NEW SLANTS

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"The Pyramid"

Pascal's Triangle is given by the coefficients of the binomial expansion $(a+b)^n$. The coefficients of a trinomial expansion $(a+b+c)^n$ take the shape of a three-dimensional pyramid:

(1) $a + b + c$

(2) $$\begin{array}{c} a + b + c \\ \hline a^2 + 2ab + 2ac + b^2 + 2bc + c^2 \end{array}$$

(3) $$\begin{array}{c} a + b + c \\ \hline a^3 + 3a^2b + 3a^2c + 3ab^2 + 6abc + 3ac^2 + b^3 + 3b^2c + 3bc^2 + c^3 \end{array}$$

$1c$

$1a \qquad 1b$

row one

$1c^2$

$2ac \qquad 2bc$

$1a^2 \qquad 2ab \qquad 1b^2$

$1c^3$

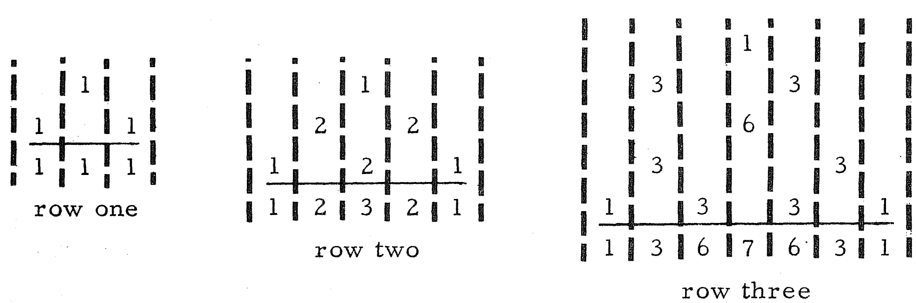
$3ac^2 \qquad 3bc^2$

$6abc$

$3a^2c \qquad 3b^2c$

$1a^3 \qquad 3a^2b \qquad 3ab^2 \qquad 1b^3$

Projecting this pyramid onto a plane:



And arranging thus:

1

1 1 1

1 2 3 2 1

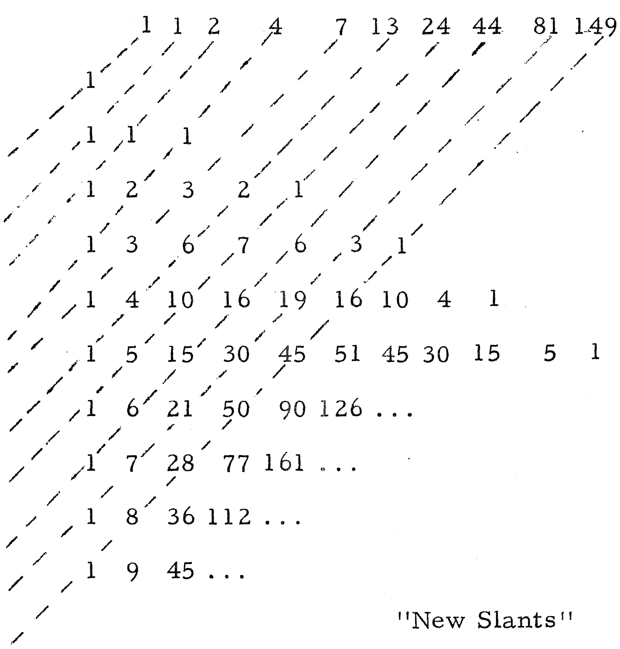
1 3 6 7 6 3 1

1 4 10 16 19 16 10 4 1

Each number is the sum of the one above it and two to the left of that.

Adding diagonals of this triangle gives the Tribonacci series (1, 1, 2, 4, 7, 13, 24, 44...) 1 . The convergent of this sequence fits the equation

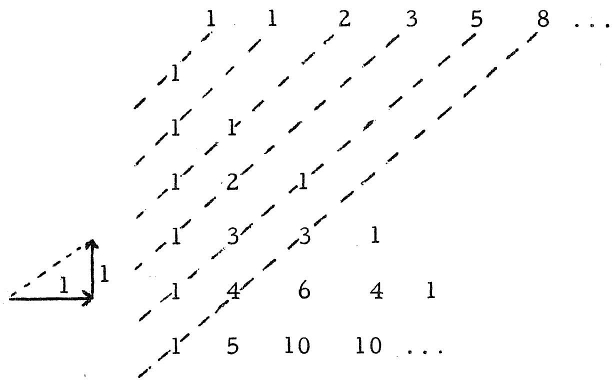
$$X = 1 + \frac{1}{X} + \frac{1}{X^2} \quad .$$



"New Slants"

Summing diagonals of Pascal's Triangle obtained by going across one column and up one row gives the Fibonacci series.2 . Its convergent, "phi," fits the equation

$$X = 1 + \frac{1}{X} \quad .$$

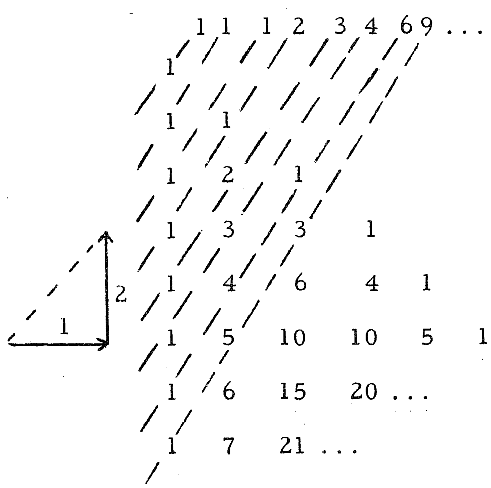


Going across one column and up two rows on the same triangle gives

1, 1, 1, 2, 3, 4, 6, 9, 13, 19, 28, 41, 60, 88 . . .

This series' convergent, 1.46. . . , fits

$$X = 1 + \frac{1}{X^2} \quad .$$



Going across one column and up three rows gives

1, 1, 1, 1, 2, 3, 4, 5, 7, 10, 14, 19, 26 . . . .

Its convergent, 1.38 . . . , fits

$$X = 1 + \frac{1}{X^3} \quad .$$

In the triangle of the 3-D expansion, going across one column and up one row gives the Tribonacci series.

Going across one column and up two rows gives

1, 1, 1, 2, 3, 5, 8, 12, 19, 30, 47, 74 . . .

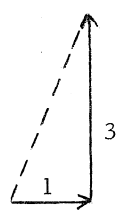
which converges upon 1.57 . . . and fits

$$X = 1 + \frac{1}{X^2} + \frac{1}{X^4}$$

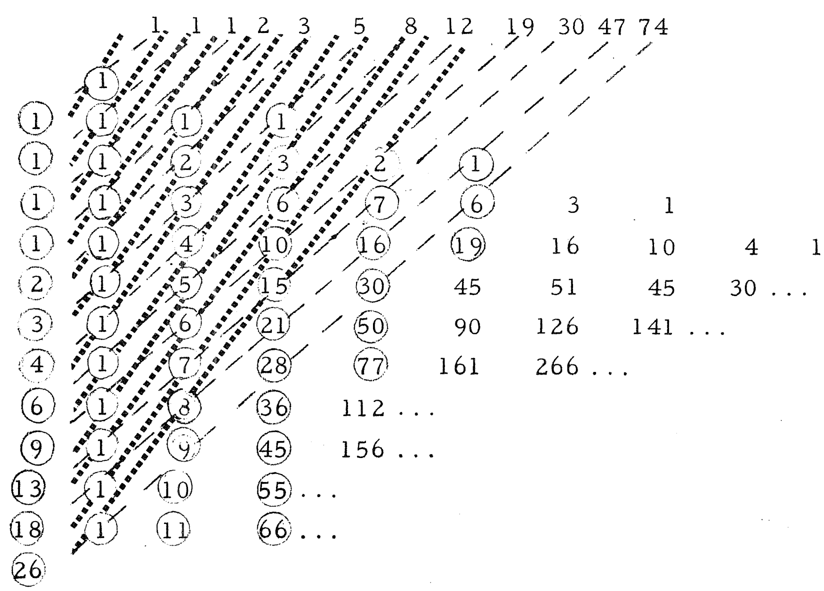
Going across one column and up three rows gives

1, 1, 1, 1, 2, 3, 4, 6, 9, 13, 18, 26, 38, 55 . . .

which converges upon 1.44 . . . and fits



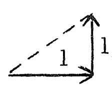
$$X = 1 + \frac{1}{X^3} + \frac{1}{X^6} .$$



A table can be made from the expansion $(a+b+c+d)^n$ so that each number is the sum of the one above it and the three to the left of that.

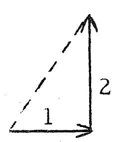
1									
1	1	1	1						
1	2	3	4	3	2	1			
1	3	6	10	12	12	10	6	3	1

Going across one column and up one row gives a series whose convergent fits



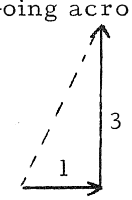
$$X = 1 + \frac{1}{X} + \frac{1}{X^2} + \frac{1}{X^3} \quad .$$

Going across one column and up two rows gives



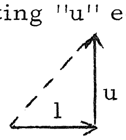
$$X = 1 + \frac{1}{X^2} + \frac{1}{X^4} + \frac{1}{X^6}$$

Going across one column and up three rows gives



$$X = 1 + \frac{1}{X^3} + \frac{1}{X^6} + \frac{1}{X^9}$$

Thus a general convergent formula is derived for diagonal series by letting "n" equal the number of terms in the expansion, and letting "u" equal the number of rows up:



$$X = 1 + \frac{1}{X_1{}^u} + \frac{1}{X_2{}^{2u}} \ldots + \frac{1}{X_{n-1}{}^{(n-1)u}} \quad .$$

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