Highly Scalable Parallel Sorting

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Outline

- Parallel sorting background
- Histogram Sort overview
- Histogram Sort optimizations
- Results
- Limitations of work
- Contributions
- Future work

Parallel Sorting

- Input
  - There are ***n*** unsorted keys, distributed evenly over ***p*** processors
  - The distribution of keys in the range is unknown and possibly skewed
- Goal
  - Sort the data globally according to keys
  - Ensure no processor has more than ***(n/p)+threshold*** keys

Scaling Challenges

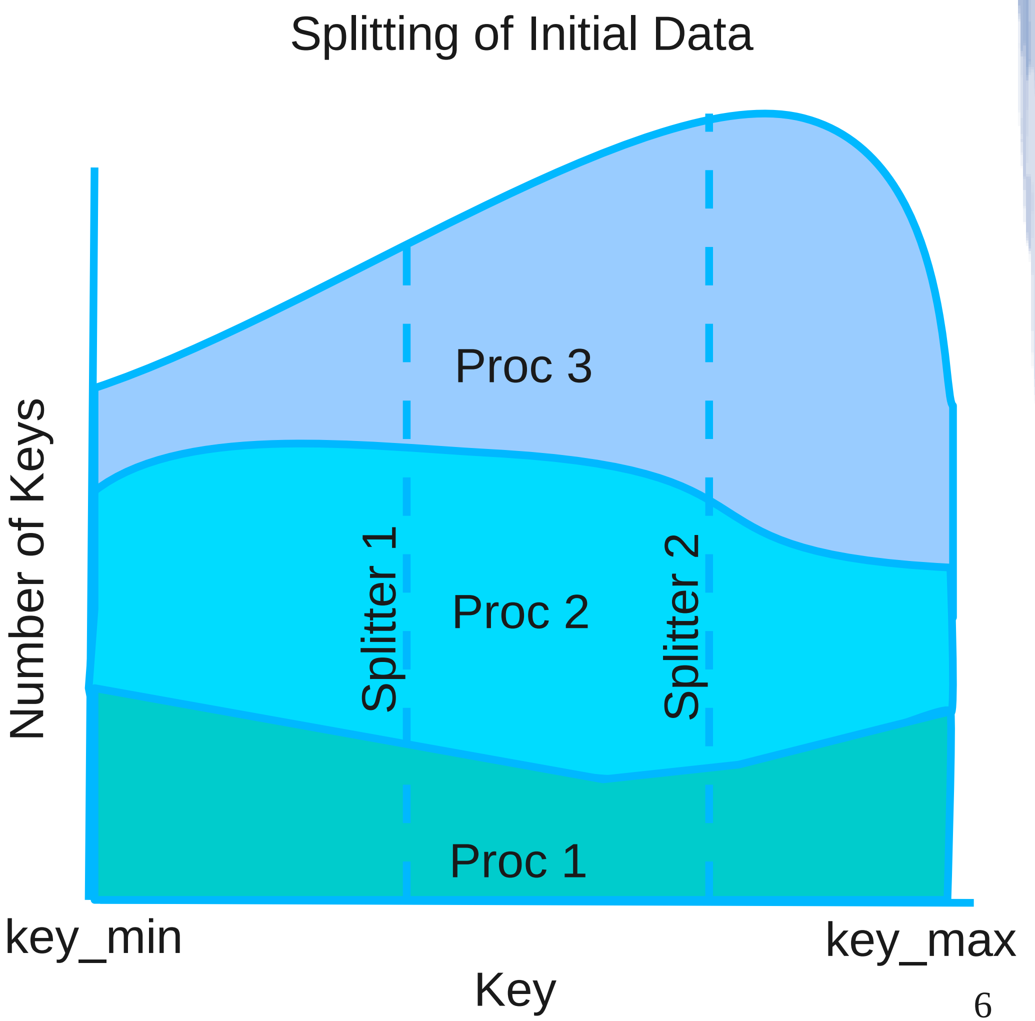
- Load balance
  - Main objective of most parallel sorting algorithms
  - Each processor needs a continuous chunk of data
- Data exchange communication
  - Can require complete communication graph
  - All-to-all contains $\boldsymbol{n}$ elements in $\boldsymbol{p}^2$ messages

Parallel Sorting Algorithms

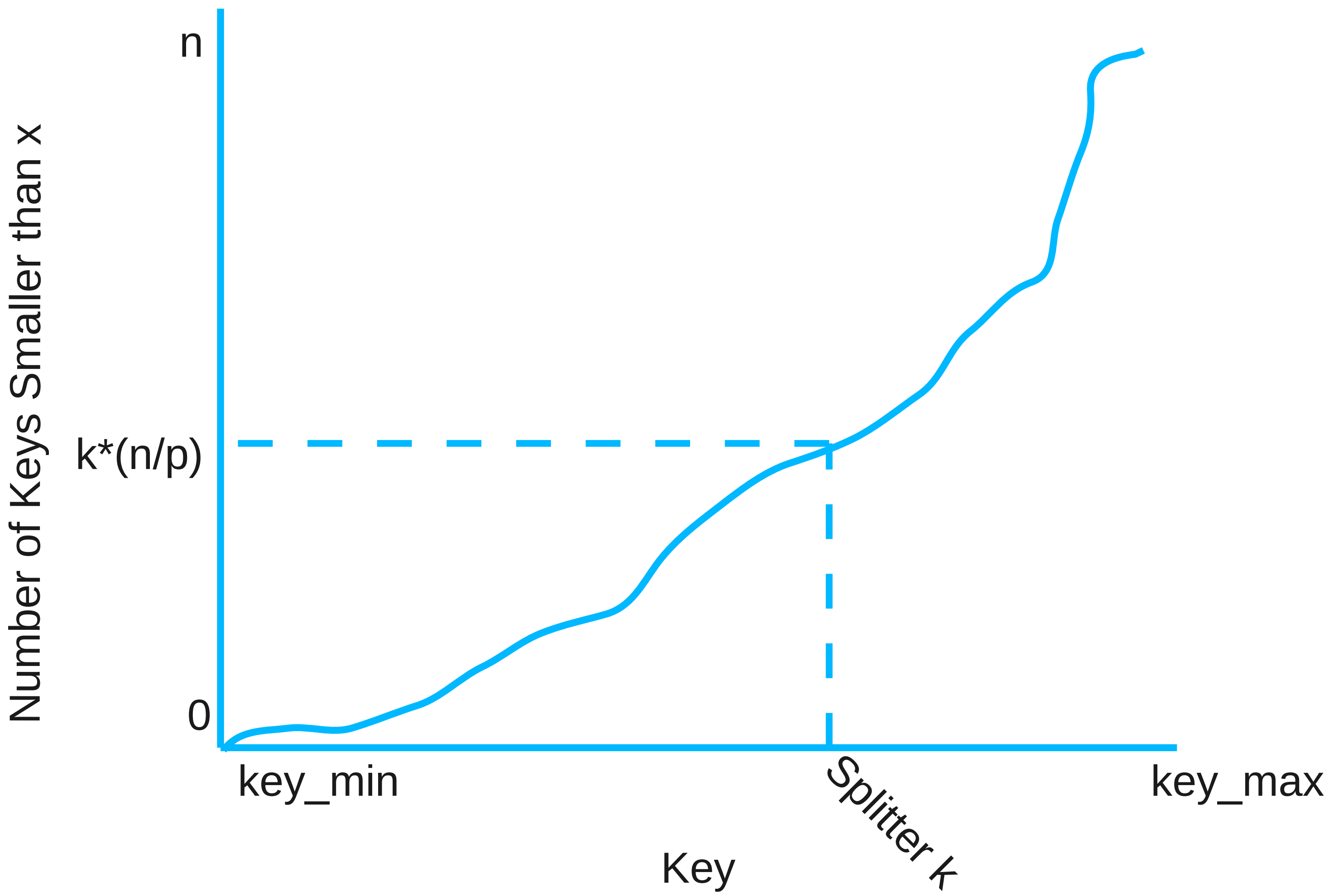
Type	Data movement
• Merge-based	
– Bitonic Sort	***$\frac{1}{2}*n*log^2(p)$***
– Cole's Merge Sort	***$O(n*log(p))$***
• **Splitter-based**	
– Sample Sort	***n***
– **Histogram Sort**	***n***
• Other	
– Parallel Quicksort	***$O(n*log(p))$***
– Radix Sort	***$O(n)$~$4*n$***

Splitter-Based Parallel Sorting

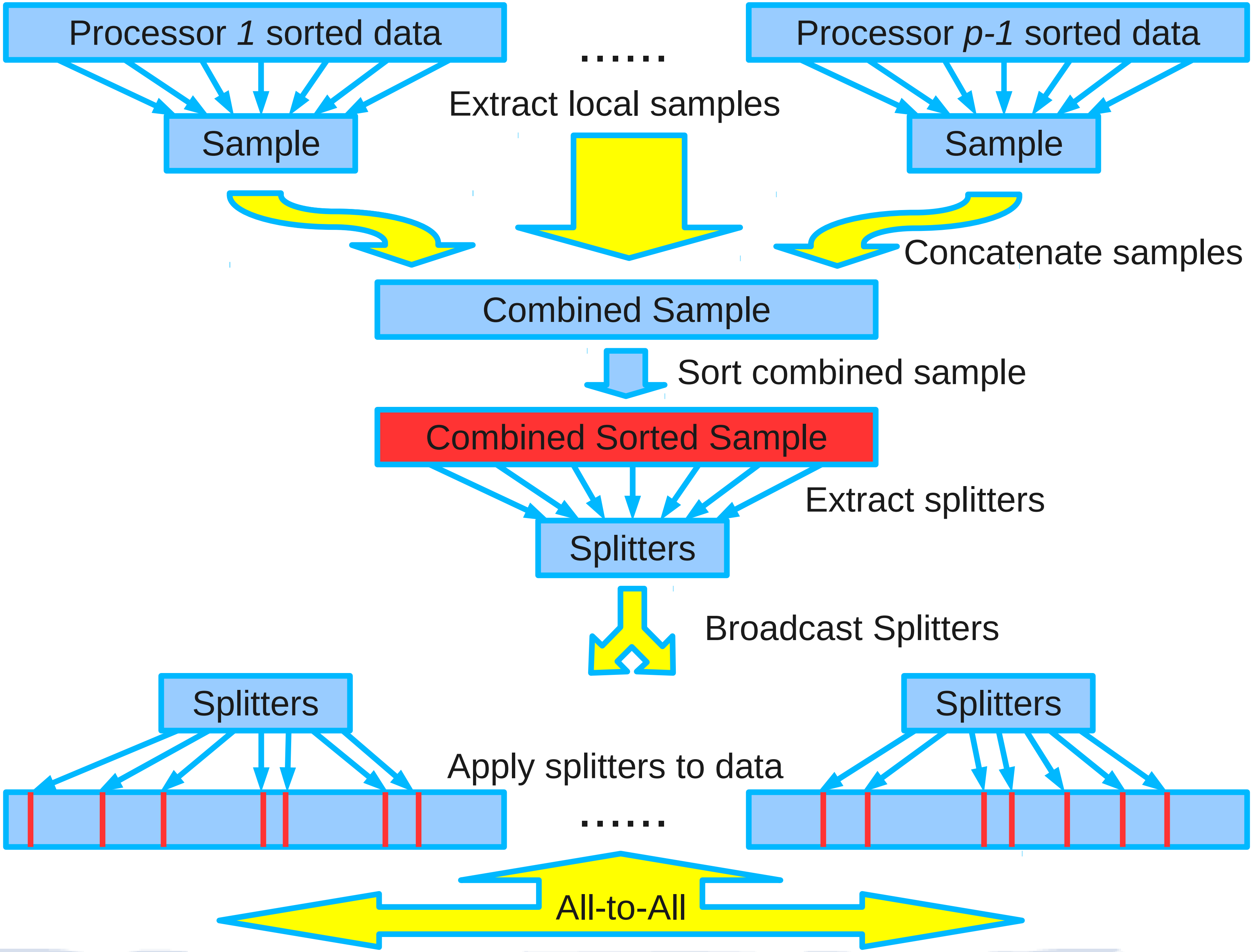
- A ***splitter*** is a key that partitions the global set of keys at a desired location
- ***p-1*** global splitters needed to subdivide the data into ***p*** continuous chunks
- Each processor can send out its local data based on the splitters
  - ***Data moves only once***
- Each processor merges the data chunks as it receives them



Splitter on Key Density Function



Sample Sort



Processor *1* sorted data
……
Processor *p-1* sorted data
Extract local samples
Sample
Sample
Concatenate samples
Combined Sample
Sort combined sample
Combined Sorted Sample
Extract splitters
Splitters
Broadcast Splitters
Splitters
Splitters
Apply splitters to data
……
All-to-All

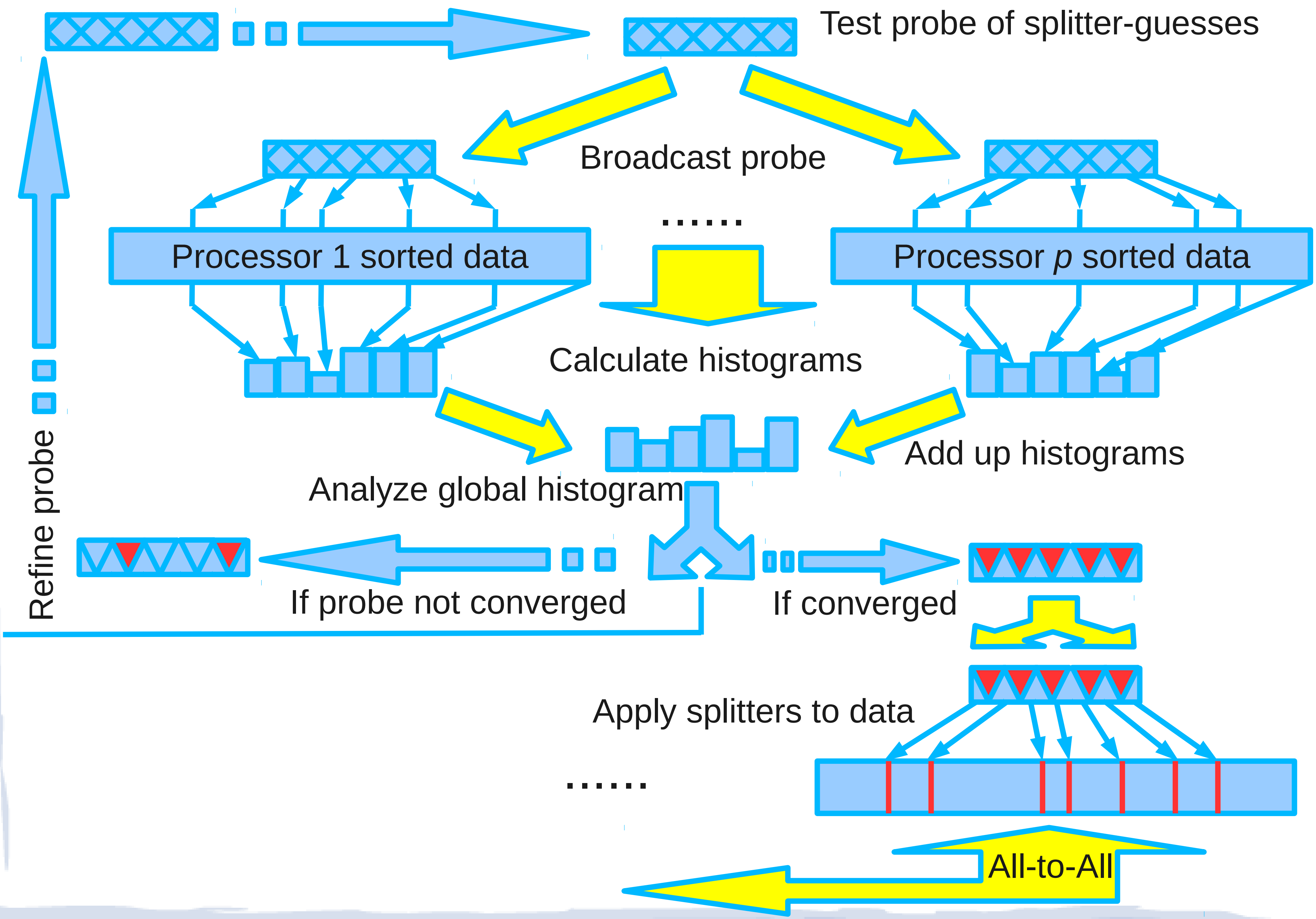
Sample Sort

- The sample is typically regularly spaced in the local sorted data ***s=p-1***
  - Worst case final load imbalance is ***2*(n/p)*** keys
  - In practice, load imbalance is typically very small
- Combined sample becomes bottleneck since ***(s*p)~p²***
  - With *64*-bit keys, if ***p = 8192***, sample is **16 GB**!

Basic Histogram Sort

- Splitter-based
- Uses iterative guessing to find splitters
  - ***O(p)*** probe rather than ***O(p²)*** combined sample
  - Probe refinement based on global histogram
    - Histogram calculated by applying splitters to data
- Kale and Krishnan, ICPP 1993
- Basis for this work

Basic Histogram Sort



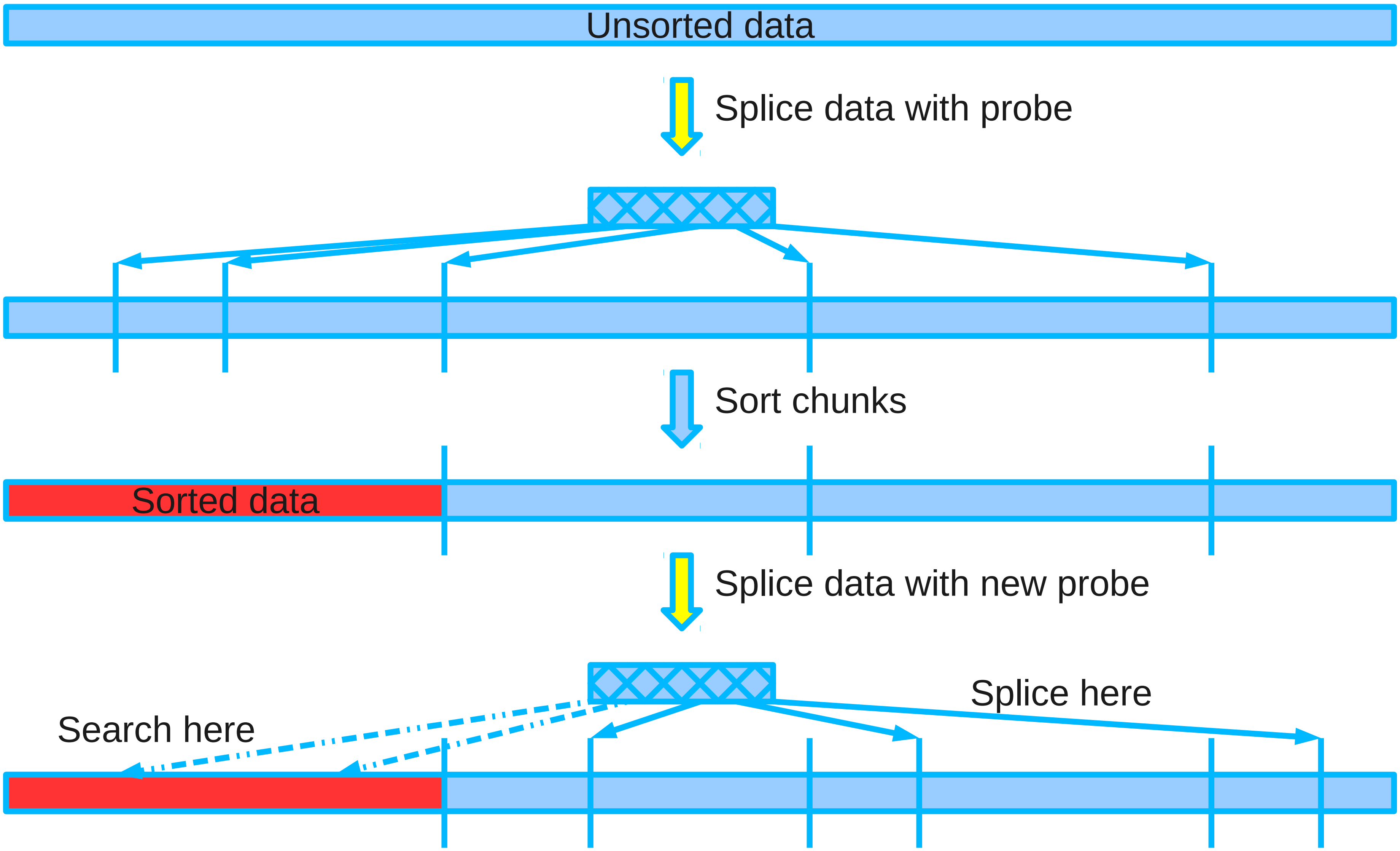
Basic Histogram Sort

- Positives
  - Splitter-based: single all-to-all data transpose
  - Can achieve arbitrarily small ***threshold***
  - Probing technique is scalable compared to sample sort, ***O(p)*** vs ***O(p²)***
  - Allows good overlap between communication and computation (to be shown)
- Negatives
  - Harder to implement
  - Running time dependent on data distribution

Sorting and Histogramming Overlap

- Don't actually need to sort local data first
- ***Splice data*** instead
  - Use splitter-guesses as Quicksort pivots
  - Each splice determines location of a guess and partitions data
- Sort chunks of data while histogramming happens

Histogramming by Splicing Data



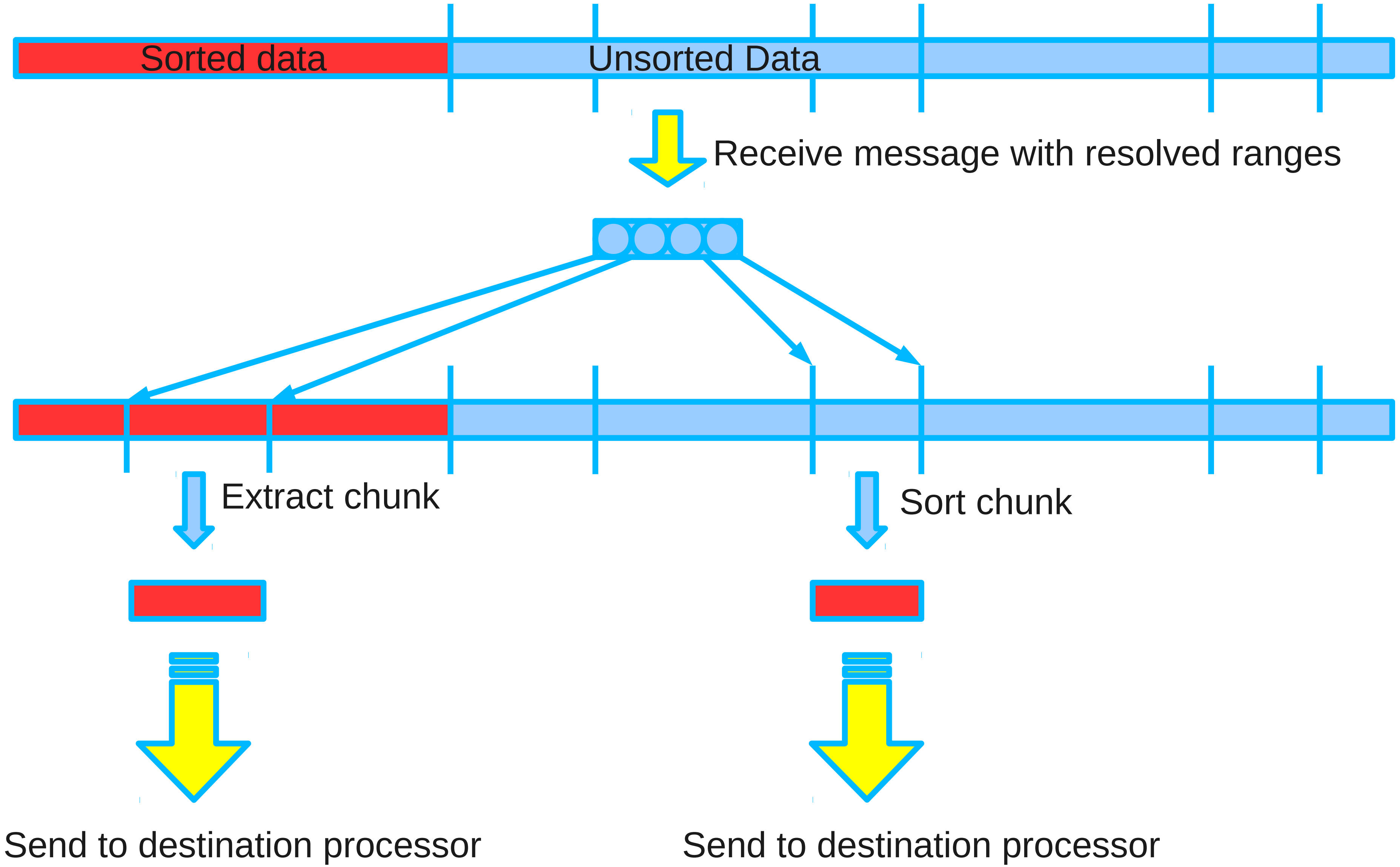
Histogram Overlap Analysis

- Probe generation work should be offloaded to one processor
  - Reduces critical path
- Splicing is somewhat expensive
  - ***O((n/p)*log(p))*** for first iteration
    - ***log(p)*** approaches ***log(n/p)*** in weak scaling
  - Small theoretical overhead (limited pivot selection)
  - Slight implementation overhead (libraries faster)
  - Some optimizations/code necessary

Sorting and All-to-All Overlap

- Histogram and local sort overlap is good but the all-to-all is the worst scaling bottleneck
- Fortunately, much all-to-all overlap available
- All-to-all can initially overlap with local sorting
  - Some splitters converge every histogram iteration
    - This is also prior to completion of local sorting
    - Can begin sending to any defined ranges

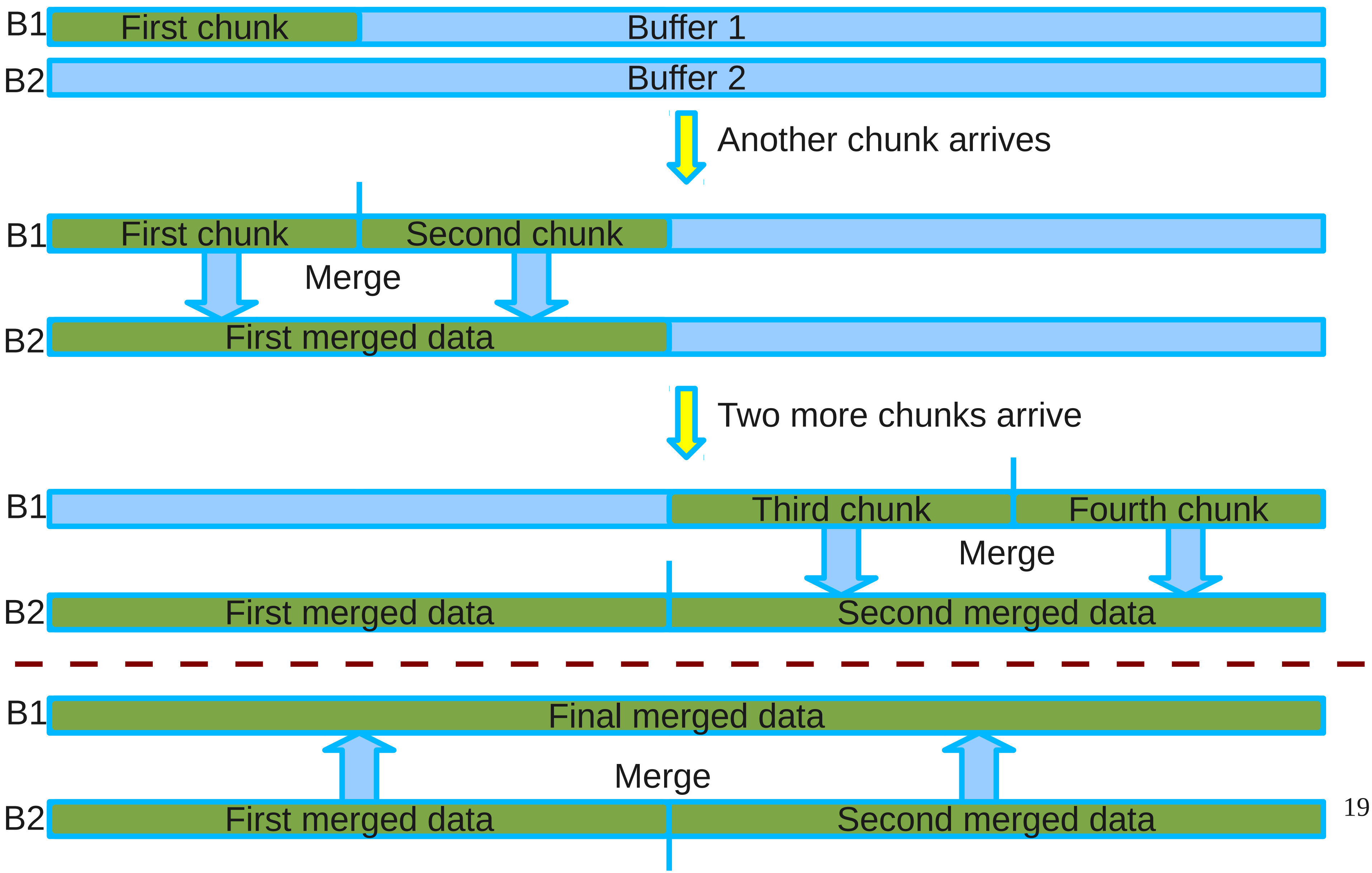
Eager Data Movement



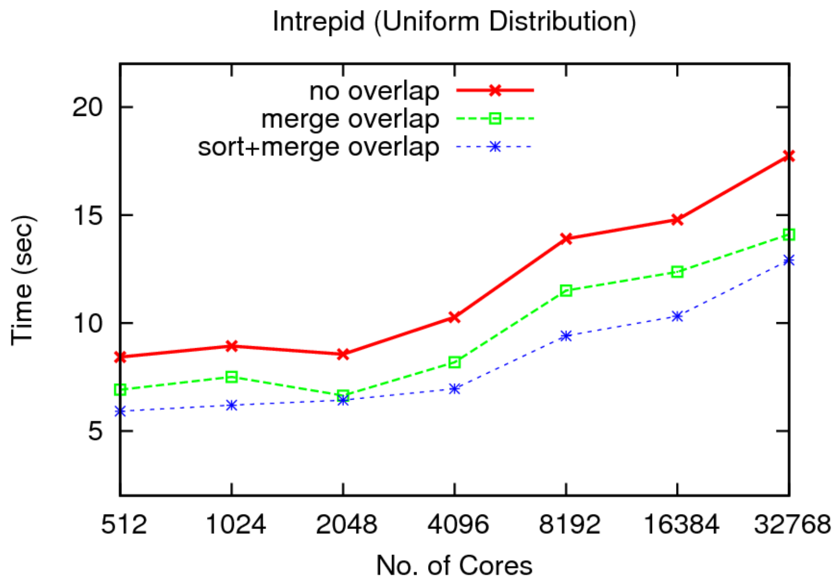
All-to-All and Merge Overlap

- The *k*-way merge done when the data arrives should be implemented as a tree merge
  - A *k*-way heap merge requires all *k* arrays
  - A tree merge can start with just two arrays
- Some data arrives much earlier than the rest
  - Tree merge allows overlap

Tree k-way Merging



Overlap Benefit (Weak Scaling)



Tests done on Intrepid (BG/P) and Jaguar (XT4) with 8 million 64-bit keys per core.

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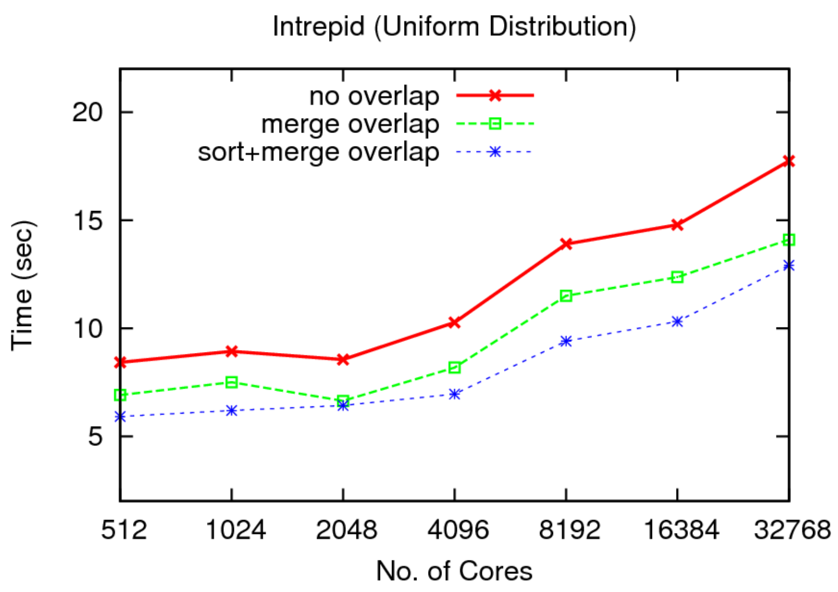

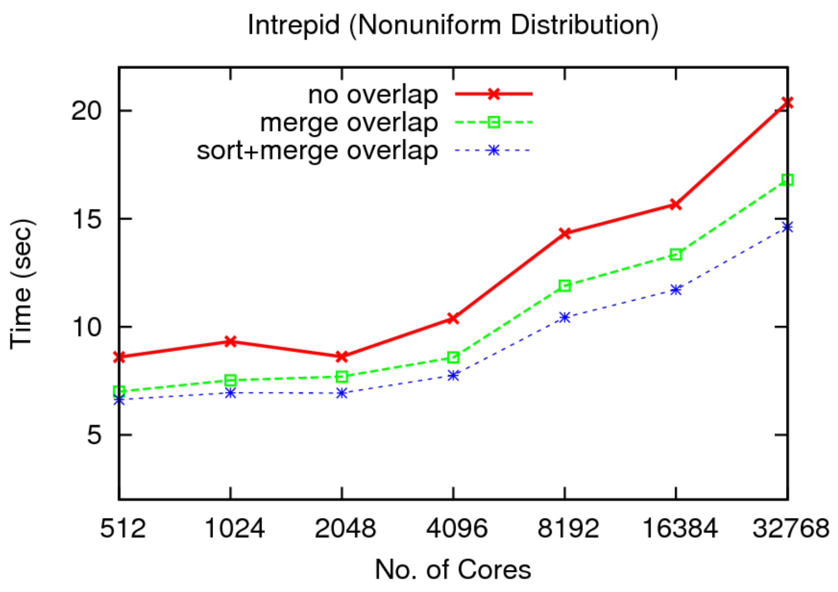

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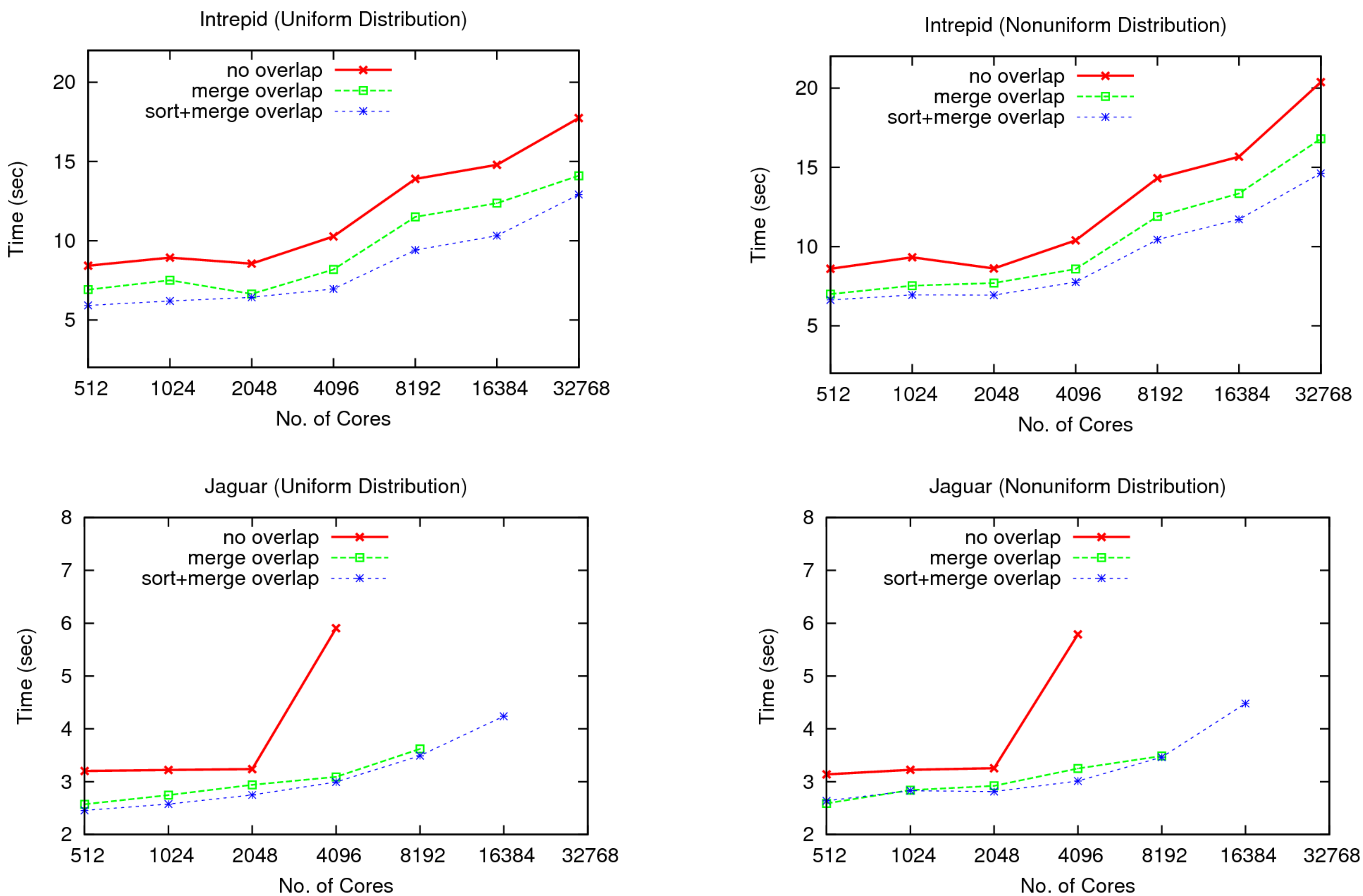

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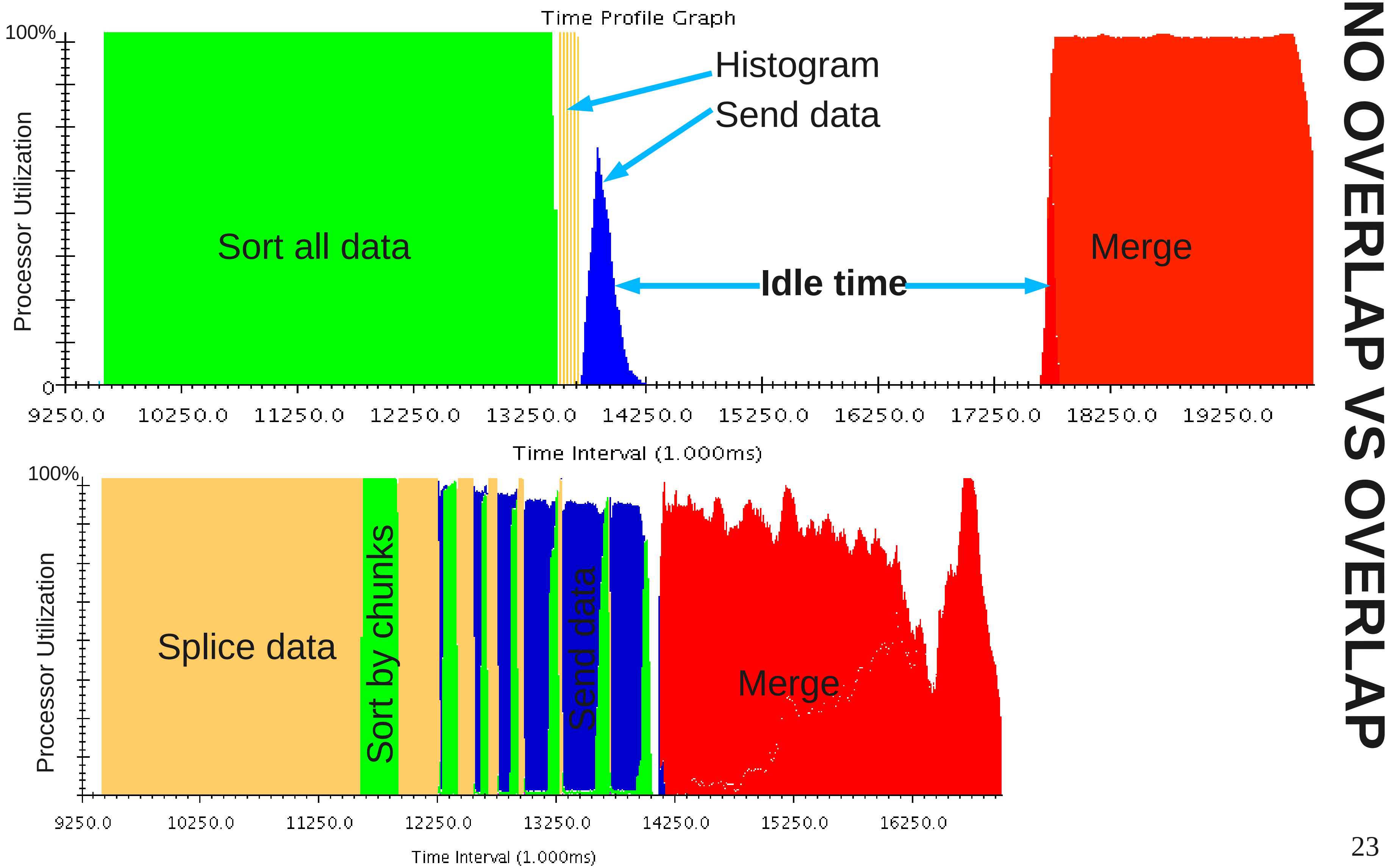
Effect of All-to-All Overlap

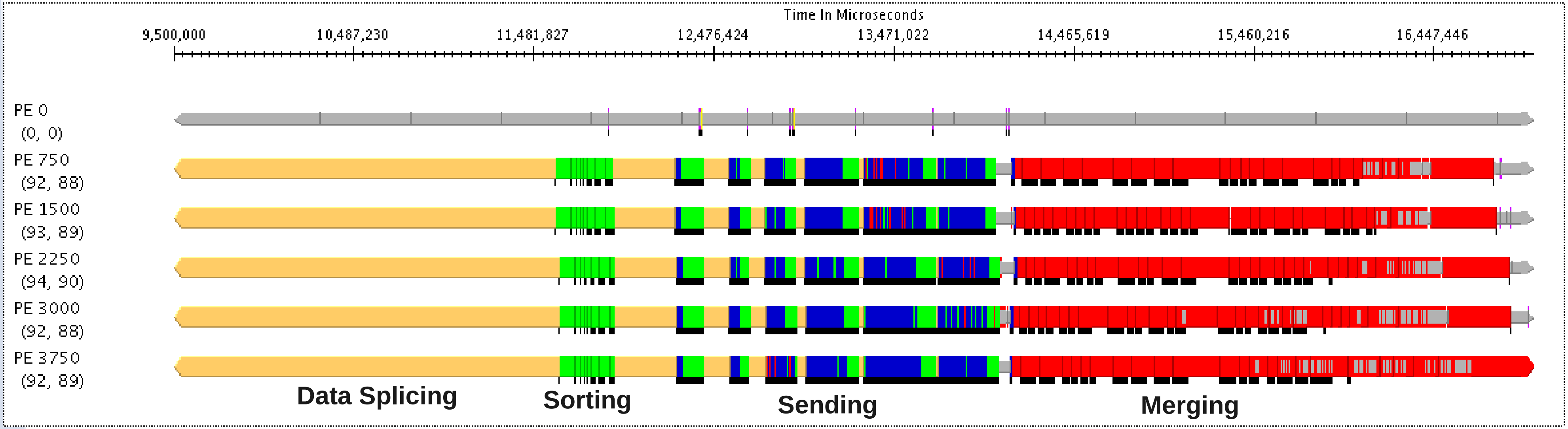


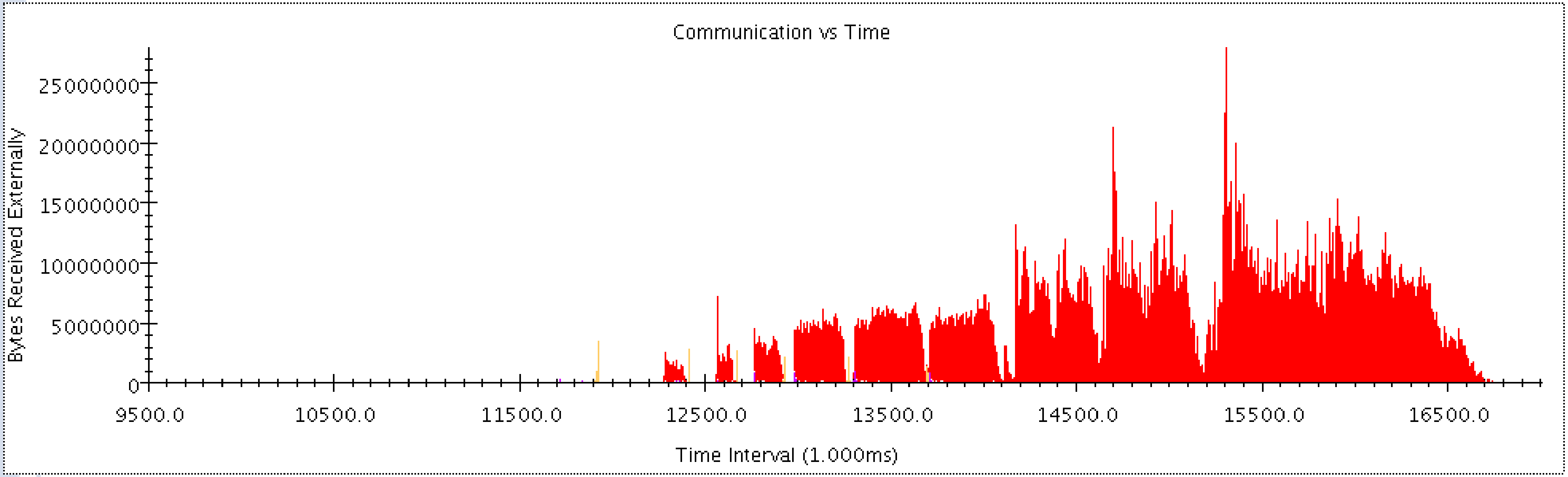
Tests done on 4096 cores of Intrepid (BG/P) with 8 million 64-bit keys per core.

All-to-All Spread and Staging

- Personalized all-to-all collective communication strategies important
  - All-to-all eventually dominates execution time
- Some basic optimizations easily applied
  - Varying order sends
    - Minimizes network contention
  - Only a subset of processors should send data to one destination at a time
    - Prevents network overload

Communication Spread





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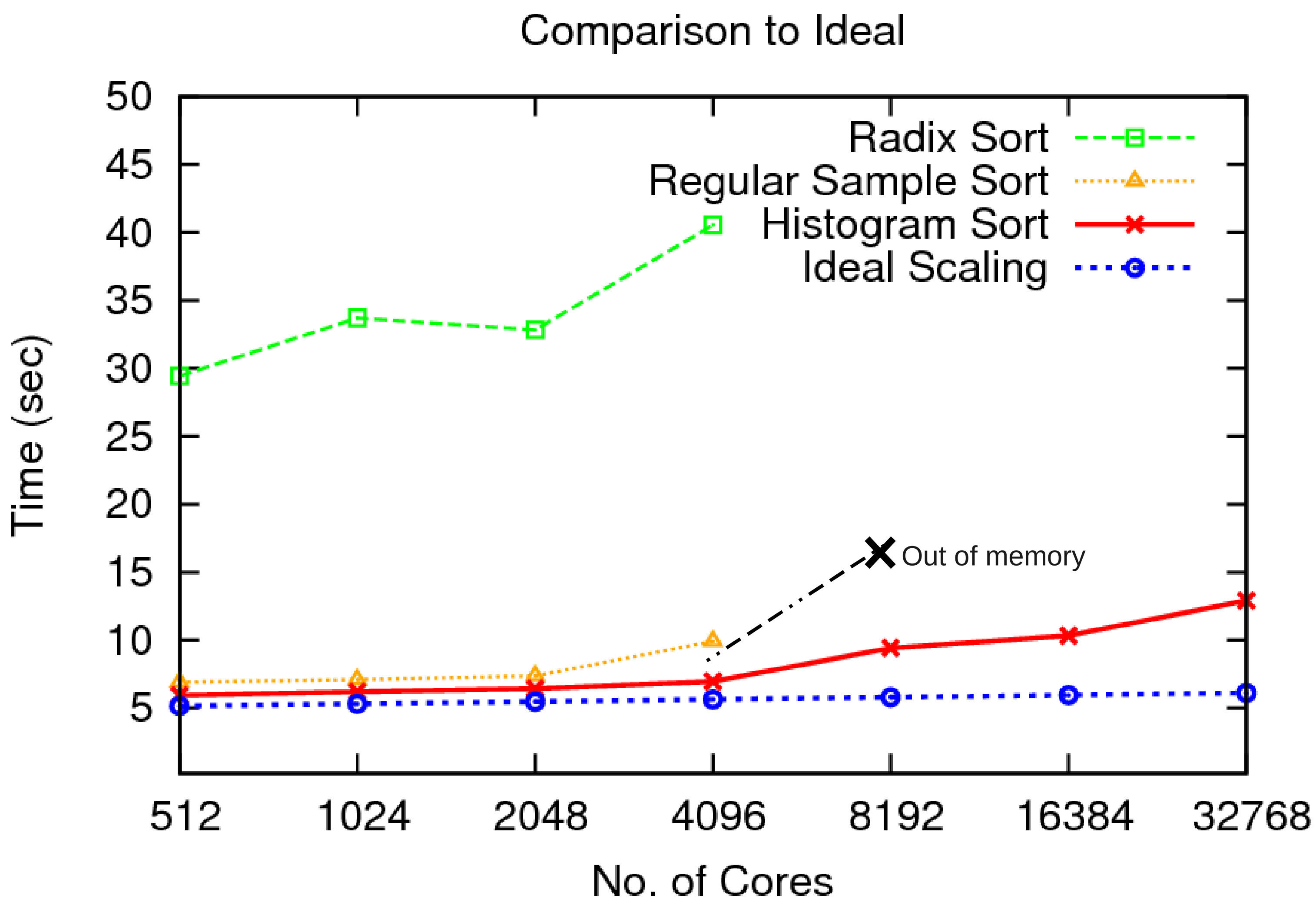
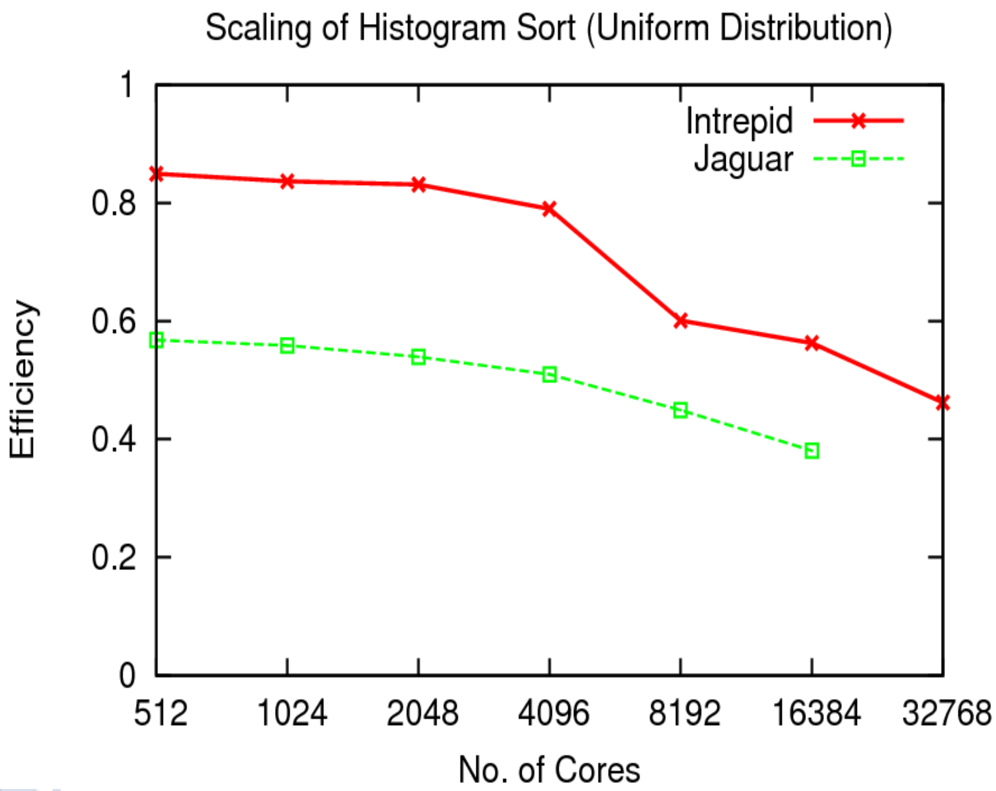
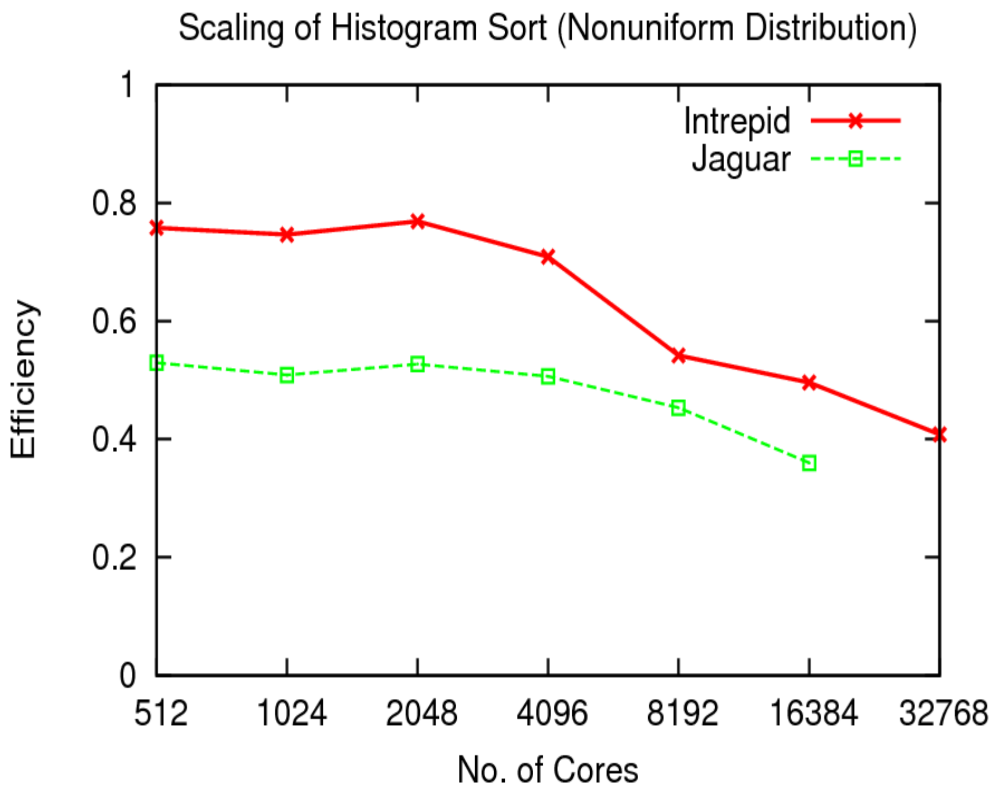
Algorithm Scaling Comparison



Tests done on Intrepid (BG/P) with 8 million 64-bit keys per core.

Histogram Sort Parallel Efficiency





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Some Limitations of this Work

- Benchmarking done with 64-bit keys rather than key-value pairs
- Optimizations presented are only beneficial for certain parallel sorting problems
  - Generally, we assumed ***n* > *p*²**
    - Splicing useless unless ***n*/*p* > *p***
    - Different all-to-all optimizations required if ***n*/*p*** is small (combine messages)
  - Communication usually cheap until ***p*>512**
- Complex implementation another issue

Future/Ongoing Work

- Write a further optimized library implementation of Histogram Sort
  - Sort key-value pairs
  - Almost completed, code to be released
- To scale past 32k cores, histogramming needs to be better optimized
  - As $\boldsymbol{p} \rightarrow \boldsymbol{n/p}$, probe creation cost matches the cost of local sorting and merging
  - One promising solution is to parallelize probing
    - Can use early determined splitters to divide probing

Contributions

- Improvements on original Histogram Sort algorithm
  - Overlap between computation and communication
  - Interleaved algorithm stages
- Efficient and well-optimized implementation
- Scalability up to tens of thousands of cores
- Ground work for further parallel scaling of sorting algorithms

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