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November 10, 1979

Dr. N. J. A. Sloane
Bell Laboratories
600 Mountain Avenue
Murray Hill, N. J. 07974

Dear Dr. Sloane,

Enclosed you will find a Xerox copy of a page from the latest issue of Mathematics Magazine which contains a new sequence for your book.

I have also enclosed a computer printout with the first 200 or so terms of this sequence.

Cordially yours,

Jeffrey Shallit

Ref MMAG 52 265, 79.
Name: A self-generating sequence.

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A504

Sequence of Sequences

May 1978

1047. Given an infinite sequence $A=\{a_n\}$ of positive integers, we define a family of sequences A_i, where $A_0=A$ and $A_i=\{b_r\}$ for $i=1,2,3,\ldots$, where b_r is the number of times that the rth lowest term of A_{i-1} occurs in A_{i-1}. For example, if $A=A_0=\{1,2,2,3,3,3,4,4,4,4,\ldots\}$, then $A_1=\{1,2,3,4,\ldots\}$ and $A_2=\{1,1,1,1,\ldots\}$.

(a) Find a non-decreasing sequence A such that the sequences A_i are all distinct.

(b)* Let $T=\{t_n\}$ be the unique non-decreasing sequence containing all the positive integers which has the property that $T_1=T_0$. Define $U=\{u_n\}$ and $V=\{v_n\}$ so that for all n, $u_n=t_{2n-1}$ and $v_n=t_{2n}$. Are the sequences U_i and V_i all distinct? [*James Propp, Great Neck, New York.*]

Solution: (a) It is easily seen that a non-decreasing sequence A for which consecutive elements differ by at most one is uniquely determined by its first element and by A_1. In fact, if a is the first element of A and $A_1=\{b_r\}$, then the first b_1 elements of A are a, the next b_2 elements are $a+1$, the next b_3 elements are $a+2$, and so on.

Therefore, defining the first element of A_i to be $i+1$ for $i>0$ produces the following sequences:

$$A=A_0=\{1,1,2,2,3,3,4,4,4,5,5,5,6,6,6,7,7,7,7,8,8,8,8,\ldots\}$$ — A5041

$$A_1=\{2,2,2,3,3,3,4,4,4,5,5,5,6,6,6,6,7,7,7,7,\ldots\}$$

$$A_2=\{3,3,3,3,4,4,4,4,5,5,5,5,6,6,6,6,7,7,7,7,\ldots\}$$

$$A_3=\{4,4,4,4,4,5,5,5,5,5,\ldots\}$$

$$A_4=\{5,5,5,5,5,5,\ldots\}$$

$$A_5=\{6,\ldots\}$$

$$A_6=\{7,\ldots\}$$

$$\vdots$$

Clearly, all these sequences are distinct.

ELI L. ISAACSON
New York University

Part (a) also solved by Jeffrey Shallit and the proposer.

Answers

Solutions to the Quickies which appear near the beginning of the Problems section.

Q662. Divide the denominator by $|z_1z_2z_3z_4z_5|$ and use $1/z=\bar{z}$ to obtain (with $z_6=z_1$) $R=|\Sigma z_iz_{i+1}|<\Sigma|z_iz_{i+1}|=5$. The idea of this problem is based on Problem E3528 of A. A. Bennett in the *American Mathematical Monthly,* 39 (1932) page 115.