A CHART FOR MEASUREMENT OF INTERFERENCE FIGURES[1]

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ABSTRACT

A simple but general chart has been developed for the solution of Mallard's formula and other relations used in the quantitative interpretation of interference figures produced by anisotropic crystals in convergent polarized light. This chart represents a general solution of the equations:

$$\frac{2D}{2} = K \sin \frac{2E}{2} = N_Y K \sin \frac{2V}{2} = NK \sin \frac{2H}{2}.$$

The chart can be used conveniently for certain types of orientation measurements as well as for the measurement of optic axial angles of biaxial crystals.

INTRODUCTION

Most mineralogists are familiar with the use of the petrographic microscope for measuring optical properties of crystalline materials. Two reference books commonly available in mineralogical libraries are Winchell (1937)[3] and Johannsen (1918), which cover the methods in some detail.

Several of the more convenient methods of measuring the optic axial angle 2V, or its equivalent in air 2E, or in water or other medium 2H, are based upon the use of a relationship known as Mallard's formula (Johannsen, 1918, art. 411). This formula is

$$D = K \sin E. \tag{1}$$

The quantity D is half the distance between the melatopes or points of emergence of the optic axes ("eyes") in a centered interference figure produced by a crystal section normal to the acute bisectrix. D is measured in terms of the linear units of the scale of an eyepiece fitted with a micrometer scale; K is a constant depending upon the optical system and the spacing of the scale divisions in the micrometer eyepiece; and E is half the apparent optic angle in air.

The relation between E, V, and N_Y is given by the following formula:

$$\sin E = N_Y \sin V. \tag{2}$$

In equation (2), E is as defined above; N_Y is the intermediate index of refraction of the crystal; and V is half the true angle between the optic axes.

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Equations (1) and (2) may be combined as follows:

$$D = K \sin E = KN_Y \sin V. \tag{3}$$

Rarely, the apparent optic angle 2H, in some other medium than air or the crystal may be desired. In this case the index of refraction of the medium will be denoted by N:

$$D = K \sin E = KN \sin H. \tag{4}$$

However, it is conventional to express these angles in terms of 2E, 2V, and 2H; likewise the distance between the two melatopes 2D, is the quantity actually measured, rather than the distance D, of one melatope from the center of the field. Therefore equations (1) to (4) will be more useful for most purposes if expressed in terms of 2D, 2H, and 2V:

$$\frac{2D}{2} = K \sin \frac{2E}{2} = KN \sin \frac{2H}{2} = KN_Y \sin \frac{2V}{2}. \tag{5}$$

The first part of equation (5) has been expressed graphically in the Schwartzmann optic angle scales (Winchell, 1937, p. 188; Johannsen, 1918, art. 413) which relate 2D and 2E. These scales are on many slide rules: D or 2D may be represented by the A scale of the slide rule; 2E, graduated in terms of E rather than of 2E, may be represented by the S scale of the slide rule. Each optical system requires its own characteristic setting of the slide rule (or of the Schwartzmann scales) to determine E or 2E from D or 2D. Two settings of the slide rule are required to convert the observed value, 2D, into the true optic angle 2V.

The Chart

Figure 1 is a logarithmic chart showing the relations expressed in equations (5). The 2D, K, and N scales are ordinary logarithmic scales similar to those used for multiplication on a slide rule; the 2E scale is a logarithmic sine scale divided like the S scale of a slide rule, but numbered differently. Like Schwartzmann's scales, this chart must be calibrated for the particular microscope and lenses used. Unlike the Schwartzmann scales, Fig. 1 may be *permanently* calibrated for *several* optical systems in one or more microscopes. One copy of this chart therefore suffices for use with all combinations of lenses in a given microscope, and may be calibrated therefor; similarly, one large copy of the chart would serve a laboratory equipped with several microscopes. This chart also differs from Schwartzmann's scales in showing 2V and 2H as well as 2E.

Calibration and Use of the Chart

The method of use of Fig. 1 is indicated briefly in the key at the top. The calibration is analogous to that used for setting the Schwartzmann

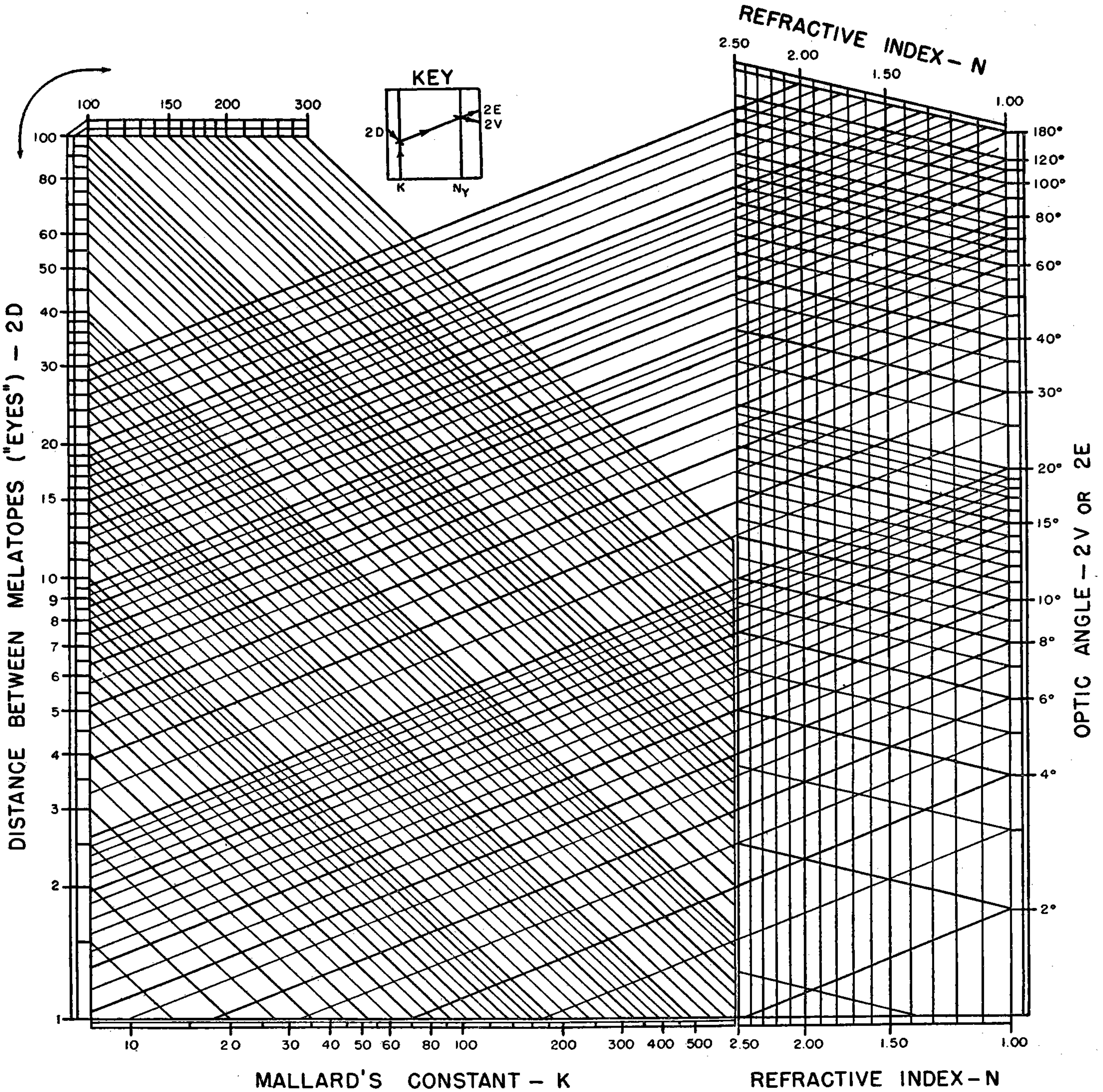

FIG. 1. Chart of $2D/2 = K \sin(2E/2) = N_Y K \sin(2V/2)$.

Measurement of optic axial angle from acute bisectrix interference figures.

1. Calibrate by drawing vertical line through Mallard's constant, K, for optical system in use.
2. Measure 2D in interference figure under study.
3. Follow lines as shown in key.

axial angle scales. The interference figure of an oriented section of a mineral of known optic angle is carefully measured and the result is used for calibrating the chart. A micrometer eyepiece is used to measure the distance 2D, between the melatopes. Sodium light should be used if possible, whenever an interference figure is to be observed and measured, for the sharpness of the isogyres is usually improved by monochromatic light, and nearly all published descriptions of natural and artificial crystalline compounds give optical properties measured in sodium light.

An example will illustrate the calibration and use of the chart. The following data were obtained for an oriented section of aragonite:

Microscope:	Spencer No. 37, Ser. No. 154974
Objective:	Spencer achromatic, 4-mm. dry
Eyepiece:	Spencer Huygenian, 10×
Micrometer:	20 divisions per millimeter
Bertrand lens:	Focused at 5.2

Diameter of field	58 (observed)
2D	18.5 (observed)
2E	31° (Winchell, 1933, p. 79)

These data were applied to the calibration of the chart as illustrated in Fig. 2. The sloping line from $2D=18.5$, and the sloping line from $2E=31°$ intersect at a point directly above $K=34.5$ on the horizontal scale. Through this point of intersection a vertical *calibration line* was drawn, extending from the K-scale upward to a terminus corresponding to $2D=58$, the diameter of the interference figure with this setup and hence the largest 2D that can be measured with it. This line was labelled to correspond with the optical system concerned. It is the calibration line for that system. By this construction, Mallard's constant K has been determined for the system, and that value could be used in equation (5) for future determinations of 2E, 2V, and 2H, in terms of 2D and the appropriate refractive index. Conversely, if K is determined by any other means, the position of the calibration line is thereby also determined.

Assuming that the chart has been calibrated for the 4-mm. objective and the 10× micrometer eyepiece, as just described for Fig. 2, it is now ready for use for the determination of 2E and 2V, and for an additional purpose which will be described below. An interference figure from a section which is oriented with the acute bisectrix nearly normal to it may now be used to measure 2E and 2V. For example, if the measured distance 2D is 45 units in the micrometer eyepiece scale, the chart is entered at $2D=45$; a line is followed down and to the right to the calibration line, and from that point a new line is followed up and to the right to the side of the chart where 2E is read: $2E=80°$. However, if an approximate estimate of the index of refraction of the material gave $N_Y = 1.60$, then instead of reading $2E = 80°$, the line may be followed up and to the right from the calibration line to the 1.60 vertical line, and thence down and to the right to $2V = 47°$. In most cases, a small error in the estimate of N_Y will not greatly affect the value of 2V; therefore

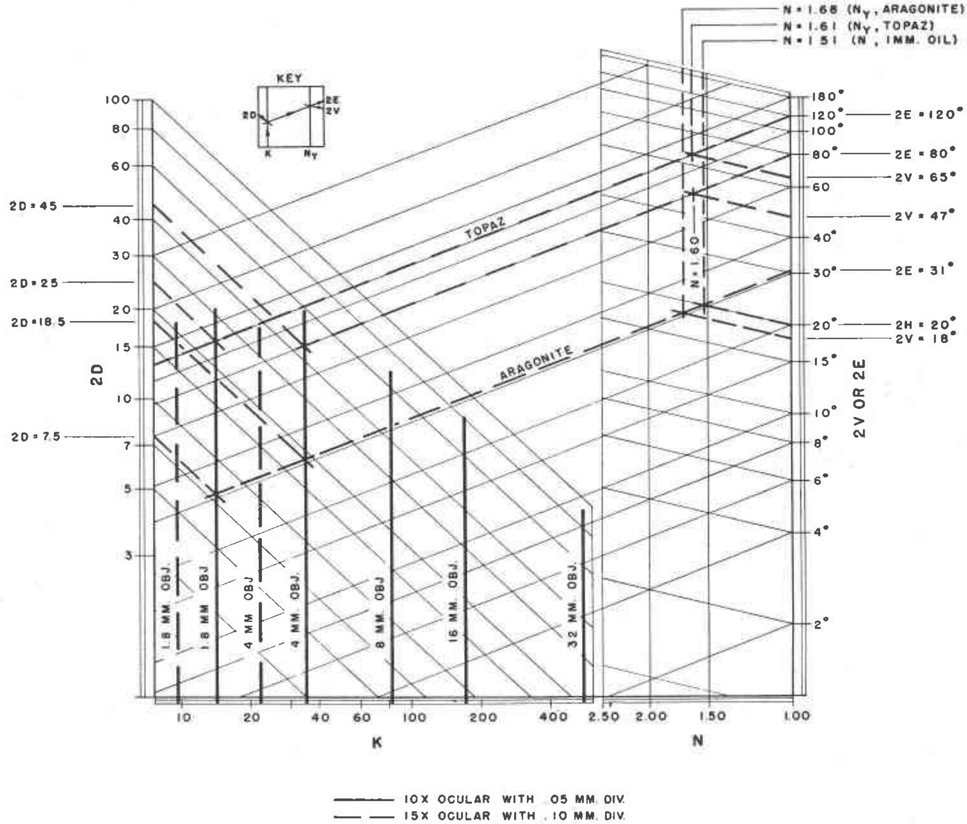

FIG. 2. Chart like Fig. 1, calibrated for use with several optical systems on Spencer petrographic microscope, serial number 154974.

a closely-graduated refractive-index scale is neither necessary nor desirable.

A further example may be taken from the calibration of the chart for a 1.8-mm. oil-immersion objective. The chart was calibrated for this objective using topaz, and the result was checked using aragonite. The following data were assembled:

Microscope:	Spencer No. 37, Ser. No. 154974
Objective:	Spencer achromatic, 1.8-mm., oil imm.
Eyepiece:	Spencer Huygenian, 10×
Micrometer:	20 divisions per millimeter
Bertrand lens:	Focused at 7.0

Diameter of field 32 (observed)

	Topaz	Aragonite
N_Y (measured)	1.613	
2V (see text)	65°	
2E		31°
2D (measured)	22.5	about 7 or 8

The optic angle of topaz varies considerably with variations in composition, but two considerations justify its use in this instance. First, the apparent optic angle 2E is about 120°; this value falls in the greatly compressed portion of the 2E scale, so that a considerable error in the numerical value of 2E (or of 2V) will introduce only a small error in the position of the corresponding point on the scale. Second, the intermediate index of refraction of the specimen had been previously measured. This value, N_Y, was 1.613; comparison with published data (Winchell, 1933, p. 199, Fig. 118) shows that topaz with $N_Y=1.613$ contains about 5 per cent of the hydroxyl end-member of the topaz series, and that for this composition $2V=65°$.

Since 2E is unknown, 2V and N_Y are used to obtain the left-sloping line for calibration of the chart: starting at $2V=65°$ follow up and to the left to the vertical line for $N_Y=1.613$, and from that point down and to the left to the intersection with the right-sloping line from $2D=22.5$. Through the point thus located, the vertical calibration line is drawn from the K-scale (at $K=14$) to the point corresponding with the diameter of the field of view. As a check, the observation of the interference figure of aragonite gave 2D = between 7 and 8; using the calibration line just determined, 2E is therefore between 29 and 33 degrees, which is good agreement with the known value of 31° for aragonite.

Instead of 2V or 2E, 2H may be read from the chart if the index of refraction of the immersion oil is known. For example, the oil used with the 1.8-mm. objective described above has an index of 1.515. The optic angle in this oil would be designated 2H, and is obtained by a procedure similar to that described above for 2V, except that the index of the oil, $N=1.515$, is used instead of the index of the crystal, N_Y. Thus for aragonite, $2H=20°$, $2V=18°$, $2E=31°$. 2H in water ($N=1.33$) would be about 24.

By a similar process the chart may be calibrated for other optical systems. This has been done as just described for our oil-immersion 1.8-mm. objective, and for our dry 4-, 8-, 16-, and 32-mm. objectives with the 10× micrometer eyepiece, and for the 1.8-mm. and the 4-mm. objectives with a 15× Ramsden eyepiece, fitted with a scale having 10 divisions per millimeter. All these calibration lines are shown in Fig. 2.

A further use for this chart (the one for which the chart was originally developed in the laboratories of the Hamilton Watch Company) is the measurement of an orientation angle, R; this angle is defined for present purposes as the true angle between the optic axis of a uniaxial crystal plate (or any other optically recognizable direction in an anisotropic crystal plate) and the normal to the plane of the section. Actually, of course, R is measured between the optical crystal direction and the optical axis of the microscope. For example, synthetic corundum jewel bearings may be studied by this means (Winchell, 1944), although other methods give more complete data. The jewel bearing often consists of a round, flat disc of corundum, polished all over, with a polished hole through its center. The relation of this hole, normal to the plane of the disc, to the crystallographic axes of the corundum, is important in the study of the processing and wearing qualities of the jewel. Using sodium light, we obtain an interference figure, measure the radial distance D, from the center of the figure to the melatope (in this case, the black cross corresponding to the optic axis of the corundum). We then enter the chart at 2D as for an optic angle determination, follow the lines down to the calibration line, then up to the appropriate refractive index line ($N_o = 1.77$), and finally down to the value of 2R; from 2R, R is easily obtained. Great advantage is obtained in using monochromatic light for this operation, since it is thereby possible to use the isochromatic curves to locate with accuracy the center of the flash figure in sections in which R is very large and the melatope is outside the field of view. From the orientation of the flash figure we obtain the complement of R, and hence the angle R itself.

A similar technique has been tried successfully, but never actually put to use by the writer, for determining the orientation of a biaxial crystal. In this case, a bisectrix and an optic axis were visible in the interference figure, and by noting for each the polar angle R and the azimuth V, which can be measured directly with the graduations on the periphery of the rotating stage, it was possible to construct in stereographic projection the complete orientation of the optical indicatrix with respect to the thin section. This included the determination of the value of 2V and the identification of the bisectrix visible in the field of view. It is probable that this method would fail if the normal to the section makes too great an angle with all three of the optic symmetry planes, and possibly also under certain other conditions, unless an objective of extremely high aperture is used. The 1.8-mm. oil-immersion objective whose calibration is shown in Fig. 2 would always produce an interference figure including at least one of the principal optical directions, X, Y, or Z, and usually also one of the optic axes (melatopes) provided that the refractive index of the specimen is low enough. Since this application would probably be rare, a complete description is hardly justified here. One of its uses would be the determination of optic orientation in laboratories where a universal stage is not available.

Discussion

A minor refinement on the use of Fig. 1 for orientation measurements would be accomplished by substituting a scale of E for scale of 2E. In so doing, each number is replaced by one which is half as large. Similarly, the scale of 2D could be replaced with a scale of D. However, the latter change would be unnecessary, since dividing the numbers by 2 would have the same effect as bodily shifting the scale upward a distance equal to the distance between any pair of numbers having a ratio of 1:2 (e.g., 10 and 20, 15 and 30, etc.). Shifting the scale requires a corresponding shift in the position of the K-scale, however. A shifted scale requires recalibration of the chart.

The chart could of course be made perfectly general in scope by drawing a family of vertical lines to represent the K-scale or Mallard's constant. Then the procedure for use would be to enter the chart at 2D and K, follow corresponding lines to their intersection, then from that intersection follow the proper line up and to the right as described above. That procedure would always involve the use of the numerical value of K, rather than a relatively isolated and easily-located calibration line. Since only a small number of fixed values of K will ordinarily be used in connection with the chart it seems better to retain it as in Fig. 1, without the complete family of vertical lines.

It should be noted that if the 2D-scale, which in its present form covers about $2\frac{1}{2}$ logarithmic cycles, should prove inadequate to cover the ranges of values necessary, this scale may be multiplied or divided by any desired factor. The K-scale should then be changed by the same factor. For example, in using a filar micrometer reading in millimeters and fractions rather than units of 0.1 or 0.05 millimeter, a more suitable range of values for 2D might be from 0.01 to 3.00 instead of from 1 to 300.

The K-scale would be renumbered accordingly, and would then read from 0.10 to 5.00 instead of from 10 to 500.

Summary

Figure 1 is an expansion and development of the principles of the Schwartzmann optic axial angle scales; this chart facilitates the prompt and easy solution of Mallard's formula and the laws of refraction, so as to give the optic axial angle in air or any medium (including the crystal itself), from a simple measurement of the distance between the melatopes or "eyes" in an acute bisectrix interference figure. The specific advantages of the chart method over the Schwartzmann scales are (1) no need to cut out, trace, or otherwise reproduce the logarithmic Schwartzmann scales to permit calibration, or to find and calibrate a slide rule containing logarithmic and logarithmic sine scales: the chart may be calibrated by drawing a single straight line through a single calibration point; (2) only one chart is needed for permanent reference purposes, and the calibration of the chart may permanently include data for more than one optical system; (3) the chart solves the problem for 2V and 2H as well as for 2E, thus eliminating a second setting for the slide rule method, or a separate calculation of 2V or 2H after obtaining 2E by the Schwartzmann method. Application of these specific advantages facilitates certain orientation measurements that otherwise would normally be made with a universal stage, or at considerably more labor with interference figures, Schwartzmann scales, and auxiliary calculations.

Acknowledgments

The chart herein described was developed in connection with investigations of the relation between orientation and processing efficiency in corundum boules, and between orientation and durability of corundum jewel bearings, in the laboratories of the Hamilton Watch Company. The writer is particularly indebted to Mr. B. L. Hummel of the Engineering Records Section for suggestions and aid in preparing the finished drawings, and for various reproductions thereof at several sizes.

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