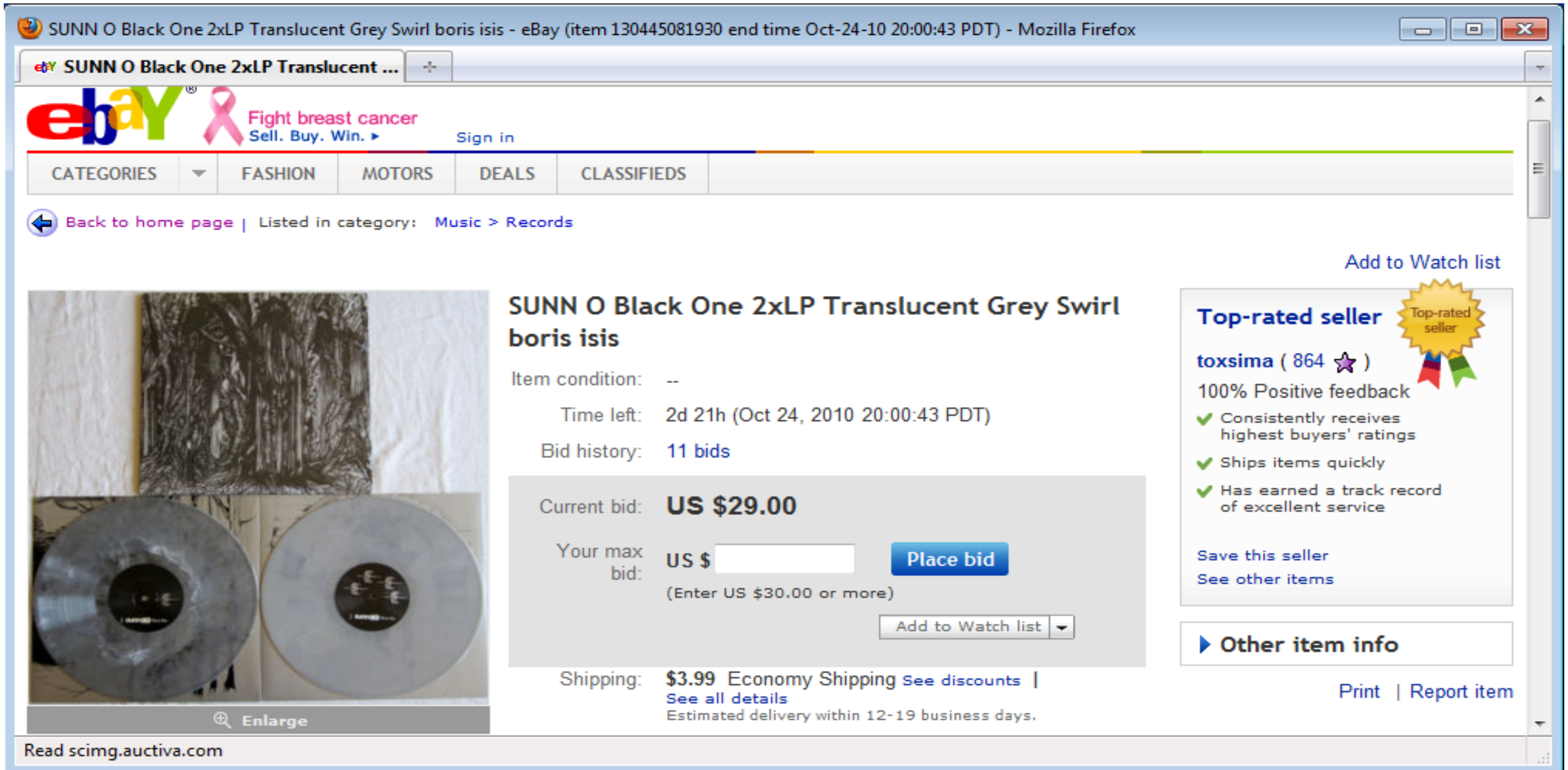
How To Think About Algorithmic Mechanism Design

[Tutorial at FOCS 2010]

Tim Roughgarden (Stanford)

An eBay Single-Good Auction



- winner = highest bidder above reserve price
- price = max{second-highest bid, reserve}

Truthful Auctions

Utility Model: bidder i has *valuation* v_i

- maximum willingness to pay
- known to bidder, unknown to seller

- *utility* = v_i - price paid; or 0 if loses auction
- submits *bid* b_i to maximize its utility

Claim: an eBay auction is *truthful*

- truthful bidding ($b_i = v_i$) is "foolproof"
- i.e., a false bid never outperforms a true bid

eBay Is Truthful

Fix player i, reserve r, other bids b_{-i}

Observation #1: bidder i effectively faces a "take-it-or-leave it" offer at a fixed price $p = \max\{\text{reserve, highest other bid}\}$.

Observation #2: truthful bidding guaranteed to maximize utility (a "dominant strategy")

- case 1: ($v \le p$) max utility = 0, achieved when $b = v$
- case 2: ($v \ge p$) max utility = $v-p$, achieved when $b = v$

Overarching Goals

- want to design "optimal" truthful mechanisms and auctions
  - for a wide range of problems
    - combinatorial auctions, scheduling, etc.
  - for different objectives (welfare, revenue)
  - often require polynomial running time as well
- general design techniques, analysis frameworks
- prove limits on what is possible

Why Truthful?

- many mechanisms "in the wild" not truthful
  - sponsored search, combinatorial auctions
  - important for practical implementations
- not clear when other mechanisms (with no dominant strategies) are fundamentally more powerful than truthful ones; sometimes have equivalence
  - e.g., "Revenue Equivalence" theorems
- truthful mechanisms definitely a good "first-cut abstraction" for foundations of mechanism design

How Theory CS Can Contribute

Unsurprising fact: very rich tradition and literature on mechanism design in economics.

- largely "Bayesian" (i.e., average-case) settings
- emphasizes exact solutions/characterizations
- usually ignores communication/computation

What we have to offer:

1. worst-case guarantees
2. approximation bounds
3. computational complexity

How To Think About Algorithmic Mechanism Design

Philosophy: designing truthful mechanisms boils down to designing algorithms in a certain "restricted computational model".

Next: focus on simple class of problems where this point is particularly clear and well understood.

Single-Parameter Problems

Outcome space: a set of vectors of the form
$(x_1, x_2, \ldots, x_n)$ [amount of "stuff" per player]

Utility Model: bidder i has private valuation v_i
(per unit of "stuff")

- utility = v_i x_i - payment
- submits bid b_i to maximize its utility

Examples: k-unit auction, "unit-demand" bidders; job scheduling on related machines

Mechanism Design Space

The essence of any truthful mechanism (formalized via the "Revelation Principle"):

- collect bid b_i from each player i
- invoke (randomized) *allocation rule*: b_i's $\longrightarrow$ x_i's
  - who gets how much (expected) stuff
- invoke (randomized) *payment rule*: b_i's $\longrightarrow$ p_i's
  - and who pays what
- truthfulness: for every i, v_i, other bids, setting $v_i = b_i$ maximizes expected utility $v_i\, x_i(\mathbf{b}) - p_i(\mathbf{b})$

Two Definitions

Implementable Allocation Rule: is a function x (from bids to expected allocations) that admits a payment rule p such that (x,p) is truthful.

- i.e., truthful bidding $[b_i := v_i]$ always maximizes a bidder's (expected) utility

Two Definitions

Implementable Allocation Rule: is a function x (from bids to expected allocations) that admits a payment rule p such that (x,p) is truthful.

- i.e., truthful bidding [$b_i := v_i$] always maximizes a bidder's (expected) utility

Monotone Allocation Rule: for every fixed bidder i, fixed other bids b_{-i}, expected allocation only increases in the bid b_i.

- example: highest bidder wins
- non-example: 2nd-highest bidder wins

Myerson's Lemma

Myerson's Lemma: [1981; also Archer-Tardos FOCS 01] *an allocation rule x is implementable if and only if it is monotone.*

Myerson's Lemma

Myerson's Lemma: [1981; also Archer-Tardos FOCS 01] *an allocation rule x is implementable if and only if it is monotone.*

Moreover: for every monotone allocation rule x, there is a unique payment rule p such that (x,p) is truthful and losers always pay 0.

Myerson's Lemma

Myerson's Lemma: [1981; also Archer-Tardos FOCS 01] *an allocation rule x is implementable if and only if it is monotone.*

Moreover: for every monotone allocation rule x, there is a unique payment rule p such that (x,p) is truthful and losers always pay 0.

Explicit formula for $p_i(b)$:

- keep b_{-i} fixed, increase z from 0 to b_i
- consider breakpoints $y_1,\ldots,y_q$ at which x_i jumps
- set $p_i(b) := \Sigma_j\, y_j \bullet$ [jump in x_i at y_j]

Myerson's Lemma (Proof Idea)

Proof idea: let x be an allocation rule, fix i and b_{-i}.
Write $x(z)$, $p(z)$ for $x_i(z, b_{-i})$, $p_i(z, b_{-i})$.

- apply purported truthfulness of (x,p) to two scenarios: true value = z, false bid = z + ε and true value = z + ε, false bid = z
- take ε to zero get
  - $p'(z) = z \circ x'(z)$ [if x differentiable at z] or
  - jump in p at z = z ∘ [jump in x at z]

Integrating from 0 to b_i, get sole candidate:

$$p_i(b) := \Sigma_j \, y_j \bullet [\text{jump in } x_i \text{ at } y_j]$$

Example: Profit Extractor

[Fiat/Goldberg/Hartline/Karlin STOC 02]

Allocation Rule: bids b + revenue target R:

- initialize S = all bidders
- while there is an i in S such that $b_i < R/|S|$:
  - remove such a bidder from S
- winners = final set S

Example: Profit Extractor

[Fiat/Goldberg/Hartline/Karlin STOC 02]

Allocation Rule: bids b + revenue target R:

- initialize S = all bidders
- while there is an i in S such that $b_i < R/|S|$:
  - remove such a bidder from S
- winners = final set S

Note: allocation rule is monotone.

By Myerson's Lemma: forms a truthful auction if and only if every winner charged price p = R/|S|

- if halts with non-empty set, raises revenue R

Revenue Maximization

Setting: k-item auction, n unit-demand bidders.

Goal: truthful auction with "optimal" revenue.

- but different auctions do better on different inputs

Revenue Maximization

Setting: k-item auction, n unit-demand bidders.

Goal: truthful auction with "optimal" revenue.

- but different auctions do better on different inputs

Approach #1: Bayesian/average-case analysis.

- "optimal" auction maximizes *expected* revenue

Approach #2: worst-case guarantee.

- "optimal" auction tricky to define, standard competitive analysis is useless
- use "Bayesian thought experiment" instead

Bayesian Profit Maximization

Example: 1 bidder, 1 item, v ~ known distribution F

- truthful auctions = posted prices p
- expected revenue of p: $p(1-F(p))$
  - given F, can solve for optimal p^*
  - e.g., $p^* = ½$ for v ~ uniform[0,1]
- but: what about k,n >1 (with i.i.d. v_i's)?

Bayesian Profit Maximization

Example: 1 bidder, 1 item, $v \sim$ known distribution F

- truthful auctions = posted prices p
- expected revenue of p: $p(1-F(p))$
  - given F, can solve for optimal p^*
  - e.g., $p^* = ½$ for $v \sim$ uniform[0,1]
- but: what about k,n >1 (with i.i.d. v_i's)?

need minor technical conditions on F

Theorem: [Myerson 81] auction with max expected revenue is Vickrey with above reserve p^*.

- note p^* is *independent of k and n*

Toward Worst-Case Analysis

Goal: prove approximation results of the form:

"Theorem: for every valuation profile v:
auction A's revenue on v is at least OPT(v)/α."
(for a hopefully small constant α)

Toward Worst-Case Analysis

Goal: prove approximation results of the form:

"Theorem: for every valuation profile v:
auction A's revenue on v is at least OPT(v)/α."
(for a hopefully small constant α)

Idea for OPT(v): sum of k largest v_i's.

Problem: too strong, not useful.

- makes all auctions A look equally bad.
- every auction A has a bad v [no finite α possible]

Bayesian Thought Experiment

Question: what would an i.i.d. Bayesian do?

- formulate prior F, run the optimal auction for F [by Myerson => Vickrey with suitable reserve]

Ambition: *design auction A that is simultaneously competitive with all Bayesian optimal auctions!*

I.e.: For every F, corresponding opt auction A_F:

A's expected revenue $\geq$ (A_F's expected revenue)/α

- [Bulow/Klemperer AER 96], [Hartline/Roughgarden EC 09], [Dhangwotnotai/Roughgarden/Yan EC 10]

Distribution-Free Benchmarks

Myerson: *for all F*, Vickrey + a reserve is optimal.

Corollary: *for all F and all* ***v****,* behavior of optimal auction for F equivalent to offering every bidder a common take-it-or-leave-it offer.

- namely: max{reserve price, (k+1)th highest bid of **v**}

Upper Bound: $RB(v) := \max_{i \leq k} i v_i$ [assume sorted v_i's]

By Design: if auction A achieves revenue RB(v)/α for every v, then it also has "simultaneous Bayesian" guarantee.

- [Goldberg/Hartline/Karlin/Saks/Wright GEB 06]
- [Hartline/Roughgarden STOC 08], [Devanur/Hartline EC 09]

Intermission

GO GIANTS!

Combinatorial Auctions (CA)

Setting: n bidders, m goods. Player i has private valuation $v_i(S)$ for each subset S of goods.

Assume: $v_i(\phi) = 0$ and v_i is

- *monotone*: S subset of T => $v_i(S) \leq v_i(T)$
- *subadditive*: $v_i(S \cup T) \leq v_i(S) + v_i(T)$
- ignore representation issues [want running time polynomial in n and m]

Facts: there is a poly-time 2-approximation for welfare $\Sigma_i v_i(S_i)$ [Feige STOC 06]. No good truthful approximation known.

Multi-Parameter Problems

Outcome space: an abstract set Ω

Utility Model: bidder i has private *valuation* $v_i(\omega)$ for each outcome ω

- *utility* = $v_i(\omega)$ - payment

Example: in a combinatorial auction, Ω = all possible allocations of goods to players

How To Think About Algorithmic Mechanism Design

Philosophy: designing truthful mechanisms boils down to designing algorithms in a certain "restricted computational model".

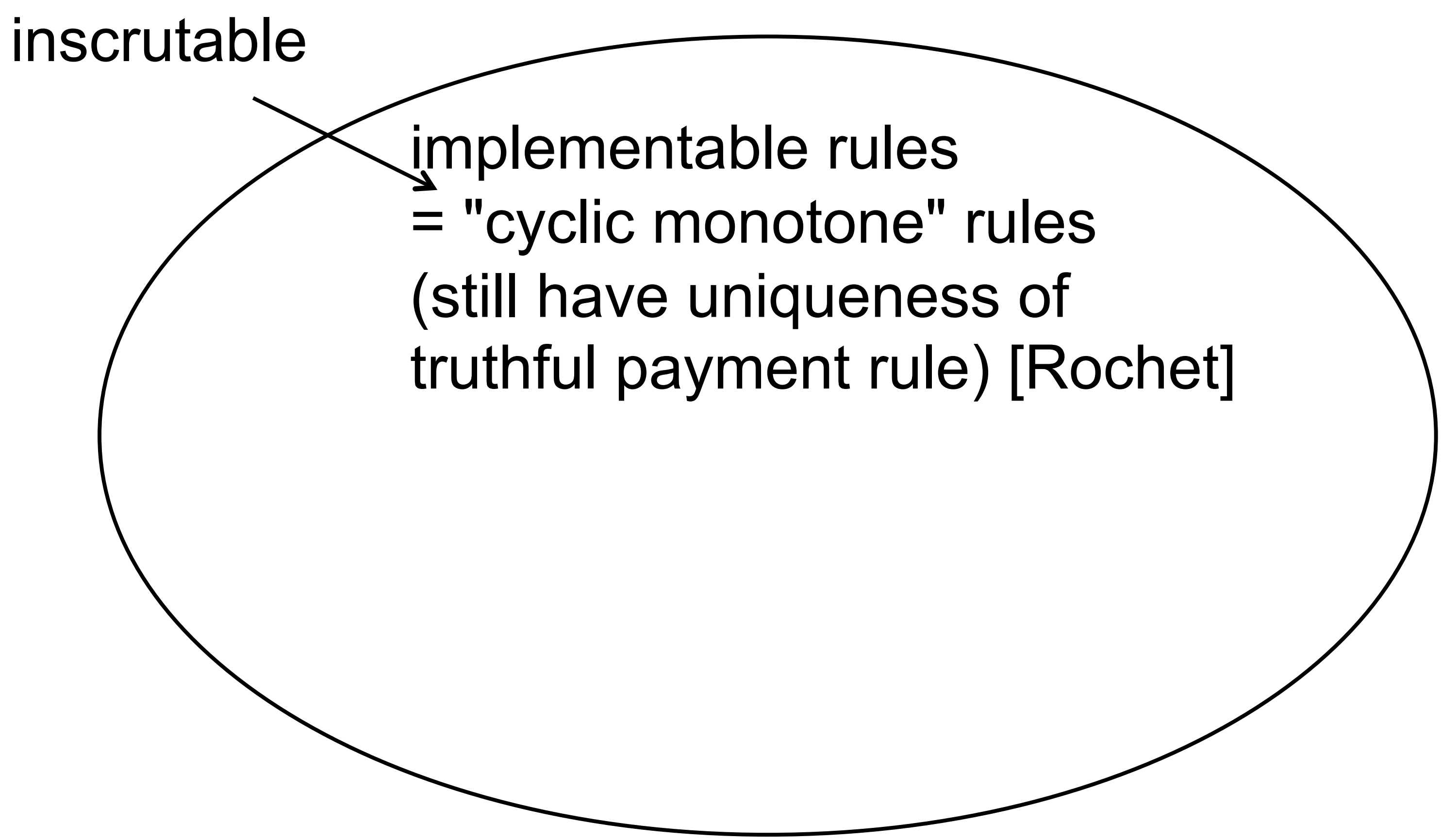
Single-Parameter Special Case:

implementable rules
= monotone rules
(Myerson's Lemma)

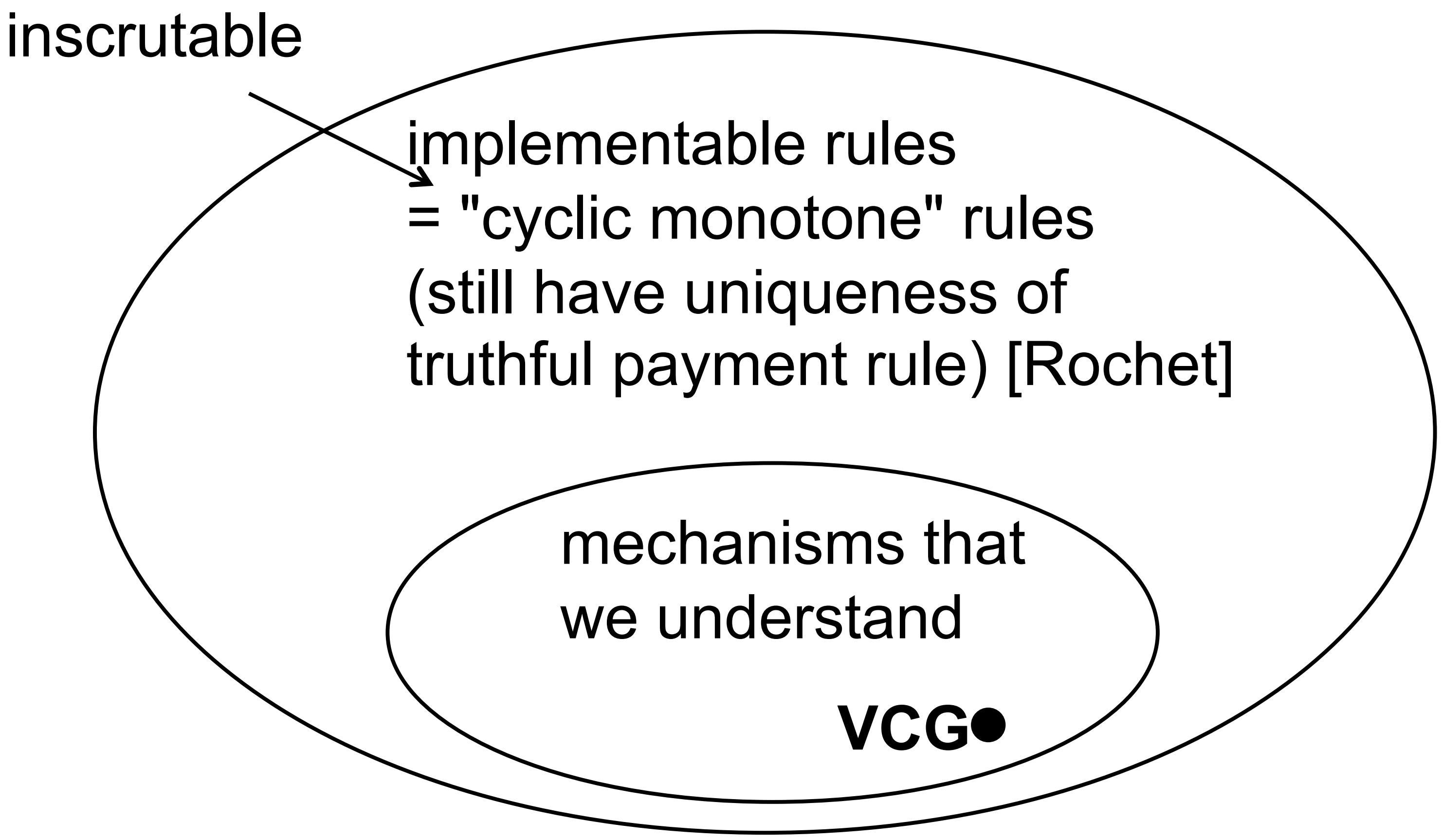
The Multi-Parameter World

implementable rules
= "cyclic monotone" rules
(still have uniqueness of
truthful payment rule) [Rochet]

The Multi-Parameter World



The Multi-Parameter World



The VCG Mechanism

Utility Model: bidder i's utility: $v_i(\omega)$ - payment

Vickrey-Clarke-Groves: (1961/71/73)

- collect bid $b_i(\omega)$ for all i, all outcomes ω in Ω
- select ω^* in argmax $\{\Sigma_i b_i(\omega)\}$
- charge $p_i = [-\Sigma_{j!=i} b_i(\omega)]$ + suitable constant
  - align private objectives with global one

Facts: truthful, maximizes welfare $\Sigma_i v_i(\omega)$ over Ω (assuming truthful bids).

Approximation Mechanisms

Assume: want to maximize welfare $\Sigma_i v_i(\omega)$

- revenue also interesting, wide open

Why Not VCG?: communication/computation lower bounds for many important problems.

- e.g., players = nodes of graph G;
- Ω = independent sets of G;
- $v_i(\omega)$ = 1 if i in ω, 0 otherwise

Goal: mechanisms that are (1) truthful; (2) run in time polynomial in natural parameters; and (3) guarantee near-optimal welfare

Approximation Mechanisms

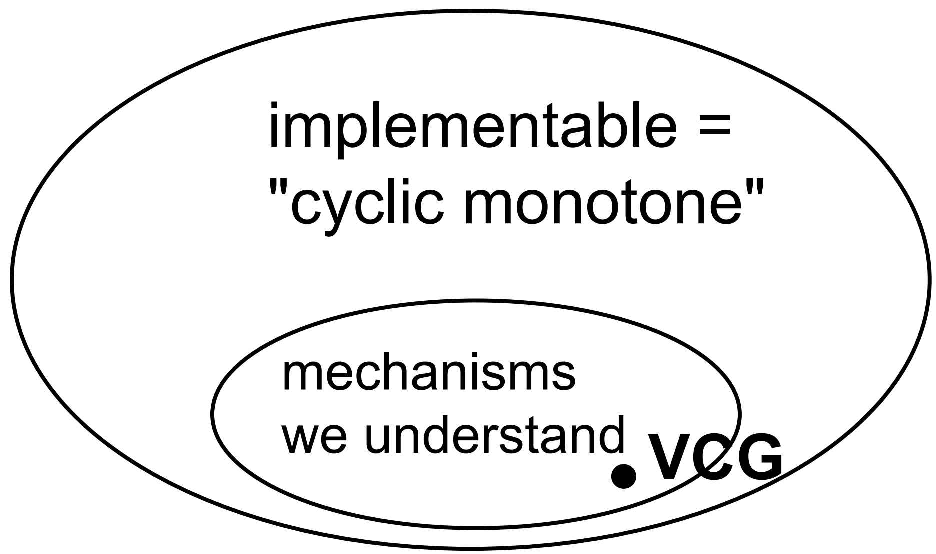
Goals: [Nisan/Ronen 99] (1) truthful; (2) run in time polynomial in natural parameters; and (3) guarantee near-optimal welfare

Best-case scenario: match approximation factor of best polynomial-time approximation algorithm (with valuations given freely as input).

Holy Grail: "black-box reduction" that turns an approximation algorithm into a truthful approximation mechanism.

Approximation Mechanisms

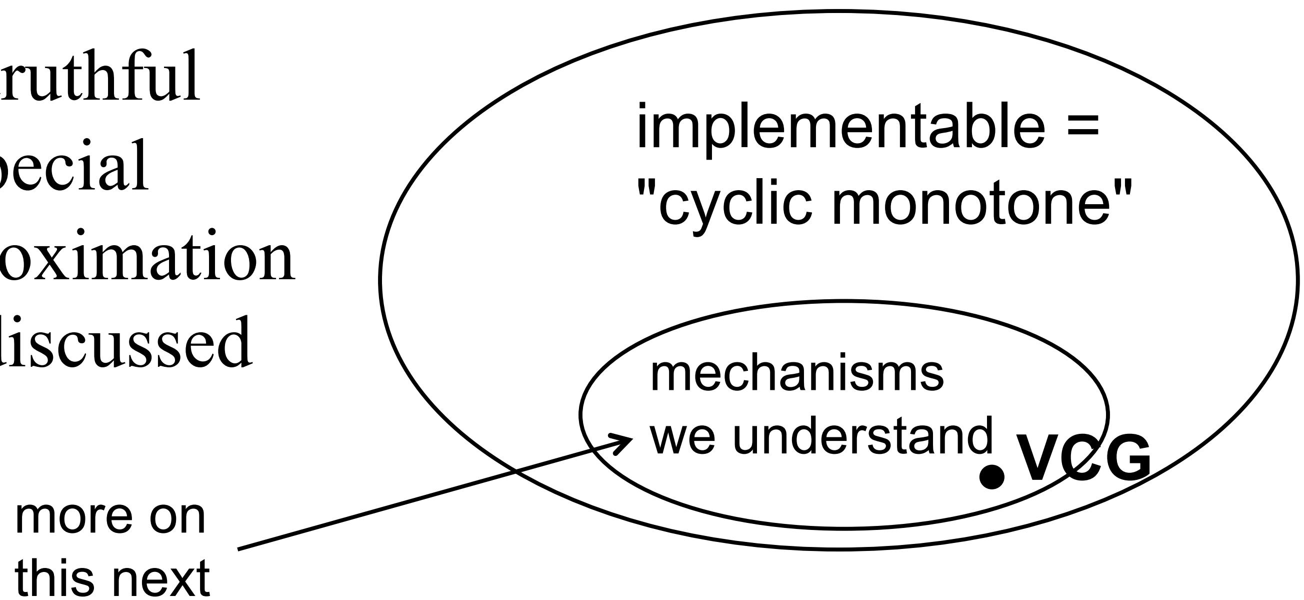
Idea: [Nisan/Ronen 00] use VCG mechanism but substitute approximation algorithm for the previous step "select ω^* in argmax $\{\Sigma_i\, b_i\,(\omega)\}$".



Approximation Mechanisms

Idea: [Nisan/Ronen 00] use VCG mechanism but substitute approximation algorithm for the previous step "select ω^* in argmax $\{\Sigma_i b_i(\omega)\}$".

Issue: only truthful for a very special type of approximation algorithm (discussed next).



VCG-Based Mechanisms

Outcome space: an abstract set Ω

Utility Model: bidder i's utility: $v_i(\omega)$ - payment

Step 1: pre-commit to a subset Ω' of Ω
Step 2: run VCG with respect to Ω'

Facts: truthful, maximizes welfare $\Sigma_i v_i(\omega)$ over Ω'

Hope: can choose Ω' to recover tractability while controlling approximation factor.

Combinatorial Auctions (CA)

Setting: n bidders, m goods. Player i has private valuation $v_i(S)$ for each subset S of goods.

Assume: $v_i(\varphi) = 0$ and v_i is

- *monotone*: S subset of T => $v_i(S) \leq v_i(T)$
- *subadditive*: $v_i(S \cup T) \leq v_i(S) + v_i(T)$
- ignore representation issues
  [want running time polynomial in n and m]

Fact: there is a 2-approximation for welfare $\Sigma_i v_i(S_i)$ [Feige STOC 06], but this allocation rule is not implementable.

VCG-Based Solution

Key Claim: for every instance, there is a $(1/2\sqrt{m})$-approximate allocation that either:

- assigns all goods to a single player; OR
- assigns at most one good to each player

VCG-Based Solution

Key Claim: for every instance, there is a (1/2√m)-approximate allocation that either:

- assigns all goods to a single player; OR
- assigns at most one good to each player

Corollary: [Dobzinski/Nisan/Schapira STOC 05] there is a truthful (1/2√m)-approximate mechanism for CAs with subadditive bidder valuations.

Proof: define Ω' as above; can optimize in poly-time via max-weight matching + case analysis.

VCG-Based Solution

Proof of Key Claim: Fix v_i's. Call a player *big* if it gets $> \sqrt{m}$ goods in the optimal allocation. (So there are at most $\sqrt{m}$ of them.)

Case 1: big players account for more than half of optimal welfare, so one big player accounts for a $1/2\sqrt{m}$ fraction. Give all goods to this player.

Case 2: otherwise, small players account for half. Give each its favorite good; by subadditivity, still have a $1/2\sqrt{m}$ fraction of optimal welfare.

Can We Do Better?

[Dobzinski/Nisan STOC 07]: Can't do much better using a deterministic VCG-based mechanism.

- results and techniques launched very active research agenda on lower bounds
  - [Papadimitriou/Schapira/Singer FOCS 08], ...

The good news: randomized mechanisms seem to hold much promise, for specific problems and for black-box reductions.

- some rigorous randomized vs. deterministic separations already known

Randomized VCG-Based Mechanisms

Step 1: precommit to subset Δ' of Δ(Ω)

- "lotteries" over outcomes

Step 2: run VCG with respect to Δ '

Facts: truthful (in expectation), maximizes expected welfare $E[\Sigma_i v_i(\omega)]$ over Δ'

Hope: can choose Δ ' to recover tractability while controlling approximation factor.

- [Lavi/Swamy FOCS 05], [Dobzinski/Dughmi FOCS 09]

A Black-Box Reduction

Theorem: [Dughmi/Roughgarden FOCS 10] If a welfare-maximization problem admits an FPTAS, then it admits a truthful FPTAS.

Proof idea: Choosing Δ ' suitably and "dualizing", the relevant optimization problem is a slightly perturbed version of the original one. Can use techniques from smoothed analysis [Roglin/Teng FOCS 09] to get expected polynomial running time.

Black-Box Reduction for Bayes-Nash Implementations

Theorem: [Hartline/Lucier STOC 10], [Bei/Hartline/Huang/Kleinberg/Malekian SODA 11] In many Bayesian settings (where valuations are drawn from known distributions), *every* approximation algorithm for welfare maximization can be transmuted into an equally good truthful (in Bayes-Nash equilibrium) approximation mechanism.

Suggestive: Bayes-Nash implementations might elude lower bounds for dominant-strategy truthful mechanisms (should such lower bounds exist).

Recap: Mechanism Design as Constrained Algorithm Design

Philosophy: designing truthful mechanisms boils down to designing algorithms in a certain "restricted computational model".

- single-parameter <=> monotone algorithms
- multi-parameter: includes all the obvious VCG variants, but what else?

Research Challenge: usefully characterize the implementable allocation rules for as many multi-parameter problems as possible.

Recap: Revenue Maximization

- Bayesian single-parameter case well solved
- worst-case guarantees for single-parameter problems: need novel analysis frameworks ("Bayesian thought experiment") but lots of recent progress

Research Challenges:

- non-i.i.d. version of Bayesian thought experiment
- (approximate) analog of Myerson's theory for multi-parameter problems (even relatively simple ones) [Bhattacharya et al STOC 10], [Chawla et al STOC 10]
- worst-case guarantees for multi-parameter problems

Recap: Welfare Maximization

- ignoring tractability, VCG works even for arbitrary multi-parameter problems
- truthful approximation mechanisms so far mostly restricted to randomized variants of VCG
- but this already enough for some interesting results

Research Challenges:

- better (randomized) approximation mechanisms for combinatorial auctions
- more general black-box reductions
- better lower bounds, especially for randomized mechanisms