Let n an integer > 0. There exists an infinity of perfect squares of the form $k.2^n - 7$, where k is an integer.

Proof :

We prove by recurrence that, for all integer *n*, there exists an integer a_n such that $a_n^2 \equiv -7 \pmod{2^n}$.

If $n \leq 3$, $a_n = 1$. Suppose there exists a_n such that $a_n^2 \equiv -7 \pmod{2^n}$ for $n \geq 3$. Then,

$$a_n^2 \equiv -7 \pmod{2^{n+1}}$$

or

$$a_n^2 \equiv 2^n - 7 \pmod{2^{n+1}}$$

In the first case, we have just have to take $a_{n+1} = a_n$

and for the second case : $a_{n+1} = a_n + 2^{n-1}$. Then :

$$a_{n+1}^2 = a_n^2 + 2^n a_n + 2^{2n-1} \equiv a_n^2 + 2^n a_n \equiv a_n^2 + 2^n \equiv -7 \pmod{2^{n+1}}$$

because $n \geq 3$ and a_n odd, we conclude this recurrence.

Finally, we observe that the sequence (a_n) is not limited because $a_n^2 \geq 2^n - 7$ for all $n \in N^*$. We conclude because, for all $p \geq n$, $a_p^2 \equiv -7 \pmod{2^n}$.