HYPERPERFECT NUMBERS WITH FIVE AND SIX DIFFERENT PRIME FACTORS

Mariano Garcia

Department of Mathematics, Touro College, 27 West 23rd Street, New York, NY 10010

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ABSTRACT

A natural number N is *hyperperfect* if there exists an integer k such that $N-1 = k[\sigma(N) - N - 1]$, where $\sigma(N)$ is the sum of the positive divisors of N. The classical perfect numbers are hyperperfect numbers corresponding to $k = 1$. In this paper we exhibit several hyperperfect numbers with five different prime factors and the first known hyperperfect number with six different prime factors.

A natural number N is said to be *hyperperfect* if there exists an integer k such that $N - 1 = k[\sigma(N) - N - 1]$, where $\sigma(N)$ is the sum of the positive divisors of N. The ordinary perfect numbers, for which $\sigma(N) = 2 \cdot N$, correspond to the case where $k = 1$.

Hyperperfect numbers have been studied by Minoli [2], [3], [4], Bear [2], te Riele [6], [7], [8], McCranie, [1], and Nash [5]. Several examples have been found of hyperperfect numbers with two, three and four different prime factors and one such number with five different prime factors was discovered be te Riele [8].

In this paper we include some new hyperperfect numbers with five different prime factors and the first known example with six different prime factors as well. These numbers were found with the aid of Rules 1, 2 and 3 that appear in [8] and a new Rule found by the author. For convenience, we state these four rules below.

First, corresponding to the positive integer k, we define M_k^* as the set of all natural numbers N satisfying the equation $N - 1 = k[\sigma(N) - N]$ and M_k as the set of all hyperperfect numbers for that value of k. We also write $\overline{a}$ for $\sigma(a)$.

Rule 1: If $a \epsilon M_k^*$ and p is a prime $= k\overline{a} + 1 - k$, then $ap \epsilon M_k$.

Rule 2: If $a \epsilon M_k^*$ and p and q are distinct primes such that $(p-k\overline{a})(q-k\overline{a}) = 1-k+k\overline{a}+k^2\overline{a}^2$, then $apq \epsilon M_k$.

Rule 3: If $a \epsilon M_k^*$ and p and q are distinct primes such that $(p - k\overline{a})(q - k\overline{a}) = 1 + k\overline{a} + k^2\overline{a}^2$, then $apq \epsilon M_k^*$.

(New) Rule A: Corresponding to a natural number a, if p is prime and k is a positive integer such that $[(\overline{a} - a)p + \overline{a}][a - (\overline{a} - a)k] = \overline{a} - a + a\overline{a}$, then $ap \epsilon M_k^*$.

The proofs of all four rules follow directly from the definitions of the sets M_k^* and M_k.

The first three of the examples that follow were obtained by starting with the product of two primes, using Rule A to obtain the product of three primes as a member of M_k^* and using Rule 2 to obtain the product of five primes as a member of M_k.

The next four examples were found by starting with a prime, using Rule A to obtain the product of two primes as a member of M_k^*, using Rule 3 to obtain the product of four primes as a member of M_k^* and using Rule 1 to obtain the product of five primes as a member of M_k.

The example consisting of the product of six primes was obtained by starting with a fixed value of k, using Rule 3 twice and then using Rule 2.

Hyperperfect numbers with five different prime factors:

($k = 1248$) 1291 · 37501 · 476132479 · 28791173123859572047·
5200604882387175116037772559

($k = 1950$) 3203 · 4987 · 34208591 · 10660774641948298 31·
110234836048892103032611805079802 1

($k = 2430$) 2689 · 25537 · 2157247 · 360118565294860859·
21980573062716770006027255774285 69

($k = 10614$) 10957 · 339091 · 39439240306867 · 27734632534386560971·
431398747818208251696567072279122454694514681 71

($k = 26772$) 36523 · 100279 · 98055842567377 · 4921404647429290225924 33·
473190510499941349285431260991180472248443319358090 9

($k = 293400$) 295411 · 43099891 · 37356349017571045 87·
2481722065276171304892829643 23·
34632352301186904553277964821120901455453111761577 91-
276882091789801

($k = 297330$) 298999 · 53266429 · 47354745819381007 51·
2985981293765889018885368472363675 1·
66959792993265791239640887007008054993187016096027 20-
5972083160650210267 1

Hyperperfect number M with six different prime factors:

($k = 22998384$)

$M = p \cdot q \cdot r \cdot s \cdot t \cdot u$, where

p = 22998427

q = 12300620431171

r = 6506126308645398457840655623

s = 137478660422370245650587710247038578405571276599361 83

t = 58194276398238994797319319270186821750600718607846222519-
95952492106441890530832343314159719058733980899738 47

u = 18962384608661284895373626306450232703958904749109475-
16705194275851074879595353555483837058005247913558036-
65045753691703961285110593835041234636889678746407110 2-
12975936960656330834440627247005979243282572792663

The number M has 418 digits.

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