

## Homework 2

*To be finished individually. Due on Tuesday, January 27, 2011. Submit in class, or to GATES460. (sliding under the door if no one is there)*

*Revised 1/20/2011*

1. (6 points) Consider the following linear program, and find all the vertices of its polytope.

$$\begin{array}{ll} \text{maximize} & x_1 + x_2 \\ \text{subject to} & 3x_1 + x_2 \leq 3 \\ & x_1 + 3x_2 \leq 3 \\ & x_1 + x_2 \leq 2 \\ & x_1, x_2 \geq 0 \end{array}$$

2. (7 points) Find the dual of the following linear program.

$$\begin{array}{ll} \text{maximize} & 6x_1 + 8x_2 + 5x_3 + 9x_4 \\ \text{subject to} & 2x_1 + x_2 + x_3 + 3x_4 > 5 \\ & x_1 + 3x_2 + x_3 + 2x_4 = 3 \\ & x_1, x_2 \geq 0 \\ & x_3, x_4 \text{ are unconstrained} \end{array}$$

3. (7 points) In a facility location problem, there is a set of facilities and a set of cities, and our job is to choose a subset of facilities to open, and to connect every city to some one of the open facilities. There is a nonnegative cost  $f_j$  for opening facility  $j$ , and a nonnegative connection cost  $c_{i,j}$  for connecting city  $i$  to facility  $j$ . Given these as input, we look for a solution that minimizes the total cost. Formulate this facility location problem as an integer programming problem, and find its linear programming relaxation.