CS364A: Algorithmic Game Theory: The Top 10 List*

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1. *The Vickrey auction.* Remember back when you first learned it and it seemed surprising or unnatural? This was our introduction to "awesome auctions" — auctions that are dominant-strategy incentive-compatible (DSIC) and run in polynomial time. Already in single-item auctions, we saw how small changes in design, such as a first-price vs. a second-price payment rule, can have major ramifications for participant behavior.

2. *Myerson's Lemma.* For single-parameter problems, DSIC mechanism design reduces to monotone allocation rule design. We saw many applications, including polynomial-time Knapsack auctions, and Myerson's Theorem stating that expected revenue maximization with respect to a prior distribution reduces to expected virtual surplus maximization.

3. *The Bulow-Klemperer Theorem.* In a single-item auction, adding an extra i.i.d. bidder is as good as knowing the underlying distribution and running an optimal auction. This result, along with the Prophet Inequality, was an important clue that simple and prior-independent auctions can be almost as good as optimal ones.

4. *The VCG Mechanism.* Charging participants their externalities yields a DSIC welfare-maximizing mechanism, even in very general settings.

5. *Spectrum auctions.* Rookie mistakes in real-world auction design can cost hundreds of millions of dollars. Examples include selling items sequentially (as in a 2000 spectrum auction in Switzerland) or simultaneously using sealed-bid (instead of ascending) auctions (as in a 1990 spectrum auction in New Zealand).

6. *Selfish routing.* Worst-case examples are always simple, with Pigou-like networks maximizing the price of anarchy (POA). The POA of selfish routing is therefore small only

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when network cost functions are highly nonlinear, corroborating empirical evidence that network over-provisioning leads to good network performance.

7. *Robust POA Bounds.* Most of the canonical POA bounds, including all of those in this course, are proved via smoothness arguments. As such, they apply not only to Nash equilibria but also extend automatically to more permissive equilibrium concepts, including coarse correlated equilibria.

8. *Potential games.* Many games of interest possess potential functions — players are inadvertently and collectively striving to optimize a potential function. In such games, pure Nash equilibria (PNE) always exist, and best-response dynamics always converges.

9. *No-regret algorithms.* No-regret algorithms exist, including simple ones with optimal regret bounds, like the multiplicative weights algorithm. When players use no-external- or no-swap-regret algorithms in games played over time, the joint history of play converges to the sets of coarse correlated equilibria (CCE) or correlated equilibria (CE), respectively. In this sense, CCE and CE are "highly tractable;" so are mixed Nash equilibria of two-player zero-sum games.

10. *Complexity of equilibrium computation.* Nash equilibria do not seem to be efficiently computable in general. Two analogs of NP-completeness —- PLS-completeness and $PPAD$-completeness — are the right tools for making precise these assertions for pure and mixed Nash equilibria, respectively.