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On Fractional Order Adaptive Observer

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Abstract: This article derives a new scheme to an adaptive observer for a class of fractional order systems. Global asymptotic convergence for joint state-parameter estimation is established for linear time invariant single-input single-output systems. For such fractional order systems, it is proved that all the signals in the resulting closed-loop system are globally uniformly bounded, the state and parameter estimation errors converge to zero. Potential applications of the presented adaptive observer include online system identification, fault detection, adaptive control of fractional order systems, etc. Numerical simulation examples are presented to demonstrate the performance of the proposed adaptive observer.

Keywords: Fractional order systems, adaptive observer, parameter estimation, continuous system identification, error compensation.

1 Introduction

Over the past decades, fractional order systems have attracted increasing attention from the control community, since many engineering plants and processes cannot be described concisely and precisely without introducing fractional order calculus[1, 2]. Due to more and more scholars devoting themselves to the fractional order field, a tremendous amount of valuable results on system identification[3, 4], controllability and observability[5, 6], stability analysis[7−9] and controller synthesis[10−12] of fractional order systems have been reported in the literature. Many fundamentals and applications of fractional order control systems can be found in [3] and the reference therein.

The reconstruction of system state from its input and output has received a great deal of attention recently. Doye et al.[13] studied the fractional order Luenberger observer and observer-based controller design. Further, it was extended to the uncertain case via linear matrix inequality approach[14]. There are some special Luenberger state observers, such as the discrete form observer[15] and proportional integral (PI) observer[16]. With the frequency distributed model[17] of a fractional order system proposed, an interesting result is obtained that the conventional state is a pseudo state of the system, not the true state. Based on the frequency distributed model, Sabatier et al.[18, 19] discussed the observability of a class of fractional order systems and proposed a class of Luenberger observers.

Despite the plentiful achievements, some critical problems still need further investigation. The system parameters should be known. For the case where no a priori knowledge of the system parameters is available, the so-called adaptive observers should be introduced. The basic idea in the approach is to use a Luenberger observer that will continuously adapt to the parameters. The related research in the integer order area has been reported in [20−23], while there exist few results in the literature investigating this point in fractional order area. There are two main reasons. Firstly, it is difficult to prove the stability of the closed-loop system. Secondly, the parameter identification belongs to the category of identification of continuous systems[24], in which there are still many problems to be solved. To the best of our knowledge, looking for an adaptive observer for fractional order system still remains open.

Motivated by these observations, this article focuses on adaptive observation for fractional order system. The theoretical contributions of this work are as follows. Firstly, a novel adaptive observation scheme is derived, which continuously adapts to the parameters in the observer. Secondly, based on the identification of continuous system, a modified parameter identified algorithm of fractional order system is proposed. The former study solves the problem "viable or not", making the impossible possible. The latter one improves the performance of the fractional order system identification.

The remainder of this article is organized as follows. Section 2 briefly provides some preliminaries for fractional order system and problem statement. A novel adaptive observation scheme is proposed in Section 3. In Section 4, two simulation examples are provided to illustrate the validity of the proposed approach. Finally, Section 5 concludes the study.

2 Preliminaries

Consider the following linear time invariant single-input single-output (LTI SISO) plant

$$G(s) = \frac{b_1 s^{(m-1)\alpha} + b_2 s^{(m-2)\alpha} + \cdots + b_m}{s^{n\alpha} + a_1 s^{(n-1)\alpha} + a_2 s^{(n-2)\alpha} + \cdots + a_n} \qquad (1)$$

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where $\alpha \in (0,1)$ is the fractional commensurate order, $u(t)$ is assumed as a piecewise continuous bounded function of time, $y(t)$ is the measureable output, $a_i (i = 1, 2, \cdots, n)$ and $b_j (j = 1, 2, \cdots, m)$ are constants but unknown, m, n are known positive integers and $m < n$, and the system is supposed to be stable.

In this study, the Caputo′s definition is adopted for fractional order derivative, i.e.,

$$ {}_cD_t^\alpha f(t) \triangleq \frac{1}{\Gamma(k-\alpha)} \int_c^t \frac{f^{(k)}(\tau)}{(t-\tau)^{\alpha-k+1}} \mathrm{d}\tau \tag{2} $$

where k is a positive integer and $k-1 \le \alpha < k$. To simplify the notation, we denote the fractional derivative of order α as D^α instead of ${}_0D_t^\alpha$ in this work.

This study aims at constructing a scheme that estimates both the plant parameters

$$ \begin{cases} a_p = [\begin{array}{cccc} a_1 & a_2 & \cdots & a_n \end{array}]^{\mathrm{T}} \\ b_p = [\begin{array}{cccc} b_1 & b_2 & \cdots & b_m \end{array}]^{\mathrm{T}} \end{cases} \tag{3} $$

as well as the system pseudo state $x(t)$ using only input $u(t)$ and output measurements $y(t)$.

To construct the above scheme, we first introduce a definition as follows.

Definition 1. Matrix A of the system

$$ D^\alpha x(t) = Ax(t) \tag{4} $$

is said to be a stable matrix if and only if it satisfies

$$ |\arg(\mathrm{spec}(A))| > \frac{\alpha\pi}{2} \tag{5} $$

where $\arg(z)$ denotes the principle argument of z and $\mathrm{spec}(A)$ denotes the spectrum of A.

3 Fractional order adaptive observer

3.1 Observer design

A minimal realization of the system (1) can be expressed as

$$ \begin{cases} D^\alpha x(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + v(t) \end{cases} \tag{6} $$

where the stable matrix $A \in \mathbf{R}^{n\times n}$, $B, C^{\mathrm{T}} \in \mathbf{R}^n$, $\{A, B\}$ is controllable and $\{A, C\}$ is observable, and the measurement noise $v(t)$ is white noise with variance σ_v^2.

On the basis of this transformation, the problem becomes how to construct a scheme that estimates both the plant parameters, i.e., A, B, C as well as the state vector $x(t)$. A good starting point for designing an adaptive observer is the Luenberger observer[22, 23]

$$ \begin{cases} D^\alpha \hat{x}(t) = A\hat{x}(t) + Bu(t) + L[y(t) - \hat{y}(t)] \\ \hat{y}(t) = C\hat{x}(t) \end{cases} \tag{7} $$

where L is chosen so that $A - LC$ is a stable matrix and guarantees that $\hat{x}(t)$ converges to $x(t)$ exponentially for any initial condition $x(0)$ and any input $u(t)$.

In order to construct the structure of the adaptive observer, the unknown matrices A, B and C in the Luenberger observer (7) should be replaced with their estimates $\hat{A}$, $\hat{B}$ and $\hat{C}$, respectively, generated by some adaptive law.

$$ \begin{cases} D^\alpha \hat{x}(t) = \hat{A}(t)\hat{x}(t) + \hat{B}(t)u(t) + L[y(t) - \hat{y}(t)] \\ \hat{y}(t) = \hat{C}(t)\hat{x}(t). \end{cases} \tag{8} $$

There are mainly 3 disadvantages in such the structure of the adaptive observer:

1) The estimate matrix $\hat{A}$ cannot be guaranteed as stable, which implies that the estimate state $\hat{x}(t)$ may not be convergent.

2) Since the state space realization result of the transfer matrix $G(s)$ is not unique, matrices A, B and C are not unique either.

3) The number of the parameters to be estimated is $n^2 + 2n$, while the number of the unknown parameters in $G(s)$ is $m + n$.

To eliminate these disadvantages, a special realization form called the observable canonical form is developed. Therefore, matrices A, B and C in (6) are given by

$$ \begin{cases} A = \left[-a_p \left| \begin{array}{c} I_{n-1} \\ \hline 0_{1\times(n-1)} \end{array} \right. \right] \\ B = \left[\begin{array}{c} 0_{(n-m)\times 1} \\ b_p \end{array} \right] \\ C = [\,\overbrace{\begin{array}{cccc} 1 & 0 & \cdots & 0 \end{array}}^{n}\,]. \end{cases} \tag{9} $$

Consequently, there are $m + n$ parameters to be estimated and we no longer need to estimate the matrix C. Then, related parameters in the adaptive observer (8) can be rewritten as

$$ \begin{cases} \hat{A}(t) = \left[-\hat{a}_p(t) \left| \begin{array}{c} I_{n-1} \\ \hline 0_{1\times(n-1)} \end{array} \right. \right] \\ \hat{B}(t) = \left[\begin{array}{c} 0_{(n-m)\times 1} \\ \hat{b}_p(t) \end{array} \right] \\ \hat{C}(t) = C \\ L(t) = a_o - \hat{a}_p(t) \end{cases} \tag{10} $$

where $\hat{a}_p(t)$ and $\hat{b}_p(t)$ are the estimates of vectors a_p and b_p, respectively, and $a_o \in \mathbf{R}^n$ is chosen such that

$$ A_o = \left[-a_o \left| \begin{array}{c} I_{n-1} \\ \hline 0_{1\times(n-1)} \end{array} \right. \right] \tag{11} $$

is a stable matrix.

Remark 1. Matrix A_o in (11) is not unique. Regarding the stability, any stable matrices can be adopted for A_o. In addition, the convergence speed of the state observation depends on A_o.

3.2 Parameter estimation

The corresponding differential equation of system (1) can be expressed as

$$\begin{aligned}&D^{n\alpha}y(t)+\sum_{i=1}^{n}a_iD^{(n-i)\alpha}y(t)-D^{n\alpha}v(t)-\\&\quad\sum_{i=1}^{n}a_iD^{(n-i)\alpha}v(t)=\sum_{j=1}^{m}b_jD^{(m-j)\alpha}u(t).\end{aligned}\tag{12}$$

To obtain a regression-like form of system (12), two methods will be introduced.

In method 1, define the differential operator $p^\alpha = D^\alpha$, $D(p^\alpha) = p^{n\alpha} + a_1p^{(n-1)\alpha} + \cdots + a_n$ and $N(p^\alpha) = b_1p^{(m-1)\alpha} + b_2p^{(m-2)\alpha} + \cdots + b_m$. Then, system (12) can be written in an alternative time-domain differential operator form as

$$D(p^\alpha)y(t)-D(p^\alpha)v(t)=N(p^\alpha)u(t).\tag{13}$$

Filtering the signals on both sides of (13) with the following stable filter

$$F(p^\alpha)=p^{n\alpha}+f_1p^{(n-1)\alpha}+\cdots+f_n\tag{14}$$

we have

$$\begin{aligned}y(t)=&\frac{F(p^\alpha)-D(p^\alpha)}{F(p^\alpha)}y(t)+\frac{N(p^\alpha)}{F(p^\alpha)}u(t)+\\&\frac{D(p^\alpha)}{F(p^\alpha)}v(t).\end{aligned}\tag{15}$$

By defining some new variabilities, (15) can be expressed in a regression-like form as

$$y(t)=\varphi^{\mathrm{T}}(t)\theta+v(t)\tag{16}$$

where

$$\begin{aligned}\varphi(t)=&\left[\frac{p^{(n-1)\alpha}y(t)}{F(p^\alpha)}\cdots\frac{y(t)}{F(p^\alpha)}\frac{p^{(m-1)\alpha}u(t)}{F(p^\alpha)}\cdots\frac{u(t)}{F(p^\alpha)}\right.\\&\left.\frac{p^{(n-1)\alpha}v(t)}{F(p^\alpha)}\cdots\frac{v(t)}{F(p^\alpha)}\right]^{\mathrm{T}}\\\theta=&\begin{bmatrix}f^{\mathrm{T}}-a_p^{\mathrm{T}} & b_p^{\mathrm{T}} & c_p^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}\\f=&\begin{bmatrix}f_1 & f_2 & \cdots & f_n\end{bmatrix}^{\mathrm{T}}\\c=&\begin{bmatrix}c_1 & c_2 & \cdots & c_n\end{bmatrix}^{\mathrm{T}}.\end{aligned}$$

In method 2, considering the generalized Poisson moment functional

$$\frac{1}{H(p^\alpha)}=\left(\frac{1}{p^\alpha+\omega}\right)^\kappa\tag{17}$$

with $\kappa \geq n$, system (12) can be rewritten as

$$\begin{aligned}&\frac{p^{n\alpha}y(t)}{H(p^\alpha)}+\sum_{i=1}^{n}a_i\frac{p^{(n-i)\alpha}y(t)}{H(p^\alpha)}-\frac{p^{n\alpha}v(t)}{H(p^\alpha)}-\\&\quad\sum_{i=1}^{n}a_i\frac{p^{(n-i)\alpha}v(t)}{H(p^\alpha)}=\sum_{j=1}^{m}b_j\frac{p^{(m-j)\alpha}u(t)}{H(p^\alpha)}.\end{aligned}\tag{18}$$

By defining $\bar{y}(t) = \frac{p^{n\alpha}y(t)}{H(p^\alpha)}$, system (18) can be written in a regression-like form as

$$\bar{y}(t)=\bar{\varphi}^{\mathrm{T}}(t)\bar{\theta}+v(t)\tag{19}$$

where the regressor and the parameter vectors are now defined by

$$\begin{aligned}\bar{\varphi}(t)=&\left[\frac{-p^{(n-1)\alpha}y(t)}{H(p^\alpha)}\cdots\frac{-y(t)}{H(p^\alpha)}\frac{p^{(m-1)\alpha}u(t)}{H(p^\alpha)}\cdots\frac{u(t)}{H(p^\alpha)}\right.\\&\left.\frac{p^{(n-1)\alpha}v(t)}{H(p^\alpha)}\cdots\frac{v(t)}{H(p^\alpha)}\right]^{\mathrm{T}}\\\bar{\theta}=&\begin{bmatrix}a_p^{\mathrm{T}} & b_p^{\mathrm{T}} & c_p^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}, c_p=\begin{bmatrix}c_1 & c_2 & \cdots & c_n\end{bmatrix}^{\mathrm{T}}.\end{aligned}$$

Since it is very similar to estimate a_p and b_p based on the least squares method through (16) and (19), respectively, only the former one is introduced.

With regard to (16), at any time instant $t = t_k, k = 1, 2, \cdots, L$, the following standard linear regression-like form can be obtained as

$$y(t_k)=\varphi^{\mathrm{T}}(t_k)\theta+v(t_k).\tag{20}$$

Now, from L available samples of the input and output signals observed at discrete times $t_1, \cdots, t_L$, not necessarily uniformly spaced, the linear least squares based parameter estimates are given by

$$\hat{\theta}(t_L)=\left[\sum_{k=1}^{L}\varphi(t_k)\varphi^{\mathrm{T}}(t_k)\right]^{-1}\left[\sum_{k=1}^{L}\varphi(t_k)y^{\mathrm{T}}(t_k)\right].\tag{21}$$

$\hat{\theta}(t_L)$ cannot converge to θ, because $\varphi(t_k)$ is correlated with $v(t_k)$. To correct the deviations of the parameter estimate, we replace $\varphi(t_k)$ with

$$\begin{aligned}\hat{\varphi}(t_k)=&\left[\frac{p^{(n-1)\alpha}y(t_k)}{F(p^\alpha)}\cdots\frac{y(t_k)}{F(p^\alpha)}\frac{p^{(m-1)\alpha}u(t_k)}{F(p^\alpha)}\cdots\frac{u(t_k)}{F(p^\alpha)}\right.\\&\left.\frac{p^{(n-1)\alpha}\hat{v}(t_k)}{F(p^\alpha)}\cdots\frac{\hat{v}(t_k)}{F(p^\alpha)}\right]^{\mathrm{T}}\end{aligned}\tag{22}$$

where $\hat{v}(t_k)$ is the estimation of $v(t_k)$.

$$\hat{v}(t_k)=y(t_{k-1})-\hat{\varphi}^{\mathrm{T}}(t_{k-1})\hat{\theta}(t_{k-1}).\tag{23}$$

Consequently, the modified solution $\hat{\theta}(t_L)$ can be expressed as

$$\hat{\theta}(t_L)=\left[\sum_{k=1}^{L}\hat{\varphi}(t_k)\hat{\varphi}^{\mathrm{T}}(t_k)\right]^{-1}\left[\sum_{k=1}^{L}\hat{\varphi}(t_k)y^{\mathrm{T}}(t_k)\right].\tag{24}$$

To estimate the parameter on line, the matrix inversion theorem is used to get the recursive least square algorithm as

$$\left\{\begin{aligned}&K(t_{k+1})=\frac{P(t_k)\hat{\varphi}(t_{k+1})}{1+\hat{\varphi}^{\mathrm{T}}(t_{k+1})P(t_k)\hat{\varphi}(t_{k+1})}\\&\varepsilon(t_{k+1})=y(t_{k+1})-\hat{\varphi}^{\mathrm{T}}(t_{k+1})\hat{\theta}(t_k)\\&\hat{\theta}(t_{k+1})=\hat{\theta}(t_k)+K(t_{k+1})\varepsilon(t_{k+1})\\&P(t_{k+1})=\left[I_{m+2n}-K(t_{k+1})\hat{\varphi}^{\mathrm{T}}(t_{k+1})\right]P(t_k)\\&\hat{v}(t_{k+1})=y(t_k)-\hat{\varphi}^{\mathrm{T}}(t_k)\hat{\theta}(t_k)\end{aligned}\right.\tag{25}$$

where $k > 0$, $\hat{v}(t_1) = 0$, $\hat{\theta}(t_1)$ can be chosen arbitrarily, and $P(t_1)$ is usually selected as $10^{3\sim 8} I_{m+2n}$.

As a result, one can get the original system parameter estimation from

$$\hat{\theta}(t_k) = \begin{bmatrix} f - \hat{a}_p(t_k) \\ \hat{b}_p(t_k) \\ \hat{c}_p(t_k) \end{bmatrix} \tag{26}$$

or

$$\hat{\bar{\theta}}(t_k) = \begin{bmatrix} \hat{a}_p(t_k) \\ \hat{b}_p(t_k) \\ \hat{c}_p(t_k) \end{bmatrix}. \tag{27}$$

Remark 2. The filter $F(p^\alpha)$ in (14) is not unique. Regarding the stability and the maximum order of $n\alpha$, any filters can be adopted for $F(p^\alpha)$.

Remark 3. The generalized Poisson moment functional $\frac{1}{H(p^\alpha)}$ in (17) is not unique. Although any $\kappa \geq n$ is feasible, one usually selects $\kappa = n$. In addition, ω can be selected to guarantee system (1) and $\frac{1}{H(p^\alpha)}$ have a similar bandwidth.

Remark 4. Actually, parameter c_p can be calculated from parameter a_p and the coefficients of $H(p^\alpha)$ or $F(p^\alpha)$. However, it is not affected that one can identify c_p and a_p independently.

Remark 5. The two methods realize the parameter adaptation by predicting the output $y(t)$ and the filtered output $\bar{y}(t)$, receptively. Additionally, the parameter estimation results depend on $H(p^\alpha)$, $F(p^\alpha)$ and other parameters. As a result, it is hard to say which one is better.

3.3 Stability analysis

For any time $t_k \leq t < t_{k+1}$, we have

$$\begin{cases} \hat{a}_p(t) = \hat{a}_p(t_k) \\ \hat{b}_p(t) = \hat{b}_p(t_k). \end{cases} \tag{28}$$

Based on the previous result, we are ready to present the adaptive observer for the fractional order system.

Theorem 1. For plant (6), if the adaptive observer is designed as (8), (10) and (25), then all the signals in the closed-loop adaptive system are global uniformly bounded. And if $u(t)$ can guarantee that $\phi(t)$ is persistent excitation, then the parameter estimation and state observation are achieved as

$$\begin{cases} \lim\limits_{t\to\infty} [\theta - \hat{\theta}(t)] = 0 \\ \lim\limits_{t\to\infty} [x(t) - \hat{x}(t)] = 0. \end{cases} \tag{29}$$

Proof. The observer equation we design can be written as

$$D^\alpha \hat{x}(t) = A_o \hat{x}(t) + [\hat{A}(t) - A_o] x(t) + \hat{b}_p(t) u(t) + L(t) v(t). \tag{30}$$

Since A_o is a stable matrix and $\hat{b}_p, \hat{A}(t), L(t), x(t), u(t), v(t)$ are bounded, it follows that $\hat{x}(t)$ is bounded, which in turn implies that all signals are bounded.

Based on the convergence properties of the least squares for the linear regression form problem, one concludes that $\hat{\theta}(t_L)$ is an unbiased estimation of θ. In other words, the equation $\lim\limits_{t\to\infty} [\theta - \hat{\theta}(t)] = 0$ holds.

Define the state observation error as

$$\tilde{x}(t) = x(t) - \hat{x}(t). \tag{31}$$

Then, $\tilde{x}(t)$ satisfies

$$D^\alpha \tilde{x}(t) = A_o \tilde{x}(t) + \tilde{b}_p(t) u(t) - \tilde{a}_p(t) [y(t) + v(t)] \tag{32}$$

where $\tilde{a}_p(t) = a_p - \hat{a}_p(t)$ and $\tilde{b}_p(t) = b_p - \hat{b}_p(t)$ are the parameter errors.

Since $\theta - \hat{\theta}(t) \to 0$ as $t \to \infty$, it follows that $\tilde{a}_p(t) \to 0$ and $\tilde{b}_p(t) \to 0$. Since $u(t), y(t)$ and $v(t)$ are bounded, the error equation consists of a homogenous part which is exponentially stable, and an input which is decaying to zero. This implies that $\tilde{x}(t) \to 0$ as $t \to \infty$. □

Remark 6. The application range of the proposed adaptive observer should be pointed out. Such an adaptive observer applies to the linear fractional order systems where the unknown parameters appear linearly in the dynamics. To estimate the unknown parameters precisely, the inputs to the systems must satisfy the conditions of persistent excitation.

4 Numerical examples

Numerical examples illustrated in this section are implemented via the piecewise numerical approximation algorithm. To get more information about the algorithm, one can refer to [25]. For all the numerical examples, consider the following LTI SISO fractional order plant

$$G(s) = \frac{s^{0.6} + 2}{s^{1.2} + 5s^{0.6} + 10}.$$

The pseudo state space realization of observable canonical form is

$$\begin{cases} D^\alpha x(t) = \begin{bmatrix} -5 & 1 \\ -10 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t) \\ y(t) = [\ 1 \quad 0\ ] x(t) + v(t). \end{cases}$$

The stable matrix A_o is selected as

$$A_o = \begin{bmatrix} -6 & 1 \\ -8 & 0 \end{bmatrix}.$$

The variance of the white noise $v(t)$ is selected as

$$\sigma_v^2 = 0.000\,1.$$

The related initial conditions are selected as

$$x(0) = \begin{bmatrix} 0.01 \\ 0.01 \end{bmatrix}, \hat{x}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \hat{v}(t_1) = 0.01$$
$$\hat{\theta}(t_1) = [\ 0.1 \quad 0.1 \quad 0.1 \quad 0.1 \quad 0.1 \quad 0.1\ ]^{\mathrm{T}}$$
$$P(t_1) = 10^4 \times \mathrm{diag}(1,\ 1,\ 1,\ 1,\ 1,\ 1).$$

The multi-sine signal is selected as the system input $u(t)$, which is shown in Fig. 1.

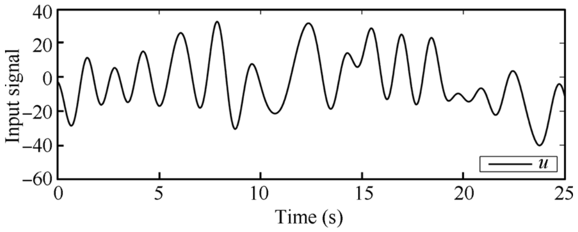

Fig. 1 System input $u(t)$ in Examples 1 and 2

Example 1. Construct an adaptive observer with the parameter identification based on the regression-like form (16). Select the stable filter

$$F\left(p^{\alpha}\right)=p^{1.2}+4p^{0.6}+4.$$

Then, one has the simulation results shown in Figs. 2−4.

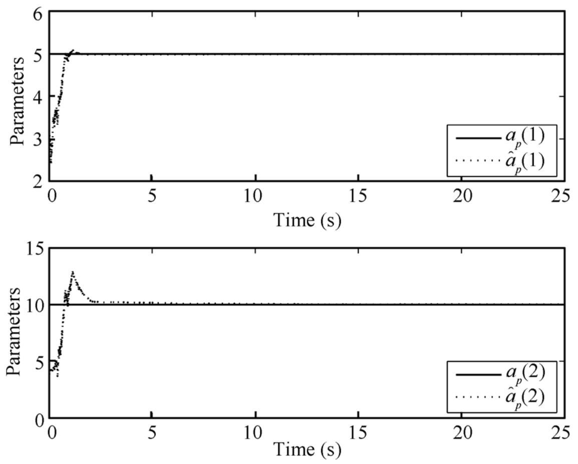

Fig. 2 Identification of parameter a_p in Example 1

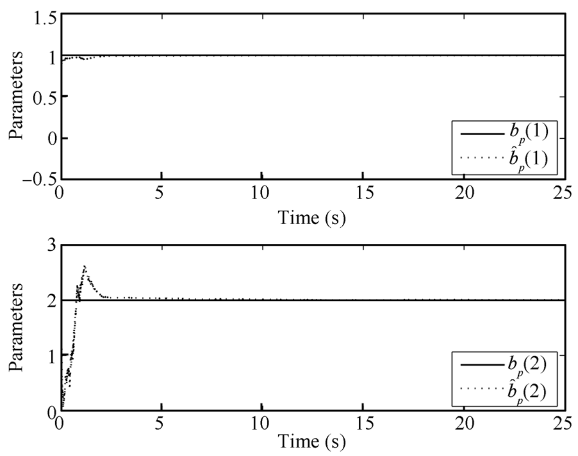

Fig. 3 Identification of parameter b_p in Example 1

Example 2. Construct an adaptive observer with the parameter identification based on the regression-like form (19). Select the generalized Poisson moment functional

$$\frac{1}{H\left(p^{\alpha}\right)}=\left(\frac{1}{p^{0.6}+2}\right)^{2}.$$

Then, one has the simulation results shown in Figs. 5−7.

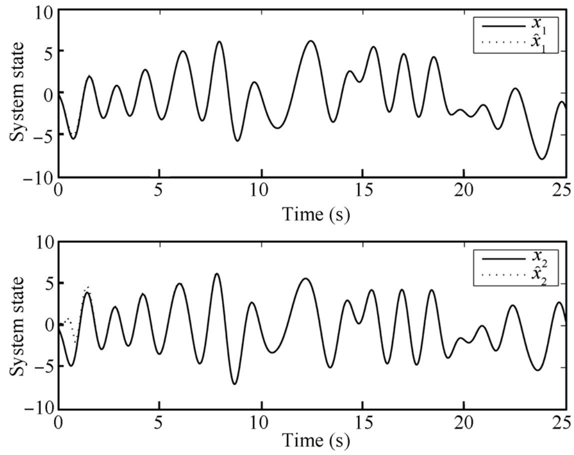

Fig. 4 Observation of system state $x(t)$ in Example 1

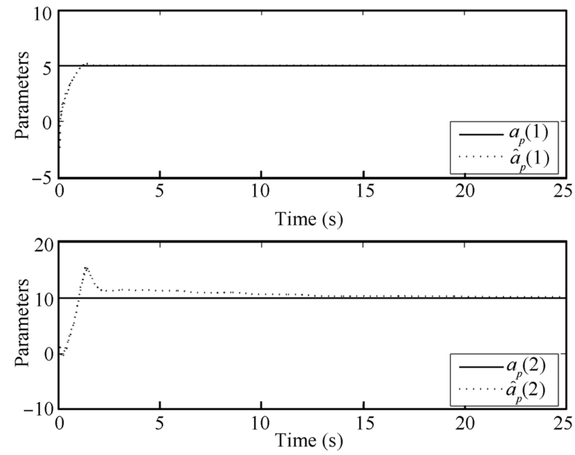

Fig. 5 Identification of parameter a_p in Example 2

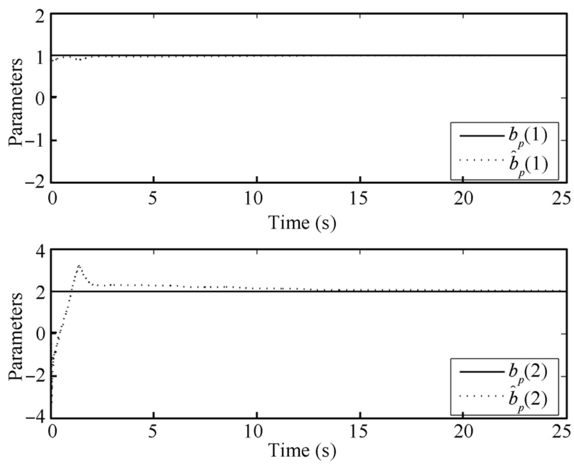

Fig. 6 Identification of parameter b_p in Example 2

It can be seen from the results of Examples 1 and 2 that both adaptive observers make the state estimations approach the actual states precisely, despite the presence of unknown system parameters and measurement noise. However, the transient responses of the adaptive observer with method 1 are better than those of the adaptive observer with method 2. The observation error of adaptive observer due to method 1 is less than that of the latter one.

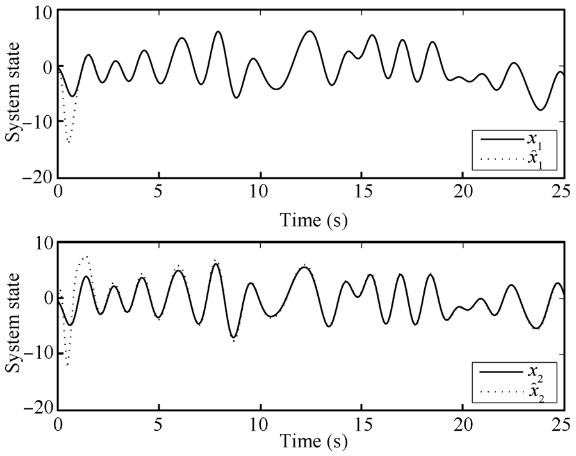

Fig. 7 Observation of system state $x(t)$ in Example 2

5 Conclusions

In this article, a new adaptive observer design scheme is proposed for LTI SISO fractional order plant. Both the state and the unknown parameters can be efficiently observed. Motivated by the noise estimation, a modified least squares solution of the parameters is developed. For the scheme of the proposed adaptive observer, the stability of the closed-loop system and the parameter convergence have been discussed in detail, with the parameters being identified rapidly and accurately. Simulation results from numerical examples are provided to demonstrate the advantages and effectiveness of the approaches proposed in this article. It is believed that the approaches provide a new avenue to solve the problem. Future research subjects will include how to extend the results to multiple-input multiple-output case and how to control the plant based on the proposed adaptive observer.

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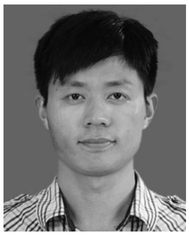

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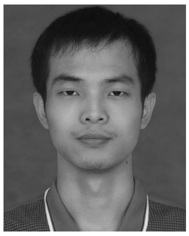

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