Energy landscape versus trajectory interpretation of neutron scattering spectra from complex systems

Seminar Center for Nonlinear Studies
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Probe the structure and dynamics of condensed matter systems with neutrons - Introduction

The Existence of a Neutron.

By J. Chadwick, F.R.S.

(Received May 10, 1932.)

Proc. R. Soc. Lond. A 1932 **136**, 692-708
doi: 10.1098/rspa.1932.0112

Properties of the neutron

Elementary properties

Mass	$m_n = 1.675 \times 10^{-27}\,\mathrm{kg}$
Electric charge	$0\,\mathrm{C}$
Spin	$\frac{1}{2}\hbar$
Magnetic moment	-1.913 nuclear magnetons

De Broglie Relations

Momentum	$\mathbf{p} = \hbar\mathbf{k} = \frac{h}{\lambda}\mathbf{n}_k$
Energy	$E = \frac{\lvert\mathbf{p}\rvert^2}{2m_n} = \frac{1}{2m_n}\frac{h^2}{\lambda^2}$

The "good relation" between energy and momentum:

For $E \approx k_B T$ and $T = 293K$ the wave length is comparable to interatomic distances, $\lambda = 1.8\,\text{Å}$. This enables the investigation of structural *and* dynamical properties of condensed matter systems on the atomic level.

Nobel prize in physics 1994

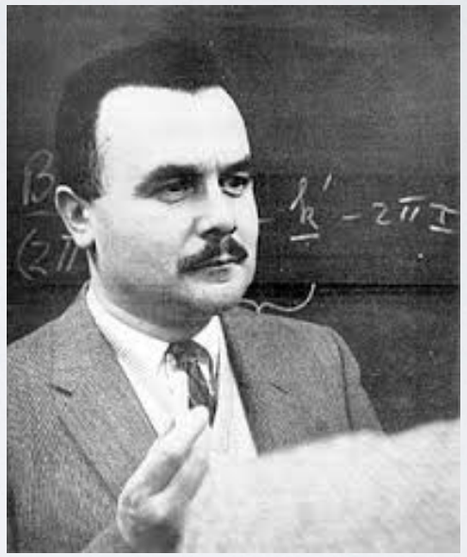
SLOW NEUTRON SPECTROSCOPY AND THE GRAND ATLAS OF THE PHYSICAL WORLD

Nobel Lecture, December 8, 1994

by

Bertram N. Brockhouse

Department of Physics, McMaster University, Hamilton, ONT, L8S 4M1, Canada



B.N. Brockhouse

EARLY DEVELOPMENT OF NEUTRON SCATTERING

Nobel Lecture, December 8, 1994

by

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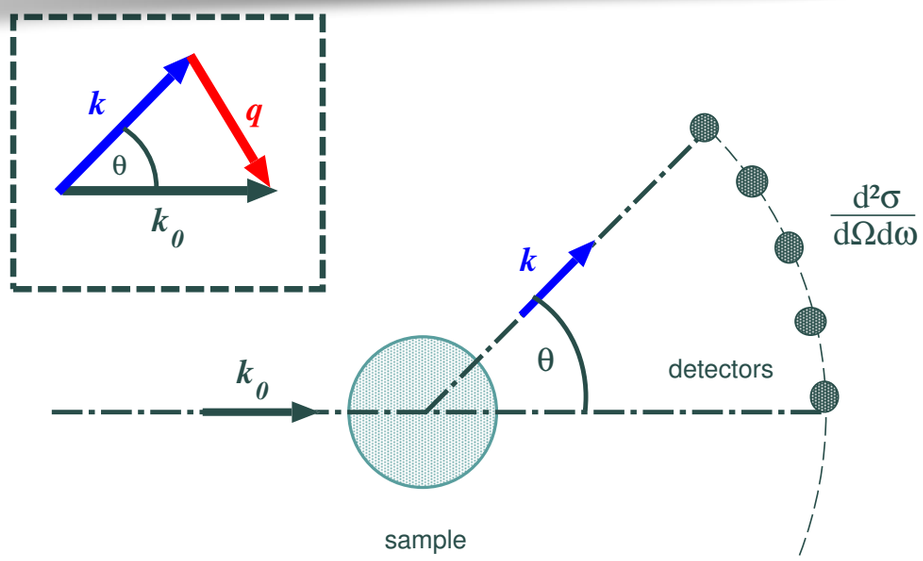
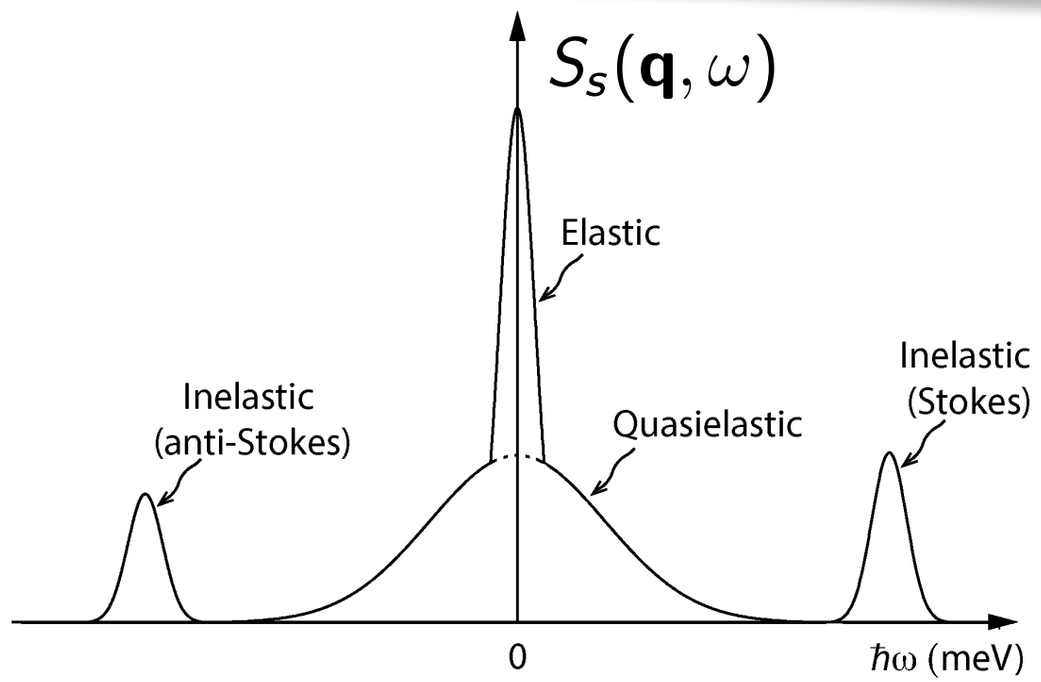
C.G. Shull

Probing atomic motions in complex systems by quasielastic neutron scattering (QENS)





Dynamic structure factor

$$S_s(\mathbf{q},\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} dt\, e^{-i\omega t} F_s(\mathbf{q},t)$$

Intermediate scattering function

$$F_s(\mathbf{q},\omega) \approx \left\langle e^{-i\mathbf{q}\cdot\hat{\mathbf{x}}_1(0)} e^{i\mathbf{q}\cdot\hat{\mathbf{x}}_1(t)} \right\rangle$$

Elastic Incoherent Structure Factor

$$EISF(\mathbf{q}) = \lim_{t\to\infty} F_s(\mathbf{q},t) = \left| \left\langle e^{i\mathbf{q}\cdot\hat{\mathbf{x}}_1} \right\rangle \right|^2$$

M. Karlsson, PCCP, 2015, 17, 26-38

Neutron scattering from hydrogen-rich (bio)polymers probes the self-correlated motions of the hydrogen atoms on the ps-ns time scale

momentum transfer:

$$\mathbf{q} = \mathbf{k}_0 - \mathbf{k} = (\mathbf{p}_0 - \mathbf{p})/\hbar$$

energy transfer:

$$\omega = (E_0 - E)/\hbar$$

The ISF is a quantum correlation function

Detailed Balance relations

$$F_s(\mathbf{q},t) = F_s(-\mathbf{q},-t+i\beta\hbar)$$
$$S_s(\mathbf{q},\omega) = e^{\beta\hbar\omega} S_s(-\mathbf{q},-\omega)$$

Recoil moment

$$\int_{-\infty}^{+\infty} d\omega\, \omega S_s(\mathbf{q},\omega) = \frac{\hbar q^2}{2m}$$

Van Hove's theory of neutron scattering

PHYSICAL REVIEW VOLUME 95, NUMBER 1 JULY 1, 1954

Correlations in Space and Time and Born Approximation Scattering in Systems of Interacting Particles

LÉON VAN HOVE
Institute for Advanced Study, Princeton, New Jersey
(Received March 16, 1954)

A REMARK ON THE TIME-DEPENDENT PAIR DISTRIBUTION

by LÉON VAN HOVE

Instituut voor theoretische fysica der Rijksuniversiteit, Utrecht, Nederland

Physica, vol. 24, no. 1, pp. 404–408, 1958.

Van Hove quantum (self) correlation function

Relate the (q,ω)-description of a scattering experiment to a (x,t) description, in which atomic motions in space and time are considered.[1]

$$S_s(\mathbf{q},\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} dt \int d^3x\, e^{i(\mathbf{q}\cdot\mathbf{x}-\omega t)} G_s(\mathbf{x},t)$$

$$G_s(\mathbf{x},t) = \frac{1}{(2\pi)^3}\int d^3q\, e^{-i\mathbf{q}\cdot\mathbf{x}} F_s(\mathbf{q},t),$$

$$= \int d^3r' \left\langle \delta(\mathbf{x}-\mathbf{x}'+\hat{\mathbf{x}}_1(0))\delta(\mathbf{x}'-\hat{\mathbf{x}}_1(t))\right\rangle = G_s^*(\mathbf{x},t+i\beta\hbar)$$

The quantum Van Hove correlation function is difficult to interpret. In a rarely cited paper[2] Van Hove showed that its imaginary part is related to the **local density perturbation of the system by the scattered neutrons.**

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[1] L. Van Hove, “Correlations in space and time and Born approximation scattering in systems of interacting particles,” Physical Review, vol. 95, no. 1, p. 249, 1954.
[2] L. Van Hove, “A remark on the time-dependent pair distribution,” Physica, vol. 24, no. 1, pp. 404–408, 1958.

Easy interpretation of G(x,t) in the "mathematical" ħ→0 limit of the scattering functions

Classical Van Hove correlation function

$$G_s(\mathbf{x},t) \overset{\hbar\to 0}{=} \langle \delta(\mathbf{x} - (\mathbf{x}_1(t) - \mathbf{x}_1(0))) \rangle_{cl}$$

Here x(t) are classical trajectories and G(x,t) becomes a classical probability density for a displacement x in time t

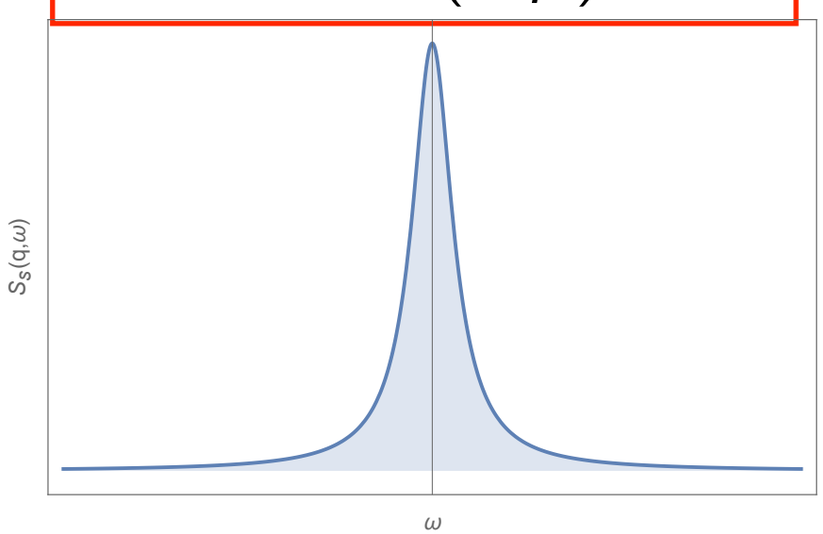
Diffusion models ("spatial motion models") for QENS

Example: free, normal diffusion

$$\partial_t G_s(\mathbf{x},t) = D\Delta G_s(\mathbf{x},t)$$

$$G(\mathbf{x},t) = \frac{e^{-\frac{|\mathbf{x}|^2}{4\mathrm{D}t}}}{2\sqrt{\pi \mathrm{D}t}}$$

$$S_s(q,\omega) = \frac{1}{\pi}\frac{Dq^2}{(Dq^2)^2 + \omega^2}$$



Challenges and limitations of classical spatial motion models

- Classical diffusion models for atomic motions do not capture the multiscale dynamics of the atoms in complex systems.

- "Impactless scattering" — vanishing recoil moment

$$\int_{-\infty}^{+\infty} d\omega\, \omega S_s(\mathbf{q},\omega) = \frac{\hbar|\mathbf{q}|^2}{2m_{\text{eff}}} \overset{\hbar\to 0}{=} 0$$

The $\hbar\to 0$ limit concerns the scattering system, but also the "kick" $\hbar q$ from the neutron to the scattering atom.

The impact of the scattered neutrons on the dynamics of the scattering atoms is by construction neglected. **The neutron is a passive probe.**

Quantum trajectory approach - Integrate the neutron kick into a trajectory-based description of neutron scattering

G. Kneller, Mol. Phys., vol. 83, no. 1, pp. 63–87, 1994.

- Describe the neutron as an active probe in a trajectory-based scenario.
- Define the "physical classical limit" of the scattering functions.

"Positionless" representation of F(q,t)

$$F_s(\mathbf{q},t) = \left\langle e^{-i\mathbf{q}\cdot\hat{\mathbf{x}}_1(0)} e^{i\mathbf{q}\cdot\hat{\mathbf{x}}_1(t)} \right\rangle = \left\langle e^{it\hat{H}'(\mathbf{q})/\hbar} e^{-it\hat{H}/\hbar} \right\rangle$$

GC Wick

Kicked Hamiltonian

$$\hat{H} = \frac{\hat{\mathbf{p}}_1^2}{2M_1} + V(\hat{\mathbf{r}}_1), \qquad \hat{H}'(\mathbf{q}) = \frac{(\hat{\mathbf{p}}_1 + \hbar\mathbf{q})^2}{2M_1} + V(\hat{\mathbf{r}}_1)$$

Proof. With $Z = \mathrm{tr}\{e^{-\beta\hat{H}}\}$

$$F_s(\mathbf{q},t) = \left\langle e^{-i\mathbf{q}\cdot\hat{\mathbf{x}}_1(0)} e^{i\mathbf{q}\cdot\hat{\mathbf{x}}_1(t)} \right\rangle$$
$$= \frac{1}{Z}\mathrm{tr}\Big\{ e^{-\beta\hat{H}} \underbrace{e^{-i\mathbf{q}\cdot\hat{\mathbf{x}}_1} e^{it\hat{H}/\hbar} e^{i\mathbf{q}\cdot\hat{\mathbf{x}}_1}}_{\hat{H}'(\mathbf{q})} e^{-it\hat{H}/\hbar} \Big\}.$$

G. C. Wick, "The scattering of neutrons by systems containing light nuclei," Physical Review, vol. 94, no. 5, pp. 1228–1242, 1954.

Propagator form of the intermediate scattering function

$$F_s(\mathbf{q},t) = \frac{1}{Z}\int\int\int dx dx' dx'' \underbrace{\langle x|e^{-\beta\hat{H}}|x'\rangle}_{K(x,x',-i\beta\hbar)} \underbrace{\langle x'|e^{it\hat{H}'(\mathbf{q})/\hbar}|x''\rangle}_{K_q(x',x'',-t)} \underbrace{\langle x''|e^{-it\hat{H}/\hbar}|x\rangle}_{K(x'',x,t)}$$

Retrieve trajectories through a **path integral representation** of the propagators

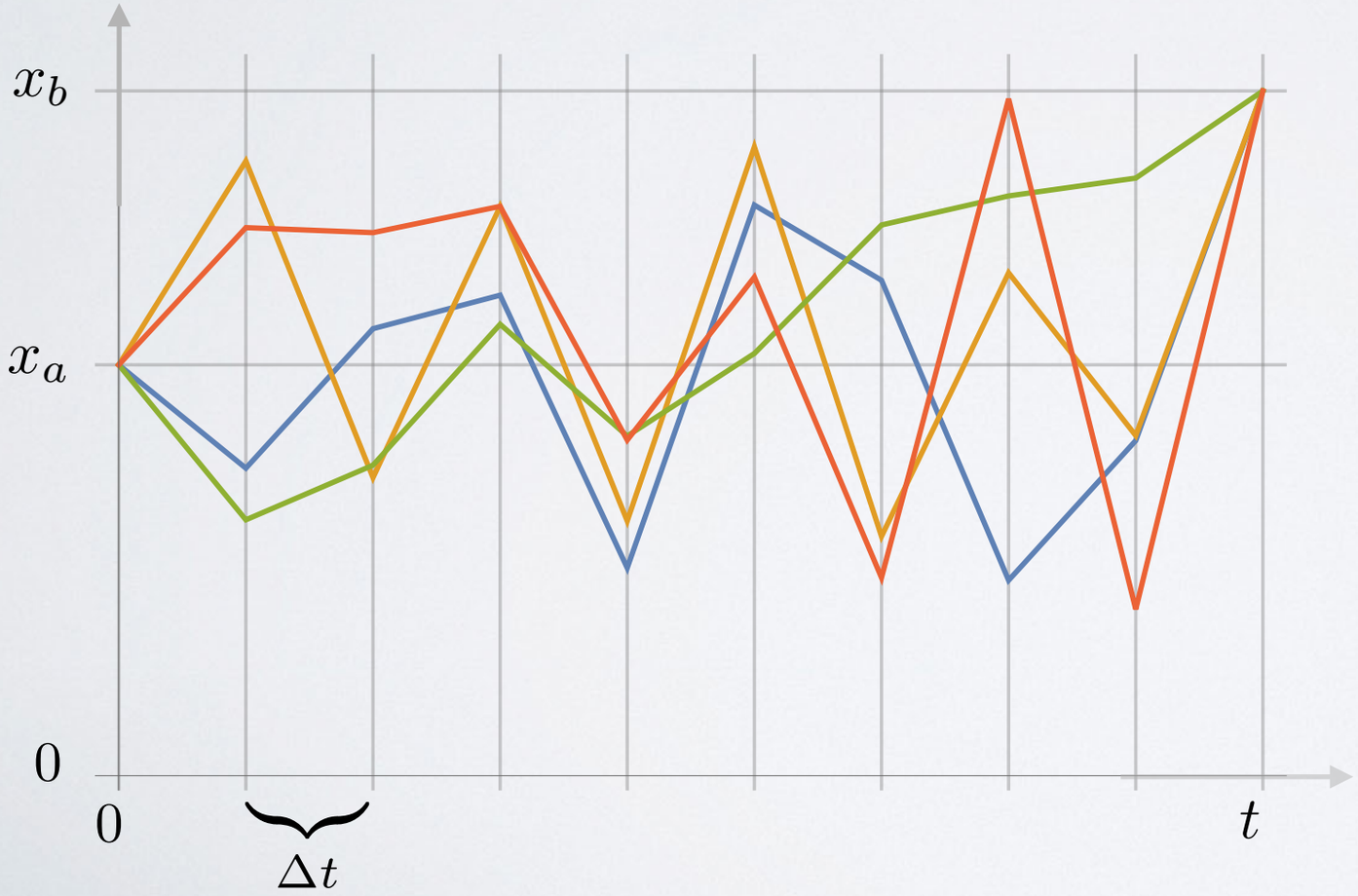
Real time propagator

Setting $\Delta t = t/n$

$$K(x_b, x_a, t) = \langle x_b | e^{-it\hat{H}/\hbar} | x_a \rangle = \langle x_b | \left(e^{-i\frac{\Delta t}{\hbar}\hat{H}} \right)^n | x_a \rangle$$

$$= \int \dots \int dx_1 \dots dx_n \langle x_b | \left(e^{-i\frac{\Delta t}{\hbar}\hat{H}} \right) | x_1 \rangle \langle x_1 | \left(e^{-i\frac{\Delta t}{\hbar}\hat{H}} \right) | x_2 \rangle \dots$$

$$\dots \langle x_n | \left(e^{-i\frac{\Delta t}{\hbar}\hat{H}} \right) | x_b \rangle \overset{n\to\infty}{\to} \int \mathcal{D}[x(\tau)] e^{iA[x(\tau)]/\hbar}$$



Path action integral

$$A[x(\tau)] = \int_0^t d\tau \underbrace{(M\dot{x}(\tau)^2/2 - V(x(\tau)}_{L(\dot{x}(\tau), x(\tau))}$$

"Kicked" real time propagator

Phase factor form

$$K_q(x_b, x_a, t) = K(x_b, x_a, t) e^{iq(x_b - x_a)}$$

Path integral form

$$K(x_b, x_a, t) = \int \mathcal{D}[x(\tau)] e^{iA_q[x(\tau)]/\hbar}$$

"Kicked" path action integral

Coupling to the neutron

$$A_q[x(\tau)] = \int_0^t d\tau \underbrace{\left(M\dot{x}(\tau)^2/2 - V(x(\tau) + \hbar q \dot{x}(\tau)\right)}_{L_q(\dot{x}(\tau), x(\tau))}$$

Imaginary time propagator

Setting $t_{th} = \beta\hbar$ and $\Delta t_{th} = t_{th}/n$

$$K(x_b, x_a, -i\beta\hbar) = \langle x_b | e^{-\beta\hat{H}} | x_a \rangle = \langle x_b | \left(e^{-\frac{\Delta t_{th}}{\hbar}\hat{H}} \right)^n | x_a \rangle$$

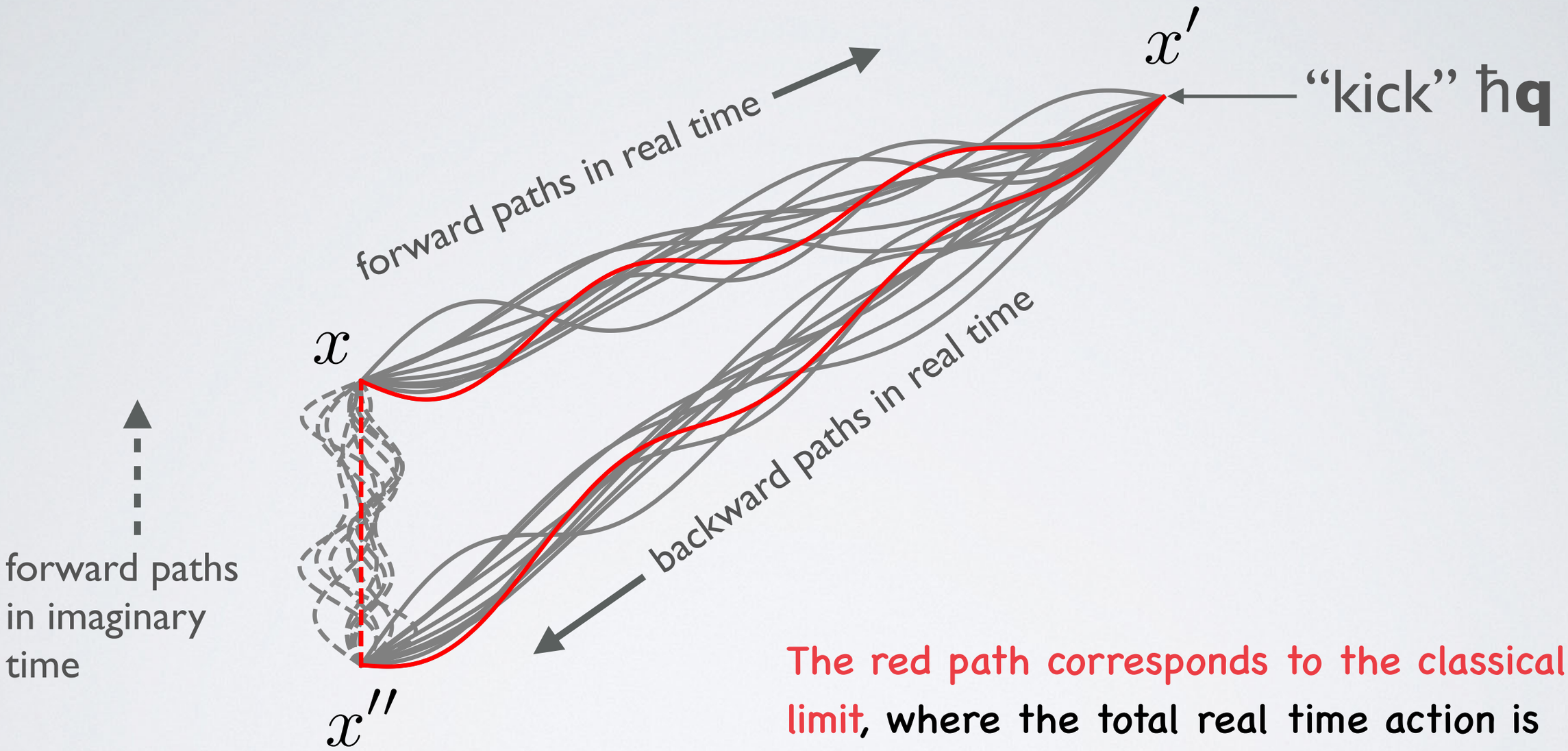
$$= \int \dots \int dx_1 \dots dx_n \langle x_b | \left(e^{-\frac{\Delta t_{th}}{\hbar}\hat{H}} \right) | x_1 \rangle \langle x_1 | \left(e^{-\frac{\Delta t_{th}}{\hbar}\hat{H}} \right) | x_2 \rangle \dots$$

$$\dots \langle x_n | \left(e^{-\frac{\Delta t_{th}}{\hbar}\hat{H}} \right) | x_b \rangle \overset{n\to\infty}{\to} \int \mathcal{D}[x(\tau)] e^{-\beta\overline{H}[x(\tau)]/\hbar}$$

Average path energy

$$\overline{H}[x(\tau)] = \frac{1}{\beta\hbar} \int_0^t d\tau \underbrace{(M\dot{x}(\tau)^2/2 + V(x(\tau))}_{H(\dot{x}(\tau), x(\tau))}$$

$$F_s(\mathbf{q},t) = \frac{1}{Z}\int\int\int dx dx' dx''\, K(x,x',-i\beta\hbar)K_q(x',x'',-t)K(x'',x,t)$$



The red path corresponds to the classical limit, where the total real time action is minimized and the high temperature/short time limit is used for the propagation in imaginary time. The "neutron kick" is taken into account.

Classical limit of the intermediate scattering function

Expressing the density matrix through the classical limit of the Wigner function and retaining only the classical path ($A \gg \hbar$) yields

$$\tilde{F}_{cl}(\mathbf{q},t) = \frac{1}{Z_{cl}} \int\int d^3p d^3x \, e^{-\beta H(\mathbf{p},\mathbf{x})} e^{-\beta \Delta V(\mathbf{p},\mathbf{x};\hbar\mathbf{q},t)} \times e^{i\Delta\Phi(\mathbf{p},\mathbf{x};\hbar\mathbf{q},t)/\hbar} e^{i\mathbf{q}\cdot(\mathbf{x}'(\mathbf{p},\mathbf{x},t)-\mathbf{x})}$$

$$\Delta V(\mathbf{p},\mathbf{x};\hbar\mathbf{q},t) = V((\mathbf{x}+\mathbf{x}'')/2) - V(\mathbf{x})$$

$$\Delta\Phi(\mathbf{p},\mathbf{x};\hbar\mathbf{q},t) = A(\mathbf{x},\mathbf{x}',t) - A(\mathbf{x}',\mathbf{x}'',t) + (\mathbf{p}_0 + \hbar\mathbf{q})\cdot(\mathbf{x}-\mathbf{x}'')$$

where $\mathbf{x}' \equiv \mathbf{x}'(\mathbf{p},\mathbf{x},t)$ and $\mathbf{x}'' \equiv \mathbf{x}''(\mathbf{p},\mathbf{x};\hbar\mathbf{q},t)$.

The standard classical limit reads

$$\lim_{\hbar\to 0} \tilde{F}_{cl}(\mathbf{q},t) = \frac{1}{Z_{cl}} \int\int d^3p d^3x \, e^{-\beta H(\mathbf{p},\mathbf{x})} e^{i\mathbf{q}\cdot(\mathbf{x}'(\mathbf{p},\mathbf{x},t)-\mathbf{x})}$$

- The intermediate scattering function F(q,t) can be written as a path integral over closed paths, one of which is the classical path.

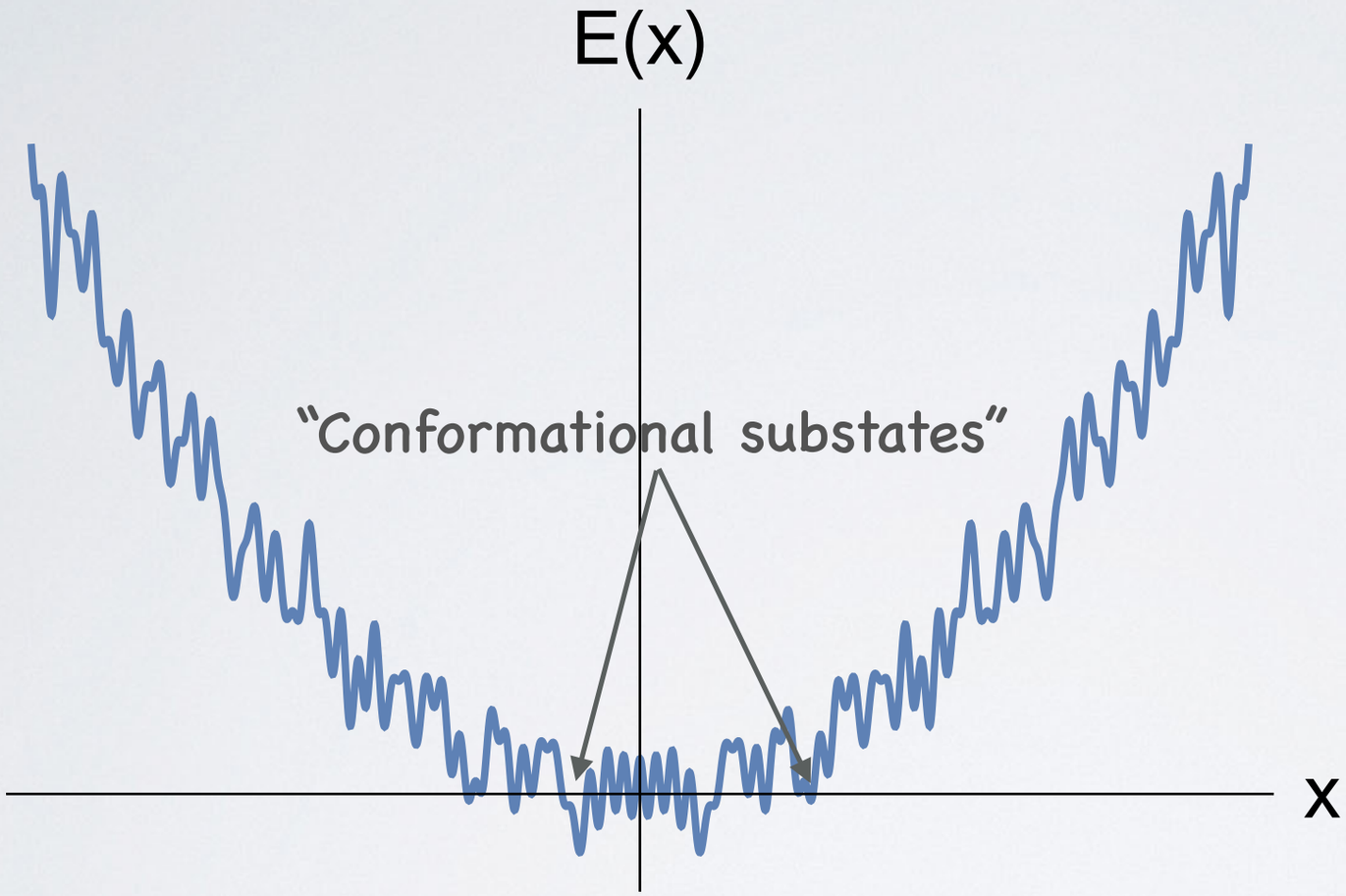
- The classical path minimizes the total action in real time and is ballistic in imaginary time (short time apprixmation).

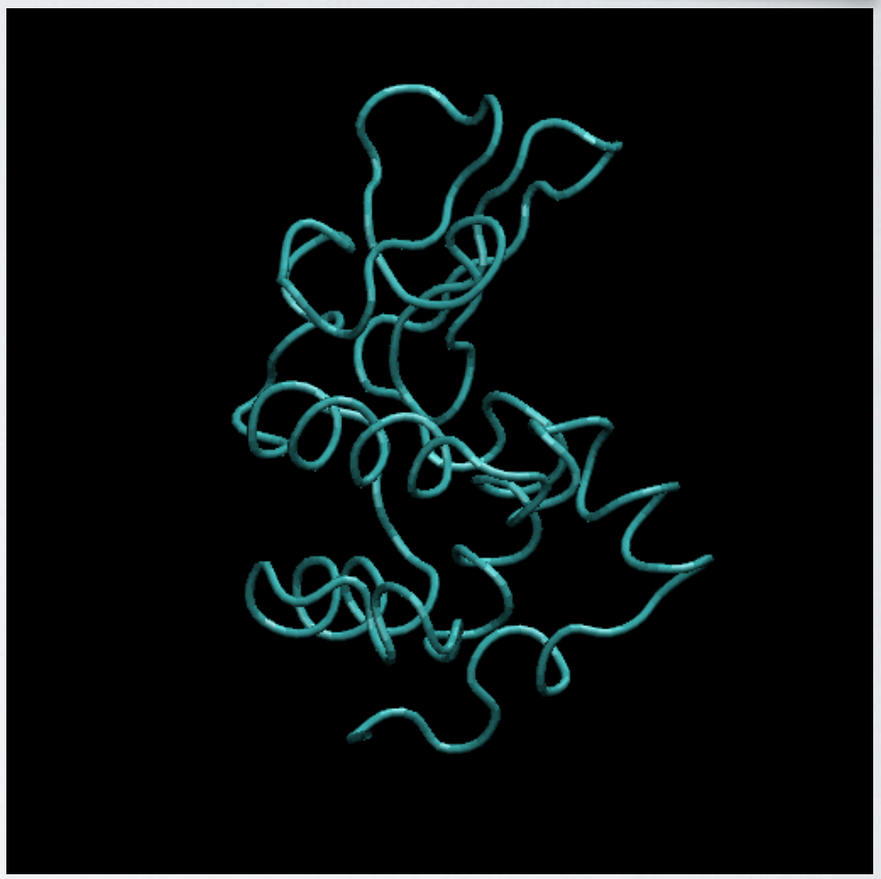
- The "physical classical limit" of F(q,t) corresponds to retaining only the total classical path, which preserves the scattering kinematics and the impact of the neutron on the scattering system.

- The complexity of calculating the scattering function is though increased compared to the mathematical classical limit $\hbar \to 0$, where the neutron is a passive probe.

Classical mechanical energy landscape description of neutron scattering — integrate multiscale dynamics

Energy landscapes – a kinetic picture of protein dynamics and kinetics





The protein jumps between different minima of the **highly multidimensional (free) energy landscape** which correspond to similar "conformational substates" (H. Frauenfelder et al, Science 254, 1598 (1991)).

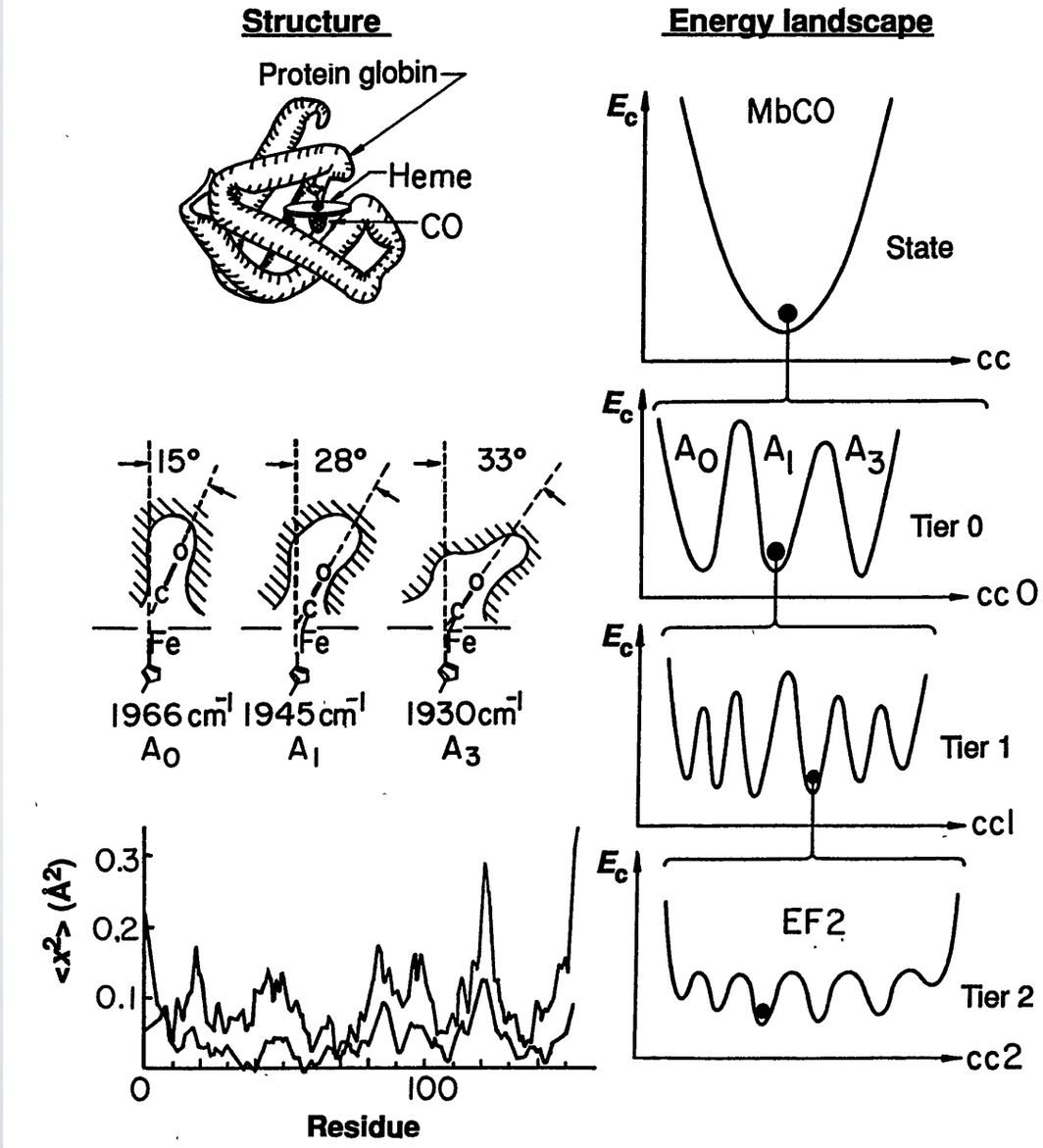
Protein dynamics through jump kinetics

The Energy Landscapes and Motions of Proteins

HANS FRAUENFELDER, STEPHEN G. SLIGAR, PETER G. WOLYNES

SCIENCE, VOL. 254

Conformational substates



Non-exponential rebinding kinetics of CO

$$N(t) = \int dH\, g(H) \exp[-k(H)t]$$

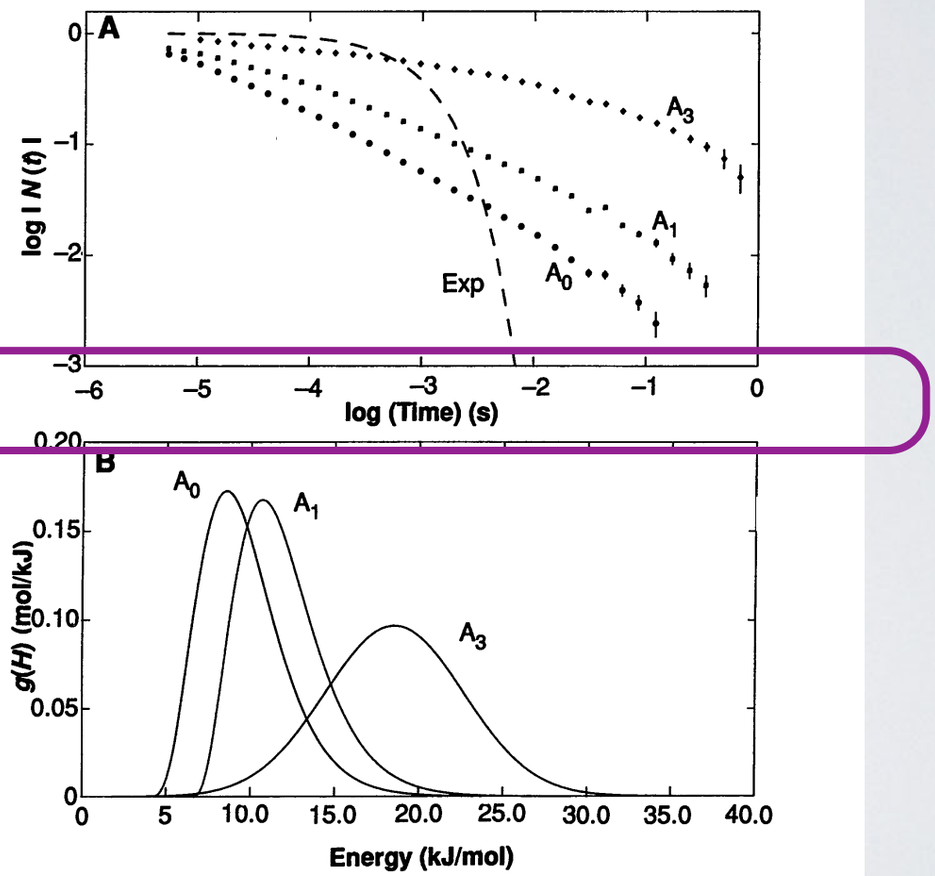


Fig. 2. Rebinding of CO to Mb after photodissociation, measured separately for the substates of tier 0 at pH 5.7. (**A**) $N(t)$ is the fraction of proteins that have not rebound a CO at the time t after photodissociation. All three substates (A_0, A_1, and A_3) rebind nonexponentially in time. (**B**) The activation enthalpy spectra, defined through Eq. 1.

Protein dynamics displays self-similarity

Relaxation and time correlation functions have a multi-exponential form:

$$\psi(t) = \int_0^\infty d\lambda\, p(\lambda) \exp(-\lambda t)$$

For complex systems these functions decay for long times slowly with a power law and exhibit thus self-similarity:

$$\psi(t) \overset{t\to\infty}{\sim} (t/\tau)^{-\beta}, \quad 0 < \beta < 1$$

$$\Rightarrow \quad \psi(\lambda t) \overset{t\to\infty}{=} \lambda^{-\beta}\psi(t)$$

CO-rebinding kinetics

46 Biophysical Journal Volume 68 January 1995 46–53

A Fractional Calculus Approach to Self-Similar Protein Dynamics

Walter G. Glöckle and Theo F. Nonnenmacher
Department of Mathematical Physics, University of Ulm, D-89069 Ulm, Germany

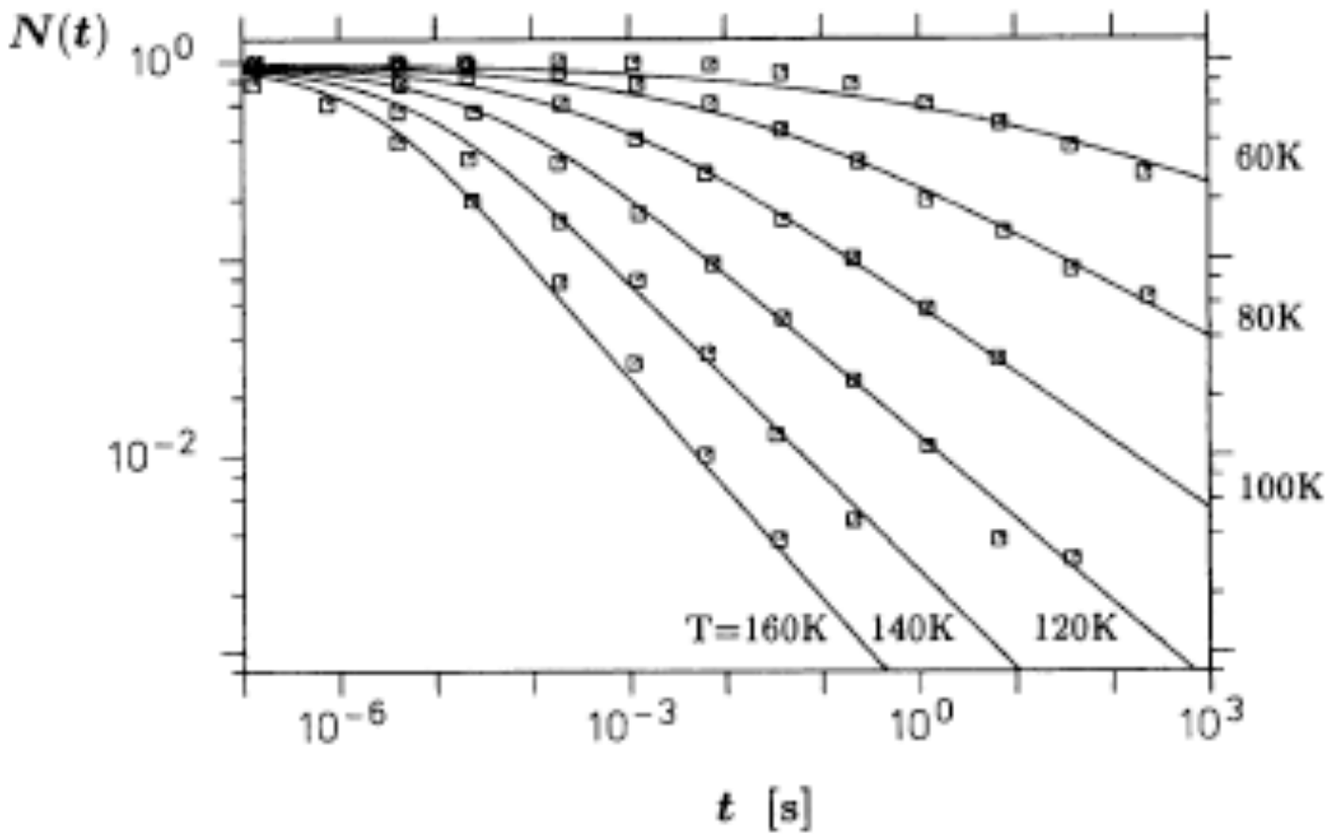


FIGURE 2 Three-parameter model Eq. 32 for rebinding of CO to Mb after photo dissociation. The parameters are $\tau_m = 8.4 \times 10^{-10}$s, $\alpha = 3.5 \times 10^{-3}\ K^{-1}$ and $k = 130$, the data points are from Austin et al. (1975).

$$N(t) = N(0) E_\beta \left((-[t/\tau]^\beta \right)$$

Mittag-Leffler function

$$E_\beta(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(1+\beta k)}$$

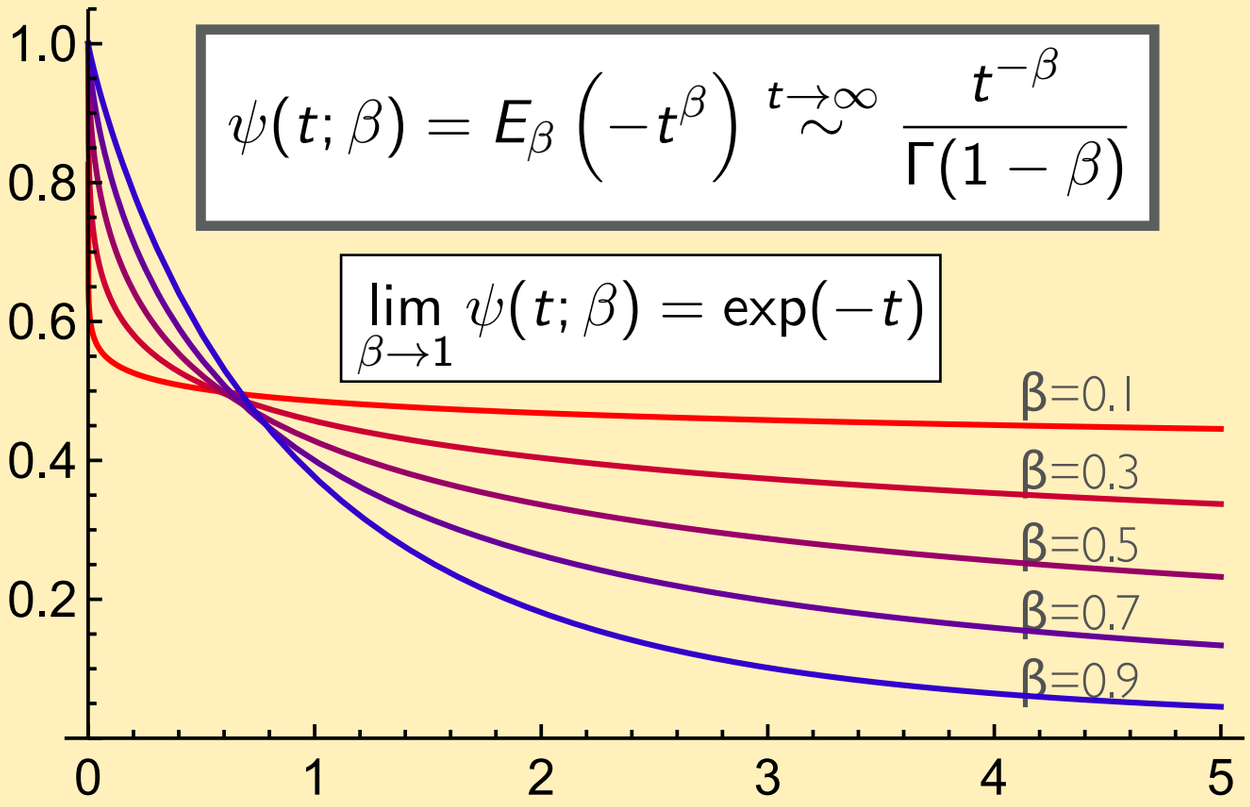
$$E_\beta\left(-t^\beta\right) \overset{t\to\infty}{\sim} \frac{t^{-\beta}}{\Gamma(1-\beta)}$$

"Stretched" ML function and relaxation spectrum

Relaxation function

$$\psi(t;\beta) = E_\beta\left(-t^\beta\right) \overset{t\to\infty}{\sim} \frac{t^{-\beta}}{\Gamma(1-\beta)}$$

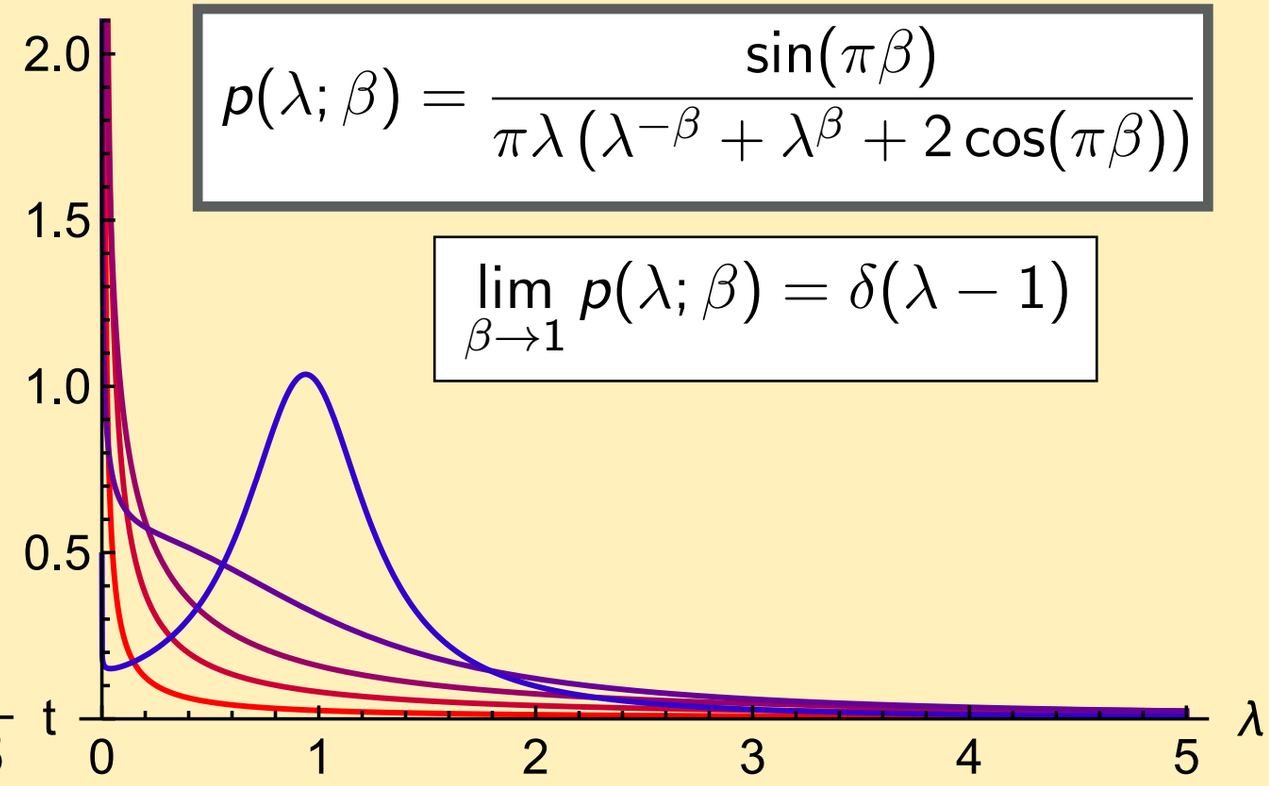
$$\lim_{\beta\to 1} \psi(t;\beta) = \exp(-t)$$



Relaxation rate spectrum

$$p(\lambda;\beta) = \frac{\sin(\pi\beta)}{\pi\lambda\left(\lambda^{-\beta}+\lambda^{\beta}+2\cos(\pi\beta)\right)}$$

$$\lim_{\beta\to 1} p(\lambda;\beta) = \delta(\lambda-1)$$



Relating relaxation rates to the "roughness" of the energy landscape

The distribution barrier heights corresponds to a distribution of rates for kinetic processes and conformational relaxation.

$$\lambda = f(\epsilon), \quad \epsilon = \frac{\Delta E}{k_B T}$$

$$p(\lambda) \rightarrow \tilde{p}(\epsilon)$$

To relate barrier heights and relaxation rates, one needs a **model**.

$$\lambda = \lambda_0 e^{-\epsilon}$$

Arrhenius

$$\lambda = \lambda_0 e^{-\epsilon^2}$$

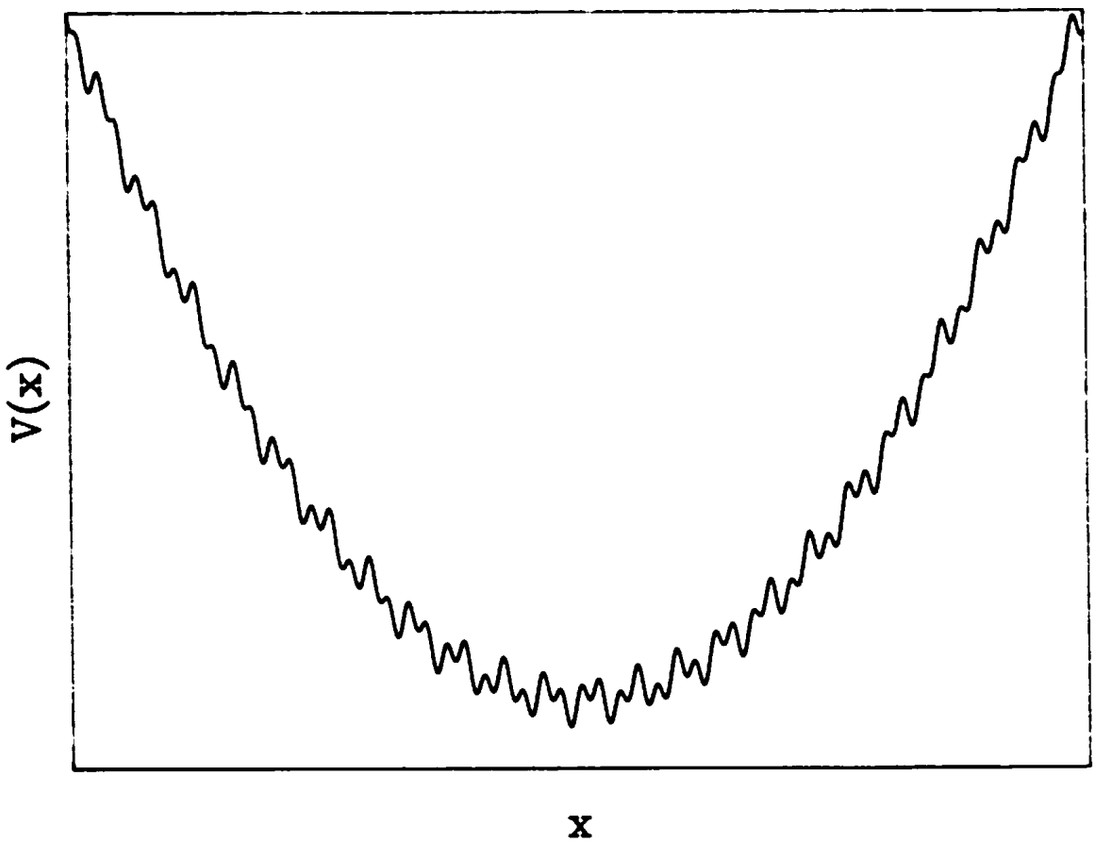
R. Zwanzig, PNAS 85,1988.

Diffusion in a rough potential

ROBERT ZWANZIG

University of Maryland, College Park, MD 20742

Proc. Natl. Acad. Sci. USA
Vol. 85, pp. 2029–2030, April 1988
Physics



$$D^* = D \exp[-(\varepsilon/k_B T)^2].$$

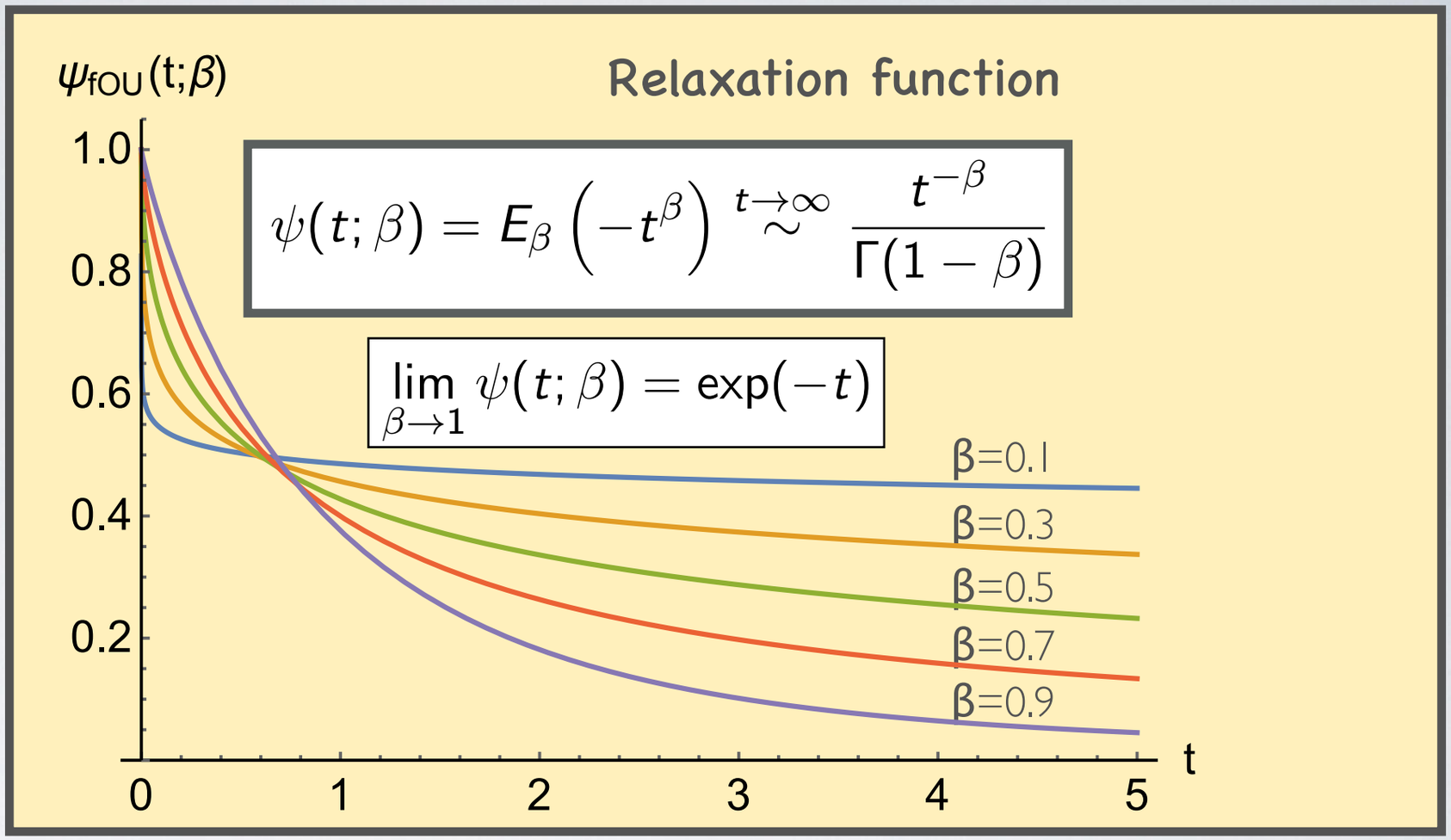
Barrier height

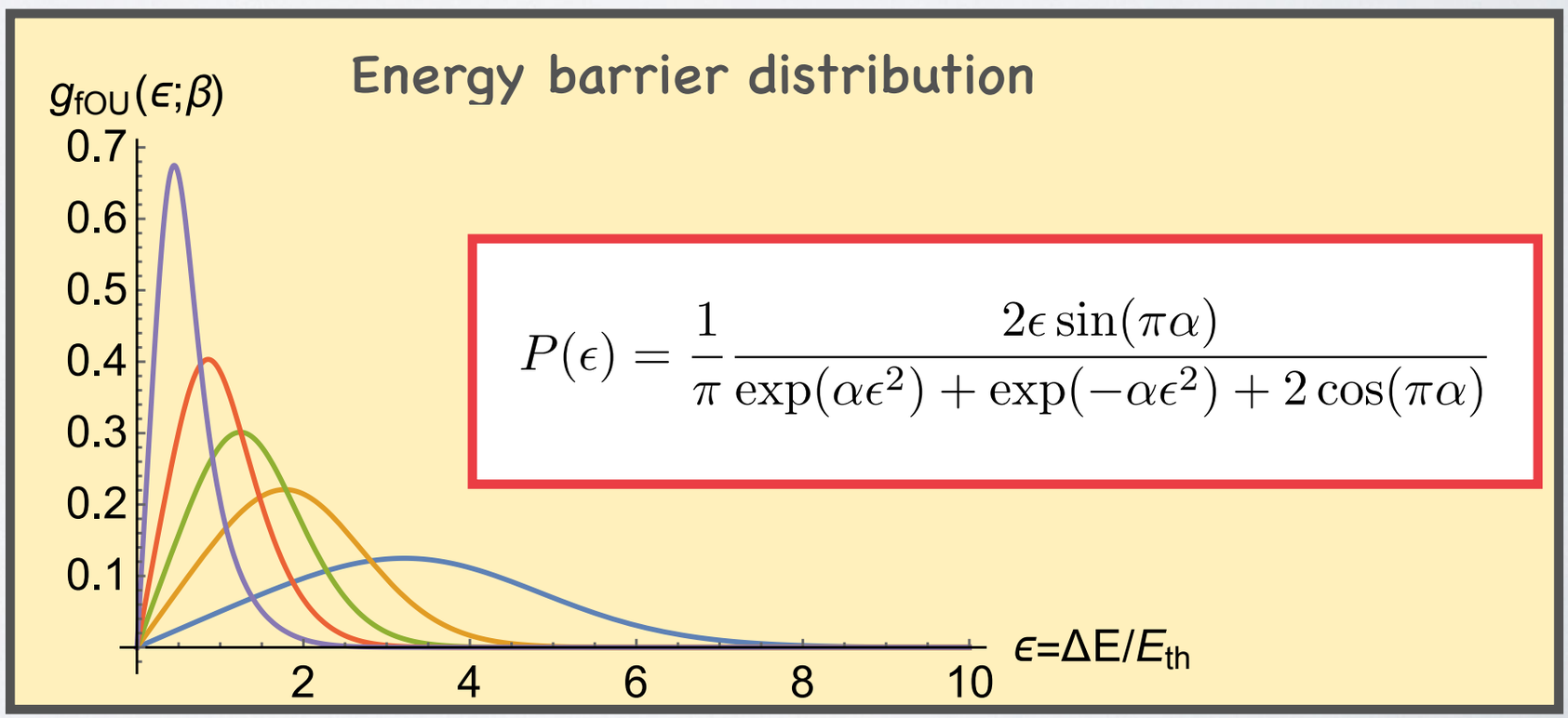
For diffusion in a harmonic potential

$$D_0 = \langle x^2 \rangle \lambda_0$$

$$\lambda = \lambda_0 e^{-\epsilon^2}$$

Saouessi, Peters & Kneller, JCP150, 2019.





- The problem with energy landscapes models is that they refer only to the ω/time domain and are not q/space-resolved. They are thus a priori not relevant for modeling neutron scattering.

- They can though be related to neutron scattering within the Gaussian approximation of F(q,t).

Gaussian approximation of $F_s(q,t)$ and diffusive motions

A. Rahman, K. Singwi, and A. Sjölander, Physical Review 126, 986 (1962).

G.R. Kneller, The Journal of Chemical Physics 145, 044103. Communication.

Cumulant expansion in q

$$F(\mathbf{q},t) = e^{\frac{\hbar q^2}{2M}t} f(\mathbf{q},t) \qquad f(\mathbf{q},t) = \exp\left(\sum_{k=1}^{\infty}(-1)^k q^{2k}\,\gamma_{2k}(t)\right)$$

$$\gamma_2(t) = \mu_2(t),$$
$$\gamma_4(t) = \mu_4(t) - \frac{1}{2}\mu_2(t)^2,$$
$$\gamma_6(t) = \frac{1}{3}\mu_2(t)^3 - \mu_4(t)\mu_2(t) + \mu_6(t),$$
$$\vdots$$

$$\mu_{2k}(t) = \int_0^t dt_1 \ldots \int_0^{t_{2k-1}} dt_{2k} \langle \hat{v}_q(t_1) \ldots \hat{v}_q(t_{2k}) \rangle$$

Gaussian approximation (moderate momentum transfers, q)

$$F(q,t) \approx \exp\left(i\frac{\hbar q^2 t}{2m}\right)\exp\left(-q^2\mu_2(t)\right)$$

$$\mu_2(t) = \frac{1}{2}\left(1 - i\tan\left(\frac{\beta\hbar}{2}\frac{d}{dt}\right)\right)\underbrace{\left\langle (x(t)-x(0))^2\right\rangle}_{MSD \equiv W(t)}$$

In the classical limit F(q,t) is completely determined by the MSD of the diffusing scattering atom

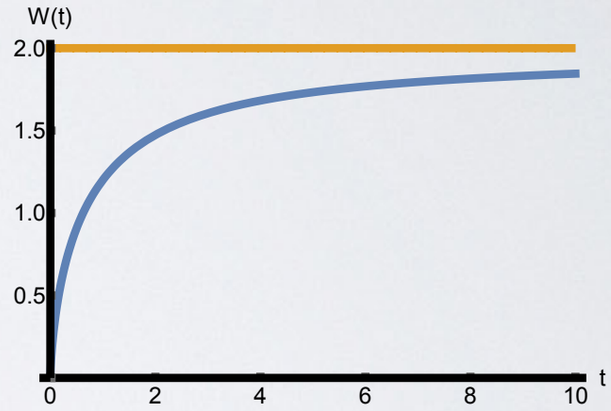
Relaxation rate/energy barrier spectra for atomic motions in proteins

Motions in proteins are confined in space (a=0) and the atomic positions can be referred to a well-defined mean positions,

$$x(t) = u(t) + \langle x \rangle$$

The MSD for confined motions can be expressed in terms of the **displacement autocorrelation function.**

$$W(t) = \left\langle (u(t) - u(0))^2 \right\rangle = 2\left(c_{uu}(0) - c_{uu}(t)\right)$$
$$c_{uu}(t) = \langle u(t)u(0) \rangle$$



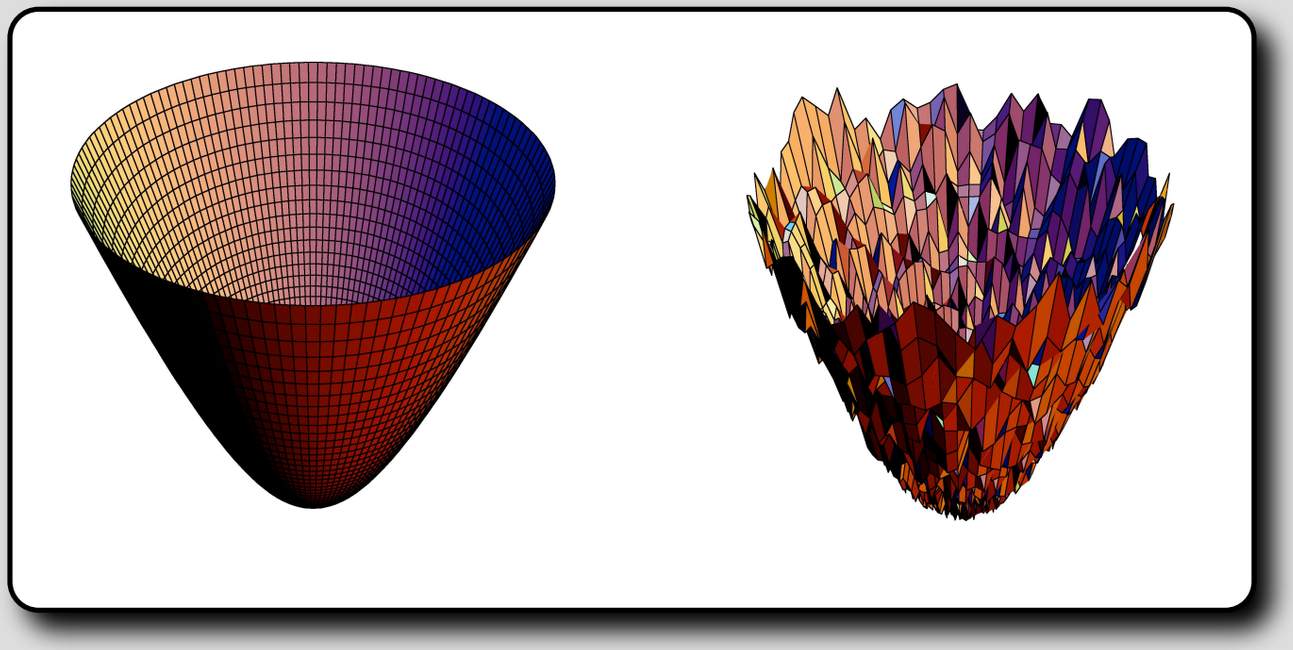
The displacement autocorrelation function has a multiexponential decay.

$$\frac{c_{uu}(t)}{c_{uu}(0)} \equiv \psi(t) = \int_0^\infty d\lambda\, p(\lambda) \exp(-\lambda t)$$

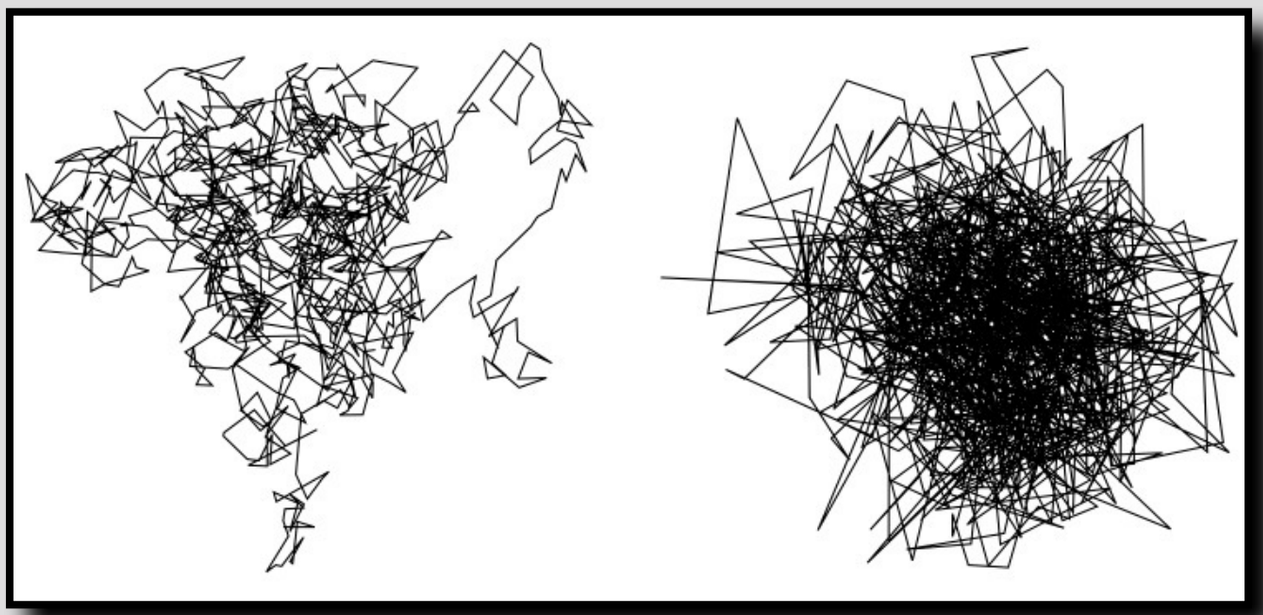
Distribution of relaxation rates/ barrier heights

Fractional Ornstein-Uhlenbeck process — a model for self-similar single-atom dynamics in proteins

Normal Ornstein-Uhlenbeck process: Diffusion in a "smooth" harmonic potential



Fractional Ornstein-Uhlenbeck process: Anomalous Diffusion in a "rugged" harmonic potential



Uhlenbeck, G. E. & Ornstein, L. S. Physical Review 36, 823 (1930).

1. Shao, Y. Physica D: Nonlinear Phenomena 83, 461–477 (1995).
2. R. Metzler, J. H. Jeon, and A. G. Cherstvy, Physical Chemistry Chemical Physics, vol. 16, pp. 24128–24164, 2014.

Fractional Fokker-Planck equation for conditional probability

$$\frac{\partial}{\partial t}p(\mathbf{u},t|\mathbf{u}_0,0) = \partial_t^{1-\beta}\mathcal{L}\,p(\mathbf{u},t|\mathbf{u}_0,0), \quad 0<\beta\leq 1$$

Time evolution operator

$$\mathcal{L} = D_\beta \frac{\partial}{\partial \mathbf{u}} \cdot \left\{ \frac{\partial}{\partial \mathbf{u}} + \frac{K\mathbf{u}}{k_B T} \right\}$$

Harmonic potential

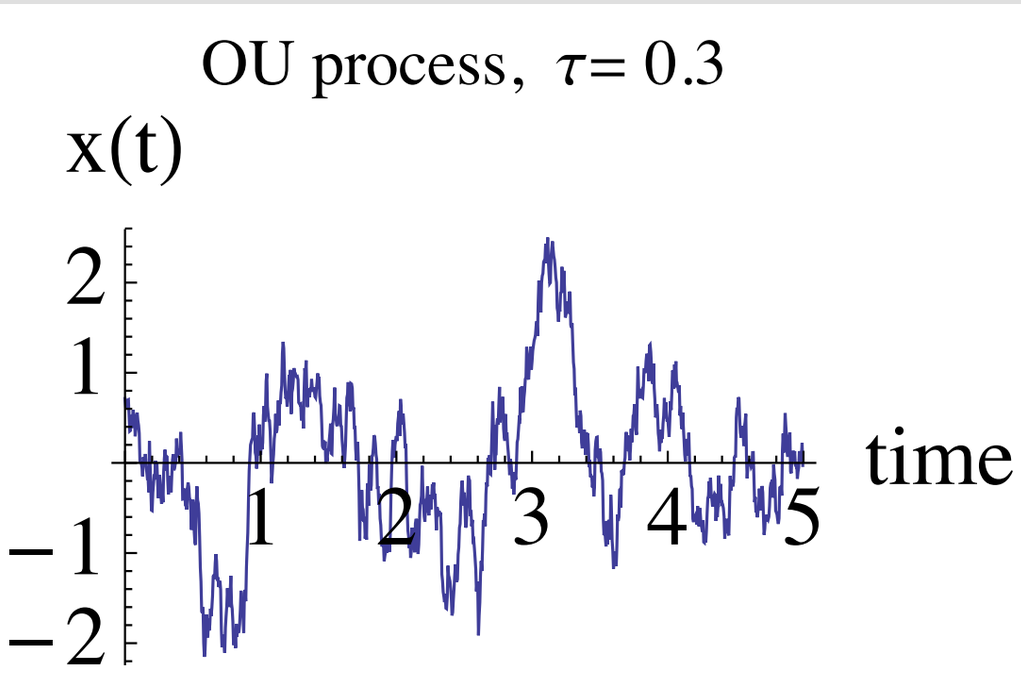
$$V(\mathbf{u}) = \frac{K}{2}|\mathbf{u}|^2$$

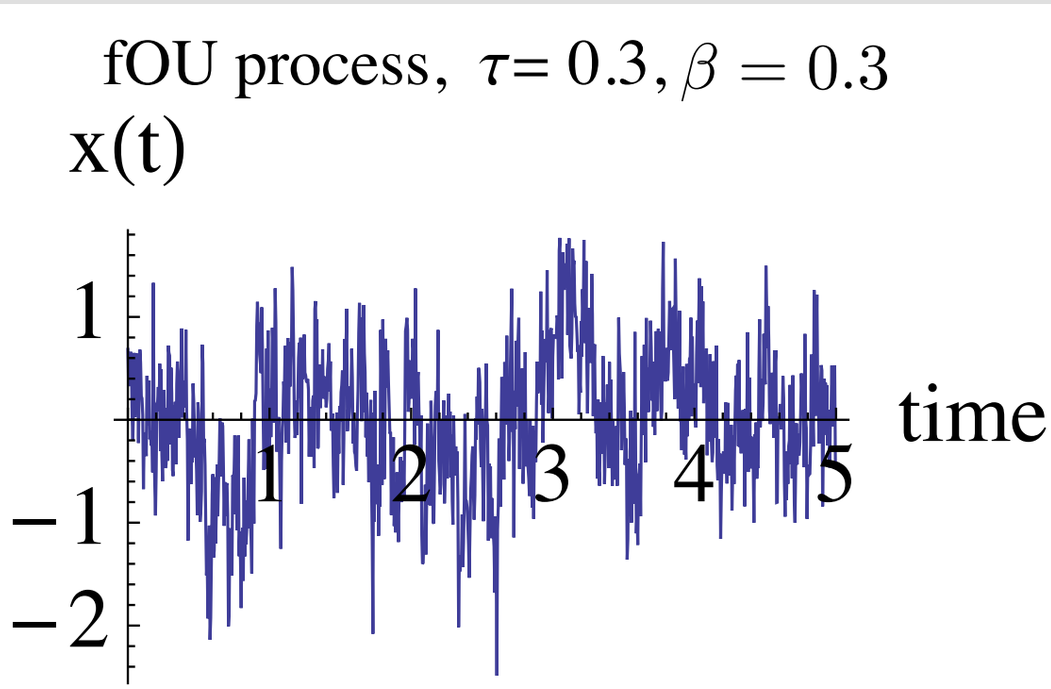
Fractional time derivative

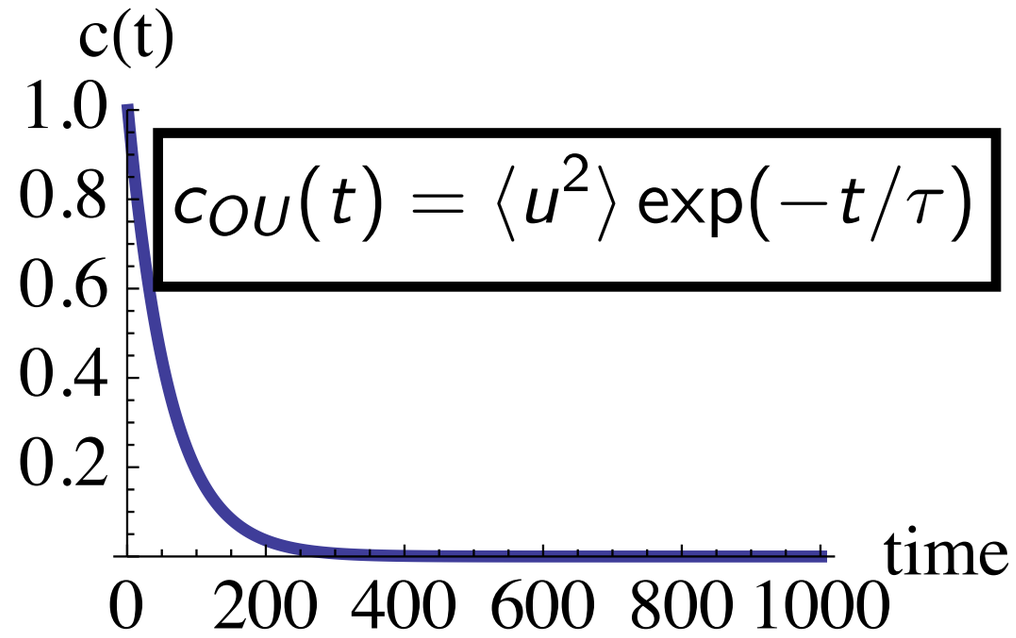
$$\partial_t^{1-\beta} g(t) = \frac{d}{dt}\int_0^t d\tau\, \frac{(t-\tau)^{\beta-1}}{\Gamma(\beta)} g(\tau)$$

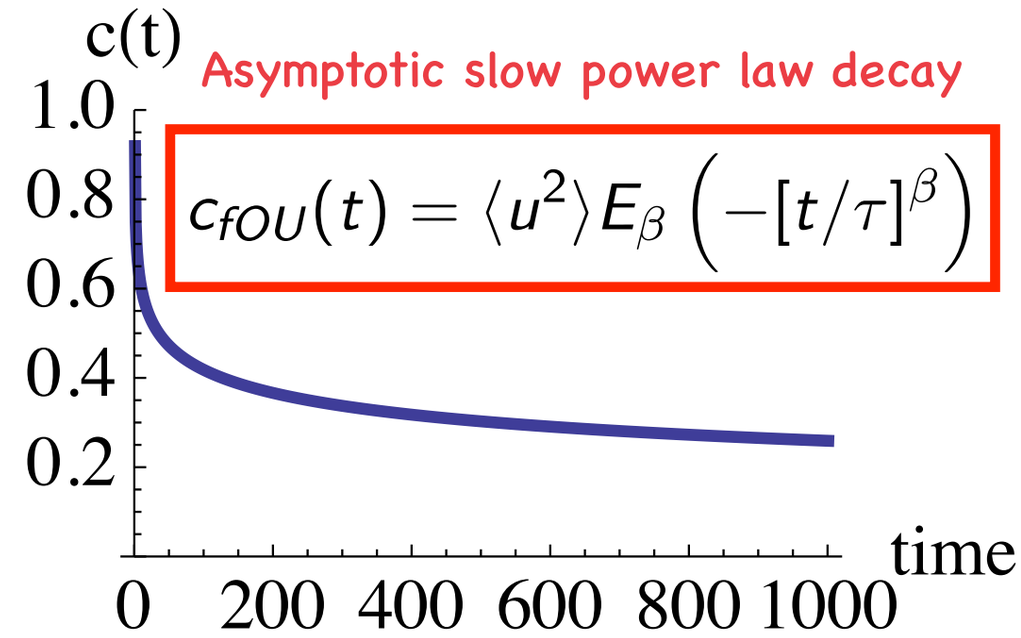
represents memory effects in the framework GLE.

Time series and autocorrelation functions for the fOU process





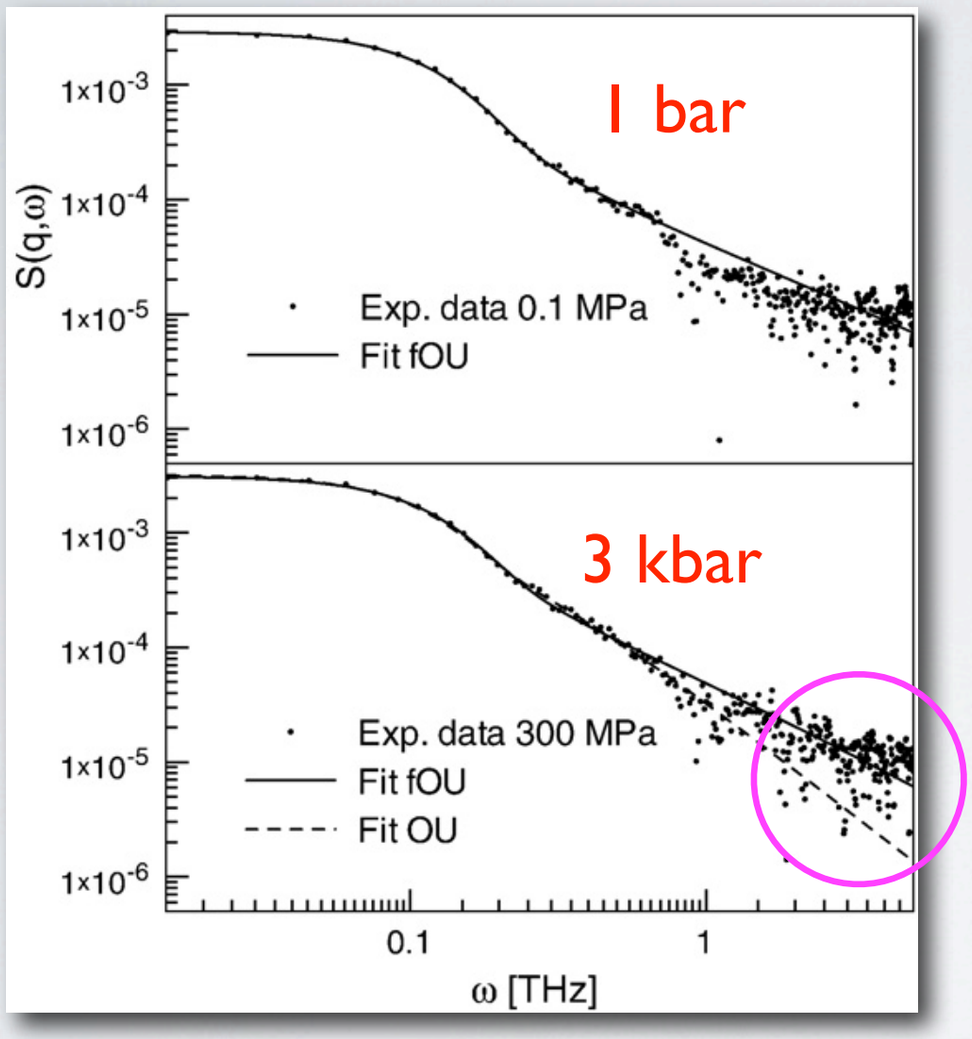


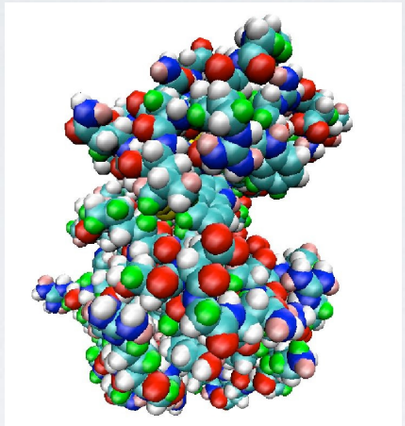


Application 1: Lysozyme under pressure by QENS and MD simulations (notation: β→α)

Neutron scattering

QENS dynamic structure factor

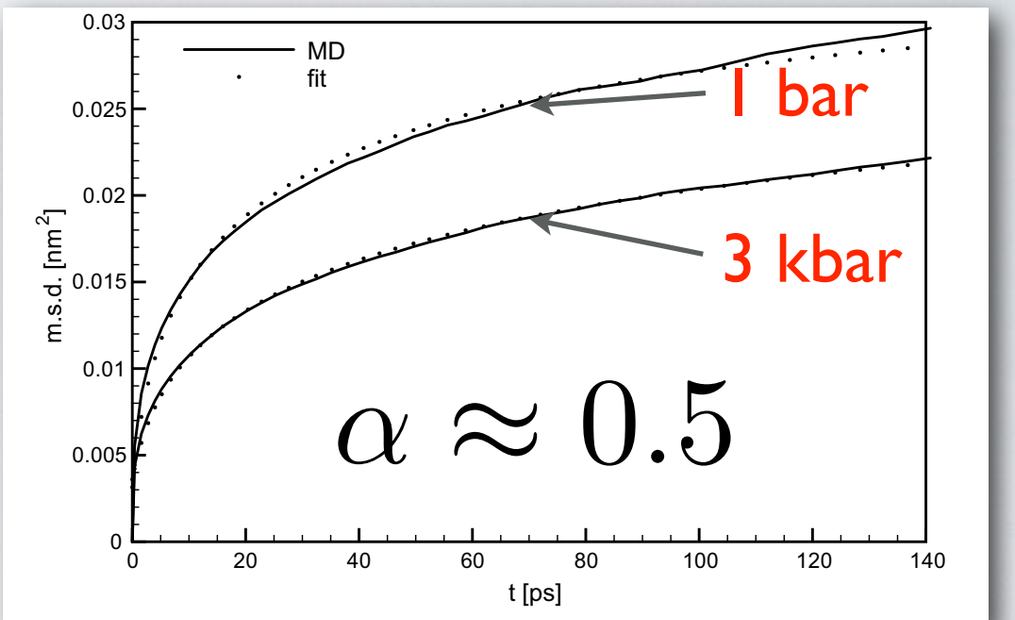




Lysozyme

MD simulation

Mean square displacement $\langle[x(t)-x(0)]^2\rangle$ of the H atoms in lysozyme MD simulation



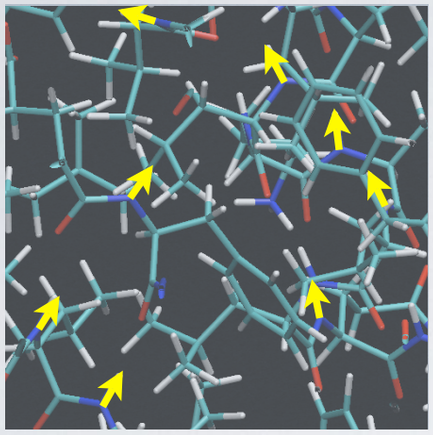
	0.1 MPa			300 MPa		
	$\langle x^2\rangle$ (nm^2)	α	τ (ps)	$\langle x^2\rangle$ (nm^2)	α	τ (ps)
MSD	6.17×10^{-3}	0.54	31.75	4.74×10^{-3}	0.54	39.08

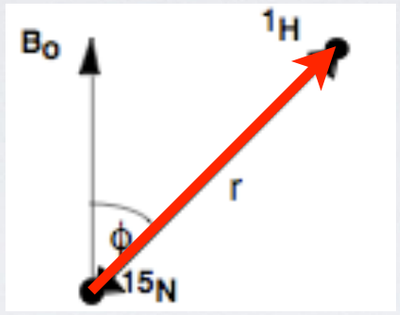
The form of the energy landscape does not change under pressure.

- Calandrini, Kneller, *J. Chem. Phys.*, vol. 128, no. 6, p. 065102, 2008.
- Calandrini et al,, *Chem. Phys.*, vol. 345, pp. 289–297, 2008.
- Kneller, Calandrini, *Biochimica et Biophysica Acta*, vol. 1804, pp. 56–62, 2010.

Application 2: Protein dynamics by NMR relaxation spectroscopy

Relaxation $^{15}N - {}^{1}H$





$$C_{ii}(t) = \langle P_2(\boldsymbol{\mu}_i(t) \cdot \boldsymbol{\mu}_i(0)) \rangle,$$

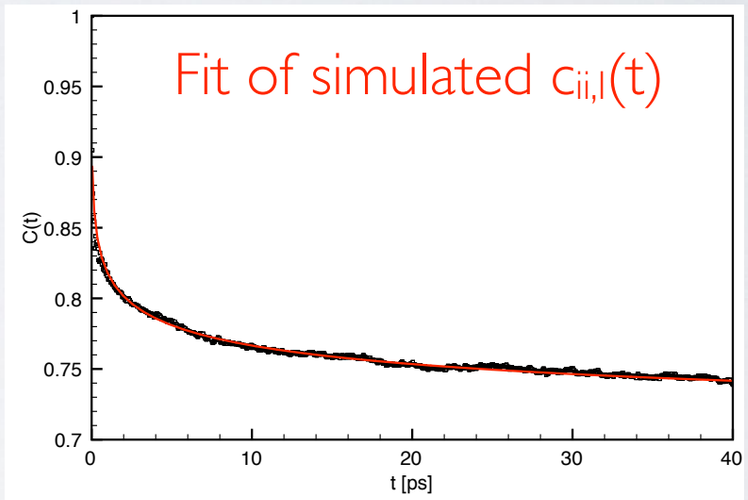
$$c_{ii}(t) = C_{ii,R}(t) C_{ii,I}(t)$$

Global rotation — Internal dynamics

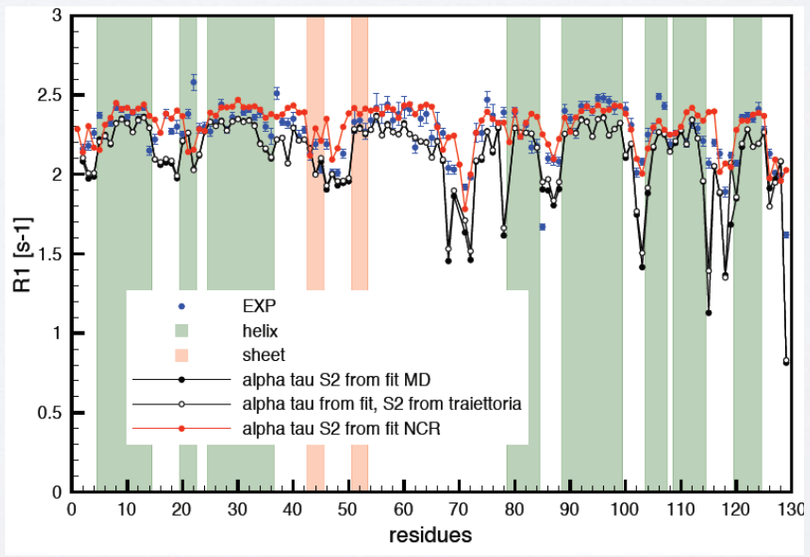
Model pour $c_{ii,I}(t)$:

$$c_{ii,I}(t) = S_{ii}^2 + (1 - S_{ii}^2) E_\alpha(-[t/\tau]^\alpha)$$

Mittag-Leffler function



Prediction of Expérimental data (T1, T2, NOE)



- Calandrini, Abergel, Kneller, *J. Chem. Phys.*, vol. 128, p. 145102, 2008.
- Calandrini, Abergel, Kneller, *J. Chem. Phys.*, vol. 133, p. 145101, 2010.

Quantum mechanical energy landscape concept for neutron scattering

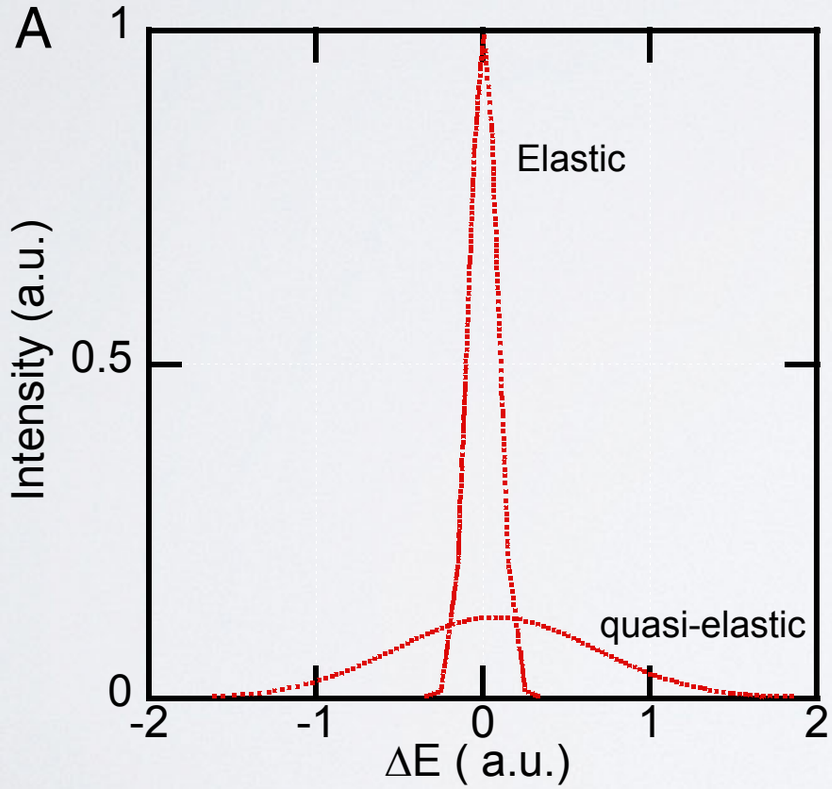
Describe the neutron as an active probe in an energy-landscape oriented interpretation of neutron scattering.

QENS from a Mössbauer perspective

Frauenfelder, Fenimore & Young, PNAS 111, 12764 (2014).

Spatial motion model

"Homogeneous" QENS spectrum from a diffusion model based on Van Hove's theory

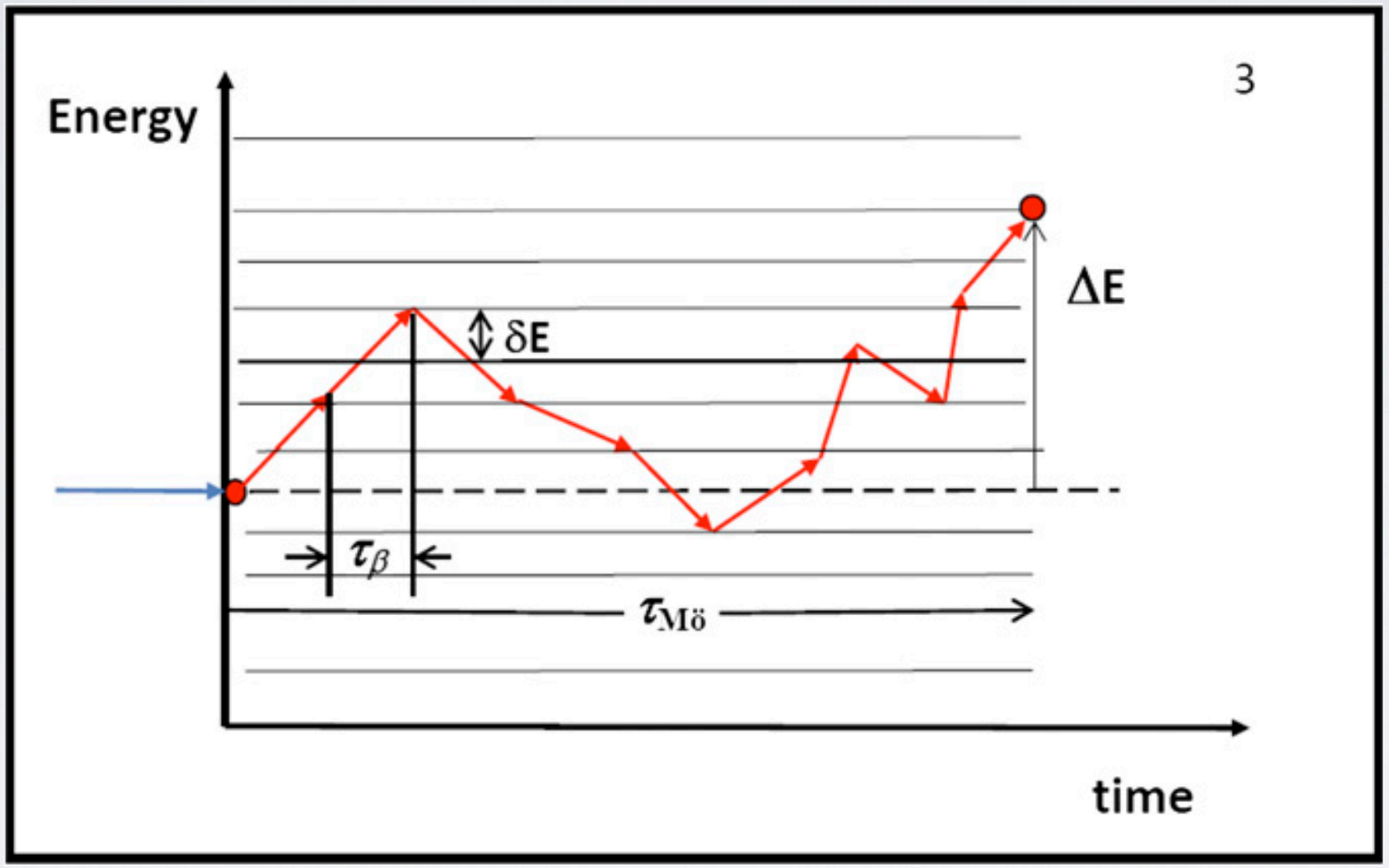


Energy landscape model

"Heterogeneous" QENS spectrum composed of many « Mössbauer lines »

No qualitative distinction between elastic and inelastic scattering

During its flight through the sample, the neutron wave packet records the net energy transition of the system from the initial energy level E to the final level E+ΔE.



- The description is essentially qualitative
- The neutron is considered as a passive probe
- Momentum transfer is not considered

The role of momentum transfer during incoherent neutron scattering is explained by the energy landscape model

Hans Frauenfelder[a,1], Robert D. Young[b], and Paul W. Fenimore[a,1]

PNAS 114, 5130 (2017).

- The neutron is an active probe : "Local heating" of the sample due to the momentum transfer.
- But: Momentum and energy transfer are not connected through scattering kinematics.

Franck–Condon picture of incoherent neutron scattering

Gerald R. Kneller[a,b,1]

PNAS, 115, pp. 9450 2018.

Based on Wick's "kicked" Hamiltonian, obtain an energy landscape formulation of neutron scattering that is based on quantum mechanical scattering theory

Use the complete sets of energy eigenstates of $\hat{H}$ and $\hat{H}'(\mathbf{q})$ to obtain a spectral representation of the intermediate scattering function,

$$F_s(\mathbf{q},t) = \frac{1}{Z}\sum_{m,n}\langle\phi_m|e^{-\beta\hat{H}}|\phi'_n(\mathbf{q})\rangle\langle\phi'_n(\mathbf{q})|e^{it\hat{H}'(\mathbf{q})/\hbar}e^{-it\hat{H}/\hbar}|\phi_m\rangle$$

$$= \frac{1}{Z}\sum_{m,n} e^{-\beta E_m} e^{i(E'_n-E_m)/\hbar}\left|\langle\phi'_n(\mathbf{q})|\phi_m\rangle\right|^2$$

Eigenvalues and eigenfunctions for the Hamiltonians

"Energy landscape"=energy spectrum

$$\hat{H}\phi_n(\mathbf{p}) = E_n\phi_n(\mathbf{p})$$
$$\hat{H}'(\mathbf{q})\phi_n(\mathbf{p}+\hbar\mathbf{Q}) = E_n\phi_n(\mathbf{p}+\hbar\mathbf{Q})$$

H′ : same energies, but shifted wave functions

→ **Momentum transfer dependent probabilities for neutron scattering induced energy transitions**

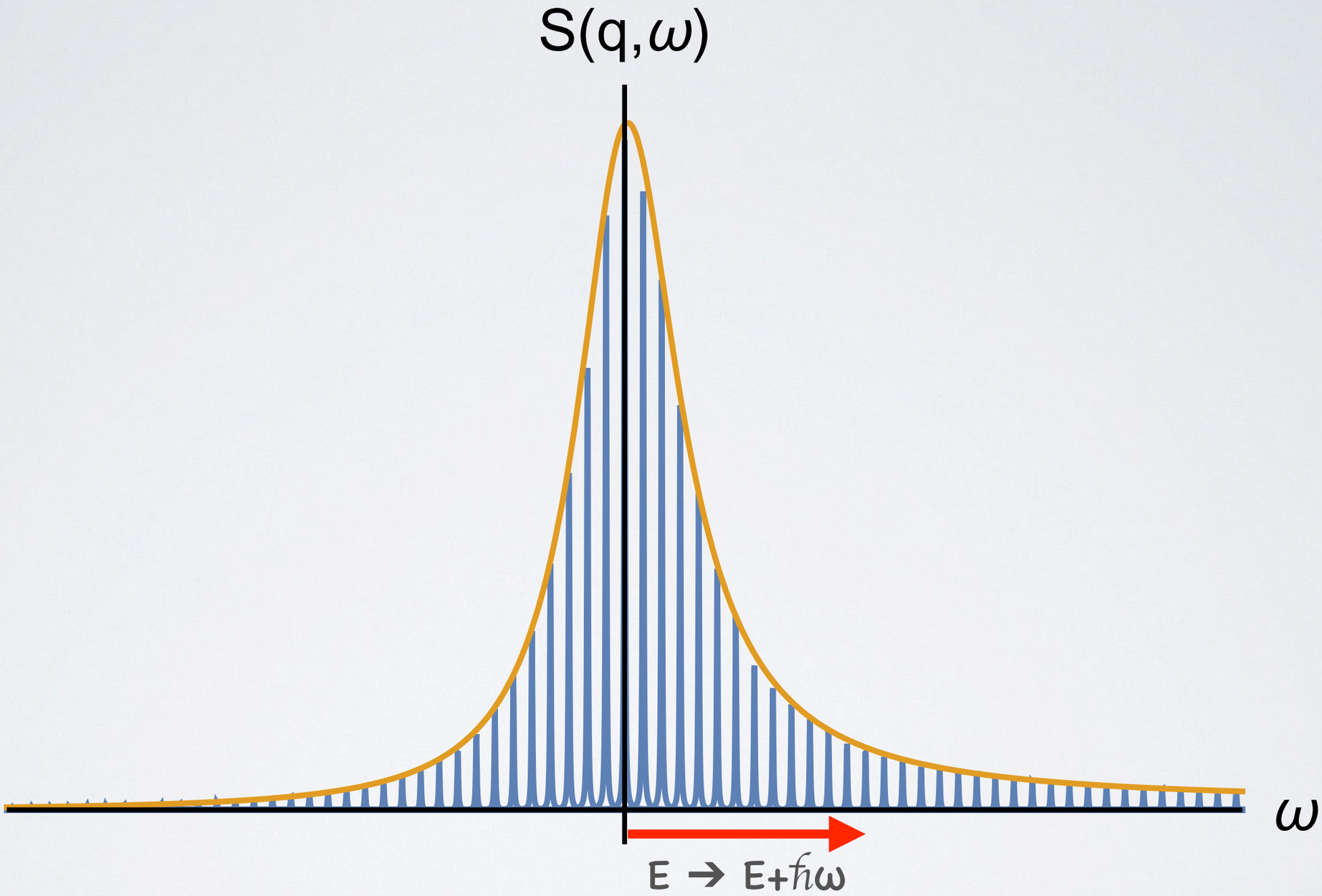
$$p_{mn}(\mathbf{q}) = \left| \int d^{3N}p\, \phi_n^*(\mathbf{p}+\hbar\mathbf{Q})\phi_m(\mathbf{p}) \right|^2$$

Franck-Condon form

→ Line spectrum for the dynamic structure factor

$$F_s(\mathbf{q},t) = \frac{1}{Z}\sum_{m,n} e^{-\beta E_m} e^{it(E_n-E_m)/\hbar} p_{mn}(\mathbf{q})$$
$$S_s(\mathbf{q},\omega) = \frac{1}{Z}\sum_{m,n} e^{-\beta E_m} p_{mn}(\mathbf{q})\delta\left(\omega - [E_n - E_m]/\hbar\right)$$

"Mößbauer" line spectrum for S(q,ω)



From the Brockhouse lecture

SLOW NEUTRON SPECTROMETRY

Theories of the physics of condensed matter involve the most basic aspects of modern physics: the principles of conservation (energy, linear momentum etc.), the chemical elements in various ionic forms, electrons, neutrons, quantum mechanics. Implementations for a particular substance in a particular setting, usually involve drastic approximations if the required quantum statistical calculations are to be possible. Happily, because the nuclear and magnetic interactions between the neutron and atom are (in some sense) weak, the very good "first Born approximation" is applicable, and the neutrons are effectively "decoupled" from the dynamics of the scattering system which can be considered in isolation. The neutron, in being scattered, "causes" transitions between the quantum states of the scattering system but does not change the states.

Detailed balance and recoil

It follows from the symmetry properties of the transition properties

$$p_{mn}(\mathbf{q}) = p_{nm}(-\mathbf{q})$$

that the detailed balance relations are fulfilled

$$F_s(\mathbf{q}, t) = F_s(-\mathbf{q}, -t + i\beta\hbar)$$
$$S_s(\mathbf{q}, \omega) = e^{\beta\hbar\omega} S_s(-\mathbf{q}, -\omega)$$

The scattering kinematics is reflected in the recoil moment

$$\int_{-\infty}^{+\infty} d\omega\, \omega S_s(\mathbf{q}, \omega) = \frac{\hbar q^2}{2m}$$

Re-interpretation of the Van Hove function

$$G_s(\mathbf{r},t) = \frac{1}{(2\pi)^3}\int d^3q\, e^{-i\mathbf{q}\cdot\mathbf{r}} F_s(\mathbf{q},t)$$

The standard form

$$= \int d^3r'\, \langle \delta(\mathbf{r}-\mathbf{r}'+\hat{\mathbf{r}}_1(0))\delta(\mathbf{r}'-\hat{\mathbf{r}}_1(t))\rangle$$

$$G_s(\mathbf{r},t) = \frac{1}{Z}\sum_{m,n} e^{-\beta E_m} G_{m\to n}(\mathbf{r},t)$$

The Franck-Condon form

Defining the overlap integrals

$$\psi_m(\mathbf{R},t) = \phi_m(\mathbf{R})e^{-iE_m t/\hbar}$$

$$T_{m\to n}(\mathbf{r},t) = \int d^3r_2 \ldots d^3r_N\, \psi_n^*(\mathbf{r},\mathbf{r}_2,\ldots,\mathbf{r}_N,t)\psi_m(\mathbf{r},\mathbf{r}_2,\ldots,\mathbf{r}_N,t).$$

The G-coefficients are given by the correlation integrals

$$G_{m\to n}(\mathbf{r},t) = \int d^3r\, T_{m\to n}(\mathbf{r}+\mathbf{r}',t)T^*_{m\to n}(\mathbf{r}',0)$$

Again the classical limit

The double sum

$$G_s(\mathbf{r},t) = \frac{1}{Z}\sum_{m,n} e^{-\beta E_m} G_{m\to n}(\mathbf{r},t)$$

translates into the single phase space average

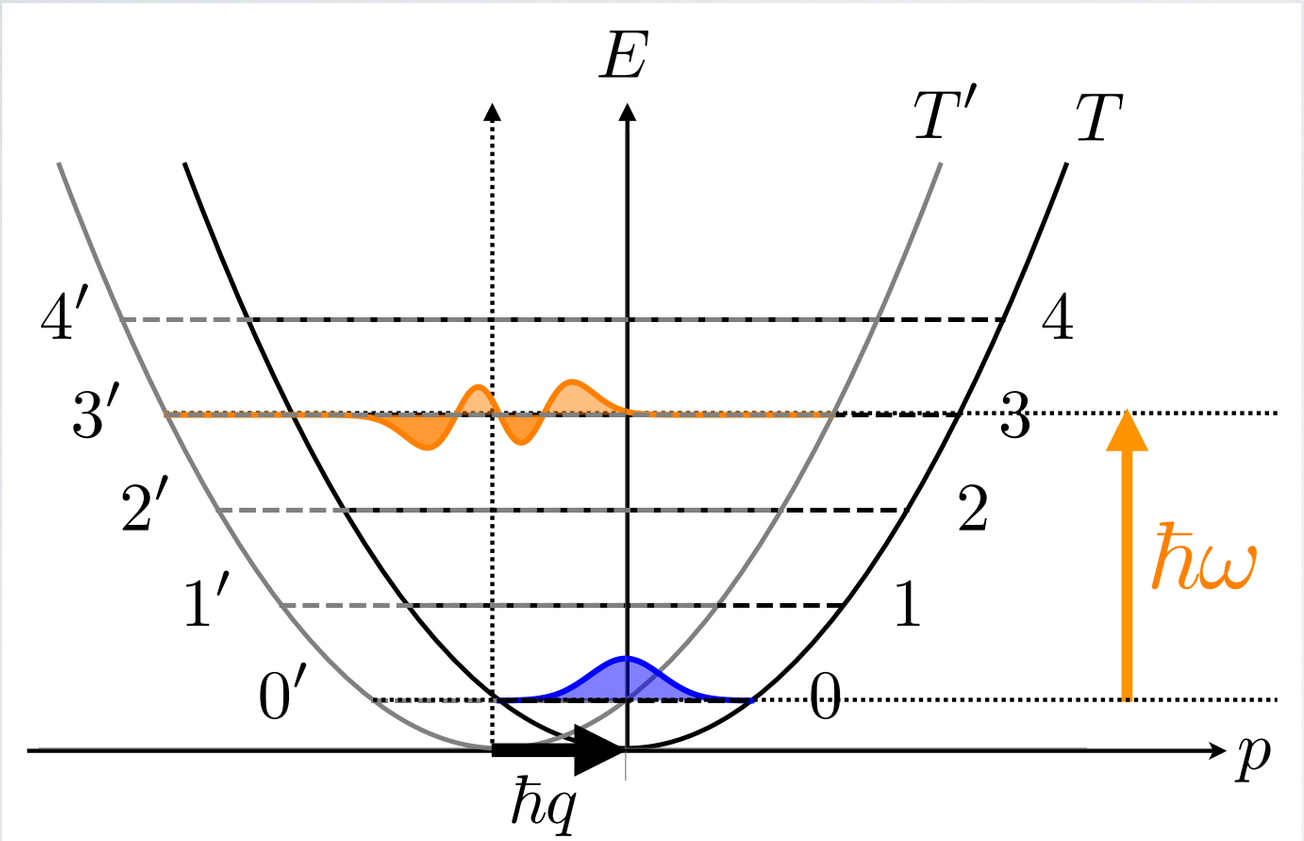
$$G_s^{(cl)}(\mathbf{r},t) = \frac{1}{Z_{cl}}\int d\Gamma(0) e^{-\beta H(\Gamma(0))}\, G^{(cl)}_{\Gamma(0)\to\Gamma(t)}(\mathbf{r},t)$$

where $G^{(cl)}_{\Gamma(0)\to\Gamma(t)}(\mathbf{r},t) = \delta(\mathbf{r} - [\mathbf{r}_1(t) - \mathbf{r}_1(0)])$

are transition "certainties", since the transition $\Gamma(0) \to \Gamma(t)$ is deterministic.

An analytical example - the harmonic oscillator

Wave functions for the 0→3′ transition



T V

$$E = \frac{p^2}{2m} + \frac{1}{2} m \Omega^2 x^2$$

$$p_{mn}(q) = e^{-\frac{y^2}{4}} (-1)^{m+n} L_m^{(n-m)}\left(\frac{y^2}{4}\right) L_n^{(m-n)}\left(\frac{y^2}{4}\right)$$

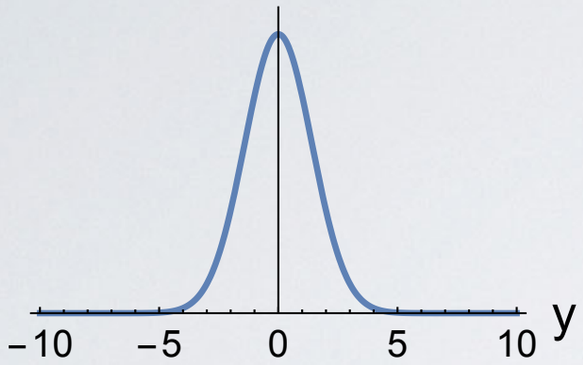
Transition probabilities

$$y(q) = \sqrt{\frac{2\hbar}{M\Omega}} q$$

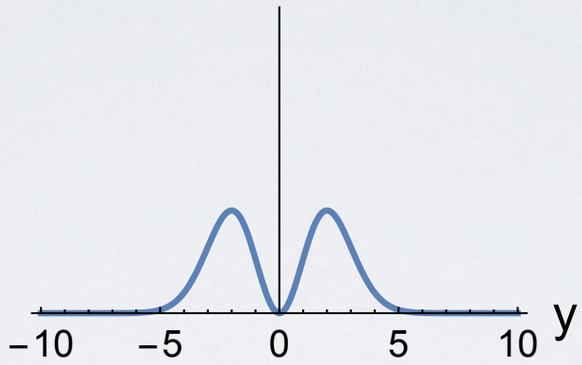
Dimensionless momentum transfer

Some transition probabilities as a function of momentum transfer

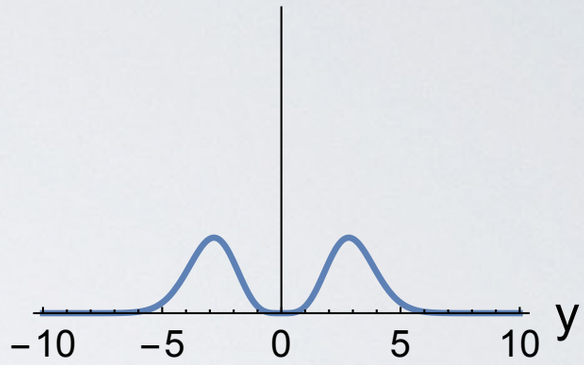
0→0



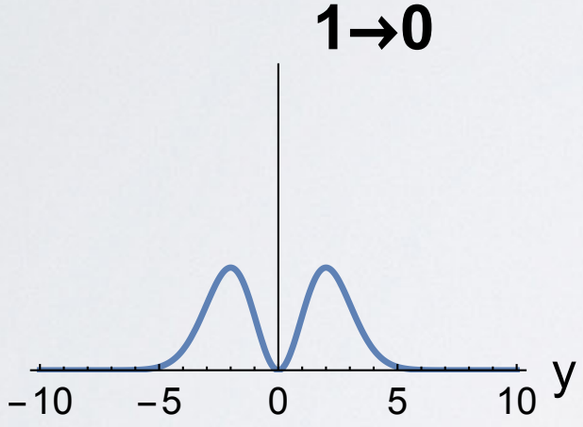
0→1



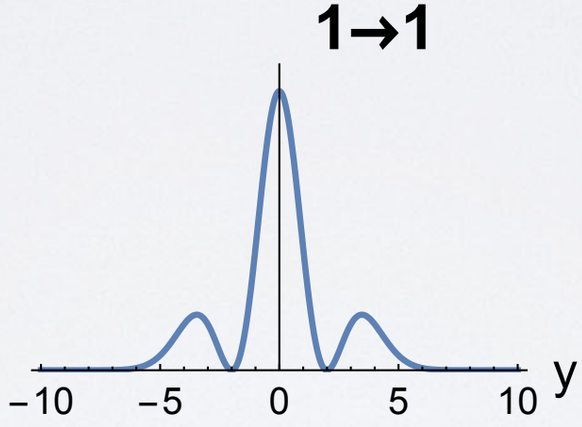
0→2



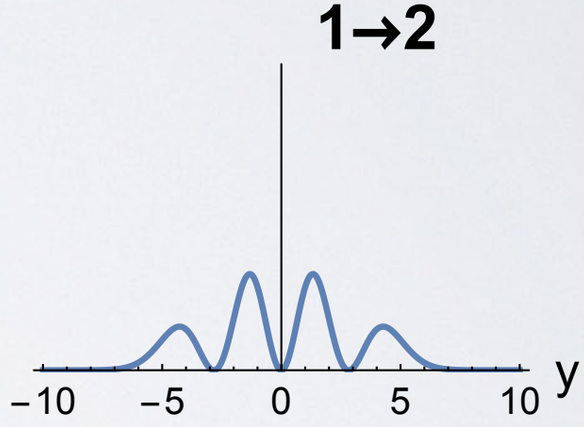
1→0



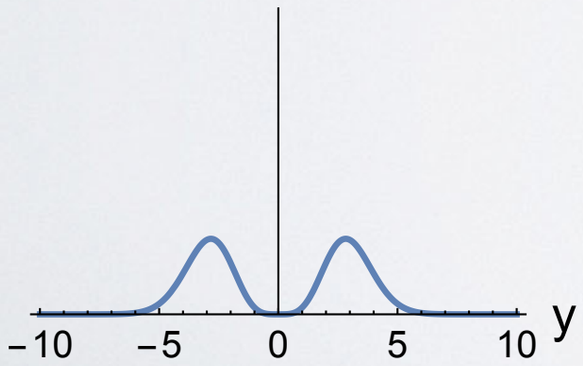
1→1



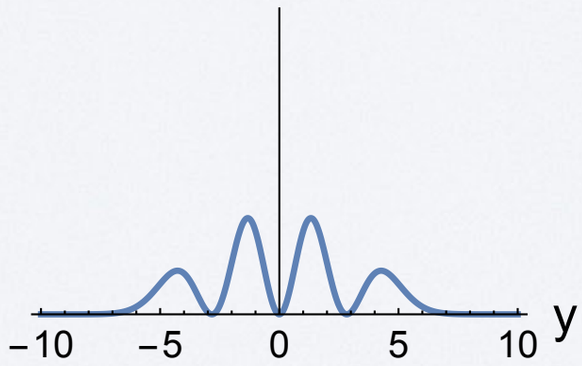
1→2



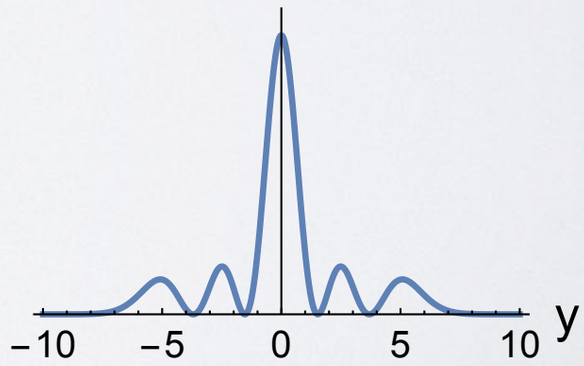
2→0



2→1



2→2



Intermediate scattering function

$$F_s(q,t) = \frac{1}{Z}\sum_{m,n} e^{-\beta\hbar\Omega(m+1/2)} e^{i(n-m)\Omega t} p_{mn}(q)$$

Textbook result (c.f. Lovesey)

$$F_s(q,t) = e^{i\frac{y(q)^2}{4}\left(\sin(\Omega t)+i(1-\cos(\Omega t))\coth\left(\frac{\beta\Omega\hbar}{2}\right)\right)}$$

Dynamic structure factor

$$S_s(q,\omega) = \frac{1}{Z}\sum_{m,n} e^{-\beta\hbar\Omega(m+1/2)}\delta\big(\omega - [n-m]\Omega\big)\, p_{mn}(q)$$

Complex systems

The quantum state variables of complex systems are quasi-continuously distributed.

$$\hat{H}|\phi(X)\rangle = E(X)|\phi(X)\rangle$$

X is a set of variables describing the state of the system

$$\langle\phi(X')|\phi(X)\rangle = \begin{cases} 1 & \text{if } X = X', \\ 0 & \text{otherwise} \end{cases}$$

Continuous counting $$m \to dm = \rho(X)d^f X$$

ρ(X) is the density of (quantum) states of the system.

For X=E ρ(E) is the (quantum) "energy landscape".

Scattering function and energy landscape

$$S_s(\mathbf{q},\omega) = \hbar \int dE\, W_{\mathrm{eq}}(E) W(E+\hbar\omega|E;\mathbf{q}).$$

$$W_{\mathrm{eq}}(E) = \rho(E)\frac{\exp(-\beta E)}{Z}$$

$$W(E'|E;\mathbf{q}) = \rho(E')|a(E'|E;\mathbf{q})|^2$$

$$a(E'|E;\mathbf{q}) = \int d^{3N}p\, \phi^*(\mathbf{p}+\hbar\mathbf{Q},E')\phi(\mathbf{p},E)$$

The scattering functions are determined by the transition probabilities and the density of states, ρ(E).

The dynamic structure factor is a continuous function in ω and for X=E it becomes a thermally weighted transition probability for transitions from $E \rightarrow E' = E + \hbar\omega$

$$S_s(\mathbf{q},\omega) = \hbar \int dE\, W_{\mathrm{eq}}(E) W(E + \hbar\omega | E; \mathbf{q}).$$

A priori continuous transition from elastic to quasielastic scattering at finite instrumental resolution.

Asymptotic analysis of QENS/ENS

GR Kneller, PNAS, 115, pp. 9450 2018.

Mathematical argument for a combined description of elastic and quasielastic scattering (e.g. protein in powders)

Generic form of F(q,t) (q-dependence omitted)

$$F_s(t) = EISF + (1 - EISF)\,R(t)$$

$$\lim_{t\to\infty} \frac{F(\lambda t)}{F(t)} = 1\ (\lambda > 0) \Rightarrow \hat{F}(s) \overset{s\to 0}{\sim} F(1/s)/s.$$

Application of a Tauberian theorem (Hardy-Littlewood & Karamata)

Resulting form for the dynamic structure factor

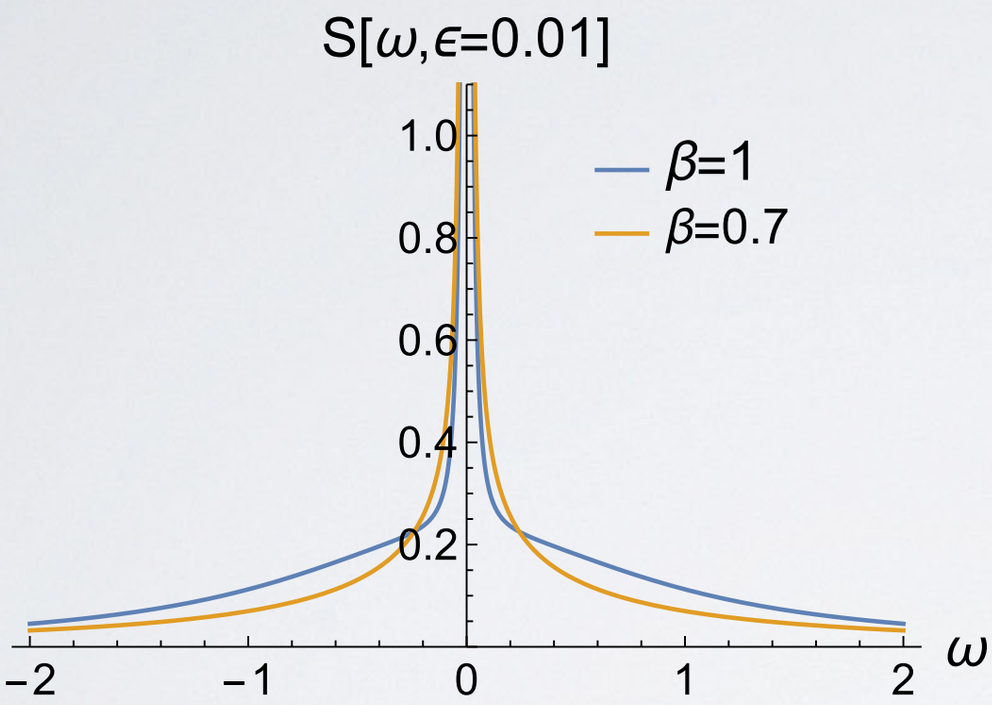
$$S_s(\omega) \overset{\omega\to 0}{\sim} \lim_{\epsilon\to 0+} \frac{1}{\pi}\Re\left\{\frac{F_s(1/(i\omega+\epsilon))}{i\omega+\epsilon}\right\}$$

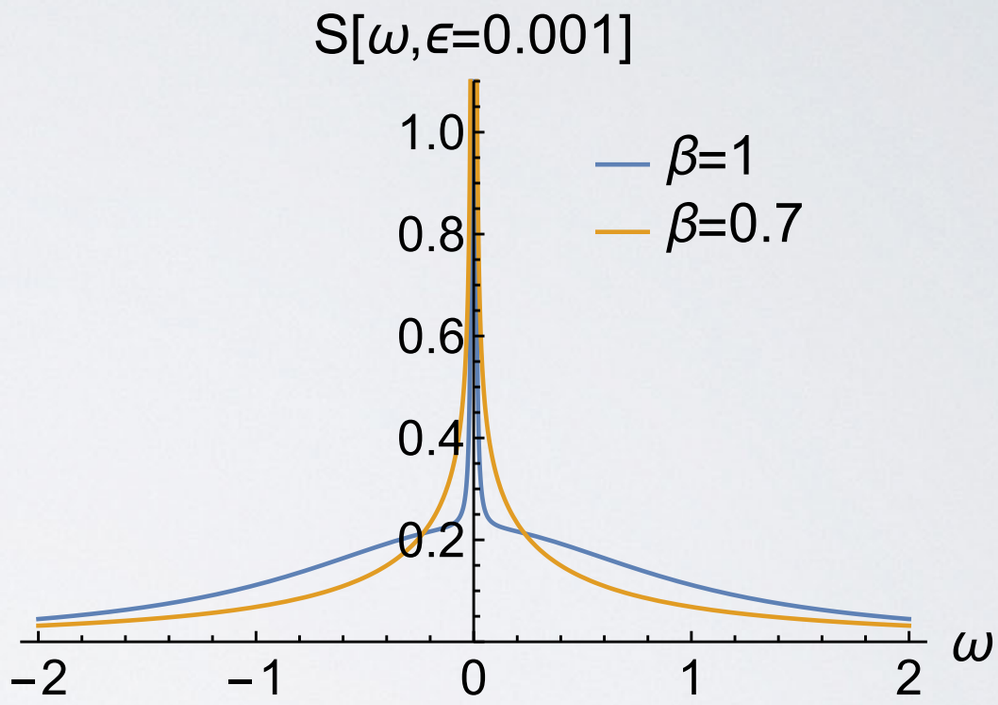
Here ε denotes finite instrumental resolution

$\hat{F}(s) = \int_0^\infty dt\, e^{-st} F(t)$ Laplace transform of F(t)

Fusion of elastic and quasielastic line

$$R(t) = E_\beta(-(|t|/\tau)^\beta), \quad 0 < \beta \leq 1$$





Here ϵ is a resolution parameter and $\beta = 1$ corresponds to exponential relaxation.

For slow power law relaxation $\propto t^{-\beta}$, with $0 < \beta < 1$, the elastic and quasielastic line are fused and the EISF must be fitted together with the parameters β, τ of the $R(t)$.

Use here the symmetrized correlation function

$$F(t) \approx F^{(+)}(t) = \frac{F(t + i\beta\hbar/2)}{F(i\beta\hbar/2)}$$

since the real symmetric part of the ISF

determines its asymptotic behavior:

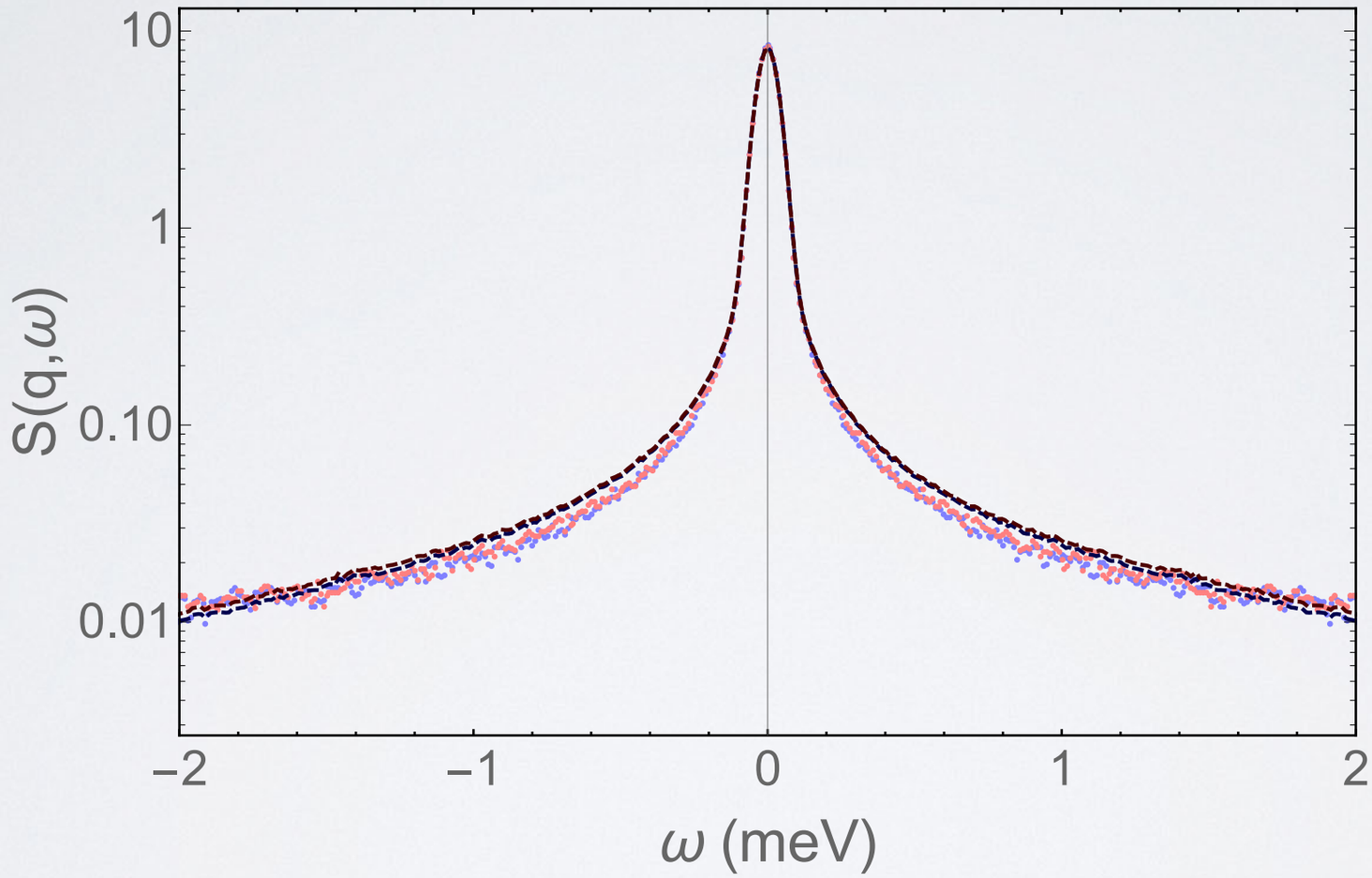
$$F^{(I)}(t) = -\tan\left(\beta\hbar\frac{d}{dt}\right) F^{(R)}(t)$$

odd

even

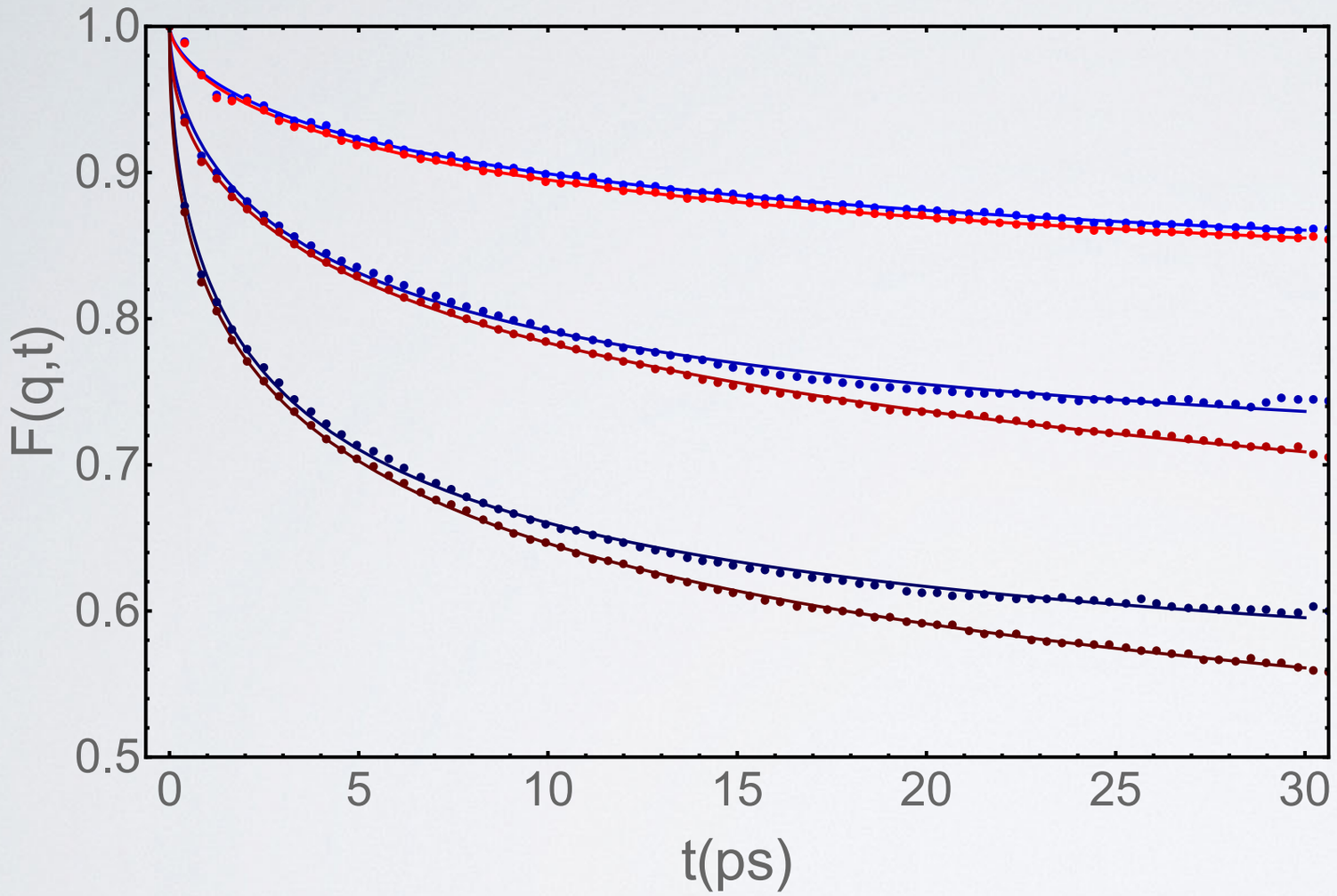
Application to Huperzine-A inhibited human Acetylcholine Esterase

M. Saouessi, J. Peters, and G. R. Kneller, *J Chem Phys*, vol. 150, p. 161104, Apr. 2019.



Experimental symmetrize dynamic structure factor for free and HupA-inhibited hAChE (blue and red dots, respectively) and corresponding reconstruction from the models (blue and red dashed line). The differences between free and HupA-inhibited hAChE are almost invisible.

Fitted intermediate scattering functions of resolution-deconvolved spectra reveal differences between free and HupA-inhibited hAChE



$$F_s(t) = EISF + (1 - EISF)R(t)$$

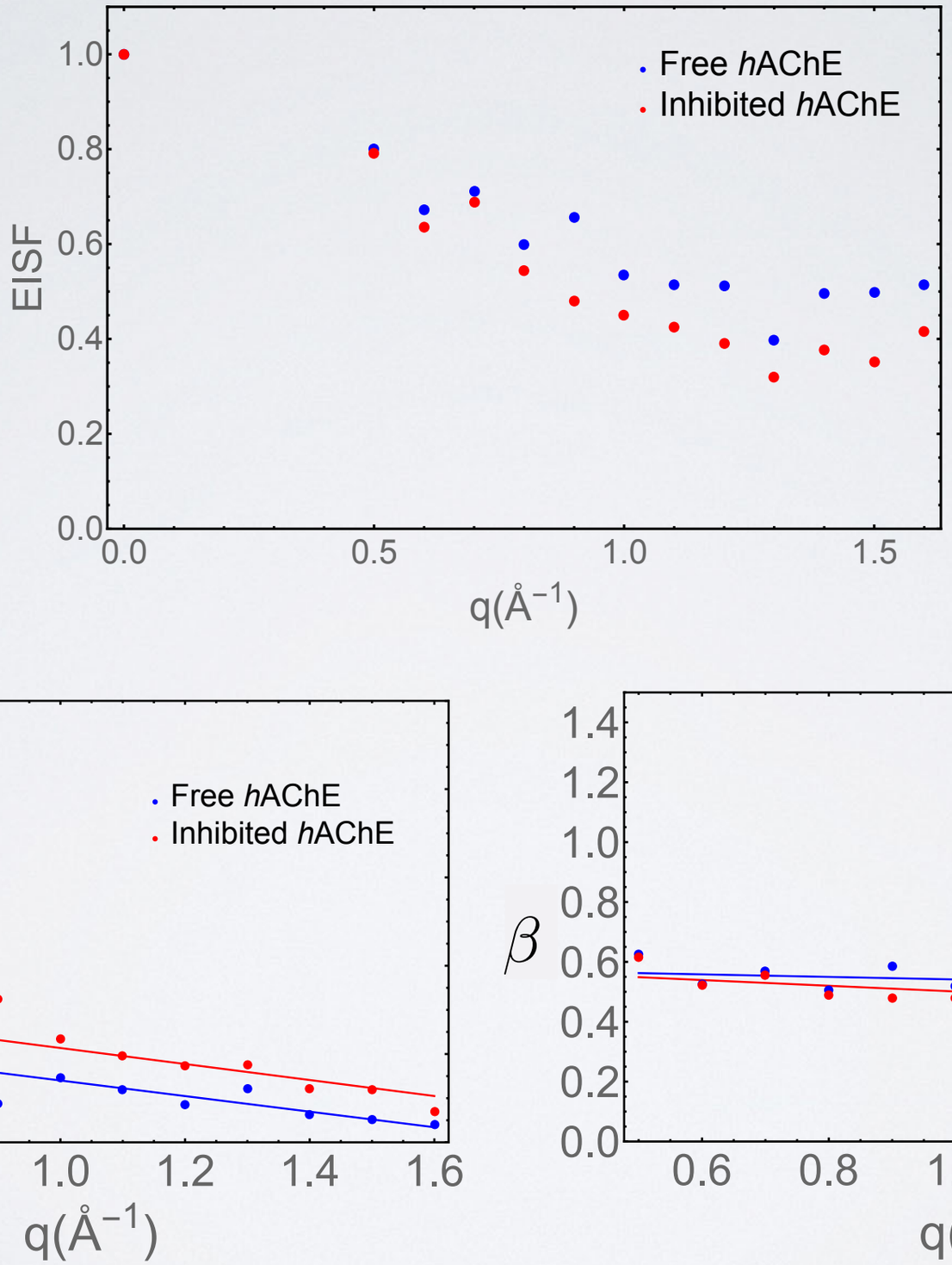
$$R(t) = E_\beta(-(|t|/\tau)^\beta), \quad 0 < \beta \leq 1$$

The EISF is a fit parameter, as β and τ.

Blue : free hAChE

Red : HupA-inhibited hAChE

Fit parameters



Q-dependent, semiclassical model for QENS

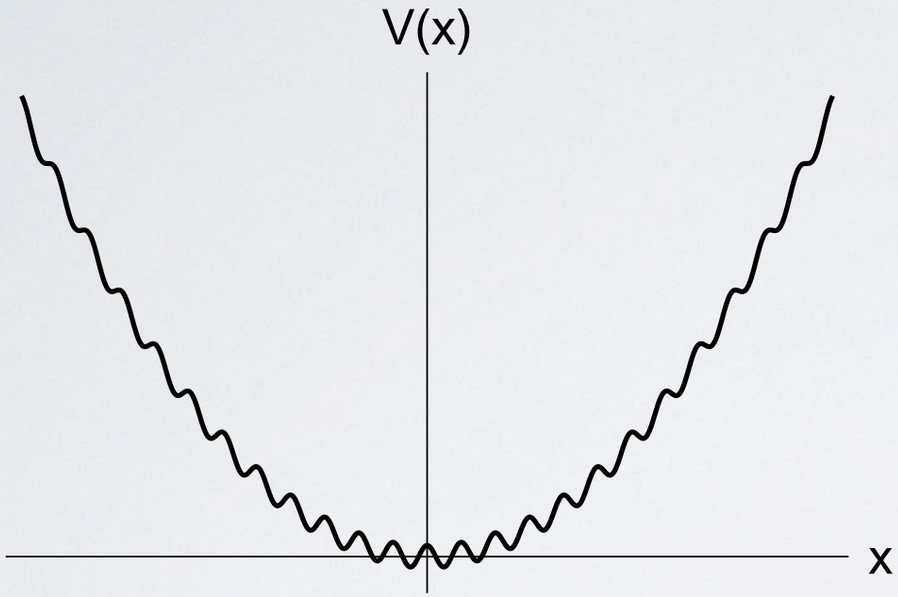
Assume $F^{(cl)}(t) \approx F^{(+)}(t)$

Introduce the q-dependent dynamical variable

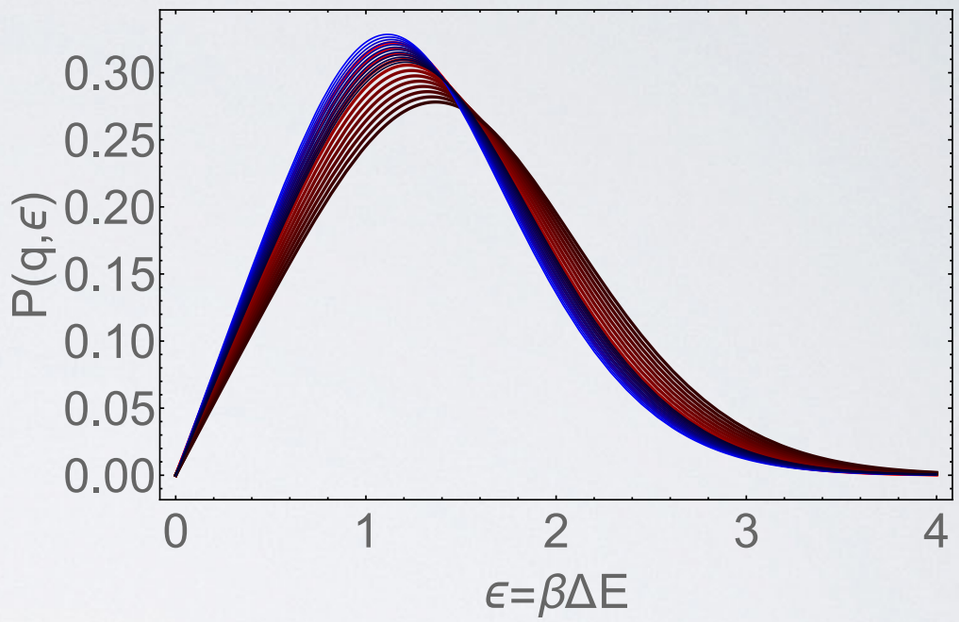
$$x(t) \equiv e^{i\mathbf{q}.\mathbf{R}(t)} - \left\langle e^{i\mathbf{q}.\mathbf{R}(t)} \right\rangle$$

which diffuses in a "rough' harmonic potential.

Rough harmonic potential and energy barrier distribution



Barrier distribution for different q-values



Use Zwanzig's theory of diffusion in a rough potential (PNAS 1988)

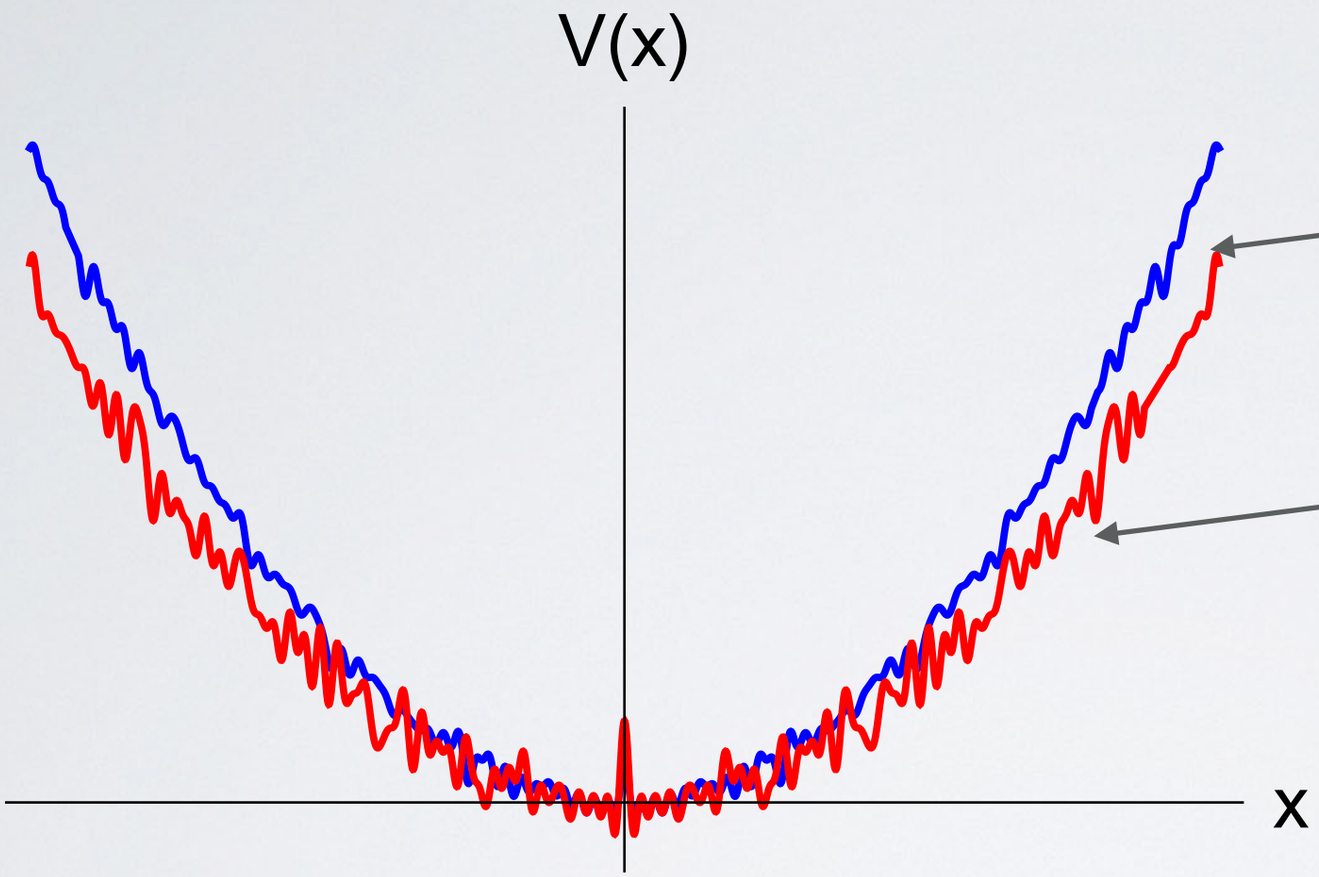
$$D = D_0 e^{-\epsilon^2}, \quad \epsilon = \frac{\Delta E}{k_B T}$$

and that $D_0 = \langle x^2 \rangle \eta_0$ in a harmonic potential, to infer that $\lambda = \eta/\eta_0 = \exp(-\epsilon^2)$

$$p(\lambda) = \frac{1}{\pi} \frac{\sin(\pi\alpha)}{\lambda(\lambda^\alpha + \lambda^{-\alpha} + 2\cos(\pi\alpha))}$$

$$P(\epsilon) = \frac{1}{\pi} \frac{2\epsilon \sin(\pi\alpha)}{\exp(\alpha\epsilon^2) + \exp(-\alpha\epsilon^2) + 2\cos(\pi\alpha)}$$

The effect of ligand binding



Widening of V(x):
"mode softening"

"Roughening" of the
potential surface

Conclusions

- Both, trajectory and energy landscape-oriented interpretations of neutron scattering from condensed matter systems can be obtained by trading the neutron as an active probe.
- In the "trajectory picture" the intermediate scattering function is written as a path integral, containing a forward and a backward path, where the latter is initiated with a momentum transfer "kick".
- In the "Franck-Condon picture" the neutron induces transitions of the system between different energy levels/quantum states and the corresponding momentum transfer-dependent transition probabilities determine the measured intensities.
- The FC picture leads to a new physical interpretation of the quantum Van Hove correlation functions and in their classical limit.
- The FC picture suggests a description of QENS, which treats elastic and quasielastic scattering as parts of one and the same spectrum.

Outlook - Energy landscape entropy

Based on the probabilistic interpretation of the dynamic structure factor

$$S_s(\mathbf{q},\omega) = \hbar \int dE\, W_{\mathrm{eq}}(E) W(E+\hbar\omega|E;\mathbf{q}).$$

one can define a Shannon entropy for the neutron scattering explored energy landscape of proteins

$$H(\mathbf{q}) = -\int_{-\infty}^{+\infty} d\omega\, S_s(\mathbf{q},\omega) \log\Big(S_s(\mathbf{q},\omega)\Big)$$

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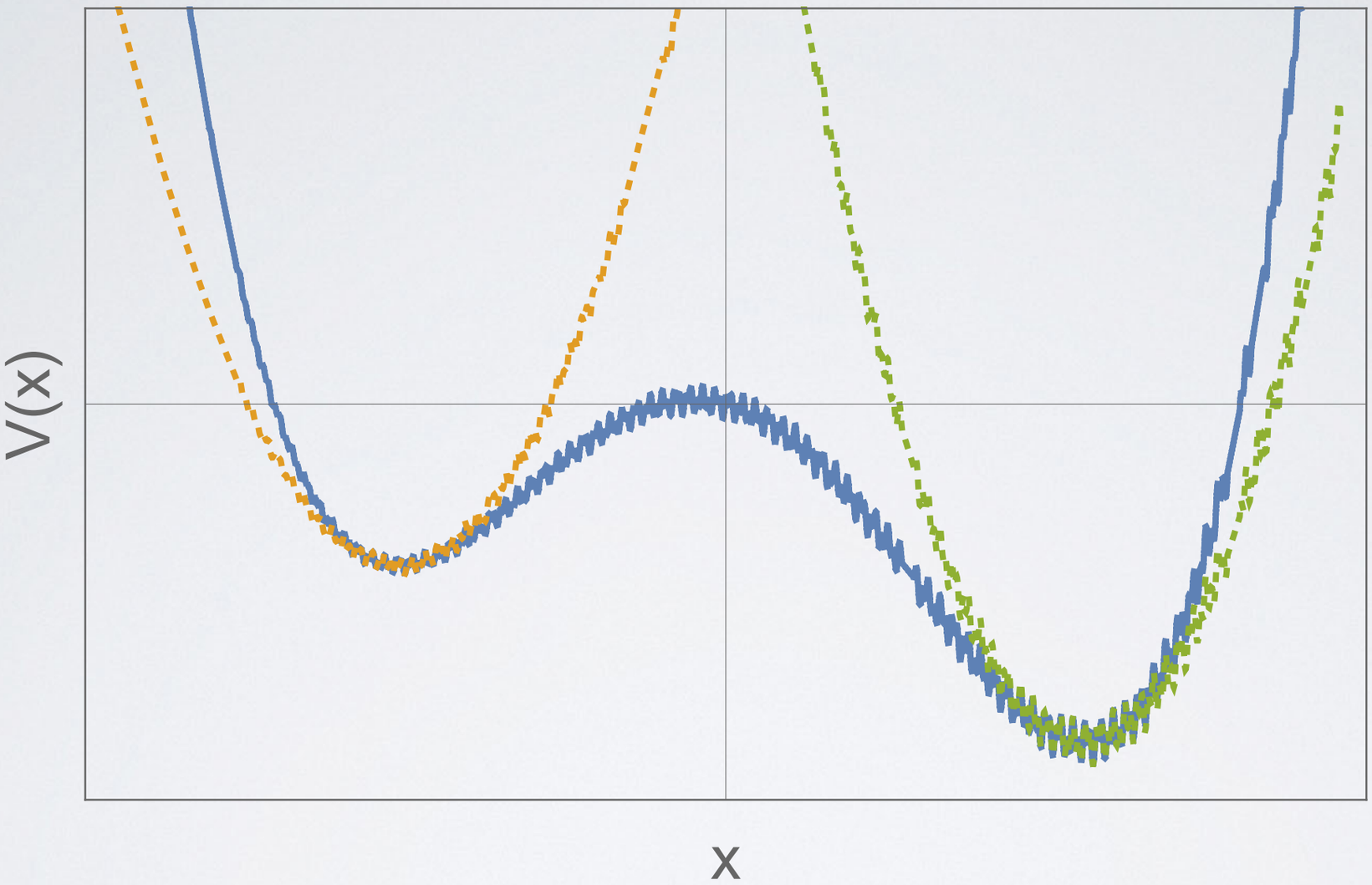
Possible explanation of the "mode softening"

hAChE ⟷ hAChE+HupA (sec. to min.)



The widened, softer potential is effectively a double-well potential. The neutrons see a superposition of the fast ps dynamics in the two wells and motional amplitudes determined by the envelope potential.