Value Through Exogenous Distribution

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Economic theory has its fair share of conundrums. Discarded as mere "chimeras" by the more pragmatic and empirically-oriented economists, these puzzles continue to fascinate those with a flair for abstraction and challenge their speculative ingenuity. Of these, the 'chimera of absolute value', long fallen into oblivion after the unsuccessful efforts, notably by Ricardo and Marx, to discover an invariant yardstick to measure value, has now been revived in Piero-Sraffa's excellently expounded book "Production of Commodities".

Sraffa resolves the Ricardian Riddle within a framework of analysis, wherein neither the output composition nor the proportions in which means of production are combined change. Thus, both the framework of investigation as also the central problem to which Sraffa particularly addresses himself are, in spirit, classical.

Those of us, accustomed to economic analysis always running in terms of variations of factor-proportions or output-combinations would be struck by the extremely rigid framework within which Sraffa's analysis is conducted. Indeed, it is a unique accomplishment of his work that within this restricted economic domain is created a nucleus of deeply significant ideas which presumably have wide-sweeping consequences.

SRAFFA begins with a simple model of production for subsistence where the total product is just sufficient to sustain the workers and to serve as means of production.* In such an economy, without surplus, there is a unique set of exchange values which restores the original distribution of products among industries, thus assuring the possibility of the continuation of the cycle of production, period after period. Technology is all important in determining relative prices. In fact, these prices are embedded in the technology itself.

With the extension of this model to production with surplus, the problem of distribution appears on the scene. Uniformity of the rate of profit and the rate of wages in all industries is assumed. As the surplus has to be distributed proportionately to the means of production advanced in each industry—and this cannot be done unless the heterogenous means of production are aggregated with the help of prices—and as prices cannot be determined before knowing the uniform rate of surplus, both prices and the rate of surplus will have to be determined simultaneously. When wage rate is fixed and consists " of the necessary subsistence of the workers, thus entering the system on the same footing as the fuel for the engines or the feed of the cattle " (page 9), technology

* "Production of Commodities by Means of Commodities", Cambridge University Press, 1960. What follows is largely an exposition of some of Sraffa's ideas contained in this book. Page numbers in the text refer to this source.

acts as the price-determinant even in this case of production with surplus. In fact, at this point, Sraffa's system resembles that of Von Neumann[1]. However, Sraffa goes a step further, in that, he relaxes the condition of a fixed wage and treats it as a variable. Distribution in Sraffa's system is not endogenously generated through production relations[2]. In conjunction with prices, production relations determine only the net surplus that is to be distributed[3]. No theory of distribution is offered in the book. His wage rate is variable which could be conceived to vary with the same ease as the profit rate. Wage is assumed to be paid post factum, so that profits and wages are surplus-sharing entities.

System of Equations

With wage introduced as a variable, the system of equations turns out as follows :

$$(A_a P_a + B_a P_b + \dots + K_a P_k)$$
$$(1+r) + L_a W = A P_a$$

$$(A_b P_a + B_b P_b + \dots + K_b P_k)$$
$$(1+r) + L_b W = B P_b$$

$$(A_k P_a + B_k P_b + \dots + K_k P_k)$$
$$(1+r) + L_k W = K P_k,$$

where the system is assumed to be in a self-replacing state. A, B ..K are the quantities of com-

modities a, b..k required to produce the quantity A of a; A_b, B_b..K_b are the quantities of commodities a, b..k required to produce the quantity B of b and so on; L_a L_b...L_k are the annual quantities of labour employed in the industries producing a, b..k respectively. The unknowns of the system are the prices P_a, P_b .... P_k of commodities a, b .... k respectively, the wage rate w and the uniform profit rate r. An additional equation defining the national income in terms of which the k prices and the wage is expressed is introduced.

$$[A-(A_a + A_b \dots + A_k)]. P_a$$
$$+ [B-(B_a + B_b \dots + B_k)]. P_b$$
$$+ \dots + [K-(K_a + K_b + K_k)].P_k$$
$$= 1$$

The system now moves with one degree of freedom. Given the wage rate or the profit rate, prices are determined simultaneously. To begin with, Sraffa assumes that the wage rate is given. We could assume perhaps that the wage rate is

[1] J Von Neumann: "A Model of General Economic Equilibrium", *Review of Economic Studies*, 1945.

[2] This is a significant departure from the widely prevalent practice of obtaining distributive shares from the production function applying the marginal method.

[3] Rather, prices and the rate of surplus are determined simultaneously given the production relations.

determined by mutual bargaining or by an external authority or by social convention. In the later portions where Sraffa prefers assuming the rate of profit as given, there is a vague reference that the rate of profit is "susceptible of being determined from outside the system of production, in particular by the level of the money rates of interest" (page 33). This transfers the burden merely from one peg to another as no explanation regarding how the level of the money rates of interest is determined appears anywhere in the book. In fact, there are "prices" in the book without any mention of money.

Importance of Distribution

Even though Sraffa is not interested in distribution per se, distribution happens to be the key factor in the mechanism which determines relative prices, subject to the given technical relations and the uniformity of the wage rate and the profit rate. Sraffa's theory of relative prices belongs to that group of theories of value determination wherein the horizontal division between classes receives the major emphasis*. Demand plays no essential role in the system. The basic proposition of these theories is that, in an economy, where all means of production are produced within the system and where constant returns to scale prevail, normal prices, corresponding to any pattern of distribution, are determined by technical relations. Either demand conditions or output composition do not play any part in the determination of these prices. Given Sraffa's framework of no changes in the scale of operation, the question regarding constancy or otherwise of returns to scale does not arise.† However, not all means of production are produced within the system. Labour and later land appear as non-produced factors. The presence of non-produced means of production seems to dist urb the determinacy of the technology-cum-distribution relative price schema, based purely on the distri bution pattern, the technical rela tions and conditions regarding the uniformity of the profit and the wage rate. The Marshallian blades

* Cf J Robinson: "Essays in the Theory of Economic Growth", McMillan & Co, London, 1962; Chapter on "Normal Prices".

† See below for further comments

of scissors, it looks as through, could no more be ignored.†† However, the fundamental dichotomy in the structure of commodities in the Sraffa system—the differentiation between the basics and the non-basics—seems to hold the clue. A basic good (in the system with no joint products) is defined as one which enters as an input in the production of all commodities directly or indirectly†. A non-basic good, defined by exclusion, may be produced by a basic good with or without non-basics, but does not itself enter as an input into a basic good. The analytical significance of this distinction is that the non-basics have no part in the determination of the system of prices, in the sense, that price changes in it would not be transmitted to other commodities‡. Any price changes, in the case of the basics, on the other hand, influence the prices and the profits of all the other commodities. With wage treated now as a variable and as a surplus-sharing entity, labour becomes a non-basic of the system and so also land, the other non-produced means of production. Relative prices are thus influenced, given the distribution pattern, by the production structure of the basics, which are produced means of production.

The 'Standard Commodity'

The problem of relative prices is intimately related to that of absolute value and Sraffa's sterling contribution lies in the ingenious concept of the Standard Commodity—the invariant measure of value. The essence of the problem is that unless we have an absolute measure of value "it is impossible to tell of any particular price fluctuation whether it arises from the peculiarities of the

†† In fact, the "marginalist" strand in Ricardian theory, it may be recalled, arises in the explanation of rent.

† This definition is later modified in the joint-products case and the general formulation of the distinction appears as: "In a system of k productive processes and k commodities (no matter whether produced singly or jointly) we say that a commodity or more generally a group of n linked commodities (where n must be smaller than k and may be equal to one) are non-basic if of the k rows (formed by the 2n quantities in which they appear in each process) not more than n rows are independent, the others being linear combinations of these". (page 51)

‡ This proposition is extended to the joint products case (pages 54-55)

commodity which is being measured or from those of the measuring standard" (page 18). The problem of valuation is thus of isolating the price movements of a product so as to observe the changes in its value as if in vacuum. Ricardo faced the same dilemma when the distributable total output was to be evaluated independently of changes in the distribution pattern. Corn was sought as a measure of absolute value as there could be a "material rate" of surplus in it, in the sense, that the same product appeared both as net output and as input. This notion, with its roots in the physiocratic doctrine of 'net product', was based on extremely over-simplified assumptions. Ricardo, dissatisfied with this measure, turned to the unit of labour time as an answer to his problem. Conscious of the limitations of this measure as well, he did not subscribe to it unreservedly. Sraffa's own interpretation regarding the logical foundation of the corn-measure as an absolute value is the genesis of the standard commodity. He wrote in his Preface to "Works of David Ricardo"*: "The advantage of Ricardo's method of approach is that, at the cost of considerable simplification, it makes possible an understanding of how the rate of profit is determined without the need of a method for reducing to a common standard a heterogeneous collection of commodities".

In order to discover such a commodity Sraffa analyses the effects of changes in the wage rate on the rate of profit and on the prices of individual commodities, on the assumption that production methods remain unchanged. With the whole national income going to wages (with profit rate reduced to zero) relative values are determined by the direct and indirect labour gone into the commodities, supporting thus the labour theory of value. When profits are assigned positive values, the simple labour theory of value is no longer valid and the key to the determination of relative price movements lies in the inequality of the proportions in which labour and means of production are employed in various industries. The relative price movements depend

* "The Works and Correspondence of David Ricardo", Vol I (ed) P. Sraffa and M H Dobb, Cambridge University Press, 1951 (p xxxii).

not only upon this labour-to-means-of-production proportion of the product but also upon the corresponding ratios for each of the means of production and, in turn. for their means of production. With any wage reduction given the uniformity assumption regarding the rate of profit in all industries, price changes are called in to redress the balance in each of the deficit industries (those with relatively low labour to means of production ratio) and the surplus industries (those with relatively higher labour to means of production proportion). The industry which enjoys the critical balancing proportion so that it is under no compulsion, arising from the conditions of production itself, to change in value consequent upon the changes in the pattern of distribution, has the unique distinction of acting as an invariant standard of value. Since the critical balancing proportion has to persist throughout the structure of direct and indirect inputs, it is evident that the balancing ratio (expressed as the value ratio of the net product to the means of production) would be equal to the rate of profit which corresponds to zero wage, or the Maximum rate of profits, called R by Sraffa. In actual system, the commodity satisfying this condition might be a composite commodity and Sraffa constructs such a commodity with the technical characteristic that it consists of the same commodities combined in the same proportion as the aggregate of their means of production. The logic of this proposition implies that only basic commodities will enter the standard commodity.

Sraffa proves that such a miniature system is embedded in an actual economy. In proving the existence and the uniqueness of the standard system, however, Sraffa seems to have obliged the literary economist more than would have been necessary.

Search for Absolute Measure of Value

The discovery of the technical properties of the invariant standard is a tribute to Sraffa's deductive logic. He has skilfully made use of this in deriving significant propositions. The search for an absolute measure of value has long been a source of frustration and yet even in theoretical economic structures,

apart from operational and empirical ones, quantitative notions have played a significant role—not purely by way of illustration but as a basis for deducing substantive propositions*. So long as the search for absolute value was in the domain of prime, non-produced factors, the factor of demand with its root in impregnable psychological factors was bound to interfere in valuation. Also, there could be no "material rate of profit" as a ratio of only two arithmetical quantities. Scepticism regarding the very possibility of finding out an absolute value measure, comparable to the measures in the physical world seems unavoidable. "Weight and length, of course, are human conventions but once the convention is established, they do not change, for practical purposes, because they refer to the *physical, non-human world*‡ (italics mine). It is precisely in the physical non-human world of technology that Sraffa's standard system is embedded. There was another difficulty with the one-factor theory of value, namely that it failed to allow for differential patterns of factor-combinations in different sectors of the economy, even under conditions of no technical change. Basic difficulties for a labour theory of value arose on two accounts—the theory of differential rent as also the problem of organic composition of capital†.

* It is interesting to recall in this connection Sraffa's observation at the Corfu Conference: "One should emphasize the distinction between two types of measurements. First, there was the one in which the statisticians were mainly interested. Second, there was measurement in theory. The statistician's measures were only approximate and provided a suitable field for work in solving index number problems. The theoretical measures required absolute precision. Any imperfections in these theoretical measures were not merely upsetting, but knocked down the whole theoretical basis" (Report on the Proceedings; Published in the Theory of Capital. (Ed.) Lutz and Hague, St. Martin's Press, 1961) page 305.

‡ J Robinson "Economic Philosophy", C A Watts & Co, London, 1962, pages 31-32.

† See Samuelson P A: "A Modern Treatment of Ricardian Theory" Part I. *Quarterly Journal of Economics*, Feb 1959; p 2. Samuelson builds a land theory of value with land as an absolute value measure, assuming homogeneous land and subsistence wages.

Problems arose also due to the non-homogeneity of these prime factors. Sraffa's standard system steers clear off these difficulties since it is based on the very fact of interconnectedness in production which the single factor theory of absolute value is incapable of handling. In fact, the labour theory of value becomes a special case of Sraffa when the rate of profit is zero and the entire net product goes to wages. If one were to add to the assumption of zero profits, another assumption that wages are fixed at subsistance level (so that wage is like any other material input) and introduce land as a non-basic one could derive a land theory of value. This again would become a particular case, valid for a particular distribution pattern. Sraffa's standard of value allows more flexibility to vary distribution patterns without landing itself into a dilemma.

With the help of the Standard Commodity Sraffa then establishes a proposition which is the pivotting point of many of the important deductions. If W is the proportion of the net product of the *Standard System* that goes to wages and R, the Maximum rate of profits, the actual rate of profit will be determined by a straight-line proportionality relation $r = R (1-W)$. This fundamental factor-price frontier (as Samuelson calls it) appears as in Fig 1:
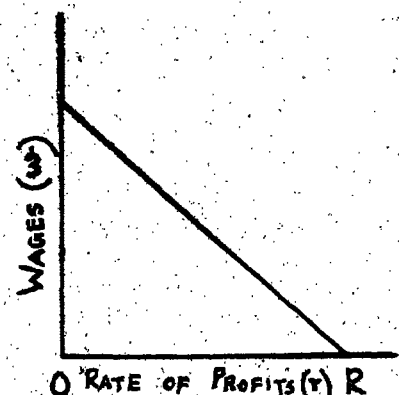

Fig 1: Relation Between Wages (as a Proportion of the Standard Net Product) and the Rate of Profits

This relation is not limited to the imaginary standard system alone but can be extended to the actual economic system. The actual economic system contains the same basic equations but in different proportions. As such, *provided that the*

wage is expressed in terms of the standard commodity, the same rate of profits which prevails in the Standard Commodity as a ratio between *quantities* of commodities, will appear as the ratio of aggregate *values* in the actual economic system. This relation forms the foundation of later deductions in both the theory of value and the theory of capital.

Employing an operation—called by Sraffa as reduction to dated quantities of labour—by which " in the equation of a commodity the different means of production used are replaced with a series of quantities of labour, each with appropriate ' date ', Sraffa resolves prices into an infinite series composed of terms containing wages and profits. Prices never resolve themselves completely into these factor shares but carry alone a commodity residue which becomes all important when wages are zero, thus setting always a finite upper limit to the Maximum rate of profits. Sraffa also establishes in the no-joint products case that irrespective of the standard in which wage is measured, an increase in the wage rate, ipso facto, implies a decline in profits. However, in the case with joint-products, this proposition does not hold generally. To a particular wage, given in any standard, there correspond several alternative rates profits. This shows the absolute necessity of measuring wage in terms of the standard commodity, if unequivocal conclusions regarding the movements of the rate of profits given the wage rate are to be drawn. Measurement of wage in terms of the standard commodity gives us definite information regarding both the direction as well as the extent of change in the rate of profits, consequent upon a change in the wage rate. No other standard possesses this predictive value.

Measurement of Capital

The most interesting use of the standard commodity (and the wage-profit relation) arises in connection with the measurement of capital. Economists have long ceased to be complacent about the average period of production as a measure of capital. Sraffa's demonstration regarding the impossibility of measuring capital, independently of distribution and prices is powerful and convincing. Resorting to the " re-

duction to dated quantities of labour " and using the unique wage-profit relation, Sraffa proves the possibility of the reversals in the direction of price movements even when methods of production remain unchanged, when the rate of profits is altered. An alternative method of calculating " correct " book-value of capital after depreciation emerges from Sraffa's discussion on fixed capital. Fixed capital is now treated as a particular case of joint-product. Durable instruments of production are treated on par with the other means of production, with the annual intake recorded on the input side and what is left of the fixed capital (now, older by a year) is entered on the output side. This treatment of capital has now become quite familiar*. Also, every year of operation of the fixed asset is treated as a separate process so that there are as many processes as are the number of years of active life of the fixed asset. The price of any given age of a durable instrument will thus result from solving the simultaneous equations of production. The difference between the prices of the instrument for two consecutive years gives the correct depreciation. This depreciation formula scores over the traditional one in many ways. In the first place, it can allow for any complex pattern of productive efficiency of the capital good during its life-time whereas the conventional formula is based upon uniform efficiency, contrary to reality. Secondly, it can make allowance for variations of inputs in every production period so that changes in efficiency of other inputs are also permitted. Thirdly, it can be applied to the cases where the same machine is used in different productive operations with varying efficiency.

The quantity of Capital which arises from the solution of the simultaneous equations is not distorted by variations in the measuring yardstick itself, since it is expressed in terms of the standard commodity which is invariant with respect to changes in the distribution pattern. Efforts to reduce the durable instrument to dated quanti-

* In his references to literature (page 95) Sraffa remarks that this method has fallen into oblivion after Marx. However, Von Neumann's model makes use of such a concept.

ties of labour are proved futile and so also the attempt to find in technology an average period of production, independent of prices and distribution. Measurement of capital under stationary state conditions, where perfect tranquillity prevails, has not been much of a problem. Even with steady growth accompanied by confident expectations, the problem has an easy enough solution, if a constant rate of profit is assumed. Sraffa has now offered an answer to the problem in a more complicated situation where the profit rate is allowed to change but technical conditions are assumed to be unaltered. Measurement of capital under even more generalised conditions of changing distribution pattern coupled with changes in technology still eludes us.

Sraffa's Contribution

Incidentally, the contribution of Sraffa's system as a fundamental analytical structure in context of measurement of economic magnitudes may be noted here. The concept of stationary state, with its invariance of structure, offered itself as a convenient scale of reference to successive approximation, it provided a firm foundation on which to superimpose change. In Sraffa's system, we enjoy an additional degree of freedom—namely, the freedom to vary distribution pattern while retaining the focal point of reference—the standard system. This is a step further towards a more realistic analytical foundation.

When Sraffa introduces multiple techniques and the process of switching over from one technique to another, complications arise, especially when a basic product is involved. The anchor of analysis—the Standard System—is itself in peril. With a new technique introduced, we move into a distinctly new economic system with its own Maximum rate of profits. The analysis becomes rather involved. With his characteristic resourcefulness, Sraffa finds certain ways out. However, this portion of the book does indicate the complications we run into if the structure of the standard system itself changes. There can be no smooth change in techniques without changing the entire configuration of relative prices. Furthermore, the

changes are not only in terms of relative prices; but are more fundamental. They involve a comparison of two different economic worlds[*].

Sraffa forewarns in his preface that no assumption regarding constancy of returns to scale is made. In fact, with no changes either in the scale of output or input-proportions this question is irrelevant[†]. However, when one thinks of incorporating growth into the system many questions relating to this assumption arise. This Standard Commodity becomes vulnerable if we do not assume the linear homogeneity condition. Even in the simplest case where we assume that output proportions do not change, namely, of balanced expansion, the Standard System is bound to be vitiated if we make no assumption regarding the constancy of returns to scale. The technical relations will change with expansion in output throwing up a different standard system with a different Maximum Rate of Profits. The ideal analytical conditions for the same standard commodity to continue its domination unruffled by change would be the Von Neumann World of proportional expansion where constant returns to scale prevail. So far as growth is concerned, a fusion of Sraffa's System with that of Von Neumann appears to have promising potentialities.

Written in an unusually compact style and embellished with chiselled logic, the book bears the imprint of sustained reflection. Unmistakably, this is the work of a master written with authority and insight. Even though Sraffa regenerates the classical approach to Economics, his researches are not of archaelogical interest. We are told that his central propositions took shape as early as the late 'twenties, though published after a lapse of over three decades. However, time has dealt kindly with Sraffa's contribution. It as relevant and sprightly today as was when conceived.

* Here, perhaps, is to be found another of the challenges to marginalism. In fact, the book, which is subtitled as a prelude to a Critique of Economic Theory, "is designed to serve as a basis for a critique of that (marginal theory of value and distribution) theory". (Preface).

† Nevertheless, while reading the paragraphs relating to the con-

struction of the standard system (pages 23-24)' and more particularly the Subsystems (page 89), one gets a feeling as though the assumption of constant returns to scale is necessary. However, such doubts could be easily warded off in the case of the Standard Commodity which is purely an auxiliary con-

struction having no physical existence in production relations. More ambiguous is the case of the Subsystems which are used to derive the direct and indirect labour content of commodities (at zero profit rate), thus implicitly attributing to the Subsystem the qualities of actual production relations.

Value through Exogenous Distribution

A Comment

Arun Bose

MAY I offer a few comments on Krishna Bharadwaj's interesting review article on Sraffa's "Producion of Commodities by Means of Commodities" (*The Economic Weekly*, August 24. 1963)?

First, the reviewer agrees that with no changes in outputs or input proportions provided for, the quesion of returns to scale is irrelevant (p 1454). But in a foot-note on the same page doubts are expressed that for construction of (a) the Standard System; and (b) the Sub-Systems, "one gets the feeling as though the assumption of constant returns to scale is necessary". Of course, the first doubt is expressed only to be disposed of immediately by recalling that Sraffa makes it quite clear that the Standard System is a purely auxiliary construction, having no physical existence. But, "More ambiguous is the case of the Sub-Systems which are used to derive the direct and indirect labour content of commodities (at zero profit rate), thus implicitly attributing to the Sub-System the qualities of actual production relations" (p 1454). Actually, there is no ambiguity involved. It can be shown without much difficulty that in the construction of the Sub-systems the problem of returns is completely bypassed by leaving (a) aggregate commodity outputs in the whole system, and (b) input proportions, unchanged. In fact the construction of Sub-Systems involves nothing more than a purely "drawing board" regrouping of the productive facilities of the actual system.

Second, on page 1452 we read that a "land theory of value" can be fitted into the Sraffa analysis as a special case by superimposing on the assumptions of zero profits and fixed subsistence wages the assumption that land is a "non-basic". But as it stands, on the assumptions made in this passage, we would get a system of production without a surplus (and with free land) where

a pure "commodity theory of value", with any one commodity chosen as a standard of value, would rule.

Third, it is far more important, at this stage, to discuss the validity and usefulness of the Sraffa theory on its own merits. than to decide which family of value theories known in the history of economic thought it belongs to. We shall be in a much better position to undertake the latter task after we have examined the many implications for theory of the Sraffa analysis and found them acceptable. But it is perhaps not entirely inappropriate to point out that an important issue is too hastily prejudged in the statement on page 1452 of the review article which reads "the labour theory of value becomes a special case of Sraffa when the rate of profit is zero and the entire net production goes to wages". Implicit in this statement is a particularly narrow definition of the labour

theory of value as a theory which says that value is determined exclusively by labour cost, with the rate of profit playing no part. But no important exponent of any of the theories usually known as labour theories of value would deny that labour cost would be the exclusive determinant of value only in the special case where the whole net product goes to labour as wages. And a theory of value which deals only with such special cases can hardly be called a theory of value applicable to actual situations. Actually, the more interesting question is how Sraffa's theory compares with the sophisticated versions of the labour theory which seek to tackle situations where the whole net product does *not* go to labour as wages, where "organic compositions of capital" differ from industry to industry, where the role of land is explicitly recognised, and where in consequence prices are *not* proportional to labour cost.

Reply

Krishna R Bharadwaj

I AM glad that Arun Bose has raised certain interesting issues connected with my review of Sraffa's book.

As far as the construction of the Sub-System is concerned, I now agree that no assumption regarding constant returns to scale is necessary. I have been doubly assured on this point by Sraffa himself in a private communication.

Regarding the land theory of value, I may point out that it was mentioned not out of any particular interest in fitting it into the Sraffa analysis. I had Samuelson's contribution (to which I have referred in the foot-note on page 1452) in mind and wanted merely to indicate, in passing, that by choosing

appropriate and convenient assumptions one could derive a land theory of value, if one were interested in it. For me, it does not seem attractive analytically.

My rather hasty treatment of the labour theory of value was not out of any prejudice against treating it on a more sophisticated basis and certainly it was not intended to cast it summarily aside as too naive for consideration. In a small article that had the modest aim of presenting some of the central propositions of Sraffa's work, a thorough discussion of all the ramifications of the labour theory of value could not be incorporated. The question raised by Bose in his last paragraph is certainly worth pursuing as an independent piece of research.