Geometric Algorithms for Objects in Motion

Dissertation Defense

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Committee:

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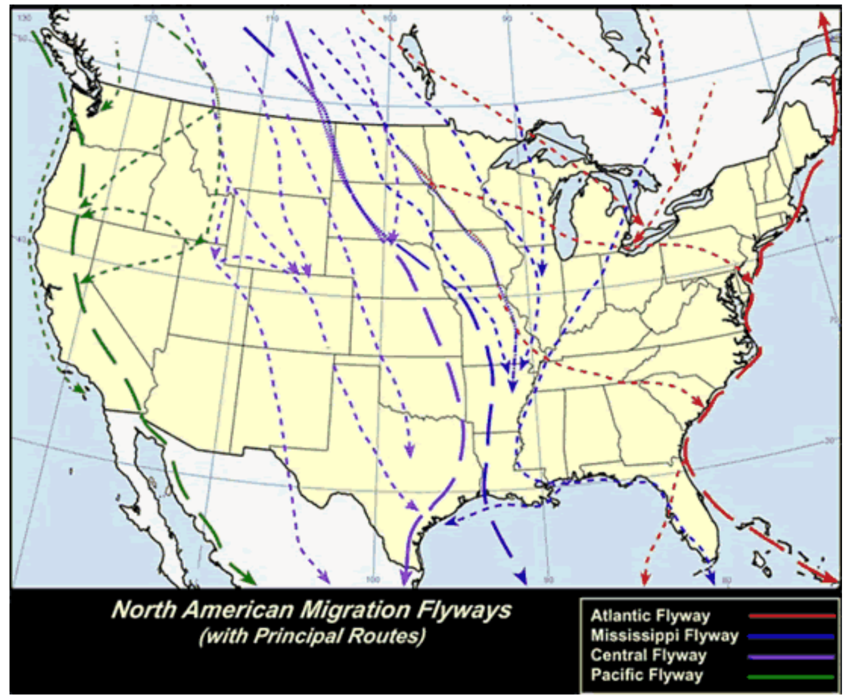
July 30, 2010

Motivation

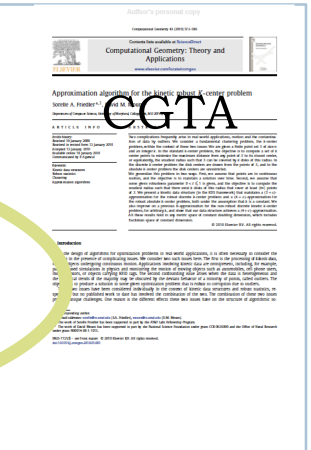

North American Migration Flyways
(with Principal Routes)
Atlantic Flyway
Mississippi Flyway
Central Flyway
Pacific Flyway

Motivation

- Computer Science
  - Graphics: Image / video segmentation and compression
  - Databases: Maintenance over time
  - Sensor Networks: Data analysis
  - Cell phone users: Motion data analysis
    - 4.6 billion subscribers worldwide (in 2009)
    - 4.1 billion text messages per day in the US (in 2009)
- Biology
  - Mathematical ecology: Migratory paths, invasive species
  - Genomic data analysis: HIV strain analysis
- Engineering
  - Traffic patterns and identification

Outline

- Kinetic Robust K-Center

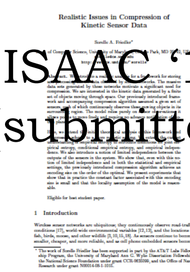


CGTA

- Sensor-Based Framework Kinetic Data Compression

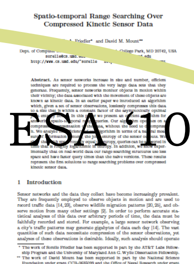
AlgoSensors '09

- Realistic Issues in Kinetic Sensor Data Compression



ISAAC '10 (submitted)

- Spatio-temporal Range Searching

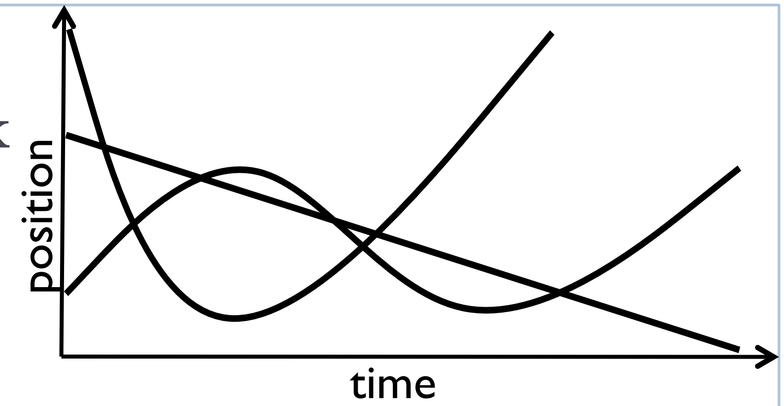


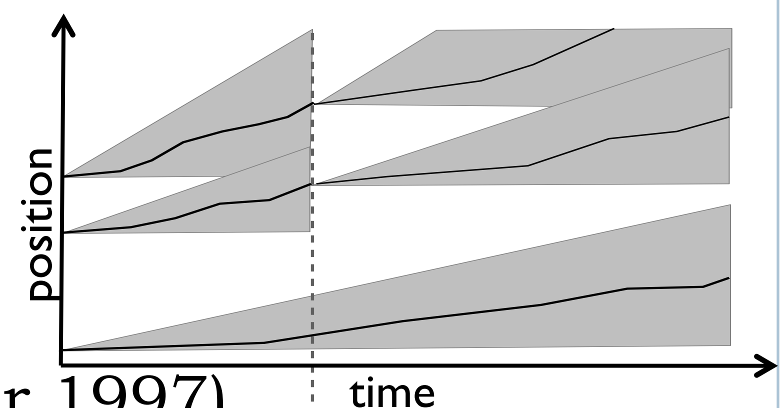
ESA '10

Kinetic Robust K-Center Problem

Existing Frameworks for Kinetic Data

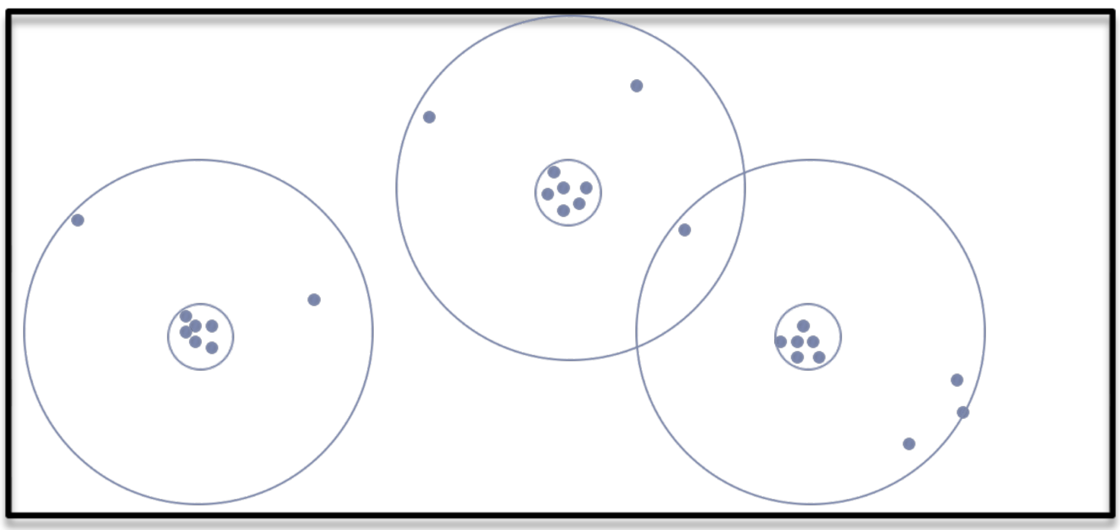
- Atallah (1983)
  - Polynomial motion of degree k
  - Motion known in advance
  - Points lie in R^d
  - Analysis in R^{d+1}
- Kahan (1991)
  - Bounds on point velocity
  - Update function provided
  - Limit queries to function
- Kinetic Data Structures (Basch, Guibas, Hershberger 1997)
  - Points have flight plans (algebraic expressions) that can change





Kinetic, Robust, *K*-Center Problem

- *k*-center problem: choose *k* centers that minimize the maximum distance from any point to its closest center
- robust *k*-center problem: allow a fraction (1-*t*) of the points to remain unclustered



Results

- Discrete: centers taken from input
- Absolute: centers any point in space

- $(3+\varepsilon)$-approximation algorithm for discrete *k*-center
  - Improves Gao *et al.* 8-approximation
  - Close to Charikar *et al.* 3-approximation
- $(4+\varepsilon)$-approximation algorithm for absolute *k*-center
  - First absolute solution
- Efficient by kinetic data structure standards

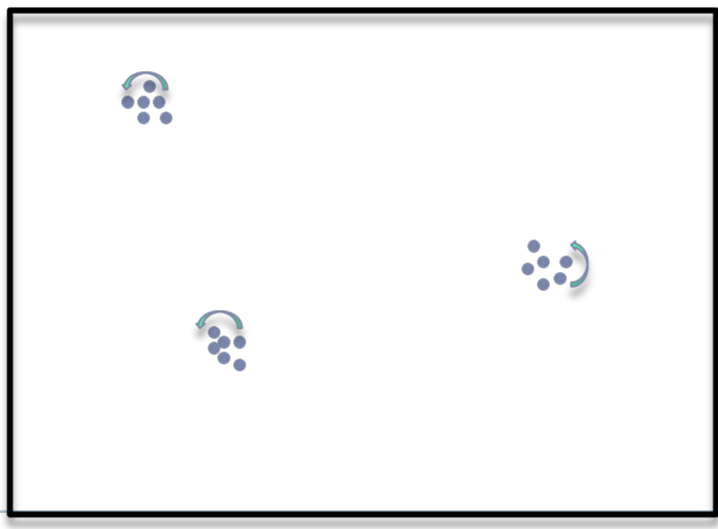
A Sensor-Based Framework for Kinetic Data Compression

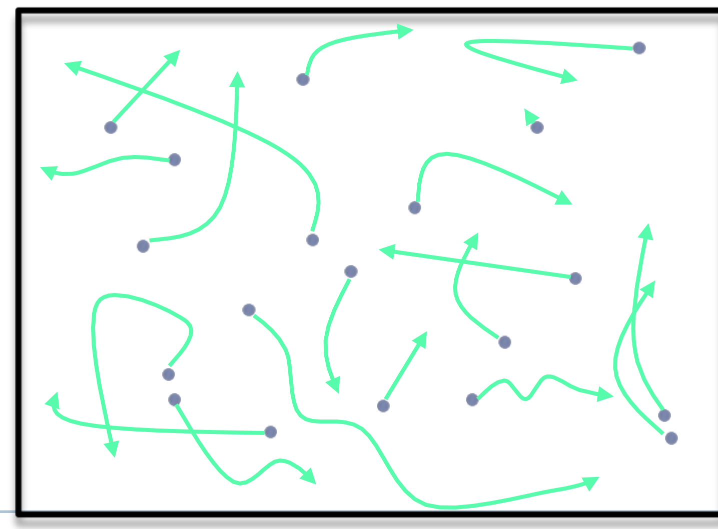
Existing Frameworks for Sensor Data

- Gandhi, Kumar, Suri (2008)
  - Sensors can count objects within their detection region
- Guitton, Skordylis, Trigoni (2007)
  - Sensors can calculate traffic flow (cars/time), occupancy (cars/area)
- Kastrinaki (2003 Survey)
  - Sensors can calculate object speed, change in angle, etc

Motivation

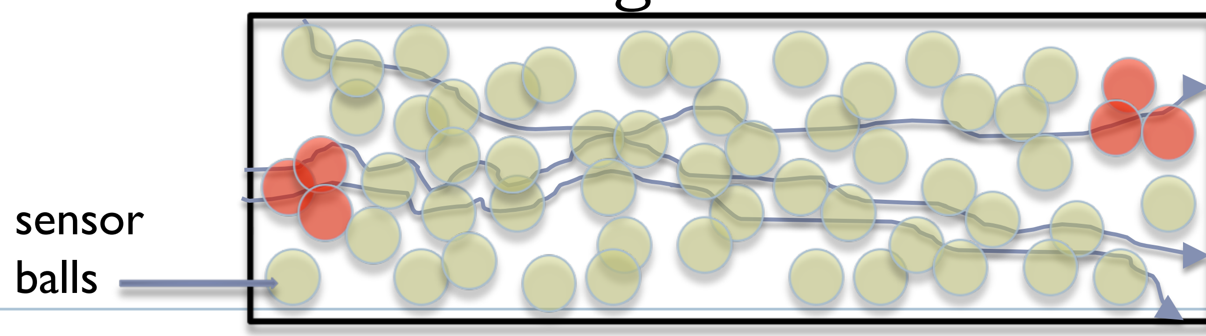
- Develop a framework for kinetic data from sensors
  - No advance object motion knowledge
  - No restrictions on object motion
  - Reasonable assumptions of what a sensor can know
  - Efficiency analysis that is <u>motion sensitive</u>





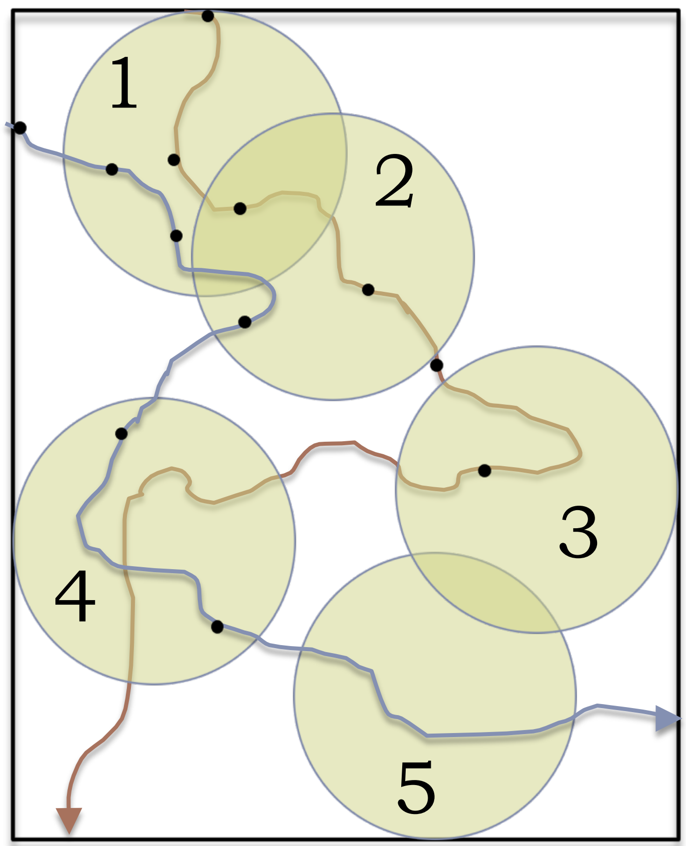
Our Framework

- Detection region around each sensor (stationary sensors)
- Point motion unrestricted
- No advance knowledge about motion
- Each sensor reports the count of points within its region at each synchronized time step
- *k*-local: Sensor outputs statistically only dependent on *k* nearest neighbors



Data Collection

Data based on underlying geometric motion



Sensor data streams

x_1	x_2	x_3	x_4	x_5
1	0	0	0	0
2	0	0	0	0
2	1	0	0	0
0	2	0	0	0
0	0	0	1	0
0	0	1	1	0

time

Motivation: Data Compression

- <u>Kinetic data</u>: data generated by moving objects
- Sensors collect data
- Large amounts of data
- Want to analyze it later
- Don't know what questions we'll want to ask in advance

- Next: Lossless data compression
- Later: Retrieval

Entropy

Consider the string generated by a random process…

- Entropy: The information content of a string or a measurement of the predictability of the random process

  $$- \Sigma_x \, pr(x) \log pr(x)$$

  - Example: A weighted coin that's always heads vs. a normal coin:

  $$-(1 \log 1) = 0 \quad \text{vs.} \quad -(½ \log ½ + ½ \log ½) = 1$$

Entropy Generalizations

- Joint entropy: The entropy for joint probabilities of a set of events occurring
- Normalized entropy: bits to encode each character, entropy/n for strings of length n

- Joint entropy chain rule ($\mathbf{X} = \{X_1, X_2, \ldots, X_S\}$):
  - $H(\mathbf{X}) = H(X_1)+H(X_2|X_1)+\ldots+H(X_S|X_1,\ldots,X_{S-1})$

- *k*-local entropy (H_k): normalized joint entropy of a set of streams that are only dependent on up to *k* streams from their *k* nearest neighbors

Finding a Data Compression Algorithm

Goal: smallest *lossless encoding* of sensor data

- **Optimal**: encoded (set) length = underlying (joint) entropy

- Idea 1: compress all strings separately
  - Not optimal
- Idea 2: compress all strings together
  - Window size needed for repetition too large to be practical

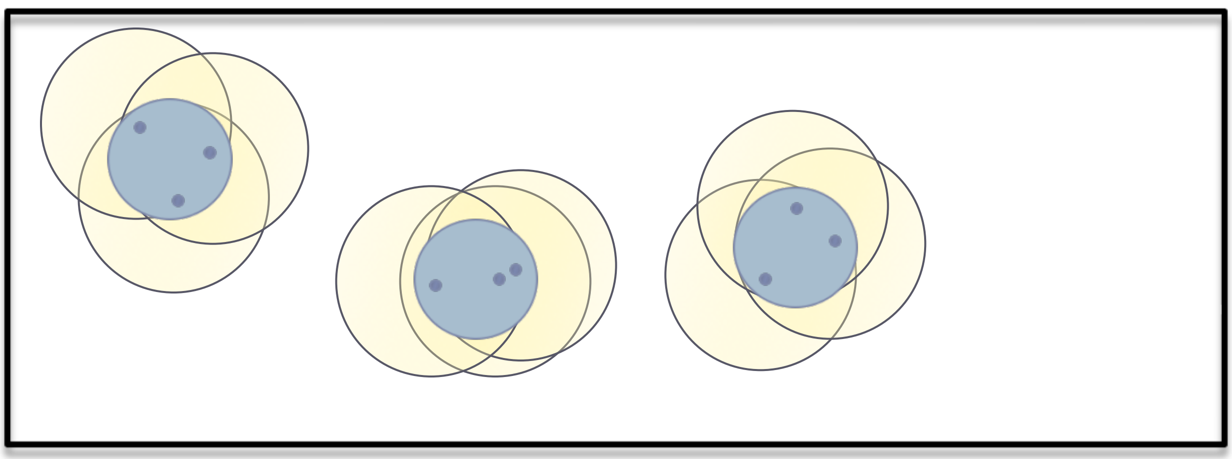
Optimal encoded sensor stream length: $H(\mathbf{X}) = H_k(\mathbf{X})$

- **Key**: compress statistically dependent
  strings together – want groups of k strings

Data Compression Algorithm: Partitioning Lemma

- *k*-clusterable: A point set that can be clustered into subsets of size at most $k+1$ so that if p and q are among each other's k nearest neighbors then they are in the same cluster.

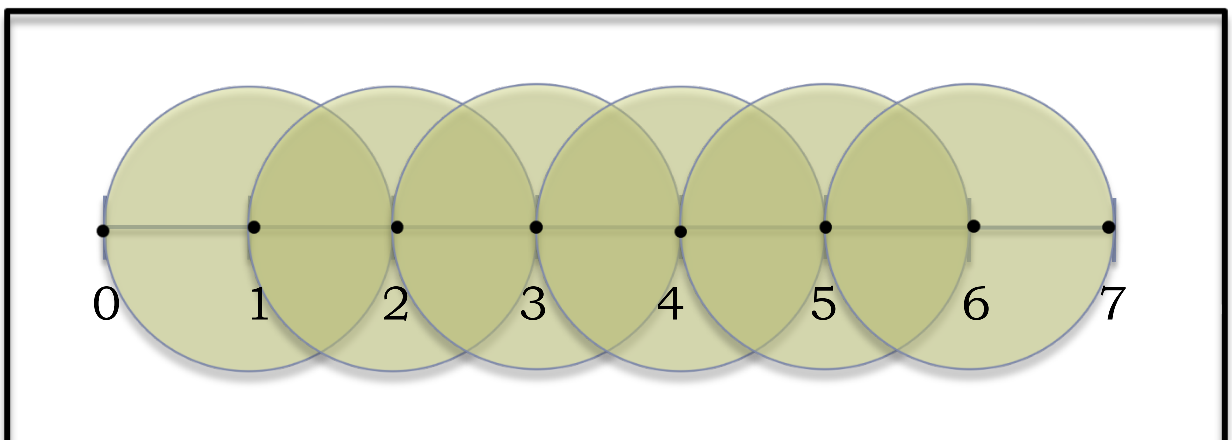
2-clusterable example



Data Compression Algorithm: Partitioning Lemma

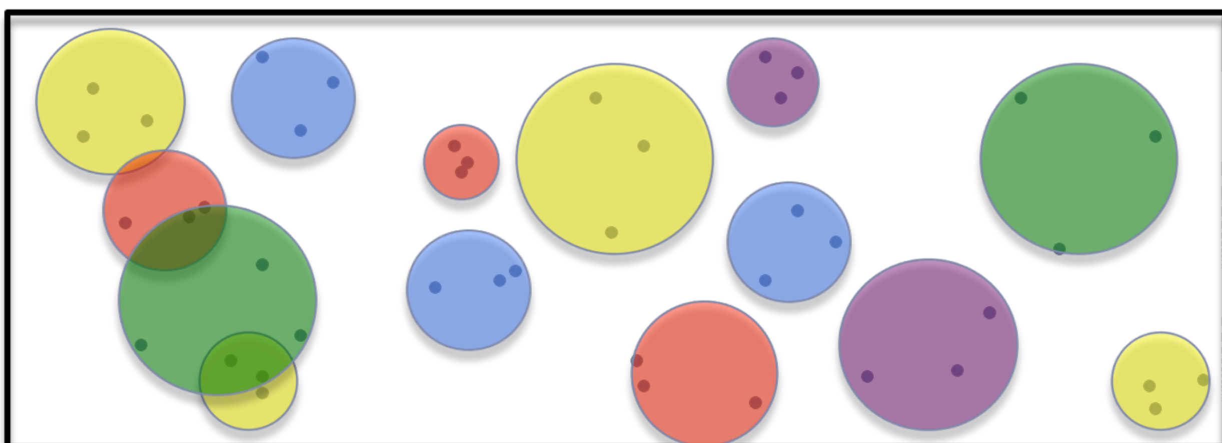
- *k*-clusterable: A point set that can be clustered into subsets of size at most $k+1$ so that if p and q are among each other's k nearest neighbors then they are in the same cluster.

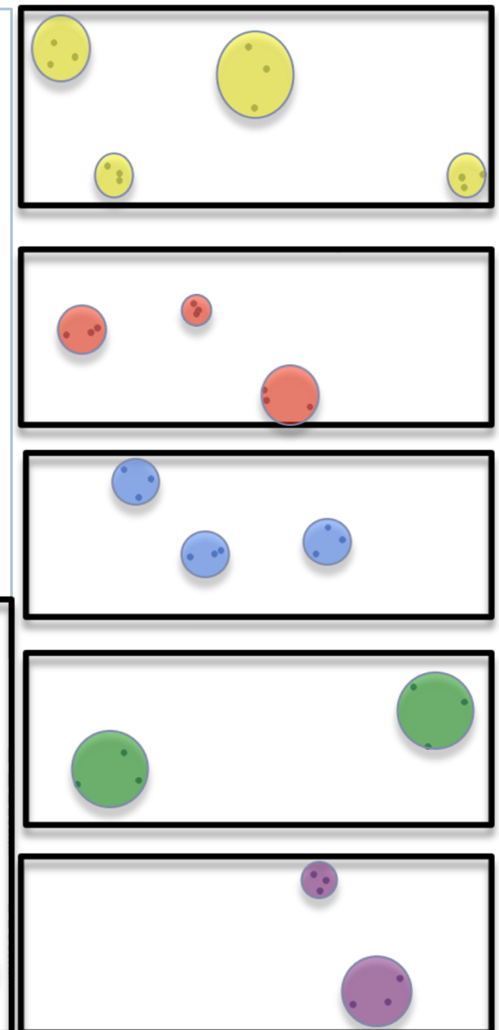
not 2-clusterable example



Data Compression Algorithm: Partitioning Lemma

- <u>Lemma</u>: There exists an integral constant c such that for all k>0 any point set can be partitioned into c partitions that are each k-clusterable.

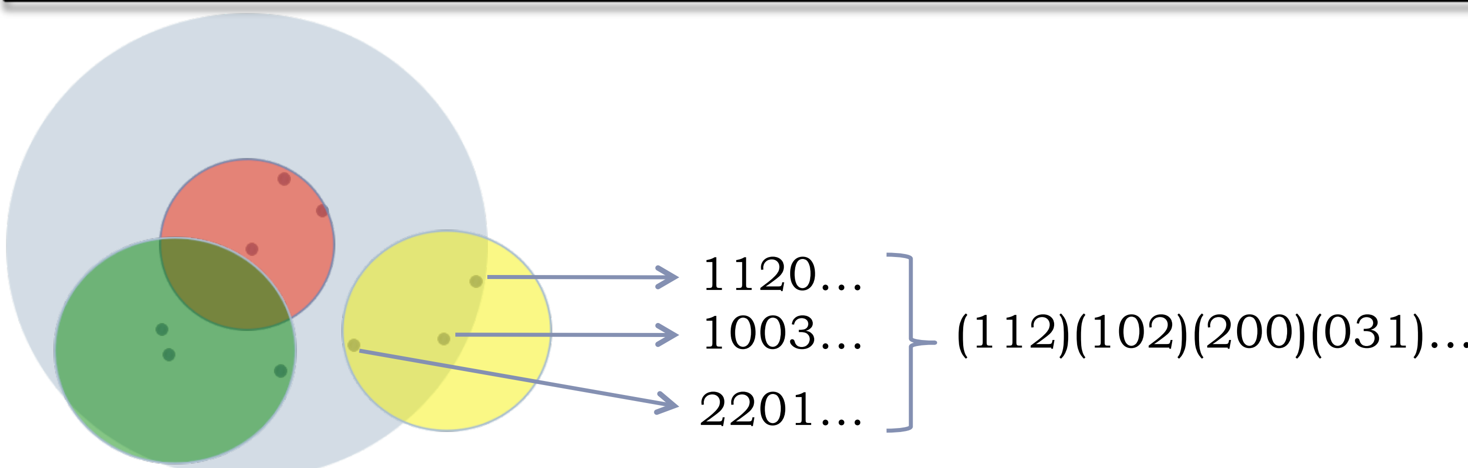




Data Compression Algorithm

- Partition and cluster the sensors, then compress

```
for each partition P_i
    for each cluster in P_i
        combine the cluster's streams into one with longer
                characters and compress it
return the union of the compressed streams
```



Data Compression Algorithm

- Proof Sketch:
  - The joint entropy of the streams is the optimal length
    - Recall: $H(\mathbf{X}) = H_k(\mathbf{X})$
  - Sensor outputs are *k*-local, so each compressed partition is the optimal length:

    statistically dependent streams are compressed together
  - There are *c* partitions, so the total length is *c* times optimal
    - c is O(1)

Realistic Issues in Compression of Kinetic Sensor Data

Two Main Issues and Solutions

Entropy	**Independence**
Shannon Entropy - assumes an underlying random process - bounds hold in the limit Empirical Entropy - relies on observed probabilities	Independence - strict independence ignores systemic patterns δ-Independence - allows limited underlying dependence between sensor outputs

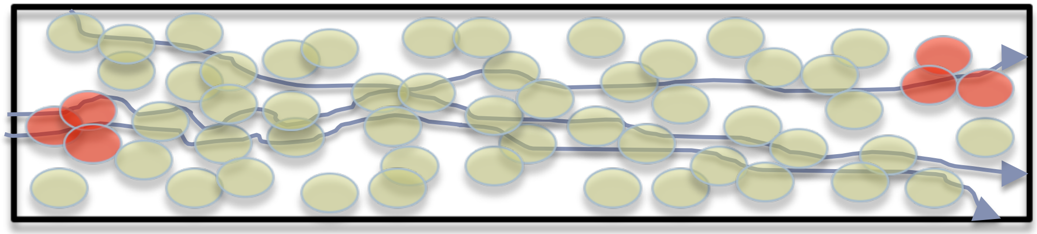
Analysis of Compression Algorithm

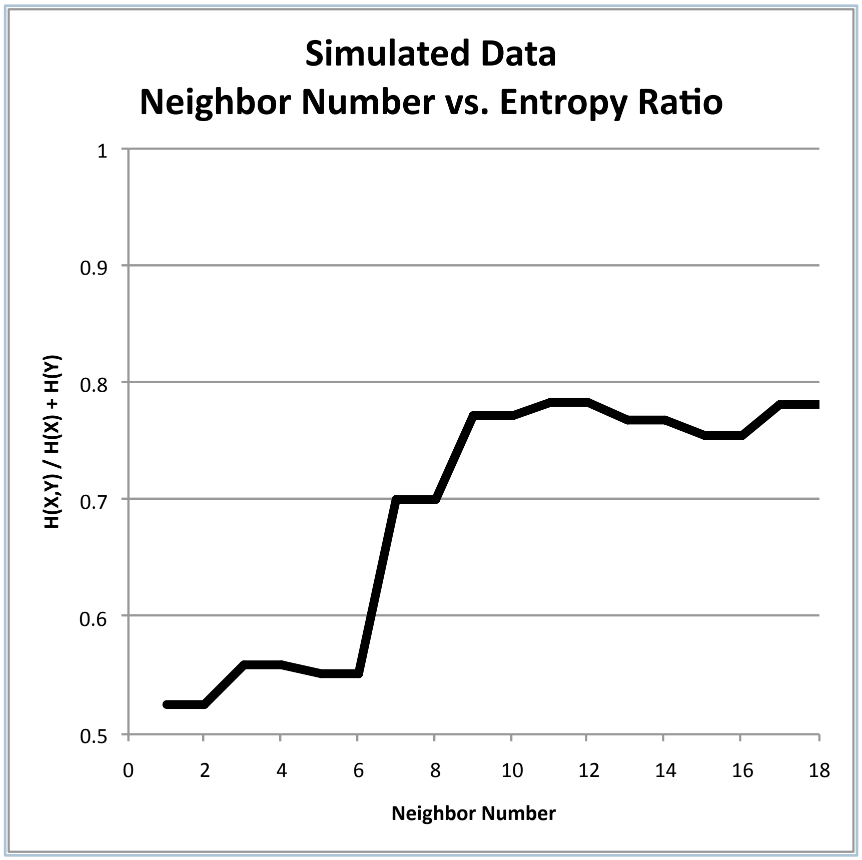
Statistical and Empirical Settings

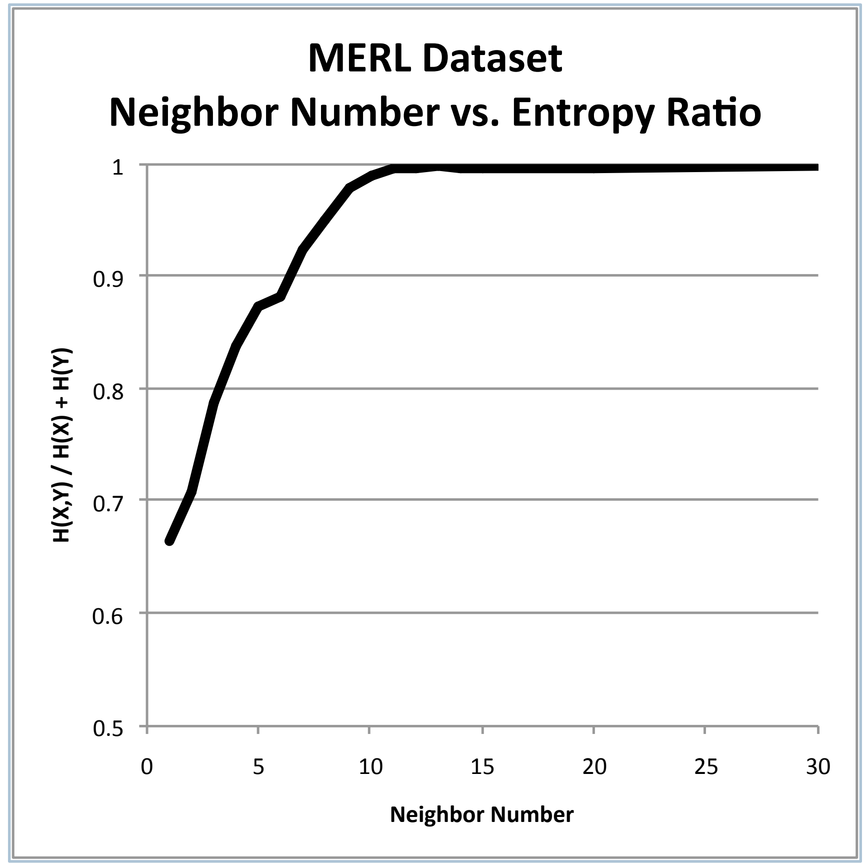
Strict Independence	δ-Independence
$O(S_{opt}(\mathbf{X}))$	$O(\max\{\delta T, S_{opt}(\mathbf{X}, \delta)\})$

- $\mathbf{X}$: set of sensor system observations
- T: length of observed time period
- δ : independence parameter
- $S_{opt}(\mathbf{X})$: optimal encoding space size for $\mathbf{X}$

Locality Experiments







Spatio-temporal Range Searching over Compressed Kinetic Sensor Data

Motivation: Data Retrieval

- Kinetic data: data generated by moving objects
- Sensors collect data
- Large amounts of data
- Want to analyze it later
- Don’t know what questions we’ll want to ask in advance
- Done: Lossless data compression
- Next: Retrieval without decompressing data

Range Searching: Our Problem

Compress and preprocess the data so as to perform…

- <u>Temporal range query</u>: Given a time interval, return an aggregation of the counts over that time interval.

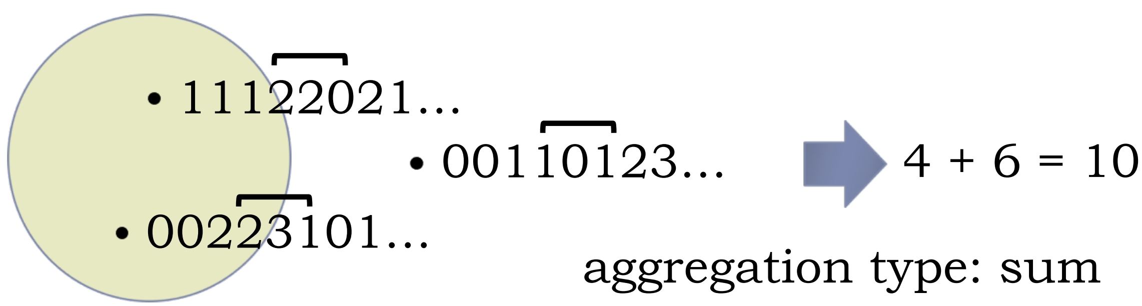
  aggregation type: sum

  t: 1 2 3 4 5 6 7 8 9 10 11

  X: 0,0,4,4,5,4,3,3,1, 1, 0

  17

- <u>Spatio-temporal range query</u>: Given a time interval and spherical spatial region, return an aggregation of the counts over that time interval and within that region.

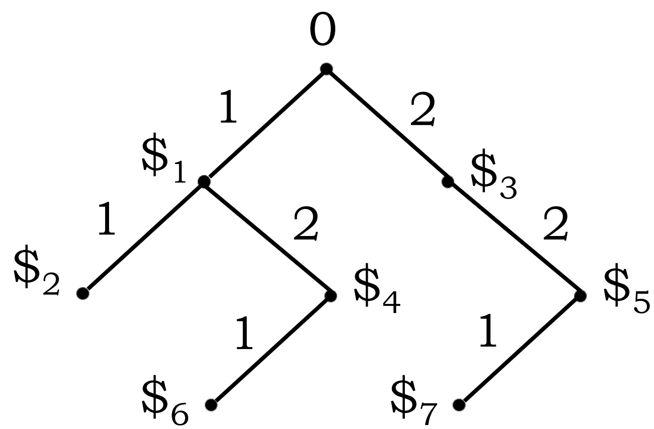


Lempel-Ziv Dictionary Compression [LZ78]

11121222121221

1 11 2 12 22 121 221

$\$_1$ $\$_2$ $\$_3$ $\$_4$ $\$_5$ $\$_6$ $\$_7$



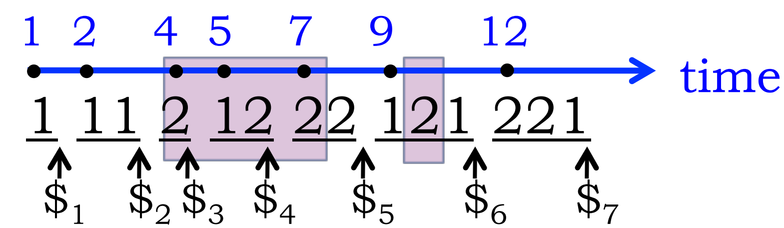
Create a trie while scanning through a string. The compressed string contains pointers to this dictionary.

(LZ78 is an optimal entropy encoding algorithm.)

Temporal Range Searching

- Create trie with accompanying pointers
- Annotate trie with aggregate values and word start times
- Given a temporal range $[t_0, t_1]$ find the anchor points $\$^0$ and $\$^1$ such that $\$^0 \leq t_0$ and $\$^1 \geq t_1$ (binary search)
- Use stored prefixes, words, and subtraction of prefixes to find aggregates

Encoded String



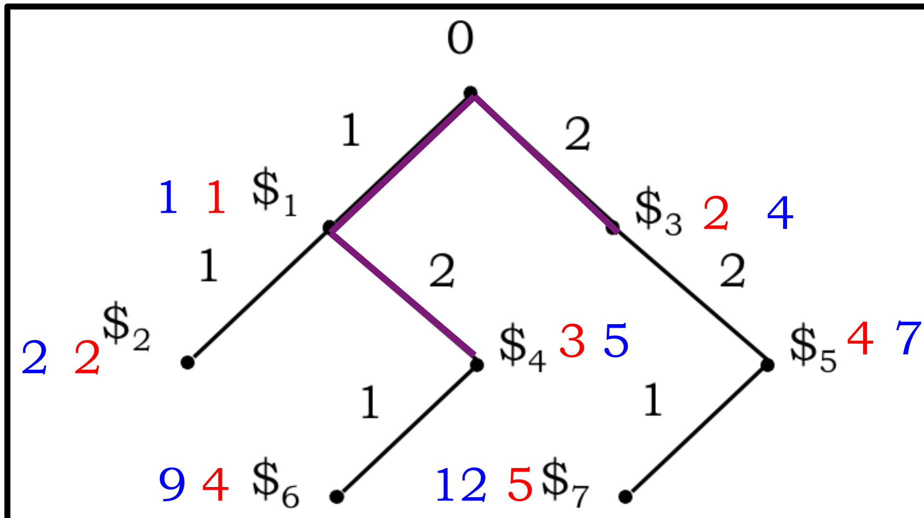
Query Examples

overlapping query: [4,7]

2 + 3 + 2 = 7

internal query: [10,10]

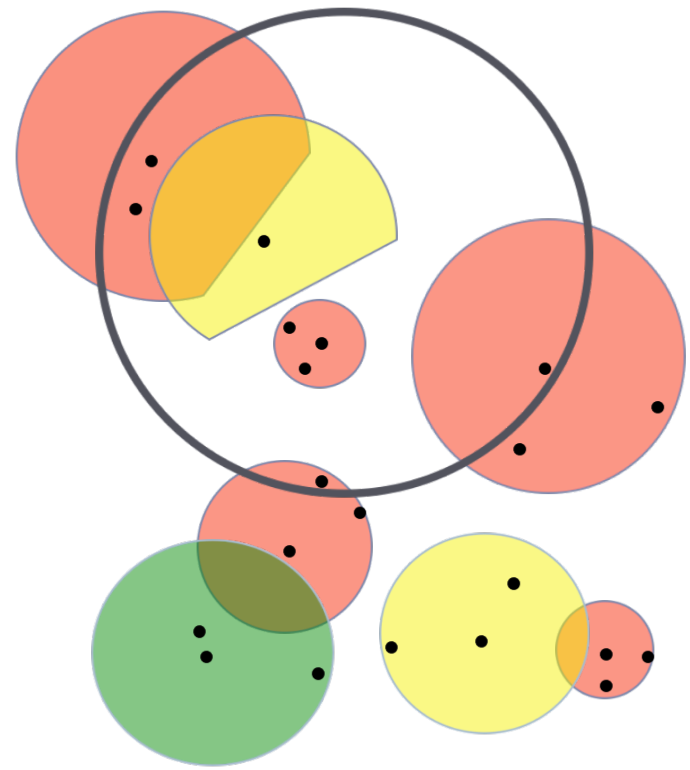
3 - 1 = 2



aggregate values word start times

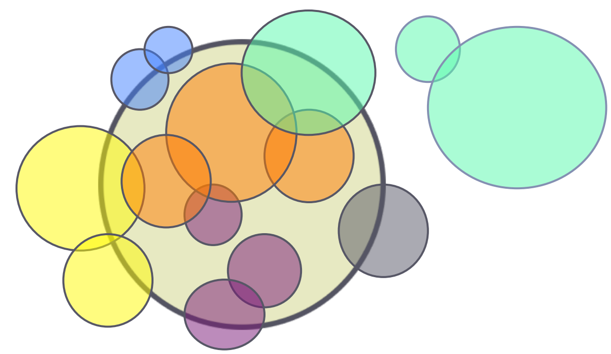
Sensor Clumps

- Recall: The sensors are partitioned, clustered, and compressed
- *Set of clumps*: A finite set of balls with a packing property limiting the number of intersections of any ball with a clump.
- Lemma: In a single partition, the nearest neighbor balls form a set of clumps that contain the sensor clusters



Range Searching Among Clumps

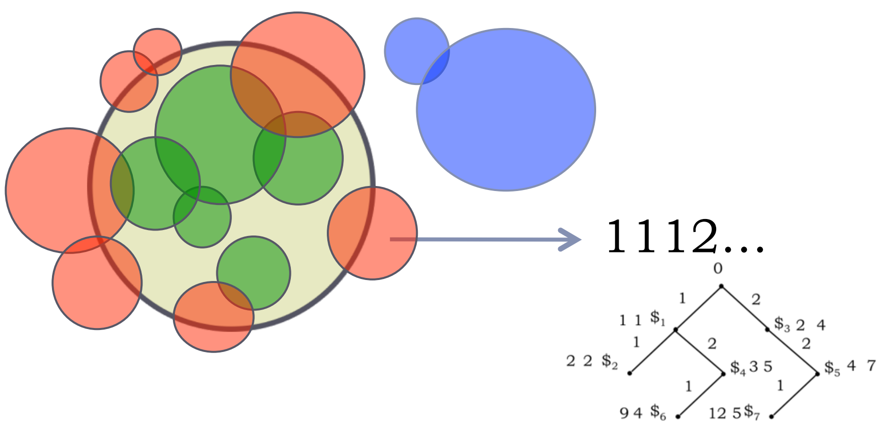
- Range Searching Among Clumps: Given any query range $\mathcal{R}$ and using a quadtree variant, we can report
  - a subset of clump subsets that form a disjoint cover of the clumps within $\mathcal{R}$
  - the subset of clumps that $\mathcal{R}$ intersects



- Lemma: A quadtree variant based data structure can answer range searching queries among clumps.

Spatio-temporal Range Searching

- <u>Main Theorem</u>: By adding an auxiliary data structure to answer temporal range queries to each node in the range searching among clumps solution we can answer spatio-temporal range queries.



- One range searching among clumps structure for each partition
- One temporal range structure for each clump and each internal quadtree node
- Get temporal sums for each clump and overlapped sensor
- Sum over all partitions

Results

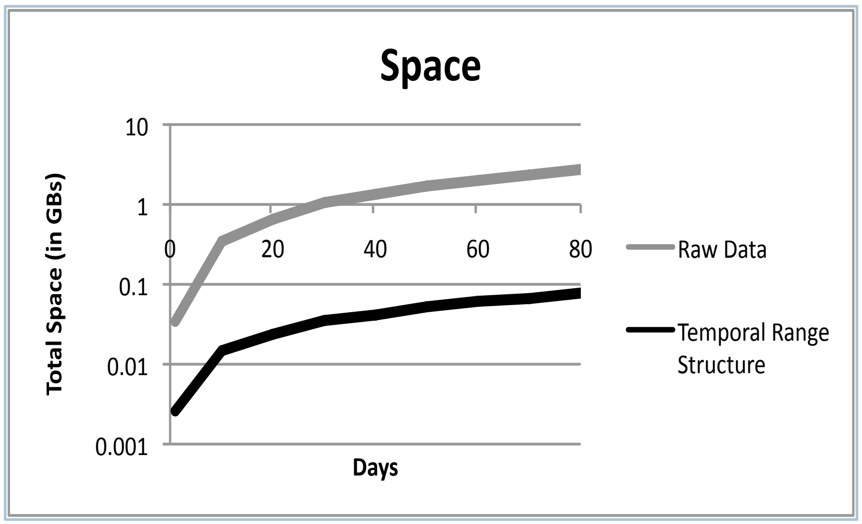
Bounds for Range Searching

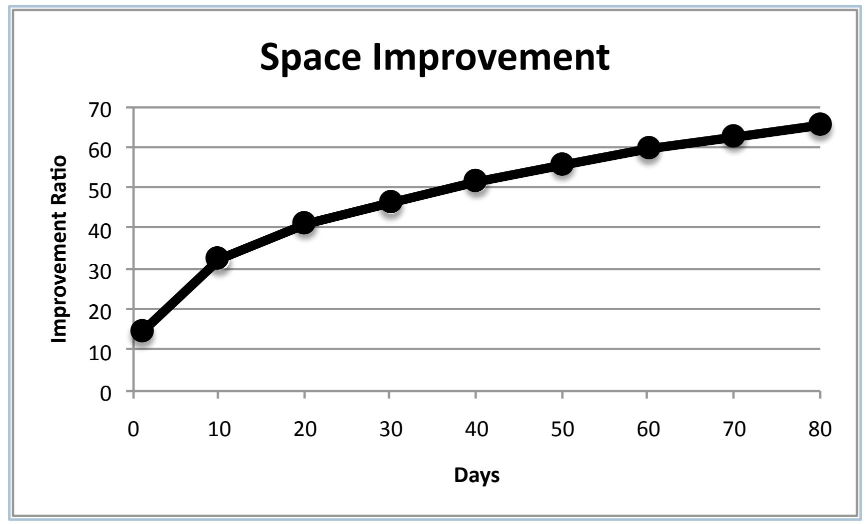
	Temporal	Spatio-temporal
Preprocessing time	$O(\mathrm{Enc}(X))$	$O(\mathrm{Enc}(\mathbf{X}))$
Query time	$O(\log T)$	$O(((1/\varepsilon^{d-1}) + \log S) \log T)$
Space	$O(\mathrm{Enc}(X))$	$O(\mathrm{Enc}(\mathbf{X}) \log S)$

- X: The set of sensor system observations
- Enc(X): The encoded size (in bits) of the compressed data
- T: The total time over which data was collected
- S: The total number of sensors
- d: The dimension of the sensor space
- ε: An error parameter (for approximate range searching)

First range searching bounds <u>over compressed data</u>

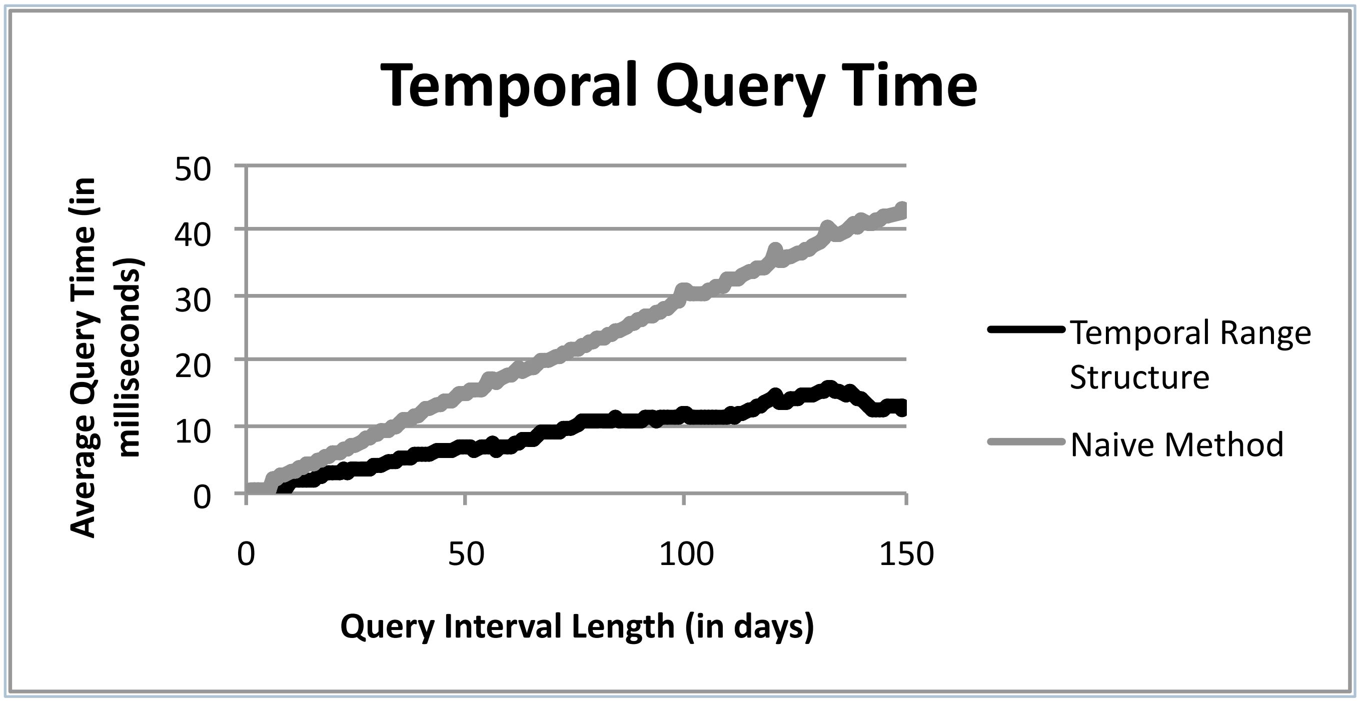
Experimental Results: Space





C. R. Wren, Y. A. Ivanov, D. Leigh, and J. Westbues.
The MERL motion detector dataset: 2007 workshop on massive datasets.
Technical Report TR 2007-069,
Mitsubishi Electronic Research Laboratories, Cambridge, MA, USA, August 2007.

Experimental Results: Time



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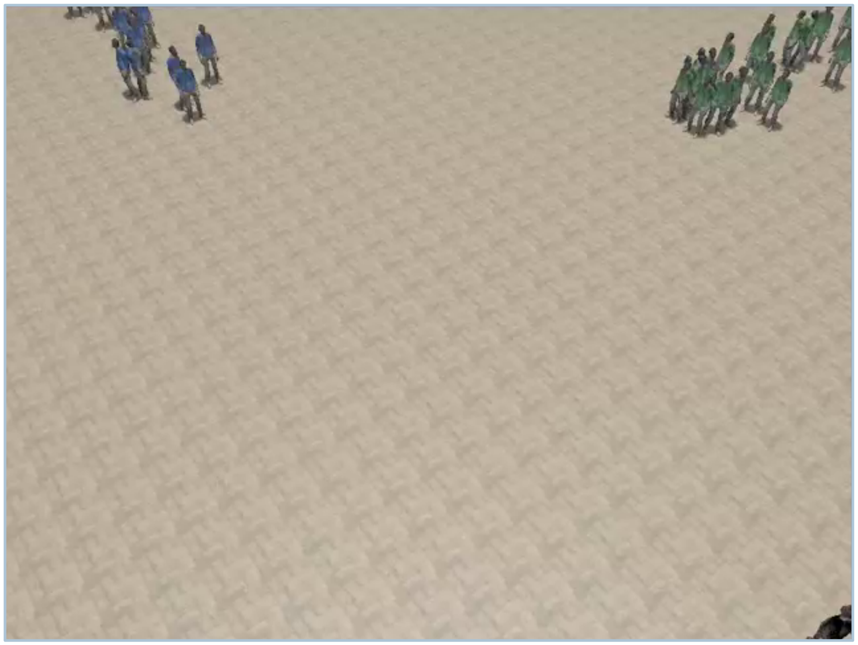
Conclusions and Open Problems

Results

- Robust clustering within the KDS model
- Framework for kinetic sensor data
  - No assumptions about object motion or advance knowledge
- Lossless compression algorithm that takes space O(optimal)
- Realistic analysis to consider empirical entropy and a limited notion of independence
- Spatio-temporal range searching over compressed kinetic sensor data
- Experimental analyses of locality, space, and query time

Spatio-temporal k-Center Problem



simulation by the UNC collision avoidance team

- $\mathbf{X} = \{X_1, \ldots, X_S\}$
- $X_i = X_{i1},\ldots,X_{ij}, \ldots , X_{iT}$
- Assign counts to k clusters $C_{ij1}, \ldots, C_{ijk}$ such that for all sensors and times i,j
  - $\Sigma_l \, C_{ijl} = X_{ij}$
- Minimize the maximum $H_\tau(\mathbf{X})$ over all $C_l = \{C_{ijl}\}_j$

Future Work: Understanding Motion

Practical	Theoretical
Relies on reasonable assumptions about motion, data, observations, etc. Reasonable to code and develop algorithms	Algorithm analyses that are motion-sensitive Allowance for both exact theoretical descriptions of motion and observations

Thank you!

Questions?