

Contents

1. Boolean Functions and Circuits	1
1.1 Introduction	1
1.2 Boolean Functions and Formulas	2
1.3 Circuit Model	7
1.4 Basic Functions and Reductions	8
1.5 Nomenclature	11
1.6 Parsing Regular and Context-Free Languages	12
1.7 Circuits for Integer Arithmetic	17
1.7.1 Circuits for Addition and Multiplication	17
1.7.2 Division Using Newton Iteration	21
1.7.3 Division Using Iterated Product	24
1.8 Synthesis of Circuits	30
1.8.1 Elementary Methods	30
1.8.2 Shannon's Method	31
1.8.3 Lupanov's Method	32
1.8.4 Symmetric Functions	34
1.9 Reducing the Fan-out	35
1.10 Relating Formula Size and Depth	39
1.11 Other Models	45
1.11.1 Switching Networks	45
1.11.2 VLSI Circuits	45
1.11.3 Energy Consumption	45
1.11.4 Boolean Cellular Automata	46
1.11.5 Branching Programs	48
1.11.6 Hopfield Nets	53
1.11.7 Communication Complexity	54
1.11.8 Anonymous Networks	54
1.12 Historical and Bibliographical Remarks	55
1.13 Exercises	56
2. Circuit Lower Bounds	61
2.1 Introduction	61
2.2 Shannon's Lower Bound	63
2.3 Nechiporuk's Bound	65

2.4	Monotonic Real Circuits	68
2.4.1	Broken Mosquito Screen	68
2.4.2	Monotonic Real Circuits Are Powerful	77
2.4.3	st -Connectivity	78
2.5	Parity and the Random Restriction Method	90
2.6	Probabilistic Methods	95
2.6.1	Håstad's Lower Bound for Parity	96
2.6.2	Depth- k Versus Depth- $(k - 1)$	99
2.6.3	Razborov's Simplification and Decision Trees	102
2.6.4	A Hybrid Switching Lemma and st -Connectivity	107
2.6.5	Hybrid Switching with the Uniform Distribution	110
2.7	Algebraic Methods	124
2.7.1	Razborov's Lower Bound for Majority over Boolean Circuits with Parity	124
2.7.2	Smolensky's Lower Bound for MOD_p Versus MOD_q	129
2.8	Polynomial Method	132
2.8.1	On the Strength of MOD_m Gates	132
2.8.2	The MOD_m -Degree of Threshold Functions	135
2.9	Method of Filters	137
2.10	Eliminating Majority Gates	140
2.11	Circuits for Symmetric Functions	141
2.11.1	Negative Results	143
2.11.2	Positive Results	145
2.12	Probabilistic Circuits	146
2.13	Historical and Bibliographical Remarks	148
2.14	Exercises	150
3.	Circuit Upper Bounds	155
3.1	Introduction	155
3.2	Definitions and Elementary Properties	156
3.3	Pólya's Enumeration Theory	162
3.4	Representability of Permutation Groups	164
3.5	Algorithm for Representing Cyclic Groups	168
3.6	Asymptotics for Invariance Groups	172
3.7	Almost Symmetric Languages	174
3.8	Symmetry and Complexity	178
3.9	Applications to Anonymous Networks	184
3.9.1	Rings	185
3.9.2	Hypercubes	185
3.10	Historical and Bibliographical Remarks	194
3.11	Exercises	194
4.	Randomness and Satisfiability	207
4.1	Introduction	207
4.2	Threshold for 2-SAT	209

4.3	Unsatisfiability Threshold for 3-SAT	212
4.3.1	A General Method and Local Maxima	213
4.3.2	Method of Single Flips	214
4.3.3	Approximating the Threshold	217
4.3.4	Method of Double Flips	217
4.3.5	Probability Calculations	218
4.4	Satisfiability Threshold for 3-SAT	224
4.4.1	Satisfiability Heuristics	224
4.4.2	Threshold	226
4.5	$(2 + p)$ -SAT	229
4.5.1	Unsatisfiability Threshold	230
4.5.2	Transition from 2-SAT to 3-SAT	232
4.6	Constraint Programming	235
4.6.1	Models of CSP	236
4.6.2	A New Model for Random CSP	238
4.6.3	The Method of Local Maxima	239
4.6.4	Threshold for Model E	241
4.7	Historical and Bibliographical Remarks	242
4.8	Exercises	243
5.	Propositional Proof Systems	247
5.1	Introduction	247
5.2	Complexity of Proofs	249
5.3	Gentzen Sequent Calculus LK	255
5.3.1	Completeness	257
5.3.2	Lower Bound for Cut-Free Gentzen	259
5.3.3	Monotonic Sequent Calculus	267
5.4	Resolution	268
5.4.1	Resolution and the PHP	271
5.4.2	Resolution and Odd-Charged Graphs	279
5.4.3	Schöning's Expander Graphs and Resolution	285
5.4.4	Width-Bounded Resolution Proofs	291
5.4.5	Interpolation and <i>st</i> -Connectivity	296
5.4.6	Phase Transition and Length of Resolution Proofs	300
5.5	Algebraic Refutation Systems	306
5.5.1	Nullstellensatz	308
5.5.2	Polynomial Calculus	316
5.5.3	Gaussian Calculus	324
5.5.4	Binomial Calculus	326
5.5.5	Lower Bounds for the Polynomial Calculus	332
5.5.6	Random CNF Formulas and the Polynomial Calculus	337
5.6	Cutting Planes CP	343
5.6.1	Completeness of CP	345
5.6.2	Cutting Planes and the PHP	348
5.6.3	Polynomial Equivalence of CP_2 and CP	353

5.6.4	Normal Form for CP Proofs	355
5.6.5	Lower Bounds for CP	359
5.6.6	Threshold Logic PTK	366
5.7	Frege Systems	370
5.7.1	Bounded Depth Frege Systems	372
5.7.2	Extended Frege Systems	393
5.7.3	Frege Systems and the PHP	398
5.8	Open Problems	403
5.9	Historical and Bibliographical Remarks	405
5.10	Exercises	406
6.	Machine Models and Function Algebras	413
6.1	Introduction	413
6.2	Machine Models	415
6.2.1	Turing Machines	415
6.2.2	Parallel Machine Model	424
6.2.3	Example Parallel Algorithms	427
6.2.4	<i>LogP</i> Model	433
6.2.5	Circuit Families	434
6.3	Some Recursion Schemes	437
6.3.1	An Algebra for the Logtime Hierarchy LH	438
6.3.2	Bounded Recursion on Notation	450
6.3.3	Bounded Recursion	458
6.3.4	Bounded Minimization	465
6.3.5	Miscellaneous	470
6.3.6	Safe Recursion	478
6.4	A Glimpse of Other Work	487
6.5	Historical and Bibliographical Remarks	488
6.6	Exercises	489
7.	Higher Types	497
7.1	Introduction	497
7.2	Type 2 Functionals	497
7.3	Some Closure Properties of \mathcal{A}_0	502
7.4	Square-Root and Multiple Recursion	511
7.5	Parallel Machine Model	527
7.6	λ -Calculi for Parallel Computable Higher Type Functionals	554
7.6.1	Introduction to Higher Types	555
7.6.2	<i>p</i> -Types	556
7.6.3	Finite Typed Lambda Calculus	558
7.7	Historical and Bibliographical Remarks	564
7.8	Exercises	565
	References	569
	Index	591