By addition of (A) and (B),

$$\alpha^2 = \alpha_1^2 + \beta_1^2,$$

which, on writing $x^{\frac{1}{8}}$ for x, is 18.

We are now in the position to prove 19; for

$$(1 + 2x + 2x^4 + \ldots)^4 - (1 - 2x + 2x^4 - \ldots)^4$$
$$= \{(\alpha + \beta)^2 + (\alpha - \beta)^2\}\{(\alpha + \beta)^2 - (\alpha - \beta)^2\}$$
$$= 8(\alpha_1 + \beta_1)^2 \alpha\beta \qquad\qquad \text{by (A)}$$
$$= 16\{1 + 2x^2 + 2x^8 + \ldots\}^2\{x^{\frac{1}{2}} + x^{\frac{9}{2}} + x^{\frac{25}{2}} + \ldots\}^2,$$

on substituting for α_1, β_1 their values, and using 17 with x^4 written for x, which transforms $\frac{1}{2}\alpha\beta$ into the second squared factor,

$$= 16(x^{\frac{1}{4}} + x^{\frac{9}{4}} + x^{\frac{25}{4}} + \ldots)^4,$$

by applying 17 again, x^2 being therein written for x; and the formula 19 is thus established by elementary algebra in a most simple and elegant manner.

The formula 18 is not required in the proof, but was no doubt added by Gauss for the sake of completeness.

Trinity College, Cambridge,
April 25, 1874.

LVII. *On the Mathematical Theory of Isomers.* By Professor CAYLEY, *F.R.S.*[*]

I CONSIDER a "diagram," viz. a set of points H, O, N, C, &c. (any number of each), connected by links into a single assemblage under the condition that through each H there passes not more than one link, through each O not more than two links, through each N not more than three links, through each C not more than four links. Of course through every point there passes at least one link, or the points would not be connected into a single assemblage.

In such a diagram each point having its full number of links is saturate, or nilvalent: in particular each point H is saturate. A point not having its full number of links is univalent, bivalent, or trivalent, according as it wants one, two, or three of its full number of links. If every point is saturate the diagram is saturate, or nilvalent; or, say, it is a "plerogram;" but if the diagram is susceptible of n more links, then it is n-valent; viz. the valency of the diagram is the sum of the valencies of the component points.

Since each H is connected by a single link (and therefore to

a point O, C, &c. as the case may be, but not to another point H), we may without breaking up the diagram remove all the points H with the links belonging to them, and thus obtain a diagram without any points H : such a diagram may be termed a "kenogram:" the valency is obviously that of the original diagram *plus* the number of removed H's.

If from a kenogram we remove every point O, C, &c. connected with the rest of the diagram by a single link only (each with the link belonging to it), and so on indefinitely as long as the process is practicable, we arrive at last at a diagram in which every point O, C, &c. is connected with the rest of the diagram by two links at least : this may be called a "mere kenogram."

Each or any point of a mere kenogram may be made the origin of a "ramification;" viz. we have here links branching out from the original point, and then again from the derived points, and so on any number of times, and never again uniting. We can thus from the mere kenogram obtain (in an infinite variety of ways) a diagram. The diagram completely determines the mere kenogram; and consequently two diagrams cannot be identical unless they have the same mere kenogram. Observe that the mere kenogram may evanesce altogether; viz. this will be the case if the diagram or kenogram is a simple ramification.

A ramification of n points C is $(2n+2)$ valent : in fact this is so in the most simple case $n=1$; and admitting it to be true for any value of n, it is at once seen to be true for the next succeeding value. But no kenogram of points C is so much as $(2n+2)$-valent; for instance, 3 points C linked into a triangle, instead of being 8-valent are only 6-valent. We have therefore plerograms of n points C and $2n+2$ points H, say plerograms $C H^{2n+2}$; and in any such plerogram the kenogram is of necessity a ramification of n points C; viz. the different cases of such ramifications are*

$n=1.$ $\qquad n=2.$ $\qquad n=3.$ $\qquad\qquad n=4.$

$\quad(\alpha)\qquad\quad(\alpha)\qquad\quad(\alpha)\qquad\quad(\alpha)\qquad(\beta)$
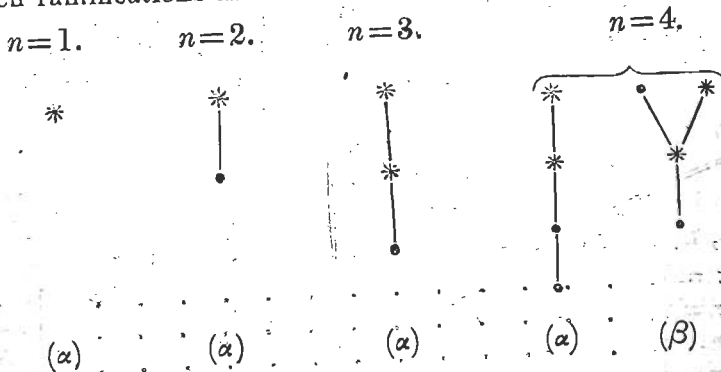
* The distinction in the diagrams of asterisks and dots is to be in the first instance disregarded; it is made in reference to what follows, the explanation as to the allotrious points.

$n=5.$ $n=6.$

(α)   (β)   (γ)

(α)   (β)   (γ)   (δ)
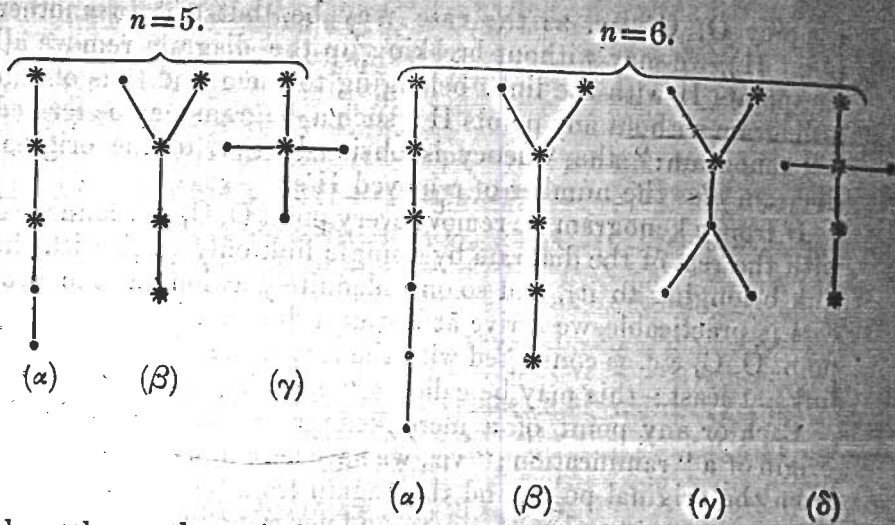
where the mathematical question of the determination of such forms belongs to the class of questions considered in my paper "On the Theory of the Analytical Forms called Trees," Phil. Mag. vol. xiii. (1857) and vol. xv. (1859), and in some papers on Partitions in the same Journal.

The different forms of univalent diagrams $C^n H^{2n+1}$ are obtained from the same ramifications by adding to each of them all but one of the $2n+2$ points H; that is, by adding to each point C except one its full number of points H, and to the excepted point one less than the full number of points H. The excepted point C must therefore be univalent at least; viz. it cannot be a saturate point, which presents itself for example in the diagrams $n=5(\gamma)$ and $n=6(\delta)$. And in order to count the number of distinct forms (for the diagrams $C^n H^{2n+1}$), we must in each of the above ramifications consider what is the number of distinct classes into which the points group themselves, or, say, the number of "allotrious" points. For instance, in the ramification $n=3$ there are two classes only; viz. a point is either terminal or medial; or, say, the number of allotrious points is $=2$: this is shown in the diagrams by means of the asterisks; so that in each case the points which may be considered allotrious are represented by asterisks, and the number of asterisks is equal to the number of allotrious points.

Thus, number of univalent diagrams $C^n H^{2n+1}$:—

$n=1$, ... 1
$n=2$, ... 1
$n=3$, ... 2
$n=4$, (α) 2; (β) 2; . . . . . . together 4
$n=5$, (α) 3; (β) 4; (γ) 1; . . . „ 8
$n=6$, (α) 3; (β) 5; (γ) 2; (δ) 3; . „ 18

where it will be observed that, $n=5(\gamma)$, and $n=6(\delta)$, the numbers of allotrious points are 2 and 4 respectively; but since in each of these cases one point is saturate, they give only the numbers 1 and 3 respectively. It might be mathematically possible to obtain a general solution; but there would be little use in this; and for even the next succeeding case, No. of bivalent diagrams $C^n H^{2n}$; the extreme complexity of the question would, it is probable, prevent the attainment of a general solution.

Passing to the chemical signification of the formulæ, and instead of the radicals $C^n H^{2n+1}$ considering the corresponding alcohols $C^n H^{2n+1}$. OH, then, $n=1, 2, 3, 4$, the numbers of known alcohols are 1, 1, 2, 4, agreeing with the foregoing theoretic number (see Schorlemmer's 'Carbon Compounds,' 1874); but $n=4$, the number of known alcohols is $=2$, instead of the foregoing theoretic number 8. It is, of course, no objection to the theory that the number of theoretic forms should exceed the number of known compounds; the missing ones may be simply unknown; or they may be only capable of existing under conceivable, but unattained, physical conditions (for instance, of temperature); and if defect from the theoretic number of compounds can be thus accounted for, the theory holds good without modification. But it is also possible that the diagrams, in order that they may represent chemical compounds, may be subject to some as yet undetermined conditions; viz. in this case the theory would stand good as far as it goes, but would require modification.

LVIII. *Proceedings of Learned Societies.*

ROYAL SOCIETY.

December 11, 1873.—Joseph Dalton Hooker, C.B., President, in the Chair.

THE following communication was read:—
"A Quantitative Investigation of certain Relations between the Gaseous, the Liquid, and the Solid States of Water-Substance." By Professor James Thomson, LL.D., Queen's College, Belfast.

In two communications made by me to the British Association, at its Meetings at Edinburgh in 1871 and at Brighton in 1872, and printed as abstracts in the Transactions of the Sections for those years, considerations were adduced on relations between the gaseous, the liquid, and the solid states of matter. The new subject of the present paper constitutes a further development of some of those previous considerations; and a brief sketch of these

L. *On the new Contact Theory of the Galvanic Cell.*
By J. A. FLEMING, *B.Sc.*, *F.C.S.**

THE contest that has for so long been waged between the sup-
porters of the two theories of the galvanic cell, the *contact*
and the *chemical,* can hardly be said to have been brought even
now to a decisive issue. For although the contact theory, as
originally proposed by Volta, received a fatal blow when the law
of conservation of energy became clearly understood, yet in its
place a *new contact theory* has arisen, supported by novel and
important experimental evidence, which has again been placed
by recent writers on electrical science in formidable opposition
to its old rival.

The old contact theory of Volta had its origin in an entire
ignorance of the science of energy. It simply referred the cur-
rent produced through the circuit of a pile to the effect of the
metallic contacts, and it ignored the thermal and chemical
changes which are also necessarily present; but it had to be
finally abandoned when once it became clearly understood that
the appearance of a current involved the disappearance of some
other energy, actual or potential, as an invariable accompani-
ment. The new contact theory may be said to have had its
source in the discovery of Sir W. Thomson, that there is un-
doubtedly a difference of potential produced when dissimilar
metals are placed in contact—a fact not only abundantly proved
by Thomson by direct experimental evidence, but, as he has