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THE DETERMINANTS OF INTEREST RATES: OLD CONTROVERSIES REOPENED

A Simple Account of the Behavior of Long-Term Interest Rates

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To a first approximation, long-term interest rates behave like short-term interest rates. For example, the yields on twenty-year Treasury bonds and on one-month Treasury bills tend to peak and to bottom out together. Thus people often speak of "the level of interest rates" without specifying maturity.

The spread between long and short rates tends to be unusually small or even negative when short rates are high relative to the experience of the last few years. Franco Modigliani and Richard Sutch (1967) showed that the relation between long and short rates can be well described by expressing the long rate as a five-year distributed lag of short rates, with the coefficients summing to about one and with substantial weight on the current short rate. Recent experience upholds this characterization except that the distributed lag has become shorter (Albert Ando and Arthur Kennickell, 1983). Equivalently, the spread between long and short rates is well explained by current and lagged short rates, with approximately equal and opposite coefficients on the current rate and the sum of lagged rates.

This moving average relation could be consistent with the simple expectations theory of the term structure, if investors look to the recent past to form expectations about future interest rates. Whether such expectations are rational depends on the time-series properties of short-term interest rates. Depending on the policy regime and its implications for the movements of short rates, the observed distributed lag might correspond to a rational expectations theory of the term structure, or a theory of overreaction or underreaction of long rates to short rates, relative to the predictions of the rational expectations model. Experimental psychologists, such as Amos Tversky and Daniel Kahneman (1974), claim to have shown that people tend to overreact in their expectations to evidence which seems superficially to be relevant, even after experience should have convinced them otherwise. This suggests that there might be policy regimes where the long rate overreacts to temporary movements in short rates. Of course, any such "overreaction" might also be reconciled with the theory of finance if certain covariances change with the short rate.

A look at the data suggests an abrupt policy shift starting with the Fed's new operating procedures in October 1979. We concentrate here on the policy regime which prevailed between the 1951 Treasury accord and 1979. Modigliani and Shiller (1973) claimed that, for the early part of the period, the observed distributed lag was approximately consistent with the time-series properties of the short rate given a simple expectations model, and Thomas Sargent (1979) was unable to reject this hypothesis with a likelihood ratio test in a vector autoregression. However, more recent work has cast doubt on the notion that the simple rational expectations model of the term structure is adequate even as a first approximation to the behavior of interest rates. It was shown by Shiller (1979) that when long-term interest rates are unusually high relative to short rates, they then tend to fall rather than rise as predicted by the expectations theory. Our 1983 article with Kermit Schoenholtz showed that when six-month bill rates are higher

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than three-month bill rates, there is no tendency for the three-month bill rate to rise subsequently. Lars Hansen and Sargent (1981) were able to reject the rational expectations theory at the 0.5 percent level with a likelihood ratio test on postwar U.S. data when an additional restriction involving the current long-term interest rate was added to Sargent's earlier formulation.

These results might be summarized as finding that the behavior of long-term interest rates is dominated by a "risk premium" which is so variable as to swamp out expectations in determining the slope of the term structure. The phrase risk premium has been defined in various ways in the term structure literature. We turn next to a discussion which will clarify the relations among these definitions. This enables us to state more formally the hypotheses that long rates overreact or underreact to short rates, and it provides a framework in which we characterize interest rate behavior.

I. "Well-Tempered" Definitions of Risk Premia

We make use here of approximations to holding-period yields and forward rates which are obtained by linearizing the exact expressions around the coupon rate on a long-term bond. These approximations were developed by ourselves and Schoenholtz. We also investigated their accuracy. Without such preliminary linearization, small differences among alternative definitions of risk premia, arising from nonlinear functions in expectations, make it difficult to consider the definitions within a single framework. The analogy with the approximation which allows a musical instrument to be tuned to more than one key at a time leads us to call our system a well-tempered one.

We chose our definitions to facilitate comparison with bond yields as commonly quoted. Bonds issued with less than a year to maturity commonly carry no coupons, but longer-term bonds generally pay coupons which bring their sale price near par. It is natural then to define the five-year ahead, ten-year forward rate, for example, as the yield on a ten-year coupon bond to be purchased at par five years hence. Such an asset can be constructed today as a portfolio of bonds with maturities up to fifteen years. Similarly, the five-year holding return on a fifteen-year bond is the yield to maturity on buying the fifteen-year bond, receiving its coupons, and selling it five years hence (when it is a ten-year bond). The fifteen-year holding yield on a five-year bond is the yield to maturity on an investment in three consecutive five-year coupon bonds, reinvesting principal (i.e., rolling over the five-year bonds) but receiving coupons.

The linear approximation to the j-period holding yield on an i-period bond is

$$(1)\quad h_t^{(i,j)} = \left(D_i R_t^{(i)} - \left(D_i - D_j\right) R_{t+j}^{(i-j)}\right)/D_j, \qquad 0 < j \le i$$

$$(2)\quad h_t^{(i,j)} = \left(1/D_j\right)\left[\sum_{k=0}^{(j-i)/i} \left(D_{ki+i} - D_{ki}\right) R_{t+ki}^{(i)}\right] \qquad 0 < i \le j,\ j/i \text{ integer}$$

The linear approximation to the n-period ahead m-period forward rate is

$$(3)\quad f_t^{(n,m)} = \left(D_{m+n} R_t^{(m+n)} - D_n R_t^{(n)}\right) / \left(D_{m+n} - D_n\right), \quad 0 < m, 0 \le n$$

where $R_t^{(i)}$ = yield to maturity on an i-period bond; $D_i = (1-g^i)/(1-g)$, $g = 1/(1+\bar{R})$, $\bar{R}$ = coupon rate. D_i is the "duration" of an i-period bond selling at par with coupon $\bar{R}$, as defined originally by Frederick Macaulay (1938). Duration is intended as a better measure than maturity of how "long" a bond is. It takes account of the fact that bonds with coupons derive much of their value from payments which are made earlier than maturity. Thus for bonds with coupons, $D_0 = 0$, $D_{i+1} - D_i = g^i$, so $D_i < i$ for $i > 1$. For pure discount bonds, $\bar{R} = 0$ and duration and time to maturity are the same.

The simple expectations theory of the term structure, with no allowance for risk, equates

$E_t h_t^{(i,j)}$ or $E_t h_t^{\prime(i,j)}$ with $R_t^{(j)}$, and $f_t^{(n,m)}$ with $E_t R_{t+n}^{(m)}$. Risk premia are deviations from this theory, which can be written either as differences between expected holding returns and yields, or as differences between forward rates and expected spot rates. We denote the former as $\phi^{(i,j)}(j \leq i)$ or $\phi^{\prime(i,j)}(j \geq i)$, and the latter as $\psi^{(n,m)}$. Then we have the holding-period risk premium:

$$\phi_t^{(i,j)} = E_t h_t^{(i,j)} - R_t^{(j)}, j \leq i \tag{4}$$

the rolling risk premium:

$$\phi_t^{\prime(i,j)} = R_t^{(j)} - E_t h_t^{\prime(i,j)}, j \geq i \tag{5}$$

and the forward rate risk premium:

$$\psi_t^{(n,m)} = f_t^{(n,m)} - E_t R_{t+n}^{(m)}. \tag{6}$$

ϕ, ϕ' and ψ all appear in the existing literature on the term structure. Our well-tempered formulation allows us to derive simple linear relationships among them. First, we can substitute (1) and (3) into (4) and (6) to show that

$$\phi_t^{(i,j)} = (D_i - D_j)\psi_t^{(j,i-j)}/D_j. \tag{7}$$

Second, we can rearrange equation (3) so that it expresses the j-period bond rate as a weighted average of forward rates of maturity i, with weights equal to those in equation (2). It is immediate that

(8)
$$\phi_t^{\prime(i,j)} = (1/D_j)\left[\sum_{k=0}^{(j-i)/i} (D_{ki+i} - D_{ki})\psi_t^{(ki,i)}\right],$$
$$0 < i \leq j$$

Finally, we can rearrange equation (1) so that it expresses the j-period bond rate at time t as a function of the i-period holding return on a j-period bond and the $(j-i)$-period bond rate at time $t+i$. By recursive substitution, we obtain the following expression:

$$\phi_t^{\prime(i,j)} = (1/D_j)\left[\sum_{k=0}^{(j-i)/i} (D_{ki+i} - D_{ki}) \times E_t \phi_{t+ki}^{(j-ki,i)}\right], 0 < i \leq j. \tag{9}$$

A natural interpretation of the notion that long rates overreact to short rates is that long bonds are a "good investment" when the short rate is high. In other words, the returns on long bonds over some holding period tend to be higher than those predicted by the expectations theory when the short rate is high: the holding period or rolling risk premium is positively correlated with the short rate.[1] In the next section we examine the relation between the one-month excess holding return on a twenty-year bond, and the one-month Treasury bill rate. We do not calculate the twenty-year excess return on a twenty-year bond, which includes the rolling risk premium, since we have only just over twenty years of data. However, we study the rolling risk premium indirectly by conducting an ARIMA analysis of the one-month bill rate.

II. The Behavior of Risk Premia

We can estimate ϕ_t by regressing the excess return $h_t^{(i,j)} - R_t^{(j)}$ on variables in the information set at t. The excess return is just $(D_i/D_j - 1)$ times the forward-spot rate difference $(f_t^{(j,i-j)} - R_{t+j}^{(i-j)})$, so equivalent results are obtained with this dependent variable.

[1] N Gregory Mankiw and Lawrence Summers (1983) interpreted overreaction as the hypothesis that the long rate behaves according to the expectations model for a bond of shorter duration This definition is consistent with ours, in that if long rates overreact in Mankiw and Summers' sense, and if the time-series process for short rates is stationary, then the holding-period risk premium is positively related to the short rate The reverse is not necessarily true, however We note that incorrect duration, whether too short or too long, could never explain the observation that the slope of the term structure gives wrong signals about the future path of interest rates.

Reuben Kessel (1965) ran regressions of forward-spot rate differences at the short end of the term structure on the short interest rate, and concluded that the forward rate premium was positively related to the short yield. Such a correlation could be taken to mean that long interest rates overreact to short rates. However, our work with more recent data shows that the effect of the short rate is, if anything, negative. Using monthly data from 1955:1 to 1979:8, and regressing the excess one-month return on a twenty-year bond over a one-month bill on the one-month bill rate, we find a coefficient of -0.479 with standard error 0.766. But the short rate has very little explanatory power (R^2 is only 0.001); it is rather the spread between long and short rates which explains excess holding returns, with an R^2 of 0.014 and a significant coefficient of 3.095, larger than unity. This is a reflection of the perverse behavior of the slope of the term structure in predicting future interest rates.[2]

There has been an uptrend in interest rates since Kessel's sample. This suggests an alternative overreaction or underreaction hypothesis that risk premia may be explained in terms of the difference between the short rate and a moving average or distributed lag of short rates. In fact, our results so far would seem to suggest just this, since as we noted in the introduction the long-short spread which explains excess returns is itself well described as a distributed lag on short rates. For our data, the estimated distributed lag places a weight of -0.805 on the current short rate and $+0.878$ on a five-year Almon cubic polynomial lag of short rates.[3] These coefficients lead us to expect that the risk premium is high when the short rate is low relative to recent experience. Nevertheless, when the excess return is regressed directly on current and lagged short rates, the point estimates are statistically insignificant with t-statistics of only about 0.1. This evidence is not inconsistent with rational forecasting in the 1955–79 period. We note however that when the sample is extended to the end of 1982, the coefficient on the current short rate becomes negative and significant at the 9 percent level, while the sum of the lag coefficients is positive and significant at the 7 percent level. This could be taken to imply that long rates have *underreacted* to short rates.

Another way to examine this issue is to conduct an ARIMA analysis of the behavior of short rates. Shiller's volatility analysis suggested that nonstationarity of interest rates might be necessary to justify the behavior of long rates; we assumed this conclusion and used monthly data over the period 1955–79 to estimate an ARIMA (1,1,1) process for the one-month bill rate. This specification has the important advantage of being time consistent, that is independent of the measurement interval. It implies that the long-short spread under the rational expectations theory of the term structure should be a function of current and lagged short rates, with the influence of lagged short rates declining geometrically at a rate equal to the *MA* parameter, and with the sum of the coefficients on lagged short rates equal to the negative of the coefficient on the current short rate. We found that the likelihood function was very flat, but was maximized by the model $(1-0.950L)\Delta R_t=(1-0.975L)u_t$. With these parameter values the rational expectations model implies that in the distributed lag equation for the spread the coefficient of the current short rate should be -0.47 and the sum of the lagged coefficients should be $+0.47$, with a very slow decay within the distributed lag. The Modigliani-Sutch distributed lag is roughly consistent with this, but has a more highly negative coefficient on the current short rate. This suggests that the rolling risk premium tends to be high when the short rate is low relative to its recent history.

When the short rate and its distributed lag are included in a regression together with the long-short spread, we find that both become

[2]We note here the curious fact that excess returns of common stock over short debt also bear a significant positive relation to the long-short spread (Campbell, 1983). This observation suggests that risk premia on different assets move together

[3]The R^2 in this regression is 0.809; however, as Llad Phillips and John Pippenger (1979) have pointed out, the residuals from this type of equation are highly serially correlated so that "spurious correlation" may exaggerate the explanatory power of the regression

significant, and the coefficient on the spread triples. The fitted values in this regression look something like a multiple of the residuals from the distributed lag equation for the spread, suggesting that the significance of the current and lagged short rates is due to the regression's trying to purge the long-short spread of the component which is explained by current and lagged short rates. When the fitted value and residual from the spread equation are included separately, only the residual is significant. It has a coefficient of 10.336 with a standard error of 3.440, while the fitted value has coefficient 1.388 and standard error 1.676. Splitting the spread into fit and residual more than doubles the R^2 to 0.032. It is also the residual which in the 1955–79 sample accounts for the violation, noted by Shiller, of variance restrictions on holding period yields. When the sample is extended to 1982, however, both the fitted value and the residual explain excess holding returns and violate the variance restrictions.

We see then that holding-period and rolling risk premia have if anything been negatively related to short rates, suggesting that long rates if anything have underreacted to short rates. If long rates had been a distributed lag on short rates, with a somewhat larger coefficient on the current short rate and smaller coefficients on lagged short rates, then excess holding returns on long bonds would have been less predictable than they in fact were. But this sort of underreaction was not primarily responsible for the failure of the expectations theory of the term structure. The independent movement of the long rate also violated the restrictions of the theory. In the 1955–79 period, it was that smaller part of the spread between long and short rates which was *not* explained by current and lagged short rates that caused excess volatility in holding period yields and destroyed the predictive power of the term structure.

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