Finding the distance between two points on the Earth's surface

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The Haversine Formula

The haversine formula is used to find the distance d between two points with longitude and latitude (ψ, ϕ).

$$d = 2r \arcsin\left(\sqrt{\sin\left(\frac{\phi_2 - \phi_1}{2}\right)^2 + \cos(\phi_1)\cos(\phi_2)\sin\left(\frac{\psi_2 - \psi_1}{2}\right)^2}\right)$$

Where r is the radius of the Earth.

Example

Find the distance between the Fermilab laboratory in Illinois, USA and CERN's Meyrin campus in Switzerland.

Fermilab is located at $41°49'55"$N, $88°15'26"$W and CERN'S Meyrin campus at $46°14'03"$N, $06°03'10"$E. In decimal format this is:

$$
\begin{aligned}
(\phi_1, \psi_1) &= 41.8319°, -88.2572° \\
(\phi_2, \psi_2) &= 46.2342°, 6.05278°
\end{aligned}
$$

These values must be converted to radians before they can be used.

$$
\begin{aligned}
(\phi_1, \psi_1) &= 0.730104, -1.54038 \\
(\phi_2, \psi_2) &= 0.806939, 0.105641
\end{aligned}
$$

Inserting these values into the haversine formula:

$$d = 2r \arcsin\left(\sqrt{\sin^2\left(\frac{0.806939 - 0.730104}{2}\right) + \cos(0.730104)\cos(0.806939)\sin^2\left(\frac{0.105641 + 1.54038}{2}\right)}\right.$$

$$= 2r\arcsin\left(\sqrt{\sin^2\left(\frac{0.076835}{2}\right) + \cos\left(0.730104\right)\cos\left(0.806939\right)\sin^2\left(\frac{1.646021}{2}\right)}\right)$$

$$= 2r\arcsin\left(\sqrt{\sin^2\left(0.0384175\right) + \cos\left(0.730104\right)\cos\left(0.806939\right)\sin^2\left(0.823011\right)}\right)$$

$$= 2r\arcsin\left(\sqrt{0.00147518 + 0.745105 \times 0.691712 \times 0.537577}\right)$$

$$= 2r\arcsin\left(\sqrt{0.00147518 + 0.277066}\right)$$

$$= 2r\arcsin\left(\sqrt{0.278541}\right)$$

$$= 2r\arcsin\left(0.527770\right)$$

$$= 2r \times 0.555973$$

Given that the radius of the Earth is $6371\,\mathrm{km}$ this gives the distance between the two points as $7084\,\mathrm{km}$.