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January 19, 1972

Dr. Neil J. A. Sloane
Bell Laboratories
600 Mountain Avenue
Murray Hill, N.J. 07974

Dear Neil:

✓ Elinor has noted another error: The terminal zero on the last term of seq. 1799, p. 115 has been dropped. The term evidently is the product of the first 22 primes, thus requiring a zero on the end.

The number of prime factors in each term gives the sequence 2, 4, 9, 22... This may or may not be your seq. 459, p. 31. We haven't checked the original reference.

Sincerely,

Herman

Herman P. Robinson

HPR:fm

We did check the original reference and couldn't fathom it.

Enclosed is an extension of seq. 1086, p. 70 This comes from exp(sin x).

Elinor has a list of alternate descriptions of ~~&~~ several sequences. Copy attached.

ent

	page	sequence	
2593	145	2262	$\Sigma (2n + 1)^3$
2309	150	2327	$\Sigma (2n + 1)^4$
2594	151	2354	$\Sigma (2n + 1)^5$
1043	63	968	sums of primes = $p_n + p_{n+1}$
1550	66	1020	$1^n + 2^n + 3^n$
1551	89	1375	$1^n + 2^n + 3^n + 4^n$
1552	102	1584	$1^n + 2^n + 3^n + 4^n + 5^n$
1553	110	1723	$1^n + 2^n + 3^n + 4^n + 5^n + 6^n$
1554	119	1850	$1^n + 2^n + 3^n + 4^n + 5^n + 6^n + 7^n$
1555	123	1914	$1^n + 2^n + 3^n + 4^n + 5^n + 6^n + 7^n + 8^n$
1556	127	1977	$1^n + 2^n + 3^n + 4^n + 5^n + 6^n + 7^n + 8^n + 9^n$
1557	129	2014	$1^n + 2^n + 3^n + 4^n + 5^n + 6^n + 7^n + 8^n + 9^n + 10^n$
2017	70	1086	exp (sin x)

Elinor Potter

1/19/72

January 24, 1972

Mr. H. P. Robinson and Ms. Elinor Potter
University of California
Lawrence Radiation Laboratory
Berkeley, California 94720

Dear Elinor and Herman:

Thank you very much for your letter of January 19 with more corrections. The number of primes in sequence 1799 is the number of primes $\leq 3^i$, $i = 1,2,\ldots$, and begins 2,4,9,22,53,..., and so is different from sequence 459. (It would have been a truly remarkable result if they had turned out to be the same, for sequence 459 is the number of solutions of certain inequalities, and ought to have nothing to do with primes. Too bad!)

It turned out there was a bug in a subroutine that was sometimes called when the first term on the second line of the sequence is too big to fit on that line. That was why the last digit was missing in a few sequences. I think I have caught all of these errors now - I had already caught Seq. 1799. But I would never have noticed it if it hadn't been for you.

Thanks very much also for Seq. 1086 and the new descriptions. Bravo!

Best regards,

MH-1216-NJAS-ls

N. J. A. Sloane

January 18, 1972

Mr. H. P. Robinson and Ms. Elinor Potter
University of California
Lawrence Radiation Laboratory
Berkeley, California 94720

Dear Elinor and Herman:

I really don't know how to begin to thank you for all the work you have put into checking my manuscript. I am very, very grateful. And future readers will also owe you a great deal, for all the errors you have found. I thought I had been very careful, checking and rechecking each sequence when it was entered; but the moral is that there are always mistakes (except perhaps for D. N. Lehmer's "Factor Table for the First Ten Millions", which is said to be error-free!).

To answer some of your questions: the type numbers (TT2 etc.) were dropped because there was too much ambiguity in them. Type 1 overlapped with type 3, type 4 with type 1, and so on. (There are explicit formulas for the primes; you can find a recurrence for just about anything you can describe mathematically; often an explicit formula is so complicated as to be useless practically, ...)

My planetary diameters were taken from a nineteenth century book of tables (a fact I had forgotten), and it is just as well you checked them!

Your suggestions are all very good, and I'm adopting almost all of them. In a couple of cases even though there is room for an extra term at the end, I can't fit it in because of the way the table is made (the last stage is a left shift of the second line by up to 6 characters, in order to improve the appearance, a device suggested by the type-setter).

It is especially nice of you to have continued out so many of the sequences. Each time this happens I feel like a stamp collector who, after years of searching through dealers' stores and auctions, has finally managed to complete a particularly rare set of stamps.

Your suggestions for giving more explicit formulas for some of the sequences is excellent.

In going through my files of sequences, I've come across a few real numbers you may want to consider for your collection. They are jotted down more or less at random.

Some Interesting Real Numbers

1. Robinson's constant $4e^{-3/2}/\pi = 0.284098...$; see "Robinson's Constant", American Mathematical Monthly, 59 (1952), pp. 296-7.

2. The first few zeros of the Riemann zeta function, and various related numbers; see reference RS7 of my book.

3. I expect you already have the coefficients c_i for the asymptotic series for $\Gamma(z)$, from Wrench, Math. Comp., July 1968. Ten further values are given by Spira, loc. cit., April 1971, p. 321.

4. Values of the Howland integrals I_k and I_k^*, see Math. Comp., April 1971, pp. 334-5 for odd values of k, and loc. cit. 15 (1961) pp. 12-18 for k even.

5. When I was looking for sequences of integers I looked at every page of the 25 volumes of Math. Tables & Aids for Computation (later Math. Comp.; henceforth abbreviated MTAC) (as well as many other journals) and I strongly recommend that you do the same. It is packed full of interesting numbers for your table. Other journals that it would be worth searching through carefully are "BIT"; Numerische Mathematik; the Canadian Journal of Mathematics; and the Canadian Bulletin of Mathematics.

6. Two other sources that you probably already know about are:

A. Fletcher, J. C. P. Miller, L. Rosenhead, and L. J. Comrie, An Index of Mathematical Tables, Second Edition, 1962. (One of the great works of this century. It is medieval in its grandeur - has the scope of the Divine Comedy.)

H. T. Davis, Tables of the Mathematical Functions, Principia Press of Trinity University, San Antonio, Texas, 3 volumes.

7. It would be useful to have all the reciprocals of integers from 2 to 999; all the fractions p/q for $p \leq q \leq 99$; all the square, cube, ... sixth roots and their reciprocals up to some point. (At present it is not easy to tell what exactly is in the table: it might be helpful to have a brief survey of the contents of the table at the beginning.)

There are of course many other important rational numbers that you could include. Some nice examples are given in my ref. RG1, page 414 etc.

8. Tables of spheroidal harmonics, by H. Bateman, ref. Messenger of Math 54 (1924) pp. 73-75 (the article is pp. 71-78).

9. Tables of values of Bernoulli polynomials etc., see back of my ref. NO1.

10. There is a table of 2n ln n, n integer, (but only to 3D) in Jnl. Res. N.B.S., 66B (1962) p. 229 etc.

11. My ref. KNAW 66 (1963) p. 737 etc. may have some nice numbers. (I don't have the whole article.) This is another journal worth looking through. The articles containing tables are not frequent, but are very good when you find them. The same is true of the Jnl. Res. N.B.S.

12. From S. Chandrasekhar, Stochastic Problems ..., Review of Modern Physics, Vol. 15, Jan. 1943; reprinted in Nelson Wax (ed.), Selected papers on noise etc., Dover, 1954:

Page 73, $H(\beta) = \frac{2}{\pi\beta} \int_0^\infty \exp[-(x/\beta)^{3/2}] x \sin x \, dx$;

$H(\frac{1}{2}) = 0.094601$, $H(1) = 0.271322$, $H(2) = 0.33918$, etc. Not many decimal places.

Page 87, a basic coefficient is $\Gamma(4/3)/(4\pi/3)^{1/3} = 0.55396$.

- to be continued -

Thank you again for all your help.

Yours sincerely,

MH-1216-NJAS-ls N. J. A. Sloane

note spelling

August 2, 1971

Mr. H. P. Robinson and Ms. Elinor Potter
University of California
Lawrence Radiation Laboratory
Berkeley, California 94720

Dear Mr. Robinson:

Thank you very much indeed for your letter of July 12, and for the hard-cover copy of UCRL-20418.

Your suggestions and comments are excellent, and in fact in the latest version of the catalog the sequences are numbered, and Lehmer's cotangent sequence and the new Mersenne prime are included. In fact I have just finished searching through Lehmer's Guide to Tables in the Theory of Numbers for sequences - it was an excellent source. Also Fletcher, Miller, Rosenhead and Comrie's Index, and many similar sources. I have spent a lot of time in the Brown University library (one of the best mathematics libraries in the world) looking up obscure references or simply searching through the shelves for sequences.

Consequently the latest version is rather fat (250 pages and 2800 sequences), and it will be a week or two before I can send you a copy (which by then will almost be the final version). If you would be willing to give it a critical reading before it goes to press, I would be very grateful (and so would Academic Press). I would really like to have your comments on it.

Do you happen to recall if Prof. Woodyard published anything on that cable splicing problem, the one that involved the sequences 1,2,5,15,52,203,877,4140,... and 1,2,4,7,14,29, 60,127,275,...? I am trying to give as many different references to the sequences as possible.

Table III of UCRL-20418 is very interesting. I have often thought of making an index to my catalog in just this way - for each integer give a list of all the sequences containing it. But just cataloging the sequences has taken enough time as it is.

Do you think Professor Lehmer would also like to see a copy of the latest catalog?

With best wishes

Yours sincerely,

MH-1216-NJAS-bk

N. J. A. Sloane

September 29, 1971

Dr. H. P. Robinson
Lawrence Radiation Laboratory
University of California
Berkeley, California 94720

Dear Herman:

Thank you for your letters of September 10 and 17. Your suggestion to have a few blank pages at the back of the book for extra sequences is excellent, and I shall pass it on to Academic Press. The sequence in your second letter was already in the table, I am happy to say. I am still adding some new sequences, though, such as the old chestnut 1,4,9,61,52,63,94,... .

Sincerely,

MH-1216-NJAS-ls N. J. A. Sloane

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September 10, 1971

Dr. Neil J. A. Sloane
Bell Laboratories
600 Mountain Avenue
Murray Hill, New Jersey 07974

Dear Neil:

This will acknowledge your letter of August 29, 1971, and make one more suggestion for your Catalog: It would be convenient to have a few blank pages, perhaps lined at the back of the book for entry of additional sequences. I know I'll be adding some.

Sincerely,

Herman

H. P. Robinson

HPR:dr

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September 17, 1971

Dr. N.J.A. Sloane
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600 Mountain AVenue
Murray Hill, N.J. 07974

Dear Neil:

Although you say you are no longer collecting sequences, they may be forced upon you. I encountered the following, which you may or may not have:

1,2,4,5,7,9,10,12,14,16,17,...

where one odd number is followed by two evens, then three odds, etc. This is type TT1. See Ian Connell, Am. Math. Monthly 67 380 (1960).

Sincerely,

Herman

H. P. Robinson

HPR:fm

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November 5, 1971

Dr. Neil J. A. Sloane
Bell Laboratories
600 Mountain Avenue
Murray Hill, N.J. 07974

Dear Neil:

I ran across two more sequences which you may or may not have:

"The Numbers Generated by Exp (arctan X)" by Richard Kelisky, Duke Math. Jour. 26, 572-581 (1959). T_n = 1,1,-1,-7,5,145,-5, -6095,-5815,433025,956375,-46676375,... and U_n = 1,1,1,-5,-17, 83,593,-2893,-36101,172195,... John Woodyard suggested the Pythagorean sequence of c in $a^2 + b^2 = c^2$: 5,13,17,25,29,37,...

Sincerely,

Herman

Herman P. Robinson

HRP:fm

ext 5068