

Palindromes' chain

Here is the idea:

- a] start the sequence **S** with $a(1)=0$
- b] $a(n)$ is the smallest integer not yet present in **S** that can be *linked* to **S**;
- c] a *link* is a palindromic pattern involving one or more digits of the new term and one or more digits of the last term(s) of **S**.

Explanation:

S=0,10,1,11,12,2,13,3,14,4,15,5,...

$a(2)=10$ because there is the palindromic link 0,10 between $a(1)$ and $a(2)$ – “10” being the smallest integer allowing such a link;

$a(3)=1$ because there is the palindromic link 10,1 between $a(2)$ and $a(3)$ – “1” being the smallest integer allowing such a link;

$a(4)=11$ because there is the palindromic link 1,1 between $a(3)$ and $a(4)$ – “11” being the smallest integer allowing such a link;

$a(5)=12$ because there is the palindromic link 1,1 between $a(4)$ and $a(5)$ – “12” being the smallest integer allowing such a link;

$a(6)=2$ because there is the palindromic link 2,2 between $a(5)$ and $a(6)$ – “2” being the smallest integer allowing such a link;

$a(7)=13$ because there is the palindromic link 12,2,1 involving $a(5)$, $a(6)$ and $a(7)$ – “13” being the smallest integer allowing such a link;

$a(8)=3$ because there is the palindromic link 3,3 between $a(7)$ and $a(8)$ – “3” being the smallest integer allowing such a link;

$a(9)=14$ because there is the palindromic link 13,3,1 involving $a(7)$, $a(8)$ and $a(9)$ – “14” being the smallest integer allowing such a link;

... etc.

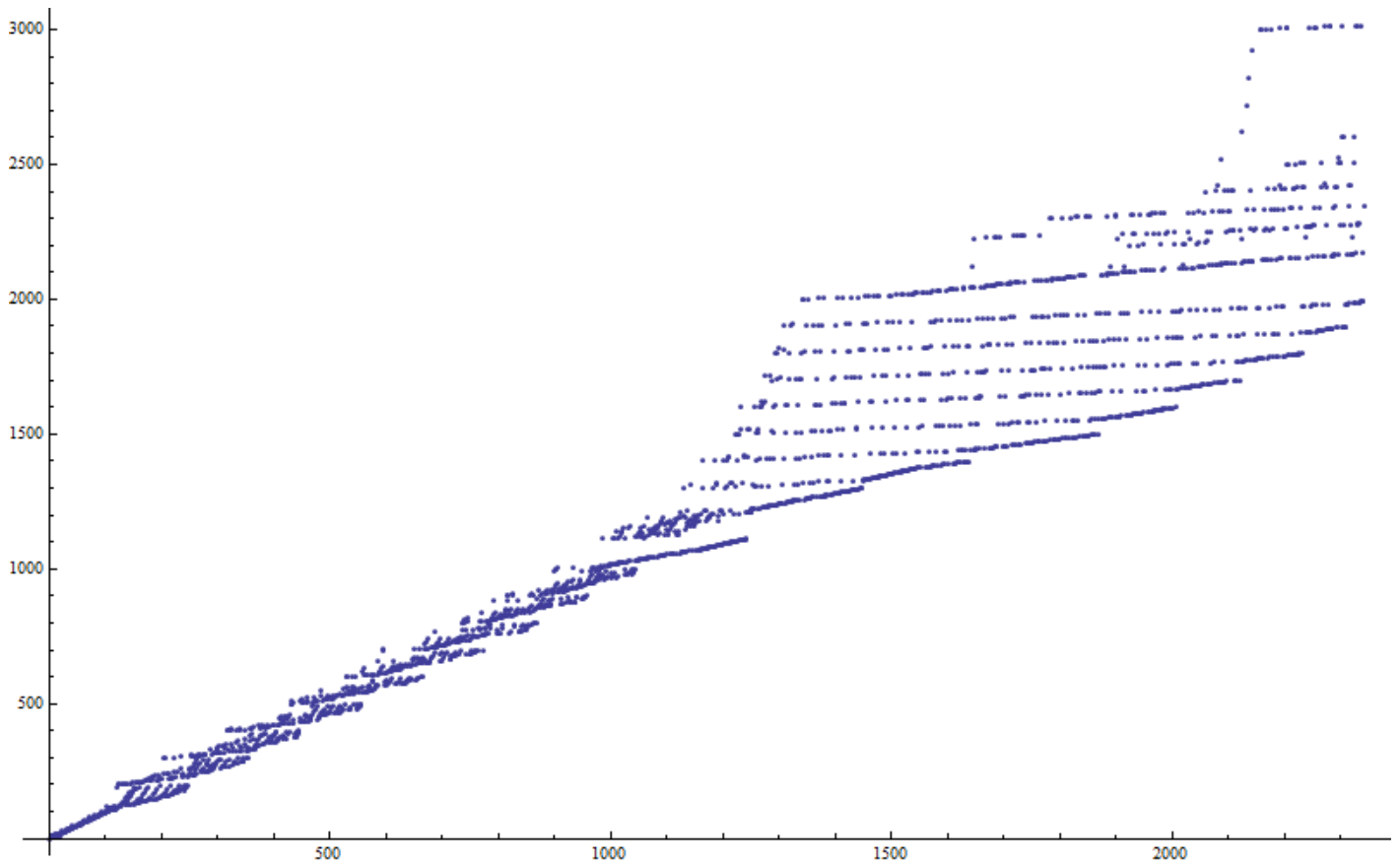
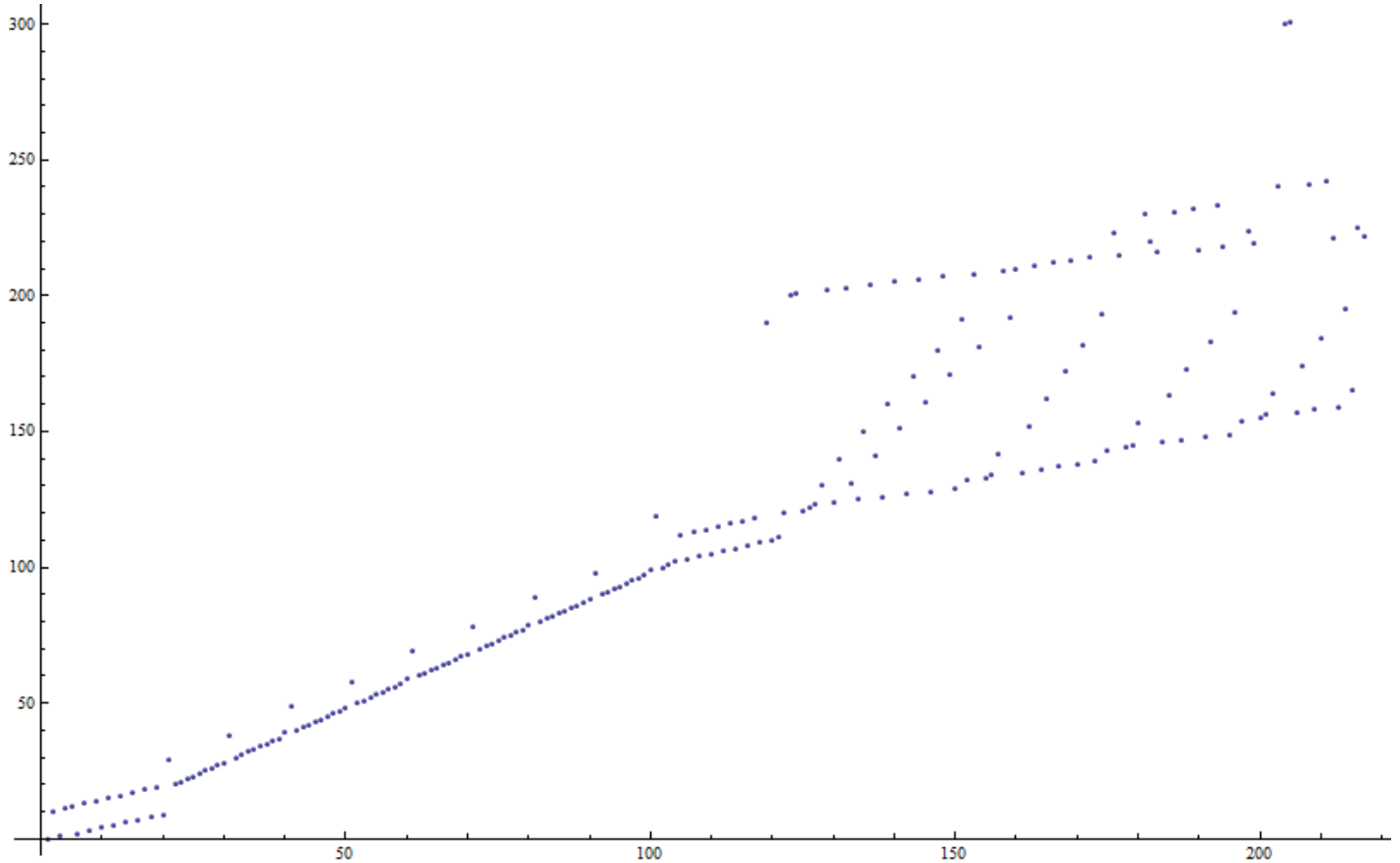
I guess **S** goes like this (but I'm not 100% sure - this has been computed by hand):

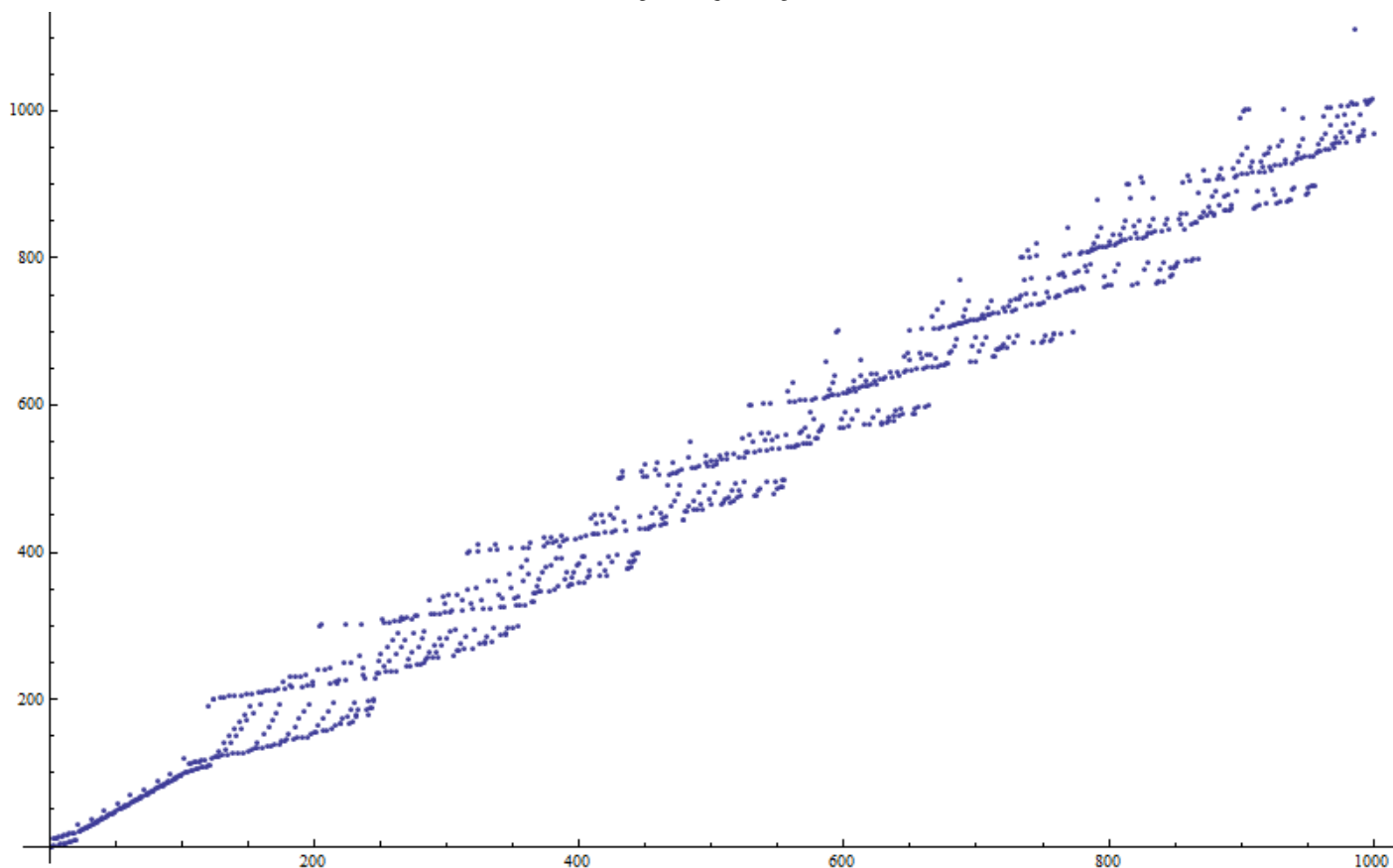
S = 0, 10, 1, 11, 12, 2, 13, 3, 14, 4, 15, 5, 16, 6, 17, 7, 18, 8, 19, 9, 29, 20, 21, 22, 23, 24, 25, 26, 27, 28, 38, 30, 31, 32, 33, 34, 35, 36, 37, 39, 49, 40, 41, 42, 43, 44, 45, 46, 47, 48, 58, 50, 51, 52, 53, 54, 55, 56, 57, 59, 69, 60, 61, 62, 63, 64, 65, 66, 67, 68, 78, 70, 71, 72, 73, 74, 75, 76, 77, 79, 89, 80, 81, 82, 83, 84, 85, 86, 87, 88, 98, 90, 91, 92, 93, 94, 95, 96, 97, 99, 190, 100, 101, 102, 112, 103, 113, 104, 114, 105, 115, 106, 116, 107, 117, 108, 118, 109,

I guess **S** is a permutation of the nonnegative integers.

Best,
É.

Jean-Marc Falcoz was quick to compute **S** and propose a few beautiful graphs (217 terms, 1000 terms and 2341 terms):





(The first 2341 terms of **S**; smallest missing integer: 1994)

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Merci **Jean-Marc** !

Best,

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[March 6th, 2014]