Dataflow Analysis

17-654/17-754
Analysis of Software Artifacts

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Overview: Analyses We've Seen

- AST walker analyses
  - e.g. assignment inside an if statement
  - Very approximate, very local
    - Misses case where accidental assignment is done outside an if
- Hoare logic
  - Useful for proving correctness
  - Requires a lot of work (even for ESC/Java)
  - Automated tool is unsound
    - So is manual proof, without a proof checker

Motivation: Dataflow Analysis

- Catch interesting errors
  - Non-local: x is null, x is written to y, y is dereferenced
- Optimize code
  - Reduce run time, memory usage…
- Soundness required
  - Safety-critical domain
    - Assure lack of certain errors
  - Cannot optimize unless it is proven safe
    - Correctness comes before performance
- Automation required
  - Dramatically decreases cost
  - Makes cost/benefit worthwhile for far more purposes

Dataflow analysis

- Tracks value flow through program
  - Can distinguish order of operations
    - Did you read the file after you closed it?
    - Does this null value flow to that dereference?
  - Differs from AST walker
    - Walker simply collects information or checks patterns
    - Tracking flow allows more interesting properties
- Abstracts values
  - Chooses abstraction particular to property
    - Is a variable null?
    - Is a file open or closed?
    - Could a variable be 0?
    - Where did this value come from?
  - More *specialized* than Hoare logic
    - Hoare logic allows any property to be expressed
    - Specialization allows automation and soundness

Zero Analysis

- Could variable x be 0?
  - Useful to know if you have an expression y/x
  - In C, useful for null pointer analysis
- Program semantics
  - η maps every variable to an integer
- Semantic abstraction
  - σ maps every variable to non zero (NZ), zero(Z), or maybe zero (MZ)
  - Abstraction function for integers α_{ZI} :
    - $\alpha_{ZI}(0) = Z$
    - $\alpha_{ZI}(n) = NZ$ for all $n \neq 0$
  - We may not know if a value is zero or not
    - Analysis is always an approximation
    - Need MZ option, too

Zero Analysis Example

σ=[]

x := 10; σ=[x$\mapsto\alpha_{ZI}$(10)]

y := x;

z := 0;

while y > -1 do

    x := x / y;

    y := y-1;

    z := 5;

Zero Analysis Example

```
x := 10;
y := x;
z := 0;
while y > -1 do
    x := x / y;
    y := y-1;
    z := 5;
```

$\sigma=[]$

$\sigma=[x\mapsto NZ]$

$\sigma=[x\mapsto NZ, y\mapsto \sigma(x)]$

Zero Analysis Example

```
x := 10;
y := x;
z := 0;
while y > -1 do
    x := x / y;
    y := y-1;
    z := 5;
```

$\sigma=[]$

$\sigma=[x\mapsto NZ]$

$\sigma=[x\mapsto NZ, y\mapsto NZ]$

$\sigma=[x\mapsto NZ, y\mapsto NZ, z\mapsto \alpha_{ZI}(0)]$

Zero Analysis Example

	σ=[]
x := 10;	σ=[x↦NZ]
y := x;	σ=[x↦NZ,y↦NZ]
z := 0;	σ=[x↦NZ,y↦NZ,z↦Z]
while y > -1 do	σ=[x↦NZ,y↦NZ,z↦Z]
x := x / y;	σ=[x↦NZ,y↦NZ,z↦Z]
y := y-1;	σ=[x↦NZ,y↦MZ,z↦Z]
z := 5;	σ=[x↦NZ,y↦MZ,z↦NZ]

Zero Analysis Example

	σ=[]
x := 10;	σ=[x↦NZ]
y := x;	σ=[x↦NZ,y↦NZ]
z := 0;	σ=[x↦NZ,y↦NZ,z↦Z]
while y > -1 do	σ=[x↦NZ,y↦MZ,z↦MZ]
x := x / y;	σ=[x↦NZ,y↦NZ,z↦Z]
y := y-1;	σ=[x↦NZ,y↦MZ,z↦Z]
z := 5;	σ=[x↦NZ,y↦MZ,z↦NZ]

Zero Analysis Example

	$\sigma=[]$
x := 10;	$\sigma=[x \mapsto NZ]$
y := x;	$\sigma=[x \mapsto NZ, y \mapsto NZ]$
z := 0;	$\sigma=[x \mapsto NZ, y \mapsto NZ, z \mapsto Z]$
while y > -1 do	$\sigma=[x \mapsto NZ, y \mapsto MZ, z \mapsto MZ]$
x := x / y;	$\sigma=[x \mapsto NZ, y \mapsto MZ, z \mapsto MZ]$
y := y-1;	$\sigma=[x \mapsto NZ, y \mapsto MZ, z \mapsto Z]$
z := 5;	$\sigma=[x \mapsto NZ, y \mapsto MZ, z \mapsto NZ]$

Zero Analysis Example

	$\sigma=[]$
x := 10;	$\sigma=[x \mapsto NZ]$
y := x;	$\sigma=[x \mapsto NZ, y \mapsto NZ]$
z := 0;	$\sigma=[x \mapsto NZ, y \mapsto NZ, z \mapsto Z]$
while y > -1 do	$\sigma=[x \mapsto NZ, y \mapsto MZ, z \mapsto MZ]$
x := x / y;	$\sigma=[x \mapsto NZ, y \mapsto MZ, z \mapsto MZ]$
y := y-1;	$\sigma=[x \mapsto NZ, y \mapsto MZ, z \mapsto MZ]$
z := 5;	$\sigma=[x \mapsto NZ, y \mapsto MZ, z \mapsto NZ]$
	Nothing more happens!

Zero Analysis Termination

- The analysis values will not change, no matter how many times we execute the loop
  - Proof: our analysis is deterministic
  - We run through the loop with the current analysis values, none of them change
  - Therefore, no matter how many times we run the loop, the results will remain the same
  - Therefore, we have computed the dataflow analysis results for any number of loop iterations
- Why does this work
  - If we simulate the loop, the data values could (in principle) keep changing indefinitely
    - There are an infinite number of data values possible
    - Not true for 32-bit integers, but might as well be true
      - Counting to 2^{32} is slow, even on today's processors
  - Dataflow analysis only tracks 2 possibilities!
    - So once we've explored them all, nothing more will change
    - This is the secret of abstraction
- We will make this argument more precise later

Using Zero Analysis

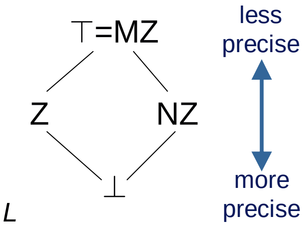
- Visit each division in the program
- Get the results of zero analysis for the divisor
- If the results are definitely zero, report an error
- If the results are possibly zero, report a warning

Defining Dataflow Analyses

- Lattice
  - Describes program data abstractly
  - Abstract equivalent of environment
- Abstraction function
  - Maps concrete environment to lattice element
- Flow functions
  - Describes how abstract data changes
  - Abstract equivalent of expression semantics
- Control flow graph
  - Determines how abstract data propagates from statement to statement
  - Abstract equivalent of statement semantics

Lattice

- A lattice is a tuple (L, ⊑, ⊔, ⊥, ⊤)
  - L is a set of abstract elements
  - ⊑ is a partial order on L
    - Means *at least as precise as*
  - ⊔ is the least upper bound of two elements
    - Must exist for every two elements in L
    - Used to merge two abstract values
  - ⊥ (bottom) is the least element of L
    - Means we haven't yet analyzed this yet
    - Will become clear later
  - ⊤ (top) is the greatest element of L
    - Means we don't know anything
- L may be infinite
  - Typically should have finite height
    - All paths from ⊥ to ⊤ should be finite
    - We'll see why later

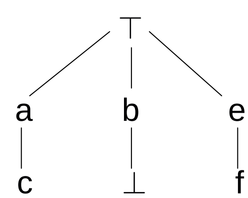


Is this a lattice?

- A lattice is a tuple (L, ⊑, ⊔, ⊥, ⊤)
  - L is a set of abstract elements
  - ⊑ is a partial order on L
  - ⊔ is the least upper bound of two elements
    - must exist for every two elements in L
  - ⊥ (bottom) is the least element of L
  - ⊤ (top) is the greatest element of L
- Yes!

Is this a lattice?

- A lattice is a tuple (L, ⊑, ⊔, ⊥, ⊤)
  - L is a set of abstract elements
  - ⊑ is a partial order on L
  - ⊔ is the least upper bound of two elements
    - must exist for every two elements in L
  - ⊥ (bottom) is the least element of L
  - ⊤ (top) is the greatest element of L
- No!
  - No bottom element
    - ⊥ is not least in the lattice order
    - It is mis-named



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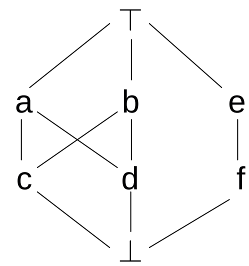
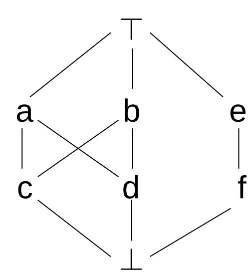


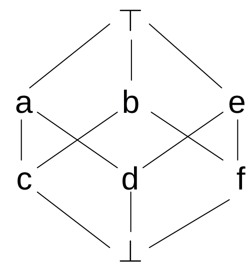
Definition: Least Upper Bounds

- x ⊔ y = z iff
  - z is an upper bound of x and y
    - x ⊑ z and y ⊑ z
  - z is the least such bound
    - ∀w∈L such that x ⊑ w and y ⊑ w we have z ⊑ w
- Also called a join
- Not a lattice
  - What is c ⊔ d?
  - a, b, and ⊤ are upper bounds
    - Assume ⊑ is transitive
  - None is least upper bound



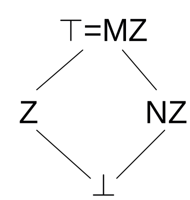
Is this a lattice?

- A lattice is a tuple (L, $\sqsubseteq$, $\sqcup$, $\bot$, $\top$)
  - L is a set of abstract elements
  - $\sqsubseteq$ is a partial order on L
  - $\sqcup$ is the least upper bound of two elements
    - must exist for every two elements in L
  - $\bot$ (bottom) is the least element of L
  - $\top$ (top) is the greatest element of L
- Yes!



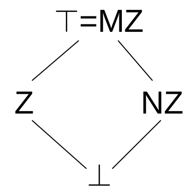
Zero Analysis Lattice

- Integer zero lattice
  - L_{ZI} = { $\bot$, Z, NZ, MZ }
  - $\bot \sqsubseteq$ Z, $\bot \sqsubseteq$ NZ, NZ $\sqsubseteq$ MZ, Z $\sqsubseteq$ MZ
    - $\bot \sqsubseteq$ MZ holds by transitivity
  - $\sqcup$ defined as join for $\sqsubseteq$
    - x $\sqcup$ y = z iff
      - z is an upper bound of x and y
      - z is the least such bound
    - Obeys laws: $\bot \sqcup \mathcal{X} = \mathcal{X}$, $\top \sqcup \mathcal{X} = \top$, $\mathcal{X} \sqcup \mathcal{X} = \mathcal{X}$
    - Also Z $\sqcup$ NZ = MZ
  - $\bot = \bot$
    - $\forall \mathcal{X} .\ \bot \sqsubseteq \mathcal{X}$
  - $\top$ = MZ
    - $\forall \mathcal{X} .\ \mathcal{X} \sqsubseteq \top$



Zero Analysis Lattice

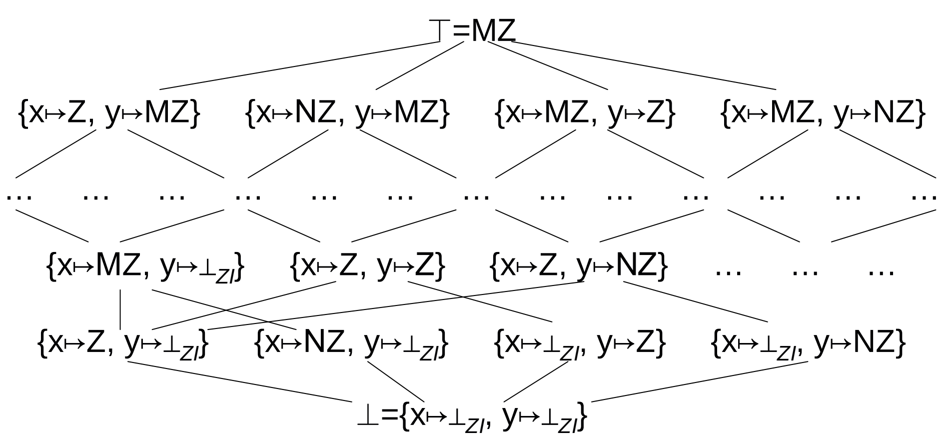
- Integer zero lattice
  - L_{ZI} = { ⊥, Z, NZ, MZ }
  - ⊥ ⊑ Z, ⊥ ⊑ NZ, NZ ⊑ MZ, Z ⊑ MZ
  - ⊔ defined as join for ⊑
  - ⊥ = ⊥
  - ⊤ = MZ



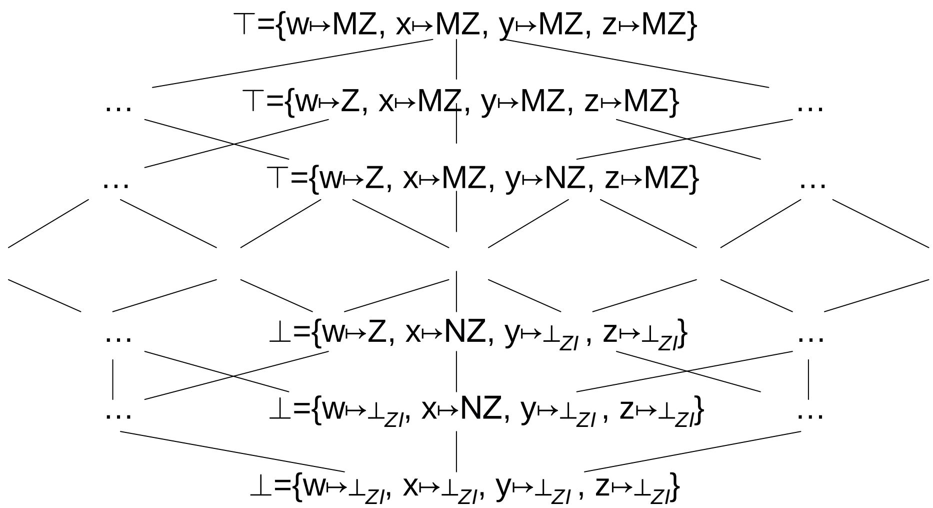
- Program lattice is a *tuple lattice*
  - L_Z is the set of all maps from **Var** to L_{ZI}
  - $\sigma_1 \sqsubseteq_Z \sigma_2$ iff $\forall x \in$ **Var** . $\sigma_1(x) \sqsubseteq_{ZI} \sigma_2(x)$
  - $\sigma_1 \sqcup_Z \sigma_2 = \{ x \mapsto \sigma_1(x) \sqcup_{ZI} \sigma_2(x) \mid x \in$ **Var** }
  - $\bot = \{ x \mapsto \bot_{ZI} \mid x \in$ **Var** }
  - $\top = \{ x \mapsto \top_{ZI} \mid x \in$ **Var** } = { x ↦ MZ | x∈ **Var** }
  - Can produce a tuple lattice from *any* base lattice
    - Just define as above

Tuple Lattices Visually

- For **Var** = { x,y }



One Path in a Tuple Lattice



Abstraction Function

- Maps each concrete program state to a lattice element
  - For tuple lattices, the function can be defined for values and lifted to tuples
- Integer Zero abstraction function α_{ZI} :
  - $\alpha_{\mathrm{ZI}}(0)$ = Z
  - $\alpha_{\mathrm{ZI}}(n)$ = NZ for all $n \neq 0$
- Zero Analysis abstraction function α_{ZA} :
  - $\alpha_{\mathrm{ZA}}(\eta) = \{x \mapsto \alpha_{\mathrm{ZI}}(\eta(x)) \mid x \in \textbf{Var}\}$
  - This is just the tuple form of $\alpha_{\mathrm{ZI}}(n)$
    - Can be done for any tuple lattice

Control Flow Graph (CFG)

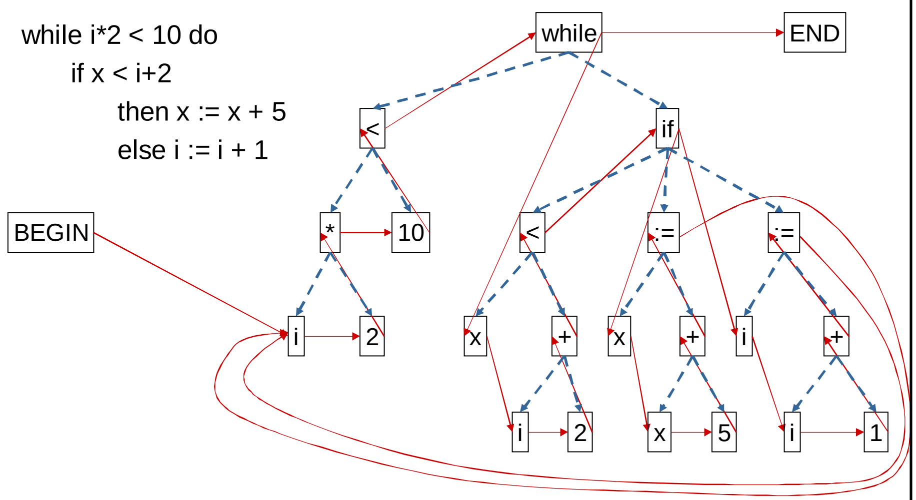
- Shows order of statement execution
  - Determines where data flows
- Decomposes expressions into primitive operations
  - Crystal: One CFG node per “useful” AST node
    - constants, variables, binary operations, assignments, if, while…
  - Loops are written out
    - Form a loop in the CFG
  - Benefit: analysis is defined one operation at a time

Intuition for Building a CFG

- Connect nodes in order of operation
  - Defined by language
- Java order of operation
  - Expressions, assignment, sequence
    - Evaluate subexpressions left to right
    - Evaluate node after children (postfix)
  - While, If
    - Evaluate condition first, then if/while
    - if branches to else and then
    - while branches to loop body and exit

Control Flow Graph Example

```
while i*2 < 10 do
    if x < i+2
        then x := x + 5
        else i := i + 1
```



Flow Functions

- Compute dataflow information after a statement from dataflow information before the statement
  - Formally, map a lattice element and a CFG node to a new lattice element
- Analysis performed on 3-address code
  - inspired by 3 addresses in assembly language: add x,y,z
- Convert complex expressions to 3-address code
  - Each subexpression represented by a temporary variable
  - x+3*y ➔ t_1:=3; t_2:= t_1*y; t_3:=x+t_2

While3Addr

copy	x = y	
binary op	x = y op z	(op ∈ {+,-,*,/,…})
literal	x = n	
unary op	x = op y	(op ∈ {-,!,++,…})
label	label *lab*	
jump	jump *lab*	
branch	btrue x *lab*	

Zero Analysis Flow Functions

- $f_{ZA}(\sigma, [x := y]) = [x \mapsto \sigma(y)]\ \sigma$
- $f_{ZA}(\sigma, [x := n])$ = if n==0

  then $[x \mapsto Z]\sigma$

  else $[x \mapsto NZ]\sigma$
- $f_{ZA}(\sigma, [x := \ldots]) = [x \mapsto MZ]\ \sigma$
  - Could be more precise, e.g.

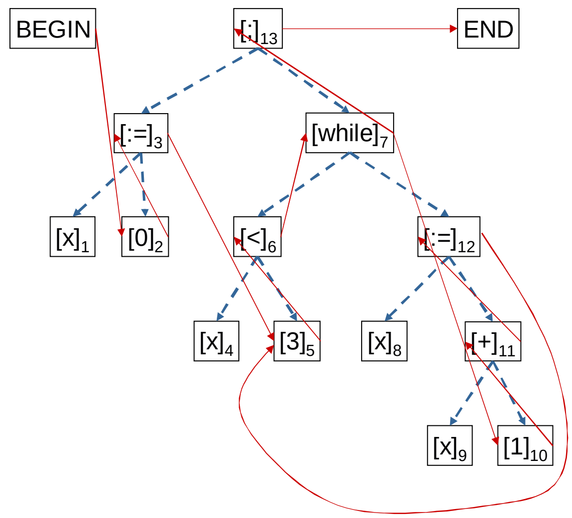
    $f_{ZA}(\sigma, [x := y + z])$ =

    if $\sigma[y]$=Z && $\sigma[z]$=Z

    then $[x \mapsto Z]\sigma$ else $[x \mapsto MZ]\sigma$
- $f_{ZA}(\sigma,$ */* any non-assignment */*$) = \sigma$

Zero Analysis Example

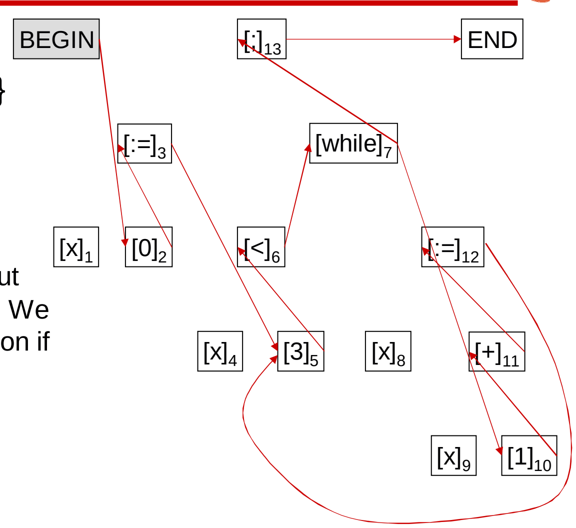
x := 0;
while x > 3 do
    x := x+1



Zero Analysis Example

Initial dataflow

$\sigma_\iota = \{ x \mapsto MZ \mid x \in \textbf{Var} \}$

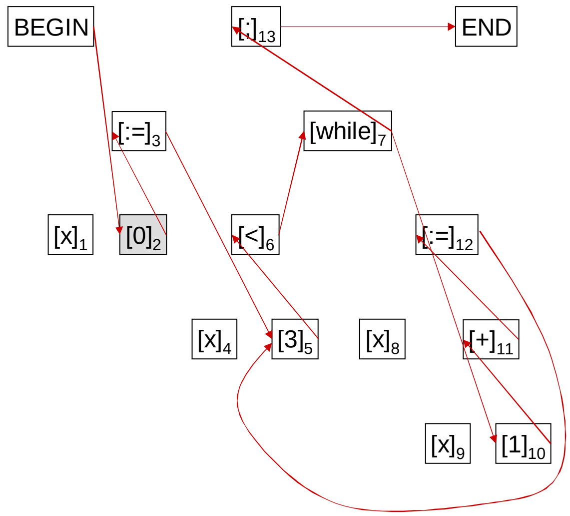


Intuition:

We know nothing about initial variable values. We could use a precondition if we had one.

Zero Analysis Example

$\sigma_\iota = \{\, x \mapsto MZ \mid x \in \mathbf{Var} \,\}$

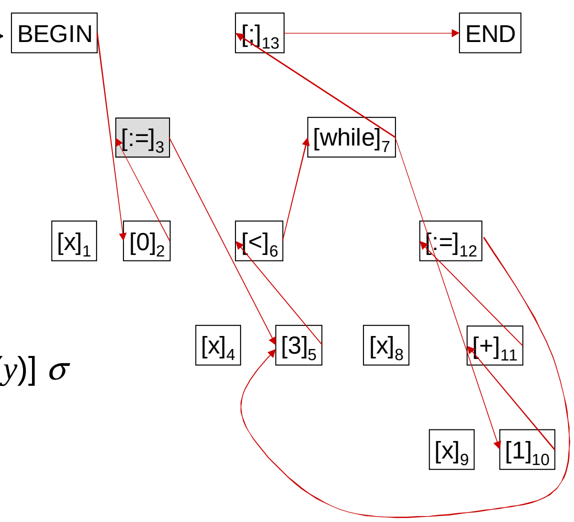


$\sigma_2 = f_{ZA}(\sigma_\iota, [t_2 := 0])$
$\quad = [t_2 \mapsto Z]\ \sigma_\iota$

$f_{ZA}(\sigma, [x := n]) =$
  if $n{=}{=}0$
    then $[x \mapsto Z]\sigma$
    else $[x \mapsto NZ]\sigma$

Zero Analysis Example

$\sigma_\iota = \{\, x \mapsto MZ \mid x \in \mathbf{Var} \,\}$
$\sigma_2 = [t_2 \mapsto Z]\ \sigma_\iota$



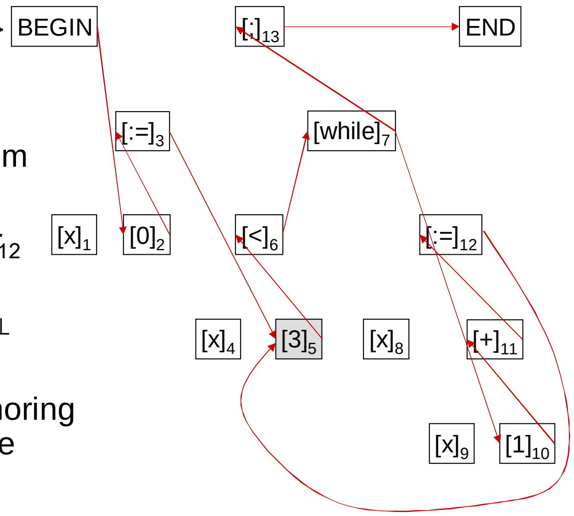
$\sigma_3 = f_{ZA}(\sigma_2, [x := t_2])$
$\quad = [x \mapsto \sigma_2(t_2)]\ \sigma_2$
$\quad = [x \mapsto Z]\ \sigma_2$
$\quad = [x \mapsto Z,\ t_2 \mapsto Z]\ \sigma_\iota$

$f_{ZA}(\sigma, [x := y]) = [x \mapsto \sigma(y)]\ \sigma$

Zero Analysis Example

σ_ι = { $x \mapsto$MZ | x$\in$**Var** }

σ_3 = [x$\mapsto$Z, $t_2\mapsto$Z] σ_ι



Input to $[3]_5$ comes from

$[:=]_3$ and $[:=]_{12}$

Input should be $\sigma_3 \sqcup \sigma_{12}$

What is σ_{12}?

Solution: assume $\perp$

Benefit: $\sigma_3 \sqcup \perp = \sigma_3$

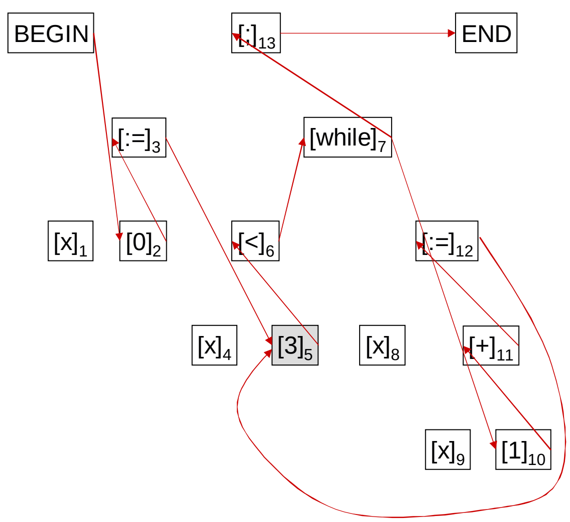
Same result as ignoring back edge first time

Zero Analysis Example

σ_ι = { $x \mapsto$MZ | x$\in$**Var** }

σ_3 = [x$\mapsto$Z, $t_2\mapsto$Z] σ_ι

σ_{12} = $\perp$



$\sigma_5 = f_{ZA}(\sigma_3 \sqcup \sigma_{12}, [t_5 := 3])$

$= f_{ZA}(\sigma_3 \sqcup \perp, [t_5 := 3])$

$= f_{ZA}(\sigma_3, [t_5 := 3])$

$= [t_5 \mapsto NZ]\ \sigma_3$

$f_{ZA}(\sigma, [x := n])$ =

if n==0

then $[x \mapsto Z]\sigma$

else $[x \mapsto NZ]\sigma$

Zero Analysis Example

$\sigma_\iota = \{ x \mapsto MZ \mid x \in \mathbf{Var} \}$

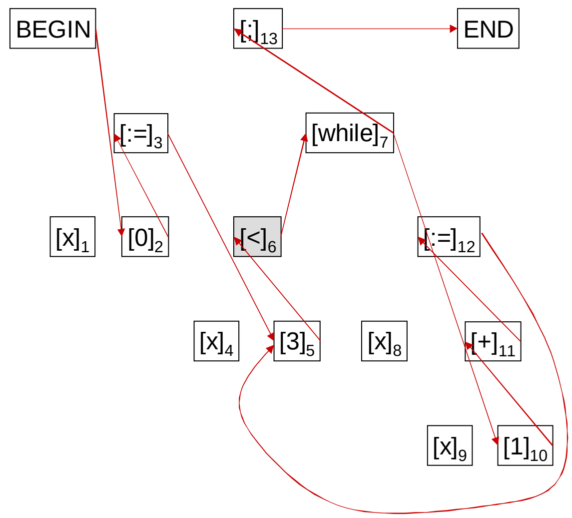
$\sigma_3 = [x \mapsto Z, t_2 \mapsto Z]\ \sigma_\iota$

$\sigma_{12} = \perp$

$\sigma_5 = [t_5 \mapsto NZ]\ \sigma_3$

$\sigma_6 = f_{ZA}(\sigma_5, [t_6 := x < t_5])$
$= \sigma_5$
$= [t_5 \mapsto NZ]\ \sigma_3$

$f_{ZA}(\sigma,$ */* any other */*$) = \sigma$



Zero Analysis Example

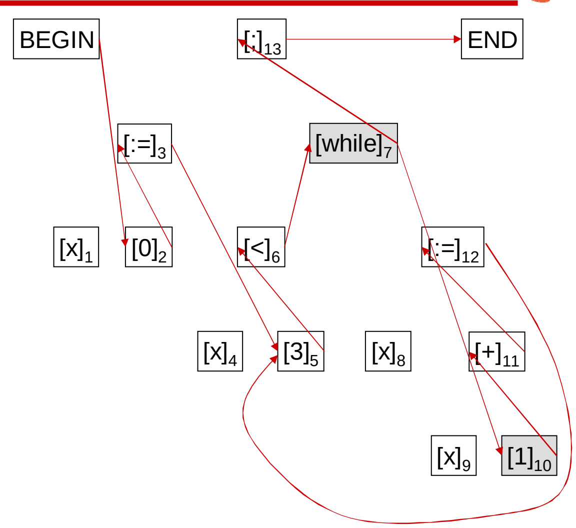
$\sigma_\iota = \{ x \mapsto MZ \mid x \in \mathbf{Var} \}$

$\sigma_3 = [x \mapsto Z, t_2 \mapsto Z]\ \sigma_\iota$

$\sigma_{12} = \perp$

$\sigma_6 = [t_5 \mapsto NZ]\ \sigma_3$

Skipping similar nodes…

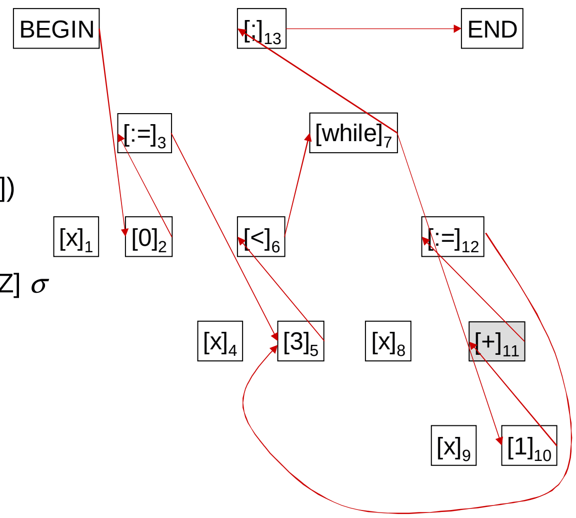


Zero Analysis Example

$\sigma_\iota = \{ x \mapsto MZ \mid x \in \textbf{Var} \}$
$\sigma_3 = [x \mapsto Z, t_2 \mapsto Z]\ \sigma_\iota$
$\sigma_{12} = \bot$
$\sigma_{10} = [t_{10} \mapsto NZ, \ldots]\ \sigma_3$

$\sigma_{11} = f_{ZA}(\sigma_{10}, [t_{11} := x + t_{10}])$
$\quad = [t_{11} \mapsto MZ]\ \sigma_{10}$

$f_{ZA}(\sigma, [x := y\ op\ z]) = [x \mapsto MZ]\ \sigma$

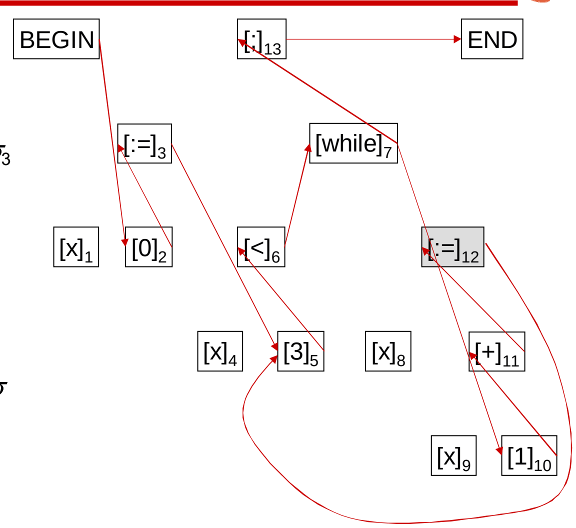


Zero Analysis Example

$\sigma_\iota = \{ x \mapsto MZ \mid x \in \textbf{Var} \}$
$\sigma_3 = [x \mapsto Z, t_2 \mapsto Z]\ \sigma_\iota$
$\sigma_{12} = \bot$
$\sigma_{11} = [t_{10} \mapsto NZ, t_{11} \mapsto MZ, \ldots]\sigma_3$

$\sigma_{12} = f_{ZA}(\sigma_{11}, [x := t_{11}])$
$\quad = [x \mapsto \sigma_{11}(t_{11})]\ \sigma_{11}$
$\quad = [x \mapsto MZ]\ \sigma_{11}$
$\quad = [x \mapsto MZ, \ldots]\ \sigma_3$

$f_{ZA}(\sigma, [x := y]) = [x \mapsto \sigma(y)]\ \sigma$

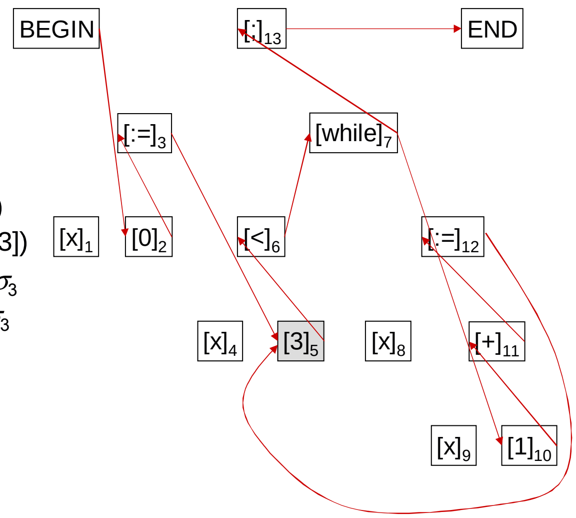


Zero Analysis Example

$\sigma_\iota = \{ x \mapsto MZ \mid x \in \mathbf{Var} \}$

$\sigma_3 = [x \mapsto Z, t_2 \mapsto Z]\ \sigma_\iota$

$\sigma_{12} = [x \mapsto MZ, \ldots]\ \sigma_3$



$$\begin{aligned}\sigma_5 &= f_{ZA}(\sigma_3 \sqcup \sigma_{12}, [t_5 := 3]) \\ &= f_{ZA}([x \mapsto MZ]\sigma_3, [t_5 := 3]) \\ &= [t_5 \mapsto NZ]\ [x \mapsto MZ, \ldots]\sigma_3 \\ &= [t_5 \mapsto NZ, x \mapsto MZ, \ldots]\ \sigma_3\end{aligned}$$

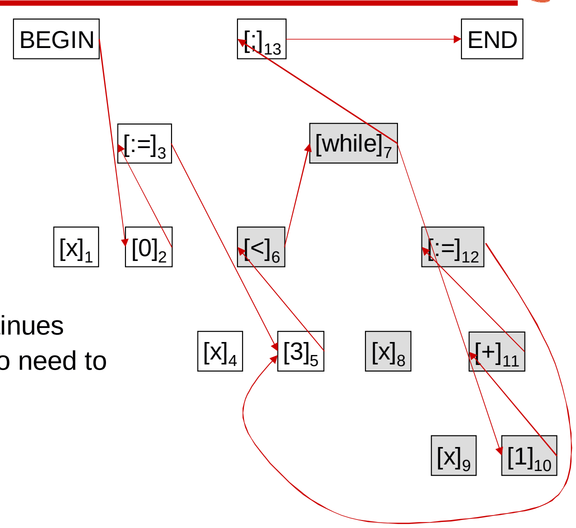
$f_{ZA}(\sigma, [x]_k) = [t_k \mapsto \sigma(x)]\ \sigma$

Zero Analysis Example

$\sigma_\iota = \{ x \mapsto MZ \mid x \in \mathbf{Var} \}$

$\sigma_3 = [x \mapsto Z, t_2 \mapsto Z]\ \sigma_\iota$

$\sigma_{12} = [x \mapsto MZ, \ldots]\ \sigma_3$



Propagation of $x \mapsto MZ$ continues

σ_{12} does not change, so no need to iterate again

Worklist Dataflow Analysis Algorithm

worklist = new Set();
for all node indexes i do
    results[i] = $\perp_A$;
results[*entry*] = ι_A;
worklist.add(*all nodes*);

Ok to just add entry node if flow functions cannot return $\perp_A$ (examples will assume this)

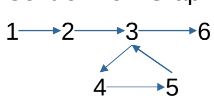
while (!worklist.isEmpty()) do
    i = worklist.pop();
    before = $\bigsqcup_{k \in pred(i)}$ results[k];
    after = f_A(before, node(i));
    if (!(after $\sqsubseteq$ results[i]))
        results[i] = after;
        for all k$\in$succ(i) do
            worklist.add(k);

Pop removes the most recently added element from the set (performance optimization)

Example of Worklist

$[a := 0]_1$
$[b := 0]_2$
while $[a < 2]_3$ do
    $[b := a]_4$;
    $[a := a + 1]_5$;
$[a := 0]_6$

Control Flow Graph



Position	Worklist	a	b
0	1	MZ	MZ
1	2	Z	MZ
2	3	Z	Z
3	4,6	Z	Z
4	5,6	Z	Z
5	3,6	MZ	Z
3	4,6	MZ	Z
4	5,6	MZ	MZ
5	3,6	MZ	MZ
3	4,6	MZ	MZ
4	6	MZ	MZ
6		Z	MZ

Worklist Algorithm Performance

- Performance
  - Visits node whenever input gets less precise
    - up to h = height of lattice
  - Propagates data along control flow edges
    - up to e = max outbound edges per node
  - Assume lattice operation cost is o
  - Overall, O(h*e*o)
    - Typically h,o,e bounded by n = number of statements in program
    - $O(n^3)$ for many data flow analyses
    - $O(n^2)$ if you assume a number of edges per node is small
  - Good enough to run on a function
    - Usually not run on an entire program at once, because n is too big