Scan S667
etc

Guy letter ✓

87-03-04

Many cops

f91

See item 8 before filing this & pp. 36-37 of enclosure.

→ 5667

~~5671~~ 5674

2386

5250

THE UNIVERSITY OF CALGARY

Faculty of SCIENCE
Department of MATHEMATICS & STATISTICS

2500 University Drive N.W., Calgary, Alberta, Canada T2N 1N4

Telephone (403) 220-5202

87-03-04

Neil J.A. Sloane,
AT&T Bell Laboratories, Room 2C-376
600 Mountain Avenue,
Murray Hill,
NJ 07974.

Dear Neil,

Will start a letter to you, but only mail it when it contains a bit more substance.

1. Sequences of numerators and denominators of convergents to continued fractions for

$$\sqrt{k^2+1} = k + \frac{1}{2k+}\frac{1}{2k+}\cdots$$

I.e. terms in recurring sequences $a_{n+1} = 2ka_n + a_{n-1}$.
For $k = 1$, you have 1064 & 552. For $k = 2$, you have 1434 & 764.
$k = 3$ (i.e. $\sqrt{10}$) might also be of interest:

5667 → 1,3,19,117,721,4443,27379,168717,1039681,6406803,39480499, 243289797,1499219281,9238605483,... ✓ 5667

5668 → 0, ~~0~~,1,6,37,228,1405,8658,53353,328776,2026009,12484830, 76934989,474094764,2921503573,... ✓ 5668

2. You have (327) "beginning of first occurrence of largest gap between primes". There might be some interest in the *ranks* of those primes:

5669 → 1,2,4,9,24,30,99,154,189,217,1183,2225,3385,.... (see also 4. below). ✓ 5669

3. Mrs. Perkins's Quilt. PCPS.

14 15 16 17 18 19 20

5670 → 1,4,6,7,8,9,9,10,10,11,11,11,11,12,12,12,12,13,13,13,13,13,13,14,14,14,14,14,14, 15,15,15,15,15,15,15,15,15,15,16,15,... unfortunately, terms > 11 aren't cast-iron. 5670

4. (87-04-23) You have almost enough to fill 2 lines of 327. However, Aaron Potler, in an 87-04-11 letter, extends the sequence with

JHC
6/91

2386 cont

(...2010733,4652353,17051707), 20831323,47326693,122164747, 189695659,191912783,436273009,1294268491,1453168141,2300942549, 3842610773,4302407359,10726904659,20678048297,22367084959, 25056082087,42652618343,127976334671,182226896239,241160624143, 297501075799,303371455241,304599508537,416608695821,461690510011,...

It might require a bit of effort to continue the sequence of *ranks* of these (see 2. above).

?5671 5. You might want to list the sizes of the gaps:
1,2,4,6,8,14,18,20,22,34,36,44,52,72,86,96,112,114,118,132,148,154,180,210,
220,222,234,248,250,282,288,292,320,336,354,382,384,394,456,464,468,474,486,490,
500,514,516,532,534,540,~~588~~... 582,588,602,652,674,716,766,778. ✓

No 6. Or the numbers of composite numbers in the gaps, one less than 5.: 1,2,3,7,13,17,19,21,33,35,43,... ✗

7. (87-04-30 07:05 MST) The imminent retirement of my secretary, in less than 9 hours time, and my desire to have your early cogent comments on 8. below, prompt me to bring this to a close. In next October's *Monthly* (better not spread this privileged info too far) will probably appear Problem 6556 by Nathan Fine (copy enclosed). I don't think it's an *Advanced* Problem myself, but parts (b) & (c) do seem to be unsolved, though not unsolvable? I enclose a rough outline of a solution of (a), in which several sequences cropped up, few of which are in the Bible. In some cases it may be more logical to list alternate members of the sequences, as well as or instead of the complete sequences. E.g. Sloane 1059 is more sensible than B_n or C_n. Some of these are *like* some of your convolved Fib sequences: not too surprising: here the Fibs. are convolved with powers of 2.

Recommended are (see enclosed sheet)

5672 D_n (001) 1,4,5,13,18,39,57,... and perhaps alternate members there of: (0) 1,4,13,39,112,... and (0) 1,5,18,57,169,... More members are quickly calculated from the formulas $D_{2n-1} = u_{2n} - 2^{n-1}$, $D_{2n} = u_{2n+1} - 2^n$.

E_n (000) 1,2,6,11,24,42,81,138,250,... and alternate members: (1) 2,11,42,138,419,... and (0) 1,6,24,81,250,... given by

$$E_{2n-1} = u_{2n+1} - 3\times2^{n-1} + 1 , \quad E_{2n} = u_{2n+2} - 2^{n+1} + 1.$$

F_n (000) 1,3,10,25,63,144,327,711,1534,... and alternates (1) 3,25,144,711,3237,... and (0) 1,10,63,327,1534,... given by

$$F_{2n} = 2^{2n-1} + 3\times2^{n-1} - u_{2n+3} , \quad F_{2n+1} = 2^{2n} + 2^{n+1} - u_{2n+4}.$$

Young & Potter MTAC 52 # Jan '89 pp 221–224

I haven't checked my arith (summation of series) at the end, but the sequence

s_n : (1) 4,13,50,135,374,910,2210,...

is probably O.~~M~~K. Send me enough money and I could probably produce a closed formula.

~~8. You may know that I'm producing a chap. on Combin. Games for the Graham-Grötschel-Lovász Handbook. I'd like to advertise lexicodes therein. I hope that limitations of space have concealed my real ignorance of the subject, but I'd like to take out insurance by getting your expert advice. Please alert me at least to the major blunders in the enclosed chunk, and tell me (other) things I ought to say or leave out. I get especially confused over technical terms and notation, e.g. $[n,k,d]$, (n,d,w) or whatever.~~

Thanks in anticipation of your usual competent help.

Yours sincerely,

Richard

RKG:jw

Richard K. Guy.

encl: Problem 6556.
Solution.
Chunk.

P.S. (87-04-30,10:40 MST) Thanks for yours of 87-04-21.

R.

October
1987

CATCH AMM Adv Prob 86-1122 6556

6556. Proposed by N. J. Fine, Deerfield Beach, Florida

(a) Consider a random walk around the edges of a square, where the probability of moving from a given vertex to either of the two adjacent vertices is 1/2. Suppose the walk stops as soon as all edges have been traversed. Find the expected path-length.

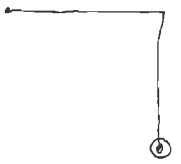
*(b) Consider a random walk around the edges of a cube, where the probability of moving from a given vertex to any one of the three adjacent vertices is 1/3. Find the expected path-length needed to traverse all edges.

*(c) Similarly with the frame of the n-dimensional cube, where each probability is 1/n.

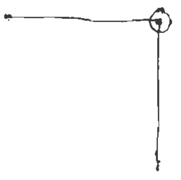
Let $a_n, b_n, \ldots$ be the probabilities that the position of the walker & the $\overset{\text{already}}{\wedge}$ traversed edges are as shown.

a_n

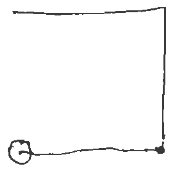
b_n



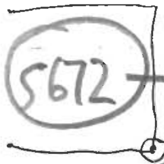
c_n



d_n



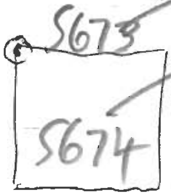
e_n



5672

5673

f_n



5674

Then
$$a_{n+1} = \tfrac{1}{2}a_n \qquad a_1 = 1$$
$$b_{n+1} = \tfrac{1}{2}a_n + c_n \qquad b_1 = 0$$
$$c_{n+1} = \tfrac{1}{2}b_n \qquad c_1 = 0$$
$$d_{n+1} = \tfrac{1}{2}b_n + \tfrac{1}{2}e_n \qquad d_1 = 0$$
$$e_{n+1} = \tfrac{1}{2}d_n + \tfrac{1}{2}e_n \qquad e_1 = 0$$
$$f_{n+1} = \tfrac{1}{2}d_n + f_n \qquad f_1 = 0$$

write $a_n = A_n/2^{n-1}$ $A_1 = 1$

$b_n = B_n/2^{n-1}$ $B_1 = 0$

etc.

$a_n + b_n + \cdots + f_n = 1$

$A_n + B_n + \cdots + F_n = 2^{n-1}$

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A_n	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
B_n	0	1	1	3	3	7	7	15	15	31	31	63	63	127	127	255	255	511	511	1023
C_n	0	0	1	1	3	3	7	7	15	15	31	31	63	63	127	127	255	255	511	511
D_n	0	0	1	1	4	5	13	18	39	57	112	169	313	482	859	1341	2328	3669	6253	9922
E_n	0	0	0	1	2	6	11	24	42	81	138	250	419	732	1214	2073	3414	5742	9411	15664
F_n	0	0	0	1	3	10	25	63	144	327	711	1534	3237	6787	14056	28971	59283	120894	245457	497167

$A_n = 1$, $B_n = 2^{\lfloor n/2 \rfloor} - 1$, $C_n = 2^{\lfloor (n-1)/2 \rfloor} - 1$, $D_n = u_{n+1} - 2^{\lfloor n/2 \rfloor}$, where u_n is the $\underline{n}$th Fibonacci number, $u_0 = 0$, $u_1 = 1$, $u_{n+1} = u_n + u_{n-1}$. $E_n = u_{n+2} - 2^{\lfloor (n+1)/2 \rfloor} - 2^{\lfloor n/2 \rfloor} + 1$

$F_n = 2^{n-1} + 2^{\lfloor n/2 \rfloor} + 2^{\lfloor (n-1)/2 \rfloor} - u_{n+3}$

Probability all edges traversed after exactly $\underline{n}$ moves is $f_n - f_{n-1} = \frac{1}{2}d_{n-1} = \dfrac{D_{n-1}}{2^{n-1}}$

Expected path length $= \sum n D_{n-1}/2^{n-1} = \sum n u_n/2^{n-1} - \sum n . 2^{\lfloor (n-1)/2 \rfloor}/2^{n-1}$

$= 56\sqrt{5}/5 - 10 \approx 15{\cdot}043961\ 347997\ 644600$. The partial sums are $s_n/2^{n-1}$

where $s_1 = 0, 0, 0, 4, 13, 50, 135, 374, 910, 2210, 5047, 11438, 25073, 54528, 116286, 246316,$
$515429, 1072762, 2215235, 4555530, \overset{s_{20}}{9319422}, \ldots$

5672
5674

- $$\boxed{D_n} = F_n - 2^{[n/2]}$$

0	1	2	3	4	5	6	7
1	1	2	3	5	8	13	21
1	1	2	2	4	4	8	8
0	0	0	1	1	4	5	13

- $$E_n = F_{n+1} - 2^{\frac{n+1}{2}} - 2^{\frac{n}{2}} + 1$$

$$F_n = 2^{n-1} + 2^{\left[\frac{n}{2}\right]} + 2^{\left[\frac{n-1}{2}\right]} - F_{n+2}$$

-