LINEAR ANALYSIS OF A MEDIUM-GRAINED GRANITE[1]

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Abstract

The design and results of an experimental evaluation of the precision of linear analysis are described. It is shown that no systematic error is introduced by duplicate or multiple measurements of the same grain, providing the traverses are evenly spaced throughout. The precision error, whether of a single linear analysis or of a mean based on a group of such analyses, varies directly with the traverse interval. The mean of a large number of linear analyses will not differ from a true Rosiwal analysis of the same group of slides.

Numerical statements of these conclusions are given for the Woodstock, Maryland, granite as analyzed by the writer on the Hurlbut and Wentworth-Hunt micrometers. The precision error of a single analysis is small in comparison to differences between thin sections; in this case it may be neglected. It is shown by comparison of Hurlbut- and Wentworth-stage results that the precision of the two instruments is of the same order.

Introduction

In the linear traverse method as described by Rosiwal (1) the unit of the sample is the individual grain, and this unit has been retained by Lincoln and Reitz (2) in their well-known investigation of the precision of the method. The continual improvement of measuring devices and the increasing reliance of petrographers on thin sections rather than polished slabs has greatly reduced the practical importance of the single grain. Partly from convenience and partly from necessity, the thin section has replaced the grain as the sample unit; information concerning grain size is not obtained in routine operation of any of the instruments used for this work. Wentworth's original suggestion (3) that the traversing interval be standardized at 1 mm. regardless of grain size has never been criticized, and has apparently found such favor that in their recent papers neither Larsen and Miller (4)* nor Postel and Lufkin (5) considered a theoretical discussion of traverse interval necessary. Yet since the Wentworth procedure often requires duplicate or multiple measurements of the same grain, it does not meet the standard set up by Rosiwal and is specifically excluded from the category of methods that might be justified by the Lincoln-Reitz analysis. Petrologists nevertheless continue to refer to their analyses as Rosiwal analyses and, if they are pressed for some rationalization of the procedure, many of them will cite the excellent work of Lincoln and Reitz.

In connection with the standardization of certain physical testing procedures the writer was recently called upon for an estimate of the miner-

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* This paper contains a full bibliography.

alogical uniformity of prepared surfaces of the Woodstock, Maryland, granite. The samples were in the form of AX diamond-drill core, and the test specimens were prepared by sawing the core into appropriate lengths. The rock was to be used as one of several provisional standards; multiple tests were to be run on it, and the spread of these results was to be considered "instrumental variation." It was obvious that this procedure would be satisfactory for tests involving the bulk composition of each specimen; but several of the tests concern properties of only the end-surfaces of the specimens, and it was not entirely clear that these small surfaces would be of sufficiently uniform composition. Some estimate of the mineralogical differences to be expected from surface to surface was desired, and it seemed that this could best be made with thin sections, since the areas of the test surfaces were of about the same order as those of standard thin sections.

A linear analysis is subject to uncertainty from three principal sources; the precision or reproducibility of each individual measurement, the variability of thin sections, and the accuracy of the final result. Most studies of the problem have attempted to treat the third factor by comparisons of mode and norm. A few have been concerned with the first factor, but there seem to be no recorded studies of the second. By and large, the petrologist may be somewhat concerned with his individual measurements, but his strongest interest is in the accuracy of the final result, judged in relation to external (usually chemical) standards, and the variability of thin sections usually has nothing but a nuisance value for him.

The orientation of the present study is very different from that of normal geological work. It is not concerned with accuracy but is primarily interested in thin-section variability (or sample variance), and this variability can be properly interpreted only if the error of individual measurements is either known or reduced sufficiently so that it may be neglected.

The immediate problem in our laboratory was to determine both the precision with which a Wentworth (6) stage could be operated and the sample variance of areas of a size that could be measured on that stage. The simplest and best test of precision consists simply in remeasuring the same slide or area a large number of times and then comparing the results. Although there may be errors in identification and manipulation, the principal source of uncertainty is in the traverse path, and any satisfactory test of precision must include this factor. What is needed is a single area in which the operator may perform successive analyses of equal length over many different traverse paths. The Wentworth instrument as marketed does not provide for the systematic spacing of traverses, and even with a piece of millimeter graph paper glued to the stage the smallest distance between adjacent traverses is 0.5 mm., so that if one is attempting to gauge the precision of analyses made with a 1-mm. traverse interval, only two analyses may be obtained from each slide. Consequently no satisfactory direct test of precision on the Wentworth stage was developed.

The Federal Geological Survey's Hurlbut (7) stage, made available to the writer through the courtesy of Dr. C. S. Ross, has been equipped by the Survey machine shop with a Spencer mechanical stage calibrated in millimeters, with a vernier reading in tenths of a millimeter. A direct single-slide test of precision was made on this stage.

On each of eleven thin sections analyses were then made with the Wentworth and Hurlbut stages, the measurements in each case being confined to a $\frac{3}{4}$ inch diameter circle drawn on the slide. From this information numerical estimates of the sample variance were obtained, and the results showed that the precision error of the Wentworth instrument, like that of the Hurlbut stage, could easily be reduced to negligible size.

A great many shorter experiments confirm the conclusions based on these major tests. Some of these are described below, but detailed accounts of most have been omitted, since often they are not germane to the main outlines of the discussion. In particular, several attempts to determine precision directly on the Wentworth stage were made before the indirect procedure described below was adopted. A good deal of information concerning the effect of orientation has been accumulated, but this is strictly a question of accuracy rather than precision or sample variance, and it is planned to treat it separately in a later paper. There is some evidence that a slight discrepancy in means may be introduced by illumination differences, so that the quartz and feldspar values determined on the Wentworth stage do not check as closely as might be desired with those obtained on the Hurlbut instrument. This effect is very persistent but so small that its influence on mean values could not be exactly determined without an amount of work out of all proportion to its importance; in any case, it affects neither sample variance nor precision.

Finally, the effect of the traverse area on the variance has been partly investigated. Much work remains to be done along these lines, however, and here it need only be said that unless otherwise stated the sample variances described below are valid only for areas of about 0.44 square inch (actually circles $\frac{3}{4}$ inch in diameter).

The argument in this paper is largely of the elementary sort that is almost self-evident once its terms are defined. A few terms that may be unfamiliar to some readers recur throughout, and in order to facilitate reference, definitions and symbols are presented in a separate section.

Terms and Symbols

The terms and symbols used in this paper are those required for almost all treatments of the precision of measurements. Some of them are common words that are given precise definitions in elementary statistics while a few have no significance except in that subject. The latter are defined first.

Variance is the mean square of the deviations about their mean. It is given by:

$$V = \frac{\Sigma(X - \bar{x})^2}{n - 1} \tag{1}$$

where $\bar{x}$ is the mean of $X_1, X_2, X_3 \cdots X_n$ measurements.

The *standard deviation* is the square root of the variance;

$$s = \sqrt{\frac{\Sigma(X - \bar{x})^2}{n - 1}} = \sqrt{V}. \tag{2}$$

The *standard error* or *standard deviation of the mean* is;

$$s_{\bar{x}} = \sqrt{\frac{\Sigma(X - \bar{x})^2}{n(n - 1)}} = \frac{s}{\sqrt{n}}. \tag{3}$$

The properties of s as a measure of variation are well-known. For normally distributed measurements of a given quantity, the range $\bar{x} \pm s$ includes about 65 per cent of all values if s and $\bar{x}$ are close approximations of the true population parameters, and on the same condition the range $\bar{x} \pm 2s$ includes 95 per cent of all similar measurements.

The standard error or error of the mean is used here for determining the number of analyses which would be required to achieve some stated precision of the mean. Its application is described below, where this question is discussed.

Variability and *variation* are used to describe a condition quantitatively expressed in terms of variance, standard deviation or standard error. *Precision* (or *precision error*) is a measure of the reproducibility of a particular measurement or group of measurements. In the former case it is here stated quantitatively as a standard deviation, in the latter as a standard error. It has nothing to do, directly, with accuracy, and the question of the accuracy of linear analysis is not touched upon in this paper.

Direct Single-Slide Precision Test on the Hurlbut Stage

On a typical thin section of Woodstock granite traverses were made at 0.1-mm. intervals from one edge of the slide to the other. The tallies on the recording dials were copied at the end of each traverse in tabular form, and the table was differenced, yielding 173 separate traverses. These were next copied onto numbered cards and the cards arranged to give the following sets of analyses:

9 analyses with traversing interval of 0.9 mm.
10 " " " " " 1.0 "
11 " " " " " 1.1 "
20 " " " " " 2.0 "
40 " " " " " 4.0 "

For each suite of analyses the average and standard deviation were computed for each of four constituents. Results are shown in Tables 1 and 2.

TABLE 1. MEANS OF MULTIPLE ANALYSES OF THE SAME SLIDE WITH DIFFERING TRAVERSE INTERVALS

Traverse interval, mm.	Number of analyses	Mean values, per cent			
		Quartz	Feldspar	Mica	Epidote
0.9	9	27.28	62.57	9.15	1.27
1.0	10	27.19	62.45	9.09	1.27
1.1	11	27.01	62.68	9.04	1.27
2.0	20	27.03	62.34	9.36	1.27
4.0	40	26.94	62.70	9.11	1.25

TABLE 2. STANDARD DEVIATIONS FOR GROUPS WHOSE MEANS ARE SHOWN IN TABLE 1

Traverse interval, mm.	Number of analyses	Standard deviation of a single analysis (per cent of whole)			
		Quartz	Feldspar	Mica	Epidote
0.9	9	0.80	1.43	1.24	0.48
1.0	10	1.34	1.13	0.93	0.53
1.1	11	0.67	2.81	0.88	0.43
2.0	20	2.63	3.56	1.78	0.65
4.0	40	4.10	4.47	2.06	0.99

The first three rows in these tables, and particularly in Table 2, can scarcely compare in reliability with the last two, being based only on small groups. It is impossible to repair this deficiency experimentally with present equipment, yet the object of this part of the study was to compare the precision obtained at 1-mm. traversing interval with that yielded by the 4-mm. interval, since the former was the smallest practical interval and the latter meets the requirement of true Rosiwal analysis for this particular rock.

It was therefore decided to treat the data for the 0.9-, 1.0- and 1.1-mm. intervals as a single group, thus obtaining thirty analyses for the range 0.9–1.1, to compare with 20 at 2 and 40 at 4 millimeters. In actual practice a range as great as 0.2 mm. in the traversing interval is probably the rule rather than the exception. The procedure used here does not duplicate routine conditions precisely, however, because of the regularity of interval within each analysis; it was adopted only because there seemed no alternative. Table 3 shows the results of this recomputation, as compared to the previously given values for the 2- and 4-mm. intervals.

TABLE 3. STANDARD DEVIATIONS OF THE COMPOSITE SMALL INTERVAL GROUP AS COMPARED TO RESULTS FOR 2- AND 4-MM. INTERVALS

Traverse interval, mm.	Number of analyses	Standard deviation (per cent of whole)			
		Quartz	Feldspar	Mica	Epidote
0.9–1.1	30	1.08	1.93	0.94	0.47
2.0	20	2.63	3.56	1.78	0.65
4.0	40	4.10	4.47	2.06	0.99

Further discussion of these results is deferred until we are in a position to compare them with the total variance developed by combined analytical error and sample variance. Since the total variance was estimated from analyses made with a 1-mm. traverse interval, only the first row of Table 3 is of interest in the discussion that follows.

TOTAL VARIATION OF A GROUP OF THIN SECTIONS

Eleven thin-section samples were cut at 1-inch intervals along a 1-foot length of AX core. The original tablet from which each thin section was ground was a circle of about 1-inch diameter, but to insure that successive analyses would be made on the same area a circle $\frac{3}{4}$ inch in diameter was drawn on each finished slide, and the measurements described below were confined to the area so outlined. A linear analysis of each slide with 1-mm. traverse interval was then made on the Hurlbut and Wentworth instruments. The results of these analyses are shown in Tables 4 and 5.

TABLE 4. HURLBUT ANALYSES OF ELEVEN THIN SECTIONS, TRAVERSE INTERVAL 1 MM.

Slide No.	Quartz, per cent	Feldspar, per cent	Mica, per cent	Epidote, per cent
1	26.14	61.44	9.80	2.62
2	24.82	64.55	8.40	2.23
3	27.91	62.85	8.09	1.15
4	27.35	63.94	7.35	1.36
5	29.60	63.63	6.00	0.77
6	32.19	55.95	9.12	2.74
7	26.42	66.82	5.80	0.96
8	34.17	55.13	9.38	1.32
9	31.23	63.59	4.32	0.86
10	35.28	53.26	9.24	2.22
11	30.71	60.10	7.74	1.45

TABLE 5. WENTWORTH STAGE ANALYSES OF ELEVEN THIN SECTIONS, TRAVERSE INTERVAL 1 MM.

Slide No.	Quartz, per cent	Feldspar, per cent	Mica, per cent	Epidote, per cent
1	29.78	59.70	8.90	1.62
2	27.32	64.02	6.52	2.14
3	32.37	58.38	8.36	0.89
4	31.60	60.51	7.12	0.77
5	31.14	63.04	4.60	1.22
6	33.81	53.80	11.11	1.28
7	26.52	67.55	4.90	1.03
8	34.41	57.78	6.78	1.03
9	31.99	61.34	6.01	0.66
10	36.83	51.92	9.87	1.38
11	29.08	62.05	8.13	0.74

In Table 6 the statistics necessary for comparison of the results are shown. These include for each constituent the mean ($\bar{x}$), the variance or mean square of the deviations (V), the standard deviation or square root of the mean square (s) and the standard error or standard deviation of the mean ($s_{\bar{x}}$).

The differences between the quartz and feldspar means as determined on the two instruments are small in relation to the standard errors of these means, but in all the work so far done by the writer the sense of this difference is the same; the mean of a group of Hurlbut quartz values is almost invariably a little lower than that of Wentworth quartz values for the same slides; conversely, Hurlbut feldspar estimates are a little higher. Comparison of the quartz values of Tables 4 and 5 shows that in only three instances (Nos. 1, 3 and 4) do the paired values differ by more than the standard deviation of either group considered separately (see Table 6), and in no case is the difference as great as $2s$. This difference is also not large in relation to the precision with which either machine may be operated, but the Hurlbut quartz value is smaller than the respective Wentworth value in more than 50 of a total of 60 such pairs accumulated to date. The discrepancy is apparently systematic rather than random even though its size can not be precisely determined with the methods described here.[3]

Table 6. Statistics Computed from Data of Tables 4 and 5

Constituent		Hurlbut stage	Wentworth stage
Quartz	$\bar{x}$	29.62	31.35
	V	11.66	9.42
	s	3.42	3.08
	$s_{\bar{x}}$	1.03	0.93
Feldspar	$\bar{x}$	61.02	60.01
	V	19.89	20.03
	s	4.45	4.52
	$s_{\bar{x}}$	1.34	1.36
Mica	$\bar{x}$	7.75	7.48
	V	3.02	4.06
	s	1.74	2.02
	$s_{\bar{x}}$	0.52	0.61
Epidote	$\bar{x}$	1.61	1.16
	V	0.50	0.19
	s	0.71	0.44
	$s_{\bar{x}}$	0.21	0.13

But this very small, non-random difference in the means for quartz and feldspar has not affected the statistics describing the variability of the sample. For the three principal constituents the differences between the Hurlbut and Wentworth estimates of V, s and $s_{\bar{x}}$ are not greater than might be expected if both runs had been made on either of the two machines, and either set of data may be used for the remaining calculations.

[3] On the assumption that the differences should be evenly distributed with regard to sign, χ^2 may be computed as $2(50-30)^2/30=26.7$, whereas the 0.01 point for 1 degree of freedom is only 6.6. Cf. Snedecor, G. W., *Statistical Methods* (1940), p. 6 and table p. 163.

The question of how extensive a measurement would be required to obtain a mean of known error has been raised intermittently since Rosiwal's original publication. For the present experiment this information may be obtained from the s values of Table 6. The standard error of the mean is given by

$$s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

and if it is desired only that the probability of $s_{\bar{x}}$ not exceeding some given value a be 65 per cent, the proper value of n is given by

$$n = \left(\frac{s}{a}\right)^2.$$

If one wishes a 95 per cent assurance that $s_{\bar{x}}$ will not exceed a, then

$$s_{\bar{x}} = 2s/\sqrt{n}$$

and

$$n = 4(s/a)^2.$$

In our case n is the number of thin sections of 0.44 square inch area which must be analyzed. Letting $a = 1\%$ of the total, the Hurlbut s values from Table 3 yield the following results:

Constituent	Number of slides (n) required to assure $s_{\bar{x}}$ not in excess of 1% of total with	
	65% probability	95% probability
Quartz	12	47
Feldspar	20	79
Mica	3	12
Epidote	1	2

To reduce the largest error, that for feldspar, to 1 per cent with 65 per cent probability would require about 8 hours' work on the Hurlbut stage or about 24 with the Wentworth instrument. It must be remembered that the slides used in the experiment are very closely spaced, and that the means and their errors apply strictly only to the original length of core. Results for another length of core drilled from the same block are very similar (see Table 8), and it may therefore be concluded that the slides represent the cores and the cores are an adequate sample of the block. Whether the block is a good sample of the quarry is a matter concerning which sufficient evidence has not been obtained.

On the other hand, we do have evidence (see Table 8 and related text) that the variability between thin sections can be materially reduced by moderate enlargement of the area of traverse. Preliminary experiments indicate that the total variance for either quartz or feldspar can be reduced by nearly half (the standard deviations reduced by about 25 per cent) if the area traversed is increased from 0.44 to 0.78 square inch. Analyses of 42 small and rather poor thin sections spaced at 6-inch intervals along continuous EX core indicate that in this particular case such an enlargement of area would compensate the increase of variance occasioned by a five- or six-fold expansion of the distance between thin section samples. This conclusion, of course, can not be applied *a priori* to any other rock or even to any other granite, but it does suggest that a few detailed studies made with thin-section areas of a square inch or more might place the whole subject of linear analysis in a new light.

Precision of a Single Analysis in Relation to Variation of Thin Sections

The V values of Table 6 are expressions of the total variation introduced both by differences between thin sections and by random errors incurred in analyzing each section. These sources of variation are evidently independent of each other, and what is required now is an estimate of how much each contributes to the total variance.

The contribution of analytical or precision error to the Hurlbut total variances is given by s^2 where s is the appropriate $0.9-1.1$ mm. entry of Table 3.[4] The variance contributions made by differences between thin sections are then obtained by subtracting these s^2 values from the Hurlbut V entries of Table 6. The first and third lines of Table 7 show the variance before and after subtraction of the portion due to analytical error.

Even in a study of variance—if, for instance, the relative effects of orientation as opposed to spacing of samples were being examined—the differences between the variances of lines 1 and 3 would scarcely be critical. Here, however, the immediate interest is the expectable range of composition of thin sections, and this is best discussed in terms of standard deviation, which is a direct index of the distribution of values.

[4] At any rate it is not larger than this. The precision test was made when the writer had had very little experience with the machine and a good deal of evidence suggests that his proficiency has increased with practice. Any one planning extensive work of this sort should test his own precision, perferably after he is thoroughly familiar with the operation of the machine; the effect of varying the speed of traverse will probably differ from operator to operator and possibly also from rock to rock. The precision indicated in Tables 2 and 3 is readily attainable.

The standard deviations before and after the extraction of analytical error are shown in lines 4 and 5 of Table 7. The largest difference is for feldspar. From the uncorrected feldspar standard deviation one would conclude that 65 percent of all similar thin sections would yield feldspar values in the range 56.6–65.5 per cent, and 95 per cent would fall between 52.1 and 69.9 per cent. Similar estimates from the corrected feldspar standard deviation would be 57.0–65.0 and 53.0–69.0 per cent, respectively. The differences between these estimates are negligible in the present case, and it may be concluded therefore that, *if a 1-mm. traverse interval is used,* the error incurred in analyzing a single slide is so much smaller than the variation between thin sections that no special account of it need be taken.

TABLE 7. EXTRACTION OF ANALYTICAL ERROR FROM TOTAL VARIANCE

	Quartz	Feldspar	Mica	Epidote
Total variance, Hurlbut	11.66	19.89	3.02	0.50
Variance due to precision error	1.17	3.72	0.88	0.22
Sample variance	10.49	16.17	2.14	0.28
Total standard deviation	3.42	4.45	1.74	0.71
Standard deviation after extraction of precision error	3.25	4.03	1.47	0.53

Table 6 indicates that this conclusion, which is based entirely on experiments with the Hurlbut stage, may be quite safely extended to the Wentworth instrument, for it will be remembered that the same sample was used in both runs. The contribution of the sample to the total variance is therefore the same, and since the total variances are very similar for the three major constituents it follows that the precision with which the Wentworth machine may be operated does not differ significantly from that which may be attained with the Hurlbut instrument.[5]

PRECISION IN RELATION TO TRAVERSE INTERVAL AND GRAIN SIZE

With the conclusion that in this case the analytical or precision error of a 1-mm. interval analysis is small enough to be neglected, the main outlines of the argument are complete. But considerable more generally applicable information may be drawn from Tables 1, 2 and 3.

[5] There is reason to suspect that the Wentworth stage is preferable for analyses involving very small grains of minor constituents, and the epidote figures in Table 6 are in this respect typical of results obtained in a good many shorter, less carefully controlled experiments. The only point of consequence here is that the precision error of the Wentworth stage, is not larger than that of the Hurlbut stage, for it has already been shown that the precision error of the latter may be safely neglected for present purposes.

On the thin section used for the precision test, no grain was measured more than once with a 4-mm. traverse interval, while with all smaller intervals duplicate or multiple measurements were unavoidable, and, of course, their frequency increased as the interval was reduced. With the 1-mm. interval nearly all quartz and feldspar and a great many mica grains were measured at least twice. Yet it is clear from Table 1 that the mean values are not significantly affected by variation of traverse interval. As long as the traverses are evenly spaced no systematic error is introduced by duplicate or multiple measurements of the same grains.

Tables 2 and 3, on the other hand, clearly indicate that the precision of a single analysis is greater the smaller the traverse interval with which it is made. Where the composition of individual thin sections is desired, the smallest practical traverse interval will give the most precise result. Using the quartz figures of Table 3 as an example, a single 1-mm. analysis will fall, with 2/1 probability, within 1.1 per cent of the true[6] composition of the slide, and the chances are 19/1 that it will not be in error by more than 2.2 per cent, while for a single 4-mm. analysis the equivalent ranges are 4.1 and 8.2 per cent.

It may be noted in passing that this direct variation of precision error and traverse interval seems independent of grain size. In the thin section used for the precision test, epidote grains were no more than a few tenths of a millimeter in maximum diameter, mica grains were on the order of 1 mm., and quartz and feldspar grain diameters were nearly all between 2 and 4 mm., with the latter mostly more and the former mostly less than 3 mm. Yet the sense of the variation of precision error with traverse interval is the same throughout; it would require a much more extensive measurement to determine whether its rate is affected by grain size, though this seems quite probable.

These conclusions deal only with the problem of determining the composition of a single thin section, and they are compatible with what might have been predicted from a consideration of the "trapezoid rule" commonly used in determining the areas of irregular figures.[7] But the

[6] "True" in the sense that it will be the mean of a very large number of analyses of the same area made at any traverse interval.

[7] $A = w(\frac{1}{2}y_0 + y_1 + y_2 + \cdots y_{n-1} + \frac{1}{2}y_n)$, cf., for instance, Marks, L., *Mechanical Engineers Handbook*, 4th Ed., McGraw Hill Co., p. 2029 (**1941**). The accuracy of the result varies directly with n and inversely with w. In our case w is the traverse interval, y is the traverse length within a single grain (or in all grains of a single mineral), and n is the number of traverses in a grain (or in all grains of a single mineral). In linear analysis one determines only the ratios of areas to each other, thus:

$$\frac{A_1}{A_2} = \frac{w_a(\frac{1}{2}y_{a_0} + y_{a_1} + y_{a_2} + \cdots y_{a_{n-1}} + \frac{1}{2}y_{a_n})}{w_b(\frac{1}{2}y_{b_0} + y_{b_1} + y_{b_2} + \cdots y_{b_{n-1}} + \frac{1}{2}y_{b_n})}.$$

object of linear analysis is nearly always the determination of the composition of some large and usually unknown volume of rock, and it is in this connection that both Rosiwal and Lincoln and Reitz specifically require that no grain be measured more than once. In Table 8 are shown results for the analysis on the Hurlbut stage of a second suite of 11 sections, prepared from the same block and in the same fashion previously described. In these analyses the entire area of the slide (approximately 1 square inch) was traversed for each analysis, and the error of the mean for each constituent is smaller than for the analyses shown in Tables 4, 5 and 6.

TABLE 8. COMPOSITION OF SECOND SUITE OF ELEVEN THIN SECTIONS

	Quartz	Feldspar	Mica	Epidote
Mean of 1-mm. traverse-interval analyses	29.4	62.0	7.2	1.4
Standard error of mean	0.8	1.0	0.5	0.1
Mean of 4-mm. traverse-interval analyses	29.8	61.1	7.8	1.3
Standard error of mean	1.7	1.7	1.1	0.2
4-mm. analyses summed as a single Rosiwal analysis	29.8	61.2	7.7	1.3

From comparison of Tables 6 and 8 it is evident that the conclusions drawn from the single-slide precision test may be safely applied in combining the results for many thin sections providing the orientation and areas of the thin sections are uniform. The error of the mean is much less for the 1- than for the 4-mm. values, while for the Rosiwal summation it is unknown.

SUMMARY

Where the composition differences between thin sections are of the order of a few per cent and the constituents being measured are present in excess of five per cent, the estimate of thin-section variability is not significantly affected by the random errors incurred in analysis of the thin sections, providing the traversing interval is 1 mm. or less.

To obtain mean values for the Woodstock granite whose standard errors would in each case be less than 1 per cent of the total with 2/1 probability would require 1-mm. traverse-interval analysis of 20 thin

If the traverse interval is held constant, as in the Wentworth procedure, $w_a = w_b$, and the areas will be related to each other as the ratio of the total traverse distances. From this all the conclusions so far stated in this section may be seen to follow.

sections of 0.44 in.^{2} area for feldspar, 12 for quartz, 3 for mica, and 1 for epidote. If thin-section areas are as much as 0.77 in.^{2} similar precision would be obtained with only 11 slides for feldspar and fewer for other constituents.

The precision error of a single analysis varies directly with the traverse interval, and its variation is independent of grain size as long as magnification is sufficient to permit ready identification of all grains. The precision of the Wentworth machine is essentially the same as that of the Hurlbut stage, except that in the writer's case the former gives somewhat better results for minor constituents present in small grains.

The mean of a group of 1-mm. linear analyses does not differ significantly from a true Rosiwal analysis of the same group of slides, or from the average of linear analyses of these slides made with a traverse interval large enough to eliminate duplicate measurements. But the precision of the 1-mm. mean is superior to that of the mean based on larger traverse interval. For the analyses reported in Table 8, for instance, the number of slides required to achieve a mean of some known precision with traverse interval of 4 mm. is between three and five times as great as would be required if a 1-mm. traverse interval were used.

The procedure suggested by Wentworth, in which traverses are regularly spaced regardless of grain size, is thus superior to the regular Rosiwal procedure, in which the maximum grain diameter sets an arbitrary lower limit to the spacing of traverses. In fact, precision readily obtained with the Wentworth procedure is of such an order as to suggest that valuable results might accrue from carefully planned variance studies of any rock suitable for linear analysis, even if no close correlation with volume, specific gravity or chemical composition were attempted.

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