Note on A00763

Consider all pairs of permutations $(\sigma, \tau) \in S_n^2$. There are $n!^2$ possible pairs. A pair (σ, τ) is achievable if σ can be transformed into τ using a double-ended priority queue. A007763(n) counts the number of achievable pairs for permutations of length n. Clearly, $a(n) \leq n!^2$. The following Java code is a brute force test as to whether a given pair should be counted:

```
private boolean isAchievable(final int[] tau, final int[] sigma) {
  final TreeSet<Integer> deq = new TreeSet<>();
  int i = 0;
  int j = 0;
  while (j < tau.length) {
    while (!deq.contains(tau[j])) {
      deq.add(sigma[i++]);
    }
    if (deq.first() == tau[j]) {
      deq.pollFirst(); // removes first
      ++j;
    } else if (deq.last() == tau[j]) {
      deq.pollLast(); // removes last
      ++j;
    } else {
      return false;
    }
  }
  return true;
}
```

If the search is restricted to a single-end priority queue (i.e., if the lines pertaining to the last element are removed), then the number of achievable permutation pairs is $(n+1)^{n-1}$ (A000272).