Linear Time Maximum Weight Independent Set On Cocomparability Graphs

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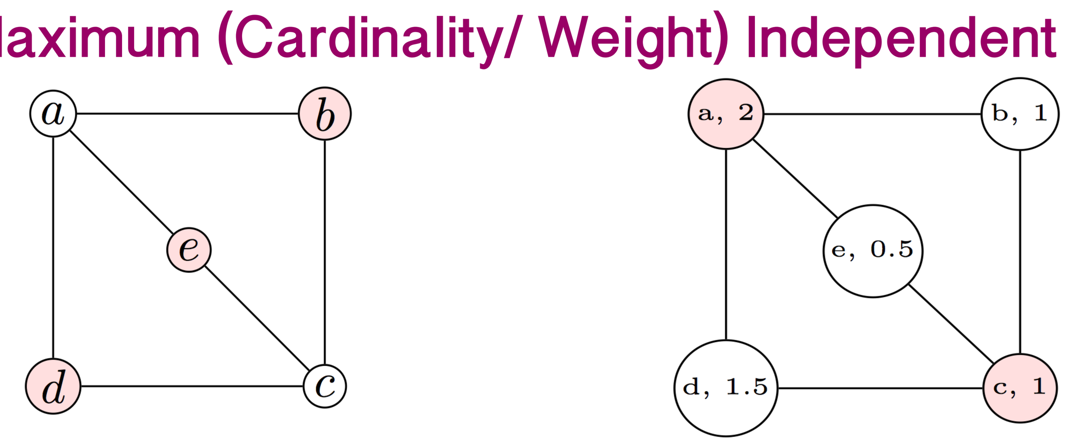
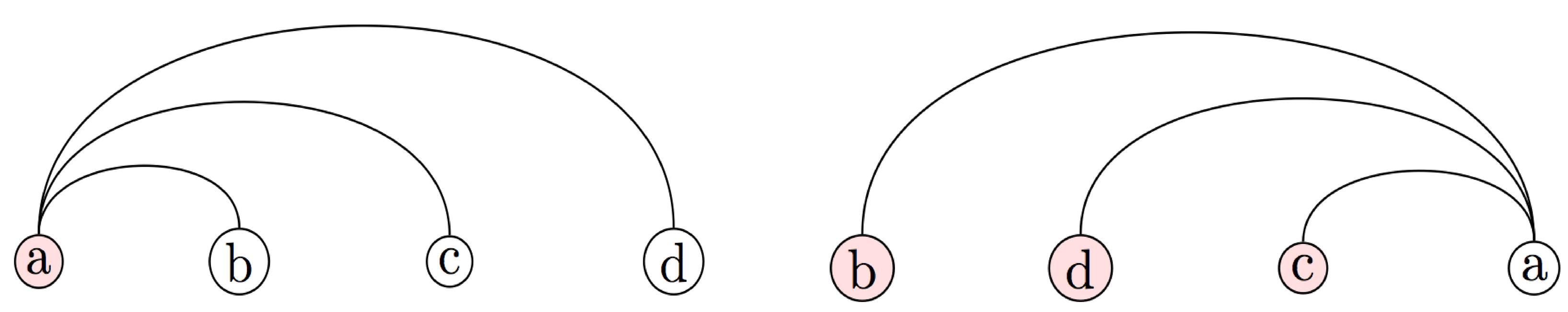
Motivation

- Given a graph G(V, E), where V is the set of vertices, and E the set of edges, consider the following problem: What is the largest subset of V where every two vertices are pairwise nonadjacent?

The Maximum (Cardinality/ Weight) Independent Set Problem



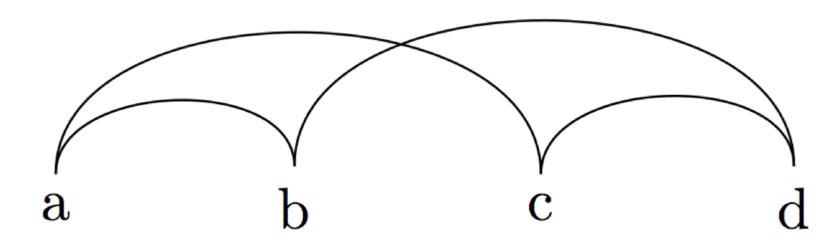
- Why do we care?
  - Scheduling. Biology. Coding Theory ...
- Very easy to trick a greedy algorithm:



- **The (Weighted) Maximum Independent Set, (W)MIS, problem is NP-hard on arbitrary graphs.**

Cocomparability Graphs

- A cocomparability graph is the complement of a comparability graph (i.e. induced by a partial order).
- **Vertex Ordering Characterization:**
  - G is a cocomparability graph iff V admits an ordering (V) where every triple a ≺ b ≺ c with ac ∈ E, ab ∈ E or bc ∈ E or both. For example:



- Such ordering is called an umbrella free ordering.
- O(m +n) to compute – McConnell & Spinrad [1].

- Cocomparability graphs are a superclass to:
  - Trapezoid and Permutation graphs
  - Cographs
  - Interval graphs

▶ Solving a problem on cocomparability graphs yields a solution to all these graph classes !

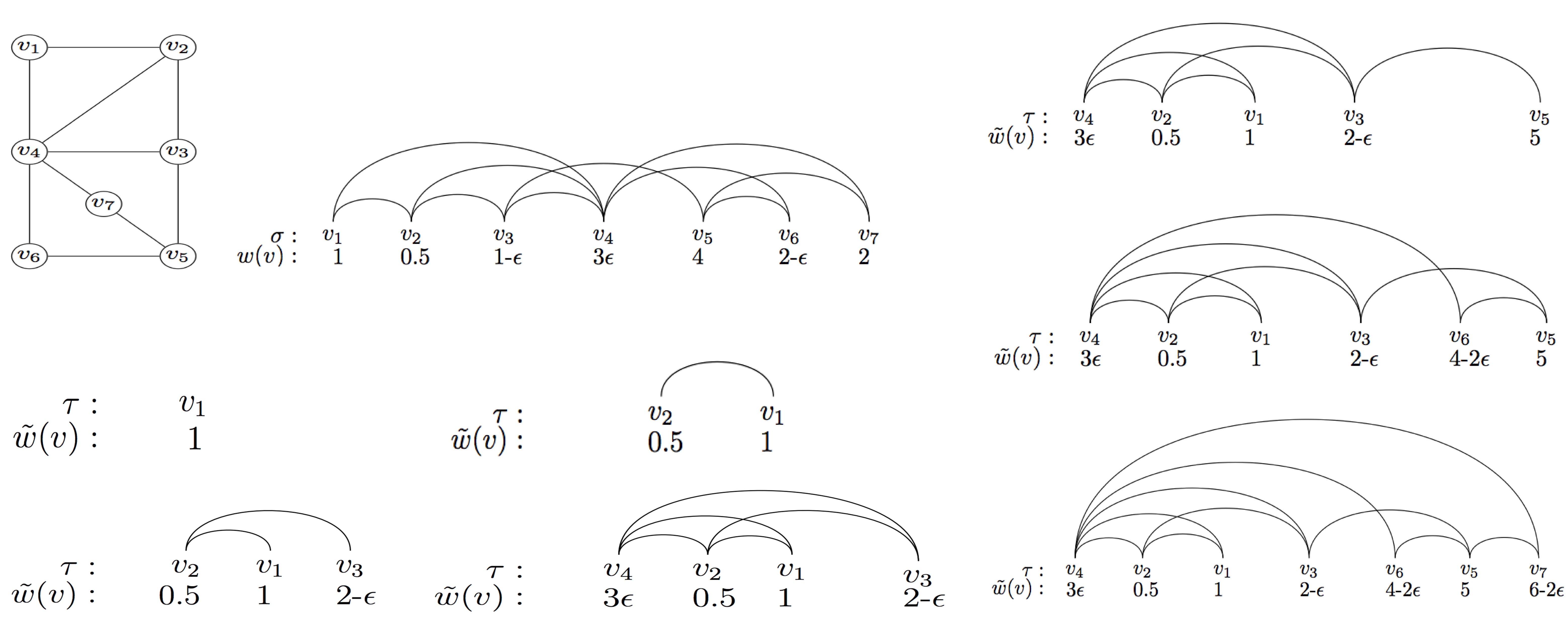
The Algorithm

- Given a cocomparability graph G(V, E):
  - Compute a valid cocomparability ordering $\boldsymbol{\sigma}$.
  - Scan $\boldsymbol{\sigma}$ from left to right to compute a new ordering $\boldsymbol{\tau}$ of V, where vertices are inserted in $\boldsymbol{\tau}$ in nondecreasing order of their (updated) weight.
  - Scan $\boldsymbol{\tau}$ from right to left to greedily collect a maximum weight independent set.

Proof of Correctness

- Associate with every vertex a set $\mathcal{S}(v_i)$, then at every iteration i:
  - For every vertex v_i, $\mathcal{S}(v_i)$ is an independent set.
  - Every $\mathcal{S}(v_i)$ is a maximum weighted independent set containing v_i in $G[v_1, ..., v_i]$.
  - Let be z_i the rightmost vertex of $\boldsymbol{\tau}_i$, then $\mathcal{S}(z_i)$ is a maximum weighted independent set in $G[v_1, ..., v_i]$.

Example



$\mathcal{IS} = \{v_7, v_6, v_3, v_1\}$

W = 6-2ϵ

Future Work

- Certify the algorithm. There exists a certifying algorithm for the unweighted case that computes a minimum clique cover of equal cardinality [2].
- Extend the algorithm to the k-colourable subgraph problem.

[1] Ross M McConnell and Jeremy P Spinrad. Modular decomposition and transitive orientation. *Discrete Mathematics*. 1999

[2] Derek G Corneil, Jérémie Dusart, Michel Habib, and Ekkehard Köhler. On the power of graph searching for cocomparability graphs. *In preparation*.