Scale Relativity and Fractal Space-Time
A new approach to Unifying Relativity and Quantum Mechanics (Laurent Nottale, 2011)

A Review by Philip Turner[1]

Since the birth of quantum theory, physicists have been challenged with the development of a unified theory of quantum mechanics and relativity, with no general consensus on the best way forward. To progress further, we have to confront deep questions about space and time, quantum theory, and cosmology, which take theory back into contact with experiment. The theory of scale relativity (ScR) described in this book offers a serious contribution to the debate on unification offering an intuitive insight into how these theories could be fundamentally linked through space-time geometry.

The theory is a result of more than 20 years of dedicated research, and an extensive body of published work. In a world of information overload, where even dedicated specialists are challenged to keep up with the latest thinking in their field, this work offers a welcome overview of historical developments and new insights. The book covers theoretical concepts, mathematical derivation of the underlying principles, outstanding questions, potential applications and numerous case studies validating a number of theoretical predictions that lend substance to the theory.

The general theoretical approach is founded on the principle that the apparently smooth geodesics of space-time at the macro-scale described by general relativity are an incomplete description of the structure of space-time at the micro-scale. In simple terms, the hypothesis states that the structure of space has both a smooth (differentiable) component at the macro-scale and a chaotic, fractal (non-differentiable) component at the micro-scale, the transition taking place at the de Broglie length scale.

At the macroscale, the fractal component and its influence, is small (and generally considered unimportant in classical physics). However, at the microscale, the fractal component and its influence dominates, with gauge fields and quantum laws originating in the underlying fractal geometry of space-time. The origins of this hypothesis can be traced back to the work of Feynman, who in 1948 suggested that the typical quantum mechanical paths that are the main contributors to his “path integral”, are non differentiable and fractal although the term fractal was only coined by Mandelbrot in the 1980’s. Mandelbrot’s insights inspired the derivation of Nottale’s new geometric description of space-time, which he introduces and then describes in detail in the first four chapters of the book. The book then goes on to describe the relations between fractal space-time, gauge fields and quantum mechanics.

Nottale notes that the full transition of a system from the classical to the quantum regime only becomes effective when three key properties have been fulfilled. The first is that the paths or trajectories are infinite in number,

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leading to a statistical, fluid like description in which the velocity is replaced by a velocity field (the concept of a single trajectory has no meaning in the theory). The second, that the trajectories are fractal curves which leads to fractal velocity fields. The final condition is that there are two fractal velocity fields each comprising a classical and a fractal component, i.e. there is a jump from a real to a complex description, which is the origin of the real and imaginary components in the wave function.

A number of important concepts highlighted below evolve from these underlying principles, which are covered in detail in the book.

- Fractal fluctuations of space-time (at the micro-scale) lead to the emergence of a "fractal field" which generates a potential energy ("quantum potential") and a quantum force which are directly analogous with the geometric origins of the gravitational field, gravitational potential and gravitational force respectively, which are described in the theory of General Relativity.

- Quantum particles are a specific manifestation of this fluid of geodesics from which the various properties of the wave-particle such as charge and spin emerge from their internal fractal geometries. The existence of quanta, upon which the concept of "particle is based" is a consequence of the quantization of these geometric properties, derived from the properties of the geodesic equation. The wave function is seen as another expression of the fluid velocity field, identifiable with a potential of velocity. In the case of more than one particle (e.g. a hydrogen atom), the particles are described by a wave function containing different subsets of geodesics, of different velocity fields and different geometric properties corresponding to the different particles. Nottale gives the analogy of the two-body problem in general relativity, where bodies with different masses, following their own geodesics contribute to the same unique space-time. Physical quantities do not exist as intrinsic properties of objects, but only as relative properties of pairs, which cannot be attributed to any of the members of the pair. This approach naturally leads on to deal with the problem of quantum entanglement, the theory offering an intuitive geometric view of entangled quantum states.

- In non-relativistic quantum mechanics (described by the Schrödinger equation), time is regarded as an invariant scalar in fractal space. However, a full fractal space-time gives rise to relativistic quantum mechanics, i.e. not only space, but the full space-time continuum, which forms the basis of a new, detailed derivation of the Klein-Gordon and Dirac equations.

- The theory offers a new interpretation of gauge transformations and gauge fields (both Abelian and non-Abelian), which are manifestations of the fractality of space-time, in the same way that gravitation is derived from its curvature. Nottale dedicates a chapter to this important area of the theory, explaining in considerable detail the construction of a new geometric description of electrodynamics (classical and quantum). Taking classical electrodynamics as an example, he shows how the Lorentz force, Maxwell's field equations and the equation of motion of a charged particle in an electromagnetic field are derived from the fractal geometry of space-time.

In searching for a theory of Quantum Gravity one is inevitable drawn to consider alternative approaches and to understand the differences between them. Nottale notes that many contributions to this field (e.g. superstring theory and loop quantum gravity) have highlighted the expected fractal structure and properties of quantum space-time. The main difference is that these quantum gravity studies assume the quantum laws are fundamental, i.e. the fractal geometry of space-time at the Planck scale is a consequence of the quantum nature of physical laws. In the ScR approach, the quantum laws are considered as manifestations of the fractality and non-differentiability of space-time, they are not separate (in analogy with Einstein's general relativity theory, in which gravitation is a manifestation of the curvature of space-time) so they do not have to be added to the geometric description. Nottale suggests that one day maybe these different approaches could meet.

The chapter on applications of the theory to elementary particle and high-energy physics is a fascinating and far-reaching account, which requires a detailed, more technical review in its own right. New contributions to our understanding of the "standard model" and a suggested mechanism of generation of the mass and charge spectrum of elementary particles are but two of many notable contributions to this field. However, it is also worth noting the outstanding challenge of unification of gauge and gravitational fields at the Planck energy scale. Nottale indicates that the problem amounts to describing a fractal and curved geometry, which become mixed and have to be accounted for on the same footing, i.e. the motion and scale covariant derivatives are not combined after they have been separately constructed. It requires the construction of a new covariance that accounts for all effects together, the quantum gravitational and gauge properties becoming mixed in a unique, extremely complicated geometric behavior that remains to be understood and to be described.

Having dealt with standard quantum theory, Nottale goes further to discuss how a new geometric theory of chaos at the microscale offers new insights into a wide range of phenomena, which are currently inadequately explained. Classical quantities remain at the heart of the quantum world as averages. Conversely quantum properties remain at the heart of the classical world. The action of fractality and irreversibility on small time scales manifests itself by the appearance of a macroscopic quantum–type potential energy, in addition to the standard classical energy balance. This potential energy leads to macroscopic quantum effects, which are normally masked by classical motion, but can be observed given the right conditions. Examples include Bose Einstein condensates and superconductivity when the de Broglie length scale becomes macroscopic as the motion of particles decrease at very low temperatures. However, it is additionally proposed that macroscopic quantum potentials play a vital role in the classical world where they are seen as generating an additional, previously unconsidered structuring force. Nottale takes an important conceptual leap here by considering the motion of a free particle in a curved space-time whose spatial part is also fractal. This fractality is not assumed, but proved from *chaotic motion* seen at time scales, which are beyond its horizon of predictability. Against this geometric backdrop, he writes the equation of motion as a geodesic equation that combines the general relativistic covariant derivative (which describes the effects of curvature) and the scale-relativistic covariant derivative (which describes the effects of fractality). The induced effects on motion (in standard space) of the internal fractal structures of geodesics (in scale space), transform Newton's fundamental equations of dynamics into a macroscopic, Schrödinger-like equation which can be interpreted as a general tendency for systems to make self organized, probabilistic structures, based on the existence of stationary, quantized solutions, in correspondence with the field, the symmetries and the boundary conditions. In other words, this new macroscopic version of the Schrödinger equation can be viewed as a fundamental equation of morphogenesis to systems in which the three specific conditions identified earlier (infinite or very large number of trajectories, fractal dimension of individual trajectories, local irreversibility) are fulfilled. Nottale gives numerous examples in his book in which various forms of these macroscopic Schrödinger equations can be constructed, which are not based on Planck's constant, but on constants that are specific to the macroscopic system and may emerge from their self organization. In such a case, some of the typical properties of genuine quantum systems such as indiscernibility of identical particles are not considered relevant.

As an example, Nottale considers the development of this "macroscopic Schrödinger approach" to gravitational structuration. He takes account of the standard approach to gravitational structure formation plus the extra potential energy to describe a number of apparently quantized gravitational systems. A nice case study illustrating the point is given by diffusing space debris in orbit around the Earth, predicted by probability peaks at 718km, 1475km and 2269km. The predictions are in close agreement with observed data at 850km and 1475km. The data adds considerable weight to the existence of the predicted probability density peaks, which shouldn't exist in the absence of the predicted self-organising underlying process.

A second case study relates to the structuration of solar systems. In 1993, Nottale published predictions of the quantized peaks of probability associated with planetary orbits. These peaks accurately predict average distances of planets to the sun in our own solar system. More interestingly, one of the most important subsequent validations of predictions made by the theory has been the subsequent discovery of new exoplanets in extra-solar planetary systems at these predicted probability peaks.

The book goes onto describe a number of additional examples of predicted and validated astrophysical phenomenon including the formation of planetary nebulae, stars and galaxies. In addition, as an example of the far reaching nature of the theory, the proposal of missing mass in the universe (dark matter), the result of the existence of a missing energy in the energy balance of the gravitational system may be explained by this additional potential energy of fractal geometric origin, negating the need for a non-baryonic dark matter hypothesis.

Moving from Astrophysics to biophysics, Nottale shows how the macroscopic Schrödinger process is particularly well adapted to the description of living systems. For example, morphologies are acquired through growth processes, which can be described in terms of an infinite family of virtual, fractal and locally irreversible, fluid-like trajectories. Their equations can therefore be written in the form of a fractal geodesic equation, which can then be integrated as a Schrödinger equation or, equivalently, in terms of hydrodynamics-type energy and continuity equations, including a quantum like potential which gives them a Schrödinger form, allowing the emergence of quantized solutions. Depending on the potential, on the boundary conditions and on the symmetry conditions, a large family of solutions (structures) can be obtained when conditions for the quantum-type regime are fulfilled. In biological systems it can have an important impact on biological structures and their evolution. Critical processes such as molecular assembly, cell formation, duplication, branching and growth into larger scale organisms such as plants and their constituent parts (e.g. flowers) can be modeled using these macroscopic Schrödinger-like equations. What becomes so far reaching from this modeling work is that at least some the “biological structures” that we see are the result of quantized growth processes driven by universal laws, with no direct genetic influence. The theory hints at a rethink and perhaps simplification of our understanding of the role of genes in growth processes. It also offers a potential future foundation of a mathematical approach to biology, which could have predictive power analogous to that of mathematical physics.

In this review it was not possible to highlight all the important ideas and details contained in this book, which offers many remarkable insights and important signposts to guide and inspire new discoveries across a broad range of scientific disciplines. Whilst a background in physics is required to comprehend the full scope of the theory, scientists from other disciplines should not be put off reading this book. Nottale does an excellent job of communicating the most important elements and fundamental laws of the theory at a conceptual level, before describing them in mathematical form. This book is unusual in transcending the boundaries between sciences. It has the hallmarks of a classic. I recommend it without hesitation.