Chapter 8

ORBITAL MECHANICS

Knowledge of orbital motion is essential for a full understanding of space operations. The vantage point of space can be visualized through the motion Kepler described and by comprehending the reasons for that motion as described by Newton. Thus, the objectives here are to gain a conceptual understanding of orbital motion and become familiar with common terms describing that motion.

A HISTORY OF THE LAWS OF MOTION[1]

Early Cosmology

This generation is far too knowledgeable to perceive the universe as early man saw it. Each generation uses the knowledge of the previous generation as a foundation to build upon in the ever-continuing search for comprehension. When the foundation is faulty, the tower of understanding eventually crumbles and a new building proceeds in a different direction. Such was the case during the dark ages in medieval Europe and the Renaissance.

The Babylonians, Egyptians and Hebrews each had various ingenious explanations for the movements of the heavenly bodies. According to the Babylonians, the Sun, Moon and stars danced across the heavenly dome entering through doors in the East and vanishing through doors in the West. The Egyptians explained heavenly movement with rivers in a suspended gallery upon which the Sun, Moon and planets sailed, entering through stage doors in the East and exiting through stage doors in the West.

Though one may view these ancient cosmologies with a certain arrogance and marvel at the incredible creativity by which they devised such a picture of the universe, their observations were amazingly precise. They computed the length of the year with a deviation of less than 0.001% from the correct value, and their observations were accurate, enabling them to precisely predict astronomical events. Although based on mythological assumptions, these cosmological theories "worked."

Greece took over from Babylon and Egypt, creating a more colorful universe. However, the 6th century BC (the century of Buddha, Confucius and Lâo Tse, the Ionian philosophers and Pythagoras) was a turning point for the human species. In the Ionian school of philosophy, rational thought was emerging from the mythological dream world. It was the beginning of the great adventure in which the Promethean quest for natural explanations and rational causes would transform humanity more radically than in the previous two hundred thousand years.

Astronomy

Many early civilizations recognized the pattern and regularity of the stars' and planets' motion and made efforts to track and predict celestial events. The invention and upkeep of a calendar required at least some knowledge of astronomy. The Chinese had a working calendar at least by the 13th or 14th century BC. They also kept accurate records for things such as comets, meteor showers, fallen meteorites and other heavenly phenomena. The Egyptians were able to roughly predict the flooding of the Nile every year: near the time when the star Sirius could be seen in the dawn

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[1]Much of this information comes from Arthur Koestler's *The Sleepwalkers*.

sky, rising just before the Sun. The Bronze Age peoples in northwestern Europe left many monuments indicating their ability to understand the movement of celestial bodies. The best known is Stonehenge, which was used as a crude calendar.

The early Greeks initiated the orbital theories, postulating the Earth was fixed with the planets and other celestial bodies moving around it; a *geocentric* universe. About 300 BC, Aristarchus of Samos suggested that the Sun was fixed and the planets, including the Earth, were in circular orbits around the Sun; a *heliocentric* universe. Although Aristarchus was more correct (at least about a heliocentric solar system), his ideas were too revolutionary for the time. Other prominent astronomers/philosophers were held in higher esteem and, since they favored the geocentric theory, Aristarchus' heliocentric theory was rejected and the geocentric theory continued to be predominately accepted.

Aristotle, one of the more famous Greek philosophers, wrote encyclopedic treatises on nearly every field of human endeavor. Aristotle was accepted as the ultimate authority during the medieval period and his views were upheld by the Roman Catholic Church, even to the time of Galileo. However, his expositions in the physical sciences in general, and astronomy in particular, were less sound than some of his other works. Nevertheless, his writings indicate the Greeks understood such phenomena as phases of the Moon and eclipses at least in the 4th century BC. Other early Greek astronomers, such as Eratosthenes and Hipparchus, studied the problems confronting astronomers, such as: How far away are the heavenly bodies? How large is the Earth? What kind of geometry best explains the observations of the planets' motions and their relationships?

The Greeks were under the influence of Plato's metaphysical understanding of the universe, which stated:

"The shape of the world must be a perfect sphere, and that all motion must be in perfect circles at uniform speed."

This circular motion was so aesthetically appealing that Aristotle promoted this circular motion into a dogma of astronomy. The mathematicians' task was now to design a system reducing the apparent irregularities of planetary motion to regular motions in perfectly fixed circles. This task would keep them busy for the next two thousand years.

Perhaps the most elaborate and fanciful system was one Aristotle constructed using fifty-four spheres to account for the motions of the seven planets.[2] Despite Aristotle's enormous prestige, this system was so contrived that it was quickly forgotten. In the 2nd century AD, Ptolemy modified and amplified the geocentric theory explaining the apparent motion of the planets by replacing the "sphere inside a sphere" concept with a "wheel inside a wheel" arrangement. According to his theory, the planets revolve about imaginary planets, which in turn revolve around the Earth. Thus, this theory employed forty wheels: thirty-nine to represent the seven planets and one for the fixed stars.

Even though Ptolemy's system was geocentric, this complex system more or less described the observable universe and successfully accounted for celestial observations. With some later modifications, his theory was accepted with absolute authority throughout the Middle Ages until it finally gave way to the heliocentric theory in the 17th century.

Modern Astronomy

Copernicus

In the year 1543, some 1,800 years after Aristarchus proposed a heliocentric system, a Polish monk named Nicolas

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[2]In this instance the seven "planets" include the Sun, Moon, Mercury, Venus, Mars, Jupiter, and Saturn.

Koppernias (better known by his Latin name, Copernicus) revived the heliocentric theory when he published *De Revolutionibus Orbium Coelestium* (*On the Revolutions of the Celestial Spheres*). This work represented an advance, but there were still some inaccuracies. For example, Copernicus thought that the orbital paths of all planets were circles with their centers displaced from the center of the sun.

Copernicus did not prove that the Earth revolves about the sun; the Ptolemic system, with some adjustments, could have accounted just as well for the observed planetary motion. However, the Copernican system had more ascetic value. Unlike the Ptolemic system, it was elegant and simple without having to resort to artful wheel upon wheel structures. Although it upset the church and other ruling authorities, Copernicus made the Earth an astronomical body, which brought unity to the universe.

Tycho De Brahe

Three years after the publication of *De Revolutionibus*, Tyge De Brahe was born to a family of Danish nobility. Tycho, as he came to be known, developed an early interest in astronomy and made significant astronomical observations as a young man. His reputation gained him royal patronage and he was able to establish an astronomical observatory on the island of Hveen in 1576. For 20 years, he and his assistants carried out the most complete and accurate astronomical observations yet made.

Tycho was a despotic ruler of Hveen, which the king could not sanction. Thus, Tycho fell from favor, leaving Hveen in 1597 free to travel. He ended his travels in Prague in 1599 and became Emperor Rudolph II's Imperial Mathematicus. It was during this time that a young mathematician, who would also become an exile from his native land, began correspondence with Tycho. Johannes Kepler joined Tycho in 1600 and, with no means of self-support, relied on Tycho for material well being.

Tycho and Kepler's relationship was far from a great friendship. It was short (eighteen months) and fraught with controversy. This brief relationship ended when Tycho De Brahe, the meticulous observer who introduced precision into astronomical measurement and transformed the science, became terminally ill and died in 1601.

Kepler

Johannes Kepler was born in Wurttemberg, Germany, in 1571. He experienced an unstable childhood that, by his own accounts, was unhappy and ridden with sickness. However, Kepler's genius propelled him through school and guaranteed his continued education.

Kepler studied theology and learned the principles of the Copernican system. He became an early convert to the heliocentric hypothesis, defending it in arguments with fellow students.

In 1594, Kepler was offered a position teaching mathematics and astronomy at the high school in Gratz. One of his duties included preparing almanacs providing astronomical and astrological data. Although he thought astrology, as practiced, was essentially quackery, he believed the stars affected earthly events.

During a lecture having no relation to astronomy, Kepler had a flash of insight; he felt with certainty that it was to guide his thoughts throughout his cosmic journey. Kepler had wondered why there were only six planets and what determined their separation. This flash of insight provided the basis for his revolutionary discoveries. Kepler believed that each orbit was inscribed within a sphere that enclosed a perfect solid[3] within which existed the next orbital sphere and so on for all the planets.

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[3]A perfect solid is a three dimensional geometric figure whose faces are identical and are regular polygons. These solids are: (1) tetrahedron bounded by four equilateral triangles, (2) cube, (3) octahedron (eight equilateral triangles), (4) dodecahedron (twelve pentagons), and (5) icosahedron (twenty equilateral triangles).

He did not believe these solids actually existed, but rather, God created the planetary orbits in relation to these perfect solids. However, Kepler made the errant connection that this was the basis of the divine plan, because there are only five regular solids and there were only six known planets.

Kepler explained his pseudo-discoveries in his first book, the _Mysterium Cosmographicum_ (_Cosmic Mystery_). Although based on faulty reasoning, this book became the basis for Kepler's later great discoveries. The scientific and metaphysical communities at the time were divided as to the worth of this first work. Kepler continued working toward proving his theory and in doing so, found fault with his enthusiastic first book. In his attempts at validation, he came to realize he could only continue with Tycho's data—but he did not have the means to travel and begin their relationship. Fortunately for the advancement of astronomy, the power of the Catholic Church in Gratz grew to a point where Kepler, a Protestant, was forced to quit his post. He then traveled to Prague where his short tumultuous relationship with Tycho began. On 4 February 1600, Kepler finally met Tycho De Brahe and became his assistant.

Tycho originally set Kepler to work on the motion of Mars, while he kept the majority of his astronomical data secret. This task was particularly difficult because Mars' orbit is the second most eccentric (of the then known planets) and defied the circular explanation. After many months and several violent outbursts, Tycho sent Kepler on a mission to find a satisfactory theory of planetary motion (the study of Mars continued to be dominant in this quest); one compatible with the long series of observations made at Hveen.

After Tycho's death in 1601, Kepler became Emperor Rudolph's Imperial Mathematicus. He finally obtained possession of the majority of Tycho's records, which he studied for the next twenty-five years of his life.

Kepler's Laws

Kepler's earth-shaking discoveries came in anything but a straightforward manner. He struggled through tedious calculations for years just to find that they led to false conclusions. Kepler stumbled upon his second law (which is actually the one he discovered first) through a succession of canceling errors. He was aware of these errors and in his explanation of why they canceled he got hopelessly lost. In the struggle for the first law (discovered second), Kepler seemed determined not to see the solution. He wrote several times telling friends that if the orbits were just an ellipse, then all would be solved, but it wasn't until much later that he actually tried an ellipse. In his frustrating machinations, he derived an equation for an ellipse in a form he did not recognize[4]. He threw out his formula (which described an ellipse) because he wanted to try an entirely new orbit: an ellipse[5].

Kepler's 1st Law
(Law of Ellipses)

The orbits of the planets are ellipses with the Sun at one focus.

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[4]In modern denotation, the formula is:
$$R = 1 + e\cos(\beta)$$
where **R** is the distance from the Sun, β the longitude referred to the center of the orbit, and _e_ the eccentricity.

[5]After accepting the truth of his elliptical hypothesis, Kepler eventually realized his first equation was also an ellipse.

Later Sir Isaac Newton found that certain refinements had to be made to Kepler's first law to account for perturbing influences. Neglecting such influences (e.g., atmospheric drag, mass asymmetry and third body effects), the law applies accurately to all orbiting bodies.

Figure 8-1 shows an ellipse where F_1 is one focus and F_2 is the other. This depiction illustrates that, by definition, an ellipse is constructed by joining all points that have the same combined distance (**D**) between the foci.
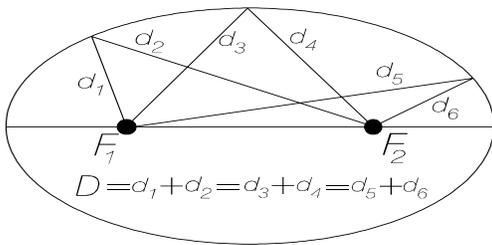

Fig. 8-1. Ellipse with axis

The maximum diameter of an ellipse is called its *major axis*; the minimum diameter is the *minor axis*. The size of an ellipse depends in part upon the length of its major axis. The shape of an ellipse is denoted by *eccentricity* (*e*) which is the ratio of the distance between the foci to the length of the major axis (see Orbit Geometry section).
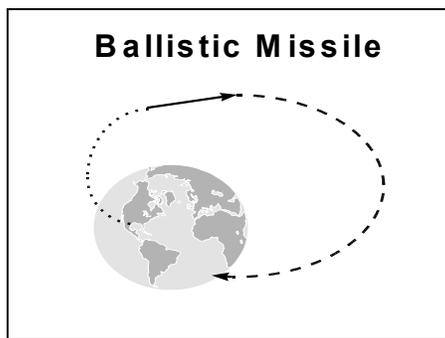
The path of ballistic missiles (not

Fig. 8-2. Ballistic Missile Path

including the powered and reentry portion) are also ellipses; however, they

happen to intersect the Earth's surface (**Fig. 8-2**).

With Kepler's second law, he was on the trail of Newton's Law of Universal Gravitation. He was also hinting at calculus, which was not yet invented.

Kepler's 2nd Law
(Law of Equal Areas)

The line joining the planet to the Sun sweeps out equal areas in equal times.

Based on his observation, Kepler reasoned that a planet's speed depended on its distance to the Sun. He drew the connection that the Sun must be the source of a planet's motive force.

With circular orbits, Kepler's second law is easy to visualize (**Fig. 8-3**). In a circular orbit an object's speed and radius both remain constant, and therefore, in a given interval of time it travels the same
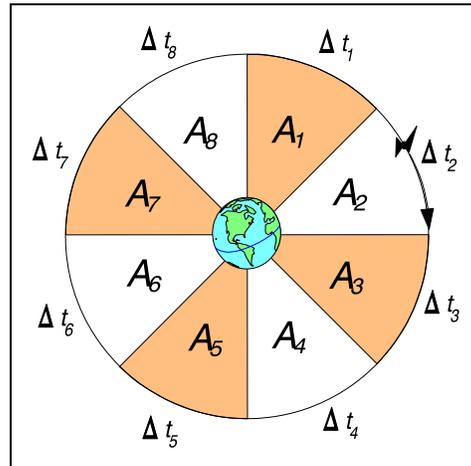

Fig. 8-3. Kepler's 2nd Law

distance along the circumference of the circle. The areas swept out over these intervals are equal.

However, closed orbits in general are not circular but instead elliptical with.non-zero eccentricity (An ellipse with zero eccentricity is a circle[6] see pg. 8-11).

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[6]That is, naturally occurring orbits have some non-zero eccentricity. A circle is a special form of an ellipse where the eccentricity is zero. Most artifi-

Kepler's second law isn't quite as obvious when applied to an ellipse. **Figure 8-4** depicts an elliptical orbit where two equal areas are swept out in equal intervals of time but are not symmetric. It is also apparent from **Fig 8-4** the closer a planet is to the Sun (also, any satellite to its prime mover, like the Earth) the faster it
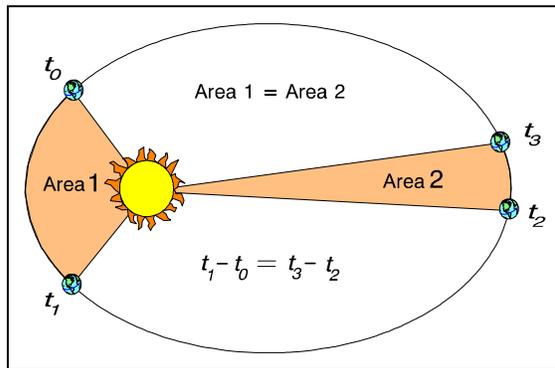

Fig. 8-4. An Elliptical Orbit

travels[7].

Kepler discovered his third law ten years after he published the first two in *Astronomia Nova* (*New Astronomy*). He had been searching for a relationship between a planet's *period* and its *distance* from the Sun since his youth. Kepler was looking at harmonic relationships in an attempt to explain the relative planetary spacing. After many false steps and with dogged persistence, he discovered his famous relationship:

Kepler's 3rd Law
(Law of Harmonics)

The squares of the periods of revolution for any two planets are to each other as the cubes

of their mean distances from the Sun.[8]

Kepler's 3rd Law directly relates the square of the period to the cube of the mean distance for orbiting objects. He believed in an underlying harmony in nature. It was a great personal triumph when he found a simple algebraic relationship, which he believed to be related to musical harmonics.

Isaac Newton

On Christmas Day 1642, the year Galileo died, there was born a male infant tiny and frail, Isaac Newton—who would alter the thought and habit of the world.

Newton stood upon the shoulders of those who preceded him; he was able to piece together Kepler's laws of planetary motion with Galileo's ideas of inertia and physical causes, synthesizing his laws of motion and gravitation. These principles are general and powerful, and are responsible for much of our technology today.

Newton took a circuitous route in formulating his hypotheses. In 1665, an outbreak of the plague forced the University of Cambridge to close for two years. During those two years, the 23-year-old genius conceived the law of gravitation, the laws of motion and the fundamental concepts of differential calculus. Due to some small discrepancies in his explanation of the Moon's motion, he tossed his papers aside; it would be 20 years before the world would learn of his momentous discoveries.

Edmund Halley asked the question that brought Newton's discoveries before the world. Halley was visiting Newton at Cambridge and posed the question: "If the Sun pulled on the planets with a force inversely proportional to the square of the

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cial satellites are predominately in orbits that are as close to circular as we can achieve.

[7]Kepler's second law is basically stating that angular momentum remains constant, but the concept of angular momentum wasn't invented when he formulated his laws.

[8]In mathematical terms: $\dfrac{P^2}{a^3} = k$, where P is the orbital period, a is the semi-major axis, which is the average orbital distance, and k is a constant.

distances, in what paths ought they to go?" To Halley's astonishment, Newton replied without hesitation: "Why in ellipses, of course. I have already calculated it and have the proof among my papers somewhere." Newton was referring to his work during the plague outbreak 20 years earlier and in this casual way, his great discovery was made known to the world.

Halley encouraged his friend to completely develop and publish his explanation of planetary motion. The result appeared in 1687 as *The Mathematical Principles of Natural Philosophy*, or simply the <u>*Principia.*</u>

Newton's Laws

As we've seen, many great thinkers were on the edge of discovery, but it was Newton that took the pieces and formulated a grand view that was consistent and capable of describing and unifying the mundane motion of a "falling apple" and the motion of the planets:[9]

Newton's 1st Law
(Inertia)

Every body continues in a state of uniform motion in a straight line, unless it is compelled to change that state by a force imposed upon it.

This concise statement encapsulates the general relationship between objects and causality. Newton combined Galileo's idea of inertia with Descartes' uniform motion (motion in a straight line) to create his first law. If an object deviates from rest or motion in a straight line with constant speed, then some force is being applied.

Newton's first law describes undisturbed motion; inertia, accordingly,

is the resistance of mass to changes in its motion. His second law describes how motion changes. It is important to define momentum before describing the second law. Momentum is a measure of an object's motion. Momentum ($\vec{p}$) is a vector quantity defined as the product of an object's mass (m) and its relative velocity ($\vec{v}$)[10].

Newton's second law describes the relationship between the applied force, the

$$\boxed{\vec{p} = m\,\vec{v}}$$

mass of the object and the resulting motion:

Newton's 2nd Law
(Momentum)

When a force is applied to a body, the time rate of change of momentum is proportional to, and in the direction of, the applied force.

When we take the *time rate of change* of an object's momentum (essentially differentiate momentum with respect to time, $d\vec{p}/dt$), this second law becomes Newton's famous equation:[11]

$$\boxed{\vec{F} = m\vec{a}}$$

Newton continued his discoveries and with his third law, completed his grand view of motion:

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[9]We still essentially see the Universe in Newtonian terms; Einstein's general relativity and quantum mechanics are a modification to Newtonian mechanics, but have yet to be unified into a single grand view.

[10]Velocity is an inertial quantity and, as such, is relative to the observer. Momentum, as measured, is also relative to the observer.

[11]The differentiation of momentum with respect to time actually gives $\vec{F} = \dot{m}\vec{v} + m\dot{\vec{v}}$ where $\dot{m}$ is the rate of change of mass and $\dot{\vec{v}}$ is the rate of change of velocity which is acceleration $\vec{a}$. In simple cases we assume that the mass doesn't change, so $\dot{m} = 0$ and the equation reduces to $\vec{F} = m\dot{\vec{v}} \Rightarrow \vec{F} = m\vec{a}$. For an accelerating booster the $\dot{m}$ term is not zero.

Newton's 3rd Law
(Action-Reaction)

For every action there is a reaction that is equal in magnitude but opposite in direction to the action.

This law hints at conservation of momentum; if forces are always balanced, then the objects experiencing the opposed forces will change their momentum in opposite directions and equal amounts.

Newton combined ideas from various sources in synthesizing his laws. Kepler's laws of planetary motion were among his sources and provided large scale examples. Newton synthesized his concept of gravity, but thought that one must be mad to believe in a force that operated across a vacuum with no material means of transport.

Newton theorized gravity, which he believed to be responsible for the "falling apples" and the planetary motion, even though he could not explain gravity or how it was transmitted. In essence, Newton developed a system that described man's experience with his environment.

Universal Gravitation

Every particle in the universe attracts every other particle with a force that is proportional to the product of the masses and inversely proportional to the square of the distance between the particles.

$$F_g = G\left(\frac{M_1 m_2}{D^2}\right)$$

Where F_g is the force due to gravity, G is the proportionality constant, M_1 and m_2 the masses of the central and orbiting bodies, and D the distance between the two bodies.

Kepler's laws of planetary motion are empirical (found by comparing vast amounts of data in order to find the algebraic relationship between them); and describe the way the planets are observed to behave. Newton proposed his laws as a basis for all mechanics. Thus Newton should have been able to derive Kepler's laws from his own, and he did:

Kepler's First Law: **If two bodies interact gravitationally, each will describe an orbit that can be represented by a conic section about the common center of mass of the pair. In particular, if the bodies are permanently associated, their orbits will be ellipses. If they are not permanently associated, their orbits will be hyperbolas.**

Kepler's Second Law: **If two bodies revolve about each other under the influence of a central force (whether they are in a closed orbit or not), a line joining them sweeps out equal areas in the orbit plane in equal intervals of time.**

Kepler's Third Law: **If two bodies revolve mutually about each other, the sum of their masses times the square of their period of mutual revolution is in proportion to the cube of their semi-major axis of the relative orbit of one about the other.**

ORBITAL MOTION

Newton's laws of motion apply to all bodies, whether they are scurrying across the face of the Earth or out in the vastness

of space. By applying Newton's laws one can predict macroscopic events with great accuracy.

Motion

According to Newton's first law, bodies remain in uniform motion unless acted upon by an external force; that uniform motion is in a straight line. This motion is known as inertial motion, referring to the property of *inertia*, which the first law describes.

Velocity is a relative measure of motion. While standing on the surface of the Earth, it seems as though the buildings, rocks, mountains and trees are all motionless; however, all of these objects are moving with respect to many other objects (Sun, Moon, stars, planets, etc.). Objects at the equator are traveling around the Earth's axis at approximately 1,000 mph; the Earth and Moon system is traveling around the Sun at 66,000 mph; the solar system is traveling around the galactic center at approximately 250,000 mph, and so on and so forth.

The only way motion can be experienced is by seeing objects change position with respect to one's location. Change in motion may be experienced by feeling the compression or tension within the body due to acceleration (sinking in the seat or being held by seat belts). In some cases, acceleration cannot be felt, as in free-fall. Acceleration is felt when the forces do not operate equally on every particle in the body; the compression or tension is sensed in the body's tissues. With this feeling and other visual clues, any change in motion that has occurred may be detected. Gravity is felt as opposing forces and the resulting compression of bodily tissues. In free-fall, acceleration is not felt because every particle in the body is experiencing the same force and so there is no tissue compression or tension; thus, no physical sensation. What is felt is the sudden change from tissue compression to a state of no compression.

According to Newton's second law, for a body to change its motion there must be

a force imposed upon it. Everyone has experience with changing objects' motion or compensating for forces that change their motion. An example is playing catch—when throwing or catching a ball, its motion is altered; thus, gravity is compensated for by throwing the ball upward by some angle allowing gravity to pull it down, resulting in an arc. When the ball leaves the hand it starts accelerating toward the ground according to Newton's laws (at sea level on the Earth the acceleration is approximately 9.81 m/s or 32.2 ft/s). If the ball is initially motionless, it will fall straight down. However, if the ball has some horizontal motion, it will continue in that motion while accelerating toward the
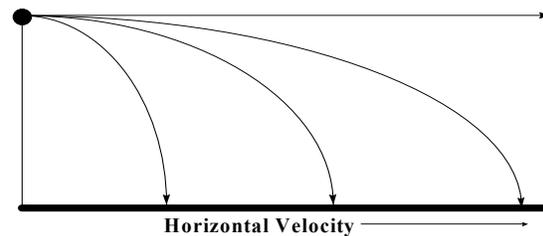

Fig. 8-5. Newton's 2nd Law

ground. **Figure 8-5** shows a ball released with varying lateral (or horizontal) velocities.

In **Figure 8-5**, if the initial height of the ball is approximately 4.9 meters (16.1 ft) above the ground, then at sea level, it would take 1 second for the ball to hit the

Table 8-1. Gravitational Effects

Horizontal Velocity	Distance (@ 1 sec)	
	Vertical	Horizontal
1	4.9	1
2	4.9	2
4	4.9	4
8	4.9	8
16	4.9	16

All values are in meters and meters/second.

ground. How far the ball travels along the ground in that one second depends on its horizontal velocity (see **Table 8-1**).

Eventually one would come to the point where the Earth's surface drops away as fast as the ball drops toward it. As **Fig. 8-6** depicts, the Earth's surface

curves down about 5 meters for every 8 km.

At the Earth's surface (without contending for the atmosphere, mountains or other structures), a satellite would have to travel at approximately 8 km/sec (or about 17,900 mph) to fall around the Earth without hitting the surface; in other words, to orbit.[12]
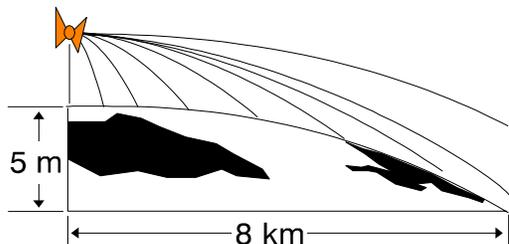

Fig. 8-6. Earth's Curvature

Figure 8-7 shows how differing velocity affects a satellite's trajectory or orbital path. The Figure depicts a satellite at an altitude of one Earth radius (6378 km above the Earth's surface). At this distance, a satellite would have to travel at
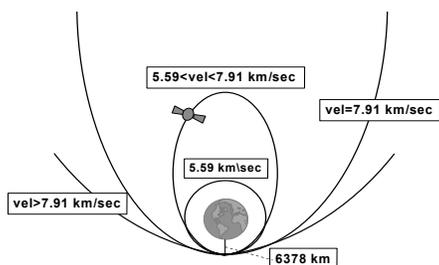

Fig. 8-7. Velocity versus Trajectory

5.59 km/sec to maintain a circular orbit and this speed is known as its *circular speed* for this altitude. As the satellite's speed increases, it falls farther and farther

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[12]Because the Earth does have an atmosphere, to stay in a "stable" orbit objects must be above the atmosphere—about 94 miles above the Earth's surface. Because the force due to gravity is inversely proportional to the square of the distance between the objects, at 94 miles an object has to travel at 7.8 km/sec (17,500 mph), while the Moon (at 249,000 miles) has to travel at only .9144 km/sec. (All these speeds are for circular orbits.)

away from the Earth and its trajectory becomes an elongating ellipse until the speed reaches 7.91 km/sec. At this speed and altitude the satellite has enough energy to leave the Earth's gravity and never return; its trajectory has now become a parabola, and this speed is known as its *escape speed* for this altitude. As the satellite's speed continues to increase beyond escape speed its trajectory becomes a flattened hyperbola. From a low Earth orbit of about 100 miles, the escape velocity becomes 11.2 km/sec. In the above description, the two specific speeds (5.59 km/sec and 7.91 km/sec) correspond to the circular and escape speeds for the specific altitude of one Earth radius.

The satellite's motion is described by Newton's three laws and his Law of Universal Gravitation. The Law of Universal Gravitation describes how the force between objects decreases with the square of the distance between the objects. As the altitude increases, the force of gravity rapidly decreases, and therefore the satellite can travel slower and still maintain a circular orbit. For the object to escape the Earth, it has to have enough kinetic energy (kinetic energy is proportional to the square of velocity) to overcome the gravitational potential energy of its position. Since gravitational potential energy is proportional to the distance between the objects, the farther the object is from the Earth, the less potential energy the satellite must overcome, which also means the less kinetic energy is needed.

ORBIT GEOMETRY

The two-body equation of motion describes conic sections. The conic section an object will follow depends on its velocity and the magnitude of the central force. If an object lacks the velocity (insufficient kinetic energy) to overcome the gravitational attraction (potential energy) then it will follow a closed path (circle or ellipse). However, if the object has enough velocity to

overcome the gravitational attraction then the object will follow an open path (parabola or hyperbola) and escape from the central force.

Figure 8-8 shows the basic geometry for the various possible conic sections. The parameters that describe the size and shape of the conics are its *semi-major axis* **a** (half of the large axis) and *eccentricity* **e** (the ratio between the separation of the foci—*linear eccentricity* **c**—and the semi-major axis).

Figure 8-9 depicts a satellite orbit with additional parameters whose conic section is an ellipse.
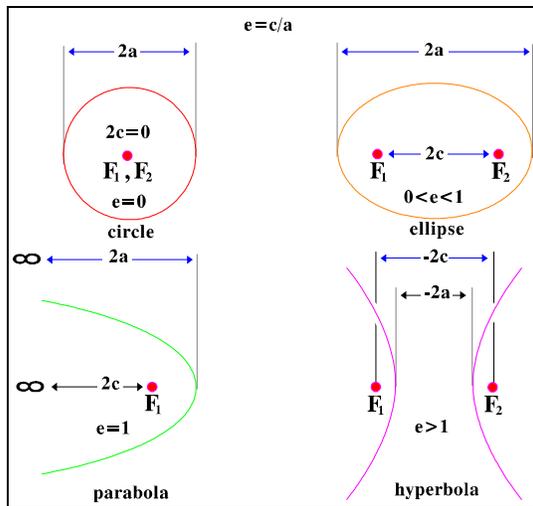
Semi-major Axis (a)—half of the distance

Fig. 8-8. Conic Section Geometry

between perigee and apogee, a measure of the orbits size, also the average distance from the attracting body.

Linear Eccentricity (c)—half of the distance between the foci.

Eccentricity (e)—ratio of the distance between the foci (**c**) to the size of the ellipse (**a**); describes the orbit's shape.

Perigee—the closest point in an orbit to the attracting body.

Apogee—the farthest point in an orbit to the attracting body.

These parameters apply to all trajectories. A circular orbit is a special case of the elliptical orbit where the foci coincide (*c* = 0). A parabolic path is a
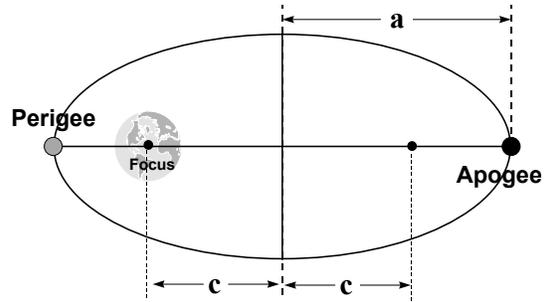

Fig. 8-9. Elliptical Geometry

transition between an elliptical and a hyperbolic trajectory. The parabolic path represents the minimum energy escape trajectory. The hyperbolic is also an escape trajectory; and represents a trajectory with excess escape velocity.

Table 8-2 shows the values for the eccentricity (discussed later) for the various types of orbits. Eccentricity is associated with the shape of the orbit. Energy is associated with the orbit's size (for closed orbits).

Table 8-2. Eccentricity Values

Conic Section	Eccentricity *(e)*
circle	$e = 0$
ellipse	$0 < e < 1$
parabola	$e = 1$
hyperbola	$e > 1$

CONSTANTS OF ORBITAL MOTION: MOMENTUM AND ENERGY

For some systems, there are basic properties which remain constant or fixed. Energy and momentum are two such properties required for a conservative system.

Momentum

Momentum is the product of mass times velocity ($\bar{p} = m\bar{v}$). This is the term for linear momentum and remains constant internal to the system in every

direction. In some instances it is more advantageous to describe motion in angular terms. For example, when dealing with spinning or rotating objects, it is simpler to describe them in angular terms An important angular property in orbital mechanics is *angular momentum.* Angular momentum is the product of linear momentum times the radius of revolution.[13] This property, like linear momentum, remains constant internal to the system for such things as orbiting objects.

A simple experiment can be performed illustrating conservation of angular momentum. Starting with some object on the end of a string, the object may be spun to impart angular momentum to the system. The amount of angular momentum depends on the object's mass and velocity, and the length of the string (radius): $\bar{h} = m(\bar{r} \times \bar{v})$. Now, if the string is shortened, the object will speed up (spin faster). From the above equation, mass (m) remains constant; angular momentum ($\bar{h}$) must remain constant as the radius ($\bar{r}$) decreases, so the object's velocity ($\bar{v}$) increases. This same principle holds true for orbiting systems. In an elliptical orbit, the radius is constantly varying and so is the orbital speed, but the angular momentum remains constant. Hence, there is greater velocity at perigee than at apogee.

Energy

A system's mechanical energy can also be conserved. Mechanical energy (denoted by E) is the sum of kinetic energy (KE) and potential energy (PE): $E = KE + PE$. Kinetic energy is the energy associated with an object's motion and potential energy is the energy associated with an object's position. Every orbit has a certain amount of mechanical energy. A circular orbit's radius and speed remain constant, so both potential and kinetic

energy remain constant. In all other orbits (elliptical, parabolic and hyperbolic) the "radius" and speed both change, and therefore, so do both the potential and kinetic energy in such a way that the total mechanical energy of the system remains constant. Again, for an elliptical orbit, this results in greater velocity at perigee than apogee. If a satellite's position and velocity is known, a satellite's orbit may be ascertained. Position determines potential energy while velocity determines kinetic energy.

COORDINATE REFERENCE SYSTEMS AND ORBITAL ELEMENTS

Reference systems are used everyday. Once an agreed upon reference has been determined, spatial information can be traded. The same must be done when considering orbits and satellite positions The reference system used depends on the situation, or the nature of the knowledge to be retrieved.

How does one know where satellites are, were or will be? Coordinate reference systems allow measurements to be defined, resulting in specific parameters which describe orbits. A set of these parameters is a satellite's *orbital element set*. Two elements are needed to define an orbit: a satellite's position and velocity. Given these two parameters, a satellite's past and future position and velocity may be predicted.

In three-dimensional spaces, it takes three parameters each to describe position and velocity. Therefore, any element set defining a satellite's orbital motion requires at least six parameters to fully describe that motion. There are different types of element sets, depending on the use. The Keplerian, or classical, element set is useful for space operations andtells us four parameters about orbits, namely:

- Orbit size
- Orbit shape
- Orientation
  - orbit plane in space
  - orbit within plane

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[13]Angular momentum is actually the vector cross product of linear momentum and the radius of revolution: $\bar{h} = \bar{r} \times \bar{p} \Rightarrow m(\bar{r} \times \bar{v})$.

- Location of the satellite

Semi-Major Axis (*a*)

. The *semi-major axis* (*a*) describes an orbit's size and is half of the distance between apogee and perigee on the ellipse. This is a significant measurement since it also equals the average radius, and thus is a measure of the mechanical energy of the orbiting object.

Eccentricity (*e*)

Eccentricity (*e*) measures the shape of an orbit and determines the positional relationship to the central body which occupies one of the foci. Recall from the orbit geometry section that eccentricity is a ratio of the foci separation (linear eccentricity, *c*) to the size (semi-major axis, *a*) of the orbit:

$$\boxed{e = c/a}$$

Size and shape relate to orbit geometry, and tell what the orbit looks like. The other orbital elements deal with orientation of the orbit relative to a fixed point in space.

Inclination (*i*)

The first angle used to orient the orbital plane is *inclination* (**i**): a measurement of the orbital plane's tilt. This is an angular measurement from the equatorial plane to the orbital plane $(0° \leq i \leq 180°)$, measured counter-clockwise at the ascending node while looking toward Earth (**Fig. 8-10**).

Inclination is utilized to define several general classes of orbits. Orbits with inclinations equal to 0° or 180° are *equatorial orbits,* because the orbital plane is contained within the equatorial plane. If an orbit has an inclination of 90°, it is a *polar orbit,* because it travels over the poles. If $0° \leq i < 90°$, the satellite orbits in the same general direction as the Earth (orbiting eastward around the Earth)

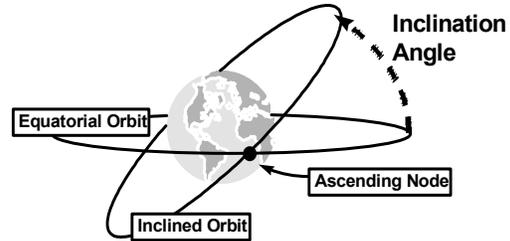Fig. 8-10. Inclination Tilt

and is in a *prograde orbit*. If $90° < i \leq 180°$, the satellite orbits in the opposite direction of the Earth's rotation (orbiting westward about the Earth) and is in a *retrograde orbit*. Inclination orients the orbital plane with respect to the equatorial plane (fundamental plane).

Right Ascension of the Ascending Node (Ω)

Right Ascension of the Ascending Node, Ω (upper case Greek letter omega), is a measurement of the orbital plane's rotation around the Earth. It is an angular measurement within the equatorial plane from the First point of Aries eastward to the ascending node $(0° \leq \Omega \leq 360°)$ (**Fig. 8-11**).

The First Point of Aries is simply a fixed point in space. The Vernal Equinox is the first day of spring (in the northern hemisphere). However, for the astronomer, it has added importance because it is a convenient way of fixing this principle direction. The Earth's
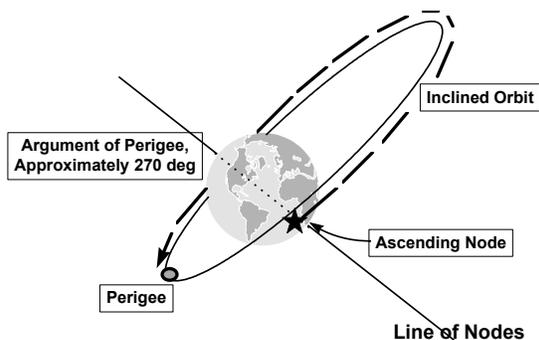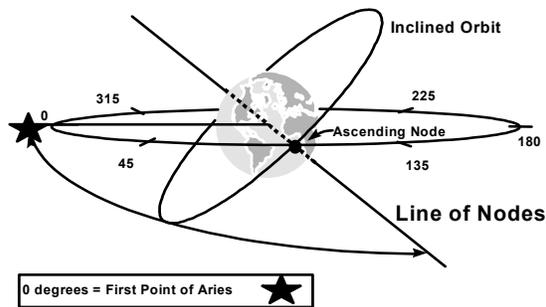

Fig. 8-13. Argument of Perigee

Fig. 8-11. Right Ascension of the Ascending Node

equatorial plane and its orbit about the Sun provide the principle direction. The Earth orbits about the Sun in the ecliptic plane, and this plane passes through the centers of both the Sun and Earth; the Earth's equatorial plane passes through the center of the Earth, which is tilted at approximately 23° to the ecliptic. The intersection of these two planes forms a line that passes though Earth's center and passes through the Sun's center twice a year: at the Vernal and Autumnal
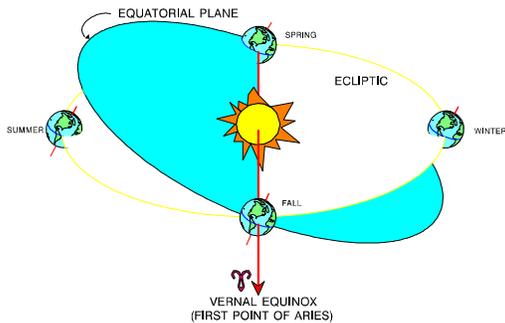

Fig. 8-12. Vernal Equinox

Equinoxes. The ancient astronomers picked the principle direction as that from the Sun's center through the Earth's center on the first day of Spring, the Vernal Equinox (**Fig. 8-12**).

The ancient astronomers called this the First Point of Aries because, at the time, this line pointed at the constellation Aries. The Earth is spinning like a top, and like a top, it wobbles on its axis. It takes approximately 25,800 years for the axis to complete one revolution. With the axis changing over time, so does the equatorial plane's orientation. The intersection between the ecliptic and the equatorial plane is rotating westward around the ecliptic. With this rotation, the principle direction points to different constellations. Presently, it is pointing towards Pisces. The orbital elements for Earth satellites are referenced to inertial space (a non-rotating principle direction) so the orbital elements must be referenced to where the principle direction was pointing at a specific time. The orbital analyst does this by reporting the orbital elements as referenced to the mean of 1950 (a popular *epoch year* reference). Most analysts have updated their systems and are now reporting the elements with respect to the mean of 2000.

Argument of Perigee (ω)

Inclination and Right Ascension fix the orbital plane in inertial space. The orbit must now be fixed within the orbital plane. For elliptical orbits, the perigee is described with respect to inertial space.

The *Argument of Perigee,* ω (lower case Greek letter omega), orients the orbit within the orbital plane. It is an angular measurement within the orbital plane from the ascending node to perigee in the direction of satellite motion ($0° \leq \omega \leq 360°$) (see **Fig. 8-13)**.

Table 8-3. Classical Orbital Elements

Element	Name	Description	Definition	Remarks
a	semi-major axis	orbit *size*	half of the long axis of the ellipse	orbital period and energy depend on orbit size
e	eccentricity	orbit *shape*	ratio of half the foci separation (c) to the semi-major axis	closed orbits: $0 \le e < 1$ open orbits: $1 \le e$
i	inclination	orbital plane's *tilt*	angle between the orbital plane and equatorial plane, measured counterclockwise at the ascending node	equatorial: $i = 0°$ or $180°$ prograde: $0° \le i < 90°$ polar: $i = 90°$ retrograde: $90° < i \le 180°$
Ω	right ascension of the ascending node	orbital plane's *rotation* about the Earth	angle, measured eastward, from the vernal equinox to the ascending node	$0° \le \Omega < 360°$ undefined when $i = 0°$ or $180°$ (equatorial orbit)
ω	argument of perigee	orbit's *orientation* in the orbital plane	angle, measured in the direction of satellite motion, from the ascending node to perigee	$0° \le \omega < 360°$ undefined when $i = 0°$ or $180°$, or $e = 0$ (circular orbit)
ν	true anomaly	satellite's *location* in its orbit	angle, measured in the direction of satellite motion, from perigee to the satellite's location	$0° \le \nu < 360°$ undefined when $e = 0$ (circular orbit)

True Anomaly (ν)

At this point all the orbital parameters needed to visualize the orbit in inertial space have been specified. The final step is to locate the satellite within its orbit. *True Anomaly*, ν (lower case Greek letter nu), is an angular measurement that describes where the satellite is in its orbit at a specified time, or Epoch. It is measured within the orbital plane from perigee to the satellite's position in the direction of motion ($0° \le \nu \le 360°$).

True Anomaly locates the satellite with respect to time and is the only orbital element that changes with time. There are various conventions for describing True Anomaly and Epoch. By fixing one, the other is also fixed. Sometimes they will choose True Anomaly to be 0° and give the Epoch as the *time of perigee passage*; or they will choose the Epoch as the moment when the satellite passes through

the ascending node and provide the value for True Anomaly.

There are different types of element sets. However, usually only orbital analysts deal with these sets. The Keplerian, or classical element, is the only element relevant to a majority of space operations. **Table 8-3** summarizes the Keplarian orbital element set, and orbit geometry and its relationship to the Earth.

ORBIT CHARACTERISTICS

Inclination (i) alone determines the four general orbit classes:

Prograde   — $0° \le i < 90°$
Retrograde — $90° < i \le 180°$
Equatorial — $i = 0°$, $180°$
Polar   — $i = 90°$

Other common orbits include those used for communication, weather and navigation:

Geostationary —Period = 23hrs56min
$$i = 0°$$
$$e = 0$$
Geosynchronous—Period = 23hrs56min
Molniya —Period = 11hrs58min
$$i = 63.4°$$
$$e = .72$$
Sun-synchronous—Period = 1hr41min
$$i = 98°$$
Semi-synchronous-Period = 11hr58min

GROUND TRACKS

The physics of two-body motion dictates the motions of the bodies will lie within a plane (two-dimensional motion). The orbital plane intersects the Earth's
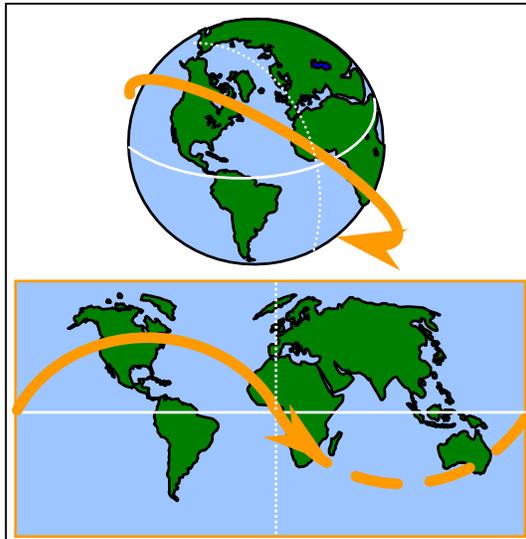

Fig. 8-14. Ground Track

surface forming a *great circle*. A satellite's ground track is the intersection of the line between the Earth's center and the satellite, and the Earth's surface; the point on the line at the surface of the Earth is called the *satellite subpoint*. The ground track, then, is the path the satellite subpoint traces on the Earth's surface over time (**Fig. 8-14**). However, the Earth does rotate on its axis at the rate of one revolution per 24 hours. With the Earth rotating under the satellite, the

intersection of the orbital plane,[14] and the Earth's surface is continually changing.

The ground track is the expression of the relative motion of the satellite in its orbit to the Earth's surface rotating beneath it. Because of this relative motion, ground tracks come in almost any form and shape imaginable. Ground track shape depends on many factors:

Inclination	i
Period	P
Eccentricity	e
Argument of Perigee	ω

Inclination defines the tilt of the orbital plane and therefore, defines the maximum latitude, both North and South of the ground track.

The period defines the ground track's *westward regression*. With a non-rotating Earth, the ground track would be a great circle. Because the Earth does rotate, by the time the satellite returns to the same place in its orbit after one revolution, the Earth has rotated eastward by some amount, and the ground track looks like it has moved westward on the Earth's surface (*westward regression)*. The orientation of the satellite's orbital plane does not change in inertial space, the Earth has just rotated beneath it. The time it takes for the satellite to orbit (its orbital period) determines the amount the Earth rotates eastward and hence its westward regression.
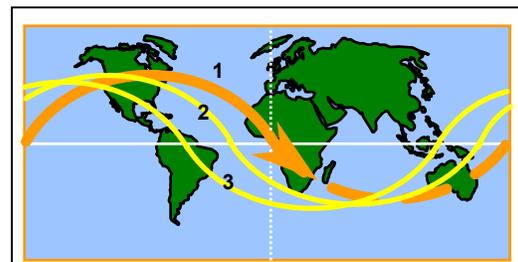

Fig. 8-15. Earth's Rotation Effects

Figure 8-15 shows the effect of the Earth's rotation on the ground track. The

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[14]Except for equatorial orbits, whose orbital plane is contained within the equatorial plane.

Earth rotates through 360° in 24 hours, giving a rotation rate of 15°/hr.[15] With a period of 90 min., a satellite's ground trace regresses 22.5° westward per revolution (15°/hr × 1.5 hrs= 22.5°). Westward regression is the angle through which the Earth has rotated underneath the satellite during the time it takes the satellite to complete one orbit.

Eccentricity affects the ground track because the satellite spends different amounts of time in different parts of its orbit (it's moving faster or slower). This means it will spend more time over certain parts of the Earth than others. This has the effect of creating an unsymmetrical ground track. It is difficult to determine how long the satellite spends in each hemisphere by simply looking at the ground trace. The time depends on both the length of the trace and the speed of the satellite.

Argument of perigee skews the ground track. For a prograde orbit, at perigee the satellite will be moving faster eastward than at apogee; in effect, tilting the ground track.

A general rule of thumb is that if the ground track has any portion in the eastward direction, the satellite is in a prograde orbit. If the ground trace does not have a portion in the eastward direction, it is either a retrograde orbit or it could be a super-synchronous prograde orbit.

Relative Motions

Because the Earth is a rotating the velocity of points on the surface is different depending on their distance from the Earth's axis of rotation. In other words, points on the equator have a greater eastward velocity than points north and south of the equator.

Figure 8-16 conceptualizes a geo-stationary orbit and its ground track. A

Fig. 8-16. Geostationary Orbit/Ground Track

satellite in a ideal geostationary orbit has the same orbital period as the Earth's rotational period, its inclination is 0° and its eccentricity is 0. The ground track will remain in the equatorial plane, the westward regression will be 360° and the satellite's speed never changes. Therefore, from the earth, the ground track will be a point on the equator.

Now take the same orbit and give it an inclination of 45°).

The period and eccentricity remain the same. The westward regression will be

Fig. 8-17. Ground Traces of Inclined, Circular, Synchronous Satellites

360° so the ground trace will retrace itself with every orbit. The ground trace will also vary between 45° North and 45° South. The apparent ground trace looks like a figure eight**(Fig. 8-17** for the simplest case. If the orbital parameters are varied (such as eccentricity and argument of perigee), the relative motions of the satellite and the

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[15]In reality, the 28-hour rotation rate is with respect to the Sun and is called the *solar day*. The rotation rate with respect to inertial space (the fixed stars, or background stars) is actually 23 hrs 56 min and is called the *sidereal day*.
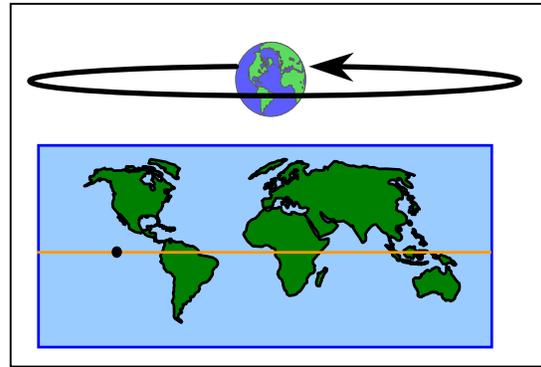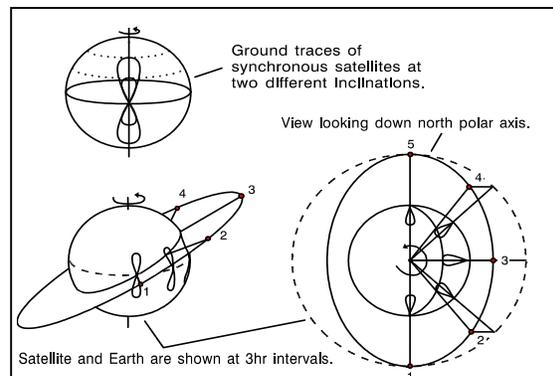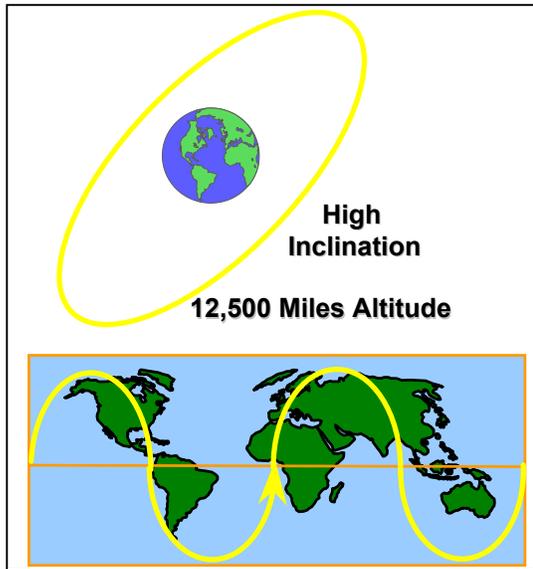
Fig. 8-18. Semi-synchronous Orbit

Fig. 8-19. Molniya Orbit

Earth's surface can become quite complicated. For orbits with small inclinations, the eccentricity and argument of perigee dominate the effect of the Earth's surface speed at different latitudes and can cause the ground track to vary significantly from a symmetric figure eight. These parameters can be combined in various ways to produce practically any ground track.

The semi-synchronous orbit (used by the Global Positioning System) also provides a unique ground track. This orbit, with its approximate 12-hour period, repeats twice a day. Since the Earth turns half way on its axis during each complete orbit, the points where the sinusoidal ground tracks cross the equator coincide pass after pass and the ground tracks repeat each day (**Fig. 8-18**).

Figure 8-19 shows a typical Molniya orbit that might be used for northern hemispheric communications. The Russians are credited with the discovery of this ingenious orbit. With the high degree of eccentricity the satellite travels slowly at apogee and can hang over the Northern Hemisphere for about two thirds of its period. Since the period is 12 hours,
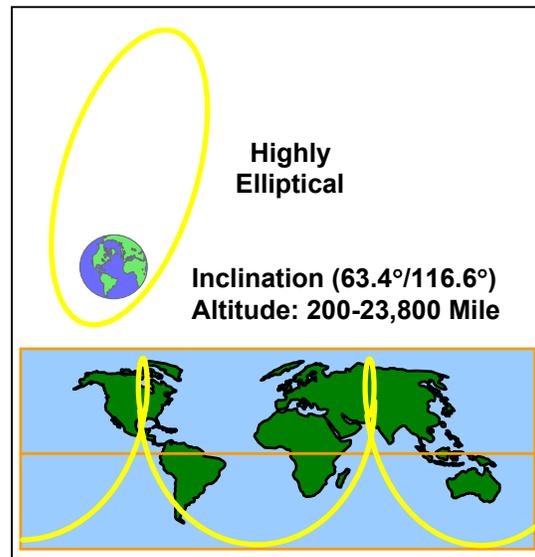
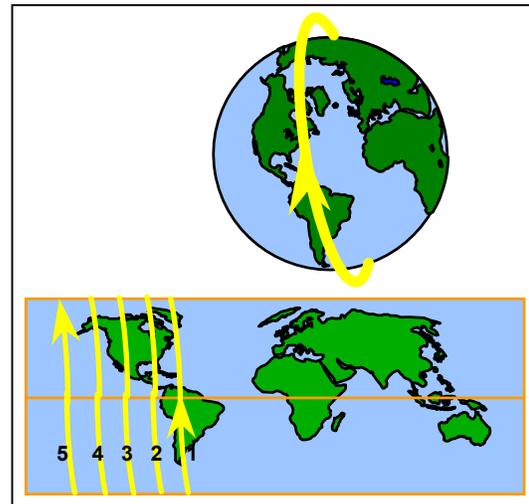the ground track retraces itself every day, much the same as the semi-synchronous orbit

Fig. 8-20. Sun-synchronous Orbit

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Figure 8-20 shows a representative sun-synchronous orbit. In this case, the orbital elements represent a DMSP (Defense Meteorological Satellite Program) satellite. The satellite is in a slightly retrograde orbit; therefore, the satellite travels east to west along the track..
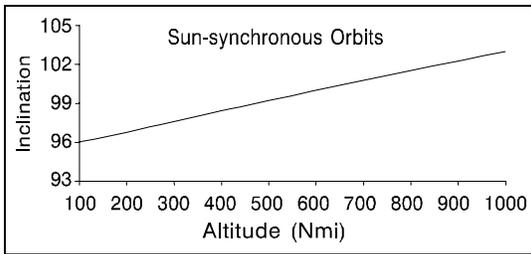
Fig. 8-21. Inclination versus Altitude

A sun-synchronous orbit is one in which the orbital plane rotates eastward around the Earth at the same rate that the Earth orbits the Sun. So, the orbit must rotate eastward around the Earth at a little less than 1°/day {(360°/year)/(365.25 days/ year) = .986°/day}. This phenomenon occurs naturally due to the oblateness of the Earth (see the section on perturbations).

Sun-synchronous orbits can be achieved at different altitudes and inclinations. However, all the inclinations for sun-synchronous satellites are greater than 90° (retrograde orbits). **Figure 8-21** plots inclination versus altitude for sun-synchronous orbits.

LAUNCH CONSIDERATIONS

The problem of launching satellites comes down to geometry and energy. If there were enough energy, satellites could be launched from anywhere at any time into any orbit. However, energy is limited and so is cost.

When a satellite is launched, it is intended to end up in a specific orbit, not only with respect to the Earth, but often with respect to an existing constellation. Also, the geometry of the planets must be taken into consideration when launching an interplanetary probe. Meeting operational constraints determines the launch window. The launch system is designed to accomplish the mission with the minimum amount of energy required because it is usually less expensive.[16]Keeping energy to a minimum restricts the launch trajectories and the launch location.

Launch site latitude and orbit inclination are two important factors affecting how much energy boosters have to supply. Orbit inclination depends on the satellite's mission, while launch site latitude is, for the most part, fixed (to our existing launch facilities).[17] Only minimum energy launches (direct launch) will be addressed. A minimum energy is one in which a satellite is launched directly into the orbital plane (i.e., no plane change or inclination maneuver).By looking at the geometry, the launch site must pass through the orbital plane to be capable of directly launching into that plane. Imagine a line drawn from the center of the Earth through the launch site and out into space. After a day, this line produces a conical configuration due to the rotation of the Earth. A satellite can be launched into any orbital plane that is tangent to, or passes through, this cone. As a result of this geometry, the lowest inclination that can be achieved by directly launching is equal to the latitude of the launch site.

If the orbital plane inclination is greater than the launch site latitude, the launch site will pass through the orbital plane twice a day, producing two launch windows per day. If the inclination of the orbital plane is equal to the launch site latitude, the launch site will be coincident with the orbital plane once a day, producing one launch window a day. If the inclination is less than the launch site latitude, the launch site will not pass through, or be coincident with the orbital plane at any time, and so there will not be any launch windows for a direct launch.

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[16]There are some situations when it is less expensive to use an existing system with extra energy because a lower class booster will not meet the mission needs. In this case, there is an extra margin and the launch windows are larger.

[17]There are various schemes to get around the problem of fixed launch sites: air launch, sea launch, and portable launch facilities, for instance.

A simplified model for determining inclination from launch site latitude and launch azimuth is:[18]

$$\cos(i) = \cos(L) \bullet \sin(Az)$$

i = inclination
L = launch site latitude
Az = launch azimuth

The cosine of the latitude reduces the range of possible inclinations and the sine of the azimuth varies the inclination within the reduced range. When viewing the Earth and a launch site, it is possible to launch a satellite in any direction (launch azimuth). The orbital plane must pass through the launch site and the center of the Earth.

For launches due east (no matter what the launch site latitude) the inclination will equal the launch site latitude. For launches on any other azimuth, the inclination will always be greater than the launch site latitude.

Just as the launch site latitude determines the minimum inclination (launching due east), it also determines the maximum inclination by launching due west. The maximum inclination is 180 minus the latitude.

The actual launch azimuths allowed (in most countries) are limited due to the safety considerations of not launching over populated areas, which further limits the possible inclinations from any launch site. However, the inclination can change after launch by performing an out of plane maneuver (see next section).

Launch Velocity

When a satellite is launched, energy is imparted to it. The two tasks of increasing the satellite's potential and kinetic energies must be accomplished. Potential energy is increased by raising the satellite above the Earth (increasing its altitude by at least 90-100 miles). In order to maintain a minimum circular orbit at that altitude, the satellite has to

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[18]This is a simplified model because it ignores the Earth's rotation, which has a small effect.

travel about 17,500 mph. Due to the Earth's rotation, additional kinetic energy may need to be supplied depending on launch azimuth to achieve this orbital velocity (17,500 mph). The starting velocity at the launch sites vary with latitude. It ranges from zero mph at the poles to 1,037 mph at the equator.

If a satellite is launched from the equator prograde (in the same direction as the earth's rotation) starting with 1,037 mph, only 16,463 mph must be supplied (17,500 mph- 1,037 mph). If launched from the equator retrograde (against the rotation of the earth), 18,537 mph must be supplied. Launching with the earth's rotation saves fuel and allows for larger payloads for any given booster.

There are substantial energy savings when locating launch sites close to the equator and launching in a prograde direction.

ORBITAL MANEUVERS

It is a rare case indeed to launch directly into the final orbit. In general, a satellite's orbit must change at least once to place it in its final mission orbit. Once a satellite is in its mission orbit, perturbations must be counteracted, or perhaps the satellite must be moved into another orbit.

As was previously mentioned, a satellite's velocity and position determine its orbit.[19] Thus, one of these parameters must be changed in order to change its orbit. The only option is to change the velocity, since position is relatively constant. By changing the velocity, the satellite is now in a different orbit. Since gravity is conservative, the satellite will always return to the point where it performed the maneuver (provided it doesn't perform another maneuver before returning).

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[19]The position and velocity correspond to the force of gravity and the satellite's momentum. Knowing the forces on the satellite and its momentum, we can apply Newton's second law and predict its future positions (in other words, we know its orbit).

Mission Considerations

Since both position and velocity determine a satellite's orbit, and many different orbits can pass through the same point, the velocity vectors must differ[20] to result in a different orbit while passing through the same point.

When an orbit is changed through its velocity vector, a *delta-v* (Δ*v*) is performed. For any single Δ*v* orbital change, the desired orbit must intersect the current orbit, otherwise it will take at least two Δ*v*s to achieve the final orbit.

When the present and desired orbits intersect, a Δ*v* is employed to change the satellite's velocity vector. The Δ*v* vector can be determined by subtracting the present vector from the desired vector. The resultant velocity vector is the Δ*v* required to get from one point to another.

PERTURBATIONS

Perturbations are forces which change the motion (orbit) of the satellite. These forces have a variety of causes/origins and effects. These forces are named and categorized in an attempt to model their effects. The major perturbations are:

- Earth's oblateness;
- Atmospheric drag;
- Third-body effects;
- Solar wind/radiation pressure;
- Electromagnetic drag.

Earth's Oblateness

The Earth is not a perfect sphere. It is somewhat misshapen at the poles and bulges at the equator. This squashed shape is referred to as *oblateness*. The North polar region is more pointed than the flatter South polar region, producing a slight "pear" shape. The equator is not a perfect circle; it is slightly elliptical. The effects of Earth's oblateness are gravitational variations or perturbations, which have a greater influence the closer a satellite is to the Earth. This bulge is often modeled with complex mathematics and is frequently referred to as the *J2 effect*.[21] For low to medium orbits, these influences are significant.

One effect of Earth's oblateness is *nodal regression.* Westward regression due to Earth's rotation under the satellite was discussed in the ground tracks section. Nodal regression is an actual rotation of the orbital plane, relative to the First Point of Aries, about the Earth (the right ascension changes). If the orbit is prograde, the orbital plane rotates westward around the Earth (right ascension decreases); if the orbit is retrograde, the orbital plane rotates eastward around the Earth (right ascension increases).

In most cases, perturbations must be counteracted. However, in the case of sun-synchronous orbits, perturbations can be advantageous. In the slightly retrograde sun-synchronous orbit, the angle between the orbital plane and a line between the Earth and the Sun needs to remain constant. As the Earth orbits eastward around the Sun, the orbital plane must rotate eastward around the Earth at the same rate. Since it takes 365 days for the Earth to orbit the Sun, the sun-synchronous orbit must rotate about the Earth at just under one degree per day. The oblateness of the Earth perturbs the orbital plane by nearly this amount.

A sun-synchronous orbit is beneficial because it allows a satellite to view the same place on Earth with the same sun angle (or shadow pattern). This is very valuable for remote sensing missions because they use shadows to measure object height.[22]

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[20] Remember that velocity is a vector — it has both a magnitude and a direction.

[21] J2 is a constant describing the size of the bulge in the mathematical formulas used to model the oblate Earth.

[22] With a constant sun angle, the shadow lengths give away any changes in height, or any shadow changes give clues to exterior configuration changes.

Another significant effect of Earth's asymmetry is *apsidal line rotation*. Only elliptical orbits have a line of apsides and so this effect only affects elliptical orbits. This effect appears as a rotation of the orbit within the orbital plane; the argument of perigee changes. At an inclination of 63.4° (and its retrograde compliment, 116.6°), this rotation is zero. The Molniya orbit was specifically designed with an inclination of 63.4° to take advantage of this perturbation. With the zero effect at 63.4° inclination, the stability of the Molniya orbit improves limiting the need for considerable on-board fuel to counteract this rotation. At a smaller inclination (but larger than 116.6°), the argument of perigee rotates eastward in the orbital plane; at inclinations between 63.4° and 116.6°, the argument of perigee rotates westward in the orbital plane. This could present a problem for non-Molniya communications satellites providing polar coverage. If the apogee point rotated away from the desired communications (rotated from the Northern to Southern Hemisphere), the satellite would be useless.

The ellipticity of the equator has an effect that shows up most notably in geostationary satellites (also in inclined geosynchronous satellites). Because the equator is elliptical, most satellites are closer to one of the lobes and experience a slight gravitational misalignment. This misalignment affects geostationary satellites more because they view the same part of the earth's surface all the time, resulting in a cumulative effect.

The elliptical force causes the subpoint of the geostationary satellite to move east or west with the direction depends on its location. There are two stable points at 75 East and 105 West, and two unstable stable points 90° out (165 East and 5 West).[23]

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[23]A stable point is like a marble in the bottom of a bowl; an unstable stable point is like a marble perfectly balanced on the top of a hill.

Atmospheric Drag

The Earth's atmosphere does not suddenly cease; rather it trails off into space. However, after about 1,000 km (620 miles), its effects become minuscule. Generally speaking, atmospheric drag can be modeled in predictions of satellite position. The current atmospheric model is not perfect because of the many factors affecting the upper atmosphere, such as the earth's day-night cycle, seasonal tilt, variable solar distance, fluctuation in the earth's magnetic field, the suns 27-day rotation and the 11-year sun spot cycle. The drag force also depends on the satellite's coefficient of drag and frontal area, which varies widely between satellites.

The uncertainty in these variables cause predictions of satellite decay to be accurate only for the short term. An example of changing atmospheric conditions causing premature satellite decay occurred in 1978-1979, when the atmosphere received an increased amount of energy during a period of extreme solar activity. The extra solar energy expanded the atmosphere, causing several satellites to decay prematurely, most notably the U.S. space station SKYLAB.

The highest drag occurs when the satellite is closest to the earth (at perigee), and has a similar effect in performing a delta-V at perigee; it decreases the apogee height, circularizing the orbit. On every perigee pass, the satellite looses more kinetic energy (negative delta-V), circularizing the orbit more and more until the whole orbit is experiencing significant drag, and the satellite spirals in.

Third Body Effects

According to Newton's law of Universal Gravitation, every object in the universe attracts every other object in the universe. The greatest third body effects come from those bodies that are very massive and/or close such as the Sun, Jupiter and the Moon. These forces ffect satellites in orbits as well. The farther a satellite is from the Earth, the greater the

third body forces are in proportion to Earth's gravitational force, and therefore, the greater the effect on the high altitude orbits.

Radiation pressure

The Sun is constantly expelling atomic matter (electrons, protons, and Helium nuclei). This ionized gas moves with high velocity through interplanetary space and is known as the solar wind. The satellites are like sails in this solar wind, alternately being speeded up and slowed down, producing orbital perturbations.

Electromagnetic Drag

Satellites are continually traveling through the Earth's magnetic field. With all their electronics, satellites produce their own localized magnetic fields which interact with the earth's, causing torque on the satellite. In some instances, this torque is advantageous for stabilization. More specifically, satellites are basically a mass of conductors. Passing a conductor through a magnetic field causes a current in the conductor, producing electrical energy. Some recent experiments used a long tether from the satellite to generate electrical power from the earth's magnetic field (the tether also provided other benefits).

The electrical energy generated by the interaction of the satellite and the earth's magnetic field comes from the satellite's kinetic energy about the earth. The satellite looses orbital energy, just as it does with atmospheric drag, which results in orbital changes. The magnetic field is strongest close to the Earth where satellites travel the fastest. Thus, this effect is largest on low orbiting satellites. However, the overall effect due to electromagnetic forces is quite small.

DEORBIT AND DECAY

So far the concern has been with placing and maintaining satellites in orbit. When no longer useful, satellites must be removed from their operational orbit. Sometimes natural perturbations such as atmospheric drag take care of disposal, but not always.

For satellites passing close to the earth (low orbit or highly elliptical orbits), satellites can be programmed to re-enter, or they may re-enter autonomously. Deliberate re-entry of a satellite with the purpose of recovering the vehicle intact is called *deorbiting*. This is usually done to recover something of value: people, experiments, film, or the vehicle itself. The natural process of spacecraft (or any debris – rocket body, payload, or piece) eventually re-entering Earth's atmosphere is called *decay*.

In some situations, the satellites are in such stable orbits that natural perturbations will not do the disposal job . In these situations, the satellite must be removed from the desirable orbit. To return a satellite to earth without destroying it takes a considerable amount of energy. Obviously, it is impractical to return old satellites to earth from a high orbit. The satellite is usually boosted into a slightly higher orbit to get it out of the way, and there it will sit for thousands of years.

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