((\text{v1} + \text{v2})/2)) \end{aligned}$$

for `df1`, `df2`, `nc` > 0.

`ncf` takes `df1`, `df2` and `nc` as shape parameters.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `ncf.pdf(x, dfn, dfd, nc, loc, scale)` is identically equivalent to `ncf.pdf(y, dfn, dfd, nc) / scale` with `y = (x - loc) / scale`.

### Examples

```
>>> from scipy.stats import ncf
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> dfn, dfd, nc = 27, 27, 0.416
>>> mean, var, skew, kurt = ncf.stats(dfn, dfd, nc, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(ncf.ppf(0.01, dfn, dfd, nc),
...                 ncf.ppf(0.99, dfn, dfd, nc), 100)
>>> ax.plot(x, ncf.pdf(x, dfn, dfd, nc),
...        'r-', lw=5, alpha=0.6, label='ncf pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = ncf(dfn, dfd, nc)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

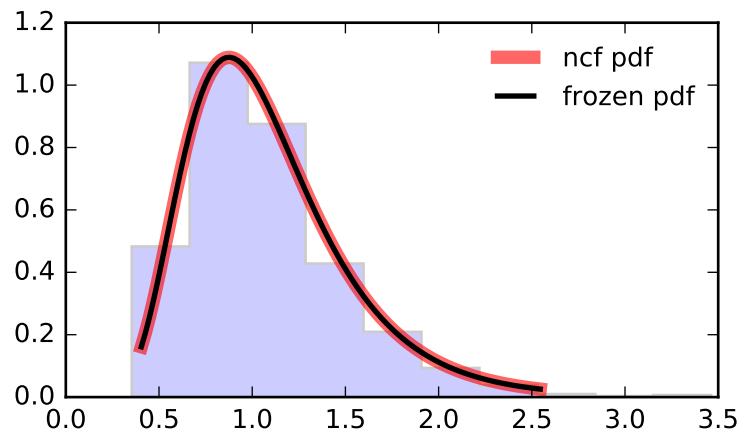
```
>>> vals = ncf.ppf([0.001, 0.5, 0.999], dfn, dfd, nc)
>>> np.allclose([0.001, 0.5, 0.999], ncf.cdf(vals, dfn, dfd, nc))
True
```

Generate random numbers:

```
>>> r = ncf.rvs(dfn, dfd, nc, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



**Methods**

<pre> rvs(dfn, dfd, nc, loc=0, scale=1, size=1,     random_state=None) pdf(x, dfn, dfd, nc, loc=0, scale=1) logpdf(x, dfn, dfd, nc, loc=0, scale=1)  cdf(x, dfn, dfd, nc, loc=0, scale=1) logcdf(x, dfn, dfd, nc, loc=0, scale=1)  sf(x, dfn, dfd, nc, loc=0, scale=1)  logsf(x, dfn, dfd, nc, loc=0, scale=1) ppf(q, dfn, dfd, nc, loc=0, scale=1)  isf(q, dfn, dfd, nc, loc=0, scale=1)  moment(n, dfn, dfd, nc, loc=0, scale=1) stats(dfn, dfd, nc, loc=0, scale=1,       moments='mv') entropy(dfn, dfd, nc, loc=0, scale=1) fit(data, dfn, dfd, nc, loc=0, scale=1) expect(func, args=(dfn, dfd, nc), loc=0,        scale=1, lb=None, ub=None,        conditional=False, **kwds) median(dfn, dfd, nc, loc=0, scale=1) mean(dfn, dfd, nc, loc=0, scale=1) var(dfn, dfd, nc, loc=0, scale=1) std(dfn, dfd, nc, loc=0, scale=1) interval(alpha, dfn, dfd, nc, loc=0,          scale=1) </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

`scipy.stats.nct` = <scipy.stats.continuous\_distns.nct\_gen object at 0x2b909bd67b50>

A non-central Student's T continuous random variable.

As an instance of the `rv_continuous` class, `nct` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**Notes**

The probability density function for `nct` is:

$$\text{nct.pdf}(x, \text{df}, \text{nc}) = \frac{\text{df}^{df/2} * \text{gamma}(\text{df}+1)}{2^{df} * \exp(\text{nc}^{df/2}) * (\text{df}+x^{df/2})^{df/2} * \text{gamma}(\text{df}/2)}$$

for  $\text{df} > 0$ .

`nct` takes `df` and `nc` as shape parameters.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `nct.pdf(x, df, nc, loc, scale)` is identically equivalent to `nct.pdf(y, df, nc) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

*Examples*

```
>>> from scipy.stats import nct
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> df, nc = 14, 0.24
>>> mean, var, skew, kurt = nct.stats(df, nc, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(nct.ppf(0.01, df, nc),
...                 nct.ppf(0.99, df, nc), 100)
>>> ax.plot(x, nct.pdf(x, df, nc),
...         'r-', lw=5, alpha=0.6, label='nct pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = nct(df, nc)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

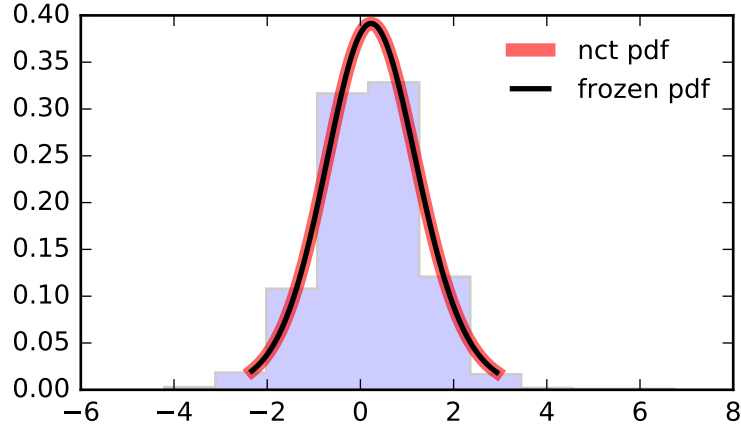
```
>>> vals = nct.ppf([0.001, 0.5, 0.999], df, nc)
>>> np.allclose([0.001, 0.5, 0.999], nct.cdf(vals, df, nc))
True
```

Generate random numbers:

```
>>> r = nct.rvs(df, nc, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre> rvs(df, nc, loc=0, scale=1, size=1,     random_state=None) pdf(x, df, nc, loc=0, scale=1) logpdf(x, df, nc, loc=0, scale=1) cdf(x, df, nc, loc=0, scale=1) logcdf(x, df, nc, loc=0, scale=1)  sf(x, df, nc, loc=0, scale=1)  logsf(x, df, nc, loc=0, scale=1) ppf(q, df, nc, loc=0, scale=1)  isf(q, df, nc, loc=0, scale=1)  moment(n, df, nc, loc=0, scale=1) stats(df, nc, loc=0, scale=1, moments='mv')  entropy(df, nc, loc=0, scale=1) fit(data, df, nc, loc=0, scale=1) expect(func, args=(df, nc), loc=0, scale=1,         lb=None, ub=None, conditional=False,         **kwds) median(df, nc, loc=0, scale=1) mean(df, nc, loc=0, scale=1) var(df, nc, loc=0, scale=1) std(df, nc, loc=0, scale=1) interval(alpha, df, nc, loc=0, scale=1) </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <math>\text{sf}</math> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <math>\text{cdf}</math> — percentiles).</p> <p>Inverse survival function (inverse of <math>\text{sf}</math>).</p> <p>Non-central moment of order <math>n</math></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
--	---

`scipy.stats.norm = <scipy.stats._continuous_distns.norm_gen object at 0x2b909baabc10>`

A normal continuous random variable.

The location (`loc`) keyword specifies the mean. The scale (`scale`) keyword specifies the standard deviation.

As an instance of the `rv_continuous` class, `norm` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `norm` is:

```
norm.pdf(x) = exp(-x**2/2)/sqrt(2*pi)
```

The survival function, `norm.sf`, is also referred to as the Q-function in some contexts (see, e.g., [Wikipedia's definition](#)).

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `norm.pdf(x, loc, scale)` is identically equivalent to `norm.pdf(y) / scale` with `y = (x - loc) / scale`.

### Examples

```
>>> from scipy.stats import norm
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> mean, var, skew, kurt = norm.stats(moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(norm.ppf(0.01),
...                 norm.ppf(0.99), 100)
>>> ax.plot(x, norm.pdf(x),
...        'r-', lw=5, alpha=0.6, label='norm pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = norm()
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

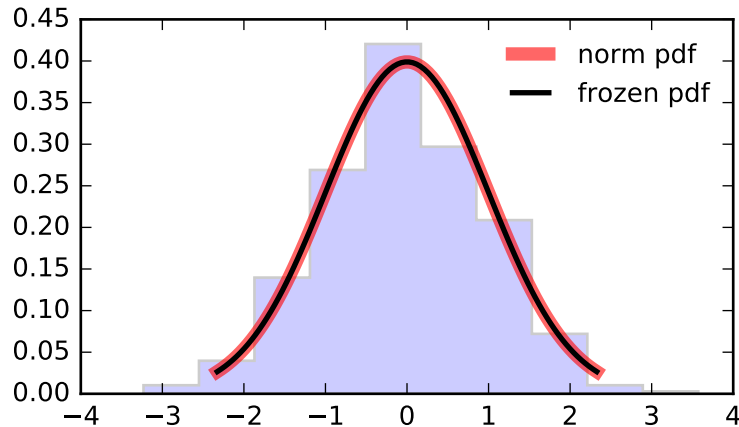
```
>>> vals = norm.ppf([0.001, 0.5, 0.999])
>>> np.allclose([0.001, 0.5, 0.999], norm.cdf(vals))
True
```

Generate random numbers:

```
>>> r = norm.rvs(size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre> rvs(loc=0, scale=1, size=1,     random_state=None) pdf(x, loc=0, scale=1) logpdf(x, loc=0, scale=1) cdf(x, loc=0, scale=1) logcdf(x, loc=0, scale=1)  sf(x, loc=0, scale=1)  logsf(x, loc=0, scale=1) ppf(q, loc=0, scale=1)  isf(q, loc=0, scale=1) moment(n, loc=0, scale=1) stats(loc=0, scale=1, moments='mv')  entropy(loc=0, scale=1) fit(data, loc=0, scale=1) expect(func, args=(), loc=0, scale=1,     lb=None, ub=None, conditional=False,     **kwds) median(loc=0, scale=1) mean(loc=0, scale=1) var(loc=0, scale=1) std(loc=0, scale=1) interval(alpha, loc=0, scale=1)                 </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

`scipy.stats.pareto` = <scipy.stats.continuous\_distns.pareto\_gen object at 0x2b909bd67e10>

A Pareto continuous random variable.

As an instance of the `rv_continuous` class, `pareto` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.



**Notes**

The probability density function for `pareto` is:

$$\text{pareto.pdf}(x, b) = b / x^{b+1}$$

for  $x \geq 1, b > 0$ .

`pareto` takes `b` as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `pareto.pdf(x, b, loc, scale)` is identically equivalent to `pareto.pdf(y, b) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

**Examples**

```
>>> from scipy.stats import pareto
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> b = 2.62
>>> mean, var, skew, kurt = pareto.stats(b, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(pareto.ppf(0.01, b),
...                 pareto.ppf(0.99, b), 100)
>>> ax.plot(x, pareto.pdf(x, b),
...         'r-', lw=5, alpha=0.6, label='pareto pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = pareto(b)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

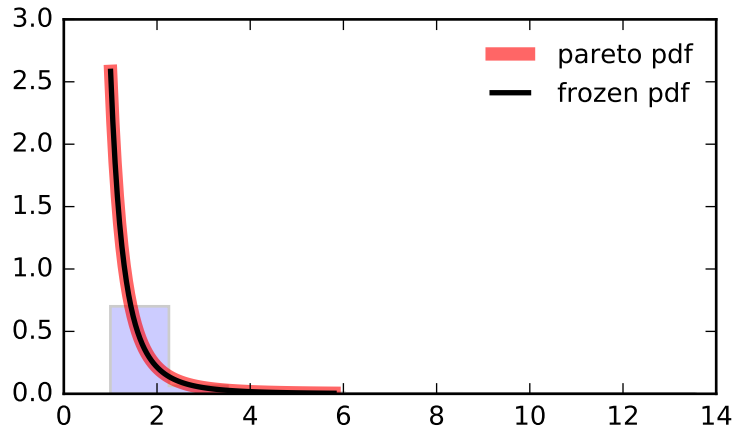
```
>>> vals = pareto.ppf([0.001, 0.5, 0.999], b)
>>> np.allclose([0.001, 0.5, 0.999], pareto.cdf(vals, b))
True
```

Generate random numbers:

```
>>> r = pareto.rvs(b, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre> rvs(b, loc=0, scale=1, size=1,     random_state=None) pdf(x, b, loc=0, scale=1) logpdf(x, b, loc=0, scale=1) cdf(x, b, loc=0, scale=1) logcdf(x, b, loc=0, scale=1)  sf(x, b, loc=0, scale=1)  logsf(x, b, loc=0, scale=1) ppf(q, b, loc=0, scale=1)  isf(q, b, loc=0, scale=1) moment(n, b, loc=0, scale=1) stats(b, loc=0, scale=1, moments='mv')  entropy(b, loc=0, scale=1) fit(data, b, loc=0, scale=1) expect(func, args=(b,), loc=0, scale=1,         lb=None, ub=None, conditional=False,         **kwds) median(b, loc=0, scale=1) mean(b, loc=0, scale=1) var(b, loc=0, scale=1) std(b, loc=0, scale=1) interval(alpha, b, loc=0, scale=1)                     </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

`scipy.stats.pearson3 = <scipy.stats.continuous_distns.pearson3_gen object at 0x2b909bd78390>`

A pearson type III continuous random variable.

As an instance of the `rv_continuous` class, `pearson3` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `pearson3` is:

$$\text{pearson3.pdf}(x, \text{skew}) = \frac{\text{abs}(\text{beta})}{\text{gamma}(\text{alpha}) * (\text{beta} * (x - \text{zeta}))^{**}(\text{alpha} - 1) * \text{exp}(-\text{beta} * (x - \text{zeta}))}$$

where:

```
beta = 2 / (skew * stddev)
alpha = (stddev * beta)**2
zeta = loc - alpha / beta
```

`pearson3` takes `skew` as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `pearson3.pdf(x, skew, loc, scale)` is identically equivalent to `pearson3.pdf(y, skew) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

### References

R.W. Vogel and D.E. McMartin, “Probability Plot Goodness-of-Fit and Skewness Estimation Procedures for the Pearson Type 3 Distribution”, *Water Resources Research*, Vol.27, 3149-3158 (1991).

L.R. Salvosa, “Tables of Pearson’s Type III Function”, *Ann. Math. Statist.*, Vol.1, 191-198 (1930).

“Using Modern Computing Tools to Fit the Pearson Type III Distribution to Aviation Loads Data”, Office of Aviation Research (2003).

### Examples

```
>>> from scipy.stats import pearson3
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> skew = 0.1
>>> mean, var, skew, kurt = pearson3.stats(skew, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(pearson3.ppf(0.01, skew),
...                 pearson3.ppf(0.99, skew), 100)
>>> ax.plot(x, pearson3.pdf(x, skew),
...         'r-', lw=5, alpha=0.6, label='pearson3 pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = pearson3(skew)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

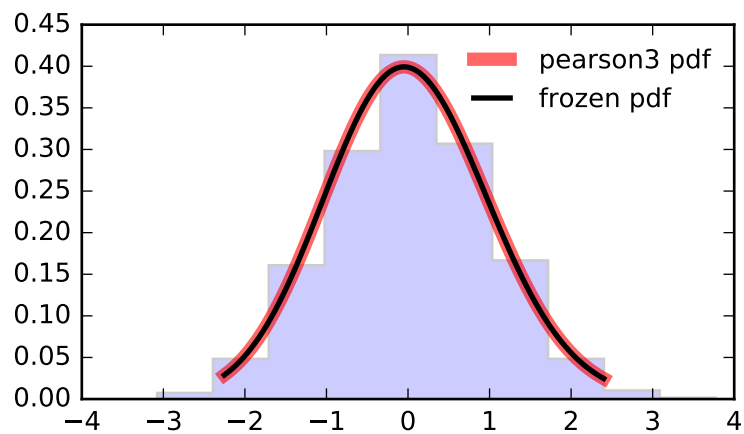
```
>>> vals = pearson3.ppf([0.001, 0.5, 0.999], skew)
>>> np.allclose([0.001, 0.5, 0.999], pearson3.cdf(vals, skew))
True
```

Generate random numbers:

```
>>> r = pearson3.rvs(skew, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



**Methods**

<code>rvs(skew, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, skew, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, skew, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, skew, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, skew, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, skew, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, skew, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, skew, loc=0, scale=1)</code>	Percent point function (inverse of <i>cdf</i> — percentiles).
<code>isf(q, skew, loc=0, scale=1)</code>	Inverse survival function (inverse of <i>sf</i> ).
<code>moment(n, skew, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(skew, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(skew, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, skew, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(skew,), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(skew, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(skew, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(skew, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(skew, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, skew, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.powerlaw = <scipy.stats._continuous_distns.powerlaw_gen object at 0x2b909bd78610>`

A power-function continuous random variable.

As an instance of the `rv_continuous` class, `powerlaw` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**Notes**

The probability density function for `powerlaw` is:

$$\text{powerlaw.pdf}(x, a) = a * x^{(a-1)}$$

for  $0 \leq x \leq 1, a > 0$ .

`powerlaw` takes *a* as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the *loc* and *scale* parameters. Specifically, `powerlaw.pdf(x, a, loc, scale)` is identically equivalent to `powerlaw.pdf(y, a) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

`powerlaw` is a special case of `beta` with  $b == 1$ .

**Examples**

```
>>> from scipy.stats import powerlaw
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> a = 1.66
>>> mean, var, skew, kurt = powerlaw.stats(a, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(powerlaw.ppf(0.01, a),
...                 powerlaw.ppf(0.99, a), 100)
>>> ax.plot(x, powerlaw.pdf(x, a),
...        'r-', lw=5, alpha=0.6, label='powerlaw pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = powerlaw(a)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

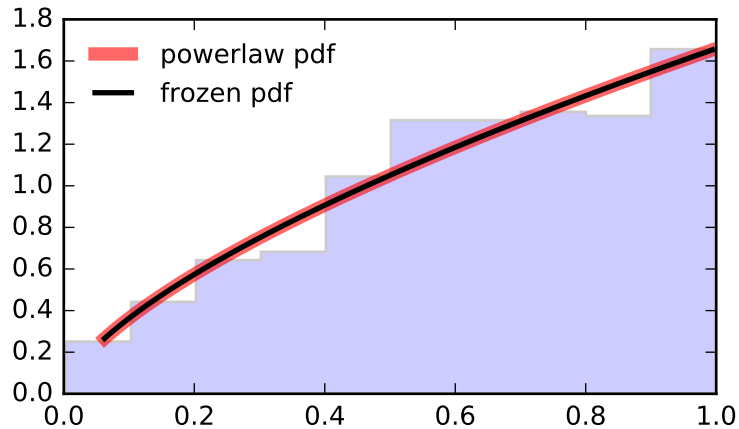
```
>>> vals = powerlaw.ppf([0.001, 0.5, 0.999], a)
>>> np.allclose([0.001, 0.5, 0.999], powerlaw.cdf(vals, a))
True
```

Generate random numbers:

```
>>> r = powerlaw.rvs(a, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<code>rvs(a, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, a, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, a, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, a, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, a, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, a, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, a, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, a, loc=0, scale=1)</code>	Percent point function (inverse of <i>cdf</i> — percentiles).
<code>isf(q, a, loc=0, scale=1)</code>	Inverse survival function (inverse of <i>sf</i> ).
<code>moment(n, a, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(a, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(a, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, a, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(a,), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(a, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(a, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(a, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(a, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, a, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.powerlognorm = <scipy.stats.continuous_distns.powerlognorm_gen object at 0x2b909bd78890>`  
 A power log-normal continuous random variable.

As an instance of the `rv_continuous` class, `powerlognorm` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `powerlognorm` is:

$$\text{powerlognorm.pdf}(x, c, s) = c / (x*s) * \text{phi}(\log(x)/s) * (\text{Phi}(-\log(x)/s))^{c-1},$$

where `phi` is the normal pdf, and `Phi` is the normal cdf, and  $x > 0, s, c > 0$ .

`powerlognorm` takes `c` and `s` as shape parameters.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `powerlognorm.pdf(x, c, s, loc, scale)` is identically equivalent to `powerlognorm.pdf(y, c, s) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

### Examples

```
>>> from scipy.stats import powerlognorm
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> c, s = 2.14, 0.446
>>> mean, var, skew, kurt = powerlognorm.stats(c, s, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(powerlognorm.ppf(0.01, c, s),
...                 powerlognorm.ppf(0.99, c, s), 100)
>>> ax.plot(x, powerlognorm.pdf(x, c, s),
...        'r-', lw=5, alpha=0.6, label='powerlognorm pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = powerlognorm(c, s)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

```
>>> vals = powerlognorm.ppf([0.001, 0.5, 0.999], c, s)
>>> np.allclose([0.001, 0.5, 0.999], powerlognorm.cdf(vals, c, s))
True
```

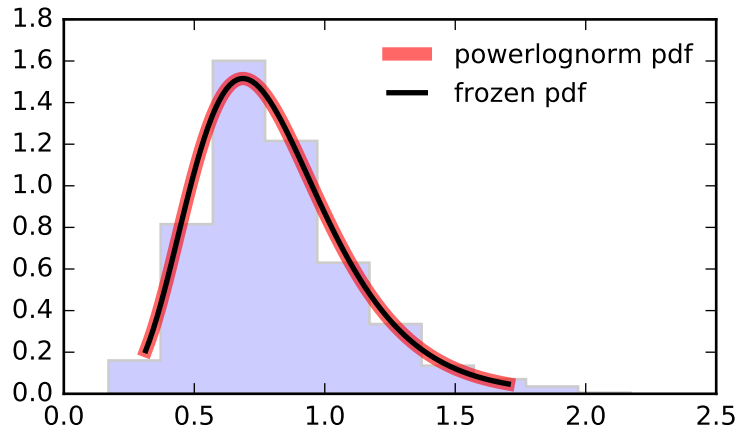
Generate random numbers:

```
>>> r = powerlognorm.rvs(c, s, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```





### Methods

<code>rvs(c, s, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, c, s, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, c, s, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, c, s, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, c, s, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, c, s, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, c, s, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, c, s, loc=0, scale=1)</code>	Percent point function (inverse of <i>cdf</i> — percentiles).
<code>isf(q, c, s, loc=0, scale=1)</code>	Inverse survival function (inverse of <i>sf</i> ).
<code>moment(n, c, s, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(c, s, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(c, s, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, c, s, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(c, s), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(c, s, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(c, s, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(c, s, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(c, s, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, c, s, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.powernorm = <scipy.stats.continuous_distns.powernorm_gen object at 0x2b909bd78b10>`  
 A power normal continuous random variable.

As an instance of the `rv_continuous` class, `powernorm` object inherits from it a collection of generic

methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `powernorm` is:

```
powernorm.pdf(x, c) = c * phi(x) * (Phi(-x))**(c-1)
```

where `phi` is the normal pdf, and `Phi` is the normal cdf, and  $x > 0, c > 0$ .

`powernorm` takes `c` as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `powernorm.pdf(x, c, loc, scale)` is identically equivalent to `powernorm.pdf(y, c) / scale` with  $y = (x - loc) / scale$ .

### Examples

```
>>> from scipy.stats import powernorm
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> c = 4.45
>>> mean, var, skew, kurt = powernorm.stats(c, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(powernorm.ppf(0.01, c),
...                 powernorm.ppf(0.99, c), 100)
>>> ax.plot(x, powernorm.pdf(x, c),
...         'r-', lw=5, alpha=0.6, label='powernorm pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = powernorm(c)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

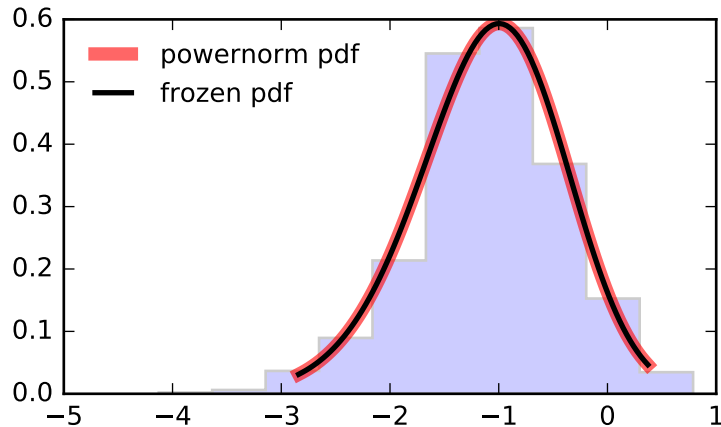
```
>>> vals = powernorm.ppf([0.001, 0.5, 0.999], c)
>>> np.allclose([0.001, 0.5, 0.999], powernorm.cdf(vals, c))
True
```

Generate random numbers:

```
>>> r = powernorm.rvs(c, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<code>rvs(c, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, c, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, c, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, c, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, c, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, c, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, c, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, c, loc=0, scale=1)</code>	Percent point function (inverse of <i>cdf</i> — percentiles).
<code>isf(q, c, loc=0, scale=1)</code>	Inverse survival function (inverse of <i>sf</i> ).
<code>moment(n, c, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(c, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(c, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, c, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(c,), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(c, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(c, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(c, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(c, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, c, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.rdist = <scipy.stats.continuous_distns.rdist_gen object at 0x2b909bd78d90>`

An R-distributed continuous random variable.

As an instance of the `rv_continuous` class, `rdist` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `rdist` is:

```
rdist.pdf(x, c) = (1-x**2)**(c/2-1) / B(1/2, c/2)
```

for  $-1 \leq x \leq 1, c > 0$ .

`rdist` takes `c` as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `rdist.pdf(x, c, loc, scale)` is identically equivalent to `rdist.pdf(y, c) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

### Examples

```
>>> from scipy.stats import rdist
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> c = 0.9
>>> mean, var, skew, kurt = rdist.stats(c, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(rdist.ppf(0.01, c),
...                 rdist.ppf(0.99, c), 100)
>>> ax.plot(x, rdist.pdf(x, c),
...         'r-', lw=5, alpha=0.6, label='rdist pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = rdist(c)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

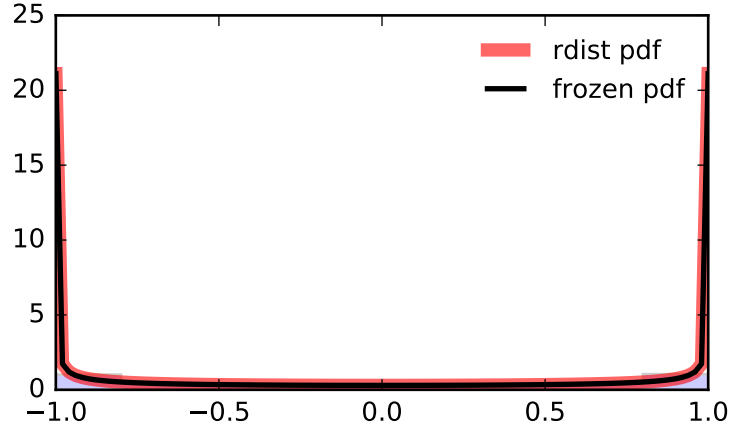
```
>>> vals = rdist.ppf([0.001, 0.5, 0.999], c)
>>> np.allclose([0.001, 0.5, 0.999], rdist.cdf(vals, c))
True
```

Generate random numbers:

```
>>> r = rdist.rvs(c, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<code>rvs(c, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, c, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, c, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, c, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, c, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, c, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, c, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, c, loc=0, scale=1)</code>	Percent point function (inverse of <i>cdf</i> — percentiles).
<code>isf(q, c, loc=0, scale=1)</code>	Inverse survival function (inverse of <i>sf</i> ).
<code>moment(n, c, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(c, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(c, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, c, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(c,), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(c, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(c, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(c, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(c, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, c, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.reciprocal = <scipy.stats.continuous_distns.reciprocal_gen object at 0x2b909bd7b250>`  
 A reciprocal continuous random variable.

As an instance of the `rv_continuous` class, `reciprocal` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `reciprocal` is:

```
reciprocal.pdf(x, a, b) = 1 / (x*log(b/a))
```

for  $a \leq x \leq b, a, b > 0$ .

`reciprocal` takes `a` and `b` as shape parameters.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `reciprocal.pdf(x, a, b, loc, scale)` is identically equivalent to `reciprocal.pdf(y, a, b) / scale` with  $y = (x - loc) / scale$ .

### Examples

```
>>> from scipy.stats import reciprocal
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> a, b = 0.00623, 1.01
>>> mean, var, skew, kurt = reciprocal.stats(a, b, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(reciprocal.ppf(0.01, a, b),
...                 reciprocal.ppf(0.99, a, b), 100)
>>> ax.plot(x, reciprocal.pdf(x, a, b),
...        'r-', lw=5, alpha=0.6, label='reciprocal pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = reciprocal(a, b)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

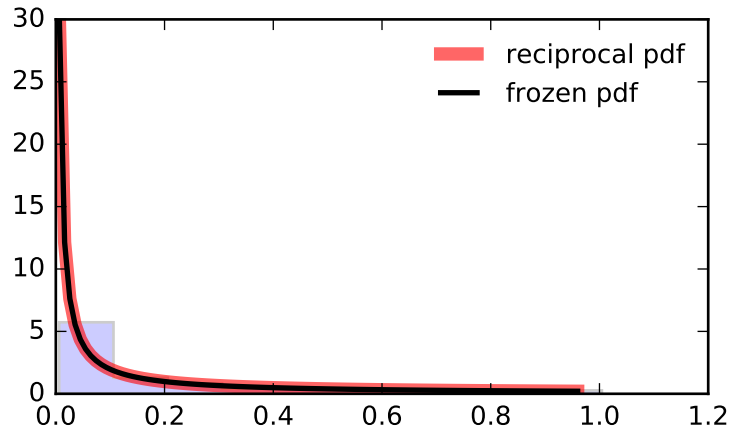
```
>>> vals = reciprocal.ppf([0.001, 0.5, 0.999], a, b)
>>> np.allclose([0.001, 0.5, 0.999], reciprocal.cdf(vals, a, b))
True
```

Generate random numbers:

```
>>> r = reciprocal.rvs(a, b, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<code>rvs(a, b, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, a, b, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, a, b, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, a, b, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, a, b, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, a, b, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, a, b, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, a, b, loc=0, scale=1)</code>	Percent point function (inverse of <i>cdf</i> — percentiles).
<code>isf(q, a, b, loc=0, scale=1)</code>	Inverse survival function (inverse of <i>sf</i> ).
<code>moment(n, a, b, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(a, b, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(a, b, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, a, b, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(a, b), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(a, b, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(a, b, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(a, b, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(a, b, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, a, b, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.rayleigh = <scipy.stats.continuous_distns.rayleigh_gen object at 0x2b909bd7b050>`  
 A Rayleigh continuous random variable.

As an instance of the `rv_continuous` class, `rayleigh` object inherits from it a collection of generic meth-

ods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `rayleigh` is:

```
rayleigh.pdf(r) = r * exp(-r**2/2)
```

for  $x \geq 0$ .

`rayleigh` is a special case of `chi` with `df == 2`.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `rayleigh.pdf(x, loc, scale)` is identically equivalent to `rayleigh.pdf(y) / scale` with  $y = (x - loc) / scale$ .

### Examples

```
>>> from scipy.stats import rayleigh
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> mean, var, skew, kurt = rayleigh.stats(moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(rayleigh.ppf(0.01),
...                 rayleigh.ppf(0.99), 100)
>>> ax.plot(x, rayleigh.pdf(x),
...        'r-', lw=5, alpha=0.6, label='rayleigh pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = rayleigh()
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

```
>>> vals = rayleigh.ppf([0.001, 0.5, 0.999])
>>> np.allclose([0.001, 0.5, 0.999], rayleigh.cdf(vals))
True
```

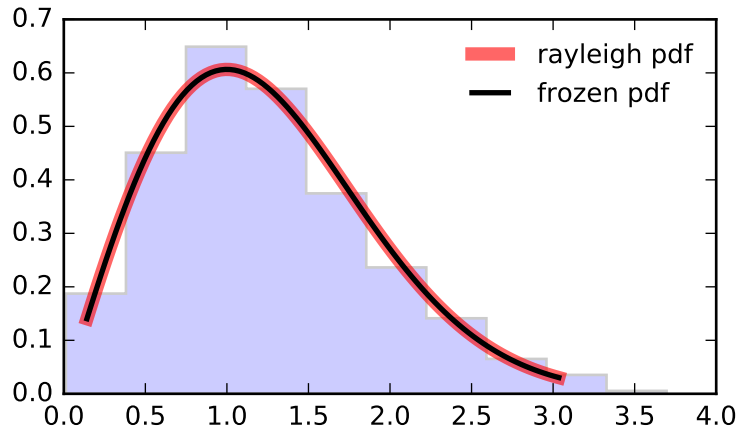
Generate random numbers:

```
>>> r = rayleigh.rvs(size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```





### Methods

<pre> rvs(loc=0, scale=1, size=1,     random_state=None) pdf(x, loc=0, scale=1) logpdf(x, loc=0, scale=1) cdf(x, loc=0, scale=1) logcdf(x, loc=0, scale=1)  sf(x, loc=0, scale=1)  logsf(x, loc=0, scale=1) ppf(q, loc=0, scale=1)  isf(q, loc=0, scale=1) moment(n, loc=0, scale=1) stats(loc=0, scale=1, moments='mv')  entropy(loc=0, scale=1) fit(data, loc=0, scale=1) expect(func, args=(), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kwds) median(loc=0, scale=1) mean(loc=0, scale=1) var(loc=0, scale=1) std(loc=0, scale=1) interval(alpha, loc=0, scale=1) </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

`scipy.stats.rice = <scipy.stats.continuous_distns.rice_gen object at 0x2b909bd7b4d0>`

A Rice continuous random variable.

As an instance of the `rv_continuous` class, `rice` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `rice` is:

```
rice.pdf(x, b) = x * exp(-(x**2+b**2)/2) * I[0](x*b)
```

for  $x > 0, b > 0$ .

`rice` takes `b` as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `rice.pdf(x, b, loc, scale)` is identically equivalent to `rice.pdf(y, b) / scale` with  $y = (x - loc) / scale$ .

The Rice distribution describes the length,  $r$ , of a 2-D vector with components  $(U+u, V+v)$ , where  $U, V$  are constant,  $u, v$  are independent Gaussian random variables with standard deviation  $s$ . Let  $R = (U**2 + V**2)**0.5$ . Then the pdf of  $r$  is `rice.pdf(x, R/s, scale=s)`.

### Examples

```
>>> from scipy.stats import rice
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> b = 0.775
>>> mean, var, skew, kurt = rice.stats(b, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(rice.ppf(0.01, b),
...                 rice.ppf(0.99, b), 100)
>>> ax.plot(x, rice.pdf(x, b),
...         'r-', lw=5, alpha=0.6, label='rice pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = rice(b)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

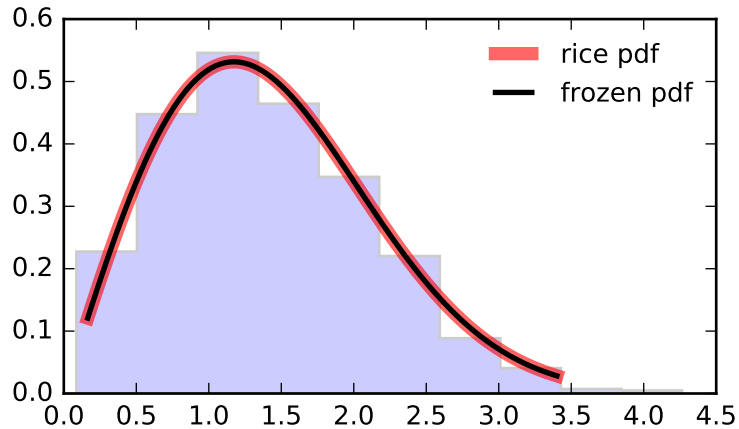
```
>>> vals = rice.ppf([0.001, 0.5, 0.999], b)
>>> np.allclose([0.001, 0.5, 0.999], rice.cdf(vals, b))
True
```

Generate random numbers:

```
>>> r = rice.rvs(b, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre>rvs(b, loc=0, scale=1, size=1,     random_state=None) pdf(x, b, loc=0, scale=1) logpdf(x, b, loc=0, scale=1) cdf(x, b, loc=0, scale=1) logcdf(x, b, loc=0, scale=1)  sf(x, b, loc=0, scale=1)  logsf(x, b, loc=0, scale=1) ppf(q, b, loc=0, scale=1)  isf(q, b, loc=0, scale=1) moment(n, b, loc=0, scale=1) stats(b, loc=0, scale=1, moments='mv')  entropy(b, loc=0, scale=1) fit(data, b, loc=0, scale=1) expect(func, args=(b,), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kwds) median(b, loc=0, scale=1) mean(b, loc=0, scale=1) var(b, loc=0, scale=1) std(b, loc=0, scale=1) interval(alpha, b, loc=0, scale=1)</pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains <i>alpha</i> percent of the distribution</p>
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`scipy.stats.recipinvgauss = <scipy.stats.continuous_distns.recipinvgauss_gen object at 0x2b909bd7b750>`  
 A reciprocal inverse Gaussian continuous random variable.

As an instance of the `rv_continuous` class, `recipinvgauss` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `recipinvgauss` is:

```
recipinvgauss.pdf(x, mu) = 1/sqrt(2*pi*x) * exp(-(1-mu*x)**2/(2*x*mu**2))
```

for  $x \geq 0$ .

`recipinvgauss` takes `mu` as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `recipinvgauss.pdf(x, mu, loc, scale)` is identically equivalent to `recipinvgauss.pdf(y, mu) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

### Examples

```
>>> from scipy.stats import recipinvgauss
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> mu = 0.63
>>> mean, var, skew, kurt = recipinvgauss.stats(mu, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(recipinvgauss.ppf(0.01, mu),
...                 recipinvgauss.ppf(0.99, mu), 100)
>>> ax.plot(x, recipinvgauss.pdf(x, mu),
...        'r-', lw=5, alpha=0.6, label='recipinvgauss pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = recipinvgauss(mu)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

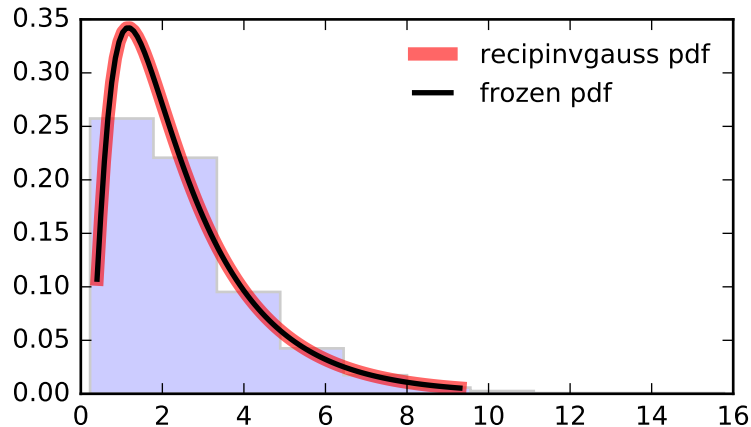
```
>>> vals = recipinvgauss.ppf([0.001, 0.5, 0.999], mu)
>>> np.allclose([0.001, 0.5, 0.999], recipinvgauss.cdf(vals, mu))
True
```

Generate random numbers:

```
>>> r = recipinvgauss.rvs(mu, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre> rvs(mu, loc=0, scale=1, size=1,     random_state=None) pdf(x, mu, loc=0, scale=1) logpdf(x, mu, loc=0, scale=1) cdf(x, mu, loc=0, scale=1) logcdf(x, mu, loc=0, scale=1)  sf(x, mu, loc=0, scale=1)  logsf(x, mu, loc=0, scale=1) ppf(q, mu, loc=0, scale=1)  isf(q, mu, loc=0, scale=1) moment(n, mu, loc=0, scale=1) stats(mu, loc=0, scale=1, moments='mv')  entropy(mu, loc=0, scale=1) fit(data, mu, loc=0, scale=1) expect(func, args=(mu,), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kwds) median(mu, loc=0, scale=1) mean(mu, loc=0, scale=1) var(mu, loc=0, scale=1) std(mu, loc=0, scale=1) interval(alpha, mu, loc=0, scale=1) </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
--	--

`scipy.stats.semicircular = <scipy.stats.continuous_distns.semicircular_gen object at 0x2b909bd7b9d0>`  
 A semicircular continuous random variable.

As an instance of the `rv_continuous` class, `semicircular` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `semicircular` is:

```
semicircular.pdf(x) = 2/pi * sqrt(1-x**2)
```

for  $-1 \leq x \leq 1$ .

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `semicircular.pdf(x, loc, scale)` is identically equivalent to `semicircular.pdf(y) / scale` with  $y = (x - loc) / scale$ .

### Examples

```
>>> from scipy.stats import semicircular
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> mean, var, skew, kurt = semicircular.stats(moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(semicircular.ppf(0.01),
...                 semicircular.ppf(0.99), 100)
>>> ax.plot(x, semicircular.pdf(x),
...         'r-', lw=5, alpha=0.6, label='semicircular pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = semicircular()
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

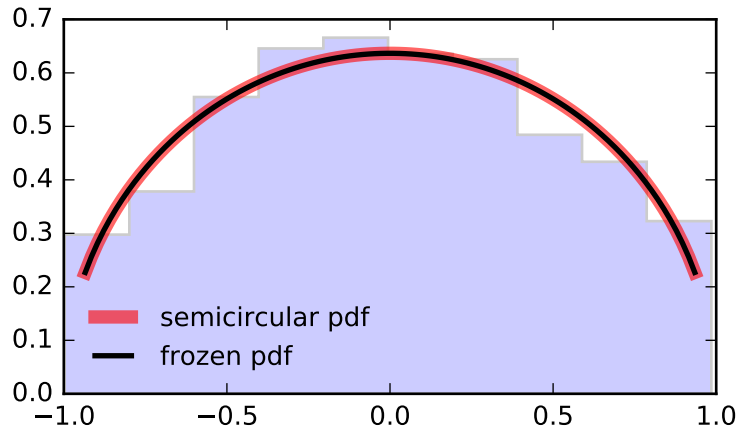
```
>>> vals = semicircular.ppf([0.001, 0.5, 0.999])
>>> np.allclose([0.001, 0.5, 0.999], semicircular.cdf(vals))
True
```

Generate random numbers:

```
>>> r = semicircular.rvs(size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre> rvs(loc=0, scale=1, size=1,     random_state=None) pdf(x, loc=0, scale=1) logpdf(x, loc=0, scale=1) cdf(x, loc=0, scale=1) logcdf(x, loc=0, scale=1)  sf(x, loc=0, scale=1)  logsf(x, loc=0, scale=1) ppf(q, loc=0, scale=1)  isf(q, loc=0, scale=1) moment(n, loc=0, scale=1) stats(loc=0, scale=1, moments='mv')  entropy(loc=0, scale=1) fit(data, loc=0, scale=1) expect(func, args=(), loc=0, scale=1,         lb=None, ub=None, conditional=False,         **kwds) median(loc=0, scale=1) mean(loc=0, scale=1) var(loc=0, scale=1) std(loc=0, scale=1) interval(alpha, loc=0, scale=1) </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <math>\text{sf}</math> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <math>\text{cdf}</math> — percentiles).</p> <p>Inverse survival function (inverse of <math>\text{sf}</math>).</p> <p>Non-central moment of order <math>n</math></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	---

`scipy.stats.skewnorm` = <scipy.stats.continuous\_distns.skew\_norm\_gen object at 0x2b909bd7bbd0>  
 A skew-normal random variable.

As an instance of the `rv_continuous` class, `skewnorm` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The pdf is:

```
skewnorm.pdf(x, a) = 2*norm.pdf(x)*norm.cdf(ax)
```

`skewnorm` takes `a` as a skewness parameter. When `a=0` the distribution is identical to a normal distribution. `rvs` implements the method of [R570].

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `skewnorm.pdf(x, a, loc, scale)` is identically equivalent to `skewnorm.pdf(y, a) / scale` with `y = (x - loc) / scale`.

### References

[R570]

### Examples

```
>>> from scipy.stats import skewnorm
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> a = 4
>>> mean, var, skew, kurt = skewnorm.stats(a, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(skewnorm.ppf(0.01, a),
...                 skewnorm.ppf(0.99, a), 100)
>>> ax.plot(x, skewnorm.pdf(x, a),
...        'r-', lw=5, alpha=0.6, label='skewnorm pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = skewnorm(a)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

```
>>> vals = skewnorm.ppf([0.001, 0.5, 0.999], a)
>>> np.allclose([0.001, 0.5, 0.999], skewnorm.cdf(vals, a))
True
```

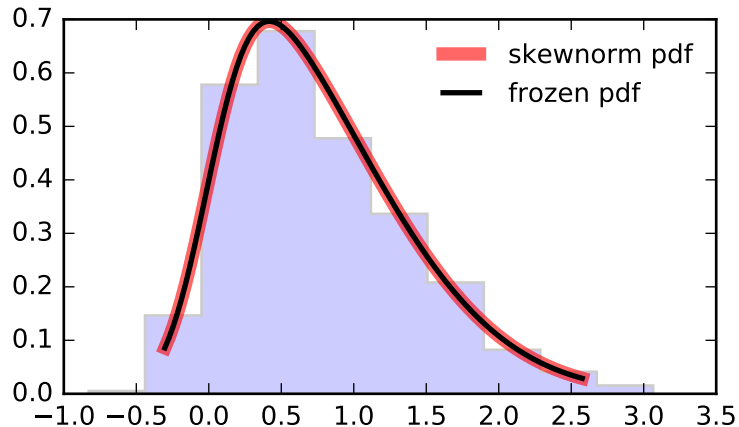
Generate random numbers:

```
>>> r = skewnorm.rvs(a, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```





### Methods

<pre> rvs(a, loc=0, scale=1, size=1,     random_state=None) pdf(x, a, loc=0, scale=1) logpdf(x, a, loc=0, scale=1) cdf(x, a, loc=0, scale=1) logcdf(x, a, loc=0, scale=1)  sf(x, a, loc=0, scale=1)  logsf(x, a, loc=0, scale=1) ppf(q, a, loc=0, scale=1)  isf(q, a, loc=0, scale=1) moment(n, a, loc=0, scale=1) stats(a, loc=0, scale=1, moments='mv')  entropy(a, loc=0, scale=1) fit(data, a, loc=0, scale=1) expect(func, args=(a,), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kwds) median(a, loc=0, scale=1) mean(a, loc=0, scale=1) var(a, loc=0, scale=1) std(a, loc=0, scale=1) interval(alpha, a, loc=0, scale=1) </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

`scipy.stats.t` = <scipy.stats.continuous\_distns.t\_gen object at 0x2b909bd678d0>

A Student's T continuous random variable.

As an instance of the `rv_continuous` class, `t` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**Notes**

The probability density function for  $t$  is:

$$t.pdf(x, df) = \frac{\text{gamma}((df+1)/2)}{\sqrt{\pi \cdot df} * \text{gamma}(df/2) * (1+x**2/df)**((df+1)/2)}$$

for  $df > 0$ .

$t$  takes  $df$  as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `t.pdf(x, df, loc, scale)` is identically equivalent to `t.pdf(y, df) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

**Examples**

```
>>> from scipy.stats import t
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> df = 2.74
>>> mean, var, skew, kurt = t.stats(df, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(t.ppf(0.01, df),
...                 t.ppf(0.99, df), 100)
>>> ax.plot(x, t.pdf(x, df),
...        'r-', lw=5, alpha=0.6, label='t pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = t(df)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

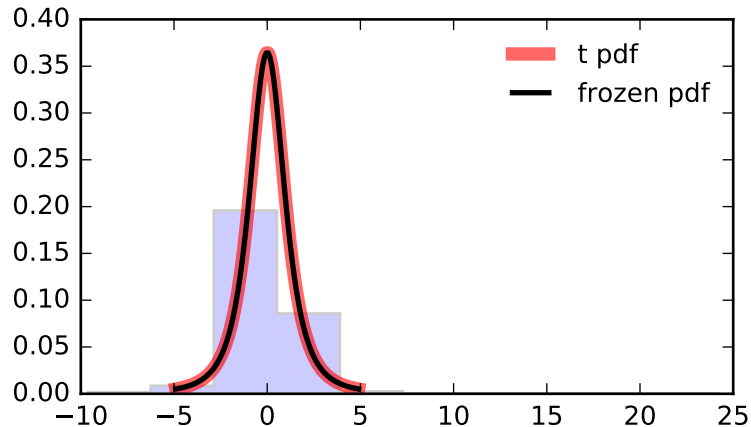
```
>>> vals = t.ppf([0.001, 0.5, 0.999], df)
>>> np.allclose([0.001, 0.5, 0.999], t.cdf(vals, df))
True
```

Generate random numbers:

```
>>> r = t.rvs(df, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre> rvs(df, loc=0, scale=1, size=1,     random_state=None) pdf(x, df, loc=0, scale=1) logpdf(x, df, loc=0, scale=1) cdf(x, df, loc=0, scale=1) logcdf(x, df, loc=0, scale=1)  sf(x, df, loc=0, scale=1)  logsf(x, df, loc=0, scale=1) ppf(q, df, loc=0, scale=1)  isf(q, df, loc=0, scale=1) moment(n, df, loc=0, scale=1) stats(df, loc=0, scale=1, moments='mv')  entropy(df, loc=0, scale=1) fit(data, df, loc=0, scale=1) expect(func, args=(df,), loc=0, scale=1,         lb=None, ub=None, conditional=False,         **kwds) median(df, loc=0, scale=1) mean(df, loc=0, scale=1) var(df, loc=0, scale=1) std(df, loc=0, scale=1) interval(alpha, df, loc=0, scale=1) </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
--	--

`scipy.stats.trapz = <scipy.stats.continuous_distns.trapz_gen object at 0x2b909bd7be90>`

A trapezoidal continuous random variable.

As an instance of the `rv_continuous` class, `trapz` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The trapezoidal distribution can be represented with an up-sloping line from `loc` to `(loc + c*scale)`, then constant to `(loc + d*scale)` and then downsloping from `(loc + d*scale)` to `(loc+scale)`.

`trapz` takes `c` and `d` as shape parameters.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `trapz.pdf(x, c, d, loc, scale)` is identically equivalent to `trapz.pdf(y, c, d) / scale` with `y = (x - loc) / scale`.

The standard form is in the range `[0, 1]` with `c` the mode. The location parameter shifts the start to `loc`. The scale parameter changes the width from 1 to `scale`.

### Examples

```
>>> from scipy.stats import trapz
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> c, d = 0.2, 0.8
>>> mean, var, skew, kurt = trapz.stats(c, d, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(trapz.ppf(0.01, c, d),
...                 trapz.ppf(0.99, c, d), 100)
>>> ax.plot(x, trapz.pdf(x, c, d),
...        'r-', lw=5, alpha=0.6, label='trapz pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = trapz(c, d)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

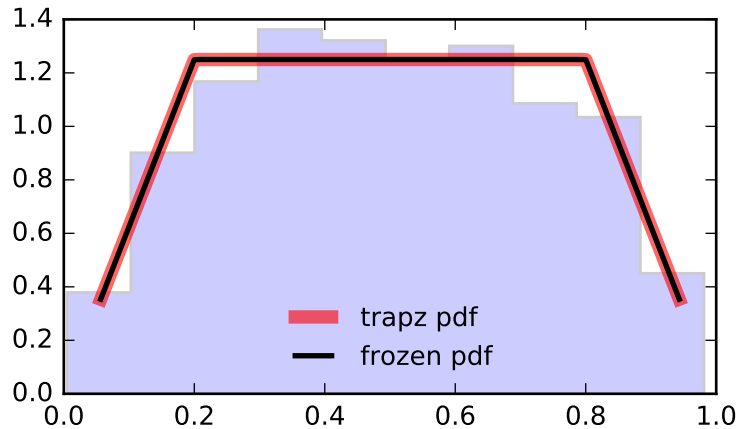
```
>>> vals = trapz.ppf([0.001, 0.5, 0.999], c, d)
>>> np.allclose([0.001, 0.5, 0.999], trapz.cdf(vals, c, d))
True
```

Generate random numbers:

```
>>> r = trapz.rvs(c, d, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<code>rvs(c, d, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, c, d, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, c, d, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, c, d, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, c, d, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, c, d, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, c, d, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, c, d, loc=0, scale=1)</code>	Percent point function (inverse of <i>cdf</i> — percentiles).
<code>isf(q, c, d, loc=0, scale=1)</code>	Inverse survival function (inverse of <i>sf</i> ).
<code>moment(n, c, d, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(c, d, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(c, d, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, c, d, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(c, d), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(c, d, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(c, d, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(c, d, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(c, d, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, c, d, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.triang = <scipy.stats.continuous_distns.triang_gen object at 0x2b909bd8d150>`

A triangular continuous random variable.

As an instance of the `rv_continuous` class, `triang` object inherits from it a collection of generic methods

(see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The triangular distribution can be represented with an up-sloping line from `loc` to `(loc + c*scale)` and then downsloping for `(loc + c*scale)` to `(loc+scale)`.

`triang` takes `c` as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `triang.pdf(x, c, loc, scale)` is identically equivalent to `triang.pdf(y, c) / scale` with `y = (x - loc) / scale`.

The standard form is in the range `[0, 1]` with `c` the mode. The location parameter shifts the start to `loc`. The scale parameter changes the width from 1 to `scale`.

### Examples

```
>>> from scipy.stats import triang
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> c = 0.158
>>> mean, var, skew, kurt = triang.stats(c, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(triang.ppf(0.01, c),
...                 triang.ppf(0.99, c), 100)
>>> ax.plot(x, triang.pdf(x, c),
...        'r-', lw=5, alpha=0.6, label='triang pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = triang(c)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

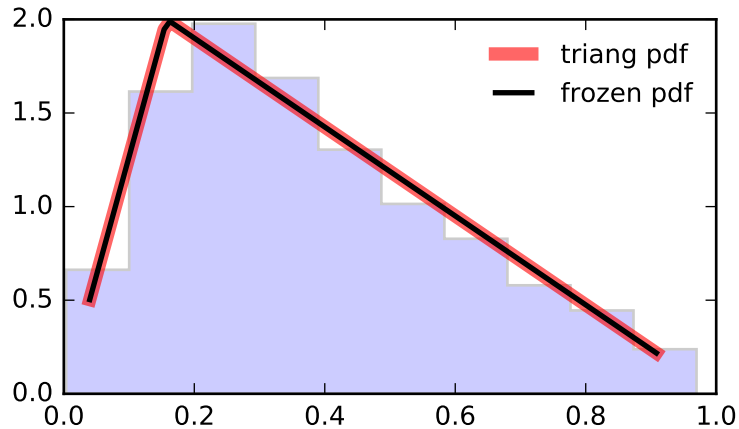
```
>>> vals = triang.ppf([0.001, 0.5, 0.999], c)
>>> np.allclose([0.001, 0.5, 0.999], triang.cdf(vals, c))
True
```

Generate random numbers:

```
>>> r = triang.rvs(c, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<code>rvs(c, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, c, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, c, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, c, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, c, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, c, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, c, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, c, loc=0, scale=1)</code>	Percent point function (inverse of <i>cdf</i> — percentiles).
<code>isf(q, c, loc=0, scale=1)</code>	Inverse survival function (inverse of <i>sf</i> ).
<code>moment(n, c, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(c, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(c, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, c, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(c,), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(c, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(c, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(c, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(c, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, c, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.truncexpon = <scipy.stats.continuous_distns.truncexpon_gen object at 0x2b909bd8d3d0>`  
 A truncated exponential continuous random variable.

As an instance of the `rv_continuous` class, `truncexpon` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `truncexpon` is:

```
truncexpon.pdf(x, b) = exp(-x) / (1-exp(-b))
```

for  $0 < x < b$ .

`truncexpon` takes `b` as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `truncexpon.pdf(x, b, loc, scale)` is identically equivalent to `truncexpon.pdf(y, b) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

### Examples

```
>>> from scipy.stats import truncexpon
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> b = 4.69
>>> mean, var, skew, kurt = truncexpon.stats(b, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(truncexpon.ppf(0.01, b),
...                 truncexpon.ppf(0.99, b), 100)
>>> ax.plot(x, truncexpon.pdf(x, b),
...        'r-', lw=5, alpha=0.6, label='truncexpon pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = truncexpon(b)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

```
>>> vals = truncexpon.ppf([0.001, 0.5, 0.999], b)
>>> np.allclose([0.001, 0.5, 0.999], truncexpon.cdf(vals, b))
True
```

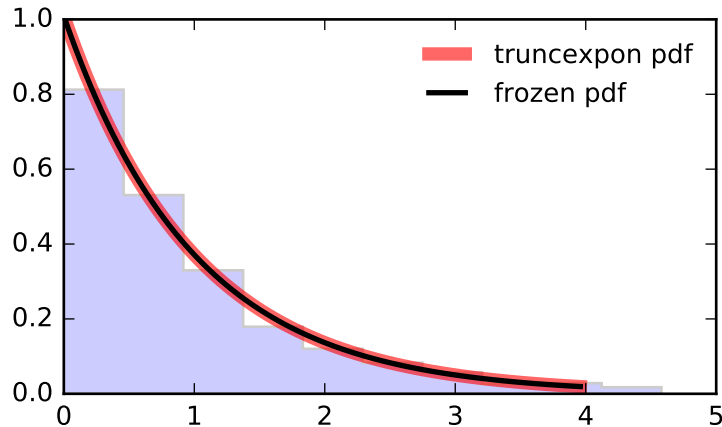
Generate random numbers:

```
>>> r = truncexpon.rvs(b, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```





### Methods

<pre> rvs(b, loc=0, scale=1, size=1,     random_state=None) pdf(x, b, loc=0, scale=1) logpdf(x, b, loc=0, scale=1) cdf(x, b, loc=0, scale=1) logcdf(x, b, loc=0, scale=1)  sf(x, b, loc=0, scale=1)  logsf(x, b, loc=0, scale=1) ppf(q, b, loc=0, scale=1)  isf(q, b, loc=0, scale=1) moment(n, b, loc=0, scale=1) stats(b, loc=0, scale=1, moments='mv')  entropy(b, loc=0, scale=1) fit(data, b, loc=0, scale=1) expect(func, args=(b,), loc=0, scale=1,         lb=None, ub=None, conditional=False,         **kwds) median(b, loc=0, scale=1) mean(b, loc=0, scale=1) var(b, loc=0, scale=1) std(b, loc=0, scale=1) interval(alpha, b, loc=0, scale=1) </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

`scipy.stats.truncnorm` = <scipy.stats.continuous\_distns.truncnorm\_gen object at 0x2b909bd8d650>  
 A truncated normal continuous random variable.

As an instance of the `rv_continuous` class, `truncnorm` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The standard form of this distribution is a standard normal truncated to the range  $[a, b]$  — notice that  $a$  and  $b$  are defined over the domain of the standard normal. To convert clip values for a specific mean and standard deviation, use:

```
a, b = (myclip_a - my_mean) / my_std, (myclip_b - my_mean) / my_std
```

`truncnorm` takes  $a$  and  $b$  as shape parameters.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `truncnorm.pdf(x, a, b, loc, scale)` is identically equivalent to `truncnorm.pdf(y, a, b) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

### Examples

```
>>> from scipy.stats import truncnorm
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> a, b = 0.1, 2
>>> mean, var, skew, kurt = truncnorm.stats(a, b, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(truncnorm.ppf(0.01, a, b),
...                 truncnorm.ppf(0.99, a, b), 100)
>>> ax.plot(x, truncnorm.pdf(x, a, b),
...        'r-', lw=5, alpha=0.6, label='truncnorm pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = truncnorm(a, b)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

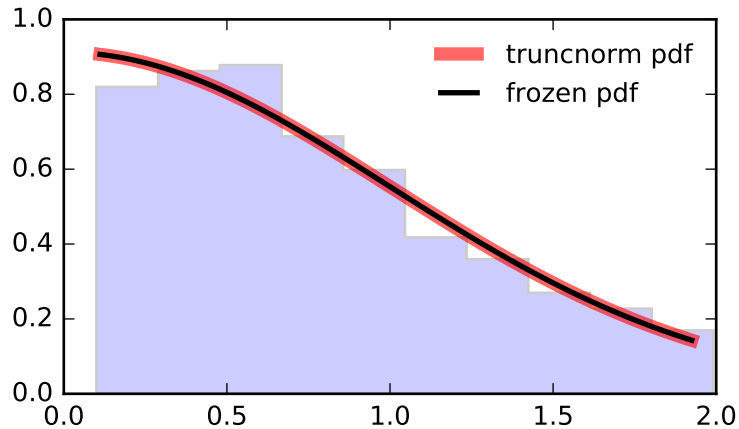
```
>>> vals = truncnorm.ppf([0.001, 0.5, 0.999], a, b)
>>> np.allclose([0.001, 0.5, 0.999], truncnorm.cdf(vals, a, b))
True
```

Generate random numbers:

```
>>> r = truncnorm.rvs(a, b, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<code>rvs(a, b, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, a, b, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, a, b, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, a, b, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, a, b, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, a, b, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, a, b, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, a, b, loc=0, scale=1)</code>	Percent point function (inverse of <i>cdf</i> — percentiles).
<code>isf(q, a, b, loc=0, scale=1)</code>	Inverse survival function (inverse of <i>sf</i> ).
<code>moment(n, a, b, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(a, b, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(a, b, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, a, b, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(a, b), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(a, b, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(a, b, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(a, b, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(a, b, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, a, b, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.tukeylambda = <scipy.stats.continuous_distns.tukeylambda_gen object at 0x2b909bd8d8d0>`  
 A Tukey-Lambda continuous random variable.

As an instance of the `rv_continuous` class, `tukeylambda` object inherits from it a collection of generic

methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

A flexible distribution, able to represent and interpolate between the following distributions:

- Cauchy (lam=-1)
- logistic (lam=0.0)
- approx Normal (lam=0.14)
- u-shape (lam = 0.5)
- uniform from -1 to 1 (lam = 1)

`tukeylambda` takes `lam` as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `tukeylambda.pdf(x, lam, loc, scale)` is identically equivalent to `tukeylambda.pdf(y, lam) / scale` with  $y = (x - loc) / scale$ .

### Examples

```
>>> from scipy.stats import tukeylambda
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> lam = 3.13
>>> mean, var, skew, kurt = tukeylambda.stats(lam, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(tukeylambda.ppf(0.01, lam),
...                 tukeylambda.ppf(0.99, lam), 100)
>>> ax.plot(x, tukeylambda.pdf(x, lam),
...         'r-', lw=5, alpha=0.6, label='tukeylambda pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = tukeylambda(lam)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

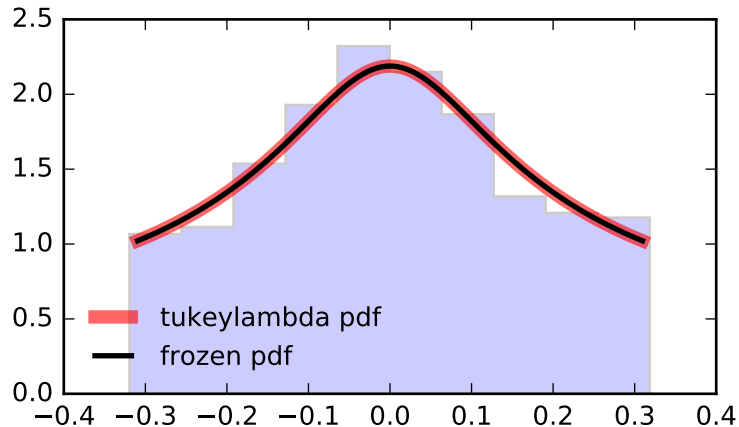
```
>>> vals = tukeylambda.ppf([0.001, 0.5, 0.999], lam)
>>> np.allclose([0.001, 0.5, 0.999], tukeylambda.cdf(vals, lam))
True
```

Generate random numbers:

```
>>> r = tukeylambda.rvs(lam, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<code>rvs(lam, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, lam, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, lam, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, lam, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, lam, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, lam, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, lam, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, lam, loc=0, scale=1)</code>	Percent point function (inverse of <i>cdf</i> — percentiles).
<code>isf(q, lam, loc=0, scale=1)</code>	Inverse survival function (inverse of <i>sf</i> ).
<code>moment(n, lam, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(lam, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(lam, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, lam, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(lam,), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(lam, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(lam, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(lam, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(lam, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, lam, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.uniform = <scipy.stats.continuous_distns.uniform_gen object at 0x2b909bd8db50>`

A uniform continuous random variable.

This distribution is constant between *loc* and *loc + scale*.

As an instance of the `rv_continuous` class, `uniform` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Examples

```
>>> from scipy.stats import uniform
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> mean, var, skew, kurt = uniform.stats(moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(uniform.ppf(0.01),
...                 uniform.ppf(0.99), 100)
>>> ax.plot(x, uniform.pdf(x),
...         'r-', lw=5, alpha=0.6, label='uniform pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = uniform()
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

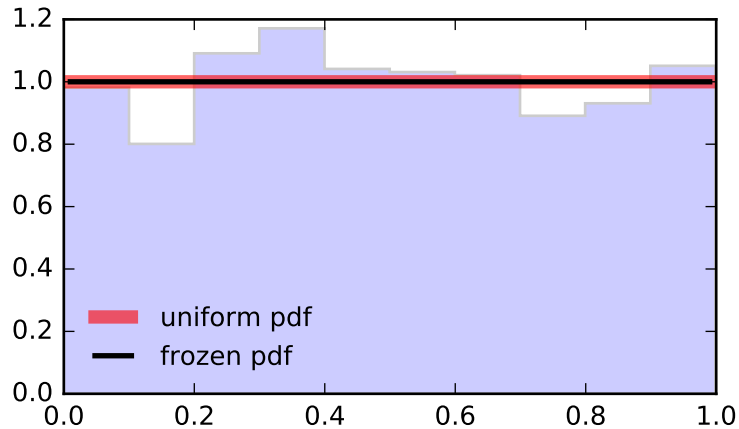
```
>>> vals = uniform.ppf([0.001, 0.5, 0.999])
>>> np.allclose([0.001, 0.5, 0.999], uniform.cdf(vals))
True
```

Generate random numbers:

```
>>> r = uniform.rvs(size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre> rvs(loc=0, scale=1, size=1,     random_state=None) pdf(x, loc=0, scale=1) logpdf(x, loc=0, scale=1) cdf(x, loc=0, scale=1) logcdf(x, loc=0, scale=1)  sf(x, loc=0, scale=1)  logsf(x, loc=0, scale=1) ppf(q, loc=0, scale=1)  isf(q, loc=0, scale=1) moment(n, loc=0, scale=1) stats(loc=0, scale=1, moments='mv')  entropy(loc=0, scale=1) fit(data, loc=0, scale=1) expect(func, args=(), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kwds) median(loc=0, scale=1) mean(loc=0, scale=1) var(loc=0, scale=1) std(loc=0, scale=1) interval(alpha, loc=0, scale=1) </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

`scipy.stats.vonmises = <scipy.stats._continuous_distns.vonmises_gen object at 0x2b909bd8dd50>`  
 A Von Mises continuous random variable.

As an instance of the `rv_continuous` class, `vonmises` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

See also:

### `vonmises_line`

The same distribution, defined on a  $[-\pi, \pi]$  segment of the real line.

#### Notes

If  $x$  is not in range or  $loc$  is not in range it assumes they are angles and converts them to  $[-\pi, \pi]$  equivalents.

The probability density function for `vonmises` is:

```
vonmises.pdf(x, kappa) = exp(kappa * cos(x)) / (2*pi*I[0](kappa))
```

for  $-\pi \leq x \leq \pi, \text{kappa} > 0$ .

`vonmises` takes `kappa` as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `vonmises.pdf(x, kappa, loc, scale)` is identically equivalent to `vonmises.pdf(y, kappa) / scale` with  $y = (x - loc) / scale$ .

#### Examples

```
>>> from scipy.stats import vonmises
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> kappa = 3.99
>>> mean, var, skew, kurt = vonmises.stats(kappa, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(vonmises.ppf(0.01, kappa),
...                 vonmises.ppf(0.99, kappa), 100)
>>> ax.plot(x, vonmises.pdf(x, kappa),
...        'r-', lw=5, alpha=0.6, label='vonmises pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = vonmises(kappa)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

```
>>> vals = vonmises.ppf([0.001, 0.5, 0.999], kappa)
>>> np.allclose([0.001, 0.5, 0.999], vonmises.cdf(vals, kappa))
True
```

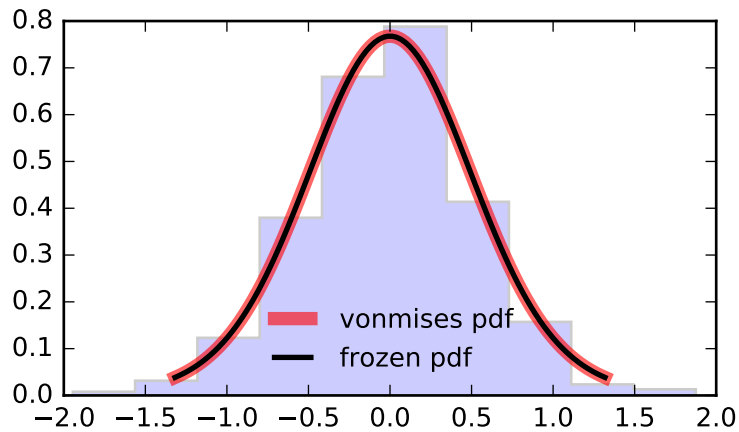
Generate random numbers:

```
>>> r = vonmises.rvs(kappa, size=1000)
```

And compare the histogram:



```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre>rvs(kappa, loc=0, scale=1, size=1,     random_state=None) pdf(x, kappa, loc=0, scale=1) logpdf(x, kappa, loc=0, scale=1) cdf(x, kappa, loc=0, scale=1) logcdf(x, kappa, loc=0, scale=1)  sf(x, kappa, loc=0, scale=1)  logsf(x, kappa, loc=0, scale=1) ppf(q, kappa, loc=0, scale=1) isf(q, kappa, loc=0, scale=1)  moment(n, kappa, loc=0, scale=1) stats(kappa, loc=0, scale=1, moments='mv')  entropy(kappa, loc=0, scale=1) fit(data, kappa, loc=0, scale=1) expect(func, args=(kappa,), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds) median(kappa, loc=0, scale=1) mean(kappa, loc=0, scale=1) var(kappa, loc=0, scale=1) std(kappa, loc=0, scale=1) interval(alpha, kappa, loc=0, scale=1)</pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <math>\text{sf}</math> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <math>\text{cdf}</math> — percentiles).</p> <p>Inverse survival function (inverse of <math>\text{sf}</math>).</p> <p>Non-central moment of order <math>n</math></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	---

`scipy.stats.vonmises_line` = <scipy.stats.continuous\_distns.vonmises\_gen object at 0x2b909bd8dfd0>  
A Von Mises continuous random variable.

As an instance of the `rv_continuous` class, `vonmises_line` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**See also:**

`vonmises_line`

The same distribution, defined on a  $[-\pi, \pi]$  segment of the real line.

**Notes**

If  $x$  is not in range or  $loc$  is not in range it assumes they are angles and converts them to  $[-\pi, \pi]$  equivalents.

The probability density function for `vonmises` is:

$$\text{vonmises.pdf}(x, \text{kappa}) = \exp(\text{kappa} * \cos(x)) / (2 * \pi * I_0(\text{kappa}))$$

for  $-\pi \leq x \leq \pi, \text{kappa} > 0$ .

`vonmises` takes `kappa` as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `vonmises_line.pdf(x, kappa, loc, scale)` is identically equivalent to `vonmises_line.pdf(y, kappa) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

**Examples**

```
>>> from scipy.stats import vonmises_line
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> kappa = 3.99
>>> mean, var, skew, kurt = vonmises_line.stats(kappa, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(vonmises_line.ppf(0.01, kappa),
...                 vonmises_line.ppf(0.99, kappa), 100)
>>> ax.plot(x, vonmises_line.pdf(x, kappa),
...         'r-', lw=5, alpha=0.6, label='vonmises_line pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = vonmises_line(kappa)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

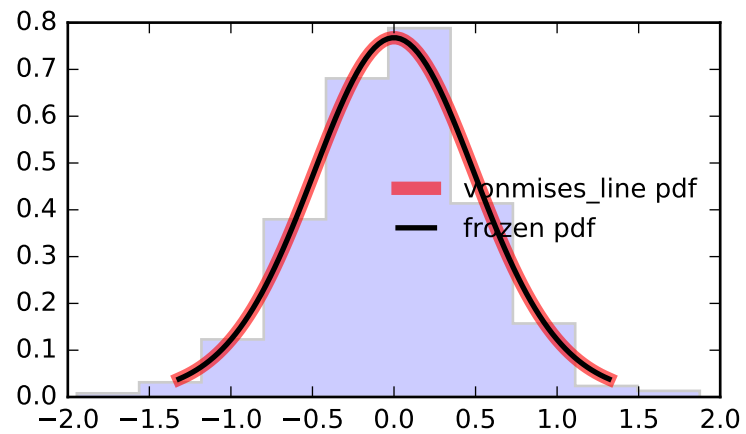
```
>>> vals = vonmises_line.ppf([0.001, 0.5, 0.999], kappa)
>>> np.allclose([0.001, 0.5, 0.999], vonmises_line.cdf(vals, kappa))
True
```

Generate random numbers:

```
>>> r = vonmises_line.rvs(kappa, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<code>rvs(kappa, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, kappa, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, kappa, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, kappa, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, kappa, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, kappa, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, kappa, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, kappa, loc=0, scale=1)</code>	Percent point function (inverse of <code>cdf</code> — percentiles).
<code>isf(q, kappa, loc=0, scale=1)</code>	Inverse survival function (inverse of <code>sf</code> ).
<code>moment(n, kappa, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(kappa, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(kappa, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, kappa, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(kappa,), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(kappa, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(kappa, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(kappa, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(kappa, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, kappa, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.wald = <scipy.stats._continuous_distns.wald_gen object at 0x2b909bd9a250>`

A Wald continuous random variable.

As an instance of the `rv_continuous` class, `wald` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `wald` is:

$$\text{wald.pdf}(x) = 1/\sqrt{2\pi x^3} * \exp(-(x-1)**2/(2*x))$$

for  $x > 0$ .

`wald` is a special case of `invgauss` with `mu == 1`.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `wald.pdf(x, loc, scale)` is identically equivalent to `wald.pdf(y) / scale` with `y = (x - loc) / scale`.

### Examples

```
>>> from scipy.stats import wald
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> mean, var, skew, kurt = wald.stats(moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(wald.ppf(0.01),
...                 wald.ppf(0.99), 100)
>>> ax.plot(x, wald.pdf(x),
...         'r-', lw=5, alpha=0.6, label='wald pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = wald()
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

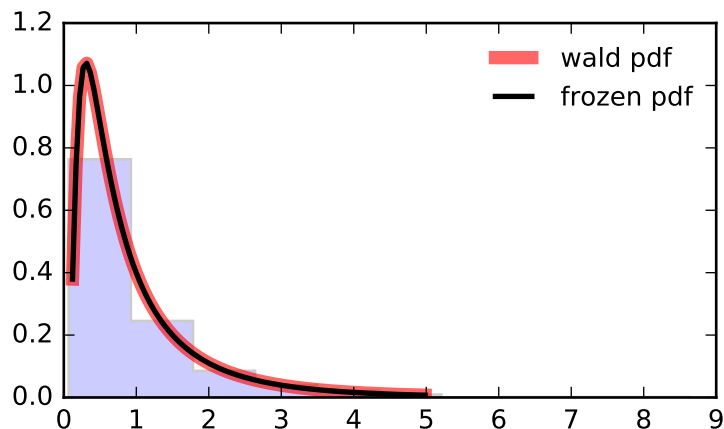
```
>>> vals = wald.ppf([0.001, 0.5, 0.999])
>>> np.allclose([0.001, 0.5, 0.999], wald.cdf(vals))
True
```

Generate random numbers:

```
>>> r = wald.rvs(size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



**Methods**

<pre> rvs(loc=0, scale=1, size=1,     random_state=None) pdf(x, loc=0, scale=1) logpdf(x, loc=0, scale=1) cdf(x, loc=0, scale=1) logcdf(x, loc=0, scale=1)  sf(x, loc=0, scale=1)  logsf(x, loc=0, scale=1) ppf(q, loc=0, scale=1)  isf(q, loc=0, scale=1) moment(n, loc=0, scale=1) stats(loc=0, scale=1, moments='mv')  entropy(loc=0, scale=1) fit(data, loc=0, scale=1) expect(func, args=(), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kwds) median(loc=0, scale=1) mean(loc=0, scale=1) var(loc=0, scale=1) std(loc=0, scale=1) interval(alpha, loc=0, scale=1) </pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

`scipy.stats.weibull_min` = <scipy.stats.continuous\_distns.frechet\_r\_gen object at 0x2b909bae4950>  
A Fréchet right (or Weibull minimum) continuous random variable.

As an instance of the `rv_continuous` class, `weibull_min` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

**See also:**

`weibull_min`

The same distribution as `frechet_r`.

`frechet_l`, `weibull_max`

**Notes**

The probability density function for `frechet_r` is:

$$\text{frechet\_r.pdf}(x, c) = c * x^{c-1} * \exp(-x^c)$$

for  $x > 0, c > 0$ .

`frechet_r` takes *c* as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the *loc* and *scale* parameters. Specifically, `weibull_min.pdf(x, c, loc, scale)` is identically equivalent to `weibull_min.pdf(y, c) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

*Examples*

```
>>> from scipy.stats import weibull_min
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> c = 1.79
>>> mean, var, skew, kurt = weibull_min.stats(c, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(weibull_min.ppf(0.01, c),
...                 weibull_min.ppf(0.99, c), 100)
>>> ax.plot(x, weibull_min.pdf(x, c),
...         'r-', lw=5, alpha=0.6, label='weibull_min pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = weibull_min(c)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

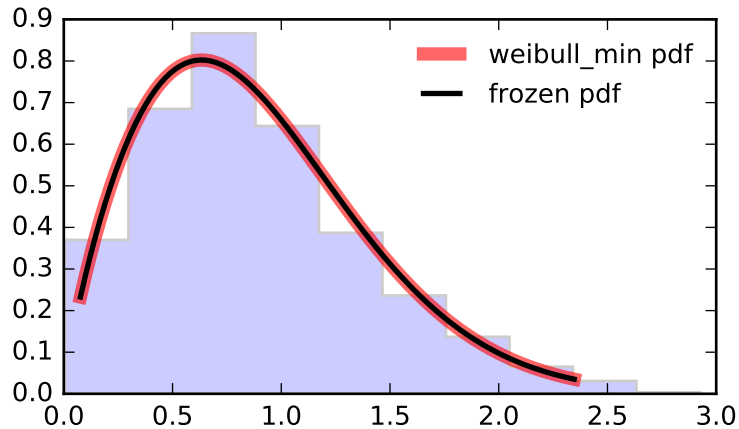
```
>>> vals = weibull_min.ppf([0.001, 0.5, 0.999], c)
>>> np.allclose([0.001, 0.5, 0.999], weibull_min.cdf(vals, c))
True
```

Generate random numbers:

```
>>> r = weibull_min.rvs(c, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<code>rvs(c, loc=0, scale=1, size=1, random_state=None)</code>	Random variates.
<code>pdf(x, c, loc=0, scale=1)</code>	Probability density function.
<code>logpdf(x, c, loc=0, scale=1)</code>	Log of the probability density function.
<code>cdf(x, c, loc=0, scale=1)</code>	Cumulative distribution function.
<code>logcdf(x, c, loc=0, scale=1)</code>	Log of the cumulative distribution function.
<code>sf(x, c, loc=0, scale=1)</code>	Survival function (also defined as $1 - \text{cdf}$ , but <i>sf</i> is sometimes more accurate).
<code>logsf(x, c, loc=0, scale=1)</code>	Log of the survival function.
<code>ppf(q, c, loc=0, scale=1)</code>	Percent point function (inverse of <i>cdf</i> — percentiles).
<code>isf(q, c, loc=0, scale=1)</code>	Inverse survival function (inverse of <i>sf</i> ).
<code>moment(n, c, loc=0, scale=1)</code>	Non-central moment of order <i>n</i>
<code>stats(c, loc=0, scale=1, moments='mv')</code>	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
<code>entropy(c, loc=0, scale=1)</code>	(Differential) entropy of the RV.
<code>fit(data, c, loc=0, scale=1)</code>	Parameter estimates for generic data.
<code>expect(func, args=(c,), loc=0, scale=1, lb=None, ub=None, conditional=False, **kwds)</code>	Expected value of a function (of one argument) with respect to the distribution.
<code>median(c, loc=0, scale=1)</code>	Median of the distribution.
<code>mean(c, loc=0, scale=1)</code>	Mean of the distribution.
<code>var(c, loc=0, scale=1)</code>	Variance of the distribution.
<code>std(c, loc=0, scale=1)</code>	Standard deviation of the distribution.
<code>interval(alpha, c, loc=0, scale=1)</code>	Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.weibull_max = <scipy.stats.continuous_distns.frechet_l_gen object at 0x2b909bae4e10>`  
 A Fréchet left (or Weibull maximum) continuous random variable.

As an instance of the `rv_continuous` class, `weibull_max` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

See also:



**`weibull_max`**

The same distribution as `frechet_1`.

`frechet_r`, `weibull_min`

**Notes**

The probability density function for `frechet_1` is:

$$\text{frechet}_1.\text{pdf}(x, c) = c * (-x)**(c-1) * \exp(-(-x)**c)$$

for  $x < 0, c > 0$ .

`frechet_1` takes `c` as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `weibull_max.pdf(x, c, loc, scale)` is identically equivalent to `weibull_max.pdf(y, c) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

**Examples**

```
>>> from scipy.stats import weibull_max
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> c = 2.87
>>> mean, var, skew, kurt = weibull_max.stats(c, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(weibull_max.ppf(0.01, c),
...                 weibull_max.ppf(0.99, c), 100)
>>> ax.plot(x, weibull_max.pdf(x, c),
...         'r-', lw=5, alpha=0.6, label='weibull_max pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = weibull_max(c)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

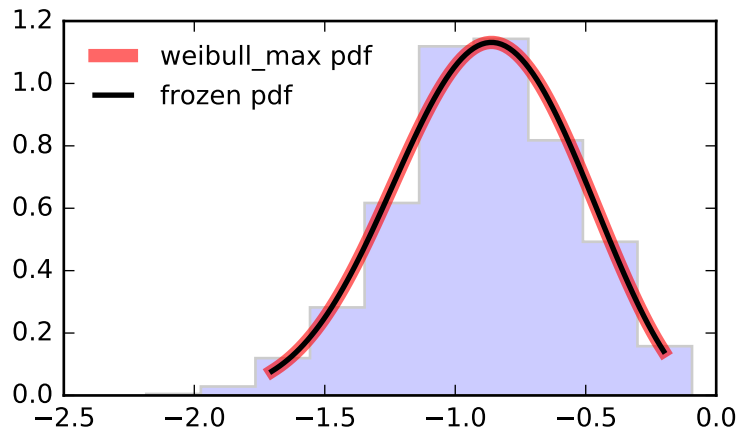
```
>>> vals = weibull_max.ppf([0.001, 0.5, 0.999], c)
>>> np.allclose([0.001, 0.5, 0.999], weibull_max.cdf(vals, c))
True
```

Generate random numbers:

```
>>> r = weibull_max.rvs(c, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre>rvs(c, loc=0, scale=1, size=1,     random_state=None) pdf(x, c, loc=0, scale=1) logpdf(x, c, loc=0, scale=1) cdf(x, c, loc=0, scale=1) logcdf(x, c, loc=0, scale=1)  sf(x, c, loc=0, scale=1)  logsf(x, c, loc=0, scale=1) ppf(q, c, loc=0, scale=1)  isf(q, c, loc=0, scale=1) moment(n, c, loc=0, scale=1) stats(c, loc=0, scale=1, moments='mv')  entropy(c, loc=0, scale=1) fit(data, c, loc=0, scale=1) expect(func, args=(c,), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kwargs) median(c, loc=0, scale=1) mean(c, loc=0, scale=1) var(c, loc=0, scale=1) std(c, loc=0, scale=1) interval(alpha, c, loc=0, scale=1)</pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	--

```
scipy.stats.wrapcauchy = <scipy.stats._continuous_distns.wrapcauchy_gen object at 0x2b909bd9a450>
```

A wrapped Cauchy continuous random variable.

As an instance of the `rv_continuous` class, `wrapcauchy` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability density function for `wrapcauchy` is:

$$\text{wrapcauchy.pdf}(x, c) = (1-c**2) / (2*pi*(1+c**2-2*c*cos(x)))$$

for  $0 \leq x \leq 2\pi, 0 < c < 1$ .

`wrapcauchy` takes `c` as a shape parameter.

The probability density above is defined in the “standardized” form. To shift and/or scale the distribution use the `loc` and `scale` parameters. Specifically, `wrapcauchy.pdf(x, c, loc, scale)` is identically equivalent to `wrapcauchy.pdf(y, c) / scale` with  $y = (x - \text{loc}) / \text{scale}$ .

### Examples

```
>>> from scipy.stats import wrapcauchy
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> c = 0.0311
>>> mean, var, skew, kurt = wrapcauchy.stats(c, moments='mvsk')
```

Display the probability density function (pdf):

```
>>> x = np.linspace(wrapcauchy.ppf(0.01, c),
...                 wrapcauchy.ppf(0.99, c), 100)
>>> ax.plot(x, wrapcauchy.pdf(x, c),
...         'r-', lw=5, alpha=0.6, label='wrapcauchy pdf')
```

Alternatively, the distribution object can be called (as a function) to fix the shape, location and scale parameters. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pdf:

```
>>> rv = wrapcauchy(c)
>>> ax.plot(x, rv.pdf(x), 'k-', lw=2, label='frozen pdf')
```

Check accuracy of cdf and ppf:

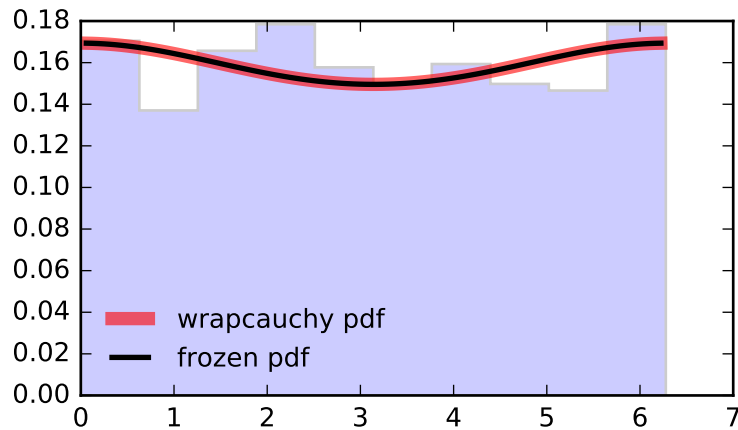
```
>>> vals = wrapcauchy.ppf([0.001, 0.5, 0.999], c)
>>> np.allclose([0.001, 0.5, 0.999], wrapcauchy.cdf(vals, c))
True
```

Generate random numbers:

```
>>> r = wrapcauchy.rvs(c, size=1000)
```

And compare the histogram:

```
>>> ax.hist(r, normed=True, histtype='stepfilled', alpha=0.2)
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



### Methods

<pre>rvs(c, loc=0, scale=1, size=1,     random_state=None) pdf(x, c, loc=0, scale=1) logpdf(x, c, loc=0, scale=1) cdf(x, c, loc=0, scale=1) logcdf(x, c, loc=0, scale=1)  sf(x, c, loc=0, scale=1)  logsf(x, c, loc=0, scale=1) ppf(q, c, loc=0, scale=1)  isf(q, c, loc=0, scale=1) moment(n, c, loc=0, scale=1) stats(c, loc=0, scale=1, moments='mv')  entropy(c, loc=0, scale=1) fit(data, c, loc=0, scale=1) expect(func, args=(c,), loc=0, scale=1,        lb=None, ub=None, conditional=False,        **kwargs) median(c, loc=0, scale=1) mean(c, loc=0, scale=1) var(c, loc=0, scale=1) std(c, loc=0, scale=1) interval(alpha, c, loc=0, scale=1)</pre>	<p>Random variates.</p> <p>Probability density function.</p> <p>Log of the probability density function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Non-central moment of order <i>n</i></p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Parameter estimates for generic data.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
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## 5.27.2 Multivariate distributions

<code>multivariate_normal</code>	A multivariate normal random variable.
<code>matrix_normal</code>	A matrix normal random variable.
<code>dirichlet</code>	A Dirichlet random variable.
<code>wishart</code>	A Wishart random variable.
<code>invwishart</code>	An inverse Wishart random variable.
<code>special_ortho_group</code>	A matrix-valued SO(N) random variable.
<code>ortho_group</code>	A matrix-valued O(N) random variable.
<code>random_correlation</code>	A random correlation matrix.

`scipy.stats.multivariate_normal` = <scipy.stats.\_multivariate.multivariate\_normal\_gen object at 0x2b909c38d750>  
A multivariate normal random variable.

The *mean* keyword specifies the mean. The *cov* keyword specifies the covariance matrix.

**Parameters**

- x** : array\_like  
Quantiles, with the last axis of *x* denoting the components.
- mean** : array\_like, optional  
Mean of the distribution (default zero)
- cov** : array\_like, optional  
Covariance matrix of the distribution (default one)
- allow\_singular** : bool, optional  
Whether to allow a singular covariance matrix. (Default: False)
- random\_state** : None or int or np.random.RandomState instance, optional  
If int or RandomState, use it for drawing the random variates. If None (or np.random), the global np.random state is used. Default is None.

**Alternatively, the object may be called (as a function) to fix the mean and covariance parameters, returning a “frozen” multivariate normal random variable:**

**rv = multivariate\_normal(mean=None, cov=1, allow\_singular=False)**

- Frozen object with the same methods but holding the given mean and covariance fixed.

### Notes

#### Setting the parameter mean to None is equivalent to having mean

be the zero-vector. The parameter *cov* can be a scalar, in which case the covariance matrix is the identity times that value, a vector of diagonal entries for the covariance matrix, or a two-dimensional array\_like.

The covariance matrix *cov* must be a (symmetric) positive semi-definite matrix. The determinant and inverse of *cov* are computed as the pseudo-determinant and pseudo-inverse, respectively, so that *cov* does not need to have full rank.

The probability density function for `multivariate_normal` is

$$f(x) = \frac{1}{\sqrt{(2\pi)^k \det \Sigma}} \exp\left(-\frac{1}{2}(x - \mu)^T \Sigma^{-1}(x - \mu)\right),$$

where  $\mu$  is the mean,  $\Sigma$  the covariance matrix, and  $k$  is the dimension of the space where  $x$  takes values.

New in version 0.14.0.

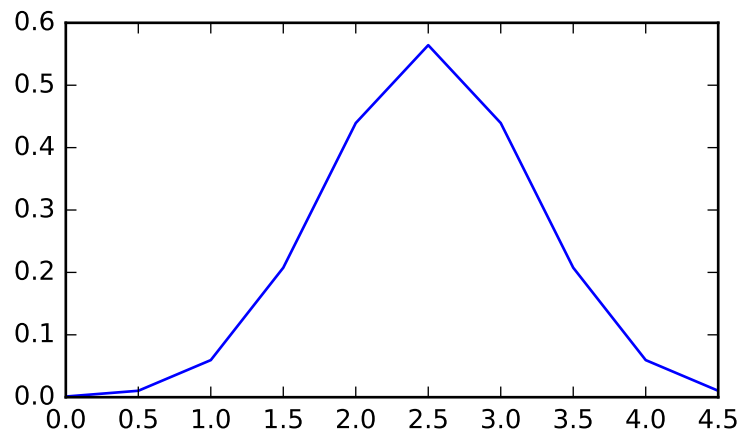
*Examples*

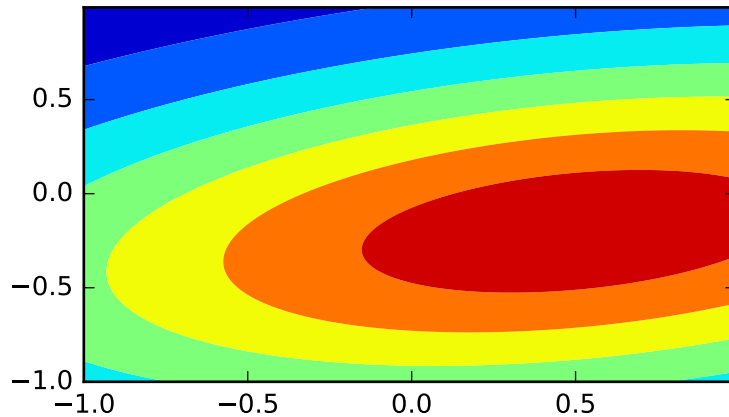
```
>>> import matplotlib.pyplot as plt
>>> from scipy.stats import multivariate_normal

>>> x = np.linspace(0, 5, 10, endpoint=False)
>>> y = multivariate_normal.pdf(x, mean=2.5, cov=0.5); y
array([ 0.00108914,  0.01033349,  0.05946514,  0.20755375,  0.43939129,
        0.56418958,  0.43939129,  0.20755375,  0.05946514,  0.01033349])
>>> fig1 = plt.figure()
>>> ax = fig1.add_subplot(111)
>>> ax.plot(x, y)
```

The input quantiles can be any shape of array, as long as the last axis labels the components. This allows us for instance to display the frozen pdf for a non-isotropic random variable in 2D as follows:

```
>>> x, y = np.mgrid[-1:1:.01, -1:1:.01]
>>> pos = np.dstack((x, y))
>>> rv = multivariate_normal([0.5, -0.2], [[2.0, 0.3], [0.3, 0.5]])
>>> fig2 = plt.figure()
>>> ax2 = fig2.add_subplot(111)
>>> ax2.contourf(x, y, rv.pdf(pos))
```





### Methods

<code>pdf(x, mean=None, cov=1, allow_singular=False)</code>	Probability density function.
<code>logpdf(x, mean=None, cov=1, allow_singular=False)</code>	Log of the probability density function.
<code>rvs(mean=None, cov=1, size=1, random_state=None)</code>	Draw random samples from a multivariate normal distribution.
<code>entropy()</code>	Compute the differential entropy of the multivariate normal.

`scipy.stats.matrix_normal = <scipy.stats.multivariate.matrix_normal_gen object at 0x2b909c38d990>`  
 A matrix normal random variable.

The *mean* keyword specifies the mean. The *rowcov* keyword specifies the among-row covariance matrix. The ‘*colcov*’ keyword specifies the among-column covariance matrix.

**Parameters** **X** : array\_like

Quantiles, with the last two axes of *X* denoting the components.

**mean** : array\_like, optional

Mean of the distribution (default: *None*)

**rowcov** : array\_like, optional

Among-row covariance matrix of the distribution (default: *I*)

**colcov** : array\_like, optional

Among-column covariance matrix of the distribution (default: *I*)

**random\_state** : None or int or `np.random.RandomState` instance, optional

If int or `RandomState`, use it for drawing the random variates. If *None* (or `np.random`), the global `np.random` state is used. Default is *None*.

**Alternatively, the object may be called (as a function) to fix the mean and covariance parameters, returning a “frozen” matrix normal random variable:**

**`rv = matrix_normal(mean=None, rowcov=1, colcov=1)`**

•Frozen object with the same methods but holding the given mean and covariance fixed.

### Notes

**If mean is set to None then a matrix of zeros is used for the mean.**

The dimensions of this matrix are inferred from the shape of `rowcov` and `colcov`, if these are provided, or set to `I` if ambiguous.

`rowcov` and `colcov` can be two-dimensional array\_likes specifying the covariance matrices directly. Alternatively, a one-dimensional array will be interpreted as the entries of a diagonal matrix, and a scalar or zero-dimensional array will be interpreted as this value times the identity matrix.

The covariance matrices specified by `rowcov` and `colcov` must be (symmetric) positive definite. If the samples in `X` are  $m \times n$ , then `rowcov` must be  $m \times m$  and `colcov` must be  $n \times n$ . `mean` must be the same shape as `X`.

The probability density function for `matrix_normal` is

$$f(X) = (2\pi)^{-\frac{mn}{2}} |U|^{-\frac{n}{2}} |V|^{-\frac{m}{2}} \exp\left(-\frac{1}{2} \text{Tr}[U^{-1}(X-M)V^{-1}(X-M)^T]\right),$$

where  $M$  is the mean,  $U$  the among-row covariance matrix,  $V$  the among-column covariance matrix.

The `allow_singular` behaviour of the `multivariate_normal` distribution is not currently supported. Covariance matrices must be full rank.

The `matrix_normal` distribution is closely related to the `multivariate_normal` distribution. Specifically, `Vec(X)` (the vector formed by concatenating the columns of `X`) has a multivariate normal distribution with mean `Vec(M)` and covariance  $V \otimes U$  (where  $\otimes$  is the Kronecker product). Sampling and pdf evaluation are  $\mathcal{O}(m^3 + n^3 + m^2n + mn^2)$  for the matrix normal, but  $\mathcal{O}(m^3n^3)$  for the equivalent multivariate normal, making this equivalent form algorithmically inefficient.

New in version 0.17.0.

### Examples

```

>>> from scipy.stats import matrix_normal

>>> M = np.arange(6).reshape(3,2); M
array([[0, 1],
       [2, 3],
       [4, 5]])
>>> U = np.diag([1,2,3]); U
array([[1, 0, 0],
       [0, 2, 0],
       [0, 0, 3]])
>>> V = 0.3*np.identity(2); V
array([[ 0.3,  0. ],
       [ 0. ,  0.3]])
>>> X = M + 0.1; X
array([[ 0.1,  1.1],
       [ 2.1,  3.1],
       [ 4.1,  5.1]])
>>> matrix_normal.pdf(X, mean=M, rowcov=U, colcov=V)
0.023410202050005054

>>> # Equivalent multivariate normal
>>> from scipy.stats import multivariate_normal
>>> vectorised_X = X.T.flatten()
>>> equiv_mean = M.T.flatten()
>>> equiv_cov = np.kron(V,U)

```



```
>>> multivariate_normal.pdf(vectorised_X, mean=equiv_mean, cov=equiv_cov)
0.023410202050005054
```

### Methods

pdf(X, mean=None, rowcov=1, colcov=1)	Probability density function.
logpdf(X, mean=None, rowcov=1, colcov=1)	Log of the probability density function.
rvs(mean=None, rowcov=1, colcov=1, size=1, random_state=None)	Draw random samples.

`scipy.stats.dirichlet` = <scipy.stats.multivariate.dirichlet\_gen object at 0x2b909c38d8d0>  
A Dirichlet random variable.

The `alpha` keyword specifies the concentration parameters of the distribution.

New in version 0.15.0.

**Parameters** `x` : array\_like

Quantiles, with the last axis of `x` denoting the components.

**alpha** : array\_like

The concentration parameters. The number of entries determines the dimensionality of the distribution.

**random\_state** : None or int or np.random.RandomState instance, optional

If int or RandomState, use it for drawing the random variates. If None (or np.random), the global np.random state is used. Default is None.

**Alternatively, the object may be called (as a function) to fix concentration parameters, returning a “frozen” Dirichlet random variable:**

**rv = dirichlet(alpha)**

- Frozen object with the same methods but holding the given concentration parameters fixed.

### Notes

Each  $\alpha$  entry must be positive. The distribution has only support on the simplex defined by

$$\sum_{i=1}^K x_i \leq 1$$

The probability density function for `dirichlet` is

$$f(x) = \frac{1}{B(\alpha)} \prod_{i=1}^K x_i^{\alpha_i - 1}$$

where

$$B(\alpha) = \frac{\prod_{i=1}^K \Gamma(\alpha_i)}{\Gamma(\sum_{i=1}^K \alpha_i)}$$

and  $\alpha = (\alpha_1, \dots, \alpha_K)$ , the concentration parameters and  $K$  is the dimension of the space where  $x$  takes values.

Note that the `dirichlet` interface is somewhat inconsistent. The array returned by the `rvs` function is transposed with respect to the format expected by the `pdf` and `logpdf`.

**Methods**

pdf(x, alpha)	Probability density function.
logpdf(x, alpha)	Log of the probability density function.
rvs(alpha, size=1, random_state=None)	Draw random samples from a Dirichlet distribution.
mean(alpha)	The mean of the Dirichlet distribution
var(alpha)	The variance of the Dirichlet distribution
entropy(alpha)	Compute the differential entropy of the multivariate normal.

`scipy.stats.wishart` = <scipy.stats.\_multivariate.wishart\_gen object at 0x2b909c38d890>  
 A Wishart random variable.

The *df* keyword specifies the degrees of freedom. The *scale* keyword specifies the scale matrix, which must be symmetric and positive definite. In this context, the scale matrix is often interpreted in terms of a multivariate normal precision matrix (the inverse of the covariance matrix).

**Parameters**

- x** : array\_like  
Quantiles, with the last axis of *x* denoting the components.
- df** : int  
Degrees of freedom, must be greater than or equal to dimension of the scale matrix
- scale** : array\_like  
Symmetric positive definite scale matrix of the distribution
- random\_state** : None or int or np.random.RandomState instance, optional  
If int or RandomState, use it for drawing the random variates. If None (or np.random), the global np.random state is used. Default is None.

**Alternatively, the object may be called (as a function) to fix the degrees of freedom and scale parameters, returning a “frozen” Wishart random variable:**

**rv = wishart(df=1, scale=1)**

- Frozen object with the same methods but holding the given degrees of freedom and scale fixed.

**See also:**

`invwishart`, `chi2`

**Notes**

The scale matrix *scale* must be a symmetric positive definite matrix. Singular matrices, including the symmetric positive semi-definite case, are not supported.

The Wishart distribution is often denoted

$$W_p(\nu, \Sigma)$$

where  $\nu$  is the degrees of freedom and  $\Sigma$  is the  $p \times p$  scale matrix.

The probability density function for `wishart` has support over positive definite matrices *S*; if  $S \sim W_p(\nu, \Sigma)$ , then its PDF is given by:

$$f(S) = \frac{|S|^{\frac{\nu-p-1}{2}}}{2^{\frac{\nu p}{2}} |\Sigma|^{\frac{\nu}{2}} \Gamma_p\left(\frac{\nu}{2}\right)} \exp(-tr(\Sigma^{-1}S)/2)$$

If  $S \sim W_p(\nu, \Sigma)$  (Wishart) then  $S^{-1} \sim W_p^{-1}(\nu, \Sigma^{-1})$  (inverse Wishart).

If the scale matrix is 1-dimensional and equal to one, then the Wishart distribution  $W_1(\nu, 1)$  collapses to the  $\chi^2(\nu)$  distribution.

New in version 0.16.0.

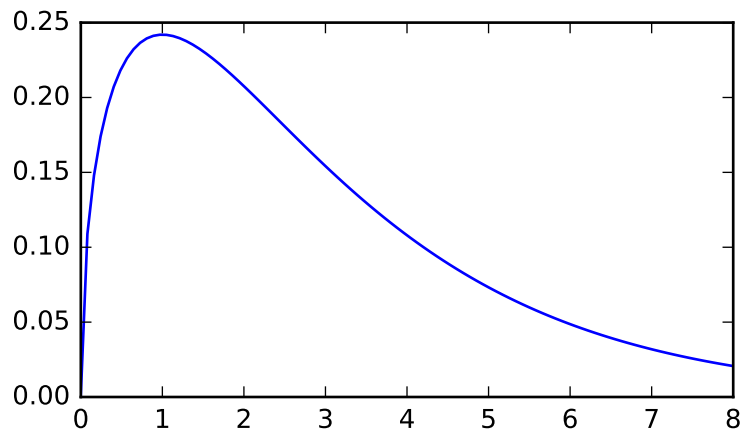
### References

[R582], [R583]

### Examples

```
>>> import matplotlib.pyplot as plt
>>> from scipy.stats import wishart, chi2
>>> x = np.linspace(1e-5, 8, 100)
>>> w = wishart.pdf(x, df=3, scale=1); w[:5]
array([ 0.00126156,  0.10892176,  0.14793434,  0.17400548,  0.1929669 ] )
>>> c = chi2.pdf(x, 3); c[:5]
array([ 0.00126156,  0.10892176,  0.14793434,  0.17400548,  0.1929669 ] )
>>> plt.plot(x, w)
```

The input quantiles can be any shape of array, as long as the last axis labels the components.



### Methods

pdf(x, df, scale)	Probability density function.
logpdf(x, df, scale)	Log of the probability density function.
rvs(df, scale, size=1, random_state=None)	Draw random samples from a Wishart distribution.
entropy()	Compute the differential entropy of the Wishart distribution.

`scipy.stats.invwishart = <scipy.stats._multivariate.invwishart_gen object at 0x2b909c38dd10>`

An inverse Wishart random variable.

The *df* keyword specifies the degrees of freedom. The *scale* keyword specifies the scale matrix, which must be symmetric and positive definite. In this context, the scale matrix is often interpreted in terms of a multivariate normal covariance matrix.

**Parameters** *x*: array\_like

Quantiles, with the last axis of *x* denoting the components.

*df*: int

Degrees of freedom, must be greater than or equal to dimension of the scale matrix

**scale** : array\_like

Symmetric positive definite scale matrix of the distribution

**random\_state** : None or int or np.random.RandomState instance, optional

If int or RandomState, use it for drawing the random variates. If None (or np.random), the global np.random state is used. Default is None.

**Alternatively, the object may be called (as a function) to fix the degrees of freedom and scale parameters, returning a “frozen” inverse Wishart random variable:**

**rv = invwishart(df=1, scale=1)**

- Frozen object with the same methods but holding the given degrees of freedom and scale fixed.

**See also:**

`wishart`

### Notes

The scale matrix *scale* must be a symmetric positive definite matrix. Singular matrices, including the symmetric positive semi-definite case, are not supported.

The inverse Wishart distribution is often denoted

$$W_p^{-1}(\nu, \Psi)$$

where  $\nu$  is the degrees of freedom and  $\Psi$  is the  $p \times p$  scale matrix.

The probability density function for `invwishart` has support over positive definite matrices  $S$ ; if  $S \sim W_p^{-1}(\nu, \Sigma)$ , then its PDF is given by:

$$f(S) = \frac{|\Sigma|^{\frac{\nu}{2}}}{2^{\frac{\nu p}{2}} |S|^{\frac{\nu+p+1}{2}} \Gamma_p\left(\frac{\nu}{2}\right)} \exp(-\text{tr}(\Sigma S^{-1})/2)$$

If  $S \sim W_p^{-1}(\nu, \Psi)$  (inverse Wishart) then  $S^{-1} \sim W_p(\nu, \Psi^{-1})$  (Wishart).

If the scale matrix is 1-dimensional and equal to one, then the inverse Wishart distribution  $W_1(\nu, 1)$  collapses to the inverse Gamma distribution with parameters shape =  $\frac{\nu}{2}$  and scale =  $\frac{1}{2}$ .

New in version 0.16.0.

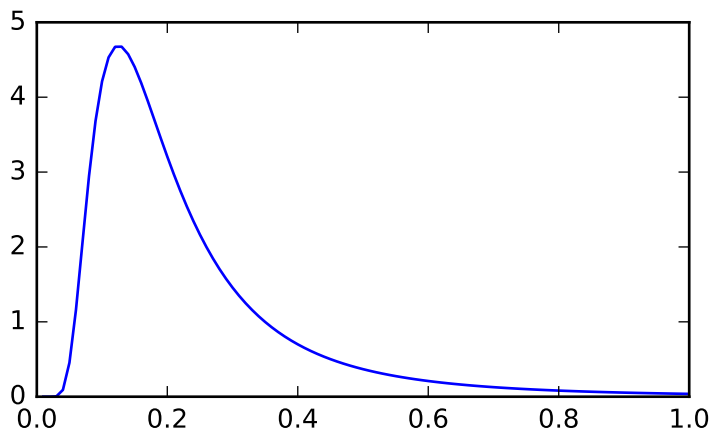
### References

[R527], [R528]

### Examples

```
>>> import matplotlib.pyplot as plt
>>> from scipy.stats import invwishart, invgamma
>>> x = np.linspace(0.01, 1, 100)
>>> iw = invwishart.pdf(x, df=6, scale=1)
>>> iw[:3]
array([ 1.20546865e-15,  5.42497807e-06,  4.45813929e-03])
>>> ig = invgamma.pdf(x, 6/2., scale=1./2)
>>> ig[:3]
array([ 1.20546865e-15,  5.42497807e-06,  4.45813929e-03])
>>> plt.plot(x, iw)
```

The input quantiles can be any shape of array, as long as the last axis labels the components.



### Methods

<code>pdf(x, df, scale)</code>	Probability density function.
<code>logpdf(x, df, scale)</code>	Log of the probability density function.
<code>rvs(df, scale, size=1, random_state=None)</code>	Draw random samples from an inverse Wishart distribution.

`scipy.stats.special_ortho_group = <scipy.stats.multivariate.special_ortho_group_gen object at 0x2b909c38ddd0>`  
A matrix-valued SO(N) random variable.

Return a random rotation matrix, drawn from the Haar distribution (the only uniform distribution on SO(n)).

The *dim* keyword specifies the dimension N.

**Parameters** **dim** : scalar  
Dimension of matrices

### Notes

This class is wrapping the `random_rot` code from the MDP Toolkit, <https://github.com/mdp-toolkit/mdp-toolkit>

Return a random rotation matrix, drawn from the Haar distribution (the only uniform distribution on SO(n)). The algorithm is described in the paper Stewart, G.W., “The efficient generation of random orthogonal matrices with an application to condition estimators”, SIAM Journal on Numerical Analysis, 17(3), pp. 403-409, 1980. For more information see [http://en.wikipedia.org/wiki/Orthogonal\\_matrix#Randomization](http://en.wikipedia.org/wiki/Orthogonal_matrix#Randomization)

See also the similar `ortho_group`.

### Examples

```
>>> from scipy.stats import special_ortho_group
>>> x = special_ortho_group.rvs(3)

>>> np.dot(x, x.T)
array([[ 1.00000000e+00,  1.13231364e-17, -2.86852790e-16],
       [ 1.13231364e-17,  1.00000000e+00, -1.46845020e-16],
       [-2.86852790e-16, -1.46845020e-16,  1.00000000e+00]])
```

```
>>> import scipy.linalg
>>> scipy.linalg.det(x)
1.0
```

This generates one random matrix from  $SO(3)$ . It is orthogonal and has a determinant of 1.

#### Methods

<code>rvs(dim=None, size=1, random_state=None)</code>	Draw random samples from $SO(N)$ .
---	------------------------------------

```
scipy.stats.ortho_group = <scipy.stats.multivariate.ortho_group_gen object at 0x2b909c38de10>
A matrix-valued  $O(N)$  random variable.
```

Return a random orthogonal matrix, drawn from the  $O(N)$  Haar distribution (the only uniform distribution on  $O(N)$ ).

The *dim* keyword specifies the dimension  $N$ .

**Parameters**    **dim** : scalar  
 Dimension of matrices

#### Notes

This class is closely related to `special_ortho_group`.

Some care is taken to avoid numerical error, as per the paper by Mezzadri.

#### References

[R551]

#### Examples

```
>>> from scipy.stats import ortho_group
>>> x = ortho_group.rvs(3)

>>> np.dot(x, x.T)
array([[ 1.00000000e+00,  1.13231364e-17, -2.86852790e-16],
       [ 1.13231364e-17,  1.00000000e+00, -1.46845020e-16],
       [-2.86852790e-16, -1.46845020e-16,  1.00000000e+00]])

>>> import scipy.linalg
>>> np.fabs(scipy.linalg.det(x))
1.0
```

This generates one random matrix from  $O(3)$ . It is orthogonal and has a determinant of +1 or -1.

#### Methods

<code>rvs(dim=None, size=1, random_state=None)</code>	Draw random samples from $O(N)$ .
---	-----------------------------------

```
scipy.stats.random_correlation = <scipy.stats.multivariate.random_correlation_gen object at 0x2b909c38df10>
A random correlation matrix.
```

Return a random correlation matrix, given a vector of eigenvalues.

The *eigs* keyword specifies the eigenvalues of the correlation matrix, and implies the dimension.

**Parameters**    **eigs** : 1d ndarray  
 Eigenvalues of correlation matrix.

### Notes

Generates a random correlation matrix following a numerically stable algorithm spelled out by Davies & Higham. This algorithm uses a single  $O(N)$  similarity transformation to construct a symmetric positive semi-definite matrix, and applies a series of Givens rotations to scale it to have ones on the diagonal.

### References

[R562]

### Examples

```
>>> from scipy.stats import random_correlation
>>> np.random.seed(514)
>>> x = random_correlation.rvs((.5, .8, 1.2, 1.5))
>>> x
array([[ 1.          , -0.20387311,  0.18366501, -0.04953711],
       [-0.20387311,  1.          , -0.24351129,  0.06703474],
       [ 0.18366501, -0.24351129,  1.          ,  0.38530195],
       [-0.04953711,  0.06703474,  0.38530195,  1.          ]])

>>> import scipy.linalg
>>> e, v = scipy.linalg.eigh(x)
>>> e
array([ 0.5,  0.8,  1.2,  1.5])
```

### Methods

<code>rvs(eigs=None, random_state=None)</code>	Draw random correlation matrices, all with eigenvalues <code>eigs</code> .
--	--

## 5.27.3 Discrete distributions

<code>bernoulli</code>	A Bernoulli discrete random variable.
<code>binom</code>	A binomial discrete random variable.
<code>boltzmann</code>	A Boltzmann (Truncated Discrete Exponential) random variable.
<code>dlaplace</code>	A Laplacian discrete random variable.
<code>geom</code>	A geometric discrete random variable.
<code>hypergeom</code>	A hypergeometric discrete random variable.
<code>logser</code>	A Logarithmic (Log-Series, Series) discrete random variable.
<code>nbinom</code>	A negative binomial discrete random variable.
<code>planck</code>	A Planck discrete exponential random variable.
<code>poisson</code>	A Poisson discrete random variable.
<code>randint</code>	A uniform discrete random variable.
<code>skellam</code>	A Skellam discrete random variable.
<code>zipf</code>	A Zipf discrete random variable.

`scipy.stats.bernoulli = <scipy.stats._discrete_distns.bernoulli_gen object at 0x2b909bd3b810>`  
A Bernoulli discrete random variable.

As an instance of the `rv_discrete` class, `bernoulli` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability mass function for `bernoulli` is:

$$\begin{aligned} \text{bernoulli.pmf}(k) &= 1-p && \text{if } k = 0 \\ &= p && \text{if } k = 1 \end{aligned}$$

for  $k$  in  $\{0, 1\}$ .

`bernoulli` takes  $p$  as shape parameter.

The probability mass function above is defined in the “standardized” form. To shift distribution use the `loc` parameter. Specifically, `bernoulli.pmf(k, p, loc)` is identically equivalent to `bernoulli.pmf(k - loc, p)`.

### Examples

```
>>> from scipy.stats import bernoulli
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> p = 0.3
>>> mean, var, skew, kurt = bernoulli.stats(p, moments='mvsk')
```

Display the probability mass function (pmf):

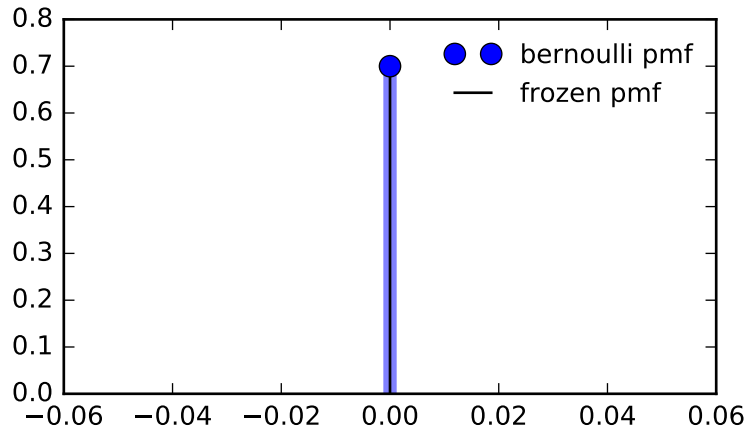
```
>>> x = np.arange(bernoulli.ppf(0.01, p),
...               bernoulli.ppf(0.99, p))
>>> ax.plot(x, bernoulli.pmf(x, p), 'bo', ms=8, label='bernoulli pmf')
>>> ax.vlines(x, 0, bernoulli.pmf(x, p), colors='b', lw=5, alpha=0.5)
```

Alternatively, the distribution object can be called (as a function) to fix the shape and location. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pmf:

```
>>> rv = bernoulli(p)
>>> ax.vlines(x, 0, rv.pmf(x), colors='k', linestyle='-', lw=1,
...          label='frozen pmf')
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```





Check accuracy of cdf and ppf:

```
>>> prob = bernoulli.cdf(x, p)
>>> np.allclose(x, bernoulli.ppf(prob, p))
True
```

Generate random numbers:

```
>>> r = bernoulli.rvs(p, size=1000)
```

### Methods

<pre>rvs(p, loc=0, size=1,     random_state=None) pmf(x, p, loc=0) logpmf(x, p, loc=0) cdf(x, p, loc=0) logcdf(x, p, loc=0) sf(x, p, loc=0)  logsf(x, p, loc=0) ppf(q, p, loc=0)  isf(q, p, loc=0) stats(p, loc=0, moments='mv')  entropy(p, loc=0) expect(func, args=(p,), loc=0,        lb=None, ub=None, conditional=False) median(p, loc=0) mean(p, loc=0) var(p, loc=0) std(p, loc=0) interval(alpha, p, loc=0)</pre>	<p>Random variates.</p> <p>Probability mass function.</p> <p>Log of the probability mass function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
--	---

`scipy.stats.binom` = <scipy.stats.\_discrete\_distns.binom\_gen object at 0x2b909bd3b610>

A binomial discrete random variable.

As an instance of the `rv_discrete` class, `binom` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability mass function for `binom` is:

```
binom.pmf(k) = choose(n, k) * p**k * (1-p)**(n-k)
```

for `k` in  $\{0, 1, \dots, n\}$ .

`binom` takes `n` and `p` as shape parameters.

The probability mass function above is defined in the “standardized” form. To shift distribution use the `loc` parameter. Specifically, `binom.pmf(k, n, p, loc)` is identically equivalent to `binom.pmf(k - loc, n, p)`.

### Examples

```
>>> from scipy.stats import binom
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> n, p = 5, 0.4
>>> mean, var, skew, kurt = binom.stats(n, p, moments='mvsk')
```

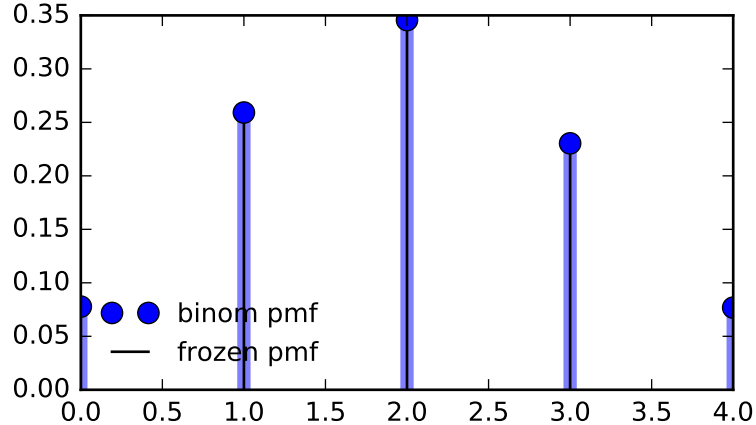
Display the probability mass function (pmf):

```
>>> x = np.arange(binom.ppf(0.01, n, p),
...               binom.ppf(0.99, n, p))
>>> ax.plot(x, binom.pmf(x, n, p), 'bo', ms=8, label='binom pmf')
>>> ax.vlines(x, 0, binom.pmf(x, n, p), colors='b', lw=5, alpha=0.5)
```

Alternatively, the distribution object can be called (as a function) to fix the shape and location. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pmf:

```
>>> rv = binom(n, p)
>>> ax.vlines(x, 0, rv.pmf(x), colors='k', linestyle='-', lw=1,
...          label='frozen pmf')
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



Check accuracy of cdf and ppf:

```
>>> prob = binom.cdf(x, n, p)
>>> np.allclose(x, binom.ppf(prob, n, p))
True
```

Generate random numbers:

```
>>> r = binom.rvs(n, p, size=1000)
```

### Methods

<pre>rvs(n, p, loc=0, size=1,     random_state=None) pmf(x, n, p, loc=0) logpmf(x, n, p, loc=0) cdf(x, n, p, loc=0) logcdf(x, n, p, loc=0) sf(x, n, p, loc=0)  logsf(x, n, p, loc=0) ppf(q, n, p, loc=0)  isf(q, n, p, loc=0) stats(n, p, loc=0, moments='mv')  entropy(n, p, loc=0) expect(func, args=(n, p), loc=0,        lb=None, ub=None, conditional=False) median(n, p, loc=0) mean(n, p, loc=0) var(n, p, loc=0) std(n, p, loc=0) interval(alpha, n, p, loc=0)</pre>	<p>Random variates.</p> <p>Probability mass function.</p> <p>Log of the probability mass function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
--	---

`scipy.stats.boltzmann = <scipy.stats._discrete_distns.boltzmann_gen object at 0x2b909bdac810>`  
A Boltzmann (Truncated Discrete Exponential) random variable.

As an instance of the `rv_discrete` class, `boltzmann` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability mass function for `boltzmann` is:

$$\text{boltzmann.pmf}(k) = (1 - \exp(-\lambda)) \exp(-\lambda * k) / (1 - \exp(-\lambda * N))$$

for  $k = 0, \dots, N-1$ .

`boltzmann` takes `lambda_` and `N` as shape parameters.

The probability mass function above is defined in the “standardized” form. To shift distribution use the `loc` parameter. Specifically, `boltzmann.pmf(k, lambda_, N, loc)` is identically equivalent to `boltzmann.pmf(k - loc, lambda_, N)`.

### Examples

```
>>> from scipy.stats import boltzmann
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> lambda_, N = 1.4, 19
>>> mean, var, skew, kurt = boltzmann.stats(lambda_, N, moments='mvsk')
```

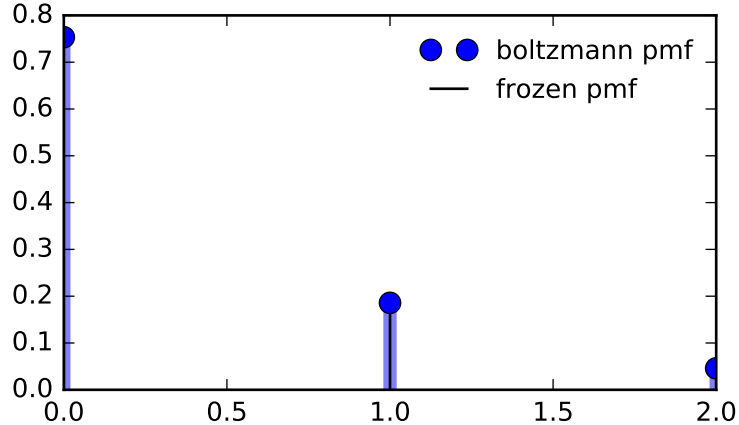
Display the probability mass function (pmf):

```
>>> x = np.arange(boltzmann.ppf(0.01, lambda_, N),
...               boltzmann.ppf(0.99, lambda_, N))
>>> ax.plot(x, boltzmann.pmf(x, lambda_, N), 'bo', ms=8, label='boltzmann pmf')
>>> ax.vlines(x, 0, boltzmann.pmf(x, lambda_, N), colors='b', lw=5, alpha=0.5)
```

Alternatively, the distribution object can be called (as a function) to fix the shape and location. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pmf:

```
>>> rv = boltzmann(lambda_, N)
>>> ax.vlines(x, 0, rv.pmf(x), colors='k', linestyle='-', lw=1,
...          label='frozen pmf')
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



Check accuracy of cdf and ppf:

```
>>> prob = boltzmann.cdf(x, lambda_, N)
>>> np.allclose(x, boltzmann.ppf(prob, lambda_, N))
True
```

Generate random numbers:

```
>>> r = boltzmann.rvs(lambda_, N, size=1000)
```

### Methods

```
rvs(lambda_, N, loc=0, size=1,
    random_state=None)
pmf(x, lambda_, N, loc=0)
logpmf(x, lambda_, N, loc=0)
cdf(x, lambda_, N, loc=0)
logcdf(x, lambda_, N, loc=0)
sf(x, lambda_, N, loc=0)

logsf(x, lambda_, N, loc=0)
ppf(q, lambda_, N, loc=0)

isf(q, lambda_, N, loc=0)
stats(lambda_, N, loc=0, moments='mv')

entropy(lambda_, N, loc=0)
expect(func, args=(lambda_, N), loc=0,
    lb=None, ub=None, conditional=False)
median(lambda_, N, loc=0)
mean(lambda_, N, loc=0)
var(lambda_, N, loc=0)
std(lambda_, N, loc=0)
interval(alpha, lambda_, N, loc=0)
```

Random variates.

Probability mass function.

Log of the probability mass function.

Cumulative distribution function.

Log of the cumulative distribution function.

Survival function (also defined as  $1 - \text{cdf}$ , but *sf* is sometimes more accurate).

Log of the survival function.

Percent point function (inverse of *cdf* — percentiles).

Inverse survival function (inverse of *sf*).

Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').

(Differential) entropy of the RV.

Expected value of a function (of one argument) with respect to the distribution.

Median of the distribution.

Mean of the distribution.

Variance of the distribution.

Standard deviation of the distribution.

Endpoints of the range that contains alpha percent of the distribution

`scipy.stats.dlaplace = <scipy.stats._discrete_distns.dlaplace_gen object at 0x2b909bdacdd0>`

A Laplacian discrete random variable.

As an instance of the `rv_discrete` class, `dlaplace` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability mass function for `dlaplace` is:

```
dlaplace.pmf(k) = tanh(a/2) * exp(-a*abs(k))
```

for  $a > 0$ .

`dlaplace` takes `a` as shape parameter.

The probability mass function above is defined in the “standardized” form. To shift distribution use the `loc` parameter. Specifically, `dlaplace.pmf(k, a, loc)` is identically equivalent to `dlaplace.pmf(k - loc, a)`.

### Examples

```
>>> from scipy.stats import dlaplace
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> a = 0.8
>>> mean, var, skew, kurt = dlaplace.stats(a, moments='mvsk')
```

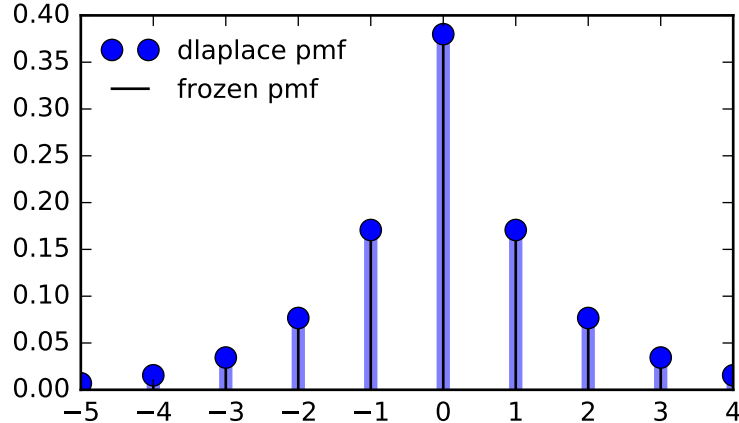
Display the probability mass function (pmf):

```
>>> x = np.arange(dlaplace.ppf(0.01, a),
...               dlaplace.ppf(0.99, a))
>>> ax.plot(x, dlaplace.pmf(x, a), 'bo', ms=8, label='dlaplace pmf')
>>> ax.vlines(x, 0, dlaplace.pmf(x, a), colors='b', lw=5, alpha=0.5)
```

Alternatively, the distribution object can be called (as a function) to fix the shape and location. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pmf:

```
>>> rv = dlaplace(a)
>>> ax.vlines(x, 0, rv.pmf(x), colors='k', linestyle='-', lw=1,
...          label='frozen pmf')
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



Check accuracy of cdf and ppf:

```
>>> prob = dlaplace.cdf(x, a)
>>> np.allclose(x, dlaplace.ppf(prob, a))
True
```

Generate random numbers:

```
>>> r = dlaplace.rvs(a, size=1000)
```

### Methods

<pre>rvs(a, loc=0, size=1,     random_state=None) pmf(x, a, loc=0) logpmf(x, a, loc=0) cdf(x, a, loc=0) logcdf(x, a, loc=0) sf(x, a, loc=0)  logsf(x, a, loc=0) ppf(q, a, loc=0)  isf(q, a, loc=0) stats(a, loc=0, moments='mv')  entropy(a, loc=0) expect(func, args=(a,), loc=0,        lb=None, ub=None, conditional=False) median(a, loc=0) mean(a, loc=0) var(a, loc=0) std(a, loc=0) interval(alpha, a, loc=0)</pre>	<p>Random variates.</p> <p>Probability mass function.</p> <p>Log of the probability mass function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
--	---

`scipy.stats.geom` = <scipy.stats.\_discrete\_distns.geom\_gen object at 0x2b909bd3bc50>

A geometric discrete random variable.

As an instance of the `rv_discrete` class, `geom` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability mass function for `geom` is:

```
geom.pmf(k) = (1-p)**(k-1)*p
```

for  $k \geq 1$ .

`geom` takes `p` as shape parameter.

The probability mass function above is defined in the “standardized” form. To shift distribution use the `loc` parameter. Specifically, `geom.pmf(k, p, loc)` is identically equivalent to `geom.pmf(k - loc, p)`.

### Examples

```
>>> from scipy.stats import geom
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> p = 0.5
>>> mean, var, skew, kurt = geom.stats(p, moments='mvsk')
```

Display the probability mass function (pmf):

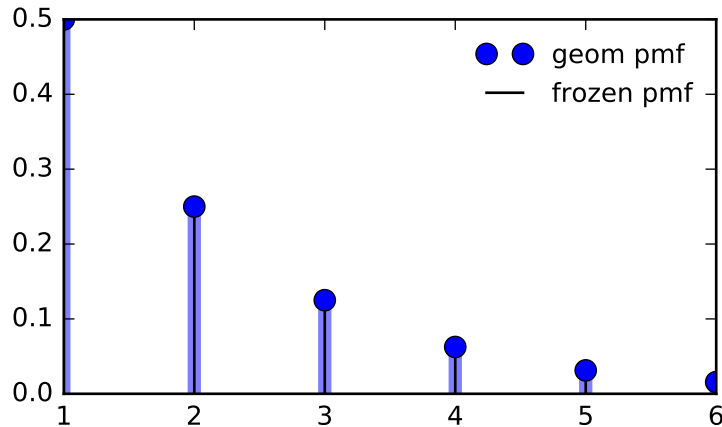
```
>>> x = np.arange(geom.ppf(0.01, p),
...               geom.ppf(0.99, p))
>>> ax.plot(x, geom.pmf(x, p), 'bo', ms=8, label='geom pmf')
>>> ax.vlines(x, 0, geom.pmf(x, p), colors='b', lw=5, alpha=0.5)
```

Alternatively, the distribution object can be called (as a function) to fix the shape and location. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pmf:

```
>>> rv = geom(p)
>>> ax.vlines(x, 0, rv.pmf(x), colors='k', linestyle='-', lw=1,
...          label='frozen pmf')
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```





Check accuracy of cdf and ppf:

```
>>> prob = geom.cdf(x, p)
>>> np.allclose(x, geom.ppf(prob, p))
True
```

Generate random numbers:

```
>>> r = geom.rvs(p, size=1000)
```

### Methods

<pre>rvs(p, loc=0, size=1,     random_state=None) pmf(x, p, loc=0) logpmf(x, p, loc=0) cdf(x, p, loc=0) logcdf(x, p, loc=0) sf(x, p, loc=0)  logsf(x, p, loc=0) ppf(q, p, loc=0)  isf(q, p, loc=0) stats(p, loc=0, moments='mv')  entropy(p, loc=0) expect(func, args=(p,), loc=0,        lb=None, ub=None, conditional=False) median(p, loc=0) mean(p, loc=0) var(p, loc=0) std(p, loc=0) interval(alpha, p, loc=0)</pre>	<p>Random variates.</p> <p>Probability mass function.</p> <p>Log of the probability mass function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
--	---

`scipy.stats.hypergeom` = <scipy.stats.\_discrete\_distns.hypergeom\_gen object at 0x2b909bd3be90>

A hypergeometric discrete random variable.

The hypergeometric distribution models drawing objects from a bin.  $M$  is the total number of objects,  $n$  is total number of Type I objects. The random variate represents the number of Type I objects in  $N$  drawn without replacement from the total population.

As an instance of the `rv_discrete` class, `hypergeom` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability mass function is defined as:

$$\text{pmf}(k, M, n, N) = \frac{\text{choose}(n, k) * \text{choose}(M - n, N - k)}{\text{choose}(M, N)},$$

for  $\max(0, N - (M-n)) \leq k \leq \min(n, N)$

The probability mass function above is defined in the “standardized” form. To shift distribution use the `loc` parameter. Specifically, `hypergeom.pmf(k, M, n, N, loc)` is identically equivalent to `hypergeom.pmf(k - loc, M, n, N)`.

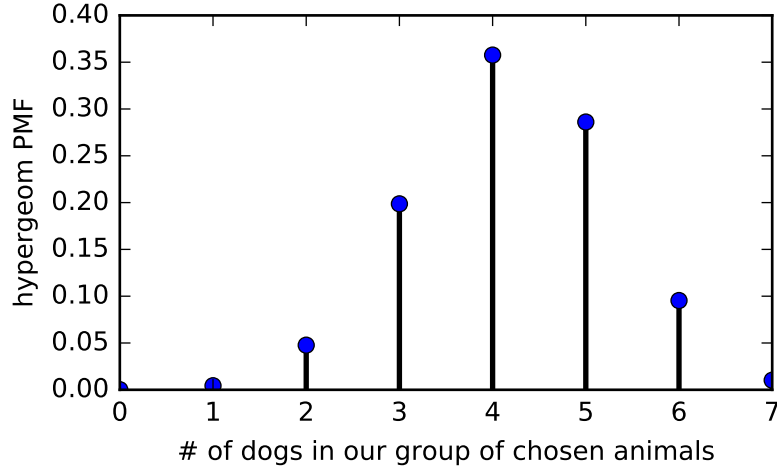
### Examples

```
>>> from scipy.stats import hypergeom
>>> import matplotlib.pyplot as plt
```

Suppose we have a collection of 20 animals, of which 7 are dogs. Then if we want to know the probability of finding a given number of dogs if we choose at random 12 of the 20 animals, we can initialize a frozen distribution and plot the probability mass function:

```
>>> [M, n, N] = [20, 7, 12]
>>> rv = hypergeom(M, n, N)
>>> x = np.arange(0, n+1)
>>> pmf_dogs = rv.pmf(x)

>>> fig = plt.figure()
>>> ax = fig.add_subplot(111)
>>> ax.plot(x, pmf_dogs, 'bo')
>>> ax.vlines(x, 0, pmf_dogs, lw=2)
>>> ax.set_xlabel('# of dogs in our group of chosen animals')
>>> ax.set_ylabel('hypergeom PMF')
>>> plt.show()
```



Instead of using a frozen distribution we can also use `hypergeom` methods directly. To for example obtain the cumulative distribution function, use:

```
>>> prb = hypergeom.cdf(x, M, n, N)
```

And to generate random numbers:

```
>>> R = hypergeom.rvs(M, n, N, size=10)
```

### Methods

<pre>rvs(M, n, N, loc=0, size=1,     random_state=None) pmf(x, M, n, N, loc=0) logpmf(x, M, n, N, loc=0) cdf(x, M, n, N, loc=0) logcdf(x, M, n, N, loc=0) sf(x, M, n, N, loc=0)  logsf(x, M, n, N, loc=0) ppf(q, M, n, N, loc=0)  isf(q, M, n, N, loc=0) stats(M, n, N, loc=0, moments='mv')  entropy(M, n, N, loc=0) expect(func, args=(M, n, N), loc=0,         lb=None, ub=None, conditional=False) median(M, n, N, loc=0) mean(M, n, N, loc=0) var(M, n, N, loc=0) std(M, n, N, loc=0) interval(alpha, M, n, N, loc=0)</pre>	<p>Random variates.</p> <p>Probability mass function.</p> <p>Log of the probability mass function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
--	---

`scipy.stats.logser = <scipy.stats._discrete_distns.logser_gen object at 0x2b909bdac190>`

A Logarithmic (Log-Series, Series) discrete random variable.

As an instance of the `rv_discrete` class, `logser` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability mass function for `logser` is:

$$\text{logser.pmf}(k) = -p^{**k} / (k * \log(1-p))$$

for  $k \geq 1$ .

`logser` takes `p` as shape parameter.

The probability mass function above is defined in the “standardized” form. To shift distribution use the `loc` parameter. Specifically, `logser.pmf(k, p, loc)` is identically equivalent to `logser.pmf(k - loc, p)`.

### Examples

```
>>> from scipy.stats import logser
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> p = 0.6
>>> mean, var, skew, kurt = logser.stats(p, moments='mvsk')
```

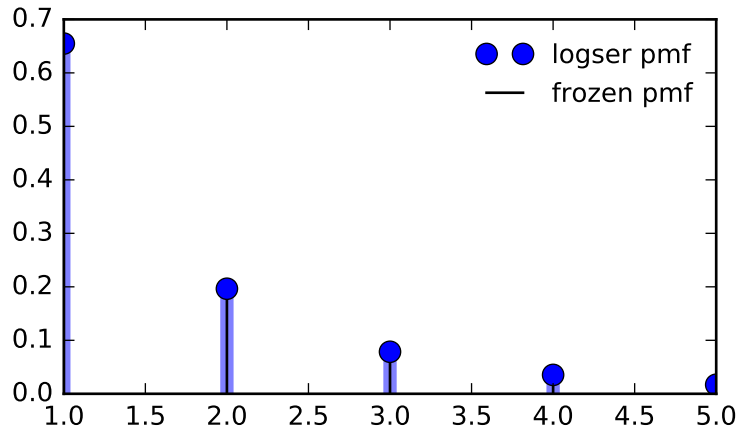
Display the probability mass function (pmf):

```
>>> x = np.arange(logser.ppf(0.01, p),
...               logser.ppf(0.99, p))
>>> ax.plot(x, logser.pmf(x, p), 'bo', ms=8, label='logser pmf')
>>> ax.vlines(x, 0, logser.pmf(x, p), colors='b', lw=5, alpha=0.5)
```

Alternatively, the distribution object can be called (as a function) to fix the shape and location. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pmf:

```
>>> rv = logser(p)
>>> ax.vlines(x, 0, rv.pmf(x), colors='k', linestyle='-', lw=1,
...          label='frozen pmf')
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



Check accuracy of cdf and ppf:

```
>>> prob = logser.cdf(x, p)
>>> np.allclose(x, logser.ppf(prob, p))
True
```

Generate random numbers:

```
>>> r = logser.rvs(p, size=1000)
```

### Methods

<pre>rvs(p, loc=0, size=1,     random_state=None) pmf(x, p, loc=0) logpmf(x, p, loc=0) cdf(x, p, loc=0) logcdf(x, p, loc=0) sf(x, p, loc=0)  logsf(x, p, loc=0) ppf(q, p, loc=0)  isf(q, p, loc=0) stats(p, loc=0, moments='mv')  entropy(p, loc=0) expect(func, args=(p,), loc=0,        lb=None, ub=None, conditional=False) median(p, loc=0) mean(p, loc=0) var(p, loc=0) std(p, loc=0) interval(alpha, p, loc=0)</pre>	<p>Random variates.</p> <p>Probability mass function.</p> <p>Log of the probability mass function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
--	---

`scipy.stats.nbinom` = <scipy.stats.\_discrete\_distns.nbinom\_gen object at 0x2b909bd3ba10>

A negative binomial discrete random variable.

As an instance of the `rv_discrete` class, `nbinom` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability mass function for `nbinom` is:

```
nbinom.pmf(k) = choose(k+n-1, n-1) * p**n * (1-p)**k
```

for  $k \geq 0$ .

`nbinom` takes `n` and `p` as shape parameters.

The probability mass function above is defined in the “standardized” form. To shift distribution use the `loc` parameter. Specifically, `nbinom.pmf(k, n, p, loc)` is identically equivalent to `nbinom.pmf(k - loc, n, p)`.

### Examples

```
>>> from scipy.stats import nbinom
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> n, p = 0.4, 0.4
>>> mean, var, skew, kurt = nbinom.stats(n, p, moments='mvsk')
```

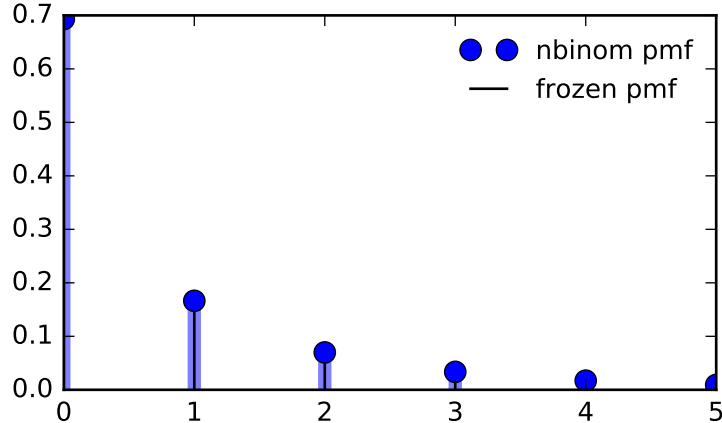
Display the probability mass function (pmf):

```
>>> x = np.arange(nbinom.ppf(0.01, n, p),
...               nbinom.ppf(0.99, n, p))
>>> ax.plot(x, nbinom.pmf(x, n, p), 'bo', ms=8, label='nbinom pmf')
>>> ax.vlines(x, 0, nbinom.pmf(x, n, p), colors='b', lw=5, alpha=0.5)
```

Alternatively, the distribution object can be called (as a function) to fix the shape and location. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pmf:

```
>>> rv = nbinom(n, p)
>>> ax.vlines(x, 0, rv.pmf(x), colors='k', linestyle='-', lw=1,
...          label='frozen pmf')
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



Check accuracy of cdf and ppf:

```
>>> prob = nbinom.cdf(x, n, p)
>>> np.allclose(x, nbinom.ppf(prob, n, p))
True
```

Generate random numbers:

```
>>> r = nbinom.rvs(n, p, size=1000)
```

### Methods

<pre>rvs(n, p, loc=0, size=1,     random_state=None) pmf(x, n, p, loc=0) logpmf(x, n, p, loc=0) cdf(x, n, p, loc=0) logcdf(x, n, p, loc=0) sf(x, n, p, loc=0)  logsf(x, n, p, loc=0) ppf(q, n, p, loc=0)  isf(q, n, p, loc=0) stats(n, p, loc=0, moments='mv')  entropy(n, p, loc=0) expect(func, args=(n, p), loc=0,        lb=None, ub=None, conditional=False) median(n, p, loc=0) mean(n, p, loc=0) var(n, p, loc=0) std(n, p, loc=0) interval(alpha, n, p, loc=0)</pre>	<p>Random variates.</p> <p>Probability mass function.</p> <p>Log of the probability mass function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
--	---

`scipy.stats.planck = <scipy.stats._discrete_distns.planck_gen object at 0x2b909bdac5d0>`

A Planck discrete exponential random variable.

As an instance of the `rv_discrete` class, `planck` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability mass function for `planck` is:

```
planck.pmf(k) = (1-exp(-lambda_)) * exp(-lambda_*k)
```

for  $k * \text{lambda\_} \geq 0$ .

`planck` takes `lambda_` as shape parameter.

The probability mass function above is defined in the “standardized” form. To shift distribution use the `loc` parameter. Specifically, `planck.pmf(k, lambda_, loc)` is identically equivalent to `planck.pmf(k - loc, lambda_)`.

### Examples

```
>>> from scipy.stats import planck
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> lambda_ = 0.51
>>> mean, var, skew, kurt = planck.stats(lambda_, moments='mvsk')
```

Display the probability mass function (pmf):

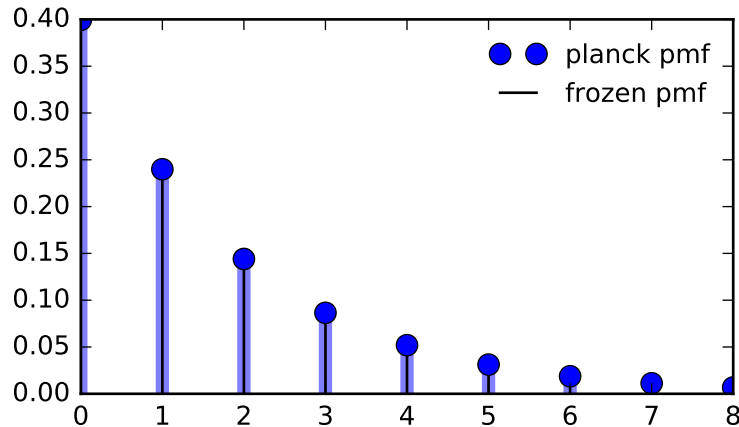
```
>>> x = np.arange(planck.ppf(0.01, lambda_),
...               planck.ppf(0.99, lambda_))
>>> ax.plot(x, planck.pmf(x, lambda_), 'bo', ms=8, label='planck pmf')
>>> ax.vlines(x, 0, planck.pmf(x, lambda_), colors='b', lw=5, alpha=0.5)
```

Alternatively, the distribution object can be called (as a function) to fix the shape and location. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pmf:

```
>>> rv = planck(lambda_)
>>> ax.vlines(x, 0, rv.pmf(x), colors='k', linestyle='-', lw=1,
...         label='frozen pmf')
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```





Check accuracy of cdf and ppf:

```
>>> prob = planck.cdf(x, lambda_)
>>> np.allclose(x, planck.ppf(prob, lambda_))
True
```

Generate random numbers:

```
>>> r = planck.rvs(lambda_, size=1000)
```

### Methods

<pre>rvs(lambda_, loc=0, size=1,     random_state=None) pmf(x, lambda_, loc=0) logpmf(x, lambda_, loc=0) cdf(x, lambda_, loc=0) logcdf(x, lambda_, loc=0) sf(x, lambda_, loc=0)  logsf(x, lambda_, loc=0) ppf(q, lambda_, loc=0)  isf(q, lambda_, loc=0) stats(lambda_, loc=0, moments='mv')  entropy(lambda_, loc=0) expect(func, args=(lambda_,), loc=0,     lb=None, ub=None, conditional=False) median(lambda_, loc=0) mean(lambda_, loc=0) var(lambda_, loc=0) std(lambda_, loc=0) interval(alpha, lambda_, loc=0)</pre>	<p>Random variates.</p> <p>Probability mass function.</p> <p>Log of the probability mass function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	---

```
scipy.stats.poisson = <scipy.stats._discrete_distns.poisson_gen object at 0x2b909bdac290>
```

A Poisson discrete random variable.

As an instance of the `rv_discrete` class, `poisson` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability mass function for `poisson` is:

```
poisson.pmf(k) = exp(-mu) * mu**k / k!
```

for  $k \geq 0$ .

`poisson` takes `mu` as shape parameter.

The probability mass function above is defined in the “standardized” form. To shift distribution use the `loc` parameter. Specifically, `poisson.pmf(k, mu, loc)` is identically equivalent to `poisson.pmf(k - loc, mu)`.

### Examples

```
>>> from scipy.stats import poisson
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> mu = 0.6
>>> mean, var, skew, kurt = poisson.stats(mu, moments='mvsk')
```

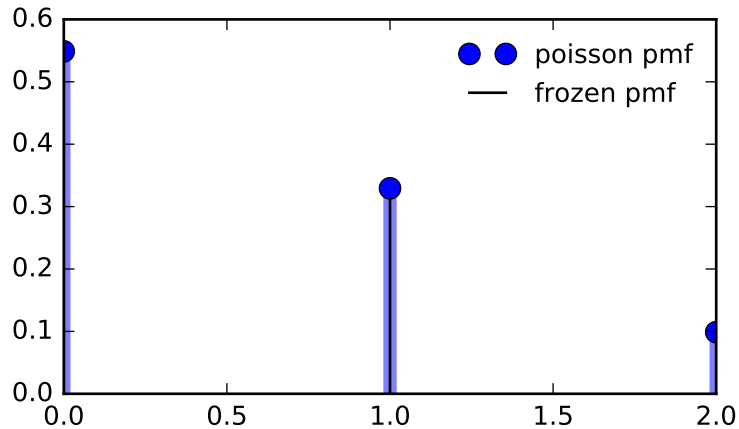
Display the probability mass function (pmf):

```
>>> x = np.arange(poisson.ppf(0.01, mu),
...               poisson.ppf(0.99, mu))
>>> ax.plot(x, poisson.pmf(x, mu), 'bo', ms=8, label='poisson pmf')
>>> ax.vlines(x, 0, poisson.pmf(x, mu), colors='b', lw=5, alpha=0.5)
```

Alternatively, the distribution object can be called (as a function) to fix the shape and location. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pmf:

```
>>> rv = poisson(mu)
>>> ax.vlines(x, 0, rv.pmf(x), colors='k', linestyle='-', lw=1,
...          label='frozen pmf')
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



Check accuracy of cdf and ppf:

```
>>> prob = poisson.cdf(x, mu)
>>> np.allclose(x, poisson.ppf(prob, mu))
True
```

Generate random numbers:

```
>>> r = poisson.rvs(mu, size=1000)
```

### Methods

<pre>rvs(mu, loc=0, size=1,     random_state=None) pmf(x, mu, loc=0) logpmf(x, mu, loc=0) cdf(x, mu, loc=0) logcdf(x, mu, loc=0) sf(x, mu, loc=0)  logsf(x, mu, loc=0) ppf(q, mu, loc=0)  isf(q, mu, loc=0) stats(mu, loc=0, moments='mv')  entropy(mu, loc=0) expect(func, args=(mu,), loc=0,        lb=None, ub=None, conditional=False) median(mu, loc=0) mean(mu, loc=0) var(mu, loc=0) std(mu, loc=0) interval(alpha, mu, loc=0)</pre>	<p>Random variates.</p> <p>Probability mass function.</p> <p>Log of the probability mass function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	---

`scipy.stats.randint` = <scipy.stats.\_discrete\_distns.randint\_gen object at 0x2b909bdaca50>

A uniform discrete random variable.

As an instance of the `rv_discrete` class, `randint` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability mass function for `randint` is:

```
randint.pmf(k) = 1./(high - low)
```

for `k = low, ..., high - 1`.

`randint` takes `low` and `high` as shape parameters.

The probability mass function above is defined in the “standardized” form. To shift distribution use the `loc` parameter. Specifically, `randint.pmf(k, low, high, loc)` is identically equivalent to `randint.pmf(k - loc, low, high)`.

### Examples

```
>>> from scipy.stats import randint
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> low, high = 7, 31
>>> mean, var, skew, kurt = randint.stats(low, high, moments='mvsk')
```

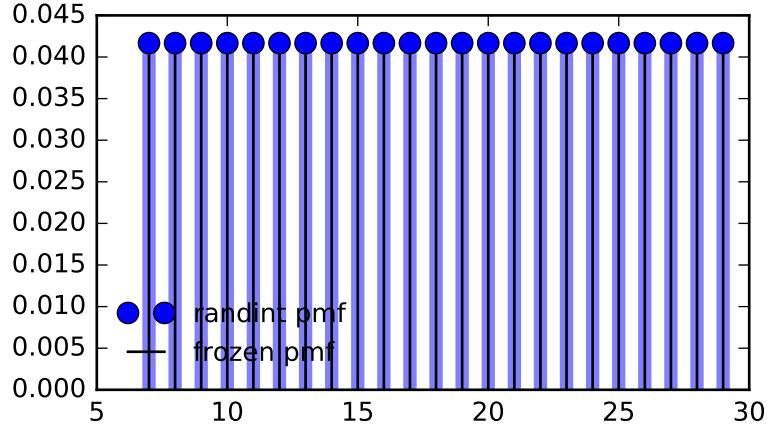
Display the probability mass function (pmf):

```
>>> x = np.arange(randint.ppf(0.01, low, high),
...               randint.ppf(0.99, low, high))
>>> ax.plot(x, randint.pmf(x, low, high), 'bo', ms=8, label='randint pmf')
>>> ax.vlines(x, 0, randint.pmf(x, low, high), colors='b', lw=5, alpha=0.5)
```

Alternatively, the distribution object can be called (as a function) to fix the shape and location. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pmf:

```
>>> rv = randint(low, high)
>>> ax.vlines(x, 0, rv.pmf(x), colors='k', linestyle='-', lw=1,
...         label='frozen pmf')
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



Check accuracy of cdf and ppf:

```
>>> prob = randint.cdf(x, low, high)
>>> np.allclose(x, randint.ppf(prob, low, high))
True
```

Generate random numbers:

```
>>> r = randint.rvs(low, high, size=1000)
```

### Methods

<pre>rvs(low, high, loc=0, size=1,     random_state=None) pmf(x, low, high, loc=0) logpmf(x, low, high, loc=0) cdf(x, low, high, loc=0) logcdf(x, low, high, loc=0) sf(x, low, high, loc=0)  logsf(x, low, high, loc=0) ppf(q, low, high, loc=0)  isf(q, low, high, loc=0) stats(low, high, loc=0, moments='mv')  entropy(low, high, loc=0) expect(func, args=(low, high), loc=0,        lb=None, ub=None, conditional=False) median(low, high, loc=0) mean(low, high, loc=0) var(low, high, loc=0) std(low, high, loc=0) interval(alpha, low, high, loc=0)</pre>	<p>Random variates.</p> <p>Probability mass function.</p> <p>Log of the probability mass function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
---	---

`scipy.stats.skellam` = <scipy.stats.discrete\_distns.skellam\_gen object at 0x2b909bdb7190>  
A Skellam discrete random variable.

As an instance of the `rv_discrete` class, `skellam` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

Probability distribution of the difference of two correlated or uncorrelated Poisson random variables.

Let  $k_1$  and  $k_2$  be two Poisson-distributed r.v. with expected values  $\lambda_1$  and  $\lambda_2$ . Then,  $k_1 - k_2$  follows a Skellam distribution with parameters  $\mu_1 = \lambda_1 - \rho \cdot \sqrt{\lambda_1 \cdot \lambda_2}$  and  $\mu_2 = \lambda_2 - \rho \cdot \sqrt{\lambda_1 \cdot \lambda_2}$ , where  $\rho$  is the correlation coefficient between  $k_1$  and  $k_2$ . If the two Poisson-distributed r.v. are independent then  $\rho = 0$ .

Parameters  $\mu_1$  and  $\mu_2$  must be strictly positive.

For details see: [http://en.wikipedia.org/wiki/Skellam\\_distribution](http://en.wikipedia.org/wiki/Skellam_distribution)

`skellam` takes  $\mu_1$  and  $\mu_2$  as shape parameters.

The probability mass function above is defined in the “standardized” form. To shift distribution use the `loc` parameter. Specifically, `skellam.pmf(k,  $\mu_1$ ,  $\mu_2$ , loc)` is identically equivalent to `skellam.pmf(k - loc,  $\mu_1$ ,  $\mu_2$ )`.

### Examples

```
>>> from scipy.stats import skellam
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> mu1, mu2 = 15, 8
>>> mean, var, skew, kurt = skellam.stats(mu1, mu2, moments='mvsk')
```

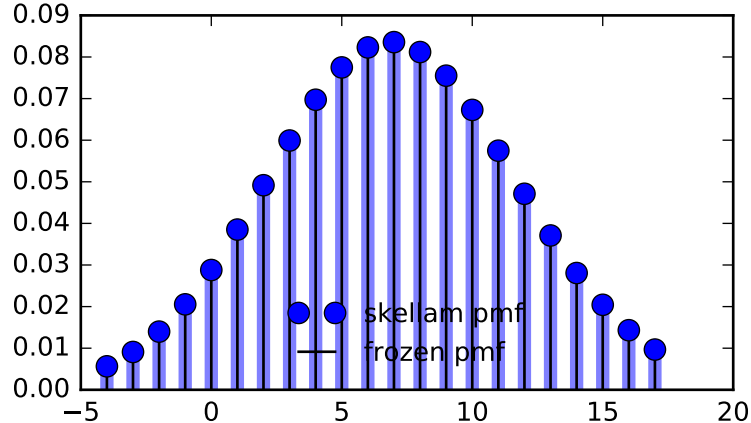
Display the probability mass function (pmf):

```
>>> x = np.arange(skellam.ppf(0.01, mu1, mu2),
...               skellam.ppf(0.99, mu1, mu2))
>>> ax.plot(x, skellam.pmf(x, mu1, mu2), 'bo', ms=8, label='skellam pmf')
>>> ax.vlines(x, 0, skellam.pmf(x, mu1, mu2), colors='b', lw=5, alpha=0.5)
```

Alternatively, the distribution object can be called (as a function) to fix the shape and location. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pmf:

```
>>> rv = skellam(mu1, mu2)
>>> ax.vlines(x, 0, rv.pmf(x), colors='k', linestyle='-', lw=1,
...          label='frozen pmf')
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```



Check accuracy of cdf and ppf:

```
>>> prob = skellam.cdf(x, mu1, mu2)
>>> np.allclose(x, skellam.ppf(prob, mu1, mu2))
True
```

Generate random numbers:

```
>>> r = skellam.rvs(mu1, mu2, size=1000)
```

### Methods

<pre>rvs(mu1, mu2, loc=0, size=1,     random_state=None) pmf(x, mu1, mu2, loc=0) logpmf(x, mu1, mu2, loc=0) cdf(x, mu1, mu2, loc=0) logcdf(x, mu1, mu2, loc=0) sf(x, mu1, mu2, loc=0)  logsf(x, mu1, mu2, loc=0) ppf(q, mu1, mu2, loc=0)  isf(q, mu1, mu2, loc=0) stats(mu1, mu2, loc=0, moments='mv')  entropy(mu1, mu2, loc=0) expect(func, args=(mu1, mu2), loc=0,        lb=None, ub=None, conditional=False) median(mu1, mu2, loc=0) mean(mu1, mu2, loc=0) var(mu1, mu2, loc=0) std(mu1, mu2, loc=0) interval(alpha, mu1, mu2, loc=0)</pre>	<p>Random variates.</p> <p>Probability mass function.</p> <p>Log of the probability mass function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
--	---

`scipy.stats.zipf` = <scipy.stats.\_discrete\_distns.zipf\_gen object at 0x2b909bdace50>

A Zipf discrete random variable.

As an instance of the `rv_discrete` class, `zipf` object inherits from it a collection of generic methods (see below for the full list), and completes them with details specific for this particular distribution.

### Notes

The probability mass function for `zipf` is:

```
zipf.pmf(k, a) = 1/(zeta(a) * k**a)
```

for  $k \geq 1$ .

`zipf` takes `a` as shape parameter.

The probability mass function above is defined in the “standardized” form. To shift distribution use the `loc` parameter. Specifically, `zipf.pmf(k, a, loc)` is identically equivalent to `zipf.pmf(k - loc, a)`.

### Examples

```
>>> from scipy.stats import zipf
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots(1, 1)
```

Calculate a few first moments:

```
>>> a = 6.5
>>> mean, var, skew, kurt = zipf.stats(a, moments='mvsk')
```

Display the probability mass function (pmf):

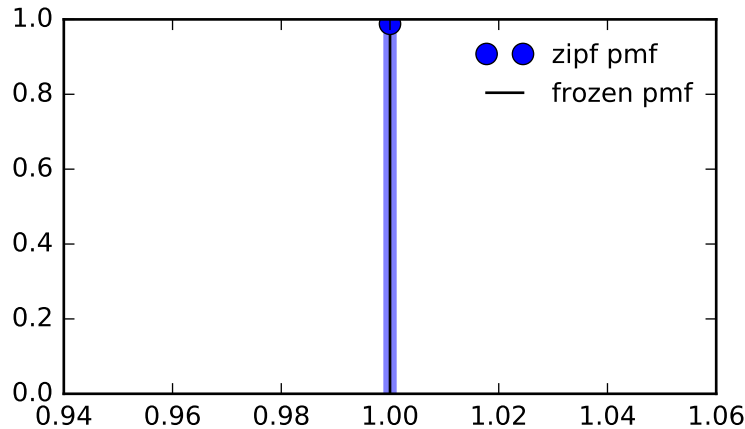
```
>>> x = np.arange(zipf.ppf(0.01, a),
...               zipf.ppf(0.99, a))
>>> ax.plot(x, zipf.pmf(x, a), 'bo', ms=8, label='zipf pmf')
>>> ax.vlines(x, 0, zipf.pmf(x, a), colors='b', lw=5, alpha=0.5)
```

Alternatively, the distribution object can be called (as a function) to fix the shape and location. This returns a “frozen” RV object holding the given parameters fixed.

Freeze the distribution and display the frozen pmf:

```
>>> rv = zipf(a)
>>> ax.vlines(x, 0, rv.pmf(x), colors='k', linestyle='-', lw=1,
...          label='frozen pmf')
>>> ax.legend(loc='best', frameon=False)
>>> plt.show()
```





Check accuracy of cdf and ppf:

```
>>> prob = zipf.cdf(x, a)
>>> np.allclose(x, zipf.ppf(prob, a))
True
```

Generate random numbers:

```
>>> r = zipf.rvs(a, size=1000)
```

### Methods

<pre>rvs(a, loc=0, size=1,     random_state=None) pmf(x, a, loc=0) logpmf(x, a, loc=0) cdf(x, a, loc=0) logcdf(x, a, loc=0) sf(x, a, loc=0)  logsf(x, a, loc=0) ppf(q, a, loc=0)  isf(q, a, loc=0) stats(a, loc=0, moments='mv')  entropy(a, loc=0) expect(func, args=(a,), loc=0,        lb=None, ub=None, conditional=False) median(a, loc=0) mean(a, loc=0) var(a, loc=0) std(a, loc=0) interval(alpha, a, loc=0)</pre>	<p>Random variates.</p> <p>Probability mass function.</p> <p>Log of the probability mass function.</p> <p>Cumulative distribution function.</p> <p>Log of the cumulative distribution function.</p> <p>Survival function (also defined as <math>1 - \text{cdf}</math>, but <i>sf</i> is sometimes more accurate).</p> <p>Log of the survival function.</p> <p>Percent point function (inverse of <i>cdf</i> — percentiles).</p> <p>Inverse survival function (inverse of <i>sf</i>).</p> <p>Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').</p> <p>(Differential) entropy of the RV.</p> <p>Expected value of a function (of one argument) with respect to the distribution.</p> <p>Median of the distribution.</p> <p>Mean of the distribution.</p> <p>Variance of the distribution.</p> <p>Standard deviation of the distribution.</p> <p>Endpoints of the range that contains alpha percent of the distribution</p>
--	---

## 5.27.4 Statistical functions

Several of these functions have a similar version in `scipy.stats.mstats` which work for masked arrays.

<code>describe(a[, axis, ddof, bias, nan_policy])</code>	Computes several descriptive statistics of the passed array.
<code>gmean(a[, axis, dtype])</code>	Compute the geometric mean along the specified axis.
<code>hmean(a[, axis, dtype])</code>	Calculates the harmonic mean along the specified axis.
<code>kurtosis(a[, axis, fisher, bias, nan_policy])</code>	Computes the kurtosis (Fisher or Pearson) of a dataset.
<code>kurtosistest(a[, axis, nan_policy])</code>	Tests whether a dataset has normal kurtosis
<code>mode(a[, axis, nan_policy])</code>	Returns an array of the modal (most common) value in the passed array.
<code>moment(a[, moment, axis, nan_policy])</code>	Calculates the nth moment about the mean for a sample.
<code>normaltest(a[, axis, nan_policy])</code>	Tests whether a sample differs from a normal distribution.
<code>skew(a[, axis, bias, nan_policy])</code>	Computes the skewness of a data set.
<code>skewtest(a[, axis, nan_policy])</code>	Tests whether the skew is different from the normal distribution.
<code>kstat(data[, n])</code>	Return the nth k-statistic ( $1 \leq n \leq 4$ so far).
<code>kstatvar(data[, n])</code>	Returns an unbiased estimator of the variance of the k-statistic.
<code>tmean(a[, limits, inclusive, axis])</code>	Compute the trimmed mean.
<code>tvar(a[, limits, inclusive, axis, ddof])</code>	Compute the trimmed variance
<code>tmin(a[, lowerlimit, axis, inclusive, ...])</code>	Compute the trimmed minimum
<code>tmax(a[, upperlimit, axis, inclusive, ...])</code>	Compute the trimmed maximum
<code>tstd(a[, limits, inclusive, axis, ddof])</code>	Compute the trimmed sample standard deviation
<code>tsem(a[, limits, inclusive, axis, ddof])</code>	Compute the trimmed standard error of the mean.
<code>variation(a[, axis, nan_policy])</code>	Computes the coefficient of variation, the ratio of the biased standard deviation to the
<code>find_repeats(arr)</code>	Find repeats and repeat counts.
<code>trim_mean(a, proportiontocut[, axis])</code>	Return mean of array after trimming distribution from both tails.

`scipy.stats.describe(a, axis=0, ddof=1, bias=True, nan_policy='propagate')`

Computes several descriptive statistics of the passed array.

**Parameters** **a** : array\_like

Input data.

**axis** : int or None, optional

Axis along which statistics are calculated. Default is 0. If None, compute over the whole array *a*.

**ddof** : int, optional

Delta degrees of freedom (only for variance). Default is 1.

**bias** : bool, optional

If False, then the skewness and kurtosis calculations are corrected for statistical bias.

**nan\_policy** : { 'propagate', 'raise', 'omit' }, optional

Defines how to handle when input contains nan. 'propagate' returns nan, 'raise' throws an error, 'omit' performs the calculations ignoring nan values. Default is 'propagate'.

**Returns**

**nobs** : int

Number of observations (length of data along *axis*).

**minmax**: tuple of ndarrays or floats

Minimum and maximum value of data array.

**mean** : ndarray or float

Arithmetic mean of data along axis.

**variance** : ndarray or float

Unbiased variance of the data along axis, denominator is number of observations minus one.

**skewness** : ndarray or float

Skewness, based on moment calculations with denominator equal to the number of observations, i.e. no degrees of freedom correction.

**kurtosis** : ndarray or float

Kurtosis (Fisher). The kurtosis is normalized so that it is zero for the normal distribution. No degrees of freedom are used.

**See also:**

`skew`, `kurtosis`

**Examples**

```
>>> from scipy import stats
>>> a = np.arange(10)
>>> stats.describe(a)
DescribeResult(nobs=10, minmax=(0, 9), mean=4.5, variance=9.1666666666666661,
               skewness=0.0, kurtosis=-1.2242424242424244)
>>> b = [[1, 2], [3, 4]]
>>> stats.describe(b)
DescribeResult(nobs=2, minmax=(array([1, 2]), array([3, 4])),
               mean=array([ 2.,  3.]), variance=array([ 2.,  2.]),
               skewness=array([ 0.,  0.]), kurtosis=array([-2., -2.]))
```

`scipy.stats.gmean(a, axis=0, dtype=None)`

Compute the geometric mean along the specified axis.

Returns the geometric average of the array elements. That is:  $n$ -th root of  $(x_1 * x_2 * \dots * x_n)$

**Parameters** **a** : array\_like

Input array or object that can be converted to an array.

**axis** : int or None, optional

Axis along which the geometric mean is computed. Default is 0. If None, compute over the whole array *a*.

**dtype** : dtype, optional

Type of the returned array and of the accumulator in which the elements are summed. If dtype is not specified, it defaults to the dtype of *a*, unless *a* has an integer dtype with a precision less than that of the default platform integer. In that case, the default platform integer is used.

**Returns** **gmean** : ndarray

see dtype parameter above

**See also:**

`numpy.mean` Arithmetic average

`numpy.average`

Weighted average

`hmean`

Harmonic mean

**Notes**

The geometric average is computed over a single dimension of the input array, `axis=0` by default, or all values in the array if `axis=None`. float64 intermediate and return values are used for integer inputs.

Use masked arrays to ignore any non-finite values in the input or that arise in the calculations such as Not a Number and infinity because masked arrays automatically mask any non-finite values.

`scipy.stats.hmean(a, axis=0, dtype=None)`

Calculates the harmonic mean along the specified axis.

That is:  $n / (1/x_1 + 1/x_2 + \dots + 1/x_n)$

**Parameters**

- a** : array\_like  
Input array, masked array or object that can be converted to an array.
- axis** : int or None, optional  
Axis along which the harmonic mean is computed. Default is 0. If None, compute over the whole array *a*.
- dtype** : dtype, optional  
Type of the returned array and of the accumulator in which the elements are summed. If *dtype* is not specified, it defaults to the dtype of *a*, unless *a* has an integer *dtype* with a precision less than that of the default platform integer. In that case, the default platform integer is used.

**Returns**

- hmean** : ndarray  
see *dtype* parameter above

**See also:**

[\*numpy.mean\*](#) Arithmetic average  
[\*numpy.average\*](#) Weighted average  
[\*gmean\*](#) Geometric mean

**Notes**

The harmonic mean is computed over a single dimension of the input array, `axis=0` by default, or all values in the array if `axis=None`. float64 intermediate and return values are used for integer inputs.

Use masked arrays to ignore any non-finite values in the input or that arise in the calculations such as Not a Number and infinity.

`scipy.stats.kurtosis` (*a*, `axis=0`, `fisher=True`, `bias=True`, `nan_policy='propagate'`)

Computes the kurtosis (Fisher or Pearson) of a dataset.

Kurtosis is the fourth central moment divided by the square of the variance. If Fisher's definition is used, then 3.0 is subtracted from the result to give 0.0 for a normal distribution.

If `bias` is False then the kurtosis is calculated using k statistics to eliminate bias coming from biased moment estimators

Use `kurtosistest` to see if result is close enough to normal.

**Parameters**

- a** : array  
data for which the kurtosis is calculated
- axis** : int or None, optional  
Axis along which the kurtosis is calculated. Default is 0. If None, compute over the whole array *a*.
- fisher** : bool, optional  
If True, Fisher's definition is used (normal ==> 0.0). If False, Pearson's definition is used (normal ==> 3.0).
- bias** : bool, optional  
If False, then the calculations are corrected for statistical bias.
- nan\_policy** : { 'propagate', 'raise', 'omit' }, optional  
Defines how to handle when input contains nan. 'propagate' returns nan, 'raise' throws an error, 'omit' performs the calculations ignoring nan values. Default is 'propagate'.

**Returns**

- kurtosis** : array  
The kurtosis of values along an axis. If all values are equal, return -3 for Fisher's definition and 0 for Pearson's definition.

**References**

[R535]

`scipy.stats.kurtosistest` (*a*, *axis=0*, *nan\_policy='propagate'*)

Tests whether a dataset has normal kurtosis

This function tests the null hypothesis that the kurtosis of the population from which the sample was drawn is that of the normal distribution:  $kurtosis = 3(n-1)/(n+1)$ .

**Parameters**

- a** : array  
array of the sample data
- axis** : int or None, optional  
Axis along which to compute test. Default is 0. If None, compute over the whole array *a*.
- nan\_policy** : {'propagate', 'raise', 'omit'}, optional  
Defines how to handle when input contains nan. 'propagate' returns nan, 'raise' throws an error, 'omit' performs the calculations ignoring nan values. Default is 'propagate'.

**Returns**

- statistic** : float  
The computed z-score for this test.
- pvalue** : float  
The 2-sided p-value for the hypothesis test

#### Notes

Valid only for  $n > 20$ . The Z-score is set to 0 for bad entries. This function uses the method described in [R536].

#### References

[R536]

`scipy.stats.mode` (*a*, *axis=0*, *nan\_policy='propagate'*)

Returns an array of the modal (most common) value in the passed array.

If there is more than one such value, only the first is returned. The bin-count for the modal bins is also returned.

**Parameters**

- a** : array\_like  
n-dimensional array of which to find mode(s).
- axis** : int or None, optional  
Axis along which to operate. Default is 0. If None, compute over the whole array *a*.
- nan\_policy** : {'propagate', 'raise', 'omit'}, optional  
Defines how to handle when input contains nan. 'propagate' returns nan, 'raise' throws an error, 'omit' performs the calculations ignoring nan values. Default is 'propagate'.

**Returns**

- mode** : ndarray  
Array of modal values.
- count** : ndarray  
Array of counts for each mode.

#### Examples

```
>>> a = np.array([[6, 8, 3, 0],
...              [3, 2, 1, 7],
...              [8, 1, 8, 4],
...              [5, 3, 0, 5],
...              [4, 7, 5, 9]])
>>> from scipy import stats
>>> stats.mode(a)
(array([[3, 1, 0, 0]]), array([[1, 1, 1, 1]]))
```

To get mode of whole array, specify `axis=None`:

```
>>> stats.mode(a, axis=None)
(array([3]), array([3]))
```

`scipy.stats.moment` (*a*, *moment=1*, *axis=0*, *nan\_policy='propagate'*)

Calculates the *n*th moment about the mean for a sample.

A moment is a specific quantitative measure of the shape of a set of points. It is often used to calculate coefficients of skewness and kurtosis due to its close relationship with them.

**Parameters**

- a** : array\_like  
data
- moment** : int or array\_like of ints, optional  
order of central moment that is returned. Default is 1.
- axis** : int or None, optional  
Axis along which the central moment is computed. Default is 0. If None, compute over the whole array *a*.
- nan\_policy** : { 'propagate', 'raise', 'omit' }, optional  
Defines how to handle when input contains nan. 'propagate' returns nan, 'raise' throws an error, 'omit' performs the calculations ignoring nan values. Default is 'propagate'.

**Returns**

- n-th central moment** : ndarray or float  
The appropriate moment along the given axis or over all values if axis is None. The denominator for the moment calculation is the number of observations, no degrees of freedom correction is done.

**See also:**

[kurtosis](#), [skew](#), [describe](#)

**Notes**

The *k*-th central moment of a data sample is:

$$m_k = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^k$$

Where *n* is the number of samples and  $\bar{x}$  is the mean. This function uses exponentiation by squares [R543] for efficiency.

**References**

[R543]

`scipy.stats.normaltest` (*a*, *axis=0*, *nan\_policy='propagate'*)

Tests whether a sample differs from a normal distribution.

This function tests the null hypothesis that a sample comes from a normal distribution. It is based on D'Agostino and Pearson's [R548], [R549] test that combines skew and kurtosis to produce an omnibus test of normality.

**Parameters**

- a** : array\_like  
The array containing the data to be tested.
- axis** : int or None, optional  
Axis along which to compute test. Default is 0. If None, compute over the whole array *a*.
- nan\_policy** : { 'propagate', 'raise', 'omit' }, optional  
Defines how to handle when input contains nan. 'propagate' returns nan, 'raise' throws an error, 'omit' performs the calculations ignoring nan values. Default is 'propagate'.

**Returns**

- statistic** : float or array

$s^2 + k^2$ , where  $s$  is the z-score returned by `skewtest` and  $k$  is the z-score returned by `kurtosistest`.

**pvalue** : float or array

A 2-sided chi squared probability for the hypothesis test.

### References

[R548], [R549]

`scipy.stats.skew` (*a*, *axis=0*, *bias=True*, *nan\_policy='propagate'*)

Computes the skewness of a data set.

For normally distributed data, the skewness should be about 0. A skewness value  $> 0$  means that there is more weight in the left tail of the distribution. The function `skewtest` can be used to determine if the skewness value is close enough to 0, statistically speaking.

**Parameters** **a** : ndarray

data

**axis** : int or None, optional

Axis along which skewness is calculated. Default is 0. If None, compute over the whole array *a*.

**bias** : bool, optional

If False, then the calculations are corrected for statistical bias.

**nan\_policy** : { 'propagate', 'raise', 'omit' }, optional

Defines how to handle when input contains nan. 'propagate' returns nan, 'raise' throws an error, 'omit' performs the calculations ignoring nan values. Default is 'propagate'.

**Returns**

**skewness** : ndarray

The skewness of values along an axis, returning 0 where all values are equal.

### References

[R569]

`scipy.stats.skewtest` (*a*, *axis=0*, *nan\_policy='propagate'*)

Tests whether the skew is different from the normal distribution.

This function tests the null hypothesis that the skewness of the population that the sample was drawn from is the same as that of a corresponding normal distribution.

**Parameters** **a** : array

The data to be tested

**axis** : int or None, optional

Axis along which statistics are calculated. Default is 0. If None, compute over the whole array *a*.

**nan\_policy** : { 'propagate', 'raise', 'omit' }, optional

Defines how to handle when input contains nan. 'propagate' returns nan, 'raise' throws an error, 'omit' performs the calculations ignoring nan values. Default is 'propagate'.

**Returns**

**statistic** : float

The computed z-score for this test.

**pvalue** : float

a 2-sided p-value for the hypothesis test

### Notes

The sample size must be at least 8.

`scipy.stats.kstat` (*data*, *n=2*)

Return the *n*th k-statistic ( $1 \leq n \leq 4$  so far).

The nth k-statistic `k_n` is the unique symmetric unbiased estimator of the nth cumulant `kappa_n`.

**Parameters** `data` : array\_like  
 Input array. Note that n-D input gets flattened.  
`n` : int, {1, 2, 3, 4}, optional  
 Default is equal to 2.  
**Returns** `kstat` : float  
 The nth k-statistic.

See also:

`kstatvar` Returns an unbiased estimator of the variance of the k-statistic.  
`moment` Returns the n-th central moment about the mean for a sample.

Notes

For a sample size `n`, the first few k-statistics are given by:

$$k_1 = \mu, k_2 = \frac{n}{n-1} m_2, k_3 = \frac{n^2}{(n-1)(n-2)} m_3, k_4 = \frac{n^2[(n+1)m_4 - 3(n-1)m_2^2]}{(n-1)(n-2)(n-3)}$$

where `math:\mu` is the sample mean, `math:m_2` is the sample variance, and `math:m_i` is the i-th sample central moment.

References

<http://mathworld.wolfram.com/k-Statistic.html>  
<http://mathworld.wolfram.com/Cumulant.html>

Examples

```
>>> from scipy import stats
>>> rndm = np.random.RandomState(1234)
```

As sample size increases, n-th moment and n-th k-statistic converge to the same number (although they aren't identical). In the case of the normal distribution, they converge to zero.

```
>>> for n in [2, 3, 4, 5, 6, 7]:
...     x = rndm.normal(size=10**n)
...     m, k = stats.moment(x, 3), stats.kstat(x, 3)
...     print("%.3g %.3g %.3g" % (m, k, m-k))
-0.631 -0.651 0.0194
0.0282 0.0283 -8.49e-05
-0.0454 -0.0454 1.36e-05
7.53e-05 7.53e-05 -2.26e-09
0.00166 0.00166 -4.99e-09
-2.88e-06 -2.88e-06 8.63e-13
```

`scipy.stats.kstatvar` (`data`, `n=2`)

Returns an unbiased estimator of the variance of the k-statistic.

See `kstat` for more details of the k-statistic.

**Parameters** `data` : array\_like  
 Input array. Note that n-D input gets flattened.  
`n` : int, {1, 2}, optional  
 Default is equal to 2.  
**Returns** `kstatvar` : float  
 The nth k-statistic variance.

See also:



**`kstat`** Returns the n-th k-statistic.  
**`moment`** Returns the n-th central moment about the mean for a sample.

**Notes**

The variances of the first few k-statistics are given by:

$$\text{var}(k_1) = \frac{\kappa^2}{n} \text{var}(k_2) = \frac{\kappa^4}{n} + \frac{2\kappa_2^2}{n-1} \text{var}(k_3) = \frac{\kappa^6}{n} + \frac{9\kappa_2\kappa_4}{n-1} + \frac{9\kappa_3^2}{n-1} + \frac{6n\kappa_2^3}{(n-1)(n-2)} \text{var}(k_4) = \frac{\kappa^8}{n} + \frac{16\kappa_2\kappa_6}{n-1} + \frac{48\kappa_3\kappa_5}{n-1}$$

`scipy.stats.tmean(a, limits=None, inclusive=(True, True), axis=None)`

Compute the trimmed mean.

This function finds the arithmetic mean of given values, ignoring values outside the given *limits*.

**Parameters**

- a** : array\_like  
Array of values.
- limits** : None or (lower limit, upper limit), optional  
Values in the input array less than the lower limit or greater than the upper limit will be ignored. When *limits* is None (default), then all values are used. Either of the limit values in the tuple can also be None representing a half-open interval.
- inclusive** : (bool, bool), optional  
A tuple consisting of the (lower flag, upper flag). These flags determine whether values exactly equal to the lower or upper limits are included. The default value is (True, True).
- axis** : int or None, optional  
Axis along which to compute test. Default is None.

**Returns**

- tmean** : float

**See also:**

`trim_mean` returns mean after trimming a proportion from both tails.

**Examples**

```
>>> from scipy import stats
>>> x = np.arange(20)
>>> stats.tmean(x)
9.5
>>> stats.tmean(x, (3, 17))
10.0
```

`scipy.stats.tvar(a, limits=None, inclusive=(True, True), axis=0, ddof=1)`

Compute the trimmed variance

This function computes the sample variance of an array of values, while ignoring values which are outside of given *limits*.

**Parameters**

- a** : array\_like  
Array of values.
- limits** : None or (lower limit, upper limit), optional  
Values in the input array less than the lower limit or greater than the upper limit will be ignored. When *limits* is None, then all values are used. Either of the limit values in the tuple can also be None representing a half-open interval. The default value is None.
- inclusive** : (bool, bool), optional

A tuple consisting of the (lower flag, upper flag). These flags determine whether values exactly equal to the lower or upper limits are included. The default value is (True, True).

**axis** : int or None, optional  
Axis along which to operate. Default is 0. If None, compute over the whole array *a*.

**ddof** : int, optional  
Delta degrees of freedom. Default is 1.

**Returns** **tvar** : float  
Trimmed variance.

**Notes**

`tvar` computes the unbiased sample variance, i.e. it uses a correction factor  $n / (n - 1)$ .

**Examples**

```
>>> from scipy import stats
>>> x = np.arange(20)
>>> stats.tvar(x)
35.0
>>> stats.tvar(x, (3,17))
20.0
```

`scipy.stats.tmin(a, lowerlimit=None, axis=0, inclusive=True, nan_policy='propagate')`

Compute the trimmed minimum

This function finds the minimum value of an array *a* along the specified axis, but only considering values greater than a specified lower limit.

**Parameters** **a** : array\_like  
array of values

**lowerlimit** : None or float, optional  
Values in the input array less than the given limit will be ignored. When lowerlimit is None, then all values are used. The default value is None.

**axis** : int or None, optional  
Axis along which to operate. Default is 0. If None, compute over the whole array *a*.

**inclusive** : {True, False}, optional  
This flag determines whether values exactly equal to the lower limit are included. The default value is True.

**nan\_policy** : {'propagate', 'raise', 'omit'}, optional  
Defines how to handle when input contains nan. 'propagate' returns nan, 'raise' throws an error, 'omit' performs the calculations ignoring nan values. Default is 'propagate'.

**Returns** **tmin** : float, int or ndarray

**Examples**

```
>>> from scipy import stats
>>> x = np.arange(20)
>>> stats.tmin(x)
0

>>> stats.tmin(x, 13)
13
```

```
>>> stats.tmin(x, 13, inclusive=False)
14
```

`scipy.stats.tmax` (*a*, *upperlimit=None*, *axis=0*, *inclusive=True*, *nan\_policy='propagate'*)  
 Compute the trimmed maximum

This function computes the maximum value of an array along a given axis, while ignoring values larger than a specified upper limit.

**Parameters**

- a** : array\_like  
array of values
- upperlimit** : None or float, optional  
Values in the input array greater than the given limit will be ignored. When upperlimit is None, then all values are used. The default value is None.
- axis** : int or None, optional  
Axis along which to operate. Default is 0. If None, compute over the whole array *a*.
- inclusive** : {True, False}, optional  
This flag determines whether values exactly equal to the upper limit are included. The default value is True.
- nan\_policy** : {'propagate', 'raise', 'omit'}, optional  
Defines how to handle when input contains nan. 'propagate' returns nan, 'raise' throws an error, 'omit' performs the calculations ignoring nan values. Default is 'propagate'.

**Returns**

- tmax** : float, int or ndarray

### Examples

```
>>> from scipy import stats
>>> x = np.arange(20)
>>> stats.tmax(x)
19

>>> stats.tmax(x, 13)
13

>>> stats.tmax(x, 13, inclusive=False)
12
```

`scipy.stats.tstd` (*a*, *limits=None*, *inclusive=(True, True)*, *axis=0*, *ddof=1*)  
 Compute the trimmed sample standard deviation

This function finds the sample standard deviation of given values, ignoring values outside the given *limits*.

**Parameters**

- a** : array\_like  
array of values
- limits** : None or (lower limit, upper limit), optional  
Values in the input array less than the lower limit or greater than the upper limit will be ignored. When limits is None, then all values are used. Either of the limit values in the tuple can also be None representing a half-open interval. The default value is None.
- inclusive** : (bool, bool), optional  
A tuple consisting of the (lower flag, upper flag). These flags determine whether values exactly equal to the lower or upper limits are included. The default value is (True, True).
- axis** : int or None, optional

Axis along which to operate. Default is 0. If None, compute over the whole array *a*.

**ddof** : int, optional  
Delta degrees of freedom. Default is 1.

**Returns** **tstd** : float

**Notes**

`tstd` computes the unbiased sample standard deviation, i.e. it uses a correction factor  $n / (n - 1)$ .

**Examples**

```
>>> from scipy import stats
>>> x = np.arange(20)
>>> stats.tstd(x)
5.9160797830996161
>>> stats.tstd(x, (3,17))
4.4721359549995796
```

`scipy.stats.tsem(a, limits=None, inclusive=(True, True), axis=0, ddof=1)`  
Compute the trimmed standard error of the mean.

This function finds the standard error of the mean for given values, ignoring values outside the given *limits*.

**Parameters** **a** : array\_like  
array of values

**limits** : None or (lower limit, upper limit), optional  
Values in the input array less than the lower limit or greater than the upper limit will be ignored. When *limits* is None, then all values are used. Either of the limit values in the tuple can also be None representing a half-open interval. The default value is None.

**inclusive** : (bool, bool), optional  
A tuple consisting of the (lower flag, upper flag). These flags determine whether values exactly equal to the lower or upper limits are included. The default value is (True, True).

**axis** : int or None, optional  
Axis along which to operate. Default is 0. If None, compute over the whole array *a*.

**ddof** : int, optional  
Delta degrees of freedom. Default is 1.

**Returns** **tsem** : float

**Notes**

`tsem` uses unbiased sample standard deviation, i.e. it uses a correction factor  $n / (n - 1)$ .

**Examples**

```
>>> from scipy import stats
>>> x = np.arange(20)
>>> stats.tsem(x)
1.3228756555322954
>>> stats.tsem(x, (3,17))
1.1547005383792515
```

`scipy.stats.variation(a, axis=0, nan_policy='propagate')`  
Computes the coefficient of variation, the ratio of the biased standard deviation to the mean.

**Parameters** **a** : array\_like

Input array.  
**axis** : int or None, optional  
 Axis along which to calculate the coefficient of variation. Default is 0. If None, compute over the whole array *a*.  
**nan\_policy** : {'propagate', 'raise', 'omit'}, optional  
 Defines how to handle when input contains nan. 'propagate' returns nan, 'raise' throws an error, 'omit' performs the calculations ignoring nan values. Default is 'propagate'.  
**Returns** **variation** : ndarray  
 The calculated variation along the requested axis.

### References

[R580]

`scipy.stats.find_repeats(arr)`  
 Find repeats and repeat counts.

**Parameters** **arr** : array\_like  
 Input array. This is cast to float64.  
**Returns** **values** : ndarray  
 The unique values from the (flattened) input that are repeated.  
**counts** : ndarray  
 Number of times the corresponding 'value' is repeated.

### Notes

In numpy >= 1.9 `numpy.unique` provides similar functionality. The main difference is that `find_repeats` only returns repeated values.

### Examples

```
>>> from scipy import stats
>>> stats.find_repeats([2, 1, 2, 3, 2, 2, 5])
RepeatedResults(values=array([ 2.]), counts=array([4]))

>>> stats.find_repeats([[10, 20, 1, 2], [5, 5, 4, 4]])
RepeatedResults(values=array([ 4.,  5.]), counts=array([2, 2]))
```

`scipy.stats.trim_mean(a, proportiontocut, axis=0)`  
 Return mean of array after trimming distribution from both tails.

If `proportiontocut = 0.1`, slices off 'leftmost' and 'rightmost' 10% of scores. The input is sorted before slicing. Slices off less if proportion results in a non-integer slice index (i.e., conservatively slices off `proportiontocut`).

**Parameters** **a** : array\_like  
 Input array  
**proportiontocut** : float  
 Fraction to cut off of both tails of the distribution  
**axis** : int or None, optional  
 Axis along which the trimmed means are computed. Default is 0. If None, compute over the whole array *a*.  
**Returns** **trim\_mean** : ndarray  
 Mean of trimmed array.

### See also:

`trimboth`  
`tmean` compute the trimmed mean ignoring values outside given *limits*.

*Examples*

```
>>> from scipy import stats
>>> x = np.arange(20)
>>> stats.trim_mean(x, 0.1)
9.5
>>> x2 = x.reshape(5, 4)
>>> x2
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11],
       [12, 13, 14, 15],
       [16, 17, 18, 19]])
>>> stats.trim_mean(x2, 0.25)
array([ 8.,  9., 10., 11.])
>>> stats.trim_mean(x2, 0.25, axis=1)
array([ 1.5,  5.5,  9.5, 13.5, 17.5])
```

---

<code>cumfreq(a[, numbins, defaultreallimits, weights])</code>	Returns a cumulative frequency histogram, using the histogram function.
<code>histogram2(*args, **kwargs)</code>	<code>histogram2</code> is deprecated!
<code>histogram(*args, **kwargs)</code>	<code>histogram</code> is deprecated!
<code>itemfreq(a)</code>	Returns a 2-D array of item frequencies.
<code>percentileofscore(a, score[, kind])</code>	The percentile rank of a score relative to a list of scores.
<code>scoreatpercentile(a, per[, limit, ...])</code>	Calculate the score at a given percentile of the input sequence.
<code>relfreq(a[, numbins, defaultreallimits, weights])</code>	Returns a relative frequency histogram, using the histogram function.

---

`scipy.stats.cumfreq(a, numbins=10, defaultreallimits=None, weights=None)`

Returns a cumulative frequency histogram, using the histogram function.

A cumulative histogram is a mapping that counts the cumulative number of observations in all of the bins up to the specified bin.

**Parameters**

- a** : array\_like  
Input array.
- numbins** : int, optional  
The number of bins to use for the histogram. Default is 10.
- defaultreallimits** : tuple (lower, upper), optional  
The lower and upper values for the range of the histogram. If no value is given, a range slightly larger than the range of the values in *a* is used. Specifically  $(a.min() - s, a.max() + s)$ , where  $s = (1/2)(a.max() - a.min()) / (numbins - 1)$ .
- weights** : array\_like, optional  
The weights for each value in *a*. Default is None, which gives each value a weight of 1.0

**Returns**

- cumcount** : ndarray  
Binned values of cumulative frequency.
- lowerlimit** : float  
Lower real limit
- binsize** : float  
Width of each bin.
- extrapoints** : int  
Extra points.

### Examples

```
>>> import matplotlib.pyplot as plt
>>> from scipy import stats
>>> x = [1, 4, 2, 1, 3, 1]
>>> res = stats.cumfreq(x, numbins=4, defaultreallimits=(1.5, 5))
>>> res.cumcount
array([ 1.,  2.,  3.,  3.])
>>> res.extrapoints
3
```

Create a normal distribution with 1000 random values

```
>>> rng = np.random.RandomState(seed=12345)
>>> samples = stats.norm.rvs(size=1000, random_state=rng)
```

Calculate cumulative frequencies

```
>>> res = stats.cumfreq(samples, numbins=25)
```

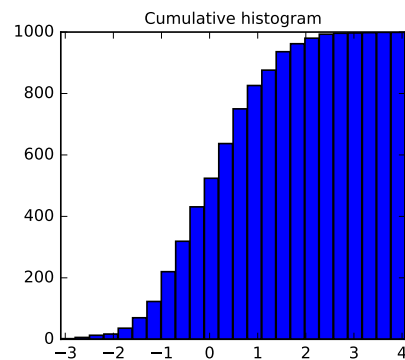
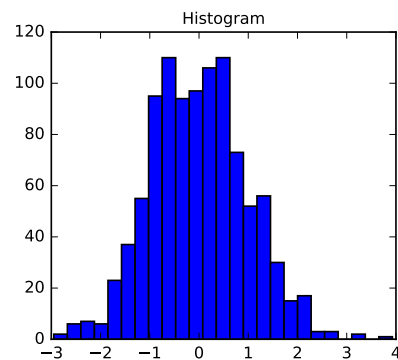
Calculate space of values for x

```
>>> x = res.lowerlimit + np.linspace(0, res.binsize*res.cumcount.size,
...                                 res.cumcount.size)
```

Plot histogram and cumulative histogram

```
>>> fig = plt.figure(figsize=(10, 4))
>>> ax1 = fig.add_subplot(1, 2, 1)
>>> ax2 = fig.add_subplot(1, 2, 2)
>>> ax1.hist(samples, bins=25)
>>> ax1.set_title('Histogram')
>>> ax2.bar(x, res.cumcount, width=res.binsize)
>>> ax2.set_title('Cumulative histogram')
>>> ax2.set_xlim([x.min(), x.max()])

>>> plt.show()
```



```
scipy.stats.histogram2(*args, **kwargs)
```

`histogram2` is deprecated! `scipy.stats.histogram2` is deprecated in scipy 0.16.0; use `np.histogram2d` instead

Compute histogram using divisions in bins.

Count the number of times values from array *a* fall into numerical ranges defined by *bins*. Range *x* is given by  $\text{bins}[x] \leq \text{range}_x < \text{bins}[x+1]$  where  $x = 0, N$  and *N* is the length of the *bins* array. The last range is given by  $\text{bins}[N] \leq \text{range}_N < \text{infinity}$ . Values less than  $\text{bins}[0]$  are not included in the histogram.

**Parameters** **a** : array\_like of rank 1

The array of values to be assigned into bins

**bins**

[array\_like of rank 1] Defines the ranges of values to use during histogramming.

**Returns** **histogram2** : ndarray of rank 1

Each value represents the occurrences for a given bin (range) of values.

`scipy.stats.histogram(*args, **kwargs)`

`histogram` is deprecated! `scipy.stats.histogram` is deprecated in scipy 0.17.0; use `np.histogram` instead

`scipy.stats.itemfreq(a)`

Returns a 2-D array of item frequencies.

**Parameters** **a** : (N,) array\_like

**Returns** **itemfreq** : (K, 2) ndarray

Input array.  
A 2-D frequency table. Column 1 contains sorted, unique values from *a*, column 2 contains their respective counts.

### Examples

```
>>> from scipy import stats
>>> a = np.array([1, 1, 5, 0, 1, 2, 2, 0, 1, 4])
>>> stats.itemfreq(a)
array([[ 0.,  2.],
       [ 1.,  4.],
       [ 2.,  2.],
       [ 4.,  1.],
       [ 5.,  1.]])
>>> np.bincount(a)
array([2, 4, 2, 0, 1, 1])

>>> stats.itemfreq(a/10.)
array([[ 0. ,  2. ],
       [ 0.1,  4. ],
       [ 0.2,  2. ],
       [ 0.4,  1. ],
       [ 0.5,  1. ]])
```

`scipy.stats.percentileofscore(a, score, kind='rank')`

The percentile rank of a score relative to a list of scores.

A `percentileofscore` of, for example, 80% means that 80% of the scores in *a* are below the given score. In the case of gaps or ties, the exact definition depends on the optional keyword, *kind*.

**Parameters** **a** : array\_like

Array of scores to which *score* is compared.

**score** : int or float

Score that is compared to the elements in *a*.

**kind** : {'rank', 'weak', 'strict', 'mean'}, optional

This optional parameter specifies the interpretation of the resulting score:



- **“rank”**: Average percentage ranking of score. In case of multiple matches, average the percentage rankings of all matching scores.
- **“weak”**: This kind corresponds to the definition of a cumulative distribution function. A percentileofscore of 80% means that 80% of values are less than or equal to the provided score.
- **“strict”**: Similar to “weak”, except that only values that are strictly less than the given score are counted.
- **“mean”**: The average of the “weak” and “strict” scores, often used in

testing. See

[http://en.wikipedia.org/wiki/Percentile\\_rank](http://en.wikipedia.org/wiki/Percentile_rank)

**Returns** `pcos` : float

Percentile-position of score (0-100) relative to *a*.

**See also:**

`numpy.percentile`

**Examples**

Three-quarters of the given values lie below a given score:

```
>>> from scipy import stats
>>> stats.percentileofscore([1, 2, 3, 4], 3)
75.0
```

With multiple matches, note how the scores of the two matches, 0.6 and 0.8 respectively, are averaged:

```
>>> stats.percentileofscore([1, 2, 3, 3, 4], 3)
70.0
```

Only 2/5 values are strictly less than 3:

```
>>> stats.percentileofscore([1, 2, 3, 3, 4], 3, kind='strict')
40.0
```

But 4/5 values are less than or equal to 3:

```
>>> stats.percentileofscore([1, 2, 3, 3, 4], 3, kind='weak')
80.0
```

The average between the weak and the strict scores is

```
>>> stats.percentileofscore([1, 2, 3, 3, 4], 3, kind='mean')
60.0
```

`scipy.stats.scoreatpercentile` (*a*, *per*, *limit*=(), *interpolation\_method*='fraction', *axis*=None)

Calculate the score at a given percentile of the input sequence.

For example, the score at *per*=50 is the median. If the desired quantile lies between two data points, we interpolate between them, according to the value of *interpolation*. If the parameter *limit* is provided, it should be a tuple (lower, upper) of two values.

**Parameters** `a` : array\_like

A 1-D array of values from which to extract score.

`per` : array\_like

Percentile(s) at which to extract score. Values should be in range [0,100].

**limit** : tuple, optional  
 Tuple of two scalars, the lower and upper limits within which to compute the percentile. Values of *a* outside this (closed) interval will be ignored.

**interpolation\_method** : {'fraction', 'lower', 'higher'}, optional  
 This optional parameter specifies the interpolation method to use, when the desired quantile lies between two data points *i* and *j*

- fraction:  $i + (j - i) * \text{fraction}$  where *fraction* is the fractional part of the index surrounded by *i* and *j*.
- lower: *i*.
- higher: *j*.

**axis** : int, optional  
 Axis along which the percentiles are computed. Default is None. If None, compute over the whole array *a*.

**Returns** **score** : float or ndarray  
 Score at percentile(s).

**See also:**

[percentileofscore](#), [numpy.percentile](#)

**Notes**

This function will become obsolete in the future. For Numpy 1.9 and higher, [numpy.percentile](#) provides all the functionality that [scoreatpercentile](#) provides. And it's significantly faster. Therefore it's recommended to use [numpy.percentile](#) for users that have numpy >= 1.9.

**Examples**

```
>>> from scipy import stats
>>> a = np.arange(100)
>>> stats.scoreatpercentile(a, 50)
49.5
```

`scipy.stats.relfreq(a, numbins=10, defaultreallimits=None, weights=None)`

Returns a relative frequency histogram, using the histogram function.

A relative frequency histogram is a mapping of the number of observations in each of the bins relative to the total of observations.

**Parameters** **a** : array\_like  
 Input array.

**numbins** : int, optional  
 The number of bins to use for the histogram. Default is 10.

**defaultreallimits** : tuple (lower, upper), optional  
 The lower and upper values for the range of the histogram. If no value is given, a range slightly larger than the range of the values in *a* is used. Specifically  $(a.min() - s, a.max() + s)$ , where  $s = (1/2)(a.max() - a.min()) / (numbins - 1)$ .

**weights** : array\_like, optional  
 The weights for each value in *a*. Default is None, which gives each value a weight of 1.0

**Returns** **frequency** : ndarray  
 Binned values of relative frequency.

**lowerlimit** : float  
 Lower real limit

**binsize** : float  
 Width of each bin.

**extrapoints** : int  
Extra points.

### Examples

```
>>> import matplotlib.pyplot as plt
>>> from scipy import stats
>>> a = np.array([2, 4, 1, 2, 3, 2])
>>> res = stats.relfreq(a, numbins=4)
>>> res.frequency
array([ 0.16666667, 0.5          , 0.16666667, 0.16666667])
>>> np.sum(res.frequency) # relative frequencies should add up to 1
1.0
```

Create a normal distribution with 1000 random values

```
>>> rng = np.random.RandomState(seed=12345)
>>> samples = stats.norm.rvs(size=1000, random_state=rng)
```

Calculate relative frequencies

```
>>> res = stats.relfreq(samples, numbins=25)
```

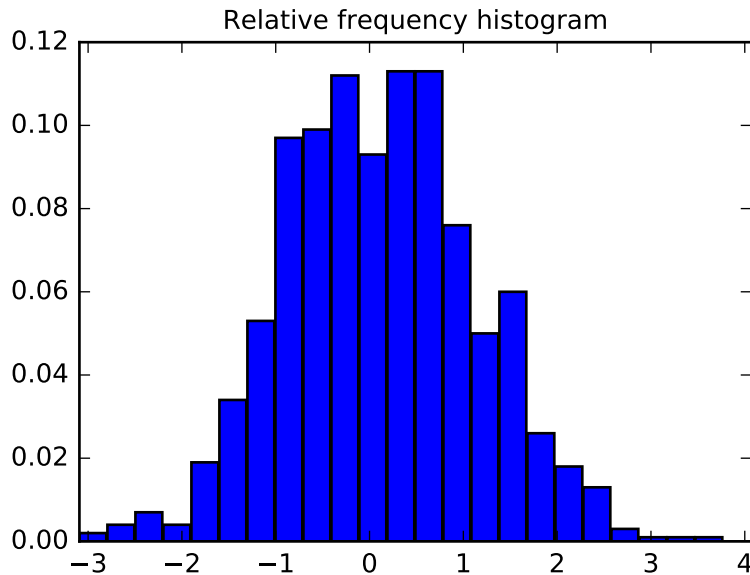
Calculate space of values for x

```
>>> x = res.lowerlimit + np.linspace(0, res.binsize*res.frequency.size,
...                                 res.frequency.size)
```

Plot relative frequency histogram

```
>>> fig = plt.figure(figsize=(5, 4))
>>> ax = fig.add_subplot(1, 1, 1)
>>> ax.bar(x, res.frequency, width=res.binsize)
>>> ax.set_title('Relative frequency histogram')
>>> ax.set_xlim([x.min(), x.max()])

>>> plt.show()
```




---

<code>binned_statistic(x, values[, statistic, ...])</code>	Compute a binned statistic for one or more sets of data.
<code>binned_statistic_2d(x, y, values[, ...])</code>	Compute a bidimensional binned statistic for one or more sets of data.
<code>binned_statistic_dd(sample, values[, ...])</code>	Compute a multidimensional binned statistic for a set of data.

---

`scipy.stats.binned_statistic(x, values, statistic='mean', bins=10, range=None)`

Compute a binned statistic for one or more sets of data.

This is a generalization of a histogram function. A histogram divides the space into bins, and returns the count of the number of points in each bin. This function allows the computation of the sum, mean, median, or other statistic of the values (or set of values) within each bin.

**Parameters** `x` : (N,) array\_like

A sequence of values to be binned.

**values** : (N,) array\_like or list of (N,) array\_like

The data on which the statistic will be computed. This must be the same shape as `x`, or a set of sequences - each the same shape as `x`. If `values` is a set of sequences, the statistic will be computed on each independently.

**statistic** : string or callable, optional

The statistic to compute (default is 'mean'). The following statistics are available:

- 'mean' : compute the mean of values for points within each bin. Empty bins will be represented by NaN.
- 'median' : compute the median of values for points within each bin. Empty bins will be represented by NaN.
- 'count' : compute the count of points within each bin. This is identical to an unweighted histogram. `values` array is not referenced.
- 'sum' : compute the sum of values for points within each bin. This is identical to a weighted histogram.
- 'function' : a user-defined function which takes a 1D array of values, and outputs a single numerical statistic. This func-

tion will be called on the values in each bin. Empty bins will be represented by function(`[]`), or NaN if this returns an error.

**bins** : int or sequence of scalars, optional

If *bins* is an int, it defines the number of equal-width bins in the given range (10 by default). If *bins* is a sequence, it defines the bin edges, including the rightmost edge, allowing for non-uniform bin widths. Values in *x* that are smaller than lowest bin edge are assigned to bin number 0, values beyond the highest bin are assigned to `bins[-1]`. If the bin edges are specified, the number of bins will be,  $(nx = \text{len}(\text{bins})-1)$ .

**range** : (float, float) or [(float, float)], optional

The lower and upper range of the bins. If not provided, range is simply  $(x.\text{min}(), x.\text{max}())$ . Values outside the range are ignored.

**Returns**

**statistic** : array

The values of the selected statistic in each bin.

**bin\_edges** : array of dtype float

Return the bin edges  $(\text{length}(\text{statistic})+1)$ .

**binnumber**: 1-D ndarray of ints

Indices of the bins (corresponding to *bin\_edges*) in which each value of *x* belongs. Same length as *values*. A binnumber of *i* means the corresponding value is between  $(\text{bin\_edges}[i-1], \text{bin\_edges}[i])$ .

**See also:**

`numpy.digitize`, `numpy.histogram`, `binned_statistic_2d`, `binned_statistic_dd`

**Notes**

All but the last (righthand-most) bin is half-open. In other words, if *bins* is `[1, 2, 3, 4]`, then the first bin is `[1, 2)` (including 1, but excluding 2) and the second `[2, 3)`. The last bin, however, is `[3, 4]`, which includes 4.

New in version 0.11.0.

**Examples**

```
>>> from scipy import stats
>>> import matplotlib.pyplot as plt
```

First some basic examples:

Create two evenly spaced bins in the range of the given sample, and sum the corresponding values in each of those bins:

```
>>> values = [1.0, 1.0, 2.0, 1.5, 3.0]
>>> stats.binned_statistic([1, 1, 2, 5, 7], values, 'sum', bins=2)
(array([ 4. ,  4.5]), array([ 1.,  4.,  7.]), array([1, 1, 1, 2, 2]))
```

Multiple arrays of values can also be passed. The statistic is calculated on each set independently:

```
>>> values = [[1.0, 1.0, 2.0, 1.5, 3.0], [2.0, 2.0, 4.0, 3.0, 6.0]]
>>> stats.binned_statistic([1, 1, 2, 5, 7], values, 'sum', bins=2)
(array([[ 4. ,  4.5], [ 8. ,  9. ]]), array([ 1.,  4.,  7.]),
 array([1, 1, 1, 2, 2]))
```

```
>>> stats.binned_statistic([1, 2, 1, 2, 4], np.arange(5), statistic='mean',
...                         bins=3)
(array([ 1.,  2.,  4.]), array([ 1.,  2.,  3.,  4.]),
 array([1, 2, 1, 2, 3]))
```

As a second example, we now generate some random data of sailing boat speed as a function of wind speed, and then determine how fast our boat is for certain wind speeds:

```
>>> windspeed = 8 * np.random.rand(500)
>>> boatspeed = .3 * windspeed**.5 + .2 * np.random.rand(500)
>>> bin_means, bin_edges, binnumber = stats.binned_statistic(windspeed,
...                 boatspeed, statistic='median', bins=[1,2,3,4,5,6,7])
>>> plt.figure()
>>> plt.plot(windspeed, boatspeed, 'b.', label='raw data')
>>> plt.hlines(bin_means, bin_edges[:-1], bin_edges[1:], colors='g', lw=5,
...           label='binned statistic of data')
>>> plt.legend()
```

Now we can use `binnumber` to select all datapoints with a windspeed below 1:

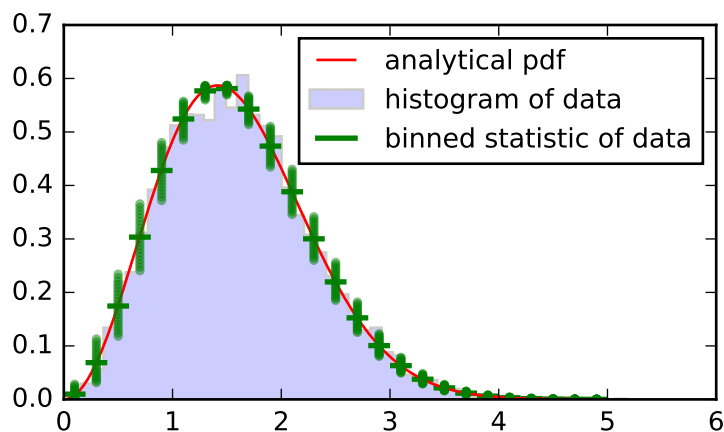
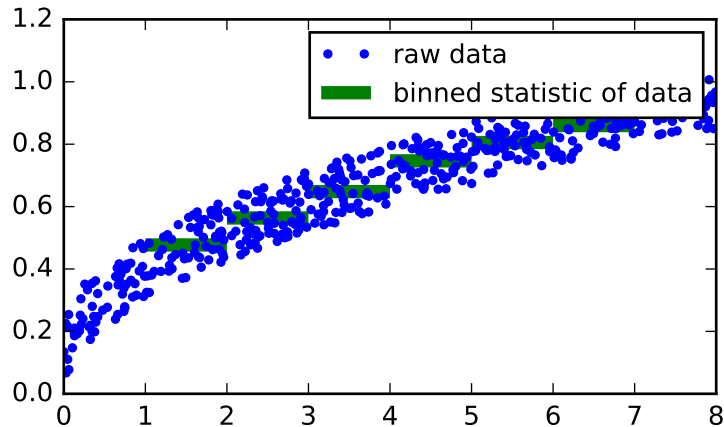
```
>>> low_boatspeed = boatspeed[binnumber == 0]
```

As a final example, we will use `bin_edges` and `binnumber` to make a plot of a distribution that shows the mean and distribution around that mean per bin, on top of a regular histogram and the probability distribution function:

```
>>> x = np.linspace(0, 5, num=500)
>>> x_pdf = stats.maxwell.pdf(x)
>>> samples = stats.maxwell.rvs(size=10000)

>>> bin_means, bin_edges, binnumber = stats.binned_statistic(x, x_pdf,
...                 statistic='mean', bins=25)
>>> bin_width = (bin_edges[1] - bin_edges[0])
>>> bin_centers = bin_edges[1:] - bin_width/2

>>> plt.figure()
>>> plt.hist(samples, bins=50, normed=True, histtype='stepfilled',
...         alpha=0.2, label='histogram of data')
>>> plt.plot(x, x_pdf, 'r-', label='analytical pdf')
>>> plt.hlines(bin_means, bin_edges[:-1], bin_edges[1:], colors='g', lw=2,
...           label='binned statistic of data')
>>> plt.plot((binnumber - 0.5) * bin_width, x_pdf, 'g.', alpha=0.5)
>>> plt.legend(fontsize=10)
>>> plt.show()
```



```
scipy.stats.binned_statistic_2d(x, y, values, statistic='mean', bins=10, range=None, expand_binnumbers=False)
```

Compute a bidimensional binned statistic for one or more sets of data.

This is a generalization of a `histogram2d` function. A histogram divides the space into bins, and returns the count of the number of points in each bin. This function allows the computation of the sum, mean, median, or other statistic of the values (or set of values) within each bin.

**Parameters** `x` : (N,) array\_like

A sequence of values to be binned along the first dimension.

`y` : (N,) array\_like

A sequence of values to be binned along the second dimension.

**values** : (N,) array\_like or list of (N,) array\_like

The data on which the statistic will be computed. This must be the same shape as `x`, or a list of sequences - each with the same shape as `x`. If `values` is such a list, the statistic will be computed on each independently.

**statistic** : string or callable, optional

The statistic to compute (default is ‘mean’). The following statistics are available:

- ‘mean’ : compute the mean of values for points within each bin. Empty bins will be represented by NaN.
- ‘median’ : compute the median of values for points within each bin. Empty bins will be represented by NaN.
- ‘count’ : compute the count of points within each bin. This is identical to an unweighted histogram. *values* array is not referenced.
- ‘sum’ : compute the sum of values for points within each bin. This is identical to a weighted histogram.
- *function* : a user-defined function which takes a 1D array of values, and outputs a single numerical statistic. This function will be called on the values in each bin. Empty bins will be represented by function([]), or NaN if this returns an error.

**bins** : int or [int, int] or array\_like or [array, array], optional

The bin specification:

- the number of bins for the two dimensions ( $nx = ny = bins$ ),
- the number of bins in each dimension ( $nx, ny = bins$ ),
- the bin edges for the two dimensions ( $x\_edge = y\_edge = bins$ ),
- the bin edges in each dimension ( $x\_edge, y\_edge = bins$ ).

If the bin edges are specified, the number of bins will be, ( $nx = len(x\_edge) - 1$ ,  $ny = len(y\_edge) - 1$ ).

**range** : (2,2) array\_like, optional

The leftmost and rightmost edges of the bins along each dimension (if not specified explicitly in the *bins* parameters):  $[[xmin, xmax], [ymin, ymax]]$ . All values outside of this range will be considered outliers and not tallied in the histogram.

**expand\_binnumbers** : bool, optional

‘False’ (default): the returned *binnumber* is a shape (N,) array of linearized bin indices. ‘True’: the returned *binnumber* is ‘unraveled’ into a shape (2,N) ndarray, where each row gives the bin numbers in the corresponding dimension. See the *binnumber* returned value, and the *Examples* section.

**Returns**

**statistic** : (nx, ny) ndarray

The values of the selected statistic in each two-dimensional bin.

**x\_edge** : (nx + 1) ndarray

The bin edges along the first dimension.

**y\_edge** : (ny + 1) ndarray

The bin edges along the second dimension.

**binnumber** : (N,) array of ints or (2,N) ndarray of ints

This assigns to each element of *sample* an integer that represents the bin in which this observation falls. The representation depends on the *expand\_binnumbers* argument. See *Notes* for details.

**See also:**

`numpy.digitize`, `numpy.histogram2d`, `binned_statistic`, `binned_statistic_dd`

**Notes**

**Binedges:** All but the last (righthand-most) bin is half-open. In other words, if *bins* is  $[1, 2, 3, 4]$ , then the first bin is  $[1, 2)$  (including 1, but excluding 2) and the second  $[2, 3)$ . The last bin, however, is  $[3, 4]$ , which *includes* 4.

**binnumber:** This returned argument assigns to each element of *sample* an integer that represents the bin in which it belongs. The representation depends on the *expand\_binnumbers* argument. If ‘False’ (default): The returned *binnumber* is a shape (N,) array of linearized indices mapping each element of *sample* to its corresponding



bin (using row-major ordering). If ‘True’: The returned *binnumber* is a shape (2,N) ndarray where each row indicates bin placements for each dimension respectively. In each dimension, a binnumber of *i* means the corresponding value is between (D\_edge[i-1], D\_edge[i]), where ‘D’ is either ‘x’ or ‘y’.

New in version 0.11.0.

### Examples

```
>>> from scipy import stats
```

Calculate the counts with explicit bin-edges:

```
>>> x = [0.1, 0.1, 0.1, 0.6]
>>> y = [2.1, 2.6, 2.1, 2.1]
>>> binx = [0.0, 0.5, 1.0]
>>> biny = [2.0, 2.5, 3.0]
>>> ret = stats.binned_statistic_2d(x, y, None, 'count', bins=[binx,biny])
>>> ret.statistic
array([[ 2.,  1.],
       [ 1.,  0.]])
```

The bin in which each sample is placed is given by the *binnumber* returned parameter. By default, these are the linearized bin indices:

```
>>> ret.binnumber
array([5, 6, 5, 9])
```

The bin indices can also be expanded into separate entries for each dimension using the *expand\_binnumbers* parameter:

```
>>> ret = stats.binned_statistic_2d(x, y, None, 'count', bins=[binx,biny],
...                                 expand_binnumbers=True)
>>> ret.binnumber
array([[1, 1, 1, 2],
       [1, 2, 1, 1]])
```

Which shows that the first three elements belong in the xbin 1, and the fourth into xbin 2; and so on for y.

`scipy.stats.binned_statistic_dd`(*sample*, *values*, *statistic*='mean', *bins*=10, *range*=None, *expand\_binnumbers*=False)

Compute a multidimensional binned statistic for a set of data.

This is a generalization of a histogramdd function. A histogram divides the space into bins, and returns the count of the number of points in each bin. This function allows the computation of the sum, mean, median, or other statistic of the values within each bin.

**Parameters** **sample** : array\_like

Data to histogram passed as a sequence of D arrays of length N, or as an (N,D) array.

**values** : (N,) array\_like or list of (N,) array\_like

The data on which the statistic will be computed. This must be the same shape as *x*, or a list of sequences - each with the same shape as *x*. If *values* is such a list, the statistic will be computed on each independently.

**statistic** : string or callable, optional

The statistic to compute (default is ‘mean’). The following statistics are available:

- ‘mean’ : compute the mean of values for points within each bin. Empty bins will be represented by NaN.

- ‘median’ : compute the median of values for points within each bin. Empty bins will be represented by NaN.
- ‘count’ : compute the count of points within each bin. This is identical to an unweighted histogram. *values* array is not referenced.
- ‘sum’ : compute the sum of values for points within each bin. This is identical to a weighted histogram.
- ‘function’ : a user-defined function which takes a 1D array of values, and outputs a single numerical statistic. This function will be called on the values in each bin. Empty bins will be represented by function([]), or NaN if this returns an error.

**bins** : sequence or int, optional

The bin specification must be in one of the following forms:

- A sequence of arrays describing the bin edges along each dimension.
- The number of bins for each dimension ( $nx, ny, \dots = bins$ ).
- The number of bins for all dimensions ( $nx = ny = \dots = bins$ ).

**range** : sequence, optional

A sequence of lower and upper bin edges to be used if the edges are not given explicitly in *bins*. Defaults to the minimum and maximum values along each dimension.

**expand\_binnumbers** : bool, optional

‘False’ (default): the returned *binnumber* is a shape (N,) array of linearized bin indices. ‘True’: the returned *binnumber* is ‘unraveled’ into a shape (D,N) ndarray, where each row gives the bin numbers in the corresponding dimension. See the *binnumber* returned value, and the *Examples* section of [binned\\_statistic\\_2d](#).

**Returns**

**statistic** : ndarray, shape( $nx1, nx2, nx3, \dots$ )

The values of the selected statistic in each two-dimensional bin.

**bin\_edges** : list of ndarrays

A list of D arrays describing the  $(nxi + 1)$  bin edges for each dimension.

**binnumber** : (N,) array of ints or (D,N) ndarray of ints

This assigns to each element of *sample* an integer that represents the bin in which this observation falls. The representation depends on the *expand\_binnumbers* argument. See *Notes* for details.

**See also:**

`numpy.digitize`, `numpy.histogramdd`, `binned_statistic`, `binned_statistic_2d`

**Notes**

**Binedges:** All but the last (righthand-most) bin is half-open in each dimension. In other words, if *bins* is [1, 2, 3, 4], then the first bin is [1, 2) (including 1, but excluding 2) and the second [2, 3). The last bin, however, is [3, 4], which *includes* 4.

**binnumber:** This returned argument assigns to each element of *sample* an integer that represents the bin in which it belongs. The representation depends on the *expand\_binnumbers* argument. If ‘False’ (default): The returned *binnumber* is a shape (N,) array of linearized indices mapping each element of *sample* to its corresponding bin (using row-major ordering). If ‘True’: The returned *binnumber* is a shape (D,N) ndarray where each row indicates bin placements for each dimension respectively. In each dimension, a binnumber of *i* means the corresponding value is between (bin\_edges[D][i-1], bin\_edges[D][i]), for each dimension ‘D’.

New in version 0.11.0.

<code>obrienttransform(*args)</code>	Computes the O’Brien transform on input data (any number of arrays).
<code>signaltonoise(*args, **kwds)</code>	<code>signaltonoise</code> is deprecated!

Continu

Table 5.282 – continued from previous page

<code>bayes_mvs(data[, alpha])</code>	Bayesian confidence intervals for the mean, var, and std.
<code>mvstdist(data)</code>	'Frozen' distributions for mean, variance, and standard deviation of data.
<code>sem(a[, axis, ddof, nan_policy])</code>	Calculates the standard error of the mean (or standard error of measurement) of the values
<code>zmap(scores, compare[, axis, ddof])</code>	Calculates the relative z-scores.
<code>zscore(a[, axis, ddof])</code>	Calculates the z score of each value in the sample, relative to the sample mean and standard deviation.
<code>iqr(x[, axis, rng, scale, nan_policy, ...])</code>	Compute the interquartile range of the data along the specified axis.

`scipy.stats.obrientransform(*args)`

Computes the O'Brien transform on input data (any number of arrays).

Used to test for homogeneity of variance prior to running one-way stats. Each array in `*args` is one level of a factor. If `f_oneway` is run on the transformed data and found significant, the variances are unequal. From Maxwell and Delaney [R550], p.112.

**Parameters** `args` : tuple of array\_like

**Returns** `obrientransform` : ndarray

Any number of arrays.  
Transformed data for use in an ANOVA. The first dimension of the result corresponds to the sequence of transformed arrays. If the arrays given are all 1-D of the same length, the return value is a 2-D array; otherwise it is a 1-D array of type object, with each element being an ndarray.

### References

[R550]

### Examples

We'll test the following data sets for differences in their variance.

```
>>> x = [10, 11, 13, 9, 7, 12, 12, 9, 10]
>>> y = [13, 21, 5, 10, 8, 14, 10, 12, 7, 15]
```

Apply the O'Brien transform to the data.

```
>>> from scipy.stats import obrientransform
>>> tx, ty = obrientransform(x, y)
```

Use `scipy.stats.f_oneway` to apply a one-way ANOVA test to the transformed data.

```
>>> from scipy.stats import f_oneway
>>> F, p = f_oneway(tx, ty)
>>> p
0.1314139477040335
```

If we require that  $p < 0.05$  for significance, we cannot conclude that the variances are different.

`scipy.stats.signaltonoise(*args, **kwargs)`

`signaltonoise` is deprecated! `scipy.stats.signaltonoise` is deprecated in scipy 0.16.0

The signal-to-noise ratio of the input data.

Returns the signal-to-noise ratio of `a`, here defined as the mean divided by the standard deviation.

**Parameters** `a` : array\_like

An array\_like object containing the sample data.

**axis** [int or None, optional] Axis along which to operate. Default is 0. If None, compute over the whole array.  
**ddof** [int, optional] Degrees of freedom correction for standard deviation. Default is 0.

**Returns** **s2n** : ndarray  
 The mean to standard deviation ratio(s) along *axis*, or 0 where the standard deviation is 0.

`scipy.stats.bayes_mvs` (*data*, *alpha*=0.9)

Bayesian confidence intervals for the mean, var, and std.

**Parameters** **data** : array\_like  
 Input data, if multi-dimensional it is flattened to 1-D by `bayes_mvs`. Requires 2 or more data points.

**alpha** : float, optional

**Returns** **mean\_cntr, var\_cntr, std\_cntr** : tuple  
 Probability that the returned confidence interval contains the true parameter. The three results are for the mean, variance and standard deviation, respectively. Each result is a tuple of the form:

`(center, (lower, upper))`

with *center* the mean of the conditional pdf of the value given the data, and (*lower, upper*) a confidence interval, centered on the median, containing the estimate to a probability *alpha*.

**See also:**

`mvsgdist`

**Notes**

Each tuple of mean, variance, and standard deviation estimates represent the (*center, (lower, upper)*) with *center* the mean of the conditional pdf of the value given the data and (*lower, upper*) is a confidence interval centered on the median, containing the estimate to a probability *alpha*.

Converts data to 1-D and assumes all data has the same mean and variance. Uses Jeffrey’s prior for variance and std.

Equivalent to `tuple((x.mean(), x.interval(alpha)) for x in mvsgdist(dat))`

**References**

T.E. Oliphant, “A Bayesian perspective on estimating mean, variance, and standard-deviation from data”, <http://scholarsarchive.byu.edu/facpub/278>, 2006.

**Examples**

First a basic example to demonstrate the outputs:

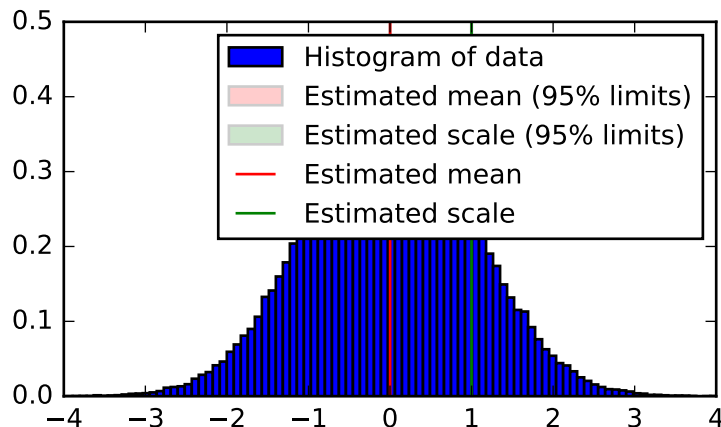
```
>>> from scipy import stats
>>> data = [6, 9, 12, 7, 8, 8, 13]
>>> mean, var, std = stats.bayes_mvs(data)
>>> mean
Mean(statistic=9.0, minmax=(7.1036502226125329, 10.896349777387467))
>>> var
Variance(statistic=10.0, minmax=(3.176724206..., 24.45910382...))
>>> std
Std_dev(statistic=2.9724954732045084, minmax=(1.7823367265645143, 4.9456146050146295))
```

Now we generate some normally distributed random data, and get estimates of mean and standard deviation with 95% confidence intervals for those estimates:

```
>>> n_samples = 100000
>>> data = stats.norm.rvs(size=n_samples)
>>> res_mean, res_var, res_std = stats.bayes_mvs(data, alpha=0.95)

>>> import matplotlib.pyplot as plt
>>> fig = plt.figure()
>>> ax = fig.add_subplot(111)
>>> ax.hist(data, bins=100, normed=True, label='Histogram of data')
>>> ax.vlines(res_mean.statistic, 0, 0.5, colors='r', label='Estimated mean')
>>> ax.axvspan(res_mean.minmax[0], res_mean.minmax[1], facecolor='r',
...           alpha=0.2, label=r'Estimated mean (95% limits)')
>>> ax.vlines(res_std.statistic, 0, 0.5, colors='g', label='Estimated scale')
>>> ax.axvspan(res_std.minmax[0], res_std.minmax[1], facecolor='g', alpha=0.2,
...           label=r'Estimated scale (95% limits)')

>>> ax.legend(fontsize=10)
>>> ax.set_xlim([-4, 4])
>>> ax.set_ylim([0, 0.5])
>>> plt.show()
```



`scipy.stats.mvstdist` (*data*)

‘Frozen’ distributions for mean, variance, and standard deviation of data.

**Parameters** **data** : array\_like

Input array. Converted to 1-D using `ravel`. Requires 2 or more data-points.

**Returns** **mdist** : “frozen” distribution object  
Distribution object representing the mean of the data

**vdist** : “frozen” distribution object  
Distribution object representing the variance of the data

**sdist** : “frozen” distribution object  
Distribution object representing the standard deviation of the data

**See also:**

`bayes_mvs`

### Notes

The return values from `bayes_mvs(data)` is equivalent to `tuple((x.mean(), x.interval(0.90)) for x in mvstdist(data))`.

In other words, calling `<dist>.mean()` and `<dist>.interval(0.90)` on the three distribution objects returned from this function will give the same results that are returned from `bayes_mvs`.

### References

T.E. Oliphant, “A Bayesian perspective on estimating mean, variance, and standard-deviation from data”, <http://scholarsarchive.byu.edu/facpub/278>, 2006.

### Examples

```
>>> from scipy import stats
>>> data = [6, 9, 12, 7, 8, 8, 13]
>>> mean, var, std = stats.mvstdist(data)
```

We now have frozen distribution objects “mean”, “var” and “std” that we can examine:

```
>>> mean.mean()
9.0
>>> mean.interval(0.95)
(6.6120585482655692, 11.387941451734431)
>>> mean.std()
1.1952286093343936
```

`scipy.stats.sem(a, axis=0, ddof=1, nan_policy='propagate')`

Calculates the standard error of the mean (or standard error of measurement) of the values in the input array.

**Parameters** **a** : array\_like

An array containing the values for which the standard error is returned.

**axis** : int or None, optional

Axis along which to operate. Default is 0. If None, compute over the whole array *a*.

**ddof** : int, optional

Delta degrees-of-freedom. How many degrees of freedom to adjust for bias in limited samples relative to the population estimate of variance. Defaults to 1.

**nan\_policy** : {'propagate', 'raise', 'omit'}, optional

Defines how to handle when input contains nan. ‘propagate’ returns nan, ‘raise’ throws an error, ‘omit’ performs the calculations ignoring nan values. Default is ‘propagate’.

**Returns** **s** : ndarray or float

The standard error of the mean in the sample(s), along the input axis.

### Notes

The default value for *ddof* is different to the default (0) used by other *ddof* containing routines, such as `np.std` and `np.nanstd`.

### Examples

Find standard error along the first axis:

```
>>> from scipy import stats
>>> a = np.arange(20).reshape(5, 4)
```

```
>>> stats.sem(a)
array([ 2.8284,  2.8284,  2.8284,  2.8284])
```

Find standard error across the whole array, using n degrees of freedom:

```
>>> stats.sem(a, axis=None, ddof=0)
1.2893796958227628
```

`scipy.stats.zmap` (*scores*, *compare*, *axis=0*, *ddof=0*)

Calculates the relative z-scores.

Returns an array of z-scores, i.e., scores that are standardized to zero mean and unit variance, where mean and variance are calculated from the comparison array.

**Parameters**

- scores** : array\_like  
The input for which z-scores are calculated.
- compare** : array\_like  
The input from which the mean and standard deviation of the normalization are taken; assumed to have the same dimension as *scores*.
- axis** : int or None, optional  
Axis over which mean and variance of *compare* are calculated. Default is 0. If None, compute over the whole array *scores*.
- ddof** : int, optional  
Degrees of freedom correction in the calculation of the standard deviation. Default is 0.

**Returns**

- zscore** : array\_like  
Z-scores, in the same shape as *scores*.

#### Notes

This function preserves ndarray subclasses, and works also with matrices and masked arrays (it uses *asanyarray* instead of *asarray* for parameters).

#### Examples

```
>>> from scipy.stats import zmap
>>> a = [0.5, 2.0, 2.5, 3]
>>> b = [0, 1, 2, 3, 4]
>>> zmap(a, b)
array([-1.06066017,  0.          ,  0.35355339,  0.70710678])
```

`scipy.stats.zscore` (*a*, *axis=0*, *ddof=0*)

Calculates the z score of each value in the sample, relative to the sample mean and standard deviation.

**Parameters**

- a** : array\_like  
An array like object containing the sample data.
- axis** : int or None, optional  
Axis along which to operate. Default is 0. If None, compute over the whole array *a*.
- ddof** : int, optional  
Degrees of freedom correction in the calculation of the standard deviation. Default is 0.

**Returns**

- zscore** : array\_like  
The z-scores, standardized by mean and standard deviation of input array *a*.

#### Notes

This function preserves ndarray subclasses, and works also with matrices and masked arrays (it uses *asanyarray* instead of *asarray* for parameters).

**Examples**

```
>>> a = np.array([ 0.7972,  0.0767,  0.4383,  0.7866,  0.8091,
...               0.1954,  0.6307,  0.6599,  0.1065,  0.0508])
>>> from scipy import stats
>>> stats.zscore(a)
array([ 1.1273, -1.247 , -0.0552,  1.0923,  1.1664, -0.8559,  0.5786,
        0.6748, -1.1488, -1.3324])
```

Computing along a specified axis, using n-1 degrees of freedom (ddof=1) to calculate the standard deviation:

```
>>> b = np.array([[ 0.3148,  0.0478,  0.6243,  0.4608],
...               [ 0.7149,  0.0775,  0.6072,  0.9656],
...               [ 0.6341,  0.1403,  0.9759,  0.4064],
...               [ 0.5918,  0.6948,  0.904 ,  0.3721],
...               [ 0.0921,  0.2481,  0.1188,  0.1366]])
>>> stats.zscore(b, axis=1, ddof=1)
array([[ -0.19264823, -1.28415119,  1.07259584,  0.40420358],
       [  0.33048416, -1.37380874,  0.04251374,  1.00081084],
       [  0.26796377, -1.12598418,  1.23283094, -0.37481053],
       [ -0.22095197,  0.24468594,  1.19042819, -1.21416216],
       [ -0.82780366,  1.4457416 , -0.43867764, -0.1792603 ]])
```

```
scipy.stats.iqr(x, axis=None, rng=(25, 75), scale='raw', nan_policy='propagate', interpolation='linear', keepdims=False)
```

Compute the interquartile range of the data along the specified axis.

The interquartile range (IQR) is the difference between the 75th and 25th percentile of the data. It is a measure of the dispersion similar to standard deviation or variance, but is much more robust against outliers [R530].

The `rng` parameter allows this function to compute other percentile ranges than the actual IQR. For example, setting `rng=(0, 100)` is equivalent to `numpy.ptp`.

The IQR of an empty array is `np.nan`.

New in version 0.18.0.

**Parameters** `x`: array\_like

Input array or object that can be converted to an array.

**axis**: int or sequence of int, optional

Axis along which the range is computed. The default is to compute the IQR for the entire array.

**rng**: Two-element sequence containing floats in range of [0,100] optional

Percentiles over which to compute the range. Each must be between 0 and 100, inclusive. The default is the true IQR: (25, 75). The order of the elements is not important.

**scale**: scalar or str, optional

The numerical value of `scale` will be divided out of the final result. The following string values are recognized:

`'raw'`: No scaling, just return the raw IQR. `'normal'`: Scale by  $2\sqrt{2}erf^{-1}(\frac{1}{2}) \approx 1.349$ .

The default is `'raw'`. Array-like `scale` is also allowed, as long as it broadcasts correctly to the output such that `out / scale` is a valid operation. The output dimensions depend on the input array, `x`, the `axis` argument, and the `keepdims` flag.

**nan\_policy**: {'propagate', 'raise', 'omit'}, optional



Defines how to handle when input contains nan. ‘propagate’ returns nan, ‘raise’ throws an error, ‘omit’ performs the calculations ignoring nan values. Default is ‘propagate’.

**interpolation** : {‘linear’, ‘lower’, ‘higher’, ‘midpoint’, ‘nearest’}, optional

Specifies the interpolation method to use when the percentile boundaries lie between two data points  $i$  and  $j$ :

- **‘linear’**       $[i + (j - i) * fraction]$ , where *fraction* is the fractional part of the index surrounded by  $i$  and  $j$ .
- **‘lower’** :  $i$ ,
- **‘higher’** :  $j$ ,
- **‘nearest’** :  $i$  or  $j$  whichever is nearest.
- **‘midpoint’** :  $(i + j) / 2$ .

Default is ‘linear’.

**keepdims** : bool, optional

If this is set to *True*, the reduced axes are left in the result as dimensions with size one. With this option, the result will broadcast correctly against the original array  $x$ .

**Returns**      **iqr** : scalar or ndarray

If *axis=None*, a scalar is returned. If the input contains integers or floats of smaller precision than `np.float64`, then the output data-type is `np.float64`. Otherwise, the output data-type is the same as that of the input.

**See also:**

`numpy.std`, `numpy.var`

**Notes**

This function is heavily dependent on the version of `numpy` that is installed. Versions greater than 1.11.0b3 are highly recommended, as they include a number of enhancements and fixes to `numpy.percentile` and `numpy.nanpercentile` that affect the operation of this function. The following modifications apply:

**Below 1.10.0** [*nan\_policy* is poorly defined.] The default behavior of `numpy.percentile` is used for ‘propagate’. This is a hybrid of ‘omit’ and ‘propagate’ that mostly yields a skewed version of ‘omit’ since NaNs are sorted to the end of the data. A warning is raised if there are NaNs in the data.

**Below 1.9.0:** `numpy.nanpercentile` does not exist.

This means that `numpy.percentile` is used regardless of *nan\_policy* and a warning is issued. See previous item for a description of the behavior.

**Below 1.9.0:** `keepdims` and `interpolation` are not supported.

The keywords get ignored with a warning if supplied with non-default values. However, multiple axes are still supported.

**References**

[R529], [R530], [R531]

**Examples**

```
>>> from scipy.stats import iqr
>>> x = np.array([[10, 7, 4], [3, 2, 1]])
>>> x
array([[10, 7, 4],
       [ 3, 2, 1]])
>>> iqr(x)
4.0
>>> iqr(x, axis=0)
array([ 3.5, 2.5, 1.5])
>>> iqr(x, axis=1)
```

```
array([ 3.,  1.])
>>> iqr(x, axis=1, keepdims=True)
array([[ 3.],
       [ 1.]])
```

---

<code>sigmaclip(a[, low, high])</code>	Iterative sigma-clipping of array elements.
<code>threshold(*args, **kwds)</code>	<code>threshold</code> is deprecated!
<code>trimboth(a, proportiontocut[, axis])</code>	Slices off a proportion of items from both ends of an array.
<code>trim1(a, proportiontocut[, tail, axis])</code>	Slices off a proportion from ONE end of the passed array distribution.

---

`scipy.stats.sigmaclip(a, low=4.0, high=4.0)`

Iterative sigma-clipping of array elements.

The output array contains only those elements of the input array *c* that satisfy the conditions

$$\text{mean}(c) - \text{std}(c) * \text{low} < c < \text{mean}(c) + \text{std}(c) * \text{high}$$

Starting from the full sample, all elements outside the critical range are removed. The iteration continues with a new critical range until no elements are outside the range.

**Parameters**

- a** : array\_like  
Data array, will be raveled if not 1-D.
- low** : float, optional  
Lower bound factor of sigma clipping. Default is 4.
- high** : float, optional  
Upper bound factor of sigma clipping. Default is 4.

**Returns**

- clipped** : ndarray  
Input array with clipped elements removed.
- lower** : float  
Lower threshold value use for clipping.
- upper** : float  
Upper threshold value use for clipping.

### Examples

```
>>> from scipy.stats import sigmaclip
>>> a = np.concatenate((np.linspace(9.5, 10.5, 31),
...                     np.linspace(0, 20, 5)))
>>> fact = 1.5
>>> c, low, upp = sigmaclip(a, fact, fact)
>>> c
array([ 9.96666667, 10.          , 10.03333333, 10.          ])
>>> c.var(), c.std()
(0.00055555555555555165, 0.023570226039551501)
>>> low, c.mean() - fact*c.std(), c.min()
(9.9646446609406727, 9.9646446609406727, 9.9666666666666668)
>>> upp, c.mean() + fact*c.std(), c.max()
(10.03535339059327, 10.03535339059327, 10.033333333333333)

>>> a = np.concatenate((np.linspace(9.5, 10.5, 11),
...                     np.linspace(-100, -50, 3)))
>>> c, low, upp = sigmaclip(a, 1.8, 1.8)
>>> (c == np.linspace(9.5, 10.5, 11)).all()
True
```

`scipy.stats.threshold(*args, **kws)`

`threshold` is deprecated! `stats.threshold` is deprecated in scipy 0.17.0

Clip array to a given value.

Similar to `numpy.clip()`, except that values less than *threshmin* or greater than *threshmax* are replaced by *newval*, instead of by *threshmin* and *threshmax* respectively.

**Parameters**    **a** : array\_like

Data to threshold.

**threshmin**    [float, int or None, optional] Minimum threshold, defaults to

**threshmax**    [float, int or None, optional] Maximum threshold, defaults

**newval**        [float or int, optional] Value to put in place of values in *a*

outside of bounds. Defaults to 0.

**Returns**        **out** : ndarray

The clipped input array, with values less than *threshmin* or greater than *threshmax* replaced with *newval*.

### Examples

```
>>> a = np.array([9, 9, 6, 3, 1, 6, 1, 0, 0, 8])
>>> from scipy import stats
>>> stats.threshold(a, threshmin=2, threshmax=8, newval=-1)
array([-1, -1,  6,  3, -1,  6, -1, -1, -1,  8])
```

`scipy.stats.trimboth(a, proportiontocut, axis=0)`

Slices off a proportion of items from both ends of an array.

Slices off the passed proportion of items from both ends of the passed array (i.e., with *proportiontocut* = 0.1, slices leftmost 10% **and** rightmost 10% of scores). The trimmed values are the lowest and highest ones. Slices off less if proportion results in a non-integer slice index (i.e., conservatively slices off ‘proportiontocut’).

**Parameters**    **a** : array\_like

Data to trim.

**proportiontocut** : float

Proportion (in range 0-1) of total data set to trim of each end.

**axis** : int or None, optional

Axis along which to trim data. Default is 0. If None, compute over the whole array *a*.

**Returns**        **out** : ndarray

Trimmed version of array *a*. The order of the trimmed content is undefined.

**See also:**

`trim_mean`

### Examples

```
>>> from scipy import stats
>>> a = np.arange(20)
>>> b = stats.trimboth(a, 0.1)
>>> b.shape
(16,)
```

`scipy.stats.trim1(a, proportiontocut, tail='right', axis=0)`

Slices off a proportion from ONE end of the passed array distribution.

If *proportiontocut* = 0.1, slices off ‘leftmost’ or ‘rightmost’ 10% of scores. The lowest or highest values are trimmed (depending on the tail). Slices off less if proportion results in a non-integer slice index (i.e., conservatively slices off *proportiontocut* ).

**Parameters**

- a** : array\_like  
Input array
- proportiontocut** : float  
Fraction to cut off of ‘left’ or ‘right’ of distribution
- tail** : { ‘left’, ‘right’ }, optional  
Defaults to ‘right’.
- axis** : int or None, optional  
Axis along which to trim data. Default is 0. If None, compute over the whole array *a*.

**Returns**

- trim1** : ndarray  
Trimmed version of array *a*. The order of the trimmed content is undefined.

---

<code>f_oneway(*args)</code>	Performs a 1-way ANOVA.
<code>pearsonr(x, y)</code>	Calculates a Pearson correlation coefficient and the p-value for testing non-correla
<code>spearmanr(a[, b, axis, nan_policy])</code>	Calculates a Spearman rank-order correlation coefficient and the p-value to test fo
<code>pointbiserialr(x, y)</code>	Calculates a point biserial correlation coefficient and its p-value.
<code>kendalltau(x, y[, initial_lexsort, nan_policy])</code>	Calculates Kendall’s tau, a correlation measure for ordinal data.
<code>linregress(x[, y])</code>	Calculate a linear least-squares regression for two sets of measurements.
<code>theilslopes(y[, x, alpha])</code>	Computes the Theil-Sen estimator for a set of points (x, y).
<code>f_value(*args, **kwds)</code>	<code>f_value</code> is deprecated!

---

`scipy.stats.f_oneway(*args)`  
Performs a 1-way ANOVA.

The one-way ANOVA tests the null hypothesis that two or more groups have the same population mean. The test is applied to samples from two or more groups, possibly with differing sizes.

**Parameters**

- sample1, sample2, ...** : array\_like  
The sample measurements for each group.

**Returns**

- statistic** : float  
The computed F-value of the test.
- pvalue** : float  
The associated p-value from the F-distribution.

**Notes**

The ANOVA test has important assumptions that must be satisfied in order for the associated p-value to be valid.

- 1.The samples are independent.
- 2.Each sample is from a normally distributed population.
- 3.The population standard deviations of the groups are all equal. This property is known as homoscedasticity.

If these assumptions are not true for a given set of data, it may still be possible to use the Kruskal-Wallis H-test (`scipy.stats.kruskal`) although with some loss of power.

The algorithm is from Heiman[2], pp.394-7.

**References**

[R512], [R513], [R514]

**Examples**

```
>>> import scipy.stats as stats
```

[R514] Here are some data on a shell measurement (the length of the anterior adductor muscle scar, standardized by dividing by length) in the mussel *Mytilus trossulus* from five locations: Tillamook, Oregon; Newport, Oregon; Petersburg, Alaska; Magadan, Russia; and Tvarminne, Finland, taken from a much larger data set used in McDonald et al. (1991).

```
>>> tillamook = [0.0571, 0.0813, 0.0831, 0.0976, 0.0817, 0.0859, 0.0735,
...             0.0659, 0.0923, 0.0836]
>>> newport = [0.0873, 0.0662, 0.0672, 0.0819, 0.0749, 0.0649, 0.0835,
...            0.0725]
>>> petersburg = [0.0974, 0.1352, 0.0817, 0.1016, 0.0968, 0.1064, 0.105]
>>> magadan = [0.1033, 0.0915, 0.0781, 0.0685, 0.0677, 0.0697, 0.0764,
...            0.0689]
>>> tvarminne = [0.0703, 0.1026, 0.0956, 0.0973, 0.1039, 0.1045]
>>> stats.f_oneway(tillamook, newport, petersburg, magadan, tvarminne)
(7.1210194716424473, 0.00028122423145345439)
```

`scipy.stats.pearsonr(x, y)`

Calculates a Pearson correlation coefficient and the p-value for testing non-correlation.

The Pearson correlation coefficient measures the linear relationship between two datasets. Strictly speaking, Pearson's correlation requires that each dataset be normally distributed, and not necessarily zero-mean. Like other correlation coefficients, this one varies between -1 and +1 with 0 implying no correlation. Correlations of -1 or +1 imply an exact linear relationship. Positive correlations imply that as x increases, so does y. Negative correlations imply that as x increases, y decreases.

The p-value roughly indicates the probability of an uncorrelated system producing datasets that have a Pearson correlation at least as extreme as the one computed from these datasets. The p-values are not entirely reliable but are probably reasonable for datasets larger than 500 or so.

<b>Parameters</b>	<b>x</b> : (N,) array_like	Input
	<b>y</b> : (N,) array_like	Input
<b>Returns</b>	<b>r</b> : float	Pearson's correlation coefficient
	<b>p-value</b> : float	2-tailed p-value

### References

<http://www.statsoft.com/textbook/glosp.html#Pearson%20Correlation>

`scipy.stats.spearmanr(a, b=None, axis=0, nan_policy='propagate')`

Calculates a Spearman rank-order correlation coefficient and the p-value to test for non-correlation.

The Spearman correlation is a nonparametric measure of the monotonicity of the relationship between two datasets. Unlike the Pearson correlation, the Spearman correlation does not assume that both datasets are normally distributed. Like other correlation coefficients, this one varies between -1 and +1 with 0 implying no correlation. Correlations of -1 or +1 imply an exact monotonic relationship. Positive correlations imply that as x increases, so does y. Negative correlations imply that as x increases, y decreases.

The p-value roughly indicates the probability of an uncorrelated system producing datasets that have a Spearman correlation at least as extreme as the one computed from these datasets. The p-values are not entirely reliable but are probably reasonable for datasets larger than 500 or so.

<b>Parameters</b>	<b>a, b</b> : 1D or 2D array_like, b is optional
	One or two 1-D or 2-D arrays containing multiple variables and observations. When these are 1-D, each represents a vector of observations of a

single variable. For the behavior in the 2-D case, see under `axis`, below. Both arrays need to have the same length in the `axis` dimension.

**axis** : int or None, optional

If `axis=0` (default), then each column represents a variable, with observations in the rows. If `axis=1`, the relationship is transposed: each row represents a variable, while the columns contain observations. If `axis=None`, then both arrays will be raveled.

**nan\_policy** : {'propagate', 'raise', 'omit'}, optional

Defines how to handle when input contains nan. 'propagate' returns nan, 'raise' throws an error, 'omit' performs the calculations ignoring nan values. Default is 'propagate'

**Returns**

**correlation** : float or ndarray (2-D square)

Spearman correlation matrix or correlation coefficient (if only 2 variables are given as parameters). Correlation matrix is square with length equal to total number of variables (columns or rows) in `a` and `b` combined.

**pvalue** : float

The two-sided p-value for a hypothesis test whose null hypothesis is that two sets of data are uncorrelated, has same dimension as rho.

### Notes

Changes in scipy 0.8.0: rewrite to add tie-handling, and axis.

### References

[R571]

### Examples

```
>>> from scipy import stats
>>> stats.spearmanr([1,2,3,4,5], [5,6,7,8,7])
(0.82078268166812329, 0.088587005313543798)
>>> np.random.seed(1234321)
>>> x2n = np.random.randn(100, 2)
>>> y2n = np.random.randn(100, 2)
>>> stats.spearmanr(x2n)
(0.059969996999699973, 0.55338590803773591)
>>> stats.spearmanr(x2n[:,0], x2n[:,1])
(0.059969996999699973, 0.55338590803773591)
>>> rho, pval = stats.spearmanr(x2n, y2n)
>>> rho
array([[ 1.          ,  0.05997   ,  0.18569457,  0.06258626],
       [ 0.05997   ,  1.          ,  0.110003 ,  0.02534653],
       [ 0.18569457,  0.110003 ,  1.          ,  0.03488749],
       [ 0.06258626,  0.02534653,  0.03488749,  1.          ]])
>>> pval
array([[ 0.          ,  0.55338591,  0.06435364,  0.53617935],
       [ 0.55338591,  0.          ,  0.27592895,  0.80234077],
       [ 0.06435364,  0.27592895,  0.          ,  0.73039992],
       [ 0.53617935,  0.80234077,  0.73039992,  0.          ]])
>>> rho, pval = stats.spearmanr(x2n.T, y2n.T, axis=1)
>>> rho
array([[ 1.          ,  0.05997   ,  0.18569457,  0.06258626],
       [ 0.05997   ,  1.          ,  0.110003 ,  0.02534653],
       [ 0.18569457,  0.110003 ,  1.          ,  0.03488749],
       [ 0.06258626,  0.02534653,  0.03488749,  1.          ]])
>>> stats.spearmanr(x2n, y2n, axis=None)
(0.10816770419260482, 0.1273562188027364)
```

```
>>> stats.spearmanr(x2n.ravel(), y2n.ravel())
(0.10816770419260482, 0.1273562188027364)

>>> xint = np.random.randint(10, size=(100, 2))
>>> stats.spearmanr(xint)
(0.052760927029710199, 0.60213045837062351)
```

`scipy.stats.pointbiserialr(x, y)`

Calculates a point biserial correlation coefficient and its p-value.

The point biserial correlation is used to measure the relationship between a binary variable,  $x$ , and a continuous variable,  $y$ . Like other correlation coefficients, this one varies between -1 and +1 with 0 implying no correlation. Correlations of -1 or +1 imply a deterministic relationship.

This function uses a shortcut formula but produces the same result as `pearsonr`.

**Parameters**

- x** : array\_like of bools  
Input array.
- y** : array\_like  
Input array.

**Returns**

- correlation** : float  
R value
- pvalue** : float  
2-tailed p-value

### Notes

`pointbiserialr` uses a t-test with  $n-1$  degrees of freedom. It is equivalent to `pearsonr`:

The value of the point-biserial correlation can be calculated from:

$$r_{pb} = \frac{\bar{Y}_1 - \bar{Y}_0}{s_y} \sqrt{\frac{N_1 N_2}{N(N-1)}}$$

Where  $Y_0$  and  $Y_1$  are means of the metric observations coded 0 and 1 respectively;  $N_0$  and  $N_1$  are number of observations coded 0 and 1 respectively;  $N$  is the total number of observations and  $s_y$  is the standard deviation of all the metric observations.

A value of  $r_{pb}$  that is significantly different from zero is completely equivalent to a significant difference in means between the two groups. Thus, an independent groups t Test with  $N-2$  degrees of freedom may be used to test whether  $r_{pb}$  is nonzero. The relation between the t-statistic for comparing two independent groups and  $r_{pb}$  is given by:

$$t = \sqrt{N-2} \frac{r_{pb}}{\sqrt{1-r_{pb}^2}}$$

### References

[R552], [R553], [R554]

### Examples

```
>>> from scipy import stats
>>> a = np.array([0, 0, 0, 1, 1, 1, 1])
>>> b = np.arange(7)
>>> stats.pointbiserialr(a, b)
(0.8660254037844386, 0.011724811003954652)
```

```
>>> stats.pearsonr(a, b)
(0.86602540378443871, 0.011724811003954626)
>>> np.corrcoef(a, b)
array([[ 1.          ,  0.8660254],
       [ 0.8660254,  1.          ]])
```

`scipy.stats.kendalltau(x, y, initial_lexsort=None, nan_policy='propagate')`

Calculates Kendall's tau, a correlation measure for ordinal data.

Kendall's tau is a measure of the correspondence between two rankings. Values close to 1 indicate strong agreement, values close to -1 indicate strong disagreement. This is the tau-b version of Kendall's tau which accounts for ties.

**Parameters**

- x, y** : array\_like  
Arrays of rankings, of the same shape. If arrays are not 1-D, they will be flattened to 1-D.
- initial\_lexsort** : bool, optional  
Whether to use lexsort or quicksort as the sorting method for the initial sort of the inputs. Default is lexsort (True), for which `kendalltau` is of complexity  $O(n \log(n))$ . If False, the complexity is  $O(n^2)$ , but with a smaller pre-factor (so quicksort may be faster for small arrays).
- nan\_policy** : {'propagate', 'raise', 'omit'}, optional  
Defines how to handle when input contains nan. 'propagate' returns nan, 'raise' throws an error, 'omit' performs the calculations ignoring nan values. Default is 'propagate'.

**Returns**

- correlation** : float  
The tau statistic.
- pvalue** : float  
The two-sided p-value for a hypothesis test whose null hypothesis is an absence of association, tau = 0.

**See also:**

[`spearmanr`](#) Calculates a Spearman rank-order correlation coefficient.

[`theilslopes`](#)

Computes the Theil-Sen estimator for a set of points (x, y).

**Notes**

The definition of Kendall's tau that is used is:

$$\tau = (P - Q) / \sqrt{((P + Q + T) * (P + Q + U))}$$

where P is the number of concordant pairs, Q the number of discordant pairs, T the number of ties only in x, and U the number of ties only in y. If a tie occurs for the same pair in both x and y, it is not added to either T or U.

**References**

W.R. Knight, "A Computer Method for Calculating Kendall's Tau with Ungrouped Data", Journal of the American Statistical Association, Vol. 61, No. 314, Part 1, pp. 436-439, 1966.

**Examples**

```
>>> from scipy import stats
>>> x1 = [12, 2, 1, 12, 2]
>>> x2 = [1, 4, 7, 1, 0]
>>> tau, p_value = stats.kendalltau(x1, x2)
>>> tau
```



```
-0.47140452079103173
>>> p_value
0.24821309157521476
```

`scipy.stats.linregress` (*x*, *y=None*)

Calculate a linear least-squares regression for two sets of measurements.

<b>Parameters</b>	<b>x, y</b> : array_like	Two sets of measurements. Both arrays should have the same length. If only <i>x</i> is given (and <i>y=None</i> ), then it must be a two-dimensional array where one dimension has length 2. The two sets of measurements are then found by splitting the array along the length-2 dimension.
<b>Returns</b>	<b>slope</b> : float	slope of the regression line
	<b>intercept</b> : float	intercept of the regression line
	<b>rvalue</b> : float	correlation coefficient
	<b>pvalue</b> : float	two-sided p-value for a hypothesis test whose null hypothesis is that the slope is zero.
	<b>stderr</b> : float	Standard error of the estimated gradient.

See also:

**`optimize.curve_fit`**

Use non-linear least squares to fit a function to data.

**`optimize.leastsq`**

Minimize the sum of squares of a set of equations.

**Examples**

```
>>> from scipy import stats
>>> np.random.seed(12345678)
>>> x = np.random.random(10)
>>> y = np.random.random(10)
>>> slope, intercept, r_value, p_value, std_err = stats.linregress(x,y)
```

```
# To get coefficient of determination (r_squared)
```

```
>>> print("r-squared:", r_value**2)
('r-squared:', 0.080402268539028335)
```

`scipy.stats.theilslopes` (*y*, *x=None*, *alpha=0.95*)

Computes the Theil-Sen estimator for a set of points (*x*, *y*).

`theilslopes` implements a method for robust linear regression. It computes the slope as the median of all slopes between paired values.

<b>Parameters</b>	<b>y</b> : array_like	Dependent variable.
	<b>x</b> : array_like or None, optional	Independent variable. If None, use <code>arange(len(y))</code> instead.
	<b>alpha</b> : float, optional	Confidence degree between 0 and 1. Default is 95% confidence. Note that <code>alpha</code> is symmetric around 0.5, i.e. both 0.1 and 0.9 are interpreted as “find the 90% confidence interval”.

**Returns**

- medslope** : float  
Theil slope.
- medintercept** : float  
Intercept of the Theil line, as  $\text{median}(y) - \text{medslope} * \text{median}(x)$ .
- lo\_slope** : float  
Lower bound of the confidence interval on *medslope*.
- up\_slope** : float  
Upper bound of the confidence interval on *medslope*.

### Notes

The implementation of `theilslopes` follows [R572]. The intercept is not defined in [R572], and here it is defined as  $\text{median}(y) - \text{medslope} * \text{median}(x)$ , which is given in [R574]. Other definitions of the intercept exist in the literature. A confidence interval for the intercept is not given as this question is not addressed in [R572].

### References

[R572], [R573], [R574]

### Examples

```
>>> from scipy import stats
>>> import matplotlib.pyplot as plt

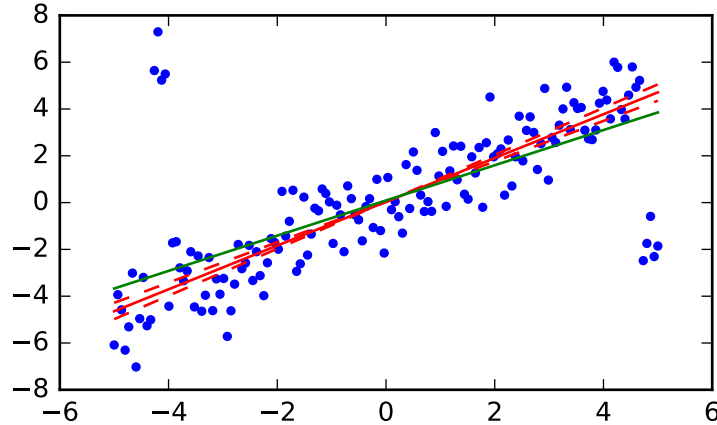
>>> x = np.linspace(-5, 5, num=150)
>>> y = x + np.random.normal(size=x.size)
>>> y[11:15] += 10 # add outliers
>>> y[-5:] -= 7
```

Compute the slope, intercept and 90% confidence interval. For comparison, also compute the least-squares fit with `linregress`:

```
>>> res = stats.theilslopes(y, x, 0.90)
>>> lsq_res = stats.linregress(x, y)
```

Plot the results. The Theil-Sen regression line is shown in red, with the dashed red lines illustrating the confidence interval of the slope (note that the dashed red lines are not the confidence interval of the regression as the confidence interval of the intercept is not included). The green line shows the least-squares fit for comparison.

```
>>> fig = plt.figure()
>>> ax = fig.add_subplot(111)
>>> ax.plot(x, y, 'b.')
>>> ax.plot(x, res[1] + res[0] * x, 'r-')
>>> ax.plot(x, res[1] + res[2] * x, 'r--')
>>> ax.plot(x, res[1] + res[3] * x, 'r--')
>>> ax.plot(x, lsq_res[1] + lsq_res[0] * x, 'g-')
>>> plt.show()
```



`scipy.stats.f_value(*args, **kws)`

`f_value` is deprecated! `stats.f_value` deprecated in scipy 0.17.0

Returns an F-statistic for a restricted vs. unrestricted model.

**Parameters** `ER` : float

*ER is the sum of squared residuals for the restricted model*

or null hypothesis

*EF*

[float]

*EF is the sum of squared residuals for the unrestricted model*

*dfR*  
*dfF*

[int] *dfR* is the degrees of freedom in the restricted model,  
[int] *dfF* is the degrees of freedom in the unrestricted model  
or alternate hypothesis

**Returns** `F-statistic` : float

`ttest_1samp(a, popmean[, axis, nan_policy])`  
`ttest_ind(a, b[, axis, equal_var, nan_policy])`  
`ttest_ind_from_stats(mean1, std1, nobs1, ...)`  
`ttest_rel(a, b[, axis, nan_policy])`  
`kstest(rvs, cdf[, args, N, alternative, mode])`  
`chisquare(f_obs[, f_exp, ddof, axis])`  
`power_divergence(f_obs[, f_exp, ddof, axis, ...])`  
`ks_2samp(data1, data2)`  
`mannwhitneyu(x, y[, use_continuity, alternative])`  
`tiecorrect(rankvals)`  
`rankdata(a[, method])`  
`ranksums(x, y)`  
`wilcoxon(x[, y, zero_method, correction])`  
`kruskal(*args, **kwargs)`  
`friedmanchisquare(*args)`  
`combine_pvalues(pvalues[, method, weights])`  
`ss(*args, **kws)`  
`square_of_sums(*args, **kws)`

Calculates the T-test for the mean of ONE group of scores.  
 Calculates the T-test for the means of *two independent* samples of scores.  
 T-test for means of two independent samples from descriptive statistics.  
 Calculates the T-test on TWO RELATED samples of scores, a and b.  
 Perform the Kolmogorov-Smirnov test for goodness of fit.  
 Calculates a one-way chi square test.  
 Cressie-Read power divergence statistic and goodness of fit test.  
 Computes the Kolmogorov-Smirnov statistic on 2 samples.  
 Computes the Mann-Whitney rank test on samples x and y.  
 Tie correction factor for ties in the Mann-Whitney U and Kruskal-Wallis H test.  
 Assign ranks to data, dealing with ties appropriately.  
 Compute the Wilcoxon rank-sum statistic for two samples.  
 Calculate the Wilcoxon signed-rank test.  
 Compute the Kruskal-Wallis H-test for independent samples  
 Computes the Friedman test for repeated measurements  
 Methods for combining the p-values of independent tests bearing upon the same hypothesis.  
`ss` is deprecated!  
`square_of_sums` is deprecated!

Continued

Table 5.285 – continued from previous page

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<code>jarque_bera(x)</code>	Perform the Jarque-Bera goodness of fit test on sample data.
-----------------------------	--

---

`scipy.stats.ttest_1samp` (*a*, *popmean*, *axis=0*, *nan\_policy='propagate'*)

Calculates the T-test for the mean of ONE group of scores.

This is a two-sided test for the null hypothesis that the expected value (mean) of a sample of independent observations *a* is equal to the given population mean, *popmean*.

**Parameters**

- a** : array\_like  
sample observation
- popmean** : float or array\_like  
expected value in null hypothesis, if array\_like than it must have the same shape as *a* excluding the axis dimension
- axis** : int or None, optional  
Axis along which to compute test. If None, compute over the whole array *a*.
- nan\_policy** : { 'propagate', 'raise', 'omit' }, optional  
Defines how to handle when input contains nan. 'propagate' returns nan, 'raise' throws an error, 'omit' performs the calculations ignoring nan values. Default is 'propagate'.

**Returns**

- statistic** : float or array  
t-statistic
- pvalue** : float or array  
two-tailed p-value

### Examples

```
>>> from scipy import stats

>>> np.random.seed(7654567) # fix seed to get the same result
>>> rvs = stats.norm.rvs(loc=5, scale=10, size=(50,2))
```

Test if mean of random sample is equal to true mean, and different mean. We reject the null hypothesis in the second case and don't reject it in the first case.

```
>>> stats.ttest_1samp(rvs, 5.0)
(array([-0.68014479, -0.04323899]), array([ 0.49961383,  0.96568674]))
>>> stats.ttest_1samp(rvs, 0.0)
(array([ 2.77025808,  4.11038784]), array([ 0.00789095,  0.00014999]))
```

Examples using axis and non-scalar dimension for population mean.

```
>>> stats.ttest_1samp(rvs, [5.0, 0.0])
(array([-0.68014479,  4.11038784]), array([ 4.99613833e-01,  1.49986458e-04]))
>>> stats.ttest_1samp(rvs.T, [5.0, 0.0], axis=1)
(array([-0.68014479,  4.11038784]), array([ 4.99613833e-01,  1.49986458e-04]))
>>> stats.ttest_1samp(rvs, [[5.0], [0.0]])
(array([[ -0.68014479, -0.04323899],
        [ 2.77025808,  4.11038784]]), array([[ 4.99613833e-01,  9.65686743e-01],
        [ 7.89094663e-03,  1.49986458e-04]]))
```

`scipy.stats.ttest_ind` (*a*, *b*, *axis=0*, *equal\_var=True*, *nan\_policy='propagate'*)

Calculates the T-test for the means of two independent samples of scores.

This is a two-sided test for the null hypothesis that 2 independent samples have identical average (expected) values. This test assumes that the populations have identical variances by default.

<b>Parameters</b>	<b>a, b</b> : array_like	The arrays must have the same shape, except in the dimension corresponding to <i>axis</i> (the first, by default).
	<b>axis</b> : int or None, optional	Axis along which to compute test. If None, compute over the whole arrays, <i>a</i> , and <i>b</i> .
	<b>equal_var</b> : bool, optional	If True (default), perform a standard independent 2 sample test that assumes equal population variances [R576]. If False, perform Welch's t-test, which does not assume equal population variance [R577]. New in version 0.11.0.
	<b>nan_policy</b> : {'propagate', 'raise', 'omit'}, optional	Defines how to handle when input contains nan. 'propagate' returns nan, 'raise' throws an error, 'omit' performs the calculations ignoring nan values. Default is 'propagate'.
	<b>Returns</b>	
	<b>statistic</b> : float or array	The calculated t-statistic.
	<b>pvalue</b> : float or array	The two-tailed p-value.

**Notes**

We can use this test, if we observe two independent samples from the same or different population, e.g. exam scores of boys and girls or of two ethnic groups. The test measures whether the average (expected) value differs significantly across samples. If we observe a large p-value, for example larger than 0.05 or 0.1, then we cannot reject the null hypothesis of identical average scores. If the p-value is smaller than the threshold, e.g. 1%, 5% or 10%, then we reject the null hypothesis of equal averages.

**References**

[R576], [R577]

**Examples**

```
>>> from scipy import stats
>>> np.random.seed(12345678)
```

Test with sample with identical means:

```
>>> rvs1 = stats.norm.rvs(loc=5, scale=10, size=500)
>>> rvs2 = stats.norm.rvs(loc=5, scale=10, size=500)
>>> stats.ttest_ind(rvs1, rvs2)
(0.26833823296239279, 0.78849443369564776)
>>> stats.ttest_ind(rvs1, rvs2, equal_var = False)
(0.26833823296239279, 0.78849452749500748)
```

`ttest_ind` underestimates p for unequal variances:

```
>>> rvs3 = stats.norm.rvs(loc=5, scale=20, size=500)
>>> stats.ttest_ind(rvs1, rvs3)
(-0.46580283298287162, 0.64145827413436174)
>>> stats.ttest_ind(rvs1, rvs3, equal_var = False)
(-0.46580283298287162, 0.64149646246569292)
```

When  $n_1 \neq n_2$ , the equal variance t-statistic is no longer equal to the unequal variance t-statistic:

```
>>> rvs4 = stats.norm.rvs(loc=5, scale=20, size=100)
>>> stats.ttest_ind(rvs1, rvs4)
(-0.99882539442782481, 0.3182832709103896)
>>> stats.ttest_ind(rvs1, rvs4, equal_var = False)
(-0.69712570584654099, 0.48716927725402048)
```

T-test with different means, variance, and n:

```
>>> rvs5 = stats.norm.rvs(loc=8, scale=20, size=100)
>>> stats.ttest_ind(rvs1, rvs5)
(-1.4679669854490653, 0.14263895620529152)
>>> stats.ttest_ind(rvs1, rvs5, equal_var = False)
(-0.94365973617132992, 0.34744170334794122)
```

`scipy.stats.ttest_ind_from_stats` (*mean1*, *std1*, *nobs1*, *mean2*, *std2*, *nobs2*, *equal\_var=True*)

T-test for means of two independent samples from descriptive statistics.

This is a two-sided test for the null hypothesis that 2 independent samples have identical average (expected) values.

<b>Parameters</b>	<p><b>mean1</b> : array_like The mean(s) of sample 1.</p> <p><b>std1</b> : array_like The standard deviation(s) of sample 1.</p> <p><b>nobs1</b> : array_like The number(s) of observations of sample 1.</p> <p><b>mean2</b> : array_like The mean(s) of sample 2</p> <p><b>std2</b> : array_like The standard deviations(s) of sample 2.</p> <p><b>nobs2</b> : array_like The number(s) of observations of sample 2.</p> <p><b>equal_var</b> : bool, optional If True (default), perform a standard independent 2 sample test that assumes equal population variances [R578]. If False, perform Welch's t-test, which does not assume equal population variance [R579].</p>
<b>Returns</b>	<p><b>statistic</b> : float or array The calculated t-statistics</p> <p><b>pvalue</b> : float or array The two-tailed p-value.</p>

**See also:**

`scipy.stats.ttest_ind`

**Notes**

New in version 0.16.0.

**References**

[R578], [R579]

`scipy.stats.ttest_rel` (*a*, *b*, *axis=0*, *nan\_policy='propagate'*)

Calculates the T-test on TWO RELATED samples of scores, a and b.

This is a two-sided test for the null hypothesis that 2 related or repeated samples have identical average (expected) values.

**Parameters**

- a, b** : array\_like  
The arrays must have the same shape.
- axis** : int or None, optional  
Axis along which to compute test. If None, compute over the whole arrays, *a*, and *b*.
- nan\_policy** : { 'propagate', 'raise', 'omit' }, optional  
Defines how to handle when input contains nan. 'propagate' returns nan, 'raise' throws an error, 'omit' performs the calculations ignoring nan values. Default is 'propagate'.

**Returns**

- statistic** : float or array  
t-statistic
- pvalue** : float or array  
two-tailed p-value

### Notes

Examples for the use are scores of the same set of student in different exams, or repeated sampling from the same units. The test measures whether the average score differs significantly across samples (e.g. exams). If we observe a large p-value, for example greater than 0.05 or 0.1 then we cannot reject the null hypothesis of identical average scores. If the p-value is smaller than the threshold, e.g. 1%, 5% or 10%, then we reject the null hypothesis of equal averages. Small p-values are associated with large t-statistics.

### References

[http://en.wikipedia.org/wiki/T-test#Dependent\\_t-test](http://en.wikipedia.org/wiki/T-test#Dependent_t-test)

### Examples

```
>>> from scipy import stats
>>> np.random.seed(12345678) # fix random seed to get same numbers

>>> rvs1 = stats.norm.rvs(loc=5, scale=10, size=500)
>>> rvs2 = (stats.norm.rvs(loc=5, scale=10, size=500) +
...        stats.norm.rvs(scale=0.2, size=500))
>>> stats.ttest_rel(rvs1, rvs2)
(0.24101764965300962, 0.80964043445811562)
>>> rvs3 = (stats.norm.rvs(loc=8, scale=10, size=500) +
...        stats.norm.rvs(scale=0.2, size=500))
>>> stats.ttest_rel(rvs1, rvs3)
(-3.9995108708727933, 7.3082402191726459e-005)
```

`scipy.stats.kstest` (*rvs*, *cdf*, *args=()*, *N=20*, *alternative='two-sided'*, *mode='approx'*)

Perform the Kolmogorov-Smirnov test for goodness of fit.

This performs a test of the distribution  $G(x)$  of an observed random variable against a given distribution  $F(x)$ . Under the null hypothesis the two distributions are identical,  $G(x)=F(x)$ . The alternative hypothesis can be either 'two-sided' (default), 'less' or 'greater'. The KS test is only valid for continuous distributions.

**Parameters**

- rvs** : str, array or callable  
If a string, it should be the name of a distribution in `scipy.stats`. If an array, it should be a 1-D array of observations of random variables. If a callable, it should be a function to generate random variables; it is required to have a keyword argument *size*.
- cdf** : str or callable  
If a string, it should be the name of a distribution in `scipy.stats`. If *rvs* is a string then *cdf* can be False or the same as *rvs*. If a callable, that callable is used to calculate the cdf.

**args** : tuple, sequence, optional  
Distribution parameters, used if *rvs* or *cdf* are strings.

**N** : int, optional  
Sample size if *rvs* is string or callable. Default is 20.

**alternative** : { 'two-sided', 'less', 'greater' }, optional  
Defines the alternative hypothesis (see explanation above). Default is 'two-sided'.

**mode** : 'approx' (default) or 'asympt', optional  
Defines the distribution used for calculating the p-value.

- 'approx' : use approximation to exact distribution of test statistic,
- 'asympt' : use asymptotic distribution of test statistic

**Returns**

**statistic** : float  
KS test statistic, either D, D+ or D-.

**pvalue** : float  
One-tailed or two-tailed p-value.

### Notes

In the one-sided test, the alternative is that the empirical cumulative distribution function of the random variable is “less” or “greater” than the cumulative distribution function  $F(x)$  of the hypothesis,  $G(x) \leq F(x)$ , resp.  $G(x) \geq F(x)$ .

### Examples

```
>>> from scipy import stats

>>> x = np.linspace(-15, 15, 9)
>>> stats.kstest(x, 'norm')
(0.44435602715924361, 0.038850142705171065)

>>> np.random.seed(987654321) # set random seed to get the same result
>>> stats.kstest('norm', False, N=100)
(0.058352892479417884, 0.88531190944151261)
```

The above lines are equivalent to:

```
>>> np.random.seed(987654321)
>>> stats.kstest(stats.norm.rvs(size=100), 'norm')
(0.058352892479417884, 0.88531190944151261)
```

### Test against one-sided alternative hypothesis

Shift distribution to larger values, so that  $\text{cdf}_{\text{dgp}}(x) < \text{norm.cdf}(x)$ :

```
>>> np.random.seed(987654321)
>>> x = stats.norm.rvs(loc=0.2, size=100)
>>> stats.kstest(x, 'norm', alternative = 'less')
(0.12464329735846891, 0.040989164077641749)
```

Reject equal distribution against alternative hypothesis: less

```
>>> stats.kstest(x, 'norm', alternative = 'greater')
(0.0072115233216311081, 0.98531158590396395)
```

Don't reject equal distribution against alternative hypothesis: greater



```
>>> stats.kstest(x, 'norm', mode='asympt')
(0.12464329735846891, 0.08944488871182088)
```

#### Testing *t* distributed random variables against normal distribution

With 100 degrees of freedom the *t* distribution looks close to the normal distribution, and the K-S test does not reject the hypothesis that the sample came from the normal distribution:

```
>>> np.random.seed(987654321)
>>> stats.kstest(stats.t.rvs(100, size=100), 'norm')
(0.072018929165471257, 0.67630062862479168)
```

With 3 degrees of freedom the *t* distribution looks sufficiently different from the normal distribution, that we can reject the hypothesis that the sample came from the normal distribution at the 10% level:

```
>>> np.random.seed(987654321)
>>> stats.kstest(stats.t.rvs(3, size=100), 'norm')
(0.131016895759829, 0.058826222555312224)
```

`scipy.stats.chisquare` (*f\_obs*, *f\_exp*=None, *ddof*=0, *axis*=0)

Calculates a one-way chi square test.

The chi square test tests the null hypothesis that the categorical data has the given frequencies.

<b>Parameters</b>	<b>f_obs</b> : array_like	Observed frequencies in each category.
	<b>f_exp</b> : array_like, optional	Expected frequencies in each category. By default the categories are assumed to be equally likely.
	<b>ddof</b> : int, optional	“Delta degrees of freedom”: adjustment to the degrees of freedom for the p-value. The p-value is computed using a chi-squared distribution with $k - 1 - \text{ddof}$ degrees of freedom, where $k$ is the number of observed frequencies. The default value of <i>ddof</i> is 0.
	<b>axis</b> : int or None, optional	The axis of the broadcast result of <i>f_obs</i> and <i>f_exp</i> along which to apply the test. If axis is None, all values in <i>f_obs</i> are treated as a single data set.
<b>Returns</b>	<b>chisq</b> : float or ndarray	Default is 0. The chi-squared test statistic. The value is a float if <i>axis</i> is None or <i>f_obs</i> and <i>f_exp</i> are 1-D.
	<b>p</b> : float or ndarray	The p-value of the test. The value is a float if <i>ddof</i> and the return value <i>chisq</i> are scalars.

**See also:**

`power_divergence`, `mstats.chisquare`

#### Notes

This test is invalid when the observed or expected frequencies in each category are too small. A typical rule is that all of the observed and expected frequencies should be at least 5.

The default degrees of freedom,  $k-1$ , are for the case when no parameters of the distribution are estimated. If  $p$  parameters are estimated by efficient maximum likelihood then the correct degrees of freedom are  $k-1-p$ . If the parameters are estimated in a different way, then the dof can be between  $k-1-p$  and  $k-1$ . However, it is also possible that the asymptotic distribution is not a chisquare, in which case this test is not appropriate.

## References

[R505], [R506]

## Examples

When just *f\_obs* is given, it is assumed that the expected frequencies are uniform and given by the mean of the observed frequencies.

```
>>> from scipy.stats import chisquare
>>> chisquare([16, 18, 16, 14, 12, 12])
(2.0, 0.84914503608460956)
```

With *f\_exp* the expected frequencies can be given.

```
>>> chisquare([16, 18, 16, 14, 12, 12], f_exp=[16, 16, 16, 16, 16, 8])
(3.5, 0.62338762774958223)
```

When *f\_obs* is 2-D, by default the test is applied to each column.

```
>>> obs = np.array([[16, 18, 16, 14, 12, 12], [32, 24, 16, 28, 20, 24]])
>>> obs.shape
(6, 2)
>>> chisquare(obs)
(array([ 2.          ,  6.66666667]), array([ 0.84914504,  0.24663415]))
```

By setting *axis=None*, the test is applied to all data in the array, which is equivalent to applying the test to the flattened array.

```
>>> chisquare(obs, axis=None)
(23.31034482758621, 0.015975692534127565)
>>> chisquare(obs.ravel())
(23.31034482758621, 0.015975692534127565)
```

*ddof* is the change to make to the default degrees of freedom.

```
>>> chisquare([16, 18, 16, 14, 12, 12], ddof=1)
(2.0, 0.73575888234288467)
```

The calculation of the p-values is done by broadcasting the chi-squared statistic with *ddof*.

```
>>> chisquare([16, 18, 16, 14, 12, 12], ddof=[0,1,2])
(2.0, array([ 0.84914504,  0.73575888,  0.5724067 ]))
```

*f\_obs* and *f\_exp* are also broadcast. In the following, *f\_obs* has shape (6,) and *f\_exp* has shape (2, 6), so the result of broadcasting *f\_obs* and *f\_exp* has shape (2, 6). To compute the desired chi-squared statistics, we use *axis=1*:

```
>>> chisquare([16, 18, 16, 14, 12, 12],
...          f_exp=[[16, 16, 16, 16, 16, 8], [8, 20, 20, 16, 12, 12]],
...          axis=1)
(array([ 3.5 ,  9.25]), array([ 0.62338763,  0.09949846]))
```

`scipy.stats.power_divergence` (*f\_obs*, *f\_exp*=None, *ddof*=0, *axis*=0, *lambda\_*=None)  
Cressie-Read power divergence statistic and goodness of fit test.

This function tests the null hypothesis that the categorical data has the given frequencies, using the Cressie-Read power divergence statistic.

**Parameters**

- f\_obs** : array\_like  
Observed frequencies in each category.
- f\_exp** : array\_like, optional  
Expected frequencies in each category. By default the categories are assumed to be equally likely.
- ddof** : int, optional  
“Delta degrees of freedom”: adjustment to the degrees of freedom for the p-value. The p-value is computed using a chi-squared distribution with  $k - 1 - \text{ddof}$  degrees of freedom, where  $k$  is the number of observed frequencies. The default value of *ddof* is 0.
- axis** : int or None, optional  
The axis of the broadcast result of *f\_obs* and *f\_exp* along which to apply the test. If axis is None, all values in *f\_obs* are treated as a single data set. Default is 0.
- lambda\_** : float or str, optional  
*lambda\_* gives the power in the Cressie-Read power divergence statistic. The default is 1. For convenience, *lambda\_* may be assigned one of the following strings, in which case the corresponding numerical value is used:

String	Value	Description
"pearson"	1	Pearson's chi-squared statistic. In this case, the function is equivalent to <code>`stats.chisquare`</code> .
"log-likelihood"	0	Log-likelihood ratio. Also known as the G-test [R557]_.
"freeman-tukey"	-1/2	Freeman-Tukey statistic.
"mod-log-likelihood"	-1	Modified log-likelihood ratio.
"neyman"	-2	Neyman's statistic.
"cressie-read"	2/3	The power recommended in [R559]_.

**Returns**

- statistic** : float or ndarray  
The Cressie-Read power divergence test statistic. The value is a float if *axis* is None or if *f\_obs* and *f\_exp* are 1-D.
- pvalue** : float or ndarray  
The p-value of the test. The value is a float if *ddof* and the return value *stat* are scalars.

**See also:**[chisquare](#)**Notes**

This test is invalid when the observed or expected frequencies in each category are too small. A typical rule is that all of the observed and expected frequencies should be at least 5.

When *lambda\_* is less than zero, the formula for the statistic involves dividing by *f\_obs*, so a warning or error may be generated if any value in *f\_obs* is 0.

Similarly, a warning or error may be generated if any value in *f\_exp* is zero when *lambda\_*  $\geq$  0.

The default degrees of freedom,  $k-1$ , are for the case when no parameters of the distribution are estimated. If  $p$  parameters are estimated by efficient maximum likelihood then the correct degrees of freedom are  $k-1-p$ . If the parameters are estimated in a different way, then the dof can be between  $k-1-p$  and  $k-1$ . However, it is also possible that the asymptotic distribution is not a chisquare, in which case this test is not appropriate.

This function handles masked arrays. If an element of *f\_obs* or *f\_exp* is masked, then data at that position is ignored, and does not count towards the size of the data set.

New in version 0.13.0.

## References

[R555], [R556], [R557], [R558], [R559]

## Examples

(See `chisquare` for more examples.)

When just `f_obs` is given, it is assumed that the expected frequencies are uniform and given by the mean of the observed frequencies. Here we perform a G-test (i.e. use the log-likelihood ratio statistic):

```
>>> from scipy.stats import power_divergence
>>> power_divergence([16, 18, 16, 14, 12, 12], lambda_='log-likelihood')
(2.006573162632538, 0.84823476779463769)
```

The expected frequencies can be given with the `f_exp` argument:

```
>>> power_divergence([16, 18, 16, 14, 12, 12],
...                  f_exp=[16, 16, 16, 16, 16, 8],
...                  lambda_='log-likelihood')
(3.3281031458963746, 0.6495419288047497)
```

When `f_obs` is 2-D, by default the test is applied to each column.

```
>>> obs = np.array([[16, 18, 16, 14, 12, 12], [32, 24, 16, 28, 20, 24]])
>>> obs.shape
(6, 2)
>>> power_divergence(obs, lambda_="log-likelihood")
(array([ 2.00657316,  6.77634498]), array([ 0.84823477,  0.23781225]))
```

By setting `axis=None`, the test is applied to all data in the array, which is equivalent to applying the test to the flattened array.

```
>>> power_divergence(obs, axis=None)
(23.31034482758621, 0.015975692534127565)
>>> power_divergence(obs.ravel())
(23.31034482758621, 0.015975692534127565)
```

`ddof` is the change to make to the default degrees of freedom.

```
>>> power_divergence([16, 18, 16, 14, 12, 12], ddof=1)
(2.0, 0.73575888234288467)
```

The calculation of the p-values is done by broadcasting the test statistic with `ddof`.

```
>>> power_divergence([16, 18, 16, 14, 12, 12], ddof=[0,1,2])
(2.0, array([ 0.84914504,  0.73575888,  0.5724067 ]))
```

`f_obs` and `f_exp` are also broadcast. In the following, `f_obs` has shape (6,) and `f_exp` has shape (2, 6), so the result of broadcasting `f_obs` and `f_exp` has shape (2, 6). To compute the desired chi-squared statistics, we must use `axis=1`:

```
>>> power_divergence([16, 18, 16, 14, 12, 12],
...                  f_exp=[[16, 16, 16, 16, 16, 8],
...                          [8, 20, 20, 16, 12, 12]],
...                  axis=1)
(array([ 3.5 ,  9.25]), array([ 0.62338763,  0.09949846]))
```

`scipy.stats.ks_2samp` (*data1*, *data2*)

Computes the Kolmogorov-Smirnov statistic on 2 samples.

This is a two-sided test for the null hypothesis that 2 independent samples are drawn from the same continuous distribution.

<b>Parameters</b>	<b>data1, data2</b> : sequence of 1-D ndarrays two arrays of sample observations assumed to be drawn from a continuous distribution, sample sizes can be different
<b>Returns</b>	<b>statistic</b> : float KS statistic
	<b>pvalue</b> : float two-tailed p-value

### Notes

This tests whether 2 samples are drawn from the same distribution. Note that, like in the case of the one-sample K-S test, the distribution is assumed to be continuous.

This is the two-sided test, one-sided tests are not implemented. The test uses the two-sided asymptotic Kolmogorov-Smirnov distribution.

If the K-S statistic is small or the p-value is high, then we cannot reject the hypothesis that the distributions of the two samples are the same.

### Examples

```
>>> from scipy import stats
>>> np.random.seed(12345678) #fix random seed to get the same result
>>> n1 = 200 # size of first sample
>>> n2 = 300 # size of second sample
```

For a different distribution, we can reject the null hypothesis since the pvalue is below 1%:

```
>>> rvs1 = stats.norm.rvs(size=n1, loc=0., scale=1)
>>> rvs2 = stats.norm.rvs(size=n2, loc=0.5, scale=1.5)
>>> stats.ks_2samp(rvs1, rvs2)
(0.20833333333333337, 4.6674975515806989e-005)
```

For a slightly different distribution, we cannot reject the null hypothesis at a 10% or lower alpha since the p-value at 0.144 is higher than 10%

```
>>> rvs3 = stats.norm.rvs(size=n2, loc=0.01, scale=1.0)
>>> stats.ks_2samp(rvs1, rvs3)
(0.10333333333333333, 0.14498781825751686)
```

For an identical distribution, we cannot reject the null hypothesis since the p-value is high, 41%:

```
>>> rvs4 = stats.norm.rvs(size=n2, loc=0.0, scale=1.0)
>>> stats.ks_2samp(rvs1, rvs4)
(0.07999999999999996, 0.41126949729859719)
```

`scipy.stats.mannwhitneyu` (*x*, *y*, *use\_continuity=True*, *alternative=None*)

Computes the Mann-Whitney rank test on samples *x* and *y*.

<b>Parameters</b>	<b>x, y</b> : array_like Array of samples, should be one-dimensional.
	<b>use_continuity</b> : bool, optional

Whether a continuity correction (1/2.) should be taken into account. Default is True.

**alternative** : None (deprecated), 'less', 'two-sided', or 'greater'  
 Whether to get the p-value for the one-sided hypothesis ('less' or 'greater') or for the two-sided hypothesis ('two-sided'). Defaults to None, which results in a p-value half the size of the 'two-sided' p-value and a different U statistic. The default behavior is not the same as using 'less' or 'greater': it only exists for backward compatibility and is deprecated.

**Returns**

**statistic** : float  
 The Mann-Whitney U statistic, equal to min(U for x, U for y) if *alternative* is equal to None (deprecated; exists for backward compatibility), and U for y otherwise.

**pvalue** : float  
 p-value assuming an asymptotic normal distribution. One-sided or two-sided, depending on the choice of *alternative*.

**Notes**

Use only when the number of observation in each sample is > 20 and you have 2 independent samples of ranks. Mann-Whitney U is significant if the u-obtained is LESS THAN or equal to the critical value of U.

This test corrects for ties and by default uses a continuity correction.

`scipy.stats.tiecorrect` (*rankvals*)

Tie correction factor for ties in the Mann-Whitney U and Kruskal-Wallis H tests.

**Parameters** **rankvals** : array\_like  
 A 1-D sequence of ranks. Typically this will be the array returned by `stats.rankdata`.

**Returns** **factor** : float  
 Correction factor for U or H.

**See also:**

- `rankdata` Assign ranks to the data
- `mannwhitneyu` Mann-Whitney rank test
- `kruskal` Kruskal-Wallis H test

**References**

[R575]

**Examples**

```
>>> from scipy.stats import tiecorrect, rankdata
>>> tiecorrect([1, 2.5, 2.5, 4])
0.9
>>> ranks = rankdata([1, 3, 2, 4, 5, 7, 2, 8, 4])
>>> ranks
array([ 1. ,  4. ,  2.5,  5.5,  7. ,  8. ,  2.5,  9. ,  5.5])
>>> tiecorrect(ranks)
0.9833333333333333
```

`scipy.stats.rankdata` (*a*, *method*='average')

Assign ranks to data, dealing with ties appropriately.

Ranks begin at 1. The *method* argument controls how ranks are assigned to equal values. See [R563] for further discussion of ranking methods.

**Parameters** **a** : array\_like

The array of values to be ranked. The array is first flattened.

**method** : str, optional  
The method used to assign ranks to tied elements. The options are ‘average’, ‘min’, ‘max’, ‘dense’ and ‘ordinal’.

‘average’: The average of the ranks that would have been assigned to all the tied values is assigned to each value.

‘min’: The minimum of the ranks that would have been assigned to all the tied values is assigned to each value. (This is also referred to as “competition” ranking.)

‘max’: The maximum of the ranks that would have been assigned to all the tied values is assigned to each value.

‘dense’: Like ‘min’, but the rank of the next highest element is assigned the rank immediately after those assigned to the tied elements.

‘ordinal’: All values are given a distinct rank, corresponding to the order that the values occur in *a*.

The default is ‘average’.

**Returns** **ranks** : ndarray  
An array of length equal to the size of *a*, containing rank scores.

### References

[R563]

### Examples

```
>>> from scipy.stats import rankdata
>>> rankdata([0, 2, 3, 2])
array([ 1. ,  2.5,  4. ,  2.5])
>>> rankdata([0, 2, 3, 2], method='min')
array([ 1,  2,  4,  2])
>>> rankdata([0, 2, 3, 2], method='max')
array([ 1,  3,  4,  3])
>>> rankdata([0, 2, 3, 2], method='dense')
array([ 1,  2,  3,  2])
>>> rankdata([0, 2, 3, 2], method='ordinal')
array([ 1,  2,  4,  3])
```

`scipy.stats.ranksums`(*x*, *y*)

Compute the Wilcoxon rank-sum statistic for two samples.

The Wilcoxon rank-sum test tests the null hypothesis that two sets of measurements are drawn from the same distribution. The alternative hypothesis is that values in one sample are more likely to be larger than the values in the other sample.

This test should be used to compare two samples from continuous distributions. It does not handle ties between measurements in *x* and *y*. For tie-handling and an optional continuity correction see `scipy.stats.mannwhitneyu`.

**Parameters** **x,y** : array\_like

**Returns** **statistic** : float  
The data from the two samples

The test statistic under the large-sample approximation that the rank sum statistic is normally distributed

**pvalue** : float

The two-sided p-value of the test

### References

[R564]

`scipy.stats.wilcoxon(x, y=None, zero_method='wilcox', correction=False)`

Calculate the Wilcoxon signed-rank test.

The Wilcoxon signed-rank test tests the null hypothesis that two related paired samples come from the same distribution. In particular, it tests whether the distribution of the differences  $x - y$  is symmetric about zero. It is a non-parametric version of the paired T-test.

**Parameters**

- x** : array\_like  
The first set of measurements.
- y** : array\_like, optional  
The second set of measurements. If *y* is not given, then the *x* array is considered to be the differences between the two sets of measurements.
- zero\_method** : string, {"pratt", "wilcox", "zsplit"}, optional
  - "pratt"**: Pratt treatment: includes zero-differences in the ranking process (more conservative)
  - "wilcox"**: Wilcox treatment: discards all zero-differences
  - "zsplit"**: Zero rank split: just like Pratt, but splitting the zero rank between positive and negative ones
- correction** : bool, optional  
If True, apply continuity correction by adjusting the Wilcoxon rank statistic by 0.5 towards the mean value when computing the z-statistic. Default is False.

**Returns**

- statistic** : float  
The sum of the ranks of the differences above or below zero, whichever is smaller.
- pvalue** : float  
The two-sided p-value for the test.

**Notes**

Because the normal approximation is used for the calculations, the samples used should be large. A typical rule is to require that  $n > 20$ .

**References**

[R581]

`scipy.stats.kruskal(*args, **kwargs)`

Compute the Kruskal-Wallis H-test for independent samples

The Kruskal-Wallis H-test tests the null hypothesis that the population median of all of the groups are equal. It is a non-parametric version of ANOVA. The test works on 2 or more independent samples, which may have different sizes. Note that rejecting the null hypothesis does not indicate which of the groups differs. Post-hoc comparisons between groups are required to determine which groups are different.

**Parameters**

- sample1, sample2, ...** : array\_like  
Two or more arrays with the sample measurements can be given as arguments.
- nan\_policy** : {'propagate', 'raise', 'omit'}, optional  
Defines how to handle when input contains nan. 'propagate' returns nan, 'raise' throws an error, 'omit' performs the calculations ignoring nan values. Default is 'propagate'.

**Returns**

- statistic** : float  
The Kruskal-Wallis H statistic, corrected for ties
- pvalue** : float  
The p-value for the test using the assumption that H has a chi square distribution

See also:

[f\\_oneway](#) 1-way ANOVA



***mannwhitneyu***

Mann-Whitney rank test on two samples.

***friedmanchisquare***

Friedman test for repeated measurements

***Notes***

Due to the assumption that H has a chi square distribution, the number of samples in each group must not be too small. A typical rule is that each sample must have at least 5 measurements.

***References***

[R533], [R534]

***Examples***

```
>>> from scipy import stats
>>> x = [1, 3, 5, 7, 9]
>>> y = [2, 4, 6, 8, 10]
>>> stats.kruskal(x, y)
KruskalResult(statistic=0.27272727272727337, pvalue=0.60150813444058948)
```

```
>>> x = [1, 1, 1]
>>> y = [2, 2, 2]
>>> z = [2, 2]
>>> stats.kruskal(x, y, z)
KruskalResult(statistic=7.0, pvalue=0.030197383422318501)
```

`scipy.stats.friedmanchisquare` (\*args)

Computes the Friedman test for repeated measurements

The Friedman test tests the null hypothesis that repeated measurements of the same individuals have the same distribution. It is often used to test for consistency among measurements obtained in different ways. For example, if two measurement techniques are used on the same set of individuals, the Friedman test can be used to determine if the two measurement techniques are consistent.

<b>Parameters</b>	<b>measurements1, measurements2, measurements3...</b> : array_like Arrays of measurements. All of the arrays must have the same number of elements. At least 3 sets of measurements must be given.
<b>Returns</b>	<b>statistic</b> : float the test statistic, correcting for ties
	<b>pvalue</b> : float the associated p-value assuming that the test statistic has a chi squared distribution

***Notes***

Due to the assumption that the test statistic has a chi squared distribution, the p-value is only reliable for  $n > 10$  and more than 6 repeated measurements.

***References***

[R520]

`scipy.stats.combine_pvalues` (pvalues, method='fisher', weights=None)

Methods for combining the p-values of independent tests bearing upon the same hypothesis.

<b>Parameters</b>	<b>pvalues</b> : array_like, 1-D Array of p-values assumed to come from independent tests.
-------------------	---

**method** : { 'fisher', 'stouffer' }, optional  
 Name of method to use to combine p-values. The following methods are available:

- "fisher": Fisher's method (Fisher's combined probability test), the default.
- "stouffer": Stouffer's Z-score method.

**weights** : array\_like, 1-D, optional  
 Optional array of weights used only for Stouffer's Z-score method.

**Returns**

**statistic**: float  
 The statistic calculated by the specified method: - "fisher": The chi-squared statistic - "stouffer": The Z-score

**pval**: float  
 The combined p-value.

**Notes**

Fisher's method (also known as Fisher's combined probability test) [R507] uses a chi-squared statistic to compute a combined p-value. The closely related Stouffer's Z-score method [R508] uses Z-scores rather than p-values. The advantage of Stouffer's method is that it is straightforward to introduce weights, which can make Stouffer's method more powerful than Fisher's method when the p-values are from studies of different size [R509] [R510].

Fisher's method may be extended to combine p-values from dependent tests [R511]. Extensions such as Brown's method and Kost's method are not currently implemented.

New in version 0.15.0.

**References**

[R507], [R508], [R509], [R510], [R511]

`scipy.stats.ss` (\*args, \*\*kws)  
 ss is deprecated! scipy.stats.ss is deprecated in scipy 0.17.0

`scipy.stats.square_of_sums` (\*args, \*\*kws)  
 square\_of\_sums is deprecated! scipy.stats.square\_of\_sums is deprecated in scipy 0.17.0

`scipy.stats.jarque_bera` (x)  
 Perform the Jarque-Bera goodness of fit test on sample data.

The Jarque-Bera test tests whether the sample data has the skewness and kurtosis matching a normal distribution.

Note that this test only works for a large enough number of data samples (>2000) as the test statistic asymptotically has a Chi-squared distribution with 2 degrees of freedom.

**Parameters** **x** : array\_like  
 Observations of a random variable.

**Returns** **jb\_value** : float  
 The test statistic.

**p** : float  
 The p-value for the hypothesis test.

**References**

[R532]

**Examples**

```
>>> from scipy import stats
>>> np.random.seed(987654321)
>>> x = np.random.normal(0, 1, 100000)
```

```
>>> y = np.random.rayleigh(1, 100000)
>>> stats.jarque_bera(x)
(4.7165707989581342, 0.09458225503041906)
>>> stats.jarque_bera(y)
(6713.7098548143422, 0.0)
```

---

<code>ansari(x, y)</code>	Perform the Ansari-Bradley test for equal scale parameters
<code>bartlett(*args)</code>	Perform Bartlett's test for equal variances
<code>levene(*args, **kwds)</code>	Perform Levene test for equal variances.
<code>shapiro(x[, a, reta])</code>	Perform the Shapiro-Wilk test for normality.
<code>anderson(x[, dist])</code>	Anderson-Darling test for data coming from a particular distribution
<code>anderson_ksamp(samples[, midrank])</code>	The Anderson-Darling test for k-samples.
<code>binom_test(x[, n, p, alternative])</code>	Perform a test that the probability of success is p.
<code>fligner(*args, **kwds)</code>	Perform Fligner-Killeen test for equality of variance.
<code>median_test(*args, **kwds)</code>	Mood's median test.
<code>mood(x, y[, axis])</code>	Perform Mood's test for equal scale parameters.

---

`scipy.stats.ansari(x, y)`

Perform the Ansari-Bradley test for equal scale parameters

The Ansari-Bradley test is a non-parametric test for the equality of the scale parameter of the distributions from which two samples were drawn.

**Parameters** `x, y` : array\_like  
arrays of sample data

**Returns** **statistic** : float  
The Ansari-Bradley test statistic

**pvalue** : float  
The p-value of the hypothesis test

**See also:**

`fligner` A non-parametric test for the equality of k variances  
`mood` A non-parametric test for the equality of two scale parameters

**Notes**

The p-value given is exact when the sample sizes are both less than 55 and there are no ties, otherwise a normal approximation for the p-value is used.

**References**

[R493]

`scipy.stats.bartlett(*args)`

Perform Bartlett's test for equal variances

Bartlett's test tests the null hypothesis that all input samples are from populations with equal variances. For samples from significantly non-normal populations, Levene's test `levene` is more robust.

**Parameters** `sample1, sample2,...` : array\_like  
arrays of sample data. May be different lengths.

**Returns** **statistic** : float  
The test statistic.

**pvalue** : float  
The p-value of the test.

**See also:**

`fligner` A non-parametric test for the equality of k variances

**levene** A robust parametric test for equality of k variances

**Notes**

Conover et al. (1981) examine many of the existing parametric and nonparametric tests by extensive simulations and they conclude that the tests proposed by Fligner and Killeen (1976) and Levene (1960) appear to be superior in terms of robustness of departures from normality and power [R496].

**References**

[R494], [R495], [R496], [R497]

`scipy.stats.levene` (\*args, \*\*kws)  
 Perform Levene test for equal variances.

The Levene test tests the null hypothesis that all input samples are from populations with equal variances. Levene’s test is an alternative to Bartlett’s test `bartlett` in the case where there are significant deviations from normality.

**Parameters** `sample1, sample2, ...` : array\_like  
 The sample data, possibly with different lengths  
**center** : {‘mean’, ‘median’, ‘trimmed’}, optional  
 Which function of the data to use in the test. The default is ‘median’.  
**proportiontocut** : float, optional  
 When `center` is ‘trimmed’, this gives the proportion of data points to cut from each end. (See `scipy.stats.trim_mean`.) Default is 0.05.  
**Returns** **statistic** : float  
 The test statistic.  
**pvalue** : float  
 The p-value for the test.

**Notes**

Three variations of Levene’s test are possible. The possibilities and their recommended usages are:

- ‘median’ : Recommended for skewed (non-normal) distributions>
- ‘mean’ : Recommended for symmetric, moderate-tailed distributions.
- ‘trimmed’ : Recommended for heavy-tailed distributions.

**References**

[R537], [R538], [R539]

`scipy.stats.shapiro` (x, a=None, reta=False)  
 Perform the Shapiro-Wilk test for normality.

The Shapiro-Wilk test tests the null hypothesis that the data was drawn from a normal distribution.

**Parameters** **x** : array\_like  
 Array of sample data.  
**a** : array\_like, optional  
 Array of internal parameters used in the calculation. If these are not given, they will be computed internally. If x has length n, then a must have length n/2.  
**reta** : bool, optional  
 Whether or not to return the internally computed a values. The default is False.  
**Returns** **W** : float  
 The test statistic.  
**p-value** : float  
 The p-value for the hypothesis test.

**a** : array\_like, optional

If *reta* is True, then these are the internally computed “a” values that may be passed into this function on future calls.

**See also:**

***anderson*** The Anderson-Darling test for normality  
***kstest*** The Kolmogorov-Smirnov test for goodness of fit.

**Notes**

The algorithm used is described in [R568] but censoring parameters as described are not implemented. For  $N > 5000$  the W test statistic is accurate but the p-value may not be.

The chance of rejecting the null hypothesis when it is true is close to 5% regardless of sample size.

**References**

[R565], [R566], [R567], [R568]

**Examples**

```
>>> from scipy import stats
>>> np.random.seed(12345678)
>>> x = stats.norm.rvs(loc=5, scale=3, size=100)
>>> stats.shapiro(x)
(0.9772805571556091, 0.08144091814756393)
```

`scipy.stats.anderson(x, dist='norm')`

Anderson-Darling test for data coming from a particular distribution

The Anderson-Darling test is a modification of the Kolmogorov-Smirnov test *kstest* for the null hypothesis that a sample is drawn from a population that follows a particular distribution. For the Anderson-Darling test, the critical values depend on which distribution is being tested against. This function works for normal, exponential, logistic, or Gumbel (Extreme Value Type I) distributions.

**Parameters** **x** : array\_like  
array of sample data  
**dist** : { 'norm', 'expon', 'logistic', 'gumbel', 'extreme1' }, optional  
the type of distribution to test against. The default is 'norm' and 'extreme1' is a synonym for 'gumbel'

**Returns** **statistic** : float  
The Anderson-Darling test statistic  
**critical\_values** : list  
The critical values for this distribution  
**significance\_level** : list  
The significance levels for the corresponding critical values in percents. The function returns critical values for a differing set of significance levels depending on the distribution that is being tested against.

**Notes**

Critical values provided are for the following significance levels:

**normal/exponential** 15%, 10%, 5%, 2.5%, 1%  
**logistic** 25%, 10%, 5%, 2.5%, 1%, 0.5%  
**Gumbel** 25%, 10%, 5%, 2.5%, 1%

If A2 is larger than these critical values then for the corresponding significance level, the null hypothesis that the data come from the chosen distribution can be rejected.

### References

[R486], [R487], [R488], [R489], [R490], [R491]

`scipy.stats.anderson_ksamp` (*samples*, *midrank=True*)

The Anderson-Darling test for k-samples.

The k-sample Anderson-Darling test is a modification of the one-sample Anderson-Darling test. It tests the null hypothesis that k-samples are drawn from the same population without having to specify the distribution function of that population. The critical values depend on the number of samples.

<b>Parameters</b>	<b>samples</b> : sequence of 1-D array_like
	Array of sample data in arrays.
	<b>midrank</b> : bool, optional
	Type of Anderson-Darling test which is computed. Default (True) is the midrank test applicable to continuous and discrete populations. If False, the right side empirical distribution is used.
<b>Returns</b>	<b>statistic</b> : float
	Normalized k-sample Anderson-Darling test statistic.
	<b>critical_values</b> : array
	The critical values for significance levels 25%, 10%, 5%, 2.5%, 1%.
	<b>significance_level</b> : float
	An approximate significance level at which the null hypothesis for the provided samples can be rejected.
<b>Raises</b>	<b>ValueError</b>
	If less than 2 samples are provided, a sample is empty, or no distinct observations are in the samples.

### See also:

[`ks\_2samp`](#) 2 sample Kolmogorov-Smirnov test  
[`anderson`](#) 1 sample Anderson-Darling test

### Notes

[R492] Defines three versions of the k-sample Anderson-Darling test: one for continuous distributions and two for discrete distributions, in which ties between samples may occur. The default of this routine is to compute the version based on the midrank empirical distribution function. This test is applicable to continuous and discrete data. If *midrank* is set to False, the right side empirical distribution is used for a test for discrete data. According to [R492], the two discrete test statistics differ only slightly if a few collisions due to round-off errors occur in the test not adjusted for ties between samples.

New in version 0.14.0.

### References

[R492]

### Examples

```
>>> from scipy import stats
>>> np.random.seed(314159)
```

The null hypothesis that the two random samples come from the same distribution can be rejected at the 5% level because the returned test value is greater than the critical value for 5% (1.961) but not at the 2.5% level. The interpolation gives an approximate significance level of 3.1%:

```
>>> stats.anderson_ksamp([np.random.normal(size=50),
... np.random.normal(loc=0.5, size=30)])
(2.4615796189876105,
```

```
array([ 0.325,  1.226,  1.961,  2.718,  3.752]),
0.03134990135800783)
```

The null hypothesis cannot be rejected for three samples from an identical distribution. The approximate p-value (87%) has to be computed by extrapolation and may not be very accurate:

```
>>> stats.anderson_ksamp([np.random.normal(size=50),
... np.random.normal(size=30), np.random.normal(size=20)])
(-0.73091722665244196,
 array([ 0.44925884,  1.3052767 ,  1.9434184 ,  2.57696569,  3.41634856]),
 0.8789283903979661)
```

`scipy.stats.binom_test(x, n=None, p=0.5, alternative='two-sided')`

Perform a test that the probability of success is  $p$ .

This is an exact, two-sided test of the null hypothesis that the probability of success in a Bernoulli experiment is  $p$ .

**Parameters**

- x** : integer or array\_like  
the number of successes, or if x has length 2, it is the number of successes and the number of failures.
- n** : integer  
the number of trials. This is ignored if x gives both the number of successes and failures
- p** : float, optional  
The hypothesized probability of success.  $0 \leq p \leq 1$ . The default value is  $p = 0.5$
- alternative** : {'two-sided', 'greater', 'less'}, optional  
Indicates the alternative hypothesis. The default value is 'two-sided'.

**Returns**

- p-value** : float  
The p-value of the hypothesis test

### References

[R498]

`scipy.stats.fligner(*args, **kws)`

Perform Fligner-Killeen test for equality of variance.

Fligner's test tests the null hypothesis that all input samples are from populations with equal variances. Fligner-Killeen's test is distribution free when populations are identical [R517].

**Parameters**

- sample1, sample2, ...** : array\_like  
Arrays of sample data. Need not be the same length.
- center** : {'mean', 'median', 'trimmed'}, optional  
Keyword argument controlling which function of the data is used in computing the test statistic. The default is 'median'.
- proportiontocut** : float, optional  
When *center* is 'trimmed', this gives the proportion of data points to cut from each end. (See `scipy.stats.trim_mean`.) Default is 0.05.

**Returns**

- statistic** : float  
The test statistic.
- pvalue** : float  
The p-value for the hypothesis test.

See also:

**bartlett** A parametric test for equality of k variances in normal samples  
**levene** A robust parametric test for equality of k variances

*Notes*

As with Levene’s test there are three variants of Fligner’s test that differ by the measure of central tendency used in the test. See [levene](#) for more information.

Conover et al. (1981) examine many of the existing parametric and nonparametric tests by extensive simulations and they conclude that the tests proposed by Fligner and Killeen (1976) and Levene (1960) appear to be superior in terms of robustness of departures from normality and power [R518].

*References*

[R516], [R517], [R518], [R519]

`scipy.stats.median_test(*args, **kwargs)`

Mood’s median test.

Test that two or more samples come from populations with the same median.

Let  $n = \text{len}(\text{args})$  be the number of samples. The “grand median” of all the data is computed, and a contingency table is formed by classifying the values in each sample as being above or below the grand median. The contingency table, along with *correction* and *lambda\_*, are passed to `scipy.stats.chi2_contingency` to compute the test statistic and p-value.

**Parameters**    **sample1, sample2, ...** : array\_like

The set of samples. There must be at least two samples. Each sample must be a one-dimensional sequence containing at least one value. The samples are not required to have the same length.

**ties** : str, optional

Determines how values equal to the grand median are classified in the contingency table. The string must be one of:

"below":

Values equal to the grand median are counted as "below".

"above":

Values equal to the grand median are counted as "above".

"ignore":

Values equal to the grand median are not counted.

The default is “below”.

**correction** : bool, optional

If True, *and* there are just two samples, apply Yates’ correction for continuity when computing the test statistic associated with the contingency table. Default is True.

**lambda\_** : float or str, optional.

By default, the statistic computed in this test is Pearson’s chi-squared statistic. *lambda\_* allows a statistic from the Cressie-Read power divergence family to be used instead. See [power\\_divergence](#) for details. Default is 1 (Pearson’s chi-squared statistic).

**Returns**    **stat** : float

The test statistic. The statistic that is returned is determined by *lambda\_*. The default is Pearson’s chi-squared statistic.

**p** : float

The p-value of the test.

**m** : float

The grand median.

**table** : ndarray

The contingency table. The shape of the table is (2, n), where n is the number of samples. The first row holds the counts of the values above the grand median, and the second row holds the counts of



the values below the grand median. The table allows further analysis with, for example, `scipy.stats.chi2_contingency`, or with `scipy.stats.fisher_exact` if there are two samples, without having to recompute the table.

**See also:**

**`kruskal`** Compute the Kruskal-Wallis H-test for independent samples.  
**`mannwhitneyu`** Computes the Mann-Whitney rank test on samples x and y.

**Notes**

New in version 0.15.0.

**References**

[R541], [R542]

**Examples**

A biologist runs an experiment in which there are three groups of plants. Group 1 has 16 plants, group 2 has 15 plants, and group 3 has 17 plants. Each plant produces a number of seeds. The seed counts for each group are:

```
Group 1: 10 14 14 18 20 22 24 25 31 31 32 39 43 43 48 49
Group 2: 28 30 31 33 34 35 36 40 44 55 57 61 91 92 99
Group 3:  0  3  9 22 23 25 25 33 34 34 40 45 46 48 62 67 84
```

The following code applies Mood's median test to these samples.

```
>>> g1 = [10, 14, 14, 18, 20, 22, 24, 25, 31, 31, 32, 39, 43, 43, 48, 49]
>>> g2 = [28, 30, 31, 33, 34, 35, 36, 40, 44, 55, 57, 61, 91, 92, 99]
>>> g3 = [0, 3, 9, 22, 23, 25, 25, 33, 34, 34, 40, 45, 46, 48, 62, 67, 84]
>>> from scipy.stats import median_test
>>> stat, p, med, tbl = median_test(g1, g2, g3)
```

The median is

```
>>> med
34.0
```

and the contingency table is

```
>>> tbl
array([[ 5, 10,  7],
       [11,  5, 10]])
```

$p$  is too large to conclude that the medians are not the same:

```
>>> p
0.12609082774093244
```

The "G-test" can be performed by passing `lambda_="log-likelihood"` to `median_test`.

```
>>> g, p, med, tbl = median_test(g1, g2, g3, lambda_="log-likelihood")
>>> p
0.12224779737117837
```

The median occurs several times in the data, so we'll get a different result if, for example, `ties="above"` is used:

```
>>> stat, p, med, tbl = median_test(g1, g2, g3, ties="above")
>>> p
0.063873276069553273

>>> tbl
array([[ 5, 11,  9],
       [11,  4,  8]])
```

This example demonstrates that if the data set is not large and there are values equal to the median, the p-value can be sensitive to the choice of *ties*.

`scipy.stats.mood(x, y, axis=0)`

Perform Mood's test for equal scale parameters.

Mood's two-sample test for scale parameters is a non-parametric test for the null hypothesis that two samples are drawn from the same distribution with the same scale parameter.

**Parameters** `x, y` : array\_like  
 Arrays of sample data.

`axis` : int, optional  
 The axis along which the samples are tested. *x* and *y* can be of different length along *axis*. If *axis* is None, *x* and *y* are flattened and the test is done on all values in the flattened arrays.

**Returns** `z` : scalar or ndarray  
 The z-score for the hypothesis test. For 1-D inputs a scalar is returned.

`p-value` : scalar ndarray  
 The p-value for the hypothesis test.

See also:

***fligner*** A non-parametric test for the equality of k variances  
***ansari*** A non-parametric test for the equality of 2 variances  
***bartlett*** A parametric test for equality of k variances in normal samples  
***levene*** A parametric test for equality of k variances

**Notes**

The data are assumed to be drawn from probability distributions  $f(x)$  and  $f(x/s) / s$  respectively, for some probability density function *f*. The null hypothesis is that  $s == 1$ .

For multi-dimensional arrays, if the inputs are of shapes  $(n_0, n_1, n_2, n_3)$  and  $(n_0, m_1, n_2, n_3)$ , then if `axis=1`, the resulting *z* and *p* values will have shape  $(n_0, n_2, n_3)$ . Note that *n1* and *m1* don't have to be equal, but the other dimensions do.

**Examples**

```
>>> from scipy import stats
>>> np.random.seed(1234)
>>> x2 = np.random.randn(2, 45, 6, 7)
>>> x1 = np.random.randn(2, 30, 6, 7)
>>> z, p = stats.mood(x1, x2, axis=1)
>>> p.shape
(2, 6, 7)
```

Find the number of points where the difference in scale is not significant:

```
>>> (p > 0.1).sum()
74
```

Perform the test with different scales:

```
>>> x1 = np.random.randn(2, 30)
>>> x2 = np.random.randn(2, 35) * 10.0
>>> stats.mood(x1, x2, axis=1)
(array([-5.7178125 , -5.25342163]), array([ 1.07904114e-08,  1.49299218e-07]))
```

---

<code>boxcox(x[, lmbda, alpha])</code>	Return a positive dataset transformed by a Box-Cox power transformation.
<code>boxcox_normmax(x[, brack, method])</code>	Compute optimal Box-Cox transform parameter for input data.
<code>boxcox_llf(lmb, data)</code>	The boxcox log-likelihood function.
<code>entropy(pk[, qk, base])</code>	Calculate the entropy of a distribution for given probability values.

---

`scipy.stats.boxcox(x, lmbda=None, alpha=None)`

Return a positive dataset transformed by a Box-Cox power transformation.

**Parameters** `x` : ndarray

Input array. Should be 1-dimensional.

**lmbda** : {None, scalar}, optional

If *lmbda* is not None, do the transformation for that value.

If *lmbda* is None, find the lambda that maximizes the log-likelihood function and return it as the second output argument.

**alpha** : {None, float}, optional

If alpha is not None, return the  $100 * (1-\alpha)\%$  confidence interval for *lmbda* as the third output argument. Must be between 0.0 and 1.0.

**Returns**

**boxcox** : ndarray

Box-Cox power transformed array.

**maxlog** : float, optional

If the *lmbda* parameter is None, the second returned argument is the lambda that maximizes the log-likelihood function.

**(min\_ci, max\_ci)** : tuple of float, optional

If *lmbda* parameter is None and alpha is not None, this returned tuple of floats represents the minimum and maximum confidence limits given alpha.

**See also:**

`probplot`, `boxcox_normplot`, `boxcox_normmax`, `boxcox_llf`

**Notes**

The Box-Cox transform is given by:

$$y = \begin{cases} (x^{*\text{lmbda}} - 1) / \text{lmbda}, & \text{for lmbda} > 0 \\ \log(x), & \text{for lmbda} = 0 \end{cases}$$

`boxcox` requires the input data to be positive. Sometimes a Box-Cox transformation provides a shift parameter to achieve this; `boxcox` does not. Such a shift parameter is equivalent to adding a positive constant to *x* before calling `boxcox`.

The confidence limits returned when alpha is provided give the interval where:

$$llf(\hat{\lambda}) - llf(\lambda) < \frac{1}{2}\chi^2(1 - \alpha, 1),$$

with `llf` the log-likelihood function and  $\chi^2$  the chi-squared function.

### References

G.E.P. Box and D.R. Cox, “An Analysis of Transformations”, Journal of the Royal Statistical Society B, 26, 211-252 (1964).

### Examples

```
>>> from scipy import stats
>>> import matplotlib.pyplot as plt
```

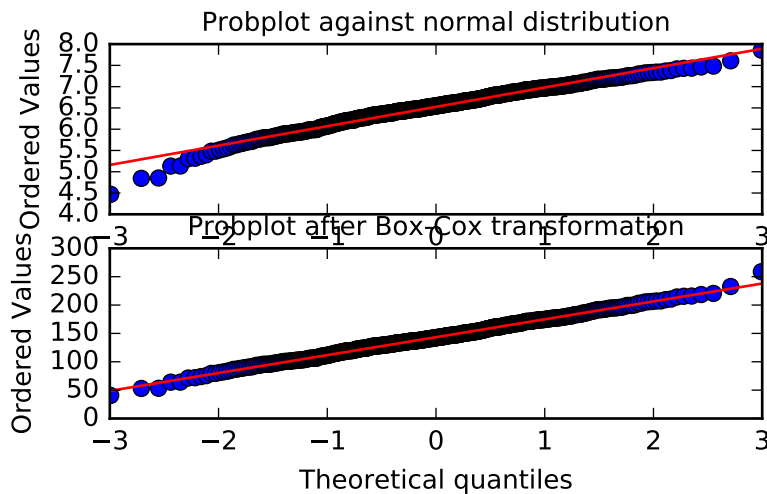
We generate some random variates from a non-normal distribution and make a probability plot for it, to show it is non-normal in the tails:

```
>>> fig = plt.figure()
>>> ax1 = fig.add_subplot(211)
>>> x = stats.loggamma.rvs(5, size=500) + 5
>>> prob = stats.probplot(x, dist=stats.norm, plot=ax1)
>>> ax1.set_xlabel('')
>>> ax1.set_title('Probplot against normal distribution')
```

We now use `boxcox` to transform the data so it's closest to normal:

```
>>> ax2 = fig.add_subplot(212)
>>> xt, _ = stats.boxcox(x)
>>> prob = stats.probplot(xt, dist=stats.norm, plot=ax2)
>>> ax2.set_title('Probplot after Box-Cox transformation')

>>> plt.show()
```



```
scipy.stats.boxcox_normmax(x, brack=(-2.0, 2.0), method='pearsonr')
Compute optimal Box-Cox transform parameter for input data.
```

**Parameters**

- `x` : array\_like  
Input array.
- `brack` : 2-tuple, optional

The starting interval for a downhill bracket search with `optimize.brent`. Note that this is in most cases not critical; the final result is allowed to be outside this bracket.

**method** : str, optional

The method to determine the optimal transform parameter (`boxcox` lambda parameter). Options are:

**'pearsonr'** (default)

Maximizes the Pearson correlation coefficient between  $y = \text{boxcox}(x)$  and the expected values for  $y$  if  $x$  would be normally-distributed.

**'mle'**

Minimizes the log-likelihood `boxcox_llf`. This is the method used in `boxcox`.

**'all'**

Use all optimization methods available, and return all results. Useful to compare different methods.

**Returns** **maxlog** : float or ndarray

The optimal transform parameter found. An array instead of a scalar for `method='all'`.

**See also:**

`boxcox`, `boxcox_llf`, `boxcox_normplot`

**Examples**

```
>>> from scipy import stats
>>> import matplotlib.pyplot as plt
>>> np.random.seed(1234) # make this example reproducible
```

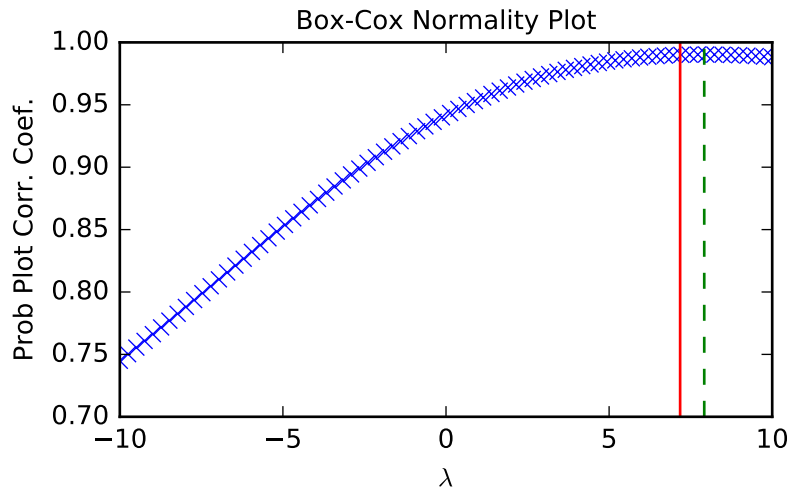
Generate some data and determine optimal lambda in various ways:

```
>>> x = stats.loggamma.rvs(5, size=30) + 5
>>> y, lmax_mle = stats.boxcox(x)
>>> lmax_pearsonr = stats.boxcox_normmax(x)

>>> lmax_mle
7.177...
>>> lmax_pearsonr
7.916...
>>> stats.boxcox_normmax(x, method='all')
array([ 7.91667384,  7.17718692])

>>> fig = plt.figure()
>>> ax = fig.add_subplot(111)
>>> prob = stats.boxcox_normplot(x, -10, 10, plot=ax)
>>> ax.axvline(lmax_mle, color='r')
>>> ax.axvline(lmax_pearsonr, color='g', ls='--')

>>> plt.show()
```



`scipy.stats.boxcox_llf(lmb, data)`

The boxcox log-likelihood function.

**Parameters** **lmb** : scalar

Parameter for Box-Cox transformation. See `boxcox` for details.

**data** : array\_like

Data to calculate Box-Cox log-likelihood for. If *data* is multi-dimensional, the log-likelihood is calculated along the first axis.

**Returns** **llf** : float or ndarray

Box-Cox log-likelihood of *data* given *lmb*. A float for 1-D *data*, an array otherwise.

**See also:**

`boxcox`, `probplot`, `boxcox_normplot`, `boxcox_normmax`

**Notes**

The Box-Cox log-likelihood function is defined here as

$$llf = (\lambda - 1) \sum_i (\log(x_i)) - N/2 \log(\sum_i (y_i - \bar{y})^2 / N),$$

where  $y$  is the Box-Cox transformed input data  $x$ .

**Examples**

```
>>> from scipy import stats
>>> import matplotlib.pyplot as plt
>>> from mpl_toolkits.axes_grid1.inset_locator import inset_axes
>>> np.random.seed(1245)
```

Generate some random variates and calculate Box-Cox log-likelihood values for them for a range of `lmbda` values:

```
>>> x = stats.loggamma.rvs(5, loc=10, size=1000)
>>> lmbdas = np.linspace(-2, 10)
>>> llf = np.zeros(lmbdas.shape, dtype=float)
```

```
>>> for ii, lambda in enumerate(lmbdas):
...     llf[ii] = stats.boxcox_llf(lambda, x)
```

Also find the optimal lambda value with `boxcox`:

```
>>> x_most_normal, lambda_optimal = stats.boxcox(x)
```

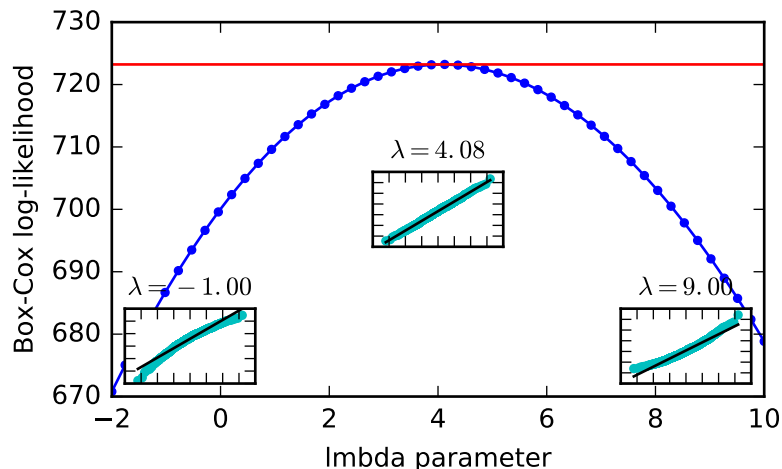
Plot the log-likelihood as function of lambda. Add the optimal lambda as a horizontal line to check that that's really the optimum:

```
>>> fig = plt.figure()
>>> ax = fig.add_subplot(111)
>>> ax.plot(lmbdas, llf, 'b.-')
>>> ax.axhline(stats.boxcox_llf(lambda_optimal, x), color='r')
>>> ax.set_xlabel('lambda parameter')
>>> ax.set_ylabel('Box-Cox log-likelihood')
```

Now add some probability plots to show that where the log-likelihood is maximized the data transformed with `boxcox` looks closest to normal:

```
>>> locs = [3, 10, 4] # 'lower left', 'center', 'lower right'
>>> for lambda, loc in zip([-1, lambda_optimal, 9], locs):
...     xt = stats.boxcox(x, lambda=lambda)
...     (osm, osr), (slope, intercept, r_sq) = stats.probplot(xt)
...     ax_inset = inset_axes(ax, width="20%", height="20%", loc=loc)
...     ax_inset.plot(osm, osr, 'c.', osm, slope*osm + intercept, 'k-')
...     ax_inset.set_xticklabels([])
...     ax_inset.set_yticklabels([])
...     ax_inset.set_title('\$\\lambda=%1.2f\$' % lambda)

>>> plt.show()
```



`scipy.stats.entropy(pk, qk=None, base=None)`

Calculate the entropy of a distribution for given probability values.

If only probabilities  $pk$  are given, the entropy is calculated as  $S = -\sum(pk * \log(pk))$ , `axis=0`.

If  $q_k$  is not None, then compute the Kullback-Leibler divergence  $S = \sum(pk * \log(pk / qk), axis=0)$ .

This routine will normalize  $pk$  and  $qk$  if they don't sum to 1.

**Parameters**

- pk** : sequence  
Defines the (discrete) distribution.  $pk[i]$  is the (possibly unnormalized) probability of event  $i$ .
- qk** : sequence, optional  
Sequence against which the relative entropy is computed. Should be in the same format as  $pk$ .
- base** : float, optional  
The logarithmic base to use, defaults to  $e$  (natural logarithm).

**Returns**

- S** : float  
The calculated entropy.

---

`chisqprob(*args, **kwds)` `chisqprob` is deprecated!  
`betai(*args, **kwds)` `betai` is deprecated!

---

`scipy.stats.chisqprob(*args, **kwds)`

`chisqprob` is deprecated! `stats.chisqprob` is deprecated in scipy 0.17.0; use `stats.distributions.chi2.sf` instead.

Probability value (1-tail) for the Chi<sup>2</sup> probability distribution.  
 Broadcasting rules apply.

**Parameters**

- chisq** : array\_like or float > 0

**Returns**

- chisqprob** : ndarray  
df : array\_like or float, probably int >= 1  
The area from  $chisq$  to infinity under the Chi<sup>2</sup> probability distribution with degrees of freedom  $df$ .

`scipy.stats.betai(*args, **kwds)`

`betai` is deprecated! `stats.betai` is deprecated in scipy 0.17.0; use `special.betainc` instead

Returns the incomplete beta function.  
 $I_x(a,b) = 1/B(a,b) * (\text{Integral}(0,x) \text{ of } t^{a-1}(1-t)^{b-1} dt)$   
 where  $a,b > 0$  and  $B(a,b) = G(a)G(b)/(G(a+b))$  where  $G(a)$  is the gamma function of  $a$ .  
 The standard broadcasting rules apply to  $a$ ,  $b$ , and  $x$ .

**Parameters**

- a** : array\_like or float > 0
- b** : array\_like or float > 0
- x** : [array\_like or float]  $x$  will be clipped to be no greater than 1.0.

**Returns**

- betai** : ndarray  
Incomplete beta function.

### 5.27.5 Circular statistical functions

---

<code>circmean(samples[, high, low, axis])</code>	Compute the circular mean for samples in a range.
<code>circvar(samples[, high, low, axis])</code>	Compute the circular variance for samples assumed to be in a range
<code>circstd(samples[, high, low, axis])</code>	Compute the circular standard deviation for samples assumed to be in the range [low to high]

---

`scipy.stats.circmean(samples, high=6.283185307179586, low=0, axis=None)`

Compute the circular mean for samples in a range.



**Parameters**

- samples** : array\_like  
Input array.
- high** : float or int, optional  
High boundary for circular mean range. Default is  $2\pi$ .
- low** : float or int, optional  
Low boundary for circular mean range. Default is 0.
- axis** : int, optional  
Axis along which means are computed. The default is to compute the mean of the flattened array.

**Returns**

- circmean** : float  
Circular mean.

`scipy.stats.circvar(samples, high=6.283185307179586, low=0, axis=None)`

Compute the circular variance for samples assumed to be in a range

**Parameters**

- samples** : array\_like  
Input array.
- low** : float or int, optional  
Low boundary for circular variance range. Default is 0.
- high** : float or int, optional  
High boundary for circular variance range. Default is  $2\pi$ .
- axis** : int, optional  
Axis along which variances are computed. The default is to compute the variance of the flattened array.

**Returns**

- circvar** : float  
Circular variance.

**Notes**

This uses a definition of circular variance that in the limit of small angles returns a number close to the ‘linear’ variance.

`scipy.stats.circstd(samples, high=6.283185307179586, low=0, axis=None)`

Compute the circular standard deviation for samples assumed to be in the range [low to high].

**Parameters**

- samples** : array\_like  
Input array.
- low** : float or int, optional  
Low boundary for circular standard deviation range. Default is 0.
- high** : float or int, optional  
High boundary for circular standard deviation range. Default is  $2\pi$ .
- axis** : int, optional  
Axis along which standard deviations are computed. The default is to compute the standard deviation of the flattened array.

**Returns**

- circstd** : float  
Circular standard deviation.

**Notes**

This uses a definition of circular standard deviation that in the limit of small angles returns a number close to the ‘linear’ standard deviation.

### 5.27.6 Contingency table functions

<code>chi2_contingency(observed[, correction, lambda_])</code>	Chi-square test of independence of variables in a contingency table.
<code>contingency.expected_freq(observed)</code>	Compute the expected frequencies from a contingency table.
Continued on next page	

Table 5.290 – continued from previous page

<code>contingency.margins(a)</code>	Return a list of the marginal sums of the array <i>a</i> .
<code>fisher_exact(table[, alternative])</code>	Performs a Fisher exact test on a 2x2 contingency table.

`scipy.stats.chi2_contingency` (*observed*, *correction=True*, *lambda\_=None*)

Chi-square test of independence of variables in a contingency table.

This function computes the chi-square statistic and p-value for the hypothesis test of independence of the observed frequencies in the contingency table [R502] *observed*. The expected frequencies are computed based on the marginal sums under the assumption of independence; see `scipy.stats.contingency.expected_freq`. The number of degrees of freedom is (expressed using numpy functions and attributes):

```
dof = observed.size - sum(observed.shape) + observed.ndim - 1
```

- Parameters**
- observed** : array\_like  
The contingency table. The table contains the observed frequencies (i.e. number of occurrences) in each category. In the two-dimensional case, the table is often described as an “R x C table”.
  - correction** : bool, optional  
If True, *and* the degrees of freedom is 1, apply Yates’ correction for continuity. The effect of the correction is to adjust each observed value by 0.5 towards the corresponding expected value.
  - lambda\_** : float or str, optional.  
By default, the statistic computed in this test is Pearson’s chi-squared statistic [R503]. *lambda\_* allows a statistic from the Cressie-Read power divergence family [R504] to be used instead. See `power_divergence` for details.
- Returns**
- chi2** : float  
The test statistic.
  - p** : float  
The p-value of the test
  - dof** : int  
Degrees of freedom
  - expected** : ndarray, same shape as *observed*  
The expected frequencies, based on the marginal sums of the table.

**See also:**

`contingency.expected_freq`, `fisher_exact`, `chisquare`, `power_divergence`

**Notes**

An often quoted guideline for the validity of this calculation is that the test should be used only if the observed and expected frequency in each cell is at least 5.

This is a test for the independence of different categories of a population. The test is only meaningful when the dimension of *observed* is two or more. Applying the test to a one-dimensional table will always result in *expected* equal to *observed* and a chi-square statistic equal to 0.

This function does not handle masked arrays, because the calculation does not make sense with missing values.

Like `stats.chisquare`, this function computes a chi-square statistic; the convenience this function provides is to figure out the expected frequencies and degrees of freedom from the given contingency table. If these were already known, and if the Yates’ correction was not required, one could use `stats.chisquare`. That is, if one calls:

```
chi2, p, dof, ex = chi2_contingency(obs, correction=False)
```

then the following is true:

```
(chi2, p) == stats.chisquare(obs.ravel(), f_exp=ex.ravel(),
                             ddof=obs.size - 1 - dof)
```

The `lambda_` argument was added in version 0.13.0 of scipy.

### References

[R502], [R503], [R504]

### Examples

A two-way example (2 x 3):

```
>>> from scipy.stats import chi2_contingency
>>> obs = np.array([[10, 10, 20], [20, 20, 20]])
>>> chi2_contingency(obs)
(2.7777777777777777,
 0.24935220877729619,
 2,
 array([[ 12.,  12.,  16.],
        [ 18.,  18.,  24.]])
```

Perform the test using the log-likelihood ratio (i.e. the “G-test”) instead of Pearson’s chi-squared statistic.

```
>>> g, p, dof, expctd = chi2_contingency(obs, lambda_="log-likelihood")
>>> g, p
(2.7688587616781319, 0.25046668010954165)
```

A four-way example (2 x 2 x 2 x 2):

```
>>> obs = np.array(
...     [[[12, 17],
...        [11, 16]],
...      [[11, 12],
...        [15, 16]]],
...     [[[23, 15],
...        [30, 22]],
...      [[14, 17],
...        [15, 16]]])
>>> chi2_contingency(obs)
(8.7584514426741897,
 0.64417725029295503,
 11,
 array([[[[ 14.15462386,  14.15462386],
           [ 16.49423111,  16.49423111]],
         [[ 11.2461395 ,  11.2461395 ],
           [ 13.10500554,  13.10500554]]],
        [[ 19.5591166 ,  19.5591166 ],
           [ 22.79202844,  22.79202844]],
         [[ 15.54012004,  15.54012004],
           [ 18.10873492,  18.10873492]]])
```

`scipy.stats.contingency.expected_freq` (*observed*)

Compute the expected frequencies from a contingency table.

Given an n-dimensional contingency table of observed frequencies, compute the expected frequencies for the table based on the marginal sums under the assumption that the groups associated with each dimension are independent.

**Parameters**    **observed** : array\_like  
 The table of observed frequencies. (While this function can handle a 1-D array, that case is trivial. Generally *observed* is at least 2-D.)

**Returns**        **expected** : ndarray of float64  
 The expected frequencies, based on the marginal sums of the table. Same shape as *observed*.

### Examples

```
>>> observed = np.array([[10, 10, 20],[20, 20, 20]])
>>> from scipy.stats import expected_freq
>>> expected_freq(observed)
array([[ 12.,  12.,  16.],
       [ 18.,  18.,  24.]])
```

`scipy.stats.contingency.margins` (*a*)

Return a list of the marginal sums of the array *a*.

**Parameters**    **a** : ndarray  
 The array for which to compute the marginal sums.

**Returns**        **margsums** : list of ndarrays  
 A list of length *a.ndim*. *margsums[k]* is the result of summing *a* over all axes except *k*; it has the same number of dimensions as *a*, but the length of each axis except axis *k* will be 1.

### Examples

```
>>> a = np.arange(12).reshape(2, 6)
>>> a
array([[ 0,  1,  2,  3,  4,  5],
       [ 6,  7,  8,  9, 10, 11]])
>>> m0, m1 = margins(a)
>>> m0
array([[15],
       [51]])
>>> m1
array([[ 6,  8, 10, 12, 14, 16]])

>>> b = np.arange(24).reshape(2, 3, 4)
>>> m0, m1, m2 = margins(b)
>>> m0
array([[[ 66]],
       [[210]])]
>>> m1
array([[[ 60],
        [ 92],
        [124]])]
>>> m2
array([[[60, 66, 72, 78]])]
```

`scipy.stats.fisher_exact` (*table*, *alternative='two-sided'*)

Performs a Fisher exact test on a 2x2 contingency table.

**Parameters**

- table** : array\_like of ints  
A 2x2 contingency table. Elements should be non-negative integers.
- alternative** : {'two-sided', 'less', 'greater'}, optional  
Which alternative hypothesis to the null hypothesis the test uses. Default is 'two-sided'.

**Returns**

- oddsratio** : float  
This is prior odds ratio and not a posterior estimate.
- p\_value** : float  
P-value, the probability of obtaining a distribution at least as extreme as the one that was actually observed, assuming that the null hypothesis is true.

**See also:**

[\*chi2\\_contingency\*](#)

Chi-square test of independence of variables in a contingency table.

**Notes**

The calculated odds ratio is different from the one R uses. This scipy implementation returns the (more common) “unconditional Maximum Likelihood Estimate”, while R uses the “conditional Maximum Likelihood Estimate”.

For tables with large numbers, the (inexact) chi-square test implemented in the function [\*chi2\\_contingency\*](#) can also be used.

**Examples**

Say we spend a few days counting whales and sharks in the Atlantic and Indian oceans. In the Atlantic ocean we find 8 whales and 1 shark, in the Indian ocean 2 whales and 5 sharks. Then our contingency table is:

	Atlantic	Indian
whales	8	2
sharks	1	5

We use this table to find the p-value:

```
>>> import scipy.stats as stats
>>> oddsratio, pvalue = stats.fisher_exact([[8, 2], [1, 5]])
>>> pvalue
0.0349...
```

The probability that we would observe this or an even more imbalanced ratio by chance is about 3.5%. A commonly used significance level is 5%—if we adopt that, we can therefore conclude that our observed imbalance is statistically significant; whales prefer the Atlantic while sharks prefer the Indian ocean.

## 5.27.7 Plot-tests

---

<code>ppcc_max(x[, brack, dist])</code>	Calculate the shape parameter that maximizes the PPCC
<code>ppcc_plot(x, a, b[, dist, plot, N])</code>	Calculate and optionally plot probability plot correlation coefficient.
<code>probplot(x[, sparams, dist, fit, plot, rvalue])</code>	Calculate quantiles for a probability plot, and optionally show the plot.
<code>boxcox_normplot(x, la, lb[, plot, N])</code>	Compute parameters for a Box-Cox normality plot, optionally show it.

---

`scipy.stats.ppcc_max(x, brack=(0.0, 1.0), dist='tukeylambda')`

Calculate the shape parameter that maximizes the PPCC

The probability plot correlation coefficient (PPCC) plot can be used to determine the optimal shape parameter for a one-parameter family of distributions. `ppcc_max` returns the shape parameter that would maximize the

probability plot correlation coefficient for the given data to a one-parameter family of distributions.

**Parameters**

- x** : array\_like  
Input array.
- brack** : tuple, optional  
Triple (a,b,c) where (a<b<c). If bracket consists of two numbers (a, c) then they are assumed to be a starting interval for a downhill bracket search (see `scipy.optimize.brent`).
- dist** : str or stats.distributions instance, optional  
Distribution or distribution function name. Objects that look enough like a stats.distributions instance (i.e. they have a `ppf` method) are also accepted. The default is `'tukeylambda'`.

**Returns**

- shape\_value** : float  
The shape parameter at which the probability plot correlation coefficient reaches its max value.

**See also:**

`ppcc_plot`, `probplot`, `boxcox`

**Notes**

The `brack` keyword serves as a starting point which is useful in corner cases. One can use a plot to obtain a rough visual estimate of the location for the maximum to start the search near it.

**References**

[R560], [R561]

**Examples**

First we generate some random data from a Tukey-Lambda distribution, with shape parameter -0.7:

```
>>> from scipy import stats
>>> x = stats.tukeylambda.rvs(-0.7, loc=2, scale=0.5, size=10000,
...                          random_state=1234567) + 1e4
```

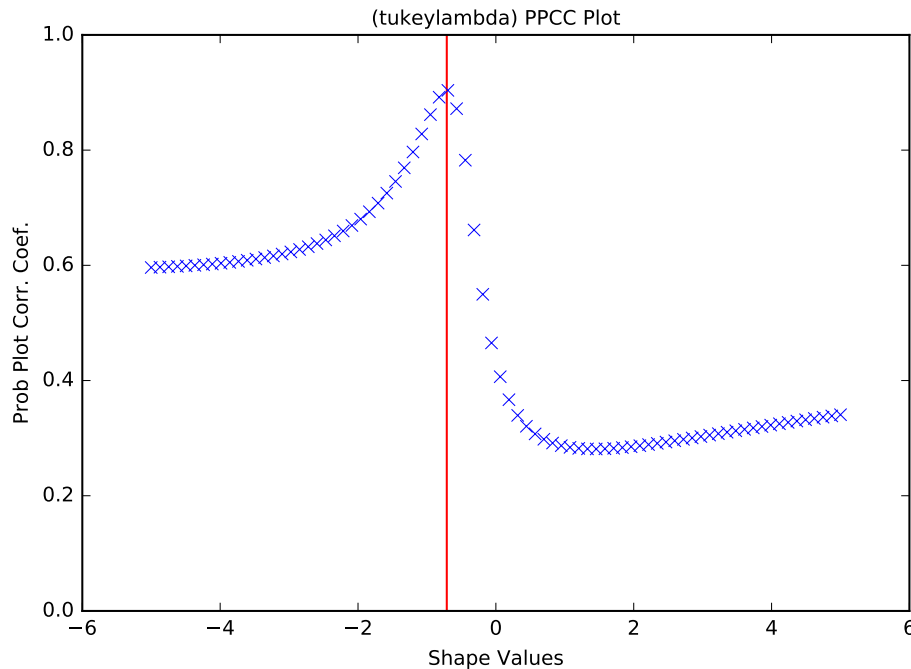
Now we explore this data with a PPCC plot as well as the related probability plot and Box-Cox normplot. A red line is drawn where we expect the PPCC value to be maximal (at the shape parameter -0.7 used above):

```
>>> import matplotlib.pyplot as plt
>>> fig = plt.figure(figsize=(8, 6))
>>> ax = fig.add_subplot(111)
>>> res = stats.ppcc_plot(x, -5, 5, plot=ax)
```

We calculate the value where the shape should reach its maximum and a red line is drawn there. The line should coincide with the highest point in the `ppcc_plot`.

```
>>> max = stats.ppcc_max(x)
>>> ax.vlines(max, 0, 1, colors='r', label='Expected shape value')

>>> plt.show()
```



`scipy.stats.ppcc_plot` (*x*, *a*, *b*, *dist*='tukeylambda', *plot*=None, *N*=80)

Calculate and optionally plot probability plot correlation coefficient.

The probability plot correlation coefficient (PPCC) plot can be used to determine the optimal shape parameter for a one-parameter family of distributions. It cannot be used for distributions without shape parameters (like the normal distribution) or with multiple shape parameters.

By default a Tukey-Lambda distribution (`stats.tukeylambda`) is used. A Tukey-Lambda PPCC plot interpolates from long-tailed to short-tailed distributions via an approximately normal one, and is therefore particularly useful in practice.

**Parameters** **x** : array\_like

Input array.

**a, b**: scalar

Lower and upper bounds of the shape parameter to use.

**dist** : str or `stats.distributions` instance, optional

Distribution or distribution function name. Objects that look enough like a `stats.distributions` instance (i.e. they have a `ppf` method) are also accepted. The default is 'tukeylambda'.

**plot** : object, optional

If given, plots PPCC against the shape parameter. *plot* is an object that has to have methods "plot" and "text". The `matplotlib.pyplot` module or a Matplotlib Axes object can be used, or a custom object with the same methods. Default is None, which means that no plot is created.

**N** : int, optional

Number of points on the horizontal axis (equally distributed from *a* to *b*).

**Returns** **svals** : ndarray

The shape values for which *ppcc* was calculated.

**ppcc** : ndarray

The calculated probability plot correlation coefficient values.

**See also:**

`ppcc_max`, `probplot`, `boxcox_normplot`, `tukeylambda`

### References

J.J. Filliben, “The Probability Plot Correlation Coefficient Test for Normality”, *Technometrics*, Vol. 17, pp. 111-117, 1975.

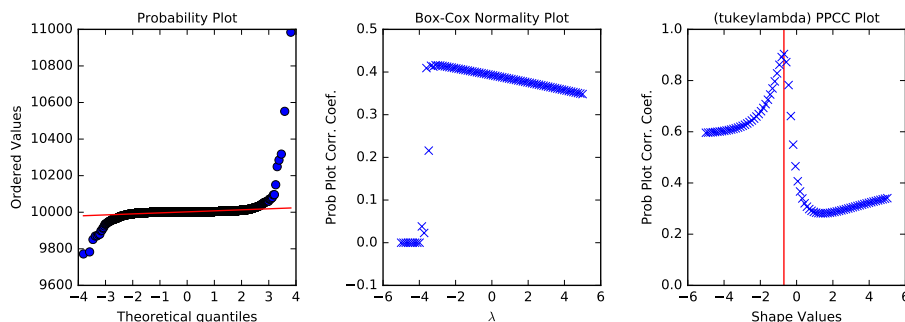
### Examples

First we generate some random data from a Tukey-Lambda distribution, with shape parameter -0.7:

```
>>> from scipy import stats
>>> import matplotlib.pyplot as plt
>>> np.random.seed(1234567)
>>> x = stats.tukeylambda.rvs(-0.7, loc=2, scale=0.5, size=10000) + 1e4
```

Now we explore this data with a PPCC plot as well as the related probability plot and Box-Cox normplot. A red line is drawn where we expect the PPCC value to be maximal (at the shape parameter -0.7 used above):

```
>>> fig = plt.figure(figsize=(12, 4))
>>> ax1 = fig.add_subplot(131)
>>> ax2 = fig.add_subplot(132)
>>> ax3 = fig.add_subplot(133)
>>> res = stats.probplot(x, plot=ax1)
>>> res = stats.boxcox_normplot(x, -5, 5, plot=ax2)
>>> res = stats.ppcc_plot(x, -5, 5, plot=ax3)
>>> ax3.vlines(-0.7, 0, 1, colors='r', label='Expected shape value')
>>> plt.show()
```



```
scipy.stats.probplot(x, sparams=(), dist='norm', fit=True, plot=None, rvalue=False)
```

Calculate quantiles for a probability plot, and optionally show the plot.

Generates a probability plot of sample data against the quantiles of a specified theoretical distribution (the normal distribution by default). `probplot` optionally calculates a best-fit line for the data and plots the results using Matplotlib or a given plot function.

**Parameters** **x** : array\_like

Sample/response data from which `probplot` creates the plot.

**sparams** : tuple, optional



Distribution-specific shape parameters (shape parameters plus location and scale).

**dist** : str or stats.distributions instance, optional  
Distribution or distribution function name. The default is 'norm' for a normal probability plot. Objects that look enough like a stats.distributions instance (i.e. they have a `ppf` method) are also accepted.

**fit** : bool, optional  
Fit a least-squares regression (best-fit) line to the sample data if True (default).

**plot** : object, optional  
If given, plots the quantiles and least squares fit. *plot* is an object that has to have methods "plot" and "text". The `matplotlib.pyplot` module or a Matplotlib Axes object can be used, or a custom object with the same methods. Default is None, which means that no plot is created.

**Returns** (**osm, osr**) : tuple of ndarrays  
Tuple of theoretical quantiles (*osm*, or order statistic medians) and ordered responses (*osr*). *osr* is simply sorted input *x*. For details on how *osm* is calculated see the Notes section.

(**slope, intercept, r**) : tuple of floats, optional  
Tuple containing the result of the least-squares fit, if that is performed by `probplot`. *r* is the square root of the coefficient of determination. If `fit=False` and `plot=None`, this tuple is not returned.

### Notes

Even if *plot* is given, the figure is not shown or saved by `probplot`; `plt.show()` or `plt.savefig('figname.png')` should be used after calling `probplot`.

`probplot` generates a probability plot, which should not be confused with a Q-Q or a P-P plot. Statsmodels has more extensive functionality of this type, see `statsmodels.api.ProbPlot`.

The formula used for the theoretical quantiles (horizontal axis of the probability plot) is Filliben's estimate:

```
quantiles = dist.ppf(val), for
    0.5**(1/n),                for i = n
    val = (i - 0.3175) / (n + 0.365), for i = 2, ..., n-1
    1 - 0.5**(1/n),          for i = 1
```

where *i* indicates the *i*-th ordered value and *n* is the total number of values.

### Examples

```
>>> from scipy import stats
>>> import matplotlib.pyplot as plt
>>> nsample = 100
>>> np.random.seed(7654321)
```

A t distribution with small degrees of freedom:

```
>>> ax1 = plt.subplot(221)
>>> x = stats.t.rvs(3, size=nsample)
>>> res = stats.probplot(x, plot=plt)
```

A t distribution with larger degrees of freedom:

```
>>> ax2 = plt.subplot(222)
>>> x = stats.t.rvs(25, size=nsample)
>>> res = stats.probplot(x, plot=plt)
```

A mixture of two normal distributions with broadcasting:

```
>>> ax3 = plt.subplot(223)
>>> x = stats.norm.rvs(loc=[0,5], scale=[1,1.5],
...                   size=(nsample//2,2)).ravel()
>>> res = stats.probplot(x, plot=plt)
```

A standard normal distribution:

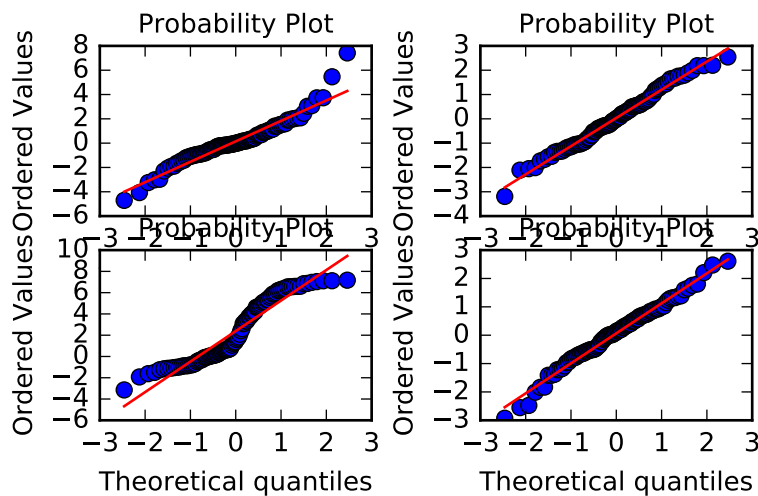
```
>>> ax4 = plt.subplot(224)
>>> x = stats.norm.rvs(loc=0, scale=1, size=nsample)
>>> res = stats.probplot(x, plot=plt)
```

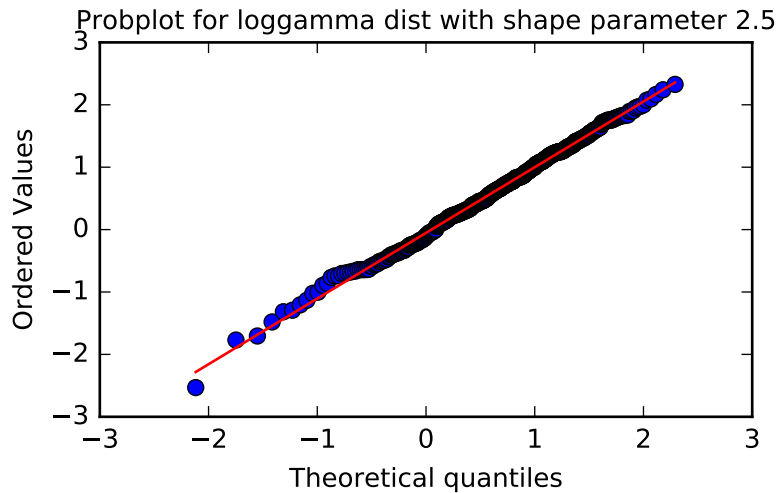
Produce a new figure with a loggamma distribution, using the `dist` and `sparams` keywords:

```
>>> fig = plt.figure()
>>> ax = fig.add_subplot(111)
>>> x = stats.loggamma.rvs(c=2.5, size=500)
>>> res = stats.probplot(x, dist=stats.loggamma, sparams=(2.5,), plot=ax)
>>> ax.set_title("Probplot for loggamma dist with shape parameter 2.5")
```

Show the results with Matplotlib:

```
>>> plt.show()
```





`scipy.stats.boxcox_normplot` (*x*, *la*, *lb*, *plot=None*, *N=80*)

Compute parameters for a Box-Cox normality plot, optionally show it.

A Box-Cox normality plot shows graphically what the best transformation parameter is to use in `boxcox` to obtain a distribution that is close to normal.

**Parameters**

- x** : array\_like  
Input array.
- la, lb** : scalar  
The lower and upper bounds for the `lambda` values to pass to `boxcox` for Box-Cox transformations. These are also the limits of the horizontal axis of the plot if that is generated.
- plot** : object, optional  
If given, plots the quantiles and least squares fit. *plot* is an object that has to have methods “plot” and “text”. The `matplotlib.pyplot` module or a Matplotlib Axes object can be used, or a custom object with the same methods. Default is `None`, which means that no plot is created.
- N** : int, optional  
Number of points on the horizontal axis (equally distributed from *la* to *lb*).

**Returns**

- lmbdas** : ndarray  
The `lambda` values for which a Box-Cox transform was done.
- ppcc** : ndarray  
Probability Plot Correlation Coefficient, as obtained from `probplot` when fitting the Box-Cox transformed input *x* against a normal distribution.

**See also:**

`probplot`, `boxcox`, `boxcox_normmax`, `boxcox_llf`, `ppcc_max`

**Notes**

Even if *plot* is given, the figure is not shown or saved by `boxcox_normplot`; `plt.show()` or `plt.savefig('figname.png')` should be used after calling `probplot`.

**Examples**

```
>>> from scipy import stats
>>> import matplotlib.pyplot as plt
```

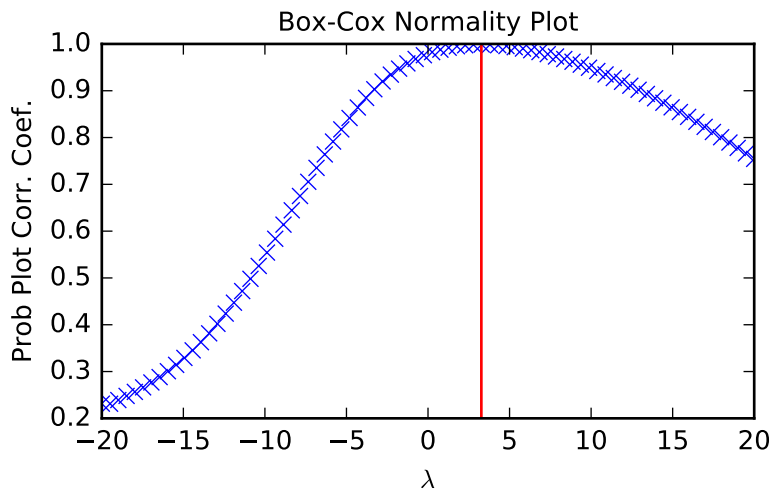
Generate some non-normally distributed data, and create a Box-Cox plot:

```
>>> x = stats.loggamma.rvs(5, size=500) + 5
>>> fig = plt.figure()
>>> ax = fig.add_subplot(111)
>>> prob = stats.boxcox_normplot(x, -20, 20, plot=ax)
```

Determine and plot the optimal lambda to transform x and plot it in the same plot:

```
>>> _, maxlog = stats.boxcox(x)
>>> ax.axvline(maxlog, color='r')

>>> plt.show()
```



## 5.27.8 Masked statistics functions

### Statistical functions for masked arrays (`scipy.stats.mstats`)

This module contains a large number of statistical functions that can be used with masked arrays.

Most of these functions are similar to those in `scipy.stats` but might have small differences in the API or in the algorithm used. Since this is a relatively new package, some API changes are still possible.

<code>argstoarray(*args)</code>	Constructs a 2D array from a group of sequences.
<code>betai(*args, **kwargs)</code>	<code>betai</code> is deprecated!
<code>chisquare(f_obs[, f_exp, ddof, axis])</code>	Calculates a one-way chi square test.
<code>count_tied_groups(x[, use_missing])</code>	Counts the number of tied values.
<code>describe(a[, axis, ddof, bias])</code>	Computes several descriptive statistics of the passed array.
<code>f_oneway(*args)</code>	Performs a 1-way ANOVA, returning an F-value and probability given any number of groups.
<code>f_value_wilks_lambda(*args, **kwargs)</code>	<code>f_value_wilks_lambda</code> is deprecated!
<code>find_repeats(arr)</code>	Find repeats in arr and return a tuple (repeats, repeat_count).
<code>friedmanchisquare(*args)</code>	Friedman Chi-Square is a non-parametric, one-way within-subjects ANOVA.
<code>kendalltau(x, y[, use_ties, use_missing])</code>	Computes Kendall's rank correlation tau on two variables <i>x</i> and <i>y</i> .

Table 5.292 – continued from previous page

<code>kendalltau_seasonal(x)</code>	Computes a multivariate Kendall's rank correlation tau, for seasonal data.
<code>kruskalwallis(*args)</code>	Compute the Kruskal-Wallis H-test for independent samples
<code>ks_twosamp(data1, data2[, alternative])</code>	Computes the Kolmogorov-Smirnov test on two samples.
<code>kurtosis(a[, axis, fisher, bias])</code>	Computes the kurtosis (Fisher or Pearson) of a dataset.
<code>kurtosistest(a[, axis])</code>	Tests whether a dataset has normal kurtosis
<code>linregress(x[, y])</code>	Calculate a linear least-squares regression for two sets of measurements.
<code>mannwhitneyu(x, y[, use_continuity])</code>	Computes the Mann-Whitney statistic
<code>plotting_positions(data[, alpha, beta])</code>	Returns plotting positions (or empirical percentile points) for the data.
<code>mode(a[, axis])</code>	Returns an array of the modal (most common) value in the passed array.
<code>moment(a[, moment, axis])</code>	Calculates the nth moment about the mean for a sample.
<code>mquantiles(a[, prob, alphap, betap, axis, limit])</code>	Computes empirical quantiles for a data array.
<code>msign(x)</code>	Returns the sign of x, or 0 if x is masked.
<code>normaltest(a[, axis])</code>	Tests whether a sample differs from a normal distribution.
<code>obrientransform(*args)</code>	Computes a transform on input data (any number of columns).
<code>pearsonr(x, y)</code>	Calculates a Pearson correlation coefficient and the p-value for testing non-co
<code>plotting_positions(data[, alpha, beta])</code>	Returns plotting positions (or empirical percentile points) for the data.
<code>pointbiserialr(x, y)</code>	Calculates a point biserial correlation coefficient and its p-value.
<code>rankdata(data[, axis, use_missing])</code>	Returns the rank (also known as order statistics) of each data point along the g
<code>scoreatpercentile(data, per[, limit, ...])</code>	Calculate the score at the given 'per' percentile of the sequence a.
<code>sem(a[, axis, ddof])</code>	Calculates the standard error of the mean of the input array.
<code>signaltonoise(*args, **kwds)</code>	<code>signaltonoise</code> is deprecated!
<code>skew(a[, axis, bias])</code>	Computes the skewness of a data set.
<code>skewtest(a[, axis])</code>	Tests whether the skew is different from the normal distribution.
<code>spearmanr(x, y[, use_ties])</code>	Calculates a Spearman rank-order correlation coefficient and the p-value to te
<code>theilslopes(y[, x, alpha])</code>	Computes the Theil-Sen estimator for a set of points (x, y).
<code>threshold(*args, **kwds)</code>	<code>threshold</code> is deprecated!
<code>tmax(a[, upperlimit, axis, inclusive])</code>	Compute the trimmed maximum
<code>tmean(a[, limits, inclusive, axis])</code>	Compute the trimmed mean.
<code>tmin(a[, lowerlimit, axis, inclusive])</code>	Compute the trimmed minimum
<code>trim(a[, limits, inclusive, relative, axis])</code>	Trims an array by masking the data outside some given limits.
<code>trima(a[, limits, inclusive])</code>	Trims an array by masking the data outside some given limits.
<code>trimboth(data[, proportiontocut, inclusive, ...])</code>	Trims the smallest and largest data values.
<code>trimmed_stde(a[, limits, inclusive, axis])</code>	Returns the standard error of the trimmed mean along the given axis.
<code>trimr(a[, limits, inclusive, axis])</code>	Trims an array by masking some proportion of the data on each end.
<code>trimtail(data[, proportiontocut, tail, ...])</code>	Trims the data by masking values from one tail.
<code>tsem(a[, limits, inclusive, axis, ddof])</code>	Compute the trimmed standard error of the mean.
<code>ttest_onesamp(a, popmean[, axis])</code>	Calculates the T-test for the mean of ONE group of scores.
<code>ttest_ind(a, b[, axis, equal_var])</code>	Calculates the T-test for the means of TWO INDEPENDENT samples of scor
<code>ttest_onesamp(a, popmean[, axis])</code>	Calculates the T-test for the mean of ONE group of scores.
<code>ttest_rel(a, b[, axis])</code>	Calculates the T-test on TWO RELATED samples of scores, a and b.
<code>tvar(a[, limits, inclusive, axis, ddof])</code>	Compute the trimmed variance
<code>variation(a[, axis])</code>	Computes the coefficient of variation, the ratio of the biased standard deviatio
<code>winsorize(a[, limits, inclusive, inplace, axis])</code>	Returns a Winsorized version of the input array.
<code>zmap(scores, compare[, axis, ddof])</code>	Calculates the relative z-scores.
<code>zscore(a[, axis, ddof])</code>	Calculates the z score of each value in the sample, relative to the sample mean
<code>compare_medians_ms(group_1, group_2[, axis])</code>	Compares the medians from two independent groups along the given axis.
<code>gmean(a[, axis, dtype])</code>	Compute the geometric mean along the specified axis.
<code>hdmedian(data[, axis, var])</code>	Returns the Harrell-Davis estimate of the median along the given axis.
<code>hdquantiles(data[, prob, axis, var])</code>	Computes quantile estimates with the Harrell-Davis method.
<code>hdquantiles_sd(data[, prob, axis])</code>	The standard error of the Harrell-Davis quantile estimates by jackknife.
<code>hmean(a[, axis, dtype])</code>	Calculates the harmonic mean along the specified axis.
<code>idealfourths(data[, axis])</code>	Returns an estimate of the lower and upper quartiles.

Table 5.292 – continued from previous page

<code>kruskal(*args)</code>	Compute the Kruskal-Wallis H-test for independent samples
<code>ks_2samp(data1, data2[, alternative])</code>	Computes the Kolmogorov-Smirnov test on two samples.
<code>median_cihs(data[, alpha, axis])</code>	Computes the alpha-level confidence interval for the median of the data.
<code>meppf(data[, alpha, beta])</code>	Returns plotting positions (or empirical percentile points) for the data.
<code>mjci(data[, prob, axis])</code>	Returns the Maritz-Jarrett estimators of the standard error of selected experim
<code>mquantiles_cimj(data[, prob, alpha, axis])</code>	Computes the alpha confidence interval for the selected quantiles of the data,
<code>rsh(data[, points])</code>	Evaluates Rosenblatt’s shifted histogram estimators for each point on the data
<code>sen_seasonal_slopes(x)</code>	
<code>trimmed_mean(a[, limits, inclusive, ...])</code>	
<code>trimmed_mean_ci(data[, limits, inclusive, ...])</code>	Selected confidence interval of the trimmed mean along the given axis.
<code>trimmed_std(a[, limits, inclusive, ...])</code>	
<code>trimmed_var(a[, limits, inclusive, ...])</code>	
<code>ttest_1samp(a, popmean[, axis])</code>	Calculates the T-test for the mean of ONE group of scores.

`scipy.stats.mstats.argmaxstoarray(*args)`

Constructs a 2D array from a group of sequences.

Sequences are filled with missing values to match the length of the longest sequence.

**Parameters** `args` : sequences

**Returns** `argstoarray` : `MaskedArray` Group of sequences.

A  $(m \times n)$  masked array, where  $m$  is the number of arguments and  $n$  the length of the longest argument.

**Notes**

`numpy.ma.row_stack` has identical behavior, but is called with a sequence of sequences.

`scipy.stats.mstats.betainc(*args, **kwargs)`

`betainc` is deprecated! `mstats.betainc` is deprecated in scipy 0.17.0; use `special.betainc` instead.

`betainc()` is deprecated in scipy 0.17.0.

For details about this function, see `stats.betainc`.

`scipy.stats.mstats.chisquare(f_obs, f_exp=None, ddof=0, axis=0)`

Calculates a one-way chi square test.

The chi square test tests the null hypothesis that the categorical data has the given frequencies.

**Parameters** `f_obs` : array\_like

Observed frequencies in each category.

`f_exp` : array\_like, optional

Expected frequencies in each category. By default the categories are assumed to be equally likely.

`ddof` : int, optional

“Delta degrees of freedom”: adjustment to the degrees of freedom for the p-value. The p-value is computed using a chi-squared distribution with  $k - 1 - ddof$  degrees of freedom, where  $k$  is the number of observed frequencies. The default value of `ddof` is 0.

`axis` : int or None, optional

The axis of the broadcast result of `f_obs` and `f_exp` along which to apply the test. If `axis` is None, all values in `f_obs` are treated as a single data set.

**Returns** `chisq` : float or `ndarray` Default is 0.

The chi-squared test statistic. The value is a float if `axis` is None or `f_obs` and `f_exp` are 1-D.

**p** : float or ndarray

The p-value of the test. The value is a float if *ddof* and the return value *chisq* are scalars.

**See also:**

`power_divergence`, `mstats.chisquare`

**Notes**

This test is invalid when the observed or expected frequencies in each category are too small. A typical rule is that all of the observed and expected frequencies should be at least 5.

The default degrees of freedom,  $k-1$ , are for the case when no parameters of the distribution are estimated. If  $p$  parameters are estimated by efficient maximum likelihood then the correct degrees of freedom are  $k-1-p$ . If the parameters are estimated in a different way, then the dof can be between  $k-1-p$  and  $k-1$ . However, it is also possible that the asymptotic distribution is not a chisquare, in which case this test is not appropriate.

**References**

[R544], [R545]

**Examples**

When just *f\_obs* is given, it is assumed that the expected frequencies are uniform and given by the mean of the observed frequencies.

```
>>> from scipy.stats import chisquare
>>> chisquare([16, 18, 16, 14, 12, 12])
(2.0, 0.84914503608460956)
```

With *f\_exp* the expected frequencies can be given.

```
>>> chisquare([16, 18, 16, 14, 12, 12], f_exp=[16, 16, 16, 16, 16, 8])
(3.5, 0.62338762774958223)
```

When *f\_obs* is 2-D, by default the test is applied to each column.

```
>>> obs = np.array([[16, 18, 16, 14, 12, 12], [32, 24, 16, 28, 20, 24]]).T
>>> obs.shape
(6, 2)
>>> chisquare(obs)
(array([ 2.          ,  6.66666667]), array([ 0.84914504,  0.24663415]))
```

By setting *axis=None*, the test is applied to all data in the array, which is equivalent to applying the test to the flattened array.

```
>>> chisquare(obs, axis=None)
(23.31034482758621, 0.015975692534127565)
>>> chisquare(obs.ravel())
(23.31034482758621, 0.015975692534127565)
```

*ddof* is the change to make to the default degrees of freedom.

```
>>> chisquare([16, 18, 16, 14, 12, 12], ddof=1)
(2.0, 0.73575888234288467)
```

The calculation of the p-values is done by broadcasting the chi-squared statistic with *ddof*.

```
>>> chisquare([16, 18, 16, 14, 12, 12], ddof=[0,1,2])
(2.0, array([ 0.84914504,  0.73575888,  0.5724067 ]))
```

$f_{obs}$  and  $f_{exp}$  are also broadcast. In the following,  $f_{obs}$  has shape (6,) and  $f_{exp}$  has shape (2, 6), so the result of broadcasting  $f_{obs}$  and  $f_{exp}$  has shape (2, 6). To compute the desired chi-squared statistics, we use `axis=1`:

```
>>> chisquare([16, 18, 16, 14, 12, 12],
...           f_exp=[[16, 16, 16, 16, 16, 8], [8, 20, 20, 16, 12, 12]],
...           axis=1)
(array([ 3.5 ,  9.25]), array([ 0.62338763,  0.09949846]))
```

`scipy.stats.mstats.count_tied_groups` ( $x$ , *use\_missing=False*)

Counts the number of tied values.

**Parameters**

- x** : sequence  
Sequence of data on which to counts the ties
- use\_missing** : bool, optional  
Whether to consider missing values as tied.

**Returns**

- count\_tied\_groups** : dict  
Returns a dictionary (nb of ties: nb of groups).

### Examples

```
>>> from scipy.stats import mstats
>>> z = [0, 0, 0, 2, 2, 2, 3, 3, 4, 5, 6]
>>> mstats.count_tied_groups(z)
{2: 1, 3: 2}
```

In the above example, the ties were 0 (3x), 2 (3x) and 3 (2x).

```
>>> z = np.ma.array([0, 0, 1, 2, 2, 2, 3, 3, 4, 5, 6])
>>> mstats.count_tied_groups(z)
{2: 2, 3: 1}
>>> z[[1,-1]] = np.ma.masked
>>> mstats.count_tied_groups(z, use_missing=True)
{2: 2, 3: 1}
```

`scipy.stats.mstats.describe` ( $a$ , *axis=0*, *ddof=0*, *bias=True*)

Computes several descriptive statistics of the passed array.

**Parameters**

- a** : array\_like  
Data array
- axis** : int or None, optional  
Axis along which to calculate statistics. Default 0. If None, compute over the whole array  $a$ .
- ddof** : int, optional  
degree of freedom (default 0); note that default ddof is different from the same routine in `stats.describe`
- bias** : bool, optional  
If False, then the skewness and kurtosis calculations are corrected for statistical bias.

**Returns**

- nobs** : int  
(size of the data (discarding missing values))
- minmax** : (int, int)  
min, max
- mean** : float



arithmetic mean  
**variance** : float  
 unbiased variance  
**skewness** : float  
 biased skewness  
**kurtosis** : float  
 biased kurtosis

**Examples**

```

>>> from scipy.stats.mstats import describe
>>> ma = np.ma.array(range(6), mask=[0, 0, 0, 1, 1, 1])
>>> describe(ma)
DescribeResult(nobs=array(3), minmax=(masked_array(data = 0,
            mask = False,
            fill_value = 999999)
, masked_array(data = 2,
            mask = False,
            fill_value = 999999)
), mean=1.0, variance=0.66666666666666663, skewness=masked_array(data = 0.0,
            mask = False,
            fill_value = 1e+20)
, kurtosis=-1.5)

```

`scipy.stats.mstats.f_oneway(*args)`

Performs a 1-way ANOVA, returning an F-value and probability given any number of groups. From Heiman, pp.394-7.

Usage: `f_oneway(*args)`, where `*args` is 2 or more arrays, one per treatment group.

**Returns**     **statistic** : float  
                                   The computed F-value of the test.  
                   **pvalue** : float  
                                   The associated p-value from the F-distribution.

`scipy.stats.mstats.f_value_wilks_lambda(*args, **kws)`

`f_value_wilks_lambda` is deprecated! `mstats.f_value_wilks_lambda` deprecated in scipy 0.17.0

*Calculation of Wilks lambda F-statistic for multivariate data, per Maxwell & Delaney p.657.*

`scipy.stats.mstats.find_repeats(arr)`

Find repeats in `arr` and return a tuple (`repeats`, `repeat_count`).

The input is cast to float64. Masked values are discarded.

**Parameters**   **arr** : sequence  
                                   Input array. The array is flattened if it is not 1D.  
**Returns**       **repeats** : ndarray  
                                   Array of repeated values.  
                   **counts** : ndarray  
                                   Array of counts.

`scipy.stats.mstats.friedmanchisquare(*args)`

Friedman Chi-Square is a non-parametric, one-way within-subjects ANOVA. This function calculates the Friedman Chi-square test for repeated measures and returns the result, along with the associated probability value.

Each input is considered a given group. Ideally, the number of treatments among each group should be equal. If this is not the case, only the first `n` treatments are taken into account, where `n` is the number of treatments of

the smallest group. If a group has some missing values, the corresponding treatments are masked in the other groups. The test statistic is corrected for ties.

Masked values in one group are propagated to the other groups.

**Returns**

- statistic** : float  
the test statistic.
- pvalue** : float  
the associated p-value.

`scipy.stats.mstats.kendalltau(x, y, use_ties=True, use_missing=False)`  
 Computes Kendall's rank correlation tau on two variables *x* and *y*.

**Parameters**

- x** : sequence  
First data list (for example, time).
- y** : sequence  
Second data list.
- use\_ties** : {True, False}, optional  
Whether ties correction should be performed.
- use\_missing** : {False, True}, optional  
Whether missing data should be allocated a rank of 0 (False) or the average rank (True).

**Returns**

- correlation** : float  
Kendall tau
- pvalue** : float  
Approximate 2-side p-value.

`scipy.stats.mstats.kendalltau_seasonal(x)`  
 Computes a multivariate Kendall's rank correlation tau, for seasonal data.

**Parameters**

- x** : 2-D ndarray  
Array of seasonal data, with seasons in columns.

`scipy.stats.mstats.kruskalwallis(*args)`  
 Compute the Kruskal-Wallis H-test for independent samples

**Parameters**

- sample1, sample2, ...** : array\_like  
Two or more arrays with the sample measurements can be given as arguments.

**Returns**

- statistic** : float  
The Kruskal-Wallis H statistic, corrected for ties
- pvalue** : float  
The p-value for the test using the assumption that H has a chi square distribution

**Notes**

For more details on `kruskal`, see `stats.kruskal`.

`scipy.stats.mstats.ks_twosamp(data1, data2, alternative='two-sided')`  
 Computes the Kolmogorov-Smirnov test on two samples.

Missing values are discarded.

**Parameters**

- data1** : array\_like  
First data set
- data2** : array\_like  
Second data set
- alternative** : {'two-sided', 'less', 'greater'}, optional  
Indicates the alternative hypothesis. Default is 'two-sided'.

**Returns**

- d** : float

**p** : float Value of the Kolmogorov Smirnov test  
Corresponding p-value.

`scipy.stats.mstats.kurtosis` (*a*, *axis=0*, *fisher=True*, *bias=True*)  
Computes the kurtosis (Fisher or Pearson) of a dataset.

Kurtosis is the fourth central moment divided by the square of the variance. If Fisher's definition is used, then 3.0 is subtracted from the result to give 0.0 for a normal distribution.

If *bias* is `False` then the kurtosis is calculated using k statistics to eliminate bias coming from biased moment estimators

Use `kurtosistest` to see if result is close enough to normal.

**Parameters**

- a** : array  
data for which the kurtosis is calculated
- axis** : int or None, optional  
Axis along which the kurtosis is calculated. Default is 0. If None, compute over the whole array *a*.
- fisher** : bool, optional  
If True, Fisher's definition is used (normal ==> 0.0). If False, Pearson's definition is used (normal ==> 3.0).
- bias** : bool, optional  
If False, then the calculations are corrected for statistical bias.

**Returns**

- kurtosis** : array  
The kurtosis of values along an axis. If all values are equal, return -3 for Fisher's definition and 0 for Pearson's definition.

#### Notes

For more details about `kurtosis`, see `stats.kurtosis`.

`scipy.stats.mstats.kurtosistest` (*a*, *axis=0*)  
Tests whether a dataset has normal kurtosis

**Parameters**

- a** : array  
array of the sample data
- axis** : int or None, optional  
Axis along which to compute test. Default is 0. If None, compute over the whole array *a*.

**Returns**

- statistic** : float  
The computed z-score for this test.
- pvalue** : float  
The 2-sided p-value for the hypothesis test

#### Notes

For more details about `kurtosistest`, see `stats.kurtosistest`.

`scipy.stats.mstats.linregress` (*x*, *y=None*)  
Calculate a linear least-squares regression for two sets of measurements.

**Parameters**

- x, y** : array\_like  
Two sets of measurements. Both arrays should have the same length. If only *x* is given (and *y=None*), then it must be a two-dimensional array where one dimension has length 2. The two sets of measurements are then found by splitting the array along the length-2 dimension.

**Returns**

- slope** : float  
slope of the regression line
- intercept** : float

intercept of the regression line  
**rvalue** : float  
 correlation coefficient  
**pvalue** : float  
 two-sided p-value for a hypothesis test whose null hypothesis is that the slope is zero.  
**stderr** : float  
 Standard error of the estimated gradient.

See also:

**`optimize.curve_fit`**

Use non-linear least squares to fit a function to data.

**`optimize.leastsq`**

Minimize the sum of squares of a set of equations.

*Notes*

Missing values are considered pair-wise: if a value is missing in *x*, the corresponding value in *y* is masked.

*Examples*

```
>>> from scipy import stats
>>> np.random.seed(12345678)
>>> x = np.random.random(10)
>>> y = np.random.random(10)
>>> slope, intercept, r_value, p_value, std_err = stats.linregress(x,y)
```

# To get coefficient of determination (r\_squared)

```
>>> print("r-squared:", r_value**2)
('r-squared:', 0.080402268539028335)
```

`scipy.stats.mstats.mannwhitneyu(x, y, use_continuity=True)`

Computes the Mann-Whitney statistic

Missing values in *x* and/or *y* are discarded.

**Parameters** **x** : sequence

Input

**y** : sequence

Input

**use\_continuity** : {True, False}, optional

Whether a continuity correction (1/2.) should be taken into account.

**Returns** **statistic** : float

The Mann-Whitney statistics

**pvalue** : float

Approximate p-value assuming a normal distribution.

`scipy.stats.mstats.plotting_positions(data, alpha=0.4, beta=0.4)`

Returns plotting positions (or empirical percentile points) for the data.

*Plotting positions are defined as  $(i-\alpha)/(n+1-\alpha-\beta)$ , where:*

- *i* is the rank order statistics
- *n* is the number of unmasked values along the given axis
- *alpha* and *beta* are two parameters.

*Typical values for alpha and beta are:*

- (0,1) :  $p(k) = k/n$ , linear interpolation of cdf (R, type 4)
- (.5,.5) :  $p(k) = (k-1/2)/n$ , piecewise linear function (R, type 5)
- (0,0) :  $p(k) = k/(n+1)$ , Weibull (R type 6)
- (1,1) :  $p(k) = (k-1)/(n-1)$ , in this case,  $p(k) = \text{mode}[F(x[k])]$ . That's R default (R type 7)
- (1/3,1/3):  $p(k) = (k-1/3)/(n+1/3)$ , then  $p(k) \sim \text{median}[F(x[k])]$ . The resulting quantile estimates are approximately median-unbiased regardless of the distribution of  $x$ . (R type 8)
- (3/8,3/8):  $p(k) = (k-3/8)/(n+1/4)$ , Blom. The resulting quantile estimates are approximately unbiased if  $x$  is normally distributed (R type 9)
- (.4,.4) : approximately quantile unbiased (Cunnane)
- (.35,.35): APL, used with PWM
- (.3175, .3175): used in `scipy.stats.probplot`

**Parameters**

**data** : array\_like  
Input data, as a sequence or array of dimension at most 2.

**alpha** : float, optional  
Plotting positions parameter. Default is 0.4.

**beta** : float, optional  
Plotting positions parameter. Default is 0.4.

**Returns**

**positions** : MaskedArray  
The calculated plotting positions.

`scipy.stats.mstats.mode(a, axis=0)`

Returns an array of the modal (most common) value in the passed array.

**Parameters**

**a** : array\_like  
n-dimensional array of which to find mode(s).

**axis** : int or None, optional  
Axis along which to operate. Default is 0. If None, compute over the whole array  $a$ .

**Returns**

**mode** : ndarray  
Array of modal values.

**count** : ndarray  
Array of counts for each mode.

### Notes

For more details, see `stats.mode`.

`scipy.stats.mstats.moment(a, moment=1, axis=0)`

Calculates the  $n$ th moment about the mean for a sample.

**Parameters**

**a** : array\_like  
data

**moment** : int, optional  
order of central moment that is returned

**axis** : int or None, optional  
Axis along which the central moment is computed. Default is 0. If None, compute over the whole array  $a$ .

**Returns**

**$n$ -th central moment** : ndarray or float  
The appropriate moment along the given axis or over all values if axis is None. The denominator for the moment calculation is the number of observations, no degrees of freedom correction is done.

*Notes*

For more details about `moment`, see `stats.moment`.

`scipy.stats.mstats.mquantiles(a, prob=[0.25, 0.5, 0.75], alphap=0.4, betap=0.4, axis=None, limit=())`

Computes empirical quantiles for a data array.

Samples quantile are defined by  $Q(p) = (1-\gamma) * x[j] + \gamma * x[j+1]$ , where  $x[j]$  is the  $j$ -th order statistic, and  $\gamma$  is a function of  $j = \text{floor}(n * p + m)$ ,  $m = \text{alphap} + p * (1 - \text{alphap} - \text{betap})$  and  $g = n * p + m - j$ .

Reinterpreting the above equations to compare to **R** lead to the equation:  $p(k) = (k - \text{alphap}) / (n + 1 - \text{alphap} - \text{betap})$

*Typical values of (alphap,betap) are:*

- (0,1):  $p(k) = k/n$ : linear interpolation of cdf (**R** type 4)
- (.5,.5):  $p(k) = (k - 1/2.) / n$ : piecewise linear function (**R** type 5)
- (0,0):  $p(k) = k / (n+1)$ : (**R** type 6)
- (1,1):  $p(k) = (k-1) / (n-1)$ :  $p(k) = \text{mode}[F(x[k])]$ . (**R** type 7, **R** default)
- (1/3,1/3):  $p(k) = (k-1/3) / (n+1/3)$ : Then  $p(k) \sim \text{median}[F(x[k])]$ . The resulting quantile estimates are approximately median-unbiased regardless of the distribution of  $x$ . (**R** type 8)
- (3/8,3/8):  $p(k) = (k-3/8) / (n+1/4)$ : Blom. The resulting quantile estimates are approximately unbiased if  $x$  is normally distributed (**R** type 9)
- (.4,.4): approximately quantile unbiased (Cunnane)
- (.35,.35): APL, used with PWM

*Parameters*

- a**: array\_like  
Input data, as a sequence or array of dimension at most 2.
- prob**: array\_like, optional  
List of quantiles to compute.
- alphap**: float, optional  
Plotting positions parameter, default is 0.4.
- betap**: float, optional  
Plotting positions parameter, default is 0.4.
- axis**: int, optional  
Axis along which to perform the trimming. If None (default), the input array is first flattened.
- limit**: tuple, optional  
Tuple of (lower, upper) values. Values of  $a$  outside this open interval are ignored.

*Returns*

- mquantiles**: MaskedArray  
An array containing the calculated quantiles.

*Notes*

This formulation is very similar to **R** except the calculation of  $m$  from `alphap` and `betap`, where in **R**  $m$  is defined with each type.

*References*

[R546], [R547]

*Examples*

```
>>> from scipy.stats.mstats import mquantiles
>>> a = np.array([6., 47., 49., 15., 42., 41., 7., 39., 43., 40., 36.])
>>> mquantiles(a)
array([ 19.2,  40. ,  42.8])
```

Using a 2D array, specifying axis and limit.

```
>>> data = np.array([[ 6.,  7.,  1.],
...                 [ 47., 15.,  2.],
...                 [ 49., 36.,  3.],
...                 [ 15., 39.,  4.],
...                 [ 42., 40., -999.],
...                 [ 41., 41., -999.],
...                 [  7., -999., -999.],
...                 [ 39., -999., -999.],
...                 [ 43., -999., -999.],
...                 [ 40., -999., -999.],
...                 [ 36., -999., -999.]])
>>> print(mquantiles(data, axis=0, limit=(0, 50)))
[[ 19.2  14.6  1.45]
 [ 40.   37.5  2.5 ]
 [ 42.8  40.05  3.55]]
```

```
>>> data[:, 2] = -999.
>>> print(mquantiles(data, axis=0, limit=(0, 50)))
[[19.200000000000003 14.6 --]
 [40.0 37.5 --]
 [42.800000000000004 40.05 --]]
```

`scipy.stats.mstats.msign(x)`

Returns the sign of *x*, or 0 if *x* is masked.

`scipy.stats.mstats.normaltest(a, axis=0)`

Tests whether a sample differs from a normal distribution.

**Parameters** **a** : array\_like

The array containing the data to be tested.

**axis** : int or None, optional

Axis along which to compute test. Default is 0. If None, compute over the whole array *a*.

**Returns**

**statistic** : float or array  
 $s^2 + k^2$ , where *s* is the z-score returned by `skewtest` and *k* is the z-score returned by `kurtosistest`.

**pvalue** : float or array

A 2-sided chi squared probability for the hypothesis test.

### Notes

For more details about `normaltest`, see `stats.normaltest`.

`scipy.stats.mstats.obrientransform(*args)`

Computes a transform on input data (any number of columns). Used to test for homogeneity of variance prior to running one-way stats. Each array in *\*args* is one level of a factor. If an `f_oneway()` run on the transformed data and found significant, variances are unequal. From Maxwell and Delaney, p.112.

Returns: transformed data for use in an ANOVA

`scipy.stats.mstats.pearsonr(x, y)`

Calculates a Pearson correlation coefficient and the p-value for testing non-correlation.

The Pearson correlation coefficient measures the linear relationship between two datasets. Strictly speaking, Pearson's correlation requires that each dataset be normally distributed. Like other correlation coefficients, this one varies between -1 and +1 with 0 implying no correlation. Correlations of -1 or +1 imply an exact linear relationship. Positive correlations imply that as  $x$  increases, so does  $y$ . Negative correlations imply that as  $x$  increases,  $y$  decreases.

The p-value roughly indicates the probability of an uncorrelated system producing datasets that have a Pearson correlation at least as extreme as the one computed from these datasets. The p-values are not entirely reliable but are probably reasonable for datasets larger than 500 or so.

**Parameters** **x** : 1-D array\_like  
 Input  
**y** : 1-D array\_like  
 Input  
**Returns** **pearsonr** : float  
 Pearson's correlation coefficient, 2-tailed p-value.

**References**

<http://www.statsoft.com/textbook/glosp.html#Pearson%20Correlation>

`scipy.stats.mstats.plotting_positions` (*data*, *alpha*=0.4, *beta*=0.4)

Returns plotting positions (or empirical percentile points) for the data.

*Plotting positions are defined as  $(i-\alpha)/(n+1-\alpha-\beta)$ , where:*

- *i* is the rank order statistics
- *n* is the number of unmasked values along the given axis
- *alpha* and *beta* are two parameters.

*Typical values for alpha and beta are:*

- (0,1) :  $p(k) = k/n$ , linear interpolation of cdf (R, type 4)
- (.5,.5) :  $p(k) = (k-1/2)/n$ , piecewise linear function (R, type 5)
- (0,0) :  $p(k) = k/(n+1)$ , Weibull (R type 6)
- (1,1) :  $p(k) = (k-1)/(n-1)$ , in this case,  $p(k) = \text{mode}[F(x[k])]$ . That's R default (R type 7)
- (1/3,1/3):  $p(k) = (k-1/3)/(n+1/3)$ , then  $p(k) \sim \text{median}[F(x[k])]$ . The resulting quantile estimates are approximately median-unbiased regardless of the distribution of  $x$ . (R type 8)
- (3/8,3/8):  $p(k) = (k-3/8)/(n+1/4)$ , Blom. The resulting quantile estimates are approximately unbiased if  $x$  is normally distributed (R type 9)
- (.4,.4) : approximately quantile unbiased (Cunnane)
- (.35,.35): APL, used with PWM
- (.3175, .3175): used in `scipy.stats.probplot`

**Parameters** **data** : array\_like  
 Input data, as a sequence or array of dimension at most 2.  
**alpha** : float, optional  
 Plotting positions parameter. Default is 0.4.  
**beta** : float, optional  
 Plotting positions parameter. Default is 0.4.  
**Returns** **positions** : MaskedArray  
 The calculated plotting positions.

`scipy.stats.mstats.pointbiserialr` (*x*, *y*)

Calculates a point biserial correlation coefficient and its p-value.

**Parameters** **x** : array\_like of bools



**Returns**

- y** : array\_like  
Input array.
- correlation** : float  
Input array.  
R value
- pvalue** : float  
2-tailed p-value

**Notes**

Missing values are considered pair-wise: if a value is missing in x, the corresponding value in y is masked.

For more details on `pointbiserialr`, see `stats.pointbiserialr`.

`scipy.stats.mstats.rankdata` (*data*, *axis=None*, *use\_missing=False*)

Returns the rank (also known as order statistics) of each data point along the given axis.

If some values are tied, their rank is averaged. If some values are masked, their rank is set to 0 if *use\_missing* is False, or set to the average rank of the unmasked values if *use\_missing* is True.

**Parameters**

- data** : sequence  
Input data. The data is transformed to a masked array
- axis** : {None,int}, optional  
Axis along which to perform the ranking. If None, the array is first flattened. An exception is raised if the axis is specified for arrays with a dimension larger than 2
- use\_missing** : bool, optional  
Whether the masked values have a rank of 0 (False) or equal to the average rank of the unmasked values (True).

`scipy.stats.mstats.scoreatpercentile` (*data*, *per*, *limit=()*, *alphap=0.4*, *betap=0.4*)

Calculate the score at the given 'per' percentile of the sequence a. For example, the score at per=50 is the median.

This function is a shortcut to `mquantile`

`scipy.stats.mstats.sem` (*a*, *axis=0*, *ddof=1*)

Calculates the standard error of the mean of the input array.

Also sometimes called standard error of measurement.

**Parameters**

- a** : array\_like  
An array containing the values for which the standard error is returned.
- axis** : int or None, optional  
If axis is None, ravel *a* first. If axis is an integer, this will be the axis over which to operate. Defaults to 0.
- ddof** : int, optional  
Delta degrees-of-freedom. How many degrees of freedom to adjust for bias in limited samples relative to the population estimate of variance. Defaults to 1.

**Returns**

- s** : ndarray or float  
The standard error of the mean in the sample(s), along the input axis.

**Notes**

The default value for *ddof* changed in `scipy` 0.15.0 to be consistent with `stats.sem` as well as with the most common definition used (like in the R documentation).

*Examples*

Find standard error along the first axis:

```
>>> from scipy import stats
>>> a = np.arange(20).reshape(5,4)
>>> print(stats.mstats.sem(a))
[2.8284271247461903 2.8284271247461903 2.8284271247461903
 2.8284271247461903]
```

Find standard error across the whole array, using n degrees of freedom:

```
>>> print(stats.mstats.sem(a, axis=None, ddof=0))
1.2893796958227628
```

scipy.stats.mstats.**signaltonoise** (\*args, \*\*kwargs)  
*signaltonoise* is deprecated! mstats.signaltonoise is deprecated in scipy 0.16.0

*Calculates the signal-to-noise ratio, as the ratio of the mean over standard deviation along the given axis.*

**Parameters**

<b>data</b> :	sequence	Input data
<b>axis</b> :	{0, int}, optional	Axis along which to compute. If None, the computation is performed on a flat version of the array.

scipy.stats.mstats.**skew** (a, axis=0, bias=True)

Computes the skewness of a data set.

**Parameters**

<b>a</b> :	ndarray	data
<b>axis</b> :	int or None, optional	Axis along which skewness is calculated. Default is 0. If None, compute over the whole array <i>a</i> .
<b>bias</b> :	bool, optional	If False, then the calculations are corrected for statistical bias.

**Returns**

<b>skewness</b> :	ndarray	The skewness of values along an axis, returning 0 where all values are equal.
-------------------	---------	---

*Notes*

For more details about *skew*, see *stats.skew*.

scipy.stats.mstats.**skewtest** (a, axis=0)

Tests whether the skew is different from the normal distribution.

**Parameters**

<b>a</b> :	array	The data to be tested
<b>axis</b> :	int or None, optional	Axis along which statistics are calculated. Default is 0. If None, compute over the whole array <i>a</i> .

**Returns**

<b>statistic</b> :	float	The computed z-score for this test.
<b>pvalue</b> :	float	a 2-sided p-value for the hypothesis test

**Notes**

For more details about `skewtest`, see `stats.skewtest`.

`scipy.stats.mstats.spearmanr` ( $x, y, use\_ties=True$ )

Calculates a Spearman rank-order correlation coefficient and the p-value to test for non-correlation.

The Spearman correlation is a nonparametric measure of the linear relationship between two datasets. Unlike the Pearson correlation, the Spearman correlation does not assume that both datasets are normally distributed. Like other correlation coefficients, this one varies between -1 and +1 with 0 implying no correlation. Correlations of -1 or +1 imply an exact linear relationship. Positive correlations imply that as  $x$  increases, so does  $y$ . Negative correlations imply that as  $x$  increases,  $y$  decreases.

Missing values are discarded pair-wise: if a value is missing in  $x$ , the corresponding value in  $y$  is masked.

The p-value roughly indicates the probability of an uncorrelated system producing datasets that have a Spearman correlation at least as extreme as the one computed from these datasets. The p-values are not entirely reliable but are probably reasonable for datasets larger than 500 or so.

<b>Parameters</b>	<b>x</b> : array_like	The length of $x$ must be $> 2$ .
	<b>y</b> : array_like	The length of $y$ must be $> 2$ .
<b>Returns</b>	<b>use_ties</b> : bool, optional	Whether the correction for ties should be computed.
	<b>correlation</b> : float	Spearman correlation coefficient
	<b>pvalue</b> : float	2-tailed p-value.

**References**

[CRCProbStat2000] section 14.7

`scipy.stats.mstats.theilslopes` ( $y, x=None, alpha=0.95$ )

Computes the Theil-Sen estimator for a set of points ( $x, y$ ).

`theilslopes` implements a method for robust linear regression. It computes the slope as the median of all slopes between paired values.

<b>Parameters</b>	<b>y</b> : array_like	Dependent variable.
	<b>x</b> : array_like or None, optional	Independent variable. If None, use <code>arange(len(y))</code> instead.
<b>Returns</b>	<b>alpha</b> : float, optional	Confidence degree between 0 and 1. Default is 95% confidence. Note that $alpha$ is symmetric around 0.5, i.e. both 0.1 and 0.9 are interpreted as “find the 90% confidence interval”.
	<b>medslope</b> : float	Theil slope.
	<b>medintercept</b> : float	Intercept of the Theil line, as <code>median(y) - medslope*median(x)</code> .
	<b>lo_slope</b> : float	Lower bound of the confidence interval on <code>medslope</code> .
	<b>up_slope</b> : float	Upper bound of the confidence interval on <code>medslope</code> .

*Notes*

For more details on `theilslopes`, see `stats.theilslopes`.

`scipy.stats.mstats.threshold(*args, **kws)`  
`threshold` is deprecated! `mstats.threshold` is deprecated in scipy 0.17.0

Clip array to a given value.

Similar to `numpy.clip()`, except that values less than `threshmin` or greater than `threshmax` are replaced by `newval`, instead of by `threshmin` and `threshmax` respectively.

**Parameters** `a` : ndarray

Input data

**`threshmin`** : [{None, float}, optional] Lower threshold. If None, set to the minimum value  
**`threshmax`** : [{None, float}, optional] Upper threshold. If None, set to the maximum value  
**`newval`** : [{0, float}, optional] Value outside the thresholds.

**Returns** `threshold` : ndarray

Returns `a`, with values less then `threshmin` and values greater `threshmax` replaced with `newval`.

`scipy.stats.mstats.tmax(a, upperlimit=None, axis=0, inclusive=True)`

Compute the trimmed maximum

This function computes the maximum value of an array along a given axis, while ignoring values larger than a specified upper limit.

**Parameters** `a` : array\_like

array of values

**`upperlimit`** : None or float, optional

Values in the input array greater than the given limit will be ignored. When `upperlimit` is None, then all values are used. The default value is None.

**`axis`** : int or None, optional

Axis along which to operate. Default is 0. If None, compute over the whole array `a`.

**`inclusive`** : {True, False}, optional

This flag determines whether values exactly equal to the upper limit are included. The default value is True.

**Returns** `tmax` : float, int or ndarray

*Notes*

For more details on `tmax`, see `stats.tmax`.

`scipy.stats.mstats.tmean(a, limits=None, inclusive=(True, True), axis=None)`

Compute the trimmed mean.

**Parameters** `a` : array\_like

Array of values.

**`limits`** : None or (lower limit, upper limit), optional

Values in the input array less than the lower limit or greater than the upper limit will be ignored. When `limits` is None (default), then all values are used. Either of the limit values in the tuple can also be None representing a half-open interval.

**`inclusive`** : (bool, bool), optional

A tuple consisting of the (lower flag, upper flag). These flags determine whether values exactly equal to the lower or upper limits are included. The default value is (True, True).

**Returns** **axis** : int or None, optional  
Axis along which to operate. If None, compute over the whole array. Default is None.

**tmean** : float

**Notes**

For more details on `tmean`, see `stats.tmean`.

`scipy.stats.mstats.tmin(a, lowerlimit=None, axis=0, inclusive=True)`  
Compute the trimmed minimum

**Parameters** **a** : array\_like  
array of values

**lowerlimit** : None or float, optional  
Values in the input array less than the given limit will be ignored. When lowerlimit is None, then all values are used. The default value is None.

**axis** : int or None, optional  
Axis along which to operate. Default is 0. If None, compute over the whole array *a*.

**inclusive** : {True, False}, optional  
This flag determines whether values exactly equal to the lower limit are included. The default value is True.

**Returns** **tmin** : float, int or ndarray

**Notes**

For more details on `tmin`, see `stats.tmin`.

`scipy.stats.mstats.trim(a, limits=None, inclusive=(True, True), relative=False, axis=None)`  
Trims an array by masking the data outside some given limits.

Returns a masked version of the input array.

**Parameters** **a** : sequence  
Input array

**limits** : {None, tuple}, optional  
If *relative* is False, tuple (lower limit, upper limit) in absolute values. Values of the input array lower (greater) than the lower (upper) limit are masked. If *relative* is True, tuple (lower percentage, upper percentage) to cut on each side of the array, with respect to the number of unmasked data. Noting *n* the number of unmasked data before trimming, the (*n*\*limits[0])th smallest data and the (*n*\*limits[1])th largest data are masked, and the total number of unmasked data after trimming is *n*\*(1.-sum(limits)) In each case, the value of one limit can be set to None to indicate an open interval. If limits is None, no trimming is performed

**inclusive** : {(bool, bool) tuple}, optional  
If *relative* is False, tuple indicating whether values exactly equal to the absolute limits are allowed. If *relative* is True, tuple indicating whether the number of data being masked on each side should be rounded (True) or truncated (False).

**relative** : bool, optional  
Whether to consider the limits as absolute values (False) or proportions to cut (True).

**axis** : int, optional  
Axis along which to trim.

### Examples

```
>>> from scipy.stats.mstats import trim
>>> z = [ 1, 2, 3, 4, 5, 6, 7, 8, 9,10]
>>> print(trim(z, (3,8)))
[-- -- 3 4 5 6 7 8 -- --]
>>> print(trim(z, (0.1,0.2),relative=True))
[-- 2 3 4 5 6 7 8 -- --]
```

`scipy.stats.mstats.trima(a, limits=None, inclusive=(True, True))`

Trims an array by masking the data outside some given limits.

Returns a masked version of the input array.

**Parameters** **a** : array\_like

Input array.

**limits** : {None, tuple}, optional

Tuple of (lower limit, upper limit) in absolute values. Values of the input array lower (greater) than the lower (upper) limit will be masked. A limit is None indicates an open interval.

**inclusive** : (bool, bool) tuple, optional

Tuple of (lower flag, upper flag), indicating whether values exactly equal to the lower (upper) limit are allowed.

`scipy.stats.mstats.trimboth(data, proportiontocut=0.2, inclusive=(True, True), axis=None)`

Trims the smallest and largest data values.

Trims the *data* by masking the `int(proportiontocut * n)` smallest and `int(proportiontocut * n)` largest values of data along the given axis, where *n* is the number of unmasked values before trimming.

**Parameters** **data** : ndarray

Data to trim.

**proportiontocut** : float, optional

Percentage of trimming (as a float between 0 and 1). If *n* is the number of unmasked values before trimming, the number of values after trimming is  $(1 - 2 * \text{proportiontocut}) * n$ . Default is 0.2.

**inclusive** : {(bool, bool) tuple}, optional

Tuple indicating whether the number of data being masked on each side should be rounded (True) or truncated (False).

**axis** : int, optional

Axis along which to perform the trimming. If None, the input array is first flattened.

`scipy.stats.mstats.trimmed_stde(a, limits=(0.1, 0.1), inclusive=(1, 1), axis=None)`

Returns the standard error of the trimmed mean along the given axis.

**Parameters** **a** : sequence

Input array

**limits** : {(0.1,0.1), tuple of float}, optional

tuple (lower percentage, upper percentage) to cut on each side of the array, with respect to the number of unmasked data.

If *n* is the number of unmasked data before trimming, the values smaller than  $n * \text{limits}[0]$  and the values larger than  $n * \text{limits}[1]$  are masked, and the total number of unmasked data after trimming is  $n * (1 - \text{sum}(\text{limits}))$ . In each case, the value of one limit can be set to None to indicate an open interval. If *limits* is None, no trimming is performed.

**inclusive** : {(bool, bool) tuple} optional

Tuple indicating whether the number of data being masked on each side should be rounded (True) or truncated (False).

**axis** : int, optional

**Returns** **trimmed\_std** : scalar or ndarray  
Axis along which to trim.

`scipy.stats.mstats.trimr` (*a*, *limits=None*, *inclusive=(True, True)*, *axis=None*)

Trims an array by masking some proportion of the data on each end. Returns a masked version of the input array.

**Parameters** **a** : sequence

Input array.

**limits** : {None, tuple}, optional

Tuple of the percentages to cut on each side of the array, with respect to the number of unmasked data, as floats between 0. and 1. Noting *n* the number of unmasked data before trimming, the ( $n \cdot \text{limits}[0]$ )th smallest data and the ( $n \cdot \text{limits}[1]$ )th largest data are masked, and the total number of unmasked data after trimming is  $n \cdot (1 - \text{sum}(\text{limits}))$ . The value of one limit can be set to None to indicate an open interval.

**inclusive** : {(True, True) tuple}, optional

Tuple of flags indicating whether the number of data being masked on the left (right) end should be truncated (True) or rounded (False) to integers.

**axis** : {None, int}, optional

Axis along which to trim. If None, the whole array is trimmed, but its shape is maintained.

`scipy.stats.mstats.trimtail` (*data*, *proportiontocut=0.2*, *tail='left'*, *inclusive=(True, True)*, *axis=None*)

Trims the data by masking values from one tail.

**Parameters** **data** : array\_like

Data to trim.

**proportiontocut** : float, optional

Percentage of trimming. If *n* is the number of unmasked values before trimming, the number of values after trimming is  $(1 - \text{proportiontocut}) * n$ . Default is 0.2.

**tail** : {'left', 'right'}, optional

If 'left' the *proportiontocut* lowest values will be masked. If 'right' the *proportiontocut* highest values will be masked. Default is 'left'.

**inclusive** : {(bool, bool) tuple}, optional

Tuple indicating whether the number of data being masked on each side should be rounded (True) or truncated (False). Default is (True, True).

**axis** : int, optional

Axis along which to perform the trimming. If None, the input array is first flattened. Default is None.

**Returns**

**trimtail** : ndarray

Returned array of same shape as *data* with masked tail values.

`scipy.stats.mstats.tsem` (*a*, *limits=None*, *inclusive=(True, True)*, *axis=0*, *ddof=1*)

Compute the trimmed standard error of the mean.

This function finds the standard error of the mean for given values, ignoring values outside the given *limits*.

**Parameters** **a** : array\_like

array of values

**limits** : None or (lower limit, upper limit), optional

Values in the input array less than the lower limit or greater than the upper limit will be ignored. When *limits* is None, then all values are used. Either

of the limit values in the tuple can also be None representing a half-open interval. The default value is None.

**inclusive** : (bool, bool), optional

A tuple consisting of the (lower flag, upper flag). These flags determine whether values exactly equal to the lower or upper limits are included. The default value is (True, True).

**axis** : int or None, optional

Axis along which to operate. If None, compute over the whole array. Default is zero.

**ddof** : int, optional

Delta degrees of freedom. Default is 1.

**Returns** **tsem** : float

**Notes**

For more details on `tsem`, see `stats.tsem`.

`scipy.stats.mstats.ttest_onesamp(a, popmean, axis=0)`

Calculates the T-test for the mean of ONE group of scores.

**Parameters** **a** : array\_like

sample observation

**popmean** : float or array\_like

expected value in null hypothesis, if array\_like than it must have the same shape as *a* excluding the axis dimension

**axis** : int or None, optional

Axis along which to compute test. If None, compute over the whole array

**Returns** **statistic** : float or array  
<sup>*a.*</sup>  
t-statistic

**pvalue** : float or array  
two-tailed p-value

**Notes**

For more details on `ttest_1samp`, see `stats.ttest_1samp`.

`scipy.stats.mstats.ttest_ind(a, b, axis=0, equal_var=True)`

Calculates the T-test for the means of TWO INDEPENDENT samples of scores.

**Parameters** **a, b** : array\_like

The arrays must have the same shape, except in the dimension corresponding to *axis* (the first, by default).

**axis** : int or None, optional

Axis along which to compute test. If None, compute over the whole arrays, *a*, and *b*.

**equal\_var** : bool, optional

If True, perform a standard independent 2 sample test that assumes equal population variances. If False, perform Welch's t-test, which does not assume equal population variance. .. versionadded:: 0.17.0

**Returns** **statistic** : float or array  
The calculated t-statistic.

**pvalue** : float or array  
The two-tailed p-value.

**Notes**

For more details on `ttest_ind`, see `stats.ttest_ind`.



`scipy.stats.mstats.ttest_onesamp` (*a*, *popmean*, *axis=0*)

Calculates the T-test for the mean of ONE group of scores.

**Parameters**

- a** : array\_like  
sample observation
- popmean** : float or array\_like  
expected value in null hypothesis, if array\_like than it must have the same shape as *a* excluding the axis dimension
- axis** : int or None, optional  
Axis along which to compute test. If None, compute over the whole array

**Returns**

- statistic** : float or array  
*a*.  
t-statistic
- pvalue** : float or array  
two-tailed p-value

#### Notes

For more details on `ttest_1samp`, see `stats.ttest_1samp`.

`scipy.stats.mstats.ttest_rel` (*a*, *b*, *axis=0*)

Calculates the T-test on TWO RELATED samples of scores, *a* and *b*.

**Parameters**

- a, b** : array\_like  
The arrays must have the same shape.
- axis** : int or None, optional  
Axis along which to compute test. If None, compute over the whole arrays,

**Returns**

- statistic** : float or array  
*a*, and *b*.  
t-statistic
- pvalue** : float or array  
two-tailed p-value

#### Notes

For more details on `ttest_rel`, see `stats.ttest_rel`.

`scipy.stats.mstats.tvar` (*a*, *limits=None*, *inclusive=(True, True)*, *axis=0*, *ddof=1*)

Compute the trimmed variance

This function computes the sample variance of an array of values, while ignoring values which are outside of given *limits*.

**Parameters**

- a** : array\_like  
Array of values.
- limits** : None or (lower limit, upper limit), optional  
Values in the input array less than the lower limit or greater than the upper limit will be ignored. When *limits* is None, then all values are used. Either of the limit values in the tuple can also be None representing a half-open interval. The default value is None.
- inclusive** : (bool, bool), optional  
A tuple consisting of the (lower flag, upper flag). These flags determine whether values exactly equal to the lower or upper limits are included. The default value is (True, True).
- axis** : int or None, optional  
Axis along which to operate. If None, compute over the whole array. Default is zero.
- ddof** : int, optional  
Delta degrees of freedom. Default is 1.

**Returns**

- tvar** : float

Trimmed variance.

**Notes**

For more details on `tvar`, see `stats.tvar`.

`scipy.stats.mstats.variation(a, axis=0)`

Computes the coefficient of variation, the ratio of the biased standard deviation to the mean.

**Parameters**

- a** : array\_like  
Input array.
- axis** : int or None, optional  
Axis along which to calculate the coefficient of variation. Default is 0. If None, compute over the whole array *a*.

**Returns**

- variation** : ndarray  
The calculated variation along the requested axis.

**Notes**

For more details about `variation`, see `stats.variation`.

`scipy.stats.mstats.winsorize(a, limits=None, inclusive=(True, True), inplace=False, axis=None)`

Returns a Winsorized version of the input array.

The `(limits[0])`th lowest values are set to the `(limits[0])`th percentile, and the `(limits[1])`th highest values are set to the `(1 - limits[1])`th percentile. Masked values are skipped.

**Parameters**

- a** : sequence  
Input array.
- limits** : {None, tuple of float}, optional  
Tuple of the percentages to cut on each side of the array, with respect to the number of unmasked data, as floats between 0. and 1. Noting *n* the number of unmasked data before trimming, the `(n*limits[0])`th smallest data and the `(n*limits[1])`th largest data are masked, and the total number of unmasked data after trimming is `n*(1.-sum(limits))` The value of one limit can be set to None to indicate an open interval.
- inclusive** : {(True, True) tuple}, optional  
Tuple indicating whether the number of data being masked on each side should be rounded (True) or truncated (False).
- inplace** : {False, True}, optional  
Whether to winsorize in place (True) or to use a copy (False)
- axis** : {None, int}, optional  
Axis along which to trim. If None, the whole array is trimmed, but its shape is maintained.

**Notes**

This function is applied to reduce the effect of possibly spurious outliers by limiting the extreme values.

`scipy.stats.mstats.zmap(scores, compare, axis=0, ddof=0)`

Calculates the relative z-scores.

Returns an array of z-scores, i.e., scores that are standardized to zero mean and unit variance, where mean and variance are calculated from the comparison array.

**Parameters**

- scores** : array\_like  
The input for which z-scores are calculated.
- compare** : array\_like  
The input from which the mean and standard deviation of the normalization are taken; assumed to have the same dimension as *scores*.

**axis** : int or None, optional  
Axis over which mean and variance of *compare* are calculated. Default is 0. If None, compute over the whole array *scores*.

**ddof** : int, optional  
Degrees of freedom correction in the calculation of the standard deviation. Default is 0.

**Returns** **zscore** : array\_like  
Z-scores, in the same shape as *scores*.

### Notes

This function preserves ndarray subclasses, and works also with matrices and masked arrays (it uses *asanyarray* instead of *asarray* for parameters).

### Examples

```
>>> from scipy.stats import zmap
>>> a = [0.5, 2.0, 2.5, 3]
>>> b = [0, 1, 2, 3, 4]
>>> zmap(a, b)
array([-1.06066017,  0.          ,  0.35355339,  0.70710678])
```

`scipy.stats.mstats.zscore(a, axis=0, ddof=0)`

Calculates the z score of each value in the sample, relative to the sample mean and standard deviation.

**Parameters** **a** : array\_like  
An array like object containing the sample data.

**axis** : int or None, optional  
Axis along which to operate. Default is 0. If None, compute over the whole array *a*.

**ddof** : int, optional  
Degrees of freedom correction in the calculation of the standard deviation. Default is 0.

**Returns** **zscore** : array\_like  
The z-scores, standardized by mean and standard deviation of input array *a*.

### Notes

This function preserves ndarray subclasses, and works also with matrices and masked arrays (it uses *asanyarray* instead of *asarray* for parameters).

### Examples

```
>>> a = np.array([ 0.7972,  0.0767,  0.4383,  0.7866,  0.8091,
...              0.1954,  0.6307,  0.6599,  0.1065,  0.0508])
>>> from scipy import stats
>>> stats.zscore(a)
array([ 1.1273, -1.247 , -0.0552,  1.0923,  1.1664, -0.8559,  0.5786,
        0.6748, -1.1488, -1.3324])
```

Computing along a specified axis, using n-1 degrees of freedom (ddof=1) to calculate the standard deviation:

```
>>> b = np.array([[ 0.3148,  0.0478,  0.6243,  0.4608],
...              [ 0.7149,  0.0775,  0.6072,  0.9656],
...              [ 0.6341,  0.1403,  0.9759,  0.4064],
...              [ 0.5918,  0.6948,  0.904 ,  0.3721],
...              [ 0.0921,  0.2481,  0.1188,  0.1366]])
>>> stats.zscore(b, axis=1, ddof=1)
```

```
array([[ -0.19264823, -1.28415119,  1.07259584,  0.40420358],
       [ 0.33048416, -1.37380874,  0.04251374,  1.00081084],
       [ 0.26796377, -1.12598418,  1.23283094, -0.37481053],
       [-0.22095197,  0.24468594,  1.19042819, -1.21416216],
       [-0.82780366,  1.4457416 , -0.43867764, -0.1792603 ]])
```

`scipy.stats.mstats.compare_medians_ms` (*group\_1*, *group\_2*, *axis=None*)

Compares the medians from two independent groups along the given axis.

The comparison is performed using the McKean-Schrader estimate of the standard error of the medians.

**Parameters**

- group\_1** : array\_like  
First dataset.
- group\_2** : array\_like  
Second dataset.
- axis** : int, optional  
Axis along which the medians are estimated. If *None*, the arrays are flattened. If *axis* is not *None*, then *group\_1* and *group\_2* should have the same shape.

**Returns**

**compare\_medians\_ms** : {float, ndarray}  
If *axis* is *None*, then returns a float, otherwise returns a 1-D ndarray of floats with a length equal to the length of *group\_1* along *axis*.

`scipy.stats.mstats.gmean` (*a*, *axis=0*, *dtype=None*)

Compute the geometric mean along the specified axis.

Returns the geometric average of the array elements. That is:  $n$ -th root of  $(x_1 * x_2 * \dots * x_n)$

**Parameters**

- a** : array\_like  
Input array or object that can be converted to an array.
- axis** : int or *None*, optional  
Axis along which the geometric mean is computed. Default is 0. If *None*, compute over the whole array *a*.
- dtype** : dtype, optional  
Type of the returned array and of the accumulator in which the elements are summed. If *dtype* is not specified, it defaults to the dtype of *a*, unless *a* has an integer dtype with a precision less than that of the default platform integer. In that case, the default platform integer is used.

**Returns**

**gmean** : ndarray  
see dtype parameter above

See also:

[`numpy.mean`](#) Arithmetic average

[`numpy.average`](#)

Weighted average

[`hmean`](#)

Harmonic mean

**Notes**

The geometric average is computed over a single dimension of the input array, *axis=0* by default, or all values in the array if *axis=None*. float64 intermediate and return values are used for integer inputs.

Use masked arrays to ignore any non-finite values in the input or that arise in the calculations such as Not a Number and infinity because masked arrays automatically mask any non-finite values.

`scipy.stats.mstats.hdmedian` (*data*, *axis=-1*, *var=False*)

Returns the Harrell-Davis estimate of the median along the given axis.

**Parameters**

- data** : ndarray  
Data array.

**axis** : int, optional  
Axis along which to compute the quantiles. If None, use a flattened array.

**var** : bool, optional  
Whether to return the variance of the estimate.

`scipy.stats.mstats.hdquantiles` (*data*, *prob*=[0.25, 0.5, 0.75], *axis*=None, *var*=False)

Computes quantile estimates with the Harrell-Davis method.

The quantile estimates are calculated as a weighted linear combination of order statistics.

**Parameters**

**data** : array\_like  
Data array.

**prob** : sequence, optional  
Sequence of quantiles to compute.

**axis** : int or None, optional  
Axis along which to compute the quantiles. If None, use a flattened array.

**var** : bool, optional  
Whether to return the variance of the estimate.

**Returns**

**hdquantiles** : MaskedArray  
A (p,) array of quantiles (if *var* is False), or a (2,p) array of quantiles and variances (if *var* is True), where *p* is the number of quantiles.

`scipy.stats.mstats.hdquantiles_sd` (*data*, *prob*=[0.25, 0.5, 0.75], *axis*=None)

The standard error of the Harrell-Davis quantile estimates by jackknife.

**Parameters**

**data** : array\_like  
Data array.

**prob** : sequence, optional  
Sequence of quantiles to compute.

**axis** : int, optional  
Axis along which to compute the quantiles. If None, use a flattened array.

**Returns**

**hdquantiles\_sd** : MaskedArray  
Standard error of the Harrell-Davis quantile estimates.

`scipy.stats.mstats.hmean` (*a*, *axis*=0, *dtype*=None)

Calculates the harmonic mean along the specified axis.

That is:  $n / (1/x_1 + 1/x_2 + \dots + 1/x_n)$

**Parameters**

**a** : array\_like  
Input array, masked array or object that can be converted to an array.

**axis** : int or None, optional  
Axis along which the harmonic mean is computed. Default is 0. If None, compute over the whole array *a*.

**dtype** : dtype, optional  
Type of the returned array and of the accumulator in which the elements are summed. If *dtype* is not specified, it defaults to the dtype of *a*, unless *a* has an integer *dtype* with a precision less than that of the default platform integer. In that case, the default platform integer is used.

**Returns**

**hmean** : ndarray  
see *dtype* parameter above

See also:

[`numpy.mean`](#) Arithmetic average

[`numpy.average`](#)

Weighted average

[`gmean`](#)

Geometric mean

*Notes*

The harmonic mean is computed over a single dimension of the input array, `axis=0` by default, or all values in the array if `axis=None`. float64 intermediate and return values are used for integer inputs.

Use masked arrays to ignore any non-finite values in the input or that arise in the calculations such as Not a Number and infinity.

`scipy.stats.mstats.idealfourths` (*data*, *axis=None*)

Returns an estimate of the lower and upper quartiles.

Uses the ideal fourths algorithm.

**Parameters**    **data** : array\_like  
                                 Input array.  
                                 **axis** : int, optional  
                                 Axis along which the quartiles are estimated. If None, the arrays are flattened.  
**Returns**        **idealfourths** : {list of floats, masked array}  
                                 Returns the two internal values that divide *data* into four parts using the ideal fourths algorithm either along the flattened array (if *axis* is None) or along *axis* of *data*.

`scipy.stats.mstats.kruskal` (\*args)

Compute the Kruskal-Wallis H-test for independent samples

**Parameters**    **sample1, sample2, ...** : array\_like  
                                 Two or more arrays with the sample measurements can be given as arguments.  
**Returns**        **statistic** : float  
                                 The Kruskal-Wallis H statistic, corrected for ties  
                                 **pvalue** : float  
                                 The p-value for the test using the assumption that H has a chi square distribution

*Notes*

For more details on `kruskal`, see `stats.kruskal`.

`scipy.stats.mstats.ks_2samp` (*data1*, *data2*, *alternative='two-sided'*)

Computes the Kolmogorov-Smirnov test on two samples.

Missing values are discarded.

**Parameters**    **data1** : array\_like  
                                 First data set  
                                 **data2** : array\_like  
                                 Second data set  
                                 **alternative** : {'two-sided', 'less', 'greater'}, optional  
                                 Indicates the alternative hypothesis. Default is 'two-sided'.  
**Returns**        **d** : float  
                                 Value of the Kolmogorov Smirnov test  
                                 **p** : float  
                                 Corresponding p-value.

`scipy.stats.mstats.median_cihs` (*data*, *alpha=0.05*, *axis=None*)

Computes the alpha-level confidence interval for the median of the data.

Uses the Hettmasperger-Sheather method.

**Parameters**    **data** : array\_like

Input data. Masked values are discarded. The input should be 1D only, or *axis* should be set to None.

**alpha** : float, optional  
Confidence level of the intervals.

**axis** : int or None, optional  
Axis along which to compute the quantiles. If None, use a flattened array.

**Returns** `median_cihs`  
Alpha level confidence interval.

`scipy.stats.mstats.mppf` (*data*, *alpha*=0.4, *beta*=0.4)

Returns plotting positions (or empirical percentile points) for the data.

*Plotting positions are defined as  $(i - \alpha) / (n + 1 - \alpha - \beta)$ , where:*

- *i* is the rank order statistics
- *n* is the number of unmasked values along the given axis
- *alpha* and *beta* are two parameters.

*Typical values for alpha and beta are:*

- (0,1) :  $p(k) = k/n$ , linear interpolation of cdf (R, type 4)
- (.5,.5) :  $p(k) = (k - 1/2) / n$ , piecewise linear function (R, type 5)
- (0,0) :  $p(k) = k / (n + 1)$ , Weibull (R type 6)
- (1,1) :  $p(k) = (k - 1) / (n - 1)$ , in this case,  $p(k) = \text{mode}[F(x[k])]$ . That's R default (R type 7)
- (1/3,1/3):  $p(k) = (k - 1/3) / (n + 1/3)$ , then  $p(k) \sim \text{median}[F(x[k])]$ . The resulting quantile estimates are approximately median-unbiased regardless of the distribution of *x*. (R type 8)
- (3/8,3/8):  $p(k) = (k - 3/8) / (n + 1/4)$ , Blom. The resulting quantile estimates are approximately unbiased if *x* is normally distributed (R type 9)
- (.4,.4) : approximately quantile unbiased (Cunnane)
- (.35,.35): APL, used with PWM
- (.3175, .3175): used in `scipy.stats.probplot`

**Parameters** **data** : array\_like  
Input data, as a sequence or array of dimension at most 2.

**alpha** : float, optional  
Plotting positions parameter. Default is 0.4.

**beta** : float, optional  
Plotting positions parameter. Default is 0.4.

**Returns** **positions** : MaskedArray  
The calculated plotting positions.

`scipy.stats.mstats.mjci` (*data*, *prob*=[0.25, 0.5, 0.75], *axis*=None)

Returns the Maritz-Jarrett estimators of the standard error of selected experimental quantiles of the data.

**Parameters** **data** : ndarray  
Data array.

**prob** : sequence, optional  
Sequence of quantiles to compute.

**axis** : int or None, optional  
Axis along which to compute the quantiles. If None, use a flattened array.

`scipy.stats.mstats.mquantiles_cimj` (*data*, *prob*=[0.25, 0.5, 0.75], *alpha*=0.05, *axis*=None)

Computes the alpha confidence interval for the selected quantiles of the data, with Maritz-Jarrett estimators.

**Parameters** **data** : ndarray  
Data array.

**prob** : sequence, optional

Sequence of quantiles to compute.  
**alpha** : float, optional  
 Confidence level of the intervals.  
**axis** : int or None, optional  
 Axis along which to compute the quantiles. If None, use a flattened array.

`scipy.stats.mstats.rsh` (*data*, *points=None*)

Evaluates Rosenblatt's shifted histogram estimators for each point on the dataset 'data'.

**Parameters** **data** : sequence  
 Input data. Masked values are ignored.  
**points** : sequence or None, optional  
 Sequence of points where to evaluate Rosenblatt shifted histogram. If None, use the data.

`scipy.stats.mstats.sen_seasonal_slopes` (*x*)

`scipy.stats.mstats.trimmed_mean` (*a*, *limits=(0.1, 0.1)*, *inclusive=(1, 1)*, *relative=True*, *axis=None*)

`scipy.stats.mstats.trimmed_mean_ci` (*data*, *limits=(0.2, 0.2)*, *inclusive=(True, True)*, *alpha=0.05*, *axis=None*)

Selected confidence interval of the trimmed mean along the given axis.

**Parameters** **data** : array\_like  
 Input data.  
**limits** : {None, tuple}, optional  
 None or a two item tuple. Tuple of the percentages to cut on each side of the array, with respect to the number of unmasked data, as floats between 0. and 1. If *n* is the number of unmasked data before trimming, then (*n* \* *limits*[0])th smallest data and (*n* \* *limits*[1])th largest data are masked. The total number of unmasked data after trimming is *n* \* (1. - sum(*limits*)). The value of one limit can be set to None to indicate an open interval.  
 Defaults to (0.2, 0.2).  
**inclusive** : (2,) tuple of boolean, optional  
 If *relative==False*, tuple indicating whether values exactly equal to the absolute limits are allowed. If *relative==True*, tuple indicating whether the number of data being masked on each side should be rounded (True) or truncated (False).  
 Defaults to (True, True).  
**alpha** : float, optional  
 Confidence level of the intervals.  
 Defaults to 0.05.  
**axis** : int, optional  
 Axis along which to cut. If None, uses a flattened version of *data*.

**Returns** **trimmed\_mean\_ci** : (2,) ndarray  
 Defaults to None.  
 The lower and upper confidence intervals of the trimmed data.

`scipy.stats.mstats.trimmed_std` (*a*, *limits=(0.1, 0.1)*, *inclusive=(1, 1)*, *relative=True*, *axis=None*, *ddof=0*)

`scipy.stats.mstats.trimmed_var` (*a*, *limits=(0.1, 0.1)*, *inclusive=(1, 1)*, *relative=True*, *axis=None*, *ddof=0*)



`scipy.stats.mstats.ttest_1samp` (*a*, *popmean*, *axis=0*)

Calculates the T-test for the mean of ONE group of scores.

**Parameters**

- a** : array\_like  
sample observation
- popmean** : float or array\_like  
expected value in null hypothesis, if array\_like than it must have the same shape as *a* excluding the axis dimension
- axis** : int or None, optional  
Axis along which to compute test. If None, compute over the whole array

**Returns**

- statistic** : float or array *a*  
t-statistic
- pvalue** : float or array  
two-tailed p-value

#### Notes

For more details on `ttest_1samp`, see `stats.ttest_1samp`.

### 5.27.9 Univariate and multivariate kernel density estimation (`scipy.stats.kde`)

---

`gaussian_kde`(*dataset*[, *bw\_method*]) Representation of a kernel-density estimate using Gaussian kernels.

---

**class** `scipy.stats.gaussian_kde` (*dataset*, *bw\_method=None*)

Representation of a kernel-density estimate using Gaussian kernels.

Kernel density estimation is a way to estimate the probability density function (PDF) of a random variable in a non-parametric way. `gaussian_kde` works for both uni-variate and multi-variate data. It includes automatic bandwidth determination. The estimation works best for a unimodal distribution; bimodal or multi-modal distributions tend to be oversmoothed.

**Parameters**

- dataset** : array\_like  
Datapoints to estimate from. In case of univariate data this is a 1-D array, otherwise a 2-D array with shape (# of dims, # of data).
- bw\_method** : str, scalar or callable, optional  
The method used to calculate the estimator bandwidth. This can be ‘scott’, ‘silverman’, a scalar constant or a callable. If a scalar, this will be used directly as *kde.factor*. If a callable, it should take a `gaussian_kde` instance as only parameter and return a scalar. If None (default), ‘scott’ is used. See Notes for more details.

#### Notes

Bandwidth selection strongly influences the estimate obtained from the KDE (much more so than the actual shape of the kernel). Bandwidth selection can be done by a “rule of thumb”, by cross-validation, by “plug-in methods” or by other means; see [R523], [R524] for reviews. `gaussian_kde` uses a rule of thumb, the default is Scott’s Rule.

Scott’s Rule [R521], implemented as `scotts_factor`, is:

$$n^{-(1/(d+4))},$$

with *n* the number of data points and *d* the number of dimensions. Silverman’s Rule [R522], implemented as `silverman_factor`, is:

```
(n * (d + 2) / 4.) ** (-1. / (d + 4)).
```

Good general descriptions of kernel density estimation can be found in [R521] and [R522], the mathematics for this multi-dimensional implementation can be found in [R521].

### References

[R521], [R522], [R523], [R524]

### Examples

Generate some random two-dimensional data:

```
>>> from scipy import stats
>>> def measure(n):
...     "Measurement model, return two coupled measurements."
...     m1 = np.random.normal(size=n)
...     m2 = np.random.normal(scale=0.5, size=n)
...     return m1+m2, m1-m2

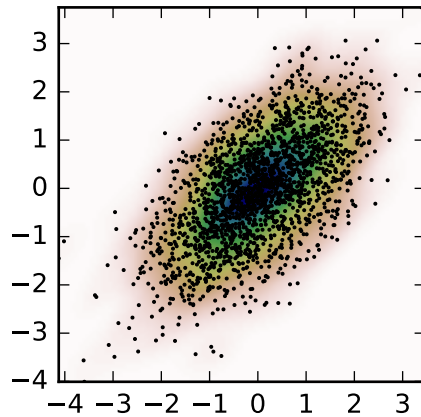
>>> m1, m2 = measure(2000)
>>> xmin = m1.min()
>>> xmax = m1.max()
>>> ymin = m2.min()
>>> ymax = m2.max()
```

Perform a kernel density estimate on the data:

```
>>> X, Y = np.mgrid[xmin:xmax:100j, ymin:ymax:100j]
>>> positions = np.vstack([X.ravel(), Y.ravel()])
>>> values = np.vstack([m1, m2])
>>> kernel = stats.gaussian_kde(values)
>>> Z = np.reshape(kernel(positions).T, X.shape)
```

Plot the results:

```
>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots()
>>> ax.imshow(np.rot90(Z), cmap=plt.cm.gist_earth_r,
...           extent=[xmin, xmax, ymin, ymax])
>>> ax.plot(m1, m2, 'k.', markersize=2)
>>> ax.set_xlim([xmin, xmax])
>>> ax.set_ylim([ymin, ymax])
>>> plt.show()
```



**Attributes**

dataset	(ndarray) The dataset with which <code>gaussian_kde</code> was initialized.
d	(int) Number of dimensions.
n	(int) Number of datapoints.
factor	(float) The bandwidth factor, obtained from <code>kde.covariance_factor</code> , with which the covariance matrix is multiplied.
covariance	(ndarray) The covariance matrix of <code>dataset</code> , scaled by the calculated bandwidth ( <code>kde.factor</code> ).
inv_cov	(ndarray) The inverse of <code>covariance</code> .

**Methods**

<code>evaluate(points)</code>	Evaluate the estimated pdf on a set of points.
<code>__call__(points)</code>	Evaluate the estimated pdf on a set of points.
<code>integrate_gaussian(mean, cov)</code>	Multiply estimated density by a multivariate Gaussian and integrate over
<code>integrate_box_1d(low, high)</code>	Computes the integral of a 1D pdf between two bounds.
<code>integrate_box(low_bounds, high_bounds[, maxpts])</code>	Computes the integral of a pdf over a rectangular interval.
<code>integrate_kde(other)</code>	Computes the integral of the product of this kernel density estimate with a
<code>pdf(x)</code>	Evaluate the estimated pdf on a provided set of points.
<code>logpdf(x)</code>	Evaluate the log of the estimated pdf on a provided set of points.
<code>resample([size])</code>	Randomly sample a dataset from the estimated pdf.
<code>set_bandwidth([bw_method])</code>	Compute the estimator bandwidth with given method.
<code>covariance_factor()</code>	Computes the coefficient ( <code>kde.factor</code> ) that multiplies the data covariance m

`gaussian_kde.evaluate(points)`  
 Evaluate the estimated pdf on a set of points.

- Parameters** `points` : (# of dimensions, # of points)-array  
 Alternatively, a (# of dimensions,) vector can be passed in and treated as a single point.
- Returns** `values` : (# of points,)-array  
 The values at each point.
- Raises** `ValueError` : if the dimensionality of the input points is different than the dimensionality of the KDE.

`gaussian_kde.__call__(points)`

Evaluate the estimated pdf on a set of points.

**Parameters** **points** : (# of dimensions, # of points)-array  
 Alternatively, a (# of dimensions,) vector can be passed in and treated as a single point.

**Returns** **values** : (# of points,)-array

**Raises** **ValueError** : The values at each point if the dimensionality of the input points is different than the dimensionality of the KDE.

`gaussian_kde.integrate_gaussian(mean, cov)`

Multiply estimated density by a multivariate Gaussian and integrate over the whole space.

**Parameters** **mean** : array\_like  
 A 1-D array, specifying the mean of the Gaussian.

**cov** : array\_like  
 A 2-D array, specifying the covariance matrix of the Gaussian.

**Returns** **result** : scalar  
 The value of the integral.

**Raises** **ValueError**  
 If the mean or covariance of the input Gaussian differs from the KDE's dimensionality.

`gaussian_kde.integrate_box_1d(low, high)`

Computes the integral of a 1D pdf between two bounds.

**Parameters** **low** : scalar  
 Lower bound of integration.

**high** : scalar  
 Upper bound of integration.

**Returns** **value** : scalar  
 The result of the integral.

**Raises** **ValueError**  
 If the KDE is over more than one dimension.

`gaussian_kde.integrate_box(low_bounds, high_bounds, maxpts=None)`

Computes the integral of a pdf over a rectangular interval.

**Parameters** **low\_bounds** : array\_like  
 A 1-D array containing the lower bounds of integration.

**high\_bounds** : array\_like  
 A 1-D array containing the upper bounds of integration.

**maxpts** : int, optional  
 The maximum number of points to use for integration.

**Returns** **value** : scalar  
 The result of the integral.

`gaussian_kde.integrate_kde(other)`

Computes the integral of the product of this kernel density estimate with another.

**Parameters** **other** : gaussian\_kde instance  
 The other kde.

**Returns** **value** : scalar  
 The result of the integral.

**Raises** **ValueError**  
 If the KDEs have different dimensionality.

`gaussian_kde.pdf(x)`

Evaluate the estimated pdf on a provided set of points.

**Notes**

This is an alias for `gaussian_kde.evaluate`. See the `evaluate` docstring for more details.

`gaussian_kde.logpdf(x)`

Evaluate the log of the estimated pdf on a provided set of points.

**Notes**

See `gaussian_kde.evaluate` for more details; this method simply returns `np.log(gaussian_kde.evaluate(x))`.

`gaussian_kde.resample(size=None)`

Randomly sample a dataset from the estimated pdf.

**Parameters** `size` : int, optional

The number of samples to draw. If not provided, then the size is the same as the underlying dataset.

**Returns** `resample` : (self.d, size) ndarray

The sampled dataset.

`gaussian_kde.set_bandwidth(bw_method=None)`

Compute the estimator bandwidth with given method.

The new bandwidth calculated after a call to `set_bandwidth` is used for subsequent evaluations of the estimated density.

**Parameters** `bw_method` : str, scalar or callable, optional

The method used to calculate the estimator bandwidth. This can be 'scott', 'silverman', a scalar constant or a callable. If a scalar, this will be used directly as `kde.factor`. If a callable, it should take a `gaussian_kde` instance as only parameter and return a scalar. If None (default), nothing happens; the current `kde.covariance_factor` method is kept.

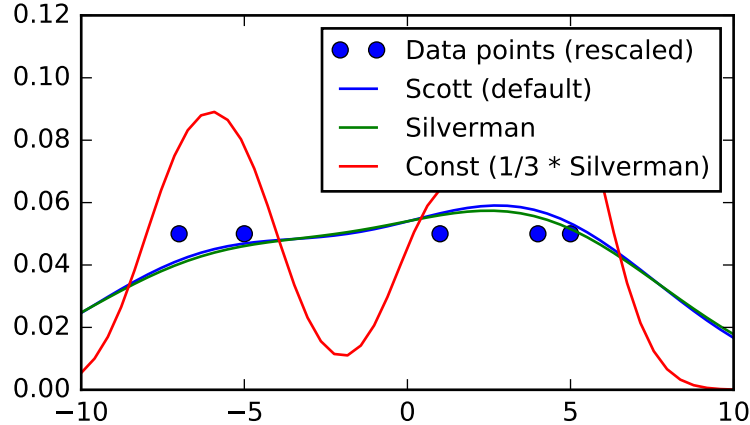
**Notes**

New in version 0.11.

**Examples**

```
>>> import scipy.stats as stats
>>> x1 = np.array([-7, -5, 1, 4, 5.])
>>> kde = stats.gaussian_kde(x1)
>>> xs = np.linspace(-10, 10, num=50)
>>> y1 = kde(xs)
>>> kde.set_bandwidth(bw_method='silverman')
>>> y2 = kde(xs)
>>> kde.set_bandwidth(bw_method=kde.factor / 3.)
>>> y3 = kde(xs)

>>> import matplotlib.pyplot as plt
>>> fig, ax = plt.subplots()
>>> ax.plot(x1, np.ones(x1.shape) / (4. * x1.size), 'bo',
...        label='Data points (rescaled)')
>>> ax.plot(xs, y1, label='Scott (default)')
>>> ax.plot(xs, y2, label='Silverman')
>>> ax.plot(xs, y3, label='Const (1/3 * Silverman)')
>>> ax.legend()
>>> plt.show()
```



```
gaussian_kde.covariance_factor()
```

Computes the coefficient (*kde.factor*) that multiplies the data covariance matrix to obtain the kernel covariance matrix. The default is `scotts_factor`. A subclass can overwrite this method to provide a different method, or set it through a call to `kde.set_bandwidth`.

For many more stat related functions install the software R and the interface package rpy.

## 5.28 Statistical functions for masked arrays (`scipy.stats.mstats`)

This module contains a large number of statistical functions that can be used with masked arrays.

Most of these functions are similar to those in `scipy.stats` but might have small differences in the API or in the algorithm used. Since this is a relatively new package, some API changes are still possible.

<code>argstoarray(*args)</code>	Constructs a 2D array from a group of sequences.
<code>betai(*args, **kws)</code>	<code>betai</code> is deprecated!
<code>chisquare(f_obs[, f_exp, ddof, axis])</code>	Calculates a one-way chi square test.
<code>count_tied_groups(x[, use_missing])</code>	Counts the number of tied values.
<code>describe(a[, axis, ddof, bias])</code>	Computes several descriptive statistics of the passed array.
<code>f_oneway(*args)</code>	Performs a 1-way ANOVA, returning an F-value and probability given any nu
<code>f_value_wilks_lambda(*args, **kws)</code>	<code>f_value_wilks_lambda</code> is deprecated!
<code>find_repeats(arr)</code>	Find repeats in arr and return a tuple (repeats, repeat_count).
<code>friedmanchisquare(*args)</code>	Friedman Chi-Square is a non-parametric, one-way within-subjects ANOVA.
<code>kendalltau(x, y[, use_ties, use_missing])</code>	Computes Kendall's rank correlation tau on two variables <i>x</i> and <i>y</i> .
<code>kendalltau_seasonal(x)</code>	Computes a multivariate Kendall's rank correlation tau, for seasonal data.
<code>kruskalwallis(*args)</code>	Compute the Kruskal-Wallis H-test for independent samples
<code>ks_twosamp(data1, data2[, alternative])</code>	Computes the Kolmogorov-Smirnov test on two samples.
<code>kurtosis(a[, axis, fisher, bias])</code>	Computes the kurtosis (Fisher or Pearson) of a dataset.
<code>kurtosistest(a[, axis])</code>	Tests whether a dataset has normal kurtosis
<code>linregress(x[, y])</code>	Calculate a linear least-squares regression for two sets of measurements.
<code>mannwhitneyu(x, y[, use_continuity])</code>	Computes the Mann-Whitney statistic
<code>plotting_positions(data[, alpha, beta])</code>	Returns plotting positions (or empirical percentile points) for the data.
<code>mode(a[, axis])</code>	Returns an array of the modal (most common) value in the passed array.
<code>moment(a[, moment, axis])</code>	Calculates the nth moment about the mean for a sample.

Table 5.295 – continued from previous page

<code>mquantiles(a[, prob, alphap, betap, axis, limit])</code>	Computes empirical quantiles for a data array.
<code>msign(x)</code>	Returns the sign of x, or 0 if x is masked.
<code>normaltest(a[, axis])</code>	Tests whether a sample differs from a normal distribution.
<code>obrientransform(*args)</code>	Computes a transform on input data (any number of columns).
<code>pearsonr(x, y)</code>	Calculates a Pearson correlation coefficient and the p-value for testing non-co
<code>plotting_positions(data[, alpha, beta])</code>	Returns plotting positions (or empirical percentile points) for the data.
<code>pointbiserialr(x, y)</code>	Calculates a point biserial correlation coefficient and its p-value.
<code>rankdata(data[, axis, use_missing])</code>	Returns the rank (also known as order statistics) of each data point along the g
<code>scoreatpercentile(data, per[, limit, ...])</code>	Calculate the score at the given ‘per’ percentile of the sequence a.
<code>sem(a[, axis, ddof])</code>	Calculates the standard error of the mean of the input array.
<code>signaltonoise(*args, **kwds)</code>	<code>signaltonoise</code> is deprecated!
<code>skew(a[, axis, bias])</code>	Computes the skewness of a data set.
<code>skewtest(a[, axis])</code>	Tests whether the skew is different from the normal distribution.
<code>spearmanr(x, y[, use_ties])</code>	Calculates a Spearman rank-order correlation coefficient and the p-value to te
<code>theilslopes(y[, x, alpha])</code>	Computes the Theil-Sen estimator for a set of points (x, y).
<code>threshold(*args, **kwds)</code>	<code>threshold</code> is deprecated!
<code>tmax(a[, upperlimit, axis, inclusive])</code>	Compute the trimmed maximum
<code>tmean(a[, limits, inclusive, axis])</code>	Compute the trimmed mean.
<code>tmin(a[, lowerlimit, axis, inclusive])</code>	Compute the trimmed minimum
<code>trim(a[, limits, inclusive, relative, axis])</code>	Trims an array by masking the data outside some given limits.
<code>trima(a[, limits, inclusive])</code>	Trims an array by masking the data outside some given limits.
<code>trimboth(data[, proportiontocut, inclusive, ...])</code>	Trims the smallest and largest data values.
<code>trimmed_stde(a[, limits, inclusive, axis])</code>	Returns the standard error of the trimmed mean along the given axis.
<code>trimr(a[, limits, inclusive, axis])</code>	Trims an array by masking some proportion of the data on each end.
<code>trimtail(data[, proportiontocut, tail, ...])</code>	Trims the data by masking values from one tail.
<code>tsem(a[, limits, inclusive, axis, ddof])</code>	Compute the trimmed standard error of the mean.
<code>ttest_onesamp(a, popmean[, axis])</code>	Calculates the T-test for the mean of ONE group of scores.
<code>ttest_ind(a, b[, axis, equal_var])</code>	Calculates the T-test for the means of TWO INDEPENDENT samples of scor
<code>ttest_onesamp(a, popmean[, axis])</code>	Calculates the T-test for the mean of ONE group of scores.
<code>ttest_rel(a, b[, axis])</code>	Calculates the T-test on TWO RELATED samples of scores, a and b.
<code>tvar(a[, limits, inclusive, axis, ddof])</code>	Compute the trimmed variance
<code>variation(a[, axis])</code>	Computes the coefficient of variation, the ratio of the biased standard deviatio
<code>winsorize(a[, limits, inclusive, inplace, axis])</code>	Returns a Winsorized version of the input array.
<code>zmap(scores, compare[, axis, ddof])</code>	Calculates the relative z-scores.
<code>zscore(a[, axis, ddof])</code>	Calculates the z score of each value in the sample, relative to the sample mean
<code>compare_medians_ms(group_1, group_2[, axis])</code>	Compares the medians from two independent groups along the given axis.
<code>gmean(a[, axis, dtype])</code>	Compute the geometric mean along the specified axis.
<code>hdmedian(data[, axis, var])</code>	Returns the Harrell-Davis estimate of the median along the given axis.
<code>hdquantiles(data[, prob, axis, var])</code>	Computes quantile estimates with the Harrell-Davis method.
<code>hdquantiles_sd(data[, prob, axis])</code>	The standard error of the Harrell-Davis quantile estimates by jackknife.
<code>hmean(a[, axis, dtype])</code>	Calculates the harmonic mean along the specified axis.
<code>idealfourths(data[, axis])</code>	Returns an estimate of the lower and upper quartiles.
<code>kruskal(*args)</code>	Compute the Kruskal-Wallis H-test for independent samples
<code>ks_2samp(data1, data2[, alternative])</code>	Computes the Kolmogorov-Smirnov test on two samples.
<code>median_cihs(data[, alpha, axis])</code>	Computes the alpha-level confidence interval for the median of the data.
<code>meppf(data[, alpha, beta])</code>	Returns plotting positions (or empirical percentile points) for the data.
<code>mjci(data[, prob, axis])</code>	Returns the Maritz-Jarrett estimators of the standard error of selected experim
<code>mquantiles_cimj(data[, prob, alpha, axis])</code>	Computes the alpha confidence interval for the selected quantiles of the data,
<code>rsh(data[, points])</code>	Evaluates Rosenblatt’s shifted histogram estimators for each point on the data
<code>sen_seasonal_slopes(x)</code>	
<code>trimmed_mean(a[, limits, inclusive, ...])</code>	
<code>trimmed_mean_ci(data[, limits, inclusive, ...])</code>	Selected confidence interval of the trimmed mean along the given axis.

Table 5.295 – continued from previous page

<code>trimmed_std(a[, limits, inclusive, ...])</code>	
<code>trimmed_var(a[, limits, inclusive, ...])</code>	
<code>ttest_1samp(a, popmean[, axis])</code>	Calculates the T-test for the mean of ONE group of scores.

`scipy.stats.mstats.argmaxtoarray(*args)`

Constructs a 2D array from a group of sequences.

Sequences are filled with missing values to match the length of the longest sequence.

**Parameters** `args` : sequences

**Returns** `argstoarray` : `MaskedArray` Group of sequences.

A ( $m \times n$ ) masked array, where  $m$  is the number of arguments and  $n$  the length of the longest argument.

#### Notes

`numpy.ma.row_stack` has identical behavior, but is called with a sequence of sequences.

`scipy.stats.mstats.betainc(*args, **kwargs)`

`betainc` is deprecated! `mstats.betainc` is deprecated in scipy 0.17.0; use `special.betainc` instead.

`betainc()` is deprecated in scipy 0.17.0.

For details about this function, see `stats.betainc`.

`scipy.stats.mstats.chisquare(f_obs, f_exp=None, ddof=0, axis=0)`

Calculates a one-way chi square test.

The chi square test tests the null hypothesis that the categorical data has the given frequencies.

**Parameters** `f_obs` : array\_like

Observed frequencies in each category.

`f_exp` : array\_like, optional

Expected frequencies in each category. By default the categories are assumed to be equally likely.

`ddof` : int, optional

“Delta degrees of freedom”: adjustment to the degrees of freedom for the p-value. The p-value is computed using a chi-squared distribution with  $k - 1 - \text{ddof}$  degrees of freedom, where  $k$  is the number of observed frequencies. The default value of `ddof` is 0.

`axis` : int or None, optional

The axis of the broadcast result of `f_obs` and `f_exp` along which to apply the test. If `axis` is None, all values in `f_obs` are treated as a single data set. Default is 0.

**Returns** `chisq` : float or ndarray

The chi-squared test statistic. The value is a float if `axis` is None or `f_obs` and `f_exp` are 1-D.

`p` : float or ndarray

The p-value of the test. The value is a float if `ddof` and the return value `chisq` are scalars.

#### See also:

`power_divergence`, `mstats.chisquare`



**Notes**

This test is invalid when the observed or expected frequencies in each category are too small. A typical rule is that all of the observed and expected frequencies should be at least 5.

The default degrees of freedom,  $k-1$ , are for the case when no parameters of the distribution are estimated. If  $p$  parameters are estimated by efficient maximum likelihood then the correct degrees of freedom are  $k-1-p$ . If the parameters are estimated in a different way, then the dof can be between  $k-1-p$  and  $k-1$ . However, it is also possible that the asymptotic distribution is not a chisquare, in which case this test is not appropriate.

**References**

[R544], [R545]

**Examples**

When just  $f\_obs$  is given, it is assumed that the expected frequencies are uniform and given by the mean of the observed frequencies.

```
>>> from scipy.stats import chisquare
>>> chisquare([16, 18, 16, 14, 12, 12])
(2.0, 0.84914503608460956)
```

With  $f\_exp$  the expected frequencies can be given.

```
>>> chisquare([16, 18, 16, 14, 12, 12], f_exp=[16, 16, 16, 16, 16, 8])
(3.5, 0.62338762774958223)
```

When  $f\_obs$  is 2-D, by default the test is applied to each column.

```
>>> obs = np.array([[16, 18, 16, 14, 12, 12], [32, 24, 16, 28, 20, 24]])
>>> obs.shape
(6, 2)
>>> chisquare(obs)
(array([ 2.          ,  6.66666667]), array([ 0.84914504,  0.24663415]))
```

By setting  $axis=None$ , the test is applied to all data in the array, which is equivalent to applying the test to the flattened array.

```
>>> chisquare(obs, axis=None)
(23.31034482758621, 0.015975692534127565)
>>> chisquare(obs.ravel())
(23.31034482758621, 0.015975692534127565)
```

$ddof$  is the change to make to the default degrees of freedom.

```
>>> chisquare([16, 18, 16, 14, 12, 12], ddof=1)
(2.0, 0.73575888234288467)
```

The calculation of the p-values is done by broadcasting the chi-squared statistic with  $ddof$ .

```
>>> chisquare([16, 18, 16, 14, 12, 12], ddof=[0,1,2])
(2.0, array([ 0.84914504,  0.73575888,  0.5724067 ]))
```

$f\_obs$  and  $f\_exp$  are also broadcast. In the following,  $f\_obs$  has shape (6,) and  $f\_exp$  has shape (2, 6), so the result of broadcasting  $f\_obs$  and  $f\_exp$  has shape (2, 6). To compute the desired chi-squared statistics, we use  $axis=1$ :

```
>>> chisquare([16, 18, 16, 14, 12, 12],
...           f_exp=[[16, 16, 16, 16, 16, 8], [8, 20, 20, 16, 12, 12]],
...           axis=1)
(array([ 3.5 ,  9.25]), array([ 0.62338763,  0.09949846]))
```

`scipy.stats.mstats.count_tied_groups` (*x*, *use\_missing=False*)

Counts the number of tied values.

**Parameters**

- x** : sequence  
Sequence of data on which to counts the ties
- use\_missing** : bool, optional  
Whether to consider missing values as tied.

**Returns**

**count\_tied\_groups** : dict  
Returns a dictionary (nb of ties: nb of groups).

### Examples

```
>>> from scipy.stats import mstats
>>> z = [0, 0, 0, 2, 2, 2, 3, 3, 4, 5, 6]
>>> mstats.count_tied_groups(z)
{2: 1, 3: 2}
```

In the above example, the ties were 0 (3x), 2 (3x) and 3 (2x).

```
>>> z = np.ma.array([0, 0, 1, 2, 2, 2, 3, 3, 4, 5, 6])
>>> mstats.count_tied_groups(z)
{2: 2, 3: 1}
>>> z[[1,-1]] = np.ma.masked
>>> mstats.count_tied_groups(z, use_missing=True)
{2: 2, 3: 1}
```

`scipy.stats.mstats.describe` (*a*, *axis=0*, *ddof=0*, *bias=True*)

Computes several descriptive statistics of the passed array.

**Parameters**

- a** : array\_like  
Data array
- axis** : int or None, optional  
Axis along which to calculate statistics. Default 0. If None, compute over the whole array *a*.
- ddof** : int, optional  
degree of freedom (default 0); note that default ddof is different from the same routine in `stats.describe`
- bias** : bool, optional  
If False, then the skewness and kurtosis calculations are corrected for statistical bias.

**Returns**

- nobs** : int  
(size of the data (discarding missing values))
- minmax** : (int, int)  
min, max
- mean** : float  
arithmetic mean
- variance** : float  
unbiased variance
- skewness** : float  
biased skewness
- kurtosis** : float  
biased kurtosis

**Examples**

```
>>> from scipy.stats.mstats import describe
>>> ma = np.ma.array(range(6), mask=[0, 0, 0, 1, 1, 1])
>>> describe(ma)
DescribeResult(nobs=array(3), minmax=(masked_array(data = 0,
            mask = False,
            fill_value = 999999)
, masked_array(data = 2,
            mask = False,
            fill_value = 999999)
), mean=1.0, variance=0.66666666666666663, skewness=masked_array(data = 0.0,
            mask = False,
            fill_value = 1e+20)
, kurtosis=-1.5)
```

`scipy.stats.mstats.f_oneway(*args)`

Performs a 1-way ANOVA, returning an F-value and probability given any number of groups. From Heiman, pp.394-7.

Usage: `f_oneway(*args)`, where `*args` is 2 or more arrays, one per treatment group.

**Returns**

- statistic** : float  
The computed F-value of the test.
- pvalue** : float  
The associated p-value from the F-distribution.

`scipy.stats.mstats.f_value_wilks_lambda(*args, **kws)`

`f_value_wilks_lambda` is deprecated! `mstats.f_value_wilks_lambda` deprecated in scipy 0.17.0

*Calculation of Wilks lambda F-statistic for multivariate data, per*

Maxwell & Delaney p.657.

`scipy.stats.mstats.find_repeats(arr)`

Find repeats in `arr` and return a tuple (`repeats`, `repeat_count`).

The input is cast to float64. Masked values are discarded.

**Parameters**

- arr** : sequence  
Input array. The array is flattened if it is not 1D.

**Returns**

- repeats** : ndarray  
Array of repeated values.
- counts** : ndarray  
Array of counts.

`scipy.stats.mstats.friedmanchisquare(*args)`

Friedman Chi-Square is a non-parametric, one-way within-subjects ANOVA. This function calculates the Friedman Chi-square test for repeated measures and returns the result, along with the associated probability value.

Each input is considered a given group. Ideally, the number of treatments among each group should be equal. If this is not the case, only the first `n` treatments are taken into account, where `n` is the number of treatments of the smallest group. If a group has some missing values, the corresponding treatments are masked in the other groups. The test statistic is corrected for ties.

Masked values in one group are propagated to the other groups.

**Returns**

- statistic** : float  
the test statistic.
- pvalue** : float  
the associated p-value.

`scipy.stats.mstats.kendalltau(x, y, use_ties=True, use_missing=False)`

Computes Kendall's rank correlation tau on two variables *x* and *y*.

**Parameters**

- x** : sequence  
First data list (for example, time).
- y** : sequence  
Second data list.
- use\_ties** : {True, False}, optional  
Whether ties correction should be performed.
- use\_missing** : {False, True}, optional  
Whether missing data should be allocated a rank of 0 (False) or the average rank (True)

**Returns**

- correlation** : float  
Kendall tau
- pvalue** : float  
Approximate 2-side p-value.

`scipy.stats.mstats.kendalltau_seasonal(x)`

Computes a multivariate Kendall's rank correlation tau, for seasonal data.

**Parameters**

- x** : 2-D ndarray  
Array of seasonal data, with seasons in columns.

`scipy.stats.mstats.kruskalwallis(*args)`

Compute the Kruskal-Wallis H-test for independent samples

**Parameters**

- sample1, sample2, ...** : array\_like  
Two or more arrays with the sample measurements can be given as arguments.

**Returns**

- statistic** : float  
The Kruskal-Wallis H statistic, corrected for ties
- pvalue** : float  
The p-value for the test using the assumption that H has a chi square distribution

**Notes**

For more details on `kruskal`, see `stats.kruskal`.

`scipy.stats.mstats.ks_twosamp(data1, data2, alternative='two-sided')`

Computes the Kolmogorov-Smirnov test on two samples.

Missing values are discarded.

**Parameters**

- data1** : array\_like  
First data set
- data2** : array\_like  
Second data set
- alternative** : {'two-sided', 'less', 'greater'}, optional  
Indicates the alternative hypothesis. Default is 'two-sided'.

**Returns**

- d** : float  
Value of the Kolmogorov Smirnov test
- p** : float  
Corresponding p-value.

`scipy.stats.mstats.kurtosis(a, axis=0, fisher=True, bias=True)`

Computes the kurtosis (Fisher or Pearson) of a dataset.

Kurtosis is the fourth central moment divided by the square of the variance. If Fisher's definition is used, then 3.0 is subtracted from the result to give 0.0 for a normal distribution.

If `bias` is `False` then the kurtosis is calculated using k statistics to eliminate bias coming from biased moment estimators

Use `kurtosistest` to see if result is close enough to normal.

**Parameters**

- a** : array  
data for which the kurtosis is calculated
- axis** : int or None, optional  
Axis along which the kurtosis is calculated. Default is 0. If None, compute over the whole array *a*.
- fisher** : bool, optional  
If True, Fisher's definition is used (normal ==> 0.0). If False, Pearson's definition is used (normal ==> 3.0).
- bias** : bool, optional  
If False, then the calculations are corrected for statistical bias.

**Returns**

- kurtosis** : array  
The kurtosis of values along an axis. If all values are equal, return -3 for Fisher's definition and 0 for Pearson's definition.

#### Notes

For more details about `kurtosis`, see `stats.kurtosis`.

`scipy.stats.mstats.kurtosistest(a, axis=0)`

Tests whether a dataset has normal kurtosis

**Parameters**

- a** : array  
array of the sample data
- axis** : int or None, optional  
Axis along which to compute test. Default is 0. If None, compute over the whole array *a*.

**Returns**

- statistic** : float  
The computed z-score for this test.
- pvalue** : float  
The 2-sided p-value for the hypothesis test

#### Notes

For more details about `kurtosistest`, see `stats.kurtosistest`.

`scipy.stats.mstats.linregress(x, y=None)`

Calculate a linear least-squares regression for two sets of measurements.

**Parameters**

- x, y** : array\_like  
Two sets of measurements. Both arrays should have the same length. If only *x* is given (and *y=None*), then it must be a two-dimensional array where one dimension has length 2. The two sets of measurements are then found by splitting the array along the length-2 dimension.

**Returns**

- slope** : float  
slope of the regression line
- intercept** : float  
intercept of the regression line
- rvalue** : float  
correlation coefficient
- pvalue** : float  
two-sided p-value for a hypothesis test whose null hypothesis is that the slope is zero.
- stderr** : float  
Standard error of the estimated gradient.

See also:

**`optimize.curve_fit`**

Use non-linear least squares to fit a function to data.

**`optimize.leastsq`**

Minimize the sum of squares of a set of equations.

*Notes*

Missing values are considered pair-wise: if a value is missing in  $x$ , the corresponding value in  $y$  is masked.

*Examples*

```
>>> from scipy import stats
>>> np.random.seed(12345678)
>>> x = np.random.random(10)
>>> y = np.random.random(10)
>>> slope, intercept, r_value, p_value, std_err = stats.linregress(x,y)
```

```
# To get coefficient of determination (r_squared)
```

```
>>> print("r-squared:", r_value**2)
('r-squared:', 0.080402268539028335)
```

`scipy.stats.mstats.mannwhitneyu(x, y, use_continuity=True)`

Computes the Mann-Whitney statistic

Missing values in  $x$  and/or  $y$  are discarded.

**Parameters**  $x$  : sequence

Input

$y$  : sequence

Input

**use\_continuity** : {True, False}, optional

Whether a continuity correction (1/2.) should be taken into account.

**Returns**

**statistic** : float

The Mann-Whitney statistics

**pvalue** : float

Approximate p-value assuming a normal distribution.

`scipy.stats.mstats.plotting_positions(data, alpha=0.4, beta=0.4)`

Returns plotting positions (or empirical percentile points) for the data.

*Plotting positions are defined as  $(i-\alpha)/(n+1-\alpha-\beta)$ , where:*

- $i$  is the rank order statistics
- $n$  is the number of unmasked values along the given axis
- $\alpha$  and  $\beta$  are two parameters.

*Typical values for  $\alpha$  and  $\beta$  are:*

- (0,1) :  $p(k) = k/n$ , linear interpolation of cdf (R, type 4)
- (.5,.5) :  $p(k) = (k-1/2.)/n$ , piecewise linear function (R, type 5)
- (0,0) :  $p(k) = k/(n+1)$ , Weibull (R type 6)
- (1,1) :  $p(k) = (k-1)/(n-1)$ , in this case,  $p(k) = \text{mode}[F(x[k])]$ . That's R default (R type 7)
- (1/3,1/3):  $p(k) = (k-1/3)/(n+1/3)$ , then  $p(k) \sim \text{median}[F(x[k])]$ . The resulting quantile estimates are approximately median-unbiased regardless of the distribution of  $x$ . (R type 8)

- (3/8,3/8):  $p(k) = (k-3/8) / (n+1/4)$ , Blom. The resulting quantile estimates are approximately unbiased if  $x$  is normally distributed (R type 9)
- (.4,.4) : approximately quantile unbiased (Cunnane)
- (.35,.35): APL, used with PWM
- (.3175, .3175): used in `scipy.stats.probplot`

**Parameters**

**data** : array\_like  
Input data, as a sequence or array of dimension at most 2.

**alpha** : float, optional  
Plotting positions parameter. Default is 0.4.

**beta** : float, optional  
Plotting positions parameter. Default is 0.4.

**Returns**

**positions** : MaskedArray  
The calculated plotting positions.

`scipy.stats.mstats.mode(a, axis=0)`

Returns an array of the modal (most common) value in the passed array.

**Parameters**

**a** : array\_like  
n-dimensional array of which to find mode(s).

**axis** : int or None, optional  
Axis along which to operate. Default is 0. If None, compute over the whole array *a*.

**Returns**

**mode** : ndarray  
Array of modal values.

**count** : ndarray  
Array of counts for each mode.

#### Notes

For more details, see `stats.mode`.

`scipy.stats.mstats.moment(a, moment=1, axis=0)`

Calculates the *n*th moment about the mean for a sample.

**Parameters**

**a** : array\_like  
data

**moment** : int, optional  
order of central moment that is returned

**axis** : int or None, optional  
Axis along which the central moment is computed. Default is 0. If None, compute over the whole array *a*.

**Returns**

**n-th central moment** : ndarray or float  
The appropriate moment along the given axis or over all values if axis is None. The denominator for the moment calculation is the number of observations, no degrees of freedom correction is done.

#### Notes

For more details about `moment`, see `stats.moment`.

`scipy.stats.mstats.mquantiles(a, prob=[0.25, 0.5, 0.75], alphap=0.4, betap=0.4, axis=None, limit=())`

Computes empirical quantiles for a data array.

Samples quantile are defined by  $Q(p) = (1-\gamma) * x[j] + \gamma * x[j+1]$ , where  $x[j]$  is the *j*-th order statistic, and  $\gamma$  is a function of  $j = \text{floor}(n * p + m)$ ,  $m = \text{alphap} + p * (1 - \text{alphap} - \text{betap})$  and  $g = n * p + m - j$ .

Reinterpreting the above equations to compare to **R** lead to the equation:  $p(k) = (k - \text{alphap}) / (n + 1 - \text{alphap} - \text{betap})$

*Typical values of (alphap,betap) are:*

- (0,1) :  $p(k) = k/n$  : linear interpolation of cdf (**R** type 4)
- (.5,.5) :  $p(k) = (k - 1/2.) / n$  : piecewise linear function (**R** type 5)
- (0,0) :  $p(k) = k / (n+1)$  : (**R** type 6)
- (1,1) :  $p(k) = (k-1) / (n-1)$  :  $p(k) = \text{mode}[F(x[k])]$ . (**R** type 7, **R** default)
- (1/3,1/3) :  $p(k) = (k-1/3) / (n+1/3)$  : Then  $p(k) \sim \text{median}[F(x[k])]$ . The resulting quantile estimates are approximately median-unbiased regardless of the distribution of  $x$ . (**R** type 8)
- (3/8,3/8) :  $p(k) = (k-3/8) / (n+1/4)$  : Blom. The resulting quantile estimates are approximately unbiased if  $x$  is normally distributed (**R** type 9)
- (.4,.4) : approximately quantile unbiased (Cunnane)
- (.35,.35) : APL, used with PWM

**Parameters**

**a** : array\_like  
Input data, as a sequence or array of dimension at most 2.

**prob** : array\_like, optional  
List of quantiles to compute.

**alphap** : float, optional  
Plotting positions parameter, default is 0.4.

**betap** : float, optional  
Plotting positions parameter, default is 0.4.

**axis** : int, optional  
Axis along which to perform the trimming. If None (default), the input array is first flattened.

**limit** : tuple, optional  
Tuple of (lower, upper) values. Values of  $a$  outside this open interval are ignored.

**Returns**

**mquantiles** : MaskedArray  
An array containing the calculated quantiles.

### Notes

This formulation is very similar to **R** except the calculation of  $m$  from `alphap` and `betap`, where in **R**  $m$  is defined with each type.

### References

[R546], [R547]

### Examples

```
>>> from scipy.stats.mstats import mquantiles
>>> a = np.array([6., 47., 49., 15., 42., 41., 7., 39., 43., 40., 36.])
>>> mquantiles(a)
array([ 19.2,  40. ,  42.8])
```

Using a 2D array, specifying axis and limit.

```
>>> data = np.array([[ 6.,  7.,  1.],
...                 [ 47., 15.,  2.],
...                 [ 49., 36.,  3.],
...                 [ 15., 39.,  4.],
...                 [ 42., 40., -999.]])
```



```

...         [ 41.,  41., -999.],
...         [  7., -999., -999.],
...         [ 39., -999., -999.],
...         [ 43., -999., -999.],
...         [ 40., -999., -999.],
...         [ 36., -999., -999.])
>>> print(mquantiles(data, axis=0, limit=(0, 50)))
[[ 19.2  14.6  1.45]
 [ 40.   37.5  2.5 ]
 [ 42.8  40.05 3.55]]

>>> data[:, 2] = -999.
>>> print(mquantiles(data, axis=0, limit=(0, 50)))
[[19.200000000000003 14.6 --]
 [40.0 37.5 --]
 [42.800000000000004 40.05 --]]

```

`scipy.stats.mstats.msign(x)`

Returns the sign of  $x$ , or 0 if  $x$  is masked.

`scipy.stats.mstats.normaltest(a, axis=0)`

Tests whether a sample differs from a normal distribution.

**Parameters** **a** : array\_like

The array containing the data to be tested.

**axis** : int or None, optional

Axis along which to compute test. Default is 0. If None, compute over the

**Returns** **statistic** : float or array

whole array  $a$ .

$s^2 + k^2$ , where  $s$  is the z-score returned by `skewtest` and  $k$  is the z-score returned by `kurtosistest`.

**pvalue** : float or array

A 2-sided chi squared probability for the hypothesis test.

### Notes

For more details about `normaltest`, see `stats.normaltest`.

`scipy.stats.mstats.obrientransform(*args)`

Computes a transform on input data (any number of columns). Used to test for homogeneity of variance prior to running one-way stats. Each array in `*args` is one level of a factor. If an `f_oneway()` run on the transformed data and found significant, variances are unequal. From Maxwell and Delaney, p.112.

Returns: transformed data for use in an ANOVA

`scipy.stats.mstats.pearsonr(x, y)`

Calculates a Pearson correlation coefficient and the p-value for testing non-correlation.

The Pearson correlation coefficient measures the linear relationship between two datasets. Strictly speaking, Pearson's correlation requires that each dataset be normally distributed. Like other correlation coefficients, this one varies between -1 and +1 with 0 implying no correlation. Correlations of -1 or +1 imply an exact linear relationship. Positive correlations imply that as  $x$  increases, so does  $y$ . Negative correlations imply that as  $x$  increases,  $y$  decreases.

The p-value roughly indicates the probability of an uncorrelated system producing datasets that have a Pearson correlation at least as extreme as the one computed from these datasets. The p-values are not entirely reliable but are probably reasonable for datasets larger than 500 or so.

**Parameters** **x** : 1-D array\_like

Input

**Returns** **y** : 1-D array\_like  
**pearsonr** : float <sup>Input</sup>  
 Pearson's correlation coefficient, 2-tailed p-value.

**References**

<http://www.statsoft.com/textbook/glosp.html#Pearson%20Correlation>

`scipy.stats.mstats.plotting_positions` (*data*, *alpha*=0.4, *beta*=0.4)

Returns plotting positions (or empirical percentile points) for the data.

*Plotting positions are defined as  $(i-\alpha)/(n+1-\alpha-\beta)$ , where:*

- *i* is the rank order statistics
- *n* is the number of unmasked values along the given axis
- *alpha* and *beta* are two parameters.

*Typical values for alpha and beta are:*

- (0,1) :  $p(k) = k/n$ , linear interpolation of cdf (R, type 4)
- (.5,.5) :  $p(k) = (k-1/2)/n$ , piecewise linear function (R, type 5)
- (0,0) :  $p(k) = k/(n+1)$ , Weibull (R type 6)
- (1,1) :  $p(k) = (k-1)/(n-1)$ , in this case,  $p(k) = \text{mode}[F(x[k])]$ . That's R default (R type 7)
- (1/3,1/3):  $p(k) = (k-1/3)/(n+1/3)$ , then  $p(k) \sim \text{median}[F(x[k])]$ . The resulting quantile estimates are approximately median-unbiased regardless of the distribution of *x*. (R type 8)
- (3/8,3/8):  $p(k) = (k-3/8)/(n+1/4)$ , Blom. The resulting quantile estimates are approximately unbiased if *x* is normally distributed (R type 9)
- (.4,.4) : approximately quantile unbiased (Cunnane)
- (.35,.35): APL, used with PWM
- (.3175, .3175): used in `scipy.stats.probplot`

**Parameters** **data** : array\_like  
 Input data, as a sequence or array of dimension at most 2.  
**alpha** : float, optional  
 Plotting positions parameter. Default is 0.4.  
**beta** : float, optional  
 Plotting positions parameter. Default is 0.4.  
**Returns** **positions** : MaskedArray  
 The calculated plotting positions.

`scipy.stats.mstats.pointbiserialr` (*x*, *y*)

Calculates a point biserial correlation coefficient and its p-value.

**Parameters** **x** : array\_like of bools  
 Input array.  
**y** : array\_like  
 Input array.  
**Returns** **correlation** : float  
 R value  
**pvalue** : float  
 2-tailed p-value

**Notes**

Missing values are considered pair-wise: if a value is missing in *x*, the corresponding value in *y* is masked.

For more details on `pointbiserialr`, see `stats.pointbiserialr`.

`scipy.stats.mstats.rankdata` (*data*, *axis=None*, *use\_missing=False*)

Returns the rank (also known as order statistics) of each data point along the given axis.

If some values are tied, their rank is averaged. If some values are masked, their rank is set to 0 if *use\_missing* is False, or set to the average rank of the unmasked values if *use\_missing* is True.

**Parameters**

- data** : sequence  
Input data. The data is transformed to a masked array
- axis** : {None,int}, optional  
Axis along which to perform the ranking. If None, the array is first flattened. An exception is raised if the axis is specified for arrays with a dimension larger than 2
- use\_missing** : bool, optional  
Whether the masked values have a rank of 0 (False) or equal to the average rank of the unmasked values (True).

`scipy.stats.mstats.scoreatpercentile` (*data*, *per*, *limit=()*, *alphap=0.4*, *betap=0.4*)

Calculate the score at the given 'per' percentile of the sequence a. For example, the score at per=50 is the median.

This function is a shortcut to `mquantile`

`scipy.stats.mstats.sem` (*a*, *axis=0*, *ddof=1*)

Calculates the standard error of the mean of the input array.

Also sometimes called standard error of measurement.

**Parameters**

- a** : array\_like  
An array containing the values for which the standard error is returned.
- axis** : int or None, optional  
If axis is None, ravel *a* first. If axis is an integer, this will be the axis over which to operate. Defaults to 0.
- ddof** : int, optional  
Delta degrees-of-freedom. How many degrees of freedom to adjust for bias in limited samples relative to the population estimate of variance. Defaults to 1.

**Returns**

- s** : ndarray or float  
The standard error of the mean in the sample(s), along the input axis.

### Notes

The default value for *ddof* changed in scipy 0.15.0 to be consistent with *stats.sem* as well as with the most common definition used (like in the R documentation).

### Examples

Find standard error along the first axis:

```
>>> from scipy import stats
>>> a = np.arange(20).reshape(5,4)
>>> print(stats.mstats.sem(a))
[2.8284271247461903 2.8284271247461903 2.8284271247461903
 2.8284271247461903]
```

Find standard error across the whole array, using n degrees of freedom:

```
>>> print(stats.mstats.sem(a, axis=None, ddof=0))
1.2893796958227628
```

`scipy.stats.mstats.signaltonoise(*args, **kwargs)`

`signaltonoise` is deprecated! `mstats.signaltonoise` is deprecated in `scipy 0.16.0`

*Calculates the signal-to-noise ratio, as the ratio of the mean over standard deviation along the given axis.*

**Parameters**

<b>data</b> : sequence	Input data
<b>axis</b>	[{0, int}, optional] Axis along which to compute. If None, the computation is performed on a flat version of the array.

`scipy.stats.mstats.skew(a, axis=0, bias=True)`

Computes the skewness of a data set.

**Parameters**

<b>a</b> : ndarray	data
<b>axis</b> : int or None, optional	Axis along which skewness is calculated. Default is 0. If None, compute over the whole array <i>a</i> .
<b>bias</b> : bool, optional	If False, then the calculations are corrected for statistical bias.

**Returns**

<b>skewness</b> : ndarray	The skewness of values along an axis, returning 0 where all values are equal.
---------------------------	---

**Notes**

For more details about `skew`, see `stats.skew`.

`scipy.stats.mstats.skewtest(a, axis=0)`

Tests whether the skew is different from the normal distribution.

**Parameters**

<b>a</b> : array	The data to be tested
<b>axis</b> : int or None, optional	Axis along which statistics are calculated. Default is 0. If None, compute over the whole array <i>a</i> .

**Returns**

<b>statistic</b> : float	The computed z-score for this test.
<b>pvalue</b> : float	a 2-sided p-value for the hypothesis test

**Notes**

For more details about `skewtest`, see `stats.skewtest`.

`scipy.stats.mstats.spearmanr(x, y, use_ties=True)`

Calculates a Spearman rank-order correlation coefficient and the p-value to test for non-correlation.

The Spearman correlation is a nonparametric measure of the linear relationship between two datasets. Unlike the Pearson correlation, the Spearman correlation does not assume that both datasets are normally distributed. Like other correlation coefficients, this one varies between -1 and +1 with 0 implying no correlation. Correlations of -1 or +1 imply an exact linear relationship. Positive correlations imply that as *x* increases, so does *y*. Negative correlations imply that as *x* increases, *y* decreases.

Missing values are discarded pair-wise: if a value is missing in *x*, the corresponding value in *y* is masked.

The p-value roughly indicates the probability of an uncorrelated system producing datasets that have a Spearman correlation at least as extreme as the one computed from these datasets. The p-values are not entirely reliable but are probably reasonable for datasets larger than 500 or so.

**Parameters**

<b>x</b> : array_like
-----------------------

**Returns**

- y** : array\_like  
The length of *x* must be > 2.
- use\_ties** : bool, optional  
The length of *y* must be > 2.
- correlation** : float  
Whether the correction for ties should be computed.  
Spearman correlation coefficient
- pvalue** : float  
2-tailed p-value.

### References

[CRCProbStat2000] section 14.7

`scipy.stats.mstats.theilslopes` (*y*, *x=None*, *alpha=0.95*)

Computes the Theil-Sen estimator for a set of points (*x*, *y*).

`theilslopes` implements a method for robust linear regression. It computes the slope as the median of all slopes between paired values.

**Parameters**

- y** : array\_like  
Dependent variable.
- x** : array\_like or None, optional  
Independent variable. If None, use `arange(len(y))` instead.
- alpha** : float, optional  
Confidence degree between 0 and 1. Default is 95% confidence. Note that *alpha* is symmetric around 0.5, i.e. both 0.1 and 0.9 are interpreted as “find the 90% confidence interval”.

**Returns**

- medslope** : float  
Theil slope.
- medintercept** : float  
Intercept of the Theil line, as  $\text{median}(y) - \text{medslope} * \text{median}(x)$ .
- lo\_slope** : float  
Lower bound of the confidence interval on *medslope*.
- up\_slope** : float  
Upper bound of the confidence interval on *medslope*.

### Notes

For more details on `theilslopes`, see `stats.theilslopes`.

`scipy.stats.mstats.threshold` (*\*args*, *\*\*kws*)

`threshold` is deprecated! `mstats.threshold` is deprecated in scipy 0.17.0

Clip array to a given value.

Similar to `numpy.clip()`, except that values less than *threshmin* or greater than *threshmax* are replaced by *newval*, instead of by *threshmin* and *threshmax* respectively.

**Parameters**

- a** : ndarray  
Input data
- threshmin** : [{None, float}, optional]  
Lower threshold. If None, set to the minimum value.
- threshmax** : [{None, float}, optional]  
Upper threshold. If None, set to the maximum value.
- newval** : [{0, float}, optional]  
Value outside the thresholds.

**Returns**

- threshold** : ndarray  
Returns *a*, with values less than *threshmin* and values greater *threshmax* replaced with *newval*.

`scipy.stats.mstats.tmax` (*a*, *upperlimit=None*, *axis=0*, *inclusive=True*)

Compute the trimmed maximum

This function computes the maximum value of an array along a given axis, while ignoring values larger than a specified upper limit.

**Parameters**

- a** : array\_like  
array of values
- upperlimit** : None or float, optional  
Values in the input array greater than the given limit will be ignored. When upperlimit is None, then all values are used. The default value is None.
- axis** : int or None, optional  
Axis along which to operate. Default is 0. If None, compute over the whole array *a*.
- inclusive** : {True, False}, optional  
This flag determines whether values exactly equal to the upper limit are included. The default value is True.

**Returns**

- tmax** : float, int or ndarray

**Notes**

For more details on `tmax`, see `stats.tmax`.

`scipy.stats.mstats.tmean` (*a*, *limits=None*, *inclusive=(True, True)*, *axis=None*)

Compute the trimmed mean.

**Parameters**

- a** : array\_like  
Array of values.
- limits** : None or (lower limit, upper limit), optional  
Values in the input array less than the lower limit or greater than the upper limit will be ignored. When limits is None (default), then all values are used. Either of the limit values in the tuple can also be None representing a half-open interval.
- inclusive** : (bool, bool), optional  
A tuple consisting of the (lower flag, upper flag). These flags determine whether values exactly equal to the lower or upper limits are included. The default value is (True, True).
- axis** : int or None, optional  
Axis along which to operate. If None, compute over the whole array. Default is None.

**Returns**

- tmean** : float

**Notes**

For more details on `tmean`, see `stats.tmean`.

`scipy.stats.mstats.tmin` (*a*, *lowerlimit=None*, *axis=0*, *inclusive=True*)

Compute the trimmed minimum

**Parameters**

- a** : array\_like  
array of values
- lowerlimit** : None or float, optional  
Values in the input array less than the given limit will be ignored. When lowerlimit is None, then all values are used. The default value is None.
- axis** : int or None, optional  
Axis along which to operate. Default is 0. If None, compute over the whole array *a*.
- inclusive** : {True, False}, optional

**Returns** `tmin` : float, int or ndarray

This flag determines whether values exactly equal to the lower limit are included. The default value is True.

### Notes

For more details on `tmin`, see `stats.tmin`.

`scipy.stats.mstats.trim(a, limits=None, inclusive=(True, True), relative=False, axis=None)`

Trims an array by masking the data outside some given limits.

Returns a masked version of the input array.

**Parameters** `a` : sequence

Input array

**limits** : {None, tuple}, optional

If *relative* is False, tuple (lower limit, upper limit) in absolute values. Values of the input array lower (greater) than the lower (upper) limit are masked.

If *relative* is True, tuple (lower percentage, upper percentage) to cut on each side of the array, with respect to the number of unmasked data.

Noting *n* the number of unmasked data before trimming, the (*n*\*limits[0])th smallest data and the (*n*\*limits[1])th largest data are masked, and the total number of unmasked data after trimming is *n*\*(1.-sum(limits)) In each case, the value of one limit can be set to None to indicate an open interval.

If limits is None, no trimming is performed

**inclusive** : {(bool, bool) tuple}, optional

If *relative* is False, tuple indicating whether values exactly equal to the absolute limits are allowed. If *relative* is True, tuple indicating whether the number of data being masked on each side should be rounded (True) or truncated (False).

**relative** : bool, optional

Whether to consider the limits as absolute values (False) or proportions to cut (True).

**axis** : int, optional

Axis along which to trim.

### Examples

```
>>> from scipy.stats.mstats import trim
>>> z = [ 1, 2, 3, 4, 5, 6, 7, 8, 9,10]
>>> print(trim(z, (3,8)))
[-- -- 3 4 5 6 7 8 -- --]
>>> print(trim(z, (0.1,0.2), relative=True))
[-- 2 3 4 5 6 7 8 -- --]
```

`scipy.stats.mstats.trimma(a, limits=None, inclusive=(True, True))`

Trims an array by masking the data outside some given limits.

Returns a masked version of the input array.

**Parameters** `a` : array\_like

Input array.

**limits** : {None, tuple}, optional

Tuple of (lower limit, upper limit) in absolute values. Values of the input array lower (greater) than the lower (upper) limit will be masked. A limit is

None indicates an open interval.

**inclusive** : (bool, bool) tuple, optional

Tuple of (lower flag, upper flag), indicating whether values exactly equal to the lower (upper) limit are allowed.

`scipy.stats.mstats.trimboth` (*data*, *proportiontocut*=0.2, *inclusive*=(*True*, *True*), *axis*=*None*)

Trims the smallest and largest data values.

Trims the *data* by masking the `int`(`proportiontocut * n`) smallest and `int`(`proportiontocut * n`) largest values of data along the given axis, where *n* is the number of unmasked values before trimming.

**Parameters**

- data** : ndarray  
Data to trim.
- proportiontocut** : float, optional  
Percentage of trimming (as a float between 0 and 1). If *n* is the number of unmasked values before trimming, the number of values after trimming is  $(1 - 2 * \text{proportiontocut}) * n$ . Default is 0.2.
- inclusive** : {(bool, bool) tuple}, optional  
Tuple indicating whether the number of data being masked on each side should be rounded (*True*) or truncated (*False*).
- axis** : int, optional  
Axis along which to perform the trimming. If *None*, the input array is first flattened.

`scipy.stats.mstats.trimmed_stde` (*a*, *limits*=(0.1, 0.1), *inclusive*=(1, 1), *axis*=*None*)

Returns the standard error of the trimmed mean along the given axis.

**Parameters**

- a** : sequence  
Input array
- limits** : {(0.1,0.1), tuple of float}, optional  
tuple (lower percentage, upper percentage) to cut on each side of the array, with respect to the number of unmasked data.  
If *n* is the number of unmasked data before trimming, the values smaller than  $n * \text{limits}[0]$  and the values larger than  $n * \text{limits}[1]$  are masked, and the total number of unmasked data after trimming is  $n * (1 - \text{sum}(\text{limits}))$ . In each case, the value of one limit can be set to *None* to indicate an open interval. If *limits* is *None*, no trimming is performed.
- inclusive** : {(bool, bool) tuple} optional  
Tuple indicating whether the number of data being masked on each side should be rounded (*True*) or truncated (*False*).
- axis** : int, optional  
Axis along which to trim.

**Returns**

- trimmed\_stde** : scalar or ndarray

`scipy.stats.mstats.trimr` (*a*, *limits*=*None*, *inclusive*=(*True*, *True*), *axis*=*None*)

Trims an array by masking some proportion of the data on each end. Returns a masked version of the input array.

**Parameters**

- a** : sequence  
Input array.
- limits** : {None, tuple}, optional  
Tuple of the percentages to cut on each side of the array, with respect to the number of unmasked data, as floats between 0. and 1. Noting *n* the number of unmasked data before trimming, the  $(n * \text{limits}[0])$ th smallest data and the  $(n * \text{limits}[1])$ th largest data are masked, and the total number of unmasked data after trimming is  $n * (1 - \text{sum}(\text{limits}))$ . The value of one limit can be set to *None* to indicate an open interval.
- inclusive** : {(True, True) tuple}, optional



Tuple of flags indicating whether the number of data being masked on the left (right) end should be truncated (True) or rounded (False) to integers.

**axis** : {None,int}, optional

Axis along which to trim. If None, the whole array is trimmed, but its shape is maintained.

`scipy.stats.mstats.trimtail` (*data*, *proportiontocut*=0.2, *tail*='left', *inclusive*=(True, True), *axis*=None)

Trims the data by masking values from one tail.

**Parameters** **data** : array\_like

Data to trim.

**proportiontocut** : float, optional

Percentage of trimming. If *n* is the number of unmasked values before trimming, the number of values after trimming is  $(1 - \text{proportiontocut}) * n$ . Default is 0.2.

**tail** : {'left','right'}, optional

If 'left' the *proportiontocut* lowest values will be masked. If 'right' the *proportiontocut* highest values will be masked. Default is 'left'.

**inclusive** : {(bool, bool) tuple}, optional

Tuple indicating whether the number of data being masked on each side should be rounded (True) or truncated (False). Default is (True, True).

**axis** : int, optional

Axis along which to perform the trimming. If None, the input array is first flattened. Default is None.

**Returns**

**trimtail** : ndarray

Returned array of same shape as *data* with masked tail values.

`scipy.stats.mstats.tsem` (*a*, *limits*=None, *inclusive*=(True, True), *axis*=0, *ddof*=1)

Compute the trimmed standard error of the mean.

This function finds the standard error of the mean for given values, ignoring values outside the given *limits*.

**Parameters** **a** : array\_like

array of values

**limits** : None or (lower limit, upper limit), optional

Values in the input array less than the lower limit or greater than the upper limit will be ignored. When *limits* is None, then all values are used. Either of the limit values in the tuple can also be None representing a half-open interval. The default value is None.

**inclusive** : (bool, bool), optional

A tuple consisting of the (lower flag, upper flag). These flags determine whether values exactly equal to the lower or upper limits are included. The default value is (True, True).

**axis** : int or None, optional

Axis along which to operate. If None, compute over the whole array. Default is zero.

**ddof** : int, optional

Delta degrees of freedom. Default is 1.

**Returns**

**tsem** : float

### Notes

For more details on `tsem`, see `stats.tsem`.

`scipy.stats.mstats.ttest_onesamp` (*a*, *popmean*, *axis*=0)

Calculates the T-test for the mean of ONE group of scores.

**Parameters** **a** : array\_like

sample observation  
**popmean** : float or array\_like  
 expected value in null hypothesis, if array\_like than it must have the same shape as *a* excluding the axis dimension  
**axis** : int or None, optional  
 Axis along which to compute test. If None, compute over the whole array

**Returns** **statistic** : float or array  
<sup>*a*</sup>  
 t-statistic  
**pvalue** : float or array  
 two-tailed p-value

**Notes**

For more details on `ttest_1samp`, see `stats.ttest_1samp`.

`scipy.stats.mstats.ttest_ind(a, b, axis=0, equal_var=True)`  
 Calculates the T-test for the means of TWO INDEPENDENT samples of scores.

**Parameters** **a, b** : array\_like  
 The arrays must have the same shape, except in the dimension corresponding to *axis* (the first, by default).  
**axis** : int or None, optional  
 Axis along which to compute test. If None, compute over the whole arrays, *a*, and *b*.  
**equal\_var** : bool, optional  
 If True, perform a standard independent 2 sample test that assumes equal population variances. If False, perform Welch's t-test, which does not assume equal population variance. .. versionadded:: 0.17.0

**Returns** **statistic** : float or array  
 The calculated t-statistic.  
**pvalue** : float or array  
 The two-tailed p-value.

**Notes**

For more details on `ttest_ind`, see `stats.ttest_ind`.

`scipy.stats.mstats.ttest_onesamp(a, popmean, axis=0)`  
 Calculates the T-test for the mean of ONE group of scores.

**Parameters** **a** : array\_like  
 sample observation  
**popmean** : float or array\_like  
 expected value in null hypothesis, if array\_like than it must have the same shape as *a* excluding the axis dimension  
**axis** : int or None, optional  
 Axis along which to compute test. If None, compute over the whole array

**Returns** **statistic** : float or array  
<sup>*a*</sup>  
 t-statistic  
**pvalue** : float or array  
 two-tailed p-value

**Notes**

For more details on `ttest_1samp`, see `stats.ttest_1samp`.

`scipy.stats.mstats.ttest_rel(a, b, axis=0)`  
 Calculates the T-test on TWO RELATED samples of scores, a and b.

**Parameters** **a, b** : array\_like  
The arrays must have the same shape.

**axis** : int or None, optional  
Axis along which to compute test. If None, compute over the whole arrays, *a*, and *b*.

**Returns** **statistic** : float or array  
t-statistic

**pvalue** : float or array  
two-tailed p-value

**Notes**

For more details on `ttest_rel`, see `stats.ttest_rel`.

`scipy.stats.mstats.tvar` (*a*, *limits=None*, *inclusive=(True, True)*, *axis=0*, *ddof=1*)

Compute the trimmed variance

This function computes the sample variance of an array of values, while ignoring values which are outside of given *limits*.

**Parameters** **a** : array\_like  
Array of values.

**limits** : None or (lower limit, upper limit), optional  
Values in the input array less than the lower limit or greater than the upper limit will be ignored. When *limits* is None, then all values are used. Either of the limit values in the tuple can also be None representing a half-open interval. The default value is None.

**inclusive** : (bool, bool), optional  
A tuple consisting of the (lower flag, upper flag). These flags determine whether values exactly equal to the lower or upper limits are included. The default value is (True, True).

**axis** : int or None, optional  
Axis along which to operate. If None, compute over the whole array. Default is zero.

**ddof** : int, optional  
Delta degrees of freedom. Default is 1.

**Returns** **tvar** : float  
Trimmed variance.

**Notes**

For more details on `tvar`, see `stats.tvar`.

`scipy.stats.mstats.variation` (*a*, *axis=0*)

Computes the coefficient of variation, the ratio of the biased standard deviation to the mean.

**Parameters** **a** : array\_like  
Input array.

**axis** : int or None, optional  
Axis along which to calculate the coefficient of variation. Default is 0. If None, compute over the whole array *a*.

**Returns** **variation** : ndarray  
The calculated variation along the requested axis.

**Notes**

For more details about `variation`, see `stats.variation`.

`scipy.stats.mstats.winsorize` (*a*, *limits=None*, *inclusive=(True, True)*, *inplace=False*, *axis=None*)

Returns a Winsorized version of the input array.

The `(limits[0])`th lowest values are set to the `(limits[0])`th percentile, and the `(limits[1])`th highest values are set to the `(1 - limits[1])`th percentile. Masked values are skipped.

**Parameters**

- a** : sequence  
Input array.
- limits** : {None, tuple of float}, optional  
Tuple of the percentages to cut on each side of the array, with respect to the number of unmasked data, as floats between 0. and 1. Noting `n` the number of unmasked data before trimming, the `(n*limits[0])`th smallest data and the `(n*limits[1])`th largest data are masked, and the total number of unmasked data after trimming is `n*(1.-sum(limits))`. The value of one limit can be set to None to indicate an open interval.
- inclusive** : {(True, True) tuple}, optional  
Tuple indicating whether the number of data being masked on each side should be rounded (True) or truncated (False).
- inplace** : {False, True}, optional  
Whether to winsorize in place (True) or to use a copy (False)
- axis** : {None, int}, optional  
Axis along which to trim. If None, the whole array is trimmed, but its shape is maintained.

**Notes**

This function is applied to reduce the effect of possibly spurious outliers by limiting the extreme values.

`scipy.stats.mstats.zmap(scores, compare, axis=0, ddof=0)`  
Calculates the relative z-scores.

Returns an array of z-scores, i.e., scores that are standardized to zero mean and unit variance, where mean and variance are calculated from the comparison array.

**Parameters**

- scores** : array\_like  
The input for which z-scores are calculated.
- compare** : array\_like  
The input from which the mean and standard deviation of the normalization are taken; assumed to have the same dimension as `scores`.
- axis** : int or None, optional  
Axis over which mean and variance of `compare` are calculated. Default is 0. If None, compute over the whole array `scores`.
- ddof** : int, optional  
Degrees of freedom correction in the calculation of the standard deviation. Default is 0.

**Returns**

- zscore** : array\_like  
Z-scores, in the same shape as `scores`.

**Notes**

This function preserves ndarray subclasses, and works also with matrices and masked arrays (it uses `asanyarray` instead of `asarray` for parameters).

**Examples**

```
>>> from scipy.stats import zmap
>>> a = [0.5, 2.0, 2.5, 3]
>>> b = [0, 1, 2, 3, 4]
>>> zmap(a, b)
array([-1.06066017,  0.          ,  0.35355339,  0.70710678])
```

`scipy.stats.mstats.zscore(a, axis=0, ddof=0)`

Calculates the z score of each value in the sample, relative to the sample mean and standard deviation.

**Parameters**

- a** : array\_like  
An array like object containing the sample data.
- axis** : int or None, optional  
Axis along which to operate. Default is 0. If None, compute over the whole array *a*.
- ddof** : int, optional  
Degrees of freedom correction in the calculation of the standard deviation. Default is 0.

**Returns**

- zscore** : array\_like  
The z-scores, standardized by mean and standard deviation of input array *a*.

### Notes

This function preserves ndarray subclasses, and works also with matrices and masked arrays (it uses *asanyarray* instead of *asarray* for parameters).

### Examples

```
>>> a = np.array([ 0.7972,  0.0767,  0.4383,  0.7866,  0.8091,
...              0.1954,  0.6307,  0.6599,  0.1065,  0.0508])
>>> from scipy import stats
>>> stats.zscore(a)
array([ 1.1273, -1.247 , -0.0552,  1.0923,  1.1664, -0.8559,  0.5786,
        0.6748, -1.1488, -1.3324])
```

Computing along a specified axis, using n-1 degrees of freedom (ddof=1) to calculate the standard deviation:

```
>>> b = np.array([[ 0.3148,  0.0478,  0.6243,  0.4608],
...              [ 0.7149,  0.0775,  0.6072,  0.9656],
...              [ 0.6341,  0.1403,  0.9759,  0.4064],
...              [ 0.5918,  0.6948,  0.904 ,  0.3721],
...              [ 0.0921,  0.2481,  0.1188,  0.1366]])
>>> stats.zscore(b, axis=1, ddof=1)
array([[ -0.19264823, -1.28415119,  1.07259584,  0.40420358],
       [ 0.33048416, -1.37380874,  0.04251374,  1.00081084],
       [ 0.26796377, -1.12598418,  1.23283094, -0.37481053],
       [-0.22095197,  0.24468594,  1.19042819, -1.21416216],
       [-0.82780366,  1.4457416 , -0.43867764, -0.1792603 ]])
```

`scipy.stats.mstats.compare_medians_ms(group_1, group_2, axis=None)`

Compares the medians from two independent groups along the given axis.

The comparison is performed using the McKean-Schrader estimate of the standard error of the medians.

**Parameters**

- group\_1** : array\_like  
First dataset.
- group\_2** : array\_like  
Second dataset.
- axis** : int, optional  
Axis along which the medians are estimated. If None, the arrays are flattened. If *axis* is not None, then *group\_1* and *group\_2* should have the same shape.

**Returns**

- compare\_medians\_ms** : {float, ndarray}  
If *axis* is None, then returns a float, otherwise returns a 1-D ndarray of floats with a length equal to the length of *group\_1* along *axis*.

`scipy.stats.mstats.gmean` (*a*, *axis=0*, *dtype=None*)

Compute the geometric mean along the specified axis.

Returns the geometric average of the array elements. That is:  $n$ -th root of  $(x_1 * x_2 * \dots * x_n)$

**Parameters**

- a** : array\_like  
Input array or object that can be converted to an array.
- axis** : int or None, optional  
Axis along which the geometric mean is computed. Default is 0. If None, compute over the whole array *a*.
- dtype** : dtype, optional  
Type of the returned array and of the accumulator in which the elements are summed. If dtype is not specified, it defaults to the dtype of *a*, unless *a* has an integer dtype with a precision less than that of the default platform integer. In that case, the default platform integer is used.

**Returns**

- gmean** : ndarray  
see dtype parameter above

**See also:**

[`numpy.mean`](#) Arithmetic average

[`numpy.average`](#)

Weighted average

[`hmean`](#)

Harmonic mean

**Notes**

The geometric average is computed over a single dimension of the input array, *axis=0* by default, or all values in the array if *axis=None*. float64 intermediate and return values are used for integer inputs.

Use masked arrays to ignore any non-finite values in the input or that arise in the calculations such as Not a Number and infinity because masked arrays automatically mask any non-finite values.

`scipy.stats.mstats.hdmedian` (*data*, *axis=-1*, *var=False*)

Returns the Harrell-Davis estimate of the median along the given axis.

**Parameters**

- data** : ndarray  
Data array.
- axis** : int, optional  
Axis along which to compute the quantiles. If None, use a flattened array.
- var** : bool, optional  
Whether to return the variance of the estimate.

`scipy.stats.mstats.hdquantiles` (*data*, *prob=[0.25, 0.5, 0.75]*, *axis=None*, *var=False*)

Computes quantile estimates with the Harrell-Davis method.

The quantile estimates are calculated as a weighted linear combination of order statistics.

**Parameters**

- data** : array\_like  
Data array.
- prob** : sequence, optional  
Sequence of quantiles to compute.
- axis** : int or None, optional  
Axis along which to compute the quantiles. If None, use a flattened array.
- var** : bool, optional  
Whether to return the variance of the estimate.

**Returns**

- hdquantiles** : MaskedArray  
A (p,) array of quantiles (if *var* is False), or a (2,p) array of quantiles and variances (if *var* is True), where *p* is the number of quantiles.

`scipy.stats.mstats.hdquantiles_sd` (*data*, *prob*=[0.25, 0.5, 0.75], *axis*=None)

The standard error of the Harrell-Davis quantile estimates by jackknife.

**Parameters**

- data** : array\_like  
Data array.
- prob** : sequence, optional  
Sequence of quantiles to compute.
- axis** : int, optional  
Axis along which to compute the quantiles. If None, use a flattened array.

**Returns**

- hdquantiles\_sd** : MaskedArray  
Standard error of the Harrell-Davis quantile estimates.

`scipy.stats.mstats.hmean` (*a*, *axis*=0, *dtype*=None)

Calculates the harmonic mean along the specified axis.

That is:  $n / (1/x_1 + 1/x_2 + \dots + 1/x_n)$

**Parameters**

- a** : array\_like  
Input array, masked array or object that can be converted to an array.
- axis** : int or None, optional  
Axis along which the harmonic mean is computed. Default is 0. If None, compute over the whole array *a*.
- dtype** : dtype, optional  
Type of the returned array and of the accumulator in which the elements are summed. If *dtype* is not specified, it defaults to the dtype of *a*, unless *a* has an integer *dtype* with a precision less than that of the default platform integer. In that case, the default platform integer is used.

**Returns**

- hmean** : ndarray  
see *dtype* parameter above

See also:

[`numpy.mean`](#) Arithmetic average

[`numpy.average`](#)

Weighted average

[`gmean`](#)

Geometric mean

**Notes**

The harmonic mean is computed over a single dimension of the input array, *axis*=0 by default, or all values in the array if *axis*=None. float64 intermediate and return values are used for integer inputs.

Use masked arrays to ignore any non-finite values in the input or that arise in the calculations such as Not a Number and infinity.

`scipy.stats.mstats.idealfourths` (*data*, *axis*=None)

Returns an estimate of the lower and upper quartiles.

Uses the ideal fourths algorithm.

**Parameters**

- data** : array\_like  
Input array.
- axis** : int, optional  
Axis along which the quartiles are estimated. If None, the arrays are flattened.

**Returns**

- idealfourths** : {list of floats, masked array}  
Returns the two internal values that divide *data* into four parts using the ideal fourths algorithm either along the flattened array (if *axis* is None) or along *axis* of *data*.

`scipy.stats.mstats.kruskal(*args)`

Compute the Kruskal-Wallis H-test for independent samples

**Parameters** `sample1, sample2, ...` : array\_like  
 Two or more arrays with the sample measurements can be given as arguments.

**Returns** `statistic` : float  
 The Kruskal-Wallis H statistic, corrected for ties

`pvalue` : float  
 The p-value for the test using the assumption that H has a chi square distribution

**Notes**

For more details on `kruskal`, see `stats.kruskal`.

`scipy.stats.mstats.ks_2samp(data1, data2, alternative='two-sided')`

Computes the Kolmogorov-Smirnov test on two samples.

Missing values are discarded.

**Parameters** `data1` : array\_like  
 First data set

`data2` : array\_like  
 Second data set

`alternative` : {'two-sided', 'less', 'greater'}, optional  
 Indicates the alternative hypothesis. Default is 'two-sided'.

**Returns** `d` : float  
 Value of the Kolmogorov Smirnov test

`p` : float  
 Corresponding p-value.

`scipy.stats.mstats.median_cihs(data, alpha=0.05, axis=None)`

Computes the alpha-level confidence interval for the median of the data.

Uses the Hettmasperger-Sheather method.

**Parameters** `data` : array\_like  
 Input data. Masked values are discarded. The input should be 1D only, or `axis` should be set to None.

`alpha` : float, optional  
 Confidence level of the intervals.

`axis` : int or None, optional  
 Axis along which to compute the quantiles. If None, use a flattened array.

**Returns** `median_cihs`  
 Alpha level confidence interval.

`scipy.stats.mstats.meppf(data, alpha=0.4, beta=0.4)`

Returns plotting positions (or empirical percentile points) for the data.

**Plotting positions are defined as  $(i-\alpha)/(n+1-\alpha-\beta)$ , where:**

- `i` is the rank order statistics
- `n` is the number of unmasked values along the given axis
- `alpha` and `beta` are two parameters.

**Typical values for alpha and beta are:**

- (0,1) :  $p(k) = k/n$ , linear interpolation of cdf (R, type 4)
- (.5,.5) :  $p(k) = (k-1/2)/n$ , piecewise linear function (R, type 5)
- (0,0) :  $p(k) = k/(n+1)$ , Weibull (R type 6)



- (1,1) :  $p(k) = (k-1)/(n-1)$ , in this case,  $p(k) = \text{mode}[F(x[k])]$ . That's R default (R type 7)
- (1/3,1/3):  $p(k) = (k-1/3)/(n+1/3)$ , then  $p(k) \sim \text{median}[F(x[k])]$ . The resulting quantile estimates are approximately median-unbiased regardless of the distribution of  $x$ . (R type 8)
- (3/8,3/8):  $p(k) = (k-3/8)/(n+1/4)$ , Blom. The resulting quantile estimates are approximately unbiased if  $x$  is normally distributed (R type 9)
- (.4,.4) : approximately quantile unbiased (Cunnane)
- (.35,.35): APL, used with PWM
- (.3175, .3175): used in `scipy.stats.probplot`

**Parameters**

**data** : array\_like  
Input data, as a sequence or array of dimension at most 2.

**alpha** : float, optional  
Plotting positions parameter. Default is 0.4.

**beta** : float, optional  
Plotting positions parameter. Default is 0.4.

**Returns**

**positions** : MaskedArray  
The calculated plotting positions.

`scipy.stats.mstats.mjci` (*data*, *prob*=[0.25, 0.5, 0.75], *axis*=None)

Returns the Maritz-Jarrett estimators of the standard error of selected experimental quantiles of the data.

**Parameters**

**data** : ndarray  
Data array.

**prob** : sequence, optional  
Sequence of quantiles to compute.

**axis** : int or None, optional  
Axis along which to compute the quantiles. If None, use a flattened array.

`scipy.stats.mstats.mquantiles_cimj` (*data*, *prob*=[0.25, 0.5, 0.75], *alpha*=0.05, *axis*=None)

Computes the alpha confidence interval for the selected quantiles of the data, with Maritz-Jarrett estimators.

**Parameters**

**data** : ndarray  
Data array.

**prob** : sequence, optional  
Sequence of quantiles to compute.

**alpha** : float, optional  
Confidence level of the intervals.

**axis** : int or None, optional  
Axis along which to compute the quantiles. If None, use a flattened array.

`scipy.stats.mstats.rsh` (*data*, *points*=None)

Evaluates Rosenblatt's shifted histogram estimators for each point on the dataset 'data'.

**Parameters**

**data** : sequence  
Input data. Masked values are ignored.

**points** : sequence or None, optional  
Sequence of points where to evaluate Rosenblatt shifted histogram. If None, use the data.

`scipy.stats.mstats.sen_seasonal_slopes` (*x*)

`scipy.stats.mstats.trimmed_mean` (*a*, *limits*=(0.1, 0.1), *inclusive*=(1, 1), *relative*=True, *axis*=None)

`scipy.stats.mstats.trimmed_mean_ci` (*data*, *limits*=(0.2, 0.2), *inclusive*=(*True*, *True*), *alpha*=0.05, *axis*=None)

Selected confidence interval of the trimmed mean along the given axis.

**Parameters**

- data** : array\_like  
Input data.
- limits** : {None, tuple}, optional  
None or a two item tuple. Tuple of the percentages to cut on each side of the array, with respect to the number of unmasked data, as floats between 0. and 1. If *n* is the number of unmasked data before trimming, then (*n* \* *limits*[0])th smallest data and (*n* \* *limits*[1])th largest data are masked. The total number of unmasked data after trimming is *n* \* (1. - sum(*limits*)). The value of one limit can be set to None to indicate an open interval.  
Defaults to (0.2, 0.2).
- inclusive** : (2,) tuple of boolean, optional  
If *relative*==False, tuple indicating whether values exactly equal to the absolute limits are allowed. If *relative*==True, tuple indicating whether the number of data being masked on each side should be rounded (True) or truncated (False).  
Defaults to (True, True).
- alpha** : float, optional  
Confidence level of the intervals.  
Defaults to 0.05.
- axis** : int, optional  
Axis along which to cut. If None, uses a flattened version of *data*.  
Defaults to None.

**Returns**

- trimmed\_mean\_ci** : (2,) ndarray  
The lower and upper confidence intervals of the trimmed data.

`scipy.stats.mstats.trimmed_std` (*a*, *limits*=(0.1, 0.1), *inclusive*=(1, 1), *relative*=True, *axis*=None, *ddof*=0)

`scipy.stats.mstats.trimmed_var` (*a*, *limits*=(0.1, 0.1), *inclusive*=(1, 1), *relative*=True, *axis*=None, *ddof*=0)

`scipy.stats.mstats.ttest_1samp` (*a*, *popmean*, *axis*=0)

Calculates the T-test for the mean of ONE group of scores.

**Parameters**

- a** : array\_like  
sample observation
- popmean** : float or array\_like  
expected value in null hypothesis, if array\_like than it must have the same shape as *a* excluding the axis dimension
- axis** : int or None, optional  
Axis along which to compute test. If None, compute over the whole array

**Returns**

- statistic** : float or array  
*a*.  
t-statistic
- pvalue** : float or array  
two-tailed p-value

**Notes**

For more details on `ttest_1samp`, see `stats.ttest_1samp`.

## 5.29 C/C++ integration (`scipy.weave`)

**Warning:** This documentation is work-in-progress and unorganized.

### 5.29.1 C/C++ integration

NOTE: this module is deprecated and will be removed from Scipy before the 1.0 release – use the standalone weave package ([‘https://github.com/scipy/weave’](https://github.com/scipy/weave)) instead.

`inline` – a function for including C/C++ code within Python  
`blitz` – a function for compiling Numeric expressions to C++  
`ext_tools` – a module that helps construct C/C++ extension modules.  
`accelerate` – a module that inline accelerates Python functions

**Note:** On Linux one needs to have the Python development headers installed in order to be able to compile things with the `weave` module. Since this is a runtime dependency these headers (typically in a `pythonX.Y-dev` package) are not always installed when installing `scipy`.

---

<code>inline(code[, arg_names, local_dict, ...])</code>	Inline C/C++ code within Python scripts.
<code>blitz(expr[, local_dict, global_dict, ...])</code>	
<code>ext_tools</code>	
<code>accelerate</code>	

`scipy.weave.inline` (*code*, *arg\_names*=[], *local\_dict*=None, *global\_dict*=None, *force*=0, *compiler*=', *verbose*=0, *support\_code*=None, *headers*=[], *customize*=None, *type\_converters*=None, *auto\_downcast*=1, *newarr\_converter*=0, *\*\*kw*)

Inline C/C++ code within Python scripts.

`inline()` compiles and executes C/C++ code on the fly. Variables in the local and global Python scope are also available in the C/C++ code. Values are passed to the C/C++ code by assignment much like variables passed are passed into a standard Python function. Values are returned from the C/C++ code through a special argument called `return_val`. Also, the contents of mutable objects can be changed within the C/C++ code and the changes remain after the C code exits and returns to Python.

`inline` has quite a few options as listed below. Also, the keyword arguments for distutils extension modules are accepted to specify extra information needed for compiling.

**Parameters**    **code** : string

A string of valid C++ code. It should not specify a return statement. Instead it should assign results that need to be returned to Python in the `return_val`.

**arg\_names** : [str], optional

A list of Python variable names that should be transferred from Python into the C/C++ code. It defaults to an empty string.

**local\_dict** : dict, optional

If specified, it is a dictionary of values that should be used as the local scope for the C/C++ code. If `local_dict` is not specified the local dictionary of the calling function is used.

**global\_dict** : dict, optional

If specified, it is a dictionary of values that should be used as the global scope for the C/C++ code. If `global_dict` is not specified, the global dictionary of the calling function is used.

**force** : {0, 1}, optional

If 1, the C++ code is compiled every time `inline` is called. This is really only useful for debugging, and probably only useful if your editing `support_code` a lot.

**compiler** : str, optional

The name of compiler to use when compiling. On windows, it understands ‘msvc’ and ‘gcc’ as well as all the compiler names understood by `distutils`. On Unix, it’ll only understand the values understood by `distutils`. (I should add ‘gcc’ though to this).

On windows, the compiler defaults to the Microsoft C++ compiler. If this isn’t available, it looks for `mingw32` (the `gcc` compiler).

On Unix, it’ll probably use the same compiler that was used when compiling Python. Cygwin’s behavior should be similar.

**verbose** : {0,1,2}, optional

Specifies how much information is printed during the compile phase of inlining code. 0 is silent (except on windows with `msvc` where it still prints some garbage). 1 informs you when compiling starts, finishes, and how long it took. 2 prints out the command lines for the compilation process and can be useful if your having problems getting code to work. Its handy for finding the name of the `.cpp` file if you need to examine it. `verbose` has no effect if the compilation isn’t necessary.

**support\_code** : str, optional

A string of valid C++ code declaring extra code that might be needed by your compiled function. This could be declarations of functions, classes, or structures.

**headers** : [str], optional

A list of strings specifying header files to use when compiling the code. The list might look like [`"<vector>", "'my_header'"`]. Note that the header strings need to be in a form than can be pasted at the end of a `#include` statement in the C++ code.

**customize** : `base_info.custom_info`, optional

An alternative way to specify `support_code`, `headers`, etc. needed by the function. See `scipy.weave.base_info` for more details. (not sure this’ll be used much).

**type\_converters** : [type converters], optional

These guys are what convert Python data types to C/C++ data types. If you’d like to use a different set of type conversions than the default, specify them here. Look in the type conversions section of the main documentation for examples.

**auto\_downcast** : {1,0}, optional

This only affects functions that have numpy arrays as input variables. Setting this to 1 will cause all floating point values to be cast as float instead of double if all the Numeric arrays are of type float. If even one of the arrays has type double or double complex, all variables maintain their standard types.

**newarr\_converter** : int, optional

Unused.

#### Other Parameters

**Relevant :mod:‘distutils‘ keywords. These are duplicated from Greg Ward’s :class:‘distutils.extension.Extension‘ class for convenience:**

**sources** : [string]

List of source filenames, relative to the distribution root (where the setup script lives), in Unix form (slash-separated) for portability. Source files may be C, C++, SWIG (.i), platform-specific resource files, or whatever else is recognized by the “`build_ext`” command as source for a Python extension.

---

**Note:** The *module\_path* file is always appended to the front of this list

---

- include\_dirs** : [string]  
List of directories to search for C/C++ header files (in Unix form for portability).
- define\_macros** : [(name, value)]  
List of macros to define; each macro is defined using a 2-tuple, where ‘value’ is either the string to define it to or None to define it without a particular value (equivalent of “#define FOO” in source or -DFOO on Unix C compiler command line).
- undef\_macros** : [string]  
List of macros to undefine explicitly.
- library\_dirs** : [string]  
List of directories to search for C/C++ libraries at link time.
- libraries** : [string]  
List of library names (not filenames or paths) to link against.
- runtime\_library\_dirs** : [string]  
List of directories to search for C/C++ libraries at run time (for shared extensions, this is when the extension is loaded).
- extra\_objects** : [string]  
List of extra files to link with (e.g. object files not implied by ‘sources’, static libraries that must be explicitly specified, binary resource files, etc.)
- extra\_compile\_args** : [string]  
Any extra platform- and compiler-specific information to use when compiling the source files in ‘sources’. For platforms and compilers where “command line” makes sense, this is typically a list of command-line arguments, but for other platforms it could be anything.
- extra\_link\_args** : [string]  
Any extra platform- and compiler-specific information to use when linking object files together to create the extension (or to create a new static Python interpreter). Similar interpretation as for ‘extra\_compile\_args’.
- export\_symbols** : [string]  
List of symbols to be exported from a shared extension. Not used on all platforms, and not generally necessary for Python extensions, which typically export exactly one symbol: “init” + extension\_name.
- swig\_opts** : [string]  
Any extra options to pass to SWIG if a source file has the .i extension.
- depends** : [string]  
List of files that the extension depends on.
- language** : string  
Extension language (i.e. “c”, “c++”, “objc”). Will be detected from the source extensions if not provided.

**See also:*****distutils.extension.Extension***

Describes additional parameters.

`scipy.weave.blitz` (*expr*, *local\_dict=None*, *global\_dict=None*, *check\_size=1*, *verbose=0*, *\*\*kw*)

**Functions**


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`assign_variable_types(variables[, ...])`

Continued on next page

Table 5.297 – continued from previous page

<code>downcast(var_specs)</code>	Cast python scalars down to most common type of arrays used.
<code>format_error_msg(errors)</code>	
<code>generate_file_name(module_name, module_location)</code>	
<code>generate_module(module_string, module_file)</code>	generate the source code file. Only overwrite
<code>indent(st, spaces)</code>	

*Classes*

<code>ext_function(name, code_block, args[, ...])</code>	<i>Methods</i>
<code>ext_function_from_specs(name, code_block, ...)</code>	<i>Methods</i>
<code>ext_module(name[, compiler])</code>	<i>Methods</i>

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- [R547] *R* quantile function: <http://stat.ethz.ch/R-manual/R-devel/library/stats/html/quantile.html>



**S**

- scipy.cluster, 295
- scipy.cluster.hierarchy, 299
- scipy.cluster.vq, 295
- scipy.constants, 317
- scipy.fftpack, 334
- scipy.fftpack.convolve, 348
- scipy.integrate, 349
- scipy.interpolate, 377
- scipy.io, 456
- scipy.io.arff, 469
- scipy.io.netcdf, 244
- scipy.io.wavfile, 468
- scipy.linalg, 471
- scipy.linalg.blas, 531
- scipy.linalg.cython\_blas, 648
- scipy.linalg.cython\_lapack, 652
- scipy.linalg.interpolative, 688
- scipy.linalg.lapack, 566
- scipy.misc, 697
- scipy.ndimage, 710
- scipy.odr, 769
- scipy.optimize, 778
- scipy.optimize.nonlin, 873
- scipy.signal, 875
- scipy.sparse, 1056
- scipy.sparse.csgraph, 1215
- scipy.sparse.linalg, 1183
- scipy.spatial, 1229
- scipy.spatial.distance, 1270
- scipy.special, 1285
- scipy.stats, 1395
- scipy.stats.mstats, 1766
- scipy.weave, 1795
- scipy.weave.ext\_tools, 1797





## Symbols

- `__call__()` (scipy.interpolate.Akima1DInterpolator method), 387
  - `__call__()` (scipy.interpolate.BPoly method), 398
  - `__call__()` (scipy.interpolate.BarycentricInterpolator method), 380
  - `__call__()` (scipy.interpolate.BivariateSpline method), 440
  - `__call__()` (scipy.interpolate.CloughTocher2DInterpolator method), 404
  - `__call__()` (scipy.interpolate.CubicSpline method), 391
  - `__call__()` (scipy.interpolate.InterpolatedUnivariateSpline method), 418
  - `__call__()` (scipy.interpolate.KroghInterpolator method), 382
  - `__call__()` (scipy.interpolate.LSQBivariateSpline method), 446
  - `__call__()` (scipy.interpolate.LSQSphereBivariateSpline method), 448
  - `__call__()` (scipy.interpolate.LSQUnivariateSpline method), 423
  - `__call__()` (scipy.interpolate.LinearNDInterpolator method), 403
  - `__call__()` (scipy.interpolate.NearestNDInterpolator method), 403
  - `__call__()` (scipy.interpolate.PPoly method), 394
  - `__call__()` (scipy.interpolate.PchipInterpolator method), 384
  - `__call__()` (scipy.interpolate.Rbf method), 405
  - `__call__()` (scipy.interpolate.RectBivariateSpline method), 410, 434
  - `__call__()` (scipy.interpolate.RectSphereBivariateSpline method), 438
  - `__call__()` (scipy.interpolate.RegularGridInterpolator method), 409
  - `__call__()` (scipy.interpolate.SmoothBivariateSpline method), 441
  - `__call__()` (scipy.interpolate.SmoothSphereBivariateSpline method), 444
  - `__call__()` (scipy.interpolate.UnivariateSpline method), 414
  - `__call__()` (scipy.interpolate.interp1d method), 379
  - `__call__()` (scipy.interpolate.interp2d method), 407
  - `__call__()` (scipy.interpolate.ppform method), 452
  - `__call__()` (scipy.optimize.LbfgsInvHessProduct method), 872
  - `__call__()` (scipy.sparse.linalg.LinearOperator method), 1149, 1184
  - `__call__()` (scipy.stats.gaussian\_kde method), 1763
  - `__call__()` (scipy.stats.rv\_continuous method), 1403
  - `__call__()` (scipy.stats.rv\_discrete method), 1412
- ## A
- A (scipy.signal.dlti attribute), 966
  - A (scipy.signal.lti attribute), 948
  - A (scipy.signal.StateSpace attribute), 951, 969
  - A (scipy.signal.TransferFunction attribute), 954, 972
  - A (scipy.signal.ZerosPolesGain attribute), 956, 974
  - `abcd_normalize()` (in module scipy.signal), 928
  - `add_points()` (scipy.spatial.ConvexHull method), 1261
  - `add_points()` (scipy.spatial.Delaunay method), 1258
  - `add_points()` (scipy.spatial.Voronoi method), 1263
  - `add_xi()` (scipy.interpolate.BarycentricInterpolator method), 380
  - `adjoint()` (scipy.optimize.LbfgsInvHessProduct method), 872
  - `adjoint()` (scipy.sparse.linalg.LinearOperator method), 1149, 1184
  - `affine_transform()` (in module scipy.ndimage), 727
  - `agm()` (in module scipy.special), 1385
  - `ai_zeros()` (in module scipy.special), 1287
  - `airy` (in module scipy.special), 1286
  - `airye` (in module scipy.special), 1286
  - Akima1DInterpolator (class in scipy.interpolate), 386
  - `alpha` (in module scipy.stats), 1414
  - `anderson()` (in module scipy.optimize), 857
  - `anderson()` (in module scipy.stats), 1709
  - `anderson_ksamp()` (in module scipy.stats), 1710
  - `anglit` (in module scipy.stats), 1416
  - `ansari()` (in module scipy.stats), 1707
  - `antiderivative()` (scipy.interpolate.Akima1DInterpolator method), 387
  - `antiderivative()` (scipy.interpolate.BPoly method), 398
  - `antiderivative()` (scipy.interpolate.CubicSpline method), 392

- antiderivative() (scipy.interpolate.InterpolatedUnivariateSpline method), 418
  - antiderivative() (scipy.interpolate.LSQUnivariateSpline method), 423
  - antiderivative() (scipy.interpolate.PchipInterpolator method), 384
  - antiderivative() (scipy.interpolate.ppform method), 452
  - antiderivative() (scipy.interpolate.PPoly method), 394
  - antiderivative() (scipy.interpolate.UnivariateSpline method), 414
  - approx\_fprime() (in module scipy.optimize), 867
  - approximate\_taylor\_polynomial() (in module scipy.interpolate), 451
  - arcsin() (scipy.sparse.bsr\_matrix method), 1060
  - arcsin() (scipy.sparse.coo\_matrix method), 1070
  - arcsin() (scipy.sparse.csc\_matrix method), 1079
  - arcsin() (scipy.sparse.csr\_matrix method), 1089
  - arcsin() (scipy.sparse.dia\_matrix method), 1098
  - arcsine (in module scipy.stats), 1418
  - arcsinh() (scipy.sparse.bsr\_matrix method), 1060
  - arcsinh() (scipy.sparse.coo\_matrix method), 1070
  - arcsinh() (scipy.sparse.csc\_matrix method), 1079
  - arcsinh() (scipy.sparse.csr\_matrix method), 1089
  - arcsinh() (scipy.sparse.dia\_matrix method), 1098
  - arctan() (scipy.sparse.bsr\_matrix method), 1060
  - arctan() (scipy.sparse.coo\_matrix method), 1070
  - arctan() (scipy.sparse.csc\_matrix method), 1079
  - arctan() (scipy.sparse.csr\_matrix method), 1089
  - arctan() (scipy.sparse.dia\_matrix method), 1098
  - arctanh() (scipy.sparse.bsr\_matrix method), 1060
  - arctanh() (scipy.sparse.coo\_matrix method), 1070
  - arctanh() (scipy.sparse.csc\_matrix method), 1079
  - arctanh() (scipy.sparse.csr\_matrix method), 1089
  - arctanh() (scipy.sparse.dia\_matrix method), 1098
  - ArffError, 470
  - argrelextrema() (in module scipy.signal), 1041
  - argrelmax() (in module scipy.signal), 1040
  - argrelmin() (in module scipy.signal), 1040
  - argstoarray() (in module scipy.stats.mstats), 1734, 1768
  - ArpackError, 1178, 1213
  - ArpackNoConvergence, 1178, 1213
  - ascent() (in module scipy.misc), 698
  - asformat() (scipy.sparse.bsr\_matrix method), 1060
  - asformat() (scipy.sparse.coo\_matrix method), 1070
  - asformat() (scipy.sparse.csc\_matrix method), 1080
  - asformat() (scipy.sparse.csr\_matrix method), 1089
  - asformat() (scipy.sparse.dia\_matrix method), 1098
  - asformat() (scipy.sparse.dok\_matrix method), 1106
  - asformat() (scipy.sparse.lil\_matrix method), 1113
  - asformat() (scipy.sparse.spmatrix method), 1118
  - asfptype() (scipy.sparse.bsr\_matrix method), 1060
  - asfptype() (scipy.sparse.coo\_matrix method), 1070
  - asfptype() (scipy.sparse.csc\_matrix method), 1080
  - asfptype() (scipy.sparse.csr\_matrix method), 1090
  - asfptype() (scipy.sparse.dia\_matrix method), 1098
  - asfptype() (scipy.sparse.dok\_matrix method), 1106
  - asfptype() (scipy.sparse.lil\_matrix method), 1113
  - asfptype() (scipy.sparse.spmatrix method), 1118
  - aslinearoperator() (in module scipy.sparse.linalg), 1151, 1185
  - assignValue() (scipy.io.netcdf\_variable method), 466
  - assoc\_laguerre() (in module scipy.special), 1364
  - astype() (scipy.sparse.bsr\_matrix method), 1060
  - astype() (scipy.sparse.coo\_matrix method), 1070
  - astype() (scipy.sparse.csc\_matrix method), 1080
  - astype() (scipy.sparse.csr\_matrix method), 1090
  - astype() (scipy.sparse.dia\_matrix method), 1098
  - astype() (scipy.sparse.dok\_matrix method), 1106
  - astype() (scipy.sparse.lil\_matrix method), 1113
  - astype() (scipy.sparse.spmatrix method), 1118
  - average() (in module scipy.cluster.hierarchy), 304
- ## B
- B (scipy.signal.dlti attribute), 966
  - B (scipy.signal.lti attribute), 948
  - B (scipy.signal.StateSpace attribute), 951, 969
  - B (scipy.signal.TransferFunction attribute), 954, 972
  - B (scipy.signal.ZerosPolesGain attribute), 956, 974
  - BadCoefficients, 927
  - band\_stop\_obj() (in module scipy.signal), 928
  - barthann() (in module scipy.signal), 999
  - bartlett() (in module scipy.signal), 1000
  - bartlett() (in module scipy.stats), 1707
  - barycentric\_interpolate() (in module scipy.interpolate), 384
  - BarycentricInterpolator (class in scipy.interpolate), 380
  - basinhopping() (in module scipy.optimize), 823
  - bayes\_mvs() (in module scipy.stats), 1676
  - bdtr (in module scipy.special), 1322
  - bdtrc (in module scipy.special), 1322
  - bdtri (in module scipy.special), 1323
  - bdtrik (in module scipy.special), 1323
  - bdtrin (in module scipy.special), 1324
  - bei (in module scipy.special), 1382
  - bei\_zeros() (in module scipy.special), 1383
  - beip (in module scipy.special), 1382
  - beip\_zeros() (in module scipy.special), 1383
  - bellman\_ford() (in module scipy.sparse.csgraph), 1137, 1219
  - ber (in module scipy.special), 1382
  - ber\_zeros() (in module scipy.special), 1383
  - bernoulli (in module scipy.stats), 1623
  - bernoulli() (in module scipy.special), 1385
  - berp (in module scipy.special), 1382
  - berp\_zeros() (in module scipy.special), 1383
  - bessel() (in module scipy.signal), 942
  - besselap() (in module scipy.signal), 928
  - besselpoly (in module scipy.special), 1310

- beta (in module `scipy.special`), 1348  
 beta (in module `scipy.stats`), 1420  
 betai() (in module `scipy.stats`), 1720  
 betai() (in module `scipy.stats.mstats`), 1734, 1768  
 betainc (in module `scipy.special`), 1348  
 betaincinv (in module `scipy.special`), 1349  
 betaln (in module `scipy.special`), 1348  
 betaprime (in module `scipy.stats`), 1422  
 bi\_zeros() (in module `scipy.special`), 1287  
 big() (in module `scipy.sparse.linalg`), 1156, 1191  
 bigstab() (in module `scipy.sparse.linalg`), 1156, 1191  
 bilinear() (in module `scipy.signal`), 909  
 binary\_closing() (in module `scipy.ndimage`), 744  
 binary\_dilation() (in module `scipy.ndimage`), 746  
 binary\_erosion() (in module `scipy.ndimage`), 748  
 binary\_fill\_holes() (in module `scipy.ndimage`), 749  
 binary\_hit\_or\_miss() (in module `scipy.ndimage`), 750  
 binary\_opening() (in module `scipy.ndimage`), 751  
 binary\_propagation() (in module `scipy.ndimage`), 753  
 binned\_statistic() (in module `scipy.stats`), 1668  
 binned\_statistic\_2d() (in module `scipy.stats`), 1671  
 binned\_statistic\_dd() (in module `scipy.stats`), 1673  
 binom (in module `scipy.special`), 1385  
 binom (in module `scipy.stats`), 1625  
 binom\_test() (in module `scipy.stats`), 1711  
 bisect() (in module `scipy.optimize`), 838  
 bisplev() (in module `scipy.interpolate`), 450  
 bisplrep() (in module `scipy.interpolate`), 449  
 BivariateSpline (class in `scipy.interpolate`), 439  
 black\_tophat() (in module `scipy.ndimage`), 755  
 blackman() (in module `scipy.signal`), 1002  
 blackmanharris() (in module `scipy.signal`), 1004  
 blitz() (in module `scipy.weave`), 1797  
 block\_diag() (in module `scipy.linalg`), 521  
 block\_diag() (in module `scipy.sparse`), 1126  
 bmat() (in module `scipy.sparse`), 1129  
 bode() (in module `scipy.signal`), 962  
 bode() (`scipy.signal.dlti` method), 966  
 bode() (`scipy.signal.lti` method), 948  
 bohman() (in module `scipy.signal`), 1005  
 boltzmann (in module `scipy.stats`), 1627  
 boxcar() (in module `scipy.signal`), 1007  
 boxcox (in module `scipy.special`), 1342  
 boxcox() (in module `scipy.stats`), 1715  
 boxcox1p (in module `scipy.special`), 1343  
 boxcox\_illf() (in module `scipy.stats`), 1718  
 boxcox\_normmax() (in module `scipy.stats`), 1716  
 boxcox\_normplot() (in module `scipy.stats`), 1731  
 boxsize (`scipy.spatial.cKDTree` attribute), 1235  
 BPoly (class in `scipy.interpolate`), 397  
 bracket() (in module `scipy.optimize`), 868  
 bradford (in module `scipy.stats`), 1424  
 braycurtis() (in module `scipy.spatial.distance`), 1247, 1279  
 breadth\_first\_order() (in module `scipy.sparse.csgraph`), 1138, 1220  
 breadth\_first\_tree() (in module `scipy.sparse.csgraph`), 1139, 1221  
 brent() (in module `scipy.optimize`), 810  
 brentn() (in module `scipy.optimize`), 836  
 brentq() (in module `scipy.optimize`), 835  
 broyden1() (in module `scipy.optimize`), 844  
 broyden2() (in module `scipy.optimize`), 846  
 brute() (in module `scipy.optimize`), 827  
 bspline() (in module `scipy.signal`), 885  
 bsr\_matrix (class in `scipy.sparse`), 1057  
 btdtr (in module `scipy.special`), 1324  
 btdtri (in module `scipy.special`), 1325  
 btdtria (in module `scipy.special`), 1326  
 btdtrib (in module `scipy.special`), 1326  
 burr (in module `scipy.stats`), 1426  
 burr12 (in module `scipy.stats`), 1429  
 buttap() (in module `scipy.signal`), 929  
 butter() (in module `scipy.signal`), 931  
 buttord() (in module `scipy.signal`), 932  
 bytescale() (in module `scipy.misc`), 699
- ## C
- C (`scipy.signal.dlti` attribute), 966  
 C (`scipy.signal.lti` attribute), 948  
 C (`scipy.signal.StateSpace` attribute), 951, 969  
 C (`scipy.signal.TransferFunction` attribute), 954, 972  
 C (`scipy.signal.ZerosPolesGain` attribute), 956, 974  
 C2F() (in module `scipy.constants`), 331  
 C2K() (in module `scipy.constants`), 330  
 c\_roots() (in module `scipy.special`), 1367  
 canberra() (in module `scipy.spatial.distance`), 1248, 1279  
 cascade() (in module `scipy.signal`), 1035  
 cauchy (in module `scipy.stats`), 1431  
 caxpy (in module `scipy.linalg.blas`), 533  
 cbrt (in module `scipy.special`), 1393  
 cc\_diff() (in module `scipy.fftpack`), 345  
 ccopy (in module `scipy.linalg.blas`), 534  
 cdf() (`scipy.stats.rv_continuous` method), 1399  
 cdf() (`scipy.stats.rv_discrete` method), 1408  
 cdist() (in module `scipy.spatial.distance`), 1242, 1273  
 cdotc (in module `scipy.linalg.blas`), 534  
 cdotu (in module `scipy.linalg.blas`), 534  
 ceil() (`scipy.sparse.bsr_matrix` method), 1060  
 ceil() (`scipy.sparse.coo_matrix` method), 1070  
 ceil() (`scipy.sparse.csc_matrix` method), 1080  
 ceil() (`scipy.sparse.csr_matrix` method), 1090  
 ceil() (`scipy.sparse.dia_matrix` method), 1098  
 center\_of\_mass() (in module `scipy.ndimage`), 732  
 central\_diff\_weights() (in module `scipy.misc`), 699  
 centroid() (in module `scipy.cluster.hierarchy`), 305  
 cg() (in module `scipy.sparse.linalg`), 1157, 1192  
 cg\_roots() (in module `scipy.special`), 1367

egbsv (in module `scipy.linalg.lapack`), 572  
 egbtrf (in module `scipy.linalg.lapack`), 574  
 egbtrs (in module `scipy.linalg.lapack`), 575  
 egebal (in module `scipy.linalg.lapack`), 576  
 egees (in module `scipy.linalg.lapack`), 578  
 egeev (in module `scipy.linalg.lapack`), 580  
 egeev\_lwork (in module `scipy.linalg.lapack`), 581  
 egegv() (in module `scipy.linalg.lapack`), 583  
 egehrd (in module `scipy.linalg.lapack`), 584  
 egehrd\_lwork (in module `scipy.linalg.lapack`), 585  
 egelsd (in module `scipy.linalg.lapack`), 589  
 egelsd\_lwork (in module `scipy.linalg.lapack`), 590  
 egelss (in module `scipy.linalg.lapack`), 586  
 egelss\_lwork (in module `scipy.linalg.lapack`), 587  
 egelsy (in module `scipy.linalg.lapack`), 591  
 egelsy\_lwork (in module `scipy.linalg.lapack`), 592  
 egemm (in module `scipy.linalg.blas`), 560  
 egemv (in module `scipy.linalg.blas`), 548  
 egeqp3 (in module `scipy.linalg.lapack`), 593  
 egeqrf (in module `scipy.linalg.lapack`), 594  
 egerc (in module `scipy.linalg.blas`), 548  
 egerqf (in module `scipy.linalg.lapack`), 595  
 egeru (in module `scipy.linalg.blas`), 549  
 egesdd (in module `scipy.linalg.lapack`), 596  
 egesdd\_lwork (in module `scipy.linalg.lapack`), 598  
 egesv (in module `scipy.linalg.lapack`), 601  
 egesvd (in module `scipy.linalg.lapack`), 599  
 egesvd\_lwork (in module `scipy.linalg.lapack`), 600  
 cgetrf (in module `scipy.linalg.lapack`), 602  
 cgetri (in module `scipy.linalg.lapack`), 602  
 cgetri\_lwork (in module `scipy.linalg.lapack`), 603  
 cgetrs (in module `scipy.linalg.lapack`), 604  
 cgges (in module `scipy.linalg.lapack`), 606  
 cggev (in module `scipy.linalg.lapack`), 609  
 cgs() (in module `scipy.sparse.linalg`), 1158, 1193  
 cgtsv (in module `scipy.linalg.lapack`), 636  
 chbevd (in module `scipy.linalg.lapack`), 609  
 chbevz (in module `scipy.linalg.lapack`), 610  
 chdtr (in module `scipy.special`), 1340  
 chdtre (in module `scipy.special`), 1340  
 chdtri (in module `scipy.special`), 1340  
 chdtriv (in module `scipy.special`), 1340  
 cheb1ap() (in module `scipy.signal`), 929  
 cheb1ord() (in module `scipy.signal`), 935  
 cheb2ap() (in module `scipy.signal`), 929  
 cheb2ord() (in module `scipy.signal`), 938  
 chebwin() (in module `scipy.signal`), 1008  
 cheby1() (in module `scipy.signal`), 933  
 cheby2() (in module `scipy.signal`), 936  
 chebyc() (in module `scipy.special`), 1366  
 chebys() (in module `scipy.special`), 1366  
 chebyshev() (in module `scipy.spatial.distance`), 1248, 1279  
 chebyt() (in module `scipy.special`), 1366  
 chebyu() (in module `scipy.special`), 1366  
 check\_format() (`scipy.sparse.bsr_matrix` method), 1060  
 check\_format() (`scipy.sparse.csc_matrix` method), 1080  
 check\_format() (`scipy.sparse.csr_matrix` method), 1090  
 check\_grad() (in module `scipy.optimize`), 868  
 cheev (in module `scipy.linalg.lapack`), 611  
 cheevd (in module `scipy.linalg.lapack`), 612  
 cheevr (in module `scipy.linalg.lapack`), 613  
 chegv (in module `scipy.linalg.lapack`), 613  
 chegvd (in module `scipy.linalg.lapack`), 614  
 chegvx (in module `scipy.linalg.lapack`), 615  
 chemm (in module `scipy.linalg.blas`), 560  
 chemv (in module `scipy.linalg.blas`), 549  
 cher (in module `scipy.linalg.blas`), 550  
 cher2 (in module `scipy.linalg.blas`), 550  
 cher2k (in module `scipy.linalg.blas`), 561  
 cherk (in module `scipy.linalg.blas`), 560  
 chi (in module `scipy.stats`), 1433  
 chi2 (in module `scipy.stats`), 1435  
 chi2\_contingency() (in module `scipy.stats`), 1722  
 chirp() (in module `scipy.signal`), 989  
 chisqprob() (in module `scipy.stats`), 1720  
 chisquare() (in module `scipy.stats`), 1697  
 chisquare() (in module `scipy.stats.mstats`), 1734, 1768  
 chndtr (in module `scipy.special`), 1341  
 chndtridf (in module `scipy.special`), 1341  
 chndtrinc (in module `scipy.special`), 1341  
 chndtrix (in module `scipy.special`), 1341  
 cho\_factor() (in module `scipy.linalg`), 495  
 cho\_solve() (in module `scipy.linalg`), 496  
 cho\_solve\_banded() (in module `scipy.linalg`), 496  
 cholesky() (in module `scipy.linalg`), 494  
 cholesky\_banded() (in module `scipy.linalg`), 494  
 circmean() (in module `scipy.stats`), 1720  
 circstd() (in module `scipy.stats`), 1721  
 circulant() (in module `scipy.linalg`), 522  
 circvar() (in module `scipy.stats`), 1721  
 cityblock() (in module `scipy.spatial.distance`), 1248, 1280  
 cKDTree (class in `scipy.spatial`), 1234  
 clange (in module `scipy.linalg.lapack`), 647  
 clarf (in module `scipy.linalg.lapack`), 616  
 clarfg (in module `scipy.linalg.lapack`), 618  
 clartg (in module `scipy.linalg.lapack`), 618  
 claswp (in module `scipy.linalg.lapack`), 620  
 clauum (in module `scipy.linalg.lapack`), 621  
 clear() (`scipy.optimize.OptimizeResult` method), 785  
 clear() (`scipy.sparse.dok_matrix` method), 1106  
 close() (`scipy.io.FortranFile` method), 461  
 close() (`scipy.io.netcdf_file` method), 465  
 close() (`scipy.spatial.ConvexHull` method), 1261  
 close() (`scipy.spatial.Delaunay` method), 1259  
 close() (`scipy.spatial.Voronoi` method), 1264  
 CloughTocher2DInterpolator (class in `scipy.interpolate`), 403



- clpmn() (in module `scipy.special`), 1359  
 ClusterNode (class in `scipy.cluster.hierarchy`), 312  
 cmplx\_sort() (in module `scipy.signal`), 930  
 coherence() (in module `scipy.signal`), 1050  
 comb() (in module `scipy.misc`), 700  
 comb() (in module `scipy.special`), 1384  
 combine\_pvalues() (in module `scipy.stats`), 1705  
 companion() (in module `scipy.linalg`), 522  
 compare\_medians\_ms() (in module `scipy.stats.mstats`), 1756, 1789  
 complete() (in module `scipy.cluster.hierarchy`), 304  
 complex\_ode (class in `scipy.integrate`), 371  
 conj() (`scipy.sparse.bsr_matrix` method), 1061  
 conj() (`scipy.sparse.coo_matrix` method), 1070  
 conj() (`scipy.sparse.csc_matrix` method), 1080  
 conj() (`scipy.sparse.csr_matrix` method), 1090  
 conj() (`scipy.sparse.dia_matrix` method), 1098  
 conj() (`scipy.sparse.dok_matrix` method), 1106  
 conj() (`scipy.sparse.lil_matrix` method), 1113  
 conj() (`scipy.sparse.spmatrix` method), 1118  
 conjtransp() (`scipy.sparse.dok_matrix` method), 1106  
 conjugate() (`scipy.sparse.bsr_matrix` method), 1061  
 conjugate() (`scipy.sparse.coo_matrix` method), 1070  
 conjugate() (`scipy.sparse.csc_matrix` method), 1080  
 conjugate() (`scipy.sparse.csr_matrix` method), 1090  
 conjugate() (`scipy.sparse.dia_matrix` method), 1099  
 conjugate() (`scipy.sparse.dok_matrix` method), 1106  
 conjugate() (`scipy.sparse.lil_matrix` method), 1113  
 conjugate() (`scipy.sparse.spmatrix` method), 1119  
 connected\_components() (in module `scipy.sparse.csgraph`), 1133, 1215  
 ConstantWarning, 320  
 construct\_dist\_matrix() (in module `scipy.sparse.csgraph`), 1143, 1225  
 construct\_fast() (`scipy.interpolate.BPoly` class method), 399  
 construct\_fast() (`scipy.interpolate.ppform` class method), 453  
 construct\_fast() (`scipy.interpolate.PPoly` class method), 396  
 cont2discrete() (in module `scipy.signal`), 985  
 convert\_temperature() (in module `scipy.constants`), 330  
 convex\_hull (`scipy.spatial.Delaunay` attribute), 1257  
 convex\_hull\_plot\_2d() (in module `scipy.spatial`), 1266  
 ConvexHull (class in `scipy.spatial`), 1259  
 convolve (in module `scipy.fftpack.convolve`), 348  
 convolve() (in module `scipy.ndimage`), 711  
 convolve() (in module `scipy.signal`), 875  
 convolve1d() (in module `scipy.ndimage`), 712  
 convolve2d() (in module `scipy.signal`), 881  
 convolve\_z (in module `scipy.fftpack.convolve`), 349  
 coo\_matrix (class in `scipy.sparse`), 1067  
 cophenet() (in module `scipy.cluster.hierarchy`), 306  
 copy() (`scipy.optimize.OptimizeResult` method), 785  
 copy() (`scipy.sparse.bsr_matrix` method), 1061  
 copy() (`scipy.sparse.coo_matrix` method), 1070  
 copy() (`scipy.sparse.csc_matrix` method), 1080  
 copy() (`scipy.sparse.csr_matrix` method), 1090  
 copy() (`scipy.sparse.dia_matrix` method), 1099  
 copy() (`scipy.sparse.dok_matrix` method), 1106  
 copy() (`scipy.sparse.lil_matrix` method), 1113  
 copy() (`scipy.sparse.spmatrix` method), 1119  
 correlate() (in module `scipy.ndimage`), 713  
 correlate() (in module `scipy.signal`), 876  
 correlate1d() (in module `scipy.ndimage`), 713  
 correlate2d() (in module `scipy.signal`), 883  
 correlation() (in module `scipy.spatial.distance`), 1248, 1280  
 correspond() (in module `scipy.cluster.hierarchy`), 316  
 cosdg (in module `scipy.special`), 1393  
 coshm() (in module `scipy.linalg`), 513  
 cosine (in module `scipy.stats`), 1437  
 cosine() (in module `scipy.signal`), 1010  
 cosine() (in module `scipy.spatial.distance`), 1249, 1280  
 cosm() (in module `scipy.linalg`), 512  
 cosm1 (in module `scipy.special`), 1393  
 cotdg (in module `scipy.special`), 1393  
 count\_neighbors() (`scipy.spatial.cKDTree` method), 1236  
 count\_neighbors() (`scipy.spatial.KDTree` method), 1231  
 count\_nonzero() (`scipy.sparse.bsr_matrix` method), 1061  
 count\_nonzero() (`scipy.sparse.coo_matrix` method), 1070  
 count\_nonzero() (`scipy.sparse.csc_matrix` method), 1080  
 count\_nonzero() (`scipy.sparse.csr_matrix` method), 1090  
 count\_nonzero() (`scipy.sparse.dia_matrix` method), 1099  
 count\_nonzero() (`scipy.sparse.dok_matrix` method), 1106  
 count\_nonzero() (`scipy.sparse.lil_matrix` method), 1114  
 count\_nonzero() (`scipy.sparse.spmatrix` method), 1119  
 count\_tied\_groups() (in module `scipy.stats.mstats`), 1736, 1770  
 covariance\_factor() (`scipy.stats.gaussian_kde` method), 1766  
 cpbsv (in module `scipy.linalg.lapack`), 622  
 cpbtrf (in module `scipy.linalg.lapack`), 623  
 cpbtrs (in module `scipy.linalg.lapack`), 624  
 cposv (in module `scipy.linalg.lapack`), 625  
 cpotrf (in module `scipy.linalg.lapack`), 626  
 cpotri (in module `scipy.linalg.lapack`), 627  
 cpotrs (in module `scipy.linalg.lapack`), 628  
 cptsv (in module `scipy.linalg.lapack`), 637  
 createDimension() (`scipy.io.netcdf_file` method), 465  
 createVariable() (`scipy.io.netcdf_file` method), 465  
 crot (in module `scipy.linalg.lapack`), 628  
 crotg (in module `scipy.linalg.blas`), 535  
 cs\_diff() (in module `scipy.fftpack`), 344  
 csc\_matrix (class in `scipy.sparse`), 1077  
 cscal (in module `scipy.linalg.blas`), 535  
 csd() (in module `scipy.signal`), 1048

- csgraph\_from\_dense() (in module scipy.sparse.csgraph), 1144, 1226
  - csgraph\_from\_masked() (in module scipy.sparse.csgraph), 1144, 1226
  - csgraph\_masked\_from\_dense() (in module scipy.sparse.csgraph), 1144, 1226
  - csgraph\_to\_dense() (in module scipy.sparse.csgraph), 1144, 1227
  - csgraph\_to\_masked() (in module scipy.sparse.csgraph), 1145, 1228
  - cspline1d() (in module scipy.signal), 885
  - cspline1d\_eval() (in module scipy.signal), 886
  - cspline2d() (in module scipy.signal), 886
  - csr\_matrix (class in scipy.sparse), 1086
  - csrot (in module scipy.linalg.blas), 535
  - csscal (in module scipy.linalg.blas), 536
  - cswap (in module scipy.linalg.blas), 536
  - csymm (in module scipy.linalg.blas), 561
  - csyr (in module scipy.linalg.blas), 550
  - csyr2k (in module scipy.linalg.blas), 561
  - csyrk (in module scipy.linalg.blas), 561
  - ctrmv (in module scipy.linalg.blas), 549
  - ctrsyl (in module scipy.linalg.lapack), 630
  - ctrtri (in module scipy.linalg.lapack), 631
  - ctrtrs (in module scipy.linalg.lapack), 632
  - cubic() (in module scipy.signal), 885
  - CubicSpline (class in scipy.interpolate), 388
  - cumfreq() (in module scipy.stats), 1662
  - cumtrapz() (in module scipy.integrate), 361
  - cunghr (in module scipy.linalg.lapack), 633
  - cungqr (in module scipy.linalg.lapack), 633
  - cungrq (in module scipy.linalg.lapack), 634
  - cunmqr (in module scipy.linalg.lapack), 634
  - curve\_fit() (in module scipy.optimize), 833
  - cut\_tree() (in module scipy.cluster.hierarchy), 314
  - cwt() (in module scipy.signal), 1037
- ## D
- D (scipy.signal.dlti attribute), 966
  - D (scipy.signal.lti attribute), 948
  - D (scipy.signal.StateSpace attribute), 951, 969
  - D (scipy.signal.TransferFunction attribute), 954, 972
  - D (scipy.signal.ZerosPolesGain attribute), 956, 974
  - dasum (in module scipy.linalg.blas), 536
  - Data (class in scipy.odr), 769
  - data (scipy.spatial.cKDTree attribute), 1235
  - daub() (in module scipy.signal), 1035
  - dawson (in module scipy.special), 1356
  - daxpy (in module scipy.linalg.blas), 536
  - dblquad() (in module scipy.integrate), 353
  - dbode() (in module scipy.signal), 977
  - dcopy (in module scipy.linalg.blas), 537
  - dct() (in module scipy.fftpack), 338
  - ddot (in module scipy.linalg.blas), 537
  - decimate() (in module scipy.signal), 902
  - deconvolve() (in module scipy.signal), 897
  - deg2rad() (scipy.sparse.bsr\_matrix method), 1061
  - deg2rad() (scipy.sparse.coo\_matrix method), 1071
  - deg2rad() (scipy.sparse.csc\_matrix method), 1080
  - deg2rad() (scipy.sparse.csr\_matrix method), 1090
  - deg2rad() (scipy.sparse.dia\_matrix method), 1099
  - Delaunay (class in scipy.spatial), 1255
  - delaunay\_plot\_2d() (in module scipy.spatial), 1266
  - den (scipy.signal.dlti attribute), 966
  - den (scipy.signal.lti attribute), 948
  - den (scipy.signal.StateSpace attribute), 951, 969
  - den (scipy.signal.TransferFunction attribute), 954, 972
  - den (scipy.signal.ZerosPolesGain attribute), 956, 974
  - dendrogram() (in module scipy.cluster.hierarchy), 308
  - depth\_first\_order() (in module scipy.sparse.csgraph), 1139, 1221
  - depth\_first\_tree() (in module scipy.sparse.csgraph), 1140, 1222
  - derivative() (in module scipy.misc), 700
  - derivative() (scipy.interpolate.Akima1DInterpolator method), 387
  - derivative() (scipy.interpolate.BPoly method), 398
  - derivative() (scipy.interpolate.CubicSpline method), 392
  - derivative() (scipy.interpolate.InterpolatedUnivariateSpline method), 419
  - derivative() (scipy.interpolate.KroghInterpolator method), 382
  - derivative() (scipy.interpolate.LSQUnivariateSpline method), 424
  - derivative() (scipy.interpolate.PchipInterpolator method), 384
  - derivative() (scipy.interpolate.ppform method), 453
  - derivative() (scipy.interpolate.PPoly method), 394
  - derivative() (scipy.interpolate.UnivariateSpline method), 415
  - derivatives() (scipy.interpolate.InterpolatedUnivariateSpline method), 420
  - derivatives() (scipy.interpolate.KroghInterpolator method), 382
  - derivatives() (scipy.interpolate.LSQUnivariateSpline method), 424
  - derivatives() (scipy.interpolate.UnivariateSpline method), 415
  - describe() (in module scipy.stats), 1650
  - describe() (in module scipy.stats.mstats), 1736, 1770
  - destroy\_convolve\_cache (in module scipy.fftpack.convolve), 349
  - det() (in module scipy.linalg), 477
  - detrend() (in module scipy.signal), 903
  - dfreqresp() (in module scipy.signal), 976
  - dft() (in module scipy.linalg), 523
  - dgamma (in module scipy.stats), 1439
  - dgbsv (in module scipy.linalg.lapack), 572

- dgbtrf (in module `scipy.linalg.lapack`), 573  
 dgbtrs (in module `scipy.linalg.lapack`), 575  
 dgebal (in module `scipy.linalg.lapack`), 576  
 dgees (in module `scipy.linalg.lapack`), 578  
 dgeev (in module `scipy.linalg.lapack`), 580  
 dgeev\_lwork (in module `scipy.linalg.lapack`), 581  
 dgegv() (in module `scipy.linalg.lapack`), 582  
 dgehrd (in module `scipy.linalg.lapack`), 584  
 dgehrd\_lwork (in module `scipy.linalg.lapack`), 585  
 dgelsd (in module `scipy.linalg.lapack`), 588  
 dgelsd\_lwork (in module `scipy.linalg.lapack`), 590  
 dgelss (in module `scipy.linalg.lapack`), 586  
 dgelss\_lwork (in module `scipy.linalg.lapack`), 587  
 dgelsy (in module `scipy.linalg.lapack`), 591  
 dgelsy\_lwork (in module `scipy.linalg.lapack`), 592  
 dgemm (in module `scipy.linalg.blas`), 562  
 dgemv (in module `scipy.linalg.blas`), 551  
 dgeqp3 (in module `scipy.linalg.lapack`), 593  
 dgeqrf (in module `scipy.linalg.lapack`), 594  
 dger (in module `scipy.linalg.blas`), 551  
 dgerqf (in module `scipy.linalg.lapack`), 595  
 dgesdd (in module `scipy.linalg.lapack`), 596  
 dgesdd\_lwork (in module `scipy.linalg.lapack`), 597  
 dgesv (in module `scipy.linalg.lapack`), 600  
 dgesvd (in module `scipy.linalg.lapack`), 598  
 dgesvd\_lwork (in module `scipy.linalg.lapack`), 600  
 dgetrf (in module `scipy.linalg.lapack`), 601  
 dgetri (in module `scipy.linalg.lapack`), 602  
 dgetri\_lwork (in module `scipy.linalg.lapack`), 603  
 dgetrs (in module `scipy.linalg.lapack`), 604  
 dggess (in module `scipy.linalg.lapack`), 605  
 dggeev (in module `scipy.linalg.lapack`), 608  
 dgtsv (in module `scipy.linalg.lapack`), 635  
 dia\_matrix (class in `scipy.sparse`), 1096  
 diagbroyden() (in module `scipy.optimize`), 860  
 diagonal() (`scipy.sparse.bsr_matrix` method), 1061  
 diagonal() (`scipy.sparse.coo_matrix` method), 1071  
 diagonal() (`scipy.sparse.csc_matrix` method), 1080  
 diagonal() (`scipy.sparse.csr_matrix` method), 1090  
 diagonal() (`scipy.sparse.dia_matrix` method), 1099  
 diagonal() (`scipy.sparse.dok_matrix` method), 1106  
 diagonal() (`scipy.sparse.lil_matrix` method), 1114  
 diagonal() (`scipy.sparse.spmatrix` method), 1119  
 diags() (in module `scipy.sparse`), 1125  
 diagsvd() (in module `scipy.linalg`), 493  
 dice() (in module `scipy.spatial.distance`), 1251, 1282  
 diff() (in module `scipy.fftpack`), 343  
 differential\_evolution() (in module `scipy.optimize`), 806, 829  
 digamma (in module `scipy.special`), 1350  
 dijkstra() (in module `scipy.sparse.csgraph`), 1135, 1218  
 dimpulse() (in module `scipy.signal`), 975  
 diric() (in module `scipy.special`), 1385  
 dirichlet (in module `scipy.stats`), 1617  
 distance\_matrix() (in module `scipy.spatial`), 1268  
 distance\_transform\_bf() (in module `scipy.ndimage`), 755  
 distance\_transform\_cdt() (in module `scipy.ndimage`), 756  
 distance\_transform\_edt() (in module `scipy.ndimage`), 756  
 dlamch (in module `scipy.linalg.lapack`), 638  
 dlange (in module `scipy.linalg.lapack`), 647  
 dlaplace (in module `scipy.stats`), 1629  
 dlarf (in module `scipy.linalg.lapack`), 616  
 dlarfg (in module `scipy.linalg.lapack`), 617  
 dlartg (in module `scipy.linalg.lapack`), 618  
 dlasd4 (in module `scipy.linalg.lapack`), 619  
 dlaswp (in module `scipy.linalg.lapack`), 619  
 dlauum (in module `scipy.linalg.lapack`), 621  
 dlsim() (in module `scipy.signal`), 975  
 dlti (class in `scipy.signal`), 964  
 dnrn2 (in module `scipy.linalg.blas`), 537  
 dok\_matrix (class in `scipy.sparse`), 1104  
 dorghr (in module `scipy.linalg.lapack`), 638  
 dorgqr (in module `scipy.linalg.lapack`), 639  
 dorgrq (in module `scipy.linalg.lapack`), 639  
 dormqr (in module `scipy.linalg.lapack`), 640  
 dot() (`scipy.optimize.LbfgsInvHessProduct` method), 872  
 dot() (`scipy.sparse.bsr_matrix` method), 1061  
 dot() (`scipy.sparse.coo_matrix` method), 1071  
 dot() (`scipy.sparse.csc_matrix` method), 1080  
 dot() (`scipy.sparse.csr_matrix` method), 1090  
 dot() (`scipy.sparse.dia_matrix` method), 1099  
 dot() (`scipy.sparse.dok_matrix` method), 1106  
 dot() (`scipy.sparse.lil_matrix` method), 1114  
 dot() (`scipy.sparse.linalg.LinearOperator` method), 1150, 1184  
 dot() (`scipy.sparse.spmatrix` method), 1119  
 dpbsv (in module `scipy.linalg.lapack`), 622  
 dpbtrf (in module `scipy.linalg.lapack`), 623  
 dpbtrs (in module `scipy.linalg.lapack`), 624  
 dposv (in module `scipy.linalg.lapack`), 625  
 dpotrf (in module `scipy.linalg.lapack`), 626  
 dpotri (in module `scipy.linalg.lapack`), 627  
 dpotrs (in module `scipy.linalg.lapack`), 628  
 dptsv (in module `scipy.linalg.lapack`), 637  
 drot (in module `scipy.linalg.blas`), 538  
 drotg (in module `scipy.linalg.blas`), 538  
 drotm (in module `scipy.linalg.blas`), 538  
 drotmg (in module `scipy.linalg.blas`), 539  
 dsbev (in module `scipy.linalg.lapack`), 641  
 dsbevd (in module `scipy.linalg.lapack`), 641  
 dsbevz (in module `scipy.linalg.lapack`), 642  
 dscal (in module `scipy.linalg.blas`), 539  
 dst() (in module `scipy.fftpack`), 341  
 dstep() (in module `scipy.signal`), 976  
 dswap (in module `scipy.linalg.blas`), 539  
 dsyev (in module `scipy.linalg.lapack`), 643  
 dsyevd (in module `scipy.linalg.lapack`), 644  
 dsyevr (in module `scipy.linalg.lapack`), 644

dsygv (in module `scipy.linalg.lapack`), 645  
 dsygvd (in module `scipy.linalg.lapack`), 646  
 dsygvx (in module `scipy.linalg.lapack`), 647  
 dsymm (in module `scipy.linalg.blas`), 562  
 dsymv (in module `scipy.linalg.blas`), 552  
 dsyr (in module `scipy.linalg.blas`), 553  
 dsyr2 (in module `scipy.linalg.blas`), 553  
 dsyr2k (in module `scipy.linalg.blas`), 563  
 dsyrk (in module `scipy.linalg.blas`), 562  
 dt (`scipy.signal.dlti` attribute), 966  
 dt (`scipy.signal.lti` attribute), 948  
 dt (`scipy.signal.StateSpace` attribute), 952, 969  
 dt (`scipy.signal.TransferFunction` attribute), 954, 972  
 dt (`scipy.signal.ZerosPolesGain` attribute), 956, 974  
 dtrmv (in module `scipy.linalg.blas`), 552  
 dtrsyl (in module `scipy.linalg.lapack`), 630  
 dtrtri (in module `scipy.linalg.lapack`), 631  
 dtrtrs (in module `scipy.linalg.lapack`), 632  
 dweibull (in module `scipy.stats`), 1441  
 dzasum (in module `scipy.linalg.blas`), 539  
 dznrm2 (in module `scipy.linalg.blas`), 540

## E

eig() (in module `scipy.linalg`), 484  
 eig\_banded() (in module `scipy.linalg`), 488  
 eigh() (in module `scipy.linalg`), 485  
 eigs() (in module `scipy.sparse.linalg`), 1166, 1201  
 eigsh() (in module `scipy.sparse.linalg`), 1168, 1204  
 eigvals() (in module `scipy.linalg`), 485  
 eigvals\_banded() (in module `scipy.linalg`), 489  
 eigvalsh() (in module `scipy.linalg`), 487  
 eliminate\_zeros() (`scipy.sparse.bsr_matrix` method), 1061  
 eliminate\_zeros() (`scipy.sparse.coo_matrix` method), 1071  
 eliminate\_zeros() (`scipy.sparse.csc_matrix` method), 1081  
 eliminate\_zeros() (`scipy.sparse.csr_matrix` method), 1090  
 ellip() (in module `scipy.signal`), 939  
 ellip\_harm() (in module `scipy.special`), 1361  
 ellip\_harm\_2() (in module `scipy.special`), 1362  
 ellip\_normal() (in module `scipy.special`), 1363  
 ellipap() (in module `scipy.signal`), 930  
 ellipse (in module `scipy.special`), 1290  
 ellipseinc (in module `scipy.special`), 1291  
 ellipj (in module `scipy.special`), 1288  
 ellipk() (in module `scipy.special`), 1288  
 ellipkinc (in module `scipy.special`), 1289  
 ellipkm1 (in module `scipy.special`), 1289  
 ellipord() (in module `scipy.signal`), 941  
 entr (in module `scipy.special`), 1344  
 entropy() (in module `scipy.stats`), 1719  
 entropy() (`scipy.stats.rv_continuous` method), 1401  
 entropy() (`scipy.stats.rv_discrete` method), 1410  
 erf (in module `scipy.special`), 1351  
 erf\_zeros() (in module `scipy.special`), 1358

erfc (in module `scipy.special`), 1352  
 erfcinv() (in module `scipy.special`), 1355  
 erfex (in module `scipy.special`), 1353  
 erfi (in module `scipy.special`), 1354  
 erfinv() (in module `scipy.special`), 1355  
 erlang (in module `scipy.stats`), 1443  
 errprint() (in module `scipy.special`), 1285  
 estimate\_rank() (in module `scipy.linalg.interpolative`), 692  
 estimate\_spectral\_norm() (in module `scipy.linalg.interpolative`), 692  
 estimate\_spectral\_norm\_diff() (in module `scipy.linalg.interpolative`), 692  
 euclidean() (in module `scipy.spatial.distance`), 1249, 1280  
 euler() (in module `scipy.special`), 1388  
 ev() (`scipy.interpolate.BivariateSpline` method), 440  
 ev() (`scipy.interpolate.LSQBivariateSpline` method), 446  
 ev() (`scipy.interpolate.LSQSphereBivariateSpline` method), 449  
 ev() (`scipy.interpolate.RectBivariateSpline` method), 411, 434  
 ev() (`scipy.interpolate.RectSphereBivariateSpline` method), 438  
 ev() (`scipy.interpolate.SmoothBivariateSpline` method), 442  
 ev() (`scipy.interpolate.SmoothSphereBivariateSpline` method), 444  
 eval\_chebyc (in module `scipy.special`), 1364  
 eval\_chebys (in module `scipy.special`), 1364  
 eval\_chebyt (in module `scipy.special`), 1364  
 eval\_chebyu (in module `scipy.special`), 1364  
 eval\_gegenbauer (in module `scipy.special`), 1365  
 eval\_genlaguerre (in module `scipy.special`), 1365  
 eval\_hermite (in module `scipy.special`), 1365  
 eval\_hermitenorm (in module `scipy.special`), 1365  
 eval\_jacobi (in module `scipy.special`), 1364  
 eval\_laguerre (in module `scipy.special`), 1364  
 eval\_legendre (in module `scipy.special`), 1364  
 eval\_sh\_chebyt (in module `scipy.special`), 1365  
 eval\_sh\_chebyu (in module `scipy.special`), 1365  
 eval\_sh\_jacobi (in module `scipy.special`), 1365  
 eval\_sh\_legendre (in module `scipy.special`), 1365  
 evaluate() (`scipy.stats.gaussian_kde` method), 1763  
 excitingmixing() (in module `scipy.optimize`), 858  
 exp1 (in module `scipy.special`), 1388  
 exp10 (in module `scipy.special`), 1393  
 exp2 (in module `scipy.special`), 1393  
 expect() (`scipy.stats.rv_continuous` method), 1401  
 expect() (`scipy.stats.rv_discrete` method), 1410  
 expected\_freq() (in module `scipy.stats.contingency`), 1723  
 expi (in module `scipy.special`), 1388  
 expit (in module `scipy.special`), 1342  
 expm() (in module `scipy.linalg`), 511  
 expm() (in module `scipy.sparse.linalg`), 1151, 1186



- expm1 (in module `scipy.special`), 1393  
 expm1() (`scipy.sparse.bsr_matrix` method), 1061  
 expm1() (`scipy.sparse.coo_matrix` method), 1071  
 expm1() (`scipy.sparse.csc_matrix` method), 1081  
 expm1() (`scipy.sparse.csr_matrix` method), 1091  
 expm1() (`scipy.sparse.dia_matrix` method), 1099  
 expm\_cond() (in module `scipy.linalg`), 517  
 expm\_frechet() (in module `scipy.linalg`), 516  
 expm\_multiply() (in module `scipy.sparse.linalg`), 1151, 1186  
 expn (in module `scipy.special`), 1388  
 expon (in module `scipy.stats`), 1444  
 exponential (in module `scipy.odr`), 777  
 exponential() (in module `scipy.signal`), 1012  
 exponnorm (in module `scipy.stats`), 1446  
 exponpow (in module `scipy.stats`), 1451  
 exponweib (in module `scipy.stats`), 1448  
 exprel (in module `scipy.special`), 1394  
 extend() (`scipy.interpolate.BPoly` method), 398  
 extend() (`scipy.interpolate.ppform` method), 453  
 extend() (`scipy.interpolate.PPoly` method), 396  
 extrema() (in module `scipy.ndimage`), 733  
 eye() (in module `scipy.sparse`), 1123
- ## F
- f (in module `scipy.stats`), 1453  
 F2C() (in module `scipy.constants`), 331  
 F2K() (in module `scipy.constants`), 332  
 f\_oneway() (in module `scipy.stats`), 1684  
 f\_oneway() (in module `scipy.stats.mstats`), 1737, 1771  
 f\_value() (in module `scipy.stats`), 1691  
 f\_value\_wilks\_lambda() (in module `scipy.stats.mstats`), 1737, 1771  
 face() (in module `scipy.misc`), 701  
 factorial() (in module `scipy.misc`), 702  
 factorial() (in module `scipy.special`), 1388  
 factorial2() (in module `scipy.misc`), 702  
 factorial2() (in module `scipy.special`), 1389  
 factorialk() (in module `scipy.misc`), 703  
 factorialk() (in module `scipy.special`), 1389  
 factorized() (in module `scipy.sparse.linalg`), 1155, 1190  
 fatiguelife (in module `scipy.stats`), 1455  
 fcluster() (in module `scipy.cluster.hierarchy`), 299  
 fclusterdata() (in module `scipy.cluster.hierarchy`), 300  
 fdtr (in module `scipy.special`), 1327  
 fdtrc (in module `scipy.special`), 1327  
 fdtri (in module `scipy.special`), 1328  
 fdtridfd (in module `scipy.special`), 1328  
 fft() (in module `scipy.fftpack`), 334  
 fft2() (in module `scipy.fftpack`), 336  
 fftconvolve() (in module `scipy.signal`), 878  
 fftfreq() (in module `scipy.fftpack`), 347  
 fftn() (in module `scipy.fftpack`), 336  
 fftshift() (in module `scipy.fftpack`), 346  
 fill\_value (`scipy.interpolate.interp1d` attribute), 379  
 filtfilt() (in module `scipy.signal`), 893  
 find() (in module `scipy.constants`), 319  
 find() (in module `scipy.sparse`), 1131  
 find\_best\_blas\_type() (in module `scipy.linalg`), 531  
 find\_best\_blas\_type() (in module `scipy.linalg.blas`), 532  
 find\_objects() (in module `scipy.ndimage`), 734  
 find\_peaks\_cwt() (in module `scipy.signal`), 1038  
 find\_repeats() (in module `scipy.stats`), 1661  
 find\_repeats() (in module `scipy.stats.mstats`), 1737, 1771  
 find\_simplex() (`scipy.spatial.Delaunay` method), 1259  
 findfreqs() (in module `scipy.signal`), 909  
 firfs() (in module `scipy.signal`), 909  
 firwin() (in module `scipy.signal`), 911  
 firwin2() (in module `scipy.signal`), 912  
 fisher\_exact() (in module `scipy.stats`), 1724  
 fisk (in module `scipy.stats`), 1457  
 fit() (`scipy.stats.rv_continuous` method), 1403  
 fit\_loc\_scale() (`scipy.stats.rv_continuous` method), 1404  
 fixed\_point() (in module `scipy.optimize`), 841  
 fixed\_quad() (in module `scipy.integrate`), 357  
 flattop() (in module `scipy.signal`), 1014  
 fligner() (in module `scipy.stats`), 1711  
 floor() (`scipy.sparse.bsr_matrix` method), 1061  
 floor() (`scipy.sparse.coo_matrix` method), 1071  
 floor() (`scipy.sparse.csc_matrix` method), 1081  
 floor() (`scipy.sparse.csr_matrix` method), 1091  
 floor() (`scipy.sparse.dia_matrix` method), 1099  
 floyd\_warshall() (in module `scipy.sparse.csgraph`), 1136, 1218  
 flush() (`scipy.io.netcdf_file` method), 465  
 fmin() (in module `scipy.optimize`), 792  
 fmin\_bfgs() (in module `scipy.optimize`), 797  
 fmin\_cg() (in module `scipy.optimize`), 795  
 fmin\_cobyla() (in module `scipy.optimize`), 803  
 fmin\_l\_bfgs\_b() (in module `scipy.optimize`), 799  
 fmin\_ncg() (in module `scipy.optimize`), 798  
 fmin\_powell() (in module `scipy.optimize`), 793  
 fmin\_slsqp() (in module `scipy.optimize`), 805  
 fmin\_tnc() (in module `scipy.optimize`), 801  
 fminbound() (in module `scipy.optimize`), 809  
 foldcauchy (in module `scipy.stats`), 1459  
 foldnorm (in module `scipy.stats`), 1461  
 FortranFile (class in `scipy.io`), 460  
 fourier\_ellipsoid() (in module `scipy.ndimage`), 725  
 fourier\_gaussian() (in module `scipy.ndimage`), 726  
 fourier\_shift() (in module `scipy.ndimage`), 726  
 fourier\_uniform() (in module `scipy.ndimage`), 726  
 fractional\_matrix\_power() (in module `scipy.linalg`), 518  
 frechet\_l (in module `scipy.stats`), 1465  
 frechet\_r (in module `scipy.stats`), 1463  
 freqresp() (in module `scipy.signal`), 961  
 freqresp() (`scipy.signal.dlti` method), 967  
 freqresp() (`scipy.signal.lti` method), 949

freqs() (in module `scipy.signal`), 914  
 freqz() (in module `scipy.signal`), 915  
 fresnel (in module `scipy.special`), 1357  
 fresnel\_zeros() (in module `scipy.special`), 1357  
 fresnelc\_zeros() (in module `scipy.special`), 1358  
 fresnels\_zeros() (in module `scipy.special`), 1358  
 friedmanchisquare() (in module `scipy.stats`), 1705  
 friedmanchisquare() (in module `scipy.stats.mstats`), 1737, 1771  
 from\_bernstein\_basis() (`scipy.interpolate.ppform` class method), 453  
 from\_bernstein\_basis() (`scipy.interpolate.PPoly` class method), 396  
 from\_derivatives() (`scipy.interpolate.BPoly` class method), 399  
 from\_mlab\_linkage() (in module `scipy.cluster.hierarchy`), 307  
 from\_power\_basis() (`scipy.interpolate.BPoly` class method), 399  
 from\_spline() (`scipy.interpolate.ppform` class method), 454  
 from\_spline() (`scipy.interpolate.PPoly` class method), 396  
 fromimage() (in module `scipy.misc`), 703  
 fromkeys() (`scipy.optimize.OptimizeResult` static method), 785  
 fromkeys() (`scipy.sparse.dok_matrix` static method), 1107  
 fromspline() (`scipy.interpolate.ppform` class method), 454  
 fsolve() (in module `scipy.optimize`), 843  
 funm() (in module `scipy.linalg`), 515

## G

gain (`scipy.signal.dlti` attribute), 966  
 gain (`scipy.signal.lti` attribute), 948  
 gain (`scipy.signal.StateSpace` attribute), 952, 969  
 gain (`scipy.signal.TransferFunction` attribute), 954, 972  
 gain (`scipy.signal.ZerosPolesGain` attribute), 956, 974  
 gamma (in module `scipy.special`), 1347  
 gamma (in module `scipy.stats`), 1482  
 gammainc (in module `scipy.special`), 1348  
 gammaincc (in module `scipy.special`), 1348  
 gammainccinv (in module `scipy.special`), 1348  
 gammaincinv (in module `scipy.special`), 1348  
 gammaln() (in module `scipy.special`), 1347  
 gammasgn (in module `scipy.special`), 1348  
 gauss\_spline() (in module `scipy.signal`), 885  
 gausshyper (in module `scipy.stats`), 1479  
 gaussian() (in module `scipy.signal`), 1016  
 gaussian\_filter() (in module `scipy.ndimage`), 714  
 gaussian\_filter1d() (in module `scipy.ndimage`), 714  
 gaussian\_gradient\_magnitude() (in module `scipy.ndimage`), 715  
 gaussian\_kde (class in `scipy.stats`), 1761  
 gaussian\_laplace() (in module `scipy.ndimage`), 715  
 gausspulse() (in module `scipy.signal`), 990

gdr (in module `scipy.special`), 1328  
 gdtrc (in module `scipy.special`), 1329  
 gdtria (in module `scipy.special`), 1329  
 gdtrib (in module `scipy.special`), 1330  
 gdtrix (in module `scipy.special`), 1331  
 gegenbauer() (in module `scipy.special`), 1366  
 general\_gaussian() (in module `scipy.signal`), 1017  
 generate\_binary\_structure() (in module `scipy.ndimage`), 758  
 generic\_filter() (in module `scipy.ndimage`), 716  
 generic\_filter1d() (in module `scipy.ndimage`), 717  
 generic\_gradient\_magnitude() (in module `scipy.ndimage`), 717  
 generic\_laplace() (in module `scipy.ndimage`), 718  
 genexpon (in module `scipy.stats`), 1475  
 genextreme (in module `scipy.stats`), 1477  
 gengamma (in module `scipy.stats`), 1484  
 genhalflogistic (in module `scipy.stats`), 1486  
 genlaguerre() (in module `scipy.special`), 1366  
 genlogistic (in module `scipy.stats`), 1468  
 gennorm (in module `scipy.stats`), 1470  
 genpareto (in module `scipy.stats`), 1472  
 geom (in module `scipy.stats`), 1631  
 geometric\_transform() (in module `scipy.ndimage`), 728  
 get() (`scipy.optimize.OptimizeResult` method), 785  
 get() (`scipy.sparse.dok_matrix` method), 1107  
 get\_blas\_funcs() (in module `scipy.linalg`), 530  
 get\_blas\_funcs() (in module `scipy.linalg.blas`), 532  
 get\_coeffs() (`scipy.interpolate.BivariateSpline` method), 440  
 get\_coeffs() (`scipy.interpolate.InterpolatedUnivariateSpline` method), 420  
 get\_coeffs() (`scipy.interpolate.LSQBivariateSpline` method), 446  
 get\_coeffs() (`scipy.interpolate.LSQSphereBivariateSpline` method), 449  
 get\_coeffs() (`scipy.interpolate.LSQUnivariateSpline` method), 425  
 get\_coeffs() (`scipy.interpolate.RectBivariateSpline` method), 411, 435  
 get\_coeffs() (`scipy.interpolate.RectSphereBivariateSpline` method), 439  
 get\_coeffs() (`scipy.interpolate.SmoothBivariateSpline` method), 442  
 get\_coeffs() (`scipy.interpolate.SmoothSphereBivariateSpline` method), 445  
 get\_coeffs() (`scipy.interpolate.UnivariateSpline` method), 415  
 get\_count() (`scipy.cluster.hierarchy.ClusterNode` method), 313  
 get\_id() (`scipy.cluster.hierarchy.ClusterNode` method), 313  
 get\_knots() (`scipy.interpolate.BivariateSpline` method), 440

- `get_knots()` (`scipy.interpolate.InterpolatedUnivariateSpline` method), 420  
`get_knots()` (`scipy.interpolate.LSQBivariateSpline` method), 446  
`get_knots()` (`scipy.interpolate.LSQSphereBivariateSpline` method), 449  
`get_knots()` (`scipy.interpolate.LSQUnivariateSpline` method), 425  
`get_knots()` (`scipy.interpolate.RectBivariateSpline` method), 411, 435  
`get_knots()` (`scipy.interpolate.RectSphereBivariateSpline` method), 439  
`get_knots()` (`scipy.interpolate.SmoothBivariateSpline` method), 442  
`get_knots()` (`scipy.interpolate.SmoothSphereBivariateSpline` method), 445  
`get_knots()` (`scipy.interpolate.UnivariateSpline` method), 415  
`get_lapack_funcs()` (in module `scipy.linalg`), 530  
`get_left()` (`scipy.cluster.hierarchy.ClusterNode` method), 313  
`get_residual()` (`scipy.interpolate.BivariateSpline` method), 440  
`get_residual()` (`scipy.interpolate.InterpolatedUnivariateSpline` method), 420  
`get_residual()` (`scipy.interpolate.LSQBivariateSpline` method), 446  
`get_residual()` (`scipy.interpolate.LSQSphereBivariateSpline` method), 449  
`get_residual()` (`scipy.interpolate.LSQUnivariateSpline` method), 425  
`get_residual()` (`scipy.interpolate.RectBivariateSpline` method), 411, 435  
`get_residual()` (`scipy.interpolate.RectSphereBivariateSpline` method), 439  
`get_residual()` (`scipy.interpolate.SmoothBivariateSpline` method), 442  
`get_residual()` (`scipy.interpolate.SmoothSphereBivariateSpline` method), 445  
`get_residual()` (`scipy.interpolate.UnivariateSpline` method), 415  
`get_right()` (`scipy.cluster.hierarchy.ClusterNode` method), 313  
`get_shape()` (`scipy.sparse.bsr_matrix` method), 1061  
`get_shape()` (`scipy.sparse.coo_matrix` method), 1071  
`get_shape()` (`scipy.sparse.csc_matrix` method), 1081  
`get_shape()` (`scipy.sparse.csr_matrix` method), 1091  
`get_shape()` (`scipy.sparse.dia_matrix` method), 1099  
`get_shape()` (`scipy.sparse.dok_matrix` method), 1107  
`get_shape()` (`scipy.sparse.lil_matrix` method), 1114  
`get_shape()` (`scipy.sparse.spmatrix` method), 1119  
`get_window()` (in module `scipy.signal`), 998  
`getcol()` (`scipy.sparse.bsr_matrix` method), 1061  
`getcol()` (`scipy.sparse.coo_matrix` method), 1071  
`getcol()` (`scipy.sparse.csc_matrix` method), 1081  
`getcol()` (`scipy.sparse.csr_matrix` method), 1091  
`getcol()` (`scipy.sparse.dia_matrix` method), 1099  
`getcol()` (`scipy.sparse.dok_matrix` method), 1107  
`getcol()` (`scipy.sparse.lil_matrix` method), 1114  
`getcol()` (`scipy.sparse.spmatrix` method), 1119  
`getdata()` (`scipy.sparse.bsr_matrix` method), 1062  
`getformat()` (`scipy.sparse.bsr_matrix` method), 1062  
`getformat()` (`scipy.sparse.coo_matrix` method), 1071  
`getformat()` (`scipy.sparse.csc_matrix` method), 1081  
`getformat()` (`scipy.sparse.csr_matrix` method), 1091  
`getformat()` (`scipy.sparse.dia_matrix` method), 1099  
`getformat()` (`scipy.sparse.dok_matrix` method), 1107  
`getformat()` (`scipy.sparse.lil_matrix` method), 1114  
`getformat()` (`scipy.sparse.spmatrix` method), 1119  
`getH()` (`scipy.sparse.bsr_matrix` method), 1061  
`getH()` (`scipy.sparse.coo_matrix` method), 1071  
`getH()` (`scipy.sparse.csc_matrix` method), 1081  
`getH()` (`scipy.sparse.csr_matrix` method), 1091  
`getH()` (`scipy.sparse.dia_matrix` method), 1099  
`getH()` (`scipy.sparse.dok_matrix` method), 1107  
`getH()` (`scipy.sparse.lil_matrix` method), 1114  
`getH()` (`scipy.sparse.spmatrix` method), 1119  
`getmaxprint()` (`scipy.sparse.bsr_matrix` method), 1062  
`getmaxprint()` (`scipy.sparse.coo_matrix` method), 1071  
`getmaxprint()` (`scipy.sparse.csc_matrix` method), 1081  
`getmaxprint()` (`scipy.sparse.csr_matrix` method), 1091  
`getmaxprint()` (`scipy.sparse.dia_matrix` method), 1099  
`getmaxprint()` (`scipy.sparse.dok_matrix` method), 1107  
`getmaxprint()` (`scipy.sparse.lil_matrix` method), 1114  
`getmaxprint()` (`scipy.sparse.spmatrix` method), 1119  
`getnnz()` (`scipy.sparse.bsr_matrix` method), 1062  
`getnnz()` (`scipy.sparse.coo_matrix` method), 1071  
`getnnz()` (`scipy.sparse.csc_matrix` method), 1081  
`getnnz()` (`scipy.sparse.csr_matrix` method), 1091  
`getnnz()` (`scipy.sparse.dia_matrix` method), 1099  
`getnnz()` (`scipy.sparse.dok_matrix` method), 1107  
`getnnz()` (`scipy.sparse.lil_matrix` method), 1114  
`getnnz()` (`scipy.sparse.spmatrix` method), 1119  
`getrow()` (`scipy.sparse.bsr_matrix` method), 1062  
`getrow()` (`scipy.sparse.coo_matrix` method), 1072  
`getrow()` (`scipy.sparse.csc_matrix` method), 1081  
`getrow()` (`scipy.sparse.csr_matrix` method), 1091  
`getrow()` (`scipy.sparse.dia_matrix` method), 1100  
`getrow()` (`scipy.sparse.dok_matrix` method), 1107  
`getrow()` (`scipy.sparse.lil_matrix` method), 1114  
`getrow()` (`scipy.sparse.spmatrix` method), 1119  
`getrowview()` (`scipy.sparse.lil_matrix` method), 1114  
`getValue()` (`scipy.io.netcdf_variable` method), 466  
`gilbrat` (in module `scipy.stats`), 1488  
`gmean()` (in module `scipy.stats`), 1651  
`gmean()` (in module `scipy.stats.mstats`), 1756, 1789  
`gmres()` (in module `scipy.sparse.linalg`), 1159, 1194  
`golden()` (in module `scipy.optimize`), 811

gompertz (in module `scipy.stats`), 1490  
 grey\_closing() (in module `scipy.ndimage`), 759  
 grey\_dilation() (in module `scipy.ndimage`), 760  
 grey\_erosion() (in module `scipy.ndimage`), 762  
 grey\_opening() (in module `scipy.ndimage`), 764  
 griddata() (in module `scipy.interpolate`), 400  
 group\_delay() (in module `scipy.signal`), 916  
 gumbel\_l (in module `scipy.stats`), 1494  
 gumbel\_r (in module `scipy.stats`), 1492

## H

H (`scipy.optimize.LbfgsInvHessProduct` attribute), 871  
 h1vp() (in module `scipy.special`), 1312  
 h2vp() (in module `scipy.special`), 1312  
 h\_roots() (in module `scipy.special`), 1368  
 hadamard() (in module `scipy.linalg`), 524  
 halfcauchy (in module `scipy.stats`), 1496  
 halfgennorm (in module `scipy.stats`), 1502  
 halflogistic (in module `scipy.stats`), 1498  
 halfnorm (in module `scipy.stats`), 1500  
 hamming() (in module `scipy.signal`), 1019  
 hamming() (in module `scipy.spatial.distance`), 1251, 1282  
 hankel() (in module `scipy.linalg`), 524  
 hankel1 (in module `scipy.special`), 1299  
 hankel1e (in module `scipy.special`), 1299  
 hankel2 (in module `scipy.special`), 1300  
 hankel2e (in module `scipy.special`), 1300  
 hann() (in module `scipy.signal`), 1021  
 hanning() (in module `scipy.signal`), 1023  
 has\_key() (`scipy.optimize.OptimizeResult` method), 785  
 has\_key() (`scipy.sparse.dok_matrix` method), 1107  
 has\_sorted\_indices (`scipy.sparse.bsr_matrix` attribute), 1058  
 has\_sorted\_indices (`scipy.sparse.csc_matrix` attribute), 1078  
 has\_sorted\_indices (`scipy.sparse.csr_matrix` attribute), 1088  
 hb\_read() (in module `scipy.io`), 467  
 hb\_write() (in module `scipy.io`), 467  
 hdmedian() (in module `scipy.stats.mstats`), 1756, 1790  
 hdquantiles() (in module `scipy.stats.mstats`), 1757, 1790  
 hdquantiles\_sd() (in module `scipy.stats.mstats`), 1757, 1790  
 he\_roots() (in module `scipy.special`), 1368  
 helmert() (in module `scipy.linalg`), 525  
 hermite() (in module `scipy.special`), 1366  
 hermitenorm() (in module `scipy.special`), 1366  
 hessenberg() (in module `scipy.linalg`), 510  
 hilbert() (in module `scipy.fftpack`), 344  
 hilbert() (in module `scipy.linalg`), 525  
 hilbert() (in module `scipy.signal`), 901  
 hilbert2() (in module `scipy.signal`), 902  
 histogram() (in module `scipy.ndimage`), 734  
 histogram() (in module `scipy.stats`), 1664

histogram2() (in module `scipy.stats`), 1663  
 hmean() (in module `scipy.stats`), 1651  
 hmean() (in module `scipy.stats.mstats`), 1757, 1791  
 hstack() (in module `scipy.sparse`), 1129  
 huber (in module `scipy.special`), 1345  
 hyp0f1 (in module `scipy.special`), 1373  
 hyp1f1 (in module `scipy.special`), 1373  
 hyp1f2 (in module `scipy.special`), 1374  
 hyp2f0 (in module `scipy.special`), 1374  
 hyp2f1 (in module `scipy.special`), 1373  
 hyp3f0 (in module `scipy.special`), 1374  
 hypergeom (in module `scipy.stats`), 1633  
 hyperu (in module `scipy.special`), 1373  
 hypsecant (in module `scipy.stats`), 1505

## I

i0 (in module `scipy.special`), 1306  
 i0e (in module `scipy.special`), 1306  
 i1 (in module `scipy.special`), 1307  
 i1e (in module `scipy.special`), 1307  
 icamax (in module `scipy.linalg.blas`), 540  
 id\_to\_svd() (in module `scipy.linalg.interpolative`), 691  
 idamax (in module `scipy.linalg.blas`), 540  
 idct() (in module `scipy.fftpack`), 340  
 idealfourths() (in module `scipy.stats.mstats`), 1758, 1791  
 identity() (in module `scipy.sparse`), 1124  
 idst() (in module `scipy.fftpack`), 342  
 ifft() (in module `scipy.fftpack`), 335  
 ifft2() (in module `scipy.fftpack`), 336  
 ifftn() (in module `scipy.fftpack`), 337  
 ifftshift() (in module `scipy.fftpack`), 347  
 ihilbert() (in module `scipy.fftpack`), 344  
 iirdesign() (in module `scipy.signal`), 918  
 iirfilter() (in module `scipy.signal`), 919  
 ilaver (in module `scipy.linalg.lapack`), 648  
 imfilter() (in module `scipy.misc`), 704  
 impulse() (in module `scipy.signal`), 958  
 impulse() (`scipy.signal.dlti` method), 968  
 impulse() (`scipy.signal.lti` method), 949  
 impulse2() (in module `scipy.signal`), 959  
 imread() (in module `scipy.misc`), 704  
 imread() (in module `scipy.ndimage`), 768  
 imresize() (in module `scipy.misc`), 705  
 imrotate() (in module `scipy.misc`), 705  
 imsave() (in module `scipy.misc`), 705  
 imshow() (in module `scipy.misc`), 706  
 inconsistent() (in module `scipy.cluster.hierarchy`), 307  
 indices (`scipy.spatial.cKDTree` attribute), 1235  
 info() (in module `scipy.misc`), 706  
 init\_convolution\_kernel (in module `scipy.fftpack.convolve`), 349  
 inline() (in module `scipy.weave`), 1795  
 insert() (in module `scipy.interpolate`), 432  
 integral() (`scipy.interpolate.BivariateSpline` method), 440



- integral() (scipy.interpolate.InterpolatedUnivariateSpline method), 420  
 integral() (scipy.interpolate.LSQBivariateSpline method), 446  
 integral() (scipy.interpolate.LSQUnivariateSpline method), 425  
 integral() (scipy.interpolate.RectBivariateSpline method), 411, 435  
 integral() (scipy.interpolate.SmoothBivariateSpline method), 442  
 integral() (scipy.interpolate.UnivariateSpline method), 416  
 integrate() (scipy.integrate.complex\_ode method), 372  
 integrate() (scipy.integrate.ode method), 371  
 integrate() (scipy.interpolate.BPoly method), 399  
 integrate() (scipy.interpolate.CubicSpline method), 392  
 integrate() (scipy.interpolate.ppform method), 454  
 integrate() (scipy.interpolate.PPoly method), 395  
 integrate\_box() (scipy.stats.gaussian\_kde method), 1764  
 integrate\_box\_1d() (scipy.stats.gaussian\_kde method), 1764  
 integrate\_gaussian() (scipy.stats.gaussian\_kde method), 1764  
 integrate\_kde() (scipy.stats.gaussian\_kde method), 1764  
 IntegrationWarning, 360  
 interp1d (class in scipy.interpolate), 378  
 interp2d (class in scipy.interpolate), 405  
 interp\_decomp() (in module scipy.linalg.interpolative), 689  
 interpn() (in module scipy.interpolate), 407  
 InterpolatedUnivariateSpline (class in scipy.interpolate), 416  
 interval() (scipy.stats.rv\_continuous method), 1403  
 interval() (scipy.stats.rv\_discrete method), 1412  
 inv() (in module scipy.linalg), 471  
 inv() (in module scipy.sparse.linalg), 1151, 1186  
 inv\_boxcox (in module scipy.special), 1343  
 inv\_boxcox1p (in module scipy.special), 1344  
 invgamma (in module scipy.stats), 1506  
 invgauss (in module scipy.stats), 1508  
 invhilbert() (in module scipy.linalg), 526  
 invpascal() (in module scipy.linalg), 528  
 invres() (in module scipy.signal), 926  
 invresz() (in module scipy.signal), 927  
 invweibull (in module scipy.stats), 1510  
 invwishart (in module scipy.stats), 1619  
 iqr() (in module scipy.stats), 1680  
 irfft() (in module scipy.fftpack), 338  
 is\_isomorphic() (in module scipy.cluster.hierarchy), 316  
 is\_leaf() (scipy.cluster.hierarchy.ClusterNode method), 313  
 is\_monotonic() (in module scipy.cluster.hierarchy), 316  
 is\_valid\_dm() (in module scipy.spatial.distance), 1246, 1277  
 is\_valid\_im() (in module scipy.cluster.hierarchy), 315  
 is\_valid\_linkage() (in module scipy.cluster.hierarchy), 315  
 is\_valid\_y() (in module scipy.spatial.distance), 1246, 1278  
 isamax (in module scipy.linalg.blas), 540  
 isf() (scipy.stats.rv\_continuous method), 1400  
 isf() (scipy.stats.rv\_discrete method), 1409  
 issparse() (in module scipy.sparse), 1132  
 isspmatrix() (in module scipy.sparse), 1132  
 isspmatrix\_bsr() (in module scipy.sparse), 1132  
 isspmatrix\_coo() (in module scipy.sparse), 1132  
 isspmatrix\_csc() (in module scipy.sparse), 1132  
 isspmatrix\_csr() (in module scipy.sparse), 1132  
 isspmatrix\_dia() (in module scipy.sparse), 1132  
 isspmatrix\_dok() (in module scipy.sparse), 1132  
 isspmatrix\_lil() (in module scipy.sparse), 1132  
 it2i0k0 (in module scipy.special), 1310  
 it2j0y0 (in module scipy.special), 1310  
 it2struve0 (in module scipy.special), 1320  
 itairy (in module scipy.special), 1287  
 itemfreq() (in module scipy.stats), 1664  
 items() (scipy.optimize.OptimizeResult method), 785  
 items() (scipy.sparse.dok\_matrix method), 1107  
 itemsize() (scipy.io.netcdf\_variable method), 467  
 iterate\_structure() (in module scipy.ndimage), 765  
 iteritems() (scipy.optimize.OptimizeResult method), 785  
 iteritems() (scipy.sparse.dok\_matrix method), 1107  
 iterkeys() (scipy.optimize.OptimizeResult method), 785  
 iterkeys() (scipy.sparse.dok\_matrix method), 1107  
 itervalues() (scipy.optimize.OptimizeResult method), 785  
 itervalues() (scipy.sparse.dok\_matrix method), 1107  
 iti0k0 (in module scipy.special), 1310  
 itilbert() (in module scipy.fftpack), 344  
 itj0y0 (in module scipy.special), 1310  
 itmodstruve0 (in module scipy.special), 1320  
 itstruve0 (in module scipy.special), 1319  
 iv (in module scipy.special), 1298  
 ive (in module scipy.special), 1298  
 ivp() (in module scipy.special), 1312  
 izamax (in module scipy.linalg.blas), 540
- ## J
- j0 (in module scipy.special), 1304  
 j1 (in module scipy.special), 1305  
 j\_roots() (in module scipy.special), 1369  
 jaccard() (in module scipy.spatial.distance), 1251, 1283  
 jacobi() (in module scipy.special), 1366  
 jarque\_bera() (in module scipy.stats), 1706  
 jn (in module scipy.special), 1292  
 jn\_zeros() (in module scipy.special), 1302  
 jn\_jnp\_zeros() (in module scipy.special), 1301  
 jnp\_zeros() (in module scipy.special), 1302  
 jnyn\_zeros() (in module scipy.special), 1302

johnson() (in module scipy.sparse.csgraph), 1138, 1220  
 johnsonsb (in module scipy.stats), 1512  
 johnsonsu (in module scipy.stats), 1515  
 js\_roots() (in module scipy.special), 1369  
 jv (in module scipy.special), 1291  
 jve (in module scipy.special), 1293  
 jvp() (in module scipy.special), 1311

## K

k0 (in module scipy.special), 1308  
 k0e (in module scipy.special), 1308  
 k1 (in module scipy.special), 1309  
 k1e (in module scipy.special), 1309  
 K2C() (in module scipy.constants), 331  
 K2F() (in module scipy.constants), 332  
 kaiser() (in module scipy.signal), 1025  
 kaiser\_atten() (in module scipy.signal), 920  
 kaiser\_beta() (in module scipy.signal), 921  
 kaiserord() (in module scipy.signal), 921  
 kappa3 (in module scipy.stats), 1520  
 kappa4 (in module scipy.stats), 1517  
 KDTree (class in scipy.spatial), 1230  
 kei (in module scipy.special), 1382  
 kei\_zeros() (in module scipy.special), 1383  
 keip (in module scipy.special), 1382  
 keip\_zeros() (in module scipy.special), 1383  
 kelvin (in module scipy.special), 1382  
 kelvin\_zeros() (in module scipy.special), 1382  
 kendalltau() (in module scipy.stats), 1688  
 kendalltau() (in module scipy.stats.mstats), 1738, 1771  
 kendalltau\_seasonal() (in module scipy.stats.mstats),  
 1738, 1772  
 ker (in module scipy.special), 1382  
 ker\_zeros() (in module scipy.special), 1383  
 kerp (in module scipy.special), 1382  
 kerp\_zeros() (in module scipy.special), 1383  
 keys() (scipy.optimize.OptimizeResult method), 785  
 keys() (scipy.sparse.dok\_matrix method), 1107  
 kl\_div (in module scipy.special), 1345  
 kmeans() (in module scipy.cluster.vq), 297  
 kmeans2() (in module scipy.cluster.vq), 298  
 kn (in module scipy.special), 1295  
 kolmogi (in module scipy.special), 1342  
 kolmogorov (in module scipy.special), 1342  
 krogh\_interpolate() (in module scipy.interpolate), 385  
 KroghInterpolator (class in scipy.interpolate), 381  
 kron() (in module scipy.linalg), 482  
 kron() (in module scipy.sparse), 1124  
 kronsum() (in module scipy.sparse), 1124  
 kruskal() (in module scipy.stats), 1704  
 kruskal() (in module scipy.stats.mstats), 1758, 1791  
 kruskalwallis() (in module scipy.stats.mstats), 1738, 1772  
 ks\_2samp() (in module scipy.stats), 1700  
 ks\_2samp() (in module scipy.stats.mstats), 1758, 1792

ks\_twosamp() (in module scipy.stats.mstats), 1738, 1772  
 ksone (in module scipy.stats), 1523  
 kstat() (in module scipy.stats), 1655  
 kstatvar() (in module scipy.stats), 1656  
 kstest() (in module scipy.stats), 1695  
 kstwobign (in module scipy.stats), 1525  
 kulsinski() (in module scipy.spatial.distance), 1252, 1283  
 kurtosis() (in module scipy.stats), 1652  
 kurtosis() (in module scipy.stats.mstats), 1739, 1772  
 kurtosistest() (in module scipy.stats), 1652  
 kurtosistest() (in module scipy.stats.mstats), 1739, 1773  
 kv (in module scipy.special), 1296  
 kve (in module scipy.special), 1297  
 kvp() (in module scipy.special), 1311

## L

L (scipy.sparse.linalg.SuperLU attribute), 1177, 1212  
 l\_roots() (in module scipy.special), 1370  
 la\_roots() (in module scipy.special), 1370  
 label() (in module scipy.ndimage), 735  
 labeled\_comprehension() (in module scipy.ndimage), 736  
 lagrange() (in module scipy.interpolate), 451  
 laguerre() (in module scipy.special), 1366  
 lambda2nu() (in module scipy.constants), 333  
 lambertw() (in module scipy.special), 1390  
 laplace (in module scipy.stats), 1527  
 laplace() (in module scipy.ndimage), 718  
 laplacian() (in module scipy.sparse.csgraph), 1133, 1216  
 LbfgsInvHessProduct (class in scipy.optimize), 871  
 leaders() (in module scipy.cluster.hierarchy), 301  
 leafsize (scipy.spatial.cKDTree attribute), 1235  
 least\_squares() (in module scipy.optimize), 813  
 leastsq() (in module scipy.optimize), 811  
 leaves\_list() (in module scipy.cluster.hierarchy), 313  
 legendre() (in module scipy.special), 1365  
 lena() (in module scipy.misc), 707  
 leslie() (in module scipy.linalg), 526  
 levene() (in module scipy.stats), 1708  
 levy (in module scipy.stats), 1529  
 levy\_l (in module scipy.stats), 1531  
 levy\_stable (in module scipy.stats), 1533  
 lfilter() (in module scipy.signal), 889  
 lfilter\_zi() (in module scipy.signal), 892  
 lfiltic() (in module scipy.signal), 891  
 lgmres() (in module scipy.sparse.linalg), 1160, 1195  
 lift\_points() (scipy.spatial.Delaunay method), 1259  
 lil\_matrix (class in scipy.sparse), 1111  
 LinAlgError, 483  
 line\_search() (in module scipy.optimize), 869  
 linear\_sum\_assignment() (in module scipy.optimize), 866  
 linearmixing() (in module scipy.optimize), 859  
 LinearNDInterpolator (class in scipy.interpolate), 402  
 LinearOperator (class in scipy.sparse.linalg), 1148, 1183  
 linkage() (in module scipy.cluster.hierarchy), 302

linprog() (in module `scipy.optimize`), 861  
 linprog\_verbose\_callback() (in module `scipy.optimize`), 863  
 linregress() (in module `scipy.stats`), 1689  
 linregress() (in module `scipy.stats.mstats`), 1739, 1773  
 lmbda() (in module `scipy.special`), 1301  
 loadarff() (in module `scipy.io.arff`), 469  
 loadmat() (in module `scipy.io`), 456  
 lobpcg() (in module `scipy.sparse.linalg`), 1171, 1206  
 log1p (in module `scipy.special`), 1393  
 log1p() (`scipy.sparse.bsr_matrix` method), 1062  
 log1p() (`scipy.sparse.coo_matrix` method), 1072  
 log1p() (`scipy.sparse.csc_matrix` method), 1081  
 log1p() (`scipy.sparse.csr_matrix` method), 1091  
 log1p() (`scipy.sparse.dia_matrix` method), 1100  
 log\_ndtr (in module `scipy.special`), 1341  
 logcdf() (`scipy.stats.rv_continuous` method), 1399  
 logcdf() (`scipy.stats.rv_discrete` method), 1408  
 loggamma (in module `scipy.special`), 1347  
 loggamma (in module `scipy.stats`), 1537  
 logistic (in module `scipy.stats`), 1535  
 logit (in module `scipy.special`), 1342  
 loglaplace (in module `scipy.stats`), 1539  
 logm() (in module `scipy.linalg`), 511  
 lognorm (in module `scipy.stats`), 1541  
 logpdf() (`scipy.stats.gaussian_kde` method), 1765  
 logpdf() (`scipy.stats.rv_continuous` method), 1398  
 logpmf() (`scipy.stats.rv_discrete` method), 1408  
 logser (in module `scipy.stats`), 1635  
 logsf() (`scipy.stats.rv_continuous` method), 1399  
 logsf() (`scipy.stats.rv_discrete` method), 1409  
 logsumexp() (in module `scipy.misc`), 707  
 lomax (in module `scipy.stats`), 1543  
 lombscargle() (in module `scipy.signal`), 1054  
 lp2bp() (in module `scipy.signal`), 930  
 lp2bs() (in module `scipy.signal`), 930  
 lp2hp() (in module `scipy.signal`), 930  
 lp2lp() (in module `scipy.signal`), 930  
 lpmn() (in module `scipy.special`), 1360  
 lpmv (in module `scipy.special`), 1358  
 lpn() (in module `scipy.special`), 1360  
 lqmn() (in module `scipy.special`), 1361  
 lqn() (in module `scipy.special`), 1360  
 lsim() (in module `scipy.signal`), 957  
 lsim2() (in module `scipy.signal`), 958  
 lsmr() (in module `scipy.sparse.linalg`), 1165, 1200  
 lsq\_linear() (in module `scipy.optimize`), 820  
 LSQBivariateSpline (class in `scipy.interpolate`), 445  
 lsqr() (in module `scipy.sparse.linalg`), 1163, 1198  
 LSQSphereBivariateSpline (class in `scipy.interpolate`), 447  
 LSQUnivariateSpline (class in `scipy.interpolate`), 421  
 lstsq() (in module `scipy.linalg`), 479  
 lti (class in `scipy.signal`), 946

lu() (in module `scipy.linalg`), 490  
 lu\_factor() (in module `scipy.linalg`), 491  
 lu\_solve() (in module `scipy.linalg`), 491

## M

m (`scipy.spatial.cKDTree` attribute), 1235  
 mahalanobis() (in module `scipy.spatial.distance`), 1249, 1281  
 mannwhitneyu() (in module `scipy.stats`), 1701  
 mannwhitneyu() (in module `scipy.stats.mstats`), 1740, 1774  
 map\_coordinates() (in module `scipy.ndimage`), 729  
 margins() (in module `scipy.stats.contingency`), 1724  
 matching() (in module `scipy.spatial.distance`), 1252, 1283  
 mathieu\_a (in module `scipy.special`), 1376  
 mathieu\_b (in module `scipy.special`), 1376  
 mathieu\_cem (in module `scipy.special`), 1377  
 mathieu\_even\_coef() (in module `scipy.special`), 1376  
 mathieu\_modcem1 (in module `scipy.special`), 1378  
 mathieu\_modcem2 (in module `scipy.special`), 1378  
 mathieu\_modsem1 (in module `scipy.special`), 1378  
 mathieu\_modsem2 (in module `scipy.special`), 1378  
 mathieu\_odd\_coef() (in module `scipy.special`), 1376  
 mathieu\_sem (in module `scipy.special`), 1377  
 matmat() (`scipy.optimize.LbfgsInvHessProduct` method), 872  
 matmat() (`scipy.sparse.bsr_matrix` method), 1062  
 matmat() (`scipy.sparse.linalg.LinearOperator` method), 1150, 1184  
 matrix\_normal (in module `scipy.stats`), 1615  
 MatrixRankWarning, 1155, 1190  
 matvec() (`scipy.optimize.LbfgsInvHessProduct` method), 872  
 matvec() (`scipy.sparse.bsr_matrix` method), 1062  
 matvec() (`scipy.sparse.linalg.LinearOperator` method), 1150, 1185  
 max() (`scipy.sparse.bsr_matrix` method), 1062  
 max() (`scipy.sparse.coo_matrix` method), 1072  
 max() (`scipy.sparse.csc_matrix` method), 1081  
 max() (`scipy.sparse.csr_matrix` method), 1091  
 max\_distance\_point() (`scipy.spatial.Rectangle` method), 1254  
 max\_distance\_rectangle() (`scipy.spatial.Rectangle` method), 1254  
 max\_len\_seq() (in module `scipy.signal`), 992  
 maxdists() (in module `scipy.cluster.hierarchy`), 307  
 maxes (`scipy.spatial.cKDTree` attribute), 1235  
 maximum() (in module `scipy.ndimage`), 737  
 maximum() (`scipy.sparse.bsr_matrix` method), 1062  
 maximum() (`scipy.sparse.coo_matrix` method), 1072  
 maximum() (`scipy.sparse.csc_matrix` method), 1082  
 maximum() (`scipy.sparse.csr_matrix` method), 1092  
 maximum() (`scipy.sparse.dia_matrix` method), 1100  
 maximum() (`scipy.sparse.dok_matrix` method), 1107

- maximum() (scipy.sparse.lil\_matrix method), 1114  
 maximum() (scipy.sparse.spmatrix method), 1119  
 maximum\_bipartite\_matching() (in module scipy.sparse.csgraph), 1143, 1225  
 maximum\_filter() (in module scipy.ndimage), 719  
 maximum\_filter1d() (in module scipy.ndimage), 720  
 maximum\_position() (in module scipy.ndimage), 739  
 maxinconsts() (in module scipy.cluster.hierarchy), 307  
 maxRstat() (in module scipy.cluster.hierarchy), 308  
 maxwell (in module scipy.stats), 1545  
 mean() (in module scipy.ndimage), 739  
 mean() (scipy.sparse.bsr\_matrix method), 1062  
 mean() (scipy.sparse.coo\_matrix method), 1072  
 mean() (scipy.sparse.csc\_matrix method), 1082  
 mean() (scipy.sparse.csr\_matrix method), 1092  
 mean() (scipy.sparse.dia\_matrix method), 1100  
 mean() (scipy.sparse.dok\_matrix method), 1108  
 mean() (scipy.sparse.lil\_matrix method), 1114  
 mean() (scipy.sparse.spmatrix method), 1120  
 mean() (scipy.stats.rv\_continuous method), 1402  
 mean() (scipy.stats.rv\_discrete method), 1411  
 medfilt() (in module scipy.signal), 888  
 medfilt2d() (in module scipy.signal), 888  
 median() (in module scipy.cluster.hierarchy), 305  
 median() (in module scipy.ndimage), 740  
 median() (scipy.stats.rv\_continuous method), 1402  
 median() (scipy.stats.rv\_discrete method), 1411  
 median\_cihs() (in module scipy.stats.mstats), 1758, 1792  
 median\_filter() (in module scipy.ndimage), 720  
 median\_test() (in module scipy.stats), 1712  
 meppf() (in module scipy.stats.mstats), 1759, 1792  
 MetaData (class in scipy.io.arff), 470  
 mielke (in module scipy.stats), 1547  
 min() (scipy.sparse.bsr\_matrix method), 1063  
 min() (scipy.sparse.coo\_matrix method), 1072  
 min() (scipy.sparse.csc\_matrix method), 1082  
 min() (scipy.sparse.csr\_matrix method), 1092  
 min\_distance\_point() (scipy.spatial.Rectangle method), 1255  
 min\_distance\_rectangle() (scipy.spatial.Rectangle method), 1255  
 minimize() (in module scipy.optimize), 779  
 minimize\_scalar() (in module scipy.optimize), 782  
 minimum() (in module scipy.ndimage), 741  
 minimum() (scipy.sparse.bsr\_matrix method), 1063  
 minimum() (scipy.sparse.coo\_matrix method), 1073  
 minimum() (scipy.sparse.csc\_matrix method), 1083  
 minimum() (scipy.sparse.csr\_matrix method), 1092  
 minimum() (scipy.sparse.dia\_matrix method), 1100  
 minimum() (scipy.sparse.dok\_matrix method), 1108  
 minimum() (scipy.sparse.lil\_matrix method), 1115  
 minimum() (scipy.sparse.spmatrix method), 1120  
 minimum\_filter() (in module scipy.ndimage), 721  
 minimum\_filter1d() (in module scipy.ndimage), 721  
 minimum\_position() (in module scipy.ndimage), 741  
 minimum\_spanning\_tree() (in module scipy.sparse.csgraph), 1141, 1223  
 minkowski() (in module scipy.spatial.distance), 1249, 1281  
 minkowski\_distance() (in module scipy.spatial), 1269  
 minkowski\_distance\_p() (in module scipy.spatial), 1269  
 minres() (in module scipy.sparse.linalg), 1161, 1196  
 mins (scipy.spatial.cKDTree attribute), 1235  
 mjci() (in module scipy.stats.mstats), 1759, 1793  
 mminfo() (in module scipy.io), 459  
 mmread() (in module scipy.io), 460  
 mmwrite() (in module scipy.io), 460  
 mode() (in module scipy.stats), 1653  
 mode() (in module scipy.stats.mstats), 1741, 1775  
 Model (class in scipy.odr), 771  
 modfresnelm (in module scipy.special), 1357  
 modfresnelp (in module scipy.special), 1357  
 modstruve (in module scipy.special), 1319  
 moment() (in module scipy.stats), 1654  
 moment() (in module scipy.stats.mstats), 1741, 1775  
 moment() (scipy.stats.rv\_continuous method), 1400  
 moment() (scipy.stats.rv\_discrete method), 1409  
 mood() (in module scipy.stats), 1714  
 morlet() (in module scipy.signal), 1036  
 morphological\_gradient() (in module scipy.ndimage), 765  
 morphological\_laplace() (in module scipy.ndimage), 767  
 mquantiles() (in module scipy.stats.mstats), 1742, 1775  
 mquantiles\_cimj() (in module scipy.stats.mstats), 1759, 1793  
 msign() (in module scipy.stats.mstats), 1743, 1777  
 multigammaln() (in module scipy.special), 1350  
 multilinear (in module scipy.odr), 777  
 multiply() (scipy.sparse.bsr\_matrix method), 1063  
 multiply() (scipy.sparse.coo\_matrix method), 1073  
 multiply() (scipy.sparse.csc\_matrix method), 1083  
 multiply() (scipy.sparse.csr\_matrix method), 1092  
 multiply() (scipy.sparse.dia\_matrix method), 1100  
 multiply() (scipy.sparse.dok\_matrix method), 1108  
 multiply() (scipy.sparse.lil\_matrix method), 1115  
 multiply() (scipy.sparse.spmatrix method), 1120  
 multivariate\_normal (in module scipy.stats), 1613  
 mvstdist() (in module scipy.stats), 1677
- ## N
- n (scipy.spatial.cKDTree attribute), 1236  
 nakagami (in module scipy.stats), 1550  
 names() (scipy.io.arff.MetaData method), 470  
 nbdtr (in module scipy.special), 1332  
 nbdtrc (in module scipy.special), 1333  
 nbdtri (in module scipy.special), 1333  
 nbdtrik (in module scipy.special), 1334  
 nbdtrin (in module scipy.special), 1334  
 nbinom (in module scipy.stats), 1637



ncf (in module `scipy.stats`), 1554  
 ncfdr (in module `scipy.special`), 1335  
 ncfdrtri (in module `scipy.special`), 1337  
 ncfdrtridf (in module `scipy.special`), 1337  
 ncfdrtridfn (in module `scipy.special`), 1337  
 ncfdrtrinc (in module `scipy.special`), 1337  
 nct (in module `scipy.stats`), 1556  
 nctdr (in module `scipy.special`), 1337  
 nctdrtridf (in module `scipy.special`), 1338  
 nctdrtrinc (in module `scipy.special`), 1339  
 nctdrtrit (in module `scipy.special`), 1339  
 ncx2 (in module `scipy.stats`), 1551  
 ndr (in module `scipy.special`), 1340  
 ndrtri (in module `scipy.special`), 1341  
 NearestNDInterpolator (class in `scipy.interpolate`), 403  
 NegativeCycleError, 1143, 1225  
 netcdf\_file (class in `scipy.io`), 463  
 netcdf\_variable (class in `scipy.io`), 465  
 newton() (in module `scipy.optimize`), 839  
 newton\_cotes() (in module `scipy.integrate`), 360  
 newton\_krylov() (in module `scipy.optimize`), 855  
 nmlf() (`scipy.stats.rv_continuous` method), 1404  
 nnls() (in module `scipy.optimize`), 820  
 nnz (`scipy.sparse.bsr_matrix` attribute), 1058  
 nnz (`scipy.sparse.coo_matrix` attribute), 1068  
 nnz (`scipy.sparse.csc_matrix` attribute), 1078  
 nnz (`scipy.sparse.csr_matrix` attribute), 1088  
 nnz (`scipy.sparse.dia_matrix` attribute), 1097  
 nnz (`scipy.sparse.dok_matrix` attribute), 1104  
 nnz (`scipy.sparse.lil_matrix` attribute), 1112  
 nnz (`scipy.sparse.linalg.SuperLU` attribute), 1177, 1212  
 nnz (`scipy.sparse.spmatrix` attribute), 1117  
 nonzero() (`scipy.sparse.bsr_matrix` method), 1063  
 nonzero() (`scipy.sparse.coo_matrix` method), 1073  
 nonzero() (`scipy.sparse.csc_matrix` method), 1083  
 nonzero() (`scipy.sparse.csr_matrix` method), 1092  
 nonzero() (`scipy.sparse.dia_matrix` method), 1100  
 nonzero() (`scipy.sparse.dok_matrix` method), 1108  
 nonzero() (`scipy.sparse.lil_matrix` method), 1115  
 nonzero() (`scipy.sparse.spmatrix` method), 1120  
 norm (in module `scipy.stats`), 1558  
 norm() (in module `scipy.linalg`), 477  
 norm() (in module `scipy.sparse.linalg`), 1152, 1187  
 normalize() (in module `scipy.signal`), 930  
 normaltest() (in module `scipy.stats`), 1654  
 normaltest() (in module `scipy.stats.mstats`), 1743, 1777  
 nquad() (in module `scipy.integrate`), 355  
 nrdrtrimn (in module `scipy.special`), 1339  
 nrdrtrisd (in module `scipy.special`), 1339  
 nu2lambda() (in module `scipy.constants`), 334  
 num (`scipy.signal.dlti` attribute), 966  
 num (`scipy.signal.lti` attribute), 948  
 num (`scipy.signal.StateSpace` attribute), 952, 970  
 num (`scipy.signal.TransferFunction` attribute), 954, 972

num (`scipy.signal.ZerosPolesGain` attribute), 956, 974  
 num\_obs\_dm() (in module `scipy.spatial.distance`), 1247, 1278  
 num\_obs\_linkage() (in module `scipy.cluster.hierarchy`), 316  
 num\_obs\_y() (in module `scipy.spatial.distance`), 1247, 1278  
 nuttall() (in module `scipy.signal`), 1027

## O

obl\_ang1 (in module `scipy.special`), 1379  
 obl\_ang1\_cv (in module `scipy.special`), 1381  
 obl\_cv (in module `scipy.special`), 1380  
 obl\_cv\_seq() (in module `scipy.special`), 1380  
 obl\_rad1 (in module `scipy.special`), 1379  
 obl\_rad1\_cv (in module `scipy.special`), 1381  
 obl\_rad2 (in module `scipy.special`), 1380  
 obl\_rad2\_cv (in module `scipy.special`), 1381  
 obrienttransform() (in module `scipy.stats`), 1675  
 obrienttransform() (in module `scipy.stats.mstats`), 1743, 1777  
 ode (class in `scipy.integrate`), 368  
 odeint() (in module `scipy.integrate`), 365  
 ODR (class in `scipy.odr`), 772  
 odr() (in module `scipy.odr`), 776  
 odr\_error (in module `scipy.odr`), 777  
 odr\_stop (in module `scipy.odr`), 777  
 OdrError, 776  
 OdrStop, 776  
 OdrWarning, 776  
 onenormest() (in module `scipy.sparse.linalg`), 1154, 1188  
 OptimizeResult (class in `scipy.optimize`), 784  
 OptimizeWarning, 786  
 order\_filter() (in module `scipy.signal`), 887  
 ordqz() (in module `scipy.linalg`), 507  
 orth() (in module `scipy.linalg`), 494  
 ortho\_group (in module `scipy.stats`), 1622  
 orthogonal\_procrustes() (in module `scipy.linalg`), 483  
 Output (class in `scipy.odr`), 775  
 output() (`scipy.signal.dlti` method), 968  
 output() (`scipy.signal.lti` method), 949

## P

p\_roots() (in module `scipy.special`), 1370  
 pade() (in module `scipy.misc`), 708  
 pareto (in module `scipy.stats`), 1560  
 ParseArffError, 470  
 parzen() (in module `scipy.signal`), 1029  
 pascal() (in module `scipy.linalg`), 527  
 pbdn\_seq() (in module `scipy.special`), 1375  
 pbdv (in module `scipy.special`), 1374  
 pbdv\_seq() (in module `scipy.special`), 1375  
 pbvv (in module `scipy.special`), 1374  
 pbvv\_seq() (in module `scipy.special`), 1375

- pbwa (in module `scipy.special`), 1375  
 pchip (in module `scipy.interpolate`), 456  
 pchip\_interpolate() (in module `scipy.interpolate`), 385  
 PchipInterpolator (class in `scipy.interpolate`), 383  
 pdf() (`scipy.stats.gaussian_kde` method), 1764  
 pdf() (`scipy.stats.rv_continuous` method), 1398  
 pdist() (in module `scipy.spatial.distance`), 1239, 1270  
 pdtr (in module `scipy.special`), 1339  
 pdtrc (in module `scipy.special`), 1340  
 pdtri (in module `scipy.special`), 1340  
 pdtrik (in module `scipy.special`), 1340  
 pearson3 (in module `scipy.stats`), 1562  
 pearsonr() (in module `scipy.stats`), 1685  
 pearsonr() (in module `scipy.stats.mstats`), 1743, 1777  
 percentile\_filter() (in module `scipy.ndimage`), 722  
 percentileofscore() (in module `scipy.stats`), 1664  
 periodogram() (in module `scipy.signal`), 1042  
 perm() (in module `scipy.special`), 1384  
 perm\_c (`scipy.sparse.linalg.SuperLU` attribute), 1177, 1212  
 perm\_r (`scipy.sparse.linalg.SuperLU` attribute), 1177, 1212  
 physical\_constants (in module `scipy.constants`), 320  
 pinv() (in module `scipy.linalg`), 480  
 pinv2() (in module `scipy.linalg`), 480  
 pinvh() (in module `scipy.linalg`), 481  
 place\_poles() (in module `scipy.signal`), 986  
 planck (in module `scipy.stats`), 1639  
 plane\_distance() (`scipy.spatial.Delaunay` method), 1259  
 plotting\_positions() (in module `scipy.stats.mstats`), 1740, 1744, 1774, 1778  
 pmf() (`scipy.stats.rv_discrete` method), 1408  
 poch (in module `scipy.special`), 1351  
 pointbiseriialr() (in module `scipy.stats`), 1687  
 pointbiseriialr() (in module `scipy.stats.mstats`), 1744, 1778  
 poisson (in module `scipy.stats`), 1641  
 polar() (in module `scipy.linalg`), 496  
 poles (`scipy.signal.dlti` attribute), 966  
 poles (`scipy.signal.lti` attribute), 948  
 poles (`scipy.signal.StateSpace` attribute), 952, 970  
 poles (`scipy.signal.TransferFunction` attribute), 954, 972  
 poles (`scipy.signal.ZerosPolesGain` attribute), 956, 974  
 polygamma() (in module `scipy.special`), 1349  
 polynomial (in module `scipy.odr`), 777  
 polynomial() (in module `scipy.odr`), 777  
 pop() (`scipy.optimize.OptimizeResult` method), 785  
 pop() (`scipy.sparse.dok_matrix` method), 1108  
 popitem() (`scipy.optimize.OptimizeResult` method), 785  
 popitem() (`scipy.sparse.dok_matrix` method), 1108  
 power() (`scipy.sparse.bsr_matrix` method), 1064  
 power() (`scipy.sparse.coo_matrix` method), 1073  
 power() (`scipy.sparse.csc_matrix` method), 1083  
 power() (`scipy.sparse.csr_matrix` method), 1093  
 power() (`scipy.sparse.dia_matrix` method), 1100  
 power() (`scipy.sparse.dok_matrix` method), 1108  
 power() (`scipy.sparse.lil_matrix` method), 1115  
 power() (`scipy.sparse.spmatrix` method), 1120  
 power\_divergence() (in module `scipy.stats`), 1698  
 powerlaw (in module `scipy.stats`), 1565  
 powerlognorm (in module `scipy.stats`), 1567  
 powernorm (in module `scipy.stats`), 1569  
 ppcc\_max() (in module `scipy.stats`), 1725  
 ppcc\_plot() (in module `scipy.stats`), 1727  
 ppf() (`scipy.stats.rv_continuous` method), 1400  
 ppf() (`scipy.stats.rv_discrete` method), 1409  
 ppform (class in `scipy.interpolate`), 452  
 PPoly (class in `scipy.interpolate`), 393  
 pprint() (`scipy.odr.Output` method), 776  
 pre\_order() (`scipy.cluster.hierarchy.ClusterNode` method), 313  
 precision() (in module `scipy.constants`), 319  
 prewitt() (in module `scipy.ndimage`), 722  
 pro\_ang1 (in module `scipy.special`), 1379  
 pro\_ang1\_cv (in module `scipy.special`), 1380  
 pro\_cv (in module `scipy.special`), 1380  
 pro\_cv\_seq() (in module `scipy.special`), 1380  
 pro\_rad1 (in module `scipy.special`), 1379  
 pro\_rad1\_cv (in module `scipy.special`), 1381  
 pro\_rad2 (in module `scipy.special`), 1379  
 pro\_rad2\_cv (in module `scipy.special`), 1381  
 probplot() (in module `scipy.stats`), 1728  
 procrustes() (in module `scipy.spatial`), 1269  
 prune() (`scipy.sparse.bsr_matrix` method), 1064  
 prune() (`scipy.sparse.csc_matrix` method), 1083  
 prune() (`scipy.sparse.csr_matrix` method), 1093  
 ps\_roots() (in module `scipy.special`), 1371  
 pseudo\_huber (in module `scipy.special`), 1346  
 psi (in module `scipy.special`), 1349
- ## Q
- qmf() (in module `scipy.signal`), 1036  
 qmr() (in module `scipy.sparse.linalg`), 1162, 1197  
 qr() (in module `scipy.linalg`), 498  
 qr\_delete() (in module `scipy.linalg`), 502  
 qr\_insert() (in module `scipy.linalg`), 503  
 qr\_multiply() (in module `scipy.linalg`), 499  
 qr\_update() (in module `scipy.linalg`), 500  
 qspline1d() (in module `scipy.signal`), 885  
 qspline1d\_eval() (in module `scipy.signal`), 886  
 qspline2d() (in module `scipy.signal`), 886  
 quad() (in module `scipy.integrate`), 350  
 quad\_explain() (in module `scipy.integrate`), 360  
 quadratic (in module `scipy.odr`), 777  
 quadratic() (in module `scipy.signal`), 885  
 quadrature() (in module `scipy.integrate`), 358  
 query() (`scipy.spatial.cKDTree` method), 1236  
 query() (`scipy.spatial.KDTree` method), 1231  
 query\_ball\_point() (`scipy.spatial.cKDTree` method), 1237

query\_ball\_point() (scipy.spatial.KDTree method), 1232  
 query\_ball\_tree() (scipy.spatial.cKDTree method), 1238  
 query\_ball\_tree() (scipy.spatial.KDTree method), 1233  
 query\_pairs() (scipy.spatial.cKDTree method), 1238  
 query\_pairs() (scipy.spatial.KDTree method), 1234  
 qz() (in module scipy.linalg), 506

## R

rad2deg() (scipy.sparse.bsr\_matrix method), 1064  
 rad2deg() (scipy.sparse.coo\_matrix method), 1073  
 rad2deg() (scipy.sparse.csc\_matrix method), 1083  
 rad2deg() (scipy.sparse.csr\_matrix method), 1093  
 rad2deg() (scipy.sparse.dia\_matrix method), 1101  
 radian (in module scipy.special), 1393  
 rand() (in module scipy.linalg.interpolative), 693  
 rand() (in module scipy.sparse), 1130  
 randint (in module scipy.stats), 1643  
 random() (in module scipy.sparse), 1130  
 random\_correlation (in module scipy.stats), 1622  
 random\_state (scipy.stats.rv\_continuous attribute), 1397  
 random\_state (scipy.stats.rv\_discrete attribute), 1407  
 rank\_filter() (in module scipy.ndimage), 723  
 rankdata() (in module scipy.stats), 1702  
 rankdata() (in module scipy.stats.mstats), 1745, 1778  
 ranksums() (in module scipy.stats), 1703  
 rayleigh (in module scipy.stats), 1575  
 Rbf (class in scipy.interpolate), 404  
 rdist (in module scipy.stats), 1571  
 read() (in module scipy.io.wavfile), 468  
 read\_ints() (scipy.io.FortranFile method), 461  
 read\_reals() (scipy.io.FortranFile method), 461  
 read\_record() (scipy.io.FortranFile method), 462  
 readsav() (in module scipy.io), 459  
 RealData (class in scipy.odr), 770  
 recipinvgauss (in module scipy.stats), 1579  
 reciprocal (in module scipy.stats), 1573  
 reconstruct\_interp\_matrix() (in module scipy.linalg.interpolative), 690  
 reconstruct\_matrix\_from\_id() (in module scipy.linalg.interpolative), 690  
 reconstruct\_path() (in module scipy.sparse.csgraph), 1146, 1228  
 reconstruct\_skel\_matrix() (in module scipy.linalg.interpolative), 691  
 Rectangle (class in scipy.spatial), 1254  
 RectBivariateSpline (class in scipy.interpolate), 409, 433  
 RectSphereBivariateSpline (class in scipy.interpolate), 435  
 RegularGridInterpolator (class in scipy.interpolate), 408  
 rel ENTR (in module scipy.special), 1345  
 relfreq() (in module scipy.stats), 1666  
 remez() (in module scipy.signal), 922  
 resample() (in module scipy.signal), 904  
 resample() (scipy.stats.gaussian\_kde method), 1765  
 resample\_poly() (in module scipy.signal), 905  
 reshape() (scipy.sparse.bsr\_matrix method), 1064  
 reshape() (scipy.sparse.coo\_matrix method), 1073  
 reshape() (scipy.sparse.csc\_matrix method), 1083  
 reshape() (scipy.sparse.csr\_matrix method), 1093  
 reshape() (scipy.sparse.dia\_matrix method), 1101  
 reshape() (scipy.sparse.dok\_matrix method), 1108  
 reshape() (scipy.sparse.lil\_matrix method), 1115  
 reshape() (scipy.sparse.spmatrix method), 1120  
 residue() (in module scipy.signal), 924  
 residuez() (in module scipy.signal), 925  
 resize() (scipy.sparse.dok\_matrix method), 1109  
 restart() (scipy.odr.ODR method), 774  
 return\_integers (scipy.stats.rv\_discrete attribute), 1407  
 reverse\_cuthill\_mckee() (in module scipy.sparse.csgraph), 1142, 1224  
 rfft() (in module scipy.fftpack), 337  
 rfftfreq() (in module scipy.fftpack), 348  
 rgamma (in module scipy.special), 1349  
 riccati\_jn() (in module scipy.special), 1317  
 riccati\_yn() (in module scipy.special), 1318  
 rice (in module scipy.stats), 1577  
 ricker() (in module scipy.signal), 1036  
 ridder() (in module scipy.optimize), 837  
 rint() (scipy.sparse.bsr\_matrix method), 1064  
 rint() (scipy.sparse.coo\_matrix method), 1073  
 rint() (scipy.sparse.csc\_matrix method), 1083  
 rint() (scipy.sparse.csr\_matrix method), 1093  
 rint() (scipy.sparse.dia\_matrix method), 1101  
 rmatvec() (scipy.optimize.LbfgsInvHessProduct method), 873  
 rmatvec() (scipy.sparse.linalg.LinearOperator method), 1150, 1185  
 rogerstanimoto() (in module scipy.spatial.distance), 1252, 1283  
 romb() (in module scipy.integrate), 364  
 romberg() (in module scipy.integrate), 358  
 root() (in module scipy.optimize), 841  
 roots() (scipy.interpolate.Akima1DInterpolator method), 387  
 roots() (scipy.interpolate.CubicSpline method), 393  
 roots() (scipy.interpolate.InterpolatedUnivariateSpline method), 421  
 roots() (scipy.interpolate.LSQUnivariateSpline method), 425  
 roots() (scipy.interpolate.PchipInterpolator method), 384  
 roots() (scipy.interpolate.ppform method), 454  
 roots() (scipy.interpolate.PPoly method), 396  
 roots() (scipy.interpolate.UnivariateSpline method), 416  
 rosen() (in module scipy.optimize), 832  
 rosen\_der() (in module scipy.optimize), 832  
 rosen\_hess() (in module scipy.optimize), 832  
 rosen\_hess\_prod() (in module scipy.optimize), 832  
 rotate() (in module scipy.ndimage), 730

round (in module `scipy.special`), 1393  
 rq() (in module `scipy.linalg`), 505  
 rsf2csf() (in module `scipy.linalg`), 509  
 rsh() (in module `scipy.stats.mstats`), 1760, 1793  
 run() (`scipy.odr.ODR` method), 774  
 russellrao() (in module `scipy.spatial.distance`), 1252, 1284  
 rv\_continuous (class in `scipy.stats`), 1395  
 rv\_discrete (class in `scipy.stats`), 1405  
 rvs() (`scipy.stats.rv_continuous` method), 1398  
 rvs() (`scipy.stats.rv_discrete` method), 1407

## S

s\_roots() (in module `scipy.special`), 1371  
 sasum (in module `scipy.linalg.blas`), 541  
 savemat() (in module `scipy.io`), 457  
 savgol\_coeffs() (in module `scipy.signal`), 921  
 savgol\_filter() (in module `scipy.signal`), 895  
 sawtooth() (in module `scipy.signal`), 994  
 saxpy (in module `scipy.linalg.blas`), 541  
 sc\_diff() (in module `scipy.fftpack`), 345  
 scasum (in module `scipy.linalg.blas`), 541  
 schur() (in module `scipy.linalg`), 508  
`scipy.cluster` (module), 295  
`scipy.cluster.hierarchy` (module), 299  
`scipy.cluster.vq` (module), 295  
`scipy.constants` (module), 317  
`scipy.fftpack` (module), 334  
`scipy.fftpack.convolve` (module), 348  
`scipy.integrate` (module), 349  
`scipy.interpolate` (module), 377  
`scipy.io` (module), 456  
`scipy.io.arff` (module), 244, 469  
`scipy.io.netcdf` (module), 244  
`scipy.io.wavfile` (module), 244, 468  
`scipy.linalg` (module), 471  
`scipy.linalg.blas` (module), 531  
`scipy.linalg.cython_blas` (module), 648  
`scipy.linalg.cython_lapack` (module), 652  
`scipy.linalg.interpolative` (module), 688  
`scipy.linalg.lapack` (module), 566  
`scipy.misc` (module), 697  
`scipy.ndimage` (module), 710  
`scipy.odr` (module), 769  
`scipy.optimize` (module), 778  
`scipy.optimize.nonlin` (module), 873  
`scipy.signal` (module), 875  
`scipy.sparse` (module), 1056  
`scipy.sparse.csgraph` (module), 1132, 1215  
`scipy.sparse.linalg` (module), 1148, 1183  
`scipy.spatial` (module), 1229  
`scipy.spatial.distance` (module), 1239, 1270  
`scipy.special` (module), 1285  
`scipy.stats` (module), 1395  
`scipy.stats.mstats` (module), 1732, 1766

`scipy.weave` (module), 1795  
`scipy.weave.ext_tools` (module), 1797  
 scnrm2 (in module `scipy.linalg.blas`), 542  
`scipy` (in module `scipy.linalg.blas`), 542  
 scoreatpercentile() (in module `scipy.stats`), 1665  
 scoreatpercentile() (in module `scipy.stats.mstats`), 1745, 1779  
 sdot (in module `scipy.linalg.blas`), 542  
 seed() (in module `scipy.linalg.interpolative`), 693  
 sem() (in module `scipy.stats`), 1678  
 sem() (in module `scipy.stats.mstats`), 1745, 1779  
 semicircular (in module `scipy.stats`), 1581  
 sen\_seasonal\_slopes() (in module `scipy.stats.mstats`), 1760, 1793  
 sepfir2d() (in module `scipy.signal`), 885  
 set\_bandwidth() (`scipy.stats.gaussian_kde` method), 1765  
 set\_f\_params() (`scipy.integrate.complex_ode` method), 372  
 set\_f\_params() (`scipy.integrate.ode` method), 371  
 set\_initial\_value() (`scipy.integrate.complex_ode` method), 372  
 set\_initial\_value() (`scipy.integrate.ode` method), 371  
 set\_integrator() (`scipy.integrate.complex_ode` method), 372  
 set\_integrator() (`scipy.integrate.ode` method), 371  
 set\_iprint() (`scipy.odr.ODR` method), 774  
 set\_jac\_params() (`scipy.integrate.complex_ode` method), 372  
 set\_jac\_params() (`scipy.integrate.ode` method), 371  
 set\_job() (`scipy.odr.ODR` method), 775  
 set\_link\_color\_palette() (in module `scipy.cluster.hierarchy`), 316  
 set\_meta() (`scipy.odr.Data` method), 770  
 set\_meta() (`scipy.odr.Model` method), 772  
 set\_meta() (`scipy.odr.RealData` method), 771  
 set\_shape() (`scipy.sparse.bsr_matrix` method), 1064  
 set\_shape() (`scipy.sparse.coo_matrix` method), 1074  
 set\_shape() (`scipy.sparse.csc_matrix` method), 1083  
 set\_shape() (`scipy.sparse.csr_matrix` method), 1093  
 set\_shape() (`scipy.sparse.dia_matrix` method), 1101  
 set\_shape() (`scipy.sparse.dok_matrix` method), 1109  
 set\_shape() (`scipy.sparse.lil_matrix` method), 1115  
 set\_shape() (`scipy.sparse.spmatrix` method), 1120  
 set\_smoothing\_factor() (`scipy.interpolate.InterpolatedUnivariateSpline` method), 421  
 set\_smoothing\_factor() (`scipy.interpolate.LSQUnivariateSpline` method), 426  
 set\_smoothing\_factor() (`scipy.interpolate.UnivariateSpline` method), 416  
 set\_solout() (`scipy.integrate.complex_ode` method), 372  
 set\_solout() (`scipy.integrate.ode` method), 371  
 set\_yi() (`scipy.interpolate.BarycentricInterpolator` method), 381  
 setdefault() (`scipy.optimize.OptimizeResult` method), 785



- setdefault() (scipy.sparse.dok\_matrix method), 1109  
 setdiag() (scipy.sparse.bsr\_matrix method), 1064  
 setdiag() (scipy.sparse.coo\_matrix method), 1074  
 setdiag() (scipy.sparse.csc\_matrix method), 1083  
 setdiag() (scipy.sparse.csr\_matrix method), 1093  
 setdiag() (scipy.sparse.dia\_matrix method), 1101  
 setdiag() (scipy.sparse.dok\_matrix method), 1109  
 setdiag() (scipy.sparse.lil\_matrix method), 1115  
 setdiag() (scipy.sparse.spmatrix method), 1121  
 seuclidean() (in module scipy.spatial.distance), 1250, 1281  
 sf() (scipy.stats.rv\_continuous method), 1399  
 sf() (scipy.stats.rv\_discrete method), 1408  
 sgbsv (in module scipy.linalg.lapack), 572  
 sgbtrf (in module scipy.linalg.lapack), 573  
 sgbtrs (in module scipy.linalg.lapack), 574  
 sgebal (in module scipy.linalg.lapack), 576  
 sgees (in module scipy.linalg.lapack), 577  
 sgeev (in module scipy.linalg.lapack), 579  
 sgeev\_lwork (in module scipy.linalg.lapack), 581  
 sgegv() (in module scipy.linalg.lapack), 582  
 sgehrd (in module scipy.linalg.lapack), 583  
 sgehrd\_lwork (in module scipy.linalg.lapack), 585  
 sgelsd (in module scipy.linalg.lapack), 588  
 sgelsd\_lwork (in module scipy.linalg.lapack), 589  
 sgelss (in module scipy.linalg.lapack), 585  
 sgelss\_lwork (in module scipy.linalg.lapack), 587  
 sgelsy (in module scipy.linalg.lapack), 590  
 sgelsy\_lwork (in module scipy.linalg.lapack), 592  
 sgemm (in module scipy.linalg.blas), 563  
 sgemv (in module scipy.linalg.blas), 553  
 sgeqp3 (in module scipy.linalg.lapack), 593  
 sgeqrf (in module scipy.linalg.lapack), 594  
 sger (in module scipy.linalg.blas), 554  
 sgerqf (in module scipy.linalg.lapack), 595  
 sgesdd (in module scipy.linalg.lapack), 596  
 sgesdd\_lwork (in module scipy.linalg.lapack), 597  
 sgesv (in module scipy.linalg.lapack), 600  
 sgesvd (in module scipy.linalg.lapack), 598  
 sgesvd\_lwork (in module scipy.linalg.lapack), 599  
 sgetrf (in module scipy.linalg.lapack), 601  
 sgetri (in module scipy.linalg.lapack), 602  
 sgetri\_lwork (in module scipy.linalg.lapack), 603  
 sgetrs (in module scipy.linalg.lapack), 603  
 sgges (in module scipy.linalg.lapack), 604  
 sggev (in module scipy.linalg.lapack), 608  
 sgtsv (in module scipy.linalg.lapack), 635  
 sh\_chebyt() (in module scipy.special), 1366  
 sh\_chebyu() (in module scipy.special), 1366  
 sh\_jacobi() (in module scipy.special), 1366  
 sh\_legendre() (in module scipy.special), 1366  
 shape (scipy.sparse.linalg.SuperLU attribute), 1177, 1212  
 shape (scipy.sparse.spmatrix attribute), 1117  
 shapiro() (in module scipy.stats), 1708  
 shichi (in module scipy.special), 1390  
 shift() (in module scipy.fftpack), 346  
 shift() (in module scipy.ndimage), 730  
 shortest\_path() (in module scipy.sparse.csgraph), 1134, 1216  
 show\_options() (in module scipy.optimize), 870  
 sici (in module scipy.special), 1390  
 sigmaclip() (in module scipy.stats), 1682  
 sign() (scipy.sparse.bsr\_matrix method), 1064  
 sign() (scipy.sparse.coo\_matrix method), 1074  
 sign() (scipy.sparse.csc\_matrix method), 1084  
 sign() (scipy.sparse.csr\_matrix method), 1093  
 sign() (scipy.sparse.dia\_matrix method), 1101  
 signaltonoise() (in module scipy.stats), 1675  
 signaltonoise() (in module scipy.stats.mstats), 1746, 1779  
 signm() (in module scipy.linalg), 514  
 simps() (in module scipy.integrate), 363  
 sin() (scipy.sparse.bsr\_matrix method), 1064  
 sin() (scipy.sparse.coo\_matrix method), 1074  
 sin() (scipy.sparse.csc\_matrix method), 1084  
 sin() (scipy.sparse.csr\_matrix method), 1094  
 sin() (scipy.sparse.dia\_matrix method), 1101  
 sinc() (in module scipy.special), 1394  
 sindg (in module scipy.special), 1393  
 single() (in module scipy.cluster.hierarchy), 304  
 sinh() (scipy.sparse.bsr\_matrix method), 1064  
 sinh() (scipy.sparse.coo\_matrix method), 1074  
 sinh() (scipy.sparse.csc\_matrix method), 1084  
 sinh() (scipy.sparse.csr\_matrix method), 1094  
 sinh() (scipy.sparse.dia\_matrix method), 1101  
 sinhm() (in module scipy.linalg), 514  
 sinm() (in module scipy.linalg), 512  
 skellam (in module scipy.stats), 1645  
 skew() (in module scipy.stats), 1655  
 skew() (in module scipy.stats.mstats), 1746, 1780  
 skewnorm (in module scipy.stats), 1583  
 skewtest() (in module scipy.stats), 1655  
 skewtest() (in module scipy.stats.mstats), 1746, 1780  
 slamch (in module scipy.linalg.lapack), 638  
 slange (in module scipy.linalg.lapack), 647  
 slarf (in module scipy.linalg.lapack), 616  
 slarfg (in module scipy.linalg.lapack), 617  
 slartg (in module scipy.linalg.lapack), 618  
 slasd4 (in module scipy.linalg.lapack), 619  
 slaswp (in module scipy.linalg.lapack), 619  
 slauum (in module scipy.linalg.lapack), 620  
 slepian() (in module scipy.signal), 1030  
 smirnov (in module scipy.special), 1341  
 smirnovi (in module scipy.special), 1341  
 SmoothBivariateSpline (class in scipy.interpolate), 441  
 SmoothSphereBivariateSpline (class in scipy.interpolate), 442  
 snrm2 (in module scipy.linalg.blas), 542  
 sobel() (in module scipy.ndimage), 723

- sokalmichener() (in module `scipy.spatial.distance`), 1252, 1284
- sokalsneath() (in module `scipy.spatial.distance`), 1253, 1284
- solve() (in module `scipy.linalg`), 472
- solve() (`scipy.interpolate.ppform` method), 454
- solve() (`scipy.interpolate.PPoly` method), 395
- solve() (`scipy.sparse.linalg.SuperLU` method), 1177, 1212
- solve\_banded() (in module `scipy.linalg`), 472
- solve\_bvp() (in module `scipy.integrate`), 372
- solve\_circulant() (in module `scipy.linalg`), 474
- solve\_continuous\_are() (in module `scipy.linalg`), 519
- solve\_discrete\_are() (in module `scipy.linalg`), 519
- solve\_discrete\_lyapunov() (in module `scipy.linalg`), 520
- solve\_lyapunov() (in module `scipy.linalg`), 521
- solve\_sylvester() (in module `scipy.linalg`), 518
- solve\_toeplitz() (in module `scipy.linalg`), 476
- solve\_triangular() (in module `scipy.linalg`), 476
- solveh\_banded() (in module `scipy.linalg`), 473
- sorghr (in module `scipy.linalg.lapack`), 638
- sorgqr (in module `scipy.linalg.lapack`), 639
- sorgrq (in module `scipy.linalg.lapack`), 639
- sormqr (in module `scipy.linalg.lapack`), 640
- sort\_indices() (`scipy.sparse.bsr_matrix` method), 1065
- sort\_indices() (`scipy.sparse.csc_matrix` method), 1084
- sort\_indices() (`scipy.sparse.csr_matrix` method), 1094
- sort\_vertices\_of\_regions()  
(`scipy.spatial.SphericalVoronoi` method), 1266
- sorted\_indices() (`scipy.sparse.bsr_matrix` method), 1065
- sorted\_indices() (`scipy.sparse.csc_matrix` method), 1084
- sorted\_indices() (`scipy.sparse.csr_matrix` method), 1094
- sos2tf() (in module `scipy.signal`), 985
- sos2zpk() (in module `scipy.signal`), 985
- sosfilt() (in module `scipy.signal`), 897
- sosfilt\_zi() (in module `scipy.signal`), 899
- sosfiltfilt() (in module `scipy.signal`), 900
- source() (in module `scipy.misc`), 709
- spalde() (in module `scipy.interpolate`), 431
- sparse\_distance\_matrix() (`scipy.spatial.cKDTree` method), 1239
- sparse\_distance\_matrix() (`scipy.spatial.KDTree` method), 1234
- SparseEfficiencyWarning, 1180
- SparseWarning, 1180
- spbsv (in module `scipy.linalg.lapack`), 621
- spbtrf (in module `scipy.linalg.lapack`), 622
- spbtrs (in module `scipy.linalg.lapack`), 623
- spdiags() (in module `scipy.sparse`), 1126
- spearmnr() (in module `scipy.stats`), 1685
- spearmnr() (in module `scipy.stats.mstats`), 1747, 1780
- special\_ortho\_group (in module `scipy.stats`), 1621
- SpecialFunctionWarning, 1285
- spectrogram() (in module `scipy.signal`), 1052
- spence (in module `scipy.special`), 1390
- sph\_harm (in module `scipy.special`), 1358
- sph\_in() (in module `scipy.special`), 1316
- sph\_inkn() (in module `scipy.special`), 1317
- sph\_jn() (in module `scipy.special`), 1315
- sph\_jnyn() (in module `scipy.special`), 1316
- sph\_kn() (in module `scipy.special`), 1316
- sph\_yn() (in module `scipy.special`), 1316
- spherical\_in() (in module `scipy.special`), 1314
- spherical\_jn() (in module `scipy.special`), 1313
- spherical\_kn() (in module `scipy.special`), 1315
- spherical\_yn() (in module `scipy.special`), 1313
- SphericalVoronoi (class in `scipy.spatial`), 1264
- spilu() (in module `scipy.sparse.linalg`), 1175, 1210
- splantider() (in module `scipy.interpolate`), 432
- splder() (in module `scipy.interpolate`), 431
- splev() (in module `scipy.interpolate`), 429
- spleval() (in module `scipy.interpolate`), 455
- spline() (in module `scipy.interpolate`), 455
- spline\_filter() (in module `scipy.ndimage`), 731
- spline\_filter() (in module `scipy.signal`), 886
- spline\_filter1d() (in module `scipy.ndimage`), 731
- splint() (in module `scipy.interpolate`), 430
- split() (`scipy.spatial.Rectangle` method), 1255
- splmake() (in module `scipy.interpolate`), 456
- splprep() (in module `scipy.interpolate`), 428
- splrep() (in module `scipy.interpolate`), 426
- spltopp() (in module `scipy.interpolate`), 456
- spilu() (in module `scipy.sparse.linalg`), 1174, 1209
- spmatrix (class in `scipy.sparse`), 1117
- sposv (in module `scipy.linalg.lapack`), 624
- spotrf (in module `scipy.linalg.lapack`), 626
- spotri (in module `scipy.linalg.lapack`), 627
- spotrs (in module `scipy.linalg.lapack`), 627
- sproot() (in module `scipy.interpolate`), 430
- spsolve() (in module `scipy.sparse.linalg`), 1154, 1189
- sptsv (in module `scipy.linalg.lapack`), 637
- squeclidean() (in module `scipy.spatial.distance`), 1250, 1281
- sqrt() (`scipy.sparse.bsr_matrix` method), 1065
- sqrt() (`scipy.sparse.coo_matrix` method), 1074
- sqrt() (`scipy.sparse.csc_matrix` method), 1084
- sqrt() (`scipy.sparse.csr_matrix` method), 1094
- sqrt() (`scipy.sparse.dia_matrix` method), 1101
- sqrtm() (in module `scipy.linalg`), 515
- square() (in module `scipy.signal`), 995
- square\_of\_sums() (in module `scipy.stats`), 1706
- squareform() (in module `scipy.spatial.distance`), 1245, 1277
- srot (in module `scipy.linalg.blas`), 543
- srotg (in module `scipy.linalg.blas`), 543
- srotm (in module `scipy.linalg.blas`), 543
- srotmg (in module `scipy.linalg.blas`), 544
- ss() (in module `scipy.stats`), 1706

- ss2tf() (in module `scipy.signal`), 984  
 ss2zpk() (in module `scipy.signal`), 985  
 ss\_diff() (in module `scipy.fftpack`), 345  
 ssbev (in module `scipy.linalg.lapack`), 640  
 ssbevd (in module `scipy.linalg.lapack`), 641  
 ssbevz (in module `scipy.linalg.lapack`), 642  
 sscal (in module `scipy.linalg.blas`), 544  
 sswap (in module `scipy.linalg.blas`), 544  
 ssyev (in module `scipy.linalg.lapack`), 643  
 ssyevd (in module `scipy.linalg.lapack`), 643  
 ssyevr (in module `scipy.linalg.lapack`), 644  
 ssygv (in module `scipy.linalg.lapack`), 645  
 ssygvd (in module `scipy.linalg.lapack`), 645  
 ssygvz (in module `scipy.linalg.lapack`), 646  
 ssymm (in module `scipy.linalg.blas`), 563  
 ssmv (in module `scipy.linalg.blas`), 554  
 ssyr (in module `scipy.linalg.blas`), 555  
 ssyr2 (in module `scipy.linalg.blas`), 555  
 ssyr2k (in module `scipy.linalg.blas`), 564  
 ssyrk (in module `scipy.linalg.blas`), 564  
 standard\_deviation() (in module `scipy.ndimage`), 742  
 StateSpace (class in `scipy.signal`), 950, 968  
 stats() (`scipy.stats.rv_continuous` method), 1400  
 stats() (`scipy.stats.rv_discrete` method), 1410  
 std() (`scipy.stats.rv_continuous` method), 1402  
 std() (`scipy.stats.rv_discrete` method), 1411  
 stdtr (in module `scipy.special`), 1340  
 stdtridf (in module `scipy.special`), 1340  
 stdtrit (in module `scipy.special`), 1340  
 step() (in module `scipy.signal`), 960  
 step() (`scipy.signal.dlti` method), 968  
 step() (`scipy.signal.lti` method), 950  
 step2() (in module `scipy.signal`), 961  
 strmv (in module `scipy.linalg.blas`), 555  
 strsyl (in module `scipy.linalg.lapack`), 629  
 strtri (in module `scipy.linalg.lapack`), 631  
 strtrs (in module `scipy.linalg.lapack`), 632  
 struve (in module `scipy.special`), 1318  
 successful() (`scipy.integrate.complex_ode` method), 372  
 successful() (`scipy.integrate.ode` method), 371  
 sum() (in module `scipy.ndimage`), 742  
 sum() (`scipy.sparse.bsr_matrix` method), 1065  
 sum() (`scipy.sparse.coo_matrix` method), 1074  
 sum() (`scipy.sparse.csc_matrix` method), 1084  
 sum() (`scipy.sparse.csr_matrix` method), 1094  
 sum() (`scipy.sparse.dia_matrix` method), 1102  
 sum() (`scipy.sparse.dok_matrix` method), 1109  
 sum() (`scipy.sparse.lil_matrix` method), 1116  
 sum() (`scipy.sparse.spmatrix` method), 1121  
 sum\_duplicates() (`scipy.sparse.bsr_matrix` method), 1065  
 sum\_duplicates() (`scipy.sparse.coo_matrix` method), 1075  
 sum\_duplicates() (`scipy.sparse.csc_matrix` method), 1084  
 sum\_duplicates() (`scipy.sparse.csr_matrix` method), 1094  
 SuperLU (class in `scipy.sparse.linalg`), 1175, 1210  
 svd() (in module `scipy.linalg`), 492  
 svd() (in module `scipy.linalg.interpolative`), 691  
 svdvals() (in module `scipy.sparse.linalg`), 1173, 1208  
 svdvals() (in module `scipy.linalg`), 493  
 sweep\_poly() (in module `scipy.signal`), 996  
 symiirorder1() (in module `scipy.signal`), 888  
 symiirorder2() (in module `scipy.signal`), 889  
 sync() (`scipy.io.netcdf_file` method), 465
- ## T
- t (in module `scipy.stats`), 1585  
 T (`scipy.optimize.LbfgsInvHessProduct` attribute), 871  
 t\_roots() (in module `scipy.special`), 1372  
 tan() (`scipy.sparse.bsr_matrix` method), 1065  
 tan() (`scipy.sparse.coo_matrix` method), 1075  
 tan() (`scipy.sparse.csc_matrix` method), 1085  
 tan() (`scipy.sparse.csr_matrix` method), 1094  
 tan() (`scipy.sparse.dia_matrix` method), 1102  
 tandg (in module `scipy.special`), 1393  
 tanh() (`scipy.sparse.bsr_matrix` method), 1065  
 tanh() (`scipy.sparse.coo_matrix` method), 1075  
 tanh() (`scipy.sparse.csc_matrix` method), 1085  
 tanh() (`scipy.sparse.csr_matrix` method), 1094  
 tanh() (`scipy.sparse.dia_matrix` method), 1102  
 tanhm() (in module `scipy.linalg`), 514  
 tanm() (in module `scipy.linalg`), 513  
 tf2sos() (in module `scipy.signal`), 980  
 tf2ss() (in module `scipy.signal`), 981  
 tf2zpk() (in module `scipy.signal`), 980  
 theilslopes() (in module `scipy.stats`), 1689  
 theilslopes() (in module `scipy.stats.mstats`), 1747, 1781  
 threshold() (in module `scipy.stats`), 1682  
 threshold() (in module `scipy.stats.mstats`), 1748, 1781  
 tiecorrect() (in module `scipy.stats`), 1702  
 tilbert() (in module `scipy.fftpack`), 343  
 tk1mbda (in module `scipy.special`), 1342  
 tmax() (in module `scipy.stats`), 1659  
 tmax() (in module `scipy.stats.mstats`), 1748, 1781  
 tmean() (in module `scipy.stats`), 1657  
 tmean() (in module `scipy.stats.mstats`), 1748, 1782  
 tmin() (in module `scipy.stats`), 1658  
 tmin() (in module `scipy.stats.mstats`), 1749, 1782  
 to\_discrete() (`scipy.signal.lti` method), 950  
 to\_mlab\_linkage() (in module `scipy.cluster.hierarchy`), 308  
 to\_ss() (`scipy.signal.StateSpace` method), 952, 970  
 to\_ss() (`scipy.signal.TransferFunction` method), 954, 972  
 to\_ss() (`scipy.signal.ZerosPolesGain` method), 956, 974  
 to\_tf() (`scipy.signal.StateSpace` method), 952, 970  
 to\_tf() (`scipy.signal.TransferFunction` method), 954, 972  
 to\_tf() (`scipy.signal.ZerosPolesGain` method), 957, 974  
 to\_tree() (in module `scipy.cluster.hierarchy`), 314  
 to\_zpk() (`scipy.signal.StateSpace` method), 952, 970

- to\_zpk() (scipy.signal.TransferFunction method), 955, 972
- to\_zpk() (scipy.signal.ZerosPolesGain method), 957, 974
- toarray() (scipy.sparse.bsr\_matrix method), 1065
- toarray() (scipy.sparse.coo\_matrix method), 1075
- toarray() (scipy.sparse.csc\_matrix method), 1085
- toarray() (scipy.sparse.csr\_matrix method), 1095
- toarray() (scipy.sparse.dia\_matrix method), 1102
- toarray() (scipy.sparse.dok\_matrix method), 1109
- toarray() (scipy.sparse.lil\_matrix method), 1116
- toarray() (scipy.sparse.spmatrix method), 1121
- tobsr() (scipy.sparse.bsr\_matrix method), 1065
- tobsr() (scipy.sparse.coo\_matrix method), 1075
- tobsr() (scipy.sparse.csc\_matrix method), 1085
- tobsr() (scipy.sparse.csr\_matrix method), 1095
- tobsr() (scipy.sparse.dia\_matrix method), 1102
- tobsr() (scipy.sparse.dok\_matrix method), 1110
- tobsr() (scipy.sparse.lil\_matrix method), 1116
- tobsr() (scipy.sparse.spmatrix method), 1122
- tocoo() (scipy.sparse.bsr\_matrix method), 1066
- tocoo() (scipy.sparse.coo\_matrix method), 1075
- tocoo() (scipy.sparse.csc\_matrix method), 1085
- tocoo() (scipy.sparse.csr\_matrix method), 1095
- tocoo() (scipy.sparse.dia\_matrix method), 1103
- tocoo() (scipy.sparse.dok\_matrix method), 1110
- tocoo() (scipy.sparse.lil\_matrix method), 1116
- tocoo() (scipy.sparse.spmatrix method), 1122
- tocsc() (scipy.sparse.bsr\_matrix method), 1066
- tocsc() (scipy.sparse.coo\_matrix method), 1075
- tocsc() (scipy.sparse.csc\_matrix method), 1085
- tocsc() (scipy.sparse.csr\_matrix method), 1095
- tocsc() (scipy.sparse.dia\_matrix method), 1103
- tocsc() (scipy.sparse.dok\_matrix method), 1110
- tocsc() (scipy.sparse.lil\_matrix method), 1116
- tocsc() (scipy.sparse.spmatrix method), 1122
- tocsr() (scipy.sparse.bsr\_matrix method), 1066
- tocsr() (scipy.sparse.coo\_matrix method), 1075
- tocsr() (scipy.sparse.csc\_matrix method), 1085
- tocsr() (scipy.sparse.csr\_matrix method), 1095
- tocsr() (scipy.sparse.dia\_matrix method), 1103
- tocsr() (scipy.sparse.dok\_matrix method), 1110
- tocsr() (scipy.sparse.lil\_matrix method), 1116
- tocsr() (scipy.sparse.spmatrix method), 1122
- todense() (scipy.optimize.LbfgsInvHessProduct method), 873
- todense() (scipy.sparse.bsr\_matrix method), 1066
- todense() (scipy.sparse.coo\_matrix method), 1076
- todense() (scipy.sparse.csc\_matrix method), 1085
- todense() (scipy.sparse.csr\_matrix method), 1095
- todense() (scipy.sparse.dia\_matrix method), 1103
- todense() (scipy.sparse.dok\_matrix method), 1110
- todense() (scipy.sparse.lil\_matrix method), 1116
- todense() (scipy.sparse.spmatrix method), 1122
- todia() (scipy.sparse.bsr\_matrix method), 1066
- todia() (scipy.sparse.coo\_matrix method), 1076
- todia() (scipy.sparse.csc\_matrix method), 1085
- todia() (scipy.sparse.csr\_matrix method), 1095
- todia() (scipy.sparse.dia\_matrix method), 1103
- todia() (scipy.sparse.dok\_matrix method), 1110
- todia() (scipy.sparse.lil\_matrix method), 1117
- todia() (scipy.sparse.spmatrix method), 1122
- todok() (scipy.sparse.bsr\_matrix method), 1066
- todok() (scipy.sparse.coo\_matrix method), 1076
- todok() (scipy.sparse.csc\_matrix method), 1086
- todok() (scipy.sparse.csr\_matrix method), 1095
- todok() (scipy.sparse.dia\_matrix method), 1103
- todok() (scipy.sparse.dok\_matrix method), 1111
- todok() (scipy.sparse.lil\_matrix method), 1117
- todok() (scipy.sparse.spmatrix method), 1122
- toeplitz() (in module scipy.linalg), 529
- toimage() (in module scipy.misc), 709
- tolil() (scipy.sparse.bsr\_matrix method), 1066
- tolil() (scipy.sparse.coo\_matrix method), 1076
- tolil() (scipy.sparse.csc\_matrix method), 1086
- tolil() (scipy.sparse.csr\_matrix method), 1095
- tolil() (scipy.sparse.dia\_matrix method), 1103
- tolil() (scipy.sparse.dok\_matrix method), 1111
- tolil() (scipy.sparse.lil\_matrix method), 1117
- tolil() (scipy.sparse.spmatrix method), 1122
- tplquad() (in module scipy.integrate), 354
- TransferFunction (class in scipy.signal), 952, 970
- transform (scipy.spatial.Delaunay attribute), 1257
- transpose() (scipy.optimize.LbfgsInvHessProduct method), 873
- transpose() (scipy.sparse.bsr\_matrix method), 1066
- transpose() (scipy.sparse.coo\_matrix method), 1076
- transpose() (scipy.sparse.csc\_matrix method), 1086
- transpose() (scipy.sparse.csr\_matrix method), 1095
- transpose() (scipy.sparse.dia\_matrix method), 1103
- transpose() (scipy.sparse.dok\_matrix method), 1111
- transpose() (scipy.sparse.lil\_matrix method), 1117
- transpose() (scipy.sparse.linalg.LinearOperator method), 1150, 1185
- transpose() (scipy.sparse.spmatrix method), 1122
- trapz (in module scipy.stats), 1587
- trapz() (in module scipy.integrate), 361
- tree (scipy.spatial.cKDTree attribute), 1236
- tri() (in module scipy.linalg), 529
- triang (in module scipy.stats), 1589
- triang() (in module scipy.signal), 1032
- tril() (in module scipy.linalg), 482
- tril() (in module scipy.sparse), 1127
- trim() (in module scipy.stats.mstats), 1749, 1783
- trim1() (in module scipy.stats), 1683
- trim\_mean() (in module scipy.stats), 1661
- trima() (in module scipy.stats.mstats), 1750, 1783
- trimboth() (in module scipy.stats), 1683
- trimboth() (in module scipy.stats.mstats), 1750, 1784



trimmed\_mean() (in module scipy.stats.mstats), 1760, 1793  
 trimmed\_mean\_ci() (in module scipy.stats.mstats), 1760, 1793  
 trimmed\_std() (in module scipy.stats.mstats), 1760, 1794  
 trimmed\_std() (in module scipy.stats.mstats), 1750, 1784  
 trimmed\_var() (in module scipy.stats.mstats), 1760, 1794  
 trimr() (in module scipy.stats.mstats), 1751, 1784  
 trimtail() (in module scipy.stats.mstats), 1751, 1785  
 triu() (in module scipy.linalg), 483  
 triu() (in module scipy.sparse), 1128  
 trunc() (scipy.sparse.bsr\_matrix method), 1067  
 trunc() (scipy.sparse.coo\_matrix method), 1076  
 trunc() (scipy.sparse.csc\_matrix method), 1086  
 trunc() (scipy.sparse.csr\_matrix method), 1096  
 trunc() (scipy.sparse.dia\_matrix method), 1104  
 truncexpon (in module scipy.stats), 1591  
 truncnorm (in module scipy.stats), 1593  
 ts\_roots() (in module scipy.special), 1372  
 tsearch() (in module scipy.spatial), 1268  
 tsem() (in module scipy.stats), 1660  
 tsem() (in module scipy.stats.mstats), 1751, 1785  
 tstd() (in module scipy.stats), 1659  
 ttest\_1samp() (in module scipy.stats), 1692  
 ttest\_1samp() (in module scipy.stats.mstats), 1760, 1794  
 ttest\_ind() (in module scipy.stats), 1692  
 ttest\_ind() (in module scipy.stats.mstats), 1752, 1786  
 ttest\_ind\_from\_stats() (in module scipy.stats), 1694  
 ttest\_onesamp() (in module scipy.stats.mstats), 1752, 1785, 1786  
 ttest\_rel() (in module scipy.stats), 1694  
 ttest\_rel() (in module scipy.stats.mstats), 1753, 1786  
 tukey() (in module scipy.signal), 1033  
 tukeylambda (in module scipy.stats), 1595  
 tvar() (in module scipy.stats), 1657  
 tvar() (in module scipy.stats.mstats), 1753, 1787  
 typecode() (scipy.io.netcdf\_variable method), 467  
 types() (scipy.io.arff.MetaData method), 470

## U

U (scipy.sparse.linalg.SuperLU attribute), 1177, 1212  
 u\_roots() (in module scipy.special), 1372  
 uniform (in module scipy.stats), 1597  
 uniform\_filter() (in module scipy.ndimage), 724  
 uniform\_filter1d() (in module scipy.ndimage), 725  
 unilinear (in module scipy.odr), 777  
 unique\_roots() (in module scipy.signal), 924  
 unit() (in module scipy.constants), 319  
 UnivariateSpline (class in scipy.interpolate), 411  
 update() (scipy.optimize.OptimizeResult method), 785  
 update() (scipy.sparse.dok\_matrix method), 1111  
 upfirdn() (in module scipy.signal), 907  
 us\_roots() (in module scipy.special), 1373  
 use\_solver() (in module scipy.sparse.linalg), 1155, 1190

## V

value() (in module scipy.constants), 318  
 values() (scipy.optimize.OptimizeResult method), 785  
 values() (scipy.sparse.dok\_matrix method), 1111  
 var() (scipy.stats.rv\_continuous method), 1402  
 var() (scipy.stats.rv\_discrete method), 1412  
 variance() (in module scipy.ndimage), 743  
 variation() (in module scipy.stats), 1660  
 variation() (in module scipy.stats.mstats), 1754, 1787  
 vectorstrength() (in module scipy.signal), 1056  
 vertex\_neighbor\_vertices (scipy.spatial.Delaunay attribute), 1258  
 vertex\_to\_simplex (scipy.spatial.Delaunay attribute), 1257  
 viewitems() (scipy.optimize.OptimizeResult method), 786  
 viewitems() (scipy.sparse.dok\_matrix method), 1111  
 viewkeys() (scipy.optimize.OptimizeResult method), 786  
 viewkeys() (scipy.sparse.dok\_matrix method), 1111  
 viewvalues() (scipy.optimize.OptimizeResult method), 786  
 viewvalues() (scipy.sparse.dok\_matrix method), 1111  
 volume() (scipy.spatial.Rectangle method), 1255  
 vonmises (in module scipy.stats), 1599  
 vonmises\_line (in module scipy.stats), 1602  
 Voronoi (class in scipy.spatial), 1261  
 voronoi\_plot\_2d() (in module scipy.spatial), 1267  
 vq() (in module scipy.cluster.vq), 296  
 vstack() (in module scipy.sparse), 1130

## W

wald (in module scipy.stats), 1604  
 ward() (in module scipy.cluster.hierarchy), 306  
 watershed\_ift() (in module scipy.ndimage), 744  
 WavFileWarning, 469  
 weibull\_max (in module scipy.stats), 1608  
 weibull\_min (in module scipy.stats), 1606  
 weighted() (in module scipy.cluster.hierarchy), 304  
 welch() (in module scipy.signal), 1045  
 white\_tophat() (in module scipy.ndimage), 767  
 whiten() (in module scipy.cluster.vq), 295  
 who() (in module scipy.misc), 710  
 whosmat() (in module scipy.io), 458  
 wiener() (in module scipy.signal), 888  
 wilcoxon() (in module scipy.stats), 1703  
 winsorize() (in module scipy.stats.mstats), 1754, 1787  
 wishart (in module scipy.stats), 1618  
 wminkowski() (in module scipy.spatial.distance), 1250, 1282  
 wofz (in module scipy.special), 1355  
 wrapcauchy (in module scipy.stats), 1610  
 write() (in module scipy.io.wavfile), 468  
 write\_record() (scipy.io.FortranFile method), 462

## X

xlog1py (in module `scipy.special`), 1393  
xlogy (in module `scipy.special`), 1393

## Y

y0 (in module `scipy.special`), 1305  
y0\_zeros() (in module `scipy.special`), 1303  
y1 (in module `scipy.special`), 1306  
y1\_zeros() (in module `scipy.special`), 1303  
y1p\_zeros() (in module `scipy.special`), 1304  
yn (in module `scipy.special`), 1293  
yn\_zeros() (in module `scipy.special`), 1303  
ynp\_zeros() (in module `scipy.special`), 1303  
yule() (in module `scipy.spatial.distance`), 1253, 1284  
yv (in module `scipy.special`), 1294  
yve (in module `scipy.special`), 1294  
yvp() (in module `scipy.special`), 1311

## Z

zaxpy (in module `scipy.linalg.blas`), 544  
zcopy (in module `scipy.linalg.blas`), 545  
zdotc (in module `scipy.linalg.blas`), 545  
zdotu (in module `scipy.linalg.blas`), 545  
zdrot (in module `scipy.linalg.blas`), 546  
zdscal (in module `scipy.linalg.blas`), 546  
zeros (`scipy.signal.dlti` attribute), 966  
zeros (`scipy.signal.lti` attribute), 948  
zeros (`scipy.signal.StateSpace` attribute), 952, 970  
zeros (`scipy.signal.TransferFunction` attribute), 954, 972  
zeros (`scipy.signal.ZerosPolesGain` attribute), 956, 974  
ZerosPolesGain (class in `scipy.signal`), 955, 973  
zeta() (in module `scipy.special`), 1392  
zetac (in module `scipy.special`), 1392  
zgbsv (in module `scipy.linalg.lapack`), 573  
zgbtrf (in module `scipy.linalg.lapack`), 574  
zgbtrs (in module `scipy.linalg.lapack`), 575  
zgebal (in module `scipy.linalg.lapack`), 577  
zgees (in module `scipy.linalg.lapack`), 579  
zgeev (in module `scipy.linalg.lapack`), 580  
zgeev\_lwork (in module `scipy.linalg.lapack`), 581  
zgegv() (in module `scipy.linalg.lapack`), 583  
zgehrd (in module `scipy.linalg.lapack`), 584  
zgehrd\_lwork (in module `scipy.linalg.lapack`), 585  
zgelsd (in module `scipy.linalg.lapack`), 589  
zgelsd\_lwork (in module `scipy.linalg.lapack`), 590  
zgelss (in module `scipy.linalg.lapack`), 587  
zgelss\_lwork (in module `scipy.linalg.lapack`), 588  
zgelsy (in module `scipy.linalg.lapack`), 592  
zgelsy\_lwork (in module `scipy.linalg.lapack`), 593  
zgemm (in module `scipy.linalg.blas`), 564  
zgemv (in module `scipy.linalg.blas`), 556  
zgeqp3 (in module `scipy.linalg.lapack`), 594  
zgerqf (in module `scipy.linalg.lapack`), 595

zgerc (in module `scipy.linalg.blas`), 556  
zgerqf (in module `scipy.linalg.lapack`), 595  
zgeru (in module `scipy.linalg.blas`), 557  
zgesdd (in module `scipy.linalg.lapack`), 597  
zgesdd\_lwork (in module `scipy.linalg.lapack`), 598  
zgesv (in module `scipy.linalg.lapack`), 601  
zgesvd (in module `scipy.linalg.lapack`), 599  
zgesvd\_lwork (in module `scipy.linalg.lapack`), 600  
zgetrf (in module `scipy.linalg.lapack`), 602  
zgetri (in module `scipy.linalg.lapack`), 603  
zgetri\_lwork (in module `scipy.linalg.lapack`), 603  
zgetrs (in module `scipy.linalg.lapack`), 604  
zgges (in module `scipy.linalg.lapack`), 607  
zggev (in module `scipy.linalg.lapack`), 609  
zgtsv (in module `scipy.linalg.lapack`), 636  
zhbevd (in module `scipy.linalg.lapack`), 610  
zhbevz (in module `scipy.linalg.lapack`), 611  
zheev (in module `scipy.linalg.lapack`), 612  
zheevd (in module `scipy.linalg.lapack`), 612  
zheevr (in module `scipy.linalg.lapack`), 613  
zhegv (in module `scipy.linalg.lapack`), 614  
zhegvd (in module `scipy.linalg.lapack`), 614  
zhegvz (in module `scipy.linalg.lapack`), 615  
zhemm (in module `scipy.linalg.blas`), 565  
zhemv (in module `scipy.linalg.blas`), 557  
zher (in module `scipy.linalg.blas`), 558  
zher2 (in module `scipy.linalg.blas`), 559  
zher2k (in module `scipy.linalg.blas`), 565  
zherk (in module `scipy.linalg.blas`), 565  
zipf (in module `scipy.stats`), 1647  
zlange (in module `scipy.linalg.lapack`), 647  
zlarf (in module `scipy.linalg.lapack`), 617  
zlarfg (in module `scipy.linalg.lapack`), 618  
zlartg (in module `scipy.linalg.lapack`), 618  
zlaswp (in module `scipy.linalg.lapack`), 620  
zlauum (in module `scipy.linalg.lapack`), 621  
zmap() (in module `scipy.stats`), 1679  
zmap() (in module `scipy.stats.mstats`), 1754, 1788  
zoom() (in module `scipy.ndimage`), 731  
zpbsv (in module `scipy.linalg.lapack`), 622  
zpbtrf (in module `scipy.linalg.lapack`), 623  
zpbtrs (in module `scipy.linalg.lapack`), 624  
zpk2sos() (in module `scipy.signal`), 982  
zpk2ss() (in module `scipy.signal`), 984  
zpk2tf() (in module `scipy.signal`), 981  
zposv (in module `scipy.linalg.lapack`), 625  
zpotrf (in module `scipy.linalg.lapack`), 626  
zpotri (in module `scipy.linalg.lapack`), 627  
zpotrs (in module `scipy.linalg.lapack`), 628  
zptsv (in module `scipy.linalg.lapack`), 638  
zrot (in module `scipy.linalg.lapack`), 629  
zrotg (in module `scipy.linalg.blas`), 546  
zscal (in module `scipy.linalg.blas`), 547  
zscore() (in module `scipy.stats`), 1679

zscore() (in module `scipy.stats.mstats`), 1755, 1788  
zswap (in module `scipy.linalg.blas`), 547  
zsymm (in module `scipy.linalg.blas`), 565  
zsyr (in module `scipy.linalg.blas`), 558  
zsyr2k (in module `scipy.linalg.blas`), 566  
zsyrk (in module `scipy.linalg.blas`), 566  
ztrmv (in module `scipy.linalg.blas`), 558  
ztrsyl (in module `scipy.linalg.lapack`), 630  
ztrtri (in module `scipy.linalg.lapack`), 631  
ztrtrs (in module `scipy.linalg.lapack`), 633  
zunghr (in module `scipy.linalg.lapack`), 633  
zungqr (in module `scipy.linalg.lapack`), 634  
zungrq (in module `scipy.linalg.lapack`), 634  
zunmqr (in module `scipy.linalg.lapack`), 635