Mathematical spline processing method for filtering and compressing data

Olena Kolganova [1], Lidiia Tereshchenko [1], Viktoriia Kravchenko [1], Viktoriia Volkogon [1], Volodymyr Shutko [1], Mykola Shutko [1] and Natalia Mykhalchyshyn [2]

[1] *National Aviation University, Lubomyr Huzar Ave., 1, Kyiv, 03058, Ukraine*
[2] *Lviv Polytechnic National University, 12 Bandera Street, 79000, Lviv, Ukraine*

Abstract

Research focuses on inventing a methodology for creating linear splines with a customized grid of glue nodes. The results aim to improve the approximation properties of the spline function for processing various digital content. In particular, the processing of satellite signals, as well as the performance of image filtering or compression using the proposed results, play a key role in the design of the compression process of digital graphics to make it cheaper to store and send.

Keywords

Hermit spline, multiple-scale analysis, image compression, digital image

1. Introduction

According to Cisco's forecasts [1], global digital transformation from 2016 to 2021 will continue to significantly affect the demands and needs of IP networks.

This is primarily indicated by: the projected growth in the number of Internet users (from 3.3 to 4.6 billion, i.e. 58% of people worldwide), a quickening growth in the use of personal devices and machine-to-machine connections, an increase in the average capacity of broadband access and an increase in video traffic.

Regarding its proportion of all IP traffic and the expansion of Internet traffic in general, the video will continue to rule. Compared to 2016, when this indicator was 67% of Internet traffic, at the end of 2021 it was already 80%. By the end of 2021, there will be almost 2 billion active consumers of video content on Internet. It eliminates those who only communicate through mobile devices. It eliminats those who only communicate through mobile devices (excluding those who use exclusively mobile communications). For comparison, this indicator was almost 1.5 billion active consumers in 2016. At the end of 2021, via the global Internet per month 3 trillion minutes of video were transmitted (equivalent to 5 million video years per month or 1 million video minutes per second).

We must also realize that regardless of our desires our daily lives are filled with instances of computer vision [31]. First, photo filtering and compression are ubiquitous in our news feeds, social networks, online shops, books. An image should be thought of as functions that represent it in the form of a matrix of pixel values. In this case, filters serve as systems for creating an improved version of the image based on a combination of pixel values of the original image. For compression purposes, images are also considered in a two-dimensional array of pixels (Fig. 1) [18-21].

CMiGIN 2022: 2nd International Conference on Conflict Management in Global Information Networks, November 30, 2022, Kyiv, Ukraine
EMAIL: kolganova@nau.edu.ua (O. Kolganova); 10118@ ukr.net (L. Tereshchenko); elzkravchenko@ukr.net (V. Kravchenko); sonechkovikaa@gmail.com (V. Volkogon); vnshutko@ukr.net (V. Shutko); nataliia.l.mykhalchyshyn@lpnu.ua (N. Mykhalchyshyn)
ORCID: 0000-0002-1301-9611 (O. Kolganova); 0000-0001-8183-9016 (L. Tereshchenko); 0000-0003-0399-7013 (V. Kravchenko); 0000-0002-6813-5724 (V. Volkogon); 0000-0002-9761-5583 (V. Shutko); 0000-0002-3531-7724 (M. Shutko); 0000-0003-3810-6460 (N. Mykhalchyshyn)

In order to reduce the volume of data arrays to facilitate their processing, storage and transmission, coding methods have become widely used [9-11, 17, 25-27]. These methods are based on approximation or partial data extraction. These include lossy compression methods (irreversible compression). More components are deleted when the compression ratio is higher, resulting in coarser images. It is the opposite of reversible data compression (lossless compression), which does not degrade data. When compared to lossless approaches, the lossy compression technique offers a significantly higher level of data reduction [8].

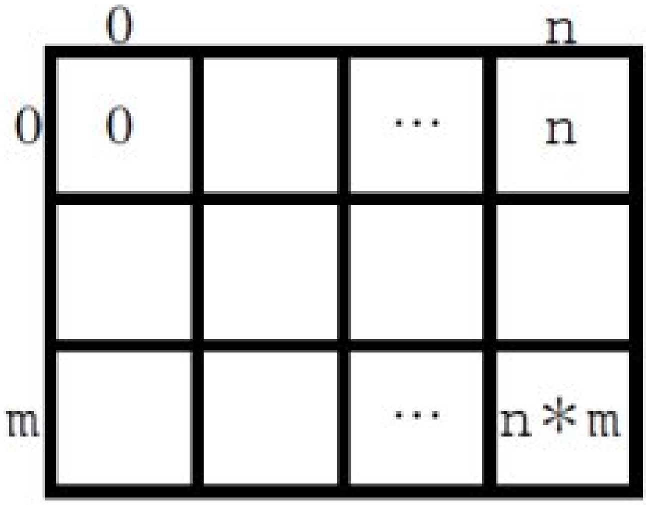


Figure 1: Image in a two-dimensional pixels' array

Lossy compression significantly reduces file size before the end user notices a deterioration in quality. Even strong compression may be desirable to further reduce the amount of data (to cut back on storage space or transmission times, for example). Most lossy compression algorithms make use of the discrete cosine transform (DCT), which was first presented by N. Ahmed, T. Natarajan, and K. R. Rao in 1974 [6].

The most typical application of this technology is to compress multimedia data, more so in streaming multimedia and Internet telephony. In comparison, compression of text files and data files (text elements and financial records) is usually done using a lossless method. For this purpose, the original lossless master file will be the basis for creating additional copies in the future. It may be useful to create a lossless master file that will then serve as a backbone for additional copying. This way, there is no need to make other lossy compressed copies of the original file, accumulating additional artifacts and data loss [12, 13, 15, 16].

We can depict a picture made up of individual pixels as a function of f to better comprehend the characteristics of images [28-30] and the technical process utilized to process them. Every pixel has a distinct meaning. Regarding grayscale images, pixel's intensity ranges from 0 to 255, where black is 0 and white is 255 (Fig. 2).

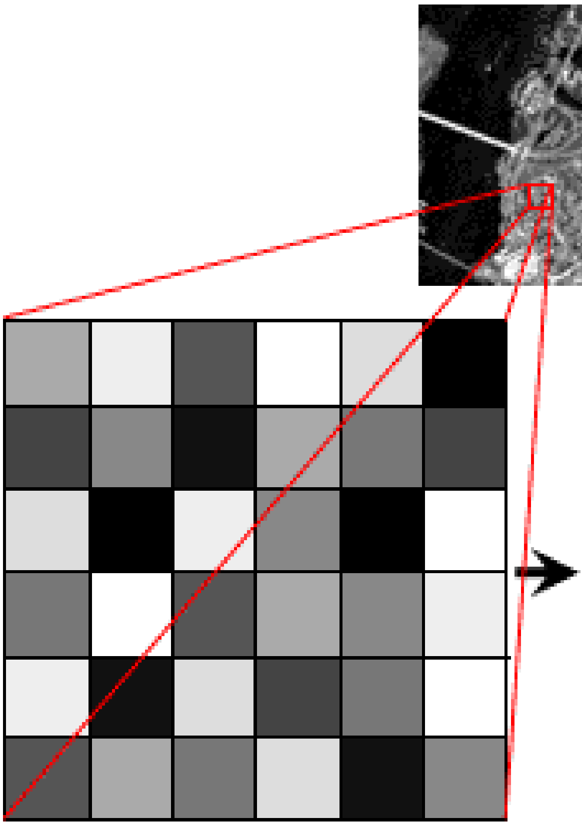
Assume that the function $f(x,t)$, is defined on a rectangular matrix with a finite range of intensity at the pixel position (x,t)

$$f:[a,b]x[c,d] \to [0,255]. \quad (1)$$

This method is simply continued in a color image. In its place, the function $f(x,t)$ is now a vector of three values:

$$f(x,y) = \begin{bmatrix} r(x,y) \\ g(x,y) \\ b(x,y) \end{bmatrix}. \quad (2)$$

Mixing red, green, and blue colors form a palette of color images according to the RGB palette. Accordingly, three channels, which we consider in the form of 1x3 vectors, represent each pixel. Since the integer values from 0 to 255 correspond to each of the specified colors, the number of color combinations is calculated as $256 \times 256 \times 256 = 16777216$. Thus, the union of the three functions will correspond to the function $f(x, t)$ (Fig. 3).



170	238	85	255	221	0
68	136	17	170	119	68
221	0	238	136	0	255
119	255	85	170	136	238
238	17	221	68	119	255
85	170	119	221	17	136

Figure 2: Pixel values in grayscale

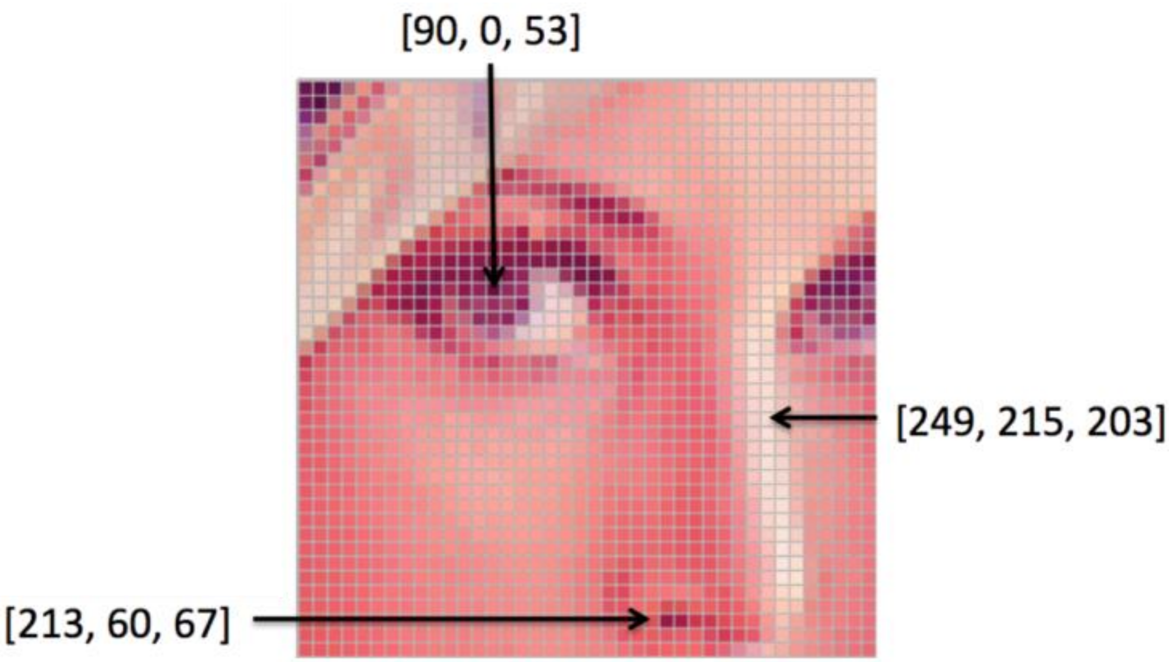


Figure 3: A matrix of pixel values as a representation of an image

Image warping and image filtering are common technologies of image processing. Realization of the range of the image by changing the value of the pixels is possible by using image filtering. In this case, the image colors change without changing the pixel positions. Changing the image area by changing the position of the pixels is possible using image warping. In this case, the points in the exposure area are matched without changing colors [11].

In the process of compressing information, the data is usually presented as a sequence of instantaneous values. The resulting data sequence is used to construct a function of some class that approximates the input signal in the sense of the selected criterion. In the future, when performing various transformations, a constructed function is used instead of a signal, which approximates it. This numerical-analytical approach is increasingly used in the modern signal processing theory, which is explained by computational considerations [23, 39, 40]. Quite important is the degree of adequacy of the numerical-analytical model constructed in this way to the real signal under study, the error of approximation of its individual characteristics [24, 41, 42]. Naturally, the approximation error depends on the selected class of functions [2-7, 22].

2. Related works and problem statement

Mathematicians have long encountered an intuitive method for using piecewise functions in approximation issues [34]. But, as N. P. Korniychuk notes, the intrusion of splines into the theory of approximation occurred due to interpolation problems, due to their good computational and approximate properties [7].

The beginning of the development of spline interpolation theory and the term spline itself is deduced from 1946 in the article by Isaac Jacob Schoenberg [3, 4]. Its intensive development took place in the 50s and 70s, and the traditional field of application of interpolation splines became computer-aided design systems at this time. However, the potential of splines is much broader than just describing some curves [32]. In the real world, many physical processes are by their very nature splines. In mechanics, it is the deformation of a flexible plate or rod fixed at individual points, or the trajectory of the object, if the force acting on it, changes stepwise (the trajectory of an artificial space object with active and inertial segments of motion, the trajectory of the aircraft with a step change of thrust and change the profile of the wing, etc.). In thermodynamics, this is the heat exchange in a rod composed of fragments with different heat transfers. In chemistry - diffusion through layers of different substances. In electricity - the propagation of electromagnetic fields through heterogeneous media. That is, the spline is not a contrived mathematical abstraction, and in many cases, it is a solution of differential equations that describe very real physical processes [38].

There are a large number of structures called splines. So let's try to make some detail in this variety.

Type of spline fragments. The fact that the spline consists of fragments of the same type is one of the key features that distinguishes it from other piece functions. The most famous splines, consist of fragments - algebraic polynomials not higher than a given degree. As a rule, these are cubic polynomials, or polynomials of non-even degrees. Linear, cubic, fifth degree. Higher degrees are rarely used, due to the complexity of the calculations and the complexities described in the previous section. Their main advantage is the simplicity of calculations and analysis. The disadvantage is that relatively few real physical processes correspond to this dependence [33].

Setting objectives. Of particular interest is the class of polynomial splines, which successfully combines the advantages of static polynomials and the ability to control the smoothness of approximations [2, 5-7].

The choice of polynomial splines as reduction functions in interpolation and approximation necessitates the resolution of a number of issues, the most important of which are the issues with selecting the degree of spline and the quantity of nodes in the interpolation grid [36, 40-42].

The spline degree used to recover the signal is based on a priori information about the differential properties of the signal, namely, the degree is selected so that the differential properties of the spline coincide with the a priori known differential properties of the signal. Due to the fact that in practice there is information only about the initial derivatives of the signal, for approximation it is advisable to use Hermitian splines with different tasks of the values of the derivatives in the interpolation nodes.

In some cases, consider functions that are close to the boundary between splines and ordinary functions and splines and piecewise functions. It:

- Splines consisting of two fragments. They have a simplified version of the construction, but special attention should be paid to boundary conditions

- A piecewise constant spline function has no continuity even values. A trivial option that does not have the main advantage of splines - smoothness. As well as the laman, has rather methodical value for developing technology of work with splines.

The grid of interpolation points can significantly affect the efficiency of calculations. Important are the cases of uniform grid and uniform grid, with the distance between the points of a multiple of the distance between the nodes of the spline. But the grid can be uneven [14].

Consider the process of creating a linear spline using this spline's modified grid of glue nodes. When using multiscale analysis to compress and filter graphical data, it is interesting to take splines into consideration [35, 37].

3. Iterative method of constructing linear splines

In practical problems of smoothing experimental data, it is often advisable to dwell on piecewise linear approximation. So, for example, according to the given observations (x_i, y_i), $i=\overline{1, N}$, where $a \le x_i \le b$, it is necessary to construct a spline $P(x)$ - r-frames broken line so that

$$\Phi = \Phi[P] = \sum_{i=1}^{N} \lambda_i (P(x_i) - y_i)^2 \Rightarrow \min \tag{3}$$

where $\lambda_i > 0$ – set weights.

We describe the spline $P(x)$ by a set of $2r$ numbers, namely: $\tilde{x}_1 < \tilde{x}_2 < \ldots < \tilde{x}_{r-1}$ - nodes in the interval $[a, b]$ (for commonality we also put, $\tilde{x}_0 = a$, $\tilde{x}_r = b$) and the value of the function in the nodes $P(\tilde{x}_k) = a_k$, $0 \le k \le r$ (Fig. 4).

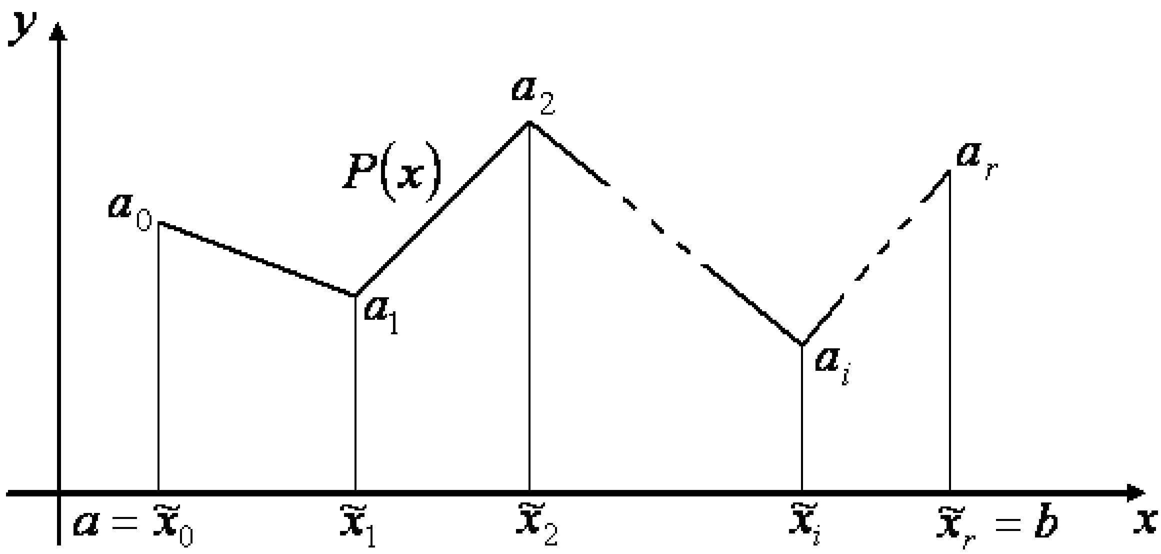


Figure 4: An example of a linear spline

Let's sign $\Delta_k = [\tilde{x}_{k-1}, \ \tilde{x}_k)$ when $1 \le k \le r-1$, $\Delta_r = [\tilde{x}_{r-1}, \ \tilde{x}_r]$

$$\Phi_k = \Phi_k[P] = \sum_{\Delta_k} \lambda_i (P(x_i) - y_i)^2 \tag{4}$$

where $\sum_{\Delta_k} -$ the amount by i, for which $x_i \in \Delta_k$.

The following iterative algorithm for solving the problem is proposed.

Step 0. The initial values $\tilde{x}_k = \tilde{x}_k^{(0)}$ are set and the corresponding values $a_k = a_k^{(0)}$ are determined, which minimize the functional Φ when $\tilde{x}_k$ are fixed.

Step 1. a) Fix the odd nodes, that is the nodes $\tilde{x}_1, \tilde{x}_3, \dots$ and values of the function in them: $a_1^{(0)}, a_3^{(0)}, \dots$ and under this condition we calculate the optimal values of $\tilde{x}_0 = a = \tilde{x}_0^{(1)}$, $\tilde{x}_2 = \tilde{x}_2^{(1)}$, $\tilde{x}_4 = \tilde{x}_4^{(1)}$, … and the corresponding values of $a_2 = a_2^{(1)}$, $a_4 = a_4^{(1)}$,….

b) For fixed even nodes, we mean $\tilde{x}_2^{(1)}, \dots, \tilde{x}_6^{(1)}, \dots$ are given, we optimize the odd nodes, so $\tilde{x}_1 = \tilde{x}_1^{(1)}$, $\tilde{x}_3 = \tilde{x}_3^{(1)}$,… and determine the value of the function in the nodes $a_1^{(1)}, a_3^{(1)}, \dots$.

The result is nodes $\tilde{x}_0^{(1)} = a$, $\tilde{x}_1^{(1)}$, $\tilde{x}_2^{(1)}$,…, $\tilde{x}_{r-1}^{(1)}$, $\tilde{x}_r^{(1)} = b$ and values in nodes: $a_0^{(1)}, a_1^{(1)}, a_2^{(1)}, \dots$. The spline built on this information will be initialized $P(x)$ and defined $\Phi[P_1]$.

Suppose the $n-1$ steps are done and we have nodes $\tilde{x}_k^{(n-1)}$ and a spline $P_{n-1}(x)$; $P_{n-1}(\tilde{x}_k^{(n-1)}) = a_k^{(n-1)}$, $0 \le k \le r$.

Let's calculate $\Phi[P_{n-1}]$. Based on the specified accuracy of calculations δ, check the condition $\Phi[P_{n-2}] - \Phi[P_{n-1}] \le \delta$. If it is executed, we stop the iterative process and consider $P(x) = P_{n-1}(x)$. Otherwise, we proceed to the n-th step, which differs from the above-described 1st step in that the superscript (0) is replaced by $(n-1)$, and (1) – by (n).

The movement of $\tilde{x}_k$ is illustrated in Figure 5.

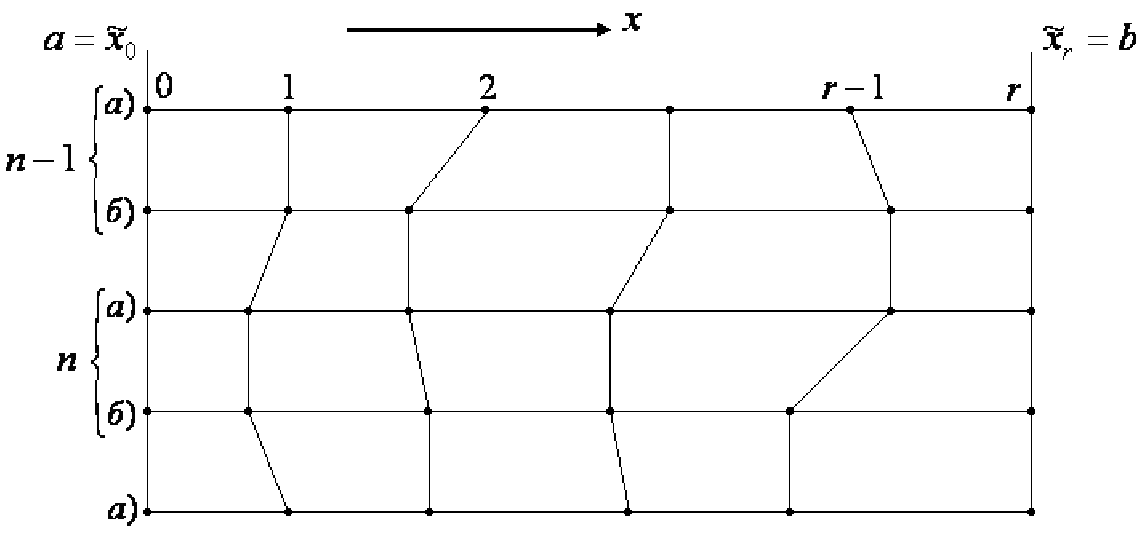


Figure 5: Movement of $\tilde{x}_k$

It remains to reveal the subalgorithm for calculating the optimal values a_k when $\tilde{x}_k$ are fixed. Concider calculation of optimal values a_k for fixed $\tilde{x}_k$. Let's write expressions for Φ_k and their derivatives.

At interval

$$P(x) = \frac{a_{k-1}(\tilde{x}_k - x) + a_k(x - \tilde{x}_{k-1})}{\tilde{x}_k - \tilde{x}_{k-1}}, \tag{5}$$

therefore

$$\Phi_k - \sum_{x_i \in [\tilde{x}_{k-1},\ \tilde{x}_k)} \lambda_i \left[\frac{a_{k-1}(\tilde{x}_k - x_i) + a_k(x_i - \tilde{x}_{k-1})}{\tilde{x}_k - \tilde{x}_{k-1}} - y_i \right]^2, \tag{6}$$

$$\frac{1}{2}\frac{\partial \Phi_k}{\partial a_{k-1}} = \sum_{x_i} \lambda_i \left[\frac{a_{k-1}(\tilde{x}_k - x_i) + a_k(x_i - \tilde{x}_{k-1})}{\tilde{x}_k - \tilde{x}_{k-1}} - y_i \right] \frac{\tilde{x}_k - x_i}{\tilde{x}_k - \tilde{x}_{k-1}} \tag{7}$$

$$\frac{1}{2}\frac{\partial \Phi_k}{\partial a_k} = \sum_{x_i} \lambda_i \left[\frac{a_{k-1}(\tilde{x}_k - x_i) + a_k(x_i - \tilde{x}_{k-1})}{\tilde{x}_k - \tilde{x}_{k-1}} - y_i \right] \frac{x_i - \tilde{x}_{k-1}}{\tilde{x}_k - \tilde{x}_{k-1}} \tag{8}$$

Because of the dependence on the value of a_k only values of Φ_k and Φ_{k+1}, then the minimum of the Φ conditions takes the form:

$$\frac{\partial \Phi_1}{\partial a_0} = 0;\ \frac{\partial \Phi_k}{\partial a_k} + \frac{\partial \Phi_{k+1}}{\partial a_k} = 0,\quad 1 \le k \le r-1;\ \frac{\partial \Phi_r}{\partial a_r} = 0. \tag{9}$$

Let’s initialize

$$q_i = \frac{x_i - \tilde{x}_{k-1}}{\tilde{x}_k - \tilde{x}_{k-1}},\quad p_i = \frac{\tilde{x}_k - x_i}{\tilde{x}_k - \tilde{x}_{k-1}}. \tag{10}$$

Obviously, $p_i + q_i = 1$.

Expressions (6), (7), (8) take a simple form

$$\Phi_k = \sum_{\Delta_k} \lambda_i (a_{k-1} p_i + a_k q_i - y_i)^2, \tag{11}$$

$$\frac{1}{2}\frac{\partial \Phi_k}{\partial a_{k-1}} = \sum_{\Delta_k} \lambda_i (a_{k-1} p_i + a_k q_i - y_i) p_i, \tag{12}$$

$$\frac{1}{2}\frac{\partial \Phi_k}{\partial a_k} = \sum_{\Delta_k} \lambda_i (a_{k-1} p_i + a_k q_i - y_i) q_i. \tag{13}$$

System (9) is disclosed as follows:

$$a_0 \sum_{\Delta_1} \lambda_i p_i^2 + a_1 \sum_{\Delta_1} \lambda_i p_i q_i = \sum_{\Delta_1} \lambda_i y_i p_i, \tag{14}$$

$$\begin{aligned} a_{k-1} \sum_{\Delta_k} \lambda_i p_i q_i + a_k \left(\sum_{\Delta_k} \lambda_i q_i^2 + \sum_{\Delta_{k+1}} \lambda_i p_i^2 \right) + a_{k+1} \sum_{\Delta_{k+1}} \lambda_i p_i q_i = \\ = \sum_{\Delta_k} \lambda_i y_i q_i + \sum_{\Delta_{k+1}} \lambda_i y_i q_i,\quad 1 \le k \le r-1, \end{aligned} \tag{15}$$

$$a_{r-1} \sum_{\Delta_r} \lambda_i p_i q_i + a_r \sum_{\Delta_r} \lambda_i q_i^2 = \sum_{\Delta_r} \lambda_i y_i q_i. \tag{16}$$

We will consider a_0 as indefinite size and we will express the a_1 through the a_0, the a_2 through the a_1 and so on to the a_r. To do this, denote

$$a_k = A_k a_0 + B_k,\quad 0 \le k \le r. \tag{17}$$

Let's put, $A_0 = 1$, $B_0 = 0$. Then with (14) we have

$$A_1 = -\sum_{\Delta_1} \lambda_i p_i^2 \Big/ \sum_{\Delta_1} \lambda_i p_i q_i, \tag{18}$$

$$B_1 = \sum_{\Delta_1} \lambda_i y_i p_i \Big/ \sum_{\Delta_1} \lambda_i p_i q_i. \tag{19}$$

Substituting (17) into (15), we obtain:

$$A_k = -\left[A_{k-2} \sum_{\Delta_{k-1}} \lambda_i p_i q_i + A_{k-1} \left(\sum_{\Delta_{k-1}} \lambda_i q_i^2 + \sum_{\Delta_k} \lambda_i p_i^2 \right) \right] \Big/ \sum_{\Delta_k} \lambda_i p_i q_i , \tag{20}$$

$$B_k = \left[\sum_{\Delta_{k-1}} \lambda_i y_i q_i + \sum_{\Delta_k} \lambda_i y_i p_i - B_{k-2} \sum_{\Delta_{k-1}} \lambda_i p_i q_i - \right.$$
$$\left. - B_{k-1} \left(\sum_{\Delta_{k-1}} \lambda_i q_i^2 + \sum_{\Delta_k} \lambda_i p_i^2 \right) \right] \Big/ \sum_{\Delta_k} \lambda_i p_i q_i , \quad 2 \le k \le r . \tag{21}$$

Thus, we consistently calculate A_k, B_k up to A_r, B_r. Now let's subst20ute $a_{r-1} = A_{r-1} a_0 + B_{r-1}$ and $a_r = A_r a_0 + B_r$ in the equation (16):

$$\left(A_{r-1} \sum\nolimits_{\Delta_r} \lambda_i p_i q_i + A_r \sum\nolimits_{\Delta_r} \lambda_i q_i^2 \right) a_0 =$$
$$= \sum\nolimits_{\Delta_r} \lambda_i y_i q_i - \left(B_{r-1} \sum\nolimits_{\Delta_r} \lambda_i p_i q_i + B_r \sum\nolimits_{\Delta_r} \lambda_i q_i^2 \right) . \tag{22}$$

From this equation we define and substitute a_0 in (17), where A_k and B_k we already know. Thus, values of a_k defined completely.

4. Conclusions

This approach makes it possible to vary the grid of interpolation points, choosing its characteristics in a specific situation to achieve the optimum outcome. The creation of a method for creating a linear spline that uses a customized grid for its glue nodes allows for a reduction in the spline's standard deviations from the approximated function. In other words, it enhances the spline function's approximation qualities, which can be utilized to process a range of digital data. We implement them specifically while processing satellite signals, filtering and compressing graphic data. This paper did not set out the task of developing a complete method for compression of digital images. The research will be continued for the creation of a mathematical approach to develop spline functions of the third degree with adaptive calculation of the gluing nodes grid of this spline. Such splines are of interest regarding their practical use in signal and image processing problems. We can conclude that this strategy offers room for advancement.

5. References

[1] Cisco Mobile Visual Networking Index (VNI) Forecast Projects 7-Fold Increase in Global Mobile Data Traffic from 2016-2021. URL: https://newsroom.cisco.com/c/r/newsroom/en/us/a/y2017/m02/cisco-mobile-visual-networking-index-vni-forecast-projects-7-fold-increase-in-global-mobile-data-traffic-from-2016-2021.html.

[2] J. Schoenberg, Contribution to the problem of approximation of equidistant data by analytic functions, Quarterly of Applied Mathematics 4 (1946) 45-99. URL: https://www.semanticscholar.org/paper/Contributions-to-the-problem-of-approximation-of-by-Schoenberg/731ecc3d6cde71cfde719ae809e2bfb1a852a1fd.

[3] P. Prystavka, O. Cholyshkina, Pyramid Image and Resize Based on Spline Model. International Journal of Image, Graphics and Signal Processing (IJIGSP), Vol.14, No.1, pp. 1-14, 2022. DOI: 10.5815/ijigsp.2022.01.01

[4] I. J. Schoenberg, Cardinal spline interpolation, Philadelphia, Society of Industrial and Applied Mathematics, 1973. URL: https://epubs.siam.org/doi/book/10.1137/1.9781611970555.
[5] X. Zhang, B. Peng, Z. Al-Huda, D. Zhai, FeatureGAN: Combining GAN and Autoencoder for Pavement Crack Image Data Augmentations. International Journal of Image, Graphics and Signal Processing (IJIGSP), Vol.14, No.5, pp. 28-43, 2022. DOI:10.5815/ijigsp.2022.05.03
[6] N. Ahmed, T. Natarajan, K. R. Rao, Discrete Cosine Transform, IEEE Transactions on Computers C-23(1) (1974) 90-93. doi: 10.1109/T-C.1974.223784.
[7] C. S. Tiwari, V. Kumar Jha, Enhancing Security of Medical Image Data in the Cloud Using Machine Learning Technique. International Journal of Image, Graphics and Signal Processing (IJIGSP), Vol.14, No.4, pp. 13-31, 2022. DOI:10.5815/ijigsp.2022.04.02
[8] Image Data Compression. URL: http://user.engineering.uiowa.edu/~dip/lecture/DataCompression.html.
[9] D. A. Huffman, A Method for the Construction of Minimum-Redundancy Codes, Proceedings of the IRE 4(9) (1952) 1098-1101. doi: 10.1109/JRPROC.1952.273898.
[10] A. Bovik, Handbook of Image and Video Processing. Communications, Networking and Multimedia) 2nd Edition, USA, Academic Press, 2005.
[11] S. Karanwal, Analysis on Image Enhancement Techniques. International Journal of Engineering and Manufacturing (IJEM). Vol.13, No.2, pp. 9-21, 2023. DOI:10.5815/ijem.2023.02.02
[12] The JPEG image format explained, BT Group, 2018. URL: https://web.archive.org/web/20190805194553/https://home.bt.com/tech-gadgets/photography/what-is-a-jpeg-11364206889349.
[13] S. Krivenko, O. Kolganova, V. Lukin, Lossy compression of images corrupted by spatially correlated noise, in: Proceedings of the 2016 13th International Conference on Modern Problems of Radio Engineering, Telecommunications and Computer Science (TCSET), Lviv, Ukraine, 2016, pp. 698-702. doi: 10.1109/TCSET.2016.7452157.
[14] V. V. Lukin, S. S. Krivenko, E. O. Kolganova, M. K. Chobanu, Acceleration and efficiency analysis of dct-based filtering, TRE 72(7) (2013). doi: 10.1615/TelecomRadEng.v72.i7.50.
[15] R. Hoffman, Data Compression in Digital Systems, Springer Science & Business Media, 2012.
[16] D. Taubman, M. Marcellin, JPEG2000: Image Compression Fundamentals, Standards and Practice: Image Compression Fundamentals, Standards and Practice. Springer Science & Business Media, 2002. URL: https://link.springer.com/book/10.1007/978-1-4615-0799-4.
[17] A. C. Bovik, The Essential Guide to Video Processing, Academic Press, 2009. doi: https://doi.org/10.1016/B978-0-12-374456-2.X0001-1.
[18] M. Shutko, V. Shutko, L. Tereshchenko, M. Zaliskyi, I. Silantieva, Two-Dimensional Spectral Detector for Baggage Inspection X-ray System, CEUR Workshop Proceedings 2300 (2018) 63-66. URL: https://ceur-ws.org/Vol-2300/Paper16.pdf.
[19] Z. Al-Ameen, A. Muttar, G. Al-Badrani, Improving the Sharpness of Digital Image Using an Amended Unsharp Mask Filter. International Journal of Image, Graphics and Signal Processing (IJIGSP), Vol.11, No.3, pp. 1-9, 2019. DOI: 10.5815/ijigsp.2019.03.01
[20] M. Zaliskyi, O. Shcherbyna, L. Tereshchenko, A. Osipchuk, O. Zharova, Shadow Image Processing of X-Ray Screening System for Aviation Security. International Journal of Image, Graphics and Signal Processing (IJIGSP), Vol.14, No.6, pp. 26-46, 2022. DOI:10.5815/ijigsp.2022.06.03
[21] F. O. Isinkaye, A. G. Aluko, O. A. Jongbo, Segmentation of Medical X-ray Bone Image Using Different Image Processing Techniques. International Journal of Image, Graphics and Signal Processing (IJIGSP), Vol.13, No.5, pp. 27-40, 2021. DOI: 10.5815/ijigsp.2021.05.03
[22] O. Solomentsev, M. Zaliskyi, Method of sequential estimation of statistical distribution parameters in control systems design, in: Proceedings of the IEEE 3rd International Conference on Methods and Systems of Navigation and Motion Control (MSNMC), Kiev, Ukraine, 2014, pp. 135-138. doi: 10.1109/MSNMC.2014.6979752.
[23] S. Shukla, R. Gupta, D. S. Rajput, Y. Goswami, V. Sharma, A Comparative Analysis of Lossless Compression Algorithms on Uniformly Quantized Audio Signals. International Journal of Image, Graphics and Signal Processing (IJIGSP), Vol.14, No.6, pp. 59-69, 2022. DOI:10.5815/ijigsp.2022.06.05

[24] V. Kasianov, The Flight Modelling: Monography,Kyiv, NAU, 2004.
[25] M. Shutko, Methods and means of increasing the reliability of measurement information processing and control of parameters of radio-electronic systems for air traffic control, Dr. thesis, KIIGA, Kyiv, Ukraine, 1991.
[26] W. K. Pratt, J. Kane, H. C. Andrews, Hadamard transform image coding, Proceedings of the IEEE 57(1) (1969) 58–68.
[27] D. Le Gall, A. Tabatabai, Sub-band coding of digital images using symmetric short kernel filters and arithmetic coding techniques, in: Proceedings of the International Conference on Acoustics, Speech, and Signal Processing, New York, NY, USA, 1988, pp. 761-764, doi: 10.1109/ICASSP.1988.196696.
[28] P. Prystavka, O. Cholyshkina, Pyramid Image and Resize Based on Spline Model. International Journal of Image, Graphics and Signal Processing (IJIGSP), Vol.14, No.1, pp. 1-14, 2022. DOI: 10.5815/ijigsp.2022.01.01
[29] S. Olszewski, Y. Tanasiichuk, V. Mashkov, V. Lytvynenko, I. Lurie, Digital Method of Automated Non-destructive Diagnostics for High-power Magnetron Resonator Blocks. International Journal of Image, Graphics and Signal Processing (IJIGSP), Vol.14, No.1, pp. 40-49, 2022. DOI: 10.5815/ijigsp.2022.01.04
[30] N. S. Awarayi, O. Appiah, B. A. Weyori, C. B. Ninfaakang, A Digital Image Watermarking Using Dwt and L-shaped Tromino Fractal Encryption. International Journal of Image, Graphics and Signal Processing (IJIGSP), Vol.13, No.3, pp. 33-43, 2021. DOI: 10.5815/ijigsp.2021.03.03
[31] C. S. Swartz, Understanding Digital Cinema: A Professional Handbook, Taylor & Francis, 2005.
[32] J. Ferguson, Multivariable Curve Interpolation, Journal of the ACM 11(2) (1964) 221–228. doi: 10.1145/321217.321225.
[33] J. H. Ahlberg, E. N. Nilson, J. L. Walsh, The Theory of Splines and Their Applications, Academic Press, 1967. URL: https://shop.elsevier.com/books/the-theory-of-splines-and-their-applications/bellman/978-1-4831-9792-0.
[34] G. Birkhoff, Fluid dynamics, reactor computations, and surface representation, in: A history of scientific computing, New York, NY, USA: Association for Computing Machinery, 1990, pp. 63–87.doi: https://doi.org/10.1145/87252.88072.
[35] R. H. Bartels, J. C. Beatty, B. A. Barsky, An Introduction to Splines for Use in Computer Graphics and Geometric Modeling, Morgan Kaufmann, 1995.
[36] G. Birkhoff, C. R de Boor, Piecewise Polynomial Interpolation and Approximation, H. L Garabedian (Ed.), Approximation of Functions, Elsevier, Amsterdam, 1965, pp. 164-190.
[37] P. Davis, B-splines and Geometric design, SIAM News 29 (5) (1997). URL: https://users.wpi.edu/~pwdavis/sinews/spline17.htm.
[38] J. Epperson, History of Splines, NA Digest 98(26) (1998). URL: https://user.it.uu.se/~ledfelt/spline1.html.
[39] J. Stoer , R. Bulirsch, Introduction to Numerical Analysis, Springer-Verlag, 2002. URL: https://link.springer.com/book/10.1007/978-0-387-21738-3.
[40] D. M. Young, Garrett Birkhoff and applied mathematics, Notices of the AMS 44(11) (1997) 1446–1449. URL: https://www.ams.org/notices/199711/comm-young.pdf.
[41] S. C. Chapra, R. P. Canale, Numerical Methods for Engineers, 5th ed., McGraw Hill College Div, 2008.
[42] C. E. Shannon, A Mathematical Theory of Communication, The Bell System Technical Journal 27 (1948) 379–423, 623–656. URL: https://people.math.harvard.edu/~ctm/home/text/others/shannon/entropy/entropy.pdf