

# A Proposal of Facial Expression Analysis using a Face Plane

Hideyuki Negishi<sup>1</sup> Hiroyuki Hase<sup>1</sup> Masaaki Yoneda<sup>1</sup> Jien Kato<sup>2</sup>

<sup>1</sup>Dept. of Intellectual Information Systems  
Engineering, Toyama University

<sup>2</sup>Dept. of Information Engineering,  
Nagoya University

## Abstract

Changes in facial expressions or differences in individual faces appear not only in distinctive features such as eyes, nose and mouth but also in the bone structure or the movement of facial muscles. We present a new concept that reflects the differences of the bone structure of a face and/or the movement of facial muscles and name this a face plane. The face plane can be described by a few parameters and reduce facial vague features. This paper presents the concept and shows some experimental results of the face expression analysis using the face plane.

## 1 Introduction

Facial analyses by image processing have been studied for a couple of decades. There are two approaches using two dimensional data and three dimensional data. The former has been mainly used until now. The extraction methods of facial parts like eyes, nose and mouth etc. using their relative position have been studied for facial recognition[1]. However, it is quite difficult to precisely extract these parts from facial images. Even though they could be extracted, it is still not easy to distinguish similar shapes in recognition. Therefore, the facial images were treated as patterns, and statistical pattern recognition methods have been applied to them without using facial features. Turk et al.[2,3] proposed *eigenface* using PCA. Simple pattern matching methods like this need the normalization of size and position. The higher order local autocorrelation which was proposed by Kurita et al.[4] is robust for the variation of face size and position. The capability and stability of those methods using two dimensional data depend on the features to be selected.

Other approaches such as recognition using a profile which is a part of the features of three dimensional data have been studied. Kaufman et al.[5] used the autocorrelation of a silhouette data. Harmon[6] obtained a good recognition result using geometrical characteristics of a profile silhouette.

Studies using three dimensional data have been reported since 1990's. Three dimensional contour lines[7], depth[8] and curvature of a face surface[9] calculated from facial distance data have been proposed. However, it is not easy to select features from three dimensional facial data.

This paper presents a new facial analysis method using three dimensional data. Our approach focuses on the movement of facial muscles from which it is difficult to extract any distinct features. The movement of facial muscles is reflected on a plane that we call "face plane"[10] which is a virtual plane across the head. The concept of the face plane is as follows; first, imagine the normal lines on the partitioned facial small areas. These normal lines point to the center of the head. Next, suppose a virtual plane inside the head. There maybe exist the optimal position and orientation of the plane that mean a convergence of the normal lines. The illustration of a face plane is shown in Fig.1. The crossing points between the normal lines and the virtual plane create a distribution. The shape and spread of the distribution will depend on the movement of the facial muscles.

In this paper, we compare two approaches to obtain the face plane. One is based on the least distance criterion(LDC in short) and the other is based on the least variance criterion(LVC in short). Lastly, we show some experimental results of facial expression analysis.

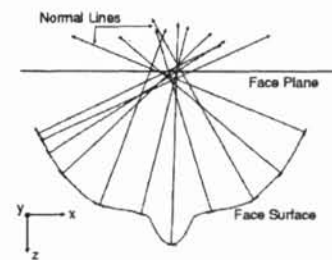


Fig.1 The concept of the face plane(Top view)

## 2 Approaches

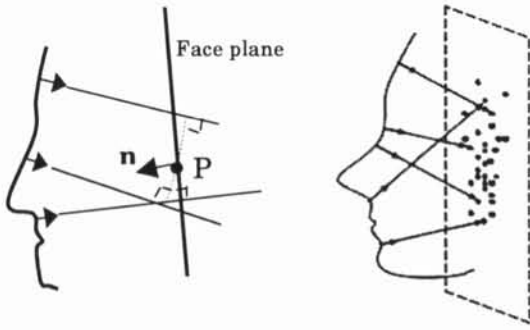
We present the concept of two methods (Fig.2).

### 2.1 Face plane based on the least distance criterion

This is a new definition of the face plane. We represent

<sup>1</sup> Address: 3190 Gofuku, Toyama 930-8555 Japan. E-mail: [negishi@pat.iis.toyama-u.ac.jp](mailto:negishi@pat.iis.toyama-u.ac.jp)

<sup>2</sup> Address: Furou-cho, Chigusa-ku Nagoya 464-8603 Japan. E-mail: [jien@nuie.nagoya-u.ac.jp](mailto:jien@nuie.nagoya-u.ac.jp)



(a) Least distance criterion (b) Least variance criterion

**Fig.2 Two approaches for getting a face plane**

the three dimensional data on the facial surface as  $\mathbf{p}_i = (a_i, b_i, c_i)^t, (i=1, \dots, N)$ . We assume that  $z$  axis is the optical axis of the range finder, a face is put in the negative direction. The  $x-y$  plane is perpendicular to the  $z$  axis, where  $x$  and  $y$  axes are the horizontal and vertical directions respectively. The  $x, y$  and  $z$  axes are in the relation of the right screw.

Now, let the normalized vector of the normal line that passes a point  $\mathbf{p}_i$  on the face be  $\mathbf{f}_i = (l_i, m_i, n_i)^t$ . Now suppose a point  $\mathbf{P}$  inside the head and orthogonal projections from the point to the normal lines. The distance  $d_i$  of the orthogonal projection from the point to the normal line  $\mathbf{f}_i$  is as follows;

$$d_i^2 = \|\mathbf{P} - \mathbf{p}_i\|^2 - (\mathbf{P} - \mathbf{p}_i, \mathbf{f}_i)^2$$

$$= (\mathbf{P} - \mathbf{p}_i)^t (E - \mathbf{f}_i \cdot \mathbf{f}_i^t) (\mathbf{P} - \mathbf{p}_i), \quad (1)$$

where  $E$  is the unit matrix and  $(\bullet, \bullet)$  is the inner product. The optimization can be carried out by differentiating  $Q = \sum_i d_i^2$  with respect to  $\mathbf{P}$ .

$$\frac{dQ}{d\mathbf{P}} = 2 \sum_i (E - \mathbf{f}_i \cdot \mathbf{f}_i^t) (\mathbf{P} - \mathbf{p}_i) = 0 \quad (2)$$

Then we can obtain the optimal point  $\mathbf{P}$ .

$$\mathbf{P} = \left[ \sum_i (E - \mathbf{f}_i \cdot \mathbf{f}_i^t) \right]^{-1} \cdot \sum_j (E - \mathbf{f}_j \cdot \mathbf{f}_j^t) \mathbf{p}_j \quad (3)$$

The face plane passes this point. But the orientation of the plane has not been determined. Therefore, we define the direction  $\mathbf{n}$  which makes the next equation the maximum.

$$S = \sum_i (\mathbf{f}_i, \mathbf{n})^2 = \mathbf{n}^t \left( \sum_i \mathbf{f}_i \cdot \mathbf{f}_i^t \right) \mathbf{n} \quad (4)$$

This results in an eigen problem. By solving this problem, we can get three eigenvalues and three eigenvectors. Let them be  $\lambda_1 \geq \lambda_2 \geq \lambda_3$  and  $\phi_1, \phi_2, \phi_3$  respectively. We can choose  $\phi_1$  as the direction  $\mathbf{n}$  of "face plane based on the least distance".

## 2.2 Face plane based on the least variance criterion

We proposed this idea before [10]. We will show it here briefly. The line equation with the direction  $\mathbf{f}_i$  passing a point  $\mathbf{p}_i$  is represented as follows:

$$\mathbf{x} = \mathbf{p}_i + t \cdot \mathbf{f}_i, \quad (5)$$

where  $\mathbf{x} = (x, y, z)^t$  is a point on the normal line,  $t$  is a real number.

Expressing equation (5) element-wise, we can obtain the next equation (6).

$$\begin{aligned} x &= a_i + t \cdot l_i \\ y &= b_i + t \cdot m_i \\ z &= c_i + t \cdot n_i \end{aligned} \quad (6)$$

Then, suppose a virtual plane in a head;

$$Ax + By + Cz + D = 0 \quad (7)$$

where  $C$  was fixed to 1. Substituting equation (6) in equation (7), we can obtain  $t$ .

$$t = - \frac{Aa_i + Bb_i + Cc_i + D}{Al_i + Bm_i + Cn_i} \quad (8)$$

Substituting  $t$  in equation (5), we can obtain the crossing point  $(x_i^*, y_i^*, z_i^*)^t$  on the plane. The distribution of the crossing points depends on the parameters  $A, B$  and  $D$  of the plane. According to the fact that most of the normal lines are pointing to the center of the head, there exists a position of the virtual plane in which the spread of the crossing points shows the minimum distribution. We call this plane "face plane based on the least variance criterion". Therefore, we define the quantity of the total distribution by equation (9).

$$V = V(x^*) + V(y^*) + V(z^*), \quad (9)$$

where  $V(x^*), V(y^*)$  and  $V(z^*)$  are the variances of the crossing points for  $x, y$  and  $z$  directions respectively.  $V(x^*)$  can be represented in the next formula.

$$V(x^*) = E[x^{*2}] - (E[x^*])^2 \quad (10)$$

$V(y^*), V(z^*)$  can be calculated in the same way.

To minimize equation (9) in the parametric space  $\{A, B, D\}$ , we derive the derivatives of  $V$ . They can be described as follows;

$$\begin{aligned} \frac{\partial V}{\partial A} &= \frac{\partial V(x^*)}{\partial A} + \frac{\partial V(y^*)}{\partial A} + \frac{\partial V(z^*)}{\partial A} \\ \frac{\partial V}{\partial B} &= \frac{\partial V(x^*)}{\partial B} + \frac{\partial V(y^*)}{\partial B} + \frac{\partial V(z^*)}{\partial B} \\ \frac{\partial V}{\partial D} &= \frac{\partial V(x^*)}{\partial D} + \frac{\partial V(y^*)}{\partial D} + \frac{\partial V(z^*)}{\partial D} \end{aligned} \quad (11)$$

Each component of the derivatives can be obtained analytically. By using the steepest descent method, we can

obtain the optimum parameters  $A, B$  and  $D$  that show the minimum value of  $V$ .

$$\begin{aligned} A^{new} &= A^{old} - k_A \cdot \frac{\partial V}{\partial A} \\ B^{new} &= B^{old} - k_B \cdot \frac{\partial V}{\partial B} \\ D^{new} &= D^{old} - k_D \cdot \frac{\partial V}{\partial D}, \end{aligned} \quad (12)$$

where  $k_A, k_B$  and  $k_D$  are coefficients related to a convergence speed.

### 3 Experiments

#### 3.1 Comparison between two approaches

We use three dimensional data acquired by a range finder. The image size is  $200 \times 200$  pixels. When the distance between a face and the range finder is 1.5 m approximately, we can get about 10,000 valid data. Small areas were made from three neighboring pixels.

For two approaches, we processed as follows; first we calculated a tentative face plane using whole data. It tends to be created between the face and the ears. But the data includes invalid data such as ears and neck. Therefore, after removing such data behind the tentative plane, we calculated face plane again using the front data of the tentative plane.

We obtained 14 normal face data and 14 smiling face data. As the face plane by the least variance approach can not be obtained deterministically so that we gave the initial values, that is  $A = B = 0$  and the mean value of  $z$  component of facial data as  $D$ . As the results, 6 samples by the least variance approach failed because the face planes created had quite different orientation or the steepest descent method diverged. However, we obtained the face plane for all samples by the least distance approach successfully.

We will show the examples for normal face by two approaches in Fig.3. The data for (a) and (b) are the same. (a) and (c) are the face plane and the distribution of crossing points by LDC respectively, (b) and (d) are by LVC.

Next, we show the coefficients of face planes for four samples in Table 1. In Table 1, the number is sample number, "n" represents that it is normal face data and "s" means smiling face data. From the results of all samples, we can say that (1)LDC is more stable than LVC, (2)the face plane by LDC tends to tilt a little around  $z$  axis than one by LVC and, (3)from the coefficient  $D$ , the position of face plane by LVC tends to be close to the facial surface.

#### 3.2 Analysis of facial expression change

As shown in Figs.3(c) and (d), the distribution of all data conceals facial expression change so that the areas on the

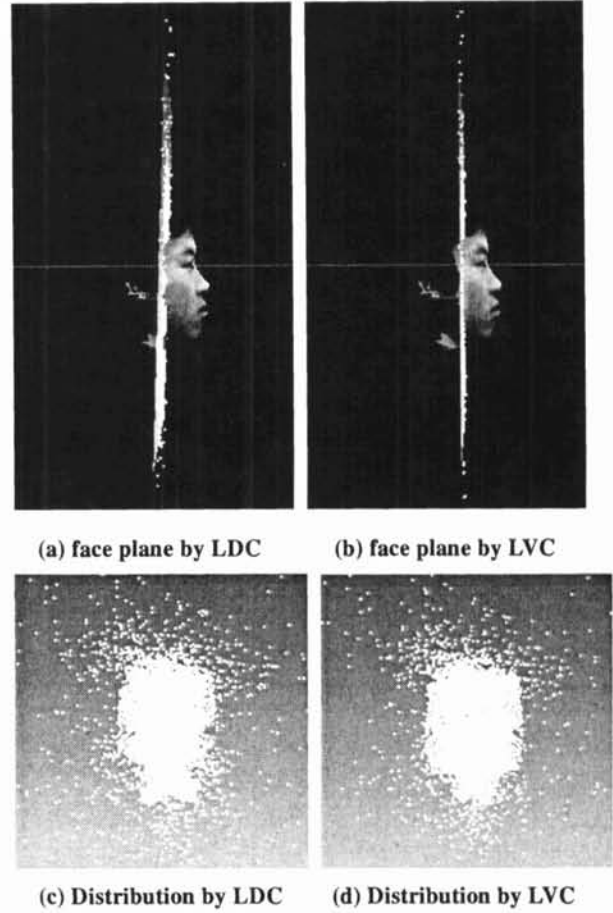


Fig.3 Face planes by two approaches

Table 1 Coefficients of face planes

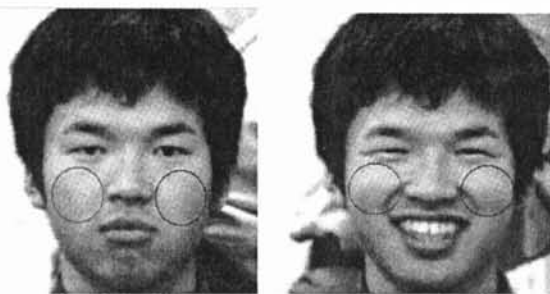
(a) The least distance criterion

	sample1-n	sample1-s	sample2-n	sample2-s
A	-0.004902	0.003087	0.034658	0.020404
B	-0.033922	-0.032154	0.053841	-0.249781
D	1677.04	1652.14	1636.04	1619.69
	sample3-n	sample3-s	sample4-n	sample4-s
A	-0.061109	-0.071945	-0.048036	-0.05748
B	0.023539	0.032623	0.093357	-0.01389
D	1631.41	1586.45	1621.46	1590.07

(b) The least variance criterion

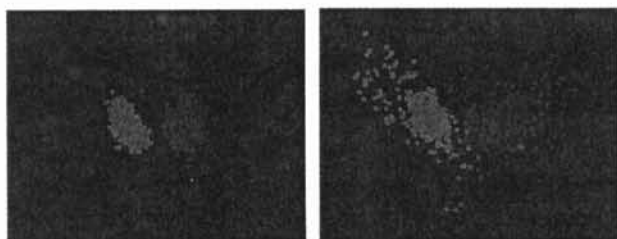
	sample1-n	sample1-s	sample2-n	sample2-s
A	-0.000708	-0.001144	-0.00875	-0.006533
B	-0.002083	0.002152	-0.001015	-0.004591
D	1669.81	1640.83	1619.85	1609.65
	Sample3-n	sample3-s	sample4-n	sample4-s
A	-0.003579	0.001516	0.011194	0.000254
B	-0.001489	0.003949	0.000231	0.003263
D	1615.86	1564.67	1610.61	1590.07

face were restricted to the left and right cheek parts (marked by two ellipses) to show their change clearly(Figs.4(a),(b)). Figs.4(c) and (d) show the restricted distributions for a normal face and a smiling face respectively. We consider the shape of distribution from now.



(a) Normal face

(b) Smiling face



(c) Distribution of normal face (d) Distribution of smiling face

Fig.4 Distributions by facial expression change

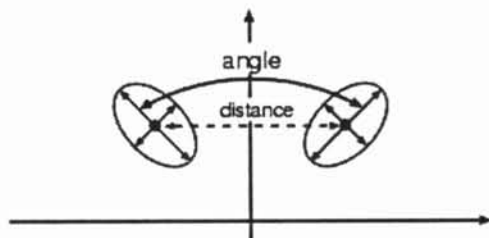


Fig.5 Measurement of distribution

Table 2 Data for four samples

	sample1-n	sample1-s	sample2-n	sample2-s
distance	45.5079	64.5299	46.9895	68.0887
v-ratio-r	7.59576	1.79653	1.81957	2.14298
v-ratio-l	8.53935	2.83728	2.46329	3.26831
angle	58.31	69.75	70.67	97.19
	sample3-n	sample3-s	sample4-n	sample4-s
distance	49.8365	60.9243	43.8131	59.3032
v-ratio-r	1.84095	1.10986	1.47392	2.96574
v-ratio-l	5.3061	3.47644	4.22524	2.42077
angle	23.13	60.96	84.65	146.08

We analyzed the facial data obtained from 14 persons. We will show four sample data from them in Table 2. In Table 2, "distance" means the distance between two gravities of distributions(see Fig.5). "v-ratio-r" and "v-ratio-l" represent the ratio of the first eigenvalue to the second eigenvalue for the right cheek and the left cheek respectively. "angle" is the angle between two eigenvectors as shown in Fig.5. As the results, the distance for a smiling face tends to be bigger than one for a normal face. The shape of distribution depends on person. In Table 2, the angle looks to be bigger for a smiling face. However, it was not definite in other data.

## 4 Summary

We proposed a new method for facial expression analysis. The usefulness of the face plane is that it can quantify the movement of facial muscles and the face plane parameters depend on a facial expression and/or a person so that we expect that this concept will be applied not only for facial expression recognition but also for person identification.

In this paper, we clarified that LDC(least distance criterion) is stable to get the face plane as compared with LVC(least variance criterion), also the distance between the right and left distributions tends to get bigger for a smiling face. Furthermore, we are planning to analyze facial expression change in detail.

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