Nonlinear Surface Impedance of HTS in High Magnetic Fields

Ruggero Vaglio, Patrick Krkotić, *Member, IEEE*, and Sergio Calatroni, *Member, IEEE*

***Abstract*—Recently envisaged high-temperature superconductor (HTS) applications rely on their radio-frequency properties in very high magnetic fields. As an example, coating of surfaces facing particle beams with HTS for reducing the beam coupling impedance is being considered for the next generation of hadron colliders, such as the FCC-hh at CERN, where HTS would be exposed to the high-frequency wakefields generated by the particle bunches and to the strong field (>16 T) of the steering magnets. Other considered applications are radio-frequency cavities for axion detection as well as capture cavities for muon colliders. In these applications, it is important to keep under control the level of the nonlinear effects, always present using superconductors. In this frame, we developed a simple model for the calculation of the radio-frequency behavior of HTS exposed to a strong external magnetic field, which takes into account the nonlinearity of the pinning potential of the Abrikosov vortices. In particular, we calculate the surface resistance as a function of the impressed radio-frequency current and the harmonic content of the electric field produced by vortex oscillation as a function of the nonlinearity level of the assumed potential. The calculations are compared with recent experimental data on YBCO tapes' nonlinear behavior.**

***Index Terms*—High-temperature superconductors, nonlinearities, surface impedance.**

I. Introduction

HIGH-temperature superconductor (HTS) applications in high energy or particle physics requiring operation of HTS at high frequency and in presence of a strong magnetic field have been discussed in the scientific literature in recent years. The most relevant example is for future colliders, such as the FCC-hh at CERN [1] or other accelerators, including CPPC in China, where the instability effects related to the wakefields produced by the image currents can be limited by HTS coating of the screen inner surface that strongly reduces the beam impedance. HTS could also play a significant role in next-generation detectors for axions particles [2], [3], where an increase in sensitivity can be achieved by increasing both the cavity quality factor and the strength of the applied magnetic field. The use of HTS in high magnetic fields is also considered in future muon colliders [4].

Manuscript received 21 September 2023; revised 16 December 2023 and 10 January 2024; accepted 23 January 2024. Date of publication 5 February 2024; date of current version 20 February 2024. *(Corresponding author: Ruggero Vaglio.)*

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Color versions of one or more figures in this article are available at https://doi.org/10.1109/TASC.2024.3361870.

Digital Object Identifier 10.1109/TASC.2024.3361870

One potential drawback related to the use of HTS in the above-mentioned applications is their nonlinear behavior as a function of the radio-frequency (RF) current intensity, that is expected to be especially relevant in high magnetic fields, as observed experimentally and mainly ascribed to the nonlinearity of the Abrikosov vortices pinning potential [5].

From a theoretical point of view, the problem of HTS nonlinearities in high magnetic fields, due to vortex (fluxon) motion in a nonlinear pinning potential has been discussed in [6], [7]. The approach was based on the Gittleman and Rosenblum (GR) original model [8], that has been proven to accurately describe the HTS surface impedance results in a high dc magnetic field (see, as an example, [9]), and sufficiently low temperature, where flux creep can be neglected [10].

In this communication, we will extend the previous results [6], [7], introducing a more general nonlinear vortex pinning potential characterized by a generic power law with an amplitude parameter describing the nonlinearity level. By comparison of the model with experimental results for the surface resistance vs RF field amplitude, it is possible to estimate the specific parameters for the considered sample and, therefore, predict other nonlinear effects, such as for example the third harmonic content of the generated electric field. This procedure will be applied to the power dependence of the surface resistance in state-of-the-art REBCO coated conductors at 8 GHz and 50 K measured with a dielectric resonator producing radio-frequency electromagnetic fields on the surface of the coated conductors with amplitudes similar to those generated by future proton collider beams [11].

II. The Nonlinear GR Model

The GR model [8] assumes that the vortex motion under a small applied RF current at an angular frequency ω is described by the following differential equation:

$$\eta\dot{x}\,(t) + kx\,(t) = \phi_o J_{rfo} \cos\omega t, \tag{1}$$

where *x(t)* is the single vortex displacement from the equilibrium position, $\eta = \phi_o B_{c2}/\rho_n$ is the flow viscosity per unit length, with ϕ_0 the flux quantum, B_{c2} the superconductor upper critical field and ρ_n the normal-state electrical resistivity, J_{rfo} is the maximum amplitude of the RF current of angular frequency ω ($\phi_o J_{rfo}$ is the maximum force per unit length exerted by the RF field on the flux lines). The effective mass per unit length *m* of the fluxons is small; therefore, the acceleration term $m\ddot{x}(t)$ is neglected.

The linear force term k is generated by a fully quadratic potential around the equilibrium vortex position. A generic nonlinear force, assuming a moderate nonlinearity, can be written as:

$$F(x) = -kx + \gamma|x|^n x, \tag{2}$$

where n could be any positive number. The $n = 1$ is the case treated in [6]; $n = 2$ is the case treated in [7]. The fluxon equation of motion becomes:

$$\eta\dot{x}(t) + kx(t) - \gamma|x|^n x = \phi_o J_{rfo}\cos\omega t. \tag{3}$$

It is convenient to rewrite this equation in adimensional units, introducing the adimensional parameters: $y = x/\xi$ (ξ is the superconductor coherence length), $\tau = \omega t$, $a_{rf} = \phi_o J_{rfo}/k\xi \propto J_{rfo}/J_c$, $r = \omega/\omega_o$, where $\omega_o = k/\eta$ is the depinning frequency and, finally, $a = \gamma\xi^n/k$ is the adimensional parameter characterizing the level of nonlinearity ($a = 0$ reproduces the linear case). Introducing these variables, we have:

$$ry'(\tau) + y(\tau) = a_{rf}\cos\tau + a|y(\tau)|^n y(\tau). \tag{4}$$

Here $y'(\tau)$ is the derivative of $y(\tau)$ with respect to the adimensional time τ. Limiting ourselves to the simple case of a thin film of thickness d, we can calculate the average dissipated power per cycle and per unit surface by integrating the overall force acting on a single vortex times the velocity, multiplying by the number of vortices per unit surface N and integrating over one period:

$$\bar{P} = Nd\phi_o J_{rfo}\frac{1}{T}\int_0^T v(t)\cos\omega t dt. \tag{5}$$

Since $\bar{P} = (1/2)R_{sf}H_{rfo}^2$ and, for a thin film, $H_{rfo} = dJ_{rfo}$, it is:

$$R_{sf} = 2R_n\frac{B_o}{B_{c2}}\frac{r}{a_{rf}}\frac{1}{2\pi}\int_0^{2\pi} y'(\tau)\cos\tau d\tau. \tag{6}$$

(Here, we made use of the adimensional units considered above, and introduced the external magnetic field $B_o = N\phi_o$ and the normal state surface resistance for a thin film $R_n = \rho_n/d$.)

Developing $y'(\tau)$ in Fourier series:

$$y'(\tau) = \sum_{-\infty}^{+\infty} A_n e^{in\tau}, A_n = \frac{1}{2\pi}\int_0^{2\pi} y'(\tau)e^{-in\tau}d\tau, \tag{7}$$

we have:

$$R_{sf} = 2R_n\frac{B_o}{B_{c2}}\frac{r}{a_{rf}}\mathrm{Re}[\mathrm{A}_1]. \tag{8}$$

Now, multiplying all terms in (4) by $e^{-i\tau}$, averaging over one period and integrating by parts, we get:

$$(r - i)A_1 = \frac{1}{2\pi}\int_0^{2\pi} a_{rf}\cos\tau e^{-i\tau}d\tau + \frac{1}{2\pi}\int_0^{2\pi} a|y(\tau)|^n y(\tau)e^{-i\tau}d\tau. \tag{9}$$

Taking the real part of A_1 we obtain the following:

$$R_{sf} = R_n\frac{B_o}{B_{c2}}\frac{r}{a_{rf}}\left[\frac{a_{rf}}{\pi}\int_0^{2\pi}\cos\tau\frac{r\cos\tau + \sin\tau}{1 + r^2}d\tau + \frac{a}{\pi}\int_0^{2\pi}|y_o(\tau)|^n y_o(\tau)\frac{r\cos\tau + \sin\tau}{1 + r^2}d\tau\right]. \tag{10}$$

TABLE I
NUMERICAL COEFFICIENTS FOR (11)

g_1	$16/3\pi$	1.70
g_2	3/2	1.50

Here, we set $y(\tau) \cong y_o(\tau)$, with $y_o(\tau)$ being the solution of the linear ((4) with $a = 0$):

$$y_o(\tau) = \frac{a_{rf}}{\sqrt{1 + r^2}}\cos(\tau - \phi),$$

with $\phi = \tan^{-1}(r)$. Inserting in (10) and solving the integrals, we finally have:

$$R_{sf} = R_{sfo}\left[1 + \frac{ag_n a_{rf}^n}{(1 + r^2)^{(1+n/2)}}\right]. \tag{11}$$

($R_{sfo} = R_n\frac{B_o}{B_{c2}}\frac{r^2}{1+r^2}$ is the surface impedance for a thin film in the linear case). Setting $y(\tau) \cong y_o(\tau)$, in (10) corresponds to the first order approximation as can be verified following the derivation reported in [7].

The numerical coefficient g_n is reported in Table I for $n = 1$ and $n = 2$. Inserting these values of the coefficient g_n the results of [6], [7] are fully reproduced. An approximate expression valid for any n is: $g_n = 1.9 - 0.2n$.

In Fig. 1, R_{sf}/R_{sfo} vs. a_{rf} (11) is plotted for different integer values of n and a for $r = 0.4$.

In Fig. 2 the value of the normalized fluxon velocity $y'(\tau)$ (proportional to the generated electric field: $E_{rf} = \omega\xi B_o\dot{y}(\tau)$) is reported, obtained by numerical solution of (4). The deformation of the induced field produced by the nonlinearity with the appearance of a third harmonic component is clearly visible and can affect the experiments using superconductors in high dc fields at high RF currents.

III. Nonlinear Surface Impedance Measurement

In order to identify the level of nonlinearity in the HTS, measurements of the dependence of the surface resistance on the RF current density in a high magnetic field were performed as described in [11], using a dielectric resonator cavity. The measurements were conducted by applying a dc magnetic field from 0 T up to 9 T in 1 T increments at a temperature of 50 K, and by sweeping the applied RF input power. The dielectric resonator used in the study was a cylindrical brass cavity loaded with a low-loss and high-permittivity rutile (TiO_2) cylinder. The resonator was designed to operate in the TE011 mode, which results in a resonance frequency of 8 GHz. The relationship between the measured surface resistance R_S and maximum RF magnetic field strength H_{rfo} on the sample surface was determined. The measurements were performed on commercially available REBCO coated conductor samples. To our knowledge, this is the first time nonlinearity measurements on HTS samples have been performed in such high dc magnetic field, previous measurements extending up to a maximum dc field of 4 T [5].

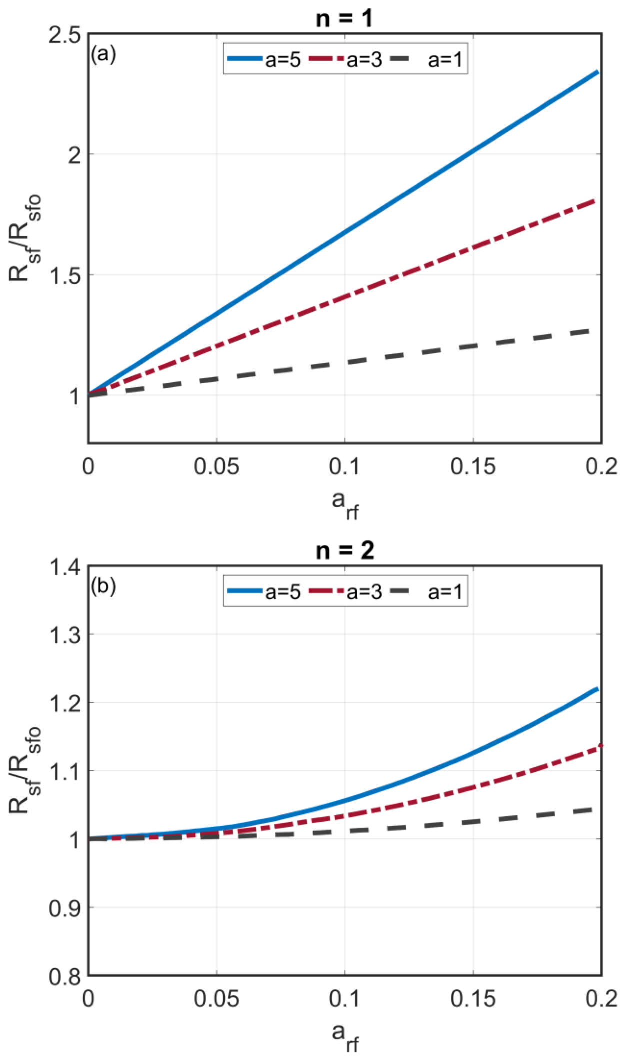

Fig. 1. Dependence of the normalized surface resistance R_{sf}/R_{sfo} vs. the normalized RF current a_{rf} for the cases (a) $n = 1$ and (b) $n = 2$. These numerical calculations are performed for $r = 0.4$ and increasing values of the nonlinearity parameter a.

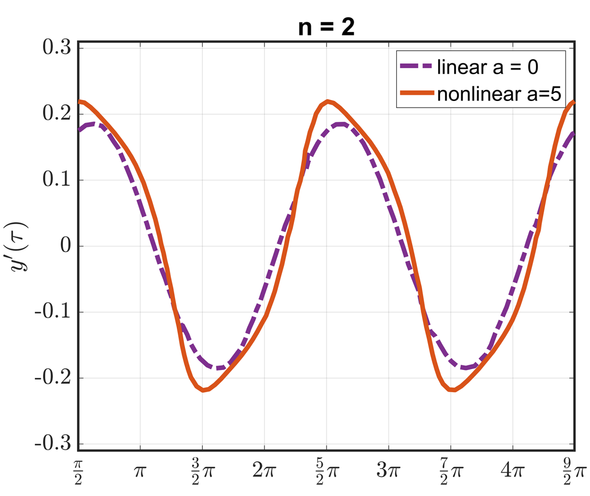

Fig. 2. Normalized fluxon velocity (continuous line) for $n = 2$, $r = 0.4$, $a = 5$, and $a_{rf} = 0.2$. The dashed line represents the solution of the linear problem ($a = 0$).

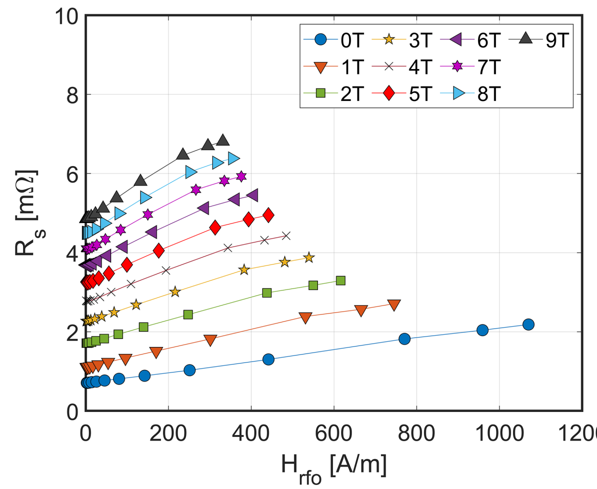

Fig. 3. Measured surface resistance at 8 GHz and 50 K as a function of maximum RF surface magnetic field strength for several dc magnetic fields. Data is shown for Theva. The lines are only guides for the eye.

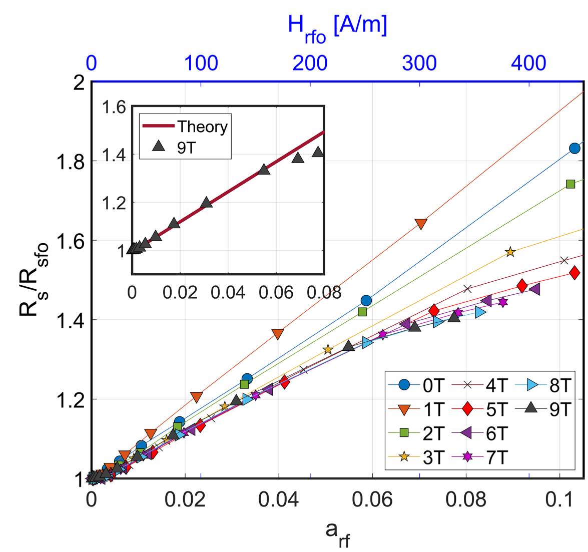

Fig. 4. Normalized measured surface resistance at 8 GHz and 50 K as a function of the adimensional parameter a_{rf} and maximum RF surface magnetic field strength for several dc magnetic fields. The lines are only guides for the eye. In the inset is shown how the experimental results at 9 T are well reproduced by (11) using the values described in the text.

The surface resistance R_S as a function of the maximum RF magnetic field strength for a gadolinium-based coated conductor provided by Theva can be seen in Fig. 3. Two primary observations can be made: firstly, R_S increases with the dc magnetic field, starting from non-zero values at zero magnetic field. This dependence follows the square-root behavior predicted by the standard GR-based models [9]. Secondly, the R_S is higher with higher H_{rfo}. Further analysis of the data presented in Fig. 4 reveals that normalizing the surface resistance values for each dc magnetic field curve to its lowest H_{rfo} measurement value R_{so}, results in curves that exhibit essentially the same slope above 3–4 T. It goes beyond the scope of this paper to perform a multiparameter fit of (11) on the available experimental data, which would require either a more consistent dataset owing to the large number of free parameters that would need to be fitted or an independent precise evaluation of several material properties in order to constrain the fit. We thus followed the approach of simply comparing the experimental data as presented in Fig. 4 with (11), under the assumption that at high field $R_{sf}/R_{sfo} \approx R_s/R_{so}$, by evaluating the adimensional parameter a_{rf} via literature values of $\xi = 1.4\,\text{nm}$ [12], $J_{rfo} = H_{rfo}/\lambda_L(50\ \text{K})$, taking $\lambda_L(0\ \text{K}) = 150\,\text{nm}$ [13], $k = 0.4 \cdot 10^5$ Nm [14]. In addition, findings in [14], [15] have shown that for a Theva sample, the depinning frequency is about $\nu_o \approx 19$ GHz ($\omega_o = 2\pi\nu_0 \cong 120 \cdot 10^9$ rad/s), field independent at high fields, thus $r \approx 0.4$. Comparing the theoretically estimated values from (11) (or Fig. 1) with the experimental values in Fig. 4 (as displayed in the inset), we can determine that $n \approx 1$ and the nonlinearity parameter $a \approx 5$, field independent for fields above 3–4 T. This means that the nonlinear force, as given by (2), and the corresponding nonlinear potential around the equilibrium position, results to be field-independent at high fields. Knowing the value of the parameters n and a, (3) can be solved numerically giving an estimation of the generated third harmonic component in the fluxon velocity and, in turn, of the generated electric field [6].

IV. Conclusion

In this paper, we have presented a very simple model, based on the GR approach and including a nonlinear force around the equilibrium fluxon position, valid for high magnetic fields (i.e., when the losses are mainly determined by fluxon oscillations). The model predicts the RF field dependence of the surface resistance and the occurrence of a third harmonic component in the generated electric field. By comparing the model results with the data, it is possible to estimate the value of the adimensional nonlinear parameter $a = \gamma\xi^n/k$. This procedure has been applied to nonlinear measurements of the surface resistance on the RF current density in a high magnetic field performed in [11], and in particular to gadolinium-based HTS coated conductor provided by Theva. The results lead to an estimation of $n = 1$ and $a = 5$. This allows a prediction, within the model assumptions, of the expected nonlinear behavior of the investigated material, and in particular of the third harmonic component in the electric field, whose effect on practical RF experiments using HTS conductors (as in the FCC beam-screen coating project) has to be taken into account.

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