Michigan Math and Science Scholars: Art and Mathematics

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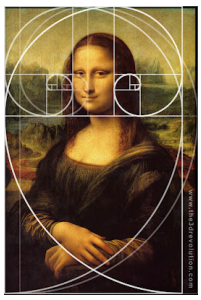
Abstract

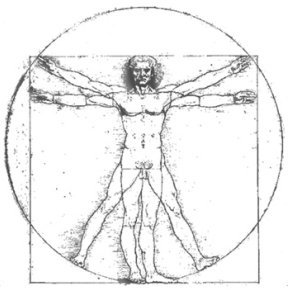
We explore math concepts in some works of art, fine and folk, including color, contrast, geometry, symmetry, perspective, curvature, dimension, self-reference, and the complementary roles of local versus global properties. Activities chosen from those below.

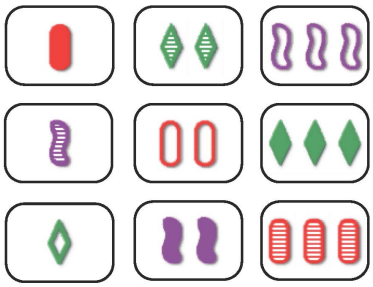
1 Fine Art

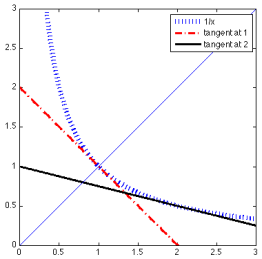
- Perspective, from left: Piero's Flaggelation; Ames room such as we'll build, with identical twins.

- Golden Ratio and symmetry, from left: da Vinci's Mona Lisa; da Vinci's Vitruvian Man; the game SET; a stretching of the curve $y = 1/x$. We'll explore some symmetries in mathematics (including non-geometric symmetries), and discuss the algebra used to study symmetries. We'll derive some calculus the painless way, using symmetries. For example, the line tangent to $xy = 1$ at $x = 1$ is easy to find; then stretching y by $1/2$ and x by 2 easily gives $\left.\frac{d(1/x)}{dx}\right|_{x=2} = -\frac{1}{4}$.



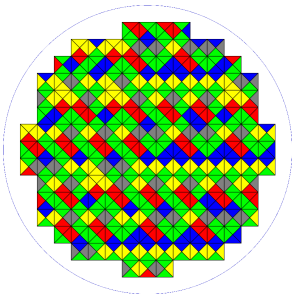
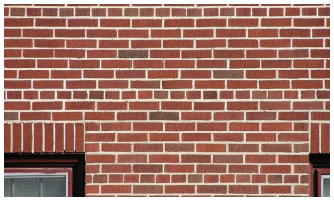
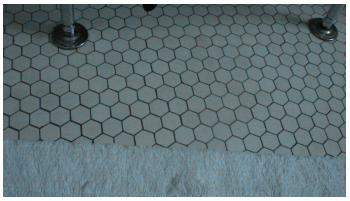




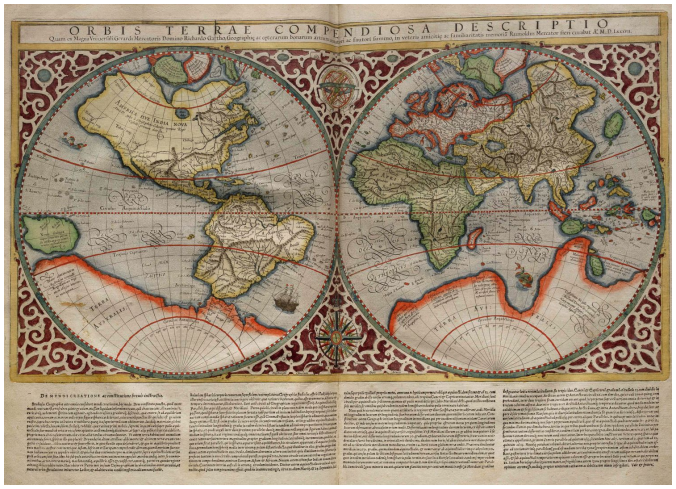


- Tiling, top, from left: Alhambra; Penrose; Wang tiles. Bottom, from left: brickwork, bathroom tile, Escher's lizards. Periodic tilings and tesselations exhibit symmetries. In general, it is undecidable whether, given a small collection of tile shapes, we can tile the plane with unlimited copies of those shapes. The technique is used to sythesize texture in images.

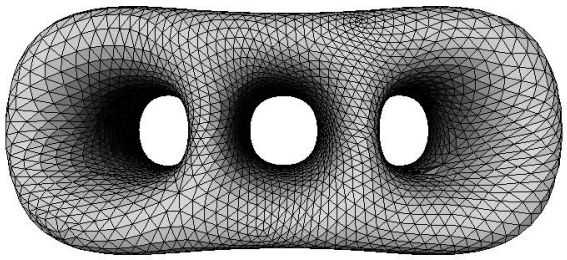


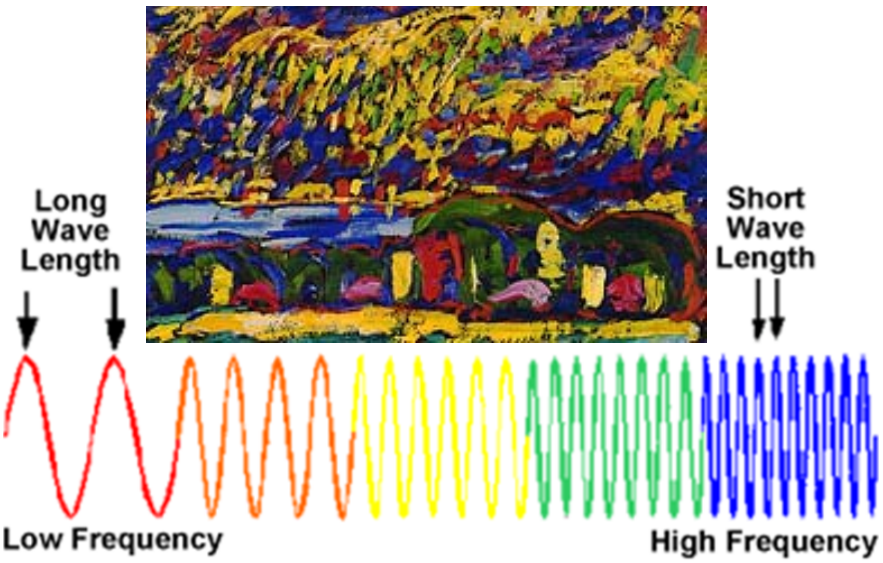

- Stereographic and other Projections: Braque's Violin and Palette, left; world map. More than one side of a solid is depicted, simultaneously.

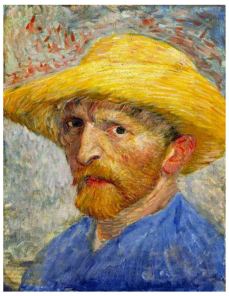
- Curvature: Moore's Two Large Forms, left; a generalized torus, right. How do we assign a curvature number at each point of a surface? What do all these curvature numbers tell us about the overall shape, e.g., how many holes in the donut? How much curvature is packed into each corner of a cube?

- Color: Hartley's Mountain Lake and a depiction of color determined by photon frequencies. We'll build spectroscopes that separate out colors, and understand the effect of red next to blue in print.

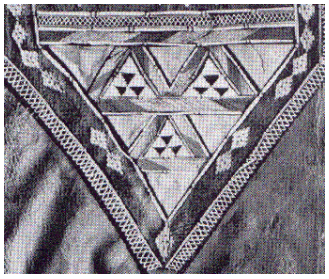


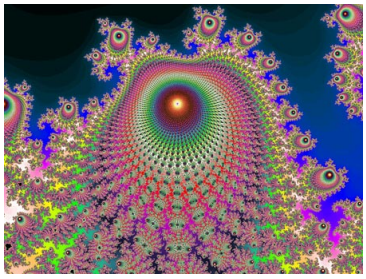
- Self-reference, from left: van Gogh self-portrait; Picasso's Artist and Model, depicting an abstract model posing for an abstract artist, who paints a traditional profile; unattributed self-portrait. Resolution of the liar paradox "this sentence is false" plays a role in what can be computed and what can be proven mathematically.



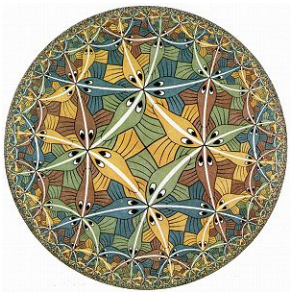
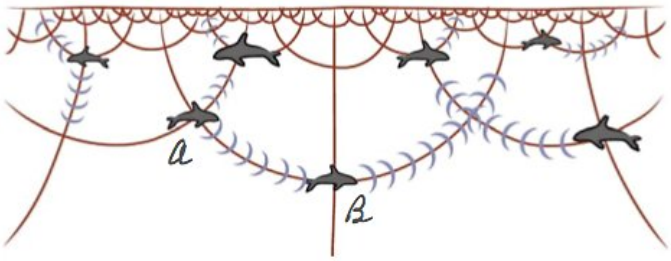
2 Folk and Pop Art

- Multiscale and fractal phenomena, from left: African textile; Sierpinski triangle cookies (which we'll build—and eat); part of the Mandelbrot set. Images look similar at the scale of feet, inches, millimeters, etc., and some of these objects have fractional dimension strictly between one and two.





- Hyperbolic Geometry: Escher's Circle Limit III, left; Shortest paths, taken by underwater sound waves that travel faster in the deep (see `http://wordplay.blogs.nytimes.com/2012/10/08/whale/`), right.

- Half-toning, photomosaic, and low resolution, from left: Lichtenstein's Girl in Mirror; Dalí's Lincoln; "qart," with error-correction technology based on polynomials over finite fields and incorporated into the image. Make your own qart with your picture, pointing to your webpage, facebook page, etc.

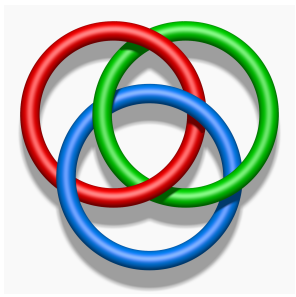


- Geometry, from left: Fuller's Montréal dome; Vi Hart's balloons; George Hart's hyperboloids. Through each point of the curved hyperboloids (that we will build), there are two distinct lines in the surface.



- Deformations, from left: Graffiti bubble letters; Macramé Owl; Loom bands; Borromean rings (removing any one ring frees the others). We'll make arrangents of knots, braids, and similar and we'll use algebra to specify knotting patterns and to characterize objects that are equivalent under stretching and moving but not cutting.

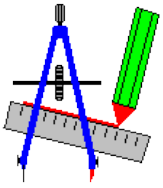
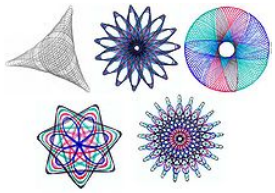


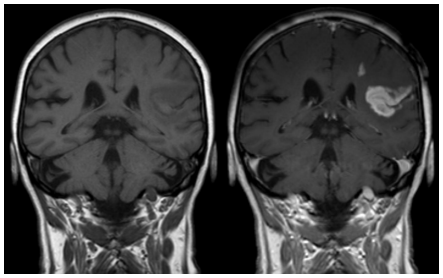
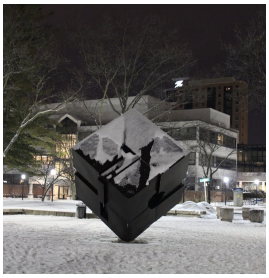
3 Engineering

- Drawing tools, from left: a planimeter, that measures the area of a plane figure by tracing the boundary; a razor scooter (makeshift planimeter); straightedge and compass, with which we can draw certain patterns, but not others; Spirograph patterns. We'll play with these.



- Chiaroscuro and interesting shadows, from left: Rembrandt's Anatomy of Dr. Tulp; tomographic scan; cube sculpture on Michigan's campus, casting a hexagonal shadow. We'll study the basics of MRI and tomography, in which a 3D subject is imaged from the shadows it casts.

- Compression, loss, and restoration: Michaelangelo's Sistine Chapel, Daniel, before and after restoration, left; an image compressed with different quality levels, right. We'll study some techniques for and effects of compression (with or without loss) and for automatic denoising.



- Authentication: From left, Vermeer's Pearl Earing and detail from van Meegeren's skillful counterfeit, Last Supper; *Ecce Homo* by Martínez and a comparatively less skillful counterfeit copy by Giménez. Art may be authenticated by expert opinion, chemical and physical analysis (we'll use the spectroscopes we built to identify substances in the work by colors they absorb or emit when they undergo chemical reactions), or mathematical and statistical analysis of artistic elements (color, multiscale nature, etc.).