Dalal's Revision in Sentential Decision Diagrams

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Belief revision denotes the operation of partially modifying a knowledge-based system to consistently incorporate new information that has become available. General well-known standard rationality principles for belief revision have been introduced by Alchourrón, Gärdenfors, and Makinson (1985). Several authors have subsequently presented specific revision operators satisfying that framework.

When considering a finite propositional language, and thus when a knowledge-based system may be identified with a single formula, Katsuno and Mendelzon (1989) have outlined an equivalent set of postulates that a revision operator should satisfy to be considered as rational. However, in such finitary setting, rationality may not be the only desideratum. More specifically, when one is interested in implementing a belief operation in real-world cases, availability of properties such as compactness and tractability of the used representation formalism may become particularly relevant.

Originally presented by Darwiche (2011), *Sentential Decision Diagrams* (SDDs) are a powerful yet compact and empirically efficient representation formalism for propositional knowledge bases. They can be regarded as a generalization of *Ordered Binary Decision Diagram* (OBDDS) (Bryant 1992), sharing the canonicity property and the fact that Boolean combinations take only polynomial time. Fast compilation schemes have been derived for SDDs (Oztok and Darwiche 2015), making the resulting models typically smaller than the corresponding OBDDs (Bova 2016).

As a matter of fact, the analysis of belief revision on specific representation formalisms received little attention. To the best of our knowledge the only study of this type has been performed by Gorogiannis and Ryan (2002) for so called *Binary Decision Diagrams* (BDDs), a very general class of models encompassing OBDDs, but not SDDs. We discuss an analogous analysis with respect to SDDs, when the specific belief operator corresponds to the one introduced by Dalal (1988). There, Dalal defines a semantic measure for minimal change, whose idea, roughly, is to change the smallest number of propositional variables, and introduces a syntactic revision operator that behaves accordingly. The technique proposed by Dalal has however several drawbacks. In particular, it requires multiple satisfiability checks, a notorious NP-complete task. We partially overcome such difficulty by adapting Dalal's revision to the particular representation language under consideration. In doing so, we show that the computation of *resolvents*, the units on which the syntactic revision operator is built, is polytime in SDDs. The same holds for the satisfiability check, because of the linear complexity of model counting in SDDs. Finally, we deliver an algorithm implementing Dalal's revision.

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