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R K Guy

&

NJAS

Corresp. Nov 1988

4 ~~3~~ pages

<u>Many</u> sequences

~~Needs more work~~

[GU5]

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88-11-28

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Dear Neil,

You won't have time to look at all of this; but say something about 1 & 2 if you can.

R

Some sequential matters.

1. I see how #425 comes from AMM 75 (1968)80 (although Sierpinski, Elem. Theory of Numbers, Warszawa, 1964 may be a better primary reference: incidentally, Schinzel recently sent me the (his) latest edition; pages 411-412 have moved on to 444 in this edition) but I don't see where #254 comes in. Perhaps I should ask Ron Graham. I'd like to use #254 in The Second Strong Law of Small Numbers, if it's easily explained. Did anyone ever find the slowest (a slower) growing sequence? A1856 A1149

2. Another one I'm not able to understand, but which I'd also probably use if I did, is #246. Apart from the Sloany 1, they're all primes, whose least positive primitive roots, 1,2,2,3,2,2,3, 2,3,2,2,... don't look very significant. The maximum primitive roots (so far) are 1,2,3,5,6,7,19,21,23,31,... corresponding to the primes 2,3,7,23,41,71,191,409,2161,5881,... which are #226 & #325. A2231 A2229 A2230

simple groups

A5180 3. You might like to include a sequence which Persi Diaconis showed me: (1),2,3,5,7,11,13,17,19,23,29,31,37,41,43, 47,53,59,60,61,... No need to tell you what it is?

?

A1317 N988 check! 4. You don't mention (perhaps no-one ever pointed it out, but Bill Watkins observed it 20 years ago) that #988 is the set of odd *n* for which regular polygons of *n* sides can be constructed with ruler and compass! I don't know how #988 is actually defined, but as I describe it, it *may* be ruled out on grounds of finiteness!

A3401 5. You *could* put in 1,2,3,4,5,6,8,10,12,15,16,17,20,24, 30,32,34,40,48,51,60,64,68,80,85,96,102,120,128,136,160,170,192,204, 240,255,256,257,272,320,340,384,408,480,510,512,514,544,640,680,768,771,816,...., set of *all* such *n*, which *is* infinite, and will fill two lines long before you have to confess any ignorance!

A381
A1611

6. #208 is much more significantly the sum of the Fibs so far, or just Fibs + 1. I haven't (yet) checked CMB 4(1961) 32 to see if the original author(s) noticed this. I was drawn to this by an article: Richard Laatsch, Measuring the abundancy of integers, *Math. Mag.* 59 (1986) 84-92; MR87e:11009. If the abundancy, $\sigma(n)/n$, of $n$ is $\geq j$, then $n$ contains at least $k$ factors, where, for $j = 2,3,4,\ldots$,
$k$ = 2,3,4,6,9,14,22,35,55,89,142,230,373,609,996,1637,2698,4461, — A5579
7198,12101,20303,34053,57148,... which stays very close to #208 for quite a while.

0 1 2

A5181
A5182

7. The ceiling of $e^{(n-1)/2}$ is 1,1,2,3,5,8,13,21,34,55,91,149,245,... and could easily be calculated to enough terms to fill two lines, if it was thought to be worth while. The floor is 0,1,1,2,4,7,12,20,33,54,90,148,244,... and hence close to #397. Compare 6. above! I note that you *have* got #695. $e^n$ A149

8. You have the Auluck partitions (#253), but not the Propper ones (Example 34 in SLSN, enclosed) which are the same, but A1524 without the insistence on contiguity in the rows above the first.

Strong law of small nos

A5168

1,1,1,2,3,5,9,15,26,45,78,135,234,406,704,1222,2120,...
See also Andrew M. Odlyzko & Herbert S. Wilf, The Editor's corner, $n$ coins in a fountain, *Amer. Math. Monthly*, 95 (1988) 840-843 and the references therein. If you count the number of Propper partitions by the number of coins in the bottom row, you get alternate Fibs.

A5183

9. You should have the sequence $n \times 2^{n-1} + 1$, which does A5183 occasionally crop up in "real life":
1,2,5,13,33,81,193,449,1025,2305,5121,11265,24577,53249,114689,245761,
524289,1114113,2359297,4980737,10485761,22020097,46137345,...

error in typing!

10. If only to contrast it with the maximum possible numbers of distinct sums of $n$ ordinals, 1,2,5,13,33,81,193,449,1089,2673,6561, 5348
15633,37249,88209,216513,531441,1266273,3017169,7189057,17537553, and from now on each is 81 times the fifth one before. I got this from JHC, but it may not be his unaided work. If he'll admit to any other reference for it, I'd be glad to know what it is.

Erdős in Vol 1 of PAMS 1930's

11. A better description of #113 is the first difference of the partition function, perhaps? A2865

A4001

12. The Conway (-Mallows) sequence 1,1,2,2,3,4,4,4,5,6,7,7,
8,8,8,8,9,10,11,12,12,13,14,14,15,15,15,16,16,16,16,16,17,18,19,20,21,21,
22,23,24,24,25,26,26,27,27,27,28,29,29,30,30,30,31,31,31,31,32,32,32,32,32,32,...
has now acquired enough notoriety to merit inclusion.

13. Also, some of the generalizations (Problem E3274, *Amer. Math. Monthly*, 95 (1988) 555):

where is E3274?

1,1,1,2,2,3,3,3,4,5,5,5,5,6,7,7,8,8,8,8,8,9,10,11,11,
12,12,12,13,13,13,13,13,13,14,15,16,16,17,18,18,19,19,19,20,20,20,20,
21,21,21,21,21,21,21,22,23,24,25,25,26,27,...

5350

f

A5707

1,1,1,1,2,2,3,3,3,4,4,4,4,5,6,6,6,6,6,7,8,8,9,9,9,9,9,9, 10,11,11,12,12,12,13,13,13,13,13,13,13,14,15,16,16,17,17,17,18,18,18,18, 19,19,19,19,19,19,19,19,20,21,...

One could go further, but out of pity for Dorothy Long, who's typing all this, I won't.

Best wishes for 1989 (but I may write again before then).

Yours sincerely,

Richard.

RKG:1

Richard K. Guy

encl: 163

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December 16, 1988

Professor Richard K. Guy
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Dear Richard:

Thank you very much for your letter of November 28.

To answer some of your questions:

Thanks for your note about #425.

#246 as I recall is to be found explicitly in the reference that I cite.

Persi's sequence (the orders of simple groups) - of course I thought of including it, but for some reason did not. (Compare #2311.)

Thanks for the offprint of SLSN, which I like very much. And, above all, thanks for all the sequences! Keep them coming!

Best wishes for 1989

**N. J. A. Sloane**

NJAS:tjp

P. S. Sequence #254: With some difficulty I tracked it down: Let $a_1 = 1$, $a_2 = 2$, and define $a_{n+1}$ from $\{ a_1, \ldots, a_n \}$ so that $a_{n+1} - a_{n-1}$ is the first number not obtainable as $a_i - a_j$ for $1 \le j < i \le n$.

Enc.
card