Morelia Test: Improving the Efficiency of the Gabriel Test and Face Routing in Ad-hoc Networks

Paul Boone[||] Edgar Chavez[*] Lev Gleitzky[†] Evangelos Kranakis[‡]
Jaroslav Opatrny[§] Gelasio Salazar[†] Jorge Urrutia[¶]

February 11, 2004

Abstract

An important technique for discovering routes between two nodes in an ad-hoc network involves application of the face routing algorithm on a planar spanner of the network. Face routing on a planar subgraph guarantees a message delivery in networks whose geometrical representation contains large holes without any node and having complex contours, where the usual greedy routing fails. Existing techniques for constructing a suitable planar spanner involve local tests that eliminate crossings between existing links by deleting some of the links. The existing tests do not test whether the deleted links actually create some crossings and some of the links are deleted needlessly. As a result, the routes in the face routing will have an unnecessarily large number of hops from source to destination. We consider a new local test for preprocessing a wireless network that produces a planar subgraph on which we can apply face routing. The test is relatively simple, requires low overhead and does not unnecessarily eliminate existing links unless it is needed to eliminate a crossing, thus reducing overhead usually associated with multiple hops.

1 Introduction

An *ad-hoc network* is a network consisting of transmitters, often called *hosts*, that is established as needed, typically without any assistance from a fixed infrastructure. Using wireless broadcasts, each host can communicate with other hosts within its transmission range. Typically, not all hosts are within the transmission range of each other. Thus, communication between two hosts in the network is in general achieved by multi-hop routing along a route where intermediate nodes cooperate by forwarding packets. Examples of such networks include sensor, piconet, bluetooth, and home/office networks, and routes in these networks have to be constructed on the fly.

In this paper we consider networks that have the following properties:

- Any host knows the geometrical coordinates (x, y) of its location.
- All hosts have the same transmission range R, *i.e.* any two hosts at distance $d \leq R$ are able to communicate directly.
- no hosts at distance grater than R communicate directly.

[||]School of Computer Science, Carleton University, Ottawa, Ontario, K1S 5B6, Canada. Research supported in part by NSERC Graduate Scholarship.

[*]Escuela de Ciencias FÃ?sico-MatemÃ¡ticas de la Universidad Michoacana de San NicolÃ¡s de Hidalgo, Mexico

[†]IICO-UASLP Av. Karakorum 1470. Lomas 4ta Seccion San Luis Potosi, SLP Mexico 78210

[‡]School of Computer Science, Carleton University, Ottawa, Ontario, K1S 5B6, Canada. Research supported in part by NSERC (Natural Sciences and Engineering Research Council of Canada) and MITACS (Mathematics of Information Technology and Complex Systems) grants.

[§]Department of Computer Science, Concordia University, 1455 de Maisonneuve Blvd West, MontrÃ©al, QuÃ©bec H3G 1M8 Canada. Research supported in part by NSERC (Natural Sciences and Engineering Research Council of Canada)

[¶]Cubiculo 222, Instituto de MatemÃ¡ticas, Universidad Nacional AutÃ³noma de MÃ©xico, D.F., Mexico City, Mexico.

- Communication links are *bidirectional.*

Ad-hoc networks satisfying the conditions above are the most common type of ad-hoc networks considered in literature.

A network can be represented by a geometric undirected graph, $G = (V, E)$, with nodes representing the hosts of the network, and an edge connecting any pair of nodes that can communicate directly.

Discovering a route between two nodes in an ad-hoc network is an important component of current research. In such systems it is vital that route discovery uses only local information and is adaptable to the network connectivity. An important technique for discovering routes between any two nodes in an ad-hoc network without the use of flooding is the face routing [7, 2]. A face routing algorithm succeeds in discovering a route in a network providing that the underlining network is planar. Since in practice, the original ad-hoc wireless network is never planar with many links crossing each other, before using the face routing we need to abstract from the original network a planar connected network spanning the entire underlying network.

There are two important issues we should be concerned in our reduction from the original wireless network to the geometric planar graph:

1. **Keep long links.** This is required so that we can prevent unnecessary large number of hops from source to destination that require extra processing at the nodes and may cause failures.

2. **Eliminate crossings.** This is required in order to create a planar underlying graph. Furthermore, the method employed must be efficient and be based on local tests and the resulting graph must be a spanner of the original network.

The first goal described above may not necessarily be consistent with minimizing power consumption. For example, consider the configuration depicted in Figure 2. If node C is inside the circle determined by the diameter AB then an elementary calculation shows that the power required to reach directly from A to B is less than the power required to reach from A to B via node C. However, multi-hopping may cause extra overhead due to local processing.

Concerning the second goal above, nodes do not have a priori knowledge of the entire network. In addition, tests must be applied locally by the individual nodes, since in many cases there is no global information available about the network and the network can keep on changing. Thus, a node will decide to keep or delete an existing edge solely based on the requirement for eliminating crossings. As a result, long links that typically are involved in many crossings may be deleted and the resulting routes will have an unnecessarily large number of hops from source to destination.

1.1 Results of the paper

In this paper we consider a method for reducing the overhead of an unnecessarily large number of hops from source to destination.

The *Morelia test* is a new local test for preprocessing a wireless network that produces a planar spanning subgraph of the original wireless network on which we can apply face routing. The Morelia test is a generalization of the Gabriel test and is relatively simple, requires low overhead and does not unnecessarily eliminate existing links if they do not create any crossing, thus reducing overhead usually associated with multiple hops. The resulting graph is planar and is a supergraph of the Gabriel graph [5].

In addition to the theoretical justifications of the Morelia test presented in this paper, we conducted simulations of our algorithms on randomly generated networks so that we can quantify the improvements given by the new method.

2 Tests for Reduction to Planarity

Here we discuss existing tests for planar reduction in wireless networks. In particular we consider the Gabriel Graph and the Relative Neighbourhood Graph.

2.1 Relative Neighbourhood Graph

A planar spanner of a network can be obtained by applying the Relative Neighbourhood Graph (RNG) test to every pair of nodes which are within the transmission range of each other. Let A, B be two nodes whose distance is less than the transmission range R of the network. Consider the region delimited by the intersection of the circles centered at A and B, respectively, where the radius of the circles is determined by the power of the stations at A and B, see Figure 1. If there is no node in the region then the link between A and B is kept. If however there is a node C in the region depicted in Figure 1, then nodes A and B remove their direct link. In particular, when A (respectively, B) is queried on routing data to B (respectively, A) the routing table at A (respectively, B) forwards the data through C (or some other similar node if more than one node is in the specified region.

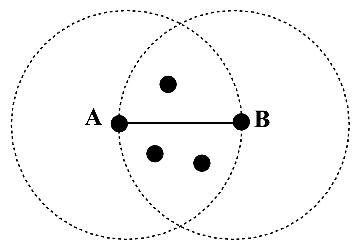

Figure 1: The RNG test for producing a planar spanner.

The RNG test suffers from the multiple hop effect because the elimination of crossings is done by elimination of longer links.

2.2 Gabriel Test

One of the most important tests for eliminating crossings in a wireless network is called Gabriel test which, similarly to the RNG test is applied to every link of the network. The main difference between the Gabriel test and the RNG test is the smaller size of the region considered for a link elimination.

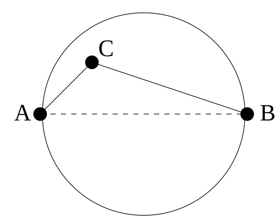

Figure 2: Eliminating an unnecessary link (dashed line AB) with the Gabriel test.

Let A, B be two nodes whose distance is less than the transmission range R of the network. In the Gabriel test, if there is no node in the circle with diameter AB then the link between A and B is kept. If however there is a node C in the circle with diameter AB, as depicted in Figure 2, then nodes A and B remove their direct link. In particular, when A (respectively, B) is queried on routing data to B (respectively, A) the routing table at A (respectively, B) forwards the data through C (or some other similar node if more than one node is in the circle with diameter AB. The advantage of doing this rerouting of data is that the resulting graph is a planar spanner on which we can apply the face routing algorithm for discovering a route from source to destination.

However, the test merely shrinks the "test region" and creates a planar spanner that keeps some of the links that would be eliminated by the RNG test. However, like the RNG test, it does not in any way prevent the multiple hop effect.

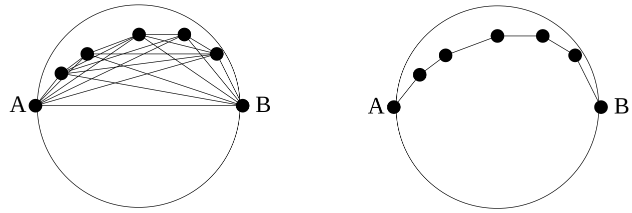

Figure 3: Multiple hop effect when eliminating a link (line segment AB) via the Gabriel test.

For example, consider a set of nodes as depicted in the left-hand side of Figure 3. All nodes are mutually reachable. However, when we apply the Gabriel test the configuration in the right-hand side of Figure 3 results. We can see that although nodes A and B could have reached each other directly in a single hop instead they must direct their data through a sequence of many hops.

We note that the multiple hop effect arises when many nodes are clustering in regions. We should note that while the multihop effect may result in slower message delivery, it may have also a positive effect, since it can decrease the power consumption. For example, if instead of using a direct link between A and B an intermediate node, say C is used (see Figure 2) which lies in the circle with diameter AB then the power consumption decreases from $d(A, B)^{-p}$ to $d(A, C)^{-p} + d(C, B)^{-p}$, for some constant $2 \leq p$. However, this is at the cost of additional overhead implied by intermediate hopping.

3 Planarity and the Multiple Hop Effect

The purpose of this section is to give an algorithm that eliminates crossings but at the same time maintains some "long" links between stations whenever possible, thus reducing the number of hops in face-routing. We introduce the *Morelia test*. The Morelia test is an extension of the Gabriel test where the algorithm checks for the presence of a crossing before eliminating a link.

3.1 Morelia Test

As mentioned in the introduction, we are concerned with the problem of routing in networks with complex topology and containing many holes. Both, the RNG and Gabriel tests eliminate some links not because they create crossing of links, but merely for the potential of being involved in crossing. As indicated in the example above, if the Gabriel or RNG test is applied to a complex network, the spanning planar subgraph that is obtained will contain holes of even larger size and the contour of the network will become more complex.

The Morelia test attempts to preserve links whenever possible and as a consequence the resulting planar graph (on which face routing is to be applied) will most likely keep the contour very similar to the original contour and the holes in the network would not grow much. Thus the resulting planar network will have smaller diameter and routes from source to destination will require less hops.

The Morelia test is similar to the Gabriel test in that given two nodes A and B it eliminates links based on the inspection of the circle with diameter AB. Unlike the Gabriel test it does not necessarily eliminate the direct link AB when it finds another node inside the circle with diameter AB. Instead, it verifies whether the nodes inside the circle create any crossing of the line AB. If no crossing is created the line AB is kept and it is removed otherwise. The verification of the existence of crossing is done in most cases by inspecting only the neighborhood of nodes A and B at the transmission distance R. In a few cases, the neighborhood of some of the nodes in the circle around AB is inspected. In all cases it is a local test that computes the neighborhood of nodes at distance at most two hops of each node A and B.

In the Morelia test of a link AB we subdivide the area of the circle with diameter AB into X_1, X_2, Y_1 and Y_2 as in Figure 4. The areas X_2 and Y_2 are determined by an arc of radius R. Furthermore, in the testing we use areas X_3 and Y_3 as indicated the Figure 4 that are outside the transmission radius R of the nodes A and B and within distance R from the link AB. For each node A let $N(A)$ be the set of nodes Z such that $d(A, Z) \leq R$.

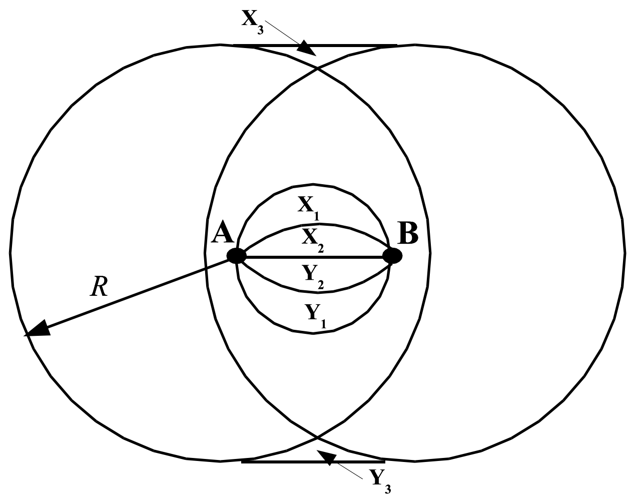

Figure 4: Morelia Test

The precise specification of the Morelia test applied to a link AB is given below (refer to Figure 4).

Morelia Test Rules.

1. If there is at least one node in $X_1 \cup X_2$ and at least one node in $Y_1 \cup Y_2$ then the link AB is removed.
2. If there is at least one node in X_1 and no node in $X_2 \cup Y_1 \cup Y_2$ then the node A checks whether any node in $N(A)$ creates a link with nodes in X_1 that crosses AB. If such a crossing occurs, link AB is removed and A sends a message to B to remove the link as well. Similarly node B performs a check of nodes in $N(B)$ for a crossing of the link AB and informs node A if a crossing is found and AB is to be removed.
3. If there is at least one node in Y_1 and no node in $Y_2 \cup X_1 \cup X_2$ (symmetric to Rule 2) then the node A checks whether any node in $N(A)$ creates a link with nodes in Y_1 that crosses AB. If such a crossing occurs, link AB is removed and A sends a message to B to remove the link as well. Similarly node B performs a check of nodes in $N(B)$ for a crossing of the link AB and informs node A if a crossing is found and AB is to be removed.
4. If there is at least one node in X_2 and no node in $Y_1 \cup Y_2$ then the node A inspects the nodes in $N(A)$ to check whether any node there creates a link with nodes in $X_1 \cup X_2$ that crosses AB. If such a crossing occurs, link AB is removed and A sends a message to B to remove the link as well. Furthermore A sends a message to nodes in X_2 with a request to send back information whether there is a node in the region Y_3. If A receives a message that a node exits in Y_3 then AB is removed and node B is informed to remove the link as well.
5. If there is at least one node in Y_2 and no node in $X_1 \cup X_2$ (symmetric to Rule 4), the node A inspects the nodes in $N(A)$ for a possible crossing of AB. If such a crossing occurs, link AB is removed and A sends a message to nodes in Y_2 with a request to send back information whether there is a node in the region X_3. If A receives a message that a node exits in X_3 then AB is removed and node B is informed to remove the link as well.

Figure 5 illustrates how, unlike the Gabriel test, the Morelia test will check for crossings prior to eliminating a link. It will eliminate link AB because it detects crossings, but it will not eliminate it when no crossing exists.

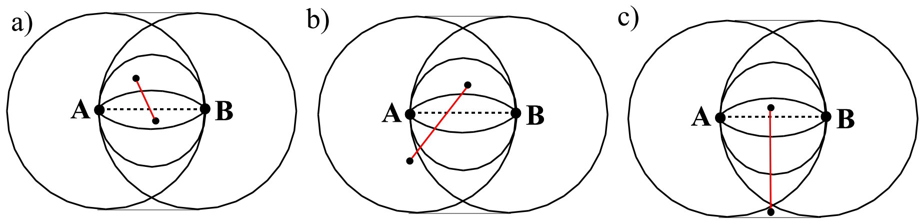

Figure 5: Examples of the Morelia test situations: a) AB deleted by rule 1, b) AB deleted by rule 2, c) AB deleted by rule 4.

Notice that in the rule 1 of the Morelia test, nodes A and B look only at nodes that are in $N(A)$ and $N(B)$ respectively, i.e., nodes that are one hop away. In rule 2, 3, and 4 node A checks nodes in $N(A)$, node B checks nodes in $N(B)$ and both A and B possibly check $N(x)$ for nodes x in $\in X_2 \cup Y_2$. Thus A or B are checking nodes that are at most two hops away. Therefore, the Morelia test is a local test.

Theorem 1 *If network N is connected then the application of the Morelia test to all links of N produces network N' which is a connected planar spanner of N. Furthermore, N' contains the Gabriel graph of N as its subgraph.*

Proof. Every edge in N that is kept by the Gabriel test is also kept by the Morelia test. Thus, N' contains the Gabriel graph of N as its subgraph. Since the Gabriel test produces a connected subgraph of N, the subnetwork N' is connected.

Assume that there is a link e in N' that crosses a link AB of N'. Since the length of any link in N is at most R, both ends of e are in $N(A) \cup N(B) \cup X_3 \cup Y_3$. If one of ends of e is in the circle with diameter AB then AB would be deleted by the Morelia test. If both ends of e are outside the circle with diameter AB then one of the ends of the edge AB must be inside the circle with diameter e, since the edges crosses each other. However, the Morelia test applied to e would eliminate e because of it being crossed by AB. Thus there can be no crossing in N'. □

In Figure 6 we show how the Morelia test keeps some of the long links. The left-hand side shows a link AB and the nodes inside the circle with the diameter AB and the right-hand side shows what edges are kept by the Morelia test.

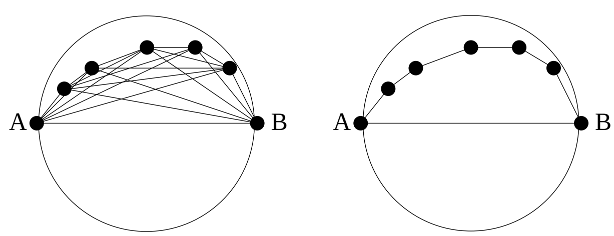

Figure 6: Multiple hop elimination (line segment AB) via the Morelia test.

3.2 The Overhead of the Morelia Test

In Rule 1 of the Morelia test node A or B needs to determine what nodes are in the circle with AB as its diameter and the complexity of it is not much greater than that of the corresponding Gabriel test.

Rule 2 for A involves nodes that are in $N(A)$ and each node of the network needs to know its neighbors anyway. To check for crossings of two line segments involves simple geometrical computation of complexity

$O(1)$, and the same applies to node B. The exchange of messages between A and B confirming deletion or retention of the edge is also simple.

Rule 3 and 4 of the test involves the existence of nodes in X_3 and Y_3 that are outside of $N(A) \cup N(B)$. However, all that is needed for A or B is to send a message to the nodes in region X_2 or Y_2 asking the question "is there any node in Y_3 or X_3?" respectively. The region X_3 or Y_3 can be specified by the three corners of the region. Thus, although these rules involve nodes that are two hops away from A and B, it does not create a significant overhead or delay. We show now that the size of the region X_2 or Y_2 is a smaller part of the circle with diameter AB. Since Rule 3 and 4 are used only when X_2 or Y_2 is the only region of the circle containing a node of the network, the probability of using these rules is also smaller.

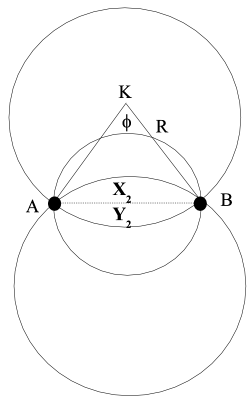

Figure 7: The arc segment X_2 (and its symmetric one Y_2) used in Rule 4.

We can easily calculate the upper bound on the ratio of $|X_2|$, the area of X_2 to $|X_1|$, the area of X_1 to get an indication on how often Rule 4 is used in comparison to the other rules.

It is easy to see that the ratio $|X_2|/|X_1|$ is getting smaller when the angle ϕ is getting smaller. Thus we get an upper bound on $|X_2|/|X_1|$ by setting $\phi = \pi/3$. In this case the length of AB equals R and the area of the circle determined by the diameter AB is exactly $\pi R^2/4$, and the area of the circular sector X_1 is equal to $\pi R^2 - \sqrt{(3)}R^2/4 = R^2(\frac{2\pi - 3\sqrt{(3)}}{12}$. Thus $\frac{|X_2|}{|X_1|} = \frac{2\pi - 3\sqrt{(3)}/12}{\pi R^2/8 - 2\pi - 3\sqrt{(3)}/12} = \frac{4\pi - 6\sqrt{(3)}}{3\pi - 4\pi - 6\sqrt{(3)}} < 0.3$.

Thus if a node is located in the circle with diameter AB and its placement is random, it will be in the area $X_2 \cup Y_2$ with probability less than 0.3.

4 Experimental Results

In this section we present our simulations designed to test the Morelia Test in comparison with the Gabriel Test. The purpose of the simulations was to give an indication on how our algorithm actually performs in the real world. We study the operation and performance of our algorithm on randomly generated graphs.

4.1 Goals

There were two main goals to achieve with our simulations.

1. To compare the average link length in the planar spanning subgraph created using the Morelia and Gabriel tests.

2. To compare the number of hops required to deliver messages with face routing on the planar spanning subgraph created by the Morelia and Gabriel tests.

4.2 Network Model

A simulation was designed that allowed the creation of a random network. A square area (called a *grid*) could be defined, nodes could be created with a specific transmission range and placed at random coordinates on the grid. We use the unit disk graph (UDG) model, where all nodes have the same transmission range. Each network simulation consisted of 50 nodes each with a transmission range of 250 units. The test area was a square grid on which the 50 nodes were randomly placed. This network setup allowed us to test the performance with different node densities by simply varying the grid size (300 units square to 1300 units square).

4.3 Analysis of Morelia Graph vs. Gabriel Graph

The Gabriel Test and the Morelia Test were run on the *same* series of random graphs. The following are the main metrics that were traced in the simulation:

1. Average length of link on each planar spanning subgraph.
2. Number of links in each planar spanning subgraph.

4.3.1 Scenario 1 - Uniform Random Graph

The first test, was to generate a series of random networks. The grid size was the only parameter that was altered for a series of trials. This had the effect of increasing or decreasing the network density (the smaller the grid size, the closer together the nodes, the greater number of links, the higher degree vertices).

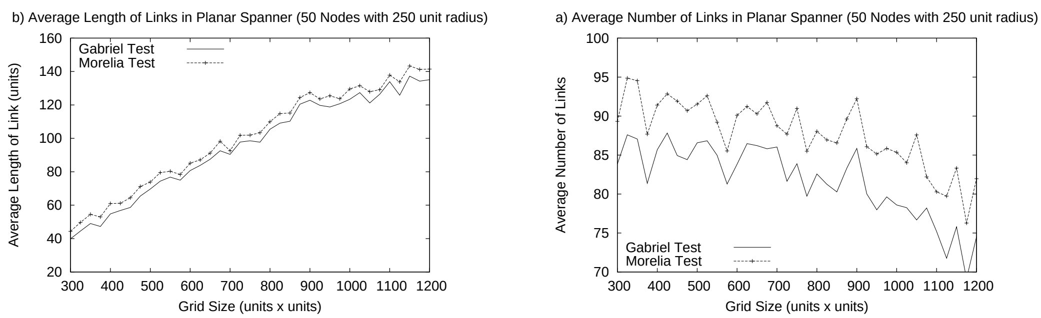

Figure 8: Average Number and Length of Links in Morelia and Gabriel Graphs.

Figure 8 a), shows the average link length of the Gabriel and Morelia graphs. The Morelia graph indeed does keep slightly longer links on average. The average increase in length was 4.86 units or 5.76%.

Figure 8 b), shows the average number of links kept in the planar spanning subgraph for both tests. The Morelia test keeps more links on average in the spanner than the Gabriel test. The average increase in links kept in the Morelia graph was 6.07 or 7.49% over the Gabriel graph.

4.3.2 Scenario 2 - Random Graph with a Sparse Region

This simulation was designed to test the planar spanning subgraph creation when there exists an area of the graph which has a less dense topology. A region 500x500 units within the test grid was defined to have a node density of one-half that of the rest of the network.

Here we report, as in the first test, the resulting average link length and number of nodes in the planar spanning subgraph.

In Figure 9 a), we see the average length of link kept in both the Gabriel and Morelia graph. The average Morelia link length has now been improved to be 5.57 units longer than the average Gabriel link.

Finally, we show in Figure 9 b), the average number of extra links kept in the two graphs. The average number of additional links kept increased by 7.67, which is an increase of 10.22%.

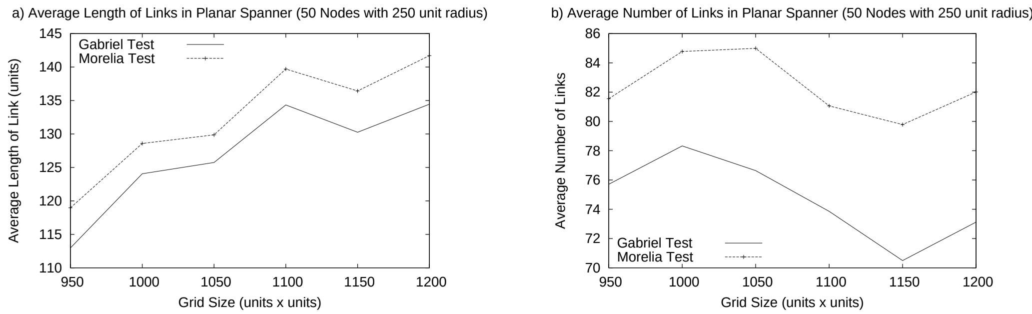

Figure 9: Average Number and Length of Links in Morelia and Gabriel Graphs.

4.4 Face Routing Performance Study

In this section we present the simulation results of the face routing algorithm *Face-2* [2] on the same series of random graphs on which the Morelia and Gabriel tests have produced a planar spanning subgraph. The *Face-2* algorithm is a modified face routing algorithm that at each iteration of a face traversal makes the decision to move to the next face when it determines a link is about to cross the line from source to target, instead of traversing the whole face and keeping track of all crossings that occur.

We want to show that, in a network with some sparse areas, the face routing algorithm will perform routing with fewer number of hops on average. Two scenarios were simulated. The first was on a uniform random graph, the latter scenario was on graphs which had a predetermined sparse region. In these simulations, the only metric studied was the *average number of hops* required for the face routing algorithm.

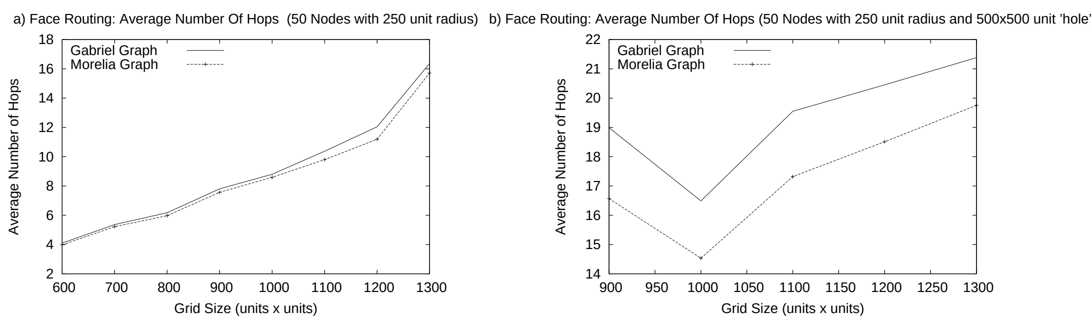

Figure 10: Face routing applied to Morelia and Gabriel Graphs.

In the first scenario, a uniform random network of 50 nodes was built. Each node had a transmission range of 250 units. As before, the grid size was the only parameter changed to vary the network density.

We see in Figure 10 a), that in a uniformly distributed network, only minor gains are made in terms of decreasing the number of hops. The average number of hops saved was 0.28 per route.

In the last scenario, we define a region 500x500 units square and set the number of nodes with the region to one-half that of the first test. Routes were chosen that would traverse the region. All other parameters remained as in the previous scenario.

In Figure 10 b) we can see that there is further improvement of the face routing algorithm. The average number of hops saved with face routing on the Morelia graph has increased to 2.03 hops per route or 10.6%.

5 Conclusion

We have shown that with the application of the Morelia test to the underlying structure of a wireless ad-hoc network we can achieve face routing with a fewer number of hops on average over the Gabriel test when the network has a sparse region. Thus the use of the Morelia test would be beneficial in any wireless ad-hoc network having a complex topology.

There are several open problems for future work in this area. One such problem is the extension of the algorithm to a network which is changing dynamically. Another area to investigate would be the improvement of the application of the Morelia test in networks which are very dense. If a network is very dense, i.e. the number of nodes inside a circle of transmission radius R is very high, the Morelia test introduces more overhead than the Gabriel test, the increase in the number of long links kept is not significant, and and thus the number of hops from source to destination remains high.

References

[1] L. Barriere, P. Fraignaud, L. Narayanan, and J. Opatrny, Robust position-based routing in wireless ad hoc networks with unstable transmission ranges; in Proceedings of the Fifth International Workshop on DialM for Mobility, 2001

[2] P. Bose, P. Morin, I. Stojmenovic, and J. Urrutia. Routing with guaranteed delivery in ad hoc wireless networks. In Proc. of Discrete Algorithms and Methods for Mobility (DIALM'99), pages 48–55, 1999.

[3] J. Broch, D. Maltz, D. Johnson, Y.-C. Hu, and J.Jetcheva. A performance comparison of multi-hop wireless ad hoc network routing protocols. In Mobile Computing and Networking, pages 85–97, 1998.

[4] S. Datta, I. Stojmenovic, and J. Wu. Internal node and shortcut based routing with guaranteed delivery in wireless networks. In Proc. IEEE Int. Conf. on Distributed Computing and Systems Workshops; Cluster Computing, to appear, pages 461–466, 2001.

[5] K. Gabriel and R. Sokal. A new statistical approach to geographic variation analysis. Systematic Zoology, 18:259–278, 1969.

[6] J. Gao, L. Guibas, J. Hershberger, L. Zhang, A. Zhu, Gemetric Spanner for Routing in Mobile Networks, in ACM MobiHOC 2001, Long Beach, CA.

[7] E. Kranakis, H. Singh, and J. Urrutia, Compass Routing on Geometric Networks, In Proceedings of 11th Canadian Conference on Computational Geometry, pp. 51–54, Vancouver, August, 1999.

[8] F. Kuhn, R. Wattenhofer, and A. Zollinger, Asymptotically Optimal Geometric Mobile Ad-Hoc Routing. In the Proceedings of the 6th International Workshop on Discrete Algorithms and Methods for Mobile Computing and Communications (DIALM), Atlanta, Georgia, September 2002.

[9] C. E. Perkins, editor, Ad Hoc Networking, Addison Wesley, 2001.

[10] G. Toussaint, The relative neighbourhood graph of a finite planar set, Pattern Recognition, 12(4):261–268, 1980