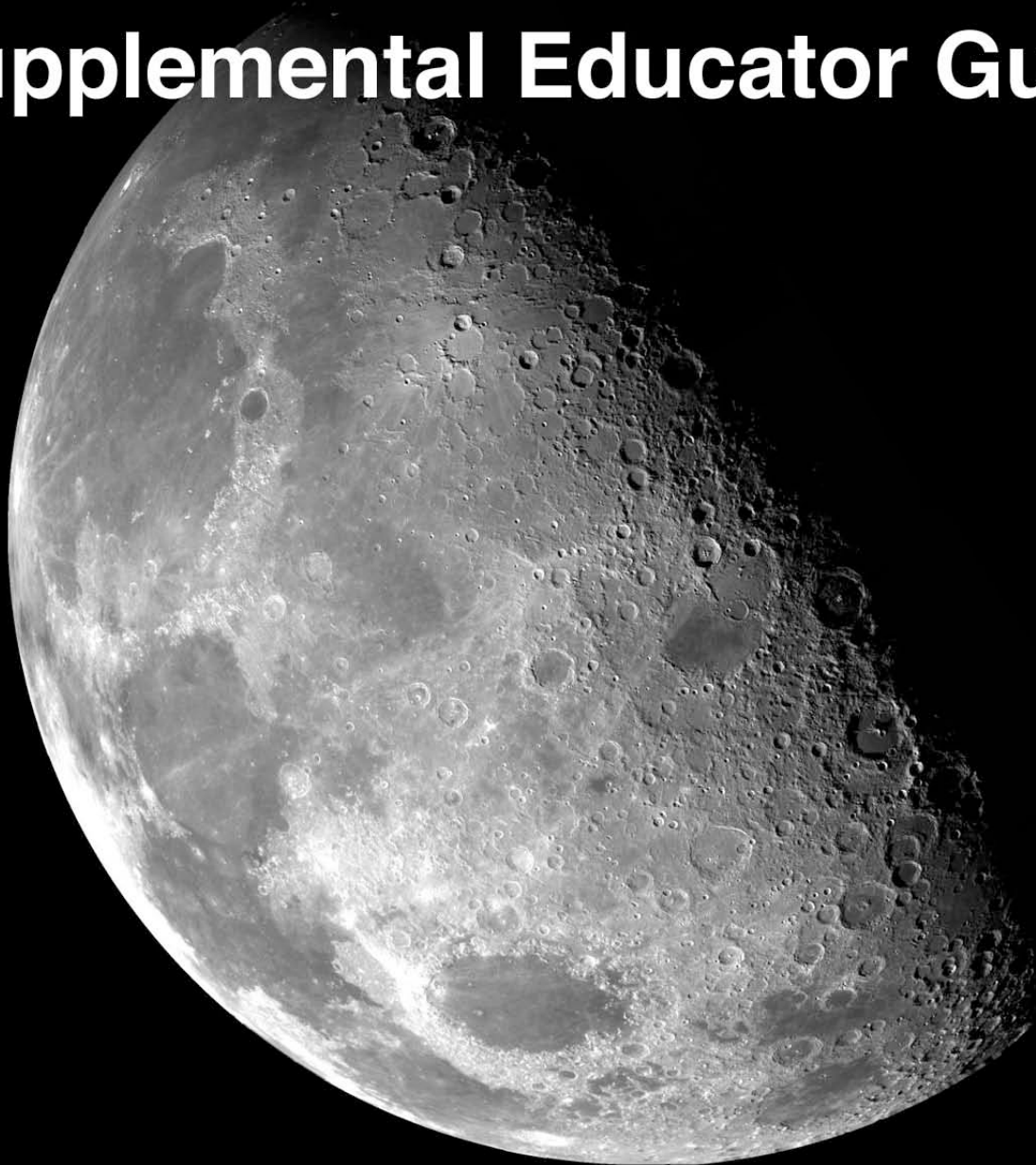# Moon Math: Craters!
Supplemental Educator Guide

A companion to the Quest
"Cratering the Moon" Challenge

Moon Math: Craters!

Grade Level: 5–8

Estimated Time: 4 main activities requiring 1 class period per activity

Copernicus Crater (Apollo 11 Mission)

Overarching Theme:

What angle of impact and what size and mass of projectile excavates the greatest amount of "stuff" (soil, matter, ejecta) for scientists to analyze?

Objectives:

- Students will correlate a crater's shape to the projectile's angle of impact.
- Students will measure the diameter and depth of a crater.
- Students will measure and/or calculate the circumference and diameter of various spherical projectiles as well as measure the masses of the projectiles.
- Students will approximate the area of the 2-dimensional, uppermost cross-section of a crater using two different methods: 1) unit square grid and 2) the formula for the area of a circle.
- Students will approximate the volume of a crater (spherical cup) using a given formula.
- Students will observe how the size and mass of a projectile and its angle of impact affects the size and shape of the impact crater.
- Students will calculate the scale of a student-made crater as compared to craters on the Earth and Moon.
- Students will organize data in a matrix, table, hierarchical list, and/or graph, and will then analyze the data and draw conclusions.

Standards:

- NM-GEO.6–8.1: Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- NM-MEA.6–8.1: Understand measurable attributes of objects and the units, systems, and processes of measurement.
- NM-MEA.6–8.2: Apply appropriate techniques, tools, and formulas to determine measurements.
- NM-DATA.6–8.1: Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer.
- NM-NUM.6–8.1: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.

Table of Contents

NOTES TO TEACHER

Guide Overview

This supplemental guide suggests activities for connecting standards-based geometry concepts (area, volume, and proportion of two- and three-dimensional objects) to an actual NASA lunar mission. It unifies the six Moon Math unit I and unit II lessons with the NASA Quest *"Cratering the Moon"* challenge.

- Moon Math lessons and *What's the Difference* software: http://quest.nasa.gov/vft/#wtd
- *"Cratering the Moon"* challenge: http://quest.nasa.gov/challenges/lcross/index.html

Supplemental Guide Format

This supplemental guide follows the 5–E lesson model: Engage, Explore, Explain, Evaluate, Extend.

- **Engage:**   set the stage (science and math in a real-world context)
- **Explore:**   apply mathematics (experimenting, calculating area and volume)
- **Explain:**   analyze results and draw conclusions (comparing, charting, ordering)
- **Evaluate:**   teacher assessment of student learning (discussion questions)
- **Extend:**   optional activities (calculating scale/proportion)

Flexibility

This guide contains four area and volume activities in the EXPLORE section and two scale activities and two geometry activities in the EXTEND section. Teachers may use all of the activities or pick and choose a select few. Teachers may also modify the activities and/or worksheets to suit their students' needs and abilities. Each activity takes about 1 class period.

Quest Challenge Participants (http://quest.nasa.gov/challenges/lcross/index.html)

This guide serves as a nice extension to the NASA Quest *"Cratering the Moon"* challenge. Participants who designed and built a launching device that will launch projectiles at varying angles into a pan of soil can use this device in Activities A and B.

Classes that did <u>not</u> design and build a launching device for the *"Cratering the Moon"* challenge can refer to the "Alternative Method" instructions highlighted in Activities A and B.

Prerequisite Concepts

In some cases, teachers may need to first teach some prerequisite concepts before implementing the activities. For example, some students may need a pre-lesson on triangles and angles to enhance their experience with Activity A.

The material in the *What's the Difference* Moon Math data set and in the Moon Math lessons can serve as good pre-lesson material to activities A–D. (http://quest.nasa.gov/vft/#wtd)

- *What's the Difference* **Moon Math data set:** view images and formulas for calculating the area of basic two-dimensional objects and the volume of basic three-dimensional objects. Includes a "Planet Weight Calculator" and a "Design a Lunar Habitat" activity.

- **Moon Math Unit I, Case Study I:** calculate the area of a rectangle and the surface area of a cylinder.

- **Moon Math Unit I, Case Study II:** calculate the volume of a rectangular solid and the volume of a cylinder.

- **Moon Math Unit II, Case Study II:** calculate the volume of two types of containers and then determine how many of each container will fill a cargo hold of a given dimension.

Finally, the ***Volume of a Sphere: A Hands-on Proof*** activity in the Teacher Resources section (pp. 46–50) is a good prerequisite to Extensions 3 and 4.

Metric

The sample problems in this guide are based on the metric system. Teachers may wish for students to use the English (or customary) system of measurement and then to practice unit conversion.

Formulas

In an effort to not confuse students, some key formulas will be written as follows:

Area of a circle $\qquad A_{circle} = \pi\ r^2$; where r is the radius of the circle


Volume of a sphere $\qquad V_{sphere} = \dfrac{4}{3}\ \pi\ r^3$; where r is the radius of the sphere

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Volume of a crater (spherical cup) $\qquad V_{crater} = \dfrac{\pi h}{6}\ (3\ a^2\ +\ h^2)$; where h is the height of the crater and a is the radius of the crater

NASA's LCROSS Mission

W ater is a vital component of human space exploration. Without water, humans cannot survive. Therefore, in order to explore other planets and moons, humans must either be able to transport or produce an adequate water supply (cost prohibitive and impractical) or have a source of water at their destination. In the Fall 2008, NASA will launch a spacecraft carrying the *Lunar CRater Observation and Sensing Satellite* (LCROSS) that will travel to the Moon in search of frozen water. First, the shepherding spacecraft will collide with the surface of a permanently shadowed polar lunar crater, resulting in a plume of ejecta that specialized instruments aboard LCROSS will analyze for the presence of ancient water ice. Subsequently the satellite itself will impact the lunar surface while Earth-based scientists and orbiting satellites analyze its resulting ejecta plume for water ice or vapor.
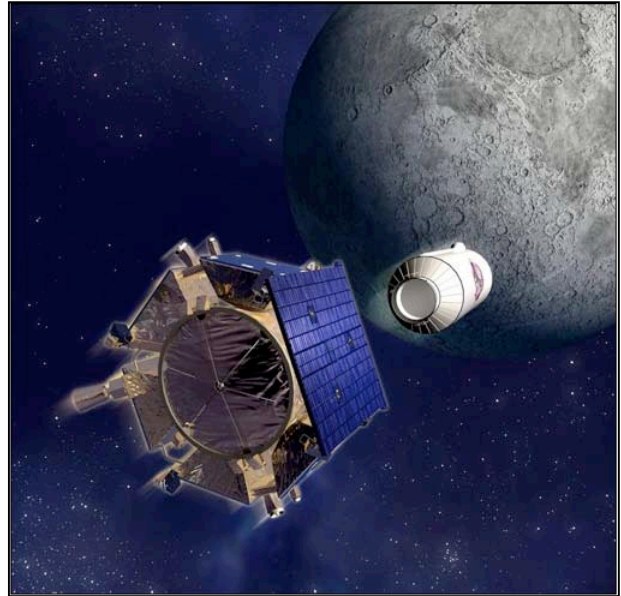
The Impact

S cientists want to analyze as much lunar ejecta as possible; therefore, the impacts of the shepherding spacecraft and LCROSS need to be large enough to excavate deep and broad layers of soil and strong enough to create a tall ejecta plume. In other words, NASA wants to maximize the area and volume of the resulting impact craters, so that they can analyze the greatest amount of "stuff" (matter, ejecta). Scientists cannot model the expected impacts on Earth at the same scale as they will occur on the lunar surface. Instead scientists will utilize Earth-based instruments and methods to *simulate* the impacts, craters, and ejecta plumes to help them determine the most effective angle and speed of impact.

Classroom Connection

S tudents role-play scientists by setting up a lunar surface test bed (pan of soil) and conducting impact experiments. As scientists, students will observe the effect that changing the angle of a projectile's impact or changing the size and mass of the projectile has on the area and volume of the impact crater. Students will apply mathematics and create hierarchical lists of data and then analyze that data and draw conclusions. Upon completing the activities, students will be able to determine the angle of impact and the size and mass of the projectile that will excavate the greatest amount of "stuff" for scientists to analyze.

Activity A: Shapes and Angles

Objectives:
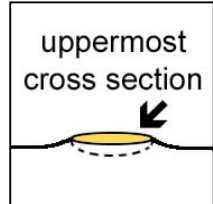
In this activity, students will:

- Identify the general two-dimensional geometric shape of the uppermost cross section of an impact crater.

- Draw a correlation between the general two-dimensional geometric shape of an impact crater and the projectile's angle of impact.

Materials:

Challenge Participants	Non-Challenge Participants
• Shallow pan (aluminum baking pan, cardboard box lid, dishpan, etc.) • Soil-like substance (dirt, sand, flour, fine kitty litter, etc.) • Spherical projectile • Launching device • Bed sheet (or other drape for spreading on the floor under the pan of soil) • *Activity A – Shapes and Angles* student worksheet (p. 53) • *Shocking Shapes* article (p. 44)	• *Activity A – Impact Crater Images* (p. 52) • *Activity A – Shapes and Angles* student worksheet (p. 53) • *Shocking Shapes* article (p. 44)

Craters are three-dimensional objects; however, in Activity A, students will focus on only the uppermost cross section of an impact crater.   Their observations will relate to the two-dimensional shape of the "top" of the crater.

(See illustration to the right. →)

uppermost cross section

Step 1: Making impact craters

Set up a testing station by filling a shallow pan with soil at least 3 inches deep. Place the pan of soil on a sheet to protect the floor from ejecta.

Then, using their launching device(s) from the "*Cratering the Moon*" challenge, students launch a standard projectile into the pan of soil at three different angles (15º, 45º, and 70º). Ideally, students should launch the projectile three times at each angle, creating a total of nine impact craters. After each launch, students should complete steps 2–5 for the resulting impact crater and then reset the pan of soil for the subsequent trial. Repeat this process for all nine impact craters.

Step 2: Identifying shapes

Analyze the impact craters (either made in step 1 or pictured on page 52) and determine which two-dimensional geometric shape best describes the uppermost cross section of each crater. Students record their observations on the **Activity A – Shapes and Angles** student worksheet on page 53.
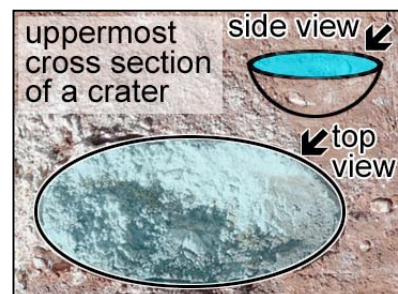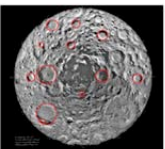
Step 3: Matching shapes with angles of impact

Based on their observations in step 2, students conclude which two-dimensional geometric shapes can be associated with each of the three angles of impact. Students record their conclusions at the bottom of **Activity A – Shapes and Angles** student worksheet on page 53.

Conclude Activity A by sharing the "***Shocking Shapes***" article with students (*See Teacher Resources p. 44*) and then discussing the questions on the following page.

1. What shape(s) would you use to describe the uppermost cross section (or "top") of each crater?

2. Do you notice any relationships between a shape and an angle of impact?

3. How does the angle of impact affect the shape of the crater? For example, do impacts at 15º create craters of similar shape? (at 45º? at 70º?)

4. How are the shapes of craters with a 70º angle of impact different than the shapes of craters with a 45º angle of impact? (with a 15º angle of impact?)

5. Examine each crater's dimensions. As the angle of impact changes, how does the relationship between a crater's greatest length and greatest width change?

EXPLORE: ACTIVITY B

Activity B: Angles and Area

Objectives:

In this activity, students will:

- Approximate the area of the uppermost cross section of an impact crater using a variety of square grids.

- Conclude which angle of impact results in the greatest area.

Materials:

Challenge Participants	Non-Challenge Participants
- Shallow pan - Soil-like substance - Spherical projectile - Launching device - Bed sheet (or other drape for spreading on the floor under the pan of soil) - Transparencies of 3 different square grids: 2 cm x 2 cm, 1 cm x 1 cm, ½ x ½ cm (templates provided on pp. 41–43) - Tape - Wet erase markers (1 light, 1 dark) ***Approximating Area Pre-lesson Activity*** student worksheet (p. 54) ***Activity B – Crater Area*** student worksheet (p. 55)	***Activity A – Impact Crater Images*** (p. 52) - Transparencies of 3 different square grids: 2 cm x 2 cm, 1 cm x 1 cm, ½ x ½ cm (templates provided on pp. 41–43) - Tape - Wet erase markers (1 light, 1 dark) ***Approximating Area Pre-lesson Activity*** student worksheet (p. 54) ***Activity B – Crater Area*** student worksheet (p. 55)
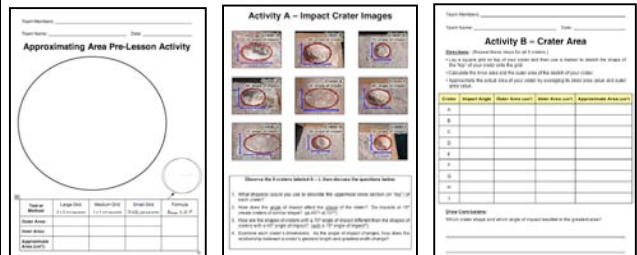
Craters are three-dimensional objects; however, in Activity B, students will focus on only the uppermost cross section of an impact crater. Their observations, measurements, and calculations will relate to the two-dimensional shape of the "top" of the crater.   (See illustration to the right. →)
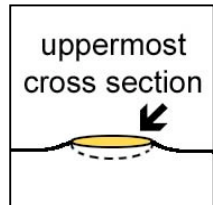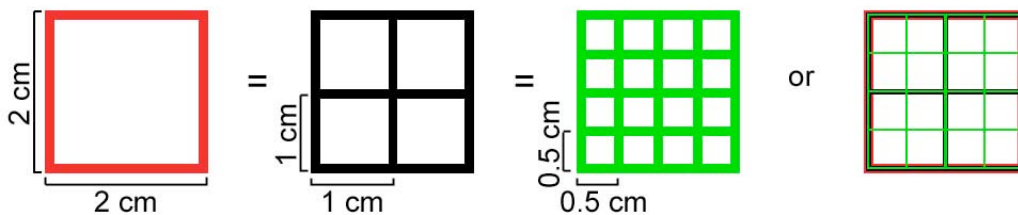
I. Approximating area <u>pre-lesson</u> activity:

The purpose of this pre-lesson activity is to give students practice in approximating the area of a near-circular shape using a square grid and to illustrate how the smaller the grid, the more accurate the approximation.

For this activity, students need to make sure that the three different square grids line up relative to each other in regard to the near-circular shape. By laying the square grids squarely on top of one another, students will see that:

- One 2 cm x 2 cm square is comprised of *four* 1 cm x 1 cm squares or *sixteen* 0.5 cm x 0.5 cm squares.

- One 1 cm x 1 cm square is comprised of *four* 0.5 x 0.5 cm squares.

* Templates for the three different square grids are provided on pages 41–43 of this guide.

In steps 1–3 outlined on page 13, students will approximate the area of the shape by determining two values with each grid. (*See example on page 12.*)

First, students will calculate the "**outer area**," which is the area of the squares that are contained within the shape <u>plus</u> the area of the squares that touch the shape's boundary.

Next, students will calculate the "**inner area**," which is the area of the squares that are wholly contained within the boundary of the shape.

The **actual area** of the shape lies between the shape's inner area and the shape's outer area. The closer these two values are to each other, the more closely they represent the shape's actual area:

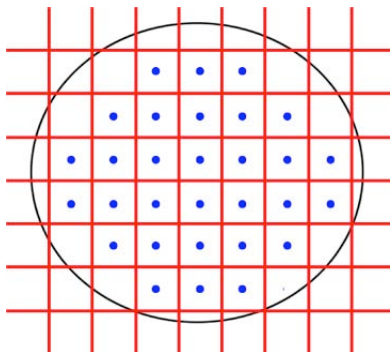**inner area ≤ actual area ≤ outer area.**

Example of finding outer area and inner area using a 2 cm x 2 cm square grid
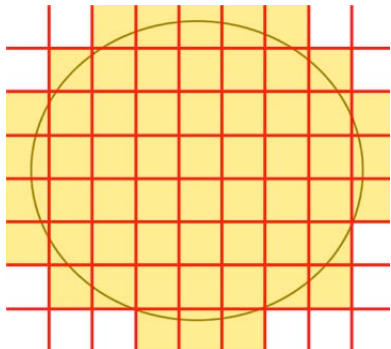
outer area

In this illustration, the squares that comprise the shape's *outer area* are marked with yellow shading. Each 2 cm x 2 cm outer area square has area 4 cm^2. There are 57 of them, so the outer area is:

$$57 \cdot 4 \text{ cm}^2 = 228 \text{ cm}^2$$

inner area

In this illustration, the squares that comprise the shape's *inner area* are marked with blue dots. Each 2 cm x 2 cm inner area square has area 4 cm^2. There are 30 of them, so the inner area is:

$$30 \cdot 4 \text{ cm}^2 = 120 \text{ cm}^2$$

We can say that the *actual area* of the shape is greater than 120 cm^2 and less than 228 cm^2.

$$120 \text{ cm}^2 \leq \text{actual area} \leq 228 \text{ cm}^2$$

More examples using actual classroom-made craters are provided on pp. 37–40.

Step 1: Using the **Approximating Area Pre-lesson Activity** student worksheet on page 54, lay the transparency of the 2 cm x 2 cm square grid on top of the shape and tape it in place. Using a light colored wet erase marker, shade the "outer area" squares, count them, compute the outer area, and record the data in the table at the bottom of the worksheet. Then, with a dark colored wet erase marker, place a dot in the "inner area" squares, count them, compute the inner area, and record the data. The actual area lies between these two values. One way to *approximate* the actual area of the shape is to *average* the value of the outer area and the value of the inner area.
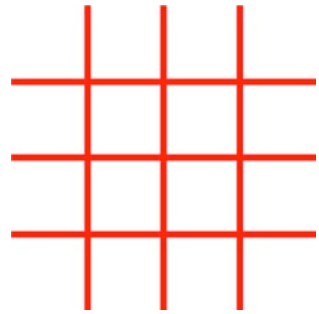
Wipe the 2 cm x 2 cm square grid transparency clean, but leave it taped to the shape.

Step 2: Lay the transparency of the 1 cm x 1 cm unit square grid on top of the shape, being sure to align it with the 2 cm x 2 cm square grid. Tape the grid in place. Shade the "outer area" squares with a light colored wet erase marker, count them, compute the outer area, and record the data. Then, with a dark colored wet erase marker, place a dot in the "inner area" squares, count them, compute the inner area, and record the data. Average the outer area and inner area values to determine the *approximate* area of the shape.
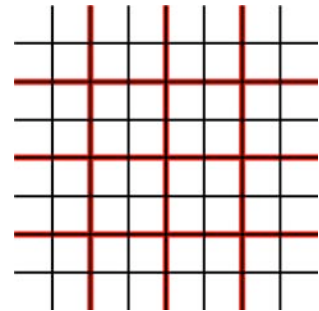
Wipe the 1 cm x 1 cm square grid transparency clean, but leave it taped to the shape.

Step 3: Lay the transparency of the 0.5 cm x 0.5 cm square grid on top of the shape, being sure to align it with the 1 cm x 1 cm unit square grid. Tape the grid in place. Shade the "outer area" squares with a light colored wet erase marker, count them, compute the outer area, and record the data. Then, with a dark colored wet erase marker, place a dot in the "inner area" squares, count them, compute the inner area, and record the data. Average the outer area and inner area values to determine the *approximate* area of the shape.
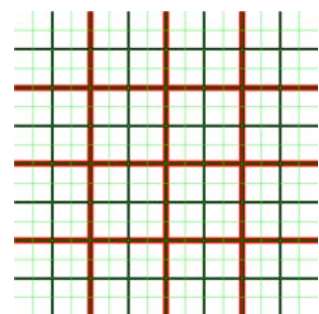
Step 4: Finally, as a class, decide that *if* this shape is a circle, what is its radius. Using this agreed upon measurement, calculate the *approximate* area of the shape using the formula for the area of a circle: $A_{circle} = \pi\ r^2$.

Activity B Pre-Lesson Activity Discussion Questions

1. How do the four approximated measurements compare?

2. How do the three grid approximations compare to the approximation using the formula for the area of a circle?

3. Which approximation seems most accurate?

II. Approximating the area of a crater:

Step 1: Set up a testing station by filling a shallow pan with soil at least 3 inches deep. Place the pan of soil on a sheet to protect the floor from ejecta.

Alternative Method (Step 1)

Students who did not participate in the challenge and who did not design and build a launching device may select 3 of the 9 impact crater pictures on page 52.

Students should select 1 image for each angle of impact: 15°, 45°, and 70°.

Then, using their launching device(s) from the "*Cratering the Moon*" challenge, students launch a standard projectile into the pan of soil at three different angles, creating a total of three craters. The first crater will have a 15° angle of impact, the second crater will have a 45° angle of impact, and the third crater will have a 70° angle of impact. (**NOTE:** *The craters should not touch one another. If necessary, use three pans—one pan for each crater.*)

15° angle of impact

45° angle of impact

70° angle of impact

Step 2: Select one square grid* that will be used to approximate the area of each crater.

Beginning with the first crater, lay the transparency of the square grid* on top of the crater and trace the shape of the crater onto the square grid using a marker.

Step 3: As was done in the pre-lesson activity, determine the crater's inner area and outer area.

Next, *approximate* the area of the uppermost cross-section ("top") of the crater by averaging the values you obtained for the inner area and the outer area of the crater. (*See examples on pp. 37–40.*)

*NOTE: If students use a grid with squares that are greater or lesser than 1 cm^2, then make sure they calculate their area approximation accordingly.

Remember, *one* 2 cm x 2 cm square is made up of *four* 1 cm x 1 cm squares. Similarly, it takes *four* 0.5 cm x 0.5 cm squares to make *one* 1 cm x 1 cm square.

Record the crater's approximate area on the **Activity B – Crater Area** student worksheet on page 55. Repeat steps 2 and 3 for the remaining two craters.

Referring to the dimensions for the 3 selected impact craters pictured on page 52, students can plot (sketch) the general geometric shape of each crater on a square grid and then *approximate* the area of each shape by determining the "inner area" and the "outer area".

NOTE: If students use a grid whose squares have area greater or lesser than 1 cm^2, then make sure they scale their sketch accordingly.

Example: Crater outline drawn on a 1 cm x 1 cm unit square grid.
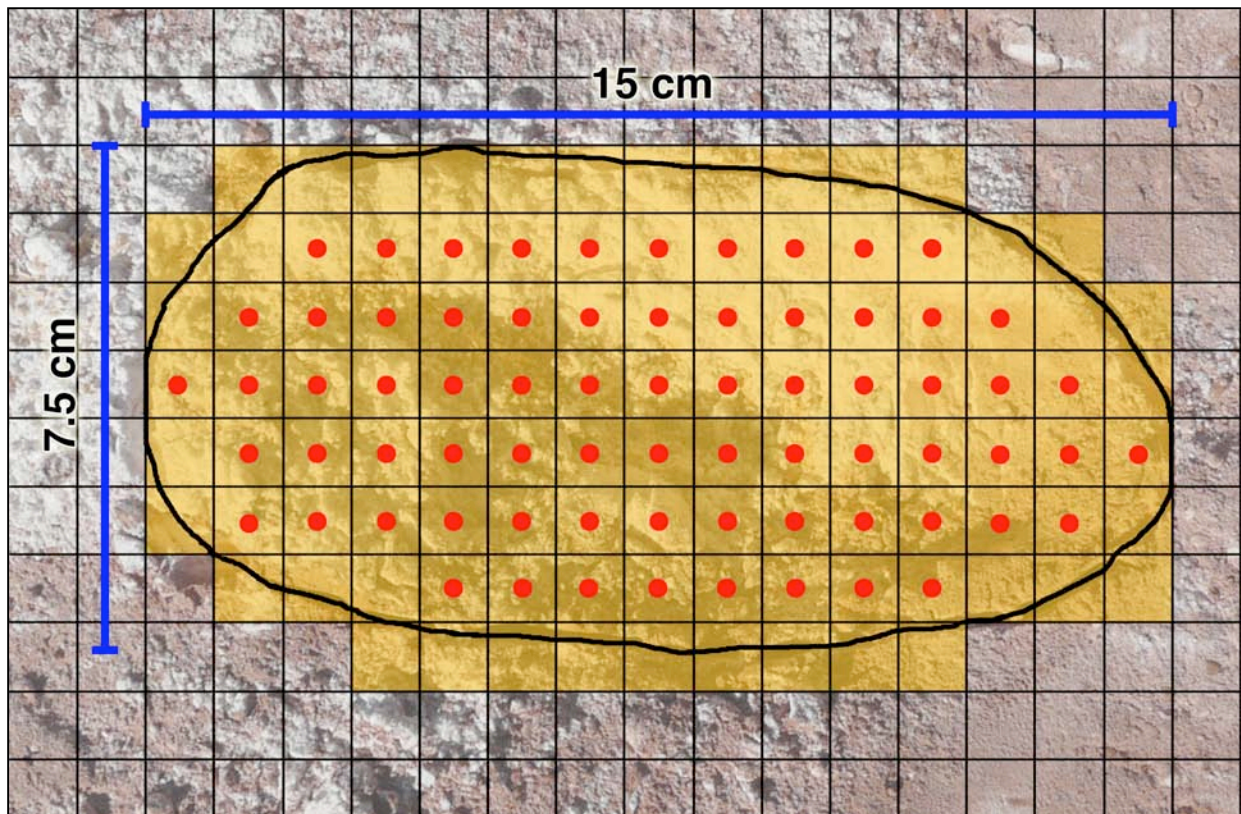
In the picture above, the squares marked with red dots are wholly contained within the shape's boundary. The total area of these squares represents the "inner area" of the shape, which is 71 cm^2.

The squares shaded in orange are either wholly contained within the shape or touch the shape's boundary. The total area of these squares is the shape's "outer area," which is 108 cm^2.

Step 4: After approximating the area of the three craters (each representing a different angle of impact: 15º, 45º, and 70º), then:

- Observe the craters and their respective drawings on the square grids.

- Analyze the calculated data.

- Conclude how the angle of impact affects the area of an impact crater.

Activity B Crater Area Discussion Questions

1. How does the size of the square grid affect the accuracy of your measurements?

2. Would the formula for the area of a circle serve as a good approximation for the crater with a 15º angle of impact? (with a 45º angle of impact? with a 70º angle of impact?) Why or why not?

3. In your experiment, which angle of impact yielded a crater with the greatest area (uppermost cross section)?

Activity C: Mass, Area, and Volume

Objectives:

In this activity, students will:

- Conduct an experiment to determine how the mass of a projectile affects the area and the volume of an impact crater.

- Measure the circumference (C) of various spherical projectiles and then calculate their diameters (d) using the formula: $d = C \div \pi$.

- Measure the mass (in grams) of various spherical projectiles using a triple beam balance (if available).

- Measure the approximate diameter of a nearly circular impact crater and approximate the crater's area using the formula: $A_{circle} = \pi\ r^2$, where r is the radius of the circle.

- Measure the depth of a nearly circular impact crater and approximate the crater's volume by using the formula for the volume of a spherical cup: $V = \dfrac{\pi\ h}{6} (3 \cdot a^2 + h^2)$, where a is the radius and h is the height of the spherical cup.

Materials:

- Shallow pan

- Soil-like substance

- Bed sheet (or other drape for spreading on the floor under the pan of soil)

- Measuring tape

- Assortment of spherical projectiles that are of similar size but of different masses

- Triple beam balance (or other means of measuring the mass of the spherical projectiles)

- *Activity C – Projectile Experiment Data Sheet* (pp. 56–57)

Step 1: Gather and measure projectiles

Gather an assortment of spherical projectiles of <u>similar size</u> but of <u>different masses</u>. Suggestions include:

• golf ball	• ping pong ball	• small ball of clay
• racquet ball	• Nerf ball	• small ball of yarn

Measure each projectile's *circumference* (in centimeters), and then calculate the projectile's *diameter* using the formula for the diameter of a sphere ($d = C \div \pi$).

If a triple beam balance is available, then measure the mass (in grams) of each projectile.

Record the name and respective measurements for each projectile on the **Activity C – Projectile Experiment Data Sheet** (p. 56).

Step 2. Conduct cratering experiment

Drop each projectile, one at a time, into a pan of soil at the "standard" angle of 90º and then measure and record the resulting crater's depth (in cm) and diameter (in cm) on the **Activity C– Projectile Experiment Data Sheet** (p. 56)**.**

Validity of the Experiment

To ensure the validity of this experiment, students should release each projectile from the same height and into the same pan, so that the only "variable" is the object being dropped. Students should:

- Drop (do *not throw*) the object at a 90º angle by holding it directly over the pan.

- Release each object from the same height. For example, a selected student stands on a stepstool and releases each object one at a time from shoulder height with arm extended perpendicular to the body. (This standard height should be recorded on the **Activity C – Projectile Experiment Data Sheet** on page 57.)

- Drop each object onto an untouched layer of soil where no other craters exist.

- If students are testing many objects and need to use more than one pan of soil to prevent the overlapping of craters, then make sure the pans of soil are identical.
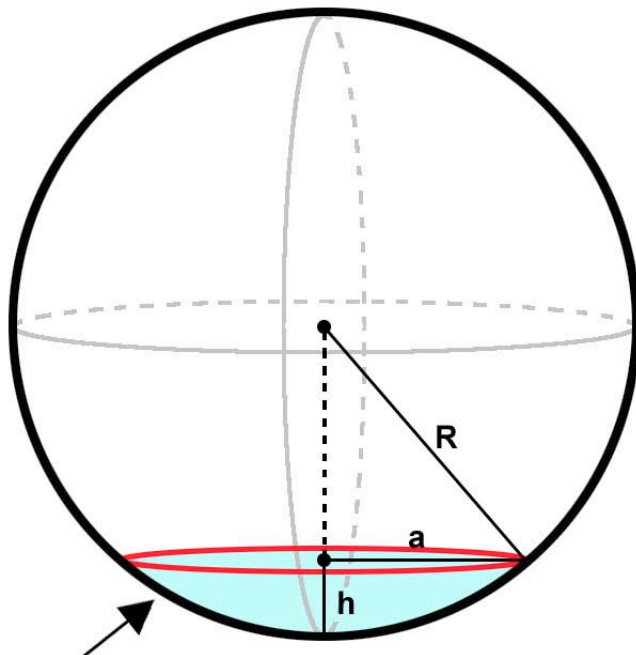

Step 3. Calculate crater area

Since a 90º impact will create a crater whose uppermost cross-section is nearly circular, students can calculate the *approximate* area of the crater's upper cross-section using the formula for the area of a circle:

$$A_{circle} = \pi\ r^2 \text{, where r is the radius of the circle.}$$

Step 4. Calculate crater volume

Again, because the crater is nearly circular, students can calculate the crater's *approximate* volume by using the formula for the volume of a spherical cup (i.e., crater):

R = radius of sphere

a = radius of crater

h = height of crater

$$V_{crater} = \frac{\pi\ h}{6}\ (\ 3\ a^2\ +\ h^2\ )$$

shaded area represents the crater (spherical cup)

Alternative Approach

For students who need more of a challenge, you can have them express the equation for the volume of a crater (spherical cup) in terms of the radius (R) of the sphere and the height (h) of the crater. This can be done using the Pythagorean theorem.

See example in the Teacher Resources section on page 45.

Guide to the sample answers on Activity C – Projectile Experiment Data Sheet (p. 56)

PROJECTILE	CRATER

PROJECTILE

Object: regulation golf ball

Circumference: 14 cm

Diameter: $d = C \div \pi$

$= 14 \text{ cm} \div \pi$

$\approx 4.5 \text{ cm}$

Mass: ≈ 46 grams

Regulation golf balls must not exceed 45.93 g

CRATER

Diameter: 5 cm

Area: $r_{circle} = d \div 2$

$= 5 \text{ cm} \div 2$

$= 2.5 \text{ cm}$

$A_{circle} = \pi \, r^2$

$= \pi \cdot (2.5 \text{ cm})^2$

$= \pi \cdot 6.25 \text{ cm}^2$

$\approx 19.6 \text{ cm}^2$

Depth (h): 2.5 cm

Volume: $V_{crater} = \dfrac{\pi \, h}{6} (3 \, r^2 + h^2)$

$= \dfrac{\pi \, 2.5 \text{ cm}}{6} [3 \, (2.5 \text{ cm})^2 + (2.5 \text{ cm})^2]$

$\approx 1.31 \text{ cm} [3 \, (6.25 \text{ cm}^2) + 6.25 \text{ cm}^2]$

$= 1.31 \text{ cm} (18.75 \text{ cm}^2 + 6.25 \text{ cm}^2)$

$= 1.31 \text{ cm} (25 \text{ cm}^2)$

$\approx 32.8 \text{ cm}^3$

Lesson Extensions

Three hands-on "proofs" related to expressing the volume of a sphere and the volume of a crater (spherical cup) are located in the Teacher Resources and EXTEND sections of this lesson. *See pages 46–50 (volume of a sphere) and pages 32–35 (volume of a crater)*.

Step 5. Order, analyze, and discuss results

On the **Activity C–Projectile Experiment Data Sheet** (p. 57), create three lists ordering the projectiles from greatest to least according to the:

> 1) Projectile's mass
>
> 2) Crater's area
>
> 3) Crater's volume

Analyze the data for any patterns from which you can draw conclusions about how a projectile's mass affects the upper cross-sectional area and the volume of its crater.

Activity C Discussion Questions

1. How do the sizes of the craters created by the different projectiles compare?

2. Did the craters vary greatly in diameter? In depth?

3. Did the more massive projectiles create "larger" and/or deeper craters?

4. How does the diameter of a projectile compare to the diameter of its crater?

Activity D: Size, Mass, Area, and Volume

Objectives:

In this activity, students will:

- Conduct an experiment to determine how the size *and* the mass of a projectile affects the area and the volume of an impact crater.

- Measure the circumference (C) of various spherical projectiles and then calculate their diameters (d) using the formula: $d = C \div \pi$.

- Measure the mass (in grams) of various spherical projectiles using a triple beam balance (if available).

- Measure the approximate diameter of a nearly circular impact crater and approximate the crater's area using the formula: $A_{circle} = \pi\ r^2$, where r is the radius of the circle.

- Measure the depth of a nearly circular impact crater and approximate the crater's volume by using the formula for the volume of a spherical cup: $V = \dfrac{\pi\ h}{6} (3 \cdot a^2 + h^2)$, where a is the radius and h is the height of the spherical cup.

Materials:

- Shallow pan

- Soil-like substance

- Bed sheet (or other drape for spreading on the floor under the pan of soil)

- Measuring tape

- Assortment of spherical projectiles that are of different size and of different masses

- Triple beam balance (or other means of measuring the mass of the spherical projectiles)

- *Activity D – Projectile Experiment Data Sheet* (pp. 58–60)

Step 1: Gather, measure, and categorize projectiles

Gather an assortment of spherical projectiles of <u>different sizes</u> and of <u>different masses</u>. Suggestions include:

• beach ball	• volley ball
• baseball / softball	• tennis ball / racquet ball
• Nerf ball	• golf ball
• ping pong ball	• stress ball
• marble / bouncing ball	• Styrofoam ball

Measure each projectile's *circumference* (in centimeters), and then calculate the projectile's *diameter* using the formula for the diameter of a sphere ($d = C \div \pi$).

If a triple beam balance is available, then measure the mass (in grams) of each projectile.

Using the size and mass measurements, categorize the projectiles in a matrix by recording the name and respective measurements of each projectile on the **Activity D – Projectile Experiment Data Sheet** (p. 58).

Step 2. Conduct cratering experiment

Drop each projectile, one at a time, into a pan of soil at the "standard" angle of 90º and then measure and record the resulting crater's depth (in cm) and diameter (in cm) on the **Activity D– Projectile Experiment Data Sheet** (p. 59).

Validity of the Experiment

To ensure the validity of this experiment, students should release each projectile from the same height and into the same pan, so that the only "variable" is the object being dropped. Students should:

- Drop (do *not throw*) the object at a 90º angle by holding it directly over the pan.

- Release each object from the same height. For example, a selected student stands on a stepstool and releases each object one at a time from shoulder height with arm extended perpendicular to the body. (This standard height should be recorded on the **Activity D – Projectile Experiment Data Sheet** on page 59.)

- Drop each object onto an untouched layer of soil where no other craters exist.

- If students are testing many objects and need to use more than one pan of soil to prevent the overlapping of craters, then make sure the pans of soil are identical.
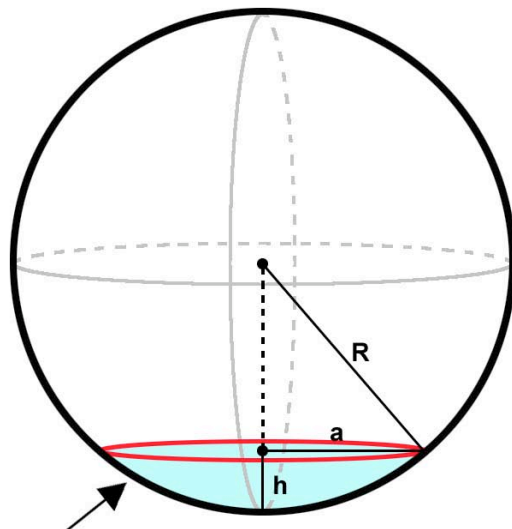

Step 3. Calculate crater area

Since a 90° impact will create a crater whose uppermost cross-section is nearly circular, students can calculate the *approximate* area of the crater's upper cross-section using the formula for the area of a circle:

$$A_{circle} = \pi \; r^2, \text{ where r is the radius of the circle.}$$

Step 4. Calculate crater volume

Again, because the crater is nearly circular, students can calculate the crater's *approximate* volume by using the formula for the volume of a spherical cup (i.e., crater):

R = radius of sphere

a = radius of crater

h = height of crater

$$V_{crater} = \frac{\pi \, h}{6} \; (3 \, a^2 + h^2)$$

shaded area represents the crater (spherical cup)

Lesson Extensions

For students who need more of a challenge, you can have them express the equation for the volume of a crater (spherical cup) in terms of the radius (R) of the sphere and the height (h) of the crater. This can be done using the Pythagorean theorem. *(See example in the Teacher Resources section on page 45.)*

Three hands-on "proofs" related to expressing the volume of a sphere and the volume of a crater (spherical cup) are located in the Teacher Resources and EXTEND sections of this lesson. *See pages 46–50 (volume of a sphere) and pages 32–35 (volume of a crater).*

Step 5: Order, analyze, and discuss results

On the **Activity D–Projectile Experiment Data Sheet** (p. 60), create four lists ordering the projectiles from greatest to least according to the:

       1) Projectile's mass

       2) Projectile's size

       3) Crater's area

       4) Crater's volume

Analyze the data for any patterns from which you can draw conclusions about how a projectile's size and/or mass affects the upper cross-sectional area and the volume of its crater.

Activity D Discussion Questions

1. How do the sizes of the craters created by the different projectiles compare?

       Projectiles of large size but low mass (example: beach ball)

       Projectiles of medium size and medium mass (example: tennis ball)

       Projectiles of small size but high mass (example: marble)

2. Did the craters vary greatly in diameter? In depth?

3. Did the more massive projectiles create "larger" and/or deeper craters?

4. Did the projectiles with greater diameters create "larger" and/or deeper craters?

5. How does the diameter of a projectile compare to the diameter of its crater?

6. Which projectile created the largest crater?

7. Which projectile created the smallest crater?

EXPLAIN

The overarching theme of this lesson is exploring what angle of impact and what size and mass of projectile excavates the greatest amount of "stuff" (soil, matter, ejecta) for scientists to analyze.

In Activities A, B, C, and D of the EXPLORE section, students performed experiments, analyzed data, and drew conclusions. Now students will review what they have learned and observed and decide which angle of impact and which projectile size and mass will result in the greatest crater size—meaning that the maximum amount of soil is displaced.

To aid their decision, students can graph or chart their data, which will help them identify any patterns. If no clear choice of projectile or angle of impact exists, then students will apply reasoning skills to help them choose the best combination of angle of impact and projectile size and mass that results in the greatest displacement of soil. Students should state their reasons for their conclusion as to which angle of impact and which projectile excavates the greatest amount of soil.

EVALUATE

Activities A and B

1. Describe the relationship between the shape of a crater and the projectile's angle of impact.

2. In your experiment, which angle of impact created a crater with the greatest upper cross-sectional area?

3. Explain one method for approximating the area of a crater's uppermost cross section.

Activities C and D

1. How does the mass of a projectile affect the area and volume of an impact crater?

2. How does the size of a projectile affect the area and volume of an impact crater?

3. What methods and/or formulas can be used to approximate the area of a crater's uppermost cross section?

4. What methods and/or formulas can be used to approximate the volume of a crater?

Extension 1: Scale of Lunar, Earth, and Classroom Craters
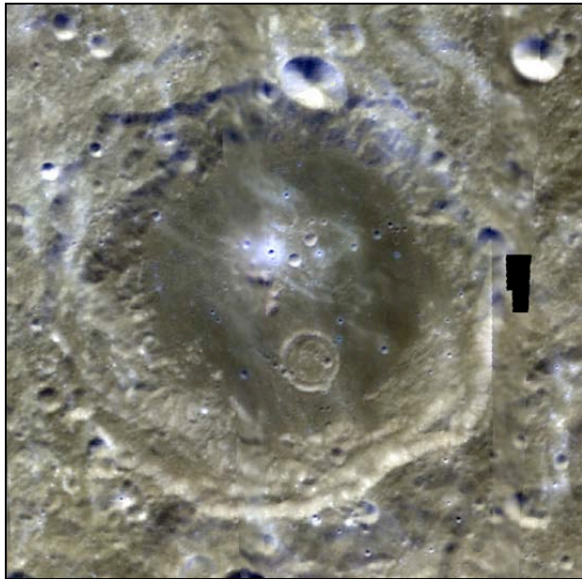
South Pole-Aitken basin on far side of Moon

The sizes of the impact craters we see on the Moon are many times larger than the size of the impact craters we can model in the classroom. The diameters of impact craters on the Moon range from 1 meter to more than 2,000,000 meters. Craters that measure more than 300 km (~186 miles) across are called basins. The largest crater on the Moon is South Pole-Aitken basin. It is located on the far side of the Moon and is approximately 2,500 km (~ 1,550 miles) in diameter and 13 km (~ 8 miles) deep.

Task 1a: Compare the size (in terms of diameter) of South Pole-Aitken basin to one of your classroom craters. (See example on p. 30)

Task 1b: Calculate how much larger (first in terms of diameter and then in terms of depth) South Pole-Aitken basin is than Earth's Meteor Crater in Arizona.   Meteor Crater measures approximately 1.2 km (~ 0.75 mile) in diameter and approximately 0.17 km (~ 0.11 mile) in depth.
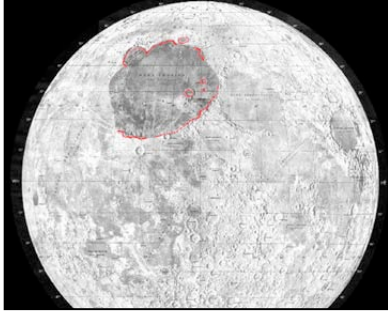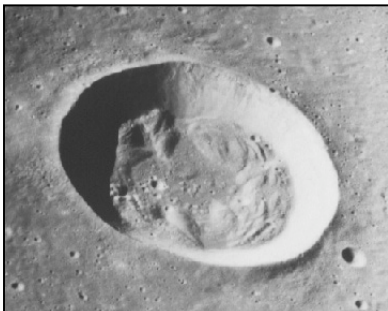
***Task 1c:*** Compare first the diameter and then the depth of a classroom-made crater to each of the five lunar craters pictured on page 29.

Meteor Crater in Arizona

Task 1d: Plot the lunar craters, Meteor crater, and the classroom craters on a line graph, first according to their diameters and then according to their depths. How do they compare?

Task 1e: Plot the lunar craters, Meteor crater, and the classroom craters on two number lines: the first for diameter and the second for depth. How do they compare? Does the spacing of the craters change from one number line to the next?

Task 1e: Compare the diameter-to-depth ratios of all of the craters. What do you notice about these ratios? Does the depth of a crater increase in proportion to its diameter?

| Crater Image	Lunar Crater Name	Approximate Diameter	Approximate Depth
	Mare Imbrium (Sea of Rains)	1,123 km (~ 697.8 miles)	12 km (~ 7.5 miles)
	Bessel Crater	16 km (~ 9.9 miles)	2 km (~ 1.2 miles)
	Copernicus Crater	93 km (~ 57.8 miles)	3.8 km (~ 2.4 miles)
	Euler Crater	28 km (~ 17.4 miles)	2.5 km (~ 1.6 miles)
	King Crater	77 km (~ 47.8 miles)	5 km (~ 3.1 miles)
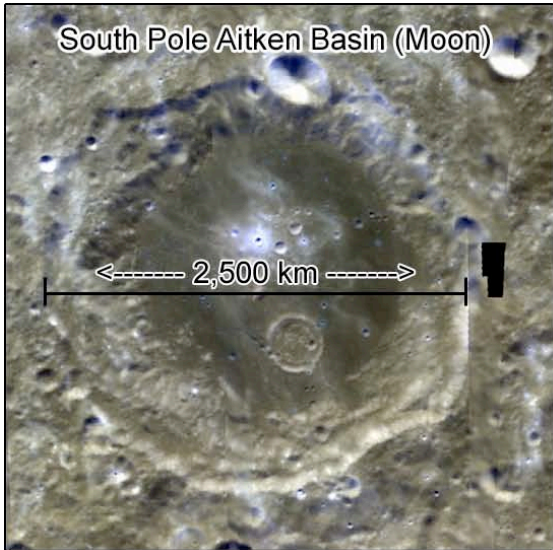
South Pole Aitken Basin (Moon)

<------- 2,500 km ------->

Compare the diameter of the Aitken Basin to the diameter of the golfball crater.

We know that:

    Diameter of the Aitken Basin   = 2,500 km

    Diameter of the golfball crater   = 5 cm

We begin by converting km to cm: 1 km = 100,000 cm
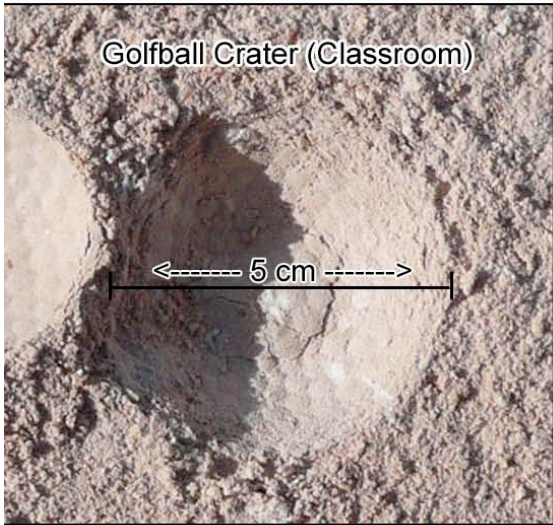
Golfball Crater (Classroom)

<------- 5 cm ------->

We will use the unit ratio: $\dfrac{100{,}000 \text{ cm}}{1 \text{ km}}$

2,500 km = _____ cm

$$= \frac{2{,}500 \text{ km}}{1} \cdot \frac{100{,}000 \text{ cm}}{1 \text{ km}}$$

$$= 2{,}500 \cdot 100{,}000 \text{ cm}$$

$$= 250{,}000{,}000 \text{ cm}$$

Now we know that:

    Diameter of the Aitken Basin = 250,000,000 cm

    Diameter of the golfball crater = 5 cm

Divide the diameter of the Aitken Basin by the diameter of the golfball crater.

    $\dfrac{250{,}000{,}000 \text{ cm}}{5 \text{ cm}}$ = 50,000,000

We can say that the diameter of the Aitken Basin is 50,000,000 (fifty million) times greater than the diameter of the golfball crater!

Extension 2: Scale of the LCROSS Crater

Examine the lunar map of craters (inset) and the image of the craters within craters covering the lunar surface below.
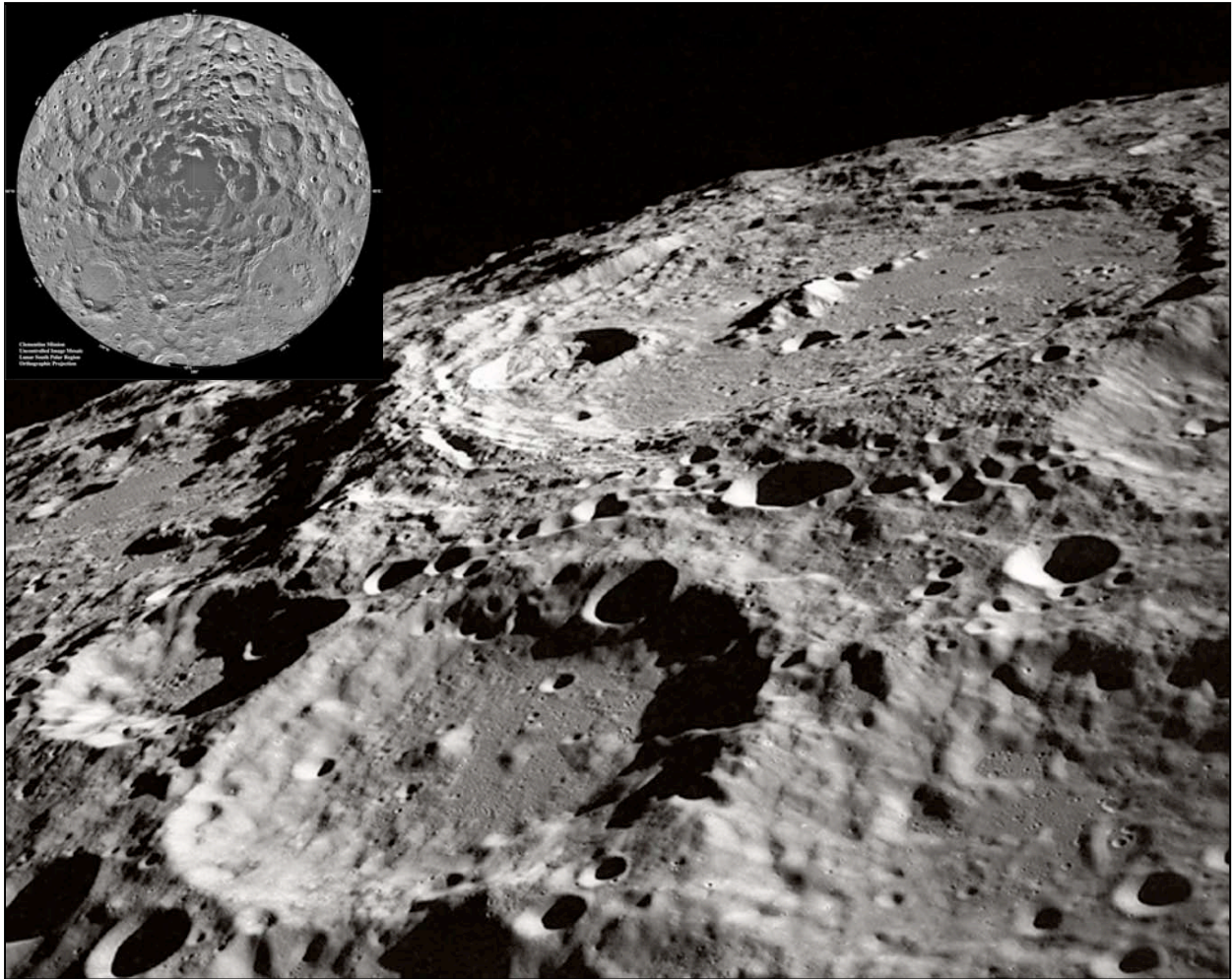
Pictured Above: Crater 302, Apollo 10

Next, watch the brief LCROSS Crater animation at http://lcross.arc.nasa.gov/CraterSizes.htm and discuss the three questions below.

Animation Discussion Questions

1. How will the crater created by the LCROSS impactor compare to other lunar craters?

2. How will the crater created by the LCROSS impactor compare to your classroom craters?

3. *IF* the large crater pictured above were 100 km in diameter (actual size is unknown), then how many LCROSS craters could you line up side by side across the 100 km diameter?


Extension 3: Hands-On Proof Using Two Cylinders

In this hands-on activity, students use a bowl to represent a crater. Then students collect or construct 2 cylinders as indicated in the equation below and fill them with sand to verify the volume of the crater.

Materials:

• Bowl that is part of a sphere. Examples: cereal bowl (pictured right), a portion cut from a sports ball, or half of a hamster exercise ball, etc.

• 2 cylinders (The size of the cylinders will depend on the size of the bowl. Students can experiment with plastic CD spindle covers, butter cookie tins, tuna or pet food cans, etc, *or* students can construct their cylinders out of heavy paper, file folders, or cardboard tubes.)

  – Cylinder 1: must have the same height and the same radius as the crater (bowl)

  – Cylinder 2: the radius and the height must equal the height of the crater (bowl)
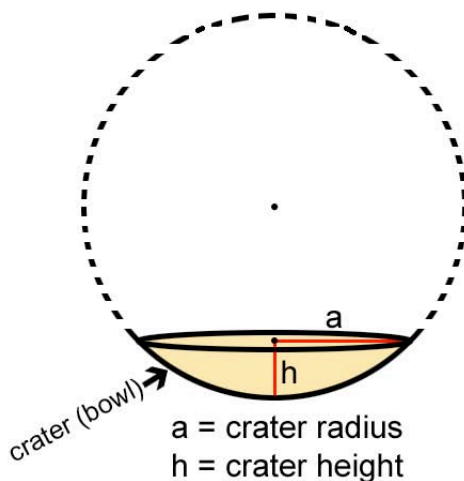
• Sand (or other fine, soil-like substance)
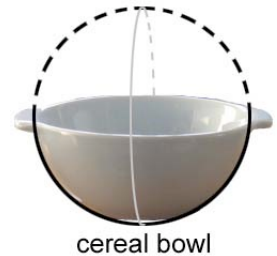

cereal bowl

sample cylinders

The amount of sand needed to fill half of a cylinder with the same height and the same radius of the bowl *plus* one-sixth of a cylinder whose height and radius is the height of the bowl will equal the volume of sand that fills the bowl (i.e., the crater). See example on following page.

a = crater radius
h = crater height

$$V_{crater} = \frac{\pi h}{6}\ (\ 3\ a^2\ +\ h^2\ )$$

$$= \frac{\pi h}{6} \cdot 3\ a^2 \qquad\qquad + \frac{\pi h}{6} \cdot h^2$$

$$= \frac{\pi h}{2} \cdot a^2 \qquad\qquad\quad + \frac{\pi h}{6} \cdot h^2$$

$$= \frac{1}{2}\pi a^2\ h \qquad\qquad\quad + \frac{1}{6}\pi h^2\ h$$

$$= \frac{1}{2} \qquad\qquad\qquad\quad + \frac{1}{6}$$

Example for Extension 3:

In this example, the crater has a radius (a) of 10 cm and a height (h) of 6 cm.
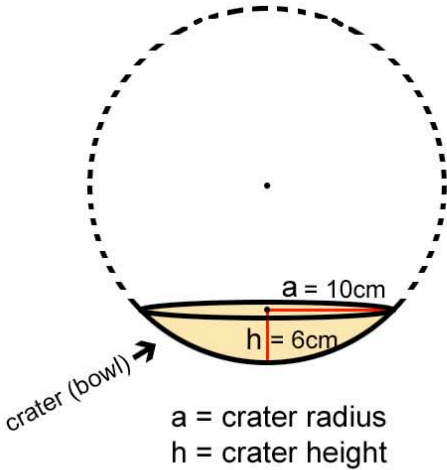
a = crater radius
h = crater height

$$V_{crater} = \frac{\pi h}{6}(3a^2 + h^2)$$

$$= \frac{\pi \, 6 \text{ cm}}{6}[3(10 \text{ cm})^2 + (6 \text{ cm})^2]$$

$$= \pi \text{ cm }[3(100 \text{ cm}^2) + 36 \text{ cm}^2]$$

$$= \pi \text{ cm }(300 \text{ cm}^2 + 36 \text{ cm}^2)$$

$$= \pi \text{ cm }(336 \text{ cm}^2)$$

$$\approx 1{,}056 \text{ cm}^3$$
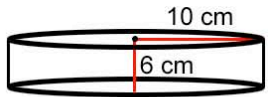
$$V_{cylinder} = \pi r^2 h$$

$$= \pi (10 \text{ cm})^2 (6 \text{ cm})$$

$$= \pi \, 100 \text{ cm}^2 (6 \text{ cm})$$

$$= \pi \, 600 \text{ cm}^3$$
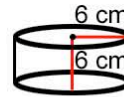
$$\approx 1{,}885.7 \text{ cm}^3$$

$$V_{cylinder} = \pi r^2 h$$

$$= \pi (6 \text{ cm})^2 (6 \text{ cm})$$

$$= \pi \, 36 \text{ cm}^2 (6 \text{ cm})$$

$$= \pi \, 216 \text{ cm}^3$$

$$\approx 678.9 \text{ cm}^3$$

$$V_{crater} = \frac{1}{2}\left[\;\right] + \frac{1}{6}\left[\;\right]$$

$$\approx \frac{1}{2}(1{,}885.7 \text{ cm}^3) + \frac{1}{6}(678.9 \text{ cm}^3)$$

$$\approx 942.9 \text{ cm}^3 + 113.1 \text{ cm}^3$$

$$= 1{,}056 \text{ cm}^3$$

Extension 4: Hands-On Proof Using a Cylinder and a Cone

In this hands-on activity, students use a bowl to represent a crater. Then students collect or construct a cylinder and a cone as indicated in the equation below and fill them with sand to verify the volume of the crater.

Materials:

• Bowl that is part of a sphere. Examples: cereal bowl (pictured right), a portion cut from a sports ball, or half of a hamster exercise ball, etc.

• Cylinder that has the same height and the same radius as the crater (bowl). Students can experiment with plastic CD spindle covers, butter cookie tins, tuna or pet food cans, etc, *or* students can construct their cylinder out of heavy paper, file folders, or cardboard tubes.

• Cone that has a radius and a height that are both equal to the height of the crater (bowl). Students can construct their cone out of heavy paper.

• Sand (or other fine, soil-like substance)
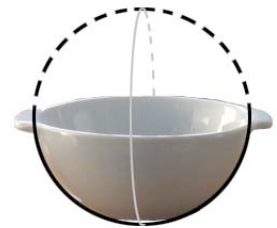
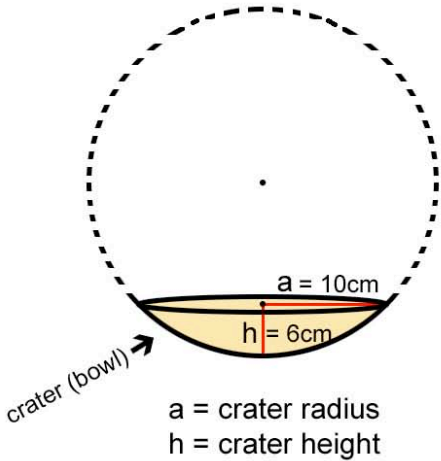cereal bowl

sample cylinders

The amount of sand needed to fill half of a cylinder with the same height and the same radius of the bowl **plus** half of a cone whose height and radius is the height of the bowl will equal the volume of sand that fills the bowl (i.e., the crater). See example on following page.

a = crater radius
h = crater height

crater (bowl)
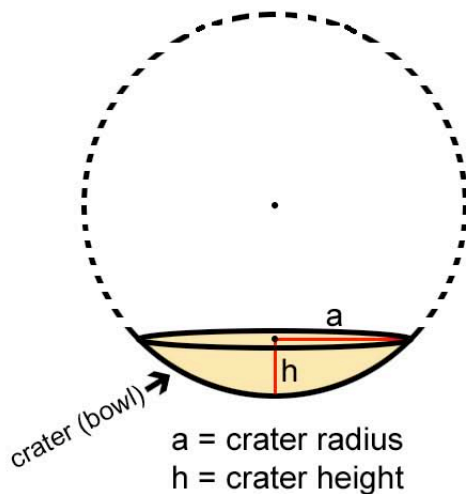
$$V_{crater} = \frac{\pi h}{6} (3 a^2 + h^2)$$

$$= \frac{\pi h}{6} \cdot 3 a^2 \qquad\qquad + \frac{\pi h}{6} \cdot h^2$$

$$= \frac{\pi h}{2} \cdot a^2 \qquad\qquad + \frac{\pi h}{6} \cdot h^2$$

$$= \frac{1}{2} \pi a^2 h \qquad\qquad + \frac{1}{6} \pi h^2 h$$

$$= \frac{1}{2} \pi a^2 h \qquad\qquad + \frac{1}{2} \cdot \frac{1}{3} \pi h^2 h$$

$$= \frac{1}{2} \quad\boxed{} \qquad + \frac{1}{2} \quad \triangledown$$

Example for Extension 4:

In this example, the crater has a radius (a) of 10 cm and a height (h) of 6 cm.

$$V_{crater} = \frac{\pi\, h}{6}\,(\,3\,a^2\,+\,h^2\,)$$

$$= \frac{\pi\, 6\ cm}{6}\,[\,3\,(10\ cm)^2\,+\,(6\ cm)^2\,]$$

$$= \pi\ cm\,[\,3\,(100\ cm^2)\,+\,36\ cm^2\,]$$
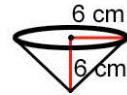
$$= \pi\ cm\,(\,300\ cm^2\,+\,36\ cm^2\,)$$

$$= \pi\ cm\,(\,336\ cm^2\,)$$

$$\approx 1{,}056\ cm^3$$

a = 10cm
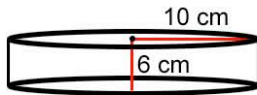
h = 6cm

crater (bowl)

a = crater radius
h = crater height

10 cm

6 cm

$$V_{cylinder} = \pi\ r^2\ h$$

$$= \pi\,(10\ cm)^2\,(6\ cm)$$

$$= \pi\ 100\ cm^2\,(6\ cm)$$

$$= \pi\ 600\ cm^3$$

$$\approx 1{,}885.7\ cm^3$$

6 cm

6 cm

$$V_{cone} = \frac{1}{3}\,\pi\ r^2\ h$$

$$= \frac{1}{3}\,\pi\,(6\ cm)^2\,(6\ cm)$$

$$= \frac{1}{3}\,\pi\ 36\ cm^2\,(6\ cm)$$

$$= \frac{1}{3}\,\pi\ 216\ cm^3$$

$$= \pi\ 72\ cm^3$$

$$\approx 226.3\ cm^3$$

10 cm

6 cm

6 cm

6 cm

$$V_{crater} = \frac{1}{2}\qquad\qquad +\ \frac{1}{2}$$

$$\approx \frac{1}{2}\,(1{,}885.7\ cm^3)\qquad +\ \frac{1}{2}\,(226.3\ cm^3)$$

$$\approx 942.9\ cm^3\qquad\qquad +\ 113.1\ cm^3$$

$$= 1{,}056\ cm^3$$

Teacher Resources

Sample crater with a 1 cm x 1 cm unit square grid overlay
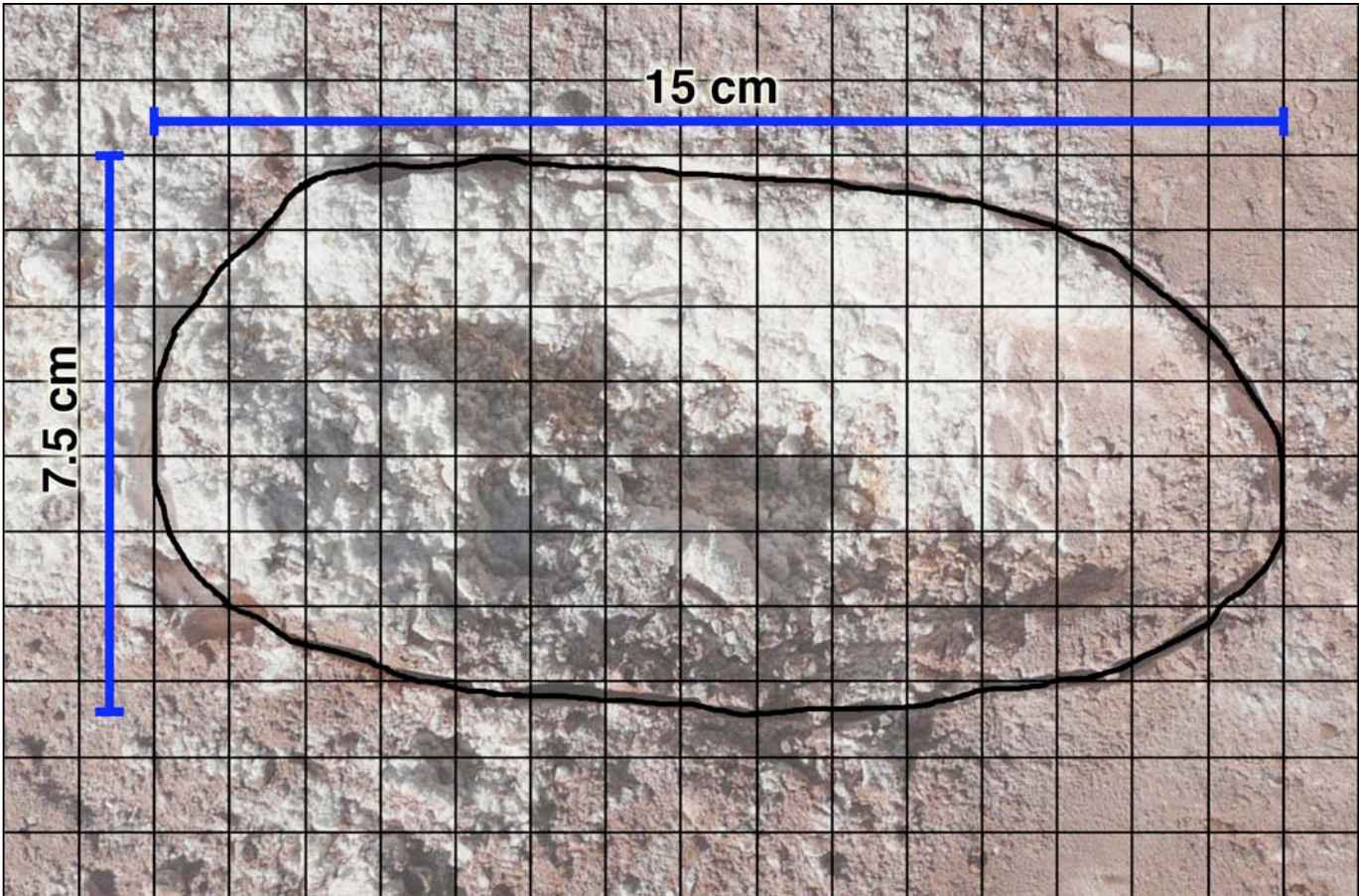
Sample crater with a 2 cm x 2 cm square grid overlay
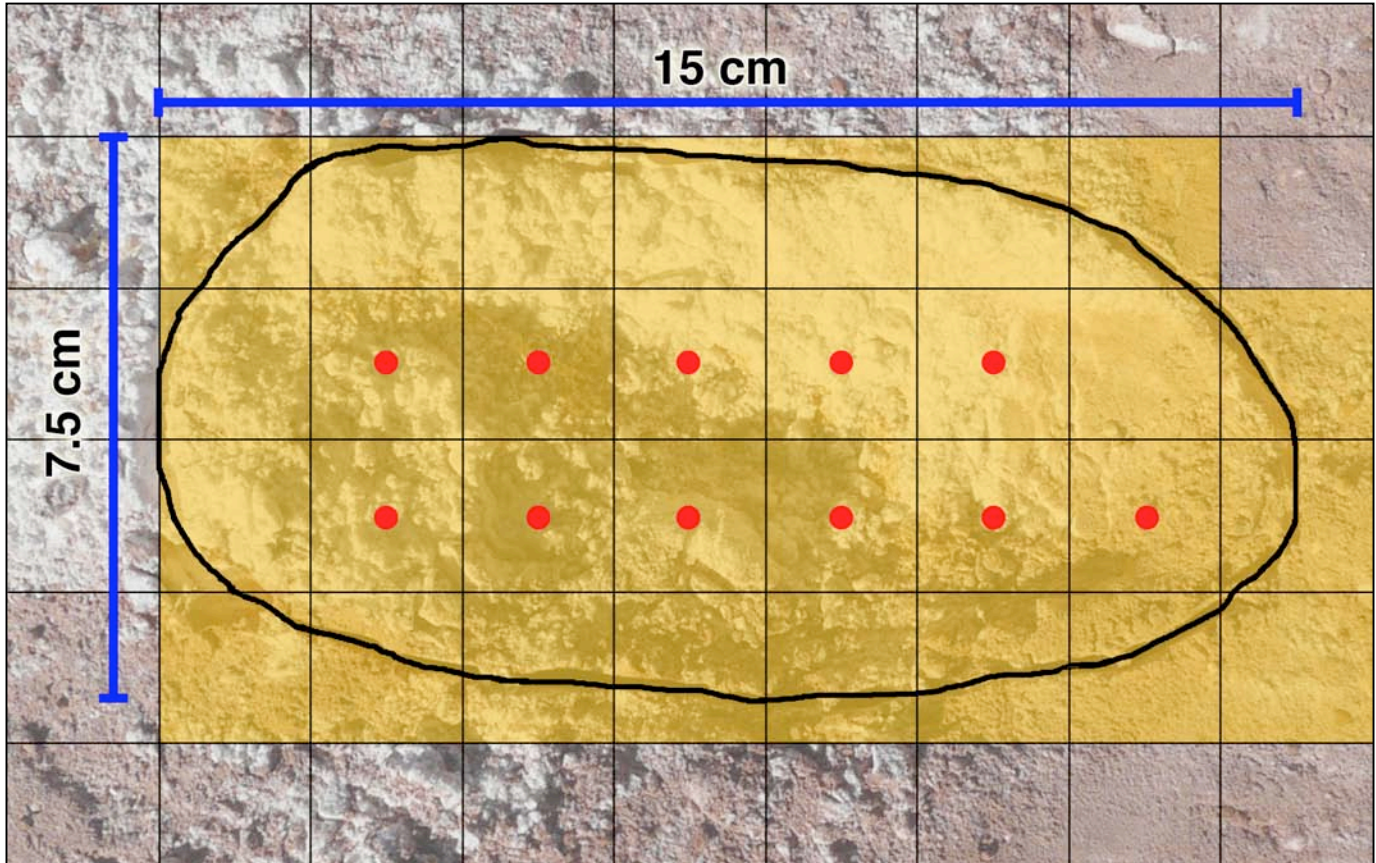
Outer area

In this example, the squares that comprise the shape's *outer area* are marked with yellow shading. Each 2 cm x 2 cm outer area square has area 4 cm^2. There are 31 of them, so the outer area is:

$$31 \cdot 4 \text{ cm}^2 = 124 \text{ cm}^2$$

Inner area

In this example, the squares that comprise the shape's *inner area* are marked with red dots. Each 2 cm x 2 cm inner area square has area 4 cm^2. There are 11 of them, so the inner area is:

$$11 \cdot 4 \text{ cm}^2 = 44 \text{ cm}^2$$

We can say that the *actual area* of the shape is greater than 44 cm^2 and less than 124 cm^2.
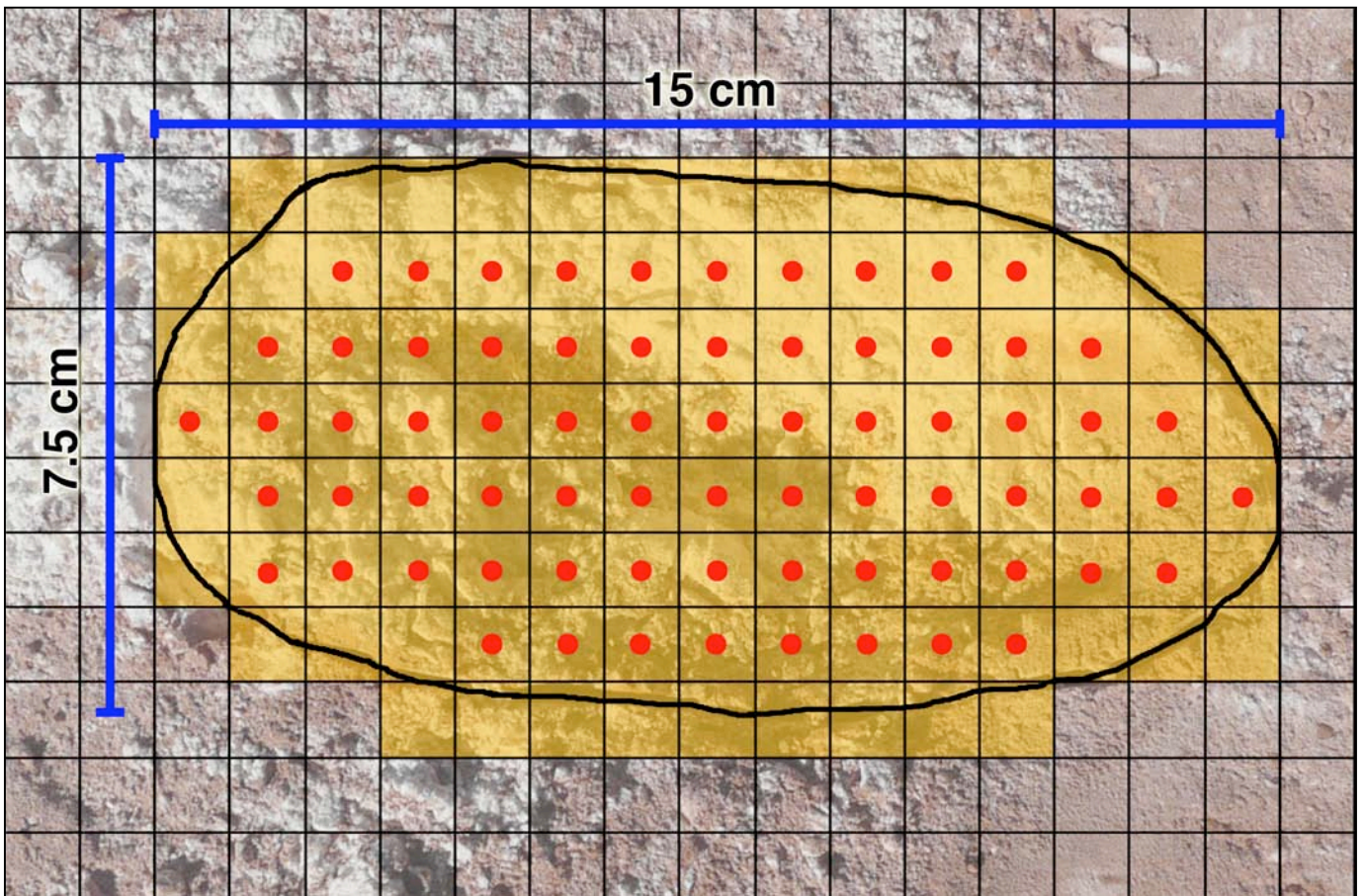
$$44 \text{ cm}^2 \leq \text{actual area} \leq 124 \text{ cm}^2$$

Furthermore, we can *approximate* the actual area of the shape by averaging the inner area and the outer area values:

$$(44 \text{ cm}^2 + 124 \text{ cm}^2) \div 2 = 84 \text{ cm}^2$$

Sample crater with a 1 cm x 1 cm unit square grid overlay

Outer area

In this example, the squares that comprise the shape's *outer area* are marked with <mark>yellow shading</mark>. Each 1 cm x 1 cm outer area square has area 1 cm^2. There are 108 of them, so the outer area is 108 cm^2.

Inner area

In this example, the squares that comprise the shape's *inner area* are marked with red dots. Each 1 cm x 1 cm inner area square has area 1 cm^2. There are 71 of them, so the inner area is 71 cm^2.

We can say that the *actual area* of the shape is greater than 71 cm^2 and less than 108 cm^2.

$$71 \text{ cm}^2 \ \leq \ \text{actual area} \ \leq \ 108 \text{ cm}^2$$

Furthermore, we can *approximate* the actual area of the shape by averaging the inner area and the outer area values:

$$(71 \text{ cm}^2 \ + \ 108 \text{ cm}^2) \ \div \ 2 \ = \ 89.5 \text{ cm}^2$$

Sample crater with a 0.5 cm x 0.5 cm square grid overlay

Outer area

In this example, the squares that comprise the shape's *outer area* are marked with <mark>yellow shading</mark>. Each 0.5 cm x 0.5 cm outer area square has area 0.25 cm^2. There are 394 of them, so the outer area is:

$$394 \cdot 0.25 \text{ cm}^2 = 98.5 \text{ cm}^2$$

Inner area

In this example, the squares that comprise the shape's *inner area* are marked with red dots. Each 0.5 cm x 0.5 cm inner area square has area 0.25 cm^2. There are 324 of them, so the inner area is:
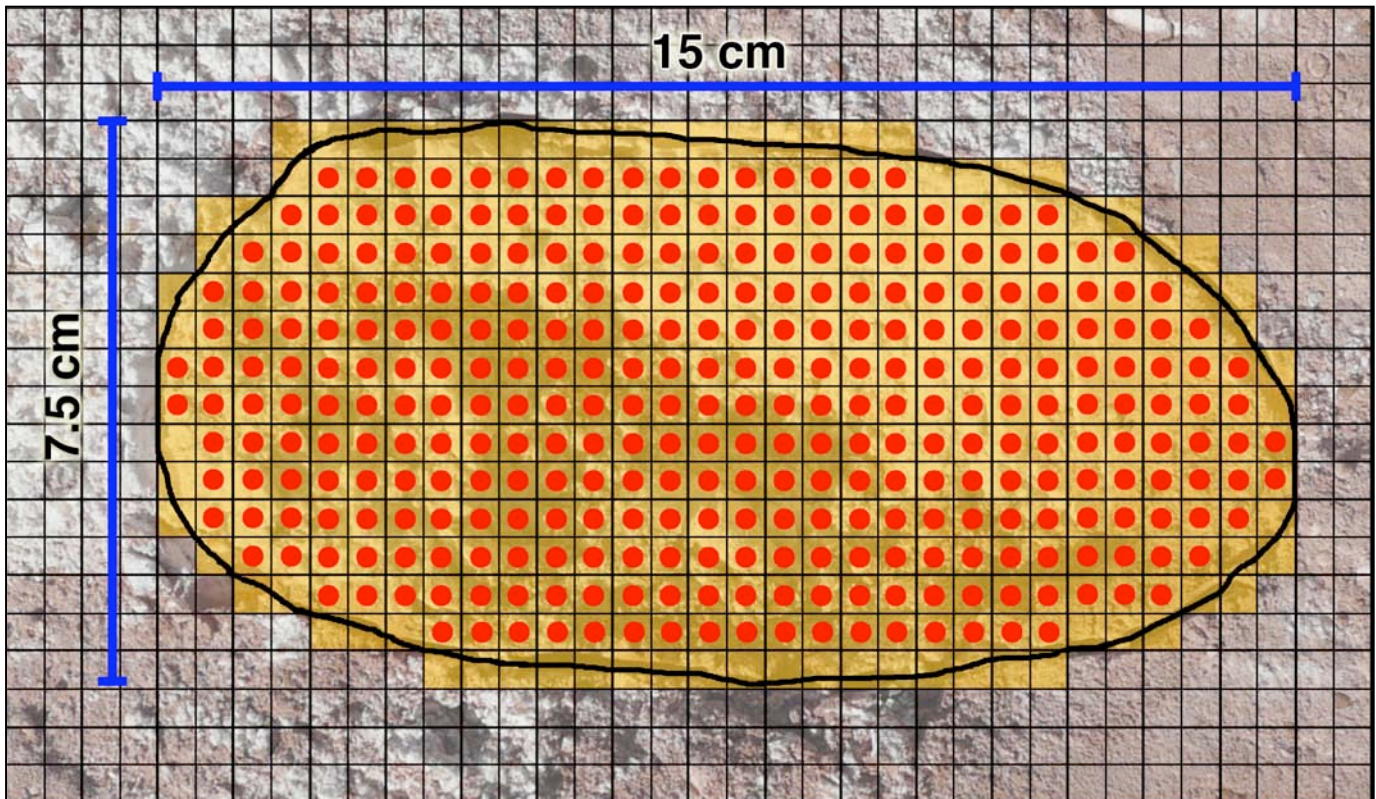
$$324 \cdot 0.25 \text{ cm}^2 = 81 \text{ cm}^2$$

We can say that the *actual area* of the shape is greater than 81 cm^2 and less than 98.5 cm^2.

$$81 \text{ cm}^2 \leq \text{ actual area } \leq 98.5 \text{ cm}^2$$

Furthermore, we can *approximate* the actual area of the shape by averaging the inner area and the outer area values:

$$(81 \text{ cm}^2 + 98.5 \text{ cm}^2) \div 2 = 89.75 \text{ cm}^2$$

Large Square Grid: 2.0 cm x 2.0 cm
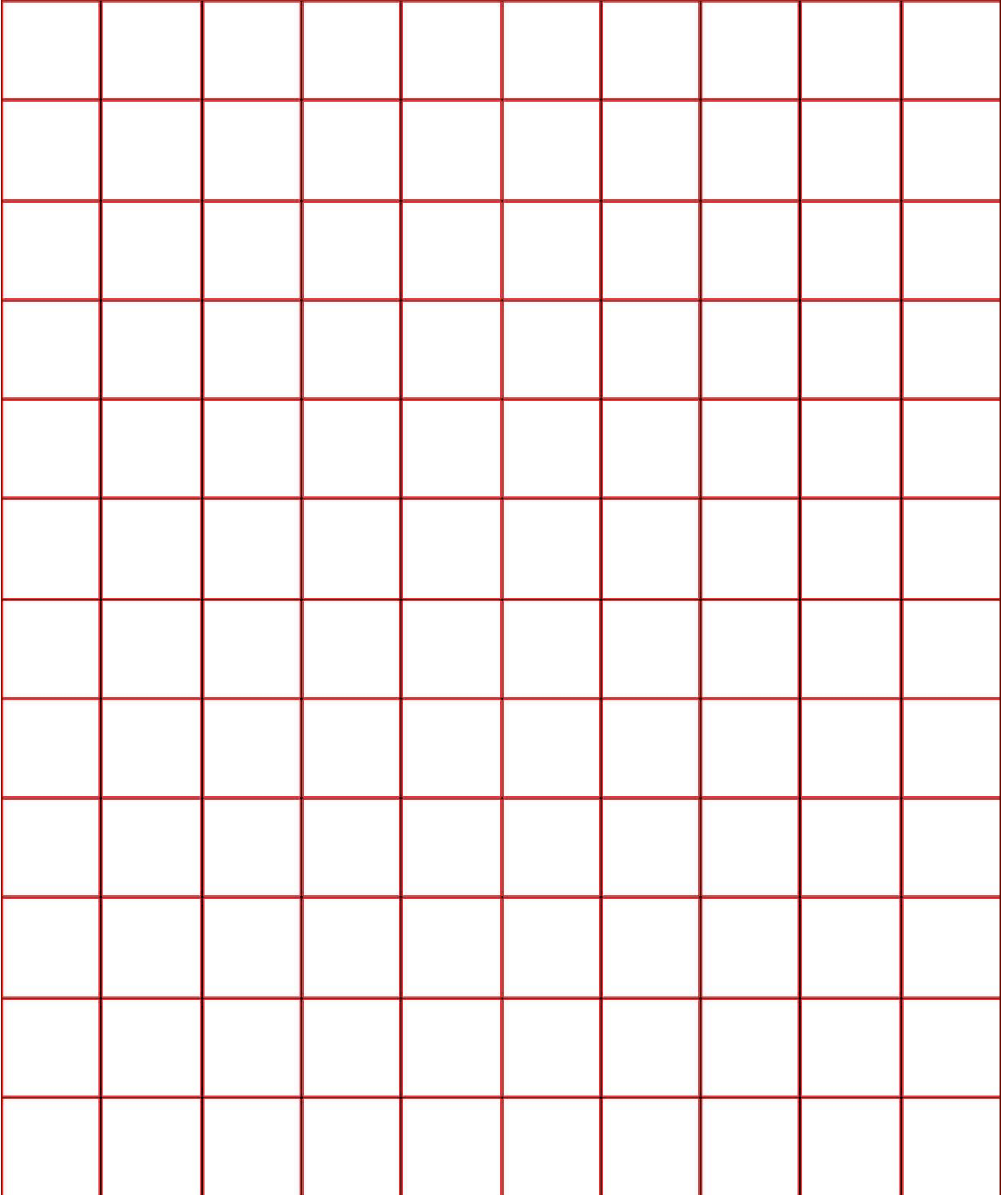
Medium Unit Square Grid: 1.0 cm x 1.0 cm

Small Square Grid: 0.5 cm x 0.5 cm

Small Square Grid: 0.5 cm x 0.5 cm

"Shocking" Shapes!

Take a close look at the craters that you have created or observed. You may notice that the shapes of the craters vary based on the angle at which they hit the surface. Some are round and some are more oval. This makes sense when you think about the path that the projectile is traveling and how the projectile itself directly creates the crater by gouging out material using its own surface.
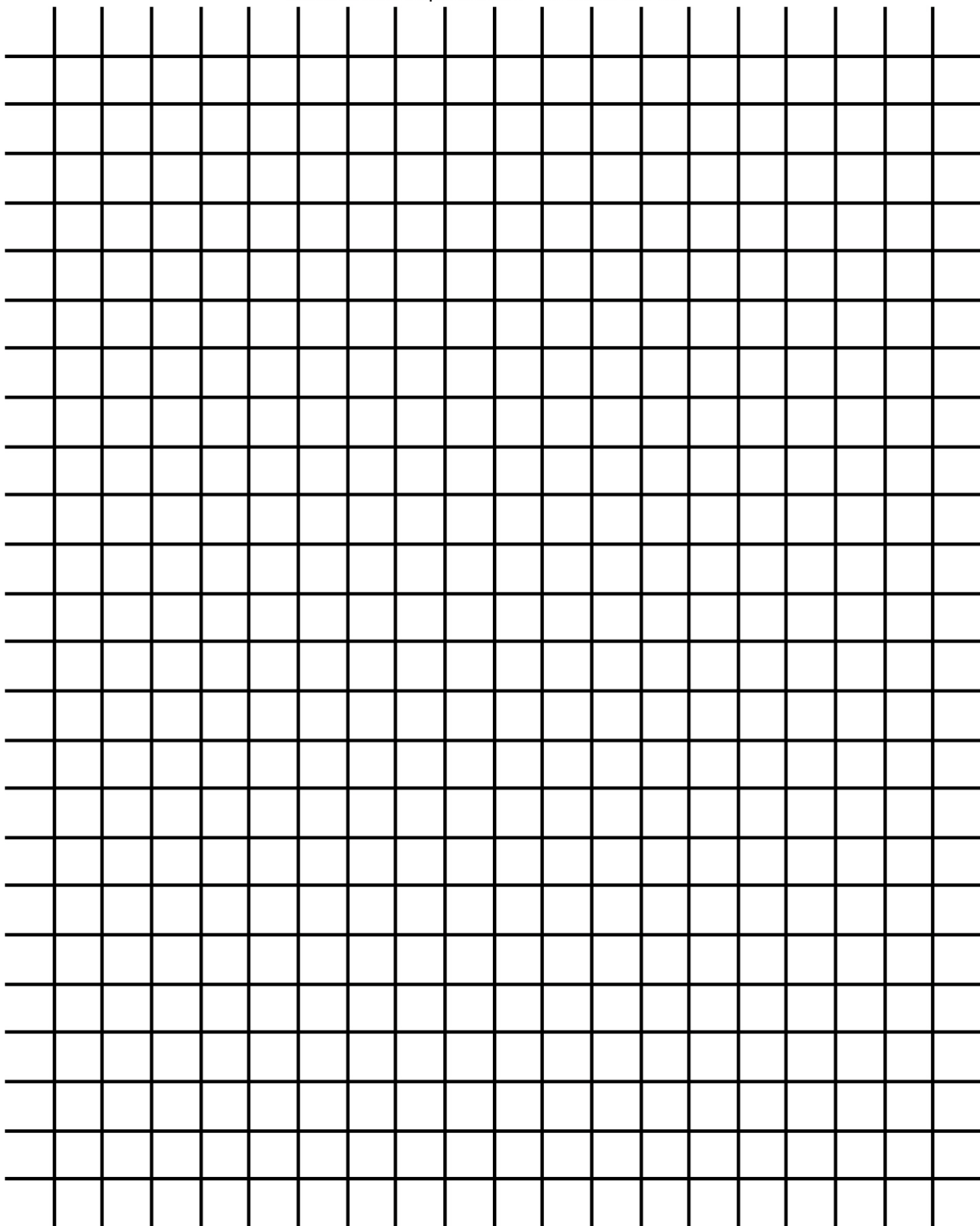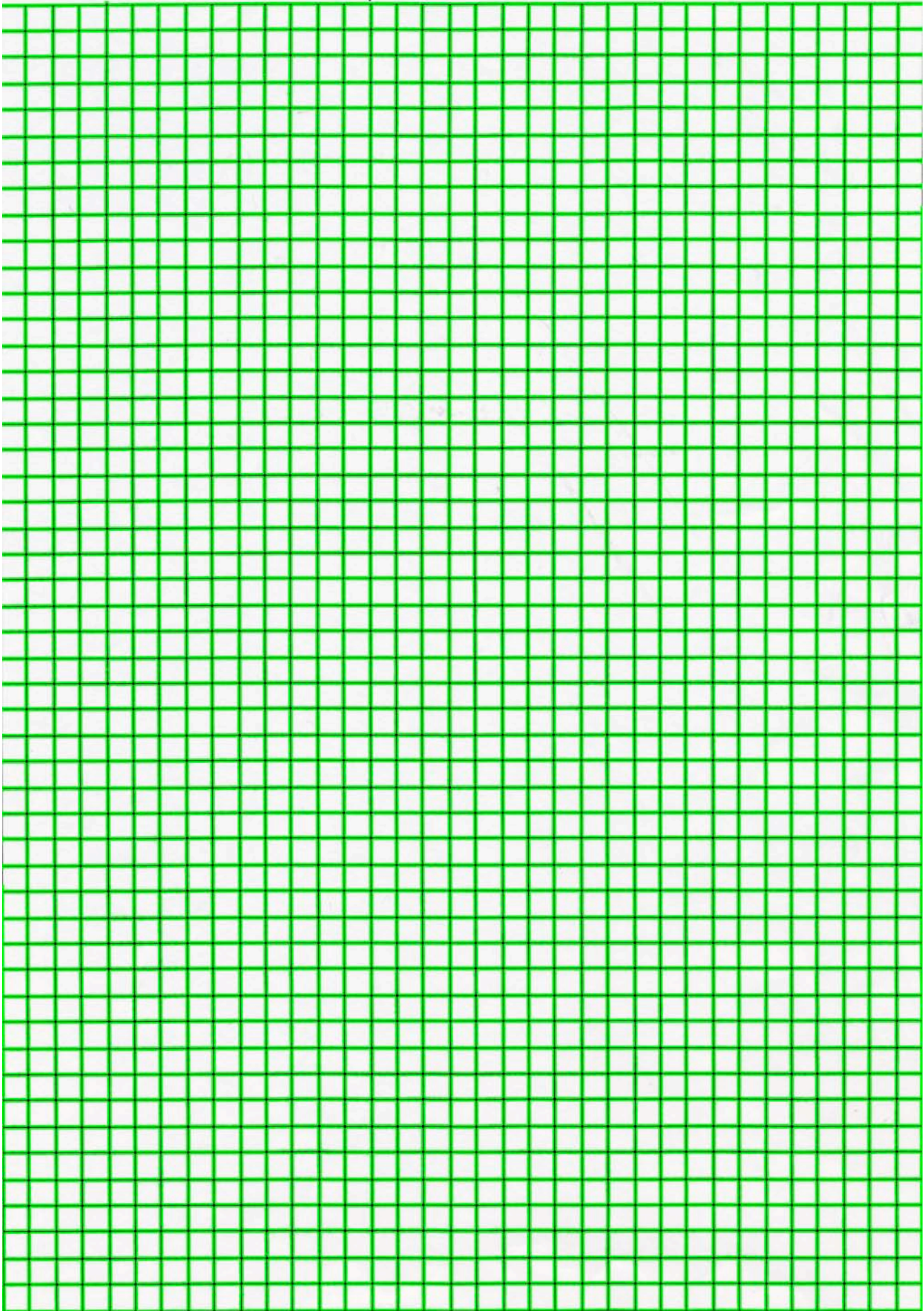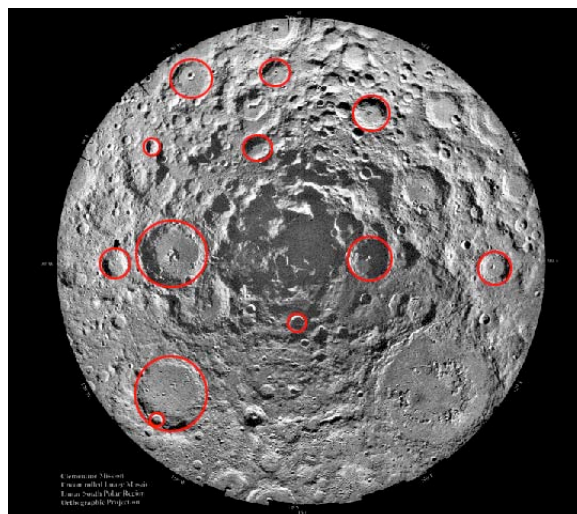
But the situation on the Moon is quite different. The lunar craters are remarkable for being uniformly round. This was a big puzzle to earlier lunar researchers. Meteoroids should strike the Moon from a wide range of angles, so if lunar craters are really impact features, why don't we see a range of crater shapes from circular to highly oval?

One critical difference between your experiments and cratering events on the Moon is the velocity at which the impactors strike the surface. A meteoroid making a crater on the Moon typically hits at speeds of thousands and even tens of thousands of kilometers per hour. At these kinds of tremendous speeds, such an impact generates a shock wave in the lunar surface. If you could watch one of these impacts from above, you would see that the shock wave spreads out across the lunar surface in a circle centered on the point of impact. The shock wave expands in a circle regardless of what angle the meteoroid strikes. You would also notice another key difference from your experiment. Your craters are probably not all that different in size from the projectile that created them. But the shock wave from a cratering event on the Moon can spread out and excavate a crater far larger than the meteoroid that generated it. An impactor only a few kilometers in diameter can create a crater over 100 kilometers across. Generating shock waves can be very effective ways of generating large craters.

Of course, lunar scientists rarely, if ever, get the chance to directly observe the formation of a crater on the Moon. However, using NASA's hypervelocity ballistics facilities, they can simulate lunar cratering events, shooting high-speed impactors at varying angles into simulated lunar surfaces, creating shockwaves and craters.

Applying the Pythagorean Theorem

Imagine the crater pictured to the right is a portion (or cup) of a greater sphere. If you know the radius (R) of this sphere, then you can express the Volume of the crater (spherical cup) in terms of the radius (R) of the sphere and the height (h) of the crater using the Pythagorean theorem.

Isidorus D Crater
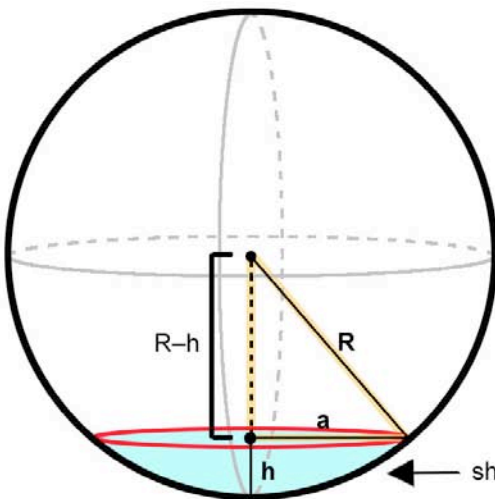
R = radius of sphere

a = radius of crater

h = height of crater

$$V_{crater} = \frac{\pi h}{6} (3 a^2 + h^2)$$

shaded area represents the crater (spherical cup)

Pythagorean theorem: $(R - h)^2 + a^2 = R^2$

Solve for a^2:

$$a^2 = R^2 - (R - h)^2$$
$$= R^2 - (R^2 - 2 R h + h^2)$$
$$= R^2 - R^2 + 2 R h - h^2$$
$$= 2 R h - h^2$$

So...

$$V_{crater} = \frac{\pi h}{6} (3 (2 R h - h^2) + h^2)$$
$$= \frac{\pi h}{6} (6 R h - 3 h^2 + h^2)$$
$$= \frac{\pi h}{6} (6 R h - 2 h^2)$$
$$= \left(\frac{\pi h}{6}\right) (2 h) (3 R - h)$$
$$= \frac{2 \pi h^2}{6} (3 R - h)$$
$$= \frac{\pi h^2}{3} (3 R - h)$$

Volume of a Sphere: A Hands-On "Proof"

In this geometric idea for a proof, students will use a cylinder, cones, and a sphere to determine the *volume* of a sphere (how much the sphere can hold).

Materials:

- **Tennis ball** (or similar sized sphere)
- **Heavy paper** for constructing a cylinder and a cone that each has the same radius (r) as the sphere (tennis ball) and that each has a height of 2r.
- **Tape** for constructing the cylinder and the cone
- **Sand** (or sugar, flour, beans, etc) for filling the cylinder and the cone

Volume of a Cylinder

Imagine that a cylinder is made up of many circles of radius (r), stacked up to height (h).

The volume (V) of a cylinder is the area of the circle of radius (r) times the height (h) of the cylinder. The area of the circle is πr^2.
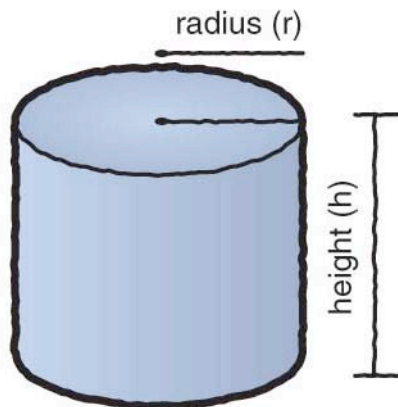
$$V_{cylinder} = \pi r^2 h$$

Volume of a Cone

If you had a cone with the same height (h) and the same radius (r) as the cylinder, it would take three cones to fill the cylinder.

Try this for yourself by constructing a cylinder and a cone with heavy paper and tape. (The cone must have the same radius (r) and the same height (h) as the cylinder.) Next, fill the cone with sand (or sugar, flour, beans, etc) and then pour the contents into the cylinder. Fill the cone two more times, each time emptying its contents into the cylinder. The volume of three cones should equal the volume of one cylinder.
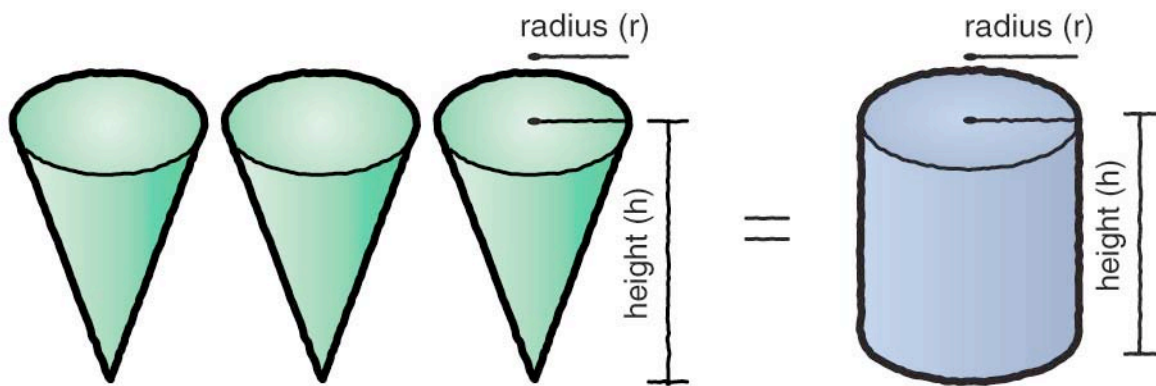
So the volume of a cone (of radius r and height h) is one-third the volume of a cylinder (of radius r and height h).
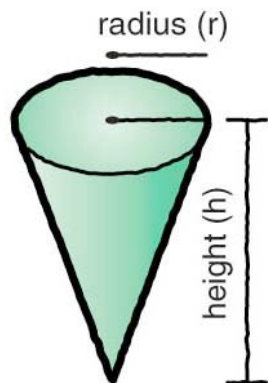
$$V_{cone} = \frac{V_{cylinder}}{3}$$

$$= \frac{\pi r^2 h}{3}$$

Volume of a Sphere

If you insert a sphere (tennis ball) with radius (r) inside a cylinder so that it fits tightly, the radius of the cylinder will be r, and the height of the cylinder will be 2r.
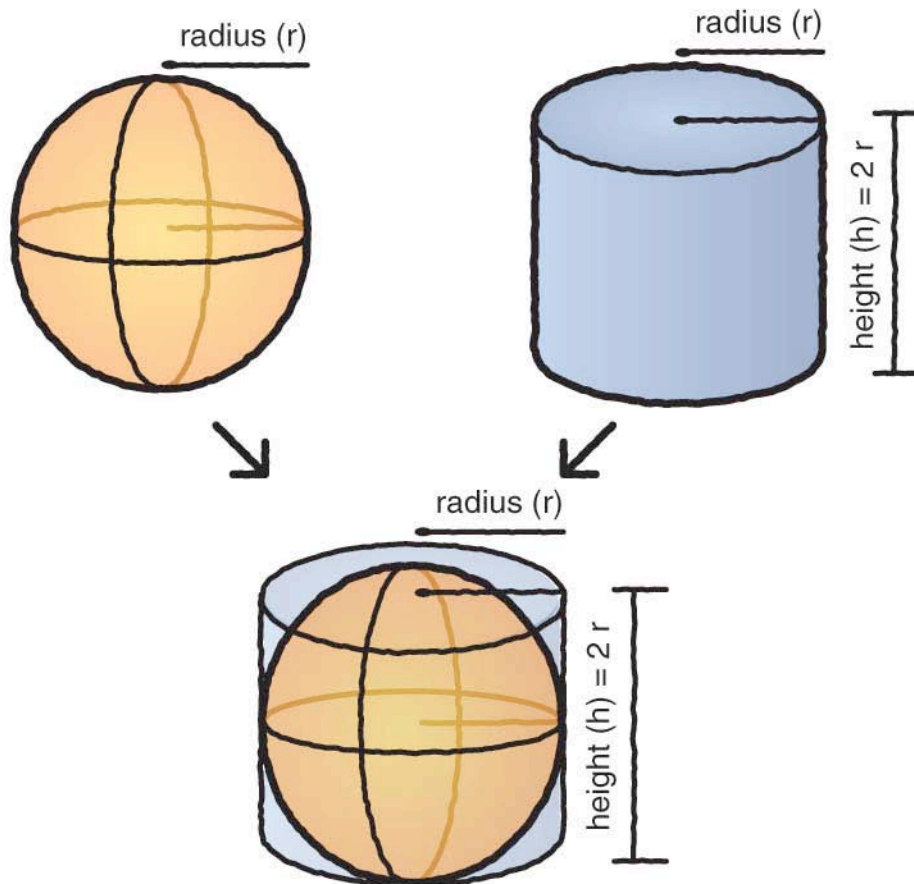
Even though the sphere has been placed inside the cylinder so that it fits tightly, you notice that there is still some space in the cylinder around the sphere that has not been filled.

We know the volume of the cylinder is the area of the circle of radius (r) multiplied by the height (h).

$$\mathbf{V_{cylinder}} = \pi\, r^2\, h$$

We know that the height (h) of the cylinder is equal to 2r.

$$\mathbf{V_{cylinder}} = \pi\, r^2\, (2r)$$

$$= 2\,\pi\, r^3$$

If we have a cone of radius (r) and height (2r), we can calculate its volume as follows:

$$\mathbf{V_{cone}} = \frac{\pi\, r^2\, h}{3}$$

$$= \frac{2\,\pi\, r^3}{3}$$
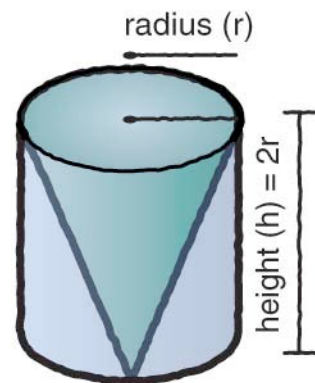
It turns out that we can fill that remaining space in the cylinder around the sphere with exactly the amount of sand that will fit into this cone. So we know that the volume of the cone with radius (r) and height (2r) plus the volume of the sphere with radius (r) is equal to the volume of the cylinder with radius (r) and height (2r).

$$\mathbf{V_{cone} + V_{sphere} = V_{cylinder}}$$
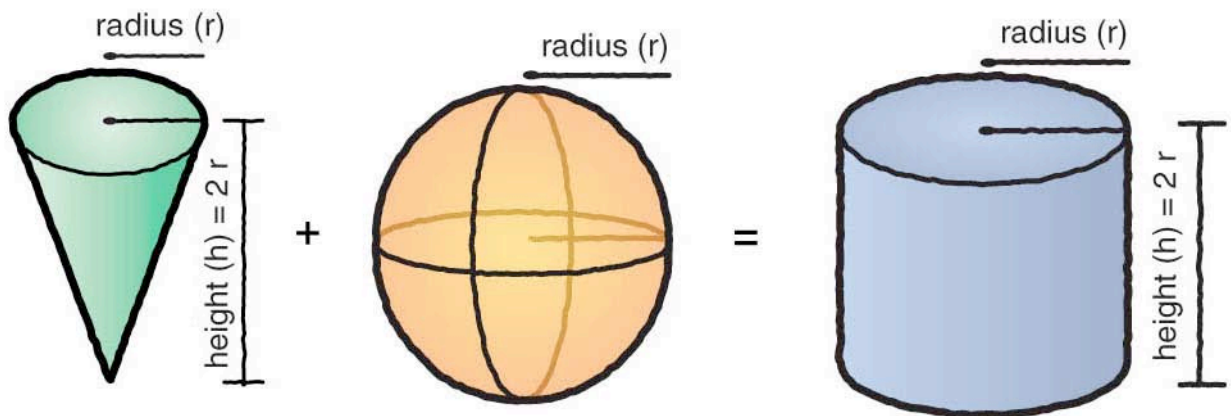
We know the formulas for the volume of the cone and the volume of the cylinder. We can use these to solve for the volume of the sphere.
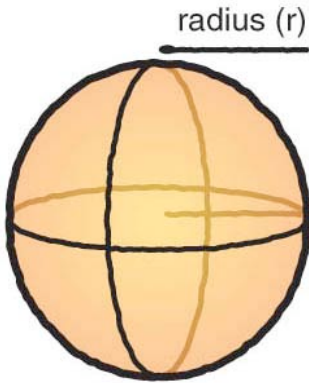
$$V_{sphere} = V_{cylinder} - V_{cone}$$

radius (r)

$$= \quad 2\pi r^3 \quad - \quad \frac{2\pi r^3}{3}$$

$$= \quad \frac{6\pi r^3}{3} \quad - \quad \frac{2\pi r^3}{3}$$

$$= \quad \frac{6\pi r^3 \quad - \quad 2\pi r^3}{3}$$

$$= \quad \frac{4\pi r^3}{3}$$

This is the formula for the volume of a sphere that can be rigorously derived using calculus.

Student Worksheets

Activity A – Impact Crater Images

Crater A
15° angle of impact
9 cm
16.5 cm

Crater B
45° angle of impact
9 cm
12.5 cm

Crater C
70° angle of impact
7.5 cm
7.5 cm

Crater D
45° angle of impact
10 cm
12 cm

Crater E
15° angle of impact
7.5 cm
14.0 cm

Crater F
45° angle of impact
9 cm
12.5 cm

Crater G
15° angle of impact
7.5 cm
11.5 cm

Crater H
70° angle of impact
7.5 cm
7.5 cm

Crater I
70° angle of impact
7.5 cm
7.5 cm
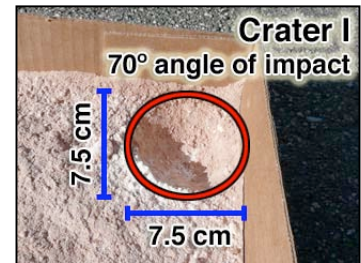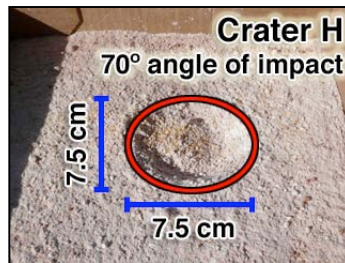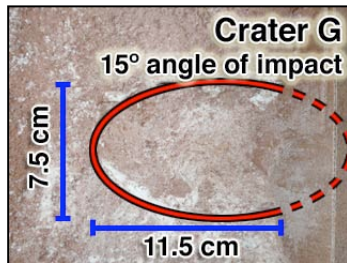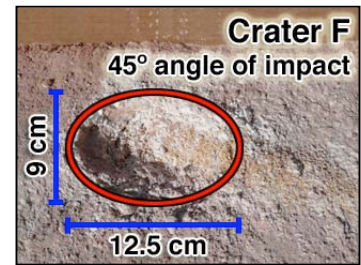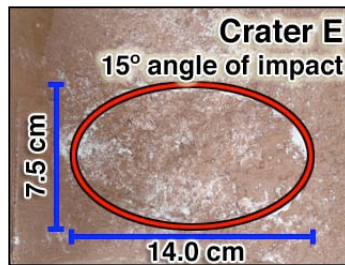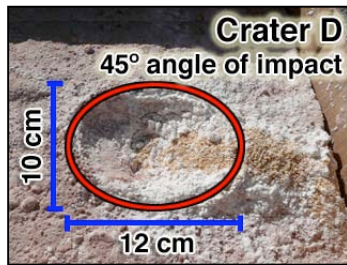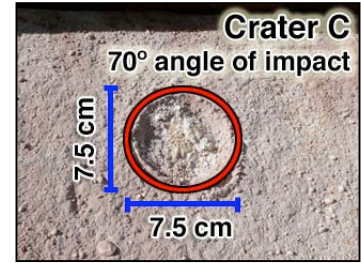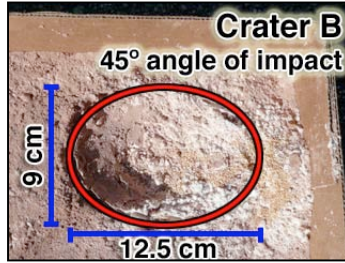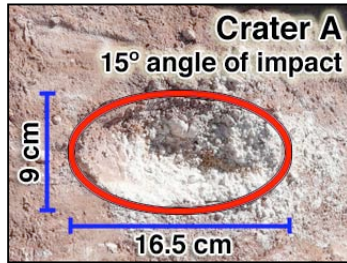
Observe the 9 craters labeled A – I, then discuss the questions below.

1. What shape(s) would you use to describe the uppermost cross section (or "top") of each crater?

2. How does the _angle_ of impact affect the _shape_ of the crater? Do impacts at 15° create craters of similar shape? (at 45°? at 70°?)

3. How are the shapes of craters with a 70° angle of impact different than the shapes of craters with a 45° angle of impact? (with a 15° angle of impact?)

4. Examine each crater's dimensions. As the angle of impact changes, how does the relationship between a crater's greatest length and greatest width change?

Team Members: _____

Team Name: _____ Date: _____

Activity A – Shapes and Angles

Directions: Examine the general shape of each crater. Record the angle of impact and the two-dimensional geometric shape of the "top" of each crater in the table below.

Crater	Angle of Impact	2-Dimensional Shape
Example	*15º*	*Long, narrow oval*
A		
B		
C		
D		
E		
F		
G		
H		
I		

Compare the angles of impact and the crater shapes in the table above. Summarize your data and record your observations below.

Angle of Impact	Shallow angle (1º – 30º)	Medium angle (31º – 60º)	Steep angle (61º – 90º)
2-Dimensional Shape			
Observations:			

Team Members: _____

Team Name: _____   Date: _____

Approximating Area Pre-Lesson Activity
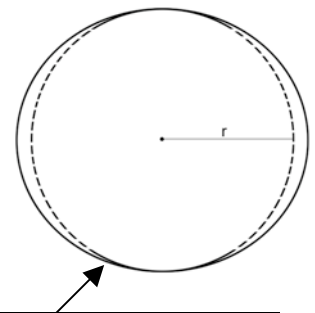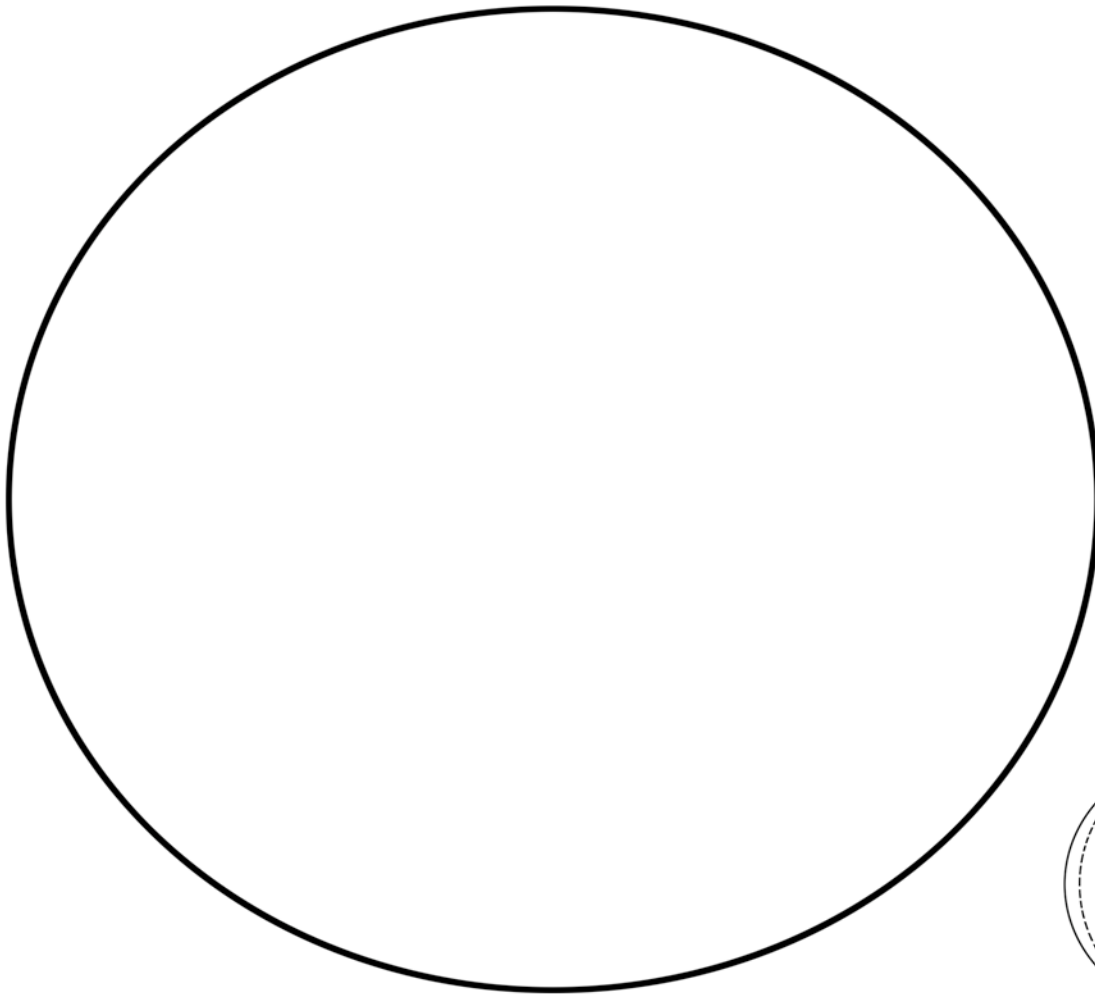
Tool or Method:	Large Grid 2 x 2 cm squares	Medium Grid 1 x 1 cm squares	Small Grid ½ x½ cm squares	Formula $A_{circle} = \pi\ r^2$
Outer Area:				
Inner Area:				
Approximate Area (cm^2):				

Team Members: _____

Team Name: _____ Date: _____

Activity B – Crater Area

Directions: (Repeat these steps for all 9 craters.)

• Lay a square grid on top of your crater and then use a marker to sketch the shape of the "top" of your crater onto the grid.

• Calculate the inner area and the outer area of the sketch of your crater.

• Approximate the actual area of your crater by averaging its inner area value and outer area value.

Crater	Impact Angle	Outer Area (cm²)	Inner Area (cm²)	Approximate Area (cm²)
A				
B				
C				
D				
E				
F				
G				
H				
I				

Draw Conclusions:

Which crater shape and which angle of impact resulted in the greatest area?

_____

_____

Team Members: _____

Team Name: _____ Date: _____

Activity C – Projectile Experiment Data Sheet

Directions:

1. Gather spherical projectiles of <u>similar size</u> but of <u>different masses</u>. List the projectiles in the table below.
2. Measure the circumference of each projectile (in cm), and then use the formula for the diameter of a sphere to calculate each projectile's diameter (in cm). Record the measurements in the table below.
3. Measure each projectile's mass (in grams). Record the measurements in the table below.

– PROJECTILE –   **– CRATER –**

Projectile Name	Circum-ference (cm)	Diameter (cm)	Mass (g)	Diameter (cm)	Area (cm^2)	Depth (cm)	Volume (cm^3)
golf ball	14 cm	$d = \frac{C}{\pi}$ 4.5 cm	46 g	d = 5.0 cm 5 cm	A = π r^2 19.6 cm^2	h = 2.5 cm 2.5 cm	32.8 cm^3

4. A) Drop each projectile one at a time from the same height at a 90º angle into a pan of soil.

    Height of drop: _____ (Each projectile will be dropped from this height.)

    B) Measure the *diameter* and the *depth* of each projectile's crater. Record your measurements in the "crater" section of the table next to the projectile's name and measurements.

    C) Approximate the upper cross-sectional *area* and the *volume* of each crater using the formulas for the area of a circle and the volume of a spherical cup. Record your calculations in the table. ***Be sure to include appropriate units of measurement.***

5. Organize your data by ranking your projectiles according to their masses and the areas and volumes of their craters. For each category, list the projectile with the greatest value first and the projectile with the least value last.

PROJECTILE MASS	CRATER AREA	CRATER VOLUME
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____

6. Analyze your data and draw conclusions.

    What is the relationship between the mass of a projectile and the crater that it forms?

    _____

    _____

Team Members: _____

Team Name: _____ Date: _____

Activity D – Projectile Experiment Data Sheet

Directions:

1. Gather spherical projectiles of <u>different</u> sizes and of <u>different</u> masses.
2. Measure the circumference of each projectile (in cm).
3. Measure the mass of each projectile (in grams).
4. Categorize the projectiles and list them in the matrix below. Record the measurements of each projectile next to its name. (*See the example below*.)

PROJECTILE SIZE

<table>
<tr><td></td><th>Large</th><th>Medium</th><th>Small</th></tr>
<tr><td rowspan="2">High</td><td></td><td></td><td>Golf ball: 46 g, d = 4.5 cm</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td>Medium</td><td></td><td></td><td></td></tr>
<tr><td>Low</td><td></td><td></td><td></td></tr>
</table>

PROJECTILE MASS

5. A) Drop each projectile one at a time from the same height at a 90º angle into a pan of soil.

   Height of drop: _____ (Each projectile will be dropped from this height.)

   B) Measure the *diameter* and the *depth* of each projectile's crater. Record your measurements in the table below next to the projectile's name.

   C) Approximate the upper cross-sectional *area* and the *volume* of each crater using the formulas for the area of a circle and the volume of a spherical cup. Record your calculations in the table. **Be sure to include appropriate units of measurement.**

EXPERIMENT DATA

Projectile Name	Crater Diameter (cm)	Crater Depth (cm)	Crater Area (cm²)	Crater Volume (cm³)
Golf ball	5 cm	2.5 cm	19.6 cm²	32.8 cm³

6. Organize your data by ranking your projectiles according to their sizes and masses and the areas and volumes of their craters. For each category, list the projectile with the greatest value first and the projectile with the least value last.

| PROJECTILE | | CRATER | |
SIZE	MASS	AREA	VOLUME
_____	_____	_____	_____
_____	_____	_____	_____
_____	_____	_____	_____
_____	_____	_____	_____
_____	_____	_____	_____
_____	_____	_____	_____
_____	_____	_____	_____
_____	_____	_____	_____

7. Analyze your data and draw conclusions.

What is the relationship between the size and mass of a projectile and the crater that it forms?

_____

_____

_____