

Bimagic chess tours on 8x8-boards

Walter Trump, 2020-09-25

The first **bimagic queen's tour** was found by **William Walkington** in July 2020.

What about bimagic or semi-bimagic rook's, king's and knight's tours on an 8x8-board?

Here semi-bimagic means that only the 8 rows (or columns) are bimagic.

Semi-bimagic rook's tours on an 8x8-board exist.

A closed rook's tour with 8 bimagic rows and 4 magic columns

1	47	48	45	35	46	2	36
4	53	32	41	54	31	3	42
5	13	16	44	55	56	28	43
6	51	50	25	7	57	26	38
59	14	15	40	8	58	27	39
60	52	49	21	9	10	22	37
61	12	33	24	34	11	23	62
64	18	17	20	19	30	29	63

The 8 rows consist of 4 pairs of complementary bimagic series.

The columns 1, 2, 3 and 4 are self-complementary and magic.

Columns 5 and 6 as well as columns 8 and 9 consist of the complementary entries.

Up to transpose and permutations of rows and columns there are 144 different rook's tours with the same set of 8 bimagic series, but only 32 are closed.

There exist at least 14 other sets of 8 suitable bimagic series.

Bimagic rook's tours on an 8x8-board do not exist.

Proof: Each of the 8 rows and each of the 8 columns consists of 8 entries given by a bimagic series. There are 38039 different bimagic series.

It is not allowed that two numbers $a < b$ with difference $b - a = 2$ are in the same row or column if the middle number $(a + b) / 2$ is not in this row, too. We need 63 rook's moves, they can be horizontal or vertical. Without loss of generality we can assume that there are at least 32 horizontal moves. Therefore the 8 rows have to consist of at least 32 pairs of consecutive numbers. A short computer program shows that there are at most 28 such pairs in valid sets of 8 disjoint rows.

Semi-bimagic king's tours on an 8x8-board do not exist.

Proof: Assume that the rows should be bimagic. Then each of the 8 rows consists of 8 entries given by a bimagic series. There are 38039 different bimagic series.

If the number x is in the first row, then for $x > 1$ the neighbor number $(x-1)$ and for $x < 64$ the neighbor number $(x+1)$ has to be in the first or in the second row, because the king can only move one step.

Let's take for example the series 1, 2, 35, 36, 45, 46, 47, 48 for the first row.

Only the neighbor numbers 3, 34, 37, 44, 49 are missing.

These 5 numbers have to occur in the second row. Now we just have to search for a series which contains these 5 numbers and 3 others. This search can be done by a computer program. Unfortunately the program does not find a suitable series.

There is only one other series where only 5 numbers are missing, it is the complement $(65 - 1)$, $(65 - 2)$, $(65 - 35)$, $(65 - 36)$, $(65 - 45)$, $(65 - 46)$, $(65 - 47)$, $(65 - 48)$ and of course it is not possible to find a series with the missing 5 neighbors.

Most of the 38039 bimagic series have more than 8 missing neighbors and cannot be used for the first row. There are only 132 series with less than 9 missing neighbors:

number of missing neighbors	number of bimagic series
5	2
6	8
7	10
8	112

In none of these 132 cases the computer finds a suitable second row.

Semi-bimagic knight's tours on an 8x8-board do not exist.

Proof: As the knight never moves straight, the first row cannot contain a consecutive pair of numbers. This condition is satisfied by 15415 bimagic series.

When 3 disjoint bimagic series are chosen for the first 3 rows, we have to check if all missing neighbors of the first row are entries of the second or third row. Only 160 arrangements of the first 3 rows satisfy this condition.

After choosing row 4, all missing neighbors of row 2 have to be in row 1, 3 or 4.

This is satisfied by 74 arrangements.

A suitable row 5 cannot be found.

The 74 suitable arrangements for the first four rows are listed in Table 1 on the next pages.

Table 1

Suitable permutations of the entries in each row allow knight moves for row 1 and 2.

64,62,32,30,21,19,17,15, 63,61,35,33,20,18,16,14, 58,56,47,31,29,22,13,04, 60,55,38,36,34,23,11,03,	63,61,35,33,20,18,16,14, 64,62,32,30,21,19,17,15, 60,58,36,34,31,25,13,03, 56,51,47,42,29,22,12,01,	60,58,35,33,31,29,08,06, 59,57,36,34,32,30,07,05, 61,56,48,28,23,20,15,09, 62,55,40,37,26,19,17,04,
64,62,32,30,21,19,17,15, 63,61,35,33,20,18,16,14, 58,56,47,31,29,22,13,04, 60,53,41,36,34,23,10,03,	63,61,35,33,20,18,16,14, 64,62,32,30,21,19,17,15, 60,58,36,34,31,25,13,03, 56,50,47,44,29,22,07,05,	60,58,35,33,31,29,08,06, 59,57,36,34,32,30,07,05, 61,56,48,28,23,20,15,09, 62,53,43,37,26,19,16,04,
64,62,32,30,21,19,17,15, 63,61,35,33,20,18,16,14, 58,56,47,31,29,22,13,04, 60,53,39,36,34,26,11,01,	63,61,35,33,20,18,16,14, 64,62,32,30,21,19,17,15, 60,58,36,34,31,25,13,03, 55,52,49,40,29,22,11,02,	60,58,35,33,31,29,08,06, 59,57,36,34,32,30,07,05, 61,56,48,28,23,20,15,09, 62,50,47,37,25,19,16,04,
64,62,32,30,21,19,17,15, 63,61,35,33,20,18,16,14, 58,56,47,31,29,22,13,04, 60,50,45,36,34,23,07,05,	63,61,35,33,20,18,16,14, 64,62,32,30,21,19,17,15, 60,58,36,34,31,25,13,03, 55,52,46,44,29,22,07,05,	60,58,35,33,31,29,08,06, 59,57,36,34,32,30,07,05, 61,56,48,28,23,20,15,09, 62,50,45,37,27,24,11,04,
64,62,32,30,21,19,17,15, 63,60,33,31,29,20,16,08, 61,57,41,38,22,18,14,09, 59,49,44,37,34,28,07,02,	63,61,35,33,20,18,16,14, 64,62,32,30,21,19,17,15, 60,58,36,34,31,25,13,03, 54,51,47,45,29,22,10,02,	60,58,35,33,31,29,08,06, 59,57,36,34,32,30,07,05, 61,56,48,28,23,20,15,09, 55,49,46,44,37,14,11,04,
63,61,35,33,20,18,16,14, 64,62,32,30,21,19,17,15, 60,58,36,34,31,25,13,03, 59,53,43,40,29,22,10,04,	60,58,35,33,31,29,08,06, 59,57,36,34,32,30,07,05, 61,56,48,28,23,20,15,09, 64,52,39,37,27,24,13,04,	60,58,35,33,31,29,08,06, 59,57,36,34,32,30,07,05, 61,56,48,28,23,20,15,09, 54,51,49,39,37,16,10,04,
63,61,35,33,20,18,16,14, 64,62,32,30,21,19,17,15, 60,58,36,34,31,25,13,03, 59,51,48,37,29,22,08,06,	60,58,35,33,31,29,08,06, 59,57,36,34,32,30,07,05, 61,56,48,28,23,20,15,09, 64,51,40,37,27,25,12,04,	60,58,35,33,31,29,08,06, 59,57,36,34,32,30,07,05, 61,51,47,38,28,14,12,09, 62,56,39,37,23,21,18,04,
63,61,35,33,20,18,16,14, 64,62,32,30,21,19,17,15, 60,58,36,34,31,25,13,03, 59,51,45,40,29,22,12,02,	60,58,35,33,31,29,08,06, 59,57,36,34,32,30,07,05, 61,56,48,28,23,20,15,09, 64,49,45,37,24,21,16,04,	60,58,35,33,31,29,08,06, 59,57,36,34,32,30,07,05, 61,51,47,38,28,14,12,09, 56,54,50,37,27,17,15,04,
63,61,35,33,20,18,16,14, 64,62,32,30,21,19,17,15, 60,58,36,34,31,25,13,03, 57,53,44,42,29,22,09,04,	60,58,35,33,31,29,08,06, 59,57,36,34,32,30,07,05, 61,56,48,28,23,20,15,09, 64,49,43,37,27,24,12,04,	60,58,35,33,31,29,08,06, 59,57,36,34,32,30,07,05, 61,50,48,38,28,15,11,09, 62,56,39,37,23,21,18,04,
63,61,35,33,20,18,16,14, 64,62,32,30,21,19,17,15, 60,58,36,34,31,25,13,03, 57,51,46,42,29,22,11,02,	60,58,35,33,31,29,08,06, 59,57,36,34,32,30,07,05, 61,56,48,28,23,20,15,09, 63,52,43,37,24,21,16,04,	60,58,35,33,31,29,08,06, 59,57,36,34,32,30,07,05, 61,50,48,38,28,15,11,09, 56,53,51,37,27,18,14,04,
63,61,35,33,20,18,16,14, 64,62,32,30,21,19,17,15, 60,58,36,34,31,25,13,03, 56,54,45,41,29,22,09,04,	60,58,35,33,31,29,08,06, 59,57,36,34,32,30,07,05, 61,56,48,28,23,20,15,09, 63,49,46,37,27,18,16,04,	60,58,35,33,31,29,08,06, 59,57,36,34,32,30,07,05, 61,47,44,42,28,26,09,03, 62,56,39,37,23,21,18,04,
63,61,35,33,20,18,16,14, 64,62,32,30,21,19,17,15, 60,58,36,34,31,25,13,03, 56,52,47,41,29,22,11,02,	60,58,35,33,31,29,08,06, 59,57,36,34,32,30,07,05, 61,56,48,28,23,20,15,09, 63,46,42,39,37,16,13,04,	60,58,35,33,31,29,08,06, 59,57,36,34,32,30,07,05, 61,47,44,42,28,26,09,03, 56,54,50,37,27,17,15,04,

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59,57,36,34,32,30,07,05, 60,58,35,33,31,29,08,06, 62,56,39,37,23,21,18,04, 61,51,47,38,28,14,12,09,	59,57,36,34,32,30,07,05, 60,58,35,33,31,29,08,06, 56,50,45,42,37,17,09,04, 61,52,41,38,28,26,13,01,	51,49,47,45,32,30,04,02, 50,48,46,44,35,33,03,01, 62,52,40,34,31,29,07,05, 61,56,43,36,24,20,11,09,
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59,57,36,34,32,30,07,05, 60,58,35,33,31,29,08,06, 56,54,50,37,27,17,15,04, 61,51,47,38,28,14,12,09,	59,57,36,34,32,30,07,05, 60,58,35,33,31,29,08,06, 56,50,45,42,37,17,09,04, 61,49,46,39,28,22,12,03,	51,49,47,45,32,30,04,02, 50,48,46,44,35,33,03,01, 62,52,40,34,31,29,07,05, 60,58,43,36,21,19,13,10,
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59,57,36,34,32,30,07,05, 60,58,35,33,31,29,08,06, 56,50,45,42,37,17,09,04, 61,53,41,38,28,22,16,01,	51,49,47,45,32,30,04,02, 50,48,46,44,35,33,03,01, 62,52,40,34,31,29,07,05, 63,54,43,36,24,18,13,09,	50,48,46,44,35,33,03,01, 51,49,47,45,32,30,04,02, 61,52,43,36,34,18,09,07, 60,58,42,31,29,20,15,05,
59,57,36,34,32,30,07,05, 60,58,35,33,31,29,08,06, 56,50,45,42,37,17,09,04, 61,53,40,38,28,25,14,01,	51,49,47,45,32,30,04,02, 50,48,46,44,35,33,03,01, 62,52,40,34,31,29,07,05, 63,54,43,36,23,19,14,08,	