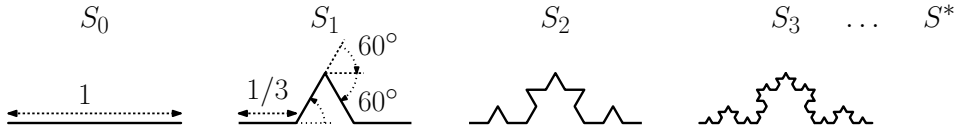
In-Class Exercise 1

Consider the series of shapes $S_0, S_1, \ldots$ shown in the figure below With each step, each line segment of length x is replaced by four segments each of length $x/3$. Let $S^* = \lim_{i \to \infty} S_i$.



Derive an L-system to generate this shape. (For the sake of consistency, please start at the left endpoint.) Be sure to include the step size, angle increment (in degrees), and the initial angle. Explain your symbols and their meanings in the drawing process. Give the start string and all the production rule(s). **Hint:** All angles are multiples of 60°.

Solution: The initial length is $d_0 = 1$ and the angle increment $\delta = 60°$ (60 degrees counterclockwise). At each stage, let's assume that the starting coordinates are $(x, y) = (0, 0)$, the starting angle is $\alpha = 0$ (i.e., directed horizontally to the right).

The length scale factor for each stage is $\sigma = 1/3$. This means that at the first generation we have segments of length $d_1 = \sigma d_0 = 1/3$, at the second generation $d_2 = \sigma d_1 = 1/9$, and generally $d_i = \sigma d_{i-1} = 1/3^i$.

The start string for generation 0 is $\omega = F$ (just a single line segment). The production rule set P consists of a single rule that takes us from generation $i - 1$ to generation i replaces each segment (F) by drawing a single segment (F), rotates up by angle δ ($+$), draws another segment (F), rotates down by -2δ ($--$), draws a segment (F), rotates up by δ ($+$), and then draws the final horizontal segment (F). Thus, we have the production rule

$$F \longrightarrow F + F - - F + F$$

In summary we have:

$$\begin{aligned}(x_0, y_0) &= (0, 0)\\ \alpha_0 &= 0\\ d_0 &= 1\\ \sigma &= 1/3\\ \omega &= F\\ P &= \{F \longrightarrow F + F - - F + F\}\end{aligned}$$