Beano!!!!

Probability-based Bingo using legumes and a pair of dice

Overview and NCTM Standard Objectives

This game is intended for 9th grade mathematics students.

This game is intended to provide students with an opportunity to develop a working knowledge of probability based on comparisons of empirical observations with theoretical probability distributions. Students must work in teams to formulate and articulate winning strategies based on conclusions drawn from the relative frequency of certain events, their intuitive grasp of likelihood and probability, and a theoretical analysis of the game's probability distribution.

This game helps teachers meet the following NCTM standards:

Content- Data Analysis and Probability
Process- Reasoning and Proof, Communication, Connections

Specifically regarding Data Analysis and Probability NCTM objectives, this game will help enable students to:

- *Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them.* This will occur through the collection of data during the game, the display of data using graphs, and the formulation of strategy based on the graphical representation of data.

- *Develop and evaluate inferences and predictions that are based on data through the use of simulations to explore the variability of sample statistics from a known population and to construct sampling distributions.* This game should help students understand how sample statistics reflect the values of population parameters and how to use sampling distributions as the basis for informal inference;

-*Understand and apply basic concepts of probability*. This game will help students understand the concepts of sample space and probability distribution, construct sample spaces and distributions in simple cases, and use simulations to construct empirical probability distributions.

Materials

one bag of dried beans (pinto, black, or lima, etc., enough for twelve beans per student)
two 6-sided dice
Activity Worksheet (one per every two students)

Overview of Game-Play

Each student is given twelve beans and receives a game-sheet with spaces numbered 2-12. The game is similar to bingo. At the start of the game, the students can place their beans on the board in whichever spaces they chose, but are not allowed to move the beans around once the game has started. The teacher will then roll a pair of dice and announce the *sum* of the numbers rolled. The students will then play "Bingo" (Beano") based on the numbered called, removing a bean from the appropriate space.   The first team to remove all their beans wins.

Activity Instructions

1. Let students divide themselves into teams of two. Have them group their desks together or sit together.

2. Give each pair of students a worksheet and 12 beans.

3. On the worksheet are the integers 2 through 12. Tell the students to distribute their beans however they wish among the numbered slots.   For example, students may choose to place all their beans on the number 4, or to distribute their beans among the numbers. Let teammates talk to each other as they decide how to distribute their beans.

Initially, students may decide to distribute their beans over the entire board equally based on the idea that every number has an equal probability of being called.   This method of distribution should change as students realize that the outcome of 2 and 12 are less likely to be called than the other numbers.

4. The teacher will now roll the pair of dice and announce the sum of the two numbers displayed on the die. This number should be systematically listed on the board. This task can also be given to a student without a partner.

5. Looking at their worksheets, the students should now remove a single bean from the column representing the number called out. If the students do not have any beans on the number that was called out, they cannot remove a bean.

6. The teacher will roll the dice again, and the cycle of removing a bean should be repeated.

7. A team wins if they are able to remove all their beans from their board. They can yell "Beano!" if they win and you may decide to give them a small prize.

8. Once a team has won, stop the game and tell the students to replace their beans on their board. Give them a few moments to discuss how they want to distribute their beans on the board. Students should observe the distribution of numbers written on the board, and formulate a strategy for placing their beans in the spots most likely to be called. Encourage teammates to talk to each other and explain their reasoning for how they should place their beans.

9. The game should be played again. Students should talk to each other, note the frequency of certain numbers, and tend to distribute their beans closer to the middle of the number line. Once a winner has been announced for the second game, the students should be directed to do the activity on the second page of their worksheet. Walk around the classroom to assist as necessary.

10. The second page of the worksheet shows a chart, where the students should list all possible combinations that can result from rolling two dice.   It also has a space for the students to graph the probabilities of each outcome from two to twelve.   Help the students fill in the matrix and graph the results. Remind them of the definition of probability, defined as:

P(desired event) = (# of desired outcomes)/(total possible outcomes)

Be sure that the students differentiate between the two dice, so that they understand that there are 36 possible outcomes. You'll need to make a distinction between the outcomes (defined as a certain number on the first dice and a certain number on the second dice) and the possible sums of the two dice (2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12).

You may choose to do the worksheet on the board after a few minutes to ensure that the students have a clear picture of the probability distribution of the possible sums.

The third page of the worksheet is a series of short answer questions–this will be completed after two more rounds of the game.

11. Have the kids place the beans on their boards again. Have them talk with their teammates about how they want to distribute their beans. Play the game until there is a winner.

A few different strategies may result from the worksheet activities. After seeing that 7 has the highest probability of being called, student may choose to place all their beans on the number 7. Students with a deeper understanding may realize that 6, 7, and 8 are commonly called and will choose to distribute their beans close to those numbers. Overall, as students develop a strategy, the teams should begin to place their beans closer to the middle of the board. Interestingly, a winning strategy will still involve some

amount of spread, and students may realize this after the third game and choose to change their distribution strategy.

12. Announce that this is the final game. Now the student will have had an opportunity to see how their strategy worked based on how their theoretical results worked out. This final game will give them a chance to combine their knowledge from the results of their previous games and their worksheet to create a final strategy. A prize or some other incentive for the winner might be useful.

13. Once the final game is finished, have the students respond to the short-answer questions on the final page of their worksheets. This can be given as homework and their responses will allow you to evaluate their conceptions of probability after the game and how their strategies developed.

Beano! Questions and Solutions for Teachers

Evaluation Question Answers

1. Based on your graph of probabilities, which outcome is most likely?

    Students should note that 7 is the most likely outcome. The probability that a 7 occurs for a given roll of the dice is 6/36 = 1/6.

2. During the actual game, which 3 outcomes appeared most often?

    It may differ between different games, but the outcomes that will generally appear most often are those near the middle. These are 7, 6 & 8, and 5 & 9. However, there may be a wide spread. The students must reference the list of outcomes that you have written on the board.

3. Describe the way your strategy changed over the course of the game:

    The answer could be something along the lines of "after a while, I put all of the beans on or near 7", or "after playing, I did not put beans on the higher and lower numbers, but closer to the middle". There should be some reference to placing beans on the numbers that appeared more likely or the numbers that were rolled most often. A winning strategy involves a combination of choosing the spread and the center of their bean distribution.

4. After playing this game, what does probability mean to you?

    This is really a chance for students to just give their opinion of probability…hopefully; they will begin to understand that probabilities such as 6/36 do not mean that an outcome will occur exactly 6 times out of every 36 throws. They should begin to get a sense of probabilities as long-term percentages and of the difference between the probability distributions that they calculated and the relative frequency of numbers that occur during a short number of trials. The students should also talk about uncertainty, the idea of probability as a way of describing likelihood, and its usefulness as a tool for making decisions

Questions during game time

1. Are any of you noticing a pattern here?
2. Does any number seem to be rolled a lot?

Debrief Questions / Summary

We have now learned some of the basics of probability. We have learned that probability = # desired / total possible.

1. Why are there certain numbers that occur more often than others? Which numbers occur least often?
2. Should you put all of your beans underneath 7, since it has the highest probability? Why should/should not you place your beans under 2 or 12? Should you spread out your beans with ~one bean underneath each number, or is there another strategy that you used?

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2	3	4	5	6	7	8	9	10	11	12

Directions

1. Take the 12 beans that have been given to you, and place them under the numbers listed above. You can place them all under one number, or spread them out however you like.
2. The dice-roller will roll a pair of dice, and then call out the sum of the two n umbers rolled. If you have a bean underneath the number he calls out, take it off the piece of paper and place it next to you on the desk.
3. The first person to remove all their beans wins! You have to shout Beano! if you win.

Probability Worksheet

Fill in the chart. Find sum of the number in the column + number in the row:
(A few boxes have been filled in for you as an example)

	1	2	3	4	5	6
1	2					
2						
3						
4						
5		7				
6						12

Now make a bar chart of the sum (x-axis) vs. the frequency of each sum (y-axis):

How many times does the sum occur in the previous chart?

2	3	4	5	6	7	8	9	10	11	12

Sum

Evaluation Questions

1. Based on your graph, which outcome is most likely? What is the probability of rolling the dice and getting that sum?

2. During the actual game, which 3 outcomes appeared most often?

3. Describe the way your strategy changed over the course of the game:

4. After playing this game, what does probability mean to you? If you want to win, should you arrange your beans to match your graph?