Scan

Add to

A1599 etc

(8 seq.)

R.K. Guy,
letter to NJAS
Add to the A-numbers
listed

√91

1599 5934
5568 5835
~~6036~~
2975
~~6037~~

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June 13, 1991

→ 6036
– ~~6039~~
6041

Neil J.A. Sloane,
A.T.&T. Bell Laboratories 2C-376,
600 Mountain Avenue,
Murray Hill NJ 07974, U.S.A.

Dear Neil,

Selfridge & I have just had faxxes from you, so it seems economical to send a combined reply.

John had already written:

"The reference is a paper

Carole B. Lacampagne & J.L. Selfridge, Large highly powerful numbers are cubeful, *Proc. Amer. Math. Soc.*, **91**(1984) 173–181.

Table 2 factors the 25 smallest highly powerfuls. We did not count 1 in our list

4	128	864	5184	31104
8	144	1296	7776	41472
16	216	1728	10368	62208
32	288	2592	15552	86400
64	432	3456	20736	108000

5934

Another reference (with a longer tabulation) is a report by

G.E. Hardy & M.V. Subbarao, Highly powerful numbers, *Congressus Numerantium*, **37**(1983) 277–307."

John didn't seem to think that I'd made a very serious omission in defining pseudoperfect numbers. Most readers would solve the problem easily and move on to the intended problem. I've set him to work calculating a few more members of the sequence A5835

6, 12, 18, 20, 24, 28, 30, 36, 40, 42, 48, 54, 56, 60, 72, 80, 84,

[A little later: Selfridge notes that any multiple of a pseudoperfect number is pseudoperfect, so that, on the one hand, Sloane has missed out 66 & 78, and on the other, it would

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I have abundant (A5101) nos) ~~but not~~ and odd primitive abundant (5934)

[A little later: Selfridge notes that any multiple of a pseudoperfect number is pseudoperfect, so that, on the one hand, Sloane has missed out 66 & 78, and on the other, it would be more interesting and economical to list only the **primitive** members of the sequence:

6, 20, 28, 88, 104, 272, 304, 350, 368, 464, 490, 496, 550, 572, 650, 748, 770, 910, 945, 1184, 1190, 1312, 1330, 1376, 1430, 1504, 1575, 1610, 1696, 1870, 1888, 1952, 2002, 2030, 2090, 2170, 2205, 2210, 6036

91

If you want **all** the pseudoperfect numbers, then I make it

6, 12, 18, 20, 24, 28, 30, 36, 40, 42, 48, 54, 56, 60, 66, 72, 78, 80, 84, 88, 90, 96, 100, 102, 104, 108, 112, 114, 120, 126, 132, 138, 140, 144, 150, 156, 160, 162, 168, 174, 176, 180, 186, 192, 196, 198, 200, 204, 208, 210, 216, 220, 222, 224, 228, 234, 240, 246, 252, 258, 260, 264, 270, 272, 276, 280, 282, 288, 294, 300, 304, 306, 308, 312, 318, 320, 324, 5835

f91

John suggests that at least equal time should be given to primitive abundant, weird (= abundant but not pseudoperfect) and harmonic numbers (not the harmonic numbers of S. 619 and S. 1157, but those also called "Ore numbers" — same ref. as in the next sentence).

For completeness, let me quote some chunks of *UPINT* **B2** p. 28 (though the page number will undoubtedly change in the 2nd edition)

"Garcia extended the list of harmonic numbers to include all 45 which are $< 10^7$, and he found more than 200 larger ones. The least one, apart from 1 and the perfect numbers, is 140. Are any of them squares, apart from 1? Are there infinitely many of them? If so, find upper and lower bounds on the number of them that are $< x$. Kanold has shown that their density is zero, and Pomerance that a harmonic number of the form $p^a q^b$ (p and q primes) is an even perfect number. If $n = p^a q^b r^c$ is harmonic, is it even?

Which values does the harmonic mean take? Presumably not 4, 12, 16, 18, 20, 22, ... ; does it take the value 23? Ore's own conjecture, that every harmonic number is even, implies that there are no odd perfect numbers!"

... ...

"Benkoski has called a number **weird** if it is abundant but not pseudoperfect. For example, 70 is not the sum of any subset of 1+2+5+7+10+14+35=74. There are 24 primitive weird numbers less than a million: 70, 836, 4030, 5830, 7192, ... Nonprimitive weird numbers include $70p$ with p prime and $p > \sigma(70) = 144$; $836p$ with $p = 421$, 487, 491, or p prime and ≥ 557; 7192×31. Some large weird numbers were found by Kravitz, and Benkoski & Erdős showed that their density is positive. Here the open questions are: are there infinitely many primitive abundant numbers which are weird? Is every odd abundant number pseudoperfect (i.e., not weird)? Can $\sigma(n)/n$ be arbitrarily large for weird n? Benkoski & Erdős conjecture "no" in answer to the last question and Erdős offers \$10 and \$25 respectively for solutions to the last two questions."

S.J. Benkoski, Problem E2308, *Amer. Math. Monthly*, **79**(1972) 774.

S.J. Benkoski & P. Erdős, On weird and pseudoperfect numbers, *Math. Comput.*, **28**(1974) 617–623; *MR* **50** #228; corrigendum, S. Kravitz, *ibid.*, **29**(1975) 673.

Mariano Garcia, On numbers with integral harmonic mean, *Amer. Math. Monthly*, **61**(1954) 89–96; *MR* **15**, 506, 1140.

Here is a list of weird numbers less than 10^6. The only weird numbers less than 222952 that are not in this list are of form $70p$ with p a prime greater than 144. We've put these in (with a *) only up to 20510*. The unstarred numbers have been called primitive weird, but their multiples are not normally weird, so the name may not be a good one. The item marked ‡ is erroneously printed in the B & E paper (p. 618) as 539774.

70, 836, 4030, 5830, 7192, 7912, 9272, 10430* 10570* 10990* 10792, 11410* 11690* 12110* 12530* 12670* 13370* 13510* 13790* 13930* 14770* 15610* 15890* 16030* 16310* 16730* 16870* 17272, 17570* 17990* 18410* 18830* 18970* 19390* 19670* 19810* 20510* 45356, 73616, 83312, 91388, 113072, 243892, 254012, 338572, 343876, 338076, 519712, 539744‡, 555616, 682592, 786208, 2975 6037(*)

John has gone over to the library in search of

L.E. Dickson, *Amer. J. Math.*, **35**(1913) 413–426

and a list of **primitive abundant** numbers

He's back! The odd primitive abundant numbers are (p. 422 of the above reference).

945, 1575, 2205, 3465, 4095, 5355, 5775, 5985, 6435, 6825, 7245, 7425, 8085, 8415, 8925, 9135, 9555, 9765, 11655, 12705, 12915, 13545, 14805, 15015, 6038

There's also

L.E. Dickson, *Quart. J. Math.*, **44**(1913) 274–277, od

which tabulates even abundant numbers < 6232, but John only consulted the first paper in compiling the following list of **primitive** non-deficient (= abundant or perfect) numbers (p. 426 of the first reference).

6, 20, 28, 70, 88, 104, 272, 304, 368, 464, 496, 550, 572, 650, 748, 836, 945, 1184, 1312, 1376, 1430, 1504, 1575, 1696, 1870, 1888, 1952, 2002, 2090, 2205, 2210, 6039

[91-06-13-12:30 I find I missed out the harmonic numbers — John has just gone over to the Library to look up (p. 95 of) the Garcia reference. Here's the result:

6, 28, 140, 270, 496, 672, 1638, 2970, 6200, 8128, 8190, 18600, 18620, 27846, 30240, 32760, 55860, 105664, 117800, 167400, 173600, 237510, 242060, 332640, 360360, 539400, 695520, 726180, 753480, 950976,] 1599

I guess that the "3 Richards" are Richard Bruce Austin, Richard Kenneth Guy & Richard Joseph Nowakowski, but I'm unable to guess the content of the paper, so the answer to your qu: was the paper published? is evidently "no!"

I won't fax my pubs list, but I will put a copy in the mail (have done so, 91-06-12-10:30).

Good idea to use section numbers in *UPINT* as I do in the Indexes. These won't change in the 2nd edition, but the page numbers will. Similarly for *UPIG*, copies of which arrived today.

The products of pairs of consecutive Catalan numbers are (A5568)

1, 2, 10, 70, 588, 5544, 56628, 613470, 6952660, 81662152, 987369656, 12228193432, 154532114800, 1986841476000, 25928281261800, 342787130211150, 4583937702039300, 61923368957373000, 844113292629453000, 11600528392993339800, 160599522947154548400, 5568

For what it's worth, the two erroneous values that I sent on 90-05-07 are just 50% larger than they should be.

Next:– to your snail-mail letter of 91-06-03.

First query (α): Mahler-Popken function (see S^4 enclosed).

I just managed to get the secretary to slit open the envelope about to wing its way to you, and insert item 152 from my bibliography, Some Suspiciously Simple Sequences, *Amer. Math. Monthly*, **93**(1986) 186–190, esp. 189 (top). Here is another excerpt from *UPINT*:

F26

Let $f(n)$ be the least number of ones that can be used to represent n using ones and any number of + and × signs (and parentheses). For example,

$$80 = (1+1+1+1+1) \times (1+1+1+1) \times (1+1+1+1)$$

so $f(80) \le 13$. It can be shown that $f(3^k) = 3k$ and $3\log_3 n \le f(n) \le 5\log_3 n$ where the logs are to base 3. Does $f(n) \sim 3\log_3 n$?

J.H. Conway & M.J.T. Guy, π in four 4's, *Eureka*, **25**(1962) 18–19.

Richard K. Guy, Some suspiciously simple sequences, *Amer. Math. Monthly*, **93**(1986) 186–190; and see **94**(1987) 965 & **96**(1989) 905.

K. Mahler & J. Popken, On a maximum problem in arithmetic (Dutch), *Nieuw Arch. Wiskunde*, (3) **1**(1953) 1–15; *MR* **14**, 852e.

Next query (β): was Richard Nowakowski's MSc thesis published? No. The details are Langford-Skolem problems, MSc thesis, The University of Calgary, 1975. We still plan to publish a paper on this. (Item 129. in guybib should be marked "in preparation"!)

At this point (91-06-12-13:00 ?) your phone call came through. No, the magnum opus hasn't yet arrived, but JLS is here until Fri, so he'll get a chance to look at it.

Query (γ) Was the paper of Joseph Zaks, On problems of Doyen, Guy & Stechkin, ever published? Almost certainly not, now that I unearth my referee's report (copy enclosed), which probably rendered the paper unnecessary (incidentally, through your good offices, or those of someone else — I couldn't have had the wit to have discovered it myself — by the time I wrote the report I knew what the magic numbers were). The sequences may be worth including, though, but how do you reference them? Perhaps (Numbers of) Paths between vertices of a complete bipartite graph GU163 (wrc). Here are some more values [n vertices in each part; starting with $n = 1$] 6040

(vertices in same part) 1, 2, 9, 82, 1313, 32826, 1181737, 57905114, 3705927297, 300180111058, 30018011105801, 3632179343801922, $a_{n+1} = n^2 a_n + 1$ makes it easy, I discover!

(vertices in different parts) 0, 2, 9, 76, 1145, 27486, 962017, 46176824, 2909139921, 232731193690, 23040388175321, 2764846581038532, $b_{n+1} = (n^2-1)b_n + n + 1$ 6041

Query (δ) Erdős, Guy, Moon is item 91. in guybib:

P. Erdős, Richard K. Guy & J.W. Moon, On refining partitions, *J. London Math. Soc.* (2), **9**(1974) 565–570; *MR* **50** #12752; *Zbl* **312**.05008; *RZh* 1975 11V331. od

6/91

Query (ϵ): GU8 is essentially one or both of the papers

79. Richard K. Guy, Sedlácek's conjecture on disjoint solutions of $x + y = z$, (*Proc. Conf. Number Theory*, Pullman WA, 1971) 221–223; *Zbl* **241**.05115.

99. Richard K. Guy, Packing $[1, n]$ with solutions of $ax + by = cz$; the unity of combinatorics, *Colloq. Internaz. Teorie Combinatorie*, Rome, 1973 *Atti Conv. Lincei*, **17**, Accad. Naz. Lincei, Rome, 1976 Tomo II, 173–179. *MR* **57** #9565; *Zbl* **361**.05012.

and in Richard Nowakowski's MSc thesis mentioned above. By luck I flipped through the Book and found S. 368 (though there are possibly others). Richard may be able to extend this (and others) for a few terms — I'll try to remember to ask him.

Query (ϕ – your knowledge of the Greek alphabet seems to be almost as lamentable as mine: I would have put ζ): if "it" refers to the Book, then what you want is

106. Richard K. Guy, Book review: A Handbook of Integer Sequences, by N.J.A. Sloane, *Zentralblatt für Math.*, **286**.10001.

Query ($\phi + 1!$ — that's factorial one): Yes:

117. Richard K. Guy, Anyone for Twopins? in D.A. Klarner (editor) The Mathematical Gardner, Prindle Weber & Schmidt, 1981, pp. 2–15.

Thanks for the A. S. & Shor paper on Dav-Sch sequences — it will be enshrined in UPINT2.

Best wishes,

Yours sincerely,

Richard.

Richard K. Guy,
Emeritus Professor of
Mathematics.

RKG/rkg

encl: ref's report on a paper of Zaks.

(Type on p— ...

Refs report on:-

ON PROBLEMS OF DOYEN, GUY AND STECHKIN

Joseph Zaks

Doyen [1] asked: "For which integers n is it true that any two distinct vertices of $K_{n,n}$ are connected by the same number of paths (known are $n = 1, 2, 3$)?" Guy added the editorial conjecture: "For $n \geq 4$, the number of paths between vertices in the same part is less than the number between vertices in different parts."

To see that the conjecture is true, let $f(n)$ [resp. $g(n)$] be the number of distinct paths between vertices of the same [resp. different] parts of $K_{n,n}$ $(n \geq 2)$. Then $f(2) = g(2) = 2$, $f(3) = g(3) = 9$, and a direct counting argument gives

$$g(n) = 1 + (n-1)^2 + (n-1)^2(n-2)^2 + (n-1)^2(n-2)^2(n-3)^2 + \dots + [(n-1)!]^2$$

$$f(n) = n + n(n-2)(n-1) + n(n-2)(n-1)(n-3)(n-2) + \dots + n!(n-2)!$$

After the term 1, the terms of $g(n)$ are greater than the corresponding ones of $f(n)$, apart from the last. But the difference between the last two terms of $g(n)$ and the last two of $f(n)$ is $(n-1)(n-2)^2(n-3)^2 \dots 4^2 3^2 2(n-4)$, so that $g(n) > f(n)$ for $n \geq 4$.

As a byproduct, note that for $n \geq 2$, $f(n)$ and $g(n)$ have the same parity as n, and since $K_{n,n}$ is Hamiltonian for $n \geq 2$, odd values of n provide counter-examples to a conjecture of Stechkin [2]

¿ A graph is Hamiltonian if and only if there is an even number of paths between any two of its vertices (possibly not counting the edge, in case of adjacent vertices)?

Note also that as $n \to \infty$,

$$g(n)/[(n-1)!]^2 = 1 + 1 + \frac{1}{(2!)^2} + \frac{1}{(3!)^2} + \cdots + \frac{1}{[(n-1)!]^2}$$

$$\sim I_0(2) = \frac{1}{\pi}\int_0^\pi e^{2\cos\theta}\, d\theta = 2.2795853023\ldots$$

$$f(n)/[(n-1)!]^2 = \frac{n}{n-1}\left(1 + \frac{2}{(2!)^2} + \frac{3}{(3!)^2} + \cdots + \frac{n-1}{[(n-1)!]^2}\right)$$

$$\sim I_1(2) = I_0'(2) = 1.5906368546\ldots$$

where I_i is the modified Bessel function of order i, so that

$$g(n)/f(n) \sim I_0(2)/I_1(2) = 1.4331274267\ldots$$

1. J. Doyen, Problem 20 in Richard K. Guy (ed.) Problems, Tagung über Graphentheorie, Oberwolfach, Aug. 1979.
2. B. Stechkin, Conjecture 2 in A. Hajnal and V.T. Sós (eds.) Combinatorics Proc. Conf. Keszthely, 1976, North-Holland, 1978.

$$\frac{n!(n-2)!}{[(n-1)!]^2} + \frac{n!(n-2)!}{1.2((n-2)!)^2}$$

$$\frac{n}{n-1}\left(1 + \frac{1}{1.2} \cdots\right.$$