Confessions of a Sequence Addict

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AofA 2017 Princeton

Outline

- About the OEIS
- Fun with digits
- Sequences from geometry
- Lexicographically Earliest Sequences ...
- The Curling Number Conjectue

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- Fun: 2, 4, 6, 3, 9, 12, 8, 10, 5, 15, ... ? (A64413)
- Addictive (better than video games)
- Accessible (free, friendly)
- Street creds (6000 citations)
- Interesting, educational
- Essential reference
- Low-hanging fruit
- Need editors

Facts about the OEIS

- Accurate information about 300000 sequences
- Definition, formulas, references, links, programs
- View as list, table, graph, music
- 75 new entries and updates every day
- 6000 articles and books cite the OEIS
- Often called one of best math sites on the Web
- Since 2010, a moderated Wiki, owned by OEIS Foundation, a 501(c)(3) public charity

Main Uses for OEIS

- To see if your sequence is new, to find references, formulas, programs
- Catalan or Collatz? (Very easy or very hard?)
- Many collaborations, very international
- Source of fascinating research problems(*)
- Has led many people into mathematics
- Fun, Escape

(*) Look for: "Conjecture", "It appears that", "It would be nice to", ...

Fun With Digits

- “Climb to a prime”
- Binary version
- Home primes
- Powertrains
- A memorable prime

NEWS FLASH: JUNE 5 2017

Math Prof loses \$1000 bet!

If $n = p_1^{e_1} p_2^{e_2} \cdots$ then $f(n) = p_1 e_1 p_2 e_2 \cdots$ but omit any $e_i = 1$.

n	1	2	3	4	5	6	7	8	9	10	11	12	..	20	
f(n)	1	2	3	22	5	23	7	23	32	25	11	223	..	225	A080670
F(n)	1	2	3	211	5	23	7	23	2213	2213	11	223	..		A195264

Still growing after 110 terms, see A195265

John Conway, 2014: Start with n, repeatedly apply f until reach 1 or a prime. Offers \$1000 for proof or disproof.

James Davis, June 5 2017:

13532385396179 = 13.53^2.3853.96179

Fixed but not a prime!

JAMES DAVIS:

TRY $n = xp$ $p \gg$ primes in x

$$f(n) = f(x)10^y + p = xp$$

$$\frac{f(x)}{x-1} \cdot 10^y = p$$

Guess $x = m10^y + 1$

$$\frac{f(x)}{m} = p$$

$m = 1407$ works! $y = 5$ $p = 96179$

$$x = 1407 \cdot 10^5 + 1 = 13 \cdot 53^2 \cdot 3853$$

$$n = 13 \cdot 53^2 \cdot 3853 \cdot 96179$$

$$= 13\,53\,2\,3853\,96179$$

BINARY VERSION: A195264 (cont.)

n:	1	2	3	4	5	...	9	...	
f(n):	1	2	3	10	5	...	14	...	A230625
F(n):	1	2	3	31	5	...	23	...	A230627

DAVID SEAL 6/13/2017:

$255987 = 3^3 \cdot 19 \cdot 499 \rightarrow 11\ 11\ 10011\ 111110011$

$= 255987$

ALSO

$1007 \rightleftarrows 1269$

As of June 17 2017, based on work of Chai Wah Wu (IBM) and David J. Seal:
there are two known loops of length 2;
217 is first number not to reach 1 or prime;
234 is first number that seems to blow up (see A287878).
No, yesterday, Sean Irvine found at step 104,
234 reaches 35074322974831751926085777660944018966290406786641

Numbers that don't reach 1 or a prime: 217, 255, 446, 558, 717, 735, 775, ...

HOME PRIMES: Jeff Heleen 1990 A37274

n:	1	2	3	4	5	6	7	8	9	...	49	
f(n):	1	2	3	22	5	23	7	222	33	...	77	(A37276)
F(n):	1	2	3	211	5	23	7	3331113965338635107 (14 steps)	311	...	? still growing after 103 steps	(A37274)

NO KNOWN CYCLES!

POWER TRAINS: John Conway, 2007

$$\text{If } n = abcde\ldots \text{ then } f(n) = a^b c^d e\ldots \text{ with } 0^0 = 1$$

f(24) = 2^4 = 16, f(623) = 6^2.3 = 108, ... (A133500)

The known fixed points are

$$1, \ldots, 9, \quad 2592 = 2^5.9^2, \text{ and}$$ (A135385)

$$n = 2^{46}\, 3^6\, 5^{10}\, 7^2 = 24547284284866560000000000$$

$$f(n) = 2^4\, 5^4\, 7^2\, 8^4\, 2^8\, 4^8\, 6^6\, 5^6 = n$$

Conjecture: no other fixed points (none below 10^100)

Perhaps all these problems have only finitely many (primitive) exceptions?

A Memorable Prime!

A Memorable Prime!

1	
121	11^2
12321	111^2
1234321	1111^2
......	
12345678987654321	111111111^2
12345678910987654321	Prime!
......	

When is 123...n-1 n n-1 ...21 a prime?

Answer: when n is 10, 2446, but next term is unknown!

When is 1234...n prime?

$$1234567 = 127 \cdot 9721$$

$$12345678910111213 = 113 \cdot 125693 \cdot 869211457$$

Conjecture: infinitely many primes

None are known!

We know the smallest one has n >340000

See A7908 for details of the search
(which seems to have stalled)

Sequences from Geometry

- Peaceable queens: A250000
- Ways to draw n circles in plane: A250001

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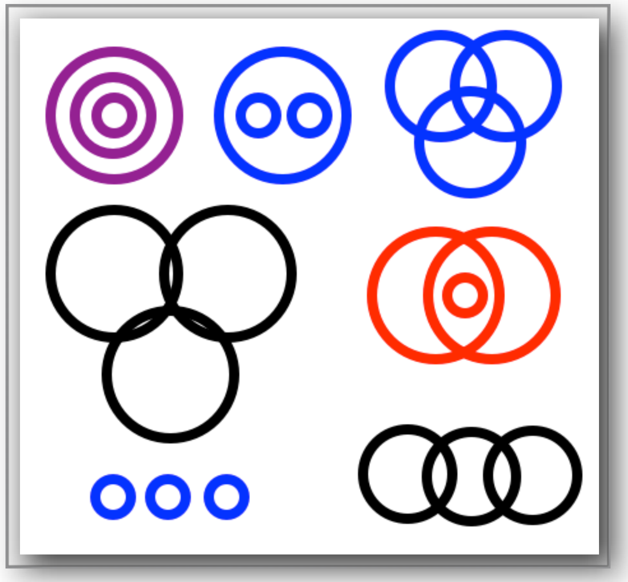
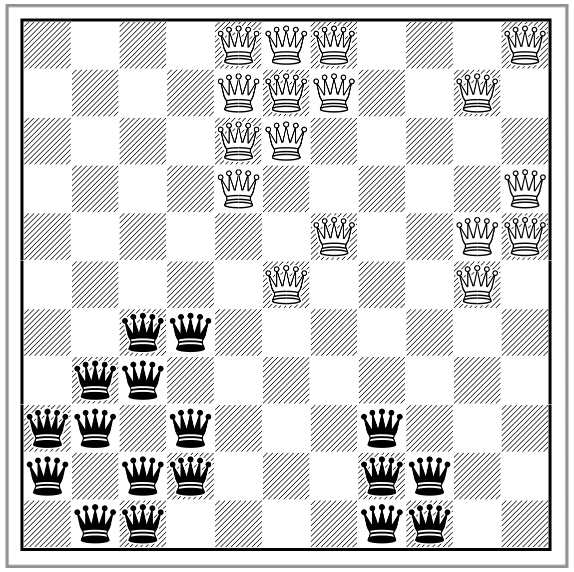
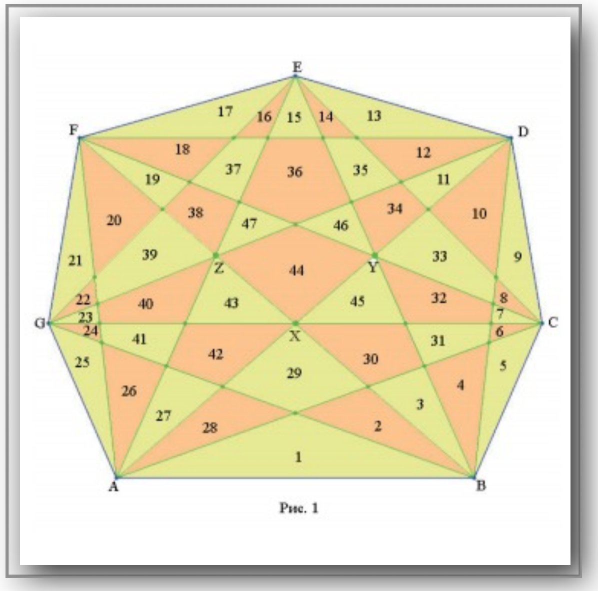
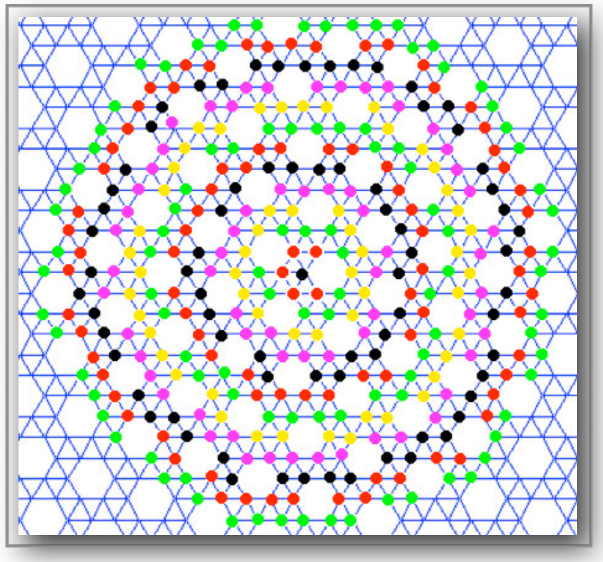
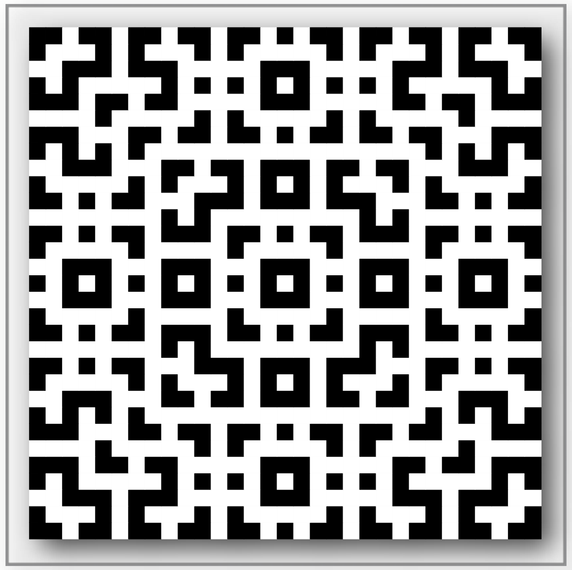
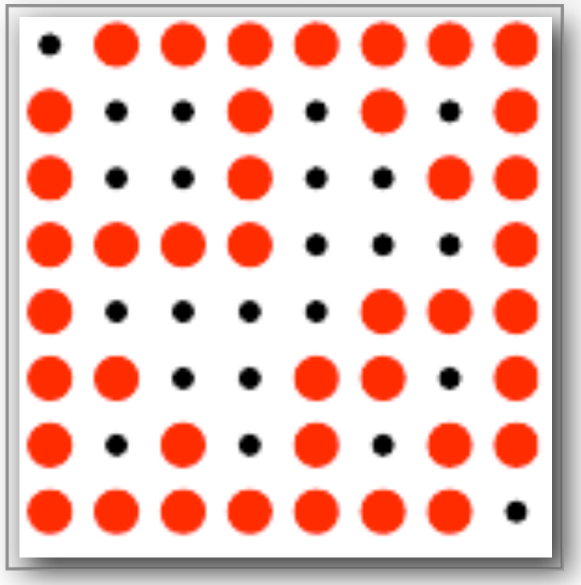


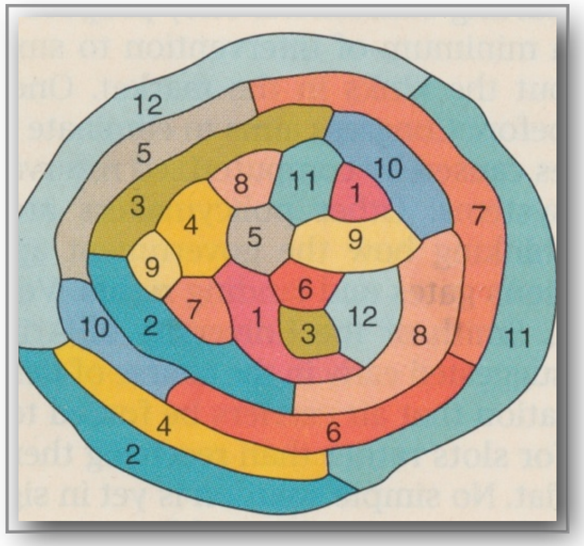


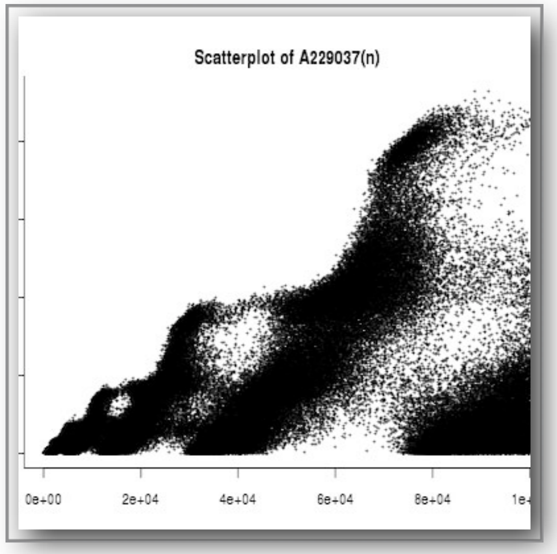












Peaceable Queens

A250000

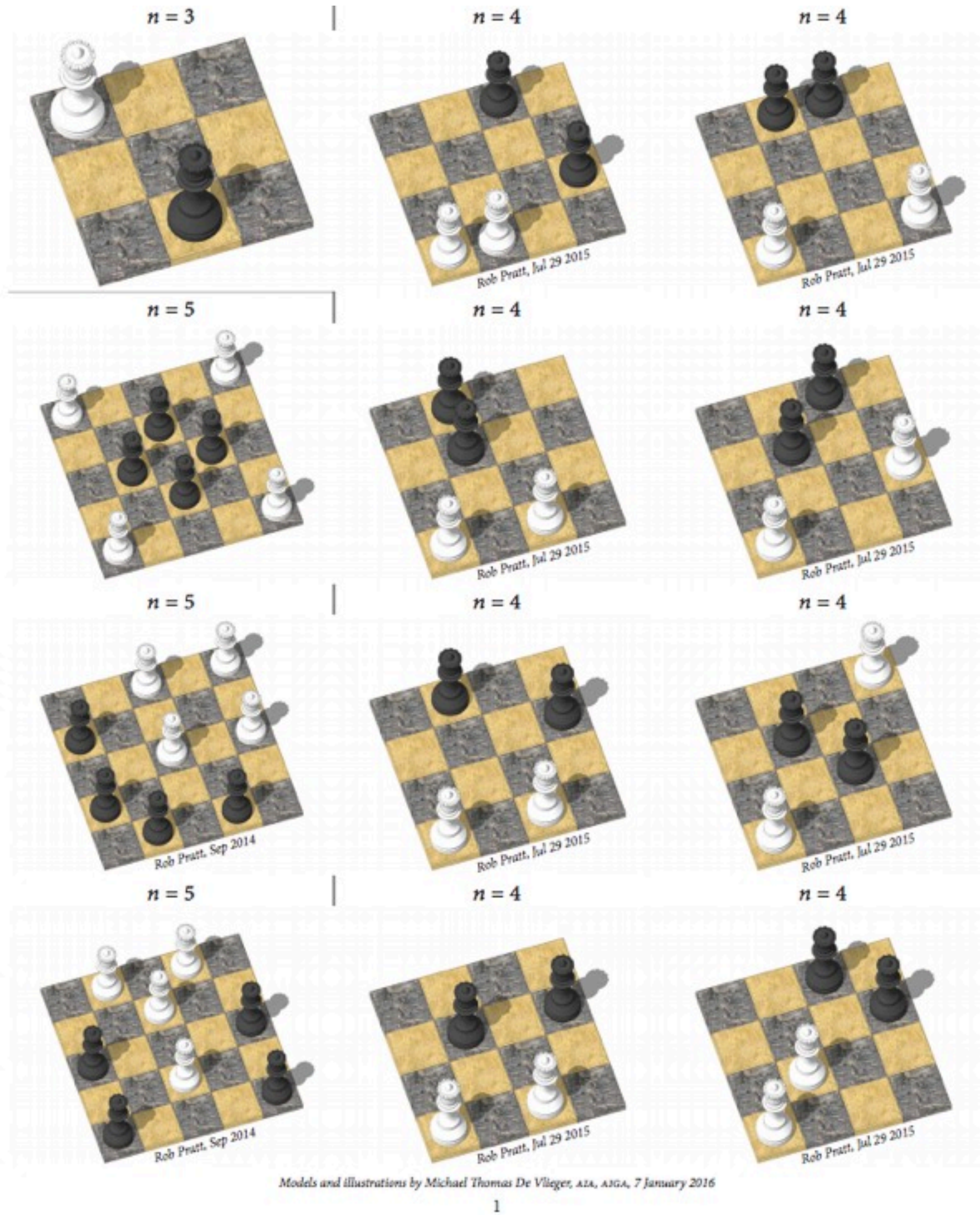
Peaceable coexisting armies of queens:
the maximum number m such that m white queens and m black queens can coexist on an n X n chessboard without attacking each other.

0, 0, 1, 2, 4, 5, 7, 9, 12, 14, 17, 21, 24

↑ 4 X 4

↑ 11 X 11

A250000



n = 3
n = 4
n = 4
Rob Pratt, Jul 29 2015
Rob Pratt, Jul 29 2015
n = 5
n = 4
n = 4
Rob Pratt, Jul 29 2015
Rob Pratt, Jul 29 2015
n = 5
n = 4
n = 4
Rob Pratt, Sep 2014
Rob Pratt, Jul 29 2015
Rob Pratt, Jul 29 2015
n = 5
n = 4
n = 4
Rob Pratt, Sep 2014
Rob Pratt, Jul 29 2015
Rob Pratt, Jul 29 2015
Models and illustrations by Michael Thomas De Vlieger, AIA, AIGA, 7 January 2016
1

a(11) = 17

(Michael De Vlieger)

A250000

$a(24) \; \geq \; 84$

Bob Selcoe
(2016)

```
------------------------
......WWWWWW............
......WWWWWW...........W
......WWWWWW..........WW
......WWWWWW.........WWW
......WWWWWW........WWWW
......WWWWW.......WWWWWW
.......WWW........WWWWWW
........W.........WWWWWW
..................WWWWWW
..................WWWWW.
..................WWWW..
..................WWW...
....BB..................
...BBB..................
..BBBB..................
.BBBBB..................
BBBBBB..........B.......
BBBBBB.........BBB......
BBBBBB........BBBB......
BBBBB........BBBBB......
BBBB........BBBBBB......
BBB.........BBBBBB......
BB..........BBBBBB......
B...........BBBBBB......
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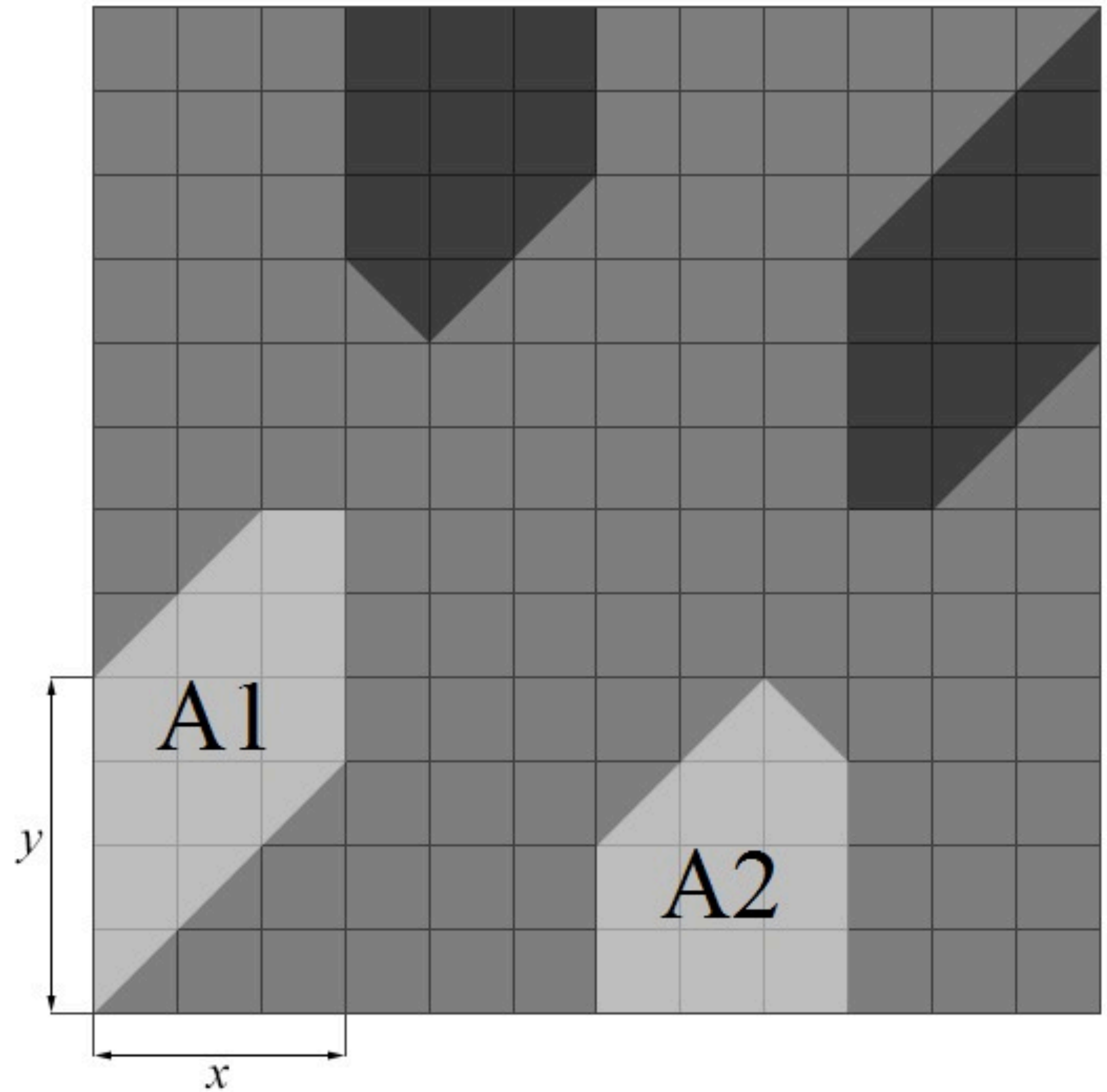
A250000

Peter Karpov



x = 1/4, y = 1/3, density = .146, Optimal?

Possible solution: a(n) = floor(7 n^2/48) except n=5, 9 ?

Number of ways to draw n circles in the affine plane

Jonathan Wild
Music Department, McGill

A250001

No. of arrangements of n circles in the plane

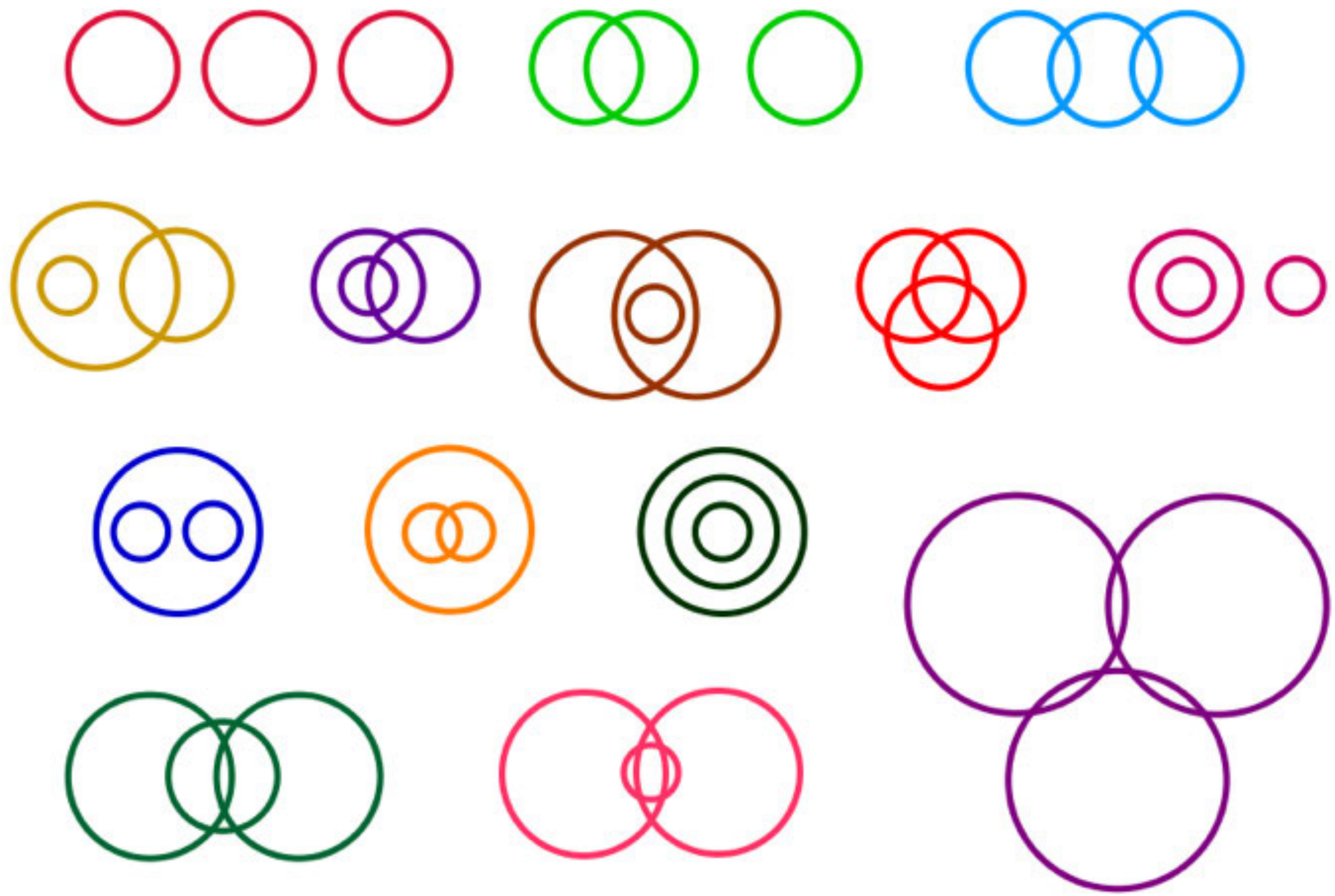
A250001

1, 2, 3, 4, 5

1, 3, 14, 173, 16951

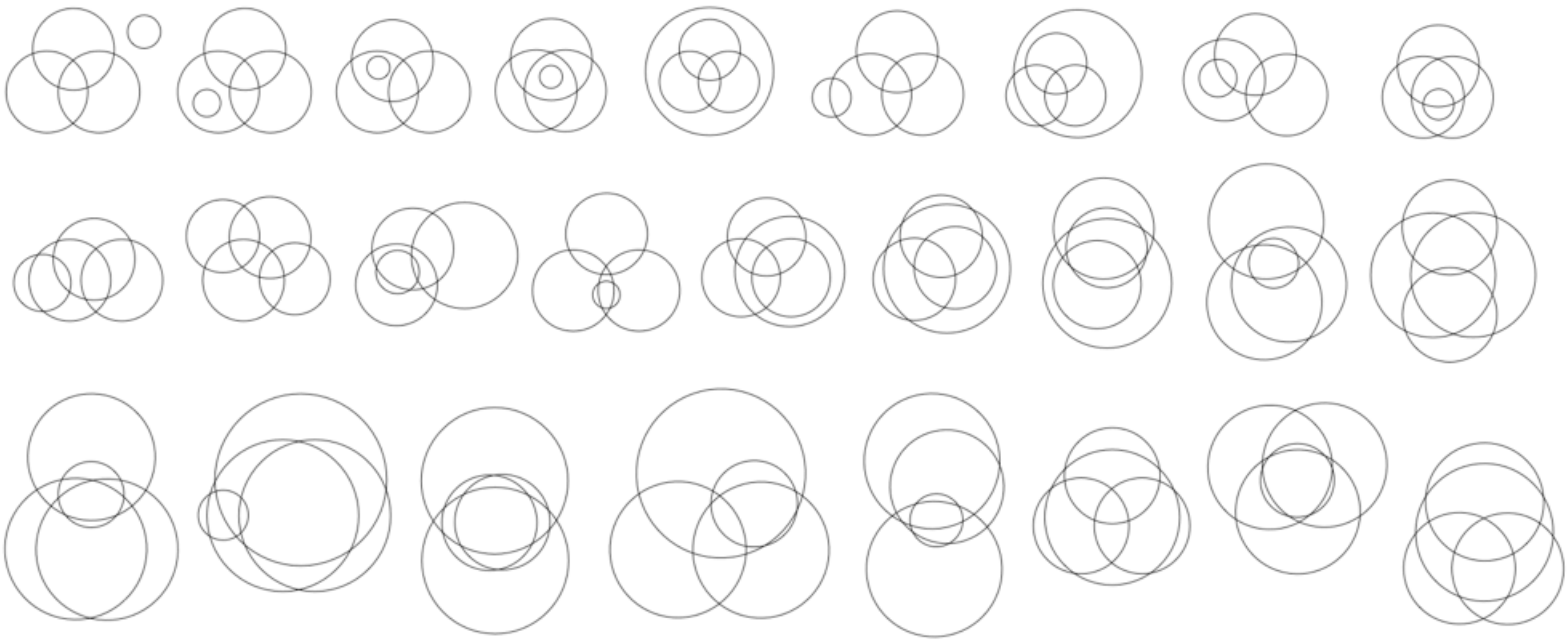
Jonathan Wild

a(3) = 14:



A250001

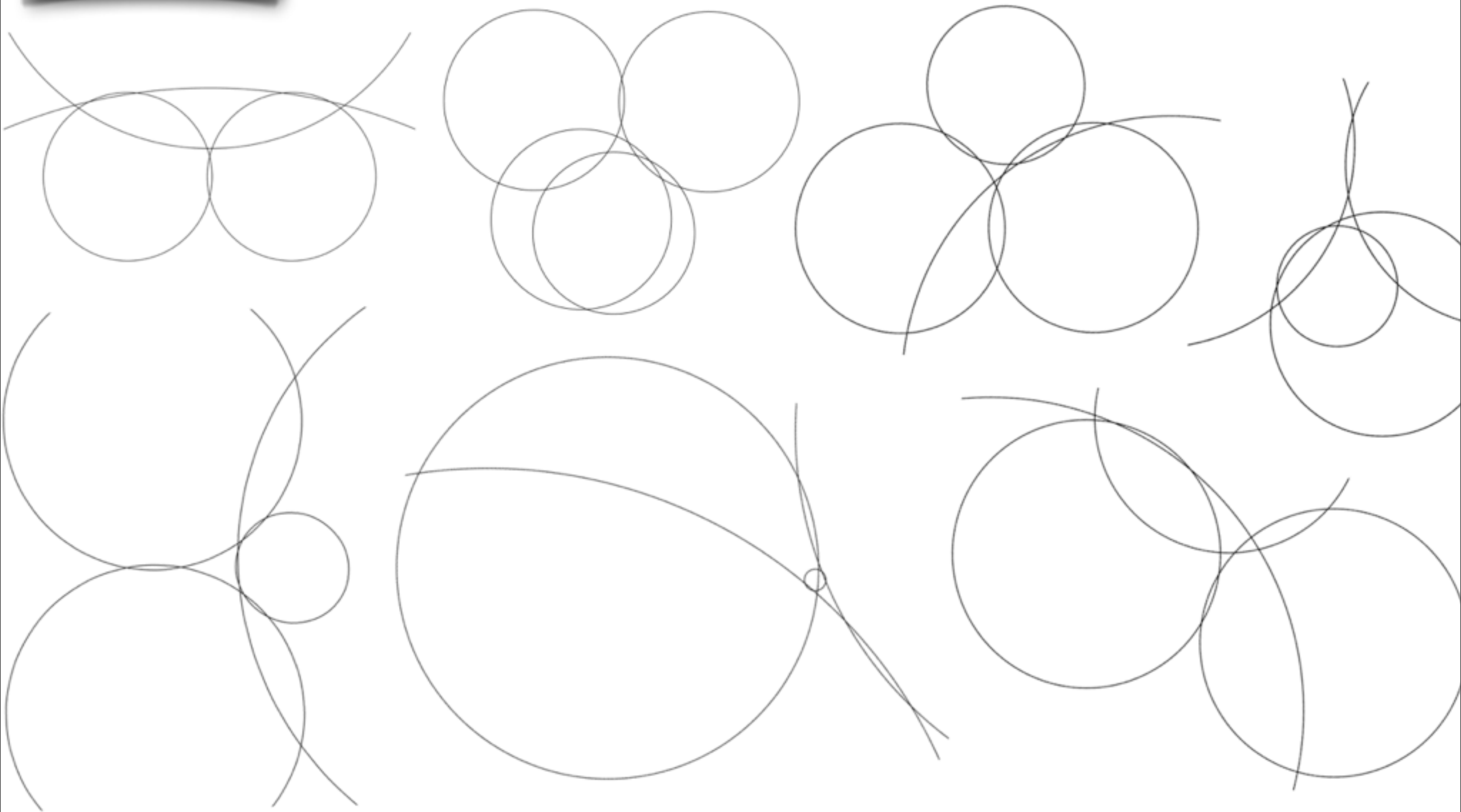
Some of the 173 arrangements of 4 circles



Counted (and drawn) by Jon Wild

A250001

More of the 173 arrangements of 4 circles



Counted (and drawn) by Jon Wild

Lexicographically Earliest Sequences

(LES sequences: A recent addiction)

- LES binary cube-free sequence
- EKG sequence
- Rémy Sigrist's sequence
- 2-dimensional LES

What is the Lexicographically Earliest Binary Cube-Free Sequence?

Axel Thue (1912):

T = 0110100110010110... is cube-free

Start with $A = 0$, repeat $A \mapsto A\bar{A}$

David W. Wilson (Feb. 2017):

What is LES binary cube-free sequence?

00100100 (oops!) 00100101... A282317

Have 10000 terms, have proof that first 999 are correct

What is the Lexicographically Earliest Binary Cube-Free Sequence? (cont.)

Does it exist?

Smallest rational number > sqrt(2)??

LES nonzero binary sequence with finite no. of 1's??

Theorem: It exists.

Proof: Let B = all 0,1 sequences

Define distance d(S,T) = 2^-i if S,T first differ at ith place

Identifies B with real interval [0,1)

Let C = cubefree sequences

Complement of C is open set in this metric space.

So C is closed set, so limit exists. QED

What is the Lexicographically Earliest Binary Cube-Free Sequence? (cont.)

Theorem:
The first 3 terms W of A282317 are correct.

Proof:

1. Use computer to show no earlier start is possible (back-tracking)
2. Claim there IS a cubefree extension of W:

Define E = WT.

If E = XXX..., |X| > |W|, say X=W Y

Then E = W Y W Y W Y..., so

T = Y W Y W Y... (Set YW = 1b say)

but Thue-Morse T is overlap-free, contradiction

alfalfa is not overlap-free

What is the Lexicographically Earliest Binary Cube-Free Sequence? (cont)

But what IS this sequence? (A282317)

0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1, 0, 1,
1, 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1, 1,
0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 1, 0, 0, 1, 0, 1, 1, 0, 0, 1, 0, 0

EKG Sequence (A64413)

1, 2, 4, 6, 3, 9, 12, 8, 10, 5, 15, ...

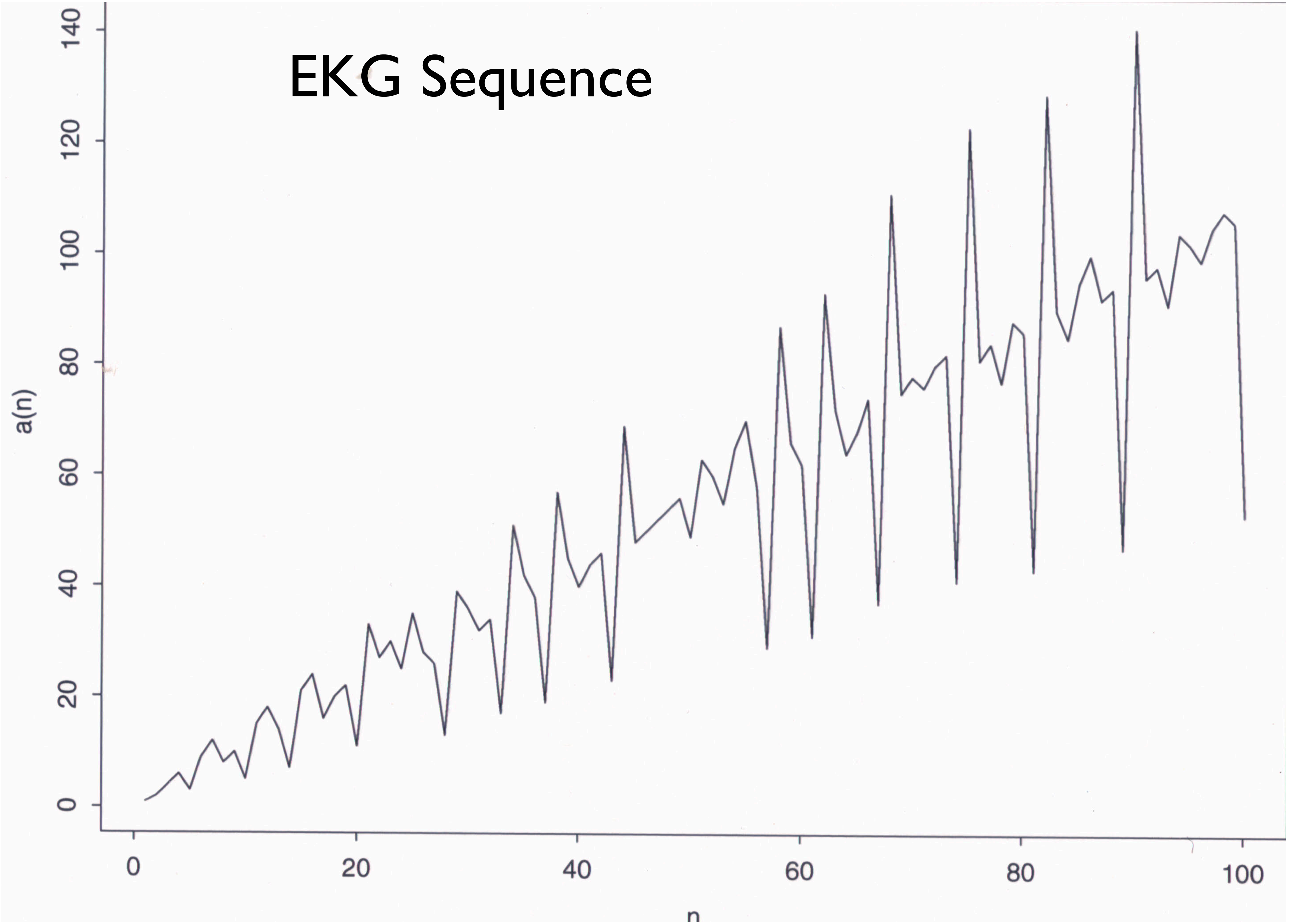
$a(1)=1$, $a(2)=2$,

$a(n) = \min k$ such that

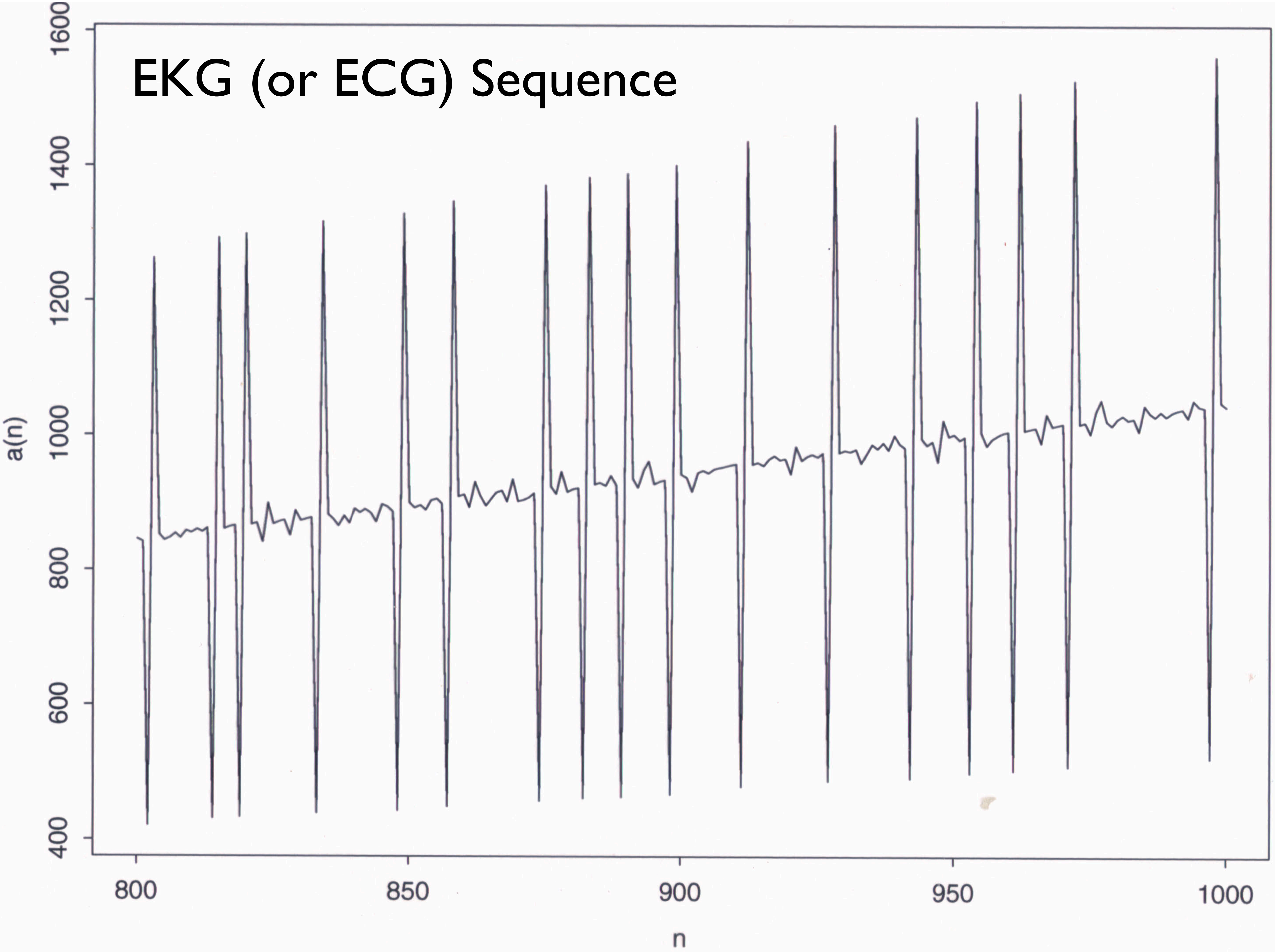
- GCD { a(n-1), k } > 1
- k not already in sequence

LES with GCD(a(n-1),a(n)) > 1 for n>2.

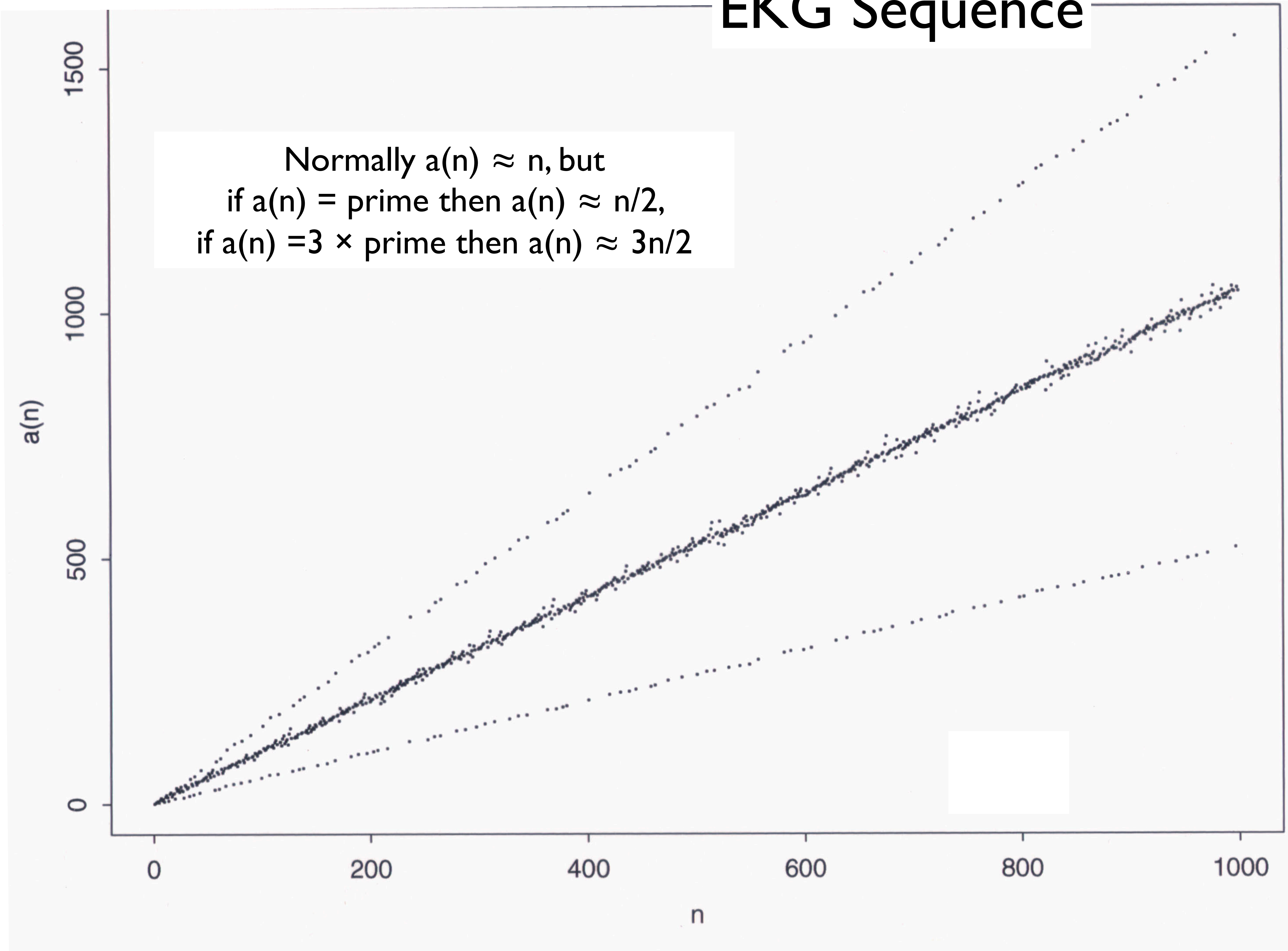
- Jonathan Ayres, 2001
- Analyzed by Lagarias, Rains, NJAS, Exper. Math., 2002


EKG Sequence
a(n)
0
20
40
60
80
100
120
140
n
0
20
40
60
80
100

EKG (or ECG) Sequence



EKG Sequence



Theorems:

- The sequence is a permutation of the natural numbers
- $c_1\, n \leq a(n) \leq c_2\, n$

Conjecture:

- $a(n) \;\sim\; n\left(1 \;+\; \dfrac{1}{3\log n}\right)$ for the main terms

EKG Sequence

LEMMA 1 IF ∞ MANY MULTIPLES OF PRIME p APPEAR, THEN ALL MULTIPLES DO.

Pf. kp not in sequence

$\exists n_0$ s.t. $n \geq n_0 \Rightarrow a(n) > kp$

$\therefore a(n) = ip \quad \therefore a(n+1) = kp.$ $\divideontimes$

LEMMA 2 IF ALL MULTIPLES OF p APPEAR THEN ALL NUMBERS DO.

Pf. k not in sequence

$a(n) = kip \qquad a(n+1) = k \qquad \divideontimes$

THEOREM $\{a(n)\}$ IS PERM. OF $\{1, 2, \ldots\}$

Pf. IF ∞ MANY DIFF^{T} PRIMES,

$\therefore \infty$ MANY $2p$'s, USE L1, L2.

IF FINITELY MANY DIFF^{T} PRIMES, ONE APPEARS ∞ OFTEN, USE L1, L2.

QED

A280864

REMY SIGRIST'S SEQUENCE

LES of positive integers such that
if a prime p divides a(n) then p
divides a(n-1) or a(n+1) but not both

n:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
a(n):	1	2	4	3	6	8	5	10	12	9	7	14	16	11	22
p(n):	–	–	2	–	3	2	–	5	2	3	–	7	2	–	11
q(n):	–	2	–	3	2	–	5	2	3	–	7	2	–	11	2

n:	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
a(n):	18	15	20	24	21	28	26	13	17	34	30	49	19	38	32
p(n):	2	3	5	2	3	7	2	13	–	17	2	15	–	19	2
q(n):	3	5	2	3	7	2	13	–	17	2	15	–	19	2	–

REMY SIGRIST'S SEQUENCE (cont.)

A280864

Conjecture: This is a permutation of the positive integers.

I can prove:

- every prime appears
- every even number appears
- infinitely many odd multiples of any odd prime p
- every number appears iff every square appears

But I cannot prove that every odd number appears

2-D LES's

- LES square array (by anti-diagonals)
- LES infinite array (spiral)

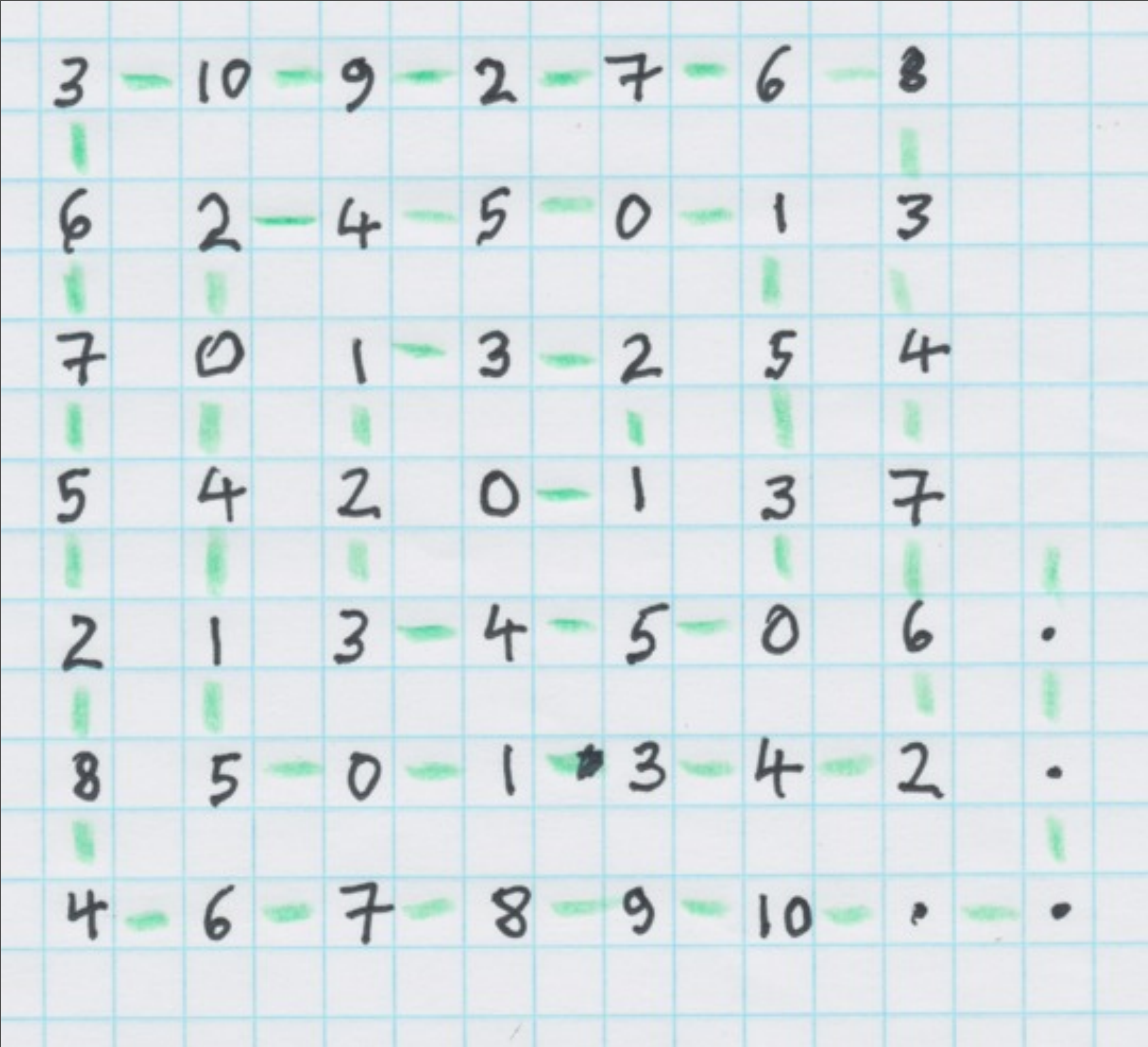
A274528

0	2	1	5	3	4	.	.
1	3	4	0	7	.	.	
2	0	5	1	.	.		
3	1	2	4	.			
4	6	0	.				
5	7	.					
6	.						
.							

Defn.: LES with no repeats
in any row, column,
or diagonal of slope +-1

Th. Every row, every column,
is perm. of nonneg. integers

Conjecture: So is every diagonal.



A274641

Defn.: LES with no repeats in any row, column, or diagonal of slope +-1

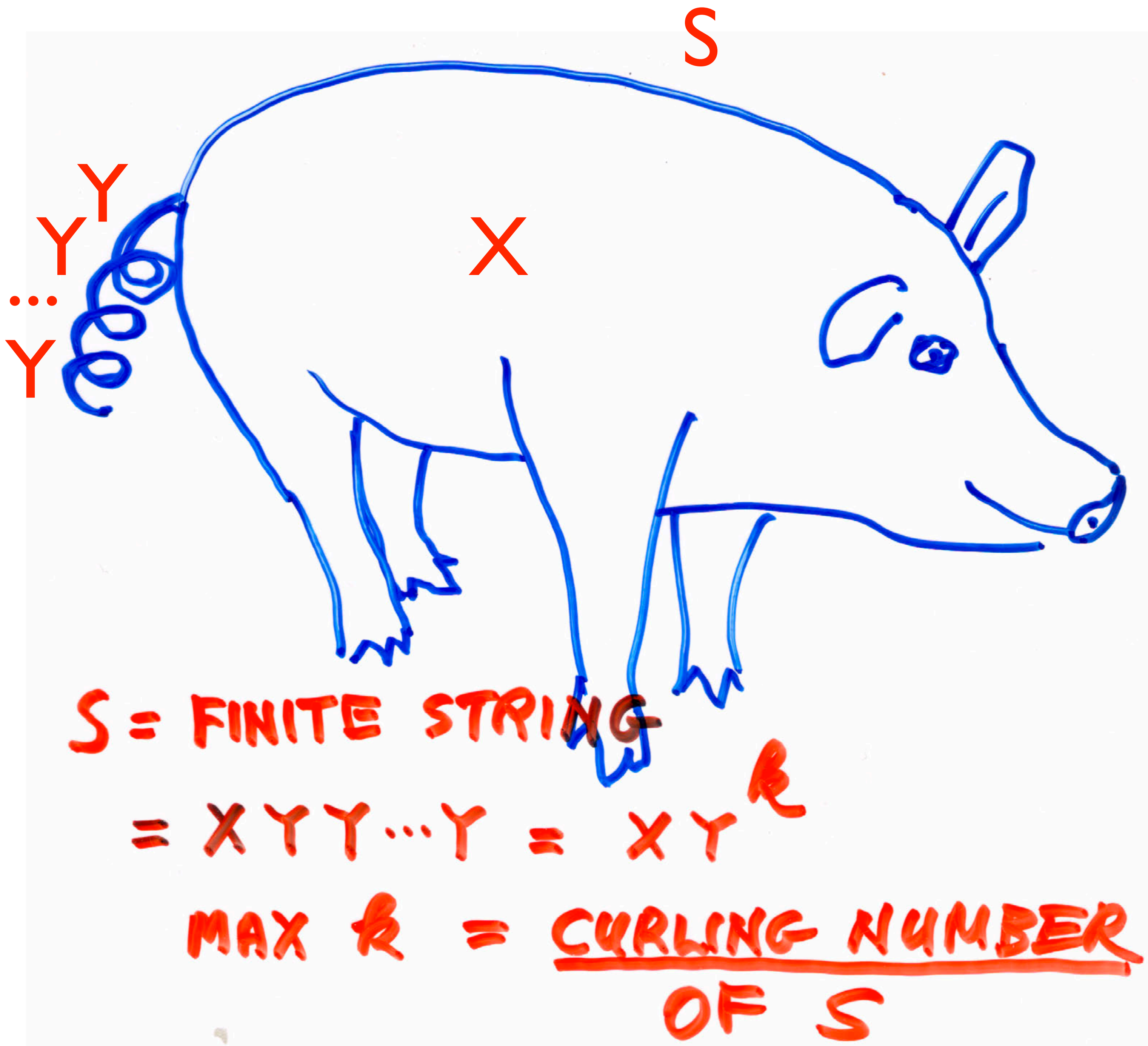
Conjecture: Every row, column, and diagonal of slope +-1 is a permutation of non-negative integers

Nothing is known!

The Curling Number Conjecture

The Curling Number Conjecture

Definition of Curling Number



$S = 7\ 5\ 2\ 2\ 5\ 2\ 2\ 5\ 2\ 2, \quad k = 3$

CURLING NUMBER CONJECTURE

- START WITH ANY FINITE STRING
- APPEND CURLING NUMBER
- REPEAT
- THEN MUST REACH A 1 !?

E.G.

START : 2 2 2 3 2 2

THEN 2 3 2 2 2 3 3 2 1 ....

BOO!

Gijswijt's Sequence

Fokko v. d. Bult, Dion Gijswijt, John Linderman,
N. J. A. Sloane, Allan Wilks (J. Integer Seqs., 2007)

Start with 1, always append curling number

1	1	<u>2</u>													
1	1	2	<u>2</u>	<u>2</u>	<u>3</u>										
1	1	2													
1	1	2	2	2	3	<u>2</u>									
1	1	2													
1	1	2	2	2	3										
1	1	2													
1	1	2	2	2	3	2	<u>2</u>	<u>2</u>	<u>3</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>3</u>	<u>3</u>	<u>2</u>
1	1	2													
.	.	.	.	.	.										

$$a(220) = 4$$

(A090822)

Gijswijt, continued

Is there a 5?

Is there a 5?

300,000 terms: no 5

Is there a 5?

300,000 terms: no 5

$2 \cdot 10^6$ terms: no 5

Is there a 5?

300,000 terms: no 5

$2 \cdot 10^6$ terms: no 5

10^{120} terms: no 5

Is there a 5?

300,000 terms: no 5

$2 \cdot 10^6$ terms: no 5

10^{120} terms: no 5

NJAS, FvdB: first 5 at about term $10^{10^{23}}$

First n appears at about term

$$2^{2^{3^{4^{5^{\cdot^{\cdot^{\cdot^{n-1}}}}}}}}$$

(F.v.d. Bult et al., J. Integer Sequences, 2007)

(A90822)

Proofs could be simplified if Curling Number Conjecture were true

How far can you get with an initial string of n 2's and 3's (before a 1 appears)?

THE UNIQUE RECORD STARTS:

LENGTH 8: 2 3 2 2 2 3 2 3 → 66

LENGTH 22:

2 3 2 2 3 2 2 3 2 3 2 2 2 3 2 3 2 2 3 2 2 3

→ 142

LENGTH 48 → 179

LENGTH 77 → 250

JOINT WORK WITH

BEN CHAFFIN

(INTEL)

Conjecture

LET $\mu(n)$ = MAX LENGTH ATTAINED STARTING WITH n 2's & 3's.

IF S ACHIEVES $\mu(n) > \mu(n-1)+1$ THEN S DOES NOT CONTAIN w^4, $w \neq \phi$.

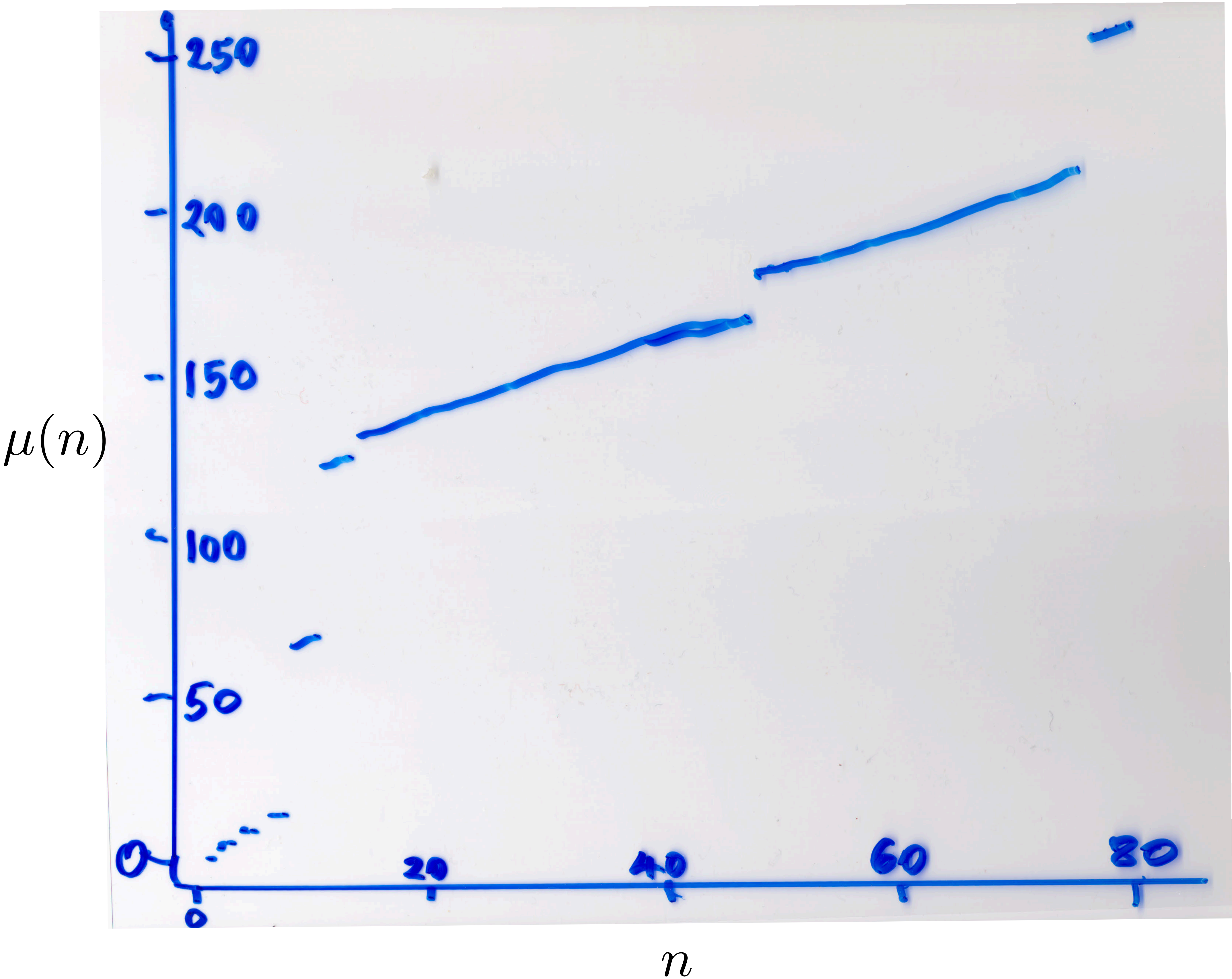
(SO NOT 2222)

Searched n <= 53

Conjecture

∴ S ALSO DOES NOT CONTAIN 33.

Searched n <= 80


$\mu(n)$
250
200
150
100
50
0
0
20
40
60
80
n

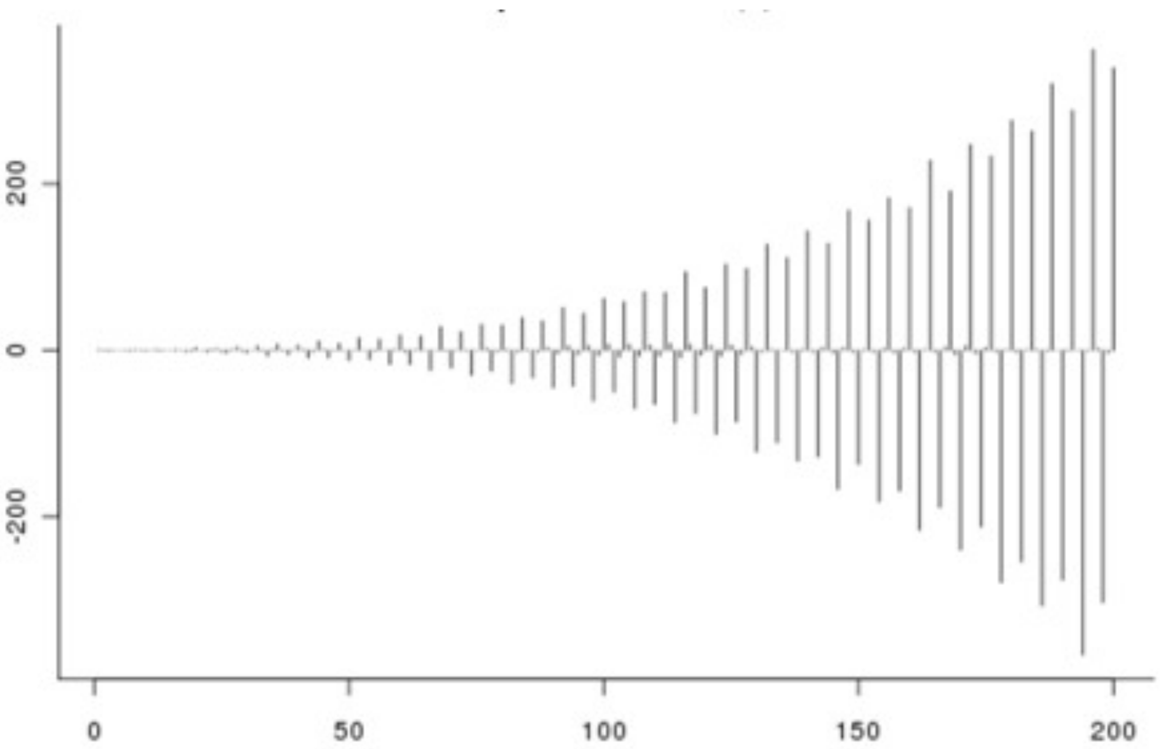
New A281488 with key-words “look” and “hear”

A281488 from Andrey Zabolotskiy
January 22 2017

$$a(n) \; = \; - \sum_{\substack{d \mid (n-2) \\ 1 \le d \le n-1}} a(d)$$

1, -1, -1, 0, 0, 0, -1, 1, 0, -1, 0, ...

A281488



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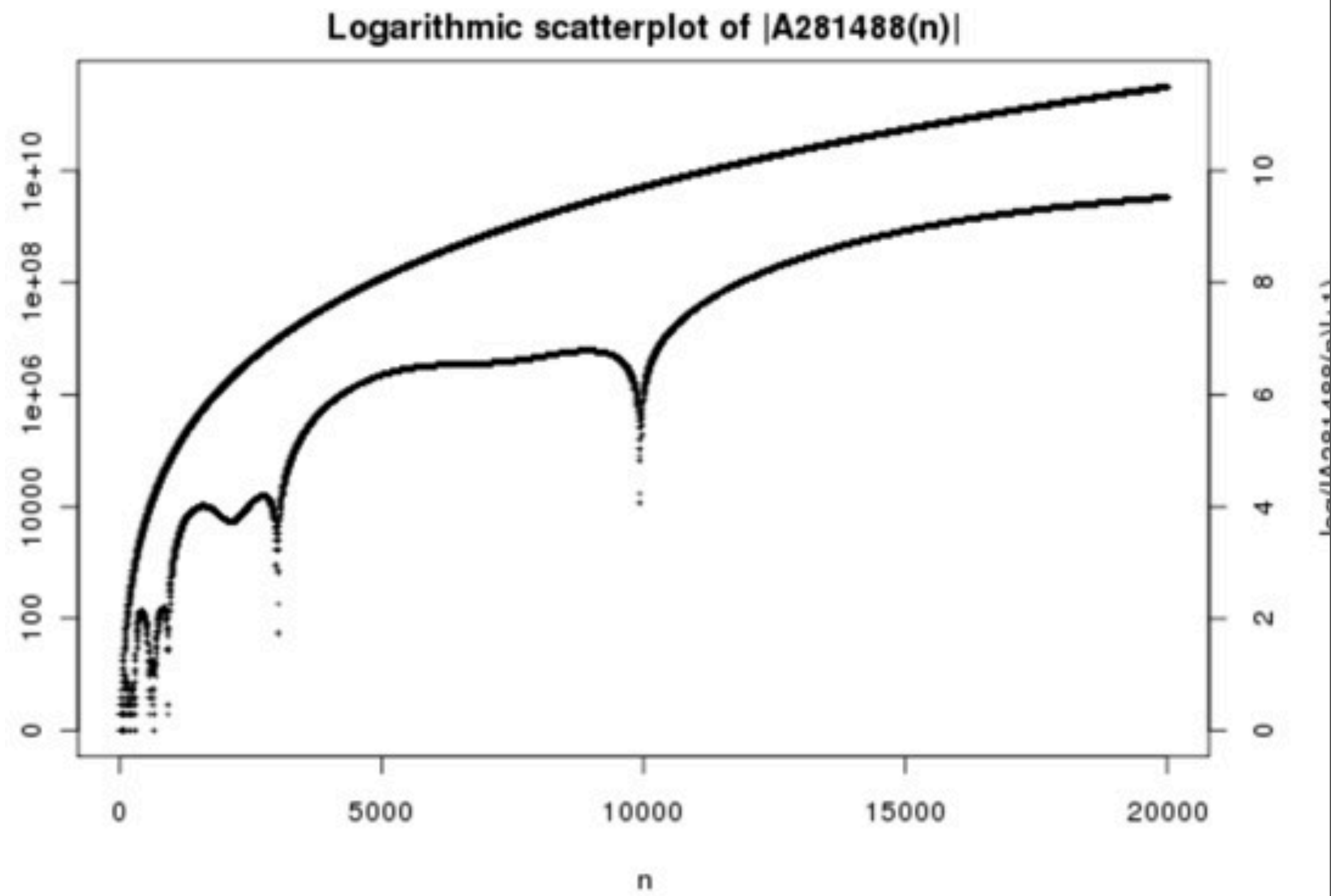

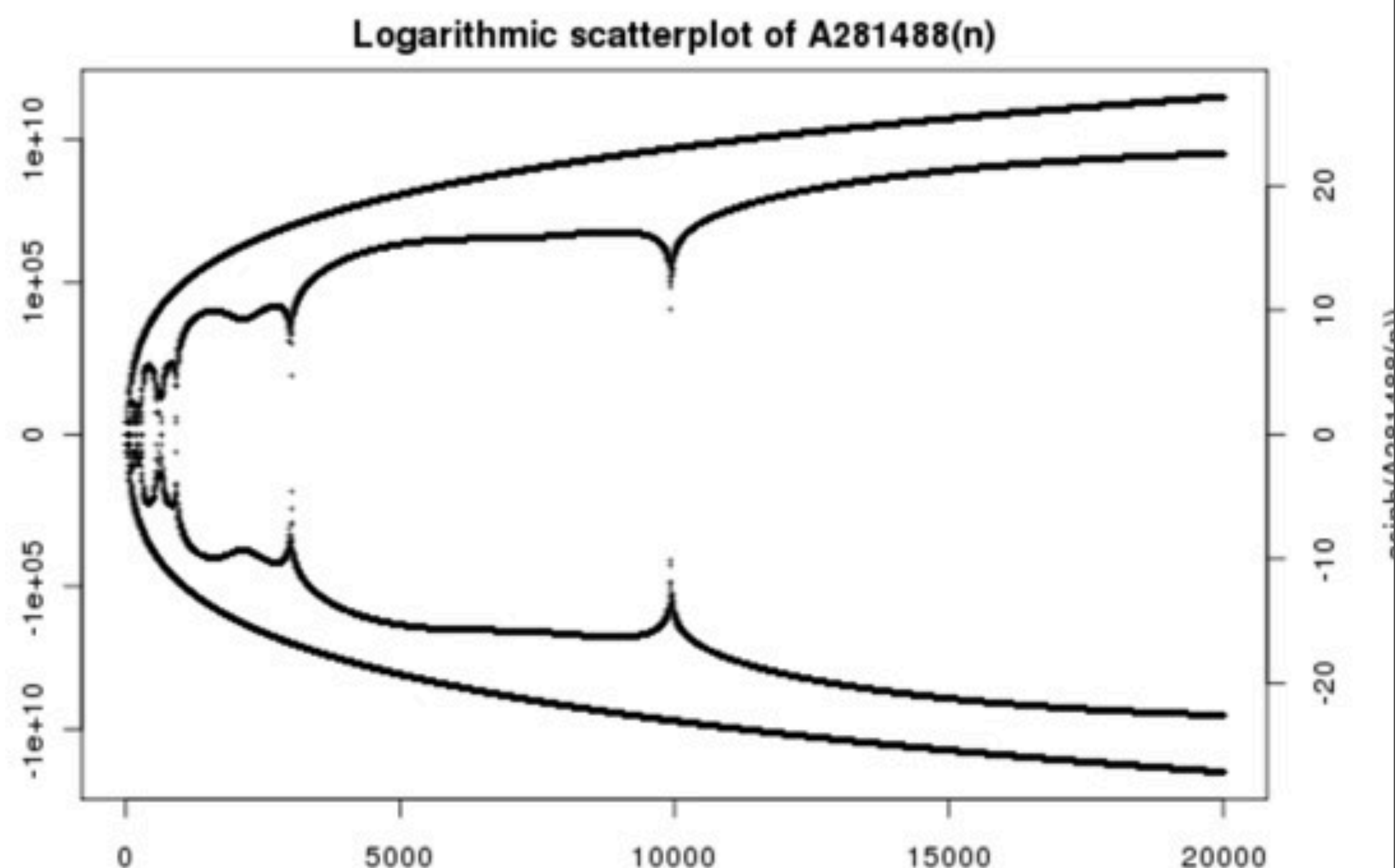

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